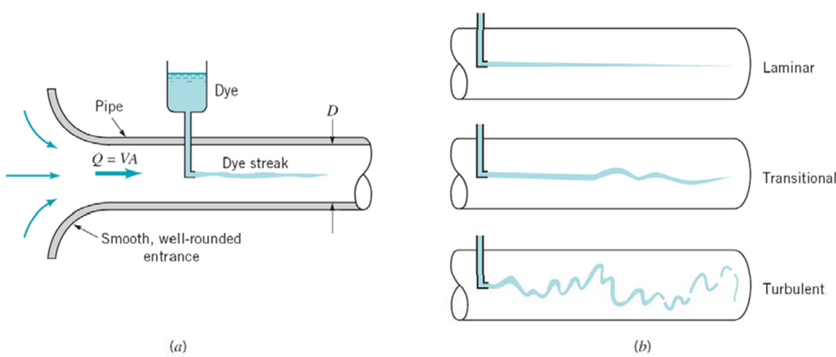
1 FLOW THROUGH PIPES

1.1 Laminar Flow

A stream line is an imaginary line with no flow normal to it, only along it. When the flow is laminar, the streamlines are parallel and for flow between two parallel surfaces we may consider the flow as made up of parallel laminar layers. In a pipe these laminar layers are cylindrical and may be called stream tubes. In laminar flow, no mixing occurs between adjacent layers and it occurs at low average velocities.

1.2 Turbulent Flow

The shearing process causes energy loss and heating of the fluid. This increases with mean velocity. When a certain critical velocity is exceeded, the streamlines break up and mixing of the fluid occurs. The diagram illustrates Reynolds coloured ribbon experiment. Coloured dye is injected into a horizontal flow. When the flow is laminar the dye passes along without mixing with the water. When the speed of the flow is increased turbulence sets in and the dye mixes with the surrounding water. One explanation of this transition is that it is necessary to change the pressure loss into other forms of energy such as angular kinetic energy as indicated by small eddies in the flow.



1.3 Laminar and Turbulent Boundary Layers

1.3.1 Critical Velocity - Reynolds Number

When a fluid flows in a pipe at a volumetric flow rate Q m^3/s the average velocity is defined

$$u_m = \frac{Q}{A} \qquad \text{A is the cross sectional area.}$$

The Reynolds number is defined as $R_e = \frac{\rho u_m D}{\mu} = \frac{u_m D}{\nu}$

If you check the units of Re you will see that there are none and that it is a dimensionless number. You will learn more about such numbers in a later section.

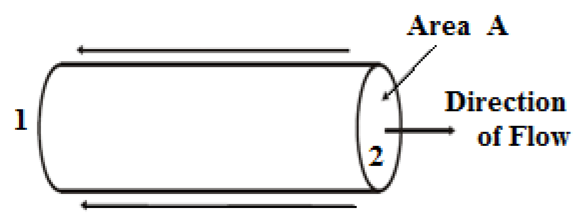
For "small enough flowrate" the dye streak will remain as a well-defined line as it flows along, with only slight blurring due to molecular diffusion of the dye into the surrounding water. For a somewhat larger "intermediate flowrate" the dye fluctuates in time and space, and intermittent bursts of irregular behavior appear along the streak. For "large enough flowrate" the dye streak almost immediately become blurred and spreads across the entire pipe. Reynolds discovered that it was possible to predict the velocity or flow rate at which the transition from laminar to turbulent flow occurred for any Newtonian fluid in any pipe. He also discovered that the critical velocity at which it changed back again was different. He found that when the flow was gradually increased, the change from laminar to turbulent always occurred at a Reynolds number of 2500 and when the flow was gradually reduced it changed back again at a Reynolds number of 2000. Normally, 2000 is taken as the critical value.

1.3.2 Head loss due to Friction:

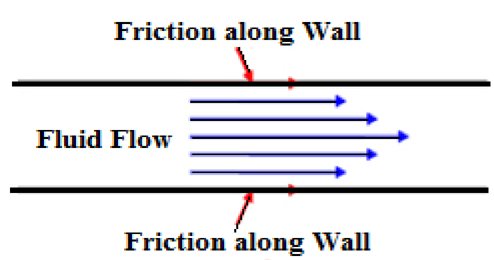
There are many types of losses of head for flowing liquids such as friction, inlet and outlet losses. The major loss is that due to frictional resistance of the pipe, which depends on the inside roughness of the pipe. The common formula for calculating the loss of head due to friction is Darcy's one.

Darcy's formula for friction loss of head:

For a flowing liquid, water in general, through a pipe, the horizontal forces on water between two sections (1) and (2) are:



$$P_1 A = P_2 A + F_R$$



f

P_1= Pressure intensity at (1).

A = Cross sectional area of pipe. P_2= Pressure intensity at (2).

F_R= Frictional Resistance at (2).

$F_R / \gamma A = (P_1 / \gamma) - (P_2 / \gamma) = h_f$

<u>Where,</u>

h_f = Loss of pressure head due to friction.

γ = Specific gravity of water.

<u>It is found experimentally that:</u>

F_R = Factor x Wetted Area x Velocity^2

$F_R = (\gamma f / 2g) \times (\pi d L) \times v^2$

<u>Where,</u> f = Friction coefficient.

d = Diameter of pipe.

L = Length of pipe.

$$h_f = \frac{(\gamma f / 2g) \times (\pi d L) \times v^2}{\gamma (\pi d^2 /4)} = \frac{4 f * L * v^2}{d * 2 g}$$

$$\mathbf{h_f = \frac{4 f L v^2}{2 g d}}$$

It may be substituted for [$v = Q / (\pi d^2 /4)$] in the last equation to get the head loss for a known discharge. Thus,

$$\mathbf{h_f = \frac{32 f L Q^2}{\pi^2 g d^5}}$$

__Note:__ $f' = 4f$

The Darcy – Weisbach equation relates the head loss (or pressure loss) due to friction along a given length of a pipe to the average velocity of the fluid flow for an incompressible fluid.

The friction factor f' is not a constant and depends on the parameters of the pipe and the velocity of the fluid flow, but it is known to high accuracy within certain flow regimes.

For given conditions, it may be evaluated using various empirical or theoretical relations, or it may be obtained from published charts.

R_e (Reynolds Number) is a dimensionless number.

$$R_e = \frac{\rho\, v\, d}{\mu}$$

For pipes,	Laminar flow,	$R_e < 2000$
	Transitional flow,	$2000 < R_e < 4000$
	Turbulent flow,	$R_e > 4000$

For laminar flow,

Poiseuille law, ($f = 64/Re$) where Re is the Reynolds number .

For turbulent flow,

Methods for finding the friction coefficient f include using a diagram such as the Moody chart, or solving equations such as the Colebrook–White equation.

Also, a variety of empirical equations valid only for certain flow regimes such as the Hazen – Williams equation, which is significantly easier to use in calculations. However, the generality of Darcy – Weisbach equation has made it the preferred one.

The only difference of (h_f) between laminar and turbulent flows is the empirical value of (f).

1.4 Minor Losses

Minor losses occur in the following circumstances.

a) Exit from a pipe into a tank.

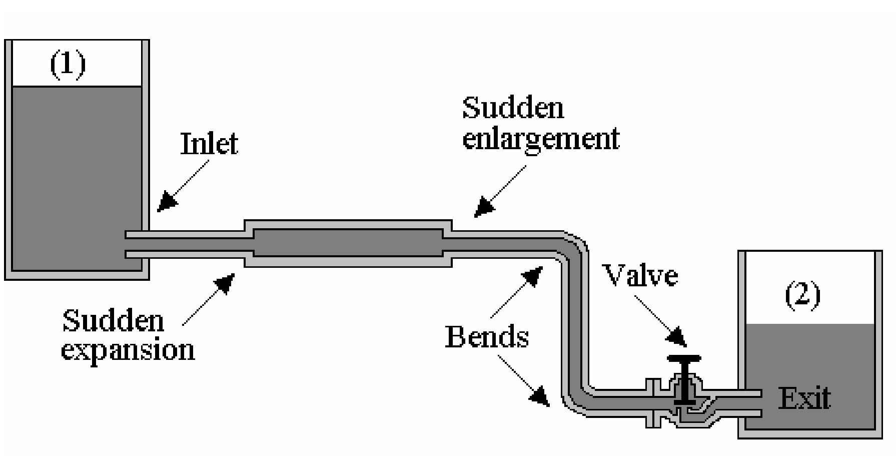
b) Entry to a pipe from a tank.

c) Sudden enlargement in a pipe.

d) Sudden contraction in a pipe.

e) Bends in a pipe.

f) Any other source of restriction such as pipe fittings and valves.



In general, minor losses are neglected when the pipe friction is large in comparison but for short pipe systems with bends, fittings and changes in section, the minor losses are the dominant factor.

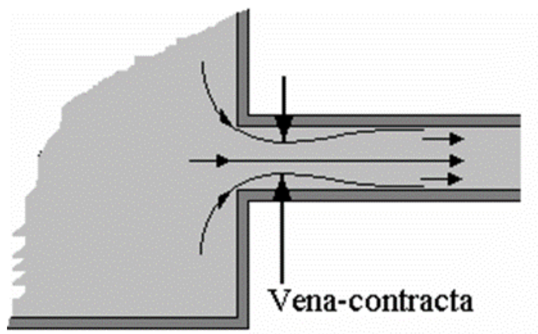
In general, the minor losses are expressed as a fraction of the kinetic head or dynamic pressure in the smaller pipe.

$$\text{Minor head loss} = k\,(v^2/2g)$$

Values of k can be derived for standard cases but for items like elbows and valves in a pipeline, it is determined by experimental methods.

1.4.1 Coefficient of Contraction Cc

The fluid approaches the entrance from all directions and the radial velocity causes the jet to contract just inside the pipe. The jet then spreads out to fill the pipe. The point where the jet is smallest is called the VENA CONTRACTA.



The coefficient of contraction C_c is defined as

$$C_c = A_j/A_o$$

A_j is the cross sectional area of the jet and A_o is the c.s.a. of the pipe. For a round pipe this becomes $C_c = d_j^2/d_o^2$.

1.4.2 Coefficient of Velocity Cv

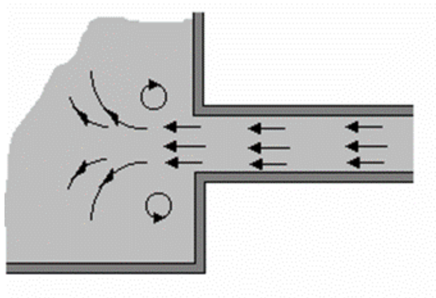
The coefficient of velocity is defined as

$$C_v = \text{actual velocity/theoretical velocity}$$

In this instance it refers to the velocity at the vena-contracta but as you will see later on, it applies to other situations also.

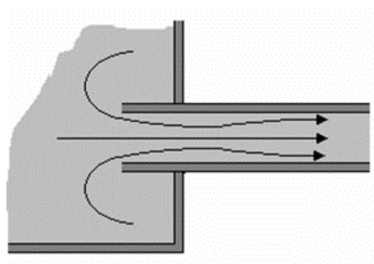
1.4.3 Exit from a Pipe Into a Tank.

The liquid emerges from the pipe and collides with stationary liquid causing it to swirl about before finally coming to rest. All the kinetic energy is dissipated by friction. It follows that all the kinetic head is lost so k = 1.0



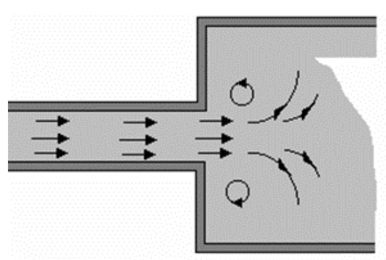
1.4.4 Entry to a Pipe from a Tank

The value of k varies from 0.78 to 0.04 depending on the shape of the inlet. A good rounded inlet has a low value but the case shown is the worst.



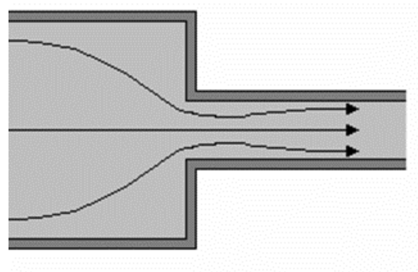
1.4.5 Sudden Enlargement

This is similar to a pipe discharging into a tank but this time it does not collide with static fluid but with slower moving fluid in the large pipe. The resulting loss coefficient is given by the following expression.



1.4.6 Sudden Contraction

This is similar to the entry to a pipe from a tank. The best case gives k = 0 and the worse case is for a sharp corner which gives k = 0.5.



1.4.7 Bends and Fittings

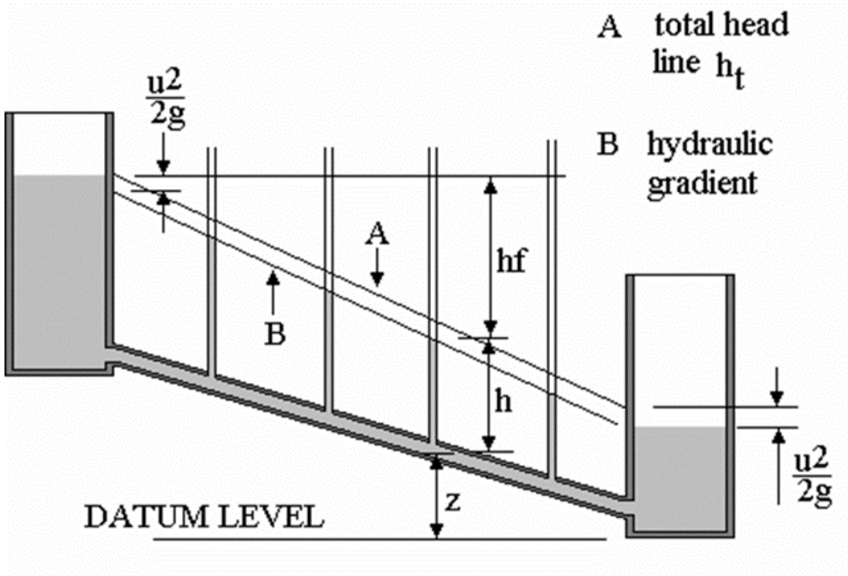
The k value for bends depends upon the radius of the bend and the diameter of the pipe. The k value for bends and the other cases is on various data sheets. For fittings, the manufacturer usually gives the k value. Often instead of a k value, the loss is expressed as an equivalent length of straight pipe that is to be added to L in the Darcy formula.

1.5 Hydraulic Gradient

Consider a tank draining into another tank at a lower level as shown. There are small vertical tubes at points along the length to indicate the pressure head (h). Relative to a datum, the total energy head is $h_T = h + z + u^2/2g$ and this is shown as line A.

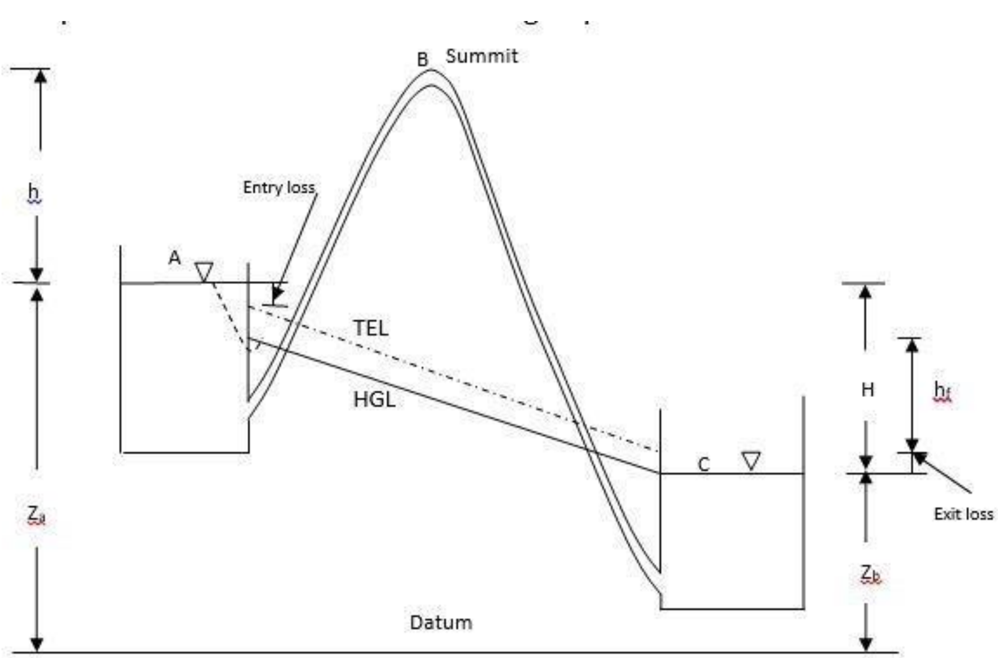
The hydraulic grade line is the line joining the free surfaces in the tubes and represents the sum of h and z only. This is shown as line B and it is always below the line of h_T by the velocity head $u^2/2g$. Note that at exit from the pipe, the velocity head is not recovered but lost as friction as the emerging jet collides with the static liquid. The free surface of the tank does not rise.

The only reason why the hydraulic grade line is not horizontal is because there is a frictional loss h_f. The actual gradient of the line at any point is the rate of change with length $i = \delta h_f/\delta L$



1.6 Siphon

A siphon is a long bent pipe which is used to carry water from a reservoir at a higher elevation to another reservoir at a lower elevation when the two reservoirs are separated by a hill or high level ground in between as shown in Fig. Since the siphon is laid over the hill or the high level ground, for some length from the entrance section it will rise above the water surface in the upper (or supply) reservoir, and then for the remaining length it will drop down to be connected to the lower reservoir.



The rising portion of the siphon is known as the 'inlet leg (or inlet limb), the highest point is known as summit and the portion between the summit and the lower reservoir is known as outlet leg (or outlet limb). As may be seen in Fig. the inlet leg (or inlet limb) of a siphon isusually smaller than the outlet leg (or outlet limb). As the siphon is also a long pipe, the loss of head due to friction will be very large and hence the other minor losses may be neglected. Further the length of the siphon maybe taken as the length of its horizontal projection. Hence the hydraulic grade line and the energy grade line (or total energy line) for a siphon, as shown in Fig.2.1 may also be obtained in the same manner as in the case of an ordinary long pipe.

It will be seen from Fig. 2.1, that the hydraulic grade line cuts the siphon at points C and D, so that some portion of the siphon is above the hydraulic grade line. The vertical distance between the hydraulic grade line and the pipe centre line represents the pressure at any section. If the hydraulic grade line is above the centre line of the pipe then the pressure is above atmospheric; and if the hydraulic grade line is below the centre line of the pipe, the pressure is negative or below atmospheric.

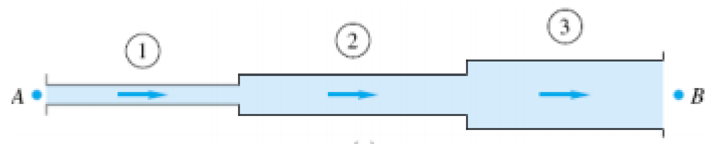
Thus for the portion of the siphon below points C and D the pressure will be above atmospheric and at points C and D the pressure of the water flowing in the siphon is equal to atmospheric pressure. For the portion of the siphon between C and D the pressure will be below atmospheric. As the highest point of the siphon above the hydraulic grade line is the summit S, the water pressure at this point is the least. Further as the vertical distance between the summit of the siphon and the hydraulic grade line increases, the water pressure at this point reduces. Theoretically this pressure may be reduced to - 10.3 m of water (if the atmospheric pressure is 10.3 m of water) or absolute vacuum, because this limit would correspond to a perfect vacuum and the flow would stop. However, in practice if the pressure is reduced to about 2.5 m of water absolute or 7.8 m of water vacuum the dissolved air or other gases would come out of the solution and collect at the summit of the siphon in sufficient quantity to form an air-lock, which will obstruct the continuity of the flow, (or the flow will completely stop).

A similar trouble may also be caused by the formation of the water vapor in the region of low pressure. Therefore the siphon should be laid so that no section of the pipe will be more than 7.8 m above the hydraulic grade line at the section. Moreover, in order to limit the reduction of the pressure at the summit the length of the inlet leg of the siphon is also required to be limited. This is so because as indicated below, if the inlet leg is very long a considerable loss of head due to friction is caused, resulting in further reduction of the pressure at the summit.

1.7 Pipes in Series and Parallel

In many pipe systems there is more than one pipe involved. The governing mechanisms for the flow in multiple pipe systems are the same as for the single pipe systems.

1.7.1 Pipes in Series



The indicated pipe system has a steady flow rate Q through three pipes with diameters D_1, D_2, & D_3. Two important rules apply to this problem.

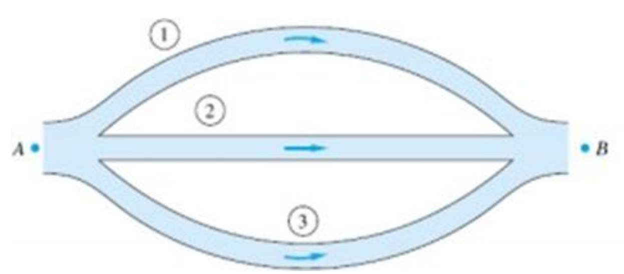
The flow rate is the same through each pipe section. $Q_1 = Q_2 = Q_3 = Q$

The total frictional head loss is the sum of the head losses through the various sections.

$$h_{f,a-b} = h_{f,1} + h_{f,2} + h_{f,3}$$

$$h_{f,a-b} = \left(f\frac{L}{D} + \sum K_i\right)_{D_1} \frac{V_1^2}{2g} + \left(f\frac{L}{D} + \sum K_i\right)_{D_2} \frac{V_2^2}{2g} + \left(f\frac{L}{D} + \sum K_i\right)_{D_3} \frac{V_3^2}{2g}$$

1.7.2 Pipes in Parallel



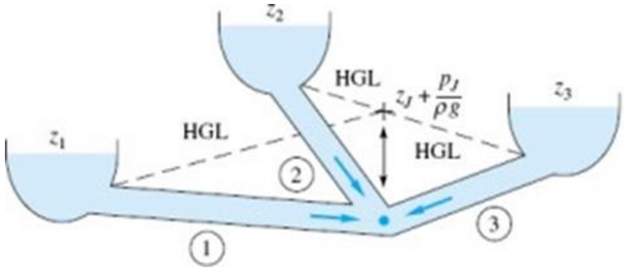
The indicated pipe system has a steady flow rate Q_1, Q_2, Q_3 through three pipes with diameters D_1, D_2, & D_3. Two important rules apply to this problem.

$$\Delta h_{A\to B} = \Delta h_1 = \Delta h_2 = \Delta h_3$$

$$Q = Q_1 + Q_2 + Q_3$$

1.8 Branched Pipes

Consider the third example of a three-reservoir pipe junction as shown in the figure. If all flows are considered positive toward the junction, then $Q_1 + Q_2 + Q_3 = 0$



$$h_J = z_J + \frac{p_J}{\rho g}$$

$$\Delta h_1 = \frac{V_1^2}{2g}\frac{f_1 L_1}{d_1} = z_1 - h_J$$

$$\Delta h_2 = \frac{V_2^2}{2g}\frac{f_2 L_2}{d_2} = z_2 - h_J$$

$$\Delta h_3 = \frac{V_3^2}{2g}\frac{f_3 L_3}{d_3} = z_3 - h_J$$

which obviously implies that one or two of the flows must be away from the junction. The pressure must change through each pipe so as to give the same static pressure p_J at the junction. In other words, let the HGL at the junction have the elevation.

1.9 Equivalent Pipe

Often a compound pipe consisting of several pipes of varying diameters and lengths is to be replaced by a pipe of uniform diameter, which is known as *equivalent pipe*. The uniform diameter of the equivalent pipe is known as the equivalent diameter of the compound pipe. The size of the equivalent pipe may be determined as follows. If L1, L2, L3 etc., are the lengths and D1, D2 and D3 etc., are the diameters respectively of the different pipes of a compound pipeline, then the total head loss in the compound pipe, neglecting the minor losses, is

$$h_L = \frac{f_1 L_1 V_1^2}{2gD_1} + \frac{f_2 L_2 V_2^2}{2gD_2} + \frac{f_3 L_3 V_3^2}{2gD_3} + \ldots$$

Again by continuity

$$Q = a_1 V_1 = a_2 V_2 = a_3 V_3$$

$$= \frac{\pi}{4} D_1^2 V_1 = \frac{\pi}{4} D_2^2 V_2 = \frac{\pi}{4} D_3^2 V_3 = \ldots$$

Assuming $f_1 = f_2 = f_3 = \ldots = f$

$$h_L = \frac{f}{2g} \frac{Q^2}{(\pi/4)^2} \left[\frac{L_1}{D_1^5} + \frac{L_2}{D_2^5} + \frac{L_3}{D_3^5} + .. \right]$$

If D is the diameter and Load is the length of the equivalent pipe then it would carry the same discharge Q if the head loss due to friction in the equivalent pipe is same as that in the compound pipe. The loss of head due to friction in the equivalent pipe is

$$h_L = \frac{fLV^2}{2gD} = \frac{f}{2g} \frac{Q^2}{(\pi/4)^2} \frac{L}{D^5}$$

Thus equating the two heads losses, we get

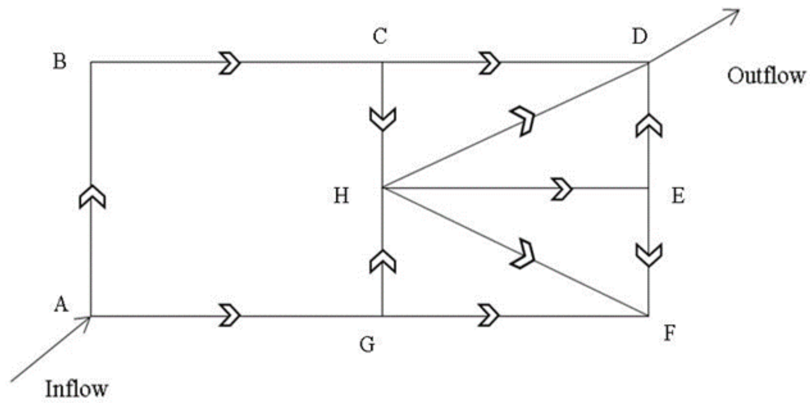
$$\frac{L}{D^5} = \left[\frac{L_1}{D_1^5} + \frac{L_2}{D_2^5} + \frac{L_3}{D_3^5} + .. \right]$$

The above equation is known as *Dupuit's equation,* which may be used to determine the size of the equivalent pipe. Thus if the length of the equivalent pipe is equal to the total length of the compound pipe i.e., L= (L_1, L_2, L_3+…), then the diameter D of the equivalent pipe may be determined by using above expression.

1.10 Pipe Networks

A group of interconnected pipes forming several loops as shown in fig below is called a network of pipes. Such networks of pipes are commonly used for municipal water distribution systems in cities.

The main problem in a pipe network is to determine the distribution of flow through the various pipes of the network such that all the conditions of flow are satisfied and all the circuits are then balanced. The conditions to be satisfied in any network of pipes are as follows:



According to the principle of continuity the flow into each junction must be equal to the flow out of the junction. For example at junction A, the inflow must be equal to the flow through AB and AG.

In each loop, the loss of head due to flow in clockwise direction must be equal to the loss of head due to flow in anticlockwise direction. For example in the loop ABCHG the sum of the head losses due to flow in AB, BC and CH (clockwise flow) must be equal to the sum of the head losses due to flow in AG and GH (anticlockwise flow).

The Darcy-Weisbach equation must be satisfied for flow in each pipe. Minor losses may be neglected if the pipe lengths are large. However if the minor losses are large, they may be taken into account by considering them in terms of the head loss due to friction in equivalent pipe lengths.According to Darcy-Weisbach equation the loss of head $h_f$through any pipe discharging at the rate of Q can be expressed as

$$h_f = rQ^n$$

Where r is a proportionality factor which can be determined for each pipe knowing the friction factors f, the length L and the diameter D of the pipe.

$$\left(r = \frac{fL}{2g(\pi/4)^2 D^5} = \frac{fL}{12.10D^5}\right);$$

and n is an exponent having numerical value ranging from 1.72 to 2.00.

1.11 Hardy – Cross Method

The pipe network problems are in general complicated and cannot be solved analytically. As such methods of successive approximations are utilised. One such method which is commonly used is ‘Hardy Cross Method’, named after its original investigator. The procedure for the solution of pipe network problems by the Hardy Cross Method is as follows:

Assume a most suitable distribution of flow that satisfies continuity at each junction.

With the assumed values of Q, compute the head losses for each pipe using $h_f = rQ^n$

Consider different loops or circuits and compute the net head loss around each circuit considering the head loss in clockwise flows as positive and in anti-clockwise flows as negative. For a correct distribution of flow the net head loss around each circuit should be equal to zero, so that the circuit will be balanced. However, in most of the cases, for the assumed distribution of flow the head loss around the circuit will not be equal to zero. The assumed flows are then corrected by introducing a correction ΔQ for the flows, till the circuit is balanced. The value of the correction ΔQ to be applied to the assumed flows of the circuit may be obtained as follows:

$$\Delta Q = -\frac{\sum rQ_0^n}{\sum rnQ_0^{n-1}}$$

In the above expression for the correction the denominator is the sum of absolute terms and hence it has no sign. Further if the head losses due to flow in the clockwise direction are more than losses due to flow in the anti-clockwise direction, then according to the sign convention adopted, ΔQ will be negative and hence it should be added to the flow in the anti- clockwise direction and subtracted from the flow in the clockwise direction. On the other hand if the head losses due to flow in the clockwise direction are less than the head losses due to flow in the anti-clockwise direction, then ΔQ will be positive hence it should be added to the flow in the clockwise direction and subtracted from the flow in the anti-clockwise direction. Moreover, for the pipes common to two circuits or loops (such as CH, GH, HF etc.) a correction from both the loops will be required to be applied.

With the corrected flows in all the pipes, a second trial calculation is made for all the loops and the process is repeated till the correction become negligible.

Numericals

Problem.

Consider the two reservoirs shown in figure 16, connected by a single pipe that changes diameter over its length. The surfaces of the two reservoirs have a difference in level of 9m. The pipe has a diameter of 200mm for the first 15m (from A to C) then a diameter of 250mm for the remaining 45m (from C to B).

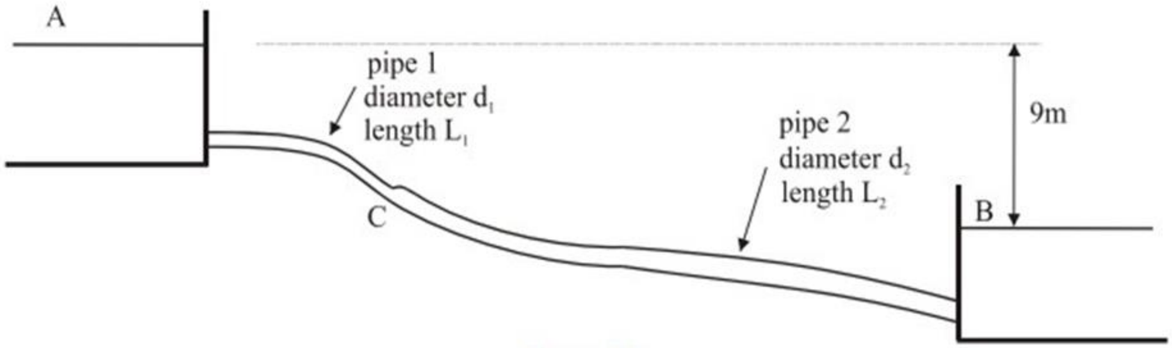


Figure 16:

For the entrance use $k_L = 0.5$ and the exit $k_L = 1.0$. The join at C is sudden. For both pipes use $f = 0.01$.

Solution.

Total head loss for the system H = height difference of reservoirs

h_{f1} = head loss for 200mm diameter section of pipe
h_{f2} = head loss for 250mm diameter section of pipe
$h_{L\ entry}$ = head loss at entry point
$h_{L\ join}$ = head loss at join of the two pipes
$h_{L\ exit}$ = head loss at exit point

So

$$H = h_{f1} + h_{f2} + h_{L\ entry} + h_{L\ join} + h_{L\ exit} = 9\text{m}$$

All losses are, in terms of Q:

$$h_{f1} = \frac{fL_1Q^2}{3d_1^5}$$

$$h_{f2} = \frac{fL_2Q^2}{3d_2^5}$$

$$h_{Lentry} = 0.5\frac{u_1^2}{2g} = 0.5\frac{1}{2g}\left(\frac{4Q}{\pi d_1^2}\right)^2 = 0.5\times 0.0826\frac{Q^2}{d_1^4} = 0.0413\frac{Q^2}{d_1^4}$$

$$h_{Lexit} = 1.0\frac{u_2^2}{2g} = 1.0\times 0.0826\frac{Q^2}{d_2^4} = 0.0826\frac{Q^2}{d_2^4}$$

$$h_{Ljoin} = \frac{(u_1 - u_2)^2}{2g} = \left(\frac{4Q}{\pi}\right)^2 \frac{\left(\frac{1}{d_1^2} - \frac{1}{d_2^2}\right)^2}{2g} = 0.0826Q^2\left(\frac{1}{d_1^2} - \frac{1}{d_2^2}\right)^2$$

Substitute these into

$$h_{f1} + h_{f2} + h_{L\ entry} + h_{L\ join} + h_{L\ exit} = 9$$

and solve for Q, to give $Q = 0.158\ m^3/s$

Problem.

Two pipes connect two reservoirs (A and B) which have a height difference of 10m. Pipe 1 has diameter 50mm and length 100m. Pipe 2 has diameter 100mm and length 100m. Both have entry loss $k_L = 0.5$ and exit loss $k_L=1.0$ and Darcy f of 0.008.

Calculate:

a) rate of flow for each pipe
b) the diameter D of a pipe 100m long that could replace the two pipes and provide the same flow.

Solution.

a)

Apply Bernoulli to each pipe separately. For pipe 1:

$$\frac{p_A}{\rho g}+\frac{u_A^2}{2g}+z_A=\frac{p_B}{\rho g}+\frac{u_B^2}{2g}+z_B+0.5\frac{u_1^2}{2g}+\frac{4flu_1^2}{2gd_1}+1.0\frac{u_1^2}{2g}$$

p_A and p_B are atmospheric, and as the reservoir surface move s slowly u_A and u_B are negligible, so

$$z_A-z_B=\left(0.5+\frac{4fl}{d_1}+1.0\right)\frac{u_1^2}{2g}$$

$$10=\left(1.0+\frac{4\times 0.008\times 100}{0.05}\right)\frac{u_1^2}{2\times 9.81}$$

$$u_1=1.731\,m/s$$

And flow rate is given by

$$Q_1=u_1\frac{\pi d_1^2}{4}=0.0034\,m^3/s$$

For pipe 2:

$$\frac{p_A}{\rho g}+\frac{u_A^2}{2g}+z_A=\frac{p_B}{\rho g}+\frac{u_B^2}{2g}+z_B+0.5\frac{u_2^2}{2g}+\frac{4flu_2^2}{2gd_2}+1.0\frac{u_2^2}{2g}$$

Again p_A and p_B are atmospheric, and as the reservoir surface move s slowly u_A and u_B are negligible, so

$$z_A - z_B = \left(0.5 + \frac{4fl}{d_2} + 1.0\right)\frac{u_2^2}{2g}$$

$$10 = \left(1.0 + \frac{4 \times 0.008 \times 100}{0.1}\right)\frac{u_2^2}{2 \times 9.81}$$

$$u_2 = 2.42\, m/s$$

And flow rate is given by

$$Q_2 = u_2 \frac{\pi d_2^2}{4} = 0.0190\, m^3/s$$

b) Replacing the pipe, we need $Q = Q_1 + Q_2 = 0.0034 + 0.0190 = 0.0224\ m^3/s$

For this pipe, diameter D, velocity u , and making the same assumptions about entry/exit losses, we have:

$$\frac{p_A}{\rho g} + \frac{u_A^2}{2g} + z_A = \frac{p_B}{\rho g} + \frac{u_B^2}{2g} + z_B + 0.5\frac{u^2}{2g} + \frac{4flu^2}{2gD} + 1.0\frac{u^2}{2g}$$

$$z_A - z_B = \left(0.5 + \frac{4fl}{D} + 1.0\right)\frac{u^2}{2g}$$

$$10 = \left(1.5 + \frac{4 \times 0.008 \times 100}{D}\right)\frac{u^2}{2 \times 9.81}$$

$$196.2 = \left(1.5 + \frac{3.2}{D}\right)u^2$$

The velocity can be obtained from Q i.e.

$$Q = Au = \frac{\pi D^2}{4}u$$

$$u = \frac{4Q}{\pi D^2} = \frac{0.02852}{D^2}$$

So

$$196.2 = \left(1.5 + \frac{3.2}{D}\right)\left(\frac{0.02852}{D^2}\right)^2$$

$$0 = 241212D^5 - 1.5D - 3.2$$

which must be solved iteratively

An approximate answer can be obtained by dropping the second term:

$$0 = 241212D^5 - 3.2$$

$$D = \sqrt[5]{3.2/241212}$$

$$D = 0.1058m$$

Writing the function

$$f(D) = 241212D^5 - 1.5D - 3.2$$

$$f(0.1058) = -0.161$$

So increase D slightly, try 0.107m

$$f(0.107) = 0.022$$

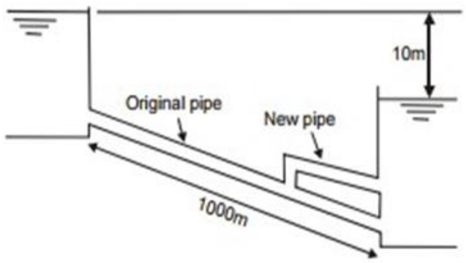
i.e. the solution is between 0.107m and 0.1058m but 0.107 if sufficiently accurate.

Problem.

A pipe joins two reservoirs whose head difference is 10m. The pipe is 0.2 m diameter, 1000m in length and has a f value of 0.008.

a) What is the flow in the pipeline?

b) It is required to increase the flow to the downstream reservoir by 30%. This is to be done adding a second pipe of the same diameter that connects at some point along the old pipe and runs down to the lower reservoir. Assuming the diameter and the friction factor are the same as the old pipe, how long should the new pipe be?



Solution.

a)

$$h_f = \frac{fLQ^2}{3d^5}$$

$$10 = \frac{0.008 \times 1000 Q^2}{3 \times 0.2^5}$$

$$Q = 0.0346\, m^3/s$$

$$Q = 34.6\, litres/s$$

b)

$$H = 10 = h_{f1} + h_{f2} = h_{f1} + h_{f3}$$

$$\therefore$$

$$h_{f2} = h_{f3}$$

$$\frac{f_2 L_2 Q_2^2}{3d_2^5} = \frac{f_3 L_3 Q_3^2}{3d_3^5}$$

as the pipes 2 and 3 are the same f, same length and the same diameter then $Q_2 = Q_3$.

By continuity $Q_1 = Q_2 + Q_3 = 2Q_2 = 2Q_3$

So

$$Q_2 = \frac{Q_1}{2}$$

and

$$L_2 = 1000 - L_1$$

Then

$$10 = h_{f1} + h_{f2}$$

$$10 = \frac{f_1 L_1 Q_1^2}{3d_1^5} + \frac{f_2 L_2 Q_2^2}{3d_2^5}$$

$$10 = \frac{f_1 L_1 Q_1^2}{3d_1^5} + \frac{f_2 (1000 - L_1)(Q_1 / 2)^2}{3d_2^5}$$

As $f_1 = f_2$, $d_1 = d_2$

$$10 = \frac{f_1 Q_1^2}{3d_1^5}\left(L_1 + \frac{(1000 - L_1)}{4} \right)$$

The new Q_1 is to be 30% greater than before so $Q_1 = 1.3 \times 0.0346 = 0.045$ m^3/s

Solve for L to give

$$L_1 = 456.7\text{m}$$

$$L_2 = 1000 - 456.7 = 543.2 \text{ m}$$

Problem. Consider the example of a reservoir feeding pipe.

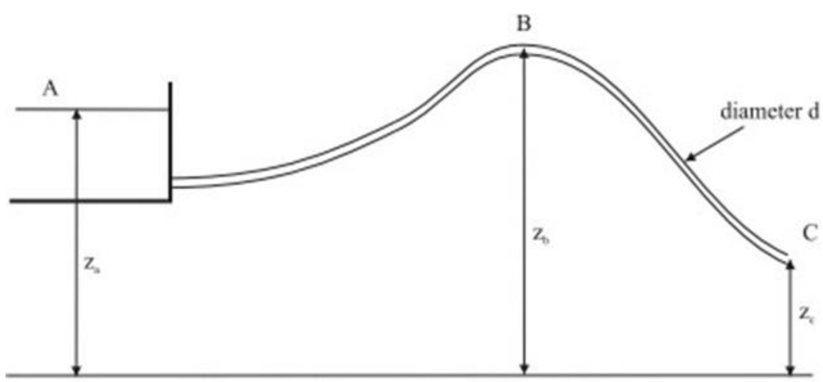


Figure 15: Reservoir feeding a pipe

The pipe diameter is 100mm and has length 15m and feeds directly into the atmosphere at point C 4m below the surface of the reservoir (i.e. $z_a - z_c = 4.0$m). The highest point on the pipe is a B which is 1.5m above the surface of the reservoir (i.e. $z_b - z_a = 1.5$m) and 5 m along the pipe measured from the reservoir. Assume the entrance and exit to the pipe to be sharp and the value of friction factor f to be 0.08. Calculate a) velocity of water leaving the pipe at point C, b) pressure in the pipe at point B.

Solution.

a)

We use the Bernoulli equation with appropriate losses from point A to C

and for entry loss $k_L = 0.5$ and exit loss $k_L = 1.0$.

For the local losses from Table 2 for a sharp entry $k_L = 0.5$ and for the sharp exit as it opens in to the atmosphere with no contraction there are no losses, so

$$h_L = 0.5\frac{u^2}{2g}$$

Friction losses are given by the Darcy equation

$$h_f = \frac{4fLu^2}{2gd}$$

Pressure at A and C are both atmospheric, u_A is very small so can be set to zero, giving

$$z_A = \frac{u^2}{2g} + z_C + \frac{4fLu^2}{2gd} + 0.5\frac{u^2}{2g}$$

$$z_A - z_C = \frac{u^2}{2g}\left(1 + 0.5 + \frac{4fL}{d}\right)$$

Substitute in the numbers from the question

$$4 = \frac{u^2}{2\times 9.81}\left(1.5 + \frac{4\times 0.08\times 15}{0.1}\right)$$

$$u = 1.26m/s$$

b)

To find the pressure at B apply Bernoulli from point A to B using the velocity calculated above. The length of the pipe is $L_1 = 5$m:

$$z_A = \frac{p_B}{\rho g} + \frac{u^2}{2g} + z_B + \frac{4fL_1u^2}{2gd} + 0.5\frac{u^2}{2g}$$

$$z_A - z_B = \frac{p_B}{\rho g} + \frac{u^2}{2g}\left(1 + 0.5 + \frac{4fL_1}{d}\right)$$

$$-1.5 = \frac{p_B}{1000\times 9.81} + \frac{1.26^2}{2\times 9.81}\left(1.5 + \frac{4\times 0.08\times 5.0}{0.1}\right)$$

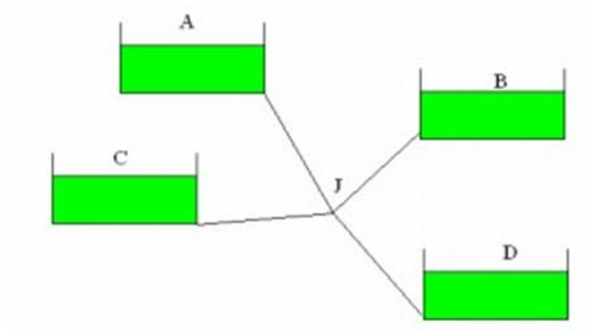
$$p_B = -28.58\times 10^3\ N/m^2$$

That is 28.58 kN/m^2 **below** atmospheric.

Problem.

The table shows the data for the network of pipes shown connecting four reservoirs to a common junction.

Reservoir	Water Level (m) above a datum	K for Pipe connecting to J (s^2/m^5)
A	50	4.0
B	45	3.0
C	40	2.0
D	30	2.0



Calculate the flow in each pipe using iteration until the final head correction at the junction is less than 0.1 m.

The height of the datum is not given so we can only calculate the combined head and height. The best guess is usually the mean height of the reservoirs which is (50 + 45 + 40 + 30)/4 = 41.25

1st ITERATION

PIPE	K	z	Δh_f	Q	Q/h_f	Guess h_J+z_J
A	4	50	8.75	1.47902	0.16903	41.25
B	3	45	3.75	1.11803	0.29814	
C	2	40	-1.25	-0.7906	0.63246	
D	2	30	-11.25	-2.3717	0.21082	
				-0.5652	1.31045	

$$\sum \delta h_f = \frac{2\,\delta Q}{\sum Q/h_f} = \frac{2 \text{ x } (-0.5652)}{1.310} = -0.863 = \text{Correct } h_J + z_J = 40.4$$

2nd ITERATION

PIPE	R	z	Δh_f	Q	Q/h_f	Guess h_J+z_J
A	4	50	9.6	1.54919	0.16137	41.25
B	3	45	4.6	1.23828	0.26919	
C	2	40	-0.4	-0.4472	1.11803	
D	2	30	-10.4	-2.2804	0.21926	
				0.05991	1.76786	

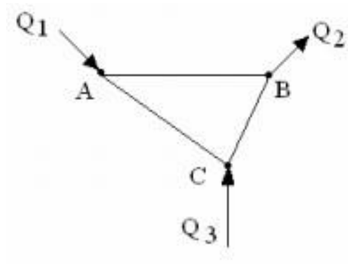
$$\sum \delta h_f = \frac{2\,\delta Q}{\sum Q/h_f} = \frac{2 \text{ x } (0.5991)}{1.76786} = 0.0678$$ This is less than 0.1 so meets the answer

$Q_A = 1.55\ m^3/s$ $Q_B = 1.24\ m^3/s$ $Q_C = -0.45\ m^3/s$ $Q_D = -2.28\ m^3/s$

Solution.

In the simple network shown $Q_1 = 0.8\ m^3/s$, $Q_2 = -1.2\ m^3/s$. The resistance of each pipe is as follows.

Pipe AB $R = 50\ s^2/m^5$
Pipe BC $R = 30\ s^2/m^5$
Pipe AC $R = 60\ s^2/m^5$



Determine the flow in the three pipes. Take n = 2

Problem.

Solution.

By conservation of flow, $Q_3 = 0.4\ m^3/s$

Guess the flow in each pipe bearing in mind the total flow at a node is zero. Clockwise is positive. The starting guess is:

Q (AB) = 0.6 Q (BC) = -0.6 Q (BC) = -0.4

First iteration

PIPE	R	Q	h_f	h_f/Q
AB	50	0.6	18	30
BC	30	-0.6	-10.8	18
AC	60	-0.2	-2.4	12
		-0.2	4.8	60

$$\delta Q = \frac{\sum h_{f1}}{2\sum h_{f1}/Q} = \frac{4.8}{2 \times 60} = 0.04$$ Correct the Q values by subtracting

Second iteration

PIPE	R	Q	h_f	h_f/Q
AB	50	0.56	15.68	28
BC	30	-0.64	-12.288	19.2
AC	60	-0.24	-3.456	14.4
		-0.32	-0.064	61.6

$$\delta Q = \frac{\sum h_{f1}}{2\sum h_{f1}/Q} = \frac{-0.32}{2 \times 61.6} = -0.000524$$ Correct the Q values by subtracting

Third iteration

PIPE	R	Q	h_f	h_f/Q
AB	50	0.56052	15.708	28.025
BC	30	-0.6395	-12.2688	19.185
AC	60	-0.2395	-3.44162	14.37
		-0.3185	-0.00005	61.58

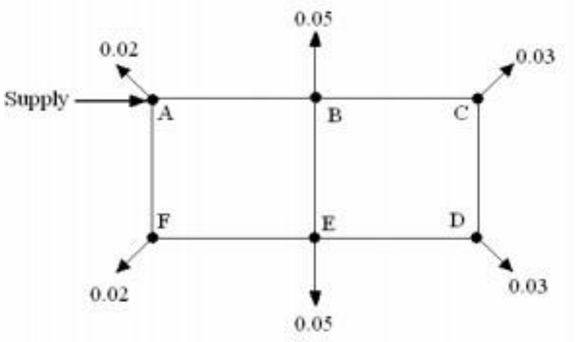
This is one iteration more than we need. The head loss is so close to zero that this is the correct answer.

Q (AB) = $0.56052\ m^3/s$, Q (BC) = $-0.6395\ m^3/s$ and Q (AC) = $-0.2395\ m^3/s$

If we check the flow into each node we will see that the original figures have been maintained.

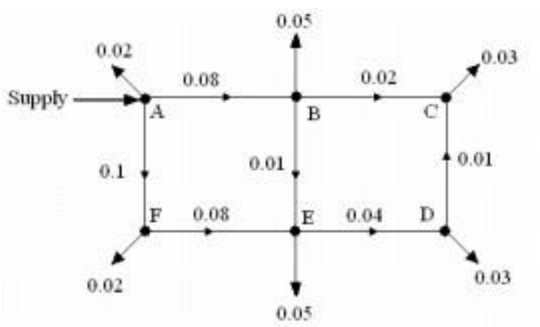
Problem.

The diagram shows a water supply network with the demands indicated at the nodes. The value of K for each pipe is 1000 s^2/m^5 except for BE which is 7500 s^2/m^5. The supply pressure head at A is 50 m above the ground elevation for the area served which is flat and level. Calculate the pressure head at each node.



Solution.

The problem must be solved as two loops with a common pipe BE.
First make a guess at the flow rates.
The supply must be
$0.02 + 0.05 + 0.03 + 0.03 + 0.05 + 0.02 = 0.2\ m^3/s$
Bear in mind that the net flow is zero at all nodes.



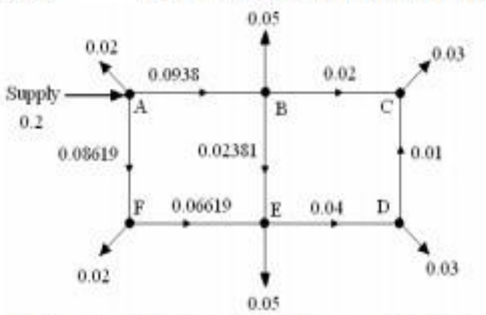
Data shown for initial guess

Start with loop ABEFA

PIPE	R	Q	h_f	h_f/Q
AB	1000	0.08	6.4	80
BE	7500	0.01	0.75	75
EF	1000	-0.08	-6.4	80
FA	1000	-0.1	-10	100
			-9.25	335

$$\delta Q = \frac{\sum h_{fl}}{2\sum h_{fl}/Q} = \frac{-9.25}{2 \times 335} = -0.01386$$

Correct all flows in this loop by adding 0.01386.



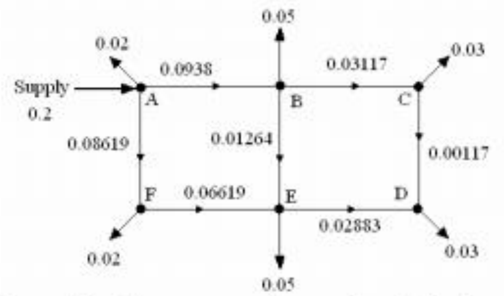
Data after first correction to left loop

Now do loop BCDEB

PIPE	R	Q	h_f	h_f/Q
BC	1000	0.02	0.4	20
CD	1000	-0.01	-0.1	10
DE	1000	-0.04	-1.6	40
BE	7500	-0.02381	-4.2504	178.5
			-5.550	248.5

$$\delta Q = \frac{\sum h_{f1}}{2\sum h_{f1}/Q} = \frac{-5.55}{2 \times 248.5} = -0.01117$$

Correct all flows in this loop by adding 0.01117.



Data after first correction to the right loop

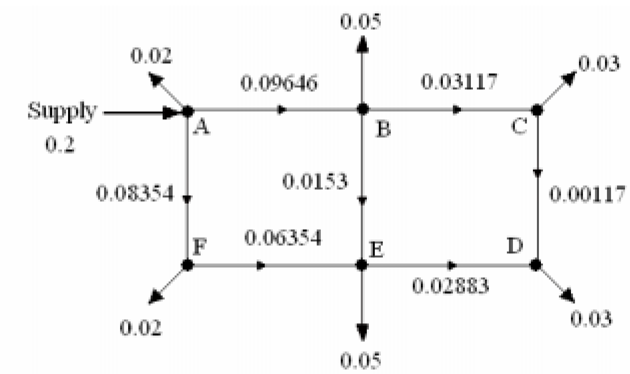
This completes the first iteration so now do loop ABEFA again.

PIPE	R	Q	hf	hf/Q
AB	1000	0.09381	8.79956	93.80597
BE	7500	0.01264	1.19829	94.80084
EF	1000	-0.0662	-4.38165	66.19403
FA	1000	-0.0862	-7.42941	86.19403
			-1.81321	340.9949

$$\delta Q = \frac{\sum h_{f1}}{2\sum h_{f1}/Q} = \frac{-1.8132}{2 \times 341} = -0.00266$$

Correct all flows by adding 0.00266

Data after second correction to the left loop



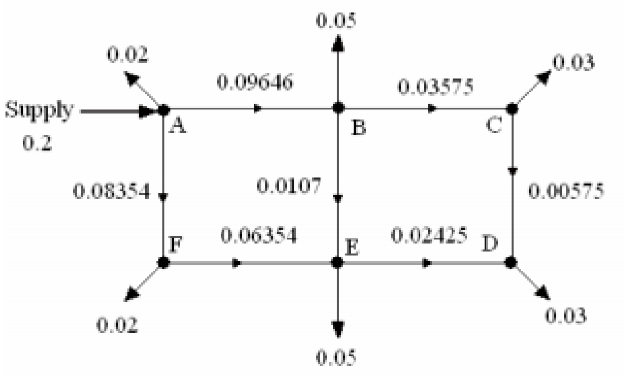
Now do loop BCDEB again.

PIPE	R	Q	h_f	h_f/Q
BC	1000	0.03117	0.971311	31.16586
CD	1000	0.00117	0.001359	1.165859
DE	1000	-0.0288	-0.83141	28.83414
BE	7500	-0.0153	-1.7554	114.7411
			-1.61414	175.907

$$\delta Q = \frac{\sum h_{f1}}{2\sum h_{f1}/Q} = \frac{1.6141}{2 \times 175.9} = -0.00459$$

Correct all flows by adding 0.00459

Data after second correction
to right loop



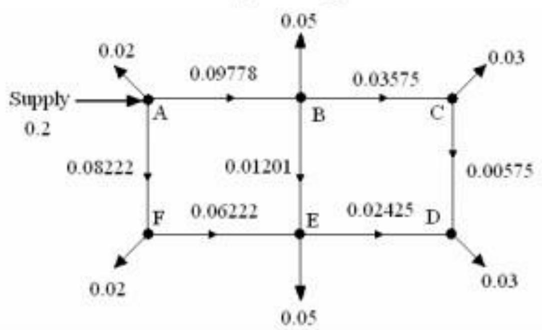
We need to keep going until h_f is very small. Our initial guess was not very good.
Do ABEFA Again

PIPE	R	Q	h_f	h_f/Q
AB	1000	0.09646	9.30543	96.46467
BE	7500	0.01071	0.8604	80.33071
EF	1000	-0.0635	-4.03674	63.53533
FA	1000	-0.0835	-6.97815	83.53533
			-0.84905	323.866

$$\delta Q = \frac{\sum h_{fl}}{2\sum h_{fl}/Q} = \frac{-0.849}{2 \times 323.866} = -0.00131$$

Correct all flows by adding 0.00131

Data after third correction
to the left loop

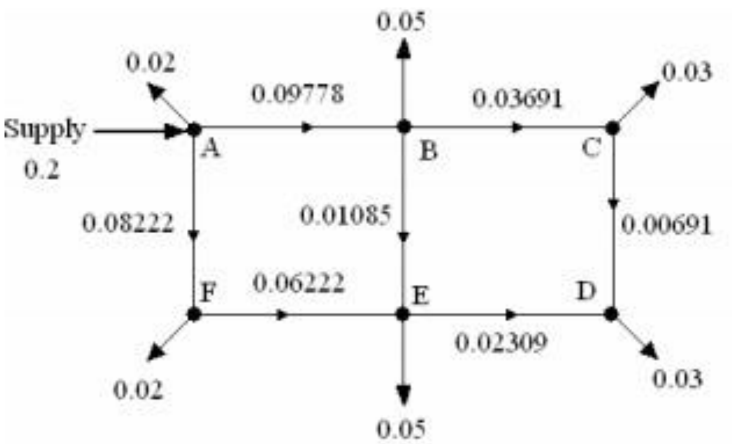


Do BCDEB again

PIPE	R	Q	h_f	h_f/Q
BC	1000	0.03575	1.278342	35.75391
CD	1000	0.00575	0.033107	5.75391
DE	1000	-0.0242	-0.58787	24.24609
BE	7500	-0.012	-1.08389	90.16178
			-0.36031	155.9157

$$\delta Q = \frac{\sum h_{fl}}{2\sum h_{fl}/Q} = \frac{-0.3603}{2 \times 155.916} = -0.00116$$

Correct all flows by adding 0.00116



Final flow rates

We have a total friction head of less than 1 metre in both loops so we will end here.
To find the pressure head at each node we must evaluate the friction heads with these flows.

PIPE	R	Q	h_f
AB	1000	0.09778	9.56004
BE	7500	0.01087	0.88554
EF	1000	-0.0622	-3.87189
FA	1000	-0.0822	-6.76087
BC	1000	0.03691	1.362302
CD	1000	0.00691	0.047739
DE	1000	-0.0231	-0.53318
BE	7500	-0.0109	-0.88554

Pressure at B = 50 – 9.6 = 40.4 m
Pressure at E = 40.4 – 0.9 = 39.5 m
Pressure at F = 50 – 6.8 = 43.2 m
Pressure at E = 43.2 – 3.9 = 39.3 m (check)
Pressure at C = 40.4 – 1.4 = 39 m
Pressure at D = 39 – 0.05 = 39 m
Pressure at D = 39.4 – 0.5 = 38.9 m (check)