1. FLOW THROUGH PIPES – VISCOUS FLOW

1.1 INTRODUCTION

Pipe flow is one of the most important, say, subjects in fluid mechanics. Since almost all, say way of life, we have to use, say some way or another way, the pipe flow; may be for water supply or may be for sewage flow or may be for say transport chemicals or a petroleum products, etc., number of applications are there for pipe flow systems. Viscous flow is also called as Laminar flow.

The simple and ordered flow is called laminar flow. In laminar flow fluid particles move along straight parallel paths in “layers” or “laminae”. In this type of flow, the molecules move in the stream they were initially and do not change their stream while flowing. That is why the flow is simple and ordered. Ordered flow is justified only when velocity is less. At low velocities, forces due to viscosity are predominant over inertial forces.

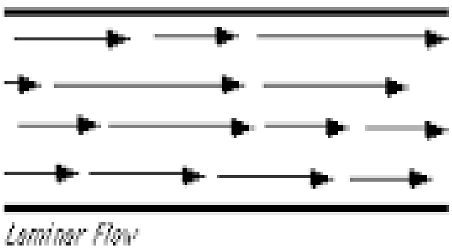


Fig. 1.1 Laminar and turbulent flow

The viscosity of fluid induces relative motion within the fluid as the fluid layers slide over each other, which in turn gives rise to shear stresses. The magnitude of viscous shear stresses so produced, varies from point to point, being maximum at boundary and gradually decreasing with increase in the distance from boundary.

The stresses so produced result in developing a resistance to flow. In order to overcome shear resistance to flow, the pressure drops from section to section in the direction of flow, so that a pressure gradient exists. Therefore, an expression relating shear and pressure gradients in laminar flow have to be studied in ordered to analyse various cases of laminar flow.

1.2 RELATION BETWEEN SHEAR AND PRESSURE GRADIENTS

Consider a free body of fluid having the form of an elementary parallelepiped of length δx width δz and thickness δy. Since there is a relative motion between different layers of fluid, the velocity distribution is non-uniform.

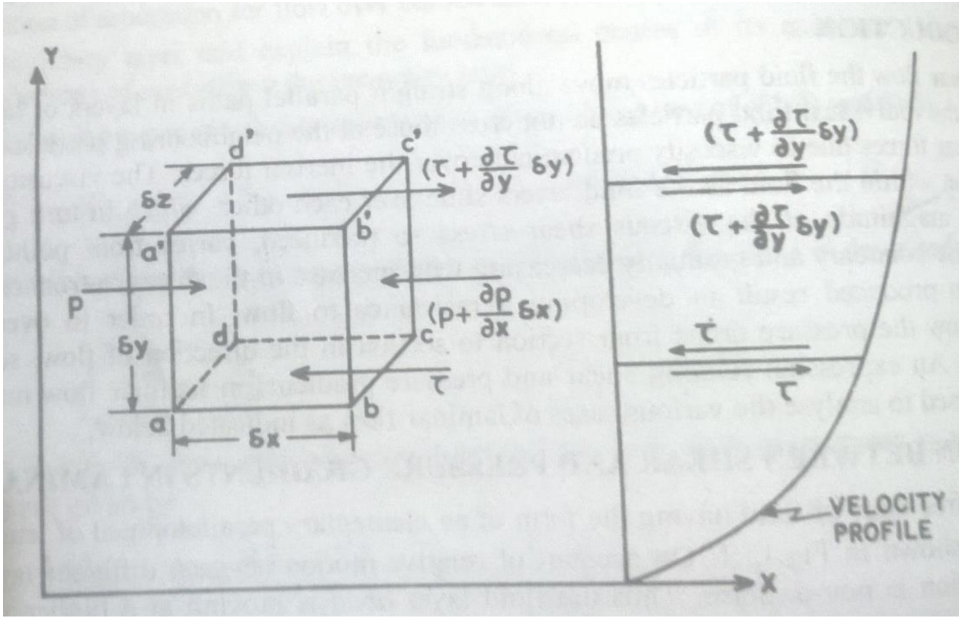


Fig. 1.2 Forces acting on parallelepiped

Thus the fluid layer abcd is moving at higher velocity than the layer below it and hence the layer abcd exerts a shear stress in the positive direction on the lower layer. On the other hand, lower layer exerts an equal and opposite shear stress on the layer abcd.

Similarly, shear stress is exerted by the a'b'c'd' on the layer below it in positive x-direction. But the magnitude of shear stresses on the layers abcd and a'b'c'd' will be different.

If τ represents the shear stress on the layer abcd then shear stresses on the layer a'b'c'd' is equal to

$$(\tau + \frac{\partial \tau}{\partial y}\delta y).$$

For a two dimensional flow there will be no shear stresses on the vertical faces abb'a' and cd'dc'. Thus the only forces acting on the parallelepiped in the direction of flow i.e, x will be pressure and shear forces.

The net shear force acting on the parallelepiped

$$= \left[\left(\tau + \frac{\partial \tau}{\partial y}\delta y\right)\delta x\delta z - \tau\delta x\delta z\right] = \left(\frac{\partial \tau}{\partial y}\right)\delta x\delta y\delta z.$$

If the pressure intensity on face add'a'is *p*, and since there exists a pressure gradient in the direction of flow, the pressure intensity on the face bcc'b' will be $(p + \frac{\partial p}{\partial x}\delta x)$.

The net shear force acting on the parallelepiped

$$= \left[p\delta y\delta z - \left(p + \frac{\partial p}{\partial x}\delta x\right)\delta y\delta z\right] = -\left(\frac{\partial p}{\partial x}\right)\delta x\delta y\delta z.$$

For a steady and uniform flow, there being no acceleration in the direction of motion, the sum of these forces in the x-direction must be equal to zero. Thus,

$$\left(\frac{\partial \tau}{\partial y}\right)\delta x\delta y\delta z - \left(\frac{\partial p}{\partial x}\right)\delta x\delta y\delta z = 0$$

or

$$\left(\frac{\partial \tau}{\partial y}\right) = \left(\frac{\partial p}{\partial x}\right)$$

1.1

Equation 1.1, indicates that in a steady uniform laminar flow the pressure gradient in the direction of flow is equal to the shear stress gradient in the normal direction. Since the acceleration is absent, the pressure gradient is independent of y and shear stress gradient is independent of x.

From Newton's law of viscosity ($= \mu \frac{\partial v}{\partial y}$) for viscous fluids, equation 1.1 can also be written as

$$\left(\mu\frac{\partial^2 v}{\partial y^2}\right) = \left(\frac{\partial p}{\partial x}\right)$$

1.2

Equation 1.2 is the differential equation which is used for the analysis of problems of steady uniform laminar flows.

1.3 FLOW OF VISCOUS FLUIDS THROUGH CIRCULAR PIPES

For flow of viscous fluids through circular pipe, the velocity distribution across a section, the shear stress distribution, drop of pressure for a given length and the ratio of maximum velocity to average velocity is to be determined. The flow through circular pipe will be laminar, if the Reynolds number ($R_e = \rho VD/\mu$) is less than 2000. The value of R_e depends upon density of fluid flowing through the pipe, average velocity of the fluid, diameter of the pipe and viscosity of the fluid.

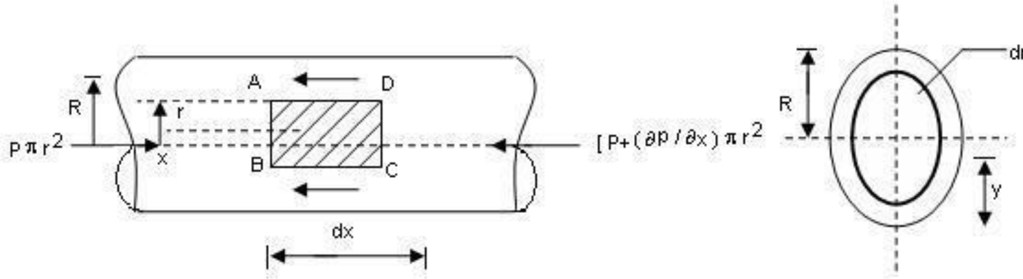


Fig. 1.3 Viscous flow through a pipe

Consider a horizontal pipe of radius R. Consider an element of radius r, sliding in a cylindrical fluid element of radius (r+dr). Let the length of the fluid element be δx. If *p* is the pressure on the face AB, then the intensity of pressure on face CD will be $(p + \frac{\partial p}{\partial x}\delta x)$. The net forces acting on the fluid element are the pressure force, $p\pi r^2$ on face AB, the pressure force, $(p + \frac{\partial p}{\partial x}\delta x)\,\pi r^2$ on face CD, the net shear force, $\tau * 2\pi r\delta x$ on the surface of the fluid element.

As there is no acceleration, the summation of all the forces in the direction of flow must be zero i.e,

$$\left[p\pi r^2 - \left(p + \frac{\partial p}{\partial x}\delta x\right)\pi r^2 - \tau\, 2\pi r\delta x\right] = 0$$

$$\tau = -\left(\frac{\partial p}{\partial x}\right)\frac{r}{2} \qquad 1.3$$

The hear stress τ across a section varies with r as $\frac{\partial p}{\partial x}$ — Hence shear stress distribution across a section is linear.

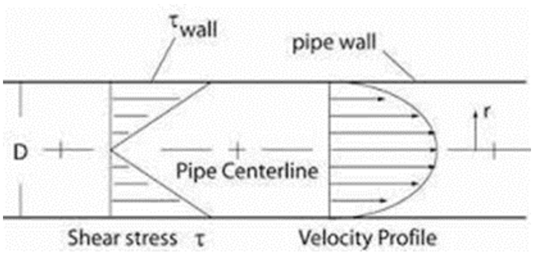


Fig. 1.4 Shear stress and velocity distribution across a section

(i) Velocity distribution

The velocity distribution across a section is obtained on substituting $\tau = \mu\frac{\partial u}{\partial y}$ in equation 1.3.

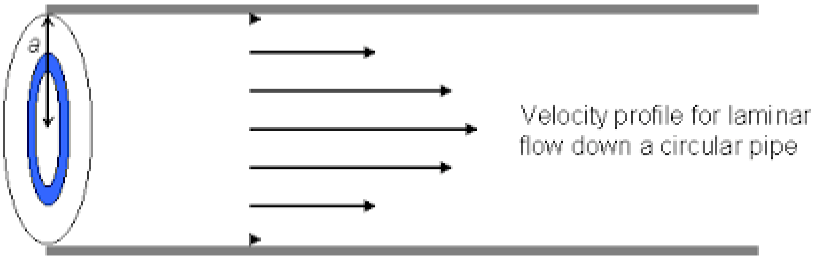


Fig. 1.5 Velocity profile across a section

But in the relation $\tau = \mu\frac{\partial}{\partial y}$, y is measured from the pipe wall. Hence y = R - r and dy = - dr. Then, $\tau = -\mu\frac{\partial u}{\partial r}$. On substituting this value in equation 1.3, we get

$$\frac{\partial u}{\partial r} = \frac{1}{2\mu}\left(\frac{\partial p}{\partial x}\right) r$$

On integrating the above equation w.r.t ., 'r', we get

$$u = \frac{1}{4\mu}\left(\frac{\partial p}{\partial x}\right) \boldsymbol{r^2} + C$$

1.4 1.4

Where c is the constant of integration and its value is obtained from the boundary condition that at r = R, u = 0.

That is, $C = -\frac{1}{4\mu}\left(\frac{\partial p}{\partial x}\right)\boldsymbol{r^2}$ and on substituting the value of C in equation 1.4,

$$= -\frac{1}{4\mu}\left(\frac{\partial p}{\partial x}\right)(\boldsymbol{R^2} - \boldsymbol{r^2})$$

1.5 1.5

In the above equation, the values of μ, $\frac{\partial p}{\partial x}$ and R are constant, which means the velocity, u varies with the square of r. Thus, the above equation is equation of parabola, indicating the velocity distribution across the section of a pipe is a parabolic.

(ii) Ratio of maximum velocity to average velocity

The velocity is maximum, when r = 0 in equation 1.5. Thus the maximum velocity, U_{max} is obtained as

$$\boldsymbol{U_{max}} = -\frac{1}{4\mu}\left(\frac{\partial p}{\partial x}\right)\boldsymbol{R^2}$$

1.6 1.6

The average velocity is obtained, ū, is obtained by dividing discharge of the fluid across the section by the area of the pipe (πR^2). The discharge (Q) across the section is obtained by considering the flow through a circular element of radius r and thickness dr. The fluid flowing per second through this element is,

$$dQ = u \times 2\pi r dr$$

$$= -\frac{1}{4\mu}\left(\frac{\partial p}{\partial x}\right)(\boldsymbol{R^2} - \boldsymbol{r^2})\, 2\pi r dr$$

On integrating the above equation within the limits 0 to R, we get

$$Q = \frac{\pi}{8\mu}\left(-\frac{\partial p}{\partial x}\right)\boldsymbol{R^4}$$

Then, from the relation $\bar{u} = \frac{Q}{Area}$, the average velocity

$$\bar{u} = \frac{1}{8\mu}\left(-\frac{\partial p}{\partial x}\right)\boldsymbol{R^2}$$

1.7 1.7

From the equations, 1.6 and 1.7, $\frac{U_{max}}{\bar{u}} = 2$. Thus, the ratio of maximum velocity to average velocity is equals to two.

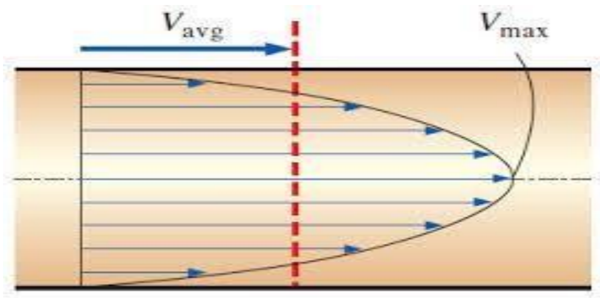


Fig. 1.6 Velocity distribution across a section

(iii) Drop of pressure for a given length (L) of a pipe

From the equation 1.7, we have

$$\bar{u} = \frac{1}{8\mu}\left(-\frac{\partial p}{\partial x}\right)\boldsymbol{R^2} \quad \text{or} \quad \left(-\frac{p}{\partial x}\right) = \frac{8\mu\bar{u}}{\boldsymbol{R^2}}$$

On integrating the above equation in between two sections 1 and 2 where the intensity of pressure is p_1 and p_2, w.r.t. x and on considering $x_2 - x_1 = L$ and $R = \frac{D}{2}$ we get,

$$(\boldsymbol{p_1 - p_2}) = \frac{32\mu\bar{u}L}{\boldsymbol{D^2}}$$

where $p_1 - p_2$ is the drop of pressure.

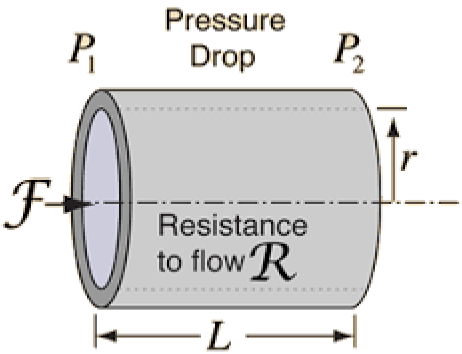


Fig. 1.7 Pressure drop across a section

Thus, loss of pressure head (h_f) is equal to $\frac{(\boldsymbol{p_1 - p_2})}{\rho g}$.

i.e., $$\boldsymbol{h_f} = \frac{32\mu\bar{u}L}{\boldsymbol{D^2}}$$

1.8 1.8

Equation 1.8 is called **Hagen Poiseuille Equation**.

1.4 FLOW BETWEEN PARALLEL FLAT PLATES – BOTH PLATES AT REST

Consider two parallel fixed plates kept at a distance 't' apart. Consider a fluid element of length δx.

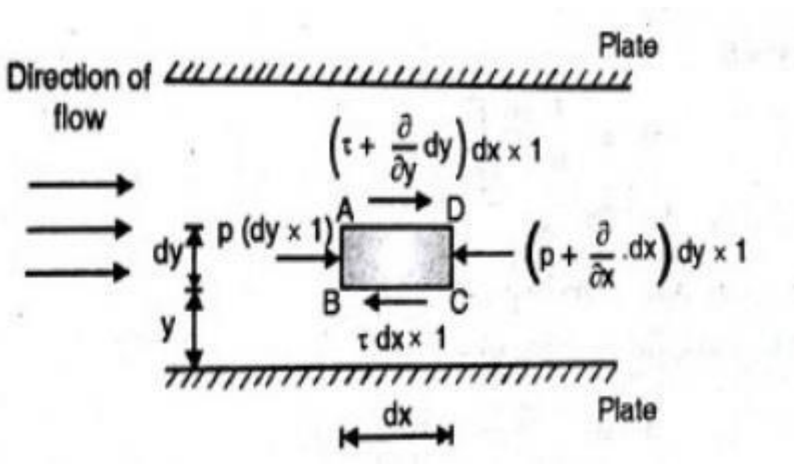


Fig. 1.8 Viscous flow between two parallel plates

If p is the pressure on the face AB, then the intensity of pressure on face CD will be $(p + \frac{\partial p}{\partial x}\delta x)$. Let τ is the shear stress acting on the face BC then the shear stress on the face AD will be $(\tau + \frac{\partial \tau}{}\delta y)$. If the width of the element in the direction perpendicular to the

∂y

paper is unity then, the forces acting on the fluid element are the pressure force, $p\delta y$ on face AB, the pressure force, $(P + \frac{\partial p}{\partial x}\delta x)\delta y$ on face CD, the shear force, $\tau\delta x$ on face BC and the shear force $(\tau + \frac{\partial \tau}{\partial y}\delta y)\,\delta x$ on face AD.

As there is no acceleration for steady and uniform flow, the summation of all the forces in the direction of flow must be zero i.e,

$$\left[p\delta y - \left(p + \frac{\partial p}{\partial x}\delta x\right)\delta y - \tau\delta x + \left(\tau + \frac{\partial \tau}{\partial y}\delta y\right)\delta x\right] = 0$$

$$\frac{\partial p}{\partial x} = \frac{\partial \tau}{\partial x}$$

The velocity distribution across a section is obtained on substituting the value of shear stress $\tau = \mu\frac{\partial u}{\partial y}$ in equation 1.9.

$$\left(\frac{\partial p}{\partial x}\right) = \mu\left(\frac{\partial^2 u}{\partial y^2}\right) \text{ or } \frac{\partial^2 u}{\partial y^2} = \frac{1}{\mu}\frac{\partial p}{\partial x}$$

On double integrating the above equation w.r.t. y, we get

$$u = \frac{1}{\mu}\left(\frac{\partial p}{\partial x}\right)\frac{y^2}{2} + C_1\ y + C_2 \qquad 1.10$$

where C_1 and C_2 are constants of integration. Their values are obtained from the two boundary conditions (i) at y = 0, u = 0 (ii) at y =t, u = 0.

On substituting first boundary condition in the equation 1.10, $C_2 = 0$. And again on solving the equation 1.10 with second boundary condition and $C_2 = 0$,

$$C_1 = \frac{-1}{2\mu}\frac{\partial p}{\partial x}t$$

From the values of C_1 and C_2, the equation 1.10 can also be written as

$$u = \frac{1}{\mu}\left(\frac{\partial p}{\partial x}\right)\frac{y^2}{2} + \left(\frac{-1}{2\mu}\left(\frac{\partial p}{\partial x}\right)t\right)y$$

$$u = \left(\frac{-1}{2\mu}\left(\frac{\partial p}{\partial x}\right)[ty - y^2]\right)$$

In the above equation, the values of μ, $\frac{\partial p}{\partial x}$ and t are constant, which means the velocity, u varies with the square of y. Hence the velocity distribution across a section of parallel plate is parabolic.

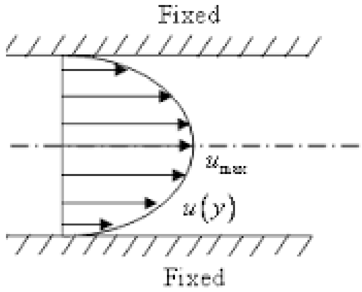


Fig. 1.9 Velocity distribution of flow between two parallel plates

(ii) Ratio of maximum velocity to average velocity

The velocity is maximum, when y = t/2. Substituting this value in equation 1.11, the maximum velocity, U_{max} is obtained as

$$\boldsymbol{U_{max}} = -\frac{1}{8\mu}\left(\frac{\partial p}{\partial x}\right)\boldsymbol{t^2} \quad 1.12$$

The average velocity is obtained, ū, is obtained by dividing discharge of the fluid across the section by the area of the section (t x 1). The discharge (Q) across the section is obtained by considering the rate of flow of fluid through the strip of thickness dy and integrating it. The rate of flow of fluid through the strip is,

$$dQ = u \times dy \times 1$$

$$= -\frac{1}{2\mu}\left(\frac{\partial p}{\partial x}\right)(\boldsymbol{ty} - \boldsymbol{y^2})dy$$

On integrating the above equation within the limits 0 to t, we get

$$Q = \frac{-1}{12\mu}\left(\frac{\partial p}{\partial x}\right)\boldsymbol{t^3}$$

Then, from the relation $\bar{u} = \frac{Q}{Area}$, the average velocity

$$\bar{u} = \frac{-1}{12\mu}\left(\frac{\partial p}{\partial x}\right)\boldsymbol{t^2} \quad 1.13$$

From the equations, 1.12 and 1.13, $\frac{U_{max}}{\bar{u}} = 3/2$ Thus, the ratio of maximum velocity to average velocity is equals to 1.5.

(iii) Drop of pressure head for a given length (L)

From the equation 1.13, we have

$$\bar{u} = \frac{-1}{12\mu}\left(\frac{\partial p}{\partial x}\right)t^2 \quad \text{or} \quad \left(\frac{\partial p}{\partial x}\right) = \frac{-12\mu\bar{u}}{t^2}$$

On integrating the above equation in between two sections 1 and 2 where the intensity of pressure is p_1 and p_2, w.r.t. x and on considering $x_2 - x_1 = L$ we get,

$$(p_1 - p_2) = \frac{12\mu\bar{u}L}{t^2}$$

where $p_1 - p_2$ is the drop of pressure.

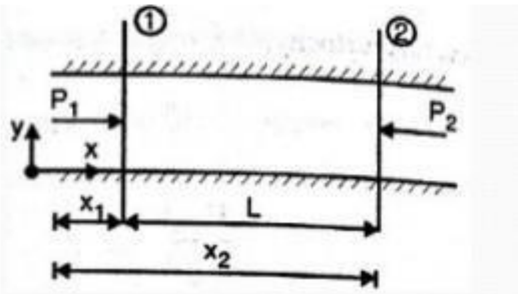


Fig. 1.10 pressure drop of flow between two parallel plates

Thus, loss of pressure head (h_f)

i.e., $$h_f = \frac{(p_1 - p_2)}{\rho g} = \frac{12\mu\bar{u}L}{\rho g t^2}$$

1.14

(iv) Shear Stress distribution

It is obtained by substituting value of velocity from the equation 1.11 into $= \mu\,\frac{\partial u}{\partial y}$, that is ,

$$\tau = \mu\left(\frac{-1}{2\mu}\left(\frac{\partial p}{\partial x}\right)[t - 2y]\right) \quad or \quad \tau = \frac{-1}{2\mu}\frac{\partial p}{\partial x}[t - 2y]$$

1.15

In the above equation, $\frac{\partial p}{\partial x}$ and t are constant. Hence τ varies linearly with y. Shear stress is maximum when y = 0 or at the walls of the plates. Shear stress is zero when y = t/2 that is at the centre line between two plates. Maximum shear stress (τ_0) is given by

$$\tau_0 = \frac{-1}{2}\frac{\partial p}{\partial x}t$$

2. FLOW THROUGH PIPES – TURBULENT FLOW

2.1 TURBULENT FLOW

Whether a flow is laminar or turbulent depends of the relative importance of fluid friction (viscosity) and flow inertia. The ratio of inertial to viscous forces is the Reynolds number. Given the characteristic velocity scale, U, and length scale, L, for a system, the Reynolds number is Re = UL/ν, where ν is the kinematic viscosity of the fluid. For most surface water systems the characteristic length scale is the basin-scale. The flow in pipes is turbulent if Reynolds number is more than 4000. The velocity distribution in turbulent flow is relatively uniform and the velocity profile of turbulent flow is much flatter than the corresponding laminar flow parabola for the same mean velocity. Shear stress in turbulent flow is sum of shear stress due to viscosity and shear stress due to turbulence,

$$\tau = \mu \frac{dv}{dy} + \eta \frac{dv}{dy}$$

Where, $\mu =$ absolute viscosity; and

$\eta =$ eddy viscosity

Turbulent shear stress by Reynolds is given as

$$\tau = \bar{\bar{\rho}}\overline{\overline{v_y}}\overline{v_x}$$

Where, v_x *and* $v_y =$ the fluctuating components of velocity in the x and y directions respectively.

According to Prandtl the shear stress in turbulent flow is given by

$$\tau = \rho l^2 \left(\frac{dv}{dy}\right)^2$$

Where, l = mixing length

According to Prandtl, mixing length is that distance in the transverse direction which must be covered by a lump of fluid particles travelling with its original mean in order to make the difference between its velocity and the velocity of the new layer equal to the mean transverse fluctuation in turbulent flow. When a fluid flows through a pipe, close to the pipe wall a boundary layer is formed which may attain a maximum thickness equal to the radius of the pipe at a certain section of the pipe at which the flow is considered to have been established. For laminar flow in a pipe laminar boundary layer will be developed for the entire length of the pipe. According to Rouse the length of the pipe x from the entrance of the pipe off to the section where the flow is established is given as

$$\frac{x}{D} = 0.07Re$$

Where, D = diameter of the pipe ; and

Re = Reynolds number

$$\left(= \frac{\rho VD}{\mu}\right)$$

According to Rouse the distance required for the establishment of a fully developed turbulent flow is given as

$$\frac{x}{D} = 50$$

The boundary may be classified as

(i) hydro-dynamically smooth boundary

(ii) hydro-dynamically rough boundary

A boundary is known as hydro-dynamically smooth boundary when (k/δ') is less than 0.25 and it is known as hydro-dynamically rough boundary when (k/δ') is greater than 6.0, where k is the average height of the irregularities projecting from the boundary surface, and δ' is the thickness of the laminar sublayer. The boundary is classified as boundary in transition when $0.25 < (k/\delta') < 6.0$.

2.2 VELOCITY DISTRIBUTION IN TURBULENT FLOW

The velocity distribution for turbulent flow hydrodynamically smooth pipes is given by

$$\frac{v}{V_*} = 5.75 \log_{10} \left(\frac{V_* y}{\nu}\right) + 5.5$$

$$V_* = \text{shear or friction velocity} = \sqrt{(\tau_0/\rho)}$$

and v = kinematic viscosity of the fluid.

Velocity distribution for turbulent flow in hydrodynamically rough pipes is given by

$$\frac{v}{V_*} = 5.75 \log_{10} \left(\frac{y}{k}\right) + 8.5$$

where

k = average height of irregularities projecting from the boundary surface.

The **mean** velocity V for turbulent flow in hydrodynamically smooth pipes of radius R is given by

$$\frac{V}{V_*} = 5.75 \log_{10} \left(\frac{V_* R}{\nu}\right) + 1.75$$

The **mean** velocity V for turbulent flow in hydrodynamically rough pipes of radius R is given by

$$\frac{V}{V_*} = 5.75 \log_{10} \left(\frac{R}{k}\right) + 4.75$$

The velocity distribution identical for both hydodynamically smooth and hydrodynamically rough pipes is given by

$$\frac{v - V}{V_*} = 5.75\, log_{10} \left(\frac{y}{k}\right) + 3.75$$

For $y = R$, i.e., at pipe axis $v = v_{max}, thus$

$$\frac{v_{max} - V}{V_*} = 3.75$$

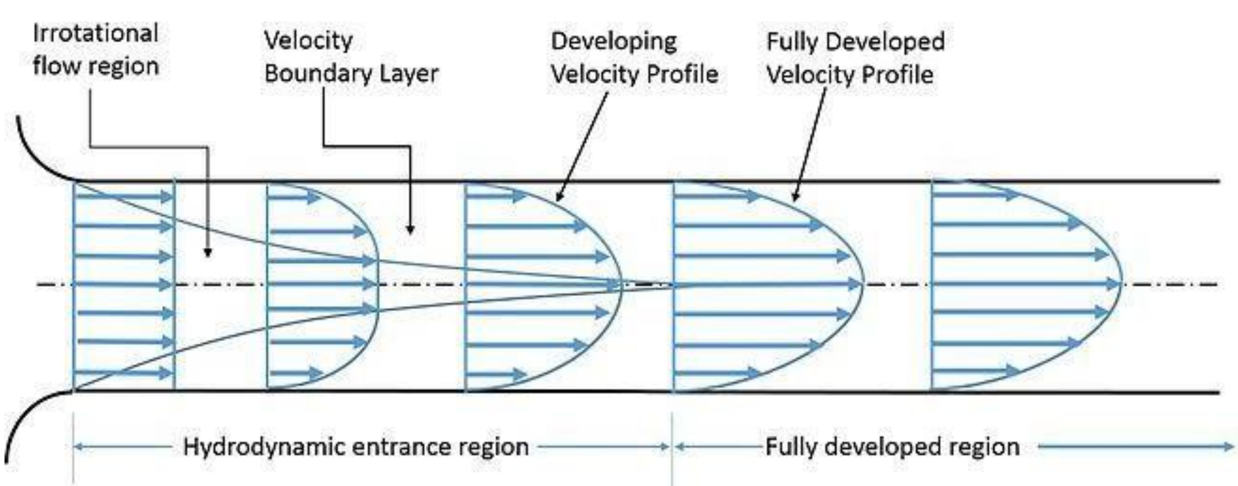
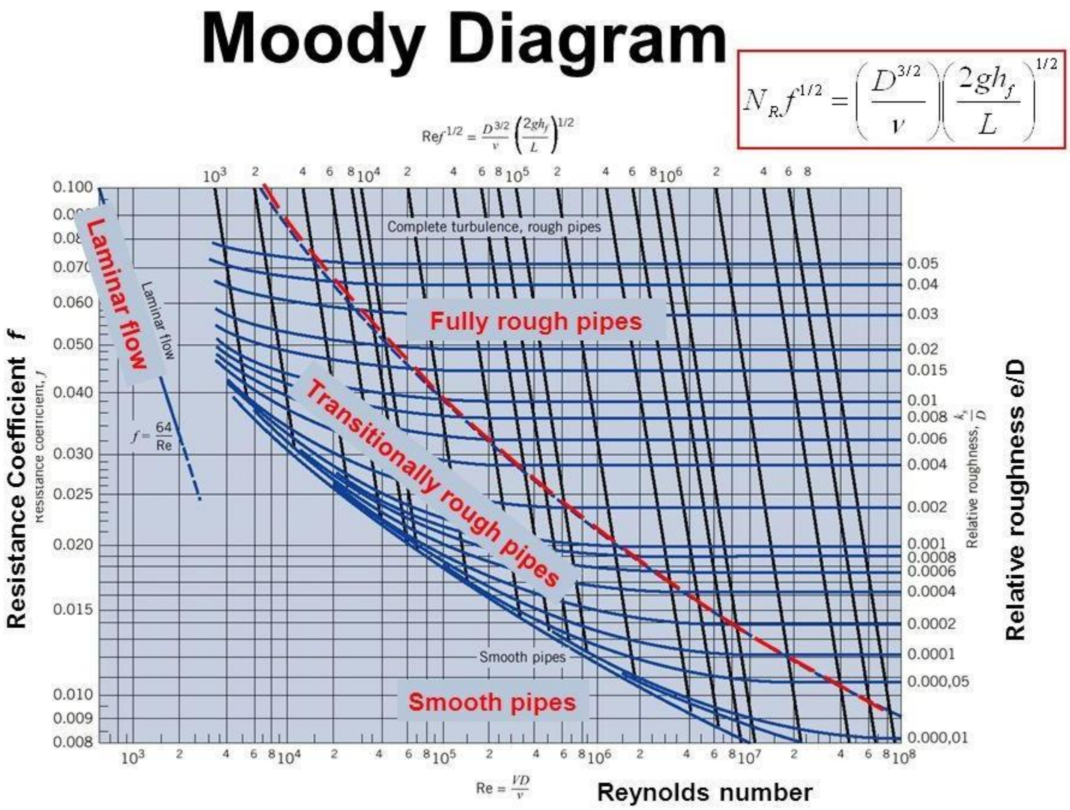


Fig. 2.1 Development of flow through pipes

2.3 THE MOODY CHART

The friction factor in fully developed turbulent pipe flow depends on the Reynolds number and the relative roughness Ɛ/D, which is the ratio of the mean height of roughness of the pipe to the pipe diameter. The functional form of this dependence cannot be obtained from a theoretical analysis, and all available results are obtained from painstaking experiments using artificially roughened surfaces (usually by gluing sand grains of a known size on the inner surfaces of the pipes). Most such experiments were conducted by Prandtl's student J. Nikuradse in 1933, followed by the works of others. The friction factor was calculated from the measurements of the flow rate and the pressure drop.



Moody chart is probably one of the most widely accepted and used charts in engineering. Although it is developed for circular pipes, it can also be used for noncircular pipes by replacing the diameter by the hydraulic diameter.

2.4 COMMERCIAL PIPES

Commercially available pipes differ from those used in the experiments in that the roughness of pipes in the market is not uniform and it is difficult to give a precise description of it. Equivalent roughness values for some commercial pipes are given in Table 2.1 as well as on the Moody chart. But it should be kept in mind that these values are for new pipes, and the relative roughness of pipes may increase with use as a result of corrosion, scale buildup, and precipitation. As a result, the friction factor may increase by a factor of 5 to 10. Actual operating conditions must be considered in the design of piping systems.

Equivalent roughness values for new commercial pipes*

	Roughness, ε	
Material	ft	mm
Glass, plastic	0 (smooth)	
Concrete	0.003–0.03	0.9–9
Wood stave	0.0016	0.5
Rubber, smoothed	0.000033	0.01
Copper or brass tubing	0.000005	0.0015
Cast iron	0.00085	0.26
Galvanized iron	0.0005	0.15
Wrought iron	0.00015	0.046
Stainless steel	0.000007	0.002
Commercial steel	0.00015	0.045

* The uncertainty in these values can be as much as ±60 percent.

Also, the Moody chart and its equivalent Colebrook equation involve several uncertainties (the roughness size, experimental error, curve fitting of data, etc.), and thus the results obtained should not be treated as "exact." It is usually considered to be accurate to ±15 percent over the entire range in the figure.

NUMERICALS:

1. A crude oil of viscosity 0.97 poise and relative density 0.9 is flowing through a horizontal circular pipe of diameter 100 mm and of length 10 m. Calculate the difference of pressure at the two ends of the pipe, if 100 kg of the oil is collected in a tank in 30 seconds.

Sol: Given μ = 0.97 poise = 0.097 Ns/m^2

Relative density = 0.9

Density of oil = ρ_o = 0.9 x 1000 = 900 kg/m^3

Diameter of the pipe = D = 100 mm = 0.1 m

Length of the pipe = L = 10 m

Mass of oil collected = M = 100 kg

Time = T = 30 seconds and then mass of oil per second = 100/30 kg/s = ρ_o x Q

Then, Q = (100 x 900)/30 = 0.0037 m^3/s

$\bar{u} = \frac{Q}{Area} = (0.0037\ x\ 4)/(\Pi\ x\ 0.1\ x\ 0.1)$ = 0.471 m/s

The difference of pressure for laminar flow is given by,

$$(\boldsymbol{p_1} - \boldsymbol{p_2}) = \frac{32\mu\bar{u}L}{\boldsymbol{D^2}}$$

$$= \frac{32\ x\ 0.097\ x\ 0.471\ x\ 10}{\mathbf{0.1^2}}$$

= **1462.8 N/m^2 = 0.1462.8 N/cm^2**

2. A fluid of viscosity 0.7 Ns/m^2 and specific gravity 1.3 is flowing through a circular pipe of diameter 100 mm. The maximum shear stress at the pipe wall is given as 196.2 N/m^2, find the i) pressure gradient, ii) the average velocity and iii) Reynolds number of the flow.

Sol: Given μ = 0.7 Ns/m^2

Relative density = 1.3

Density of oil = ρ_o = 1.3 x 1000 = 1300 kg/m^3

Diameter of the pipe = D = 100 mm = 0.1 m

Shear stress = τ_o = as 196.2 N/m^2

i) pressure gradient $\frac{\partial p}{\partial x}$, Maximum shear stress is given by, $\tau = -\left(\frac{\partial p}{\partial x}\right)\frac{r}{2} = -\left(\frac{\partial p}{\partial x}\right)\frac{D}{2} = \left(\frac{\partial p}{\partial x}\right)\frac{0.1}{2}$

Then, $\left(\frac{\partial p}{\partial x}\right) = -\frac{196.2\ x\ 4}{0.1} = \mathbf{-7848}$ **N/m² per m.**

ii) the average velocity $\bar{u} = \frac{1}{2}U_{max} = \left[-\frac{1}{4\mu}\left(\frac{\partial p}{\partial x}\right)R^2\right] / 2$

= [1 x 7848 x 0.05²] / [8 x 0.7]

= **3.5 m/s**

iii) Reynolds number of the flow $R_e = \frac{\rho \bar{u} D}{\mu}$

= (1300 x 3.50 x 0.1) / (0.7) = **650**

3. Determine the pressure gradient, shear stress at the two horizontal parallel plate and discharge per metre width for the laminar flow of oil with a maximum velocity of 2 m/s between two horizontal parallel fixed plates which are 100 mm apart. Take μ = 2.4525 Ns/m².

Sol: Given U_{max} = 2 m/s

t = 100 mm = 0.1 m

μ = 2.4525 Ns/m²

i) pressure gradient $\frac{\partial p}{\partial x}$,

Maximum velocity is given by the equation

$$\boldsymbol{U_{max}} = -\frac{1}{8\mu}\left(\frac{\partial p}{\partial x}\right)\boldsymbol{t^2}$$

$$2 = -\frac{(0.1\ x\ 0.1)}{(8\ x\ 2.4525)}\left(\frac{\partial p}{\partial x}\right)$$

$\left(\frac{\partial p}{\partial x}\right)$ = **- 3924 N/m² per m**

ii) Shear stress at wall,

$$\boldsymbol{\tau_0} = \frac{-1}{2}\frac{\partial p}{\partial x}t = -\ (1\ x\ 3924\ x\ 0.1) / (2) = \mathbf{196.2\ N/\ m^2}$$

iii) Discharge per metre width Q = mean velocity x area

= (2/3) x U_{max} x (t x 1) = (2/3) x 2 x 0.1 x 1 = **0.133 m³/s**.

Exercise problems:

1. An oil of viscosity 0.1 Ns/m^2 and relative density 0.9 is flowing through a circular pipe of diameter 50 mm and a length of 300 m. The rate of flow of fluid through the pipe is 3.5 litres/sec. Find the pressure drop in a length of 300 m and also the shear stress at the pipe wall.

2. A laminar flow is taking place in a pipe of diameter 200 mm. The maximum velocity is 1.5 m/s. Find the mean velocity and radius at which this occurs. Also calculate the velocity at 4 cm from the wall of the pipe.

3. Power is required per kilometre of a line to overcome the viscous resistance to the flow of glycerine through a horizontal pipe of diameter 100 mm at the rate of 100 lit/s? Take viscosity as 8 poise and kinematic viscosity as 6 stokes.

4. Water at 15º flows between two larger parallel plates at a distance of 1.6 mm apart. Determine i) maximum velocity, ii) pressure drop per unit length and iii) shear stress at the walls of the plates if the average velocity is 0.2 m/s. The viscosity of water at 15º is given as 0.01 poise.

5. There is a horizontal crack 40 mm wide and 2.5 mm deep in a wall of thickness 100 mm. Water leaks through the crack. Find the rate of leakage through the crack, if the difference of pressure between the two ends of the crack is 0.02943 N/cm^2. Take viscosity of water equals to 0.01 poise.

6. Calculate the pressure gradient along flow, the average velocity and the discharge of an oil of viscosity 0.02 Ns/m^2. flowing between two stationary plates 1 m wide maintained 10 mm apart. The velocity midway between the plates is 2 m/s.

Problem.

Determine the wall shearing stress in a pipe of diameter 100 mm which carries water. The velocities at the pipe centre and 30 mm from the pipe centre are 2 m/s and 1.5 m/s respectively. The flow in pipe is given as turbulent.

Solution.

Dia. of pipe, $D = 100 \text{ mm} = 0.10 \text{ m}$

∴ Radius, $R = \frac{0.10}{2} = 0.05 \text{ m}$

Velocity at centre, $u_{max} = 2 \text{ m/s}$

Velocity at 30 mm or 0.03 m from centre = 1.5 m/s

∴ Velocity (at $r = 0.03$ m), $u = 1.5$ m/s

Let the wall shearing stress $= \tau_0$

For turbulent flow, the velocity distribution in terms of centre line velocity (u_{max}) is given by equation (10.18) as

$$\frac{u_{max} - u}{u_*} = 5.75 \log_{10}\left(\frac{R}{y}\right)$$

where $u = 1.5$ m/s at $y = (R - r) = 0.05 - 0.03 = .02$ m

$$\therefore \quad \frac{2.0 - 1.5}{u_*} = 5.75 \log_{10} \frac{.05}{.02} = 2.288 \text{ or } \frac{0.5}{u_*} = 2.288$$

$$\therefore \quad u_* = \frac{0.5}{2.288} = 0.2185 \text{ m/s}$$

Using the relation $u_* = \sqrt{\tau_0 / \rho}$, where ρ *for* water = 1000 kg/m^3

$$\therefore \quad 0.2185 = \sqrt{\frac{\tau_0}{1000}} \text{ or } \frac{\tau_0}{1000} = 0.2185^2 = 0.0477$$

$$\tau_0 = 0.0477 \times 1000 = \mathbf{47.676 \ N/m^2}. \textbf{ Ans.}$$

Problem.

For turbulent flow in a pipe of diameter 300 mm, find the discharge when the centre-line velocity is 2.0 m/s and the velocity at a point 100 mm from the centre as measured by pitot-tube is 1.6 m/s.

Solution.

Dia. of pipe, $D = 300$ mm $= 0.3$ m

$\therefore$ Radius, $R = \frac{0.3}{2} = 0.15$ m

Velocity at centre, $u_{max} = 2.0$ m/s

Velocity (at $r = 100$ mm $= 0.1$ m), $u = 1.6$ m/s

Now $y = R - r = 0.15 - 0.10 = 0.05$ m

$\therefore$ Velocity (at $r = 0.1$ m or at $y = 0.05$ m), $u = 1.6$ m/s

The velocity in terms of centre-line velocity is given by equation (10.18) as

$$\frac{u_{max} - u}{u_*} = 5.75 \log_{10} (R/y)$$

Substituting the values, we get $\frac{2.0 - 1.6}{u_*} = 5.75 \log_{10} \frac{.15}{.05}$ $\left[\because \begin{array}{l} y = .05 \text{ m} \\ R = 0.15 \text{ m} \end{array}\right]$

$$= 5.75 \log_{10} 3.0 = 2.7434$$

or $$\frac{0.4}{u_*} = 2.7434$$

$\therefore$ $$u_* = \frac{0.4}{2.7434} = 0.1458 \text{ m/s} \quad ...(i)$$

Using equation (10.26) which gives relation between velocity at any point and average velocity, we have

$$\frac{u - \overline{U}}{u_*} = 5.75 \log_{10} (y/R) + 3.75$$

at $y = R$, velocity u becomes $= u_{max}$

$\therefore$ $$\frac{u_{max} - \overline{U}}{u_*} = 5.75 \log_{10} (R/R) + 3.75 = 5.75 \times 0 + 3.75 = 3.75$$

But $u_{max} = 2.0$ and u_* from $(i) = 0.1458$

$\therefore$ $$\frac{2.0 - \overline{U}}{0.1458} = 3.75$$

or $$\overline{U} = 2.0 - .1458 \times 3.75 = 2.0 - 0.5467 = 1.4533 \text{ m/s}$$