FLOW THROUGH ORIFICES AND MOUTH PIECES

4.0 INTRODUCTION

Orifice is a small opening of any cross-section (such as circular, triangular, rectangular etc.) on the side or at the bottom of a tank, through which a fluid is flowing. A mouthpiece is a short length of a pipe which is two to three times its diameter in length, fitted in a tank or vessel containing the fluid. Orifices as well as mouthpieces are used for measuring the rate of flow of fluid.

4.1 CLASSIFICATIONS OF ORIFICES

The orifices are classified on the basis of their size, shape, nature of discharge and shape of the upstream edge. The following are the important classifications:

1. Depending upon the size of orifice and head of liquid from the centre of the orifice

The orifices are classified as small orifice or large orifice. If the head of liquid from the centre of orifice is more than five times the depth of orifice, the orifice is called small orifice. And if the head of liquids is less than five times the depth of orifice, it is known as large orifice.

2. Depending upon their cross-sectional areas

The orifices are classified as (i) Circular orifice, (ii) Triangular orifice, (iii) Rectangular orifice and (iv) Square orifice

3. Depending upon the shape of upstream edge of the orifices.

The orifices are classified as (i) Sharp-edged orifice and (ii) Bell-mouthed orifice

4. Depending upon the nature of discharge.

The orifices are classified as (i) Free discharging orifices and (ii) Drowned or sub-merged orifices. The sub-merged orifices are further classified as (a) Fully sub-merged orifices and (b) Partially sub-merged orifices.

4.3 FLOW THROUGH AN ORIFICE

Consider a tank fitted with a circular orifice in one of its sides as shown in Fig. 7.1. Let H be the head of the liquid above the centre of the orifice. The liquid flowing through the orifice forms a jet of liquid whose area of cross-section is less than that of orifice. The area of jet of fluid goes on decreasing and at a section C-C, the area is minimum. This section is approximately at a distance of half of diameter of the orifice. At this section, the streamlines are straight and parallel to each other and perpendicular to the plane of the orifice. This section is called Vena-contracta. Beyond this section, the jet diverges and is attracted in the downward direction by the gravity.

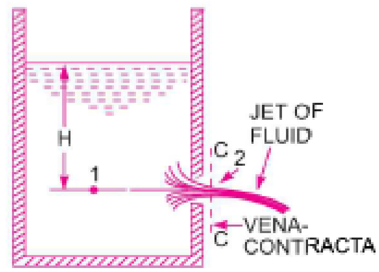
$$\frac{p_1}{\rho g} + \frac{v_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{v_2^2}{2g} + z_2$$

But $$z_1 = z_2$$

$\therefore$ $$\frac{p_1}{\rho g} + \frac{v_1^2}{2g} = \frac{p_2}{\rho g} + \frac{v_2^2}{2g}$$

Now $$\frac{p_1}{\rho g} = H$$

$$\frac{p_2}{\rho g} = 0 \quad \text{(atmospheric pressure)}$$



Consider two points 1 and 2 as shown in Fig. 7.1. Point 1 is inside the tank and point 2 at the vena-contracta. Let the flow is steady and at a constant head H. Applying Bernoulli's equation at points 1 and 2.

v_1 is very small in comparison to v_2, as area of tank is very large as compared to the area of the jet of liquid.

$$H + 0 = 0 + \frac{v_2^2}{2g}$$

$$v_2 = \sqrt{2gH}$$

This is theoretical velocity. Actual velocity will be less than this value.

4.4 HYDRAULIC CO-EFFICIENTS

1. Co-efficient of velocity, Cv
2. Co-efficient of contraction, Cc.
3. Co-efficient of discharge, Cd.

4.4.1 Co-efficient of Velocity:

It is defined as the ratio between the actual velocity of a jet of liquid at vena-contracta and the theoretical velocity of jet. It is denoted by Cv,

$$C_v = \frac{\text{Actual velocity of jet at vena-contracta}}{\text{Theoretical velocity}}$$

$$= \frac{V}{\sqrt{2gH}}, \text{ where } V = \text{actual velocity}, \sqrt{2gH} = \text{Theoretical velocity}$$

The value of C, varies from 0.95 to 0.99 for different orifices, depending on the shape, size of the orifice and on the head under which flow takes place. Generally the value of C, = 0.98 is taken for sharp-edged orifices.

4.4.2 Co-efficient of Contraction

It is defined as the ratio of the area of the jet at vena-contracta to the area of the orifice. It is denoted by Cc.

a = area of orifice and

ac = area of jeat at vena-contracta.

$$a = \text{area of orifice and}$$
$$a_c = \text{area of jet at vena-contracta.}$$
$$C_c = \frac{\text{area of jet at vena-contracta}}{\text{area of orifice}}$$
$$= \frac{a_c}{a}$$

Cc varies from 0.61 to 0.69 depending on shape and size of the orifice and head of liquid under which flow takes place. In general, the value of Cc, may be taken as 0.64.

4.4.3 Co-efficient of Discharge

It is defined as the ratio of the actual discharge from an orifice to the theoretical discharge from the orifice. It is denoted by Cd. If Q is actual discharge and Oth is the theoretical discharge then mathematically, Cd is given as

$$C_d = \frac{Q}{Q_{th}} = \frac{\text{Actual velocity} \times \text{Actual area}}{\text{Theoretical velocity} \times \text{Theoretical area}}$$
$$= \frac{\text{Actual velocity}}{\text{Theoretical velocity}} \times \frac{\text{Actual area}}{\text{Theoretical area}}$$
$$C_d = C_v \times C_c$$

The value of C, varies from 0.61 to 0.65. For general purpose the value of C, is taken as 0.62.

Problem 4.1 The head of water over an orifice of diameter 40 mm is 10 m. Find the actual discharge and actual velocity of the jet at vena-contracta. Take Cd = 0.6 and C, = 0.98.

Problem 4.2 The head of water over the centre of an orifice of diameter 20 mm is 1 m. The actual discharge through the orifice is 0.85 litrels. Find the co-efficient of discharge.

4.5 EXPERIMENTAL DETERMINATION OF HYDRAULIC CO EFFICIENTS

4.5.1 Determination of Co-efficient of Discharge (Cd).

The water is allowed to flow through an orifice fitted to a tank under a constant head, H as shown in Fig The water is collected in a measuring tank for a known time, t. The height of water in the measuring tank is noted down. Then actual discharge through orifice,

$$Q = \frac{\text{Area of measuring tank} \times \text{Height of water in measuring tank}}{\text{Time } (t)}$$

theoretical discharge = area of orifice × $\sqrt{2gH}$

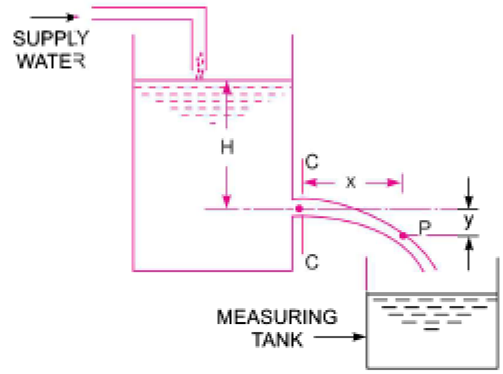


Fig. 7.2 *Value of C_d.*

$$C_d = \frac{Q}{a \times \sqrt{2gH}}$$

7.5.2 Determination of Co-efficient of Velocity (Cv).

Let C-C represents the venacontracta of a jet of water coming out from an orifice under constant head H as shown in Fig. Consider a liquid particle which is at vena-contracta at any time and takes the position at P along the jet in time 't'.

Let x = horizontal distance travelled by the particle in time t

y = vertical distance between P and C-C

V = actual velocity of jet at vena-contracta.

Then horizontal distance, x = V x t

and vertical distance, $$y = \frac{1}{2} g t^2$$

From equation (*i*), $$t = \frac{x}{V}$$

Substituting this value of '*t*' in (*ii*), we get

$$y = \frac{1}{2} g \times \frac{x^2}{V^2}$$

$$V^2 = \frac{gx^2}{2y}$$

∴ $$V = \sqrt{\frac{gx^2}{2y}}$$

But theoretical velocity,

$$V_{th} = \sqrt{2gH}$$

∴ Co-efficient of velocity, $$C_v = \frac{V}{V_{th}} = \sqrt{\frac{gx^2}{2y}} \times \frac{1}{\sqrt{2gH}} = \sqrt{\frac{x^2}{4yH}}$$

$$= \frac{x}{\sqrt{4yH}}.$$

4.5.3 Determination of Co-efficient of Contraction (C.).

Cd can be determined as,

Cd = Cv x Cc.

$$C_c = \frac{C_d}{C_v}$$

Problem 7.3 A jet of water, issuing from a sharp-edged vertical orifice under a constant head of 10.0 cm, at a certain point, has the horizontal and vertical co-ordinates measured from the vena-contracta as 20.0 cm and 10.5 cm respectively. Find the value of Cy. Also find the value of Cif Ca=0.60.

Problem 7.4 The head of water over an orifice of diameter 100 mm is 10 m. The water coming out from orifice is collected in a circular tank of diameter 1.5 m. The rise of water level in this tank is 1.0 m in 25 seconds. Also the co-ordinates of a point on the jet, measured from vena-contracta are 4.3 m horizontal and 0.5 m vertical. Find the co-efficients, Cd, C, and Ce

Problem 7.5 Water discharge at the rate of 98.2 litres/s through a 120 mm diameter vertical sharp-edged orifice placed under a constant head of 10 metres. A point, on the jet, measured from the vena-contracta of the jet has co-ordinates 4.5 metres horizontal and 0.54 metres vertical. Find the co-efficient C, C and Cd of the orifice.

Problem 7.6 A 25 mm diameter nozzle discharges 0.76 mof water per minute when the head is 60 m. The diameter of the jet is 22.5 mm. Determine : (i) the values of co-efficients Cc Cy and Ca and (ii) the loss of head due to fluid resistance.

Problem 7.7 A pipe, 100 mm in diameter, has a nozzle attached to it at the discharge end, the diameter of the nozzle is 50 mm. The rate of discharge of water through the nozzle is 20 litres/s and the pressure at the base of the nozzle is 5.886 N/cm-. Calculate the co-efficient of discharge. Assume that the base of the nozzle and outlet of the nozzle are at the same elevation.

Problem 7.8 A tank has two identical orifices on one of its vertical sides. The upper orifice is 3 m below the water surface and lower one is 5 m below the water surface. If the value of C, for each orifice is 0.96, find the point of intersection of the two jets.

Problem 7.9 A closed vessel contains water upto a height of 1.5 m and over the water surface there is air having pressure 7.848 N/cm (0.8 kgf/cm') above atmospheric pressure. At the bottom of the vessel there is an orifice of diameter 100 mm. Find the rate of flow of water from orifice. Take Ca = 0.6.

Problem 7.10 A closed tank partially filled with water upto a height of 0.9 m having an orifice of diameter 15 mm at the bottom of the tank. The air is pumped into the upper part of the tank. Determine the pressure required for a discharge of 1.5 litres/s through the orifice. Take Cd = 0.62.

4.6 FLOW THROUGH LARGE ORIFICES

If the head of liquid is less than 5 times the depth of the orifice, the orifice is called large orifice. In case of small orifice, the velocity in the entire cross-section of the jet is considered to be constant and discharge can be calculated by Q = $\sqrt{2gh}$ x a x Cd. But in case of a large orifice, the velocity is not constant over the entire cross-section of the jet and hence Q cannot be calculated by Q = $\sqrt{2gh}$ x a x Cd

4.6.1 Discharge Through Large Rectangular Orifice.

Consider a large rectangular orifice in one side of the tank discharging freely into atmosphere under a constant head, H as shown in Fig. 7.7. Let

H1 = height of liquid above top edge of orifice

H2 = height of liquid above bottom edge of orifice

b = breadth of orifice

d = depth of orifice = H2 –H1

Cd= co-efficient of discharge.

Consider an elementary horizontal strip of depth dh' at a depth of 'h' below the free surface of the liquid in the tank as shown in Fig

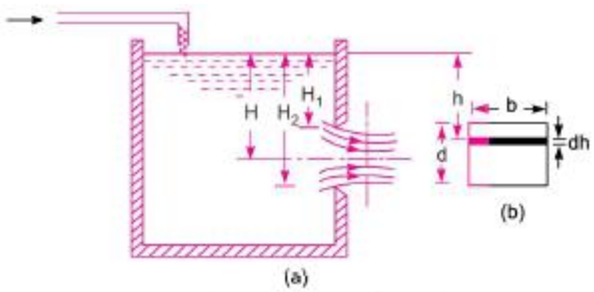


Fig. 7.7 *Large rectangular orifice.*

Area of strip = b x dh

and theoretical velocity of water through strip = $\sqrt{2gh}$.

Discharge through elementary strip is given

dQ = Cd X Area of strip x Velocity

= Cd x b x dh x $\sqrt{2gh}$

By integrating the above equation between the limits H, and H, the total discharge through the whole orifice is obtained

$$Q = \int_{H_1}^{H_2} C_d \times b \times \sqrt{2gh}\, dh$$

$$= C_d \times b \times \sqrt{2g} \int_{H_1}^{H_2} \sqrt{h}\, dh = C_d \times b \times \sqrt{2g} \left[\frac{h^{3/2}}{3/2} \right]_{H_1}^{H_2}$$

$$= \frac{2}{3} C_d \times b \sqrt{2g} \left[H_2^{3/2} - H_1^{3/2} \right].$$

Problem 7.11 Find the discharge through a rectangular orifice 2.0 m wide and 1.5 m deep fitted to a water tank. The water level in the tank is 3.0 m above the top edge of the orifice. Take Ca = 0.62.

Problem 7.12 A rectangular orifice, 1.5 m wide and 1.0 m deep is discharging water from a tank. If the water level in the tank is 3.0 m above the top edge of the orifice, find the discharge through the orifice. Take the co-efficient of discharging for the orifice = 0.6.

Problem 7.13 A rectangular orifice 0.9 m wide and 1.2 m deep is discharging water from a vessel. The top edge of the orifice is 0.6 m below the water surface in the vessel. Calculate the discharge through the orifice if Ca = 0.6 and percentage error if the orifice is treated as a small orifice.

7.7 DISCHARGE THROUGH FULLY SUB-MERGED ORIFICE

Fully sub-merged orifice is one which has its whole of the outlet side sub-merged under liquid so that it discharges a jet of liquid into the liquid of the same kind. It is also called totally drowned orifice. Fig. 7.8 shows the fully sub-merged orifice. Consider two points (1) and (2), point 1 being in the reservoir on the upstream side of the orifice and point 2 being at the vena-contracta as shown in Fig. 7.8.

Let H1 = Height of water above the top of the orifice on the upstream side,

H2 = Height of water above the bottom of the orifice,

H = Difference in water level,

b = Width of orifice,

Cd = Co-efficient of discharge.

Height of water above the centre of orifice on upstream side

$$= H_1 + \frac{H_2 - H_1}{2} = \frac{H_1 + H_2}{2}$$

Height of water above the centre of orifice on downstream side

$$= \frac{H_1 + H_2}{2} - H$$

Applying Bernoulli's equation at (1) and (2), we get

$$\frac{p_1}{\rho g} + \frac{V_1^2}{2g} = \frac{p_2}{\rho g} + \frac{V_2^2}{2g}$$

Now $\frac{p_1}{\rho g} = \frac{H_1 + H_2}{2}, \frac{p_2}{\rho g} = \frac{H_1 + H_2}{2} - H$ and V_1 is negligible

$$\therefore \quad \frac{H_1 + H_2}{2} + 0 = \frac{H_1 + H_2}{2} - H + \frac{V_2^2}{2g}$$

$$\therefore \quad \frac{V_2^2}{2g} = H$$

$$\therefore \quad V_2 = \sqrt{2gH}$$

Area of orifice $= b \times (H_2 - H_1)$

$\therefore$ Discharge through orifice $= C_d \times$ Area $\times$ Velocity

$$= C_d \times b\,(H_2 - H_1) \times \sqrt{2gH}$$

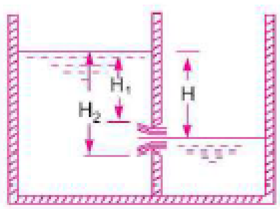
$$\therefore \quad Q = C_d \times b\,(H_2 - H_1) \times \sqrt{2gH}\,.$$

Problem 7.14 Find the discharge through a fully sub-merged orifice of width 2 m if the difference of water levels on both sides of the orifice be 50 cm. The height of water from top and bottom of the orifice are 2.5 m and 2.75 m respectively. Take Ca = 0.6.

Problem 7.15 Find the discharge through a totally drowned orifice 2.0 m wide and I m deep, if the difference of water levels on both the sides of the orifice be 3 m. Take Cg = 0.62.

4.8 DISCHARGE THROUGH PARTIALLY SUB-MERGED ORIFICE

Partially sub-merged orifice is one which has its outlet side partially sub-merged under liquid as shown in Fig. 7.9. It is also known as partially drowned orifice. Thus the partially sub-merged orifice has two portions. The upper portion behaves as an orifice discharging free while the lower portion behaves as a sub-merged orifice. Only a large orifice can behave as a partially sub-merged orifice. The total discharge Q through partially sub-merged orifice is equal to the discharges through free and the sub-merged portions.



Discharge through the sub-merged portion is given by equation (7.9)

$$Q_1 = C_d \times b \times (H_2 - H) \times \sqrt{2gH}$$

Discharge through the free portion is given by equation (7.8) as

$$Q_2 = \frac{2}{3} C_d \times b \times \sqrt{2g}\,[H_2^{3/2} - H_1^{3/2}]$$

$\therefore$ Total discharge

$$Q = Q_1 + Q_2$$

$$= C_d \times b \times (H_2 - H) \times \sqrt{2gH} + \frac{2}{3} C_d \times b \times \sqrt{2g}\,[H_2^{3/2} - H^{3/2}].$$

4.9 TIME OF EMPTYING A TANK THROUGH AN ORIFICE AT ITS BOTTOM

Consider a tank containing some liquid upto a height of H. Let an orifice is fitted at the bottom of the tank. It is required to find the time for the liquid surface to fall from the height H1 to a height H2.

Let A = Area of the tank

a = Area of the orifice

H1 = Initial height of the liquid

H2 = Final height of the liquid

T = Time in seconds for the liquid to fall from H, to H2.

Let at any time, the height of liquid from orifice is h and let the liquid surface fall by a small height dh in time dT. Then

Volume of liquid leaving the tank in time, dT = A x dh

Also the theoretical velocity through orifice, V = $\sqrt{2gh}$

Discharge through orifice/sec,

dQ = Cd x Area of orifice x Theoretical velocity = Cd.a. $\sqrt{2gh}$

Discharge through orifice in time interval

$$dT = Cd.a.\sqrt{2gh}\,.\,dT$$

As the volume of liquid leaving the tank is equal to the volume of liquid flowing through orifice in time dT, we have

$$A\,(-\,dh) = Cd.a.\sqrt{2gh}\,.dT$$

- ve sign is inserted because with the increase of time, head on orifice decreases.

$$-Adh = C_d\,.\,a\,.\,\sqrt{2gh}\;.\,dT \text{ or } dT = \frac{-A\,dh}{C_d\,.\,a\,.\sqrt{2gh}} = \frac{-A(h)^{-1/2}}{C_d\,.\,a\,.\sqrt{2g}}dh$$

By integrating the above equation between the limits H1 and H2 , the total time, T is obtained as

$$\int_0^T dT = \int_{H_1}^{H_2} \frac{-Ah^{-1/2}\,dh}{C_d\,.\,a\,.\sqrt{2g}} = \frac{-A}{C_d\,.\,a\,.\sqrt{2g}}\int_{H_1}^{H_2} h^{-1/2}\,dh$$

$$T = \frac{-A}{C_d\,.a\,.\sqrt{2g}}\left[\frac{h^{-\frac{1}{2}+1}}{-\frac{1}{2}+1}\right]_{H_1}^{H_2} = \frac{-A}{C_d\,.a\,.\sqrt{2g}}\left[\frac{\sqrt{h}}{\frac{1}{2}}\right]_{H_1}^{H_2}$$

$$= \frac{-2A}{C_d\,.a\,.\sqrt{2g}}\left[\sqrt{H_2}-\sqrt{H_1}\right] = \frac{2A\left[\sqrt{H_1}-\sqrt{H_2}\right]}{C_d\,.a\,.\sqrt{2g}}$$

For emptying the tank completely, H2 = 0 and hence

$$T = \frac{2A\sqrt{H_1}}{C_d \, . \, a \, . \sqrt{2g}}.$$

Problem 7.17 A circular tank of diameter 4 m contains water upto a height of 5 m. The tank is provided with an orifice of diameter 0.5 m at the bottom. Find the time taken by water (i) to fall from 5 m to 2 m (ii) for completely emptying the tank. Take Ca = 0.6.

Problem 7.18 A circular tank of diameter 1.25 m contains water upto a height of 5 m. An orifice of 50 mm diameter is provided at its bottom. If Ca = 0.62, find the height of water above the orifice after 1.5 minutes.

4.12 CLASSIFICATION OF MOUTHPIECES

1. Depending upon their position

(i) External mouthpiece or (ii) Internal mouthpiece

2.Based on shape

(i) Cylindrical mouthpiece or (ii) Convergent mouthpiece or (iii) Convergent-divergent

3. The mouthpieces are classified as (i) Mouthpieces running full or (ii) Mouthpieces running free, depending upon the nature of discharge at the outlet of the mouthpiece. This classification is only for internal mouthpieces which are known Borda's or Re-entrant mouthpieces. A mouthpiece is said to be running free if the jet of liquid after contraction does not touch the sides of the mouthpiece. But if the jet after contraction expands and fills the whole mouthpiece it is known as running full.

4.13 FLOW THROUGH AN EXTERNAL CYLINDRICAL MOUTHPIECE

A mouthpiece is a short length of a pipe which is two or three times its diameter in length. If this pipe is fitted externally to the orifice, the mouthpiece is called external cylindrical mouthpiece and the discharge through orifice increases.

Consider a tank having an external cylindrical mouthpiece of cross-sectional area a1, attached to one of its sides as shown in Fig. 7.13. The jet of liquid entering the mouthpiece contracts to form a vena-contracta at a section C-C. Beyond this section, the jet again expands and fill the mouthpiece completely.

Let H = Height of liquid above the centre of mouthpiece

Vc = Velocity of liquid at C-C section

Ac = Area of flow at vena-contracta

V1 = Velocity of liquid at outlet

a1 = Area of mouthpiece at outlet

Cc = Co-efficient of contraction. Applying continuity equation at C-C and (1)-(1), we get

$$a_c \times v_c = a_1 v_1$$

$$v_c = \frac{a_1 v_1}{a_c} = \frac{v_1}{a_c / a_1}$$

$$\frac{a_c}{a_1} = C_c = \text{Co-efficient of contraction}$$

Taking $C_c = 0.62$, we get $\frac{a_c}{a_1} = 0.62$

$$\therefore \qquad v_c = \frac{v_1}{0.62}$$

The jet of liquid from section *C-C* suddenly enlarges at section (1)-(1). Due to sudden enlargement, there will be a loss of head, h_L* which is given as $h_L = \frac{(v_c - v_1)^2}{2g}$

But $v_c = \frac{v_1}{0.62}$ hence $h_L = \frac{\left(\frac{v_1}{0.62} - v_1\right)^2}{2g} = \frac{v_1^2}{2g}\left[\frac{1}{0.62} - 1\right]^2 = \frac{0.375\, v_1^2}{2g}$

Applying Bernoulli's equation to point *A* and (1)-(1)

$$\frac{p_A}{\rho g} + \frac{v_A^2}{2g} + z_A = \frac{p_1}{\rho g} + \frac{v_1^2}{2g} + z_1 + h_L$$

where $z_A = z_1$, v_A is negligible,

$\frac{p_1}{\rho g}$ = atmospheric pressure = 0

$$\therefore \qquad H + 0 = 0 + \frac{v_1^2}{2g} + .375\,\frac{v_1^2}{2g}$$

$$\therefore \qquad H = 1.375\,\frac{v_1^2}{2g}$$

$$\therefore \qquad v_1 = \sqrt{\frac{2gH}{1.375}} = 0.855\sqrt{2gH}$$

Theoretical velocity of liquid at outlet is $v_{th} = \sqrt{2gH}$

$\therefore$ Co-efficient of velocity for mouthpiece

$$C_v = \frac{\text{Actual velocity}}{\text{Theoretical velocity}} = \frac{0.855\sqrt{2gH}}{\sqrt{2gH}} = 0.855.$$

Cc for mouthpiece = 1 as the area of jet of liquid at outlet is equal to the area of mouthpiece at outlet.

Thus Cd= Cc x Cv = 1.0 x 0.855 = 0.855

Thus the value of C, for mouthpiece is more than the value of Cd for orifice, and so discharge through mouthpiece will be more.

Problem 7.24 Find the discharge from a 100 mm diameter external mouthpiece, fitted to a side of a large vessel if the head over the mouthpiece is 4 metres.

Solution. Given : Dia. of mouthpiece = 100 m = 0.1 m .. Area,

Problem 7.25 An external cylindrical mouthpiece of diameter 150 mm is discharging water under a constant head of 6 m. Determine the discharge and absolute pressure head of water at vena-contracta. Take C, = 0.855 and C for vena-contracta = 0.62. Atmospheric pressure head = 10.3 m of water.

4.14 FLOW THROUGH A CONVERGENT-DIVERGENT MOUTHPIECE

If a mouthpiece converges upto vena-contracta and then diverges as shown in Fig. 7.15 then that type of mouthpiece is called Convergent-Divergent Mouthpiece. As in this mouthpiece there is no sudden enlargement of the jet, the loss of energy due to sudden enlargement is eliminated. The coefficient of discharge for this mouthpiece is unity. Let H is the head of liquid over the mouthpiece.

Applying Bernoulli's equation to the free surface of water in tank and section C-C, we have

$$\frac{p}{\rho g} + \frac{v^2}{2g} + z = \frac{p_c}{\rho g} + \frac{v_c^2}{2g} + z_c$$

Taking datum passing through the centre of orifice, we get

$$\frac{p}{\rho g} = H_a,\ v = 0,\ z = H,\ \frac{p_c}{\rho g} = H_c,\ z_c = 0$$

$$\therefore \quad H_a + 0 + H = H_c + \frac{v_c^2}{2g} + 0 \quad \text{...}(i)$$

$$\therefore \quad \frac{v_c^2}{2g} = H_a + H - H_c \quad \text{...}(ii)$$

or

$$v_c = \sqrt{2g(H_a + H - H_c)}$$

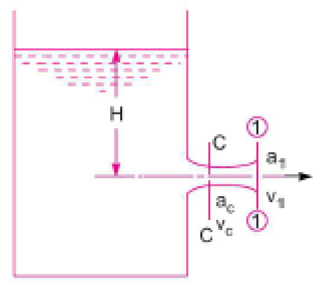


Fig. 7.15 *Convergent-divergent mouthpiece.*

Now applying Bernoulli's equation at sections *C-C* and (1)-(1)

$$\frac{p_c}{\rho g} + \frac{v_c^2}{2g} + z_c = \frac{p_1}{\rho g} + \frac{v_1^2}{2g} + z_1$$

But

$$z_c = z_1 \text{ and } \frac{p_1}{\rho g} = H_a$$

$$\therefore \quad H_c + \frac{v_c^2}{2g} = H_a + \frac{v_1^2}{2g}$$

Also from (*i*), $H_c + v_c^2/2g = H + H_a$

$\therefore$ $H_a + v_1^2/2g = H + H_a$

$$\therefore \quad v_1 = \sqrt{2gH} \quad \text{...}(iii)$$

Now by continuity equation, $a_c v_c = v_1 \times a_1$

$$\therefore \quad \frac{a_1}{a_c} = \frac{v_c}{v_1} = \frac{\sqrt{2g(H_a + H - H_c)}}{\sqrt{2gH}} = \sqrt{\frac{H_a}{H} + 1 - \frac{H_c}{H}}$$

$$= \sqrt{1 + \frac{H_a - H_c}{H}}$$

The discharge, Q is given as $Q = a_c \times \sqrt{2gH}$

where a_c = area at vena-contracta.

Problem 7.26 A convergent-divergent mouthpiece having throat diameter of 4.0 cm is discharging water under a constant head of 2.0 m, determine the maximum outer diameter for maximum discharge. Find maximum discharge also. Take H = 10.3 m of water and Her = 2.5 m of water (absolute).

Problem 7.27 The throat and exit diameters of convergent-divergent mouthpiece are 5 cm and 10 cm respectively. It is fitted to the vertical side of a tank, containing water. Find the maximum head of a water for steady flow. The maximum vacuum pressure is 8 m of water and take atmospheric pressure = 10.3 m water.

Problem 7.28 A convergent-divergent mouthpiece is fitted to the side of a tank. The discharge through mouthpiece under a constant head of 1.5 m is 5 litres/s. The head loss in the divergent portion is 0.10 times the kinetic head at outlet. Find the throat and exit diameters, if separation pressure is 2.5 m and atmospheric pressure head = 10.3 m of water.

7.15 FLOW THROUGH INTERNAL OR RE-ENTRANT ON BORDA'S MOUTHPIECE

A short cylindrical tube attached to an orifice in such a way that the tube projects inwardly to a tank, is called an internal mouthpiece. It is also called Re-entrant or Borda's mouthpiece. If the length of the tube is equal to its diameter, the jet of liquid comes out from mouthpiece without touching the sides of the tube as shown in Fig. 7.16. The mouthpiece is known as running free. But if the length of the tube is about 3 times its diameter, the jet comes out with its diameter equal to the diameter of mouthpiece at outlet as shown in Fig. 7.17. The mouthpiece is said to be running full.

(i) Borda's Mouthpiece Running Free.

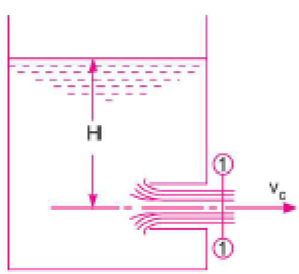
Fig. 7.16 shows the Borda's mouthpiece running free.

Let H = height of liquid above the mouthpiece,

a = area of mouthpiece,

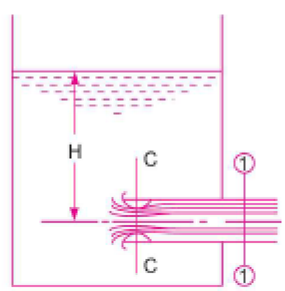
ac = area of contracted jet in the mouthpiece,

ae = velocity through mouthpiece.



RUNNING FREE

Fig. 7.16



RUNNING FULL

Fig. 7.17

The flow of fluid through mouthpiece is taking place due to the pressure force exerted by the fluid on the entrance section of the mouthpiece. As the area of the mouthpiece is 'a' hence total pressure force on entrance = ρ.g.a. h

where h = distance of C.G. of area 'a' from free surface = H.

= ρ.g.a. h

According to Newton's second law of motion, the net force is equal to the rate of change of momentum.

Now mass of liquid flowing/sec = ρ x ac x vc

The liquid is initially at rest and hence initial velocity is zero but final velocity of fluid is vc Rate of change of momentum = mass of liquid flowing/sec x [final velocity - initial velocity]

= ρ ac, x vc [vc - 0] = ρ ac vc2

Equating (i) and (ii), we get

ρ g.a.H. = ρ ac vc2

Applying Bernoulli's equation to free surface of liquid and section (1)-(1) of Fig. 7.16

$$\frac{p}{\rho g} + \frac{v^2}{2g} + z = \frac{p_1}{\rho g} + \frac{v_1^2}{2g} + z_1$$

Taking the centre line of mouthpiece as datum, we have

$$z = H,\ z_1 = 0,\ \frac{p}{\rho g} = \frac{p_1}{\rho g} = p_{atmosp.} = 0,$$

$$v_1 = v_c, \qquad v = 0$$

$$0 + 0 + H = 0 + \frac{v_c^2}{2g} + 0 \text{ or } H = \frac{v_c^2}{2g}$$

$$v_c = \sqrt{2gH}$$

Substituting the value of vc in (iii), we get

$$\rho g \,.\, a \,.\, H. = \rho \,.\, a_c \,.\, 2g \,.\, H$$

or $$a = 2a_c \text{ or } \frac{a_c}{a} = \frac{1}{2} = 0.5$$

$\therefore$ Co-efficient of contraction, $C_c = \frac{a_c}{a} = 0.5$

Since there is no loss of head, co-efficient of velocity, $C_v = 1.0$

$\therefore$ Co-efficient of discharge, $C_d = C_c \times C_v = 0.5 \times 1.0 = 0.5$

$\therefore$ Discharge $Q = C_d\, a\sqrt{2gH}$

$$= 0.5 \times a\sqrt{2gH}$$

(ii) Borda's Mouthpiece Running Full.

Fig. 7.17 shows Borda's mouthpiece running full.

Let H = height of liquid above the mouthpiece,

v1 = velocity at outlet or at (1)-(1) of mouthpiece,

a = area of mouthpiece,

ac = area of the flow at C-C,

vc = velocity of liquid at vena-contracta or at C-C.

The jet of liquid after passing through C-C, suddenly enlarges at section (1)-(1). Thus there will be a loss of head due to sudden enlargement.

$$h_L = \frac{(v_c - v_1)^2}{2g}$$

Now from continuity, we have $a_c \times v_c = a_1 \times v_1$

$\therefore$ $$v_c = \frac{a_1}{a_c} \times v_1 = \frac{v_1}{a_c / a_1} = \frac{v_1}{C_c} = \frac{v_1}{0.5} \qquad \{\because C_c = 0.5\}$$

or $$v_c = 2v_1$$

Substituting this value of v_c in (*i*), we get $h_L = \frac{(2v_1 - v_1)^2}{2g} = \frac{v_1^2}{2g}$

Applying Bernoulli's equation to free surface of water in tank and section (1)-(1), we get

$$\frac{p}{\rho g} + \frac{v^2}{2g} + z = \frac{p_1}{\rho g} + \frac{v_1^2}{2g} + z_1 + h_L$$

Taking datum line passing through the centre line of mouthpiece

$$0 + 0 + H = 0 + \frac{v_1^2}{2g} + 0 + \frac{v_1^2}{2g}$$

$\therefore$ $$H = \frac{v_1^2}{2g} + \frac{v_1^2}{2g} = \frac{v_1^2}{g}$$

$\therefore$ $$v_1 = \sqrt{gH}$$

Here v_1 is actual velocity as losses have been taken into consideration,

But theoretical velocity, $v_{th} = \sqrt{2gH}$

$\therefore$ Co-efficient of velocity, $C_v = \frac{v_1}{v_{th}} = \frac{\sqrt{gH}}{\sqrt{2gH}} = \frac{1}{\sqrt{2}} = 0.707$

As the area of the jet at outlet is equal to the area of the mouthpiece, hence co-efficient of contraction = 1

$\therefore$ $$C_d = C_c \times C_v = 1.0 \times .707 = 0.707$$

$\therefore$ Discharge, $$Q = C_d \times a \times \sqrt{2gH} = 0.707 \times a \times \sqrt{2gH} \qquad \text{...(7.20)}$$

Problem 7.29 An internal mouthpiece of 80 mm diameter is discharging water under a constant head of 8 metres. Find the discharge through mouthpiece, when

(i) The mouthpiece is running free, and (ii) The mouthpiece is running full.

NOTCHES AND WEIRS

8.1. INTRODUCTION

A notch is a device used for measuring the rate of flow of a liquid through a small channel or a tank. It may be defined as an opening in the side of a tank or a small channel in such a way that the liquid surface in the tank or channel is below the top edge of the opening.

A weir is a concrete or masonary structure, placed in an open channel over which the flow occurs. It is generally in the form of vertical wall, with a sharp edge at the top, running all the way across the open channel. The notch is of small size while the weir is of a bigger size. The notch is generally made of metallic plate while weir is made of concrete or masonary structure.

1. Nappe or Vein. The sheet of water flowing through a notch or over a weir is called Nappe or Vein.

2. Crest or Sill. The bottom edge of a notch or a top of a weir over which the water flows, is known as the sill or crest.

8.2 CLASSIFICATION OF NOTCHES AND WEIRS

1. According to the shape of the opening :

(a) Rectangular notch, (b) Triangular notch,

(c) Trapezoidal notch, and(d) Stepped notch.

2. According to the effect of the sides on the nappe :

(a) Notch with end contraction.

(b) Notch without end contraction or suppressed notch.

Weirs are classified according to the shape of the opening, the shape of the crest, the effect of the sides on the nappe and nature of discharge. The following are important classifications.

(a) According to the shape of the opening :

(i) Rectangular weir,

(ii) Triangular weir, and

(iii) Trapezoidal weir (Cipolletti weir)

(b) According to the shape of the crest :

(i) Sharp-crested weir,

(ii) Broad-crested weir,

(iii) Narrow-crested weir, and

(iv) Ogee-shaped weir.

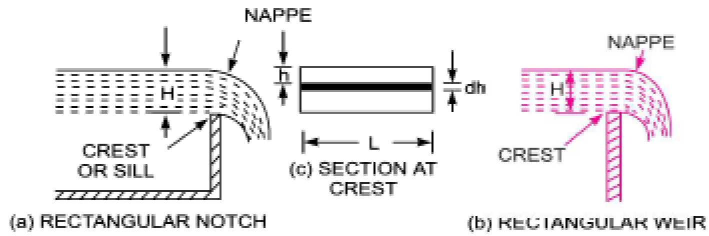
(c) According to the effect of sides on the emerging nappe :

(i) Weir with end contraction, and

(ii) Weir without end contraction.

8.3 DISCHARGE OVER A RECTANGULAR NOTCH OR WEIR

The expression for discharge over a rectangular notch or weir is the same.



Rectangular notch and weir.

Consider a rectangular notch or weir provided in a channel carrying water as shown in Fig. 8.1.

H = Head of water over the crest

L = Length of the notch or weir

For finding the discharge of water flowing over the weir or notch, consider an elementary horizontal strip of water of thickness dh and length L at a depth h from the free surface of water as shown in Fig. 8.1(c).

The area of strip = L x dh

and theoretical velocity of water flowing through strip = $\sqrt{2gh}$

The discharge dQ, through strip is

dQ = Cd X Area of strip x Theoretical velocity

= Cd x L x dh x $\sqrt{2gh}$

where Cd = Co-efficient of discharge.

The total discharge, Q for the whole notch or weir is determined by integrating equation between the limits 0 and H.

$$Q = \int_0^H C_d \,.\, L \,.\, \sqrt{2gh}\ .\, dh = C_d \times L \times \sqrt{2g} \int_0^H h^{1/2}\, dh$$

$$= C_d \times L \times \sqrt{2g} \left[\frac{h^{1/2+1}}{\frac{1}{2}+1} \right]_0^H = C_d \times L \times \sqrt{2g} \left[\frac{h^{3/2}}{3/2} \right]_0^H$$

$$= \frac{2}{3} C_d \times L \times \sqrt{2g}\ [H]^{3/2}.$$

Problem 8.1 Find the discharge of water flowing over a rectangular notch of 2 m length when the constant head over the notch is 300 mm. Take Ca = 0.60.

Problem 8.2 Determine the height of a rectangular weir of length 6 m to be built across a rectangular channel. The maximum depth of water on the upstream side of the weir is 1.8 m and discharge is 2000 litres/s. Take Ca = 0.6 and neglect end contractions.

Problem 8.3 The head of water over a rectangular notch is 900 mm. The discharge is 300 litres/s. Find the length of the notch, when Ca = 0.62.

8.4 DISCHARGE OVER A TRIANGULAR NOTCH OR WEIR

The expression for the discharge over a triangular notch or weir is the same. It is derived as:

Let H = head of water above the V- notch

θ= angle of notch

Consider a horizontal strip of water of thickness 'dh' at a depth of h from the free surface of water as shown in Fig. 8.3.

From Fig. 8.3 (*b*), we have

$$\tan\frac{\theta}{2} = \frac{AC}{OC} = \frac{AC}{(H-h)}$$

$$\therefore \quad AC = (H-h)\tan\frac{\theta}{2}$$

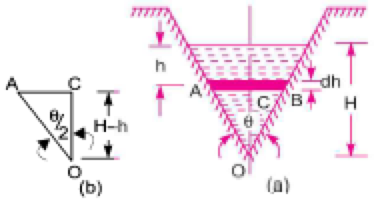


Fig. 8.3 *The triangular notch.*

Width of strip $= AB = 2AC = 2\,(H-h)\tan\frac{\theta}{2}$

$\therefore$ Area of strip $= 2\,(H-h)\tan\frac{\theta}{2} \times dh$

The theoretical velocity of water through strip $= \sqrt{2gh}$

$\therefore$ Discharge, through the strip,

$$dQ = C_d \times \text{Area of strip} \times \text{Velocity (theoretical)}$$

$$= C_d \times 2\,(H-h)\tan\frac{\theta}{2} \times dh \times \sqrt{2gh}$$

$$= 2C_d\,(H-h)\tan\frac{\theta}{2} \times \sqrt{2gh} \times dh$$

$\therefore$ Total discharge,

$$Q = \int_0^H 2C_d\,(H-h)\tan\frac{\theta}{2} \times \sqrt{2gh} \times dh$$

$$= 2C_d \times \tan\frac{\theta}{2} \times \sqrt{2g}\int_0^H (H-h)h^{1/2}\,dh$$

$$= 2 \times C_d \times \tan\frac{\theta}{2} \times \sqrt{2g}\int_0^H (Hh^{1/2} - h^{3/2})\,dh$$

$$= 2 \times C_d \times \tan\frac{\theta}{2} \times \sqrt{2g}\left[\frac{Hh^{3/2}}{3/2} - \frac{h^{5/2}}{5/2}\right]_0^H$$

$$= 2 \times C_d \times \tan\frac{\theta}{2} \times \sqrt{2g}\left[\frac{2}{3}H.H^{3/2} - \frac{2}{5}H^{5/2}\right]$$

$$= 2 \times C_d \times \tan\frac{\theta}{2} \times \sqrt{2g}\left[\frac{2}{3}H^{5/2} - \frac{2}{5}H^{5/2}\right]$$

$$= 2 \times C_d \times \tan\frac{\theta}{2} \times \sqrt{2g}\left[\frac{4}{15}H^{5/2}\right]$$

$$= \frac{8}{15}C_d \times \tan\frac{\theta}{2} \times \sqrt{2g} \times H^{5/2}$$

For a right-angled V-notch, if $C_d = 0.6$

$$\theta = 90°, \quad \therefore \quad \tan\frac{\theta}{2} = 1$$

Discharge,

$$Q = \frac{8}{15} \times 0.6 \times 1 \times \sqrt{2 \times 9.81} \times H^{5/2}$$
$$= 1.417\ H^{5/2}.$$

Problem 8.4 Find the discharge over a triangular notch of angle 60° when the head over the V-notch is 0.3 m. Assume Cd = 0.6.

Problem 8.5 Water flows over a rectangular weir 1 m wide at a depth of 150 mm and afterwards passes through a triangular right-angled weir. Taking Ca for the rectangular and triangular weir as 0.62 and 0.59 respectively, find the depth over the triangular weir.

Problem 8.5A Water flows through a triangular right-angled weir first and then over a rectangular weir of 1 m width. The discharge co-efficients of the triangular and rectangular weirs are 0.6 and 0.7 respectively. If the depth of water over the triangular weir is 360 mm, find the depth of water over the rectangular weir.

Problem 8.6 A rectangular channel 2.0 m wide has a discharge of 250 litres per second, which is measured by a right-angled V-notch weir. Find the position of the apex of the notch from the bed of the channel if maximum depth of water is not to exceed 1.3 m. Take Ca = 0.62.

8.5 ADVANTAGES OF TRIANGULAR NOTCH OR WEIR OVER RECTANGULAR NOTCH OR WEIR

A triangular notch or weir is preferred to a rectangular weir or notch due to following reasons :

1. The expression for discharge for a right-angled V-notch or weir is very simple.

2. For measuring low discharge, a triangular notch gives more accurate results than a rectangular notch.

3. In case of triangular notch, only one reading, i.e., H is required for the computation of discharge.

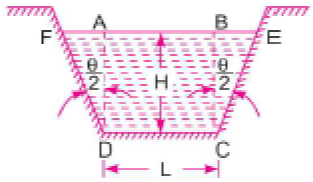
4. Ventilation of a triangular notch is not necessary.

8.6 DISCHARGE OVER A TRAPEZOIDAL NOTCH OR WEIR

As shown in Fig. 8.4, a trapezoidal notch or weir is a combination of a rectangular and triangular notch or weir. Thus the total discharge will be equal to the sum of discharge through a rectangular weir or notch and discharge through a triangular notch or weir.

Let H = Height of water over the notch

L = Length of the crest of the notch



Cd1 = Co-efficient of discharge for rectangular portion ABCD of Fig. 8.4.

Cd2 = Co-efficient of discharge for triangular portion [FAD and BCE]

The discharge through rectangular portion ABCD is given by (8.1)

or

The discharge through two triangular notches FDA and BCE is equal to the discharge through a single triangular notch of angle and it is given by equation (8.2) as

$$Q_2 = \frac{8}{15} \times C_{d_2} \times \tan\frac{\theta}{2} \times \sqrt{2g} \times H^{5/2}$$

Discharge through trapezoidal notch or weir FDCEF = Q, + Q2

$$= \frac{2}{3} C_{d_1} L \sqrt{2g} \times H^{3/2} + \frac{8}{15} C_{d_2} \times \tan\theta/2 \times \sqrt{2g} \times H^{5/2}.$$

Problem 8.7 Find the discharge through a trapezoidal notch which is 1 m wide at the top and 0.40 m at the bottom and is 30 cm in height. The head of water on the notch is 20 cm. Assume Cafor rectangular portion = 0.62 while for triangular portion = 0.60.

8.10 VELOCITY OF APPROACH

Velocity of approach is defined as the velocity with which the water approaches or reaches the weir or notch before it flows over it. Thus if V, is the velocity of approach, then an additional head ha equal to

$$\frac{V_a^2}{2g}$$

Due to velocity of approach, is acting on the water flowing over the notch. Then initial height of water over the notch becomes (H+ ha) and final height becomes equal to ha. Then all the formulae are changed taking into consideration of velocity of approach.

The velocity of approach, Va, is determined by finding the discharge over the notch or weir neglecting velocity of approach. Then dividing the discharge by the cross-sectional area of the channel on the upstream side of the weir or notch, the velocity of approach is obtained. Mathematically,

$$V_a = \frac{Q}{\text{Area of channel}}$$

This velocity of approach is used to find an additional head

$$\left(h_a = \frac{V_a^2}{2g}\right).$$

.Again the discharge is calculated and above process is repeated for more accurate discharge. Discharge over a rectangular weir, with velocity of approach

$$= \frac{2}{3} \times C_d \times L \times \sqrt{2g}\ [(H_1 + h_a)^{3/2} - h_a^{3/2}]$$

Problem 8.15 Water is flowing in a rectangular channel of l m wide and 0.75 m deep. Find the discharge over a rectangular weir of crest length 60 cm, if the head of water over the crest of weir is 20 cm and water from channel flows over the weir. Take Ca = 0.62. Neglect end contractions. Take velocity of approach into consideration.

Problem 8.16 Find the discharge over a rectangular weir of length 100 m. The head of water over the weir is 1.5 m. The velocity of approach is given as 0.5 m/s. Take Ca = 0.60.

Problem 8.17 A rectangular weir of crest length 50 cm is used to measure the rate of flow of water in a rectangular channel of 80 cm wide and 70 cm deep. Determine the discharge in the channel if the water level is 80 mm above the crest of weir. Take velocity of approach into consideration and value of Cg = 0.62.

Problem 8.18 A suppressed rectangular weir is constructed across a channel of 0.77 m width with a head of 0.39 m and the crest 0.6 m above the bed of the channel. Estimate the discharge over it. Consider velocity of approach and assume Ca = 0.623.

Problem 8.19 A sharp crested rectangular weir of 1 m height extends across a rectangular channel of 3 m width. If the head of water over the weir is 0.45 m, calculate the discharge. Consider velocity of approach and assume Ca = 0.623.

8.11 EMPIRICAL FORMULAE FOR DISCHARGE OVER RECTANGULAR WEIR

The discharge over a rectangular weir is given by

$$Q = \frac{2}{3} C_d \sqrt{2g} \times L \times [H^{3/2}] \text{ without velocity of approach} \quad ...(i)$$

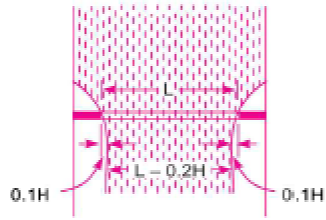
$$= \frac{2}{3} C_d \sqrt{2g} \times L \times [(H + h_a)^{3/2} - h_a^{3/2}] \text{ with velocity of approach} \quad ...(ii)$$

Equations (i) and (ii) are applicable to the weir or notch for which the crest length is equal to the width of the channel. This type of weir is called Suppressed weir. But if the weir is not suppressed, the effect of end contraction will be taken into account.

(a) Francis's Formula.

Francis on the basis of his experiments established that end contraction decreases the effective length of the crest of weir and hence decreases the discharge. Each end contraction reduces the

NO crest length by 0.1 x H, where H is the head over the weir. For a rectangular weir there are two end contractions only and hence effective length L = (L-0.2 H)



$$Q = \frac{2}{3} \times C_d \times [L - 0.2 \times H] \times \sqrt{2g}\ H^{3/2}$$

$C_d = 0.623$, $g = 9.81$ m/s^2, then

$$Q = \frac{2}{3} \times .623 \times \sqrt{2 \times 9.81} \times [L - 0.2 \times H] \times H^{3/2}$$

$$= 1.84\ [L - 0.2 \times H]H^{3/2}$$

If end contractions are suppressed, then

$$H = 1.84\ LH^{3/2}$$

If velocity of approach is considered, then

$$Q = 1.84\ L\ [(H + h_a)^{3/2} - h_a^{3/2}]$$

(b) Bazin's Formula.

On the basis of results of a series of experiments, Bazin's proposed the following formula for the discharge over a rectangular weir as

$$Q = m \times L \times \sqrt{2g} \times H^{3/2}$$

$$\text{where } m = \frac{2}{3} \times C_d = 0.405 + \frac{.003}{H}$$

H = height of water over the weir If velocity of approach is considered, then

$$Q = m_1 \times L \times \sqrt{2g}\ [(H + h_a)^{3/2}]$$

$$\text{where } m_1 = 0.405 + \frac{.003}{(H + h_a)}\ .$$

Problem 8.20 The head of water over a rectangular weir is 40 cm. The length of the crest of the weir with end contraction suppressed is 1.5 m. Find the discharge using the following formulae : (i) Francis's Formula and (ii) Bazin's Formula.

Problem 8.21 A weir 36 metres long is divided into 12 equal bays by vertical posts, each 60 cm wide. Determine the discharge over the weir if the head over the crest is 1.20 m and velocity of approach is 2 metres per second.

Problem 8.22 A discharge of 2000 m/s is to pass over a rectangular weir. The weir is divided into a number of openings each of span 10 m. If the velocity of approach is 4 m/s, find the number of openings needed in order the head of water over the crest is not to exceed 2 m.

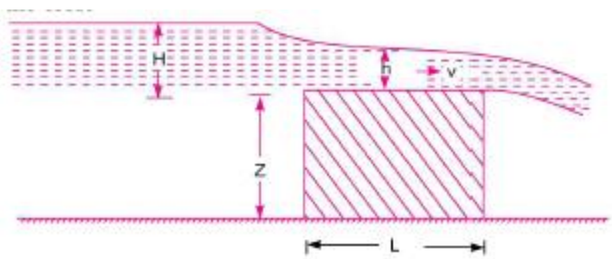
Problem 8.23 Find the discharge over a cipolletti weir of length 2.0 m when the head over the weir is 1 m. Take Ca = 0.62.

Problem 8.24 A cipolletti weir of crest length 60 cm discharges water. The head of water over the weir is 360 mm. Find the discharge over the weir if the channel is 80 cm wide and 50 cm deep. Take Ca = 0.60. Solution.

8.13 DISCHARGE OVER A BROAD-CRESTED WEIR

A weir having a wide crest is known as broad-crested weir.

Let H = height of water above the crest



If 2L > H, the weir is called broad-crested weir

If 2L < H, the weir is called a narrow-crested weir Fig. 8.10 shows a broad-crested weir.

Let h = head of water at the middle of weir which is constant

v = velocity of flow over the weir

Applying Bernoulli's equation to the still water surface on the upstream side and running water at the end of weir,

$$0 + 0 + H = 0 + \frac{v^2}{2g} + h$$

$$\therefore \quad \frac{v^2}{2g} = H - h$$

$$\therefore \quad v = \sqrt{2g\,(H - h)}$$

$\therefore$ The discharge over weir $Q = C_d \times \text{Area of flow} \times \text{Velocity}$

$$= C_d \times L \times h \times \sqrt{2g\,(H - h)}$$

$$= C_d \times L \times \sqrt{2g\left(Hh^2 - h^3\right)}$$

The discharge will be maximum, if $(Hh^2 - h^3)$ is maximum

or $$\frac{d}{dh}(Hh^2 - h^3) = 0 \text{ or } 2h \times H - 3h^2 = 0 \text{ or } 2H = 3h$$

$$\therefore \quad h = \frac{2}{3}H$$

Q_{max} will be obtained by substituting this value of h in equation (8.18) as

$$Q_{max} = C_d \times L \times \sqrt{2g\left[H \times \left(\frac{2}{3}H\right)^2 - \left(\frac{2}{3}H\right)^3\right]}$$

$$= C_d \times L \times \sqrt{2g}\sqrt{H \times \frac{4}{9} \times H^2 - \frac{8}{27}H^3}$$

$$= C_d \times L \times \sqrt{2g}\sqrt{\frac{4}{9}H^3 - \frac{8}{27}H^3} = C_d \times L \times \sqrt{2g}\sqrt{\frac{(12-8)H^3}{27}}$$

$$= C_d \times L \times \sqrt{2g}\sqrt{\frac{4}{27}H^3} = C_d \times L \times \sqrt{2g} \times 0.3849 \times H^{3/2}$$

$$= .3849 \times \sqrt{2 \times 9.81} \times C_d \times L \times H^{3/2} = 1.7047 \times C_d \times L \times H^{3/2}$$

$$= 1.705 \times C_d \times L \times H^{3/2}. \qquad \text{...(8.19)}$$