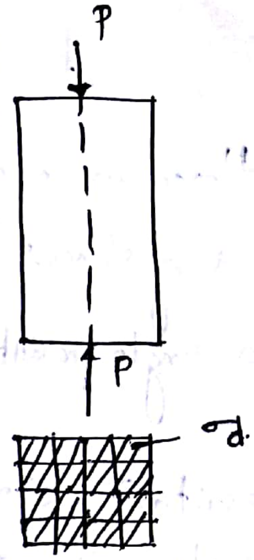
UNIT-III

Direct & Bending stresses

Combined bending & direct stresses:

Consider the case of column subjected by a compressive load 'P' acting along the axis of the column as shown in fig.



This load will cause direct compressive stress whose intensity will be uniform across the cross-section of the column.

let σ_d = Intensity of the stress

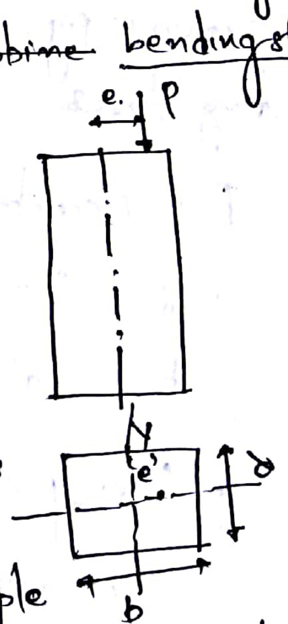
A = Area of c/s

P = Load acting on the column

Then stress

$$\sigma_d = \frac{P}{A}$$

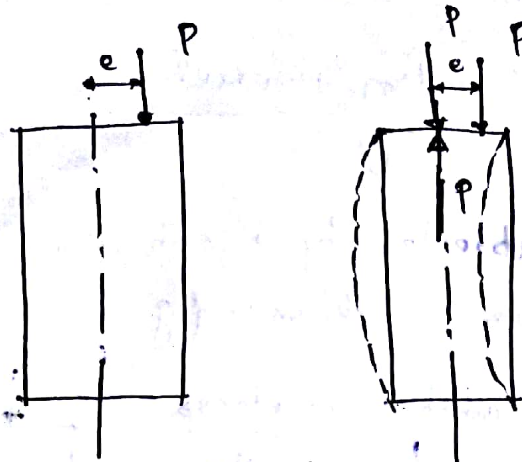
Now consider this case of column subjected to compressive load 'p' acting at distance of 'e' from line of action as shown in fig. The eccentric load will cause direct stress & ~~combine~~ bending stress.



This is proved as discussed below.

1. The load 'p' is applied along the line of axis of the column, two equal & opposite force P. These forces are acting now on the column. And hence this load will cause direct stress

2. The forces shown in figure will form a couple whose moment will be Pe. This couple will produce a bending stress.



Hence an eccentric load will produce a direct stress as well as a bending stress. By adding these two stresses ~~algebrically~~ algebraically a single resultant stress can be obtained

Resultant stress when a column of rectangular section is subjected to an eccentric load:

A column of rectangular section subjected to an eccentric load is shown in fig. Let the load is eccentric w.r.t the axis X-Y as shown in fig. that an eccentric load causes direct stress as well as bending stress.

P = Eccentric load on column ; e = Eccentricity of load

σ_d = Direct stress σ_b = Bending stress

b = width of column; d = Depth of column

Area of column section $A = bd$

Now moment due to eccentric load P is given by

M = load × eccentricity

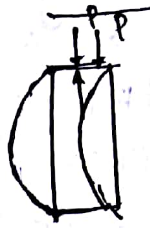
$$\boxed{M = P \times e}$$

The direct stress 'σ_d' is given by

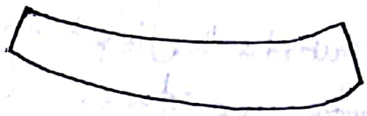
$$\sigma_d = \frac{\text{Load}}{\text{Area}} = \frac{P}{A}$$

The stress is uniform along the c/s of the column

The bending stress σ_b due to moment at any point of the column section at a distance 'y' from the NA Y-Y is given by

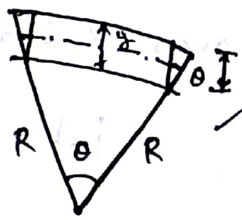


$$\frac{M}{I} = \frac{\sigma_b}{\pm y}$$



$$\Rightarrow \quad \sigma_b = \pm \frac{M}{I} y$$

where I = Moment of inertia of the column section about the NA Y-Y $= \frac{db^3}{12}$



Substituting the value of 'I' in above equation

$$\sigma_b = \pm \frac{M}{\frac{db^3}{12}} \times y = \pm \frac{12M}{db^3} y$$

The bending stress depends upon the value of 'y' from the axis Y-Y

The bending stress at the extreme is obtained by substituting $y = \frac{b}{2}$ in the above equation.

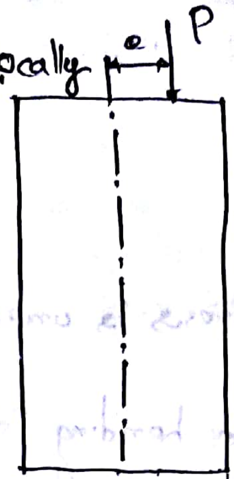
$$\sigma_b = \pm \frac{12M}{b^3 d} \cdot y \Rightarrow \sigma_b = \pm \frac{12M}{b^3 \cdot d} \cdot \frac{b}{2}$$

$$\sigma_b = \pm \frac{6M}{db^2}$$

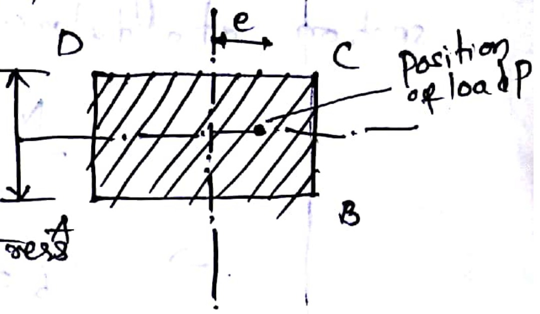
$$\sigma_b = \pm \frac{6pe}{db^2}$$

$$\sigma_b = \pm \frac{6pe}{A \cdot b} \quad \left[\because \text{Area} = b \times d = A\right]$$

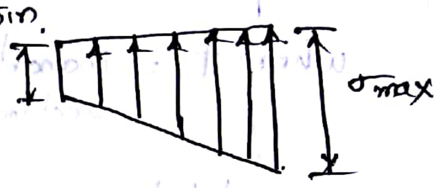
The resultant stress at any point will be the algebraically sum of direct stress & bending stress.



If y is taken positive on the same side of Y-Y as the load, then bending stress will be of the same type as the direct stress. Here, direct stress is compressive & hence bending stress will also be compressive towards the right of the axis Y-Y. Similarly bending stress will be tensile towards the left of the axis Y-Y. Taking compressive stress as (+)ve & tensile stress as (−)ve we can find the max, & min, stress at the extremities of the section.



The stress will be maximum along layer BC & min, along the layer AD



let σ_{max} = Maximum stress (i.e., stress along BC)

σ_{min} = Minimum stress (i.e., stress along AD)

Then σ_{max} = Direct stress + Bending stress

$$= \sigma_d + \sigma_b$$

$$= \frac{P}{A} + \frac{6Pe}{A \cdot b}$$

$$= \frac{P}{A}\left(1 + \frac{6e}{b}\right)$$

σ_{min} = Direct stress − Bending stress

$$= \sigma_d - \sigma_b$$

$$= \frac{P}{A} - \frac{6Pe}{Ab} = \frac{P}{A}\left(1 - \frac{6e}{b}\right)$$

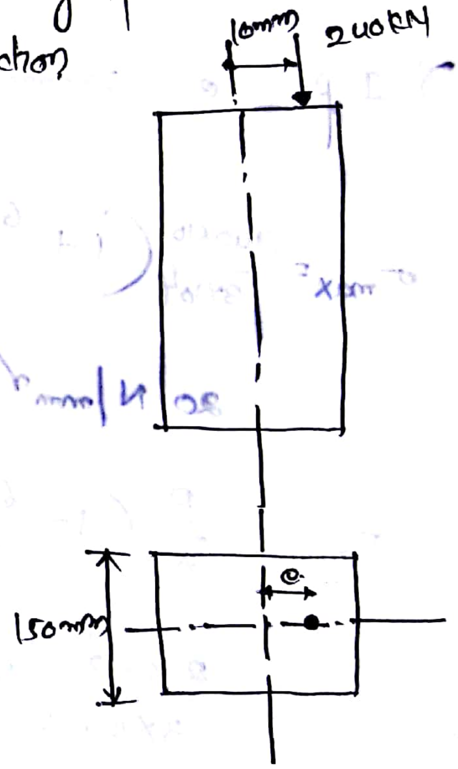
These stresses are shown in fig. The resultant stress along the width of the column will vary by a straight line law.

If σ_{min} is (−)ve then the stress along the layer AB will be tensile.

If σ_{min} is zero then there will be no tensile stress along the width of the column.

If σ_{min} is (+)ve then there will be only compressive stress along the width of column.

Example 1: A rectangular column of width 200 mm & thickness 150 mm carries a point load of 240 kN at an eccentricity of 10 mm as shown in fig. Determine the max, & min, stresses on the section.



Sol: width $b = 200\,mm$; $d = 150\,mm$

$A = b \times d = 200 \times 150 = 30000\,mm^2$

Eccentric load $P = 240\,kN$

$= 240 \times 10^3\,N$

Eccentricity $e = 10\,mm$

σ_{max} = Max, stress

σ_{min} = Min, stress

(i) $\sigma_{max} = \frac{P}{A}\left[1 + \frac{6e}{b}\right]$

$= \frac{240 \times 10^3}{3 \times 10^4}\left[1 + \frac{6 \times 10}{200}\right]$

$= 8(1 + 0.3) = 10.4\,N/mm^2$

(ii) $\sigma_{min} = \frac{P}{A}\left(1 - \frac{6e}{b}\right)$

$= \frac{240 \times 10^3}{3 \times 10^4}\left[1 - \frac{6 \times 10}{200}\right] = 8(1 - 0.3) = 5.6\,N/mm^2$

(ii) If $\sigma_{min} = 0$, calculate eccentricity; Determine max. stress.

$$\sigma_{min} = \frac{P}{A}\left(1 - \frac{6e}{b}\right)$$

$$0 = \frac{240\times10^3}{3\times10^4}\left(1 - \frac{6e}{200}\right)$$

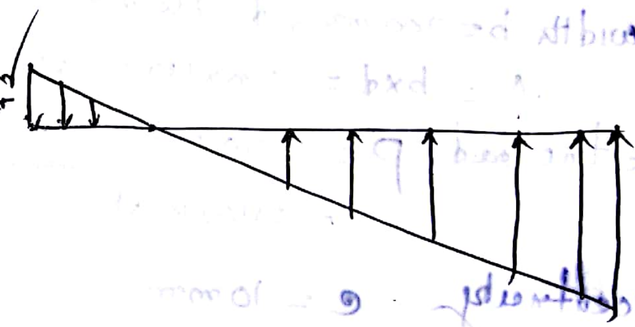
$$\Rightarrow 1 - \frac{6e}{200} = 0 \Rightarrow e = \frac{200}{6} = 33.33\ mm$$

$$\sigma_{max} = \frac{P}{A}\left(1 + \frac{6e}{b}\right)$$

$$= \frac{240\times10^3}{3\times10^4}\left(1 + \frac{\frac{6\times200}{6}}{200}\right) = 8(1+1) = 16\ N/mm^2$$

(iii) If $e = 50\ mm$; Determine max, minimum stresses.

$$\sigma_{max} = \frac{240\times10^3}{3\times10^4}\left(1 + \frac{6\times50}{200}\right) = 20\ N/mm^2$$



$$\sigma_{min} = \frac{P}{A}\left(1 - \frac{6e}{b}\right)$$

$$= \frac{240\times10^3}{3\times10^4}\left(1 - \frac{6\times50}{200}\right) = 8(1-1.5) = -4\ N/mm^2$$

'Negative sign' means tensile stress

Resultant stress when a column of Rectangular section is subjected to a load which is eccentric to both axes:

A column of rectangular section ABCD, subjected to a load which is eccentric to both axes.

let P = Eccentric load on column

e_x = Eccentricity of load about X-X axis

e_y = Eccentricity of load about Y-Y axis

b = width of column

d = Depth of column

σ_d = Direct stress

σ_{bx} = Bending stresses due to eccentricity e_x

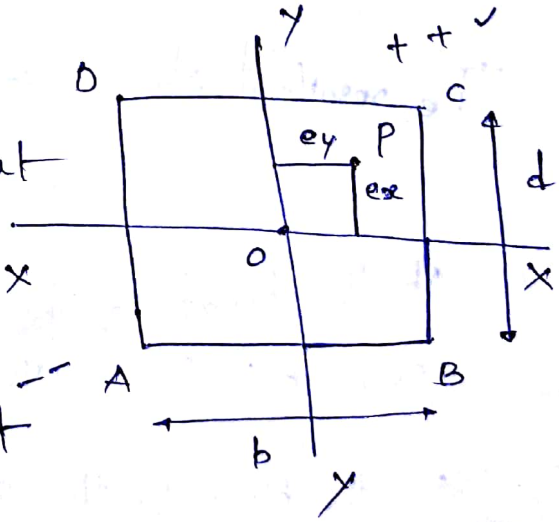
σ_{by} = Bending stresses due to eccentricity e_y

M_x = Moment of load about X-X axis

$= P \times e_x$

M_y = Moment of load about Y-Y axis

$= P \times e_y$



I_{xx} = Moment of Inertia about X-X axis

$$= \frac{bd^3}{12}$$

I_{yy} = Moment of inertia about Y-Y axis

$$= \frac{db^3}{12}$$

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Now the eccentric load is equivalent to a central load P, together with a bending moment $P \times e_y$ about Y-Y & a bending moment $P \times e_x$ about X-X

(i) The direct stress 'σ_d' is given by

$$\sigma_d = \frac{P}{A}$$

(ii) The bending stress due to eccentricity 'e_y' is given by

$$\sigma_{by} = \frac{M_y x}{I_{yy}} = \frac{P e_y x}{I_{yy}}$$

In the above equation x varies from $-\frac{b}{2}$ to $+\frac{b}{2}$

(iii) The bending stress due to eccentricity 'e_x' is given by

$$\sigma_{bx} = \frac{M_x y}{I} = \frac{P \times e_x y}{I_{xx}}$$

In the above equation, y varies from $-\frac{d}{2}$ + $\frac{d}{2}$

The resultant stress at any point on the section

$$= \sigma_d \pm \sigma_{by} \pm \sigma_{bx}.$$

(i) At the point C, the coordinates x & y are (+ve) hence the resultant stress will be maximum.

(ii) At the point A, the coordinates x & y are (-ve) & hence the resultant stress will be minimum.

(iii) At the point B x is +ve y is -ve & hence resultant stress

$$= \frac{P}{A} + \frac{M_y x}{I_{yy}} - \frac{M_x y}{I_{xx}}$$

(iv) At the point D, x is -ve & y is +ve & hence

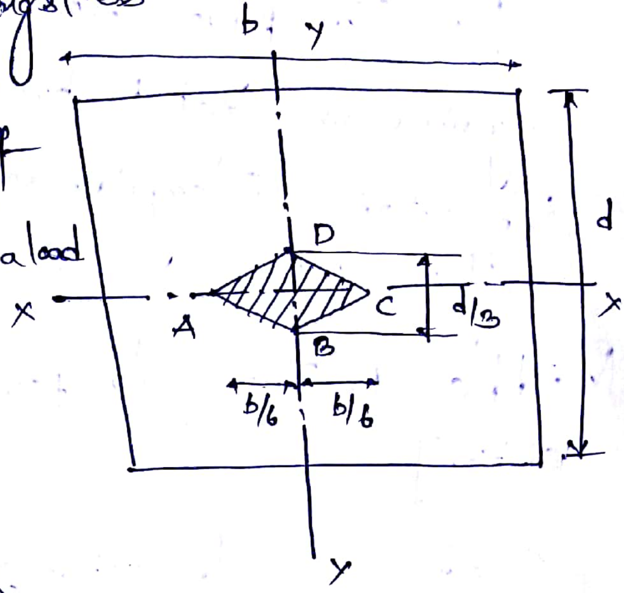
$$\frac{P}{A} - \frac{M_y x}{I_{yy}} + \frac{M_x y}{I_{xx}}$$

Core of section: It is the part of the column section in which the load can be applied without causing tensile stress anywhere in the section.

Middle third rule for rectangular sections (i.e., Kernel of section)

The cement concrete columns are weak in tension. Hence the load must be applied on these columns in such a way that there is no tensile stress anywhere in the section. But when an eccentric load is acting on a column, it produces direct stress as well as bending stress.

The resultant stress at any point in the section is the algebraic sum of the direct & bending stress.



Consider a rectangular section of width 'b' & depth 'd' as shown in fig.

Let this section is subjected to a load which is eccentric to the axis Y-Y

Then from equation, we have

minimum stress

$$\sigma_{min} = \frac{P}{A}\left(1 - \frac{6e}{b}\right)$$

If σ_{min} is $-ve$ stress will be tensile

If σ_{min} is zero (or possitive) then there will be no tensile stress along the width of the column

Hence for no tensile stress along the width of column

but $\sigma_{min} \geq 0$

$$\frac{P}{A}\left(1 - \frac{6e}{b}\right) \geq 0 \quad \text{(or)} \quad \left(1 - \frac{6e}{b}\right) \geq 0$$

$$1 \geq \frac{6e}{b} \quad \text{or} \quad \frac{b}{6} \geq e$$

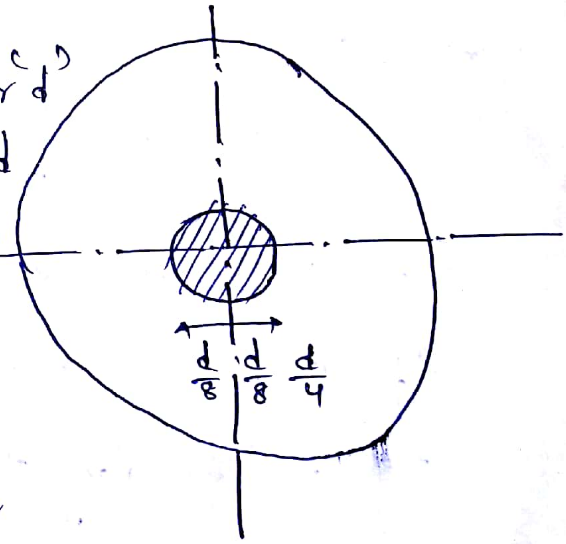
$$e \leq \frac{b}{6}$$

The above result shows that the eccentricity 'e' must be less than or equal to $\frac{b}{6}$. Hence the greatest eccentricity of the load is $\frac{b}{6}$ from the axis Y-Y.

Similarly, if the load had been eccentric with respect to the axis X-X, the condition that tensile stress will not occur is when the eccentricity of the load w.r.t this axis X-X does not exceed $\frac{d}{6}$. This figure ABCD within which the load may be applied anywhere so as not to produce tensile stress in any part of the entire rectangular section, is called Core or kernal of the section.

Middle quarter rule for circular sections [i.e., kernel of section]

Consider a circular section of diameter 'd' as shown in fig. Let this section is subjected to a load which is eccentric to the axis Y-Y



let P = Eccentric load

e = Eccentricity of the load

A = Area of the section $= \frac{\pi}{4} d^2$

Now direct stress

$$\sigma_d = \frac{P}{A} = \frac{P}{\left(\frac{\pi}{4} d^2\right)} = \frac{4P}{\pi d^2}$$

Moment, $M = Pe$

Bending stress (σ_b) is given by

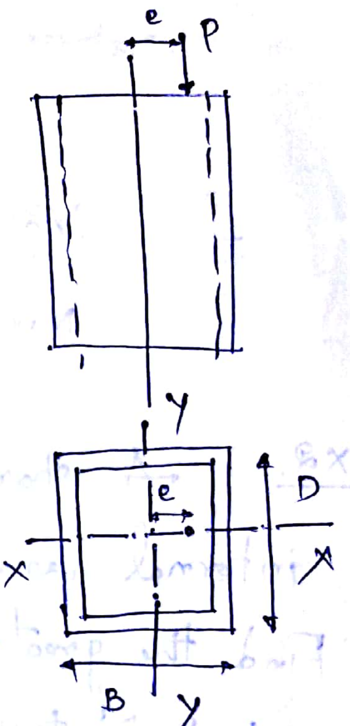
$$\frac{M}{I} = \frac{\sigma_b}{y} \quad (or) \quad \boxed{\sigma_b = \frac{My}{I}}$$

1. A hollow rectangular column of external depth 1m & external width 0.8m is 10cm thick. Calculate the max, & min stress in the section of the column if a vertical load of 200kN is acting with an eccentricity of 15cm ~~as shown in fig~~. about centre of width.

Sol: $B = 0.8m = 800mm$; $D = 1.0m = 1000mm$

$t = 10cm = 100mm$; $b = B - (2\times100)$

$= 800 - 200 = 600mm$



$d = D - 2t$

$= 1000 - (2\times100) = 800mm$

$\therefore$ ~~Area~~ $A = (B\times D) - (b\times d)$

$= (800\times1000) - (600\times800)$

$= 320000\ mm^3$

M.O.I is about y-y axis is given by

$$I = \frac{1000\times800^3}{12} - \frac{800\times600^3}{12}$$

$= 42.66\times10^9 - 14.4\times10^9$

$= 28.26\times10^9\ mm^4$

Eccentre load $P = 200kN = 200,000N$

$e = 15cm = 150mm$

Bending moment caused by couple

$M = P\times e = 200,000\times150$

$= 3\times10^6\ N.mm$

The Bending stress is given by

$$\frac{M}{I} = \frac{\sigma_b}{y}$$

$$\sigma_b = \frac{My}{I}$$

$$\sigma_b = \frac{M(\pm 400)}{I}$$

$$= \pm \frac{3\times10^6}{28.26\times10^9} \times 400$$

$$= \pm 0.4246 \text{ N/mm}^2$$

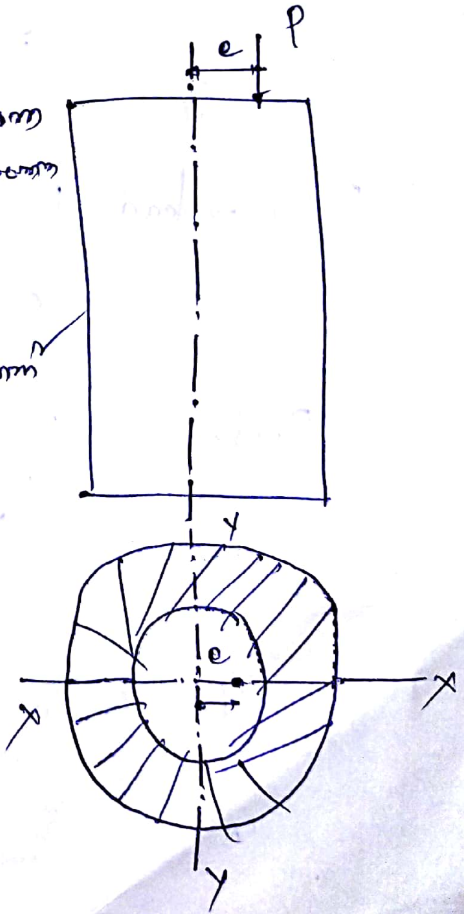
Direct stress is given by $\sigma_d = \frac{P}{A} = \frac{2\times10^5}{3.2\times10^5} = 0.625 \text{ N/mm}^2$

$\therefore$ Max. stress $= \sigma_d + \sigma_b = 1.0496 \text{ N/mm}^2$

Min. stress $= \sigma_d - \sigma_b = 0.625 - 0.4246$

$= 0.2004 \text{ N/mm}^2$

Ex 2: A short column of external diameter 40cm & internal diameter 20cm carries an eccentric load of 80kN. Find the greatest eccentricity which the load can have without producing tension on the c/s.



Sol: $D = 40\text{cm} = 0.40\text{m} = 400\text{mm}$, $d = 20\text{cm} = 0.20\text{m} = 200\text{mm}$

Area of c/s $= \frac{\pi}{4}(D^2 - d^2)$

$= \frac{\pi}{4}(400^2 - 200^2) = 30000\pi \text{ mm}^2$

~~Momen~~ M.O.I about Y-Y axis

$$I = \frac{\pi}{64}(D^4 - d^4)$$

$$= \frac{\pi}{64}(400^4 - 200^4)$$

$$= 3.75\times10^8 \pi \text{ mm}^4$$