Reinforced Cement Concrete (R.C.C)

Introduction :-

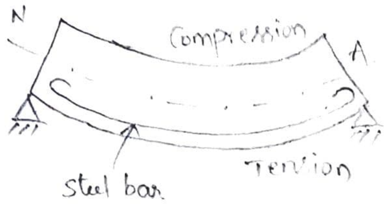
* The plain cement concrete (PCC) is very strong in compression but weak in tension.

* The tensile strength of concrete is about 10 to 15% of its compressive strength.

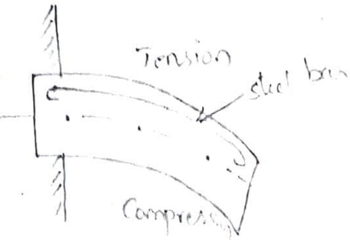
* If a beam is made of PCC, it has a very low load carrying capacity since its low tensile strength limits the overall strength.

* The PCC is reinforced by placing steel bars in tension zone of the concrete beam, so that the compressive bending stress is carried by concrete and tensile bending stress is entirely carried by steel reinforcing bars.

* Figure shows a simply supported beam subjected to transverse loads. The bottom portion below neutral axis is in tension and above neutral axis portion of beam is in compression. The steel bars are placed at a suitable depth below neutral axis to carry the tension.



* Figure shows a cantilever beam bending downwards. Since the tensile zone is above the neutral axis, steel bars are provided at some suitable height above the neutral axis.



* In both cases, the steel reinforcement is provided in the tension zone only, such beams are known as "singly reinforced beams".

* However if reinforcement is provided in compression zone also to carry the compressive stress. It is known as "Doubly reinforced beam".

* The zone of action of steel and concrete in a reinforced section is dependent on following factors:

(i) Bond between concrete and steel bars

(ii) obsense of corrosion of steel bars embedded into concrete

(iii) Partially equal thermal expansion of both concrete and steel.

Objective and Basic requirements of structural Design:-

* The objective of the structural Design is to plan a structure which meets the basic requirements of structural science and the user.

* The basic requirements of structural Design are

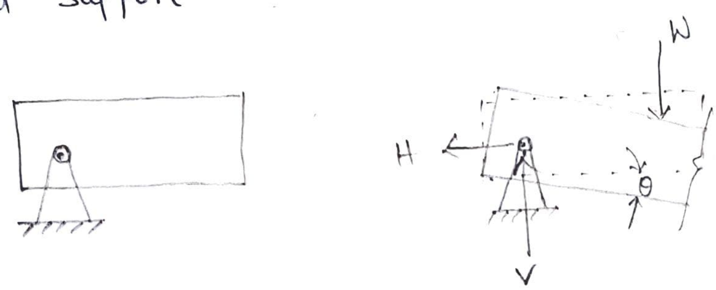
1. Safety
2. Serviceability
3. Durability
4. Economy
5. Easthetics
6. Feasibility, Practicability and Acceptability.

Types of Supports :-

1. Simple support (or) Roller support.

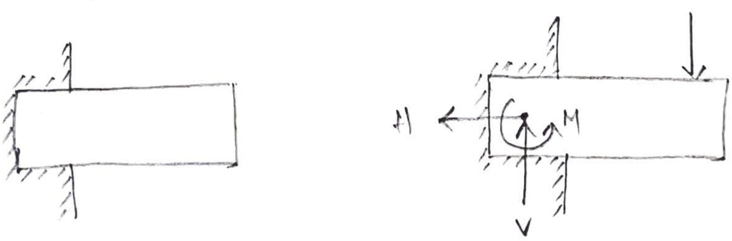
No. of unknown reactions = 1

2. Hinged support



No. of unknown reactions = 2

3. Fixed support



No. of unknown reactions = 3

Types of beams :-

1. Cantilever beam

2. Simply supported beam

3. Fixed beam

4. Continuous beam

5. over hanging beam

Single

Double

UNIT - I

Design Principles :-

* Since the inception of the Concept of Reinforced Concrete in the last twenties of 19^{th} century, the following design ~~phy~~ philosophies have been evolved for Design of R.C.C structures.

1. Working stress Method (WSM)
2. Ultimate load Method (ULM)
3. Limit State Method (LSM)

1. Working stress Method :-

* The working stress method is based on classical elastic theory and is basically developed for elastic materials.
* The working stress method is logically not applicable for Concrete structures.
* Due to non linear stress - strain relation ship, modulus of elasticity also varies, therefore constant value of modular ratio cannot be used
* It does not predict true margin of safety.
* Additional load carrying capacity in the Plastic region is not taken into account.
* It considers ultimate stress as the limit of Safety is a function of ultimate strain and not ultimate stress.
* Factor of Safety does not Predict true margin of safety.

* The effect of Creep and shrinkage of Concrete is totally ignored.

* Failure load Computed by this method in majority of the cases is less than that obtained by experimental results at Collapse.

* The permissible stress (working stress) in Concrete and steel is obtained by dividing the stress with a factor of safety (3 to 4 for Concrete and 1.8 to 2 for steel).

2. Ultimate Load Method (ULM) :-

* In this method, the structural element is proportioned to withstand the ultimate load which is obtained by enhancing the service load by some factor referred as "load factor" for giving a desired margin of safety.

Merits

* The stress block parameters are defined by actual stress-strain curve.

* The calculated failure load matches with experimental results.

* It utilizes the reserve strength in plastic region.

* It takes the ultimate strain as the failure criteria.

* The load causing collapse is taken as the limit of safety.

* The method allows selection of different load factors.

Demerits

* It totally neglects the serviceability criteria of deflection and cracking.

* The effect on deflection due to creep and shrinkage are neglected.

* The use of high strength deform bars affect in increase of deflection and crack width.

3. Limit state Method (LSM) :-

* The working stress method though ensures satisfactory Performance at working loads, is unrealistic and irrational at ultimate state and hence does not give true margin of Safety.

* Ultimate load method though Provides realistic assessment of degree of Safety with the actual behavior of the structure at or near the ultimate state, it does not guarantee the satisfactory performance of the structure at service loads.

* The limit state method ensures the safety at ultimate load and serviceability at working load.

* It Considers the actual behaviour of the structure during the entire loading upto collapse.

* The method based on statistical probabilistic Principles.

* The limit state method thus, makes a Judious Combination of the ultimate load method and working stress avoiding demerits of both.

Types of limit states :-

* Limit states are grouped into two major categories.

1. Limit state of collapse
2. Limit state of serviceability

1. Limit state of Collapse :-

* It relates to stability and ultimate strength of the structure
* The structural failure can be any of following types.

(i) Collapse of one or more members occuring as a result of force coming on the member exceeding its strength

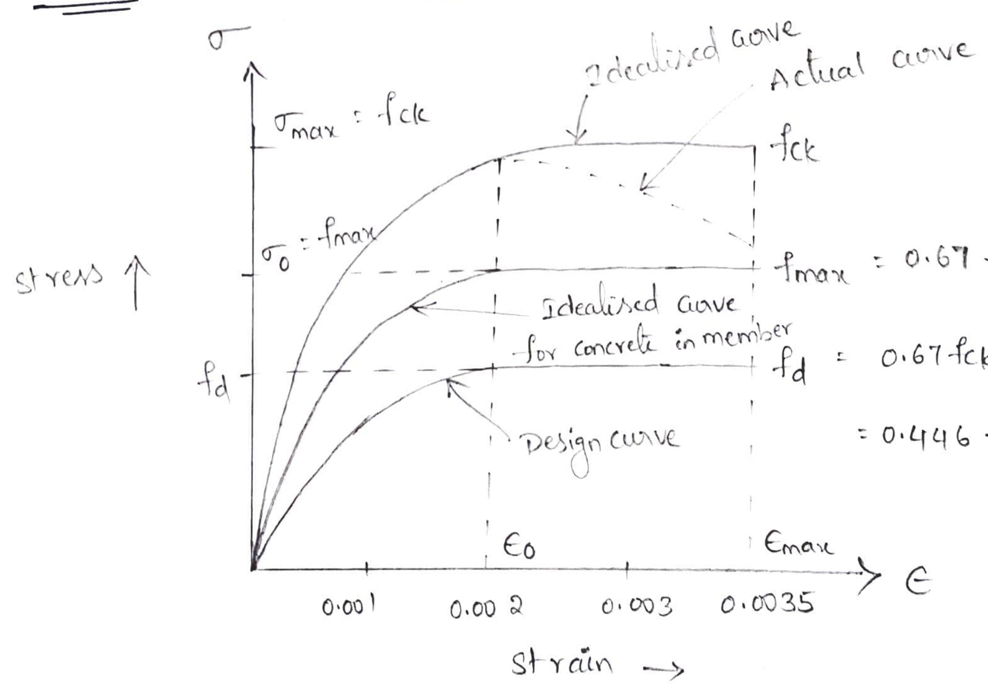
(a) Flexure (b) Compression (c) Shear (d) Torsion.

(ii) Displacement of structure due to lack of equilibrium b/n the external forces and the resisting reactions

(a) Deflection (b) Cracking (c) Overturning

(d) Sinking (e) vibration (f) Fire resistance (g) Durability.

Idealized Stress - Strain curve of Concrete :- (Fig 21, Pg 69)



The equation of the idealised stress - strain curve is given by

$$\sigma = \left[\frac{2\epsilon}{\epsilon_0} \left(\frac{\epsilon}{\epsilon_0}\right)^2\right] \sigma_0 \quad \text{for} \quad 0 < \epsilon < \epsilon_0$$

and $\sigma = \sigma_0$ for $\epsilon_0 \leq \epsilon \leq \epsilon_{cu}$

ϵ_0 = strain at which Parabolic Part ends = 0.002

* The effect of size and shape of Test specimen with that of structural member is taken into account.

The idealised stress-strain curve for concrete in member is obtained by, $\sigma_0 = \frac{f_{ck}}{1.5} = 0.67 f_{ck}$ ($\because$ Material safety factor = 1.5)

In addition, a partial safety factor of 1.5 is applied to get the design curve,

$\therefore$ The design stress, $f_d = \frac{0.67 f_{ck}}{1.5} = 0.446 f_{ck}$

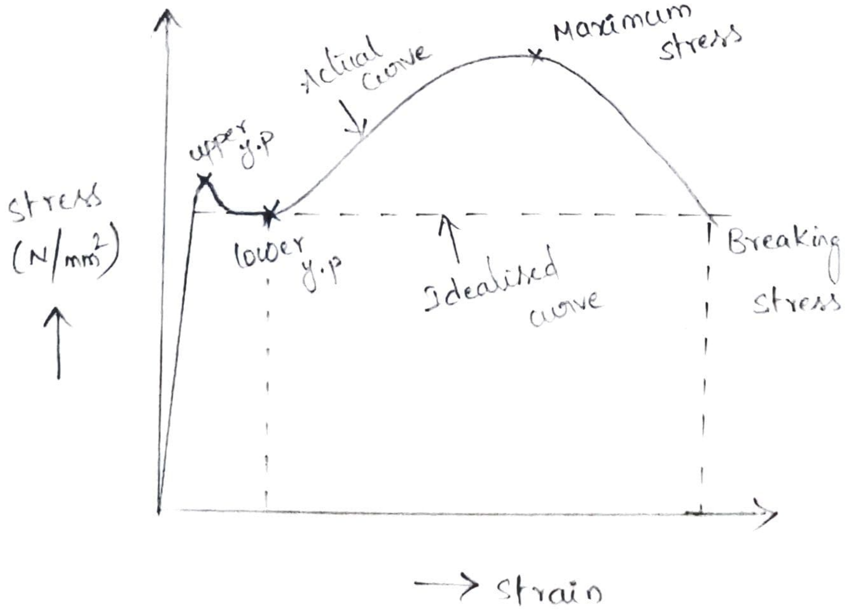
* Short-term modulus of elasticity of concrete, $E_c = 5000\sqrt{f_{ck}}$ N/mm²

f_{ck} = characteristic strength of concrete.

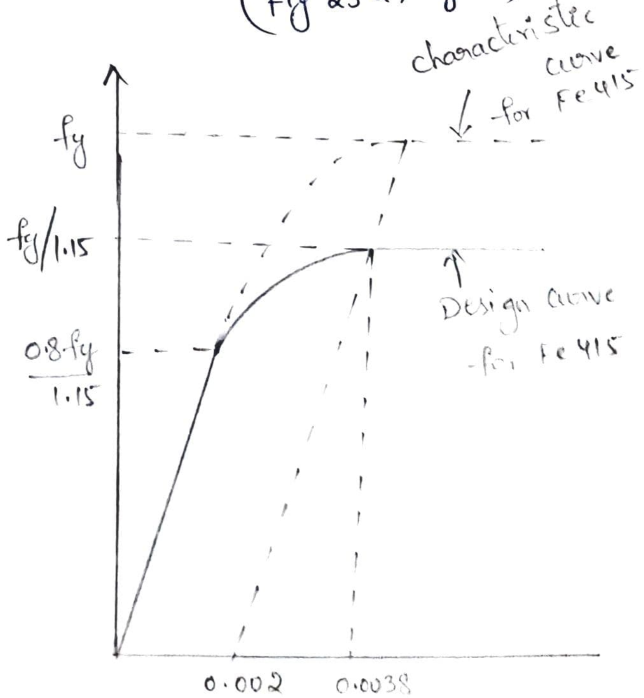
Stress - Strain Curve for steel :-

1. Mild steel (Fe 250)
2. HYSD (Fe 415 and Fe 500)
3. Corrosion resistant steel

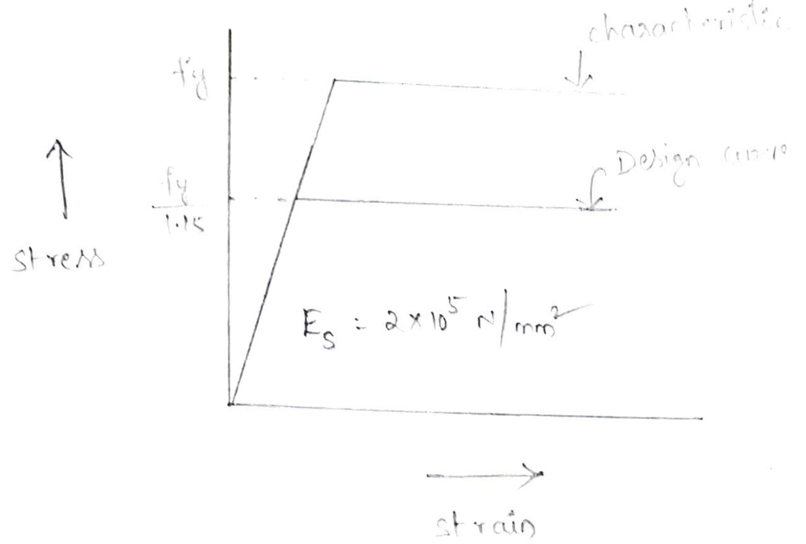
(Fig 23 a, Pg 70)



Stress - Strain curve for Mild steel



Stress - strain curve for Fe 415



(Fig 23 B , Pg 70)

* characteristic yield strength, $f_y = 250\ N/mm^2$ (Fe 250)

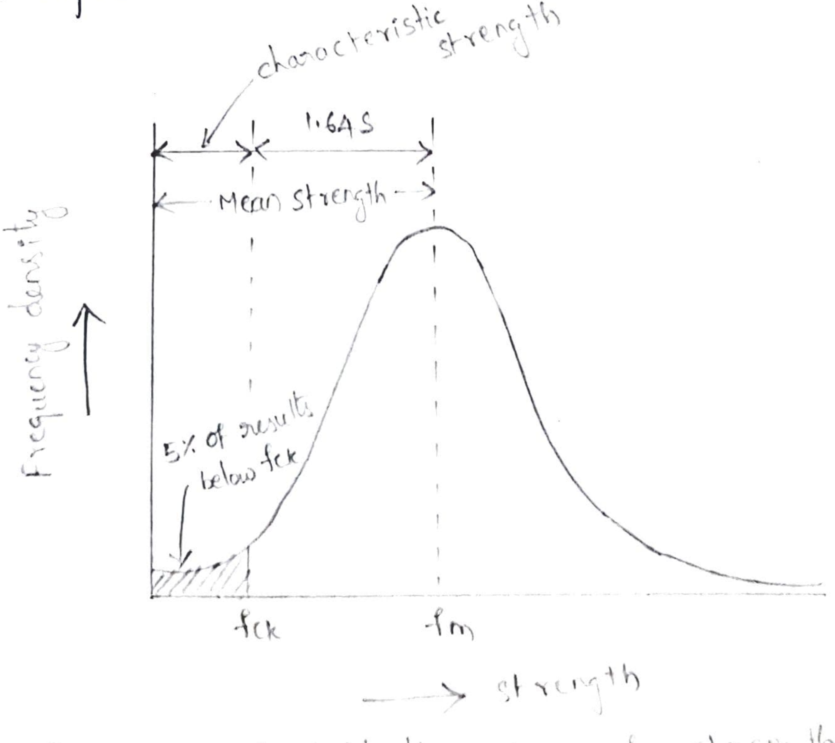
$f_y = 415\ N/mm^2$ (Fe 415)

$= 500\ N/mm^2$ (Fe 500)

* Design yield strength , $f_{yd} = 0.87\ f_y$

Characteristic Strength :-

* The characteristic strength of material is that value of material strength below which not more than a minimum acceptable percentage of test results are expected to fall.



Frequency distribution curve for strength

* Most of the design codes, including I.S. Code, prescribe this minimum acceptable percentage equal to 5% for reinforced concrete structures.

* It means that characteristic strength has 95% reliability or there is only 5% probability of actual strength being less than the characteristic strength.

Eg:- Let us take M20 grade concrete. $f_{ck} = 20 N/mm^2$. If 100 cubes are casted in M20 grade, 95 samples should be greater than $20 N/mm^2$ and remaining 5 samples less than $20 N/mm^2$ is acceptable.

characteristic load :- (cl 36.2, Pg 67, IS:456-2000)

* It is defined as that value of load which has 95% probability of not being exceeded during the life span of structure

$$f_K = f_m + 1.64 S$$

where, f_m = Mean strength

S = standard deviation

Types of loads :- (Pg 32, cl. 19.2, IS: 456-2000)

1. Dead load - IS: 875-1987 (Part-I)
2. Live load or Imposed load - IS: 875-1987 (Part-II)
3. Wind load - IS: 875-1987 (Part-III)
4. Earthquake load - IS: 1893-1984, IS: 1893-2002

Partial safety factors:-

* Since the safety of the structure depends on each of the two principle design factors (loads and material strength) which are not the functions of each other, hence two different safety factors, one for the load and one for the material strength are used instead of single safety factor.

* Because each of the two safety factors contribute partially to safety they are called as partial safety factors.

* It is denoted by 'γ_f'.

Partial safety factors for loads:- (cl 36.3.2, Pg 68)

* Partial safety factor for load is a load enhancing factor which when multiplied to characteristic load gives a load known as "Design load".

* Partial safety factor for the load is given by,

$$\gamma_f = F_d / F_k \Rightarrow F_d = \gamma_f F_k$$

where F_d = Design load

F_k = characteristic load.

* The partial safety factor for loads, which are simply known as load factor, depends upon

a. The type of load and the load combination

b. The type of limit state.

* The various load factors recommended by IS : 456 are (Table 18, Pg 68)

Load Combination	Limit state of Collapse			Limit state of Serviceability		
	DL	LL	WL	DL	LL	WL
DL + LL	1.5	1.5	-	1.0	1.0	-
DL + WL	1.5 or 0.9	-	1.5	1.0	-	1.0
DL + LL + WL	1.2	1.2	1.2	1.0	0.8	0.8

* If Earth quake load occurs use the factors similar to wind load.

Partial safety factors for material strength :- (cl 36.3.1, Pg 68)

* Partial safety factor for material strength is a strength reduction factor when applied to the characteristic strength gives a strength known as "Design strength".

* It is denoted as "γ_m".

$$\gamma_m = f_{ck}/f_d \Rightarrow f_d = \frac{f_{ck}}{\gamma_m}$$

where, f_d = Design strength

f_{ck} = characteristic strength.

* Partial safety factor γ_m for material strength recommended by IS : 456 are given as

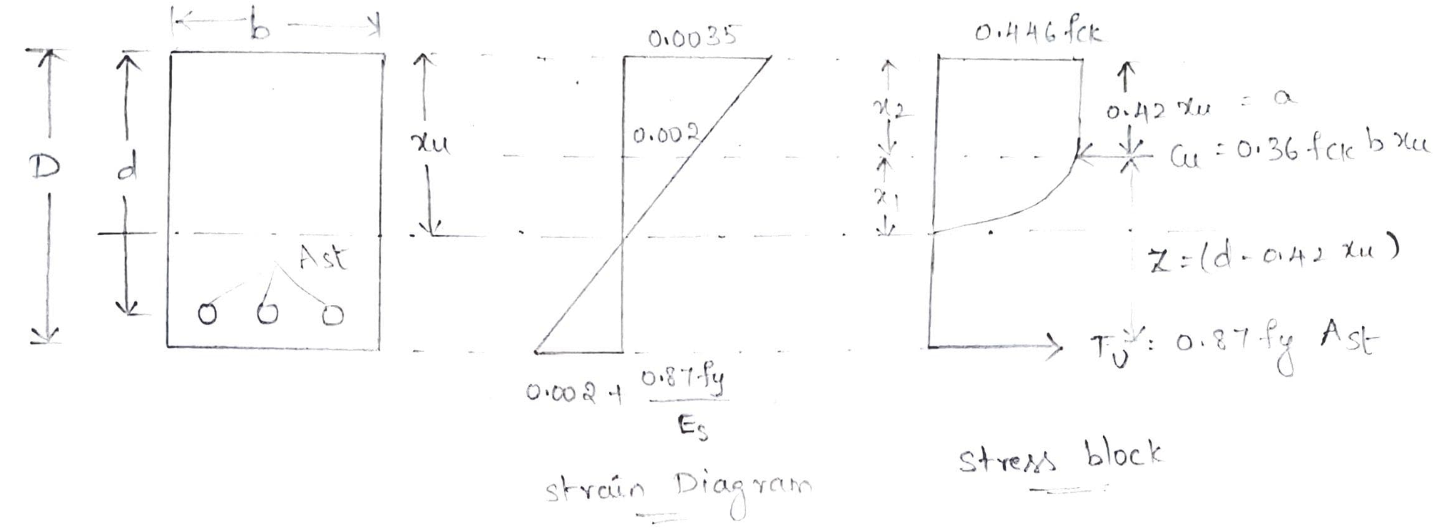
a. For Concrete - 1.5

b. For steel - 1.15

Differences b/n working stress method and Limit state Method :-

working stress method	Limit state method.
1. General factors of safety are used.	1. Partial safety factors are used.
2. The stress - strain relation of steel and concrete under working load is a straight line	2. The stress - strain curve of steel and concrete under limit state are non - linear.
3. working loads are used	3. Design load are used which obtained by multiplying the characteristic load by Partial safety factor.
4. Deflection and cracking are likely to exceed the permissible limits.	4. Deflection and cracking are checked in this method.

Stress - Block Parameters :- (Fig 22, Pg 69)



* Maximum compressive stress in Concrete in the outermost Compression fibre by considering partial safety factor $= \frac{0.67\, f_{ck}}{1.5}$

$$= 0.446\, f_{ck}$$

* Depth of Parabolic portion of the stress block can be obtained from the strain diagram.

From similar triangles, $\frac{x_1}{0.002} = \frac{x_u}{0.0035}$

$$\Rightarrow x_1 = \frac{4}{7} x_u$$

Similarly, $\frac{x_2}{(0.0035 - 0.002)} = \frac{x_u}{0.0035}$

$$\Rightarrow x_2 = \frac{3}{7} x_u$$

* Force of compression in the parabolic stress block

C_1 = Area of parabolic stress block x width

$$= (2/3 \times 0.446\, f_{ck} \times x_1) \times b$$

$$= 2/3 \times 0.446\, f_{ck} \times \frac{4}{7} x_u \times b = 0.17\, f_{ck}\, b\, x_u$$

* Force of Compression in the rectangular stress block,

C_2 = Area of rectangular stress block × width

$= 0.446\, f_{ck}\, x_2 \times b$

$= 0.446\, f_{ck} \times \frac{3}{7} x_u \times b = 0.191\, f_{ck}\, b x_u$

* Total Compressive force, $C_u = C_1 + C_2$

$= 0.17\, f_{ck}\, b x_u + 0.191\, f_{ck}\, b x_u$

$= 0.361\, f_{ck}\, b x_u$

* Let 'a' be the distance of the line of action of force of compression of stress block from the extreme compression fibre.

$$a = \left[C_1 \left(x_2 + \frac{3}{8} x_1\right) + C_2 \left(\frac{x_2}{2}\right)\right] \Big/ (C_1 + C_2)$$

$$= \frac{0.17\, f_{ck}\, b x_u \left(x_2 + \frac{3}{8} x_1\right) + 0.191\, f_{ck}\, b x_u \left(\frac{x_2}{2}\right)}{0.361\, f_{ck}\, b x_u}$$

$$= \frac{0.17\, x_2 + 0.063\, x_1 + 0.095\, x_2}{0.361}$$

$$= \frac{0.063 \times \frac{4}{7} x_u + 0.265 \left(\frac{3}{7} x_u\right)}{0.361} = \frac{0.149\, x_u}{0.361} = 0.414\, x_u$$

$$\simeq 0.42\, x_u$$

* The depth of neutral axis is obtained by considering the equilibrium of forces.

$C_u = T_u$

Resultant force in Compression, $C_u = 0.36\, f_{ck}\, b x_u$

Resultant force in Tension, $T_u = 0.87\, f_y\, A_{st}$

$0.36\, f_{ck}\, b x_u = 0.87\, f_y\, A_{st}$

$$\boxed{x_u = \frac{0.87\, f_y\, A_{st}}{0.36\, f_{ck}\, b}}$$ (Pg 96, Annex G 1.1.a)

$$\frac{x_u}{d} = \frac{0.87\, f_y A_{st}}{0.36\, f_{ck}\, bd}$$

f_y	$\frac{x_{umax}}{d}$
250	0.53
415	0.48
500	0.46

(Pg 70)

From similar triangles of strain diagram

$$\frac{x_{umax}}{0.0035} = \frac{d - x_{umax}}{0.87\frac{f_y}{E_s} + 0.002}$$

$$\frac{x_{umax}}{d} = \frac{700}{1100 + 0.87 f_y}$$

$(E_s = 2\times10^5\ MPa)$

* Lever arm, $z = d - 0.42\, x_u$
(distance b/n point of application of tensile force and compressive force)

* Moment of resistance with respect to concrete = Compressive force × lever arm

$$= 0.36\, f_{ck}\, b\, x_u \times z$$

* Moment of resistance with respect to steel = Tensile force × lever arm

$$= 0.87\, f_y\, A_{st} \times z$$

* If $\frac{x_u}{d}$ equal to limiting value, $\frac{x_{umax}}{d}$ – Balanced section

If $\frac{x_u}{d}$ less than the limiting value – Under reinforced section

If $\frac{x_u}{d}$ greater than the limiting value – Over reinforced section

Balanced Section :-

* The strain in steel and Concrete reach their maximum values simultaneously.

* In balanced section $\varepsilon_c = \varepsilon_{cu}$ and $\varepsilon_s = \varepsilon_{sy}$

* The percentage of steel in this section is known as critical or limiting steel percentage ($P_{t\,lim}$). The depth of neutral axis $x_u = x_{umax}$

Under reinforced section :-

* The strain in steel reaches its yield strain before Concrete reaches its ultimate value.

* Failure takes place by yielding of steel reinforcement.

* The percentage of steel (P_t) is less than critical or limiting percentage.

The actual neutral axis is above balanced neutral axis and $x_u < x_{umax}$.

Over reinforced section :-

* The strain in Concrete reaches first before steel reinforcement reaches its yield strain due to higher percentage of steel

* Failure takes place due to crushing of Concrete.

* It should be noted that compression failure is sudden and therefore not desirable.

* The Code recommends that if x_u/d found to be greater than limiting value, the beam should be redesigned.

Limiting Moment of resistance :- (Balanced section). (Pg 96, Annex- G.1.1.c)

* strain in both Concrete and steel reaches at the same time, so stress block is fully developed.

* The moment of resistance M_u reaches its limiting value $M_{u,lim}$ which can be determined by taking moment of compressive force about the centre of tensile reinforcement.

$$M_{u,lim} = 0.36 f_{ck}\, b\, x_{umax} (d - 0.42\, x_{umax})$$

$$= 0.36 f_{ck} \frac{x_{umax}}{d} \left(1 - 0.42 \frac{x_{umax}}{d}\right) b d^2$$

For Fe 250, $\frac{x_{umax}}{d} = 0.53$

$$M_{u,lim} = 0.36 f_{ck} \times 0.53 (1 - 0.42 \times 0.53) bd^2$$

$$= 0.148 f_{ck}\, bd^2$$

For M_{15}, $M_{u,lim} = 2.225\, bd^2$

* The limiting moment of resistance for different grades of steel and concrete are

Grade of Concrete	Fe 250	Fe 415	Fe 500
1. General	$0.148 f_{ck} bd^2$	$0.138 f_{ck} bd^2$	$0.133 f_{ck} bd^2$
2. M_{15}	$2.22 bd^2$	$2.07 bd^2$	$1.995 bd^2$
M_{20}	$2.96 bd^2$	$2.76 bd^2$	$2.66 bd^2$
M_{25}	$3.70 bd^2$	$3.45 bd^2$	$3.33 bd^2$
M_{30}	$4.44 bd^2$	$4.14 bd^2$	$3.99 bd^2$

(Grade of steel: Fe 250, Fe 415, Fe 500)

Moment of resistance of under reinforced section :- (Pg 96, Annex G.1.1.b)

* The strain in steel at the limit state of collapse will be more than $\frac{0.87 f_y}{E_s}$ + 0.002 and the stress in steel will fully reach its maximum value of $0.87 f_y$.

* The moment of resistance is found based on maximum stress in steel.

M_u = Tensile force × lever arm

$$= 0.87 f_y A_{st} (d - 0.42 x_u) = 0.87 f_y A_{st} d \left(1 - 0.42 \frac{x_u}{d}\right)$$

substituting value of $\frac{x_u}{d}$,

$$M_u = 0.87 f_y A_{st} d \left(1 - 0.42 \frac{0.87 f_y A_{st}}{0.36 f_{ck} bd}\right)$$

$$= 0.87 f_y A_{st} d \left(1 - 1.005 \frac{f_y A_{st}}{f_{ck} bd}\right)$$

$$\boxed{M_u = 0.87 f_y A_{st} d \left[1 - \frac{f_y A_{st}}{f_{ck} bd}\right]}$$

Clear Cover :-

1. For slabs - 15 mm
2. For beams - 25 mm
3. For columns - 40 mm
4. For Footings - 50 mm , at sea shore - 75 mm.

Minimum Reinforcement :- (Pg 46, cl 26.5.1.1) (For under reinforced section)

* Minimum area of Tension reinforcement should not be less than the following.

$$P_t = \frac{A_{st}}{bd}\% = \frac{0.85}{f_y}\%$$

For Fe 250, $P_{t\,lim} = 0.34\%$
For Fe 415, $P_{t\,lim} = 0.205\%$
For Fe 500, $P_{t\,lim} = 0.170\%$

Maximum Reinforcement :- (Pg 46, cl 26.5.1.1)

* The maximum area of tension reinforcement shall not exceed 4% of gross cross sectional area (i.e 0.04 bD) to avoid difficulty in placing and compacting concrete properly in the form work.

Effective span :- (Pg 34, cl 22.2)

* For a simply supported beam or slab, the effective span of a member that is not built integrally with its supports is least of

(i) L_{eff} = clear span + Effective depth

(ii) L_{eff} = Centre to centre distance of supports.

Types of Problems :-

1. Analysis problems – Determination of moment of resistance of given section
2. Design Problems – Design a given section to resist a given ultimate design moment

Procedure for finding Moment of Resistance :-

* The moment of resistance of a singly rein forced rectangular section is determined as follows.

a. Determine the depth of neutral axis, $\frac{x_u}{d} = \frac{0.87 f_y A_{st}}{0.36 f_{ck} b d}$

b. If the value of x_u/d is equal to limiting value,

$$M_{u\,lim} = 0.36 f_{ck} \frac{x_{umax}}{d}\left(1 - 0.42 \frac{x_{umax}}{d}\right) b d^2$$

c. If the value of x_u/d is less than the limiting value,

$$M_u = 0.87 f_y A_{st}\, d \left(1 - \frac{f_y A_{st}}{f_{ck} b d}\right)$$

d. If x_u/d is greater than limiting value, the section is over-reinforced

The moment of resistance of such section is limited to $M_{u,lim}$ and the section is designed as doubly-reinforced section.

Procedure for design of singly reinforced section :-

1. Determine the limiting moment of resistance,

$$M_{u\,Lim} = 0.36 f_{ck} \frac{x_{umax}}{d}\left(1 - 0.42 \frac{x_{umax}}{d}\right) b d^2$$

2. If depth of section is unknown, Determine 'd' by equating $M_u = M_{u\,lim}$

where M_u = ultimate moment or Design moment.

3. If $M_u = M_{ulim}$, The area of steel reinforcement is found from

$$M_u = M_{ulim} = 0.87 f_y A_{st} (d - 0.42 x_{umax})$$

If $M_u < M_{ulim}$, The area of steel reinforcement is found from

$$M_u = 0.87 f_y A_{st} d \left(1 - \frac{f_y A_{st}}{f_{ck} bd}\right)$$

If $M_u > M_{u,Lim}$, the section is designed as doubly reinforced section.

Steel requirement at limiting Condition :-

$$C = T$$

$$0.36\, f_{ck}\, b\, x_{umax} = 0.87\, f_y\, A_{st\,lim}$$

$$A_{st\,lim} = \frac{0.36\, f_{ck}\, b\, x_{umax}}{0.87\, f_y}$$

$$= 0.414\, \frac{f_{ck}}{f_y}\, \frac{x_{umax}}{d}\, bd$$

For $Fe\,250$ & M_{20}, $p_{t\,lim} = 1.755\%$

For M_{20}, $Fe\,415$, $p_{t\,lim} = 0.958\%$

For M_{20}, $Fe\,500$, $p_{t\,lim} = 0.762\%$

Limiting percentage of steel, $p_{t\,lim} = \frac{A_{st\,lim}}{bd} \times 100 = \boxed{41.4\, \frac{f_{ck}}{f_y}\, \frac{x_{umax}}{d}}$

For an under reinforced section :-

$$M_u = 0.87\, f_y\, A_{st}\, d \left(1 - \frac{f_y\, A_{st}}{f_{ck}\, bd}\right)$$

$$p_t = \frac{A_{st}}{bd} \times 100$$

$$\therefore\ M_u = 0.87\, f_y\, \frac{p_t}{100}\, bd^2 \left(1 - \frac{p_t}{100}\, \frac{f_y}{f_{ck}}\right)$$

$$\frac{M_u}{0.87\, f_y\, bd^2} = \frac{p_t}{100} - \frac{f_y}{f_{ck}} \left(\frac{p_t}{100}\right)^2$$

$$\frac{f_y}{f_{ck}} \left(\frac{p_t}{100}\right)^2 - \frac{p_t}{100} + \frac{M_u}{0.87\, f_y\, bd^2} = 0$$

Solving the above equation,

$$\frac{p_t}{100} = \frac{1 - \sqrt{1 - \frac{4.6}{f_{ck}}\, \frac{M_u}{bd^2}}}{2\, \frac{f_y}{f_{ck}}}$$

$$\boxed{p_t = 50 \left[\frac{1 - \sqrt{1 - \frac{4.6}{f_{ck}}\, \frac{M_u}{bd^2}}}{\frac{f_y}{f_{ck}}}\right]}$$

Basic Assumptions for theory of bending of R.C members at limit state of Collapse :-

(i) A normal section plane before bending remain plane after bending right upto collapse.

(ii) The ultimate state of collapse is said to have reached in flexure when the maximum compressive strain in concrete in the outermost fibre reaches the ultimate crushing strain ε_{cu} (i.e 0.0035).

(iii) Concrete under tension is ignored. Tension is assumed to be carried entirely by reinforcement.

(iv) The distribution of compressive stress in concrete across the section is defined by an idealised stress - strain curve of concrete.

(v) Perfect bond exists b/n steel and concrete right upto collapse.

(vi) The design stress in steel reinforcement is obtained from the strain at reinforcement level using idealised stress - strain curve for the type of reinforcement used.

(vii) According to I.S. Code, the maximum strain in steel in tension shall not be less than $0.002 + \dfrac{0.87 f_y}{E_s}$ at collapse

Analysis Problems:-

1) A singly reinforced beam 200 mm wide is 400 mm deep to the centre of the tensile reinforcement. Determine the limiting moment of resistance of beam section, and also the limiting area of reinforcement. Use M20 concrete and Fe 250 steel.

Sol:- Given data:-

$b = 200\,mm$

$d = 400\,mm$

$f_{ck} = 20\,N/mm^2, \quad f_y = \overset{250}{\cancel{415}}\,N/mm^2$

Limiting moment of resistance, $M_{u\,lim} = 0.\overset{14}{\cancel{13}}8\, f_{ck}\, b d^2$

$= 0.\overset{14}{\cancel{13}}8 \times 20 \times 200 \times 400^2$

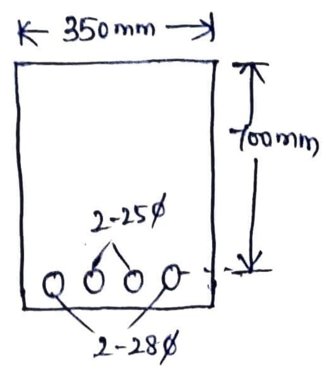
$= 95.36\,KNm.$

$$P_{t\,lim} = 41.4\, \frac{f_{ck}}{f_y} \cdot \frac{x_{u\,max}}{d} = 41.4 \times \frac{20}{250} \times 0.53$$

$= 1.755\,\%.$

$$A_{st\,lim} = \frac{1.755}{100} \times 200 \times 400 = 1404\,mm^2$$

2) A RC rectangular section having a breadth of 350 mm is reinforced with two bars of 25 mm diameter and two bars of 28 mm diameter at an effective depth of 700 mm adopting M20 grade concrete and Fe 415 steel. Determine the ultimate moment of resistance of section.



Sol:- Given data:-

$b = 350\,mm, \quad d = 700\,mm$

$A_{st} = 2 \times \pi/4 \times 25^2 + 2 \times \pi/4 \times 28^2 = 2213\,mm^2$

$f_{ck} = 20\,N/mm^2, \quad f_y = 415\,N/mm^2$

$$\frac{x_u}{d} = \frac{0.87\, f_y\, A_{st}}{0.36\, f_{ck}\, b d} = \frac{0.87 \times 415 \times 2213}{0.36 \times 20 \times 350 \times 700} = 0.452$$

For M_{20} & Fe415 , $\frac{x_{umax}}{d} = 0.48$

$\frac{x_u}{d} < \frac{x_{umax}}{d}$ — Section is under reinforced.

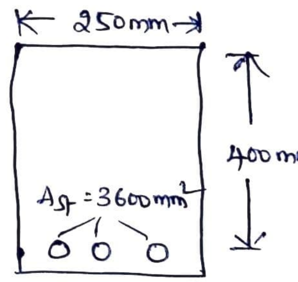
$$M_u = 0.87 f_y A_{st} d \left[1 - \frac{f_y A_{st}}{f_{ck} bd}\right]$$

$$= 0.87 \times 415 \times 2213 \times 700 \left[1 - \frac{415 \times 2213}{20 \times 350 \times 700}\right]$$

$$= \underline{454.4} \text{ KNm}$$

3) A singly reinforced beam having width of 250 mm reinforced with steel bars of area 3600 mm^2 at an effective depth of 400mm. If M_{20} grade concrete and Fe415 steel are used, compute the ultimate flexural strength of section.

Sol:- Given data :-



b = 250 mm , d = 400 mm

A_{st} = 3600 mm^2 , f_{ck} = 20 N/mm^2 , f_y = 415 N/mm^2

Depth of neutral axis ,

$$\frac{x_u}{d} = \frac{0.87 f_y A_{st}}{0.36 f_{ck} bd} = \frac{0.87 \times 415 \times 3600}{0.36 \times 20 \times 250 \times 400} = 1.805$$

For M_{20} & Fe415 , $\frac{x_{umax}}{d} = 0.48$

$\frac{x_u}{d} > \frac{x_{umax}}{d}$ — It is a over reinforced section.

For Fe415 , $M_{ulim} = 0.138 f_{ck} bd^2$

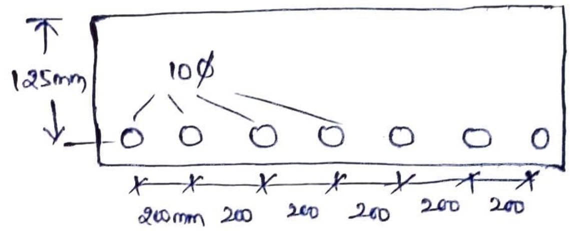
$$= 0.138 \times 20 \times 250 \times 400^2$$

$$= \underline{110.4} \text{ KNm}$$

4) A RC slab 150 mm thick is reinforced with 10 mm diameter bars at 200 mm c/c located at an effective depth of 125 mm. Use M_{20} and Fe_{415}. Find the ultimate moment of resistance.

Sol:- Given data:-

$a_{st} = \pi/4 \times 10^2 = 78.55\ mm^2$



$d = 125mm$, $f_{ck} = 20 N/mm^2$

$f_y = 415\ N/mm^2$

$$\text{Spacing} = \frac{a_{st}}{A_{st}} \times 1000 \qquad (\text{Assume } b = 1000\ mm)$$

$$A_{st} = \frac{a_{st}}{\text{spacing}} \times 1000 = \frac{78.55}{200} \times 1000 = 392.75\ mm^2 \simeq 393\ mm^2$$

$$\frac{x_u}{d} = \frac{0.87 f_y A_{st}}{0.36 f_{ck} bd} = \frac{0.87 \times 415 \times 393}{0.36 \times 20 \times 1000 \times 125} = 0.157$$

For Fe 415 & M_{20}, $\frac{x_{umax}}{d} = 0.48$

$\frac{x_u}{d} < \frac{x_{umax}}{d}$ - It is a under reinforced section

Moment of resistance, $M_u = 0.87 f_y A_{st} d \left[1 - \frac{f_y A_{st}}{f_{ck} bd}\right]$

$$= 0.87 \times 415 \times 393 \times 125 \left[1 - \frac{415 \times 393}{20 \times 1000 \times 125}\right]$$

$$= 16.58\ kNm$$

Design Problems:-

1) Determine the area of steel required for a singly reinforced section having a breadth of 300 mm and effective depth of 675 mm to support a factored moment of 185 KNm. Adopt M_{20} and Fe_{415} grade Concrete and steel.

Sol:- Given data:-

$b = 300mm$, $d = 675$ mm

Factored moment = ultimate moment = M_u = 185 KN m

$f_{ck} = 20 N/mm^2$, $f_y = 415 N/mm^2$

Limiting moment of resistance, $M_{u\,lim} = 0.138 f_{ck} b d^2$

$= 0.138 \times 20 \times 300 \times 675^2$

$= 377.2$ KNm.

[$M_u < M_{u\,lim}$ → under reinforced section
$M_u > M_{u\,lim}$ → over reinforced section]

$M_u < M_{u\,lim}$ - It is a under reinforced section.

$$M_u = 0.87 f_y A_{st} d \left[1 - \frac{f_y A_{st}}{f_{ck} b d}\right]$$

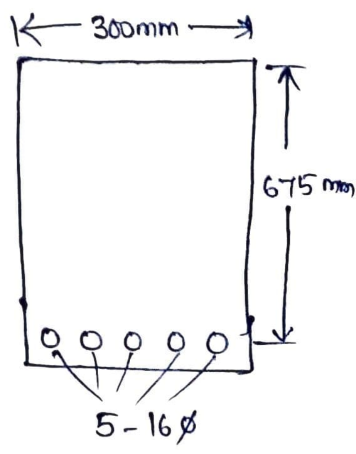
$$185 \times 10^6 = 0.87 \times 415 A_{st} \times 675 \left[1 - \frac{415 \times A_{st}}{20 \times 300 \times 675}\right]$$

$$24.97 A_{st}^2 - 243708.75 A_{st} + 185 \times 10^6 = 0$$

$$A_{st} = 829.6 = 830 \text{ mm}^2$$

Assume diameter of steel bar, d = 16 mm

$$\text{No. of bars} = \frac{A_{st}}{a_{st}} = \frac{830}{\pi/4 \times 16^2} = 4.12 \simeq 5$$



So provide 5 bars of 16 mm diameter.

Area of steel provided $= \pi/4 \times 5 \times 16^2 = 1005\ mm^2$

2) Determine the minimum effective depth required and the area of reinforcements for the rectangular beam having a width of 300 mm to resist the ultimate moment of 200 KNm using M20 grade concrete and Fe 415 steel.

Sol:- Given data:-

$b = 300mm$, $M_u = 200\ KN\ m$.

$f_{ck} = 20\ N/mm^2$, $f_y = 415\ N/mm^2$

Let $M_u = M_{u\,lim}$

$$200 \times 10^6 = 0.138\ f_{ck}\ bd^2 = 0.138 \times 20 \times 300 \times d^2$$

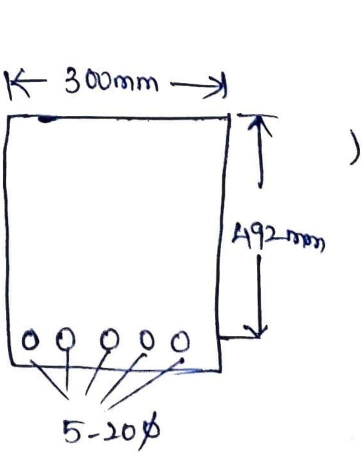
$$\Rightarrow d^2 = 241545.89 \Rightarrow d = 491.4mm \simeq 492\ mm$$

$$M_u = 0.87\ f_y\ A_{st}\ d\left[1 - \frac{f_y\ A_{st}}{f_{ck}\ bd}\right]$$

$$200 \times 10^6 = 0.87 \times 415\ A_{st} \times 492\left[1 - \frac{415 \times A_{st}}{20 \times 300 \times 492}\right]$$

$$24.97\ A_{st}^2 - 177636.6\ A_{st} + 200 \times 10^6 = 0$$

$$A_{st} = 1403\ mm^2$$



Assume 20 mm dia steel bars.

$$\text{No. of bars} = \frac{1403}{\pi/4 \times 20^2} = 4.46 \simeq 5$$

So provide 5 bars of 20 mm diameters

Area of steel provided $= 5 \times \pi/4 \times 20^2 = 1571\ mm^2$

3) Design a suitable rectangular simply supported beam over a span of 6m carries a live load of 20 KN/m. Design the beam for flexure using M15 Concrete and Mild steel reinforcement.

Soln Given data:-

span, $l = 6m$

live load, $w = 20$ KN/m

$f_{ck} = 15 N/mm^2$, $f_y = 250 N/mm^2$

Let us assume a section 250 x 500 mm.

Self weight of beam = $0.25 \times 0.5 \times 25 = 3.125$ KN/m

Total load = $20 + 3.125 = 23.125$ KN/m

As per IS:456-2000, load factor for dead load and live load is 1.5.

Ultimate load, $w_u = 23.125 \times 1.5 = 34.687$ KN/m

Ultimate moment, $M_u = \frac{w_u l^2}{8} = \frac{34.687 \times 6000^2}{8} = 156.093$ kNm.

$M_{u\,lim} = 0.148 f_{ck} b d^2$ (for M15 & Fe 250)

$156.093 \times 10^6 = 0.148 \times 15 \times 250 \times d^2$

$d = 530.33$ mm.

Assume the diameter of bar = 16mm ϕ and clear cover = 25mm.

Overall depth, $D = d + \frac{\phi}{2} + \text{Cover}$

$= 530.33 + 8 + 25$

$= 563.33 \simeq 564$ mm.

Drequired > Dprovided → D = 600 mm.

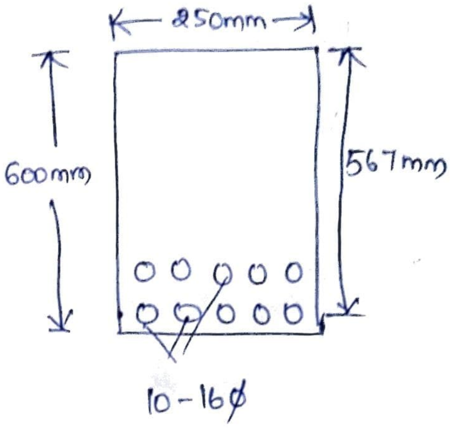
Effective depth, $d = 600 - 8 - 25 = 567$ mm.

Limiting percentage of reinforcement,

$$P_{t\,lim} = 41.4 \frac{f_{ck}}{f_y} \times \frac{x_{umax}}{d}$$

$$= 41.4 \times \frac{15}{250} \times 0.53$$

$$= 1.315\,\%$$



$$A_{st\,lim} = \frac{P_{t\,lim} \times bd}{100} = \frac{1.315 \times 250 \times 567}{100}$$

$$= 1869 \text{ mm}^2$$

$$\text{No. of bars} = \frac{A_{st}}{a_{st}} = \frac{1869}{\pi/4 \times 16^2} = 9.294 \approx 10$$

∴ Provide 10 bars of 16 mm diameter.

Area of steel provided $= 10 \times \pi/4 \times 16^2 = 2010.88 \text{ mm}^2$

Doubly Reinforced Beams:-

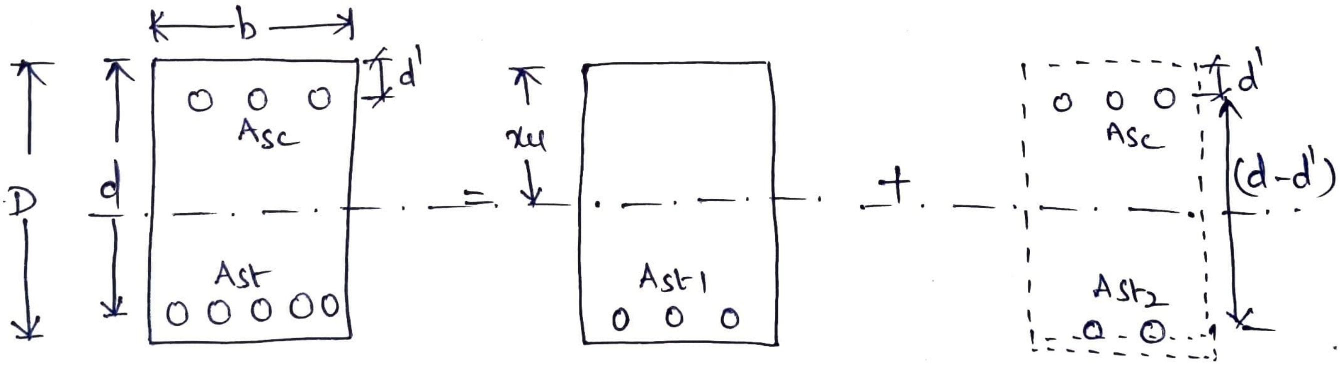
* A section with steel on tension as well as on Compression side is known as a "Doubly reinforced section".

* A Doubly reinforced section is normally required under the following circumstances

(i) Sectional dimensions are restricted due to requirements of head room, appearance etc.. and the strength of given singly reinforced section is inadequate.

(ii) Compression steel ~~someti~~ is provided sometimes to reduce the deflection i.e to increase stiffness and also to increase the rotation capacity.

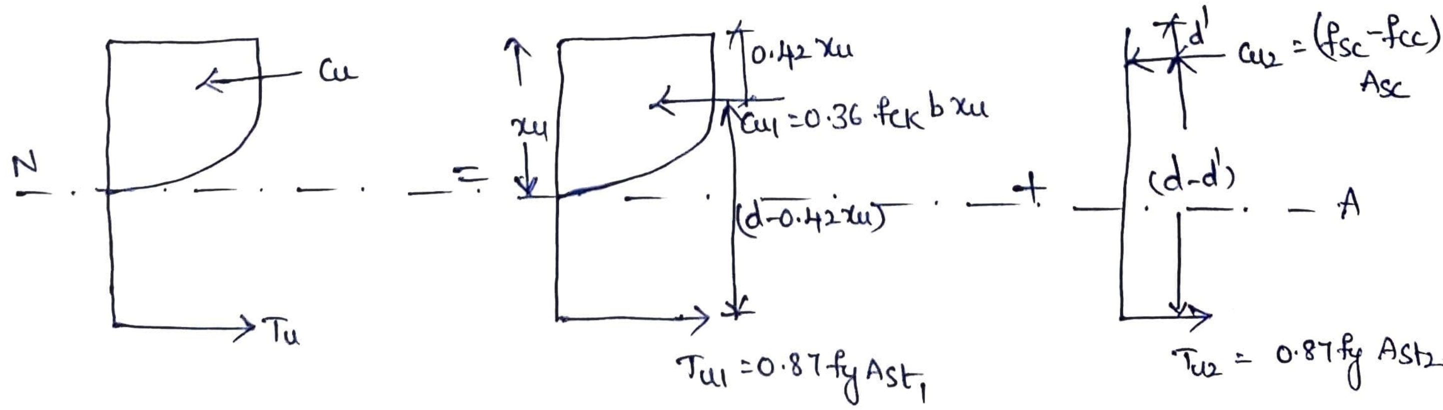
Properties of Doubly reinforced rectangular sections :-



section subjected to moment Mu

Sec I - Resisting moment Mu1

sect-II - Balanced resisting moment Mu2



where f_{sc} = stress in Compression steel

f_{cc} = Compression stress in Concrete at the level of Compressive steel

A_{sc} = Area of Compression reinforcement

* A doubly reinforced beam is assumed to be made up of two ~~beam~~ sections 1 and 2 as shown in Figure.

* Section - I :-

which is a singly reinforced section with Concrete resisting Compression C_{u1} balanced by Tensile force T_{u1} Provided by tension steel A_{st_1}. This section is assumed to resist a part of moment 'M_{u1}' out of total moment 'M_u'

* Section - II :-

An imaginary section (shown dotted) Consisting of Compression steel providing additional Compressive force C_{u2} which is balanced by tensile force T_{u2} Provided by tensile steel A_{st_2}. This Section is assumed to resist balance moment $M_{u2} = M_u - M_{u1}$

* while deriving the properties, the following additional assumptions made for theory of failure for a singly reinforced section.

(i) stress - strain curve for steel is same in tension and Compression.

(ii) Yield stress "f_y" in steel and modulus of elasticity of steel is same in tension and Compression.

Depth of neutral axis :-

* with no external force acting on the section, by equilibrium of longitudinal internal forces.

Total Compression = Total Tension

$$C_u = T_u \Rightarrow C_{u1} + C_{u2} = T_{u1} + T_{u2}$$

$$0.36\, f_{ck}\, b\, x_u + (f_{sc} - f_{cc}) A_{sc} = 0.87 f_y A_{st1} + 0.87 f_y A_{st2}$$

$$(0.36\, f_{ck}\, b\, x_u - f_{cc} A_{sc}) + f_{sc} A_{sc} = 0.87 f_y A_{st}$$

$$x_u = \frac{0.87 f_y A_{st} - (f_{sc} - f_{cc}) A_{sc}}{0.36\, f_{ck}\, b}$$

$$x_u = \frac{0.87 f_y A_{st} - f_{sc} A_{sc}}{0.36\, f_{ck}\, b}$$

where,

f_{cc} = Stress in Concrete in Compression at level of A_{sc} = $0.45 f_{ck}$.

Ultimate moment of Resistance :-

* The ultimate moment of resistance of doubly reinforced section is obtained by taking moments of C_{u1} and C_{u2} about the centroid of tension steel.

$$M_u = M_{u1} + M_{u2}$$

M_{u1} = Compressive force × Lever arm = $C_{u1} \times (d - 0.42\, x_u)$

$= 0.36\, f_{ck}\, b\, x_u\, (d - 0.42\, x_u)$

$M_{u2} = C_{u2} \times (d - d') = (f_{sc} - f_{cc}) A_{sc} (d - d')$

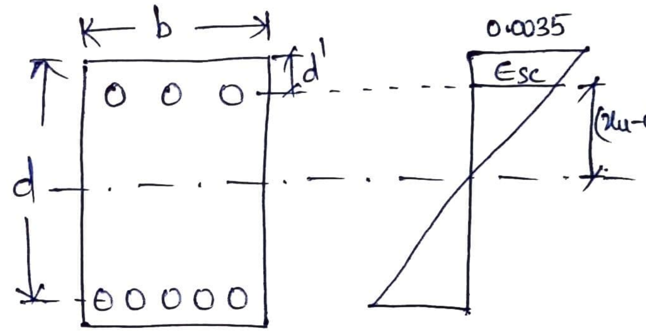
∴ Total moment of resistance,

$$M_u = 0.36\, f_{ck}\, b\, x_u\,(d - 0.42\, x_u) + (f_{sc} - f_{cc})\, A_{sc}\,(d - d')$$

$$= 0.36\, f_{ck}\, b\, x_u\,(d - 0.42\, x_u) + f_{sc}\, A_{sc}\,(d - d')$$

Stress in Compression steel reinforcement (f_{sc}) :-

* The stress in compression steel is depends upon the strain ϵ_{sc} at the level of compression steel.



From similar triangles,

$$\epsilon_{sc} = 0.0035 \times \frac{(x_u - d')}{x_u}$$

$$= 0.0035\left(1 - \frac{d'}{x_u}\right)$$

For balanced design, $x_u = x_{umax}$

$$\epsilon_{sc} = 0.0035\left(1 - \frac{d'}{x_{umax}}\right)$$

Minimum and Maximum reinforcement :- (Pg 46, cl 26.5.1.1)

* The minimum and maximum amount of tension reinforcement in a doubly reinforced beam are same as singly reinforced beam.

* Minimum reinforcement, $\frac{A_{st\,min}}{bd} = \frac{0.85}{f_y}$

* Maximum A_{st} = 4% gross c/s area = $0.04\, bD$

Design of Doubly Reinforced Sections :-

1. Determine the limiting moment of resistance Mulim for the given cross section using the equation for singly reinforced beam.

$$M_{ulim} = 0.36 f_{ck} \frac{x_{umax}}{d} \left(1 - 0.42 \frac{x_{umax}}{d}\right) bd^2$$

If $M_u < M_{ulim}$, It is designed for under reinforced section,

$$M_u = 0.87 f_y A_{st} d \left(1 - \frac{f_y A_{st}}{f_{ck} bd}\right)$$

2. If the factored moment Mu exceeds the Mulim, a doubly reinforced section is required to be design for additional moment

$$M_{u2} = M_u - M_{ulim}$$

This moment is resisted by an internal couple consisting of Compressive steel and tensile force T_2 due to the additional tension steel in sec 2.

$$M_u - M_{ulim} = (f_{sc} - f_{cc}) A_{sc} (d - d')$$

Since f_{cc} is very less Compared to f_{sc} Hence ~~finally~~ f_{cc} is neglected.

$$\therefore M_u - M_{ulim} = f_{sc} A_{sc} (d - d') \rightarrow (1)$$

3. The additional area of tension steel A_{st2} is obtained by considering the equilibrium of force of Compression C_2 in Compression steel and force of Tension T_2 in the additional tension steel

$$f_{sc} A_{sc} - \cancel{f_{cc} A_{sc}}^{\,0} = 0.87 f_y A_{st2} \rightarrow (2)$$

4. Area of Tension steel, $A_{st} = A_{st1} + A_{st2}$

From Total compression = Total Tension

$$\cancel{0.8}\ 0.36\, f_{ck}\, b\, x_{umax} = 0.87\, f_y\, A_{st1}$$

$$A_{st1} = \frac{0.36\, f_{ck}\, b\, x_{umax}}{0.87\, f_y}$$

From eq (1) $\Rightarrow$ $$A_{sc} = \frac{M_u - M_{ulim}}{f_{sc}\,(d - d')}$$

From eq (2) $\Rightarrow$ $$A_{st2} = \frac{f_{sc}\, A_{sc}}{0.87\, f_y}$$

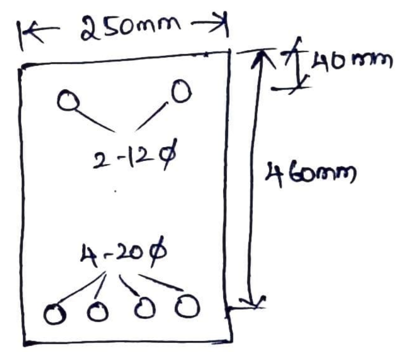
Analysis problem :-

1. Find the moment of resistance of a beam 250 x 500mm overall depth. It is reinforced with 2 bars of 12 mm diameter in Compression zone and 4 bars of 20 mm diameter in Tension zone. Each at an effective cover of 40mm.

(i) Assume M_{15} and Fe_{250}

(ii) Assume M_{15} and Fe_{415}.

sol:- Given data :-



b = 250mm

overall depth D = 500mm

Effective depth, d = 500 - 40 = 460 mm

$A_{st} = 4 \times \pi/4 \times 20^2 = 1256.8 \text{ mm}^2$

$A_{sc} = 2 \times \pi/4 \times 12^2 = 226.22 \text{ mm}^2$

(i) For M_{15} and Fe_{250}.

$f_{ck} = 15$ MPa

$f_y = 250$ MPa

Total Compression = Total Tension

$C_1 + C_2 = T$

$0.36 f_{ck}\, b\, x_u + (f_{sc} - f_{cc}) A_{sc} = 0.87 f_y A_{st}$

$f_{sc} = \epsilon_{sc} E_s = 0.0035 \left(1 - \frac{d'}{x_u}\right) \times 2 \times 10^5$.

$= 700 \left(1 - \frac{40}{x_u}\right)$

$$\therefore\ 0.36 \times 15 \times 250 \times x_u + 700\left(1 - \frac{40}{x_u}\right) \times 226.22 = 0.87 \times 250 \times 1256.8$$

$$1350\, x_u + \left(700 - \frac{28000}{x_u}\right) 226.22 = 273354$$

$$1350\, x_u + 158354 - \frac{6334160}{x_u} = 273354$$

$$1350\, x_u^2 + 158354\, x_u - 6334160 = 273354\, x_u$$

$$\Rightarrow 1350\, x_u^2 - 115000\, x_u - 6334160 = 0$$

$$\underline{x_u = 123.25 \text{ mm}}$$

$$f_{sc} = 0.0035\left(1 - \frac{40}{123.25}\right) \times 2 \times 10^5 = 472.81 \text{ N/mm}^2$$

But $0.87 f_y = 217.5 \text{ N/mm}^2$

$$\therefore\ \underline{f_{sc} = 217.5 \text{ N/mm}^2}$$

$\therefore$ ultimate moment of resistance,

$$M_u = 0.36 f_{ck}\, b\, x_u (d - 0.42 x_u) + f_{sc} A_{sc} (d - d')$$

$$x_{umax} = 0.53 d = 243.8 \text{ mm}$$

$$x_u < x_{umax} \Rightarrow x_u = 123.25 \text{ mm}$$

$$\therefore\ M_u = 0.36 \times 15 \times 250 \times 123.25 (460 - 0.42 \times 123.25) + 217.5 \times 226.22 (460 - 40)$$

$$= \underline{88.59 \text{ kNm}}$$

(ii) For M15 and Fe415 grade steel

$f_{ck} = 15\ N/mm^2$

$f_y = 415\ N/mm^2$

$0.36\ f_{ck}\ b\ x_u + (f_{sc} - f_{cc}) A_{sc} = 0.87\ f_y\ A_{st}$

$f_{sc} = 700\left(1 - \frac{40}{x_u}\right)$

$f_{cc} = 0.446\ f_{ck} = 0.446 \times 15 = 6.69\ N/mm^2$

$0.36 \times 15 \times \cancel{250}\ x_u + \left[700\left(1 - \frac{40}{x_u}\right) - 6.69\right] \times 226.22 = 0.87 \times 415 \times 1256.8$

$1350\ x_u + \left(693.31 - \frac{28000}{x_u}\right) 226.22 = 453767.64$

$1350\ x_u + 156840.58 - \frac{6334160}{x_u} = 453767.64$

$\Rightarrow 1350\ x_u^2 - 296927.06\ x_u - 6334160 = 0$

$x_u = 239.53\ mm$

$f_{sc} = 700\left(1 - \frac{40}{239.53}\right) = 583.1\ N/mm^2$

But $f_{sc} = 0.87\ f_y = 0.87 \times 415 = 361.05\ N/mm^2$

$\Rightarrow f_{sc} = 361.05\ N/mm^2$

$x_{umax} = 0.48 \times 460 = 220.8\ mm$

$x_u > x_{umax} \Rightarrow x_u = 220.8\ mm$

$\therefore\ M_u = 0.36 \times 15 \times 250 \times 220.8\ (460 - 0.42 \times 220.8) + (361.05 - 6.69) \times 226.22\ (460 - 40)$

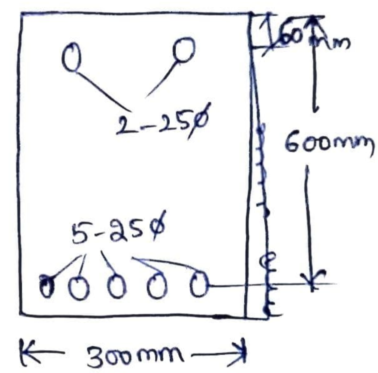
$= \cancel{123.39}\ 109.47 + 33.66$

$= \cancel{157.05}\ 143.13\ KNm.$

working moment $= \frac{M_u}{1.5} = \frac{\cancel{157.05}\ 143.13}{1.5} = \cancel{104.7}\ 95.42\ KNm.$

2. Calculate the ultimate moment of resistance of a doubly reinforced beam of rectangular section having width of 300 mm and reinforced with 5 bars of 25mm dia and at an effective depth of 600mm. The Compression steel ~~is made up~~ Consists of 2 bars of 25 mm dia at an effective Cover of 60mm. Adopt M_{20} and Fe_{415} grade Concrete and steel.

Sol:- Given data:-



Breadth, $b = 300mm$

Effective depth, $d = 600mm$

Effective Cover, $d' = 60mm$

$f_{ck} = 20 \, N/mm^2$, $f_y = 415 \, N/mm^2$

$A_{st} = 5 \times \pi/4 \times 25^2 = 2455 \, mm^2$

$A_{sc} = 2 \times \pi/4 \times 25^2 = 981.74 \, mm^2$

IS: 456 - 200:-

Trail - I:-

For Fe 415 grade steel, $\frac{x_{umax}}{d} = 0.48$

$\Rightarrow x_{umax} = 0.48 \times d = 0.48 \times 600 = 288 \, mm$

$\epsilon_{sc} = 0.0035 \left(1 - \frac{d'}{x_{umax}}\right)$

$= 0.0035 \left(1 - \frac{60}{288}\right) = 0.00277$

$\epsilon_{sy \, min} = \frac{0.87 f_y}{E_s} = \frac{0.87 \times 415}{2 \times 10^5} = 0.0018$

$\epsilon_{sc} > \epsilon_{sy \, min} \rightarrow$ over reinforced section.

From the stress- strain diagram (Pg 70)

$$f_{sc} = 0.85\,f_y = 0.85 \times 415 = 352.75\ N/mm^2$$

$$A_{st_2} = \frac{f_{sc}\,A_{sc}}{0.87\,f_y} = \frac{352.75 \times 981.74}{0.87 \times 415} \quad (Pg\ 94)$$

$$= 959.17\ mm^2$$

$$A_{st_1} = A_{st} - A_{st_2} = 2455 - 959.17 = 1495.83\ mm^2$$

$$x_u = \frac{0.87\,f_y\,A_{st_1}}{0.36\,f_{ck}\,b}$$

$$= \frac{0.87 \times 415 \times 1495.83}{0.36 \times 20 \times 300} = 250mm$$

$x_u < x_{umax}$

Trial - II :-

Assume $x_u = 250$ mm

$$\epsilon_{sc} = 0.0035\left(1 - \frac{60}{250}\right) = 0.00266$$

$$f_{sc} = 0.86\,f_y \quad (\text{From Pg } 70)$$

$$= 0.86 \times 415 = 356.9$$

$$= 357\ N/mm^2$$

$$A_{st_2} = \frac{f_{sc}\,A_{sc}}{0.87\,f_y}$$

$$= \frac{357 \times 981.74}{0.87 \times 415}$$

$$= 970.72 \simeq 971\ mm^2$$

$A_{st1} = 2455 - 971 = 1484\ mm^2$

$$x_u = \frac{0.87 f_y A_{st1}}{0.36 f_{ck} b} = \frac{0.87 \times 415 \times 1484}{0.36 \times 20 \times 300}$$

$$= 248.05\ mm$$

$x_{ulim} = 248\ mm$ which is nearly equal to assumed value.

Moment of resistance,

$$M_u = 0.36 f_{ck}\, b\, x_u (d - 0.42 x_u) + (f_{sc} - f_{cc}) A_{sc} (d - d')$$

$$= 0.36 \times 20 \times 300 \times 248 (600 - 0.42 \times 248) + (357 - 0.446 \times 20) \times 981.74 (600 - 60)$$

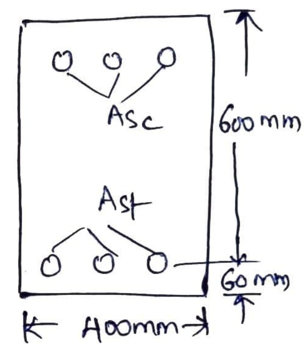
$$= 265611571.2 + 184530992$$

$$= \underline{450.14\ kNm}$$

Design Problems:-

1. A rectangular R.C beam of width 400 mm and effective ~~width~~ depth of 600 mm is to be designed to support an ultimate moment of 600 KN m. Use M20 and Fe 415 grade concrete and steel. Effective cover = 60 mm.

Sol:- Given data:-



Breadth, $b = 400\,mm$

Effective depth, $d = 600\,mm$

effective cover, $d' = 60mm$

$f_{ck} = 20\,N/mm^2$

$f_y = 415\,N/mm^2$

$M_u = 600\,KNm$

Limiting moment of resistance, $M_{u\,lim} = 0.138\, f_{ck}\, b d^2$

$$= 0.138 \times 20 \times 400 \times 600^2$$

$$= 397.4\ kNm.$$

$M_u > M_{ulim}$ → It is a over reinforced section.

Total Compression = Total Tension (Section - I)

$$0.87 f_y A_{st1} = 0.36 f_{ck}\, b\, x_{umax}$$

$$A_{st1} = \frac{0.36 f_{ck}\, b\, x_{umax}}{0.87 f_y} \quad \left[\text{or } M_{ulim} = 0.87 f_y A_{st1} d \left(1 - \frac{f_y A_{st1}}{f_{ck} b d}\right)\right]$$

$$= \frac{0.36 \times 20 \times 400 \times (0.48 \times 600)}{0.87 \times 415}$$

$$= 2297.3\ mm^2$$

$$M_u - M_{ulim} = 600 - 397 = 203 \text{ kNm}$$

$$M_u - M_{ulim} = f_{sc} A_{sc} (d - d')$$

$$\varepsilon_{sc} = 0.0035 \left(1 - \frac{d'}{x_{umax}}\right)$$

$$= 0.0035 \left(1 - \frac{60}{0.48 \times 600}\right) = 0.00277$$

$$f_{sc} = \overset{0.87}{\cancel{0.86}} f_y \quad (\text{From Pg } 70)$$

$$= \overset{0.87}{\cancel{0.86}} \times 415 = \overset{361.05}{\cancel{356.9}} \text{ N/mm}^2$$

$$M_u - M_{ulim} = \overset{361.05}{\cancel{356.9}} \times A_{sc} (600 - 60) = 203 \times 10^6$$

$$\Rightarrow A_{sc} = \overset{1041.2}{\cancel{1053.3}} \text{ mm}^2$$

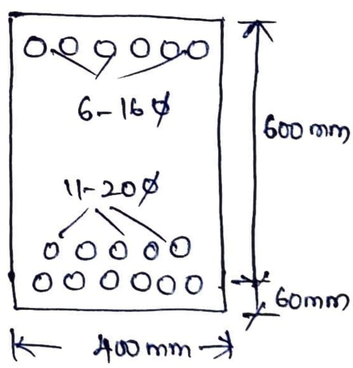
$$A_{st2} = \frac{f_{sc} A_{sc}}{0.87 f_y}$$

$$= \frac{\overset{361.05}{\cancel{356.9}} \times 1053.3}{0.87 \times 415} = 1041.2 \text{ mm}^2$$

$$A_{st} = A_{st1} + A_{st2} = 2297.3 + 1041.2$$

$$= 3338.5 \text{ mm}^2$$

$$A_{sc} = 1053.3 \text{ mm}^2$$



Assume the diameter of bar as 20mm in tension zone.

$$\text{No. of bars, } n = \frac{A_{st}}{a_{st}} = \frac{3338.5}{\frac{\pi}{4} \times 20^2} = 11 \text{ bars}$$

Assume the diameter of bar as 16mm in Compression zone.

$$\text{No. of bars, } n = \frac{A_{sc}}{a_{sc}} = \frac{1053.3}{\frac{\pi}{4} \times 16^2} = 6 \text{ bars}.$$

2. Design the reinforcement for a doubly reinforced concrete beam section to support a factored moment of 1000 KNm. ~~As~~ Take b = 400 mm, d = 550 mm, d' = 50 mm, f_{ck} = 30 N/mm², f_y = 415 N/mm²

Sol:- Given data:-

b = 400 mm

d = 550 mm

d' = 50 mm , f_{ck} = 30 N/mm², f_y = 415 N/mm²

M_u = 1000 KNm

Limiting moment of resistance, $M_{ulim} = 0.138\, f_{ck}\, bd^2$

$= 0.138 \times 30 \times 400 \times 550^2$

$= 500.94$ KNm.

$M_u > M_{ulim}$ → It is a over reinforced section. So design the beam as doubly reinforced section.

Total Compression = Total tension

$$A_{st1} = \frac{0.36\, f_{ck}\, b\, x_{umax}}{0.87\, f_y}$$

$$= \frac{0.36 \times 30 \times 400 \times (0.48 \times 550)}{0.87 \times 415} = 3158.78 \text{ mm}^2$$

$M_u - M_{ulim} = 1000 - 500.94 = 499.06$ KNm

$M_u - M_{ulim} = f_{sc}\, A_{sc}\, (d - d')$

$$\epsilon_{sc} = 0.0035\left(1 - \frac{d'}{x_{umax}}\right)$$

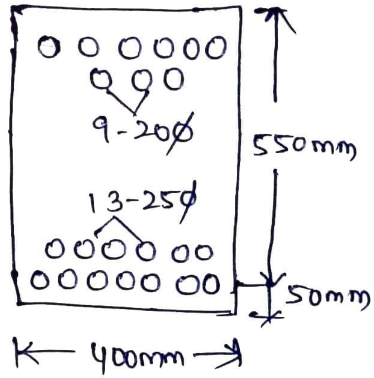
$$= 0.0035\left(1 - \frac{50}{0.48 \times 550}\right) = 0.00283$$

$f_{sc} = 0.86\, f_y$ (From Pg 70)

$= 0.86 \times 415 = 356.9\ N/mm^2$

$M_u - M_{ulim} = 499.06 \times 10^6 = 356.9 \times A_{sc}\,(550-50)$

$\Rightarrow A_{sc} = 2796.63\ mm^2$



$$A_{st_2} = \frac{f_{sc}\, A_{sc}}{0.87\, f_y} = \frac{356.9 \times 2796.63}{0.87 \times 415}$$

$= 2764.45\ mm^2$

$A_{st} = A_{st_1} + A_{st_2} = 3158.7 + 2764.45 = 5923\ mm^2$

$A_{sc} = 2796.6\ mm^2$

Assume the diameter of the bar as 25mm in tension zone.

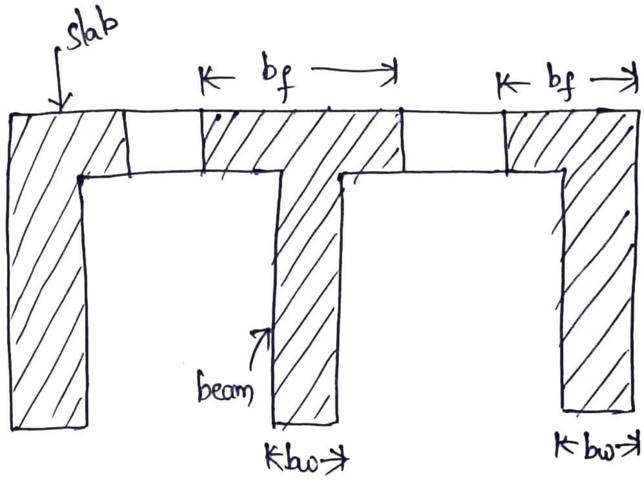
$$\text{No. of bars} = \frac{5923}{\pi/4 \times 25^2} = 13 \text{ bars}$$

Assume the diameter of the bar as 20 mm in Compression zone.

$$\text{No. of bars} = \frac{2796.6}{\pi/4 \times 20^2} = 9 \text{ bars.}$$

Flanged Beams :-

* The reinforced Concrete floors, roofs, bridge decks consists of beams and slabs which are always cast monolithically.



* Since the stirrups and bent up bars of beam are also extended into the slab, it results in an integral Connection between the slab and beam due to which certain portion of the slab acts along with the beam in resisting compression, in sagging moment regions (positive bending) and it acts like a flange of beam.

* The total resulting action is a flanged section in the span region.

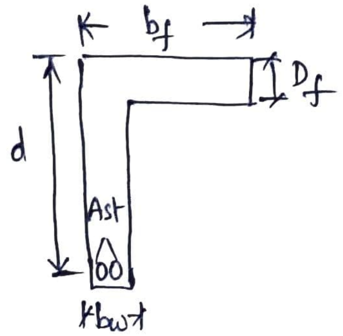
* where as at supports, the slab is in tension and part of the rib or web is in Compression.

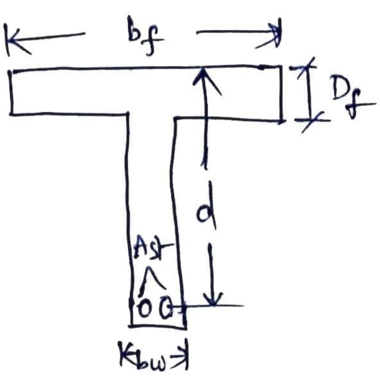
* ~~Since~~ The slab thus perform two functions

  (i) It transfers the load to the beam by spanning across the beam

  (ii) It assist the beam in transferring the load longitudinally.

* The two actions are slab action and the beam action causing normal stresses in the slab at right angles to each other.

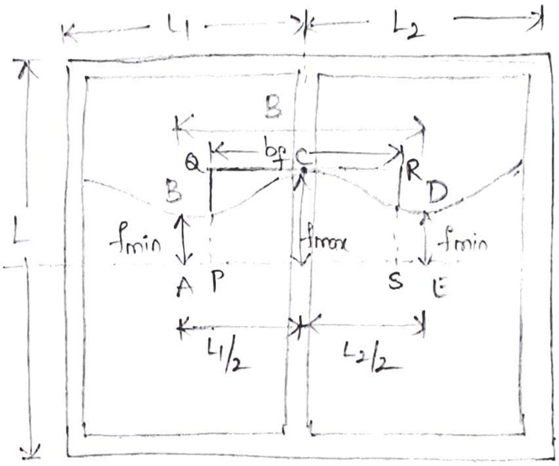




Effective width of flange :- (Pg 37)

* The moment of resistance is calculated using the effective width of flanged sections instead of actual width.

* The actual width of the flanged beam is equal to the distance b/n the middle points of the adjacent spans of the slab.

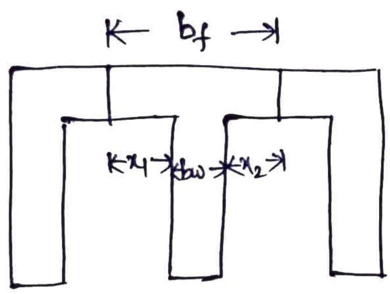


Area ABCDE = Area PQRS

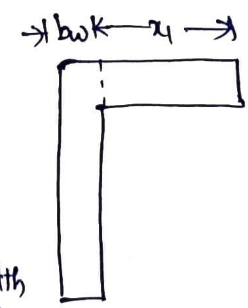
* The effective width of flange is given by

(a) For T – beams

$$b_f = L_0/6 + b_w + 6D_f \leq \text{actual width } (B)$$



$$\therefore \; b_f = x_1 + b_w + x_2$$



(b) For L - beams

$$b_f = L_0/12 + b_w + 3D_f \leq \text{actual width } (B)$$

$$= b_w + x_1$$

(c) For isolated beams

T - beam, $b_f = \left(\dfrac{l_0}{\left(\frac{l_0}{b}\right) + 4} \right) + b_w$

$= B$

} lesser value should be taken

L-beam, $b_f = \left(\dfrac{0.5\, l_o}{\frac{l_o}{b} + 4} \right) + b_w$ } lesser value should be taken.

$= B$

where, b_f = Effective width of flange

l_o = Distance b/n points of zero moments in the beam

b_w = breadth of the web

D_f = Thickness of flange

B = actual width of flange.

Note:- For Continuous beams and frames 'l_o' may be assumed as 0.7 times the effective span.

Types of Problems:- or Properties of flanged section :-

* In case of flanged section, following different cases arise depending on the depth of flange D_f in relation to the depth of neutral axis x_u and in relation to the rectangular part $3x_u/7$ of rectangular - parabolic stress distribution.

case (i):- Neutral axis lies within the flange i.e $x_u < D_f$

case (ii):- Neutral axis lies outside the flange i.e inside the web $x_u > D_f$.

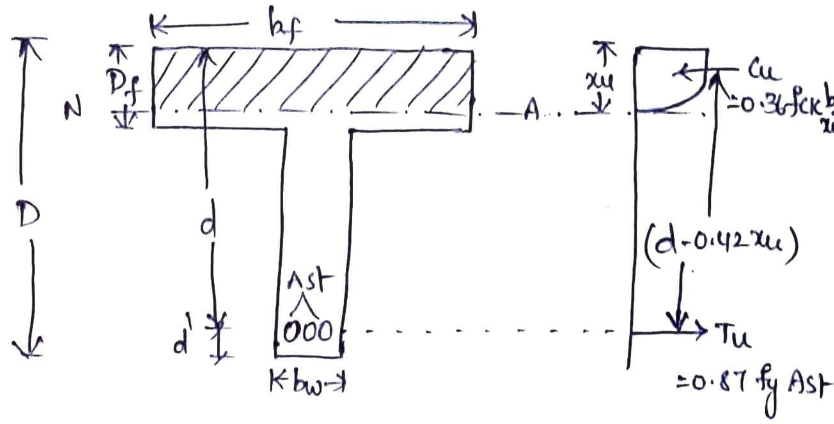
(a) $x_u > D_f$, $\dfrac{D_f}{d} < 0.2$, $\dfrac{3x_u}{7} < D_f$ under reinforced

(b) $x_u > D_f$, $\dfrac{D_f}{d} < 0.2$, $\dfrac{3x_u}{7} > D_f$

Case (iii):- $x_u > D_f$, $\dfrac{D_f}{d} > 0.2$ - Balanced section

Case (i) :- Neutral axis lying inside the flange ($x_u < D_f$)

* The flanged beam is considered as a rectangular beam of width $b = b_f$



* The expressions for x_u, M_{ur} and A_{st} for singly reinforced rectangular beam can be used by replacing 'b' by 'b_f'.

* Depth of neutral axis, $x_u = \dfrac{0.87 f_y A_{st}}{0.36 f_{ck} b_f}$

* Moment of resistance, $M_{ur} = 0.36 f_{ck} b_f x_u (d - 0.42 x_u)$

(or)

$$M_{ur} = 0.87 f_y A_{st} (d - 0.42 x_u)$$

$$= 0.87 f_y A_{st} d \left(1 - \frac{f_y A_{st}}{f_{ck} b d}\right)$$

* The bifurcation b/n case - I and case - II is $x_u = D_f$

i.e the neutral axis is at the Junction of flange and web.

$\therefore$ Depth of neutral axis $x_u = D_f$

Moment of resistance, $M_{ur} = 0.36 f_{ck} b_f D_f (d - 0.42 D_f)$

$$A_{st} = \frac{0.36 f_{ck} b_f D_f}{0.87 f_y}$$

Case (ii):- Neutral axis is lying in web, $x_u > D_f$

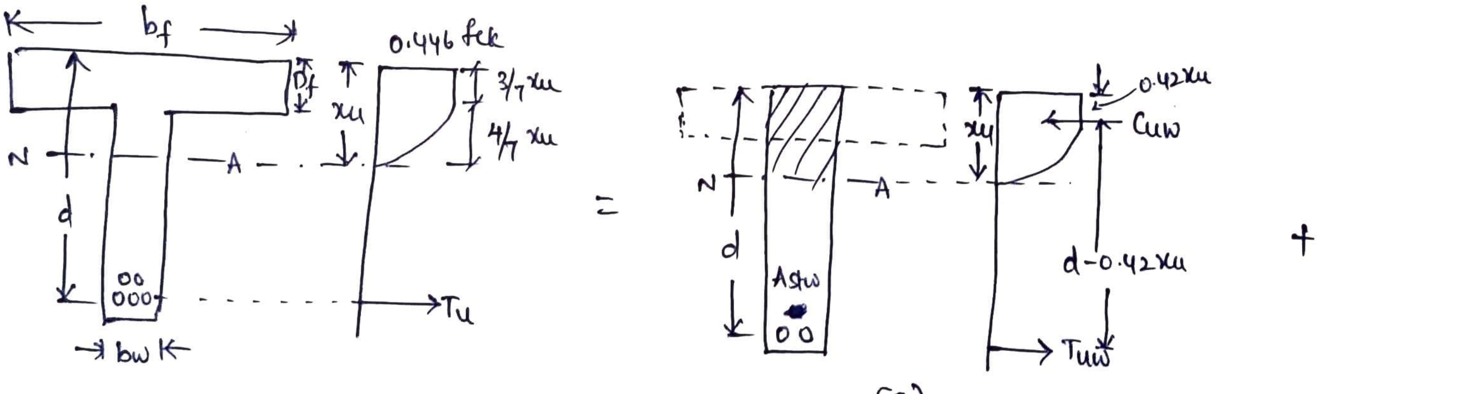
* In this case the shape of concrete under compression is itself a T-section

* For convenience in calculations, it is divided into two parts. One consists of concrete in web portion of width 'b_w' and depth 'x_u', and the other consists of projecting flanges of width $(b_f - b_w)$ and depth D_f.

(a) $\frac{D_f}{d} < 0.2$ and $\frac{3x_u}{7} < D_f$

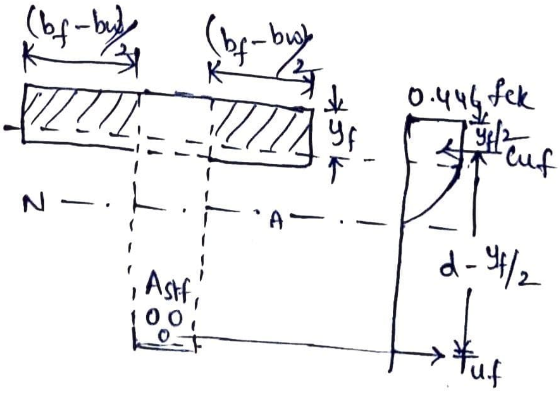
i.e Depth of rectangular part of stress block is less than the depth of the flange.

* The stress distribution across the depth of flange is rectangular and partly parabolic.



(a)

* In fig (a), the stress distribution across the section can be taken equivalent to web subjected to rectangular-parabolic stress-distribution for a depth equal to x_u



(b)

* In fig (b), the outstanding portion of the flange $(b_f - b_w)$ subjected to rectangular and partly parabolic stress distribution with reduced depth of rectangular stress block, i.e y_f.

$y_f = (0.15\,x_u + 0.65\,D_f) < D_f$

* Depth of neutral axis -

By considering equilibrium of forces $C_u = T_u$

$C_{uw} + C_{uf} = T_u$

C_{uw} = Compression resisted by web $= 0.36\,f_{ck}\,b_w\,x_u$

C_{uf} = Compression resisted by flange $= 0.446\,f_{ck}\,(b_f - b_w)\,y_f$

$\therefore\; 0.36\,f_{ck}\,b_w\,x_u + 0.446\,f_{ck}\,(b_f - b_w)\,y_f = 0.87\,f_y\,A_{st} \rightarrow ①$

Substituting $y_f = 0.15\,x_u + 0.65\,D_f$

$0.36\,f_{ck}\,b_w\,x_u + 0.446\,f_{ck}\,(b_f - b_w)\,(0.15\,x_u + 0.65\,D_f) = 0.87\,f_y\,A_{st}$

$= \cancel{0.87 f_y A_{st}}$

$0.36\,f_{ck}\,b_w\,x_u + 0.446 \times 0.15\,f_{ck}\,(b_f - b_w)\,x_u + 0.446 \times 0.65\,f_{ck}\,D_f\,(b_f - b_w)$

$= 0.87\,f_y\,A_{st}$

$$x_u = \frac{0.87\,f_y\,A_{st} - 0.446 \times 0.65\,f_{ck}\,D_f\,(b_f - b_w)}{0.36\,f_{ck}\,b_w + 0.446 \times 0.15\,f_{ck}\,(b_f - b_w)}$$

* Moment of resistance

By taking moment of Compressive force about C.G of tension

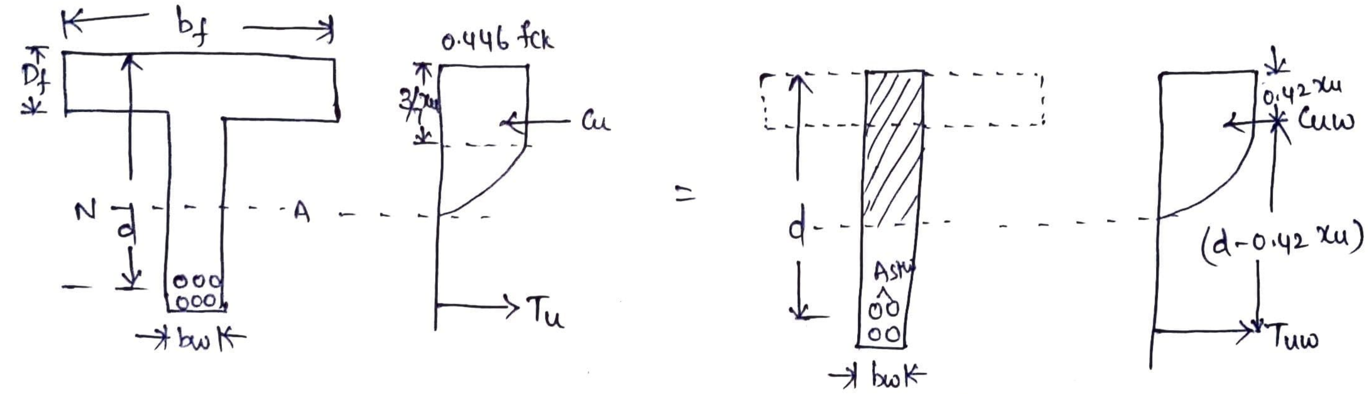
$M_{ur} = 0.36\,f_{ck}\,b_w\,x_u\,(d - 0.42\,x_u) + 0.446\,f_{ck}\,(b_f - b_w)\,y_f\,(d - y_f/2)$

* Area of steel,

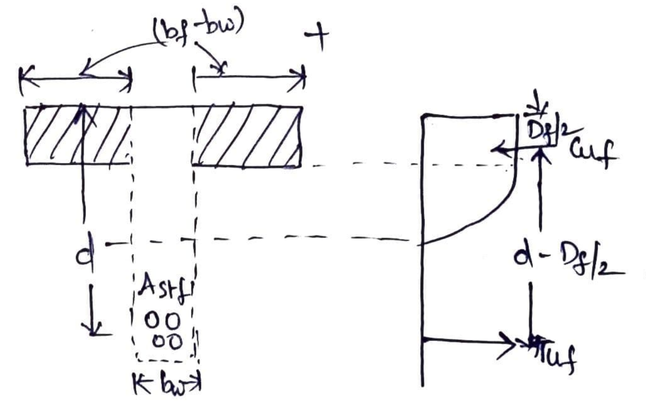
From eq ①, $A_{st} = \dfrac{0.36\,f_{ck}\,b_w\,x_u + 0.446\,f_{ck}\,(b_f - b_w)\,y_f}{0.87\,f_y}$.

(b) Depth of rectangular part of stress block greater than the depth of flange

$$\frac{3x_u}{7} > D_f \,, \quad \frac{D_f}{d} < 0.2$$



* The stress distribution across the depth of flange is rectangular, hence y_f is replaced by D_f.



* Depth of neutral axis

$$C_{uw} + C_{uf} = T_u$$

$$0.36 f_{ck} \cdot b_w x_u + 0.446 f_{ck} (b_f - b_w) D_f = 0.87 f_y A_{st}$$

$$x_u = \frac{0.87 f_y A_{st} - 0.446 f_{ck} (b_f - b_w) D_f}{0.36 f_{ck} b_w}$$

* Moment of resistance,

$$M_{ur} = 0.36 f_{ck} b_w x_u (d - 0.42 x_u) + 0.446 f_{ck} (b_f - b_w) D_f (d - D_f/2)$$

* Area of steel,

$$A_{st} = \frac{0.36 f_{ck} b_w x_u + 0.446 f_{ck} (b_f - b_w) D_f}{0.87 f_y}$$

Case (iii):- For a balanced Section, $x_u = x_{umax}$

when $x_u = x_{umax}$, the case is usually either case-2 (a) or (b) based on value of D_f/d and $\frac{x_{umax}}{d}$.

* Taking $x_u = x_{umax}$, $\frac{x_{umax}}{D_f} = \frac{7}{3}$

$$\Rightarrow \frac{x_{umax}}{d} = \frac{D_f}{d} \cdot \frac{7}{3} \Rightarrow \frac{D_f}{d} = \frac{3}{7} \frac{x_{umax}}{d} = 0.197 \frac{x_{umax}}{d}.$$

* If $D_f/d \leq 0.2$, equation for case 2(b), by considering $x_u = x_{umax}$.

$$M_{ur\,max} = 0.36 f_{ck} b_w x_{umax} (d - 0.42 x_{umax}) + 0.446 f_{ck} (b_f - b_w) D_f (d - D_f/2)$$

* If $\frac{D_f}{d} > 0.2$ and $x_u > D_f$, equation for case 2(a) with $x_u = x_{umax}$ is used

$$M_{ur\,max} = 0.36 f_{ck} b_w x_{umax} (d - 0.42 x_{umax}) + 0.446 f_{ck} (b_f - b_w) y_f (D - y_f/2)$$

where $y_f = 0.15 x_{umax} + 0.65 D_f$ but $\nless D_f$

Analysis Problems:-

1. Determine the ultimate flexural strength of T-beam having the following sectional properties.

width of flange, b_f = 800mm, Depth of flange, D_f = 150 mm

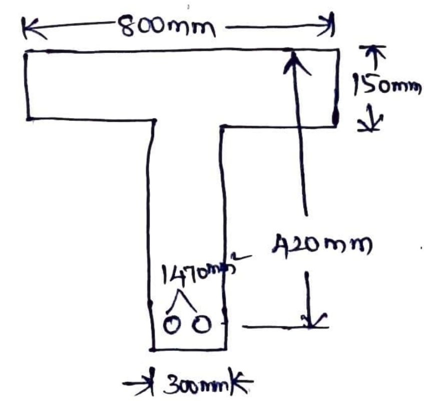
width of rib or web, b_w = 300mm, Effective depth (d) = 420mm

Area of steel, A_{st} = 1470 mm², use M_{20} and Fe 415 steel.

Sol:- Given data:-

b_f = 800 mm, D_f = 150mm, b_w = 300mm, d = 420mm, A_{st} = 1470 mm²

f_{ck} = 20 N/mm², f_y = 415 N/mm²



Let us assume that the neutral axis lies within the flange (i.e $x_u < D_f$)

$$\frac{x_u}{d} = \frac{0.87\, f_y A_{st}}{0.36\, f_{ck}\, b_f d} = \frac{0.87 \times 415 \times 1470}{0.36 \times 20 \times 800 \times 420}$$

$$= 0.219 \Rightarrow x_u = 92.14 \text{ mm}$$

$\therefore x_u < D_f$ (92.14 < 150mm)

Assumption is Correct.

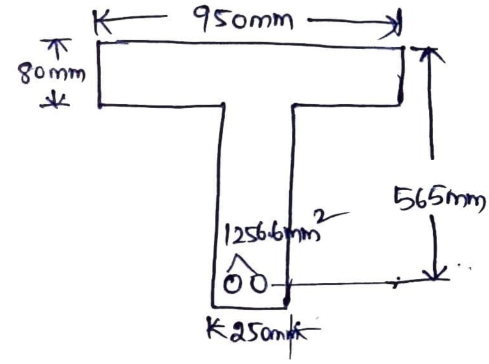
Moment of resistance, $M_{ur} = 0.87\, f_y A_{st}\, d \left(1 - \frac{f_y A_{st}}{f_{ck}\, b d}\right)$

$$= 0.87 \times 415 \times 1470 \times 420 \left(1 - \frac{415 \times 1470}{20 \times 800 \times 420}\right)$$

$$= 202.67 \text{ kNm}$$

2. A singly reinforced T-beam of flange width 950mm, Thickness of flange 80 mm, width of rib 250mm, effective depth 565 mm, $A_{st} = 1256.6\,mm^2$. use M_{15} and Fe 415 grade Concrete and steel.

Sol:-



Neutral axis depth,

$$x_u = \frac{0.87\, f_y\, A_{st}}{0.36\, f_{ck}\, b_f}$$

$$= \frac{0.87 \times 415 \times 1256.6}{0.36 \times 15 \times 950} = 88.84\ mm$$

$\therefore\ x_u > D_f$, i.e $88.84 > 80\ mm$

$\therefore$ The neutral axis lies outside the flange.

$$\frac{D_f}{d} = \frac{80}{565} = 0.141 < 0.2$$

Neutral axis depth is determined by using equilibrium of forces.

$C_1 + C_2 = T$ assuming $\cancel{\frac{D_f}{x_u} < 0.43}$ $\frac{3x_u}{7} > D_f$

$$0.36\, f_{ck}\, b_w\, x_u + 0.446\, f_{ck}\,(b_f - b_w)\, D_f = 0.87\, f_y\, A_{st}$$

$$0.36 \times 15 \times 250\, x_u + 0.446 \times 15\,(950 - 250) \times 80 = 0.87 \times 415 \times 1256.6$$

$\Rightarrow x_u = \cancel{56.07}$ 58.5 mm

$\cancel{\frac{D_f}{x_u} = \frac{80}{56.07} = 1.426 > 0.43}$

$\frac{3x_u}{7} = \cancel{24.03}$ 25.07 < 80 mm

$\therefore\ 0.36\, f_{ck}\, b_w\, x_u + 0.446\, f_{ck}\,(b_f - b_w)\, y_f = 0.87\, f_y\, A_{st}$

$y_f = 0.15\, x_u + 0.65\, D_f$

$= 0.15\, x_u + 0.65 \times 80 = 0.15\, x_u + 52$

$\therefore 0.36 \times 15 \times 250\, x_u + 0.446 \times 15\, (950-250) \times (0.15\, x_u + 52) = 0.87 \times 415 \times 1256.6$

$1350\, x_u + 708.75\, x_u + 245700 = 453695.43$

$x_u = \cancel{101.03}\ 102.4$ mm

$\frac{3x_u}{7} = 43.29 < 80\text{mm}\ (D_f)$

$\therefore y_f = 0.15 \times 101.03 + 0.65 \times 80 = \cancel{67.15}\ 36$ mm

Moment of resistance,

$M_{ur} = 0.36\, f_{ck}\, b_w \cancel{d}\, x_u (d - 0.42\, x_u) + 0.446\, f_{ck}\, (b_f - b_w)\, y_f\, (d - y_f/2)$

$= 0.36 \times 15 \times 250 \times \cancel{565} \times \cancel{101.03}\ 102.4\ (565 - 0.42 \times \cancel{101.03}\ 102.4) + 0.446 \times 15$

$(950-250) \times \cancel{67.15}\ 36\ (565 - \frac{\cancel{67.15}\ 36}{2})$

$= 71273228.97 + 168612516.8$

$= \cancel{239.88}\ 252.44$ KNm

3\. Calculate the ultimate moment of resistance of T-beam having the followin sectional properties. width of flange b_f = 1300 mm, D_f = 100mm, b_w = 325m d = 600mm, A_{st} = 4000mm²: use M_{20} and Fe 415 materials.

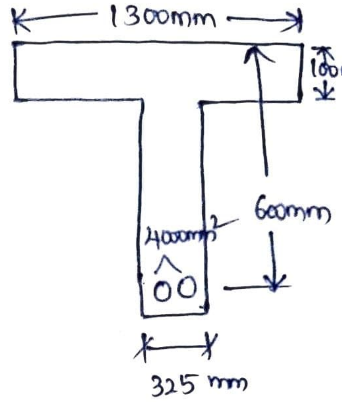
sol:- Given data:-

b_f = 1300mm d = 600 mm f_{ck} = 20N/mm²

D_f = 100mm A_{st} = 4000mm² f_y = 415 N/mm²

b_w = 325 mm

Neutral axis depth, x_u by assuming $x_u < D_f$



$$x_u = \frac{0.87\ f_y\ A_{st}}{0.36\ f_{ck}\ b_f} = \frac{0.87 \times 415 \times 4000}{0.36 \times 20 \times 1300}$$

$$= 154.3 \text{ mm}$$

$\therefore x_u > D_f$ (i.e $154.3 > 100$ mm)

$$\frac{D_f}{d} = \frac{100}{600} = 0.167 < 0.2$$

Assuming that $\frac{3x_u}{7} > D_f$.

The neutral axis depth can be determined by equilibrium of forces.

$C_1 = 0.36\ f_{ck}\ b_w\ x_u = 0.36 \times 20 \times 325\ x_u = 2.34\ x_u$ kN

$C_2 = 0.446 \times f_{ck}\ (b_f - b_w)\ D_f = 0.446 \times 20\ (1300 - 325) \times 100$

$= 877.5$ kN

$T = 0.87\ f_y\ A_{st} = 0.87 \times 415 \times 4000 = 1444.2$ kN

$\therefore 2.34\ x_u + 877.5 = 1444.2$

$x_u = 242.18$ mm

$\frac{3x_u}{7} = 103.79 > D_f$

Moment of resistance, $M_{ur} = 0.36\ f_{ck}\ b_w x_u\ (d - 0.42\ x_u) + 0.446\ f_{ck}\ (b_f - b_w)\ D_f\ (d - D_f/2)$

$M_{ur} = 0.36 \times 20 \times 325 \times 242.18\ (600 - 242.18 \times 0.42) + 0.446 \times 20\ (1300 - 325) \times 100\ (600 - \frac{100}{2})$

$= 282378367.4 + 482625000$

$= 765$ kNm

4. A singly reinforced T- beam has a flange width 900mm, $D_f = 150mm$, $b_w = 300mm$, $d = 650mm$, $A_{st} = 4000 mm^2$. use M_{20} and Fe415 reinforcement. Find the ultimate flexural strength.

Sol:-

Assuming $x_u < D_f$

Neutral axis depth, $x_u = \dfrac{0.87 f_y A_{st}}{0.36 f_{ck} b_f}$

$$= \frac{0.87 \times 415 \times 4000}{0.36 \times 20 \times 900} = 222.8 \text{ mm}$$

$\therefore x_u > D_f$ (i.e $222.8 > 150mm$)

$$\frac{D_f}{d} = \frac{150}{650} = 0.23 > 0.2$$

Neutral axis depth is determined by equilibrium of forces.

$$C_1 + C_2 = T$$

$$C_1 = 0.36 f_{ck} b_w x_u = 0.36 \times 20 \times 300 x_u = 2160 x_u$$

$$C_2 = 0.446 f_{ck} (b_f - b_w) y_f$$

$$y_f = 0.15 x_u + 0.65 D_f = 0.15 x_u + 0.65 \times 150$$

$$= 0.15 x_u + 97.5$$

$$C_2 = 0.446 \times 20 (900 - 300)(0.15 x_u + 97.5)$$

$$= 810 x_u + 526500$$

$$T = 0.87 f_y A_{st} = 0.87 \times 415 \times 4000 = 1444200 N$$

$\therefore$ $2160 x_u + 810 x_u + 526500 = 1444200$

$$x_u = 309 \text{ mm}$$

$$\frac{3x_u}{7} = 132.42 < D_f = 150\ mm$$

$\therefore$ Moment of resistance, ~~Mur~~

$$M_{ur} = 0.36\ f_{ck}\ b_w\ x_u\left(d - 0.42\ x_u\right) + 0.446\ f_{ck}\ (b_f - b_w)\ y_f\ (d - y_f/2)$$

$$y_f = 0.15 \times 309 + 0.65 \times 150 = 143.85\ mm$$

$$M_{ur} = 0.36 \times 20 \times 300 \times 309\left(650 - 0.42 \times 309\right) + 0.446 \times 20\left(900 - 300\right) \times 143.85 \left(650 - \frac{143.85}{2}\right)$$

$$= 347215636.8 + 449042879.3$$

$$= \underline{\underline{796.58\ KNm}}.$$

Design Problems:-

1. Determine the area of tensile reinforcement required in a flanged beam having the sectional properties to support a factored moment of 300 kNm. width of flange = 750mm, width of rib = 300 mm, thickness of flange = 120mm, Effective depth = 600 mm, use M_{20} and Fe 415 reinforcement.

Sol:-

$b_f = 750mm$, $b_w = 300mm$, $D_f = 120mm$, $d = 600mm$

$M_u = 300$ kNm, $f_y = 415\ N/mm^2$, $f_{ck} = 20\ N/mm^2$

$$\frac{D_f}{d} = \frac{120}{600} = 0.2 \qquad \left(\frac{D_f}{d} \leq 0.2\right)$$

Limiting moment of resistance, by assuming $x_u < D_f$

$$M_{ulim} = 0.36\, f_{ck}\, b_f\, x_{umax}\, (d - 0.42\, x_{umax})$$

$$= 0.36 \times 20 \times 750 \times (0.48 \times 600)(600 - 0.42 \times 0.48 \times 600)$$

$$= 745 \text{ KNm}$$

$M_u < M_{ulim}$ → It is a under reinforced section.

$$M_u = 0.36\, f_{ck}\, b_f\, x_u\, (d - 0.42\, x_u)$$

$$300 \times 10^6 = 0.36 \times 20 \times 750\, x_u\, (600 - 0.42\, x_u)$$

$$2268\, x_u^2 - 3240000\, x_u + 300 \times 10^6 = 0$$

$$x_u = 99.5 \text{ mm}$$

$x_u < D_f$, Assumption is Correct.

$$M_u = 0.87\, f_y\, A_{st}\, d \left(1 - \frac{f_y A_{st}}{f_{ck}\, b_f\, d}\right)$$

$$300 \times 10^6 = 0.87 \times 415\, A_{st} \times 600 \left(1 - \frac{415 \times A_{st}}{20 \times 750 \times 600}\right)$$

$$9.989\, A_{st}^2 - 216630\, A_{st} + 300 \times 10^6 = 0$$

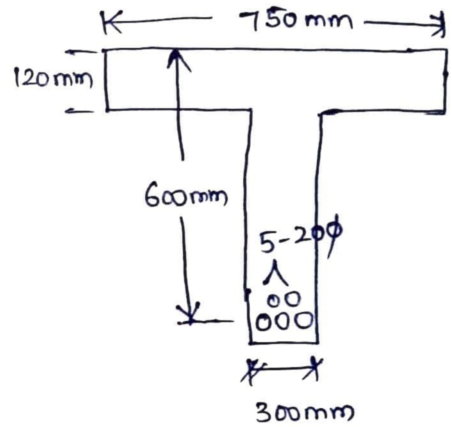
$$A_{st} = 1486.77\ mm^2$$

(or)

$$A_{st} = \frac{0.36\, f_{ck}\, b_f\, x_u}{0.87\, f_y}$$

$$= \frac{0.36 \times 20 \times 750 \times 99.5}{0.87 \times 415}$$

$$= 1488.16\ mm^2$$



Assume 20 mm dia bars,

$$\text{No. of bars} = \frac{A_{st}}{a_{st}} = \frac{1486.77}{\pi/4 \times 20^2} = 4.73 \simeq 5$$

2. A T-beam has the following sectional properties. $b_f = 2000mm$, $D_f = 150mm$, $b_w = 300mm$, $d = 1000mm$. use M_{20} and Fe415 grade Concrete and steel. Calculate the limiting capacity of the section and the corresponding area of tension reinforcement.

Sol:-

$b_f = 2000mm$, $D_f = 150mm$, $b_w = 300mm$, $d = 1000mm$

$f_{ck} = 20\ N/mm^2$, $f_y = 415\ N/mm^2$

$$\frac{D_f}{d} = \frac{150}{1000} = 0.15 < 0.2$$

Assuming $x_u = x_{umax} = 0.48 \times 1000$

$= 480$ mm

$\frac{D_f}{x_u} = \frac{150}{480} = 0.31 < 0.43$, $\frac{3x_u}{7} = \frac{3 \times 480}{7} = 205.71$ mm $> D_f = 150$mm

$x_u > D_f$ $(480 > 150$ mm$)$

$\therefore M_{ulim} = 0.36 f_{ck} b_w x_{umax}(d - 0.42 x_{umax}) + 0.446 f_{ck} (b_f - b_w) D_f (d - D_f/2)$

$= 0.36 \times 20 \times 300 \times 480 (1000 - 0.42 \times 480) + 0.446 \times 20 (2000 - 300) \times 150 (1000 - 150/2)$

$= 827781120 + 2122875000$

$= 2950.65$ KNm

$C_{uw} = 0.36 f_{ck} b_w x_u$, $T_{uw} = 0.87 f_y A_{stw}$

$C_{uf} = 0.446 f_{ck} (b_f - b_w) D_f$, $T_{uf} = 0.87 f_y A_{stf}$

$C_{uw} = T_{uw}$

$\Rightarrow 0.36 \times 20 \times 300 \times 480 = 0.87 \times 415 \times A_{stw}$

$\Rightarrow A_{stw} = 2871.6 \text{ mm}^2$

$C_{uf} = T_{uf}$

$0.446 f_{ck} (b_f - b_w) D_f = 0.87 f_y A_{stf}$

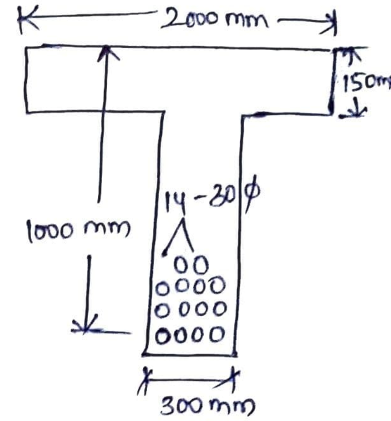
$0.446 \times 20 (2000 - 300) \times 150 = 0.87 \times 415 \times A_{stf}$

$A_{stf} = 6356.4 \text{ mm}^2$

∴ Area of reinforcement, $A_{st} = A_{stw} + A_{st.f}$

$= 2871.6 + 6356.4$

$= 9228 \text{ mm}^2$



Assume 30 mm dia bar.

No. of bars $= \dfrac{9228}{\pi/4 \times 30^2} = \cancel{18.79}\ 13.05 \simeq 14$ bars

3. Design the tension reinforcement for a T-beam. $b_f = 1500\text{mm}$, $b_w = 300\text{mm}$, $d = 750\text{mm}$, $D_f = 200\text{mm}$, $f_{ck} = 20 \text{ N/mm}^2$, $f_y = 415 \text{ N/mm}^2$. Ultimate Design bending moment, $M_u = 1600$ KNm.

Sol:- $\dfrac{D_f}{d} = \dfrac{200}{750} = 0.266 > 0.2$, $x_{umax} = 0.48 \times 750 = 360$ mm

$M_{ulim} = 0.36 f_{ck} b_w x_{umax} (d - 0.42 x_{umax}) + 0.446 f_{ck} (b_f - b_w) y_f (d - y_f/2)$

$= 0.36 \times 20 \times 300 \times 360 (750 - 0.42 \times 360) + 0.446 \times 20 (1500 - 300) \times 184 (750 - \frac{184}{2})$

$= 465626880 + 1307577600$

$= 1773.2$ KNm

$y_f = 0.15 x_u + 0.65 D_f$

$= 0.15 \times 360 + 0.65 \times 200 = 184$ mm

$M_u < M_{ulim}$ - It is a under reinforced section.

* From the value of M_u, Compute the neutral axis depth (x_u) by replacing x_{umax} with x_u.

$M_u = 0.36\, f_{ck}\, b_w\, x_u \left(d - 0.42\, x_u\right) + 0.446\, f_{ck}\, (b_f - b_w)\, y_f\, (d - y_f/2)$

$y_f = 0.15\, x_u + 0.65\, D_f$

$= 0.15\, x_u + 130$

$\therefore 1600 \times 10^6 = 0.36 \times 20 \times 300\, x_u \left(750 - 0.42\, x_u\right) + 0.446\, f_{ck}\, (1500 - 300)$
$(0.15\, x_u + 130)\left(750 - \frac{0.15\, x_u + 130}{2}\right)$

$1600 \times 10^6 = 1620000\, x_u - 907.2\, x_u^2 + 1109700\, x_u - 121.5\, x_u^2 - 96174000 - 105300\, x_u$

$\Rightarrow 1028.7\, x_u^2 - \cancel{2624}00\, x_u + 63826\,0000 = 0$

$x_u = 272.25\,mm \simeq 273\,mm$

$C_{uw} = T_{uw}$

$0.36\, f_{ck}\, b_w\, x_u = 0.87\, f_y\, A_{stw}$

$0.36 \times 20 \times 300 \times 273 = 0.87 \times 415\, A_{stw}$

$A_{stw} = 1633.23\, mm^2$

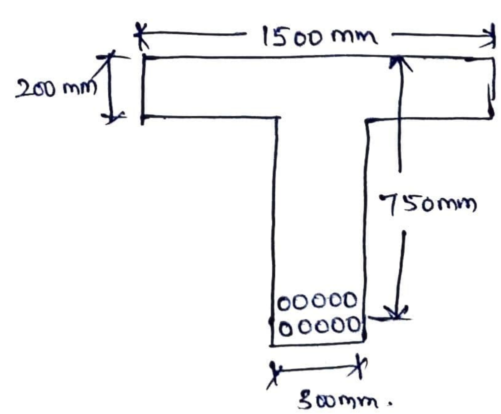
$C_{uf} = T_{uf} \Rightarrow 0.446\, f_{ck}\, (b_f - b_w)\, y_f = 0.87\, f_y\, A_{stf}$

$0.446 \times 20\,(1500 - 300)\,(0.15 \times 273 + 0.65 \times 200) = 0.87 \times 415 \times A_{stf}$

$A_{stf} = 5113.58\, mm^2$

$\therefore A_{st} = A_{stw} + A_{stf}$

$= 6746.81\, mm^2$



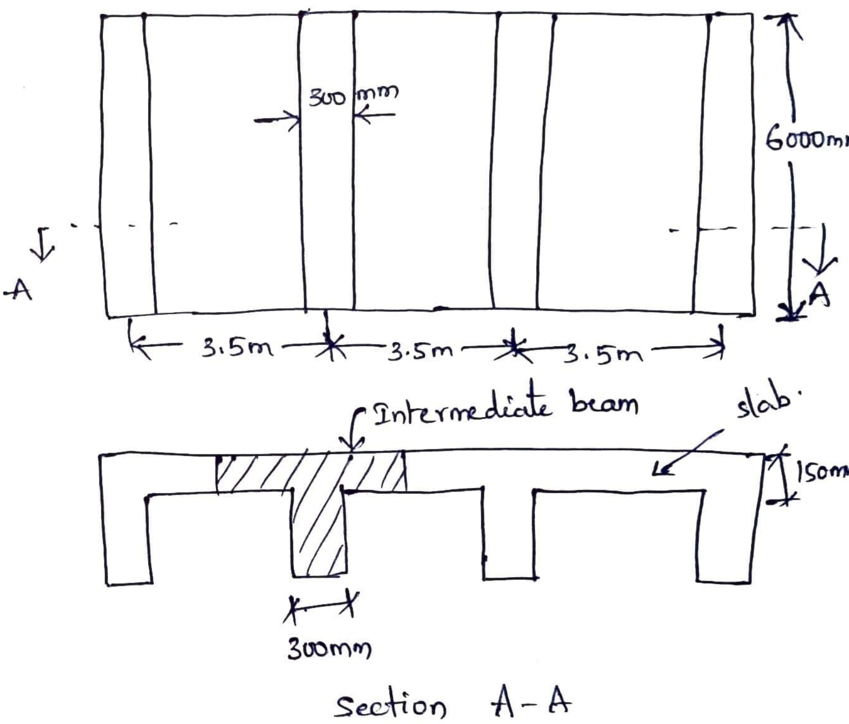
Assume the diameter of bar as 30 mm.

No. of bars $= \dfrac{6746.81}{\pi/4 \times 30^2} \simeq 10$ bars

$\therefore$ 10 bars of 30 mm dia bars Provided in tension zone.

4. A T-beam floor consists of 15 cm thick R.C slab monolithic thick with 30 cm wide beams. The beams are spaced at 3.5m c/c and their effective span is 6m as shown in figure. If the super imposed load on the slab is 5 KN/m². Design an intermediate beam using M_{20} and Fe 415 materials.

Sol:-



Section A-A

$$b_f = \frac{l_o}{6} + b_w + 6 D_f$$

$$= \frac{6000}{6} + 300 + 6 \times 150$$

$= 2.2m$ } lesser value

$b_f = 3.5m$

∴ Effective width of flange,

$b_f = 2200$ mm.

Loads:-

Dead load of slab = $1 \times 0.15 \times 25 = 3.75$ KN/m² (Assume b = 1m)

Super imposed load = 5 KN/m²

∴ Total load = 8.75 KN/m²

Load per meter length of the beam = load on slab × c/c distance b/n beams

$= 8.75 \times 3.5$

$= 30.62$ KN/m

Let us adopt overall depth 'D' = 400 mm and effective cover, d' = 40 mm

∴ Effective depth = 400 − 40 = 360 mm.

self weight of the web of the beam = width of web × depth of web × density of concrete

$= 0.3 \times (0.4 - 0.15) \times 25$

$= 1.875$ KN/m

$\therefore$ The total load on beam per unit run = $30.62 + 1.875$

$= 32.495$ KN/m

$\therefore$ Factored bending moment $= 1.5 \dfrac{wl^2}{8}$

$= \dfrac{1.5 \times 32.495 \times 6^2}{8}$

$= 219.2$ KNm

Assume $x_u < D_f$, $x_u = \dfrac{0.87 f_y A_{st}}{0.36 f_{ck} b_f} = \dfrac{0.87 \times 415 A_{st}}{0.36 \times 20 \times 2200}$

$= 0.022 A_{st}$

$M_u = 0.87 f_y A_{st} (d - 0.42 x_u)$

$219.2 \times 10^6 = 0.87 \times 415 \times A_{st} (360 - 0.42 \times 0.022 A_{st})$

$3.336 A_{st}^2 - 129978 A_{st} + 219.2 \times 10^6 = 0$

$\therefore A_{st} = 1766.5 \text{ mm}^2$

Minimum $A_{st} \Rightarrow \dfrac{A_{st\,min}}{b_w d} = \dfrac{0.85}{f_y}$

$\Rightarrow A_{st\,min} = \dfrac{0.85 \times 300 \times 360}{415} = 221.2 \text{ mm}^2$

Maximum $A_{st} = 0.04\, b_w D = 0.04 \times 300 \times 400 = 4800 \text{ mm}^2$

$\therefore A_{st} = 1766.5 \text{ mm}^2$

5) Calculate the amount of steel required in a T-beam to develop a moment of resistance of 300 KNm at working loads. The dimensions of the beam are $b_f = 750$ mm, $D_f = 100$ mm, $d = 500$ mm, $b_w = 200$ mm, $d' = 50$ mm, use M_{20} grade concrete and Fe_{415} steel reinforcement.

Sol:-

Ultimate moment of resistance, $M_u = 1.5 \times 300$

$= 450$ KNm

$$\frac{D_f}{d} = \frac{100}{500} = 0.2$$

$x_{umax} = 0.48 \times 500 = 240$ mm

Let us assume that section is balanced and neutral axis lies in the web.

Factored bending moment, M_{ulim} =

$$0.36\, f_{ck}\, b_w x_{umax} (d - 0.42\, x_{umax}) + 0.446\, f_{ck} (b_f - b_w) D_f (d - D_f/2)$$

$$= 0.36 \times 20 \times 200 \times 0.48 \times 500 (500 - 0.42 \times 0.48 \times 500) + 0.446 \times 20 (750 - 200) \times 100 (500 - 100/2)$$

$= 360.71$ KNm

$M_u > M_{ulim}$ → It is over reinforced section.

Hence Compression steel is required. Area of tension steel corresponding to a moment equal to 360 KNm is given by

$$M_u = 0.87\, f_y\, A_{st_1} (d - 0.42\, x_{umax})$$

$$360 \times 10^6 = 0.87 \times 415\, A_{st_1} (500 - 0.42 \times 240)$$

$\Rightarrow A_{st} = 2497.7\ mm^2$

The remaining moment is to be resisted by Compression steel Asc and additional steel A_{st2}. If effective Cover to Compression steel is 50 mm.

$$\varepsilon_{sc} = 0.0035 \left(1 - \frac{50}{240}\right) = 0.0027$$

$$f_{sc} = 0.87\, f_y = 0.87 \times 415 = 361 \ N/mm^2$$

$$M_u - M_{ulim} = f_{sc} A_{sc} (d - d')$$

$$(450 - 360) \times 10^6 = 361 \times A_{sc} (500 - 50)$$

$$A_{sc} = 554.01 \ mm^2$$

$$A_{st2} = \frac{f_{sc} A_{sc}}{0.87 f_y} = \frac{361 \times 554.01}{0.87 \times 415} = 554.01 \ mm^2$$

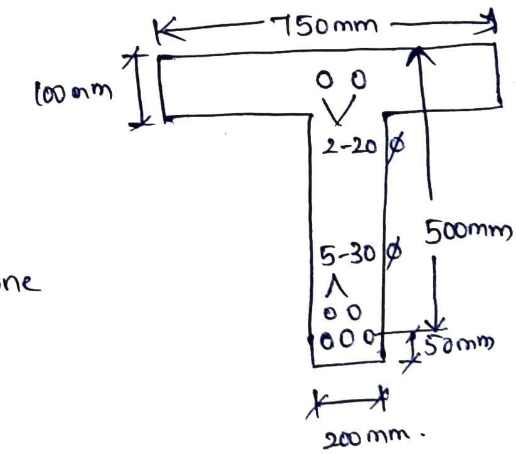
$$\therefore A_{st} = A_{st1} + A_{st2} = 2497.7 + 554.01 = 3051.7 \ mm^2$$

Assume 30 mm dia bars in tension zone.

$$\text{No. of bars} = \frac{3051.7}{\pi/4 \times 30^2} = 5 \text{ bars}$$

Assume 20 mm dia bars in Compression zone

$$\text{No. of bars} = \frac{554.01}{\pi/4 \times 20^2} = 2 \text{ bars}$$



$\therefore$ 5 bars of 30mm ø provided in tension zone and 2 bars of 20mm dia provided in Compression zone.