UNIT - II

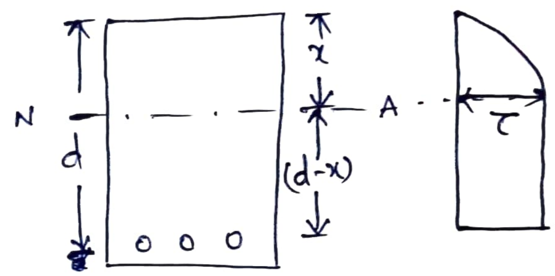
Design for shear, Torsion and Bond

* Shear in a beam is caused due to the change of bending moment in the span.

* For a homogeneous material, the shear stress at any section is given by $\tau = \frac{VA\bar{y}}{Ib}$

Shear stress in R.C beam:-

* Under elastic condition, the total shear 'V' is given by the area of stress diagram,



shear distribution

$$V = \frac{2}{3}x \cdot \tau b + (d-x)\tau b$$

$$= \tau b\,(d - x/3)$$

$$\tau = V/bz, \quad z = \text{lever arm} = (d - x/3)$$

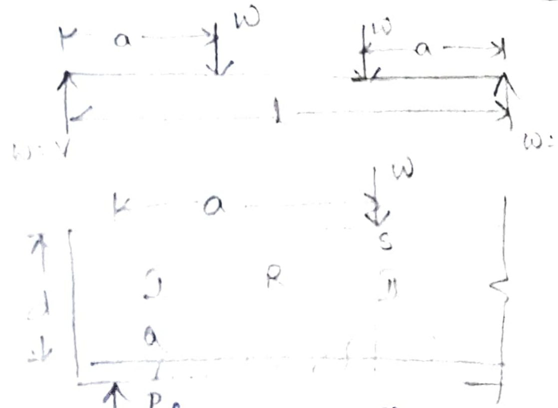
* For simplicity, a simple concept of nominal stress have been introduced and expressed as,

$$\tau_v = \frac{V_u}{bd} \Rightarrow V_u = \tau_v\, bd$$

Shear failure :-

* A simply supported beam symmetrically loaded by two equal concentrated loads acting at distance 'a' from supports.

Shear force, $V = W$



Bending moment, $M = W \times a = V \times a$

$$a = \frac{M}{V}$$

* The distance 'a' represents the dominance of shear over flexure called as "shear span".

* For large values of a/d (>6), failure is only due to flexure.

* For a/d in between 2.5 to 3 and 6, failure is a combined effect of shear and bending. Crack initiates in vertical direction due to flexure and propagates in the inclined direction towards loading point known as flexure - shear crack or diagonal crack.

  - Diagonal tension failure.

* Small values of a/d $(1 < a/d < 2.5$ to $3)$, shear is predominant, the diagonal crack is developed along with flexure crack. The shear failure is occured by one of the following modes.

  Shear, bond failure

  (a) Anchorage failure

  (b) crushing failure at support

  (c) Flexural failure

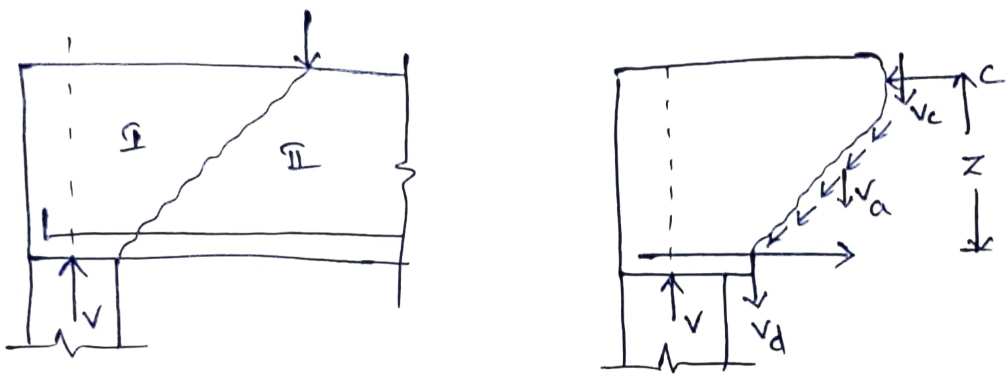
  (d) Tension cracking failure at support

  (e) Arch rib crushing failure.

* deep beam $(a/d < 1)$, shear is more predominant.

Ultimate shear strength of beam without shear reinforcement

* The load at which the diagonal crack forms is taken as the strength of beam in shear.
* The mechanism formed in beam at ultimate state of shear due to formation of the diagonal crack is known as mechanism of shear transfer at ultimate state.



* The load causing diagonal crack is resisted by beam, consisting of
  (i) shear resisted by dowel action, V_d (15 to 30%) of the flexural reinforcement.
  (ii) shear carried by the vertical component of the interface shear 'V_a' along the crack due to aggregate interlock. (33 to 50%)
  (iii) shear resisted by uncracked concrete 'V_c' in compression zone. (20 to 40%)

∴ ~~total~~ shear resisting capacity, $V = V_d + V_a + V_c$

Factors effecting shear resistance of a R.C member :-

1. characteristic strength of Concrete
2. Percentage of longitudinal tension reinforcement
3. shear span to depth ratio (a/d)
4. Compressive force
5. Effect of cross section
6. Effect of shear reinforcement

Shear strength of beam with shear reinforcement :-

* When the shear acting on the section is greater than the shear capacity of concrete (V_{uc}), the diagonal crack emerges.

* In order to prevent the formation of the diagonal cracking and its widening, additional shear resistance is required to hold together the two parts of the beam.

* The additional shear resistance is provided by the transverse reinforcement.

* This transverse reinforcement is known as web steel (or) shear reinforcement

* Normally, the shear reinforcement consists of,

  (a) Vertical stirrups, or links, at right angles to the longitudinal reinforcement

  (b) Inclined stirrups making an angle greater than or equal to 45° with longitudinal steel.

  (c) Inclined bent up bars obtained by bending some of the main longitudinal bars at an angle of 45° or more.

  (d) Combinations of (a), (b) and (c)

* In case of beam with shear reinforcement, The total shear carried by concrete and web steel, is

$$V_u = V_{uc} + V_{us}$$

V_{uc} = Ultimate shear resisted by concrete member without shear reinforcement

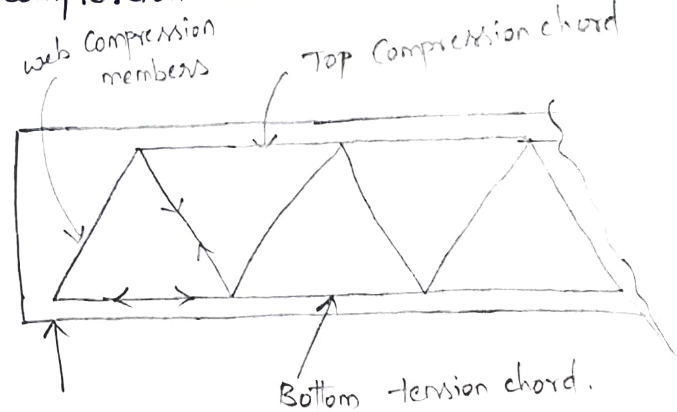
V_{us} = Ultimate shear carried by shear reinforcement.

Shear resisted by shear reinforcement :-

* The ultimate shear resistance (V_{us}) offered by the shear reinforcement is obtained by considering the truss analogy.

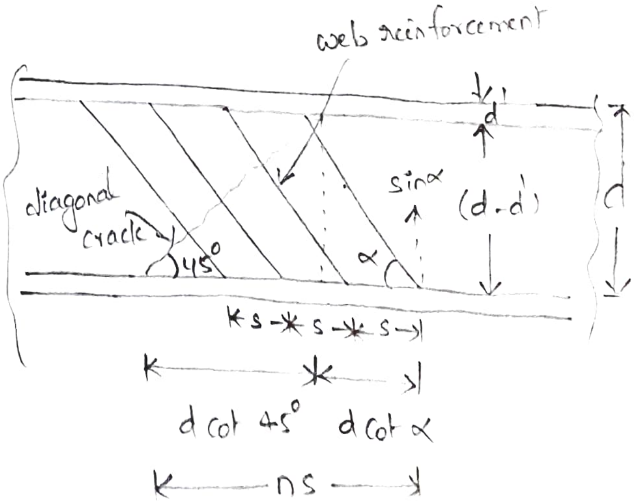
* According to this analogy, the R.C member is assumed to be made up of an analogous truss in which concrete in compression zone acts as a compression chord, the longitudinal tensile reinforcement acts as a tension chord, Transverse reinforcement forms a compression web members.

* The diagonal crack is assumed to extend at an angle of 45° with the horizontal.



Truss analogy for R.C beams

* Figure shows the diagonal crack intersecting 'n' number of web reinforcing bars inclined at an angle 'α' with the axis of member and spaced at 's' apart.



* At failure, 'n' number of web bars crossing the diagonal crack reach its ultimate strength 'f_{yd}'.

* For equilibrium,

shear carried by shear reinforcement = Sum of the vertical components of tensile forces developed in shear reinforcement

$\therefore V_{us} = n A_{sw} f_{yd} \sin\alpha$

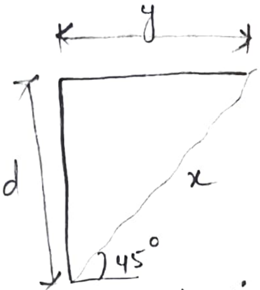
where, V_{us} = Ultimate shear carried by shear reinforcement.

A_{sw} = Total c/s area of steel in web within a distance 's'

f_{yd} = Design yield stress of steel ($= 0.87 f_y$)

n = number of links or bars crossed by the crack.

α = Inclination of the web steel with the axis of the member.



* From geometry,

$$ns = (d-d')\cot 45^\circ + (d-d')\cot\alpha$$

$$= (d-d')(1+\cot\alpha)$$

$$n = d(1+\cot\alpha)/s$$

$\therefore \tan 45 = \dfrac{d}{y}$

$y = \dfrac{d}{\tan 45^\circ}$

$= d\cot 45^\circ$

$\tan 45^\circ = \dfrac{\sin 45^\circ}{\cos 45^\circ}$

$\sin 45^\circ = \dfrac{d}{x}$

$\cos 45^\circ = \dfrac{y}{x}$

$$\therefore V_{us} = \frac{d(1+\cot\alpha) A_{sw} \times 0.87 f_y \sin\alpha}{s}$$

$$= \frac{0.87 f_y A_{sw} d}{s} (\sin\alpha + \cos\alpha)$$

This equation can be used to find shear strength of inclined stirrups.

* For Vertical stirrups, $\alpha = 90^\circ$

$$V_{usv} = \frac{0.87 f_y A_{sv} d}{S_v}$$

$$\therefore \text{spacing of vertical stirrup, } S_v = \frac{0.87 f_y A_{sv} d}{V_{usv}}$$

* Maximum spacing of stirrup $\ngtr$ 0.75d or $\ngtr$ 300 mm – whichever is less.

* Amount of minimum shear reinforcement,

$$\frac{A_{sv}}{bs} \geq \frac{0.4}{0.87 f_y}$$

$$A_{sv} = \frac{0.4\, bs}{0.87 f_y} \Rightarrow s = \frac{0.87 f_y A_{sv}}{0.4\, b}$$

* when τ_v exceeds τ_c given in Table - ~~20~~ 19, Pg - 73, shear reinforcement shall be provided.

Table : 20.

Concrete grade	M_{15}	M_{20}	M_{25}	M_{30}	M_{35}	M_{40}
$\tau_{c\,max}$ N/mm²	2.5	2.8	3.1	3.5	3.7	4.0

* the design strength of concrete τ_{c} for different values of percentage of tension steel 'P_t' and grades of concrete are given in Table - 19, Pg - 73

* τ_v should be always less than τ_c max given in table 20, Pg 73.

1. A simply supported beam 300 mm wide, 600 mm effective depth carries a UDL of 70 KN/m including self weight over an effective span of 6m. The reinforcement consists of 5 bars of 25 mm diameter out of these 2 bars ~~can~~ are be safely bent at 1m distance from support. Design the shear reinforcement for the beam. use M15 and Fe 415 reinforcement.

Sol:- Given data:-

$b = 300\,mm$, $d = 600\,mm$, $l = 6m$

$f_{ck} = 15\,N/mm^2$, $f_y = 415\,N/mm^2$, $w = 70\,KN/m$

shear force $= \frac{wL}{2} = \frac{70 \times 6}{2} = 210\,KN$

Factored shear force $= 210 \times 1.5 = 315\,KN$

$A_{st} = 3 \times \pi/4 \times 25^2 = 1472.81\,mm^2$

$A_{sv} = 2 \times \pi/4 \times 25^2 = 981.87\,mm^2$

$p_t = \frac{100\,A_{st}}{bd} = \frac{100 \times 1472.81}{300 \times 600} = 0.81$

$p_t - 0.75$, $\tau_c = 0.54\,N/mm^2$

$p_t - 1.00$, $\tau_c = 0.60\,N/mm^2$

for $p_t - 0.81$, $\tau_c = 0.54 + \frac{0.60 - 0.54}{1 - 0.75}(0.81 - 0.75)$

$= 0.556\,N/mm^2$

Shear resisted by Concrete, $V_{uc} = \tau_c \times bd$

$= 0.556 \times 300 \times 600$

$= 100.4\,KN$.

$-(70 \times 1.5 \times 0.6)$

Nominal shear stress, $\tau_v = \dfrac{V_u}{bd} = \dfrac{315 \cancel{\times 10^3}}{300 \times 600} = \cancel{1.75}\ 1.4\ N/mm^2$

(As per IS code, the critical shear occurs at a distance d: 600mm)

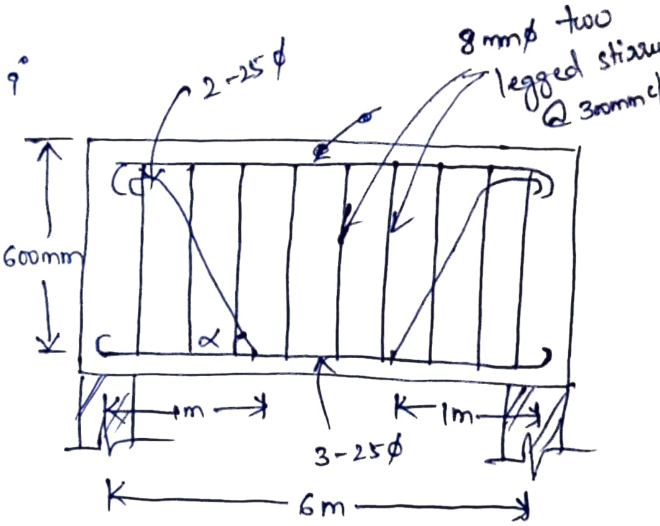
$\tau_v > \tau_c$ — Shear reinforcement has to be provided.

$\tau_{c\,max} = 2.8\ N/mm^2 > \tau_v$

Shear resisted by bent up bars, $V_{us\alpha} = 0.87\, f_y\, A_{sv} \sin\alpha$

$V_{us\alpha} = 0.87 \times 415 \times 981.87 \times \sin 30.9^\circ$

$= 182.05$ kN.



$V_{uc} + V_{us\alpha} = 103.4 + 182.05$

$= 285.45$ kN.

$V_{usv} = V_u - (V_{uc} + V_{us\alpha})$

$= 315 - 285.45 = 29.55$ kN.

$\tan\alpha = \dfrac{600}{1000} \Rightarrow \alpha = 30.9^\circ$

Assume 8 mm dia two legged stirrups, $A_{sv} = \pi/4 \times 8^2 \times 2 = 100.54\ mm^2$

Shear supported by vertical stirrups, $V_{usv} = \dfrac{0.87 f_y A_{sv} d}{S_v}$

$29.55 \times 10^3 = \dfrac{0.87 \times 415 \times 100.54 \times 600}{S_v}$

$\Rightarrow S_v = 737.05$ mm

Spacing is lesser of

(i) $S_v = 737.05$ mm

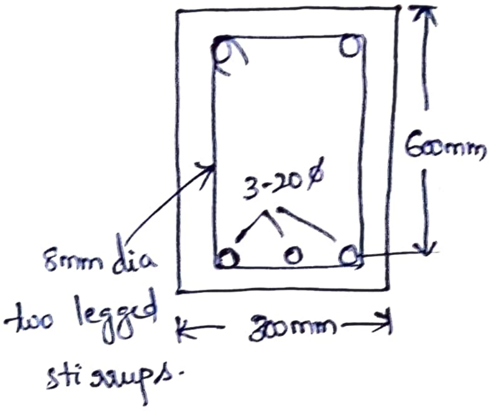
(ii) $0.75\, d = 0.75 \times 600 = 450$ mm

(iii) 300 mm.

$\therefore$ Provide $S_v = 300$ mm.

2. A reinforced beam has a support section with a width of 300 mm and effective depth of 600 mm. The support section is reinforced with 3 bars of 20 mm dia and 8 mm dia two legged stirrups at a spacing of 200 mm is provided as shear reinforcement near supports. Use M_{20} and Fe_{415} materials. Estimate the shear strength of the support section.

Sol:-



$b = 300$ mm, $d = 600$ mm

$A_{st} = 3 \times \pi/4 \times 20^2 = 942.6 \text{ mm}^2$

$S_v = 200$ mm

$f_y = 415 \text{ N/mm}^2$, $f_{ck} = 20 \text{ N/mm}^2$

$A_{sv} = \pi/4 \times 8^2 \times 2 = 100.5 \text{ mm}^2$

$$p_t = \frac{100 A_{st}}{bd} = \frac{100 \times 942.6}{300 \times 600} = 0.523$$

From Table, 19, Pg 73.

$0.5 \rightarrow 0.48$

$0.75 \rightarrow 0.56$

For 0.523, $\tau_c = 0.486 \text{ N/mm}^2$

shear resistance by concrete, $V_{uc} = \tau_c \times bd$

$= 0.486 \times 300 \times 600 = 87.55$ kN.

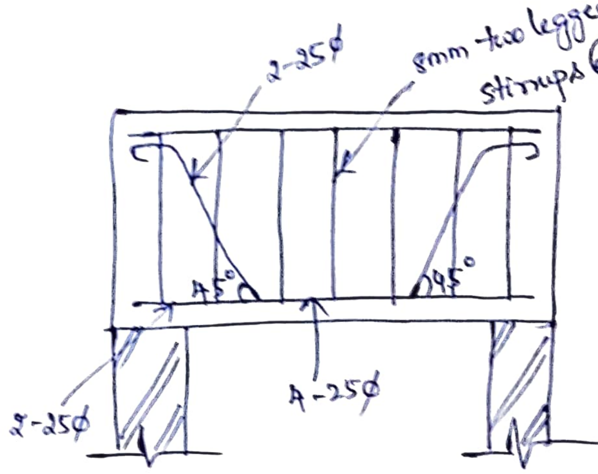
$$\text{shear resisted by stirrups, } V_{us} = \frac{0.87 f_y A_{sv} d}{S_v} = \frac{0.87 \times 415 \times 100.5 \times 600}{200}$$

$= 108.8$ kN

$\therefore$ Total shear resistance, $V_u = V_{uc} + V_{us} = 87.55 + 108.8$

$= 196.3$ kN

3. A reinforced concrete beam of rectangular section has a width of 250mm and effective depth of 500mm, the beam is reinforced with 4 bars of 25 mm dia on tension side, two of the tension bars are bent at 45° near the support section. In addition, the beam has 8 mm diameter two legged stirrups at 150 mm c/c is provided near the support. use M20 and Fe 415 materials and estimate the ultimate shear strength at the support section.



sol:- $b = 250mm$, $d = 500mm$

$A_{st} = 2 \times \pi/4 \times 25^2 = 981.87 \ mm^2$

$A_{sv\alpha} = \pi/4 \times 2 \times 25^2 = 981.87 \ mm^2$

$A_{sv} = 2 \times \pi/4 \times 8^2 = 100.5 \ mm^2$

$p_t = \frac{100 A_{st}}{bd} = \frac{100 \times 981.87}{250 \times 500} = 0.78$

0.75 → 0.56

1.00 → 0.62

For $p_t = 0.78$, $\tau_c = 0.567 \ N/mm^2$

shear resisted by Concrete, $V_{uc} = \tau_c bd$

$= 0.567 \times 250 \times 500$

$= 70.875 \ KN$

shear resisted by vertical stirrups, $V_{us} = \frac{0.87 f_y A_{sv} d}{S_v}$

$= \frac{0.87 \times 415 \times 100.5 \times 500}{150}$

$= 120.95 \ KN$

Shear resisted by bent up bars : $V_{us\alpha} = 0.87 f_y A_{sv\alpha} \sin\alpha$

$= 0.87 \times 415 \times 981.87 \times \sin 45°$

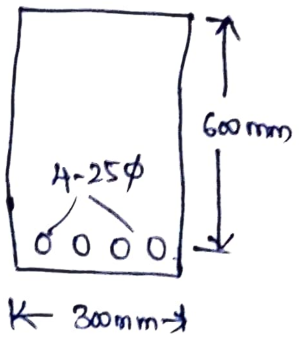
$= 250.67$ kN

$\therefore$ Total shear resistance $= 70.875 + 120.95 + 250.67$

$= 442.49$ kN

4. A reinforced concrete beam of rectangular section 300 x 600mm effective depth is reinforced with 4 bars of 25 mm diameter. The beam has to resist a factored shear force of 400 kN at support section. Use M_{20} and Fe415 materials. Design the vertical stirrups ~~of~~ for the section.

Sol:- $b = 300$mm, $d = 600$ mm



$A_{st} = 4 \times \pi/4 \times 25^2 = 1963.75$ mm^2

Factored shear force $= 400$ kN $= V_u$

$p_t = \frac{100 A_{st}}{bd} = \frac{100 \times 1963.75}{300 \times 600} = 1.09$

$1 \longrightarrow 0.62$

$1.25 \longrightarrow 0.67$

for $p_t = 1.09$, $\tau_c = 0.638$ N/mm^2

shear resisted by concrete, $V_{uc} = \tau_c bd$

$= 0.638 \times 300 \times 600 = 114.84$ kN

shear resisted by stirrups, $V_{us} = V_u - V_{uc}$

$= 400 - 114.84$

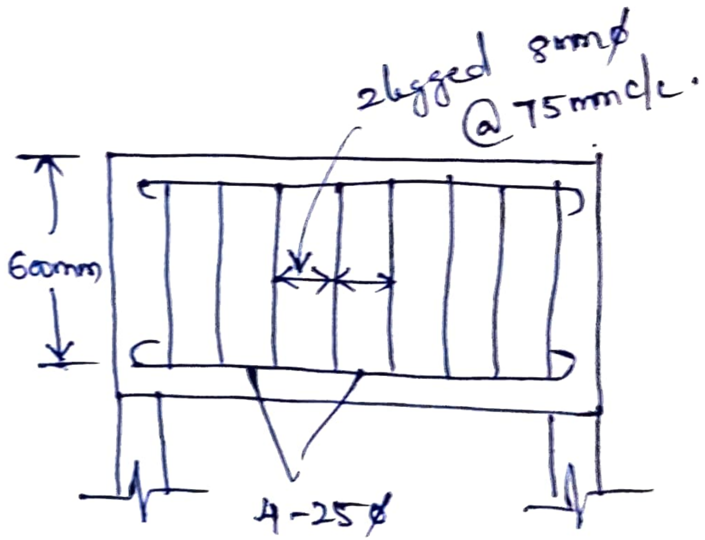
$= 285.16$ kN

$$V_{us} = \frac{0.87\ f_y\ A_{sv}\ d}{S_v}$$

Assume 8mm diameter, 2 legged stirrup.

$$A_{sv} = \pi/4 \times 8^2 \times 2 = 100.5\ mm^2$$

$$\therefore 285.16 \times 10^3 = \frac{0.87 \times 415 \times 100.5 \times 600}{S_v}$$



$$\left.\begin{aligned} S_v &= 76.34\ mm \\ 0.75\,d &= 0.75 \times 600 = 450\,mm \\ &300\,mm \end{aligned}\right\}\ \text{lesser}$$

$$\therefore S_v = 76.34 \simeq 75\ mm$$

Provide 8mm diameter two legged stirrups at 75 mm c/c.

Bond :-

* The force which prevents the relative movement between the two Constituent materials is known as "Bond".
* The force which prevents the impending shear action b/n two at their interface is known as "Interfacial shear".
* "Bond stress" is the shear stress acting parallel to the bar on the interface b/n the bar and concrete.

Types of Bond :-

1. Anchorage bond - The bond required to be developed for transfering the force from the bar to the surrounding concrete.
2. Development bond - The bond required to be developed for transfering the force to the bar from the surrounding concrete.

Factors affecting bond resistance :-

1. Grade of Concrete - Bond strength increases with increase in grade of Concrete
   - when actual stresses in concrete exceeds the tensile strength of concrete, crack develops.
2. Diameter of bar - Bond decreases with increase in bar diameter.
   - Bars of larger dia lead to greater cracking.
3. Bar profile condition - Deformed bars gives greater bond resistance because of the interlocking of ribs with surrounding concrete.
4. Nature of force in bar - the bond strength depends upon the nature of force (Compression or Tension) in the bar.

5. Grouping of bars or Bundling - For bundled bars, the surface area coming in contact with concrete is reduced with the result bond resistance decreases.

6. Bends and hooks in bars - The bond resistance increases with increase in bend angle.

7. Curtailment of bars in tension zone - The curtailment of bar creates the condition of stress concentration and differential strain at the point of cut-off, resulting in loss of shear and bond.

8. Cover and proximity of adjacent bars - The proximity of bars and inadequate cover causes side splitting and/or horizontal splitting and loss of bond resulting in ultimate total cracking.

9. Repeated or cyclic loading and vibrations - In case of repeated or cyclic loading and vibrations, the frictional resistance diminishes, resulting in deterioration of bond.

Anchorage length :-

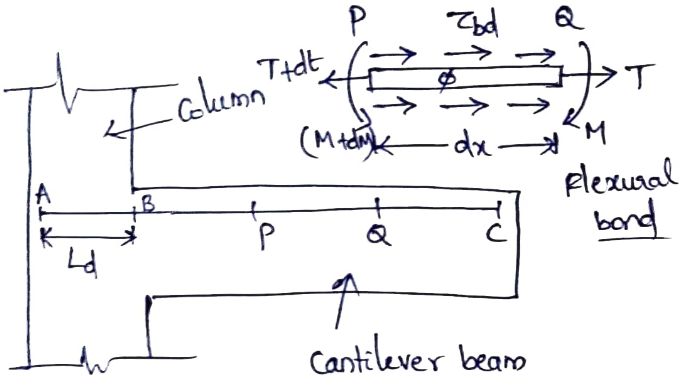
* Anchorage length is a length required to transfer the force from steel to concrete.
* Anchorage bond stress arises when a bar carrying certain force 'F' is terminated.
* Let the length required to transfer a force 'F' in the bar to the surrounding concrete by means of bond, before it is terminated ~~is~~ be 'Ld'.

* The bond considerations require that the bar must extend beyond that section by length 'L_d' before it is terminated, so that it does not get pulled out.

* This length 'L_d' of embedment of bar beyond the theoritical termination point is known as "Anchorage length".

* Consider a uniformly loaded cantilever designed to carry given load.

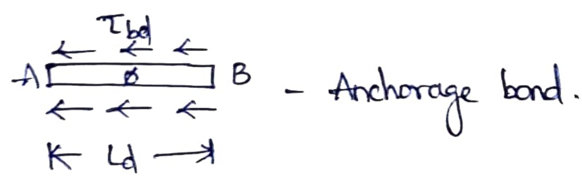
* Let the main tension reinforcement at support consists of only one bar of diameter 'ϕ'.



* The maximum bending moment and hence maximum tension occurs at the face of support i.e point B.

* The maximum tension 'T' which occurs at B is given by

$$T = \sigma_s \times \frac{\pi \phi^2}{4}$$



* This force must be transferred from steel to concrete through bond acting over the perimeter of the bar in a length $AB = L_d$.

For equilibrium, $\tau_{bd} \times \pi \phi \times L_d = \sigma_s \, \pi/_4 \times \phi^2$

$$\therefore L_d = \frac{\sigma_s}{4\tau_{bd}} \phi = K \phi$$

where, τ_{bd} = Average design bond stress acting over the surface area.

σ_s : Actual stress in bar at the section considered at design load.

ϕ = Nominal diameter of bar.

L_d = Anchorage length = Development length.

* In limit state design of collapse, the design stress in bar = $0.87 f_y$

$$\therefore L_d = \frac{0.87 f_y}{4 \tau_{bd}} \phi$$

The design bond stress, τ_{bd} values are given in ~~Table~~ cl. 26.2.1.1, Pg. 43

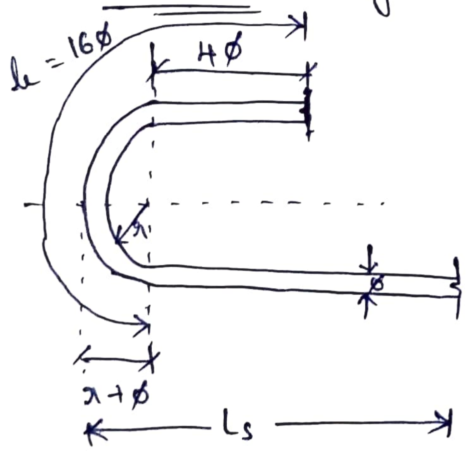
* For deformed bars confirming to IS-1786, these values shall be increased to 60%.

Development length :-

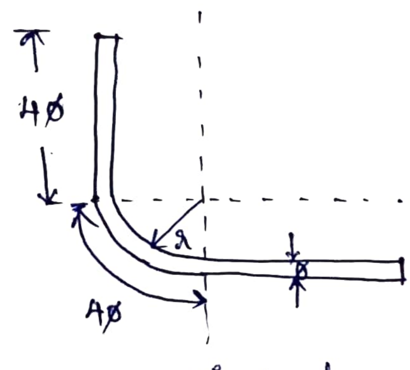
* The development length is defined as the length of bar required on either side of section to develop the required stress in steel at the section through bond.

* In the cantilever beam, the length BC of the bar must be sufficient to develop the stress from zero at C to its maximum value at B.

Equivalent Anchorage lengths of standard hooks and bends :-



U Hook



90° bend

$r = K\phi$

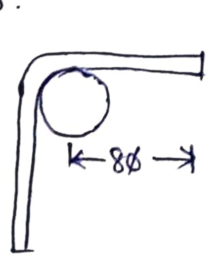
K = 2 for plain bars

K = 4 for HYSD bars

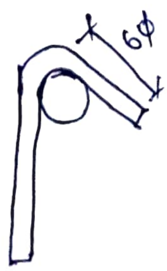
cl. 26.2.2.1 b) i)

For every 45°, anchorage value of bend = 4ϕ.

Stirrups :-



90° bend



135° bend

180° bend

1. A cantilever beam having a width of 200 mm and effective depth of 300 mm supports a UDL and is reinforced with 4 bars of 16 mm ϕ. If the factored load is 80 KN. Calculate (i) The Maximum local bond stress, (ii) The anchorage length required, (iii) If the anchorage length provided is 900 mm. Find the average bond stress. Use M_{20} grade concrete and Fe 415 steel reinforcement.

Sol:- Given data :-

$b = 200mm$, $d = 300mm$

$A_{st} = 4 \times \pi/4 \times 16^2 = 804.35 \ mm^2$

Factored load = 80 KN, $f_{ck} = 20 \ N/mm^2$, $f_y = 415 \ N/mm^2$

Bond stress, $\tau_{bd} = 1.2 \ N/mm^2$ (For M_{20} grade Concrete)

For HYSD bars τ_{bd} is increased upto 60%.

$\therefore \ \tau_{bd} = 1.2 \times 1.6 = 1.92 \ N/mm^2$

(i) Maximum local bond stress, $\tau_0 = \dfrac{V}{\Sigma_0 \, d}$

Σ_0 = Perimeter of A_{st} $= \pi d n$

$= \pi \times 16 \times 4 = 201 \ mm$

$$\tau_0 = \frac{80 \times 10^3}{201 \times 300} = 1.32 \ N/mm^2$$

(ii) Anchorage length or development length, $L_d = \dfrac{\sigma_{st} \, \phi}{4 \, \tau_{bd}}$

$$= \frac{0.87 \times 415 \times 16}{4 \times 1.92} = 752.18 \ mm.$$

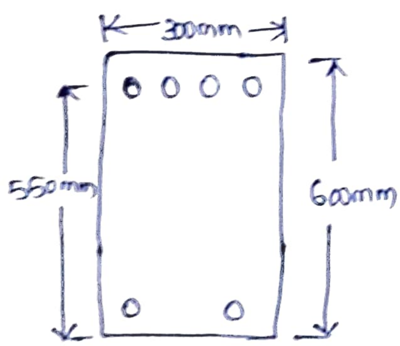
(iii) $L_d = 900\ mm$

$$L_d = \frac{\phi\ \sigma_{st}}{4\ \tau_{bd}}$$

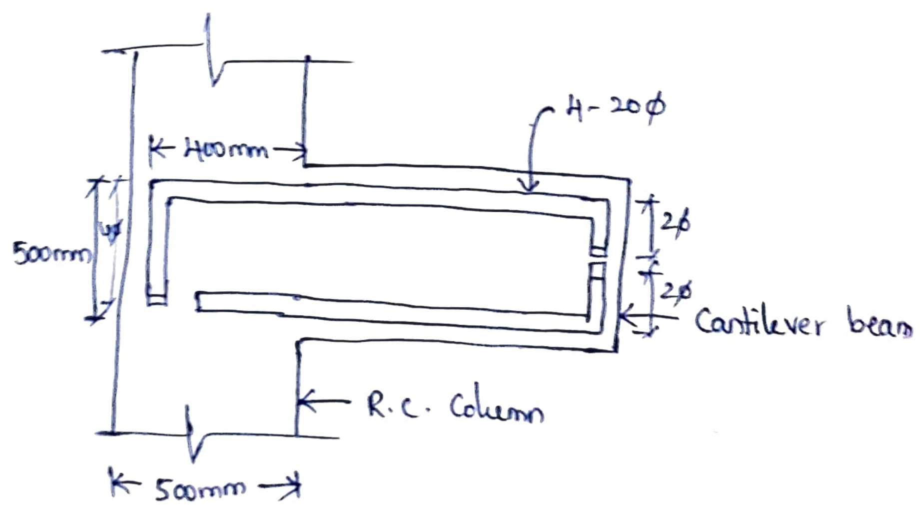
$$\tau_{bd} = \frac{16 \times 0.87 \times 415}{4 \times 900}$$

$$= 1.6\ N/mm^2$$

2. A R.C Cantilever beam of rectangular section 300 x 600mm deep is built into a column 500 mm wide as shown in figure. The cantilever beam is subjected to a hogging moment of 200 KNm at the Junction of the beam and Column. Design suitable reinforcement in beam and check for the required anchorage length. Use M_{20} and Fe_{415} reinforcement.



Section of the face of support



Sol:- Given data :-

$b = 300\ mm$, $d = 550\ mm$, $M_u = 200\ KNm$

$$M_{ulim} = 0.138\ f_{ck}\ b\ d^2$$

$$= 0.138 \times 20 \times 300 \times 550^2$$

$$= 250.47\ KNm$$

$M_u < M_{ulim}$ → under reinforced section.

$$M_u = 0.87 f_y A_{st} d \left(1 - \frac{f_y A_{st}}{f_{ck} bd}\right)$$

$$200 \times 10^6 = 0.87 \times 415 \times A_{st} \times 550 \left(1 - \frac{415 \times A_{st}}{20 \times 300 \times 550}\right)$$

$$A_{st} = 1181 \text{ mm}^2$$

use 20 mm diameter bars.

$$\text{No. of bars}, n = \frac{1181}{\pi/4 \times 20^2} = 4$$

Provide 4 bars of 20 mm diameter.

$$\text{Anchorage length}, L_d = \frac{\phi \, \sigma_{st}}{4 \tau_{bd}}$$

$$= \frac{20 \times 0.87 \times 415}{4 \times 1.2 \times 1.6}$$

$$= 940.23 \text{ mm}$$

The bars are extended into the column to a length of 400mm with a 90° bend and 500mm length as shown in figure.

$$\text{Development length provided} = 500 + 400 + 8 \times 20$$

$$= 1060 \text{ mm} > 940 \text{mm}$$

∴ The development length provided is safe.

Torsion :-

* Torsion may occur due to eccentric loading not lying in the bending plane.

* Torsion is of two types

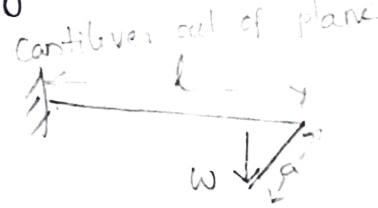
  (a) Compatibility torsion

  (b) Equilibrium torsion

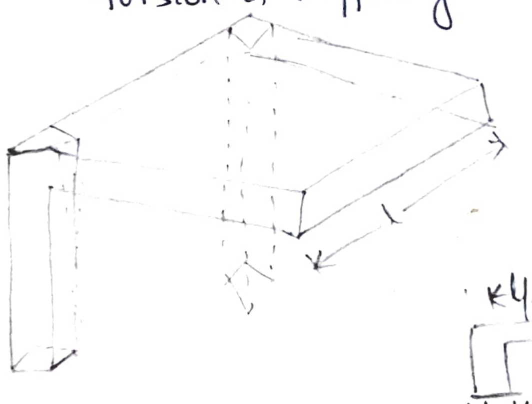
(a) Compatibility torsion - It is induced in a member due to compatibility of rotations at the joint of inter connected members.

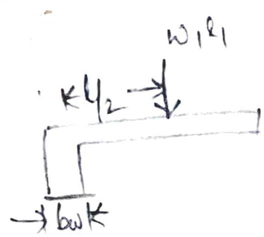
(b) Equilibrium torsion :-

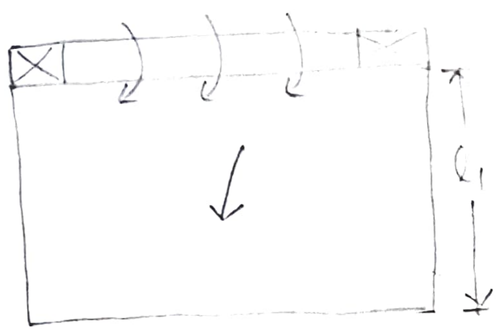
* It is torsion induced as a result of maintaining equilibrium of structure.



Eg:- 1. Cantilever slab inducing torsion in supporting beams.



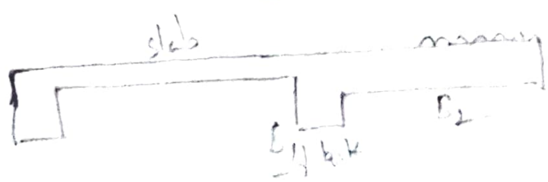




slab of length l_1 subjected to UDL of w_1 per metre.

The twisting moment on the beam ① of width b_w $= (w_1 \times l_1)\left(\frac{l_1}{2} + \frac{b_w}{2}\right)$

2. Beam B_2 projecting from beam B_1



* Torsion produces shear stress, which in turn produces diagonal cracks.
* Concrete being weak in tension, a plain concrete beam when subjected to torsion fails suddenly along a diagonal crack running spirally on the surface of member.

Plain Concrete beam under Pure torsion:-

a. Circular Sections:-

$$\frac{T}{J} = \frac{\tau}{R} = \frac{G\theta}{L}$$

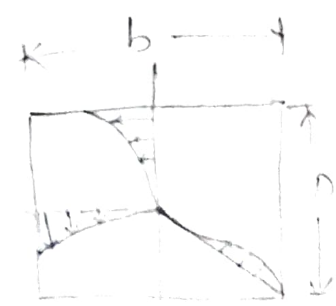
$$\tau_{max} = \frac{T \times D/2}{\pi D^4/32} = \frac{16T}{\pi D^3}$$

$$T = \frac{\pi D^3}{16} f_{cr}$$

$\frac{\pi D^3}{16}$ = Torsional section modulus

f_{cr} = Modulus of rupture (or) Cracking strength of concrete.

b. Non- Circular Sections:-



* The magnitude of the absolute maximum shear stress is given by

$$\tau_{max} = T/Z_T$$

where, Z_T = Elastic torsional section modulus.

c. Flanged Sections:-

$$\tau_{max} = \frac{T \times b_w}{\frac{b_w^3 d}{3} + \frac{(b_f - b_w) D_f^3}{3}}$$

Torsional Reinforcement :-

* Reinforcement in the form of a 45° rectangular spiral is most effective in resisting torque in case of a member of rectangular section.
* Left hand and right hand spirals will be required to be provided for beams which are likely subjected to the reversal of torsional moments.
* Provision of only longitudinal bars without stirrups increase torsional strength to the extent of 15%.
* The most appropriate system of torsion reinforcement shall consists of longitudinal bars together with closed transverse stirrups.

Ultimate strength of R.C members Under Pure Torsion :-

* Three approaches are available to calculate the ultimate strength of R.C members under pure torsion.

a. Equilibrium Theory - By Lessing based on Russian Code.

b. Skew bending theory - followed by ACI code and IS : 456

c. Space truss analogy - followed by IS : 456

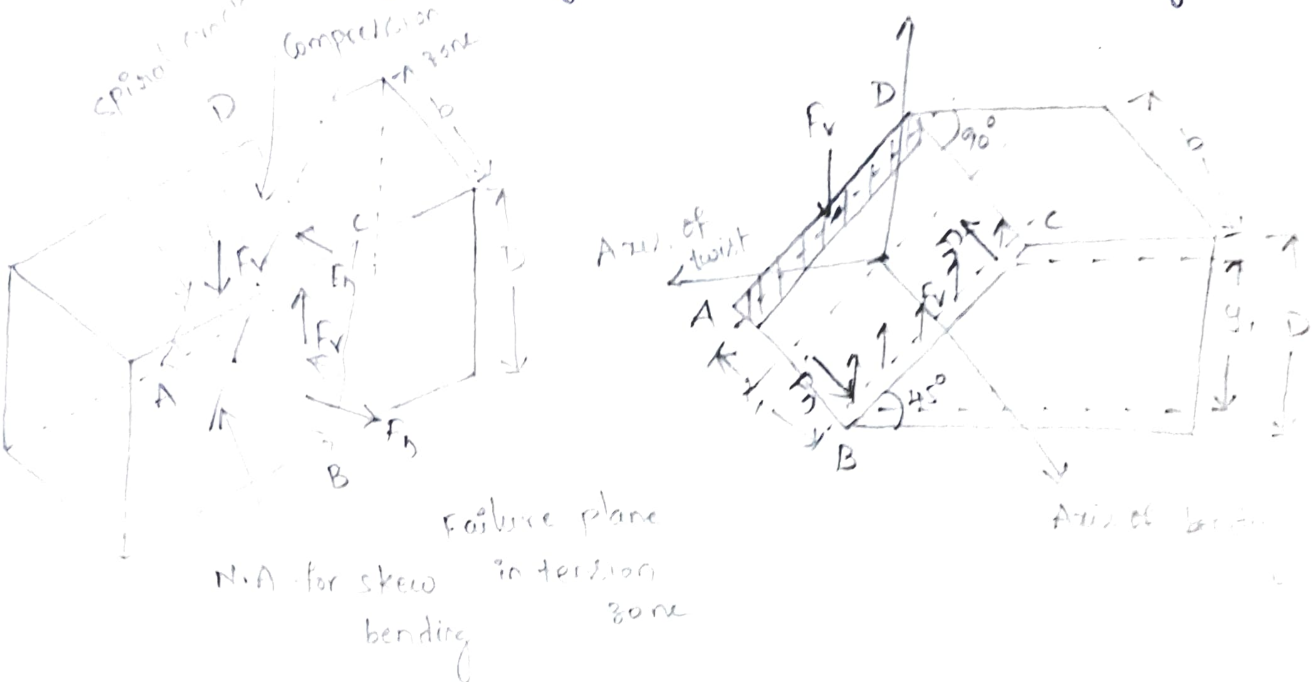
1. Skew bending theory :-

* The skew-bending theory based on a assumption that a R.C member subjected to torsion fails by bending on a skew-surface which is not plane but a warped one.

* The failure is by development of such spiral cracks at close spacing.

* On cracking, the torsional resistance of concrete drops, transferring the remaining torque to steel.

* The failure is by yielding of reinforcement and crushing of concrete.



* The ultimate torsional strength of R.C member is obtained by idealising warped surface to a skew (Inclined) plane and considering equilibrium of forces.

* The failure is considered basically due to bending about an inclined axis in the skew plane.

* Since the failure plane is inclined at 45° to the axis of member, the longitudinal component of crack length is equal to depth 'D' ~~and~~

* Number of stirrups within the length is y_1/s

  where s: spacing of stirrups

* At collapse, each vertical leg of stirrups yields.

* Total tensile force 'F_v' in vertical stirrups within the fracture plane, is

$$F_v = 0.87\, f_y \frac{A_{sv}}{2} \cdot y_1/s$$

where, A_{sv} = Area of two legs of stirrups.

* The Couple formed by these vertical Tensile - Compressive force is the torsional resistance offered by vertical legs of stirrups.

$$T_{sv} = F_v\, x_1 = 0.87\, f_y \frac{A_{sv}}{2} \cdot \frac{y_1}{s} \cdot x_1$$

* The horizantal legs of stirrups crossing the crack on top and bottom faces do not yield.

Assuming the stress in these legs equal to 'f_s', torsional resistance offered by horizantal legs can be obtained as

$$T_{sh} = F_h \times y_1 = f_s \times \frac{A_{sv}}{2} \times \frac{x_1}{s} \cdot y_1$$

* The total torque provided by stirrup is

$$T_s = T_{sv} + T_{sh}$$

$$= 0.87\, f_y \frac{A_{sv}}{2} \cdot \frac{y_1}{s} \cdot x_1 + f_s \frac{A_{sv}}{2} \cdot \frac{x_1}{s} \cdot y_1$$

$$= \left[\frac{0.87\, f_y + f_s}{2}\right] A_{sv} \frac{x_1 y_1}{s}$$

$$= \alpha\, A_{sv} \frac{x_1 y_1}{s}$$ - when inclination of failure plane is 45°.

where, x_1 = Center to center distance b/n vertical legs of stirrups.

* The torque resisted by concrete is about 50% of the cracking torque 'T_{cr}'

Considered as 40%.

$$T_c = 0.4\, T_{cr}$$

* The total torque resisted by reinforcement concrete member is

$$T = T_c + T_s$$

* Torsional members are designed in such a way that the volume of longitudinal and transverse reinforcement is equal.

$$A_{sl} \times s = A_{sv}\,(x_1 + y_1)$$

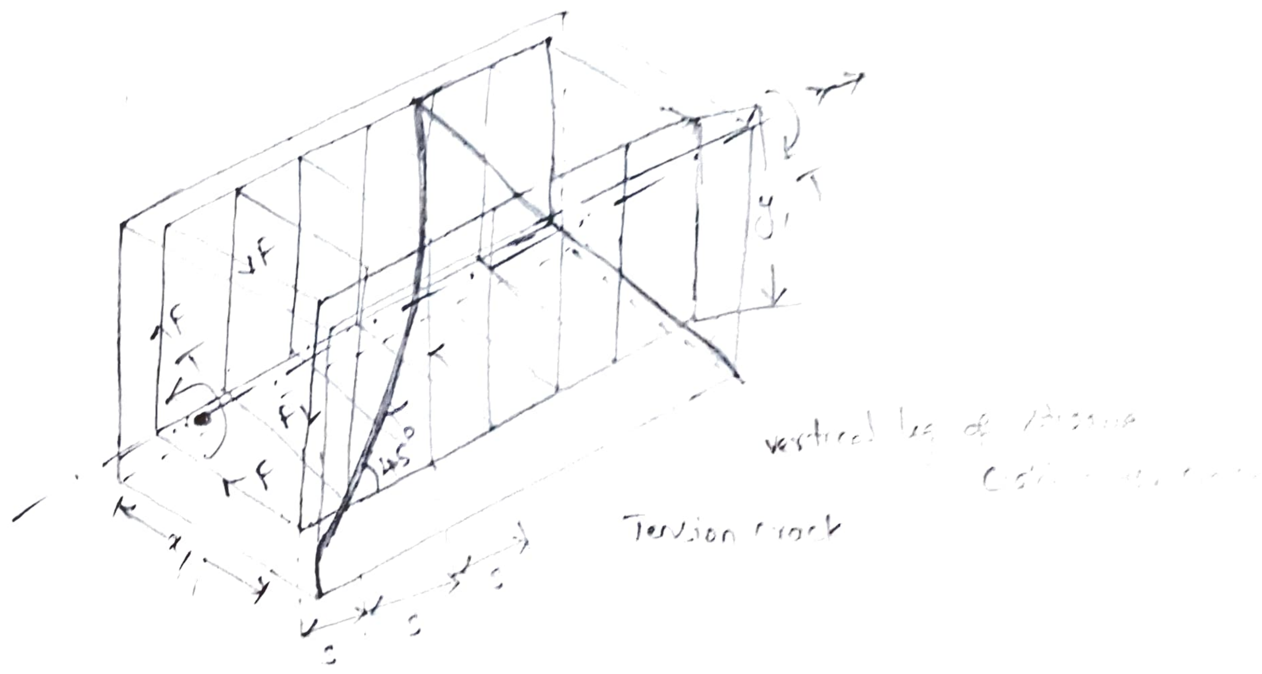
where A_{sl} = Area of longitudinal reinforcement for torsion.

* If inclination of failure plane is not 45°,

$$T_s = x_1 y_1 \sqrt{\frac{A_{sv}\, f_{yv}}{s} \times \frac{A_{sl}\, f_{yl}}{(x_1 + y_1)}}$$

Space - Truss analogy:-

* This analogy is developed by Rausch and Morsch in Russia.
* It assumes traditional rectangular space truss model consisting of longitudinal corner bars acting as stringers, the legs of stirrups acting as vertical and horizontal tension members, and the diagonal concrete portion b/n longitudinal stringers acting as diagonal struts.
* This theory assumes yielding of stirrups prior to crushing of concrete i.e an under-reinforced section.



* It is desirable that the longitudinal bars and transverse links yield simultaneously.
* To achieve this, the product of steel volume and yield strength per unit length should be equal for both types of steel.

$$A_{sl}\, f_{yl} \cdot s = A_{sv}\, f_{yv}\,(x_1+y_1) \Rightarrow A_{sl} = \frac{f_{yv}}{f_{yl}}\,\frac{x_1+y_1}{s}\,A_{sv}$$

Moment of force F about the centre line $= F\, x_1/2$ for vertical leg

$= F\, y_1/2$ for horizantal leg.

* Total torsional moment provided by one closed link is given by the sum of the moments due to each leg of the link about the centre line of the section.

$$T = \left(F \cdot \frac{x_1}{2}\right) \times 2 + \left(F \cdot \frac{y_1}{2}\right) \times 2 = F x_1 + F y_1$$

* The torsional resistance of steel (T_s) is given by

$$T_s = \left(f_{yv} \times \frac{A_{sv}}{2}\right) \cdot \frac{y_1}{s} \cdot x_1 + f_{yv} \times \frac{A_{sv}}{2} \times \frac{x_1}{s} \cdot y_1$$

$$= A_{sv}\, f_{yv}\, \frac{x_1 y_1}{s}$$

$$\therefore A_{sv} = \frac{T_s \times s}{f_{yv}\, x_1\, y_1}$$

where, $f_{yv} = 0.87\, f_y$

$$\therefore A_{sv} = \frac{T_s \times s}{0.87\, f_y\, b_1\, d_1}$$ (As per IS code $x_1 = b_1$, $y_1 = d_1$)

* These equations for T_s holds good only if spacing of stirrups does not exceed

(i) x_1

(ii) $\frac{(x_1 + y_1)}{4}$

(iii) 300 mm

whichever is less.

* Minimum longitudinal bar diameter $\phi\ \frac{s}{16}$ to prevent bulging of concrete b/n the stirrups.

Combined Torsion and ~~Bending~~ Shear :-

* In most cases, torsion is usually accompanied by transverse shear.
* The main difference b/n the two, is that transverse shear produces diagonal tensile stresses on the vertical faces only, while torsion produces diagonal tensile stresses on all four faces of rectangular section spiraling round the beam.
* Equivalent shear due to the combined effect of torsion and shear is

$$V_{ue} = V_u + 1.6\, T_u/b \longrightarrow (1) \quad (\text{cl } 41.3.1,\ \text{Pg } 75)$$

where, V_{ue} = equivalent ultimate vertical shear

V_u = Actual ultimate vertical shear acting on the section

T_u = " " Torsional moment " "

b = Breadth of section.

* The transverse reinforcement is required to carry a shear force equal to

$$V_{use} = V_{ue} - V_{uc}$$

Substituting eq (1),

$$V_{use} = (V_u + 1.6\, T_u/b) - V_{uc}$$

$$= (V_u - V_{uc}) + 1.6\, T_u/b$$

or $$V_{use} = V_{us} + 1.6\, T_u/b \longrightarrow (2)$$

where, V_{us} = shear to be carried by reinforcement for vertical shear only.

* Let transverse reinforcement be 'A_{sv}' at spacing 's'.

$$\therefore V_{use} = 0.87 f_y A_{sv} d/s \rightarrow (3)$$

Dividing both sides of eq (2) with eq (3)

$$1 = \frac{V_{us}}{0.87 f_y A_{sv} d/s} + \frac{1.6 T_u / b}{0.87 f_y A_{sv} d/s}$$

$$= \frac{V_{us}}{0.87 f_y A_{sv} d/s} + \frac{1.6 T_u}{0.87 f_y A_{sv} bd/s} \rightarrow (4)$$

* shear strength of reinforcement in absence of torsion, i.e $T_u = 0$

$$V_{uso} = 0.87 f_y A_{sv} d/s$$

* shear strength of reinforcement in pure torsion in absence of shear i.e $V_{us} = 0$

$$T_{uo} = 0.87 f_y A_{sv} b_1 d_1/s$$

* Substituting these new terms in eq (4)

$$\frac{V_{us}}{V_{uso}} + \frac{1.6 T_u}{T_{uo}} = 1$$

This is the interaction formula for Combined Torsion and shear.

* Some design codes follow simple interaction equation

$$\frac{V_{us}}{V_{uso}} + \frac{T_u}{T_{uo}} = 1$$

* If $V_{us} = 0.4\, V_u$

$$\frac{0.4\, V_u}{V_{uso}} + \frac{T_u}{T_{uo}} = 1$$

Substituting the values of V_{uso} and T_{uo}

$$\frac{V_u}{2.5\,(0.87 f_y\, A_{sv} d/s)} + \frac{T_u}{(0.87 f_y\, A_{sv}\, b_1 d_1/s)} = 1$$

$$\Rightarrow A_{sv} = \frac{V_u\, s}{2.5\, d_1\,(0.87 f_y)} + \frac{T_u\, s}{0.87 f_y\, b_1 d_1} \quad \text{(cl 41.4.3, Pg 75)}$$

* The first term represents the shear reinforcement shall be designed to ~~reinforced~~ resist shear not less than $0.4\, V_u$.

* The second term represents the reinforcement required for pure torsion.

---

* The longitudinal reinforcement shall be designed to resist equivalent bending moment, M_{e1} given as

$$M_{e1} = M_u + M_t \quad \text{(cl 41.4, Pg 75)}$$

where, M_u = Bending moment at the cross section

$$M_t = T_u \left(\frac{1 + D/b}{1.7}\right)$$

T_u = Torsional moment.

* If $M_t > M_u$, longitudinal reinforcement will be provided for a moment $M_{e2} = M_t - M_u$.

1. A rectangular beam 300 mm wide and 400 mm deep is reinforced with 2 no's of 12 mm dia bars at top and 2 no.s of 16 mm dia bars at the bottom, each provided at an effective cover of 40mm. Assuming concrete of M_{20} grade ~~conc~~ and steel of Fe 415 grade, Determine the resistance of beam in pure torsion.

Sol:- Given data:-

$b = 300$ mm

$D = 400$ mm, $d = 400 - 40 = 360$ mm, $d' = 40$ mm

$A_{st} = 2 \times \pi/4 \times 16^2 = 402.17 \text{ mm}^2$

$A_{sc} = 2 \times \pi/4 \times 12^2 = 226.19 \text{ mm}^2$

$f_{ck} = 20 \text{ N/mm}^2$

$f_y = 415 \text{ N/mm}^2$

Since the beam section is under pure torsion, the torsional moment of resistance (T_u) will be governed by M_{e1} or M_{e2} whichever is lesser. However since top steel has lesser area, M_{e2} will govern the torsional moment of resistance.

$\therefore$ Percentage of steel reinforcement, $p_t = \dfrac{100 A_{st}}{bd}$

$$= \frac{100 \times 226.19}{300 \times 360} = 0.209$$

$$\frac{x_u}{d} = \frac{0.87 f_y A_{st}}{0.36 f_{ck} bd} = \frac{0.87 \times 415 \times 226.19}{0.36 \times 20 \times 300 \times 360} = 0.105$$

for Fe415, $\dfrac{x_{umax}}{d} = 0.48$

$\frac{x_u}{d} < \frac{x_{umax}}{d} \rightarrow$ It is a under reinforced section.

$$p_{t\,lim} = 0.414 \frac{f_{ck}}{f_y} \cdot \frac{x_{umax}}{d} \times 100 = 0.414 \times \frac{20}{415} \times 0.48 \times 100 = 0.955\%$$

$p_t < p_{t\,lim} \rightarrow$ It is under reinforced section.

$$\therefore M_{ur} = M_{e2} = 0.87 f_y A_{st} d \left(1 - \frac{f_y A_{st}}{f_{ck}\, b d}\right)$$

$$= 0.87 \times 415 \times 226.19 \times 360 \left(1 - \frac{415 \times 226.19}{20 \times 300 \times 360}\right)$$

$$= 28.12 \times 10^6 \text{ N mm}$$

$M_{e2} = M_t - M_u$, $M_u = 0$

$\therefore M_t = M_{e2} = 28.12$ kNm

$$T_u = \frac{1.7\, M_t}{1 + D/b} = \frac{1.7 \times 28.12 \times 10^6}{1 + 400/300} = \underline{20.489 \text{ kNm}}.$$

2. Find the reinforcement required for a rectangular beam section for the follo[wing] data:

Size of beam : 300 x 600mm, Concrete Mix : M_{20}, steel grade : Fe 415, Factored moment M_u : 115 kNm, Factored torsion, T : 45 kNm, Factored shear force, S_u : 95 kN

Sol:- Providing 25 mm ø bars at a clear cover of 25 mm

$\therefore$ Effective Cover = 25 + 12.5 = 37.5 mm

Effective depth = 600 − 37.5 = 562.5 mm

$\therefore$ Equivalent bending moment, $M_{e1} = M_u + M_t$

$$M_t = T_u \frac{(1+D/b)}{1.7} = 45 \frac{(1+600/300)}{1.7} = 79.41 \text{ KNm}$$

$$\therefore M_{e1} = 115 + 79.41 = 194.41 \text{ KNm.}$$

$$M_{ulim} = 0.36 f_{ck} \frac{x_{umax}}{d}\left(1 - 0.42 \frac{x_{umax}}{d}\right) bd^2$$

$$= 0.36 \times 20 \times 0.48 \left(1 - 0.42 \times 0.48\right) \times 300 \times 562.5^2$$

$$= 261.91 \text{ KNm}$$

$M_{e1} < M_{ulim}$ → The section is under reinforced

$$M_u = M_{e1} = 0.87 f_y A_{st}\, d \left(1 - \frac{f_y A_{st}}{f_{ck}\, bd}\right)$$

$$194.41 \times 10^6 = 0.87 \times 415 \times A_{st} \times 562.5 \left(1 - \frac{415 \times A_{st}}{20 \times 300 \times 562.5}\right)$$

$$194.41 \times 10^6 = 203090.62\, A_{st} - 24.97\, A_{st}^2$$

$$A_{st} = 1088.1 \text{ mm}^2$$

Assume 20 mm dia bar.

$$\therefore \text{No. of bars} = \frac{1088.1}{\pi/4 \times 20^2} = 3.46 \simeq 4$$

$\therefore$ Provide 4 bars of 20 mm dia $(4 \times \pi/4 \times 20^2 = 1256.8 \text{ mm}^2)$

$M_t < M_u$ - No need of M_{e2}.

$\therefore$ Provide only 2 bars of 12 mm dia on the Compression face to act as a holding bars for transverse reinforcement.

Transverse reinforcement:-

$$\text{Equivalent ultimate shear, } V_e = V_u + 1.6 \frac{T_u}{b}$$

$$= 95 + 1.6 \times \frac{45}{0.30} = 335 \text{ kN.}$$

$$p_t = \frac{100\,A_{st}}{bd} = \frac{100 \times 1256}{300 \times 562.5} = 0.74$$

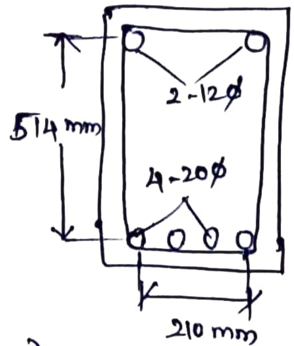
critical shear stress, $\tau_c = 0.48 + \frac{0.56-0.48}{0.75-0.5}(0.74-0.5)$

$= 0.556\ N/mm^2$

Nominal shear stress, $\tau_v = \frac{V_u}{bd} = \frac{335 \times 10^3}{300 \times 562.5} = 1.99\ N/mm^2$

$\tau_v > \tau_c$ - shear reinforcement is necessary.

$$A_{sv} = \frac{T_u\,S_v}{b_1 d_1\, 0.87 f_y} + \frac{V_u\,S_v}{2.5\, d_1\, 0.87 f_y}$$



Assume 10 mm dia stirrups,

$b_1 = 300 - 2\left(25 + 10 + \frac{20}{2}\right) = 210\,mm$

$d_1 = 600 - \left(25 + 10 + \frac{20}{2}\right) - \left(25 + 10 + \frac{12}{2}\right)$

$= 514\ mm$

$A_{sv} = 2 \times \pi/4 \times 10^2 = 157.1\ mm^2$

$$157.1 = \left(\frac{45 \times 10^6}{210 \times 514 \times 0.87 \times 415} + \frac{95 \times 10^3}{2.5 \times 514 \times 0.87 \times 415}\right) S_v$$

$157.1 = (1.15 + 0.2)\, S_v$

$S_v = 116.37 \cong 115\ mm$

Total transverse reinforcement shall not be less than

$$\frac{(\tau_{ve} - \tau_c)\, b\, S_v}{0.87\, f_y} = \frac{(1.99 - 0.556) \times 300 \times 115}{0.87 \times 415}$$

$= 137\ mm^2$

Hence safe.

As per code requirement, the spacing is least of

$\frac{3}{7} x_u < D_f$ $\frac{D_f}{d} < 0.2$

$3/4 x_u > D_f$ $\frac{D_f}{d} < 0.2$

D_f

(i) $x_1 = 300 - 2(25 + 5) = 240$ mm

(ii) $\frac{x_1 + y_1}{4} = \frac{240 + 540}{4} = 195$ mm

$y_1 = 600 - 2(25 + 5) = 540$ mm

(iii) 300 mm

i.e $S = 195$ mm

Hence 10 mm dia two legged stirrups @ 115 mm c/c satisfy codal requirement

Side face reinforcement:-

Since depth of beam is more than 450 mm, side face reinforcement is necessary.

Area of side reinforcement $= 0.001\, bD = 0.001 \times 300 \times 600$

$= 180\ mm^2$

Hence area of steel on each face $= 90\ mm^2$.

Hence provide 1 - 12 mm ϕ bar on each face at mid depth.

Area of steel provided $= 1 \times \pi/4 \times 12^2 = 113\ mm^2$

3. A reinforced concrete rectangular beam 300 mm wide and 425 mm deep is reinforced with 4 bars of 20 mm dia at bottom and 4 bars of 12 mm dia at top. Determine the strength of the beam section in torsion, if it is also subjected to an ultimate bending moment of 60 KNm and ultimate shear of 15 KN. Assume M_{20} grade concrete, Fe 415 steel and an effective cover of 40 mm from each face.

Sol:- Given data:-

$b = 300$ mm $\quad$ $f_{ck} = 20$ N/mm²

$D = 425$ mm $\quad$ $f_y = 415$ N/mm²

$M_u = 60$ KNm

$V_u = 15$ KN

step - 1. Computation of M_{e1} with bottom steel as Tension steel

The section is doubly reinforced, $A_{st} = 4 \times \pi/4 \times 20^2 = 1256.63$ mm²

$A_{sc} = 4 \times \pi/4 \times 12^2 = 452.39$ mm²

$d = 425 - 40 = 385$ mm

$x_{umax} = 0.48\, d = 0.48 \times 386 = 184.4$ mm.

Tensile force, $T_u = 0.87 f_y A_{st} = 0.87 \times 415 \times 1256.63 = 453706$ N

Compressive force, $C_u = 0.36 f_{ck}\, b x_u + (f_{sc} - f_{cc}) A_{sc}$

$= 0.36 \times 20 \times 300 \times 150 + (361 - 0.446 \times 20)\, 452.39$

$= 483277.4$ N.

Assume $x_u = 150$ mm

$\varepsilon_{sc} = 0.0035 \left(\frac{x_u - d'}{x_u} \right) = 0.0035 \left(1 - \frac{40}{150} \right) = 0.00257$

$\therefore f_{sc} = 0.87 f_y = 361$ N/mm²

$T_u < C_u$

Assume $x_u = 136$ mm

$\epsilon_{sc} = 0.0035 \left(1 - \frac{40}{136}\right) = 0.00247$

$\therefore f_{sc} = 361 \ N/mm^2$

$C_u = 0.36 \times 20 \times 300 \times 136 + (361 - 0.446 \times 20) \times 452.39$

$= 453037 \ N.$

$T_u \cong C_u$

$\therefore x_u = 136$ mm

$M_{ur} = M_{e1} = 0.36 \ f_{ck} \ b \ x_u (d - 0.42 x_u) + (f_{sc} - f_{cc}) A_{sc} (d - d')$

$= 0.36 \times 20 \times 300 \times 136 (385 - 0.42 \times 136) + (361 - 0.446 \times 20) \times 452.39 (385 - 40)$

$= 96.318 \times 10^6 + 54.95 \times 10^6$

$= 151.26 \times 10^6 \ N \ mm = 151.26 \ KNm$

Step-2 - Computation of M_{e2} with top steel as Tension steel.

$A_{st} = 452.39 \ mm^2$, $A_{sc} = 1256.63 \ mm^2$

Since the section is heavily reinforced on compression side, x_u will be very small than '$x_{u max}$'.

Let $x_u = 70$ mm, $\frac{3}{7} x_u = \frac{3}{7} \times 70 = 30mm < d' = 40mm$

$\epsilon_{cu} = 0.0035 \left(1 - \frac{d'}{x_u}\right) = 0.0035\left(1 - \frac{40}{70}\right) = 0.0015$

$f_{sc} = 0.84 \ f_y$ (From Fig 23 a, Pg 70)

$= 348.6 \ N/mm^2.$

Since '3/7 x_u' is less than d', the stress in concrete around compressive steel will be less than $0.446 f_{ck} = 0.446 \times 20 = 8.92$ N/mm²

Let us assume the stress in concrete = 7 N/mm²

$$C_u = 0.36 \times 20 \times 300 \times 70 + (348.6 - 7) \times 1256.63$$

$$= 580464.8 \text{ N}$$

$$T_u = 0.87 \times 415 \times 452.39 = 163335 \text{ N}$$

$C_u > T_u$, Reduce value of x_u

$$x_u = 45 \text{ mm}$$

$$\epsilon_{cu} = 0.0035\left(1 - \frac{40}{45}\right) = 0.000388$$

Since ϵ_{cu} is quite low, $f_{sc} = 0.000388 \times 2 \times 10^5 = 77.77$ N/mm²

$$C_u = 0.36 \times 20 \times 300 \times 45 + (77.77) \times 1256.63$$

(Neglecting area of concrete).

$$= \cancel{186131.7 \text{ N}} \quad 194928.1 \text{ N}$$

$C_u > T_u$, assume $x_u = 43.5$ mm

$$f_{sc} = 0.0035\left(1 - \frac{40}{43.5}\right) \times 2 \times 10^5 = 56.32 \text{ N/mm}^2$$

$$C_u = 0.36 \times 20 \times 300 \times 43.5 + (56.32) \times 1256.63$$

$$= 164733.4 \text{ N.}$$

$C_u \cong T_u$, $\therefore x_u = 43.5$ mm

$$M_{ur} = M_{e2} = \overset{0.36 f_{ck} b x_u}{C_u}(d - 0.42 x_u) + f_{sc}(d - d') A_{sc}$$

$$= 0.36 \times 20 \times 300 \times 43.5 (385 - 0.42 \times 43.5) + 56.32 \times 1256.63 (385 - 40)$$

$$= 58.89 \text{ kNm}$$

step 3 :- Computation of M_t on the basis of Me_1 and Me_2

$$Me_1 = Mu + M_t$$

$$M_t = Me_1 - Mu = 151.26 - 60 = 91.26 \text{ KNm}$$

$$Me_2 = M_t - Mu$$

$$M_t = Me_2 + Mu = 58.89 + 60 = 118.89 \text{ KNm}$$

$\therefore$ M_t will be lesser of values obtained from Me_1 and Me_2

$$M_t = 91.26 \text{ KNm.}$$

step 4 :- Computation of torsional moment (T_u)

$$T_u = \frac{1.7 M_t}{1 + D/b} = \frac{1.7 \times 91.26}{1 + 425/300}$$

$$= 63.78 \text{ KNm}$$