UNIT - III

DESIGN OF SLABS AND BEAMS

Slabs :-

* slabs are plate elements forming floors and roofs of building and carrying distributed loads primarily by flexure.

* A slab may be supported by beams or walls

classification of slabs:-

a. On basis of shape : Rectangular, circular and other shapes.

b. On the basis of spanning directions : spanning in one - direction - one way slab

spanning in two orthogonal direction - Two way slab

c. Circular slabs

d. Flat slabs

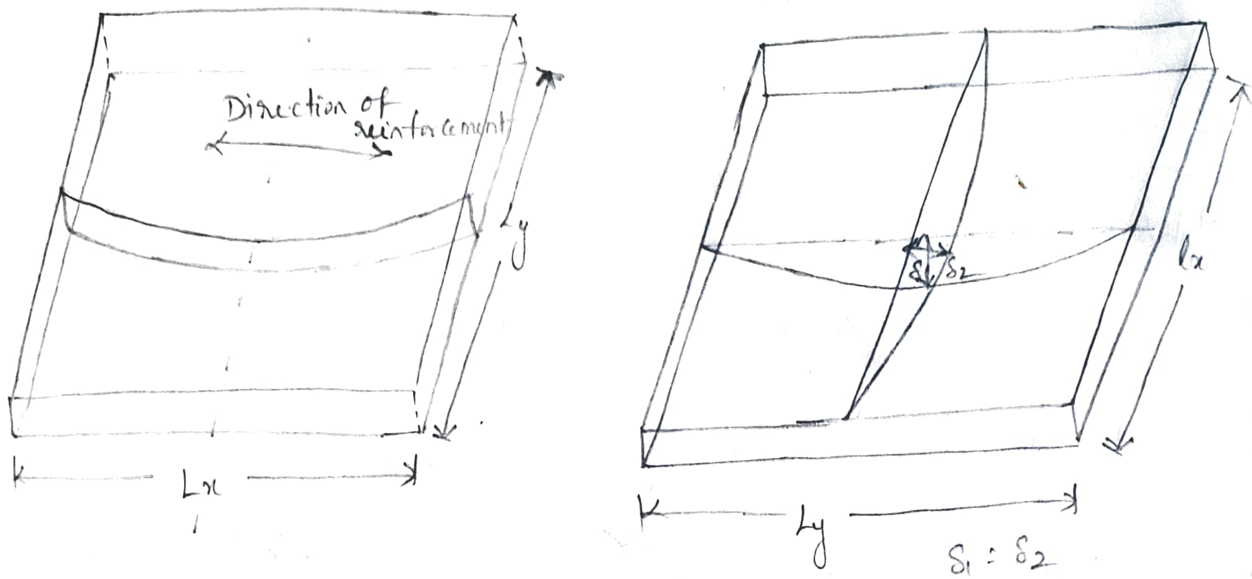
e. Grid floors or Ribbed slabs.

* In practical design, rectangular slabs are most Common. They are (i) one - way slab, (ii) Two - way slab (iii) Flat slab.

* when a slab supported only on two opposite parallel edges, it spans only in the direction perpendicular to two supporting edges. It has only one single curvature and reinforced in only one direction called as "one-way slab".

* A slab supported on four edges, The deflected surface is of double curvature. The load carried in both directions to the four supporting edges. the slab is called as "Two-way slab".

*



* In the case of a rectangular slab supported on all four sides, two-way bending is predominant only when $Ly/Lx < 2$. When $Ly/Lx > 2$, Practically entire slab excepting a small portion near short edges spans only across short span and is therefore designed as one-way slab.

Effective span of slab :- (Pg 34, cl 22.2)

* For simply supported beam or slab which is not built integrally with its supports,

Effective span, L = c/c distance b/n supports (or) clear span + Effective depth } whichever is less.

Design of one way slab :-

* The slab supported on two opposite edges and also the slab having $L_y/L_x > 2$ is designed as one-way slab.

Steps :-

1. **Span :-** Depending on the end conditions determine the effective span.

Assume effective depth $= L/22$ to $L/28$ for Fe415 steel

$= L/28$ to $L/34$ for Fe500 steel

The overall depth of slab = Effective depth + effective Cover

Effective Cover = clear Cover + half bar diameter.

2. **Loads :-**

Total ultimate load $= w_u = 1.5\,(DL + LL)$

Dead load = self weight + floor finish.

3. **Design moments :-**

For simply supported slab, $M_u = \dfrac{w_u L^2}{8}$

shear force, $V_u = \dfrac{w_u \times L}{2}$

4. **Check for section :-**

Calculate M_{ulim} and Compare with M_u to check whether the section is under (or) over (or) Balanced section.

5. **Main steel :-**

$M_u = 0.87\, f_y\, A_{st}\, d \left(1 - \dfrac{f_y A_{st}}{f_{ck}\, b d}\right)$ by assuming $b = 1\,m$

calculate A_{st}.

$A_{st\,min} = 0.12\% \ bD$ for HYSD bars

$= 0.15\% \ bD$ for mild steel

spacing $s = \dfrac{1000 \ a_{st}}{A_{st}}$

Maximum spacing $\leq 3d$ or 300 mm } whichever is less.

6. Distribution steel :-

$A_{st\,min} = 0.12\% \ bD$ for HYSD bars

$= 0.15\% \ bD$ for Mild steel bars

Maximum spacing $\leq 5d$, 300 mm } whichever is less.

7. check for shear :-

calculate design shear stress τ_c for corresponding $p_t = \dfrac{100 \ A_{st}}{bd}$

(Table 9.4.1 b)

$V_{uc} = k \ \tau_c \ bd$

k = factor for increase in shear resistance due to membrane action of slab (cl 40.2.1.1, Pg 72)

If $V_{uc} > V_u$ – safe

otherwise increase the thickness of slab

8. check for deflection :- (cl 23.2, Pg 37)

$$\left(\frac{L}{d}\right)_{max} = \left(\frac{L}{d}\right)_{basic} \times k_c \times k_t \times k_f$$

$\left(\frac{L}{d}\right)_{basic}$ = 7 for cantilever

= 20 for simply supported

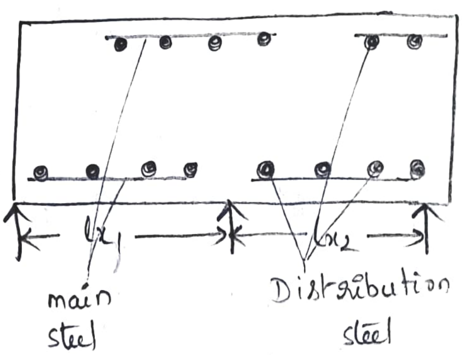
= 26 for continuous.

K_t = Modification factor for Tension reinforcement (Fig 4, Pg 38)

K_c = Modification factor for compression reinforcement (Fig 5, Pg 39)

K_f = Reduction factor for ratios of span to effective depth for flanged beams (Fig 6, Pg 39)

Codal Provisions:-

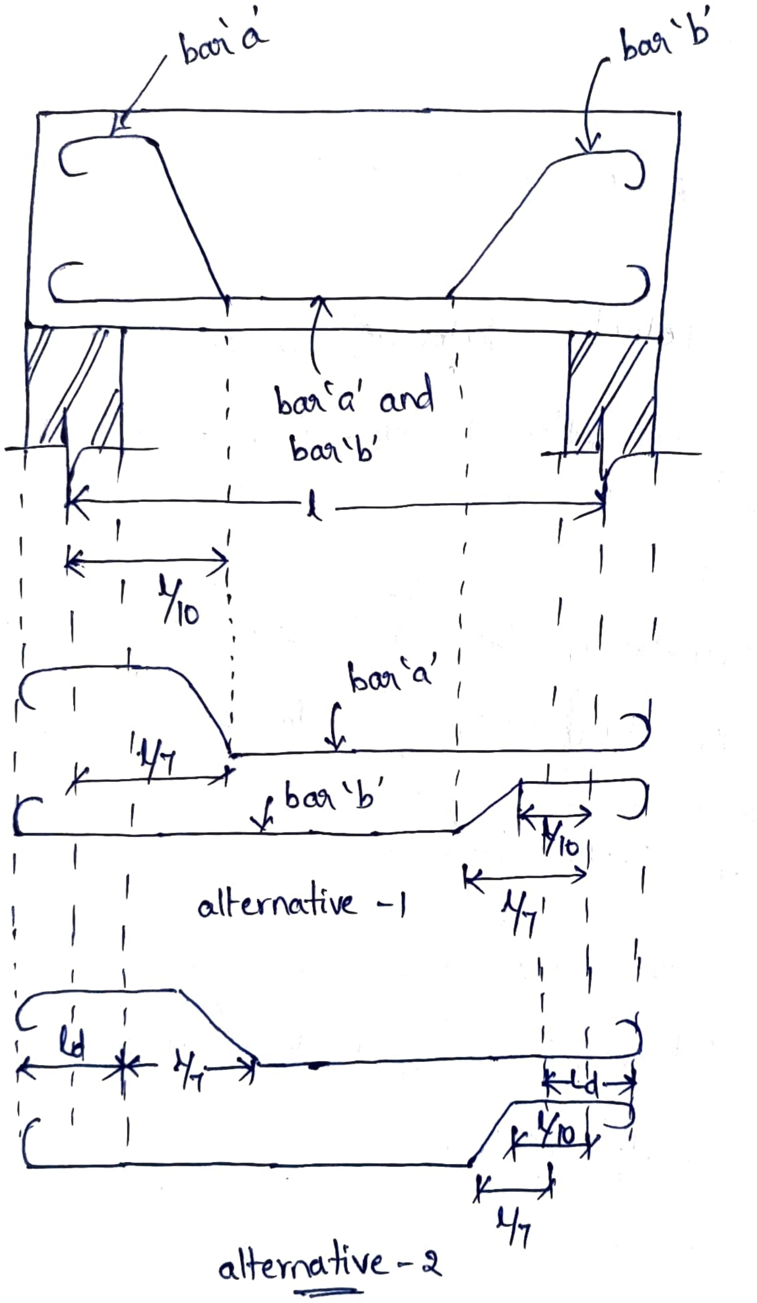


* The maximum diameter of bar shall not be exceed $\frac{1}{8}$th of the total thickness of slab.

* At least $\frac{1}{3}$rd of the maximum positive reinforcement should extend along the same face of the slab in the support to a length equal to $L_d/3$. Some Positive reinforcement should also bent up near the support to take up negative bending moment which may develop due to partial fixity.

* (i) Alternate bars are bent up at a distance of 0.15 l (or) l/7 from the centre of support, so that bars are available at the top face, for atleast a length equal to 0.1 l from the centre of support.

(ii) Alternate bars are bent up at a distance of 0.15 l from the face of the support when bending moment reduces to less than half its maximum value. This will ensure in majority of the cases, that length of bars available at the upper face is more than 0.1 l from the centre of support.



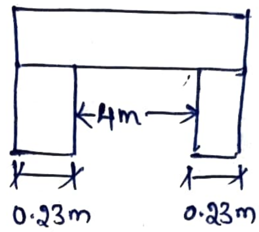
Bar bending scheme of one way slab

1. Design a simply supported R.C.C slab for an office floor having clear dimensions of 4m × 10m with ~~230~~ 230 mm thick walls around. Use M_{20} and Fe_{415} reinforcement. Live load = 4 kN/m², floor finish = 0.6 kN/m²

Sol:- $L_y = 10m$, $L_x = 4m$, L.L = 4 kN/m², F.F = 0.6 kN/m²

$f_{ck} = 20\ N/mm^2$, $f_y = 415\ N/mm^2$, wall thickness = 230 mm

$\frac{l_y}{l_x} = \frac{10}{4} = 2.5 > 2$ — one way slab



Step-I:- Thickness of slab (or) span of slab:-

Assuming effective depth, $d = \frac{L}{25} = \frac{4000}{25} = 160\ mm$

Overall depth, $D = d + \phi/2 +$ clear cover

Assume a clear cover of 20mm and using 10mm diameter bars

The overall depth, $D = 160 + \frac{10}{2} + 20 = 185\ mm$

Effective span = clear span + Effective depth = $4 + 0.16 = 4.16m$

= c/c distance b/n supports = $4 + \frac{0.23}{2} + \frac{0.23}{2} = 4.23m$

∴ Effective span = 4.16m

Step-II:- Loads:-

Self weight of slab = $0.185 \times 1 \times 25$ (Assume b = 1m)

= 4.625 kN/m²

Total load = D.L + L.L + F.F

= $4.625 + 4 + 0.6 = 9.225$ kN/m²

Ultimate load = $1.5 \times 9.225 = 13.83$ kN/m² step-3:- Design moments:-

Ultimate bending moment $= \frac{W_u l^2}{8}$

$$M_u = \frac{13.83 \times 4.16^2}{8} = 29.91 \text{ kNm}$$

shear force, $V_u = \frac{W_u \times L}{2} = \frac{13.83 \times 4.16}{2} = 28.76$ kN

step-4:-

Limiting moment of resistance, $M_{ulim} = 0.138 f_{ck} b d^2$

$$= 0.138 \times 20 \times 1000 \times 160^2$$

$$= 70.65 \text{ kNm}$$

$M_u < M_{ulim}$ → Under reinforced section.

step-5:- Main steel:-

$$M_u = 0.87 f_y A_{st} d \left(1 - \frac{f_y A_{st}}{f_{ck} b d}\right)$$

$$29.91 \times 10^6 = 0.87 \times 415 \times A_{st} \times 160 \left(1 - \frac{415 A_{st}}{20 \times 1000 \times 160}\right)$$

$$29.91 \times 10^6 = 57768 A_{st} - 7.49 A_{st}^2$$

$$A_{st} = 558 \text{ mm}^2$$

$A_{st\,min}$ = 0.12% gross c/s area

$$= \frac{0.12}{100} \times 1000 \times 185 = 222 \text{ mm}^2$$

Use 10 mm diameter bars, the spacing of bars is computed as

$$S = \frac{a_{st}}{A_{st}} \times 1000$$

$$= \frac{\pi/4 \times 10^2}{558} \times 1000 = 140.7 \text{ mm}$$

∴ Provide a spacing of 130 mm c/c and alternate bars are bent up at supports.

Step-6 :- Distribution steel :-

$$A_{st\,min} = \frac{0.12}{100} \times 1000 \times 185 = 222\ mm^2$$

(For one way slabs distribution reinforcement is minimum.)

Provide 8mm diameter bars at spacing $S = \frac{\pi/4 \times 8^2}{222} \times 1000$

$= 226.45\ mm$

∴ Provide 8mm dia bars @ 200 mm c/c.

Step-7 :- check for shear :-

Nominal shear stress, $\tau_v = \frac{V_u}{bd} = \frac{28.76 \times 10^3}{1000 \times 160} = 0.179\ N/mm^2$

$p_t = \frac{100 A_{st}}{bd} = \frac{100 \times 558/2}{1000 \times 160}$ (∵ Half the bars are alternative and they are bent up).

$= 0.174$

$p_t = 0.15 \longrightarrow 0.28$

$p_t = 0.25 \longrightarrow 0.36$

(Pg -73)

for $p_t = 0.174$, $\tau_c = 0.299\ N/mm^2$

$\tau_v < \tau_c$

Strength of Concrete $= \tau_c \times K$

For $D = 185mm$, $K = 1.23$

$D = 175\ mm \longrightarrow K = 1.25$

$D = 200\ mm \longrightarrow K = 1.2$

(Pg 72)

$\tau_c K = 0.299 \times 1.23 = 0.36\ N/mm^2$

$\tau_v < K \tau_c$

Hence the slab is safe in shear.

Step-8 :- Check for deflection :- (Pg 37)

$$\left(\frac{L}{d}\right)_{max} = \left(\frac{L}{d}\right)_{basic} \times K_t \times K_c \times K_f$$

$(L/d)_{basic} = 20$ – For simply supported ~~beam~~. slab.

$$f_s = 0.58\, f_y \frac{\text{Area of c/s steel required}}{\text{Area of c/s steel provided}}$$

$= 0.58 \times 415 = 240.7\ N/mm^2$

$\therefore K_t = 0.18$

$K_c = 1$

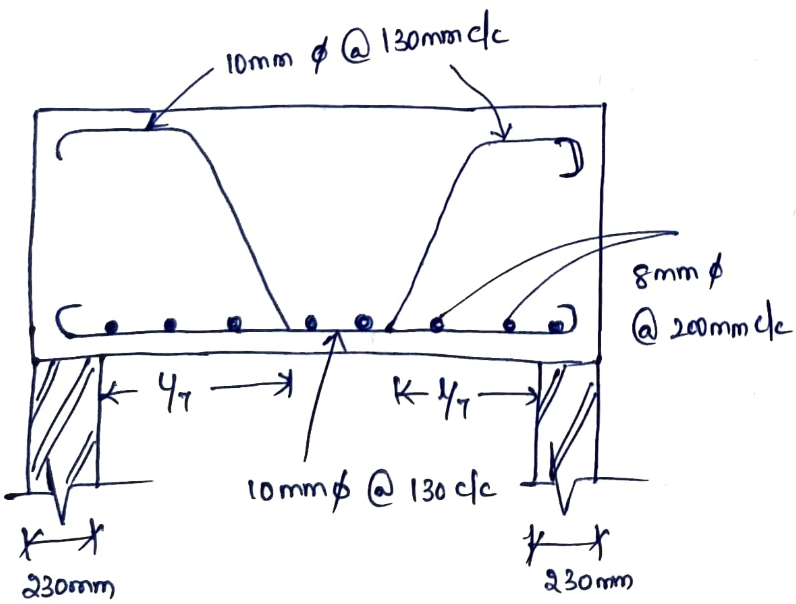
$K_f = 1$

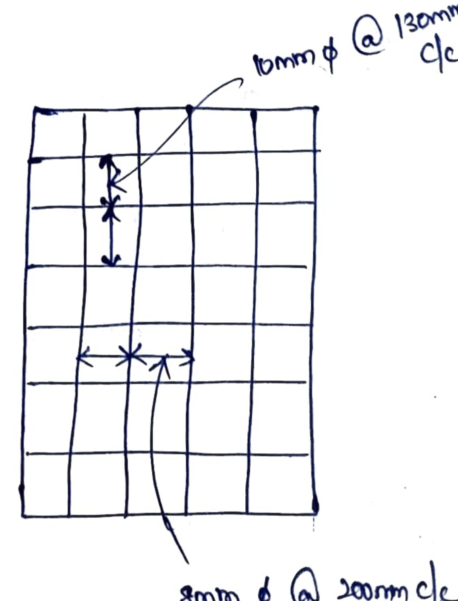
$(L/d)_{max} = 20 \times 1.8 \times 1 \times 1 = 36$

$(L/d)_{prov} = \dfrac{4160}{160} = 26$

$(L/d)_{max} > (L/d)_{prov}$

Hence the section is satisfied deflection criteria.

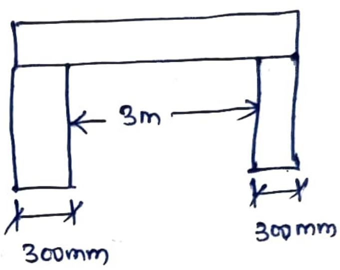




2. Design a R.C slab for a room having inside dimensions 3m × 7m. The thickness of support is 300mm. The slab carries 75mm thick lime concrete at its top, the unit weight of which may be taken as 20 kN/m². The live load on the slab may be taken as 2 kN/m². Assume the slab to be simply supported at the ends. Use M_{20} and Fe_{415} grade concrete and reinforcement.

Sol:- $L_y = 7m$, $L_x = 3m$, live load = 2 kN/m²

$f_{ck} = 20 N/mm^2$, $f_y = 415 N/mm^2$, wall thickness = 300mm



Step-I:- span of slab:-

Assuming effective depth = $\frac{span}{25} = \frac{3000}{25} = 120$ mm

overall depth = $d + \phi/2$ + clear cover

Assume a clear cover of 20mm and using 10mm dia bars.

overall depth = D = 120 + 20 + 5 = 145 mm

Effective span = c/c distance b/n supports = 3.3m

= clear span + Effective depth = 3 + 0.12 = 3.12m

∴ Effective span = 3.12m.

step 2:- Loads:-

Self weight of slab = 0.145 × 1 × 25 = 3.625 kN/m²

Live load = 2 kN/m²

Floor finish = 0.075 × 1 × 20 = 1.5 kN/m²

Total load = 7.125 KN/m^2

ultimate load = $7.125 \times 1.5 = 10.687\ KN/m^2$

step - 3 :- Design moments :-

Ultimate moment, $M_u = \frac{w_u l^2}{8} = \frac{10.687 \times 3.12^2}{8} = 13\ KNm$

shear force, $V_u = \frac{w_u l}{2} = \frac{10.687 \times 3.12}{2} = 16.67\ KN$

step - 4 :-

Limiting moment of resistance, $M_{ulim} = 0.138\ f_{ck}\ bd^2$

$M_{ulim} = 0.138 \times 20 \times 1000 \times 120^2 = 39.74\ KNm$

$M_u < M_{ulim}$ → under reinforced section.

step - 5 :- Main steel :-

$M_u = 0.87\ f_y\ A_{st}\ d \left(1 - \frac{f_y A_{st}}{f_{ck}\ bd}\right)$

$= 0.87 \times 415\ A_{st} \times 120 \left(1 - \frac{415\ A_{st}}{20 \times 1000 \times 120}\right)$

$13 \times 10^6 = 43326\ A_{st} - 7.49\ A_{st}^2$

$A_{st} = 317.47\ mm^2$

$A_{st\,min} = \frac{0.12}{100} \times 1000 \times 145 = 174\ mm^2$

use 10mm diameter bars the spacing of bars is computed as

$S = \frac{a_{st}}{A_{st}} \times 1000 = \frac{\pi/4 \times 10^2}{317.47} \times 1000 = 247.42\ mm$

Adopt spacing of 240mm c/c. Alternate bars are bent up at supports.

step 6 :- Distribution steel :-

$$Ast_{min} = \frac{0.12}{100} \times 1000 \times 145 = 174 \text{ mm}^2$$

Provide 8 mm diameter bars, $S = \frac{\pi/4 \times 8^2}{174} \times 1000 = 288.9$ mm

∴ provide 8 mm dia bars at 280 mm c/c.

step 7 :- check for shear :-

Nominal shear stress, $\tau_v = \frac{V_u}{bd}$

$$= \frac{16.67 \times 10^3}{1000 \times 120} = 0.138 \text{ N/mm}^2$$

$$P_t = \frac{100 \, Ast}{bd} = \frac{100 \times 317.47/2}{1000 \times 120}$$

$= 0.132$

for $P_t = 0.132$, $\tau_c = 0.28 \text{ N/mm}^2$

$\tau_v < \tau_c$

strength of concrete $= \tau_c k$

• for D = 145mm, k = 1.30

∴ $\tau_c k = 0.28 \times 1.3 = 0.364 \text{ N/mm}^2$

$\tau_v < \tau_c k$

Hence the slab is safe in shear.

step 8 :- check for deflection :-

$$(l/d)_{max} = (l/d)_{basic} \times K_t \times K_c \times K_f$$

$$(l/d)_{basic} = 20$$

For K_t , $f_s = 0.58 f_y = 240.7 \ N/mm^2$

$K_t = 0.183$

$K_c = 1$

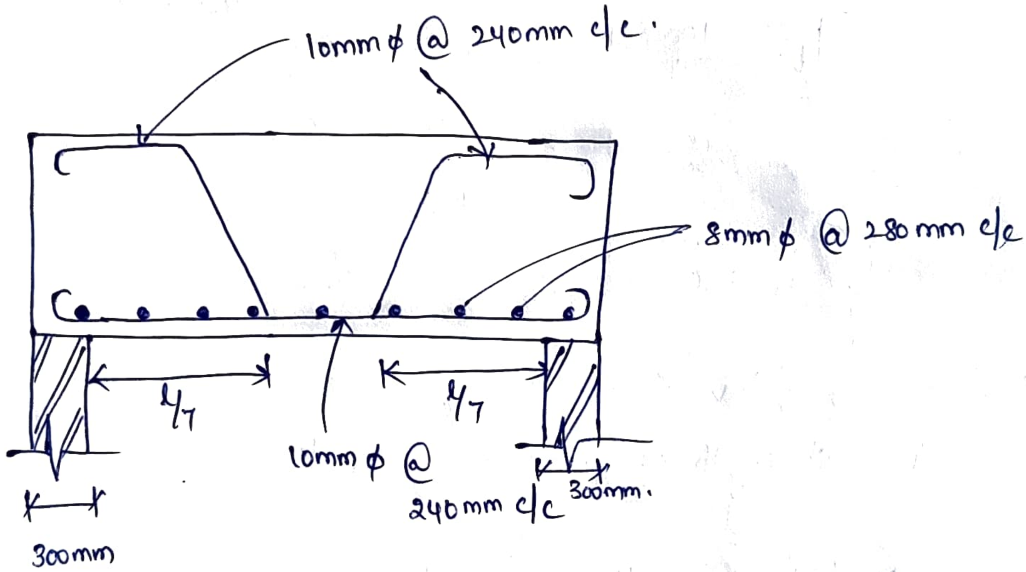
$K_f = 1$

$(l/d)_{max} = 20 \times 1.83 \times 1 \times 1 = 36.6$

$(l/d)_{prov} = \dfrac{3120}{120} = 26$

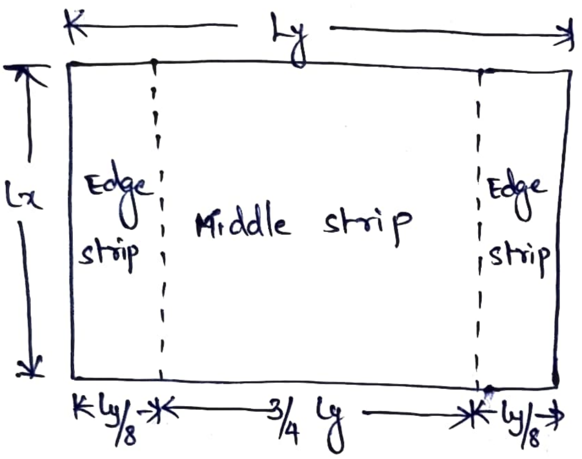
$(l/d)_{max} > (l/d)_{prov}$

Hence the deflection criteria is satisfied.

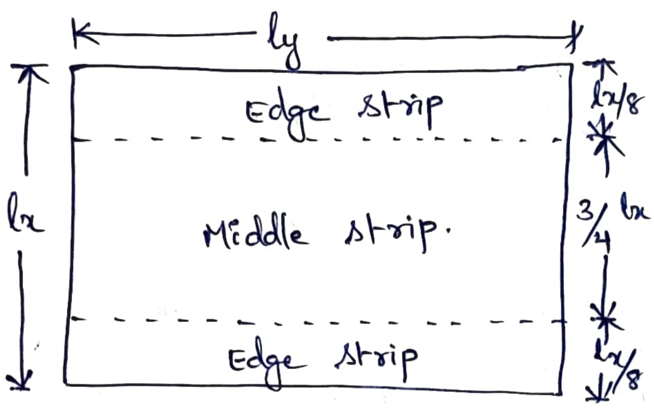


Two Way Slab :-

* For the purpose of design of slab and provision of reinforcement it is divided into middle strip and edge strip. (Pg 90)



For span lx



For span ly

* when the two way slab is loaded, the corners get lifted up.
* If corners are held down by fixity at wall support etc... The bending moment and deflection are further reduced, thus requiring thinner slabs.
* In that case special torsional reinforcement at the corners has to be provided to check the cracking of the corners.

* The two way slabs are divided in to three types.
  (i) slabs simply supported on four edges with corners not held down and carrying UDL
  (ii) slabs simply supported on four edges with corners held down carrying UDL

(iii) slabs with edges fixed (or) continuous and carrying UDL

* The analysis of two way slabs may be done by the following methods.

1) Grashoff - Ranking method. - For slabs with corners free to lift
2) Pigeaud's method
3) Marcus method
4) IS code method

} slabs with edges fixed (or) continuous and carrying UDL.

* when the corners of a slab are prevented from lifting the slab may be designed by using the bending moment equation.

$$M_x = \alpha_x \, w \times l_x^2$$

$$M_y = \alpha_y \, w \, l_x^2$$

where, α_x and α_y are bending moment coefficients (Pg 91, Table 26)

M_x and M_y are moments on strips of unit width spanning l_x and l_y

l_x and l_y are lengths of shorter and longer spans.

1. Pb) Design a two way slab for a room size 4m x 5m with discontinuous and simply supported on all the edges and all the sides with corners prevented with lifting of support subjected to a live load of 4 KN/m². Use M_{20} and Fe 415 reinforcement.

Sol:- $l_x = 4m$

$l_y = 5m$

live load = $4\ KN/m^2$

Condition :- Two way slab simply supported on all sides with provision for torsion at corners.

$\frac{L_y}{L_x} = 5/4 = 1.25 < 2 \rightarrow$ Two way slab.

~~α_x~~ For $\frac{L_y}{L_x} = 1.2$, $\alpha_x = 0.072$ (From Table 26, Pg 91)

For $L_y/L_x = 1.3$, $\alpha_x = 0.079$

$\therefore$ For $\frac{L_y}{L_x} = 1.25$, $\alpha_x = \underline{0.0755}$

$\alpha_y = \underline{0.056}$

step - I :- Span :-

Effective ~~overall~~ depth, $d = \frac{\text{span}}{25} = \frac{4000}{25} = 160\ mm$

~~Adopt effective depth, d=~~

Adopt 20 mm clear cover and 10 mm dia bar

overall depth, $D = d + \text{clear cover} + \phi/2 = 160 + 20 + 10/2 = 185\ mm$

Effective span, L_{eff} = clear span + effective depth

$= 4 + 0.16 = 4.16\,m$

Loads:- self weight of slab = $0.185 \times 1 \times 25 = 4.625\ kN/m^2$

Live load = $4\ kN/m^2$

Assume floor finish $= 0.6\ kN/m^2$

$\therefore$ Total load $= 4.625 + 4 + 0.6 = 9.225\ kN/m^2$

Ultimate load $= 9.225 \times 1.5 = 13.837\ kN/m^2$

Ultimate bending moment, $M_x = \alpha_x W_u l_x^2$

$= 0.0755 \times 13.837 \times 4.16^2$

$= 18.07\ kNm$

$M_y = \alpha_y W_u l_x^2$

$= 0.056 \times 13.837 \times 4.16^2$

$= 13.41\ kNm$

shear force, $V_u = \dfrac{W_u l_x}{2} = \dfrac{13.837 \times 4.16}{2} = 28.78\ kN.$

check for depth :-

$M_{ulim} = 0.138\, f_{ck}\, b d^2$

$18.07 \times 10^6 = 0.138 \times 20 \times 1000 \times d^2$

$d = 80.91\,mm < 160\,mm$

Hence safe.

Reinforcement in long and short spans :-

$$M_{ux} = 0.87 f_y A_{st_x} d_x \left(1 - \frac{f_y A_{st_x}}{f_{ck} b d_x}\right)$$

$$18.07 \times 10^6 = 0.87 \times 415 \times A_{st-x} \times 160 \left(1 - \frac{415 \times A_{st-x}}{20 \times 1000 \times 160}\right)$$

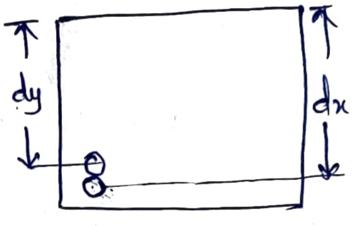
$$18.07 \times 10^6 = 57768 A_{st-x} - 7.49 A_{st-x}^2$$

$$A_{st-x} = 326.63 \text{ mm}^2$$

use 10mm diameter bars, spacing $= \frac{\pi/4 \times 10^2}{326.63} \times 1000 = 240.48$ mm

Adopt 10mm dia bars at a spacing of 200mm c/c.

$$M_{uy} = 0.87 \times f_y A_{st_y} d_y \left(1 - \frac{f_y A_{st_y}}{f_{ck} b d_y}\right)$$



$$13.41 \times 10^6 = 0.87 \times 415 \times A_{sty} \times 150 \left(1 - \frac{415 \times A_{sty}}{20 \times 1000 \times 150}\right)$$

$$13.41 \times 10^6 = \cancel{57768}\ 54157.5\ A_{sty} - 7.49 A_{sty}^2$$

$$A_{sty} = 256.72 \text{ mm}^2$$

use 10mm diameter bars, Spacing $= \frac{\pi/4 \times 10^2}{256.72} \times 1000 = 305.97$ mm

∴ Adopt 10mm diameter bars at a spacing of 300mm c/c.

Check for shear :-

Considering the shorter span 'l_x' and unit width of slab.

The shear stress is given by, $\tau_v = \frac{V_u}{bd} = \frac{28.78 \times 10^3}{1000 \times 160} = 0.179 \text{ N/mm}^2$

$$p_t = \frac{100 A_{st}}{bd} = \frac{100 \times 326.63}{1000 \times 160} = 0.204\%$$

For $p_t = 0.15 \rightarrow \tau_c = 0.28\ N/mm^2$

For $p_t = 0.25 \rightarrow \tau_c = 0.36\ N/mm^2$

for $p_t = 0.204$, $\tau_c = 0.28 + \dfrac{0.36-0.28}{0.25-0.15}(0.204-0.15)$

$= 0.32\ N/mm^2$

shear strength of Concrete $= \tau_c \times K$

Overall depth, $D = 185\ mm$

For $D = 175\ mm \rightarrow K = 1.25$

For $D = 200\ mm \rightarrow K = 1.2$

For $D = 185\ mm$, $K = 1.23$

$\therefore\ K\tau_c = 0.32 \times 1.23 = 0.393\ N/mm^2$

$\tau_v < K\tau_c$

$\therefore$ The slab is safe in shear.

check for deflection :- (Pg 37)

Considering unit width of slab in the shorter span direction l_x.

$(L/d)_{max} = (L/d)_{basic} \times K_c \times K_t \times K_f$

$(L/d)_{basic} = 20$

For K_t, $f_s = 0.58 f_y = 0.58 \times 415 = 240.7\ N/mm^2$

$\therefore\ K_t = 1.6$

$K_c = 1$ ($\because$ There is no Compression reinforcement)

$K_f = 1$ ($\because$ It is a rectangular slab)

$(l/d)_{max} = 20 \times 1.6 \times 1 \times 1 = 32$

$(l/d)_{provided} = \dfrac{4.16 \times 10^3}{160} = 26$

$(l/d)_{max} > (l/d)_{prov}$

Hence the slab is safe against deflection.

<u>Check for Cracking</u> :-

(i) steel provided is more than the minimum percentage of 0.12%

(ii) spacing of main steel $< 3 \times d = 3 \times 160 = 480$ mm

(iii) Diameter of reinforcement less than $\dfrac{D}{8} = \dfrac{185}{8} = 23.12$

Hence cracks will be within permissible limits as per IS:456.

<u>Torsion reinforcement at Corners</u> :-

Area of reinforcement in each of the four layers $= 0.75 \times 326.63$

$= 244.97\ mm^2$

Distance over which torsion reinforcement is provided $= \frac{1}{5}\, l_x$

$= \dfrac{4000}{5} = 800$ mm.

Provide 6mm dia bars, spacing $= \dfrac{\pi/4 \times 6^2}{244.97} \times 1000 = \cancel{106.02}$ 115.43 mm.

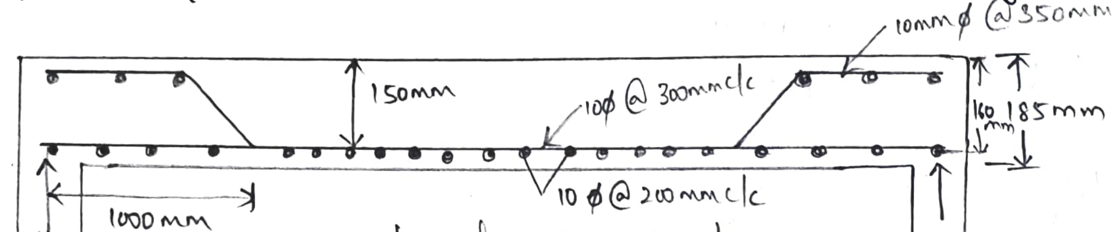
$\therefore$ Provide 6mm dia bars at 100mm c/c for a length of 800mm at all four corners in four layers.

Reinforcement in edge strips :-

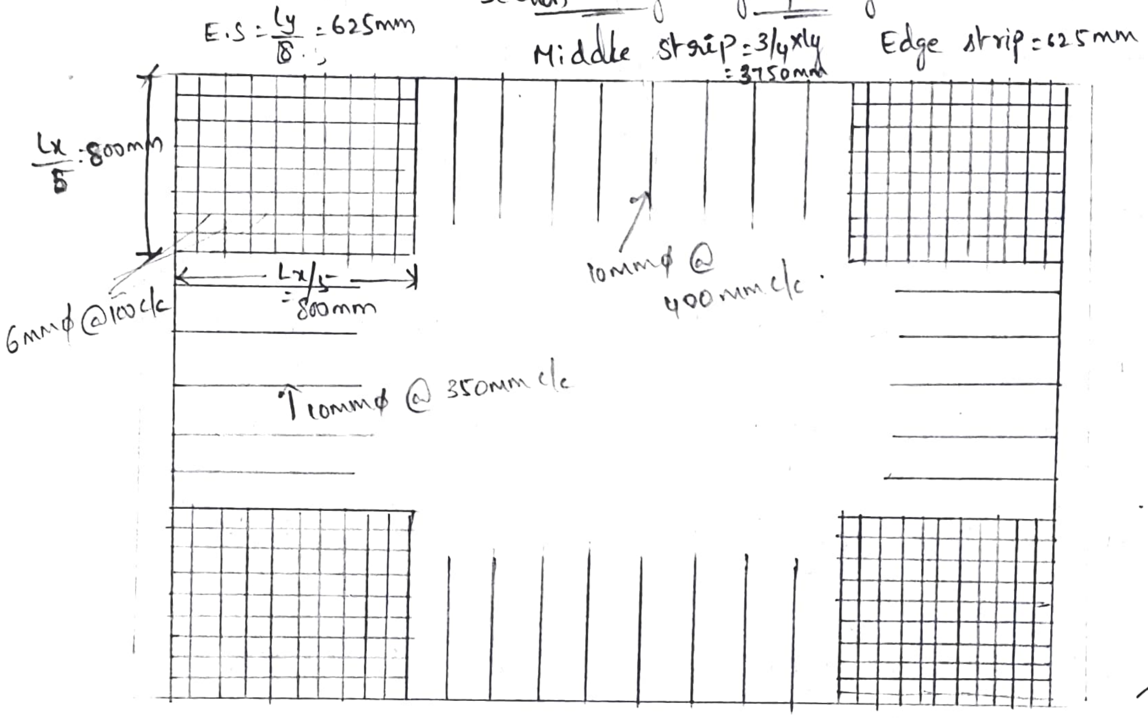
$A_{st} = 0.12\%$ of Gross c/s area

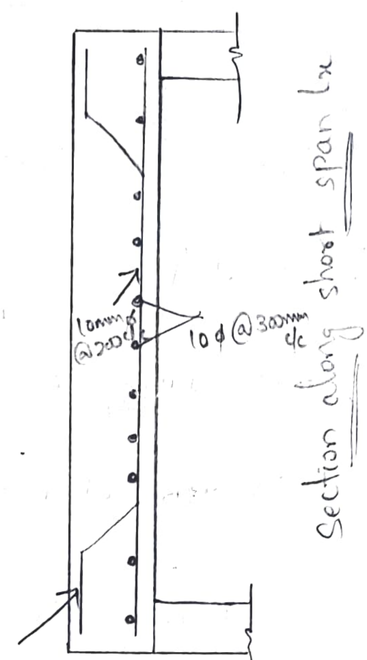
$= \frac{0.12}{100} \times 1000 \times 185 = 222\ mm^2/m$

Provide 10mm dia bars @ 300 mm c/c in all edge strips.



Section along long span Ly

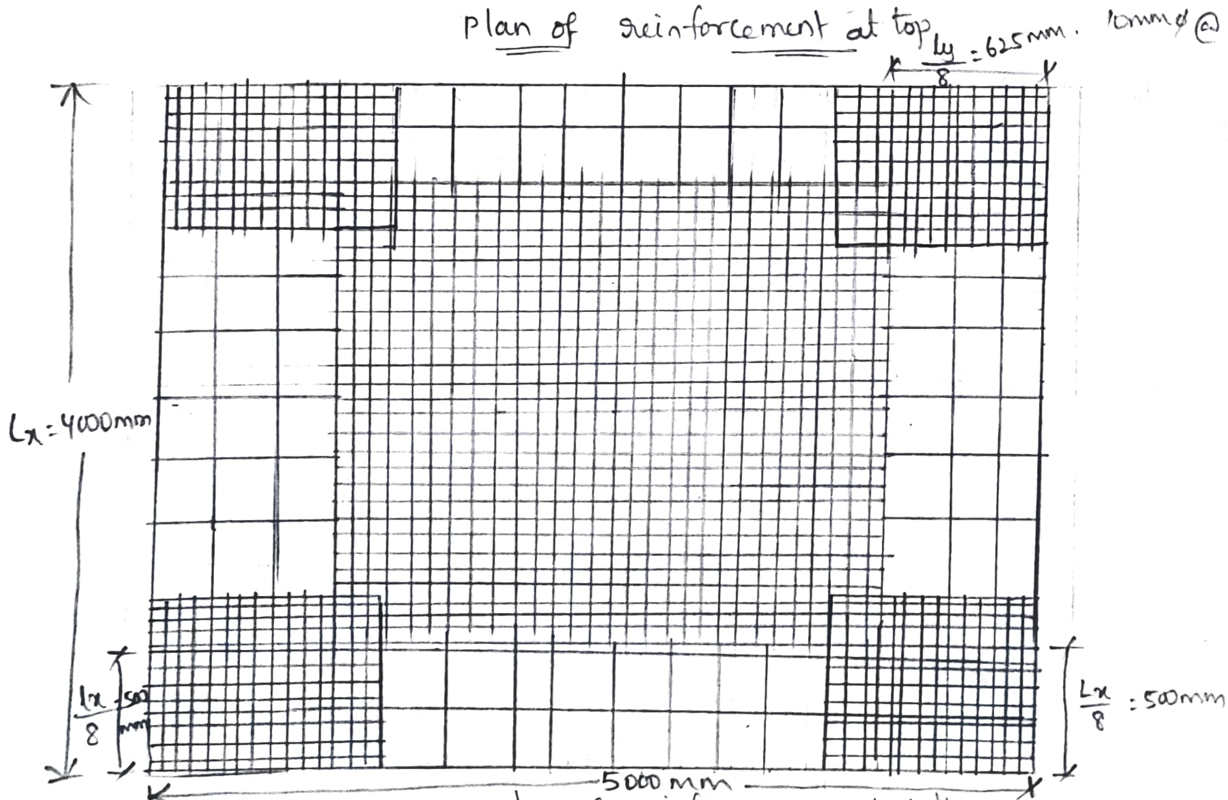




Section along short span Lx

Plan of reinforcement at top

10mmφ @ 400 c/c



Plan of reinforcement at bottom

2) Design a two way slab for an office floor to suit the following data. size of the floor - 4m x 6m, edge conditions - Two adjacent edges discontinuous. use M_{20} grade concrete and Fe_{415} reinforcement.

Sol:- $\frac{L_y}{L_x} = \frac{6000}{4000} = 1.5 < 2$

It is a two way slab.

Edge condition - Two adjacent edge discontinuous

Span :- Effective depth, $d = \frac{span}{25} = \frac{4000}{25} = 160$ mm

Assume 20mm clear cover and 10mm diameter bar.

overall depth, $D = 160 + 20 + 10/2 = 185$ mm.

Effective span = clear span + effective depth.

$L_{ex} = 4000 + 160 = 4.16$ m

$L_{ey} = 6000 + 160 = 6.16$ m

Loads :- Dead load = $1.000 \times 0.185 \times 25 = 4.625$ KN/m^2

Assume live load = 4 KN/m^2

floor finish = 1.5 KN/m^2

$\therefore$ Total load = $4.625 + 4 + 1.5 = 10.125$ KN/m^2

Design load = $10.125 \times 1.5 = 15.187$ KN/m^2.

Ultimate design moments :-

(i) short span moment Coefficients

(a) Negative moment Coefficient, $\alpha_x = 0.0075$

(b) positive moment Coefficient, $\alpha_x = 0.056$

(ii) Long span moment Coefficients

(a) Negative moment Coefficient, $\alpha_y = 0.047$

(b) positive moment Coefficient, $\alpha_y = 0.035$

$$M_{ux}(-ve) = \alpha_x w_u l_x^2$$

$$= 0.075 \times 15.187 \times 4.16^2 = 19.71 \text{ KNm}$$

$$M_{ux}(+ve) = 0.056 \times 15.187 \times 4.16^2 = 14.71 \text{ KNm}$$

$$M_{uy}(-ve) = \alpha_y w_u l_x^2$$

$$= 0.047 \times 15.187 \times 4.16^2 = 12.35 \text{ KNm}$$

$$M_{uy}(+ve) = 0.035 \times 15.187 \times 4.16^2 = 9.198 \text{ KNm.}$$

$$\text{shear force} = \frac{w_u \times l_x}{2} = \frac{15.187 \times 4.16}{2} = 32.21 \text{ KN.}$$

check for depth :-

$$M_{ulim} = 0.138 f_{ck} b d^2$$

$$19.71 \times 10^6 = 0.138 \times 20 \times 1000 \times \cancel{160} d^2$$

$$d = 85.12 \text{ mm} < 160 \text{ mm}$$

$\therefore$ The assumed depth is safe.

Reinforcement in Long and short span :-

For shorter span :- $d = 160\ mm$

(a) Negative bending moment,

$$19.71 \times 10^6 = 0.87 \times 415 \times A_{stx} \times 160 \left(1 - \frac{415\ A_{stx}}{20 \times 1000 \times 160}\right)$$

$$= 57768\ A_{stx} - 7.49\ A_{st}^2$$

$$A_{stx} = 357.79\ mm^2$$

Use 10mm dia bars, $\text{spacing} = \dfrac{\pi/4 \times 10^2}{357.79} \times 1000 = 219.5\ mm$

∴ Provide 10mm dia bars @ 200mm c/c.

(b) Positive bending moment, $d = 160\ mm.$

$$14.71 \times 10^6 = \cancel{0.87}\ 57768\ A_{stx} - 7.49\ A_{st}^2$$

$$A_{stx} = 263.65\ mm^2$$

use 10mm dia bars, $\text{Spacing} = \dfrac{\pi/4 \times 10^2}{263.65} \times 1000 = 297.93\ mm$

∴ Provide 10mm dia bars @ 250mm c/c.

For Longer span :- $d = 150mm.$

(a) Negative bending moment,

$$12.35 \times 10^6 = 0.87 \times 415 \times A_{sty} \times 150 \left(1 - \frac{415\ A_{sty}}{20 \times 1000 \times 150}\right)$$

$$= 54157.5\ A_{sty} - 7.49\ A_{st}^2$$

$$A_{sty} = 235.72\ mm^2$$

Use 10mm dia bars, $\text{Spacing} = \dfrac{\pi/4 \times 10^2}{235.72} \times 1000 = 333.2\ mm.$

∴ Provide 10mm dia bars @ 300 mm c/c.

(b) positive bending moment

$$9.198 \times 10^6 = 54157.5\, A_{sty} - 7.49\, A_{sty}^2$$

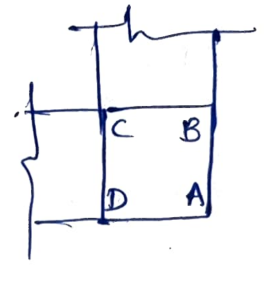
$$A_{sty} = 174.02\ mm^2$$

use 10mm dia bars, spacing $= \dfrac{\pi/4 \times 10^2}{174.02} \times 1000 = 451.38\ mm$

∴ Provide 10mm dia bars @ 400 mm c/c.

Location	Ast req (mm²)	spacing of 10mm ø bars (mm)
shorter span		
(a) -ve B.M	357.79	200
(b) +ve B.M	263.65	250
Longer span		
(a) -ve B.M	235.72	300
(b) +ve B.M	174.02	400

Torsion reinforcement at Corners :-



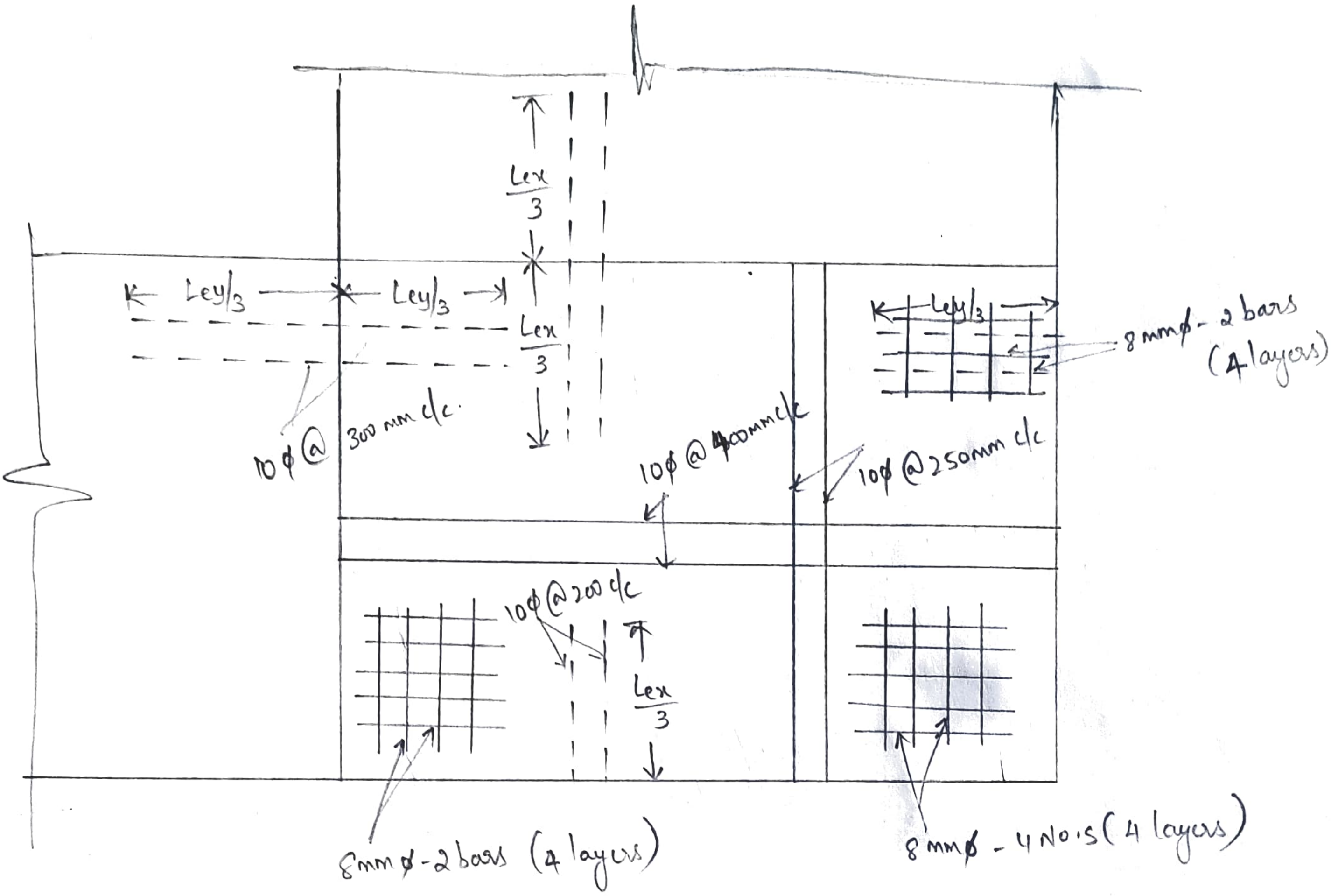
Area of torsion steel in each of four layers at A

$= 0.75\, A_{stx}$

$= 268.34\ mm^2$

Provide four layers of reinforcement at A with four bars of 8mm diameter in each layer over a length of $l_x/5 = \frac{4000}{5} = 800\ mm$ in each direction from the corner.

At Corner B and D, 50% of torsion steel i.e 2 bars of 8mm diameter in each of four layers. At 'C' Torsion steel is not required.



Reinforcement details

Continuous Slab :-

(Pb) Design a Continuous one way slab for an office floor, the slab is Continuous over T-beams spaced at 4m interval. Assume the live load of 4 KN/m^2. Use M_{20} and Fe 415 reinforcement.

Sol: Given data :-

Live load = 4 KN/m^2

f_{ck} = 20 N/mm^2

f_y = 415 N/mm^2

L = 4m

Depth of slab :-

Since the slab is Continuous slab, the span to depth ratio is assumed as 30.

Effective depth, d = $\frac{4000}{30}$ = 133.33

Adopt effective depth, d = 140 mm and

Overall depth = D = 160 mm

Loads :-

Self weight of slab, = 0.16 × 1 × 25 = 4 KN/m^2

Live load = 4 KN/m^2

Assume floor finish = 1 KN/m^2

∴ Total load = 4 + 4 + 1 = 9 KN/m

Effective span = 4m.

Moment and shear forces (Pg 36) :-

Referring Table-12 of IS : 456 - 2000

		Support	Midspan	Support	Midspan	Support	Midspan	Support	Midspan	Support
B.M	DL		$\frac{1}{12}$	$-\frac{1}{10}$	$\frac{1}{16}$	$-\frac{1}{12}$	$\frac{1}{16}$	$-\frac{1}{10}$	$\frac{1}{12}$	
	LL		$+\frac{1}{10}$	$-\frac{1}{9}$	$\frac{1}{12}$	$-\frac{1}{9}$	$\frac{1}{12}$	$-\frac{1}{9}$	$\frac{1}{10}$	
S.F	DL	0.4		0.6 | 0.55		0.5		0.55 | 0.6		0.4
	LL	0.45		0.6 | 0.6		0.6		0.6 | 0.6		0.45

Maximum negative moment at support next to end support

$$M_{u\,-ve} = \frac{g l^2}{10} + \frac{q l^2}{9}$$

$$= 1.5 \left(\frac{5 \times 4^2}{10} + \frac{4 \times 4^2}{9} \right) = \underline{22.67 \text{ KNm}}$$

Maximum positive bending moment at middle of end span

$$M_{u\,+ve} = 1.5 \left(\frac{g l^2}{12} + \frac{q l^2}{10} \right)$$

$$= 1.5 \left(\frac{5 \times 4^2}{12} + \frac{4 \times 4^2}{10} \right) = \underline{19.6 \text{ KNm.}}$$

Maximum shear force is computed as

$$V_u = 1.5 \left(0.6 (g + q) l \right)$$

$$= 1.5 \left[0.6 \times 9 \times 4 \right] = \underline{32.4 \text{ KN}}$$

check for depth :-

$$M_{u\,lim} = 0.138\ f_{ck}\ bd^2$$

$$22.67 \times 10^6 = 0.138 \times 20 \times 1000 \times d^2$$

$$d = 90.63\ mm < 140\ mm$$

Hence safe.

Reinforcement :-

$$M_u = 0.87\ f_y\ A_{st}\ d \left(1 - \frac{f_y A_{st}}{f_{ck}\ bd}\right)$$

$$22.67 \times 10^6 = 0.87 \times 415\ A_{st} \times 140 \left(1 - \frac{415 \times A_{st}}{20 \times 1000 \times 140}\right)$$

$$= 50547\ A_{st} - 7.49\ A_{st}^2$$

$$A_{st} = 483.07\ mm^2$$

Use 10 mm diameters bars spacing $s = \frac{a_{st}}{A_{st}} \times 1000$

$$= \frac{\pi/4 \times 10^2}{483.07} \times 1000 = 162.6\ mm.$$

Provide 10 mm diameter bars @ 150 mm c/c at supports. The same reinforcement is provided for positive moment at mid span.

Distribution reinforcement = 0.12% of gross c/s area

$$= \frac{0.12}{100} \times 1000 \times 160 = 192\ mm^2$$

use 10mm dia bars, spacing, $s = \frac{a_{st}}{A_{st}} \times 1000 = \frac{\pi/4 \times 10^2}{192} \times 1000 = 409.11\ mm.$

For spacing, the three considerations are

(i) $0.75\ d = 0.75 \times 140 = 105\ mm$

(ii) 409.1 mm

(iii) 300 mm

} which is less. } check

Provide 10 mm diameter bars @ 100 mm c/c.

check for shear:-

$$\tau_v = \frac{V_u}{bd} = \frac{32.4 \times 10^3}{1000 \times 140} = 0.23\ N/mm^2$$

$$P_t = \frac{100\ Ast_{prov}}{bd} = \frac{100 \times 523.67}{1000 \times 140} = 0.374\%$$

$$Ast_{prov} = \frac{a_{st}}{S} \times 1000 = \frac{\pi/4 \times 10^2}{150} \times 1000 = 523.67\ mm$$

$P_t = 0.25 \rightarrow \tau_c = 0.36\ N/mm^2$

$P_t = 0.5 \rightarrow \tau_c = 0.48\ N/mm^2$

For $P_t = 0.374\%$, $\tau_c = 0.43\ N/mm^2$

$D = 150 \rightarrow K = 1.3$

$D = 175 \rightarrow K = 1.25$

For $D = 160mm$, $K = 1.28$

$$K\tau_c = 1.28 \times 0.43 = 0.55\ N/mm^2.$$

$$\tau_v < K\tau_c$$

Hence the slab is safe in shear.

check for deflection:-

$$(l/d)_{max} = (l/d)_{basic} \times K_t \times K_c \times K_f$$

$(l/d)_{basic} = 26$ for Continuous slab

$K_c = 1$

$K_t = 1.4$

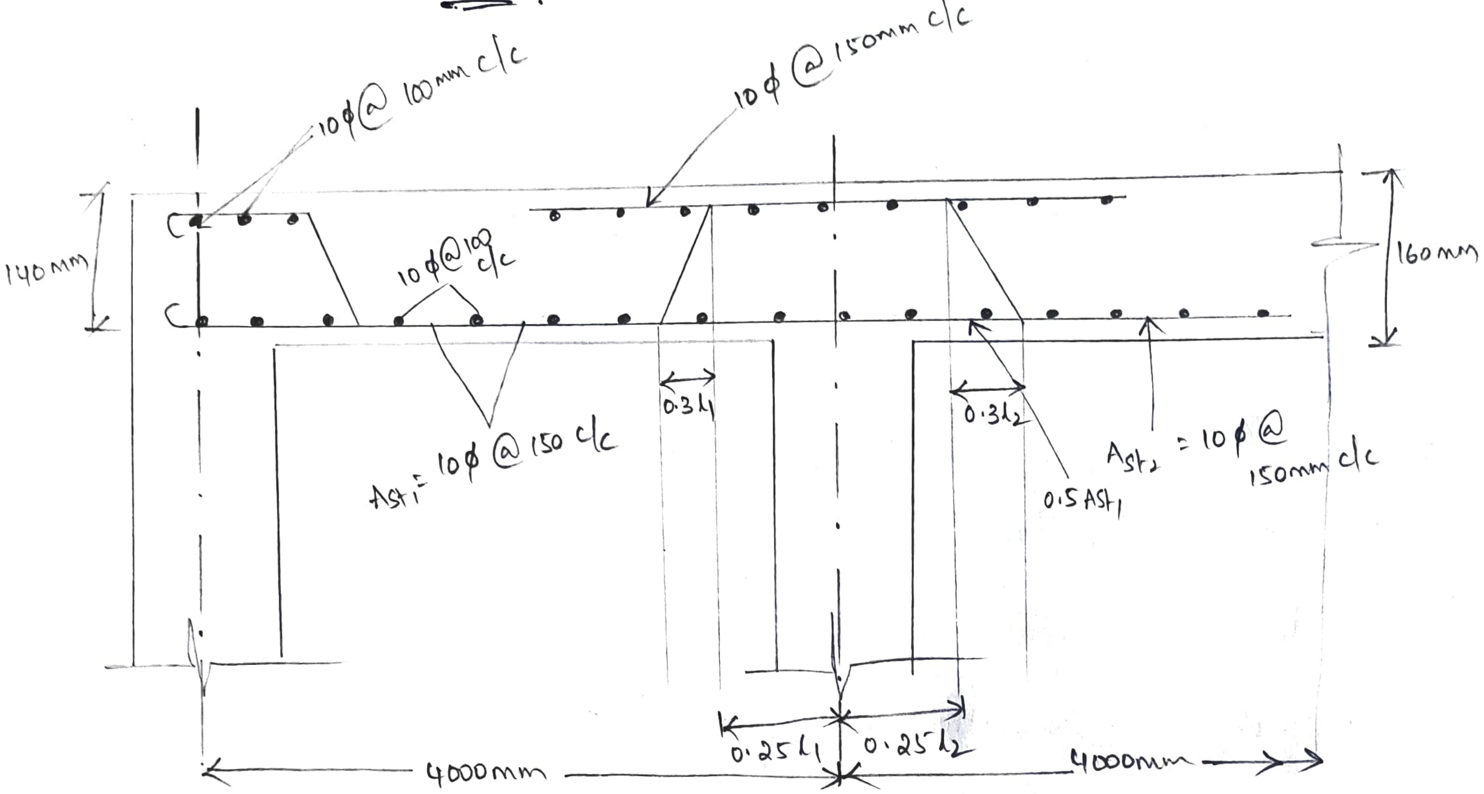
$K_f = 1$

$$(l/d)_{max} = 26 \times 1 \times 1.4 \times 1 = 36.4$$

$$(l/d)_{prov} = \frac{4000}{140} = 28.57$$

$(l/d)_{max} > (l/d)_{prov}$

Hence the slab is safe in deflection.



Continuous beams :-

* Beams supported by more than two supports and which cover more than one span are called "Continuous beams."

* The continuous beams are often more economical than simply supported beams of same span. The maximum factored bending moment in a continuous beam is much less than that in a corresponding S.S. beams.

* The continuous beams may be analysed by the following methods.

1. Castigliano's theorem
2. slope - deflection method
3. Moment distribution method
4. kani's method
5. stiffness method
6. Flexibility method.

* Maximum deflection $= \frac{\text{span}}{250}$ - Not applicable.

* span to effective depth ration $\not> 26$

* For spans greater than 10m, the value of $\frac{\text{span}}{d}$ is multiplied by $\frac{10\text{ m}}{\text{span}}$

* In general, continuous beams carry heavy loads and consequently the span to effective depth ratio for practical design is 10 to 15.

(Fb) Design a Continuous reinforced beam of rectangular section to support a dead load of 10 KN/m and live load of 12 KN/m over three spans of 6m each. The ends are simply supported. Use M_{20} grade Concrete and Fe 415 steel.

Sol:: Given data:-

Dead load = 10 KN/m

live load = 12 KN/m

span, L = 6m

$f_{ck} = 20 \, N/mm^2$, $f_y = 415 \, N/mm^2$

Cross Sectional Dimensions:-

$$\text{Effective, } d = \frac{\text{span}}{10} = \frac{6000}{10} = 600\,mm$$

Overall depth = 650mm

$$\text{Breadth of beam, } b = d/2 = \frac{600}{2} = 300mm.$$

Loads:-

Self weight = $0.3 \times 0.65 \times 25 = 4.875$ KN/m

Dead load = 10 KN/m

Live load = 12 KN/m = q

Assume floor finish = 0.125 KN/m

Total dead load = $10 + 0.125 + 4.875 = 15$ KN/m = g

Bending moment and shear forces:-

Referring to bending moments and shear force coefficients, Negative bending moment at interior support is computed as

$$M_{u-ve} = 1.5\left(\frac{gl^2}{10} + \frac{ql^2}{9}\right)$$

$$= 1.5\left(\frac{15\times6^2}{10} + \frac{12\times6^2}{9}\right) = 153 \text{ kNm}$$

$$M_{u+ve} = 1.5\left(\frac{gl^2}{12} + \frac{ql^2}{10}\right)$$

$$= 1.5\left(\frac{15\times6^2}{12} + \frac{12\times6^2}{10}\right) = 132.3 \text{ kNm}$$

Maximum shear force at the support section is given by

$$V_u = 1.5 \times 0.6\,(g+q)\,l$$

$$= 1.5 \times 0.6\,(15+12)\times 6 = 145.8 \text{ kN.}$$

Check for depth :-

$$M_{ulim} = 0.138\, f_{ck}\, b d^2$$

$$153\times10^6 = 0.138 \times 20 \times 300 \times d^2$$

$$d = 429.86 \text{ mm} < 600\text{mm}$$

Hence safe.

Reinforcement :-

$$M_u = 0.87\, f_y\, A_{st}\, d\left(1 - \frac{f_y A_{st}}{f_{ck}\, b d}\right)$$

$$153\times10^6 = 0.87\times415\, A_{st} \times 600\left(1 - \frac{415\, A_{st}}{20\times300\times600}\right)$$

$$= 216630\, A_{st} - 24.97\, A_{st}^2$$

$$A_{st} = 775.6 \text{ mm}^2$$

$$\simeq 780 \text{ mm}^2$$

Use 2 bars of 25mm diameter on the tension side at supports

$$A_{st\,prov} = \pi/4 \times 25^2 \times 2 = 981.8 \simeq 982\ mm^2$$

For positive bending moment the area of steel required is $675\ mm^2$.

For positive bending moment, $132.3 \times 10^6 = 0.87 \times 415 \times A_{st} \times 600 \left(1 - \frac{415\ A_{st}}{20 \times 300 \times 600}\right)$

$$= 216630\ A_{st} - 24.97\ A_{st}^2$$

$$A_{st} = 661.09 \simeq 675\ mm^2$$

Hence Provide 2 bars of 22mm diameter on the tension side at mid span.

$$A_{st\,prov} = 2 \times \pi/4 \times 22^2 = 760.36\ mm^2$$

<u>shear reinforcement</u> :-

$$\tau_v = \frac{V_u}{bd} = \frac{145.8 \times 10^3}{300 \times 600} = 0.81\ N/mm^2$$

$$P_t = \frac{100\ A_{st}}{bd} = \frac{100 \times 982}{300 \times 600} = 0.54\ \%$$

$$P_t = 0.5 \rightarrow \tau_c = 0.48\ N/mm^2$$

$$P_t = 0.75 \rightarrow \tau_c = 0.56\ N/mm^2$$

$$\tau_c = 0.48 + \frac{0.56 - 0.48}{0.75 - 0.5}(0.54 - 0.5)$$

$$= 0.492\ N/mm^2$$

$$\tau_v > \tau_c$$

$\therefore$ shear reinforcement is required.

Shear resisted by Concrete, $V_{uc} = \tau_c bd$

$= 0.492 \times 300 \times 600$

$= 88.56$ KN

Shear resisted by vertical stirrups, $V_{us} = V_u - V_{uc}$

$= 145.8 - 88.56$

$= 57.24$ kN.

$$V_{us} = \frac{0.87 f_y A_{sv} d}{S_v}$$

Assume 2 legged 8 mm diameter stirrups.

$A_{sv} = 2 \times \pi/4 \times 8^2 = 100.55\ mm^2$

The spacing near supports,

$$57.24 \times 10^3 = \frac{0.87 \times 415 \times 100.55 \times 600}{S_v}$$

$S_v = 380.52$ mm.

Provide 2 legged 8 mm dia stirrups at 300 mm c/c through out the beam.

Check for deflection:-

$(L/d)_{max} = (L/d)_{basic} \times K_t \times K_c \times K_f$

$(L/d)_{basic} = 26$

$K_c = 1$

$K_f = 1$

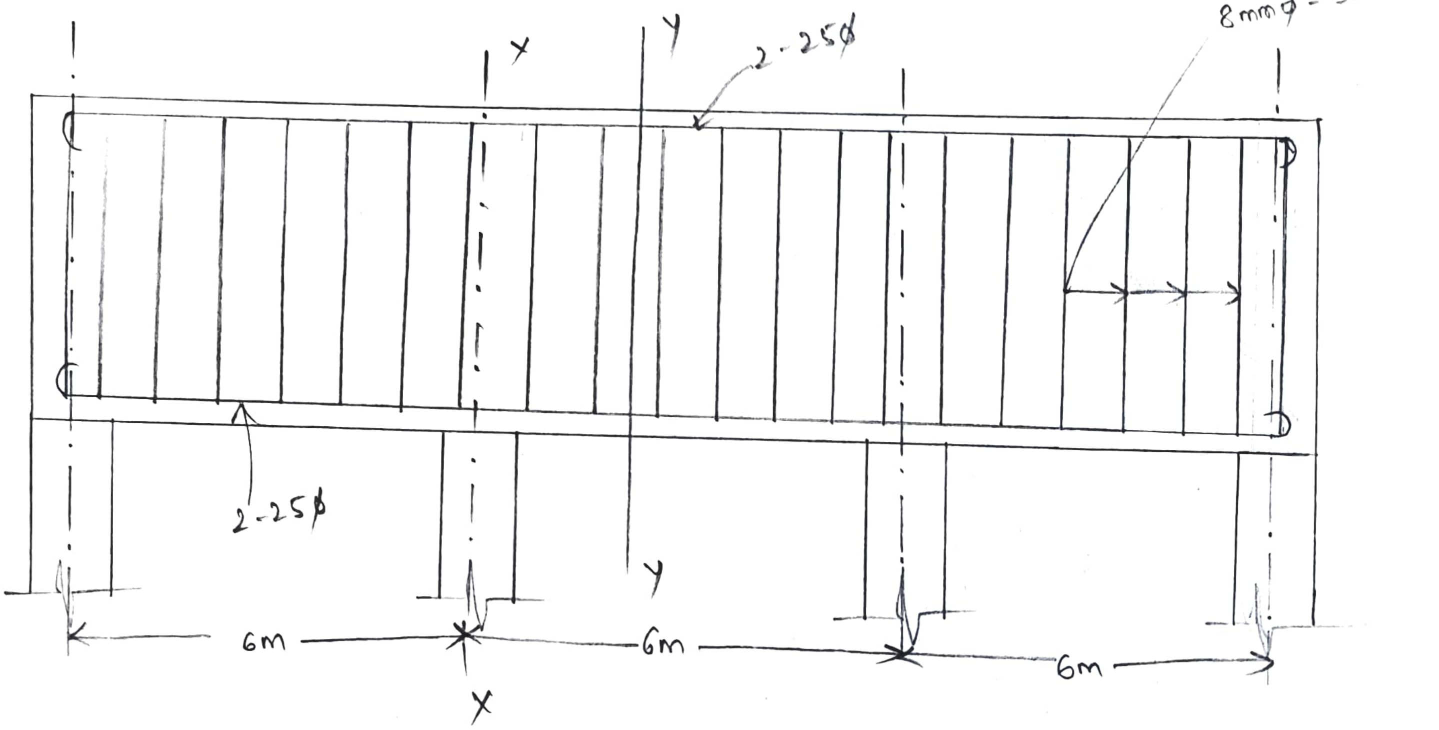
$K_t = 1.2$

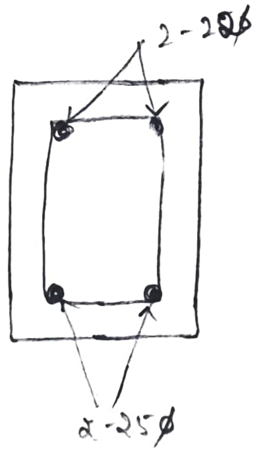
$(L/d)_{max} = 26 \times 1 \times 1 \times 1.2 = 31.2$

$(L/d)_{prov} = \frac{6000}{600} = 10$

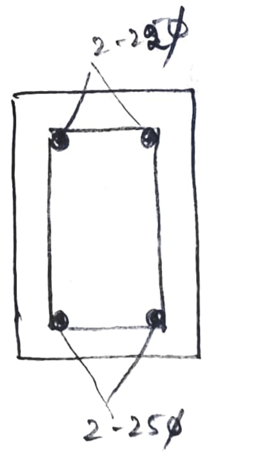
$(L/d)_{max} > (L/d)_{prov}$

Hence the beam is safe in deflection





Section Y-Y



Section X-X