UNIT - 5

YIELD LINE THEORY

* The yield line theory is an ultimate load theory for design of R.C. slabs.
* yield line is defined as "a line in the plane of the slab across which reinforcing bars have yielded and about which excessive deformations (plastic rotations) under constant ultimate moment, Continues to occur leading to failure.
* this theory was introduced by "Ingerslev" and extended and advance by "Johansen", Hence it is called as Johansen's yield line theory
* The yield line theory is a powerful tool in analysis as it enables determination of failure moment in slabs of rectangular as well as irregular shapes ~~of~~ for different support conditions and loadings.

Behaviour of slab upto failure :-

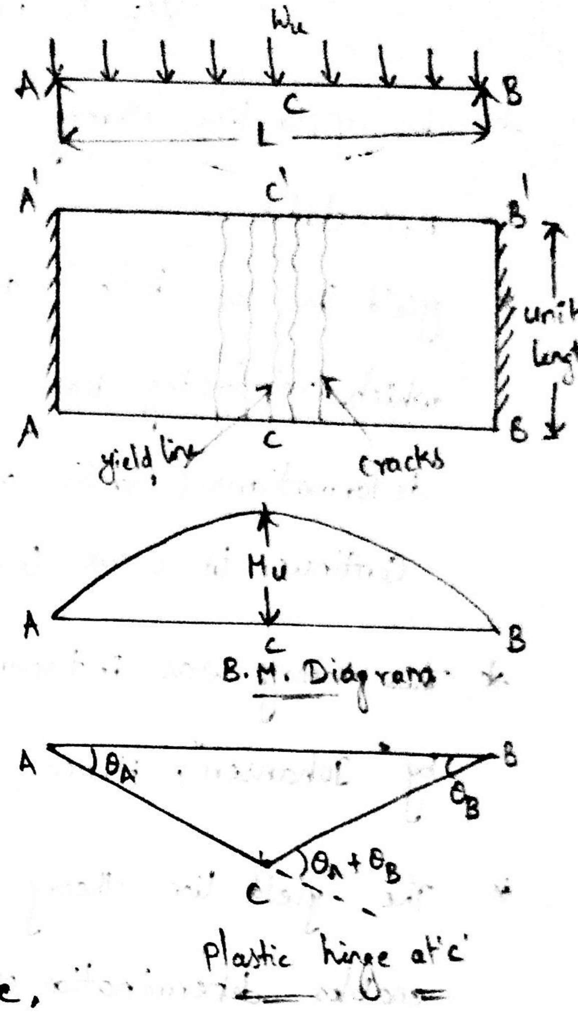
(i) One Way slab :-

* one way slab is Considered to be made up of Series of Parallel strips placed side by side, each strip acting like a beam.
* A R.C beam has to be under reinforced for the application of limit analysis, so that a plastic hinge can form at ultimate state.

* Thus, the slab is also designed as under - reinforced members.

* Consider a uniformly loaded one-way under reinforced slab, simply supported along opposite Parallel edges, every strip of the slab (Say AB), develops a plastic hinge at the Point of Maximum bending moment on loading, at ultimate state.

* The Plastic hinges in all strips, join together and form a yield line cc'.



* The resultant deformations due to plastic rotation $\theta_A + \theta_B$ at c' are so large, that the elastic deformations are neglected at AA' and BB'.

* At this stage, the collapse mechanism have formed and is represented by arrangement of yield lines in slab are known as yield line pattern.

* For a one-way Supported slab, it Consists of a single yield line Parallel to supporting edges

[Called positive yield line due to Sagging (positive) B.M]

* The ultimate moment capacity of one way slab is same as that of beam of uniform width.

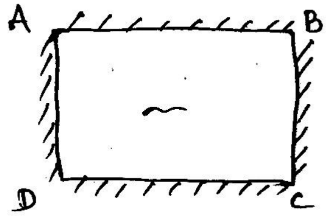
(ii) Two way slab :-

* Consider, an under reinforced rectangular R.C slab, simply supported over its edges uniformly loaded upto failure.

* Let the slab be orthotropically loaded.

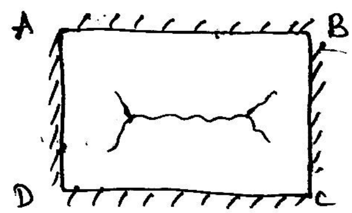
* On initial loading, the slab behaves elastically. But as the load goes on increasing, cracking of concrete on tension side starts in the mid span in the the direction parallel to longer span at the point of maximum bending moment.

* With the increase in load, maximum moment along short span reaches ultimate, the tension steel at cracked section yields.



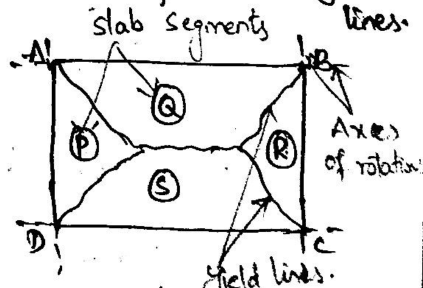
(a) Initial yielding

* As the load further increased, the yielded portion of slab acts as a plastic hinge and carries no further moment and consequently redistribution of moment takes place to the adjacent section.



(b) Development of yield lines.

* This causes the reinforcement in the adjacent region to yield on further loading.



(c) Formation of mechanism

* The lines of intense cracking, across which tension steel has yielded continue to grow and reach the corners of the slab forming a collapse mechanism at ultimate state.

* The system of yield lines which divide the slab into segments are called yield line pattern.

* Plastic deformations occur along the yield lines while the region b/n the yield lines still behave elastically.

* Since the elastic deformations are very small, assume that the slab segments between the yield lines remain plane and all excessive plastic deformations take place in the yield lines only.

Assumptions of yield line theory :-

* The failure mode from yield line theory is arrived on the basis of certain assumptions given below.

1. The failure due to complete yielding of reinforcing steel along the yield lines. Thus, the slab is designed as under reinforced and the yield line occurs on the tension face.

2. The bending moment and twisting moments at ultimate state are uniformly distributed along the yield line.

3. At failure, the yield line divides the slab into individual segments. The slab, deforms plastically but the individual segments behave elastically.

4. The elastic deformations are much lesser than plastic deformations, which can be considered as negligible. The plastic deformation takes place in the yield lines only, and the slab segment b/n yield lines remain plane and rotates as a plane segment at collapse.

Guide lines for predicting yield line pattern :-

1. Yield lines are straight lines (They are Intersections of Planar segments of the slab)

2. Yield lines end at the slab boundary. (otherwise, the failure mechanism won't be complete)

3. Yield lines (or) Yield lines produced pass through the intersection of axes of rotations of adjacent elements.

4. Axes of rotations generally lie along lines of supports and pass over the columns. (If the edge is fixed, a negative yield line may form along the support due to negative (or) hogging bending moment.

* The symbols, signs and Conventions used for showing boundary Conditions, axes of rotations and yield lines are shown below.

―――――― - Free (or) unsupported edge

////////// - Simply supported edge

xxxxxxxxxx - Fixed (or) Continuous edge

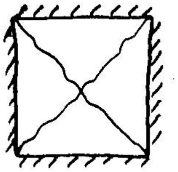
■ - Column

~~~~~~~~ - Positive moment yield line

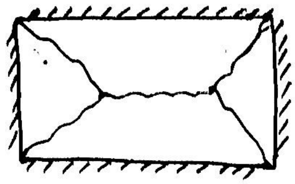
- - - - - - - - - Negative moment yield line

—·—·—·— - Axis of Rotation
~~~~~~~~

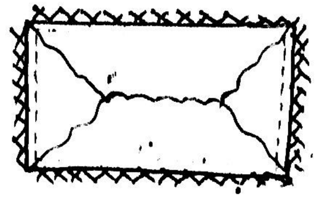
* Illustrations of yield line patterns of slabs for various shapes and boundary conditions are,



(i)



(ii)



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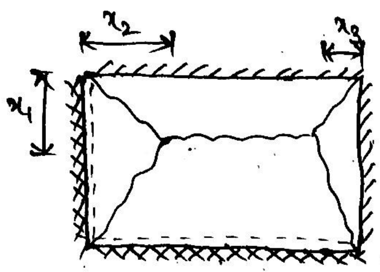

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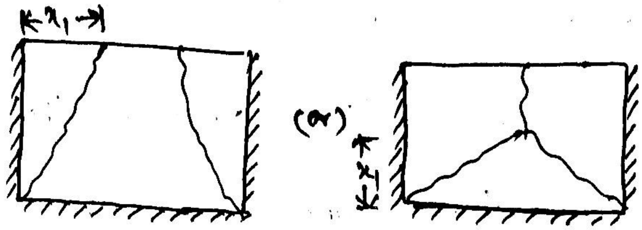

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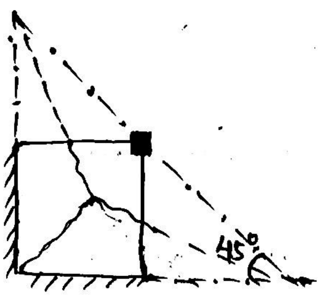

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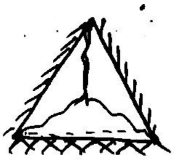

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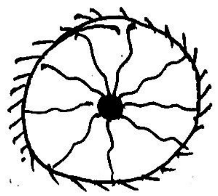

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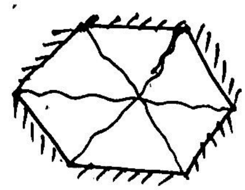
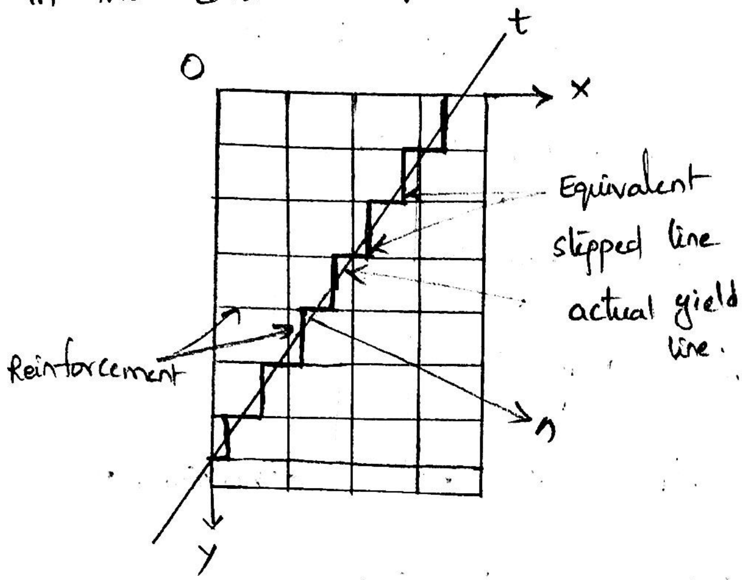


(ix)

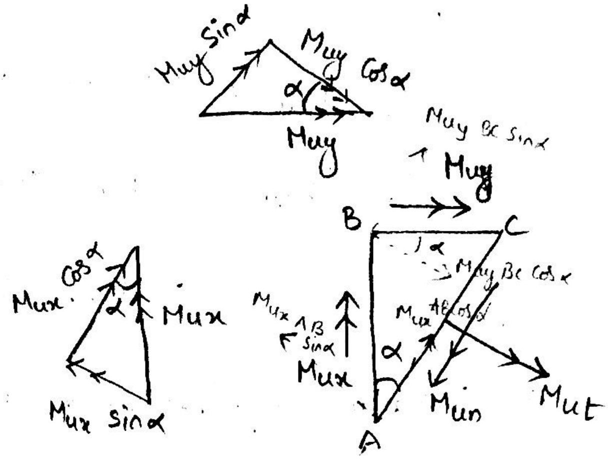
Yield Criterion: -

* The yield criterion defines the strength of a given slab element subjected to general moment field.
* The Johansen's stepped yield criterion has been considered to be sufficiently accurate in general use when in plane forces in the slab are absent.
* It assumes that a straight yield line can be replaced by a stepped yield line and with small stepped length perpendicular

to the principal directions of reinforcement and all reinforcement crossing the yield line yields due to the principal moments acting in the direction of reinforcement.



(a)



(b)

* Fig (a) shows, the equivalent stepped line in which 'x' and 'y' are the principal directions of reinforcement and the yield line in the direction 't' makes an angle in the anticlockwise direction with the y-axis.
* The ultimate moment of resistance per unit width in the orthogonal x, y directions are Mux and Muy respectively.
* The moment of resistance per unit width Mun and Mut in the directions normal and tangential to the yield line can be found by considering the equilibrium of a small triangular element as shown in Fig (b).

Total B.M acting on side AB = momet per unit width AB

= Mux · AB

" " " Bc = Muy · Bc

Total bending moment acting on side AC $= M_{un} \cdot AC$

" Twisting " " " " $= M_{ut} \cdot AC$

* Equilibrium of moment in the tangential direction

$$M_{un}\, AC = M_{ux}\, AB \cos\alpha + M_{uy}\, BC \sin\alpha$$

$$M_{un} = M_{ux} \cos\alpha \frac{AB}{AC} + M_{uy} \sin\alpha \frac{BC}{AC}$$

$$= M_{ux} \cos^2\alpha + M_{uy} \sin^2\alpha$$

* Similarly, equilibrium of moments in the normal direction, 'n'

$$M_{ut}\, AC = M_{ux}\, AB \sin\alpha - M_{uy}\, BC \cos\alpha$$

$$M_{ut} = (M_{ux} - M_{uy}) \sin\alpha \cos\alpha$$

* If $M_{ux} = M_{uy}$

$$M_{un} = M_{ux} = M_{uy} \quad \text{and} \quad M_{ut} = 0$$

* This means, when ultimate moment of resistance per unit width is equal in all directions, the torsional moment at yield line is zero. Such slabs are called "isotropic slabs".

* If $M_{ux} \neq M_{uy}$, both bending and torsional moments both exists. such slabs are called "orthotropic slabs".

Methods of Analysis and Basic Principles behind Yield line theory :-

* There may be more than one possible yield line pattern for a given slab and each pattern in general will contain unknown dimensions which locate the position of yield lines.

* All possible yield line patterns need to be analysed and the correct pattern is the one which gives the lowest ultimate load and highest ultimate moment.

* The yield line theory is based on upper bound approach.

* The upper bound approach states that the ultimate load computed on the basis of an assumed yield line pattern is either greater than (or) at the best equal to the true collapse load.

* Generally, two methods are used for determining the ultimate strength of slab using yield line analysis.

* (i) Virtual work method

  (ii) Equilibrium method

* The virtual work method is based on the principle of virtual work and both methods make use of the equilibrium equations to establish the equilibrium of various segments formed by the yield line.

(a) Virtual Work Method :-

* The principle of virtual work states that " If a deformable structure in equilibrium under the action of a system of external forces is subjected to a virtual deformation compatible with its condition of support, the work done by these forces on the displacements associated with the virtual deformation is equal to the work done by the internal stresses on the stra associated with this deformation.

∴ work done by external forces = work done by internal force (or energy absorbed by hinges)

$$W_E = W_i$$

where W_E = Work done by external forces

W_I = Energy absorbed by plastic hinges in undergoing corresponding rotations.

* If 'w_u' is the uniformly distributed external load,

$$W_E = \iint w_u \, \delta_{xy} \, dx \, dy = \Sigma W_u \Delta$$

where, δ_{xy} = Virtual displacement at any point

W_u = Resultant load on each segment

Δ = Corresponding displacement at centroid of load in each segment

If M_{un} = ultimate moment across an yield line

$$W_I = \Sigma M_{un} \theta_n l_0$$

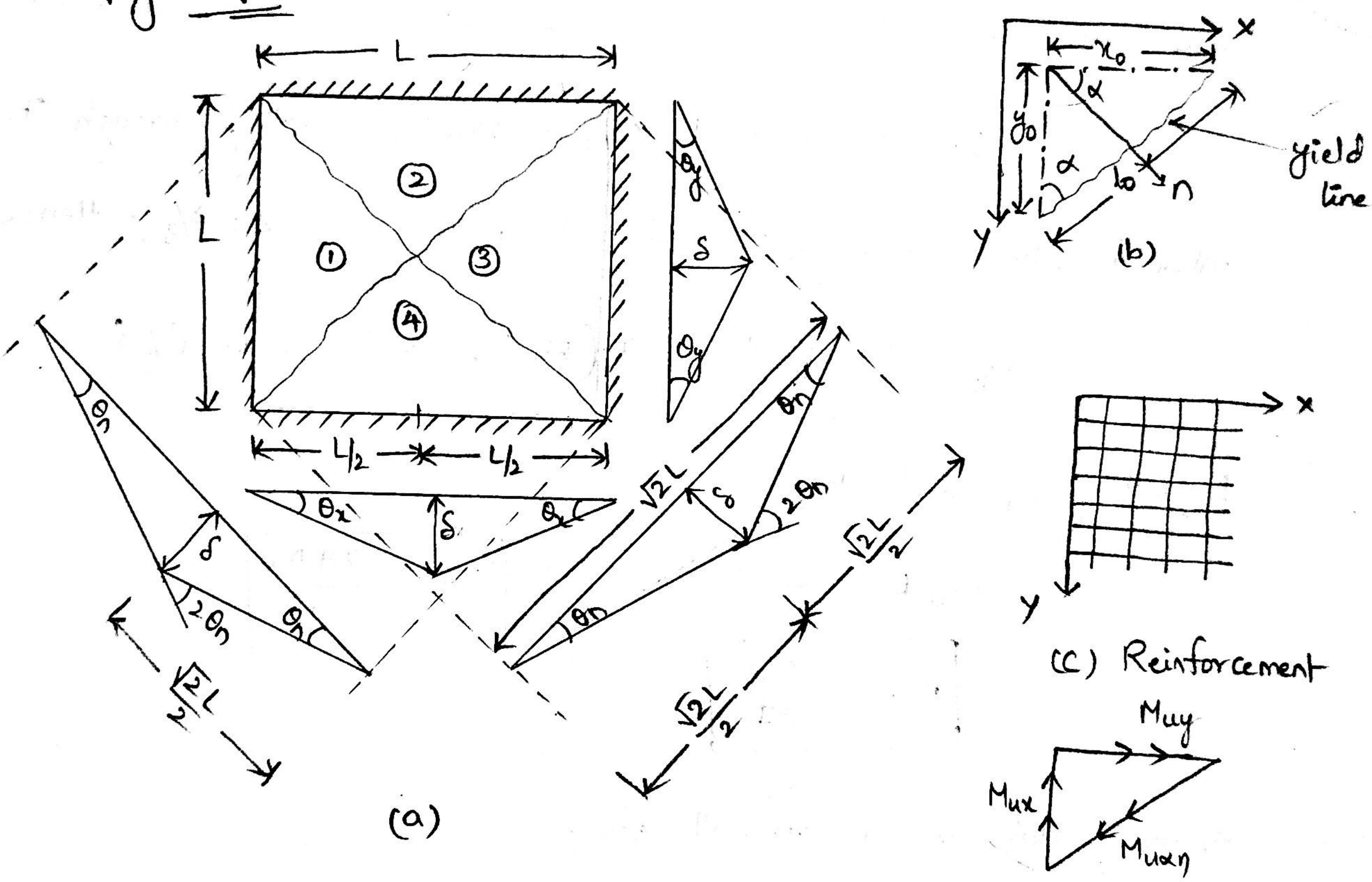
where, l_0 = length of yield line

θ_n = Relative rotation of two adjacent planes, & perpendicular to yield line.

By equating $W_E = W_I$

$$\Sigma W_u \Delta = \Sigma M_{u\alpha n} \theta_n l_0 .$$

1. Simply supported square slab :-



(c) Reinforcement

(d) Moment vectors

SQUARE SLAB : FREELY SUPPORTED

* A square slab is isotropically reinforced, Hence moment of resistance $M_{u\alpha n}$ is same in all the directions.

* Figure shows a slab of span 'L', having length of diagonal equal to $\sqrt{2}L$.

* If 'δ' is the virtual displacement at the middle, the total rotation

of the diagonal segments $= 2\theta_n = 2 \dfrac{\delta}{\sqrt{2}L/2} = \boxed{\dfrac{2\sqrt{2}\,\delta}{L} = \theta_n}$

Length of each diagonal yield line $= \sqrt{2}L$

Hence, for the two diagonal yield lines,

$$W_I = \Sigma M_{un}\,\theta_n L_o = 2 \times m_u \frac{2\sqrt{2}\,\delta}{L} \times \sqrt{2}L$$

$$= 8\, m_u \delta$$

There are four segments, each segment carrying equal total load

$$W_u = w_u \times \tfrac{1}{2} L \times L/2 = w_u L^2/4$$ acting through its

Centroid which undergoes a virtual displacement $\Delta = \delta/3$. Hence

(C.G of each triangle $= h/3$)

$$W_E = \Sigma W_u \Delta = 4\left(w_u L^2/4\right) \times \delta/3 = w_u L^2/3\ \delta$$

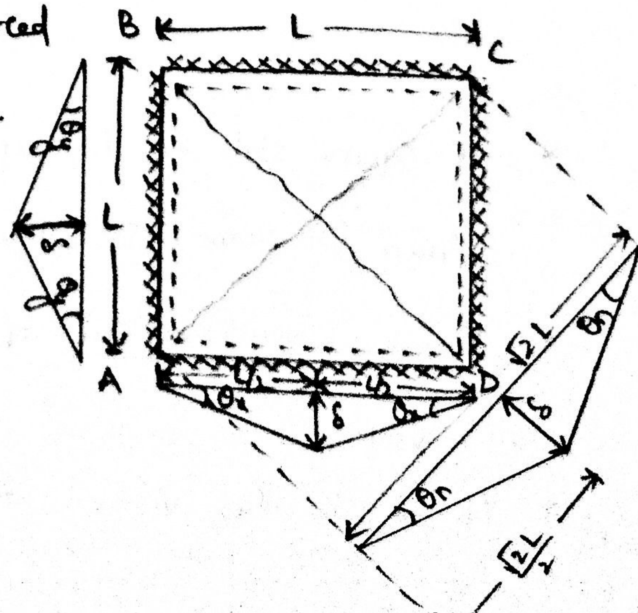
Equating W_E and W_I

$$w_u L^2/3 = 8\, m_u \delta \Rightarrow \boxed{w_u = \frac{24\, m_u}{L^2}}$$

$$\boxed{m_u = \frac{w_u L^2}{24}}$$

2. Square slab fixed on all edges:-

* Figure shows an isotropically reinforced Square slab fixed at all four supports.



* Due to this, negative yield lines will also be formed along the fixed edges.
* the rotation $\theta_x = \theta_y = \delta/(L/2)$

* The length of each negative yield line $= L$

* Hence, internal work done by negative yield lines AB, BC, CD and DA

$$\Sigma M_{un} \theta_n l_0 = 4\, m_u L \cdot \delta/(L/2) = 8\, m_u \delta$$

* Internal work done along positive yield lines along the diagonals AC and BD is

$$\Sigma M_{un} \theta_n l_0 = 2 \times m_u \frac{2\sqrt{2}}{L} \delta \times \sqrt{2} L = 8\, m_u \delta$$

* Total internal work done, $W_I = 8\, m_u \delta + 8\, m_u \delta = 16\, m_u \delta$

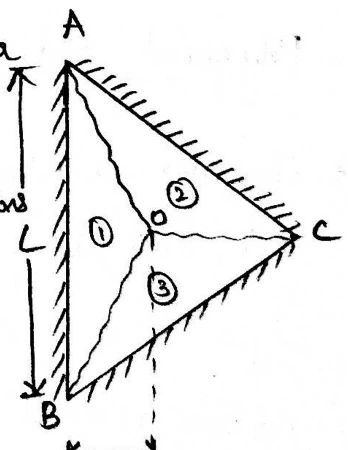
* External work done, $W_E = \Sigma W_u A = w_u L^2 \cdot \delta/3$ (– from case (i))

$$\therefore 16\, m_u \delta = w_u L^2 \cdot \delta/3$$

$$\Rightarrow \boxed{m_u = \frac{w_u L^2}{48}} \quad (\text{or}) \quad \boxed{w_u = \frac{48\, m_u}{L^2}}$$

3. Equilateral triangular slab isotropically reinforced :–

* Figure shows isotropically reinforced triangular slab.



(a)

* The ultimate moment of resistance in all directions is 'm_u'.

* when it is simply supported along all the three edges, the three yield lines will divide the slab in three symmetrically placed sector.

* The distance of point 'O' from edge AB will be $\frac{L}{2\sqrt{3}}$.

* For sector 1, $W_I = m_u \theta l_0 = m_u \cdot \frac{\delta}{L/2\sqrt{3}} \cdot L = 2\sqrt{3}\, m_u \delta$

* Also, $W_E = \Sigma\, w_u\, \Delta$

$$= \tfrac{1}{2} L \times \frac{L}{2\sqrt{3}} \times w_u\, \delta/3 = \frac{w_u\, \delta L^2}{12\sqrt{3}}$$

$$\therefore W_I = W_E$$

$$2\sqrt{3}\, m_u\, \delta = \frac{w_u\, \delta L^2}{12\sqrt{3}}$$

$$\boxed{w_u = \frac{72\, m_u}{L^2}} \quad \text{(or)} \quad \boxed{m_u = \frac{w_u L^2}{72}}$$

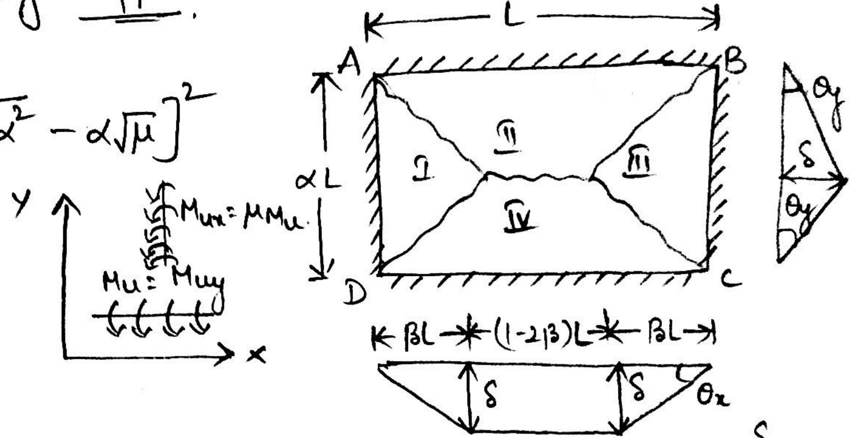
4. Rectangular slab, simply supported :-

$$m_u = \frac{w_u\, \alpha^2 L^2}{24}\left[\sqrt{3+\mu\alpha^2} - \alpha\sqrt{\mu}\right]^2$$

For square slab,

$\alpha = 1$ and $\mu = 1$

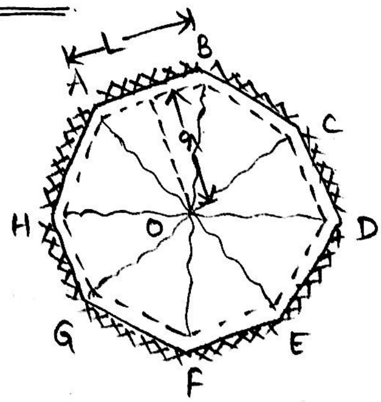
$$\therefore m_u = \frac{w_u L^2}{24}$$



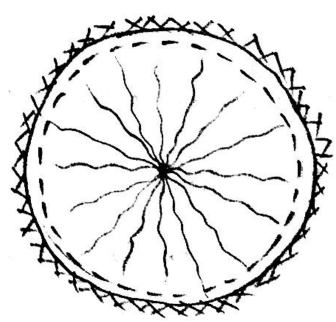
$$\theta_x = \frac{\delta}{\beta L}$$

5. Polygonal and circular slabs :-

* Consider a polygonal slab, fixed along its edges with isotropic positive moments 'm_u' and isotropic negative moment 'm_u''.



(a) Polygonal slab



(b) Circular slab

* Let the length of each side be 'L', and let the perpendicular distance of each side from centre be 'r', which is also the radius of the inscribed circle.

* Let us give a virtual displacement 'δ' at the apex 'o'.

* For any sector ABO

$$M_I = (m_u + m_u')\, L \frac{\delta}{r} \longrightarrow (1)$$

$$M_E = \tfrac{1}{2} L r \frac{\delta}{3} w_u \longrightarrow (2)$$

* Equating the two, we get

$$w_u\, l r\, \delta/6 = (m_u + m_u')\, L \cdot \delta/r$$

$$\Rightarrow w_u = \frac{6(m_u + m_u')}{r^2}$$

* For circular slabs, there will be ~~[illegible]~~ innumerable number of positive yield lines, such that number of sides 'n' will tend to be infinity and 'L' will tend to be zero, r will be the radius of the circle and for any sector ABO,

$$w_u = \frac{6(m_u + m_u')}{r^2}$$

* For circular slabs, simply supported along edges, $m_u' = 0$

$$w_u = \frac{6 m_u}{r^2} \quad (\text{or}) \quad m_u = \frac{w_u r^2}{6}$$

* For hexagonal slab, $r = \frac{\sqrt{3}L}{2}$

$$\therefore w_u = \frac{6(m_u + m_u')}{(\sqrt{3}L/2)^2} = \frac{8(m_u + m_u')}{L^2}$$

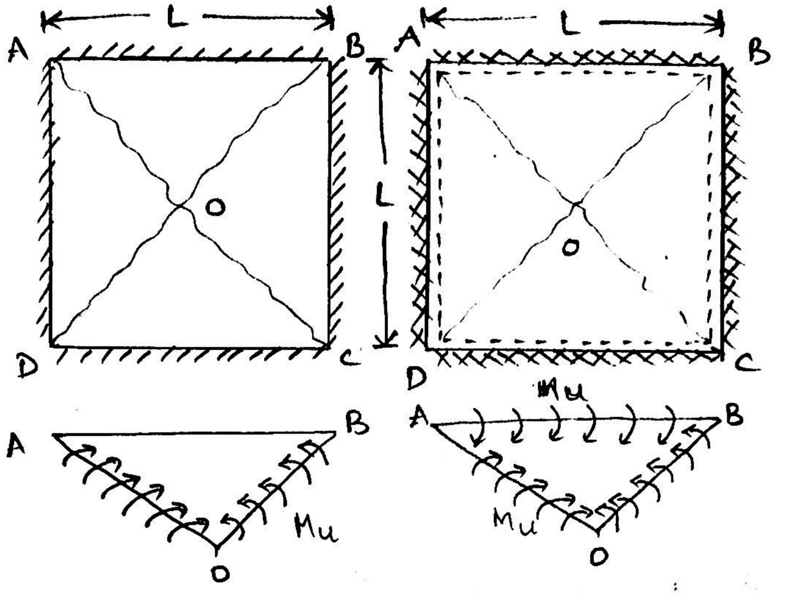
* If hexagonal slab is simply supported

$$w_u = \frac{8 m_u}{L^2} \quad (\text{or}) \quad m_u = \frac{w_u L^2}{8}$$

(b) Analysis by Equilibrium Method :-

1. Isotropically Reinforced Square slab :-

* Fig (a) shows a simply supported square slab. 'mu' will be same along all the directions. The yield lines will be along the two diagonals.



(a) simply supported (b) Fixed.

* Considering the equilibrium of the triangular sector 'ABO' and taking moments about edge AB, we get

$$m_u \cdot L = \frac{1}{2} \times L \times \frac{L}{2} \times w_u \times \left(\frac{L}{2} \times \frac{1}{3}\right)$$

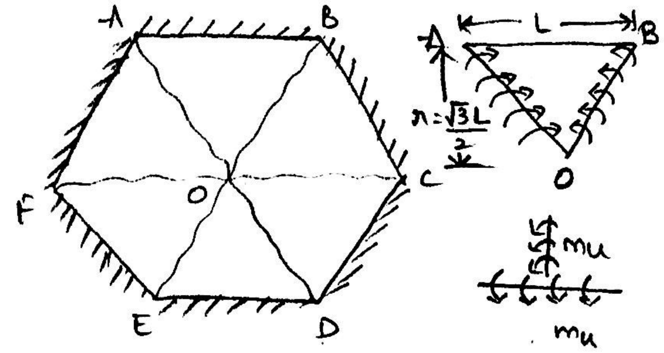
$$\boxed{m_u = \frac{w_u L^2}{24}}$$

* Fig (b) shows a fixed slab, positive yield lines will be along the diagonals and negative yield lines will be developed along fixed edges
* Considering the equilibrium of sector ABO, and taking moments about edge AB, we get

$$m_u L + m_u L = \frac{1}{2} \times L \times \frac{L}{2} \times w_u \times \left(\frac{1}{3} \times \frac{L}{2}\right)$$

$$\boxed{m_u = \frac{w_u L^2}{48}}$$

(2) Isotropically reinforced Hexagonal slab :-



* Consider the equilibrium of sector ABO, and take moments about AB.

* The value of perpendicular

$$r = \frac{\sqrt{3}L}{2}$$

$$\therefore m_u L = \frac{1}{2} \times L \times r \times \frac{r}{3} \times w_u$$

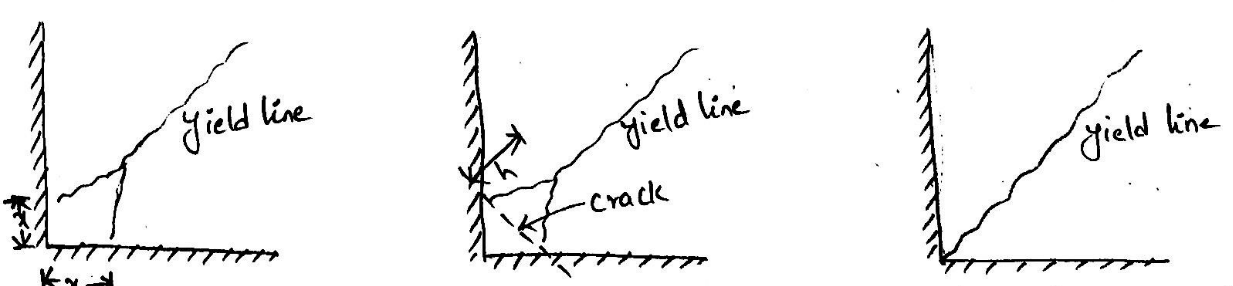
$$= \frac{w_u}{6} r^2 = \frac{w_u}{6}\left(\frac{\sqrt{3}L}{2}\right)^2$$

$$\boxed{m_u = \frac{w_u L^2}{8}}$$

Corner Levers :-

* The elastic analysis of plates shows that the corners of a simply supported plate tend to lift off the support if not held down.

* In such plate, the axis of rotation, cutting the corners diagonally, is formed and the positive yield line tends to fork near the corner. These are known as "Corner levers".



(a) up lift Portion

(b) Corner restrained no top steel

(c) Corner restrained top steel Present

* If the slab is held down without Provision of top steel, the slab tends to crack along the diagonal.

* However with provision of necessary top steel and corners held down, the corner levers do not form and the diagonal yield line reaches the corner without formation of crack.

* The overall effect of these corner levers is to reduce the ultimate load capacity of the slab.

* For an isotropically reinforced, uniformly loaded square slab simply supported along its edges, with corners held down and no top steel, the reduction in ultimate load is 9.6 %.

* Hence, the effect is more significant for triangular slabs with acute angle corners.

Circular slabs :-

* A circular slab can be regarded as a polygonal slab with sides tending to infinity.

* Therefore, consider an isotropically reinforced regular sided polygonal slab fixed along the edges and loaded by a uniformly distributed load of intensity 'w_u'.

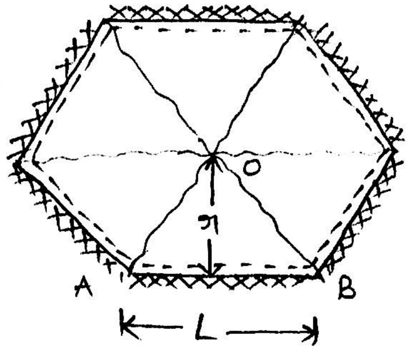
* Let the positive and negative ultimate moment of resistance per unit width be equal to "m_u" and "m_u'".

* Let the length of each side of polygon be 'L' and radius of inscribed circle be 'r'.

* ~~Con~~

* Consider the equilibrium of segment AOB,

Taking moments about the axis of rotation AB,



$$(m_u + m_u') L = w_u \cdot L \frac{r}{2} \cdot \frac{r}{3}$$

$$= w_u L \frac{r^2}{6}$$

$$\boxed{w_u = \frac{6(m_u + m_u')}{r^2}}$$

* The above equation can be applied to a wide range of cases of regular sided slabs fixed along the edges by substituting appropriate values for 'r'.
* For example, equilateral triangular slab having $n = 3$, $r = \frac{L}{2\sqrt{3}}$
* Square slab having $n = 4$, $r = L/2$
* Hexagonal slab having $n = 6$, $r = \sqrt{3}L/2$
* Circular slab having $n = \infty$, $r = r$

$\therefore$ For circular slab, $w_u = 6(m_u + m_u')/r^2$

1. A reinforced Concrete slab 5m x 5m is simply supported along the four edges and is reinforced with 10 mm dia. Fe415 steel bars @ 150 mm c/c both ways. The average effective depth of the slab is 100mm and the overall depth of slab is 130mm. The slab carries a flooring of 50mm thick having unit weight of 22 KN/m³. Determine the maximum permissible service load, if M20 Concrete is used.

write given data first.

Sol:-

$$A_{st} = \frac{1000 \times \pi/4 \times 10^2}{150} = 523.6 \text{ mm}^2/\text{m}$$

$$\therefore M_u = 0.87 f_y A_{st} d \left(1 - \frac{f_y A_{st}}{f_{ck} b d}\right)$$

$$= 0.87 \times 415 \times 523.6 \times 100 \left(1 - \frac{415 \times 523.6}{20 \times 1000 \times 100}\right)$$

$$= 16.85 \times 10^6 \text{ N mm}$$

$$M_{u,lim} = 0.138 f_{ck} b d^2$$

$$= 0.138 \times 20 \times 1000 \times 100^2 = 27.61 \times 10^6 \text{ N mm}$$

$M_u < M_{ulim} \rightarrow$ The slab is under reinforced.

$$\therefore m_u = M_u = 16.85 \times 10^6 \text{ N mm} = 16.85 \text{ KNm}$$

For Simply supported square slab, $m_u = \frac{W_u L^2}{24} \Rightarrow W_u = \frac{24\, m_u}{L^2}$

$$W_u = \frac{24 \times 16.85}{5^2} = 16.176 \text{ KN/m}^2$$

$$\therefore \text{Service load} = \frac{16.176}{1.5} = 10.78 \text{ KN/m}^2$$

Dead load of slab $= 0.13 \times 25 = 3.25$ KN/m²

Dead load of finishing $= 0.05 \times 22 = 1.1$ KN/m²

Total dead load = 3.25 + 1.1 = 4.35 KN/m^2

∴ Permissible service load = 10.78 − 4.35 = 6.43 KN/m^2.

2. A Square slab of side length 4m is simply supported at the ends and carries a service live load of 3 KN/m^2. Design the slab using M_{20} grade Concrete and Fe_{415} grade steel bars.

Sol:- Given data :- L = 4m

f_{ck} = 20 N/mm^2, f_y = 415 N/mm^2

live load = 3 KN/m^2

Minimum effective depth = $\frac{\text{Span}}{35} = \frac{4000}{35}$ = 114 mm

Let us Provide an overall depth of 150mm.

Using 15 mm clear Cover and 10 mm dia. bars,

Avg. Effective depth, d = $150 - 15 - \frac{10}{2}$ = 130 mm

∴ Self weight of slab = 0.15 × 25 = 3750 N/m^2

Assume 50mm thick floor finish of unit weight 22 KN/m^3.

weight of floor finish = 0.05 × 22 = 1100 N/m^2

Live load = 3000 N/m^2

Total Service load = 3750 + 1100 + 3000 = 7850 N/m^2

Ultimate load, W_u = 1.5 × 7850 = 11775 N/m^2

Ultimate capacity of slab, ~~$W_u = \frac{24\, m_u}{L^2}$~~ $m_u = \frac{W_u L^2}{24}$

$= \frac{11775 \times 4^2}{24}$ = 7850 Nm = 7.85 × 10^6 N mm limiting moment, $M_{u,lim} = 0.138\, f_{ck}\, bd^2$

$= 0.138 \times 20 \times 1000 \times 130^2 = 46.64 \times 10^6$ Nmm

$m_u < M_{ulim}$ → It is under reinforced slab.

$\therefore \quad M_u = m_u = 0.87\, f_y\, A_{st}\, d \left(1 - \frac{f_y A_{st}}{f_{ck}\, bd}\right)$

$7.85 \times 10^6 = 0.87 \times 415 \times A_{st} \times 130 \left(1 - \frac{415 \times A_{st}}{20 \times 1000 \times 130}\right)$

$7.85 \times 10^6 = 46936.5\, A_{st} - 7.49\, A_{st}^2$

$A_{st} = 171.96 \text{ mm}^2$

$A_{st\,min} = 0.12\%$ gross c/s area

$= \frac{0.12}{100} \times 1000 \times 150 = 180 \text{ mm}^2$

$\therefore A_{st} = 180 \text{ mm}^2$

using 8 mm dia bars, spacing $= \frac{\pi/4 \times 8^2}{180} \times 1000 = 279.28$ mm

∴ Provide 8 mm dia bars @ 250 mm c/c both ways.

3. A reinforced Concrete slab 4 m × 6m is reinforced with 10mm dia. bars @ 150mm spacing in the short direction and 200 mm spacing in the long direction. The slab is 100 mm thick with average effective depth of 80mm. Determine the safe permissible service live load. The dead load of flooring may be assumed as 1 kN/m². Use M20 Concrete and Fe415 steel bars.

Sol:- Given data:- $L = 6$ m

$\alpha L = 4$ m $\Rightarrow \alpha = \frac{4}{6} = 0.667$

$D = 100$ mm

$d = 80$ mm

$$Ast_y = \frac{1000 \times \pi/4 \times 10^2}{150} = 524 \text{ mm}^2$$

$$Ast_x = \frac{1000 \times \pi/4 \times 10^2}{200} = 393 \text{ mm}^2$$

$$m_{uy} = 0.87 \, f_y \, Ast_y \, d \left(1 - \frac{f_y \, Ast_y}{f_{ck} \, bd}\right)$$

$$= 0.87 \times 415 \times 524 \times 80 \left(1 - \frac{415 \times 524}{20 \times 1000 \times 80}\right)$$

$$= 13.078 \times 10^6 \text{ N mm} = m_u$$

$$m_{ux} = 0.87 \times 415 \times 393 \times 80 \left(1 - \frac{415 \times 393}{20 \times 1000 \times 80}\right)$$

$$= 10.194 \times 10^6 \text{ N mm}$$

$$m_{ux} = \mu \, m_u \Rightarrow \mu = \frac{10.194 \times 10^6}{13.078 \times 10^6} = 0.779$$

For Rectangular slab, ~~with~~ with simply supported ends.

$$m_u = \frac{W_u \alpha^2 L^2}{24} \left[\sqrt{3 + \mu\alpha^2} - \alpha\sqrt{\mu}\right]^2$$

$$\Rightarrow W_u = \frac{24 \, m_u}{\alpha^2 L^2 \left[\sqrt{3 + \mu\alpha^2} - \alpha\sqrt{\mu}\right]^2}$$

$$= \frac{24 \times 13.078}{0.667^2 \times 6^2 \times \left[\sqrt{3 + 0.779 \times 0.667^2} - 0.667 \times \sqrt{0.779}\right]^2}$$

$$= 12.729 \text{ KN/m}^2$$

$$\text{Service load} = \frac{12.729}{1.5} = 8.486 \text{ KN/m}^2$$

Dead load of slab $= 0.1 \times 25 = 2.5 \text{ KN/m}^2$

Dead load of flooring $= 1 \text{ KN/m}^2$

$\therefore$ Service live load $= 8.486 - 2.5 - 1 = 4.986 \text{ KN/m}^2$

4. For the slab in Pb (3), If the yield lines are inclined at 45° to either direction of reinforcements, find the ultimate moments $m_{u\alpha n}$, $m_{u\alpha t}$ for unit length along yield line (use M20 Concrete and Fe415 grade steel).

Sol:- From Pb (3), $M_{ux} = 10.194$ KNm

$M_{uy} = 13.078$ KNm.

$$m_{u\alpha n} = m_{ux} \cos^2\alpha + m_{uy} \sin^2\alpha$$

$$m_{u\alpha t} = (m_{ux} - m_{uy}) \sin\alpha \cos\alpha$$

$\alpha = 45° \Rightarrow \cos\alpha = \sin\alpha = 0.707$

$$\therefore M_{u\alpha n} = 10.194 \times 0.707^2 + 13.078 \times 0.707^2 = \underline{11.636 \text{ KNm}}$$

$$m_{u\alpha t} = (10.194 - 13.078)\, 0.707^2 = \underline{-1.442} \text{ KNm.}$$

5. A rectangular slab 3.5 x 5 m in size, simply supported at the edges. The slab is expected to carry a service live load of 3 KN/m² and a floor finishing load of 1 KN/m². Use M20 Concrete and Fe415 steel. Design the slab if (a) it is isotropically reinforced, (b) If it is orthotropically reinforced with $\mu = 0.75$.

Sol:- Given data:- $L = 5$m , $\alpha L = 3.5$m

$\Rightarrow \alpha = 3.5/5 = 0.7$

$$\text{Effective Depth of slab} = \frac{\text{span}}{35} = \frac{3500}{35} = 100 \text{ mm}$$

Using 10mm dia bars both ways, using 15 mm clear cover.

Overall depth = $100 + 15 + 10/2 = 120$ mm.

Dead weight of slab = $0.12 \times 25 = 3\ KN/m^2$

Dead weight of flooring = $1\ KN/m^2$

live load = $3\ KN/m^2$

$\therefore$ Total service load = $3+1+3 = 7\ KN/m^2$

Ultimate design load = $1.5 \times 7 = 10.5\ KN/m^2$.

(i) isotropically reinforced slab :- $\mu = 1$

Ultimate moment is given by $m_u = \dfrac{w_u \alpha^2 l^2}{24}\left[\sqrt{3+\mu\alpha^2} - \alpha\sqrt{\mu}\right]^2$

$$= \frac{10.5 \times 0.7^2 \times 5^2}{24}\left[\sqrt{3+1\times 0.7^2} - 0.7\times\sqrt{1}\right]^2$$

$$= 7.313\ KNm/m$$

The reinforcement is given by

$$7.313 \times 10^6 = 0.87 \times 415 \times A_{st} \times 100\left(1 - \frac{415 \times A_{st}}{20 \times 1000 \times 100}\right)$$

$$= 36105\ A_{st} - 7.49\ A_{st}^2$$

$A_{st} = 211.86\ mm^2$

Using $\overset{10}{8}$ mm dia bars, spacing = $\overset{370}{237.28}$ mm

$\therefore$ Provide $\overset{10}{8}$ mm dia bars @ $\overset{350}{230}$ mm c/c in both ways.

(ii) orthotropically reinforced slab :- $\mu = 0.75$

$$m_u = \frac{10.5 \times 0.7^2 \times 5^2}{24}\left[\sqrt{3+0.75\times 0.7^2} - 0.7\sqrt{0.75}\right]^2$$

$$= 8.093\ KN m/m$$

The reinforcement is given by

$$8.093 \times 10^6 = 0.87 \times 415 \times A_{st} \times 100\left(1 - \frac{415\ A_{st}}{20 \times 1000 \times 100}\right)$$

$$= 36105\ A_{st} - 7.49\ A_{st}^2$$

$A_{st} = 235.67\ mm^2$

Using $\overset{10}{\cancel{8}}$ mm dia bars, spacing in short direction $= \dfrac{\pi/4 \times \overset{10^2}{\cancel{8^2}} \times 1000}{235.67}$

$= \overset{333}{213.31}$ mm

Spacing in long direction $= \overset{333}{213.31}/0.75 = \overset{444}{284.42}$ mm.

$\therefore$ Provide 8mm dia. bars @ $\overset{300}{200}$ mm c/c in short direction and @ $\overset{400}{280}$ mm c/c in long direction.

6) A hexagonal slab is simply supported along the edges with a side length of 3.5m. It is isotropically reinforced with 10mm dia. bars @ 125mm c/c both ways at an average effective depth of 120mm. The overall depth of slab is 150mm. Calculate (i) ultimate load capacity of slab, (ii) safe permissible service live load. Use M_{20} Concrete and Fe 415 steel. Take the load of floor finishing as 1.25 kN/m².

Sol:- Given data :- $L = 3.5\,m$

$$A_{st} = \frac{\pi/4 \times 10^2 \times 1000}{125} = 628.3\ mm^2$$

$$m_u = M_u = 0.87 \times 415 \times 628.3 \times 120 \left(1 - \frac{415 \times 628.3}{20 \times 1000 \times 120}\right)$$

$$= 24.264\ KNm$$

Ultimate load capacity, $w_u = \dfrac{8\,m_u}{L^2} = \dfrac{8 \times 24.264}{3.5^2} = 15.845\ KN/m^2$

Total service load $= 15.845/1.5 = 10.564\ KN/m^2$

Dead load of slab $= 0.15 \times 25 = 3.75\ KN/m^2$

load of floor finishing $= 1.25\ KN/m^2$

Permissible service live load $= 10.564 - 3.75 - 1.25 = 5.564\ kN/m^2$

7. Design a reinforced circular slab for the following data.

(i) Diameter of slab = 5.5m

(ii) Service live load = 4 KN/m^2

(iii) Floor finishing load = 1 KN/m^2

(iv) Grade of Concrete = M_{20} and Grade of steel = Fe_{415}. The slab is simply supported along the edge.

Sol:- Effective depth of slab = $\frac{\text{span}}{35} = \frac{5500}{35} = 157\,mm \simeq 160\,mm$

Using 10mm dia. bars and a clear cover of 15mm,

overall depth = $160 + 15 + 10/2 = 180\,mm$.

Self weight of slab = $0.18 \times 25 = 4.5$ KN/m^2

Dead load of flooring = 1 KN/m^2

Live load = 4 KN/m^2

∴ Total service load = 9.5 KN/m^2

Ultimate load = $1.5 \times 9.5 = 14.25$ KN/m^2

For circular slab, $M_u = \frac{W_u a^2}{6} = \frac{14.25 \times (5.5/2)^2}{6} = 17.96$ kNm.

$M_{u\,lim} = 0.138\, f_{ck}\, b d^2 = 0.138 \times 20 \times 1000 \times 160^2 = 70.65$ kNm.

$M_u < M_{u\,lim}$ — The slab is under reinforced.

$$17.96 \times 10^6 = 0.87 \times 415 \times A_{st} \times 160 \left(1 - \frac{415 \times A_{st}}{20 \times 1000 \times 160}\right)$$

$$= 57768\, A_{st} - 7.49\, A_{st}^2$$

$A_{st} = 324.55\,mm^2$

Using 10mm dia. bars, spacing = $\frac{\pi/4 \times 10^2}{324.55} \times 1000 = 242\,mm$

∴ Provide 10mm dia bars @ 240 mm c/c.