UNIT- VI

PRESTRESSED CONCRETE

* Prestressed Concrete is basically concrete in which internal stresses of a suitable magnitude and distribution are introduced so that the stresses resulting from external loads are counter-acted to a desired degree.

* In reinforced Concrete members, the prestress is commonly introduced by tensioning the steel reinforcement.

Materials used in Pre stressed Concrete :-

(1) High strength Concrete

(2) High tensile steel

Need for High strength Concrete and high tensile steel :-

* The early attempts to use mild steel in pre stressed Concrete were not successful as a working stress of 120 N/mm² in mild steel is more (or) less Completely lost due to elastic deformation, creep and shrinkage of concrete.

* The normal loss of steel is generally about 100 to 240 N/mm² and it is apparant that if this loss of stress is to be a small portion of the initial stress, the stress in the steel in the initial stages must be very high about 1200 to 2000 N/mm².

* These high stress ranges are possible only with the use of high strength steel.

* High strength concrete is necessary in pre stressed concrete as the material offers high resistance in tension, shear, bond and bearing.

* In the zone of anchorages, the bearing stresses being higher, high strength concrete is invariably prefered to minimise the cost.

* High strength concrete is less liable to shrinkage cracks and has a high modulus of elasticity and smaller ultimate creep strain resulting in a smaller loss of pre stress in steel.

* Use of high strength concrete result in the reduction of c/s dimensio[n] of pre stressed concrete structural elements with a reduced dead weight.

* Longer spans become technically and economically practicable.

Terminology :-

(1) Tendon :-

A streched element used in a concrete member of structur[e] to impart pre stress to the concrete.

Generally high tensile wires, bars, cables (or) strands are used as tendons.

2. Anchorage :-

A device generally used to enable the tendon to impart and maintain pre stress in the concrete. The commonly used anchorages are

(i) Freyssinet system

(ii) Magnel blaton

(iii) Gliford - Udall system

(iv) Lion - hardt

(v) Lee - Mecall systems.

3. Pre - tensioning :-

A method of pre ~~tensioning~~ stressing in which the tendons are tensioned before the concrete is placed. In this method the pre-stress imparted to concrete by bond between steel and concrete.

4. Post - Tensioning :-

A method of pre stressing concrete by tensioning the tendons against hardened concrete. In this method the prestress is imparted to concrete by bearing.

5. Full pre - stressing :-

Pre stressed concrete in which tensile stresses in the concrete are entirely obviated at working loads by having sufficiently high pre stress in the members.

6. Partial Pre - stressing :-

The degree of pre stress applied to concrete in which tensile stresses to a limited degree are permitted in concrete under working loads. In this case in addition to tension steel a considerable proportion of untensioned steel is generally used to limit the crack under service loads.

7. Circular pre stressing :-

The term refers to pre stressing in round members such as tanks and pipes.

Advantages of pre stressed concrete :-

(i) Pre stressed concrete offers great technical advantage in comparison with other forms of construction, such as reinforced concrete & steel.

(ii) In the case of fully pre stressed members, which are free from tensile stresses under working loads. The cross section is more effectively utilised when compared with R.C section, which is cracked under working loads.

(iii) Within certain limits a permanent dead load may be counteracted by increasing the eccentricity of the prestressing force in a pre stressed structural element. Thus effective saving in the use of materials.

(iv) The material cost in the pre stressed concrete is much less than in equivalent to R.C members.

(v) The deformation of Pre stressed Concrete members is much less Compared to R.C members.

(vi) Pre stressed Concrete has high fatigue strength and resiliance.

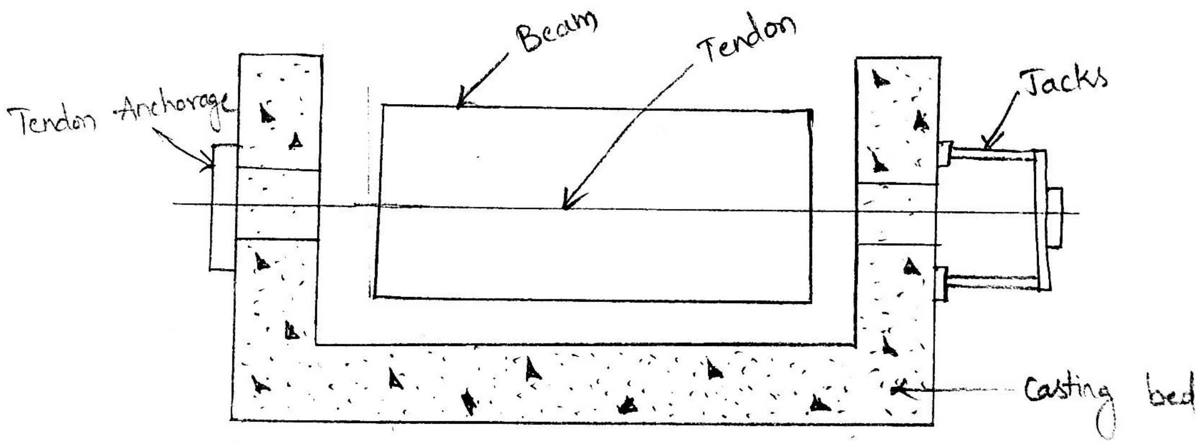
Disadvantages :-

(i) It requires skilled labour and good quality control.

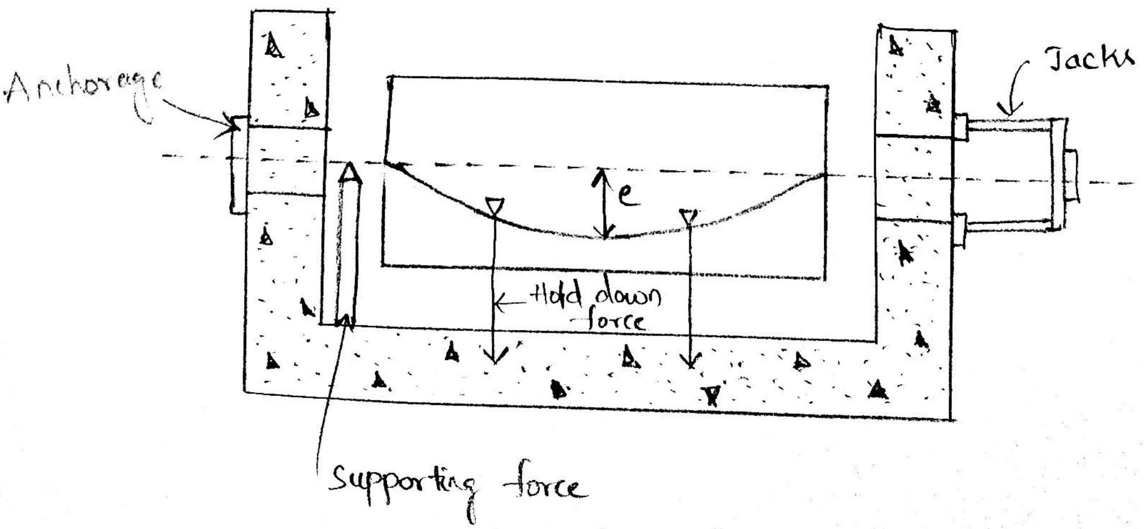
(ii) It needs special techniques to apply pre stressing forces and anchoring the wires.

Pre tensioning Systems :-

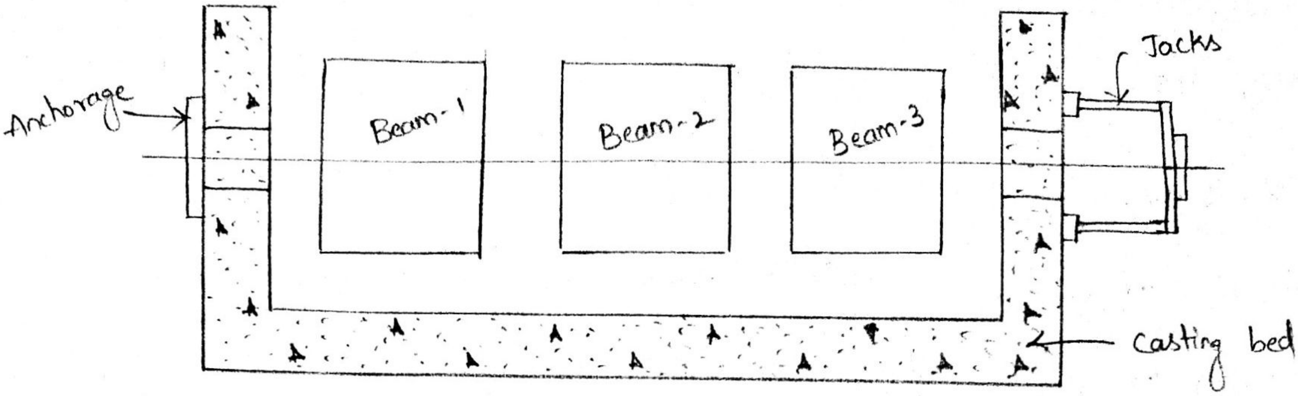
(1) Beam with straight tendon :-



(2) Beam with variable tendon eccentricity :-

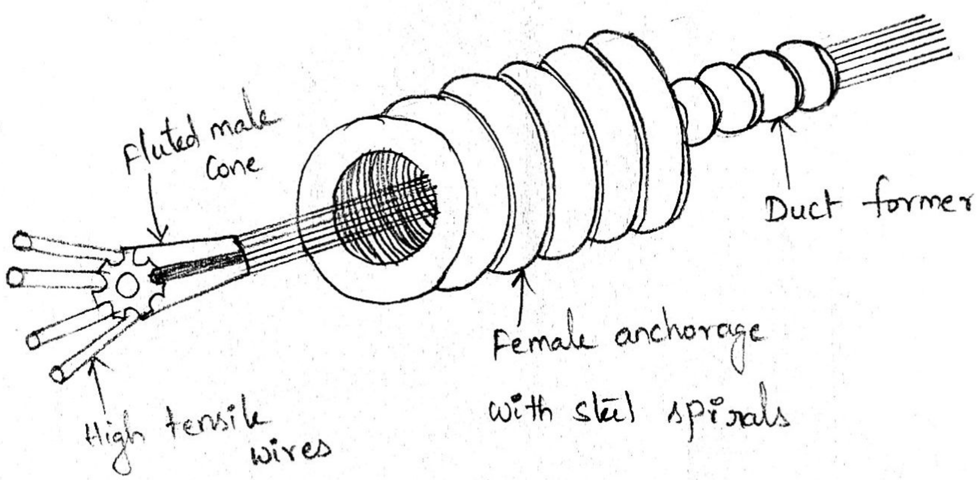


(3) Hoyer's long line system of pre stressing :-

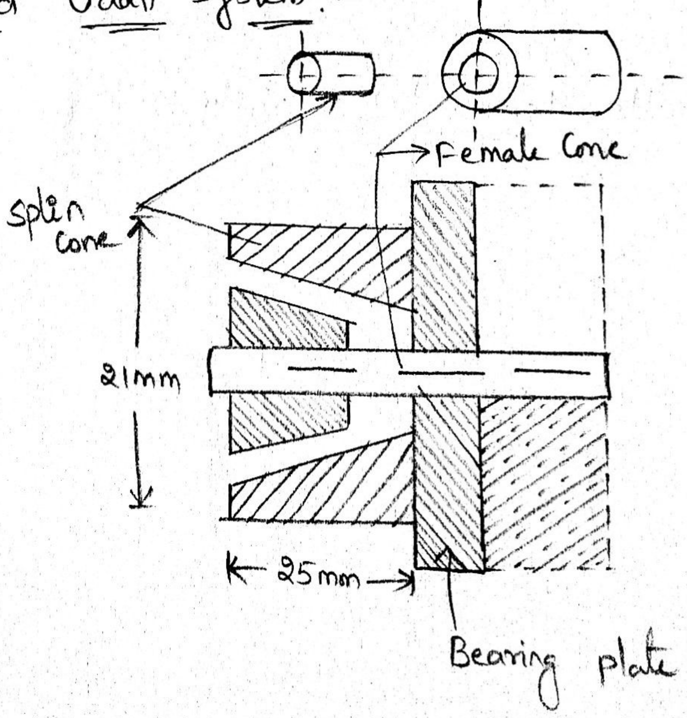


Type of anchorages :-

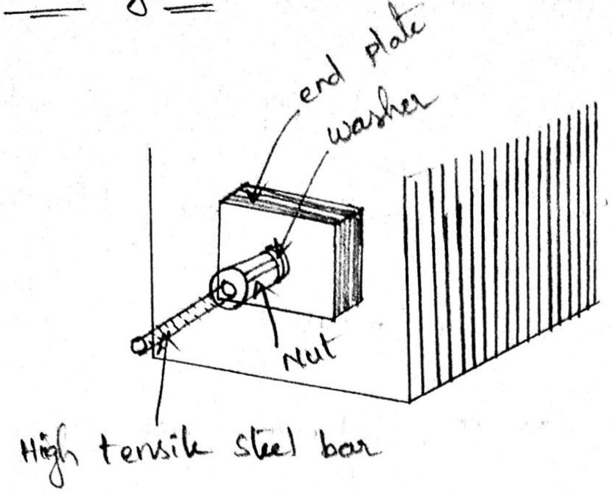
(1) Freyssinet system :-

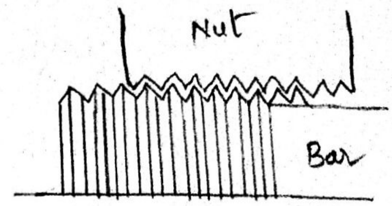


(2) Gifford Udall system :-



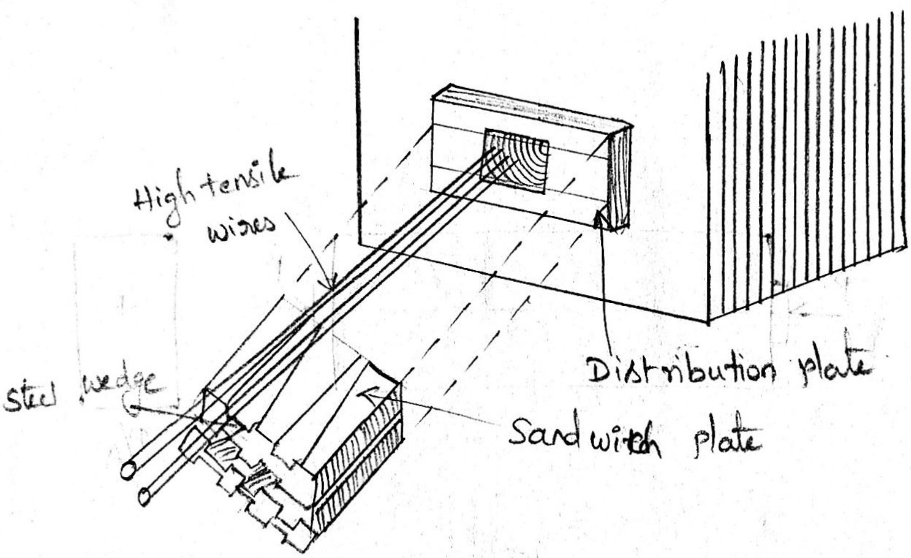
(3) Lee - Mecall System :-



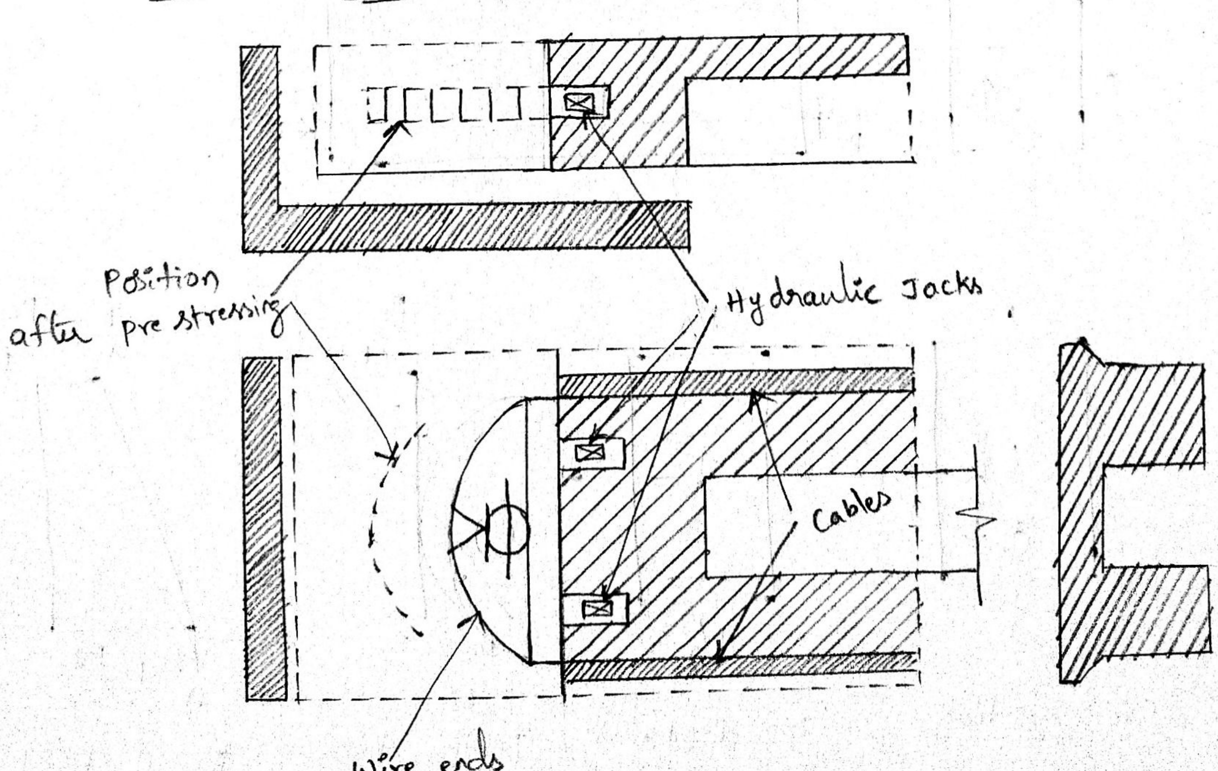


Enlarged detail of thread

(4) Magnel - Blaton system :-



(5) Baur - leon hardt system :-



Analysis of Pre stress and bending stresses :-

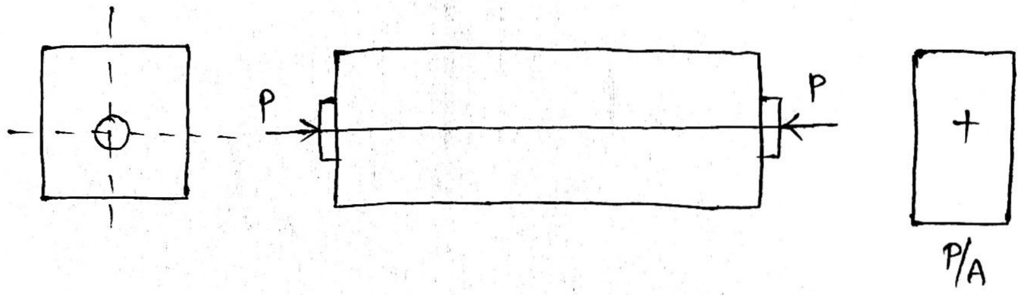
The analysis of stresses developed in a prestressed concrete structural element is based on the following assumptions.

(i) Concrete is homogeneous elastic material

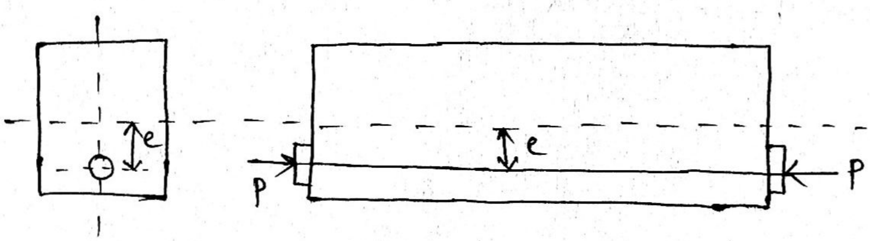
(ii) with in the range of working stress both concrete and steel behave elastically.

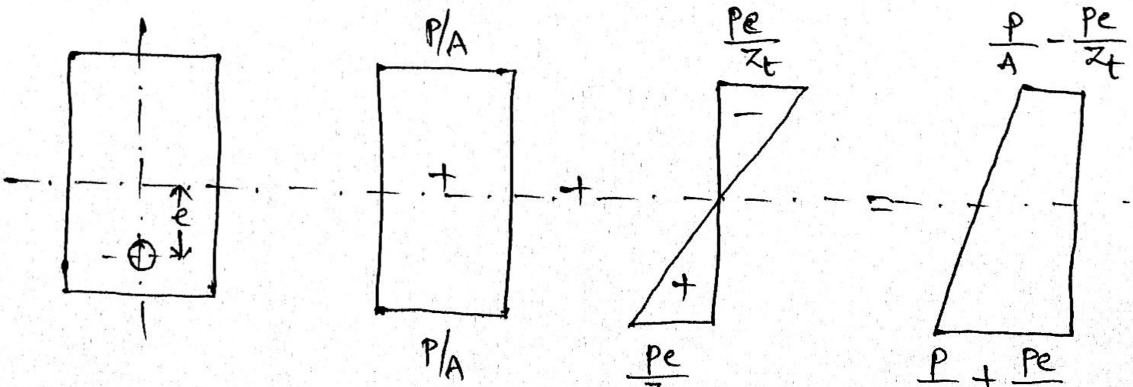
(iii) A Plane section is assumed to be remain plane after bending.

(1) Concentric tendon :-



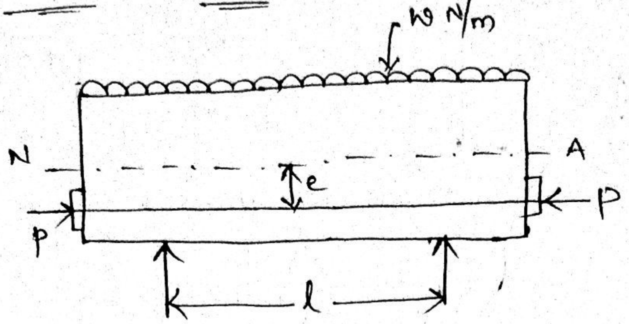
(2) Eccentric tendon :-

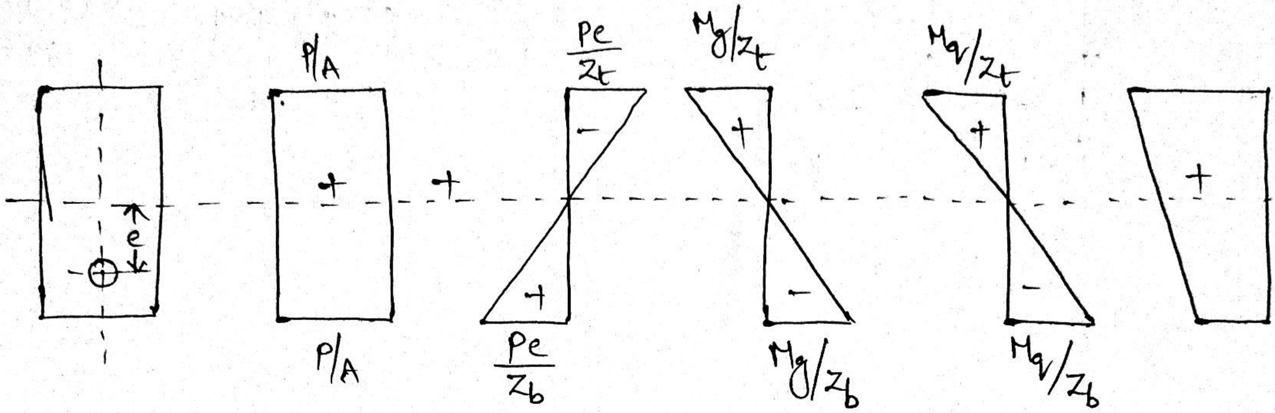




Cross Section   Direct stress   Bending stress   Resultant stress.

(3) Resultant stress at a Section :-





Cross section — Direct stress — Bending stress — Dead load stress — Live load stress — Resultant Stress

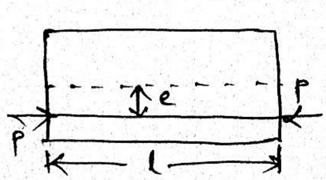
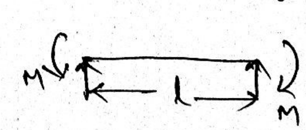
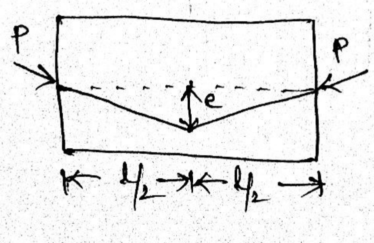
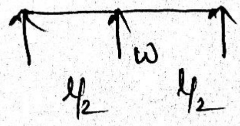
$\underbrace{\text{Direct stress} \quad \text{Bending stress}}_{\text{Pre stress}}$

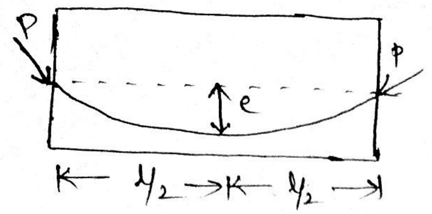
Resultant stress at top $= \dfrac{P}{A} - \dfrac{Pe}{Z_t} + \dfrac{M_g}{Z_t} + \dfrac{M_q}{Z_t}$

Resultant stress at bottom $= \dfrac{P}{A} + \dfrac{Pe}{Z_b} - \dfrac{M_g}{Z_b} - \dfrac{M_q}{Z_b}$

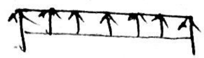
$M_q = \dfrac{ql^2}{8}$ and $M_g = \dfrac{gl^2}{8}$

Tendon profile and equivalent loads in prestressed Concrete beams :-

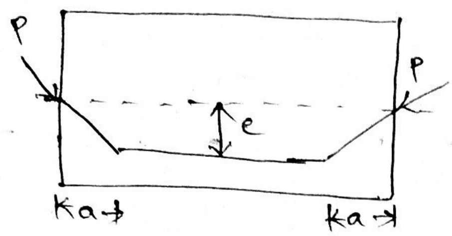
S.NO	Tendon profile	Equivalent moment (or) Load	Equivalent loading	Camber
1.		$M = Pe$		$\dfrac{Ml^2}{8EI}$
2.		$w = \dfrac{4\,Pe}{l}$		$\dfrac{wl^3}{48EI}$

3. 

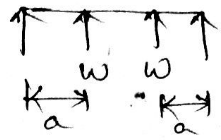
$w = \frac{8Pe}{l}$



$\frac{5wl^4}{384EI}$

4. 

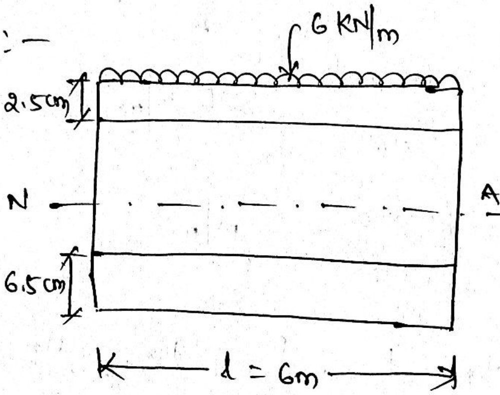
$w = \frac{Pe}{al}$

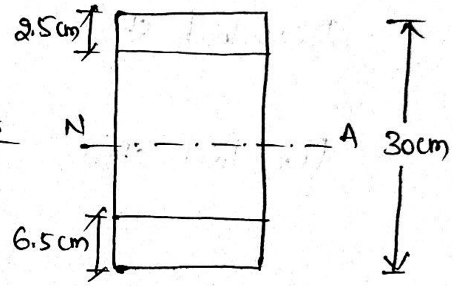


$\frac{a(3-4a^2)wl^3}{24EI}$

1) A Rectangular Concrete beam of cross section 30 cm deep and 20 cm wide is pre stressed by means of 15 wires of 5 mm diameter located at 6.5 cm from the bottom of the beam and 3 wires of 5 mm diameter at 2.5 cm from the top. Assume the pre stress in the steel as 840 N/mm². Calculate the stresses at the extreme fibre of the mid span section when the beam is simply supported over a span of 6m. If a uniformly distributed load of 6 KN/m is imposed, evaluate maximum working stress in Concrete. The density of Concrete = 24 KN/m³.

Sol:- Given data :-





$b = 20$ cm

$d = 30$ cm

Span $l = 6$m

UDL = 6 KN/m

Pre stress in steel = 840 N/mm²

Density of Concrete = 24 KN/m³

Distance of centroid of the pre stressing force from the base,

$$y = \frac{15 \times 65 + 3(300 - 25)}{15 + 3} = 100 \text{ mm}$$

Eccentricity, $e = 150 - 100 = 50$ mm

Prestressing force $P = 840 \times 18 \times \pi/4 \times 5^2 = 2.96 \times 10^5 = 3 \times 10^5$ N

Area of cross section, $A = 300 \times 200 = 6 \times 10^4$ mm²

Moment of Inertia, $I = \frac{bd^3}{12} = \frac{200 \times 300^3}{12} = 45 \times 10^7 \ mm^4$

Z_t and $Z_b = \frac{45 \times 10^7}{150} = 3 \times 10^6 \ mm^3$

Self weight of beam $= 0.2 \times 0.3 \times 24 = 1.44 \ KN/m = g$

Self weight moment $(M_g) = \frac{gl^2}{8} = \frac{1.44 \times 6^2}{8} = 6.48 \ KNm$

Live load bending moment $(M_q) = \frac{6 \times 6^2}{8} = 27 \ KN \ m$

Direct stress due to pre stress $= \frac{P}{A} = \frac{300 \times 10^3}{6 \times 10^4} = 5 \ N/mm^2$

Bending stress at bottom and top $= \frac{Pe}{Z_b \ (or) \ Z_t} = \frac{300 \times 50}{3 \times 10^6} = 5 \ N/mm^2$

Dead load stress $= \frac{6.48 \times 10^6}{3 \times 10^6} = 2.16 \ N/mm^2$

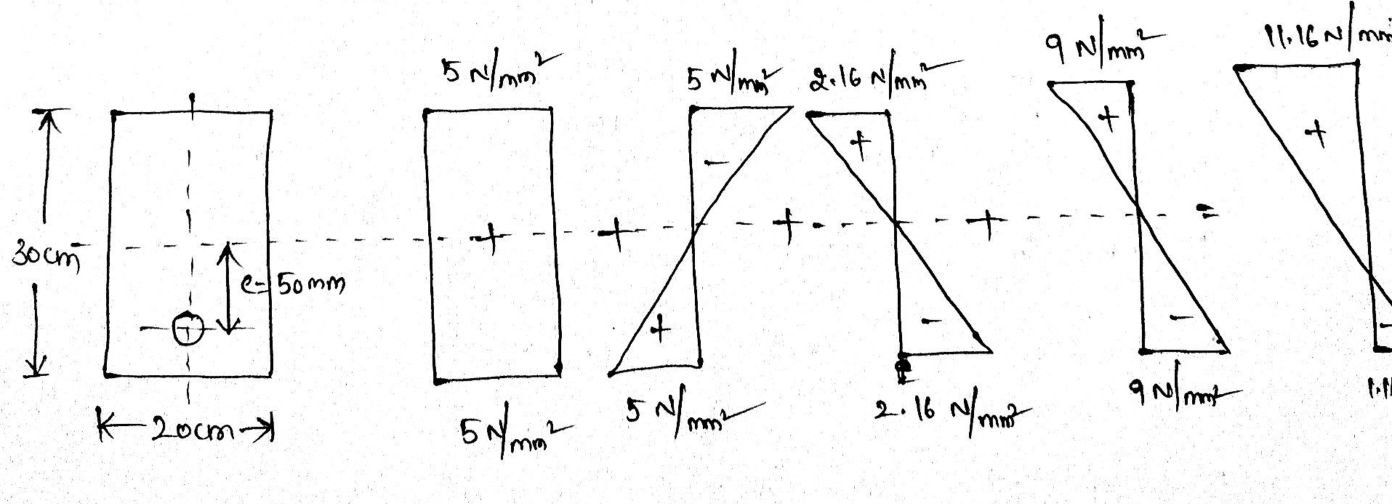
Live load stress $= \frac{27 \times 10^6}{3 \times 10^6} = 9 \ N/mm^2$

Resultant stress at top $= \frac{P}{A} - \frac{Pe}{Z_t} + \frac{M_g}{Z_t} + \frac{M_q}{Z_t}$

$= 5 - 5 + 2.16 + 9 = 11.16 \ N/mm^2$

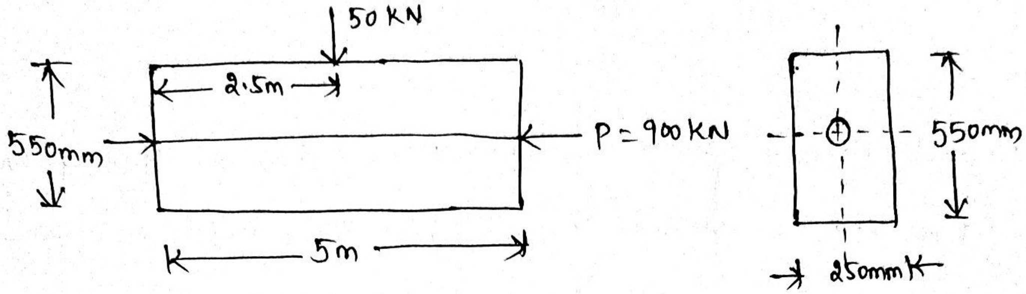
Resultant stress at bottom $= \frac{P}{A} + \frac{Pe}{Z_t} - \frac{M_g}{Z_t} - \frac{M_q}{Z_t}$

$= 5 + 5 - 2.16 - 9 = -1.16 \ N/mm^2$



2) A Prestressed Concrete beam 250 mm X 550mm in section and has a span of 5m and is Subjected to a central point load of 50 KN excluding the self weight. The prestressing tendon is located along the longitudinal centroidal axis, provides an effective prestressing force of 900 kN. Find the resultant stresses at the mid span section.

Sol:-



Gross c/s area, $A = 0.25 \times 0.550 = 137.5 \times 10^3 \ mm^2$

$Z_t = Z_b = \frac{bd^3}{12} / \frac{d}{2} = \frac{bd^2}{6} = \frac{250 \times 550^2}{6} = 12.6 \times 10^6 \ mm^3$

Maximum B.M due to external load $= \frac{WL}{4} = \frac{50 \times 5}{4} = 62.5 \ kNm$

Self weight $= 0.25 \times 0.55 \times 25 = 3.437 \ kN/m$

Dead load moment, $M_g = \frac{wl^2}{8} = \frac{3.437 \times 5^2}{8} = 10.74 \ kNm$

Direct stress $= \frac{P}{A} = \frac{900 \times 10^3}{137.5 \times 10^3} = 6.545 \ N/mm^2$

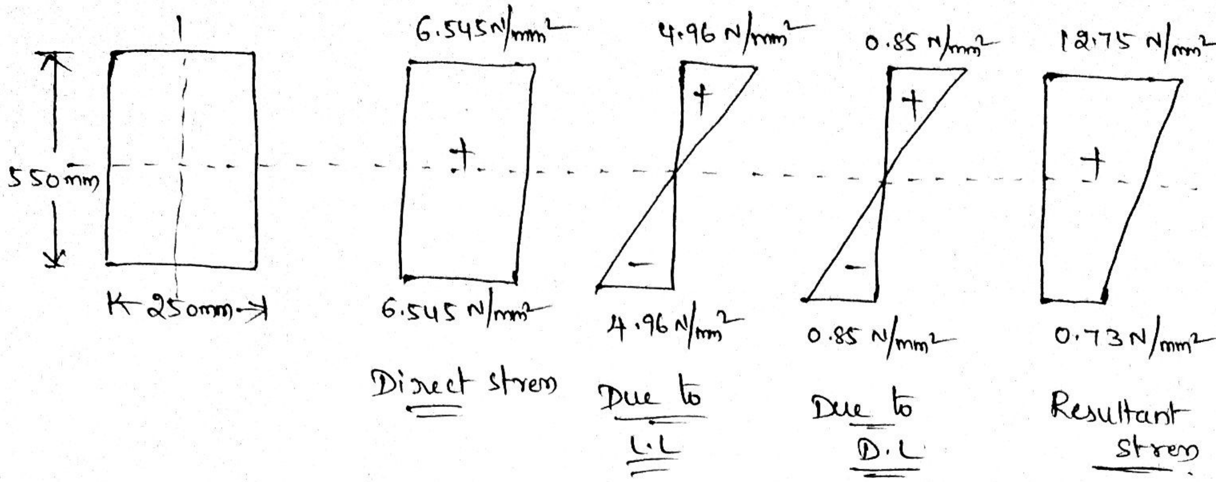
Dead load stress $= \frac{M}{Z} = \frac{10.74 \times 10^6}{12.6 \times 10^6} = 0.85 \ N/mm^2$

Live load stress $= \frac{62.5 \times 10^6}{12.6 \times 10^6} = 4.96 \ N/mm^2$

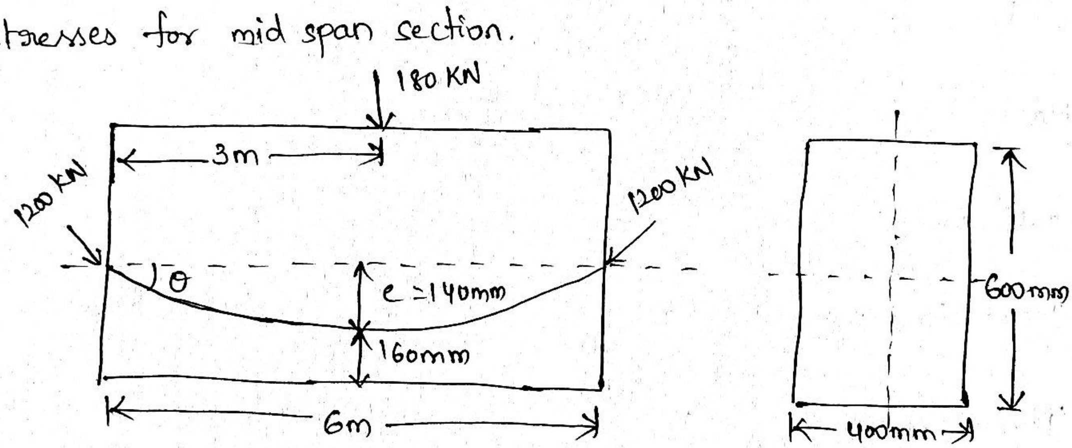
Resultant stress at top $= 6.54 + 0.85 + 4.96 = 12.35 \ N/mm^2$

Resultant stress at bottom $= 6.54 - 0.85 - 4.96$

$= 0.73 \ N/mm^2$



3) A Prestressed concrete beam is 400mm x 600mm has a span of 6m. The beam is pre stressed with a tendon which is shown in figure. The external load on the beam is a concentrated load of 180 KN at mid span. If the effective pre stressing force is 1200 KN. Calculate the extreme stresses for mid span section.



Sol: Given data :-

$b = 400mm$, $d = 600mm$, $l = 6m$, $W = 180 KN$, $P = 1200 KN$.

Gross c/s area $= 400 \times 600 = 24 \times 10^4 \, mm^2$

$$Z = \frac{bd^2}{6} = \frac{400 \times 600^2}{6} = 24 \times 10^6 \, mm^3$$

$$\tan\theta = \frac{140}{3000} = 0.0467 \Rightarrow \theta = 2.67°$$

Net down ward live load $W_q = W - 2P\sin\theta$

$= 180 - 2 \times 1200 \times \sin 2.67°$

$= 68.12$ KN

Bending moment due to live load = $\frac{WL}{4} = \frac{68.12 \times 6}{4}$

$= 102.18$ KNm

Self weight = $0.4 \times 0.6 \times 25 = 6$ KN/m

Bending moment due to dead load = $\frac{\cancel{g}l^2}{8} = \frac{6 \times 6^2}{8} = 27$ KNm

Direct stress = $\frac{P \cos\theta}{A}$ (Due to axial pre stressing force)

$= \frac{1200 \times \cos 2.67° \times 10^3}{24 \times 10^4}$

$= 5$ N/mm^2

Stress due to dead load = $\frac{M_g}{Z} = \frac{27 \times 10^6}{24 \times 10^6}$

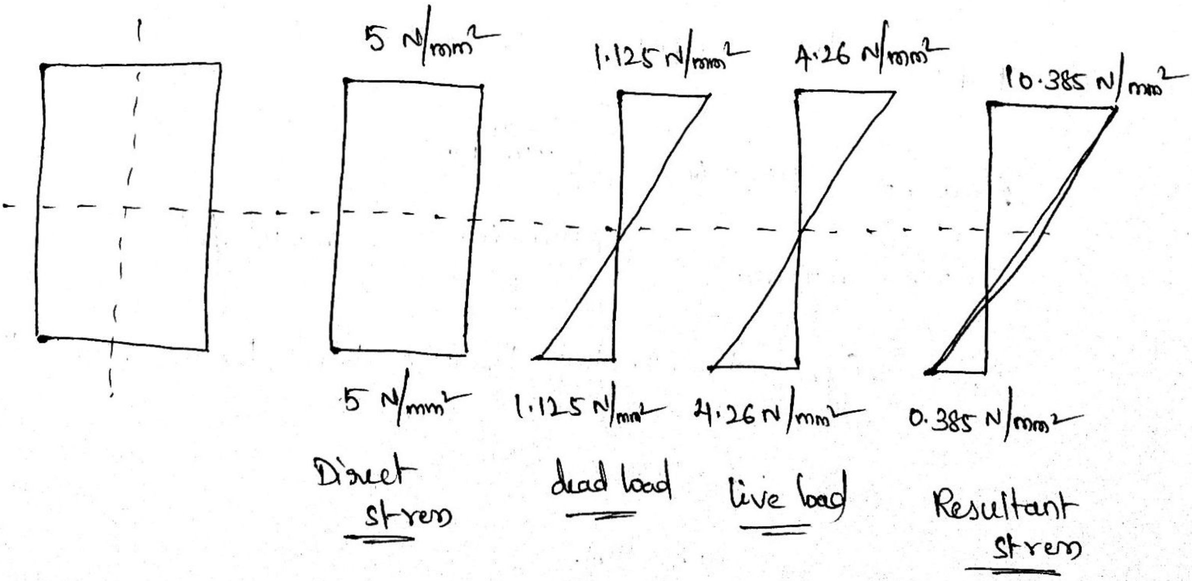
$= 1.125$ N/mm^2

Live load stress = $\frac{M_q}{Z} = \frac{102.18 \times 10^6}{24 \times 10^6}$

$= 4.26$ N/mm^2

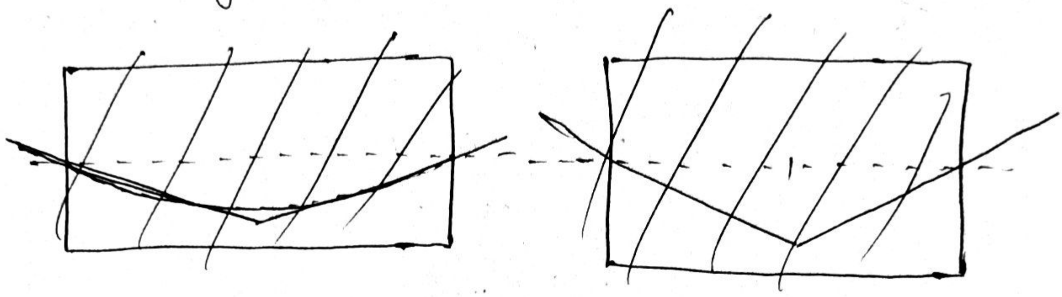
Resultant stress at top = $5 + 1.125 + 4.26 = 10.385$ N/mm^2

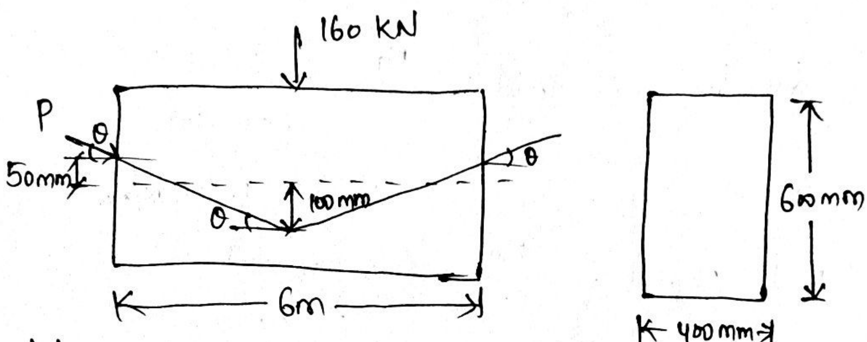
Resultant stress at bottom = $5 - 1.125 - 4.26 = -0.385$ N/mm^2



4) A pre stressed Concrete beam 400 x 600 mm in section has a span of 6m is pre stressed with bend tendons as shown in figure. The beam carries a point load of magnitude 160 KN at the centre. Find the stress distribution for the end span section and mid span section. Take pre-stressing force as 1000 KN.

Sol:-





Given data:-

Section = 400 x 600 mm

$L = 6m$

$P = 1000$ KN

$W = 160$ KN

$\tan\theta = \frac{150}{3000} = 0.05 \Rightarrow \theta = 2.86°$

Area, $A = 400 \times 600 = 24 \times 10^4 \ mm^2$

Section modulus, $Z = \frac{400 \times 600^2}{6} = 24 \times 10^6 \ mm^3$

Upward force transmitted by the bend tendon, $W_1 = 2P \sin\theta$

$= 2 \times 1000 \times \sin 2.86°$

$= 99.87$ KN.

Net downward live load = 160 − 99.87 = 60.13 KN

Bending moment due to live load = $\frac{60.13 \times 6}{4}$ = 90.3 KNm

Self weight of beam = 0.4 × 0.6 × 25 = 6 KN/m

Moment due to dead load = $\frac{6 \times 6^2}{8}$ = 27 KNm

(i) Analysis of stress due to Pre stressing force at mid span section:-

Direct stress = P/A = $\frac{1000 \times 10^3 \cos 2.86}{24 \times 10^4}$ = 4.16 N/mm²

Bending stress due to eccentricity of prestressing force

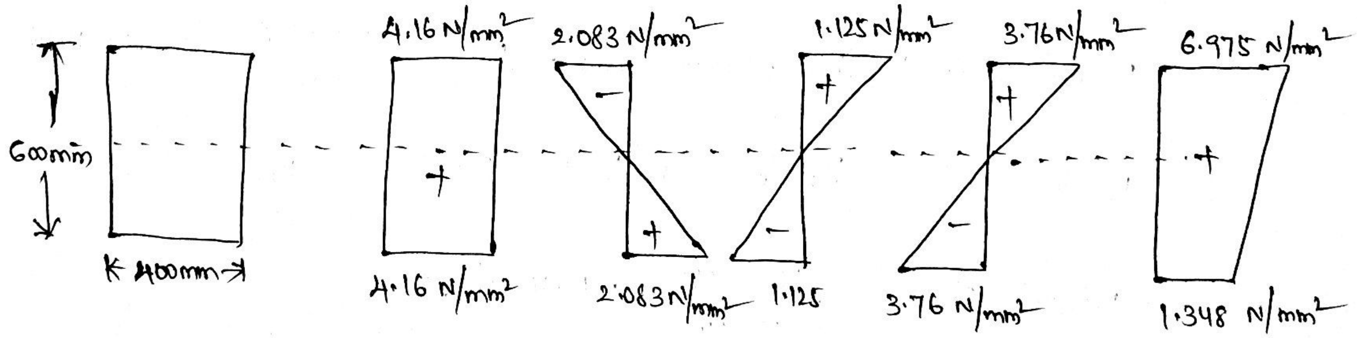
= $\frac{1000 \times 50 \times 10^3 \cos 2.86^\circ}{24 \times 10^6}$ = 2.083 N/mm²

Stress due to dead load = $\frac{27 \times 10^6}{24 \times 10^6}$ = 1.125 N/mm²

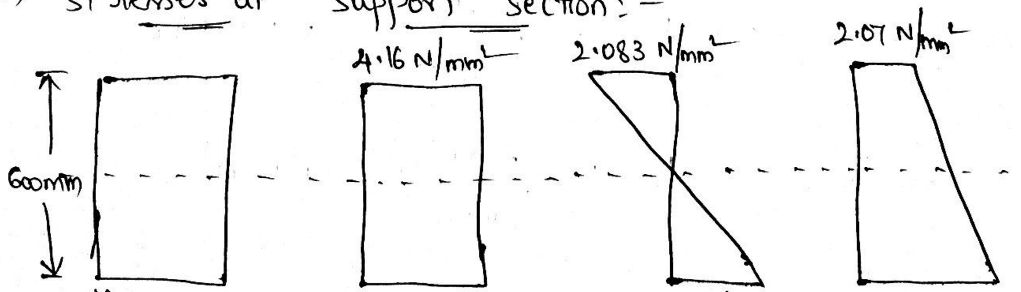
Stress due to live load = $\frac{90.3 \times 10^6}{24 \times 10^6}$ = 3.76 N/mm²

Resultant stress at top = 4.16 − 2.083 + 1.125 + 3.76 = 6.975 N/mm²

Resultant stress at bottom = 4.16 + 2.083 − 1.125 − 3.76 = 1.348 N/mm²



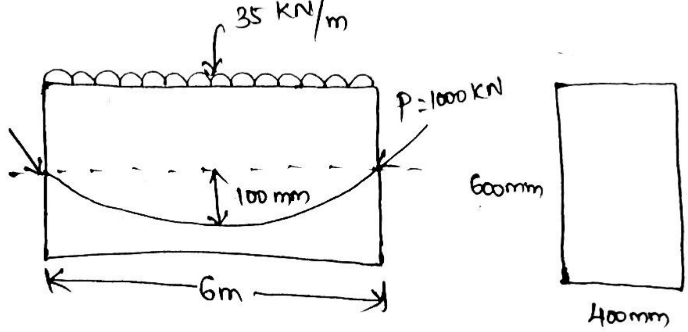
(ii) Stresses at Support Section:-



Resultant at top = 4.16 − 2.083 = 2.07 N/mm²

Resultant at bottom = 4.16 + 2.083 = 6.243 N/mm²

5) A Prestressed concrete beam provided with a tendon having a Parabolic profile as shown in figure. If the total external load on the beam is 35 KN/m on the whole span, calculate the extreme stresses for the mid span section. The tendon carries a prestressing force of 1000 KN.



Sol:- Given data :-

$W = 35 \text{ KN/m}$

$P = 1000 \text{ KN}$

Area, $A = 24 \times 10^4 \text{ mm}^2$

section modulus, $Z = 24 \times 10^6 \text{ mm}^3$

Self weight $= 0.4 \times 0.6 \times 25 = 6 \text{ KN/m}$

Upward force transmitted by tendon $= \frac{8ep}{l^2}$

$= \frac{8 \times 100 \times 1000 \times 10^3}{6000^2} = 22.22 \text{ KN/m}$

Net downward load $= 35 - 22.22 = 12.78 \text{ KN/m}$

Bending moment due to live load $= \frac{wl^2}{8} = \frac{12.78 \times 6^2}{8} = 57.51 \text{ KNm}$

Bending moment due to dead load $= \frac{6 \times 6^2}{8} = 27 \text{ KNm}$

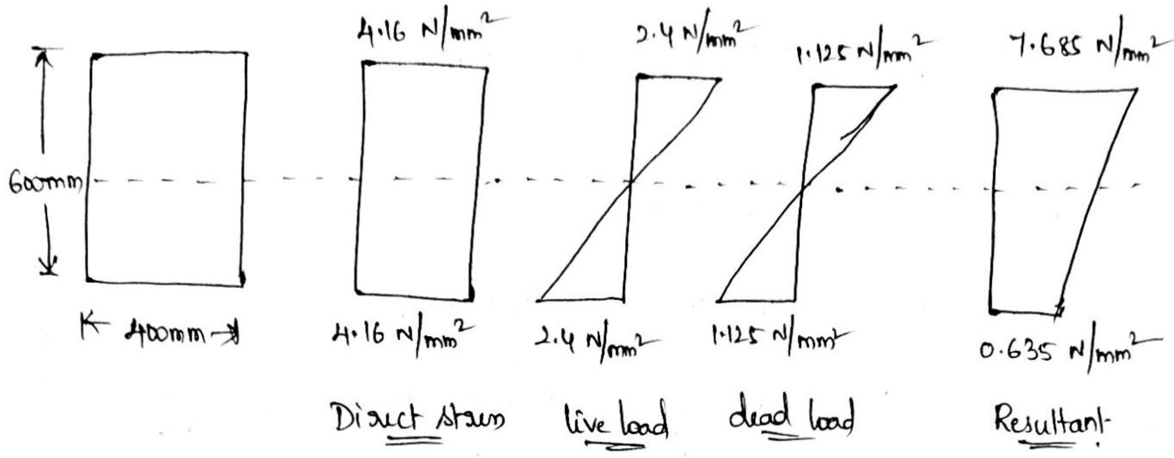
Direct stress $= \frac{1000 \times 10^3}{24 \times 10^4} = 4.16 \text{ N/mm}^2$

stress due to live load $= \frac{57.51 \times 10^6}{24 \times 10^6} = 2.4 \text{ N/mm}^2$

stress due to dead load $= \frac{27 \times 10^6}{24 \times 10^6} = 1.125 \text{ N/mm}^2$

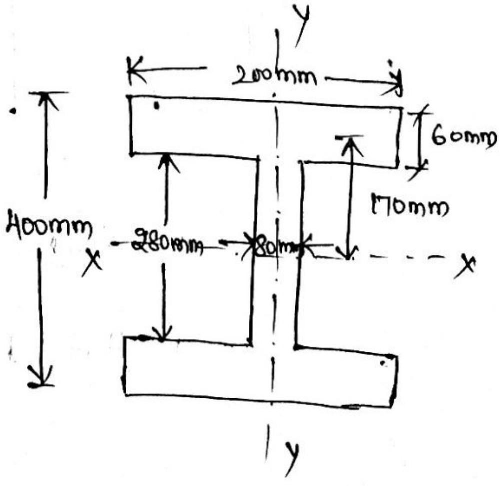
Resultant stress at top $= 4.16 + 2.4 + 1.125 = 7.685 \ N/mm^2$

Resultant stress at bottom $= 4.16 - 2.4 - 1.125 = 0.635 \ N/mm^2$



6) A beam of symmetrical I - section has a span of 8m and having flange width of 200mm and thickness of 60mm. The overall depth of the beam is 400mm. The thickness of web is 80mm. The beam is Pre stressed by a parabolic cable with an eccentricity of 150mm at the centre and zero at the supports with an effective pre stressing force 100 KN. The live load on the beam is 2 KN/m. Draw the stress distribution at mid span section for following condition.
(i) Pre stressing force + self weight (ii) pre stressing force + self weight + live load.

Sol:- Given data :-



$P = 100 \ KN$

$e = 150 \ mm$

Area, $A = 2 \times 200 \times 60 + 80 \times 280$

$= 46400 \ mm^2$

$I_{yy} = 2 \times \dfrac{60 \times 200^3}{12} + \dfrac{280 \times 80^3}{12}$

$= 91.94 \times 10^6 \ mm^4$

$$I_{xx} = \left[\frac{2\times 200\times 60^3}{12} + 2\times 200\times 60\times 170^2\right] + \frac{80\times 280^3}{12}$$

$$= 847.14\times 10^6 \text{ mm}^4$$

$$Z_{xx} = \frac{I_{xx}}{y} = \frac{847.14\times 10^6}{200} = 423.57\times 10^4 \text{ mm}^3$$

Self weight = Area x unit weight

$$= 46.4\times 10^3\times 10^{-6}\times 25 = 1.16 \text{ KN/m}$$

Net upward force $= \frac{8Pe}{l^2} = \frac{8\times 0.15\times 100}{8^2} = 1.875$ KN/m

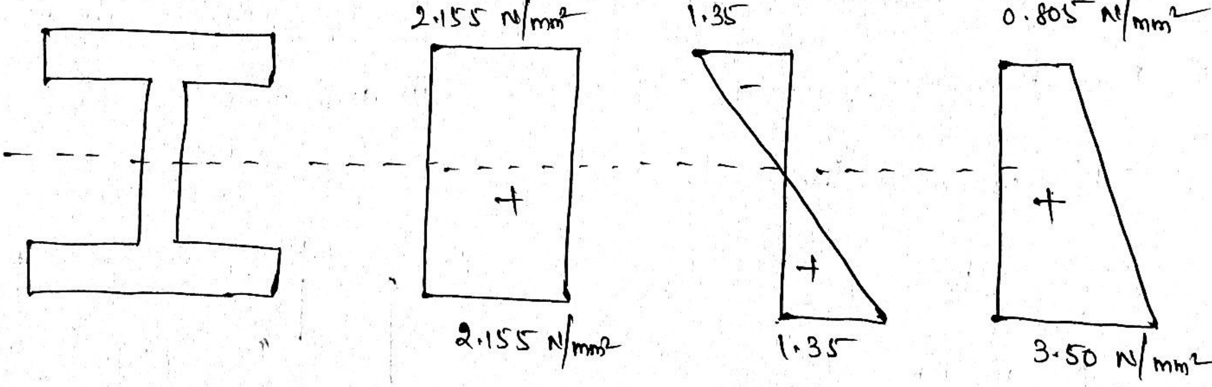
(i) Considering pre stressing force + self weight :-

Total load = $w = 1.875 - 1.16 = 0.715$ KN/m

Bending moment $= \frac{wl^2}{8} = \frac{0.715\times 8^2}{8} = 5.72$ KNm

Bending stress $= \frac{M}{Z} = \frac{5.72\times 10^6}{423.57\times 10^4} = 1.35$ N/mm²

Direct stress $= \frac{100\times 10^3}{46.4\times 10^3} = 2.155$ N/mm²



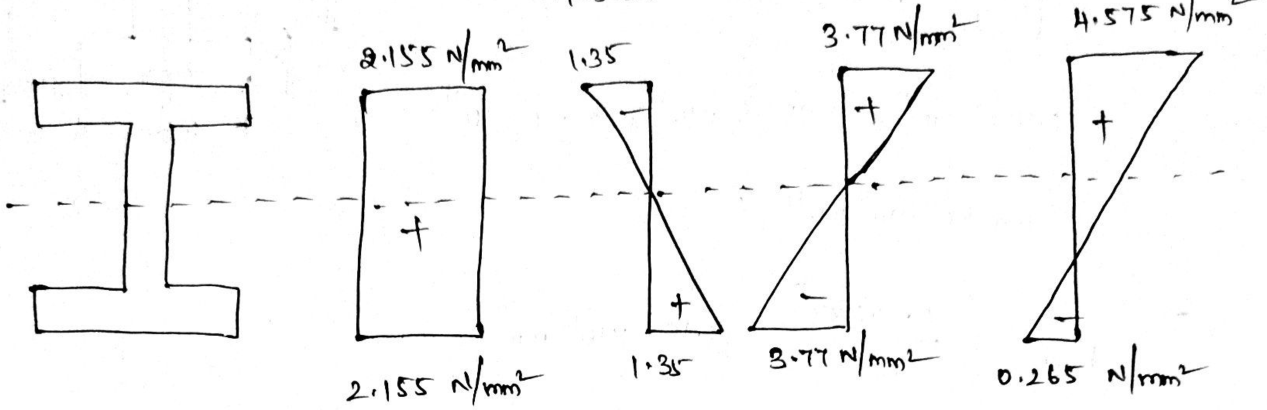
Resultant stress at top = $2.155 - 1.35 = 0.805$ N/mm²

Resultant stress at bottom = $2.155 + 1.35 = 3.50$ N/mm²

(ii) live load = 2 KN/m

live load moment = $\frac{2 \times 8^2}{8}$ = 16 KNm

live load stress = $\frac{16 \times 10^6}{423.57 \times 10^4}$ = 3.77 N/mm²



Resultant stress at top = 2.155 − 1.135 + 3.77 = 4.575 N/mm²

Resultant stress at bottom = 2.155 + 1.35 − 3.77 = −0.265 N/mm²

7) A prestressed Concrete I-beam has the following dimensions of top flange 750mm x 200mm, bottom flange 400 x 300mm, web 500 x 150mm. It is supported over a span of 30m and carries a UDL of 4 KN/m exclusive of its self weight. It is prestressed with 120 wires of 5mm diameter with their centroid 100mm above the bottom edge and is initially tensioned to 1000 MPa. Assuming 15% loss in prestress, find the extreme fibre stresses at mid span at various stages.

Sol:-

$A = 750 \times 200 + 500 \times 150 + 400 \times 300$

$= 3.45 \times 10^5 \text{ mm}^2$

$$\bar{y} = \frac{(750 \times 200)(800 + \frac{200}{2}) + (500 \times 150)(300 + \frac{500}{2}) + (400 \times 300)(300/2)}{3.45 \times 10^5}$$

= 563.04 mm from bottom flange

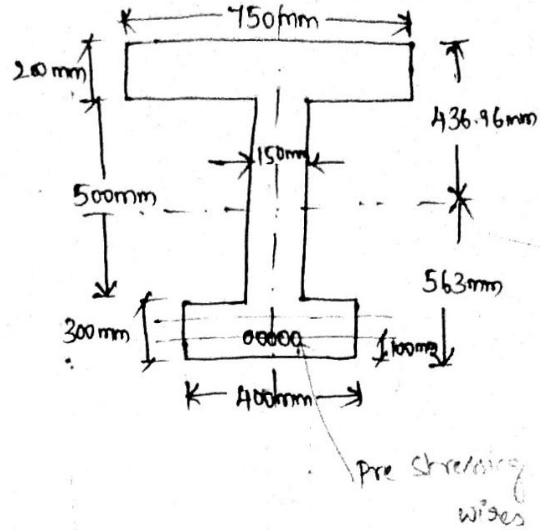
$$I_{xx} = \left[\frac{750\times200^3}{12} + 750\times200\times(900-563)^2\right]$$

$$+\left[\frac{150\times500^3}{12} + 150\times500\times(563-550)^2\right]$$

$$+\left[\frac{400\times300^3}{12} + 400\times300\times(563-150)^2\right]$$

$$= 1.753\times10^{10} + 0.1575\times10^{10} + 2.136\times10^{10}$$

$$= 4.046\times10^{10}\ mm^4$$



$$Z_{xx\,bottom} = \frac{4.046\times10^{10}}{563} = 7.187\times10^7\ mm^3.$$

$$y_t = 1000 - 563.04 = 436.96\ mm$$

$$Z_{xx\,top} = \frac{4.046\times10^{10}}{436.96} = 9.26\times10^7\ mm^3$$

Initial prestressing force $= 1000\times\pi/4\times5^2\times120$

$= 2.356\times10^3$ KN

loss

Net Prestressing force $= 2356.5 - (2356.5\times0.15)$

$= \cancel{2000}\ 2003.02$ KN

Self weight of the beam $= 3.45\times10^5\times10^{-6}\times25$

$= 8.625$ KN/m

Direct Compressive stress due to prestressing force

$$f_1 = \frac{P}{A} = \frac{2003.02\times10^3}{3.45\times10^5} = 5.8\ N/mm^2$$

Stresses due to eccentricity

(i) at the top fibre $= \dfrac{Pe}{Z_t} = \dfrac{2003.02\times10^3\times436.96}{9.26\times10^7} = 9.45\ N/mm^2$

(ii) at the bottom fibre $= \frac{Pe}{Z_b} = \frac{2003.02 \times 10^3 \times 463.04}{7.187 \times 10^7}$ ← (563.04 − 100)

$= 12.9 \ N/mm^2$

Bending moment due to dead load, $M_g = \frac{8.625 \times 30^2}{8}$

$= 970.31$ KNm

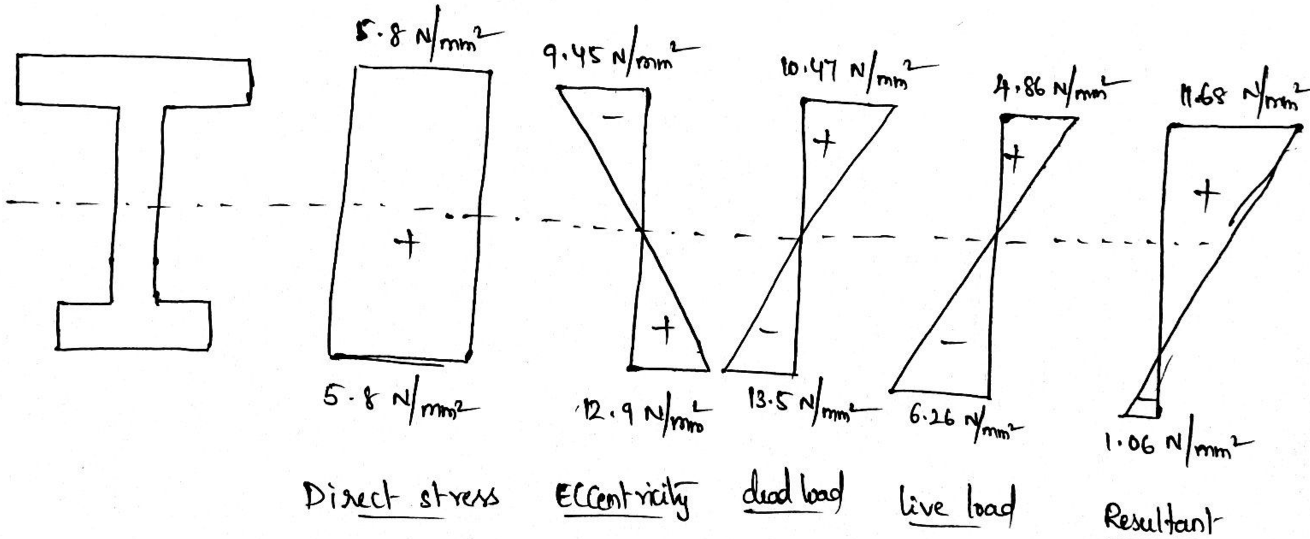
Live load bending moment, $M_q = \frac{4 \times 30^2}{8} = 450$ KNm

Bending stress due to live load at top $= \frac{450 \times 10^6}{9.26 \times 10^7} = 4.86 \ N/mm^2$

Bending stress due to live load at bottom $= \frac{450 \times 10^6}{7.187 \times 10^7} = 6.26 \ N/mm^2$

Bending stress due to dead load at top $= \frac{970.31 \times 10^6}{9.26 \times 10^7} = 10.47 \ N/mm^2$

Bending stress due to dead load at bottom $= \frac{970.31 \times 10^6}{7.187 \times 10^7} = 13.5 \ N/mm^2$



Resultant stress at top $= 5.8 - 9.45 + 10.47 + 4.86 = 11.68 \ N/mm^2$

Resultant stress at bottom $= 5.8 + 12.9 - 13.5 - 6.26 = -1.06 \ N/mm^2$

Losses in pre stress :- (Pg NO. 32)

The losses in pre stress may be classified as follows.

1. Loss of Pre stress during tensioning process
2. Loss of pre stress at the anchoring stage
3. Losses occuring subsequently.

Types of losses of Pre stress:-

S.NO	Pre tensioning	Post tensioning.
1.	Elastic deformation of Concrete	No loss due to elastic deformation if all the wires are simultaneously tensioned. If the wires are ~~successi~~ Successively tensioned, there will be loss of prestress due to elastic deformation of Concrete.
2.	Relaxation of stress in steel	Relaxation of Stress in steel
3.	Shrinkage of Concrete	Shrinkage of Concrete
4.	Creep of Concrete	Creep of Concrete
5.		Friction
6.		Anchorage slip.

Loss of Pre stress during tensioning process:-

1. Loss of stress due to wave (or) wobling effect:-

The length effect means the extent of the friction met with in a straight tendon due to slight imperfections of the duct.

This loss may be calculated as

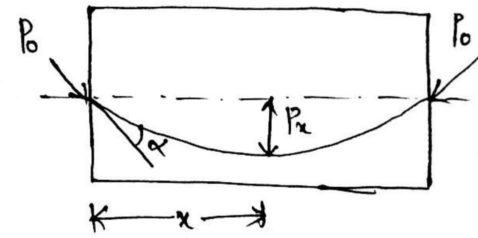
$$P_x = P_0 e^{-Kx}$$

where, P_x = The magnitude of prestressing force in the tendon at any distance 'x' from the Jack (or) tensioning end

P_0 = Prestressing force at the Jacking end.

K = friction coefficient for wave effect (or) wobble correction factor varies from 0.15 per 100 m for normal conditions and 1.5 per 100 m for thin wall ducts and where heavy vibrations are encountered and in other adverse conditions.

2. <u>Losses due to curvature effect</u>:-



$$P_x = P_0 (1 - Kx - \mu\alpha)$$

μ = Coefficient of friction b/n concrete and steel (or) cable and duct

= 0.55 for steel ~~having~~ moving on smooth concrete

= 0.35 for steel moving on ~~moving~~ fixed duct.

3. <u>Loss of prestress at the Anchoring stage</u>:-

Loss of stress = strain × Young's modulus of steel

$$\text{loss} = \frac{\text{Effective slip}}{l} \times \text{Young's modulus of steel}$$

$$= \frac{\Delta l}{l} \times E$$

4. losses occuring Subsequently :-

(i) Due to shrinkage of Concrete :-

Loss = Total residual shrinkage strain × young's modulus of steel

$= \epsilon_{cs} \times E_s$

$\epsilon_{cs} = 0.0003$ for pre tensioning

$= \dfrac{0.0002}{\log_{10}(t+2)}$ for post tensioning

where, t = age of Concrete at transfer in days.

(ii) Due to Creep of Concrete :-

Creep loss = Creep strain × young's modulus of steel

$E_s = m E_c$ = creep Coefficient

(iii) loss of stress in steel due to elastic shortening of concrete :-

loss = Elastic strain × young's modulus of steel

$= \dfrac{\text{elastic stress}}{\text{young's modulus of Concrete}} \times E_s$

$= \dfrac{P_0}{A E_c} \times E_s = m \times f_c \quad \left(\because f_c = \dfrac{P_0}{A}\right)$

(iv) Due to creep in steel :-

This approximately varies between 1 to 5% of initial stress

Normally 3% of initial stress is Considered.

loss = 3% of f_s = 3% $\dfrac{P}{A_s}$

Creep Coefficient method :-

$$\text{Creep Coefficient}, \phi = \frac{\text{creep strain}}{\text{elastic strain}}$$

$$= \frac{\epsilon_c}{\epsilon_e}$$

$$\epsilon_c = \phi\, \epsilon_e = \phi \frac{f_c}{E_c}$$

$$\Rightarrow \epsilon_c \times E_s = \phi \frac{f_c}{E_c} \times E_s \quad (\because \text{multiplying with } E_s \text{ both sides})$$

$$\Rightarrow \epsilon_c \times E_s = \phi f_c' m$$

'ϕ' value for watery situations (wet condition) is equal to 1.5.

For dry conditions $\phi = 4$.

Ultimate creep strain method :-

Loss of stress in steel due to creep of concrete $= \epsilon_{cc} \times f_c \times E_s$

where ϵ_{cc} = ultimate creep strain in concrete

f_c = Compressive stress in concrete at the level of steel

E_s = young's modulus of steel

1) A Post tensioned beam 400 x 600 mm, 10m long is provided with straight tendons which are tensioned to 1050 N/mm² at the Jacking end. Find the loss of prestress due to wobbling effect at mid span and at the remote end from the Jack. Take K = 0.3 for 100m.

Sol:-

Given data:-

Span of beam = 10m

$P_0 = 1050\ N/mm^2$

$K = 0.3$ for 100m

Prestress due to wobbling (or) wave effect at any distance 'x' is given by $P_x = P_0 e^{-Kx}$

$= P_0 e^0 \times e^{-Kx}$ $(\because e^0 = 1)$

$= P_0 (1 - Kx)$ $(\because e^{-Kx}$ is negligible, so $e^{-Kx} = -Kx)$

At mid span:-

$x = 5m$

$P_5 = 1050\left(1 - \frac{0.3}{100} \times 5\right) = 1034.25\ N/mm^2$

$\therefore$ loss $= 1050 - 1034.25 = 15.75\ N/mm^2$

At remote end:-

$x = 10m$

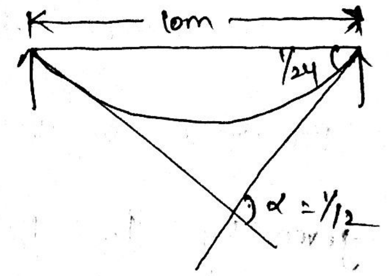
$P_{10} = 1050\left(1 - \frac{0.3}{100} \times 10\right)$

$= 1018.5\ N/mm^2$

$\therefore$ loss $= 1050 - 1018.5 = 31.5\ N/mm^2$

2) A post tensioned cable of a beam 10m long is initially tensioned to a stress of 1000 mpa at one end. If the tendons are curved, So that the slope is $1/24$ at each end with a c/s area of $600 mm^2$. calculate the loss of prestress due to friction. Take the Coefficient of friction between the duct surface and the cable = 0.3. Friction Coefficient for wave effect = 0.0015/m

sol:- slope = $1/24$



Area, $A = 600 mm^2$

$\mu = 0.3$, $K = 0.0015/m$

$P_0 = 1000 N/mm^2$, $l = 10m$

$P_x = P_0 e^{-(\mu\alpha + kx)}$

$= P_0 (1 - \mu\alpha - kx)$

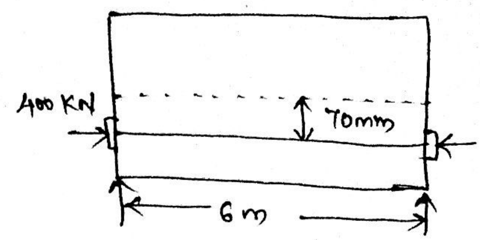
$P_x = 1000 (1 - 0.3 \times 1/12 - 0.0015 \times 10)$

$= 960 N/mm^2$

loss = $1000 - 960 = \underline{40 N/mm^2}$

3) In a prestressed Concrete beam of cross section $200 \times 300 mm$ and span 6m, an internal prestressing force of 400 KN is applied on both sides of beam at an eccentricity of 70 mm by the tendons of area $400 mm^2$. Take young's modulus of steel = $2 \times 10^5 N/mm^2$. $E_c = 0.333 \times 10^5 N/mm^2$, Anchorage slip = 1.5mm, Creep Coefficient = 1, shrinkage of Concrete = 0.0002, creep in steel = 3%, $K = 0.0015/m$. Find the total Percentage loss of Prestress in tendons.

Sol: (i) loss of prestress during tensioning process i.e due to length effect and curvature effect (or) simply due to friction is equal to zero.



(Since the tension is applied on both ends)

(ii) losses due to slip at anchorage

$$= \frac{\Delta l}{l} \times E = \frac{1.5}{6000} \times 2 \times 10^5 = 50 \ N/mm^2$$

(iii) loss due to shrinkage of concrete

$$= \epsilon_{cs} \times E_s = 0.0003 \times 2 \times 10^5 = 40 \ N/mm^2$$

(iv) loss due to creep of concrete

Creep coefficient $\phi = 1$

$$\epsilon_s \times E_s = \phi \ f_c \ m$$

$$m = \frac{E_s}{E_c} = \frac{2 \times 10^5}{0.333 \times 10^5} = 6$$

$f_c' =$ stress in concrete at the level of prestressing steel after the elastic shortening of concrete.

$$P_o = \frac{400 \times 10^3}{400} = 1000 \ N/mm^2 = \text{prestress after elastic shortening}$$

$$f_c = \frac{P}{A_c} + \frac{Pe}{Z}$$

$$Z = \frac{bd^2}{6} = \frac{200 \times 300^2}{6} = 3 \times 10^6 \ mm^3$$

$$f_c = \frac{400 \times 10^3}{200 \times 300} + \frac{400 \times 10^3 \times 70}{3 \times 10^6} = 16 \ N/mm^2$$

$$\text{loss} = \phi \ m \ f_c = 1 \times 6 \times 16 = 96 \ N/mm^2$$

(V) loss of prestress due to elastic shortening of Concrete

$$\text{loss} = m f_c$$

$$= 6 \times 16 = 96 \ N/mm^2$$

(vi) loss of prestress due to creep in steel

$$= \frac{3}{100} \times \frac{P}{A_s} = \frac{3}{100} \times \frac{400 \times 10^3}{400} = 30 \ N/mm^2$$

Total loss = 0 + 50 + 40 + 96 + 30 + 96

= 312 N/mm²

$$\text{Percentage of loss} = 100 - \frac{1000 - 312}{1000} \times 100 = 31.2\%$$

$$\left(= \frac{312}{1000} \times 100 = 31.2\% \right)$$

4) Calculate the loss of prestress in the tendon due to shrinkage in a post tensioned beam, if the age of Concrete at 15 days. Take $E = 2 \times 10^5 \ N/mm^2$.

Sol:

$$\epsilon_{cs} = \frac{0.0002}{\log(t+2)} = \frac{0.0002}{\log(15+2)} = 1.62 \times 10^{-4}$$

$$\text{loss} = f_c = 1.62 \times 10^{-4} \times 2 \times 10^5$$

$$= 32.4 \ N/mm^2$$

5) A Pre tensioned Concrete beam 100 x 300 mm is Pre stressed with steel wires carrying an initial force of 150 kN. at an eccentricity of 50mm. Take young's modulus of steel = 210 kN/mm^2, $E_c = 35$ kN/mm^2. Estimate the Percentage loss of pre stress due to elastic shortening of Concrete. If the area of steel wires is 188 mm^2.

Sol:- loss of prestress due to elastic shortening

$$= \text{elastic strain} \times E_s$$

$$= \frac{\text{Elastic stress}}{E_c} \times E_s$$

$$= \frac{\frac{Pe}{Z} + P/A_c}{E_c} \times E_s = \left(\frac{P}{A_c} + \frac{Pe}{Z}\right) \times m$$

$$= f_c\, m$$

$$= \left(\frac{150 \times 10^3}{100 \times 300} + \frac{150 \times 1000 \times 50}{100 \times 300^2/6}\right) \times \frac{2.1 \times 10^5}{0.35 \times 10^5}$$

$$= (5+5) \times 6 = 60 \text{ N/mm}^2$$

$$\text{Stresses in steel wires} = \frac{150 \times 10^3}{188} = 797.87 \text{ N/mm}^2$$

$$\text{Percentage loss} = \frac{60}{797.87} \times 100 = 7.52\%$$

6) A post tensioned Pre stressed Concrete beam 16m span is subjected to an initial pre stress of 1458 kN, transfered at 28 days. Profile of the cable is Parabolic with an eccentricity of 520mm at the centre of the span. Take $A = 2.42 \times 10^5$ mm^2, moment of inertia $= 5.3 \times 10^{10}$ mm^4, $A_t = 1386$ mm^2, $f_s = 1059$ N/mm^2

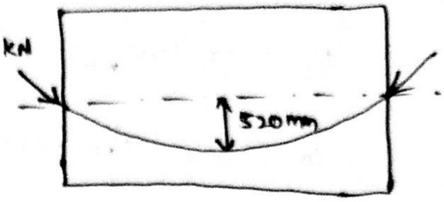
at transfer, $E_s = 2.1 \times 10^5$ N/mm², $E_c = 0.382 \times 10^5$ N/mm², $\mu = 0.25$

$K = 0.0015$/m, slip = 2.5 mm. Find All losses of Pre stress.

Sol:

Initial stress in tendons

$$= \frac{1458 \times 10^3}{1386} = 1051.95 \text{ N/mm}^2$$



(i) loss of Pre stress due to elastic shortening

$$= m \times f_c$$

Dead load $= 2.42 \times 10^5 \times 25 \times 10^3 \times 10^{-9} = 6.125$ N/mm

Dead load Bending moment $= \frac{6.125 \times (16 \times 10^3)^2}{8} = 196 \times 10^6$ N mm

Bending moment due to eccentricity $= -1458 \times 10^3 \times 520$ $(P \times e)$

$= -758.16 \times 10^6$ Nmm

Net bending moment $= -758.16 \times 10^6 + 196 \times 10^6$

$=$ ~~-572.3~~ 562.16×10^6 N mm

Elastic strain at end span $= \frac{P_o}{A E_c} = \frac{1458 \times 10^3}{2.42 \times 10^5 \times 0.382 \times 10^5}$

$= 1.57 \times 10^{-4}$.

Elastic strain at mid span $= \frac{P_o}{A E_c} + \frac{Me}{I E_c} = 1.57 \times 10^{-4} + \frac{562.16 \times 10^6 \times 520}{5.3 \times 10^{10} \times 0.382 \times 10^5}$

$= 3.02 \times 10^{-4}$

Average elastic strain $= \frac{1.57 + 3.02}{2} \times 10^{-4} = 2.295 \times 10^{-4}$

$\therefore$ loss of pre stress $= m \times f_c =$ Elastic strain $\times E_s$

$= 2.295 \times 10^{-4} \times 2.1 \times 10^5 = 48.19$ N/mm²

(ii) loss of stress due to shrinkage,

$= \epsilon_{cs} \times E_s$

$\epsilon_{cs} = \dfrac{0.0002}{\log(t+2)} = \dfrac{0.0002}{\log(28+2)} = 1.35 \times 10^{-4}$

loss $= 1.35 \times 10^{-4} \times 2.1 \times 10^{5} = 28.43\ N/mm^2$

(iii) loss of stress due to creep in Concrete

Assume creep Coefficient, $\phi = 1.5$

Creep strain due to creep in concrete

$= \phi \times$ elastic strain $= 1.5 \times 2.295 \times 10^{-4}$

$= 3.492 \times 10^{-4}$.

loss of pre stress $= 3.492 \times 10^{-4} \times 2.1 \times 10^{5}$

$= 72.28\ N/mm^2$

(iv) loss of pre stress due to creep in steel

$= 3\%$ of f_s

$= \dfrac{3}{100} \times \dfrac{P}{A_s} = \dfrac{3}{100} \times 1051.95$

$= 31.55\ N/mm^2$

(v) loss of stress due to slip in anchorage

$= \dfrac{\Delta l}{l} \times E = \dfrac{2.5}{16000} \times 2.1 \times 10^{5}$

$= 32.81\ N/mm^2$

(vi) loss due to friction,

slope of cable at end $= \alpha = \dfrac{4e}{l} = \dfrac{4 \times 520}{16000} = 0.13$

Loss of prestress = $(1 - Kx - \mu\alpha) f_o$

$$f_8 = [1 - (0.0015 \times 8) + (0.25 \times 0.13)] \times 1051.95$$

$$= 1005.14 \ N/mm^2$$

$$loss = 1051.95 - 1005.14 = 46.81 \ N/mm^2$$

Total loss of prestress = $48.19 + 28.43 + 72.28 + 31.55 + 32.81 + 46.81$

$$= 260.07 \ N/mm^2$$

Percentage loss = $\frac{260.07}{1051.95} \times 100 = 24.72\%$

7) A concrete beam of rectangular section 100 mm wide and 300 mm deep is prestressed by 5 wires of 7 mm diameter located at an eccentricity of 50mm, the initial prestress in the wires being 1200 N/mm^2. Estimate the loss of prestress in the steel due to creep of concrete using ultimate creep strain method and creep coefficient method. Take $E_s = 210 \ kN/mm^2$, $E_c = 35 \ kN/mm^2$, $\epsilon_{cc} = 41 \times 10^{-6}$, $\phi = 1.6$

Sol:- Ultimate creep strain method :-

$$Loss = \epsilon_{cc} f_c E_s$$

$$f_c = \frac{P}{A} + \frac{Pe}{Z}$$

$$P = \pi/4 \times 7^2 \times 5 \times 1200 = 230.9 \ kN$$

$$A = 300 \times 100 = 3 \times 10^4 \ mm^2$$

$$Z = \frac{bd^2}{6} = \frac{100 \times 300^2}{6} = 1.5 \times 10^6 \ mm^3$$

$$I = \frac{bd^3}{12} = \frac{100 \times 300^3}{12} = 225 \times 10^6 \ mm^3$$

$$m = \frac{210}{35} = 6$$

$$\cancel{\therefore loss = 41 \times 10^{-6} \times 15.39}$$

$$f_c = \frac{230.9 \times 10^3}{3 \times 10^4} + \frac{230.9 \times 10^3 \times 50}{1.5 \times 10^6} = 15.39 \ N/mm^2$$

$$\therefore \ loss = 41 \times 10^{-6} \times 15.39 \times 210 \times 10^3$$

$$= \underline{132.53} \ N/mm^2$$

Creep Coefficient method:-

$$loss = \phi \times m \times f_c$$

$$= 1.6 \times 6 \times 15.39$$

$$= \underline{147.7} \ N/mm^2$$