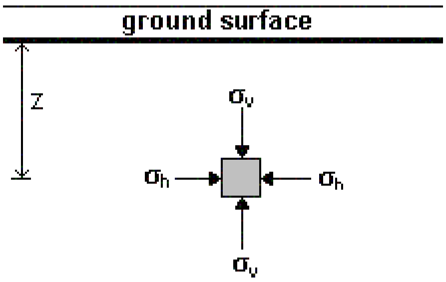
Unit – III

PRINCIPLE OF EFFECTIVE STRESS

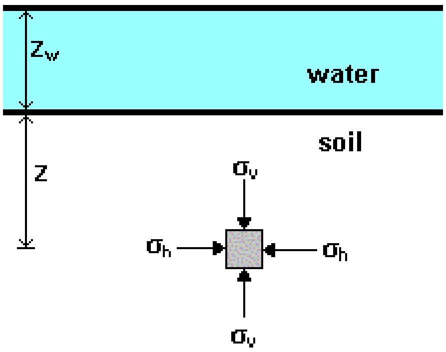
The **principle of effective stress** was enunciated by **Karl Terzaghi** in the year 1936. This principle is **valid only for saturated soils.**

Total Stress (σ)
When a load is applied to soil, it is carried by the solid grains and the water in the pores. The **total vertical stress** acting at a point below the ground surface is due to the weight of everything that lies above, including soil, water, and surface loading. Total stress thus increases with depth and with unit weight. Total stress is parameter which can be computed or even measured with suitable instruments such as a pressure cell.

Vertical total stress at depth z, $\boldsymbol{\sigma_v = \gamma.Z}$



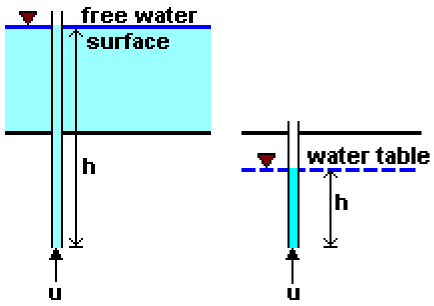
Below a water body, the total stress is the sum of the weight of the soil up to the surface and the weight of water above this. $\boldsymbol{\sigma_v = \gamma.Z + \gamma_w.Z_w}$



The total stress may also be denoted by σ_z or just σ. It varies with changes in water level and with excavation.

Pore Water Pressure (u)
The pressure of water in the pores of the soil is called **pore water pressure (u).** The magnitude of pore water pressure depends on:

- the depth below the water table.
- the conditions of seepage flow.



Pore water pressure at any point is simply equal to the depth (h) of the point below the groundwater table. Under hydrostatic conditions, no flow takes place, and the pore pressure at a given point is given by

$$u = \gamma_w . h$$

where h = depth below water table or overlying water surface

It is convenient to think of pore water pressure as the pressure exerted by a column of water in an imaginary standpipe inserted at the given point. A stand pipe or a piezometer is used to measure the pore water pressure.

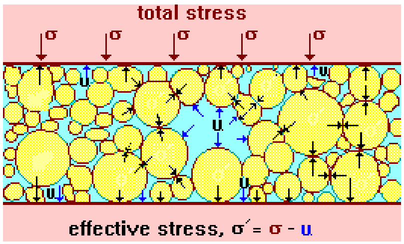
The natural level of ground water is called the **water table** or the **phreatic surface.** Under conditions of no seepage flow, the water table is horizontal. The magnitude of the pore water pressure at the water table is zero. Below the water table, pore water pressures are positive.

Effective Stress ($\bar{\sigma}$ or σ')

At any point in a soil mass, the effective stress is related to total stress (σ) and pore water pressure (u) as

$$\bar{\sigma} = \sigma - u$$

Both the total stress and pore water pressure can be measured at any point. All measurable effects of a change of stress, such as compression and a change of shearing resistance, are exclusively due to changes in effective stress.



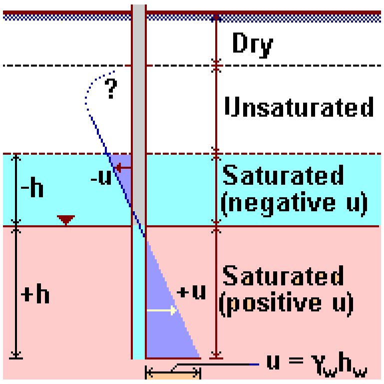
In a saturated soil system, as the voids are completely filled with water, the pore water pressure acts equally in all directions. Hence, pore water pressure is also called **neutral stress**. It has no shear stress component.

The effective stress is not the exact contact stress between particles but the distribution of load carried by the soil particles over the area considered. It cannot be measured and can only be computed.

If the total stress is increased due to additional load applied to the soil, the pore water pressure initially increases to counteract the additional stress. The entire additional load is initially borne by the pore water pressure. This increase in pressure within the pores might cause water to drain out of the soil mass, and the load is gradually transferred to the solid grains. This will lead to decrease in pore water pressure and increase of effective stress.

EFFECTIVE STRESS IN UNSATURATED ZONE

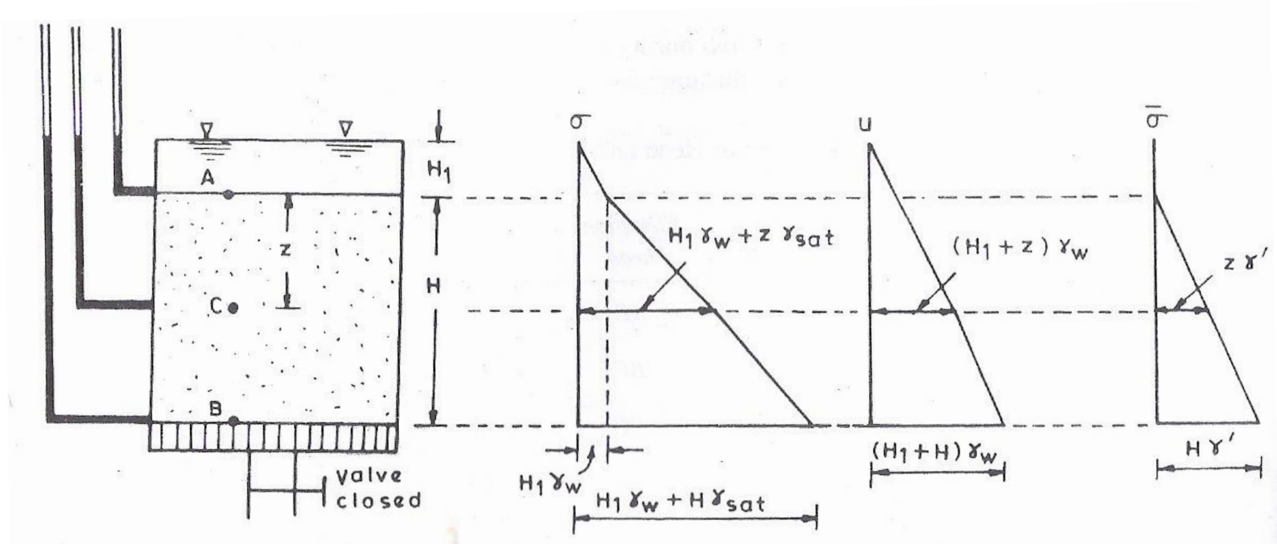
Above the water table, when the soil is saturated, pore pressure will be negative (less than atmospheric). The height above the water table to which the soil is saturated is called the **capillary rise**, and this depends on the grain size and the size of pores. In coarse soils, the capillary rise is very small.



Between the top of the saturated zone and the ground surface, the soil is partially saturated, with a consequent reduction in unit weight . The pore pressure in a partially saturated soil consists of two components:
Pore water pressure = u_w
Pore air pressure = u_a

Water is incompressible, whereas air is compressible. The combined effect is a complex relationship involving partial pressures and the degree of saturation of the soil.

EFFECTIVE STRESS UNDER NO FLOW CONDITION

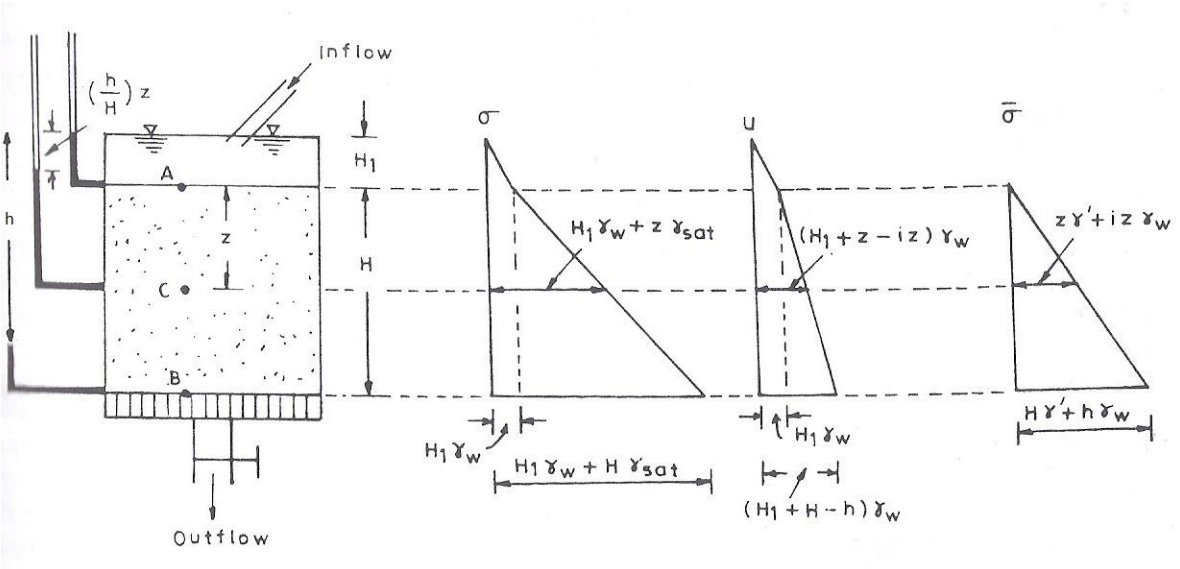


Variation of σ, u and $\bar{\sigma}$ with depth for 'no flow condition

When no flow is taking place in the soil, the condition is known as no-flow condition. Figure above shows a tank filled with submerged soil, with no seepage occurring, because the valve at the bottom is closed. Hence, the water levels in the standpipes inserted at the top, bottom and any intermediate position of the soil layer are the same.

At A: Total stress, $\sigma_A = H_1\gamma_w$ Pore water pressure, $u_A = H_1\gamma_w$ Effective stress, $\bar{\sigma}_A = \sigma_A - u_A = 0$	**At B:** Total stress, $\sigma_B = H\gamma_{sat} + H_1\gamma_w$ Pore water pressure, $u_B = (H + H_1)\gamma_w$ Effective stress, $\bar{\sigma}_B = \sigma_B - u_B = H\gamma_{sat} + H_1\gamma_w - (H + H_1)\gamma_w$ $or\ \bar{\sigma}_B = H(\gamma_{sat} - \gamma_w) = H\gamma'$

EFFECTIVE STRESS UNDER DOWNWARD FLOW CONDITION



Variation of σ, u and $\bar{\sigma}$ with depth for downward flow

When water flows through the soil, it exerts drag forces called seepage forces on the individual grains of soil. **Seepage forces act in the direction of flow.** The presence of seepage forces will cause changes in the pore water pressures and effective stresses in the soil. Figure above shows the variation of total stress, pore water pressure and effective stress with depth for down ward flow condition.

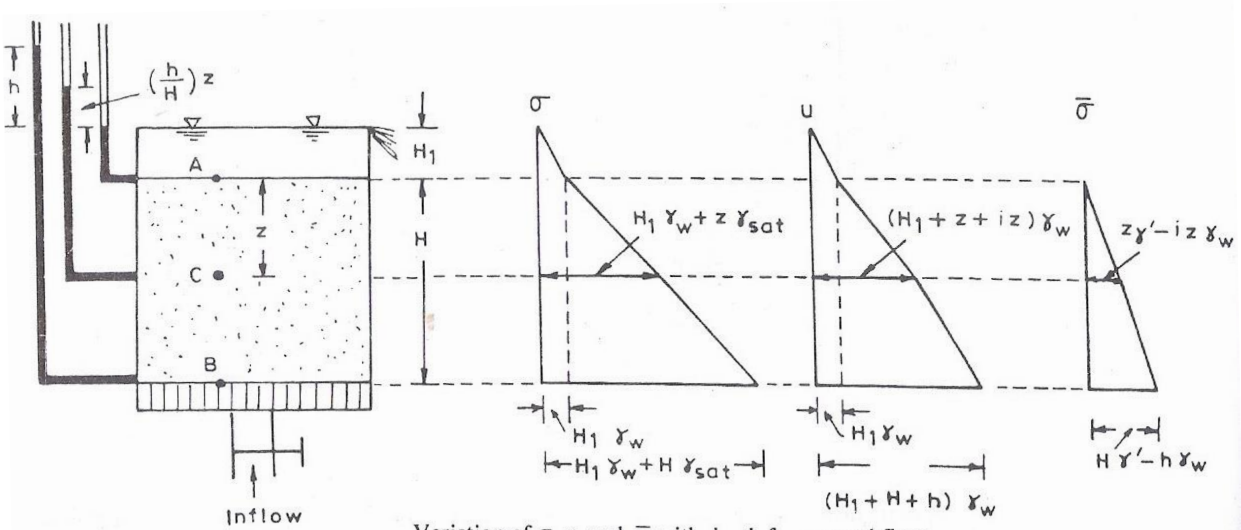
At A: Total stress, $\sigma_A = H_1\gamma_w$ Pore water pressure, $u_A = H_1\gamma_w$ Effective stress, $\overline{\sigma}_A = \sigma_A - u_A = 0$	**At B:** Total stress, $\sigma_B = H\gamma_{sat} + H_1\gamma_w$ Pore water pressure, $u_B = (H + H_1 - h)\gamma_w$ Effective stress, $\overline{\sigma}_B = \sigma_B - u_B = H\gamma_{sat} + H_1\gamma_w - (H + H_1 - h)\gamma_w$ $or\ \overline{\sigma}_B = H(\gamma_{sat} - \gamma_w) + h\gamma_w = H\gamma' + h\gamma_w$

For no flow condition, the effective stress at B is $H\gamma'$ whereas for downward flow condition the effective stress at B is $H\gamma' + h\gamma_w$. Hence, the effective stress increases by $h\gamma_w$ when there is a downward flow. **Thus, downward seepage means an increase in effective stress.** This additional stress is due to the frictional force or drag acting on the surface of grains which form the walls of the pores.

Over a length of flow H, the head loss is h. Therefore, over a length of flow z, the head loss is $\left(\frac{h}{H}\right)z$. Therefore, at point C, at a depth z below the soil surface, the increase in effective stress is $\left(\frac{h}{H}\right)z\gamma_w\ or\ iz\gamma_w$. Seepage pressure is thus equal to $iz\gamma_w$.

At C:
Total stress, $\sigma_C = z\gamma_{sat} + H_1\gamma_w$
Pore water pressure, $u_C = \left[z + H_1 - \left(\frac{h}{H}\right)z\right]\gamma_w$
Effective stress,
$\overline{\sigma}_C = \sigma_C - u_C = z\gamma_{sat} + H_1\gamma_w - \left(z + H_1 - \left(\frac{h}{H}\right)z\right)\gamma_w$
$or\ \overline{\sigma}_C = z\gamma' + iz\gamma_w$

EFFECTIVE STRESS UNDER UPWARD FLOW CONDITION



Variation of σ, u and $\overline{\sigma}$ with depth for upward flow

The figure above shows the condition of upward seepage. It is caused by opening the valve located below the tank.

At A:	**At B:**
Total stress, $\sigma_A = H_1\gamma_w$ Pore water pressure, $u_A = H_1\gamma_w$ Effective stress, $\overline{\sigma}_A = \sigma_A - u_A = 0$	Total stress, $\sigma_B = H\gamma_{sat} + H_1\gamma_w$ Pore water pressure, $u_B = (H + H_1 + h)\gamma_w$ Effective stress, $\overline{\sigma}_B = \sigma_B - u_B = H\gamma_{sat} + H_1\gamma_w - (H + H_1 + h)\gamma_w$ $or\ \overline{\sigma}_B = H(\gamma_{sat} - \gamma_w) - h\gamma_w = H\gamma' - h\gamma_w$ $or\ \overline{\sigma}_B = H\gamma' - \left(\frac{h}{H}\right)H\gamma' = H\gamma' - iH\gamma_w$

In the case of upward flow, seepage pressure acts in the upward direction, and reduces the effective stress at the level of point B by $iH\gamma_w$.

QUICK SAND CONDITION

For the upward flow condition, the effective stress at the level of point B is given by,

$$\overline{\sigma}_B = H\gamma' - iH\gamma_w$$

When the effective stress is reduced to zero, the above expression may be written as

$$H\gamma' - iH\gamma_w = 0$$

$$or\ H\gamma' = iH\gamma_w$$

here, the seepage pressure becomes equal to the effective pressure so that the effective stress throughout the soil is reduced to zero.

$$or\quad i = i_{cr} = \frac{\gamma'}{\gamma_w}$$

i_{cr} is called the *critical hydraulic gradient*. When upward flow takes place the critical hydraulic gradient, a soil such as coarse silt or fine sand looses all its shearing strength and it cannot support any load. The soil is said to have become ‘quick’ or ‘alive’ and *boiling* will occur. In such a situation, effective stress is reduced to zero and the soil behaves like a very viscous liquid. Such a state is known as **quick sand condition.**

If the void ratio of the natural deposit is known, i_{cr} can be computed as follows:

$$\gamma_{sat} = \left(\frac{G+e}{1+e}\right)\gamma_w$$

$$\gamma_{sat} - \gamma_w = \left(\frac{G+e}{1+e}\right)\gamma_w - \gamma_w$$

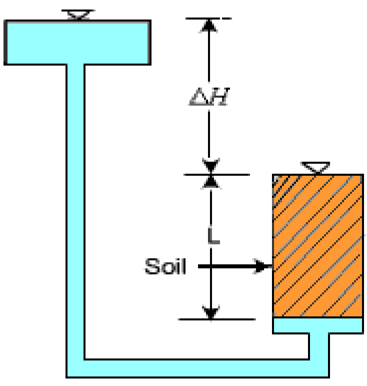
$$\gamma' = \left(\frac{G-1}{1+e}\right)\gamma_w$$

$$or\quad i_{cr} = \frac{\gamma'}{\gamma_w} = \frac{G-1}{1+e}$$

Assuming G = 2.65, i_{cr} will vary from 1.1 for e = 0.5 (dense sand) to 0.83 for e =1 (loose sand). For usual void ratios in sand soils of 0.6 to 0.7, the critical hydraulic gradient will just be about 1.

Seepage forces affect sands more than clays because sands do not possess cohesion, while fine silts and clays have some inherent cohesion which holds the grains together even at critical hydraulic gradient. Boiling does not occur in coarse sands and gravels either, because these soils are highly pervious and large discharges are required to produce $i_{cr} = 1$.

Alternate approach for critical hydraulic gradient



At the bottom of the soil column,

$$\sigma = \gamma_{sat}L$$

$$u = (L + \Delta H)\gamma_w$$

During quick sand condition, the effective stress is reduced to zero.

$$\gamma_{sat}L = (L + \Delta H)\gamma_w$$

Simplifying, $\frac{\gamma'}{\gamma_w} = i_{cr} = \frac{\Delta H}{L}$

where i_{cr} = **critical hydraulic gradient**

THE IMPORTANCE OF EFFECTIVE STRESS

At any point within the soil mass, the magnitudes of both total stress and pore water pressure are dependent on the ground water position. With a shift in the water table due to seasonal fluctuations, there is a resulting change in the distribution in pore water pressure with depth.

Changes in water level ***below ground*** result in changes in effective stresses below the water table. A rise increases the pore water pressure at all elevations thus causing a decrease in effective stress. In contrast, a fall in the water table produces an increase in the effective stress.

Changes in water level ***above ground*** do not cause changes in effective stresses in the ground below. A rise of water level above ground surface increases both the total stress and the pore water pressure by the same amount, and consequently effective stress is not altered.

In some analyses it is better to work with the *changes* of quantity, rather than in absolute quantities. The effective stress expression then becomes:

$$\Delta\sigma' = \Delta\sigma - \Delta u$$

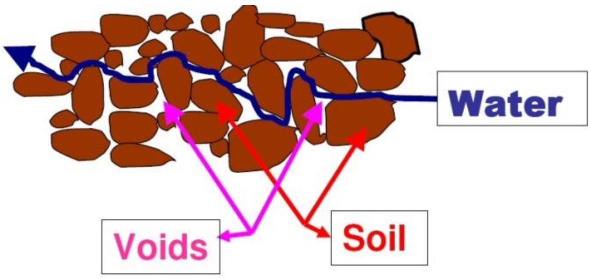
If both total stress and pore water pressure change by the same amount, the effective stress remains constant.

Total and effective stresses must be distinguishable in all calculations. Ground movements and instabilities can be caused by changes in total stress, such as caused by loading by foundations and unloading due to excavations. They can also be caused by changes in pore water pressures, such as failure of slopes after rainfall.

Boiling may occur when excavations are made below water table and water is pumped out from the excavation pit to keep the area free from water. Boiling can be prevented by lowering the water table at the site before excavation or alternately, by increasing the length of upward flow. Boiling condition may also occur when a pervious sand stratum underying a clay soil is in artesian pressure condition.

PERMEABILITY

Flow takes place only when voids are present and they are interconnected. Cork is an example of a material that has voids but these voids are not interconnected. Hence flow cannot take place through a material like cork.

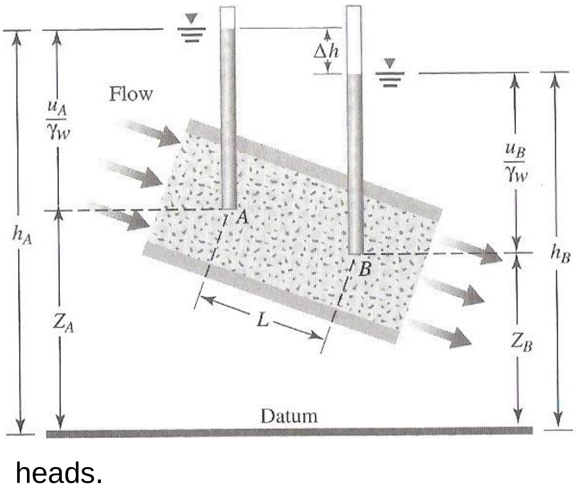


Soils consist of discrete particles, the void spaces between the particles are interconnected and may be viewed as a highly complex and intricate network of irregular tubes. The resistance to flow is greater when the pores are of smaller size and the pore channels irregular. The resistance to flow is much less when the soils have larger voids i.e., more or less regular flow channels. Thus, even while are soils are permeable, the degree of perviousness is different. Gravels are more pervious than sands, sand more pervious than silts and silts more pervious than clays.

Permeability of a soil is the ease with which water flows through that soil. The study of the flow of water through permeable soil media is necessary for

i. estimating the quantity of underground seepage under various hydraulic conditions
ii. investigating problems involving the pumping of water for underground construction
iii. making stability analyses of earth dams and earth retaining structures that are subject to seepage forces
iv. problems involving excavations of open cuts in sands below water table
v. subgrade drainage
vi. rate of consolidation of compressible soils

Pressure, Elevation and Total Heads



According to Bernoulli's equation **Total head** consists of **three components:** elevation head, pressure head and velocity head.

As seepage velocity in soils is normally low, velocity head is ignored, and total head becomes equal to the piezometric head. Due to the low seepage velocity and small size of pores, the flow of water in the pores is steady and laminar in most cases. Water flows from zones of higher potential to zones of lower potential. Thus flow takes place between two points in soil due to the difference in total heads.

Elevation head (***Z***) is the vertical distance of a given point above or below a datum plane.

At point **A,** the pore water pressure ($\boldsymbol{u_A}$) can be measured from the height of water in a standpipe located at that point. The pressure head is the pore water pressure ($\boldsymbol{u_A}$) at that point divided by the unit weight of water, γ_w.

The height of water level in the standpipe above the datum is the **piezometric head (h).**

For flow of water through a porous soil medium, velocity head can be neglected because seepage velocity is small.

The total head at any point can be adequately represented by

$$h = \frac{u}{\gamma_w} + Z$$

Figure above shows the relationship among pressure, elevation and total heads for the flow of water through soil. Open standpipes called *piezometers* are installed at points A and B. The levels to which water rises in the piezometers tubes situated at points A and B are known as piezometric levels of points *A* and *B*, respectively. Pressure head at a point is the height of the vertical column of water in the piezometers installed at that point.

The loss of head between the two points *A* and *B*, can be given by

$$\Delta h = h_A - h_B = \left(\frac{u_A}{\gamma_w} + Z_A\right) - \left(\frac{u_B}{\gamma_w} + Z_B\right)$$

The head loss, Δh, can be expressed in a non-dimensional form as

$$i = \frac{\Delta h}{L}$$

where i = hydraulic gradient
L = distance between points *A* and *B* – that is, the length of flow over which the loss of head occurred.

The hydraulic gradient, *i*, is the head lost in flow due to friction per unit length of flow. The potential driving the water flow is the hydraulic gradient between the two points, which is equal to the head drop per unit length. In steady state seepage, the gradient remains constant.

At low velocities, the flow through soils remains laminar. For laminar flow, the velocity *v* bears a linear relationship to the hydraulic gradient, *i*. Thus,

$$v \propto i$$

DARCY'S LAW

The flow of free water (water which moves under the influence of gravity) in soils is governed by Darcy's law. Darcy published a simple equation for the discharge velocity of water through saturated soils, which may be expressed as

$$v = k\,i$$

where v = **discharge velocity** or **superficial velocity**, which is the quantity of water flowing in unit time through a unit gross cross sectional area of soil at right angles to the direction of flow

k = **coefficient of permeability** also known as **hydraulic conductivity**

Thus, **Darcy's law** states that there is a linear relationship between flow velocity (**v**) and hydraulic gradient (**i**) for any given saturated soil under steady laminar flow conditions.

If the hydraulic gradient is unity, the coefficient of permeability is equal to the velocity of flow.

Thus, **the coefficient of permeability is defined as the velocity of flow which would occur under unit hydraulic gradient**. It has the dimensions of velocity and is measured in mm/s, m/s or m/day.

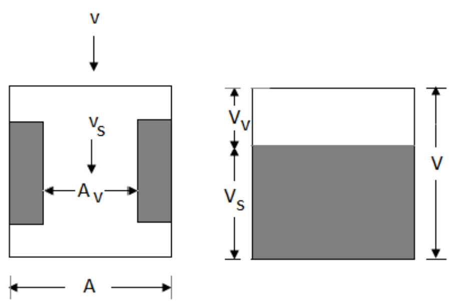
The actual velocity of water (that is the **seepage velocity,** $\boldsymbol{v_s}$) through the void spaces is greater than v.

The discharge q is obtained by multiplying the velocity of flow v by the gross cross-sectional area (area of both the solids and voids) of soil A normal to the direction of flow. Thus,

$$\boldsymbol{q = kiA}$$

RELATIONSHIP BETWEEN SEEPAGE VELOCITY AND DISCHARGE VELOCITY

The total cross-sectional area of soil consists of voids and solids. As the flow can take place only through voids, the actual velocity or seepage velocity (v_s) through the voids is much greater than the discharge velocity (v).



A relationship between the discharge velocity and the seepage velocity can be established referring to the figure, where a longitudinal section through a sample of soil in which the voids and solid particles are segregated.

From the continuity of flow,

$$\boldsymbol{q = vA = v_s A_v}$$

or $\boldsymbol{v_s = v \; x \; \frac{A}{A_v}}$

Multiplying the numerator and denominator by the length of the soil sample, L,

$$\boldsymbol{v_s = v \; x \; \left(\frac{AxL}{A_v xL}\right) = v \; x \; \frac{V}{V_v} = \frac{v}{n} = \frac{ki}{n} = k_p i}$$

where $\boldsymbol{A_V}$ = Area of voids on a cross section normal to the direction of flow, $\boldsymbol{n}$ = porosity and $\boldsymbol{k_p}$
is the **coefficient of percolation.**

The value of coefficient of percolation ($\boldsymbol{k_p}$) is always greater than the coefficient of permeability(**k**)

PERMEABILITY OF DIFFERENT SOILS

Permeability (**k**) is an engineering property of soils and is a function of the soil type. Its value depends on the average size of the pores and is related to the distribution of particle sizes, particle shape and soil structure. The ratio of permeabilities of typical sands/gravels to those of typical clays is of the order of $\mathbf{10^6}$. A small proportion of fine material in a coarse-grained soil can lead to a significant reduction in permeability.

For different soil types as per grain size, the orders of magnitude for permeability are as follows:

Soil	k (cm/sec)
Gravel	10^0
Coarse sand	10^0 to 10^{-1}
Medium sand	10^{-1} to 10^{-2}
Fine sand	10^{-2} to 10^{-3}
Silty sand	10^{-3} to 10^{-4}
Silt	1×10^{-5}
Clay	10^{-7} to 10^{-9}

FACTORS AFFECTING PERMEABILITY

The Poisueille equation for the rate of flow through a tube of any geometrical cross-section is

$$q = d_e^2 \frac{\gamma_w}{\eta} \frac{e^3}{1+e} CiA$$

Comparing the above equation with Darcy's law gives

$$k = Cd_e^2 \frac{\gamma_w}{\eta} \frac{e^3}{1+e}$$

Where C is composite shape factor dependent on grain shape and d_e is representative grain size.

From the equation above, **the factors affecting the permeability are**

i. **Particle shape and size**
- Permeability varies with the shape (factor C in the above equation) and size of the soil particles.
- Permeability varies with the square of particle diameter.
- Smaller the grain-size the smaller the voids and thus lower the permeability.
- A relationship between permeability and grain-size is more appropriate in case of sands and silts.
- Allen Hazen proposed the following empirical equation, $k = CD_{10}^2 \ cm/s$

  C is a constant that varies from 1.0 to 1.5 and D_{10} is the effective size in mm

ii. **Void ratio**
- For a given soil, greater the void ratio, the higher is the value of coefficient of permeability.
- It causes an increase in the percentage of cross-sectional area available for flow.

iii. **Degree of saturation**
- Higher the degree of saturation, higher is the permeability.
- In the case of certain sands the permeability may increase three-fold when the degree of saturation increases from 80% to 100%.

iv. **Adsorbed water**
- Permeability depends on the thickness adsorbed water. Adsorbed water is not free to move under gravity.
- It causes an obstruction to flow of water in the pores and hence reduces the permeability.

v. Soil structure

- Fine-grained soils with a flocculated structure have a higher coefficient of permeability than those with a dispersed structure.
- Remoulding of a natural soil reduces the permeability
- Permeability parallel to stratification is much more than that perpendicular to stratification

vi. Presence of entrapped air and other foreign matter.

- Entrapped air reduces the permeability of a soil.
- Organic foreign matter may choke flow channels thus decreasing the permeability

vii. Properties of permeant (water)

- Coefficient of permeability (k) is directly proportional to unit weight of water γ_w and inversely proportional to its viscosity (η).
- k increases with an increase in temperature due to reduction in the viscosity.

MEASUREMENT OF PERMEABILITY

The coefficient of permeability can be determined in three ways:

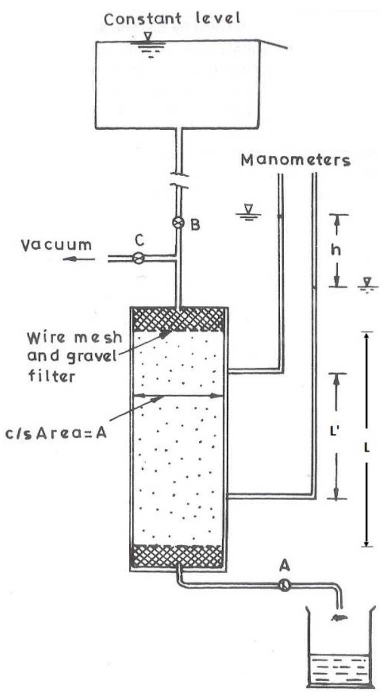
- Laboratory tests
- Field tests
- Empirical approach

Laboratory methods

In the laboratory, it is determined using either constant head or variable head test.

Constant Head Test - IS:2720 (Part XVII)

The test is conducted in an instrument known as permeameter. Constant head permeters are specially suited to the testing of pervious, coarse grained soils, since adequate, measurable discharge is needed for the accurate determination of permeability by this method. The permeameter consiss of a metallic mould, 100 mm internal diameter, 127.3 mm effective height and 1000 ml capacity. The sample is placed between two porous discs and the whole assembly is placed in a constant head chamber filled with water to the brim at the start of the test.



A typical set up is shown in figure. Water is allowed to flow through the soil sample from a *constant head reservoir* designed to keep the keep water level constant by overflow. The quantity of water flowing out of the sample in to *the constant head chamber* spills over the chamber and the discharge Q during a given time t is collected in a measuring cylinder.

Since the presence of entrapped air in the soil can affect the permeability, de-aired water is supplied to the reservoir and then vacuum is applied to the soil sample before commencing the test. It is essential that the sample is fully saturated.

If the cross-sectional area of the specimen is *A* and Q is the volume of water collected in time t, the discharge is given by

$$q = kiA$$

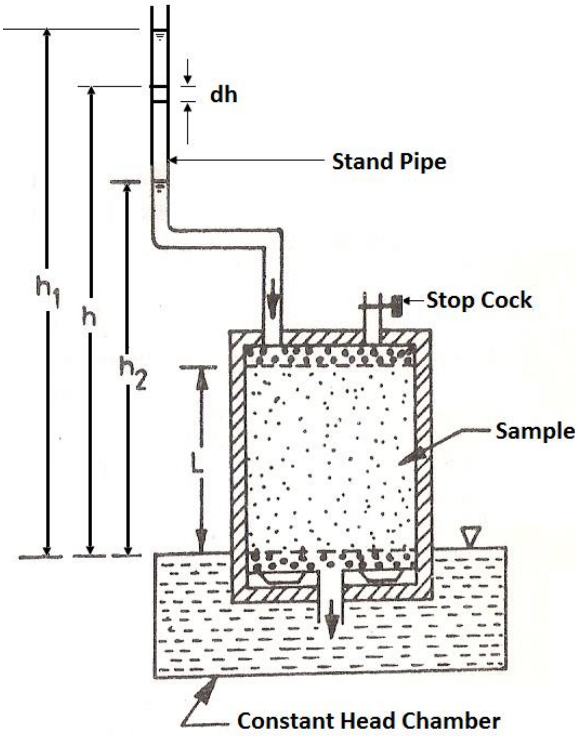
$$q = k\frac{h}{L}A$$

$$k = \frac{qL}{Ah} = \frac{QL}{Aht}$$

Falling Head Test or Variable Head Test

This method is used to determine the permeability of fine-grained soils such as fine sands and silts. In such soils, the permeability is too small to enable accurate measurement of discharge using a constant head permeameter. The permeameter mould is the same as that used in the constant head test. A vertical graduated standpipe of known diameter is fitted to the top of permeameter. The sample is placed between two porous discs and the whole assemble is placed in a constant head chamber filled with water to the brim at the start of the test.

The test is started by allowing water in the standpipe to flow through the sample to the constant head chamber from which it overflows and spills out. As the water flows through the soil, the water level in the standpipe falls. The time required for water level to fall from a known initial head (h_1) to the known final head (h_2) is determined. The head is measured with reference to the level of water in the constant head chamber.



Consider the instant when the head is h. For the infinitesimal small time dt, the head falls by dh. Let the discharge through the sample be q. Let the cross-sectional area of the standpipe be a. From the continuity of flow,

$$a.dh = -q.dt$$

From Darcy's law, *q = kiA.* Substituting for q,

$$a.dh = -(A.k.i)dt$$

$$a.dh = -Ak\frac{h}{L}dt$$

Rearranging the terms,

$$\frac{Akdt}{aL} = -\frac{dh}{h}$$

Integrating,

$$\frac{Ak}{aL}\int_{t_1}^{t_2} dt = -\int_{h_1}^{h_2}\frac{dh}{h}$$

$$\frac{Ak}{aL}(t_2 - t_1) = \log_e\frac{h_1}{h_2}$$

$$k = \frac{aL}{At}\log_e\frac{h_1}{h_2}$$

$$k = \frac{2.30aL}{At}\log_{10}\frac{h_1}{h_2}$$

where $t = (t_2 - t_1)$, the time interval during which the head reduces from h_1 to h_2.

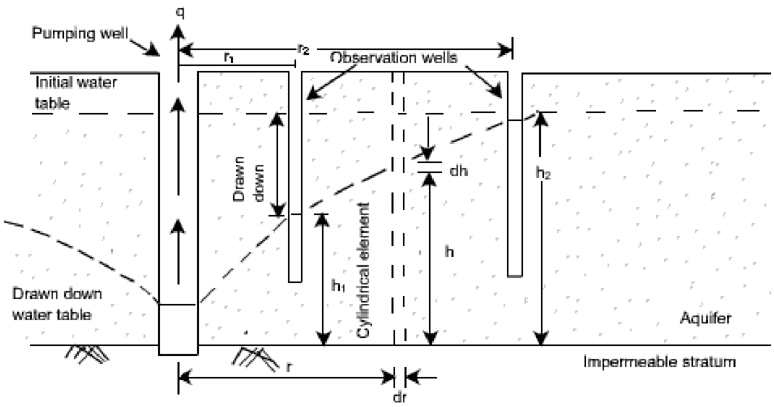
FIELD TESTS FOR PERMEABILITY

Field or *in-situ* measurement of permeability avoids the difficulties involved in obtaining and setting up undisturbed samples in a permeameter. It also provides information about bulk permeability, rather than merely the permeability of a small sample.

A field permeability test consists of pumping out water from a main well and observing the resulting drawdown surface of the original horizontal water table from at least two observation wells. When a steady state of flow is reached, the flow quantity and the levels in the observation wells are noted.

Two important field tests for determining permeability are: Unconfined flow pumping test, and confined flow pumping test.

Unconfined Flow Pumping Test



In this test, the pumping causes a drawdown in an unconfined (i.e. open surface) soil stratum, and generates a radial flow of water towards the pumping well. The steady-state heads $\mathbf{h_1}$ and $\mathbf{h_2}$ in observation wells at radii $\mathbf{r_1}$ and $\mathbf{r_2}$ are monitored till the flow rate **q** becomes steady.

The rate of radial flow through any **cylindrical surface** around the pumping well is equal to the amount of water pumped out. Consider such a surface having radius **r**, thickness **dr** and height **h.** The hydraulic gradient is

$$\boldsymbol{i = \frac{dh}{dr}}$$

Area of flow, *A*, is

$$\boldsymbol{A = 2\pi r h}$$

From Darcy's Law,

$$\boldsymbol{q = k.i.A}$$

$$\boldsymbol{q = k.\frac{dh}{dr}.2\pi r h}$$

Arranging and integrating,

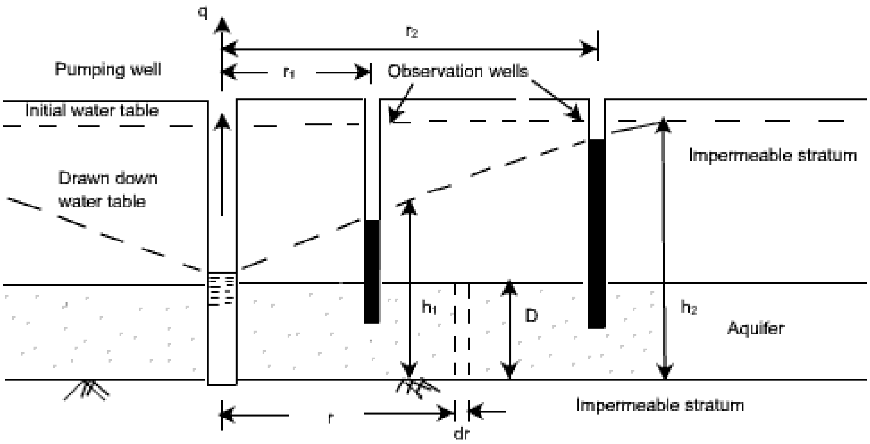
$$\int_{r_1}^{r_2} \frac{dr}{r} = \frac{2\pi k}{q} \int_{h_1}^{h_2} h\,dh$$

$$\log_e \frac{r_2}{r_1} = \frac{\pi k}{q}\left(h_2^2 - h_1^2\right)$$

$$k = \frac{q.\log_e \frac{r_2}{r_1}}{\pi\left(h_2^2 - h_1^2\right)}$$

$$k = \frac{2.303q \log_{10} \frac{r_2}{r_1}}{\pi\left(h_2^2 - h_1^2\right)}$$

Confined Flow Pumping Test



Artesian conditions can exist in a aquifer of thickness **D** confined both above and below by impermeable strata. In this, the drawdown water table is above the upper surface of the aquifer.

For a **cylindrical surface** at radius **r**,

$$q = k.\frac{dh}{dr}.2\pi r D$$

Integrating,

$$\int_{r_1}^{r_2} \frac{dr}{r} = \frac{2\pi k D}{q} \int_{h_1}^{h_2} dh$$

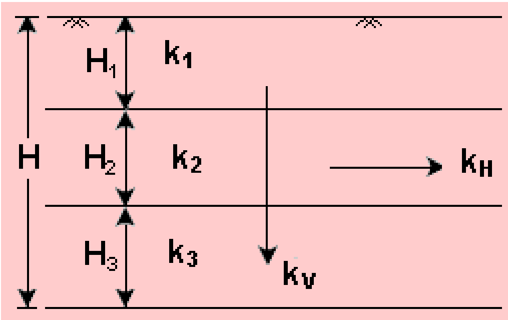
$$\log_e \frac{r_2}{r_1} = \frac{2\pi k D}{q}(h_2 - h_1)$$

$$k = \frac{2.303q \log_{10} \frac{r_2}{r_1}}{2\pi D(h_2 - h_1)}$$

PERMEABILITY OF STRATIFIED SOILS

When a soil deposit consists of a number of horizontal layers having different permeabilities, the average value of permeability can be obtained separately for both vertical flow and horizontal flow, as $\mathbf{k_V}$ and $\mathbf{k_H}$ respectively.

Consider a stratified soil having horizontal layers of thickness $\mathbf{H_1}$, $\mathbf{H_2}$, $\mathbf{H_3}$, etc. with coefficients of permeability $\mathbf{k_1}$, $\mathbf{k_2}$,$\mathbf{k_3}$, etc.



For vertical flow
The flow rate **q** through each layer per unit area is the same.

$$q = q_1 = q_2 = \ldots\ldots\ldots\ldots$$

Let **i** be the equivalent hydraulic gradient over the total thickness **H** and let the hydraulic gradients in the layers be $\mathbf{i_1}$, $\mathbf{i_2}$, $\mathbf{i_3}$, etc. respectively.

$$k_V.i = k_1.i_1 = k_2.i_2 = \ldots\ldots\ldots\ldots$$ where $\mathbf{k_V}$ = Average vertical permeability

$$k_V.\frac{h}{H} = k_1.\frac{h_1}{H_1} = k_2.\frac{h_2}{H_2} = \ldots\ldots\ldots\ldots$$

The total head drop **h** across the layers is

$$h = h_1 + h_2 + \ldots\ldots\ldots\ldots$$

$$h = \frac{k_V.h}{H}.\frac{H_1}{k_1} + \frac{k_V.h}{H}.\frac{H_2}{k_2} + \ldots\ldots\ldots$$

$$k_V = \frac{H}{\frac{H_1}{k_1} + \frac{H_2}{k_2} + \ldots\ldots\ldots\ldots}$$

Horizontal flow
When the flow is horizontal, the hydraulic gradient is the same in each layer, but the quantity of flow is different in each layer.

$$i = i_1 = i_2 = i_3 = \ldots\ldots$$

The total flow is

$$q = q_1 + q_2 + q_3 \ldots\ldots$$

Considering unit width normal to the cross-section plane,

$$k_H i.H = k_1 i_1.H_1 + k_2 i_2.H_2 + \ldots\ldots\ldots\ldots$$

$$k_H = \frac{1}{H}(k_1.H_1 + k_2 H_2 + \ldots\ldots\ldots\ldots)$$