SEEPAGE IN SOILS

Seepage is the flow of water under gravitational forces in a permeable medium. The flow of water through soil, in many instances, is not in one direction only, nor is it uniform over the entire area perpendicular to the flow. In the kind of seepage that takes place around sheet pile walls, under masonry dams and other water retaining structures and through earth dams, embankments etc., the flow condition is two dimensional. In a two dimensional flow, the velocity components in the horizontal and vertical directions vary from point to point within the cross-section of the soil mass. The three dimensional flow is the most general flow situation but the analysis of such problems is too complex to be practical and, hence, flow situations are simplified to the two dimensional flow.

In two-dimensional flow, ground water flow is generally calculated by the use of graphs referred to as flow nets. The concept of low net is based on Laplace's equation of continuity, which governs the steady flow condition for a given point in the soil mass.

The study of two dimensional flow of water is used to

- Calculate flow under and within earth structures.
- Calculate seepage stresses, porewater pressure distribution, uplift forces, hydraulic gradients, and the critical hydraulic gradient.
- Determine the stability of simple geotechnical systems subjected to two-dimensional flow of water.

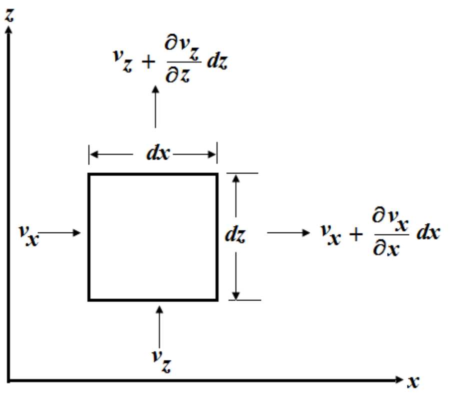
TWO DIMENSIONAL FLOW – LAPLACE'S EQUATION

Assumptions

1. The soil mass is fully saturated.
2. Darcy's law is valid.
3. The soil mass is homogeneous and isotropic.
4. Both the soil grains forming the skeleton and pore fluid are incompressible i.e. no compression or expansion takes place during the flow.
5. Flow conditions do not change with time, that is, steady state conditions exist.

Consider the flow of water into an element in a saturated soil, with dimensions *dx* and *dz* in the horizontal and vertical directions as shown in figure below. The third dimension is along y-axis is very large. For convenience, it is taken as unity.

Let,



Velocity at the inlet face in the horizontal direction = $\boldsymbol{v_x}$
Velocity at the inlet face in the vertical direction = $\boldsymbol{v_z}$
Velocity at the outlet face in the horizontal direction = $\boldsymbol{v_x + \frac{\partial v_x}{\partial x} . dx}$
Velocity at the outlet face in the vertical direction = $\boldsymbol{v_z + \frac{\partial v_z}{\partial z} . dz}$

As the flow is steady and the soil is incompressible, the discharge entering the element is equal to that leaving the element.

Thus,

$$\boldsymbol{v_x dz + v_z dz = \left(v_x + \frac{\partial v_x}{\partial x} . dx\right) dz + \left(v_z + \frac{\partial v_z}{\partial z} . dz\right) dx}$$

$$\left(\frac{\partial v_x}{\partial x} + \frac{\partial v_z}{\partial z}\right) dxdz = 0$$

As dx.dz ≠ 0,

$$\left(\frac{\partial v_x}{\partial x} + \frac{\partial v_z}{\partial z}\right) = 0$$

The above equation is the **continuity equation** for two-dimensional flow.

Let *h* be the total head at any point. The horizontal and vertical components of the hydraulic gradient are

$$i_x = -\frac{\partial h}{\partial x} \text{ and } i_z = -\frac{\partial h}{\partial z}$$

The minus indicates that the head decreases in the direction of flow.

From Darcy's law

$$v_x = -k_x \frac{\partial h}{\partial x} \text{ and } v_z = -k_z \frac{\partial h}{\partial z}$$

Substituting for v_x and v_z, the equation may be written as

$$-k_x \frac{\partial^2 h}{\partial x^2} - k_z \frac{\partial^2 h}{\partial z^2} = 0$$

$$k_x \frac{\partial^2 h}{\partial x^2} + k_z \frac{\partial^2 h}{\partial z^2} = 0$$

As the soil is isotropic, $k_x = k_z$. Therefore, the continuity equation for two-dimensional flow, known as **Laplace equation** may be written as,

$$\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial z^2} = 0$$

The Laplace equation can be solved if the boundary conditions at the inlet and exit are known. The equation represents two families of curves which are orthogonal to each other. One family represents the flow lines. ***A flow line is a line along which the water particle will travel from upstream to the downstream side in the permeable soil medium.*** The other family represents the equipotential lines. ***An equipotential line is a line along which the total head at all points is equal.***

FLOW NETS

A combination of a number flow lines and equipotential lines is called a **flow net.** A flow net is graphical representation of a flow field (solution of Laplace's equation). Flow nets are constructed for the calculation of groundwater flow and the evaluation of heads in the permeable soil media.

CHARACTERISTICS OF FLOW NETS

1. Flow lines or stream lines represent flow paths of particles of water
2. Flow lines and equipotential line are orthogonal to each other
3. The area between two flow lines is called a flow channel
4. The rate of flow in a flow channel is constant (Δq)
5. Flow cannot occur across flow lines
6. An equipotential line is a line joining points with the same head
7. The velocity of flow is normal to the equipotential line
8. The difference in head between two equipotential lines is called the potential drop or head loss) (Δh)
9. A flow line cannot intersect another flow line.
10. An equipotential line cannot intersect another equipotential line
11. There can be no flow along an equipotential line as there is no hydraulic gradient.

Two flow lines can never meet. If they did, it implies that the water flowing in the flow path between them has disappeared, which is a physical impossibility.

Two equipotential lines can never meet. If they did, it implies that at the point where they meet, there are two potentials in the porewater., which is an absurdity.

The hydraulic gradient between any two equipotential lines is the drop in potential head (head loss) between those lines divided by the distance travelled. Thus, as the equipotential lines come closer to each other in a flow net, the length of flow for the same drop in head becomes smaller and the gradient of flow becomes higher.

GUIDELINES FOR DRAWING FLOW NETS

- Draw the cross section of the structure, soil mass and water elevations to a suitable scale.

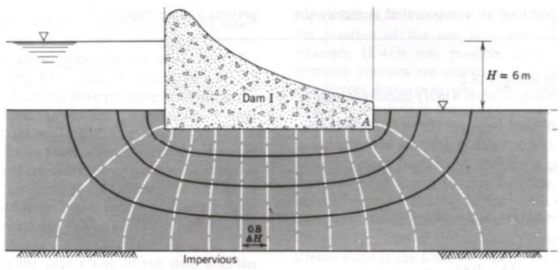
- Identify the equipotential and flow boundaries. ***The soil and impermeable boundary interfaces are flow lines. The soil and permeable boundary interfaces are equipotential lines.***

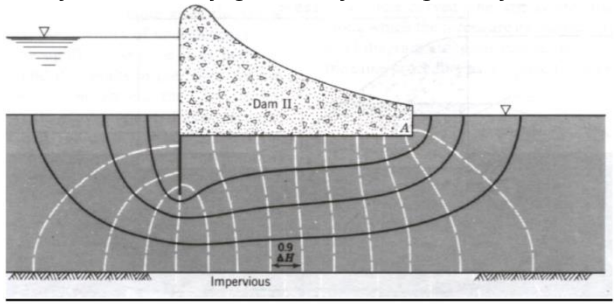
- Draw a few flow lines and equipotential lines. Sketch intermediate flow lines and equipotential lines by smooth curves adhering to right-angle intersections such that area between a pair of flow lines and a pair of equipotential lines is approximately a curvilinear square grid.

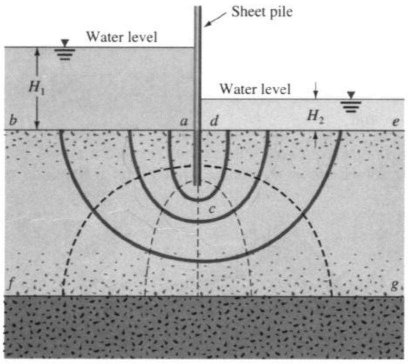
- Where flow direction is a straight line, flow lines are equal distance apart and parallel. Also, the flownet in confined areas between parallel boundaries usually consists of flow lines and equipotential lines that are elliptical in shape and symmetrical

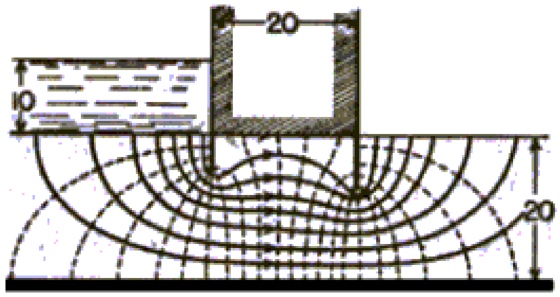
- Try to avoid making sharp transition between straight and curved sections of flow and equipotential lines. Transitions must be gradual and smooth. Continue sketching until a problem develops.

- Successive trials will result in a reasonably consistent flow net. In most cases, 3 to 8 flow lines are usually sufficient. Depending on the number of flow lines selected, the number of equipotential lines will automatically be fixed by geometry and grid layout.





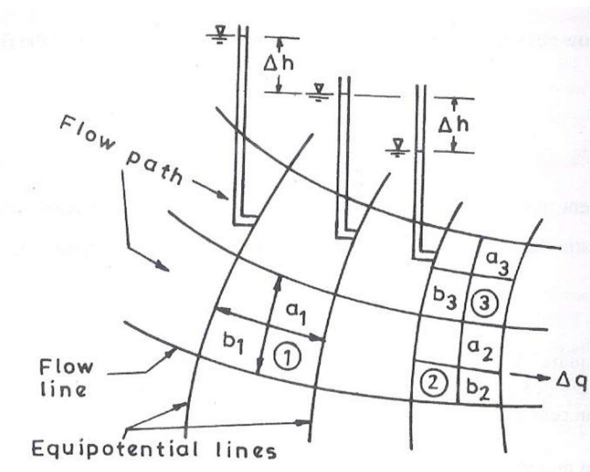




USES OF FLOW NETS

Head loss and seepage calculation

Refer to fig shown below. Consider two fields 1 and 2 lying between two successive flow lines. The average dimensions of the two fields along and normal to the flow are b_1, a_1 and b_2, a_2, respectively. Consider a third field 3 in such a way that fields 2 and 3 lie within the same equipotential lines. The dimensions of field 3 are b_3, a_3.



Let Δq_1, Δq_2, and Δq_3 be the rate of flow through fields 1 , 2 and 3, respectively, and Δh_1, Δh_2, and Δh_3 be the head loss across the fields 1, 2 and 3.

Consider the flow per unit width perpendicular to the plane of the section. From Darcy's law,

$$\Delta q_1 = k\frac{\Delta h_1}{b_1}(a_1\ x\ 1)$$

$$\Delta q_2 = k\frac{\Delta h_2}{b_2}(a_2\ x\ 1)$$

$$\Delta q_3 = k\frac{\Delta h_3}{b_3}(a_3\ x\ 1)$$

Fields 1 and 2 are within the same flow channel. Hence $\Delta q_1 = \Delta q_2$, since there can be no flow across the flow lines.

Since fields 2 and 3 are within the same equipotential lines, $\Delta h_2 = \Delta h_3$. If all the fields are elementary squares,

$$\frac{a_1}{b_1} = \frac{a_2}{b_2} = \frac{a_3}{b_3} = 1$$

Thus,

$$\Delta q_1 = \Delta q_2 = \Delta q_3 = \Delta q$$

$$\Delta h_1 = \Delta h_2 = \Delta h_3 = \Delta h$$

Thus, in a flow net constructed such that all its fields are elementary squares, the quantity of flow through each field will be equal and the head loss across each field will also be equal.

Thus,

$$\Delta q = k.\Delta h$$

If H is the total head loss during the flow, and n_d = number of equipotential drops, then

$$\Delta h = \frac{H}{n_d}$$

$$\therefore \ \Delta q = k\frac{H}{n_d}$$

If the number of flow channels in a flow net is n_f, the rate of flow q through all the flow channels per unit length is

$$q = \Delta q \ x \ n_f$$

$$q = kH\frac{n_f}{n_d}$$

It is not necessary to make all fields elementary squares in flow net. It is only necessary to make $\frac{a}{b}$ ratio the same for all the fields. If $\frac{a}{b} = n$

$$q = kH\frac{n_f}{n_d}n$$

Rate of flow, q, is function of

(i) permeability k, which is an engineering property of soil
(ii) Loss of head h, a hydraulic condition
(iii) $\frac{n_f}{n_d}$ ratio, called the shape factor

The graphical solution of Laplace's equation, namely the flow net, enables to determine n_f and n_d, and thus the shape factor $\frac{n_f}{n_d}$, which is function only of the boundary conditions that govern a given flow problem.

The flow net would not change

i. If the soil were to be replaced (change in k)
ii. If the head loss during flow were to be different
iii. Even if the upstream and downstream water levels were to be reversed. In this case the direction of flow would be reversed.

It is only when the geometry of the flow space (boundary conditions) is altered that the flow net would be different and would yield a different $\frac{n_f}{n_d}$ ratio.

For a given set of boundary conditions, the flow net would be unique.

Maximum hydraulic gradient or exit hydraulic gradient and piping

An unstable condition arises when seepage lines emerge vertically upwards on the downstream boundary. The hydraulic gradient of flow will be maximum adjacent to the toe of a dam, and at the base for a sheet pile because it is here that the curvilinear squares are th smallest in size and thus the length of flow for a certain constant drop in head will be smallest.

The maximum hydraulic gradient is $\boldsymbol{i_{max}} = \frac{\Delta \boldsymbol{h}}{\boldsymbol{L_{min}}}$

L_{min} = minimum length of an equipotential drop. Usually, L_{min} occurs at exits. .

Piping Effects

Soils can be eroded by flowing water. Erosion can occur underground, beneath the hydraulic structures, if there are cavities, cracks in rock, or high exit gradient induced instability at toe of the dam, such that soil particles can be washed into them and transported away by high velocity seeping water. This type of underground erosion progresses and creates an open path for flow of water; it is called "piping". Preventing piping is a prime consideration in the design of safe dams. Briefly the processes associated with initiation of piping in dams are as follows:

- Upward seepage at the toe of the dam on the downstream side causes local instability of soil in that region leading erosion.
- A process of gradual erosion and undermining of the dam may begin, this type of failure known as piping, has been a common cause for the total failure of earth dams
- The initiation of piping starts when exit hydraulic gradient of upward flow is close to critical hydraulic gradient

Factor of safety against piping is defined as,

$$\boldsymbol{FS = \frac{i_{critical}}{i_{exit}}}$$

Where i_{exit} is the maximum exit gradient and $i_{critical}$ is the critical hydraulic gradient. Exit gradient must never come close to the critical hydraulic gradient. The maximum exit gradient can be determined from the flow net. A factor of safety of at least 6 is considered adequate for the safe performance of the structure against piping failure. Exit gradient can be reduced to a considerable extent by providing vertical cut off walls at the downstream end of the base of the dam.

Critical hydraulic gradient occurs when effective stress is zero.

$$i = i_{cr} = \frac{\gamma'}{\gamma_w}$$

But

$$\gamma' = \left(\frac{G-1}{1+e}\right)\gamma_w$$

Therefore,

$$i_{cr} = \left(\frac{G-1}{1+e}\right)$$

For the usual void ratios in sand of 0.60 to 0.70, the critical hydraulic gradient is just about 1.

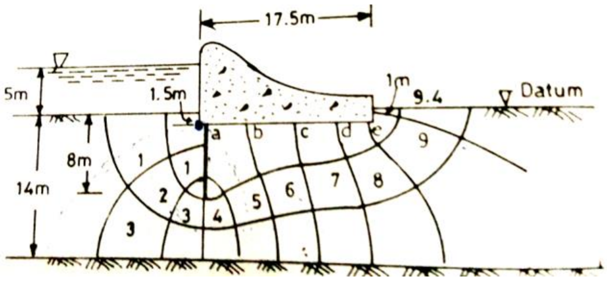
Porewater pressure

The porewater pressure at a given point is calculated by multiplying the pressure head at that point with the unit weight of water.

Uplift pressure

Uplift pressure at any given point is the pore water pressure acting vertically upward due to residual pressure head at that point. The uplift force is calculated by working out the area of the uplift pressure distribution diagram. The total uplift force acts opposite the force of gravity due to the weight of the dam and hence reduces the stability of the hydraulic structure. Uplift pressures along the base of masonry dams can be reduced by providing vertical cut off walls at the upstream end of the base of the dam.

Example 1.



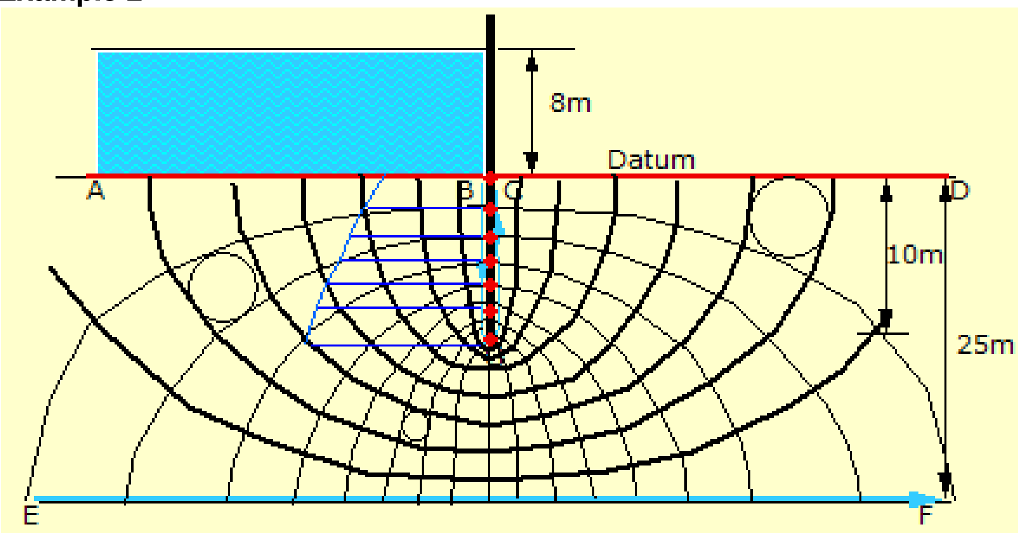
Elevation head for points at upstream end of toe, a, b, c, d, e = -1.5m (-ve sign as elevation is measures below datum)

Total number of drops = 9.4 (given in figure)
$\therefore$5m head is lost in 9.4 drops
Head loss per drop = 5/9.4 = 0.53 m

Upstream end of toe
No. of drops at upstream end = 0.5 (approx.)
Head loss = 0.5 x 0.53 =0.26 m
Total head available = 5 – 0.26 = 4.74 m
Point a
No. of drops at a = 4.5 (approx.) (point a is between 4^{th} and 5^{th} equipotential lines)
Head loss = 4.5 x 0.53 =2.38 m
Total head available = 5 – 2.38 = 2.62 m
Point b
No. of drops at b = 5
Head loss = 5 x 0.53 =2.65 m
Total head available = 5 – 2.65 = 2.35 m

Point	**No. of drops**	**Head loss**	**Total Head (m) (5m – head loss)**	**Pressure Head (m) (Total head – elv. Head)**	**Uplift pressure (kPa) (Pr. Head x γ_w)**
Upstream end of toe	0.5		4.73	4.73 – (-1.5) = 6.23	61.11
a	4.5		2.62	2.62 – (-1.5) = 4.12	40.42
b	5.0		2.35	3.85	37.77
c	6.0		1.82	3.32	32.57
d	7.0		1.29	2.79	27.37
e	8.0		0.76	2.26	22.17

Example 2



Total number of drops = 18
$\therefore$8m head is lost in 18 drops
Head loss per drop = 8/18 = 0.44 m

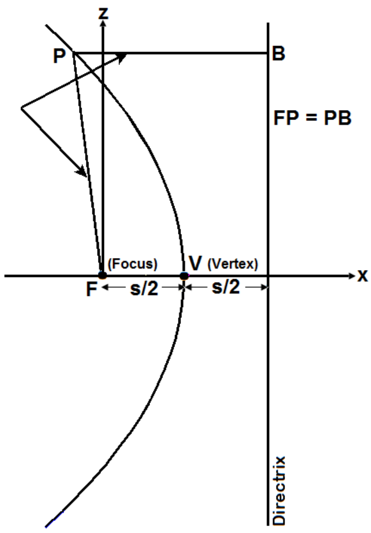
Point (red dots)	No. of drops	Head loss (m)	Total Head (m) (8m – head loss)	Elevation head (m)	Pressure Head (m) (Total head – elv. Head)	Uplift pressure (kPa) (Pr. Head x γ_w)
2	1	1 x 0.44 = 0.44	7.56	-2	7.56-(-2) = 9.56	93.78
3	2	2 x 0.44 = 0.88	7.12	-3.8	7.12-(-3.8) = 10.92	107.12
4	3	1.32	6.68	-5.2	11.88	116.54
5	4	1.76	6.24	-7	13.24	129.88
6	5.5	2.42	5.58	-8.4	13.98	137.14
7	8	3.52	4.48	-10	14.48	142.05

Seepage Flow through Homogeneous Earth Dams

Flow through earth dams is an important design consideration. In order to ensure that the pore water pressure at the downstream end does not lead to instability and the exit hydraulic gradient does not lead to piping, the flow net must be drawn and analysed.

In the case of earth dams, the top boundary flow line, which is a free water surface, is not evident from the geometrical boundaries and therefore, the flow space is not fully defined. The major exercise is to locate the position of the top flow line of seepage in the cross section. The top flow line is called the phreatic line. The pressure on the phreatic line is zero.

Casagrande showed that the phreatic surface can be approximated by a parabola with corrections at the points of entry and exit. The assumed parabola representing the uncorrected phreatic line is called the basic parabola. The properties of the regular parabola which are essential to obtain phreatic line are depicted in figure below.



Every point on the parabola is equidistant from the focus and directrix.

Therefore, **FP = PB**

Also, the distance between the focus and vertex, and vertex and directrix is equal.

Therefore, **FV = p =s/2**

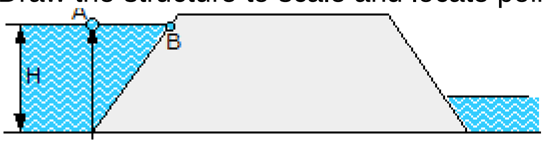
For a point P(x, z),

$$x^2 + z^2 = (2p + x)^2$$

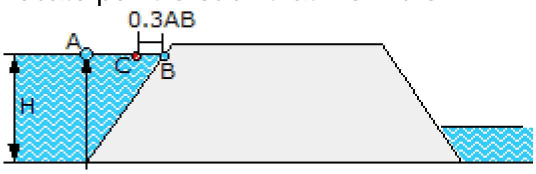
$$x = \frac{z^2 - 4p^2}{4p}$$

PHREATIC LINE FOR AN EARTH DAM WITHOUT TOE FILTER

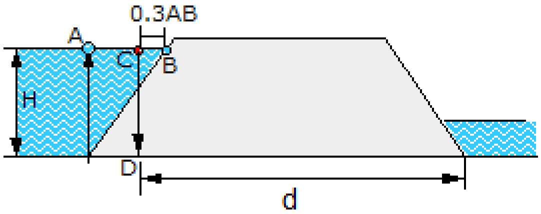
1. Draw the structure to scale and locate points A and B as shown



2. Locate point C such that BC = 0.3AB



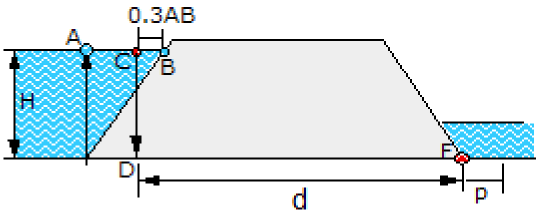
3. Project a vertical line from C as shown



4. Locate the focus, F, at the toe of the dam and calculate the focal distance p as

$$p = \frac{\sqrt{d^2 + H^2} - d}{2}$$

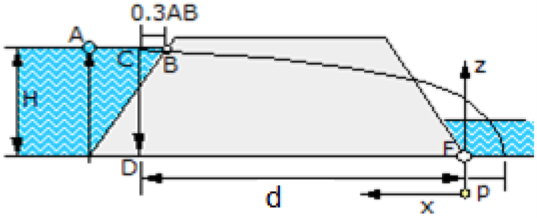
The directrix is located at a distance 2p from the focus F.



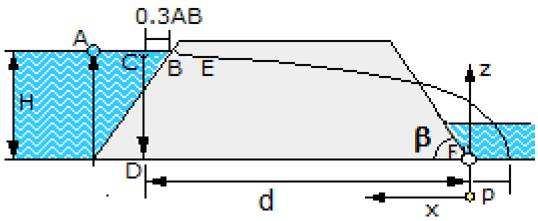
5. Construct basic parabola from

$$z = 2\sqrt{p(p + x)}$$

Choose arbitrary values of x and compute z. Join all the points to get the basic parabola.



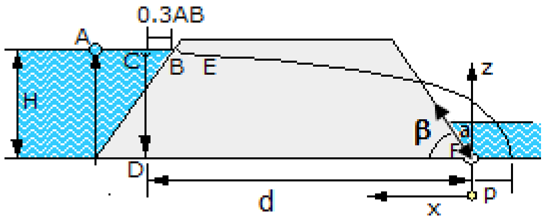
6. Upstream Correction: Sketch the section BE such that it is normal to the upstream sloping surface (i.e. equipotential line)



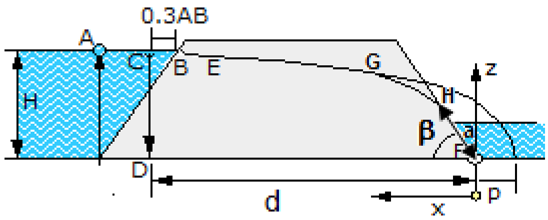
7. Calculate the distance 'a' from

$$for\ \beta \leq\ 30^o, \qquad a = \frac{d}{cos\beta} - \sqrt{\frac{d^2}{cos^2\beta} - \frac{H^2}{sin^2\beta}}$$

$$for\ 30^o < \beta < \ 60^o, \qquad a = \sqrt{d^2 + H^2} - \sqrt{d^2 - H^2cot^2\beta}$$



8. Downstream Correction: Sketch in a transition section GH



The curve BEGHF is the phreatic line for an earth dam without toe filter.

9. The discharge can be computed as

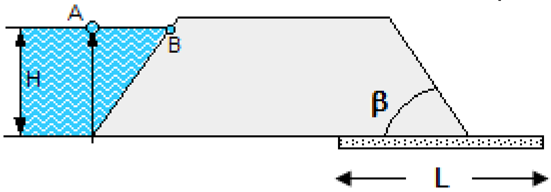
$$for\ \beta < \ 30^o, \qquad q = \ k(a\ sin\beta)tan\beta$$

$$for\ 30^o < \beta < \ 60^o, \qquad q = \ k\ asin^2\beta$$

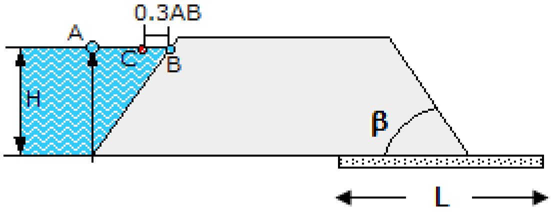
Where k = coefficient of permeability of the earth dam material.

PHREATIC LINE FOR AN EARTH DAM WITH TOE FILTER

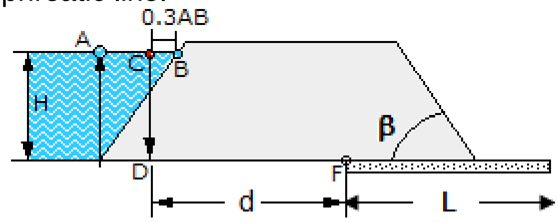
1. Draw the structure to scale and locate points A and B as shown.



2. Locate point C such that BC = 0.3AB

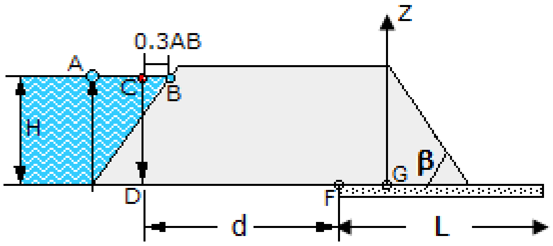


3. Project a vertical line from C as shown. Select F as the focus of the parabolic phreatic line.



4. Locate point G on the directrix at a distance 2p from the focal point.

$$p = \frac{\sqrt{d^2 + H^2} - d}{2}$$

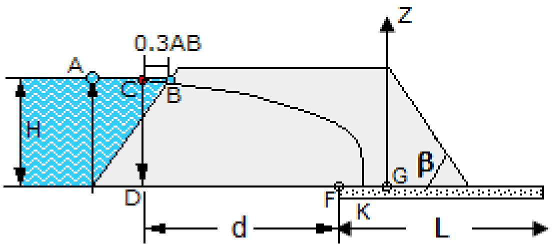


Select the base of the dam and directrix as x and z-axes.

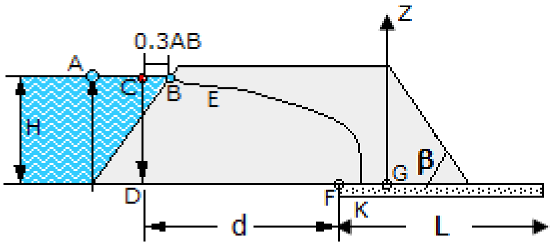
5. Construct basic parabola from

$$z = 2\sqrt{p(p + x)}$$

Choose arbitrary values of x and compute z. Join all the points to get the basic parabola.



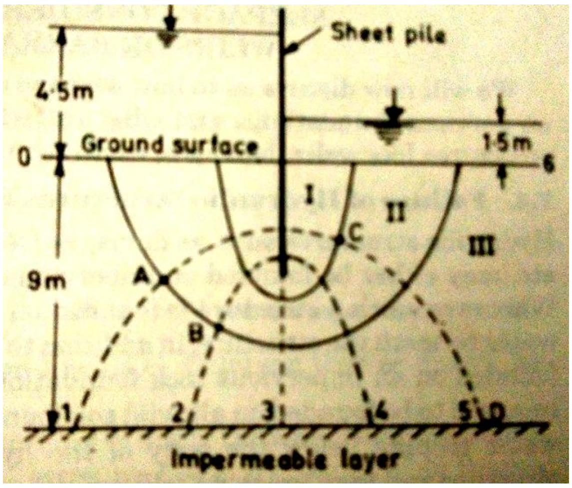
6. Upstream Correction: Sketch the section BE such that it is normal to the upstream sloping surface (i.e. equipotential line). BEK is the phreatic line (top flow line)



7. Let the distance between focus, F, and directrix, G, be S.
   i.e., $\boldsymbol{S = 2p}$
   The quantity of seepage per unit length of earth dam with toe filter is
   $$\boldsymbol{q = kS = k\ x\ (2\ x\ p) = 2kp}$$
   Where k is the permeability of the earth dam material.

Problems
A flow net for a flow around a single row of sheet piles in a permeable soil is shown.
Given k = 5 x 10^{-2} mm/s, determine

i. How high (above the ground surface) will the water rise, if piezometers are placed at points A, B, C and D?
ii. What will be the rate of seepage through the flow channel II per unit length of the sheet pile?
iii. What is the total rate of seepage through the permeable layer per unit length?



The no. of flow channels, n_f = 3
The no of drops, n_d = 6
Diff. of u/s head and d/s head =4.5-1.5 = 3 m
Loss of head per drop = 3/6 = 0.5 m

<u>Point A</u>
A is located on equipotential line 1
∴ No. of drops, n_d = 1
Loss in head up to A = n_d x head loss per drop
= 1 x 0.5 = 0.5 m
∴ The water in piezometer at A will rise up to 4.5 – 0.5 = 4 m above ground surface.

<u>Point B</u>
B is located on equipotential line 2
∴ No. of drops, n_d = 2
Loss in head up to A = n_d x head loss per drop
=2 x 0.5 = 1.0 m
∴ The water in piezometer at B will rise up to 4.5 – 1.0 = 3.5 m above ground surface.

Point C

C is located on equipotential line 5

$\therefore$ No. of drops, n_d = 5

Loss in head up to A = n_d x head loss per drop

= 5 x 0.5 = 2.5 m

$\therefore$ The water in piezometer at C will rise up to
4.5 – 2.5 = 2 m above ground surface.

Point D

D is located on equipotential line 5

$\therefore$ Head loss at D will be the same as at C

$\therefore$ The water in piezometer at D will rise up to
4.5 – 2.5 = 2 m above ground surface.

ii) The discharge through a flow channel in a flow net is

$$\Delta \boldsymbol{q} = \boldsymbol{k}\frac{\boldsymbol{H}}{\boldsymbol{n_d}}$$

n = 1 for square fields

Given: k = 5 x 10^{-2} mm/s = 5 x 10^{-5} m/s, H = 3m, n_f = 3, n_d = 6

$$\therefore \boldsymbol{q} = \mathbf{5}\ \boldsymbol{x}\ \mathbf{10^{-5}}\ \boldsymbol{x}\ \frac{\mathbf{3}}{\mathbf{6}} = \mathbf{2.5}\ \boldsymbol{x}\ \mathbf{10^{-5}}\ \boldsymbol{m^3/s}$$

iii) Total seepage = $\Delta \boldsymbol{q}\ \boldsymbol{x}\ \boldsymbol{n_f} = \mathbf{2.5}\ \boldsymbol{x}\mathbf{10^{-5}}\ \boldsymbol{x}\ \mathbf{3} = \mathbf{7.5}\ \boldsymbol{x}\mathbf{10^{-5}}$