UNIT-II

MATHEMATICAL MODELING OF PHYSICAL SYSTEMS

In this chapter, let us discuss the **differential equation modeling** of mechanical systems. There are two types of mechanical systems based on the type of motion.

- Translational mechanical systems
- Rotational mechanical systems

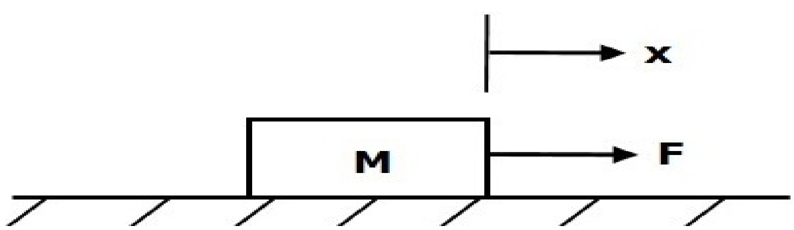
Modeling of Translational Mechanical Systems

Translational mechanical systems move along a **straight line**. These systems mainly consist of three basic elements. Those are mass, spring and dashpot or damper.

If a force is applied to a translational mechanical system, then it is opposed by opposing forces due to mass, elasticity and friction of the system. Since the applied force and the opposing forces are in opposite directions, the algebraic sum of the forces acting on the system is zero. Let us now see the force opposed by these three elements individually.

<u>MASS:</u>

Mass is the property of a body, which stores kinetic energy. If a force is applied on a body having mass M, then it is opposed by an opposing force due to mass. This opposing force is proportional to the acceleration of the body. Assume elasticity and friction are negligible.



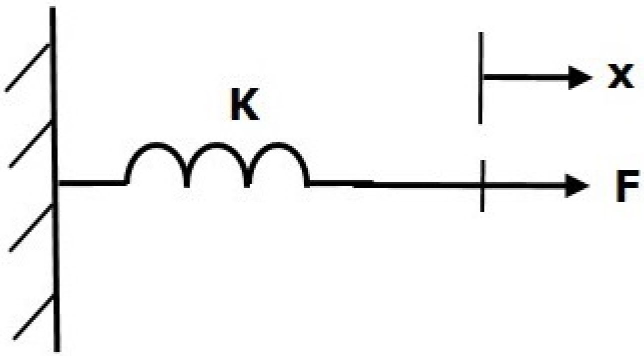
$$F_m \propto a$$
$$\Rightarrow F_m = Ma = M\,d^2x/dt^2$$
$$F = F_m = M\,d^2x/dt^2$$

Where,

- F is the applied force
- Fm is the opposing force due to mass
- M is mass
- a is acceleration
- x is displacement

SPRING

Spring is an element, which stores potential energy. If a force is applied on spring K, then it is opposed by an opposing force due to elasticity of spring. This opposing force is proportional to the displacement of the spring. Assume mass and friction are negligible.



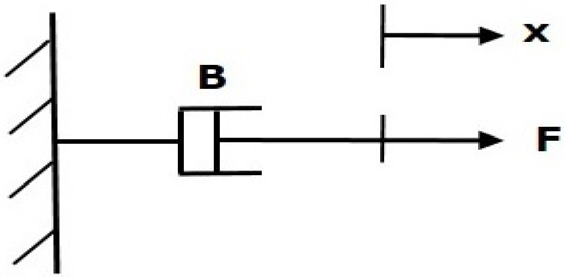
$$F \propto x$$
$$\Rightarrow F_k = Kx$$
$$F = F_k = Kx$$

Where,

- F is the applied force
- F_k is the opposing force due to elasticity of spring
- K is spring constant
- x is displacement

DASHPOT

If a force is applied on dashpot B, then it is opposed by an opposing force due to friction of the dashpot. This opposing force is proportional to the velocity of the body. Assume mass and elasticity are negligible



$$F_b \propto v$$
$$\Rightarrow F_b = Bv = B\mathrm{d}x/\mathrm{d}t$$
$$F = F_b = B\mathrm{d}x/\mathrm{d}t$$

Where,

- Fb is the opposing force due to friction of dashpot
- B is the frictional coefficient
- v is velocity
- x is displacement

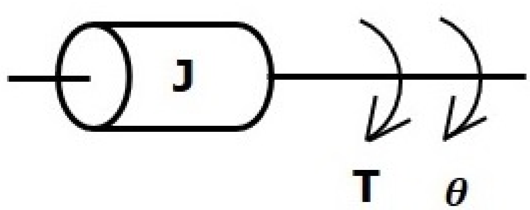
Modeling of Rotational Mechanical Systems

- Rotational mechanical systems move about a fixed axis. These systems mainly consist of three basic elements. Those are moment of inertia, torsional spring and dashpot.
- If a torque is applied to a rotational mechanical system, then it is opposed by opposing torques due to moment of inertia, elasticity and friction of the system. Since the applied torque and the opposing torques are in opposite directions, the algebraic sum of torques acting on the system is zero. Let us now see the torque opposed by these three elements individually.

Moment of Inertia

In translational mechanical system, mass stores kinetic energy. Similarly, in rotational mechanical system, moment of inertia stores kinetic energy.

If a torque is applied on a body having moment of inertia J, then it is opposed by an opposing torque due to the moment of inertia. This opposing torque is proportional to angular acceleration of the body. Assume elasticity and friction are negligible.



$$T_j \propto \alpha$$
$$\Rightarrow T_j = J\alpha = J\frac{d^2\theta}{dt^2}$$
$$T = T_j = J\frac{d^2\theta}{dt^2}$$

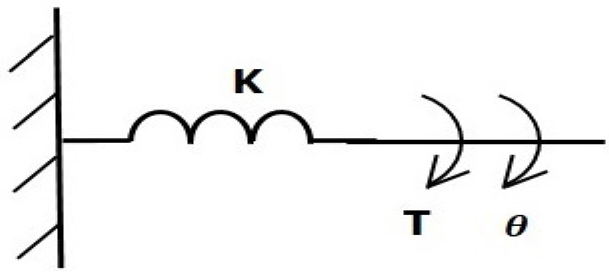
Where,

- T is the applied torque
- Tj is the opposing torque due to moment of inertia
- J is moment of inertia
- α is angular acceleration
- θ is angular displacement

TORSIONAL SPRING

In translational mechanical system, spring stores potential energy. Similarly, in rotational mechanical system, torsional spring stores potential energy.

If a torque is applied on torsional spring K, then it is opposed by an opposing torque due to the elasticity of torsional spring. This opposing torque is proportional to the angular displacement of the torsional spring. Assume that the moment of inertia and friction are negligible.



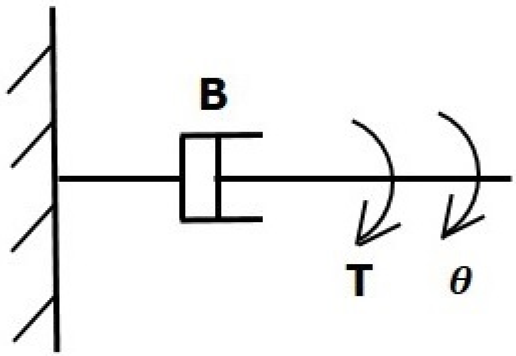
$$T_k \propto \theta$$
$$\Rightarrow T_k = K\theta$$
$$T = T_k = K\theta$$

Where,

- T is the applied torque
- Tk is the opposing torque due to elasticity of torsional spring
- K is the torsional spring constant
- θ is angular displacement

Dashpot

If a torque is applied on dashpot B, then it is opposed by an opposing torque due to the rotational friction of the dashpot. This opposing torque is proportional to the angular velocity of the body. Assume the moment of inertia and elasticity are negligible.



$$T_b \propto \omega$$
$$\Rightarrow T_b = B\omega = B d\theta dt$$
$$T = T_b = B d\theta dt$$

Where,

- T_b is the opposing torque due to the rotational friction of the dashpot
- B is the rotational friction coefficient
- ω is the angular velocity
- θ is the angular displacement

Two systems are said to be analogous to each other if the following two conditions are satisfied.

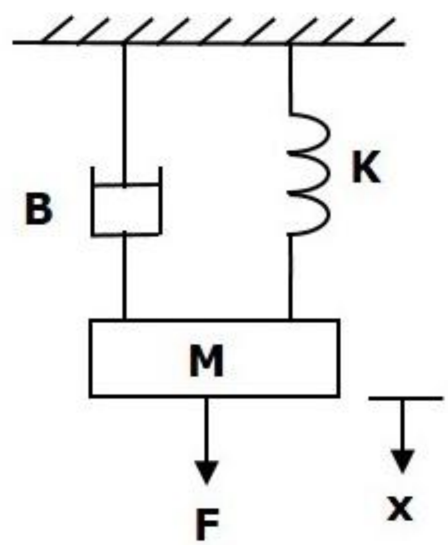
- The two systems are physically different
- Differential equation modelling of these two systems are same

Electrical systems and mechanical systems are two physically different systems. There are two types of electrical analogies of translational mechanical systems. Those are force voltage analogy and force current analogy.

Force Voltage Analogy

In force voltage analogy, the mathematical equations of translational mechanical system are compared with mesh equations of the electrical system.

Consider the following translational mechanical system as shown in the following figure.

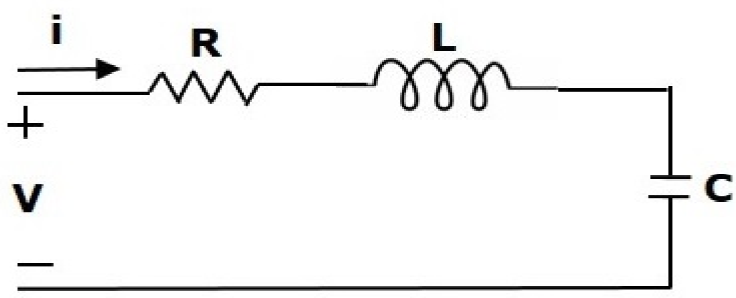


The **force balanced equation** for this system is

$$F = F_m + F_b + F_k$$

$$\Rightarrow F = M(d^2x/dt^2) + B(dx/dt) + K*x$$ **(Equation 1)**

Consider the following electrical system as shown in the following figure. This circuit consists of a resistor, an inductor and a capacitor. All these electrical elements are connected in a series. The input voltage applied to this circuit is V volts and the current flowing through the circuit is i Amps.



Mesh equation for this circuit is

$$V=Ri+L\ didt+1_c\int idt$$ **(Equation 2)**

Substitute, i=dq/dt in Equation 2.

$$V=R\ (dq/dt)+L(d^2q/dt^2)+qC$$

$$\Rightarrow V=L(d^2q/dt^2)+R(dq/dt)+(1_c)q$$ **(Equation 3)**

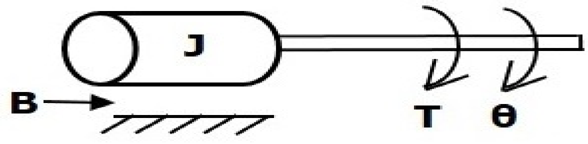
By comparing Equation 1 and Equation 3, we will get the analogous quantities of the translational mechanical system and electrical system. The following table shows these analogous quantities.

Translational Mechanical System	Electrical System
Force(F)	Voltage(V)
Mass(M)	Inductance(L)
Frictional Coefficient(B)	Resistance(R)
Spring Constant(K)	Reciprocal of Capacitance $(1/c)$
Displacement(x)	Charge(q)
Velocity(v)	Current(i)

Similarly, there is torque voltage analogy for rotational mechanical systems. Let us now discuss about this analogy.

Torque Voltage Analogy

In this analogy, the mathematical equations of rotational mechanical system are compared with mesh equations of the electrical system.
Rotational mechanical system is shown in the following figure.



The torque balanced equation is

$T = Tj + Tb + Tk$

$\Rightarrow T = J d^2\theta/dt^2 + B d\theta/dt + k\theta$ **(Equation 4)**

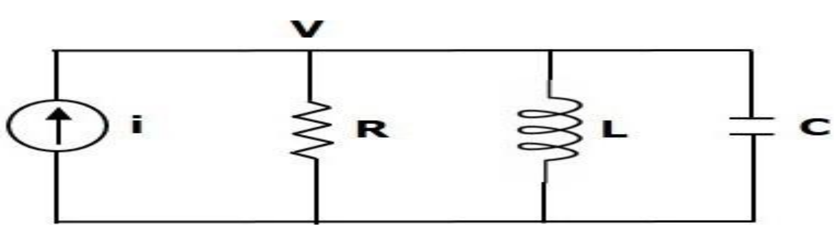
By comparing Equation 4 and Equation 3, we will get the analogous quantities of rotational mechanical system and electrical system. The following table shows these analogous quantities.

Rotational Mechanical System	**Electrical System**
Torque(T)	Voltage(V)
Moment of Inertia(J)	Inductance(L)
Rotational friction coefficient(B)	Resistance(R)
Torsional spring constant(K)	Reciprocal of Capacitance $(1c)$
Angular Displacement(θ)	Charge(q)
Angular Velocity(ω)	Current(i)

Force Current Analogy

In force current analogy, the mathematical equations of the translational mechanical system are compared with the nodal equations of the electrical system.

Consider the following electrical system as shown in the following figure. This circuit consists of current source, resistor, inductor and capacitor. All these electrical elements are connected in parallel.



The nodal equation is

$i = V*R + (1/L)\int V dt + C*dV/dt$ **(Equation 5)**

Substitute, $V = d\Psi/dt$ in Equation 5.

$$i=(1/R)(d\Psi/dt)+(1/L)\Psi+C(d^2\Psi/dt^2)$$

$\Rightarrow i=C(d^2\Psi/dt^2)+(1/R)(d\Psi/dt)+(1/L)\Psi$ **(Equation 6)**

By comparing Equation 1 and Equation 6, we will get the analogous quantities of the translational mechanical system and electrical system. The following table shows these analogous quantities.

Translational Mechanical System	Electrical System
Force(F)	Current(i)
Mass(M)	Capacitance(C)
Frictional coefficient(B)	Reciprocal of Resistance$(1R)$
Spring constant(K)	Reciprocal of Inductance$(1L)$
Displacement(x)	Magnetic Flux(ψ)
Velocity(v)	Voltage(V)

Similarly, there is a torque current analogy for rotational mechanical systems. Let us now discuss this analogy.

Torque Current Analogy

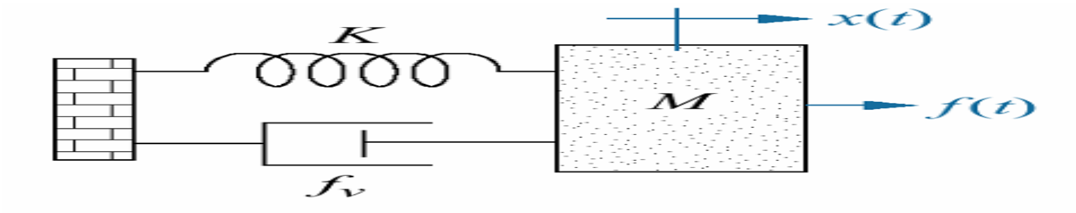
In this analogy, the mathematical equations of the rotational mechanical system are compared with the nodal mesh equations of the electrical system.

By comparing Equation 4 and Equation 6, we will get the analogous quantities of rotational mechanical system and electrical system. The following table shows these analogous quantities.

Rotational Mechanical System	Electrical System
Torque(T)	Current(i)
Moment of inertia(J)	Capacitance(C)
Rotational friction coefficient(B)	Reciprocal of Resistance$(1R)$
Torsional spring constant(K)	Reciprocal of Inductance$(1L)$
Angular displacement(θ)	Magnetic flux(ψ)
Angular velocity(ω)	Voltage(V)

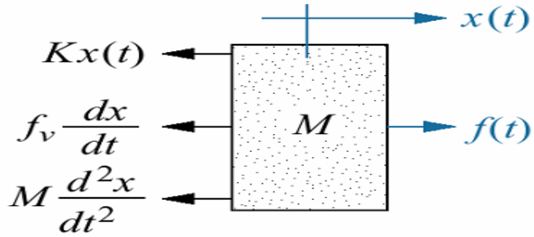
Example:

Find the transfer function X(s) / F(s) for the system given below

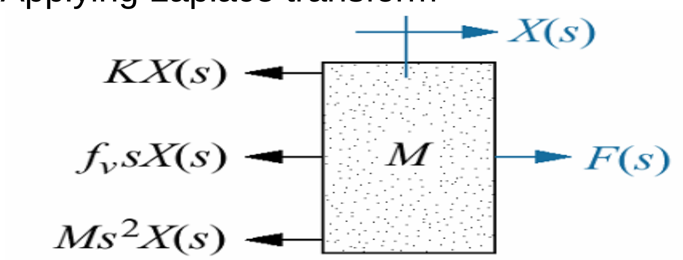


Solution:

Step1: Free-body diagram of mass, spring, and damper system



Applying Laplace transform



We now write the differential equation of motion using Newton's law

$$M \frac{d^2 x(t)}{dt^2} + f_v \frac{dx(t)}{dt} + Kx(t) = f(t)$$

Taking the Laplace transform, assuming zero initial conditions

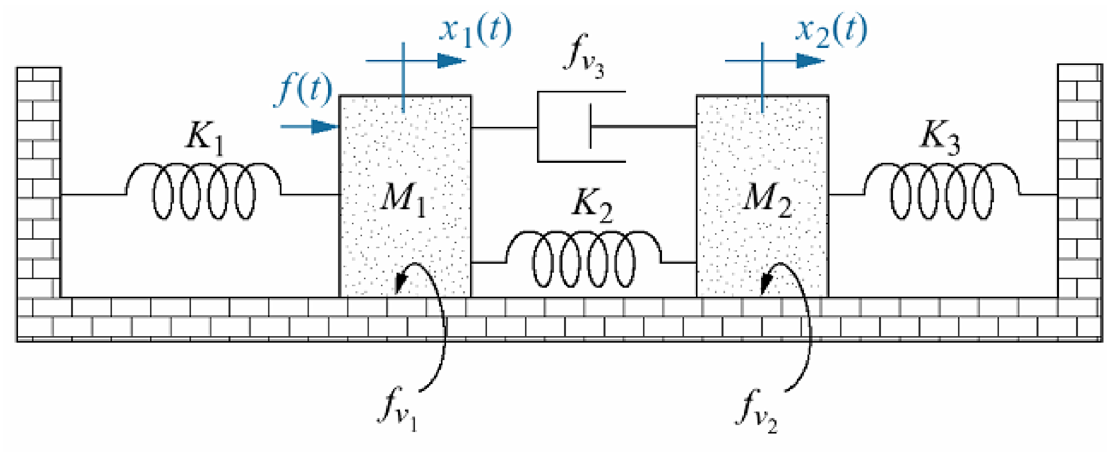
$$Ms^2 X(s) + f_v sX(s) + KX(s) = F(s)$$

$$(Ms^2 + f_v s + K) X(s) = F(s)$$

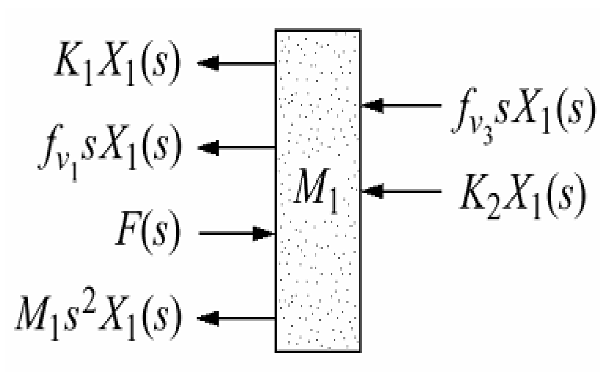
Solving for transfer function yields

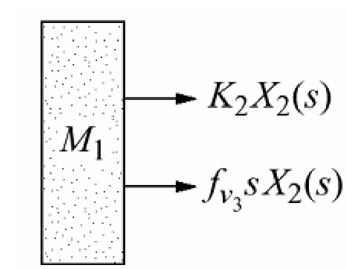
$$G(s) = \frac{X(s)}{F(s)} = \frac{1}{Ms^2 + f_v + K}$$

2. Find the transfer function X(s)/F(s)

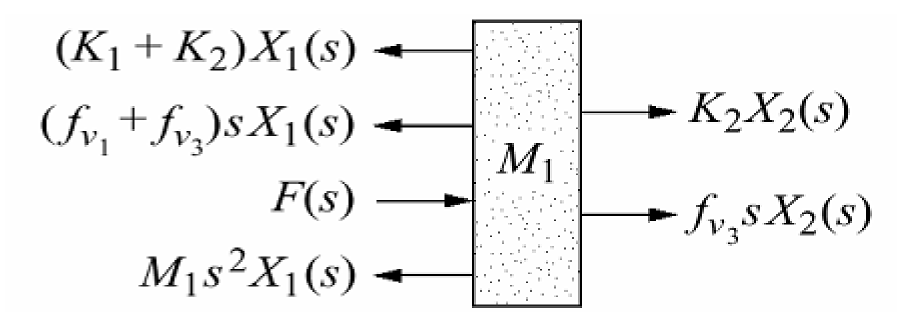


Forces on M1 due only to motion of M





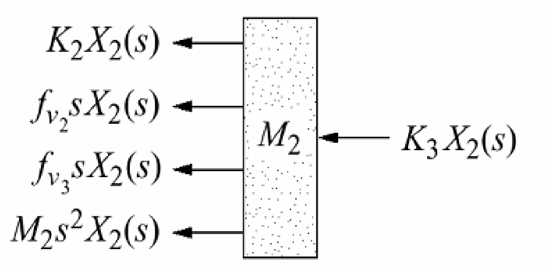
Final force on mass1

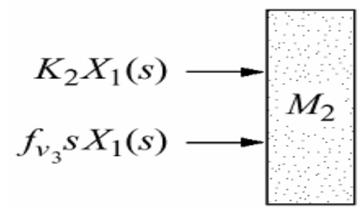


We now write the differential equation of motion using Newton's law

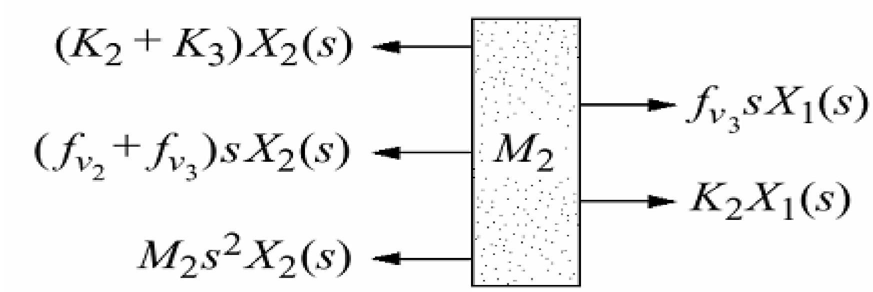
$$[M_1s^2 + (f_{v1}+f_{v3})s + (K_1+K_2)]\, \mathbf{X_1(s)} - (f_{v3}s+K_2)\mathbf{X_2(s)} = F(s)$$

Forces on M_2





Total force on M2

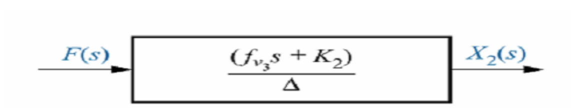


We now write the differential equation of motion using Newton's law

$$-(f_{v3}s + K_2)X_1(s) + [M_2s^2 + (f_{v2}+f_{v3})s + (K_2 + K_3)]X_2(s) = 0$$

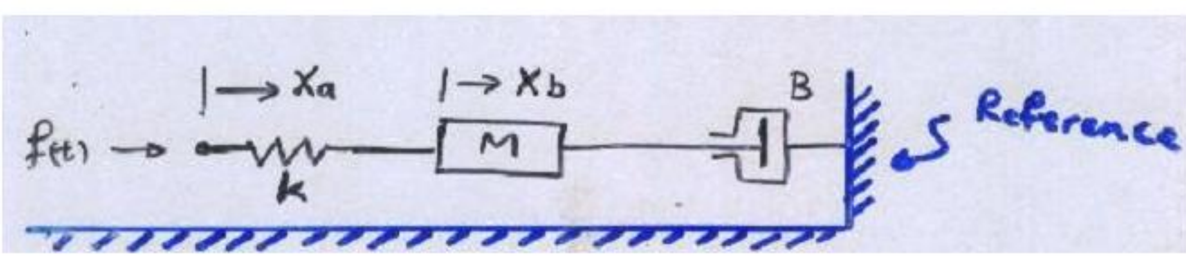
By solving above 2 equations

The transfer function $X_2(s)/F(s)$ is

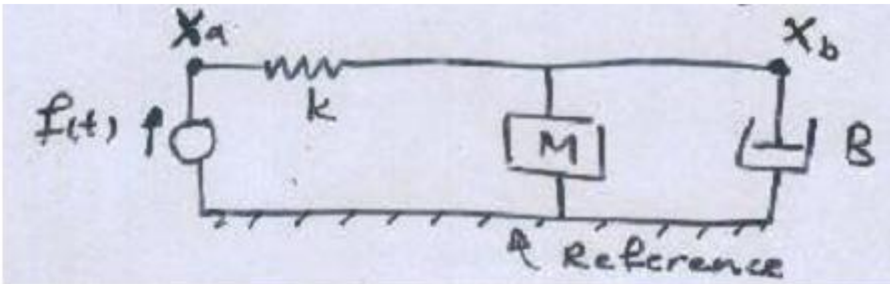


$$\Delta = \begin{vmatrix} [M_1s^2 + (f_{v1}+f_{v3})s + (K_1+K_2)] & -(f_{v3}s+K_2) \\ -(f_{v3}s+K_2) & [M_2s^2 + (f_{v2}+f_{v3})s + (K_2+K_3)] \end{vmatrix}$$

3. Find the T.F. of simple mass-spring-damper mechanical system



To draw the mechanical network, the points xa, xband the reference are located. The complete mechanical network is drawn in fig. below.



For nodes a & b

$$f = f_k = k(x_a - x_b) \qquad (1)$$

$$f_k = f_m + f_B = MD^2x_b + BDx_b \qquad (2) \text{ where } D=d/dt$$

$$or \quad f_k = f_m + f_B = M\ddot{x}_b + BD\,\dot{x}_b$$

It is possible to obtain one equation relating x_a to f, x_b to x_a, or x_b to f by combining equation (1) & (2)

$$k(MD^2 + BD)x_a = (MD^2 + BD + k)f \qquad (3)$$ *check*

$$(MD^2 + BD + k)x_b = kx_a \qquad (4)$$

$$(MD^2 + BD)x_b = f \qquad (5)$$

Let $$G_1 = \frac{x_a}{f} = \frac{MD^2 + BD + k}{k(MD^2 + BD)} \qquad (6)$$

$$G_2 = \frac{x_b}{x_a} = \frac{k}{MD^2 + BD + k} \qquad (7)$$

$$G = \frac{x_b}{f} = \frac{1}{MD^2 + BD} \qquad (8)$$

Or by taking L.T. to equation 3,4,5 with all initial condition equal to zero. The transfer functions are

$$G_1(s) = \frac{x_a(s)}{f(s)} = \frac{Ms^2 + Bs + k}{k(Ms^2 + Bs)} \qquad (9)$$

$$G_2(s) = \frac{x_b(s)}{x_a(s)} = \frac{k}{Ms^2 + Bs + k} \qquad (10)$$

$$G = \frac{x_b(s)}{f(s)} = \frac{1}{Ms^2 + Bs} \qquad (11)$$

4.

Automobile suspension some system of one wheel

M_1= mass of the automobile

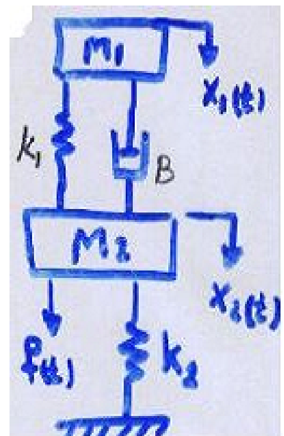
B = the shock absorber

k_1 =the spring

k_2= elestance of the tire

M_2= mass of the wheel

Two independent displacement exist, so we must write Two equations



$$M_1 \frac{d^2x_1}{dt^2} = -B(\frac{dx_1}{dt} - \frac{dx_2}{dt}) - k_1(x_1 - x_2)$$

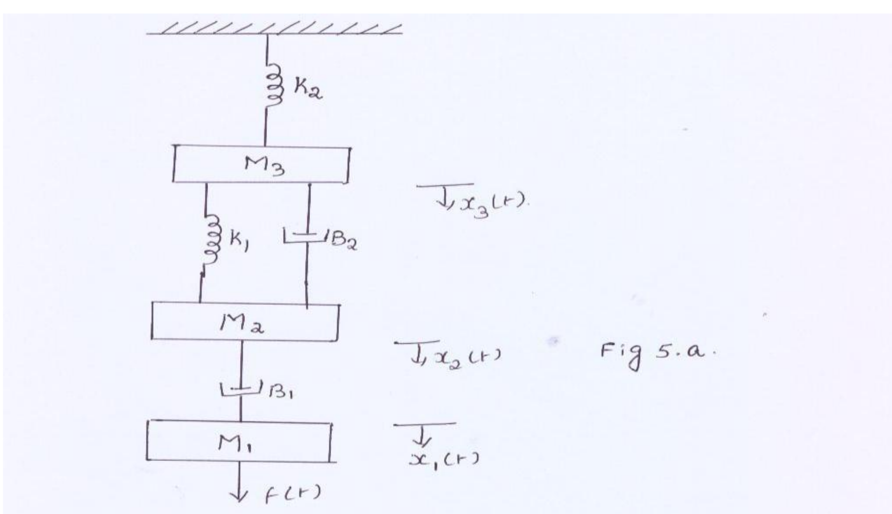
$$M_1 \frac{d^2x_2}{dt^2} = f(t) - B(\frac{dx_2}{dt} - \frac{dx_1}{dt}) - k_1(x_2 - x_1) - k_2x_2$$

$M_1s^2X_1(s) + B(sX_1(s)-sX_2(s))+k_1(X_1(s)-X_2(s)) =0$

$M_2s^2X_2(s) + B(sX_2(s)-sX_1(s))+k_1(X_2(s)-X_1(s))+k_2X_2(s) =F(s)$

$$T(s) = \frac{X_1(s)}{F(s)} = \frac{B_s + k_1}{M_1M_2s^4 + B(M_1 + M_2)s^3 + (K_1M_2 + k_2M_1)s^2 + k_2Bs + k_1k_2}$$

5.Obtain the analogous electrical network for the system shown in fig.5. (AU:Nov./Dec.-2007)



The Mass M1 is under the displacement x1(t).

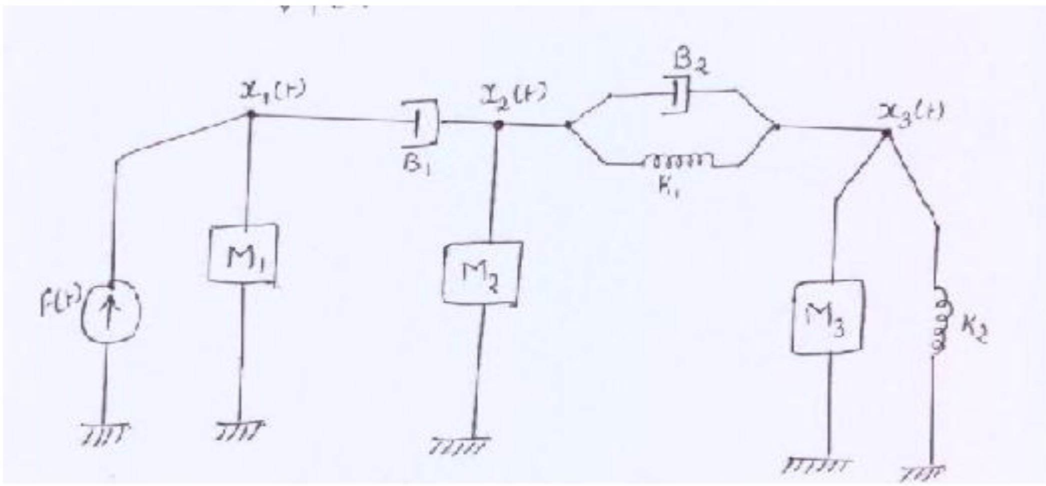
The friction B1 is responsible to change the displacement from x1(t) to x2(t)

The Mass M2 is under the displacement x2(t).

The friction B2 and spring K1 are responsible to change the displacement from x2(t) to x3(t)

The Mass M3 and spring K2 are under the influence of displacement x3(t).

The equivalent Mechanical system is shown in fig.5.a.



The equilibrium equations are

$F(t) = M1(d2x1(t)/dt2) + B1d(x1(t) - x2(t))/dt$ -- (1)

$0 = B1d(x2(t) - x1(t))/dt + M2(d2x2(t)/dt2) + K1(x2(t) - x3(t)) + B2d(x2(t) - x3(t))/dt$ -------- (2)

$0 = K1(x3(t) - x2(t)) + B2d(x3(t) - x2(t))/dt + M3(d2x3(t)/dt2) + K3x3(t)$ --------------------- (3)

Using force- voltage analogy ,

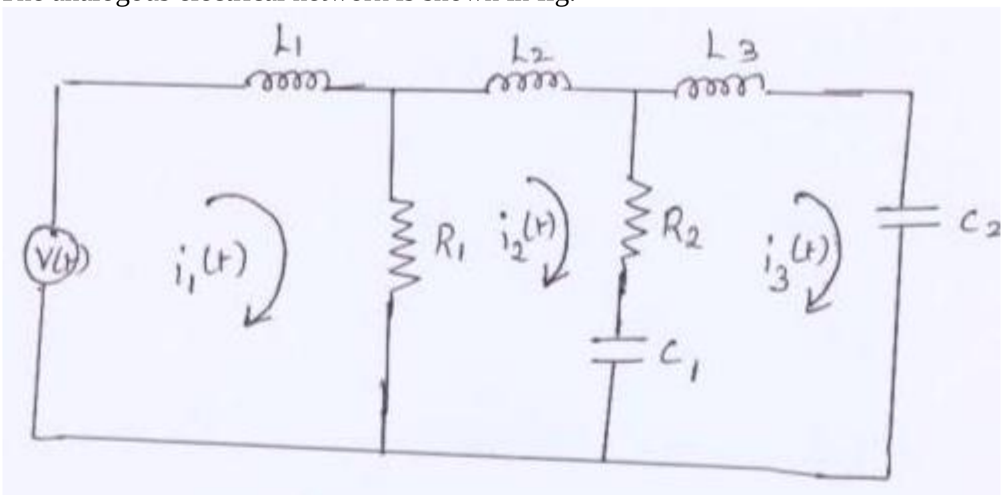
Mass is replaced by inductance, friction or dashpot is replaced by resistance, spring is replaced by reciprocal of capacitance, displacement is replaced by charge. Rate of change of displacement is replaced by current, force is replaced by voltage.

$V(t) = L_1 di_1(t)/dt + R_1 (i_1(t) – i_2(t))$ ------------------ (4)

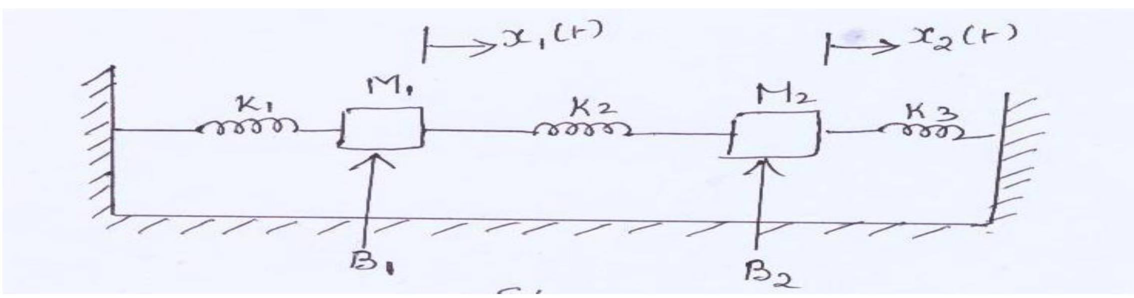
$0 = R_1 (i_2(t) – i_1(t)) + L_2 di_2(t)/dt + 1/C_1 \int (i_2(t) – i_3(t))dt + R_2 (i_2(t) – i_3(t))$------- (5)

$0 = 1/C_1\int(i_3(t) – i_2(t))\, dt + R_2\, (i_3(t) – i_2 +(t)) + L_3 di_3(t)/dt + 1/C_2\int(i_3(t)dt$.------- (6)

The analogous electrical network is shown in fig.

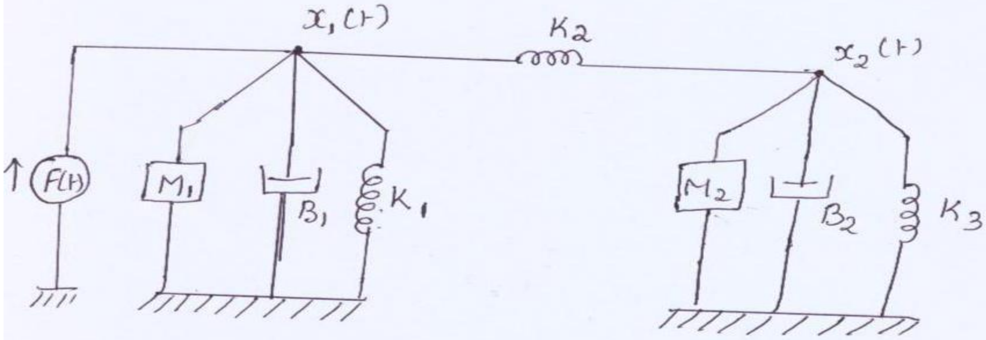


6.Draw the equivalent mechanical system of the system shown in fig.. write the set of equilibrium equations for it and obtain electrical analogous circuits using i) F-V analogy ii)F-I analogy.



As shown in fig.6. M_1,K_1,and B_1 are under the displacement x_1 as K_1 and B_1 are with respect to rigid support. K_2 is between x_1 and x_2 as it is responsible for the change in displacement. While M_2 , K_3 and B_2 are under the displacement x_2. Hence the equivalent mechanical system is as shown in

fig



The equilibrium equations are

$$F(t) = M_1(d^2x_1(t)/ dt^2) + B_1dx_1(t)/dt + K_1x_1(t) + K_2(x_1(t)- x_2(t)) \text{ ------(1)}$$

$$0 = M_2(d^2x_2(t)/ dt^2) + B_2dx_2(t)/ dt + K_2(x_2(t)- x_1(t)) + K_3x_2(t) \text{ ------(2)}$$

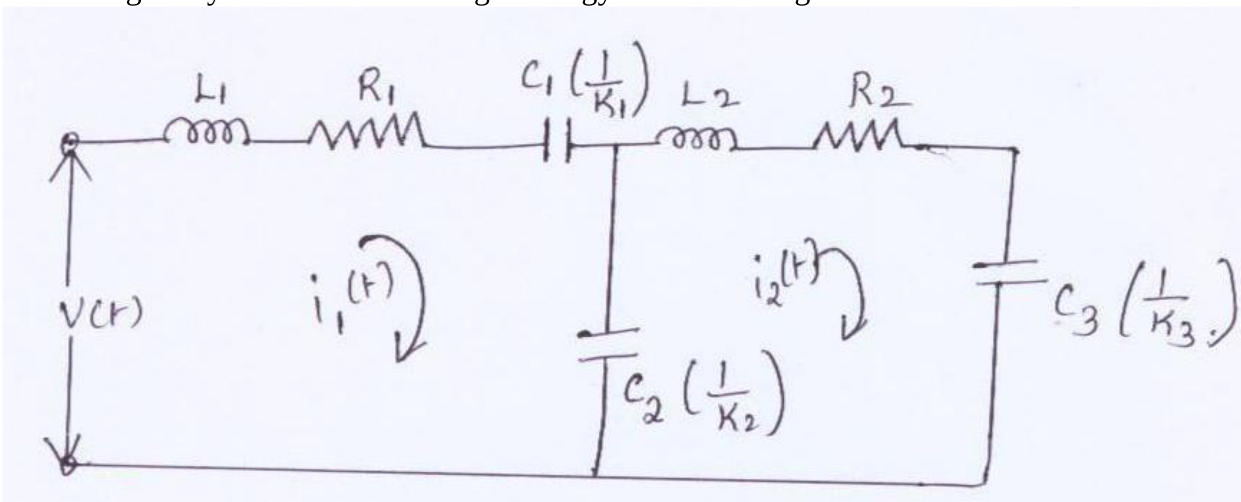
Using force- voltage analogy ,

Mass is replaced by inductance, friction or dashpot is replaced by resistance, spring is replaced by reciprocal of capacitance, displacement is replaced by charge. Rate of change of displacement is replaced by current, force is replaced by voltage.

$$V(t) = L_1di_1(t)/dt + R_1\, i_1(t) + 1/C_1\int(i_1(t)\, dt + 1/C_2\int(i_1(t) – i_2(t))\, dt \text{ ----------(3)}$$

$$0 = L_2di_2(t)/dt + R_1\, i_2(t) +1/C_2\int(i_2(t) – i_1(t))dt + 1/C_3\int(i_2(t)\, dt\text{---4)}$$

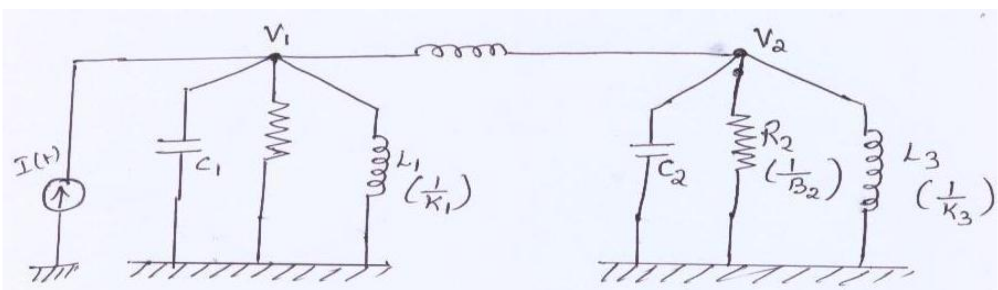
The analogous system for force voltage analogy is shown in fig.6.c.



Using force- current analogy ,

Mass is replaced by capacitance, friction or dashpot is replaced by reciprocal of resistance, spring is replaced by reciprocal of inductance, displacement is replaced by flux. Rate of change of displacement is replaced by voltage, force is replaced by current.

The analogous system for force current analogy is shown in fig



$I(t) = C_1 dV_1(t)/dt + 1/R_1\, V_1(t) + 1/L_1 \int (V_1(t)\, dt + 1/L_2 \int (V_1(t) – V_2(t))\, dt$.. (5)

$0 = C_2 dV_2(t)/dt + 1/R_2\, V_2(t) + 1/L_2 \int (V_2(t) – V_1(t))dt + 1/L_3 \int (V_2(t)\, dt$------6)

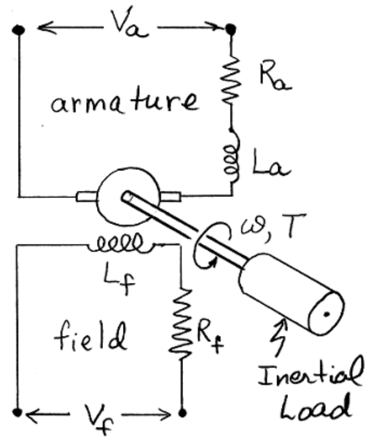
Armature Controlled DC Motor Transfer Functions

In a armature-current controlled DC motor, the field current i_f is held constant, and the armature current is controlled through the armature voltage V_a. In this case, the ***motor torque increases linearly with the armature current***.

$$\boxed{T_m = K_{ma}\, i_a} \quad (1)$$

K_{ma} is a ***constant*** that depends on the chosen motor. The ***transfer function*** from the input armature current to the resulting motor torque is

$$\boxed{\frac{T_m}{I_a}(s) = K_{ma}} \quad (2)$$



DC Motor

The ***voltage/current relationship*** for the armature side of the motor is

$$\boxed{V_a = V_R + V_L + V_b = R_a i_a + L_a\left(di_a/dt\right) + V_b} \quad (3)$$

V_b represents the "***back EMF***" ***induced by the rotation of the armature windings in a magnetic field***. V_b is proportional to the rotational speed ω, i.e. $\boxed{V_b(s) = K_b\omega(s)}$.

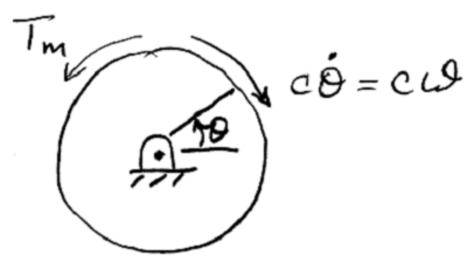
Taking Laplace transforms of Eq. (3) gives

$$\boxed{V_a(s) - V_b(s) = \left(R_a + L_a s\right)I_a(s)} \quad \text{or} \quad \boxed{V_a(s) - K_b\omega(s) = \left(R_a + L_a s\right)I_a(s)} \quad (4)$$

Applying ***Newton's Law*** (by summing moments) for the rotational motion of the motor gives

$$\boxed{\sum M = T_m - c\omega = J\dot{\omega}} \quad \text{(CCW positive)}$$

$$\boxed{J\dot{\omega} + c\omega = T_m} \quad (5)$$

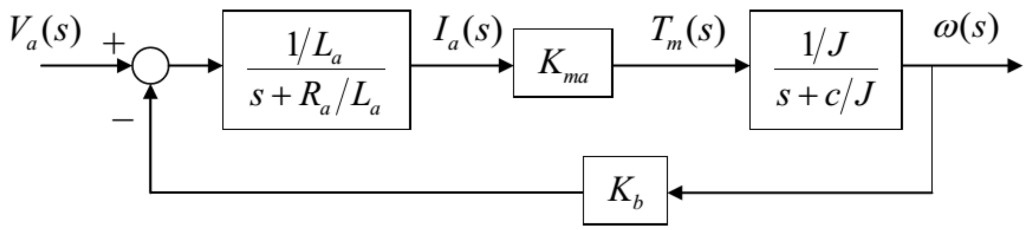


Free Body Diagram of the Inertial Load

Thus, the ***transfer function*** from the input motor torque to rotational speed changes is

$$\boxed{\frac{\omega}{T_m}(s)=\frac{(1/J)}{s+(c/J)}} \qquad (1^{st} \text{ order system}) \qquad (6)$$

Together, Eqs. (2), (4) and (6) can be represented by the ***closed loop block diagram***:



Block diagram reduction gives the transfer function from the input armature voltage to the resulting speed change.

$$\boxed{\frac{\omega}{V_a}(s)=\frac{(K_{ma}/L_aJ)}{(s+R_a/L_a)(s+c/J)+(K_bK_{ma}/L_aJ)}} \qquad (2^{nd} \text{ order system}) \qquad (7)$$

If we assume the ***time constant of the electrical circuit is small*** compared to the time constant of the load dynamics, the transfer function of Eq. (7) may be reduced to a first order transfer function

$$\boxed{\frac{\omega}{V_a}(s)=\frac{K_{ma}/R_aJ}{s+(cR_a+K_bK_{ma})/R_aJ}} \qquad (1^{st} \text{ order system}) \qquad (8)$$

The transfer function from the input armature voltage to the resulting ***angular position*** change is found by multiplying Eqs. (7) and (8) by $1/s$.

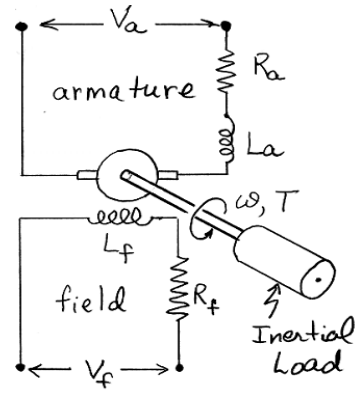
$$\boxed{\frac{\theta}{V_a}(s)=\frac{K_{ma}/R_aJ}{s(s+(cR_a+K_bK_{ma})/R_aJ)}} \qquad (2^{nd} \text{ order system}) \qquad (9)$$

Note that this transfer function also represents a second order differential equation with inertia and damping, but no stiffness (same form as for a hydraulic cylinder!).

Field ContrOlled DC Motor

The figure at the right represents a DC motor attached to an inertial load. The voltages applied to the field and armature sides of the motor are represented by V_f and V_a. The resistances and inductances of the field and armature sides of the motor are represented by R_f, L_f, R_a, and L_a. The torque generated by the motor is proportional to i_f and i_a the currents in the field and armature sides of the motor.

$$\boxed{T_m = K\, i_f\, i_a} \tag{1.1}$$



DC Motor

Field-Current Controlled:

In a field-current controlled motor, the armature current i_a is held constant, and the field current is controlled through the field voltage V_f. In this case, the motor torque increases linearly with the field current. We write

$$T_m = K_{mf}\, i_f$$

By taking Laplace transforms of both sides of this equation gives the transfer function from the input current to the resulting torque.

$$\boxed{\frac{T_m(s)}{I_f(s)} = K_{mf}} \tag{1.2}$$

For the field side of the motor the voltage/current relationship is

$$\begin{aligned} V_f &= V_R + V_L \\ &= R_f i_f + L_f\left(di_f/dt\right) \end{aligned}$$

The transfer function from the input voltage to the resulting current is found by taking Laplace transforms of both sides of this equation.

$$\boxed{\frac{I_f(s)}{V_f(s)} = \frac{\left(1/L_f\right)}{s + \left(R_f/L_f\right)}} \qquad (1^{st} \text{ order system}) \qquad (1.3)$$

The transfer function from the input voltage to the resulting motor torque is found by combining equations (1.2) and (1.3).

$$\boxed{\frac{T_m(s)}{V_f(s)} = \frac{T_m(s)}{I_f(s)}\frac{I_f(s)}{V_f(s)} = \frac{\left(K_{mf}/L_f\right)}{s + \left(R_f/L_f\right)}} \qquad (1^{st} \text{ order system}) \qquad (1.4)$$

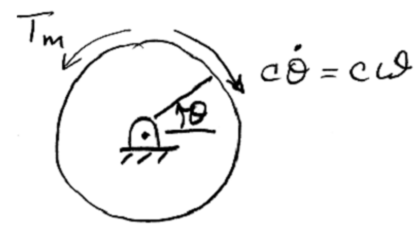
So, a step input in field voltage results in an exponential rise in the motor torque.

An equation that describes the rotational motion of the inertial load is found by summing moments

$$\sum M = T_m - c\omega = J\dot{\omega} \qquad \text{(counterclockwise positive)}$$

or

$$\boxed{J\dot{\omega} + c\omega = T_m}$$



Free Body Diagram of the Inertial Load

Thus, the transfer function from the input motor torque to rotational speed changes is

$$\boxed{\frac{\omega(s)}{T_m(s)} = \frac{\left(1/J\right)}{s + \left(c/J\right)}} \qquad (1^{st} \text{ order system}) \qquad (1.5)$$

Combining equations (1.4) and (1.5) gives the transfer function from the input field voltage to the resulting speed change

$$\boxed{\frac{\omega(s)}{V_f(s)} = \frac{\omega(s)}{T_m(s)}\frac{T_m(s)}{V_f(s)} = \frac{\left(K_{mf}/L_f J\right)}{\left(s + c/J\right)\left(s + R_f/L_f\right)}} \qquad (2^{nd} \text{ order system}) \qquad (1.6)$$

Finally, since $\omega = d\theta/dt$, the transfer function from input field voltage to the resulting rotational position change is

$$\boxed{\frac{\theta(s)}{V_f(s)} = \frac{\theta(s)}{\omega(s)}\frac{\omega(s)}{V_f(s)} = \frac{\left(K_{mf}/L_f J\right)}{s\left(s + c/J\right)\left(s + R_f/L_f\right)}} \qquad (3^{rd} \text{ order system}) \qquad (1.7)$$

Servo Motor

A servo motor is one of the widely used variable speed drives in industrial production and process automation and building technology worldwide. A servo motor is a linear or rotary actuator that provides fast precision position control for closed-loop position control applications. Unlike large industrial motors, a servo motor is not used for continuous energy conversion.

Types of Servo Motors

asically, servo motors are classified into AC and DC servo motors depending upon the nature of supply used for its operation. Brushed permanent magnet DC servo motors are used for simple applications owing to their cost, efficiency and simplicity.

These are best suited for smaller applications. With the advancement of microprocessor and power transistor, AC servo motors are used more often due to their high accuracy control

DC Servo Motors

A DC servo motor is an assembly of four major components, namely a DC motor, a position sensing device, a gear assembly, and a control circuit. The below figure shows the parts that consisting in RC servo motors in which small DC motor is employed for driving the loads at precise speed and position. A DC reference voltage is set to the value corresponding to the desired output. This voltage can be applied by using another potentiometer, control pulse width to voltage converter, or through timers depending on the control circuitry.

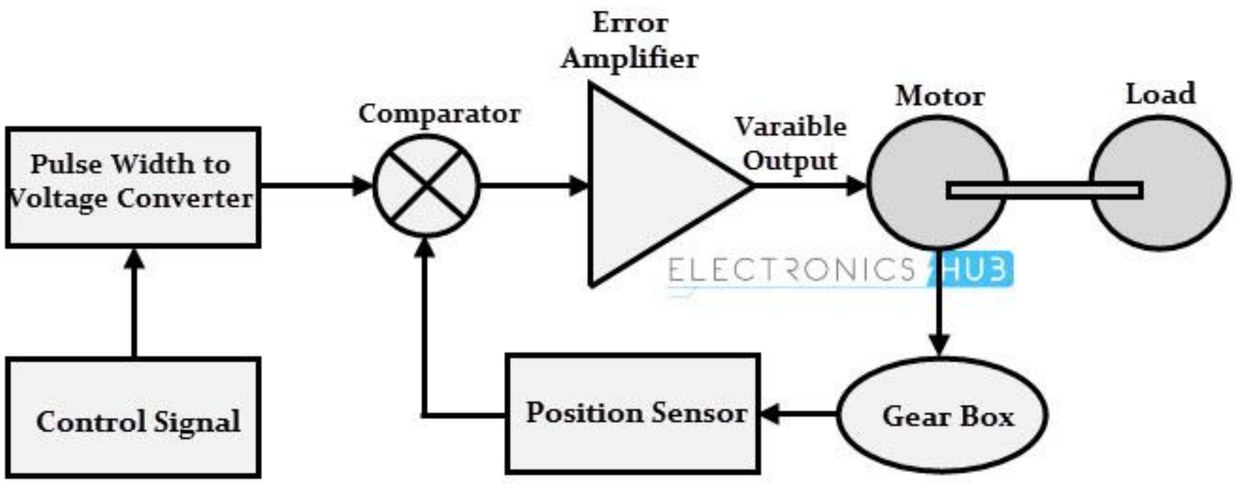
The dial on the potentiometer produces a corresponding voltage which is then applied as one of the inputs to error amplifier.

In some circuits, a control pulse is used to produce DC reference voltage corresponding to desired position or speed of the motor and it is applied to a pulse width to voltage converter.

In this converter, the capacitor starts charging at a constant rate when the pulse high. Then the charge on the capacitor is fed to the buffer amplifier when the pulse is low and this charge is further applied to the error amplifier.

So the length of the pulse decides the voltage applied at the error amplifier as a desired voltage to produce the desired speed or position.

In digital control, microprocessor or microcontroller are used for generating the PWM pluses in terms of duty cycles to produce more accurate control signals.



The feedback signal corresponding to the present position of the load is obtained by using a position sensor. This sensor is normally a potentiometer that produces the voltage corresponding to the absolute angle of the motor shaft through gear mechanism. Then the feedback voltage value is applied at the input of error amplifier (comparator).

The error amplifier is a negative feedback amplifier and it reduces the difference between its inputs. It compares the voltage related to current position of the motor (obtained by potentiometer) with desired voltage related to desired position of the motor (obtained by pulse width to voltage converter), and produces the error either a positive or negative voltage.

This error voltage is applied to the armature of the motor. If the error is more, the more output is applied to the motor armature.

As long as error exists, the amplifier amplifies the error voltage and correspondingly powers the armature. The motor rotates till the error becomes zero. If the error is negative, the armature voltage reverses and hence the armature rotates in the opposite direction.

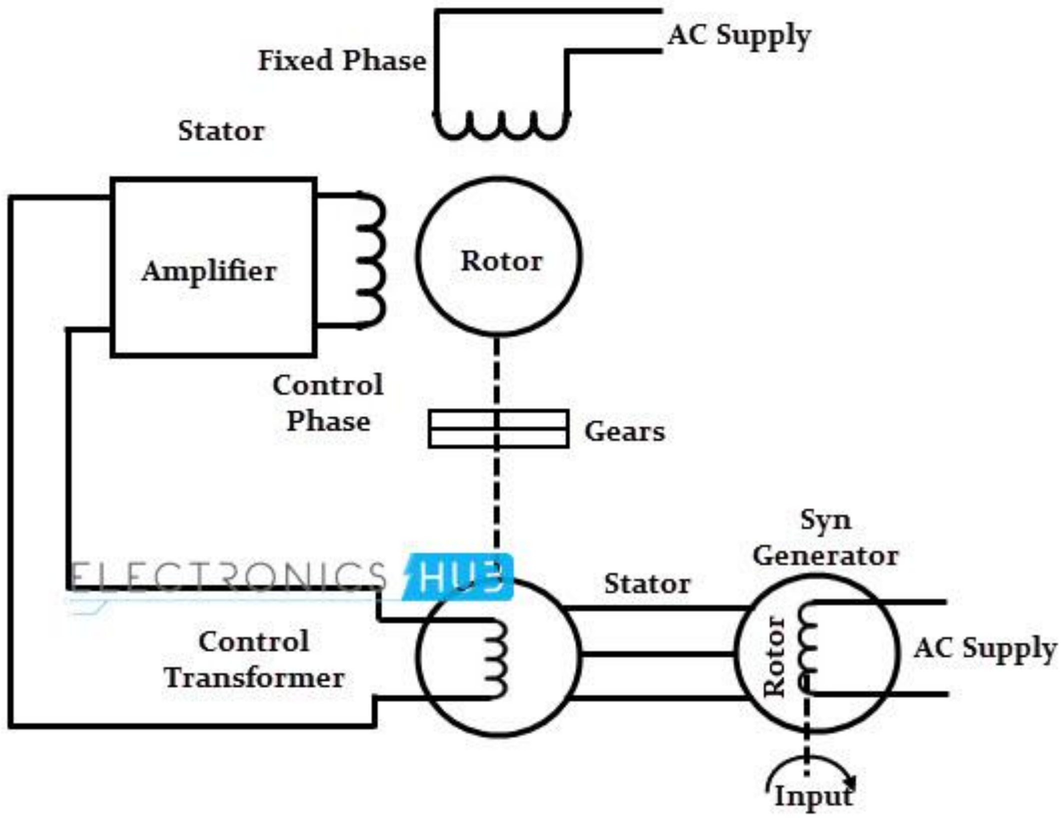
AC Servo Motors

AC servo motors are basically two-phase squirrel cage induction motors and are used for low power applications. Nowadays, three phase squirrel cage induction motors have been modified such that they can be used in high power servo systems.

The main difference between a standard split-phase induction motor and AC motor is that the squirrel cage rotor of a servo motor has made with thinner conducting bars, so that the motor resistance is higher.

Working Principle of AC Servo Motor

The schematic diagram of servo system for AC two-phase induction motor is shown in the figure below. In this, the reference input at which the motor shaft has to maintain at a certain position is given to the rotor of synchro generator as mechanical input theta. This rotor is connected to the electrical input at rated voltage at a fixed frequency.



he three stator terminals of a synchro generator are connected correspondingly to the terminals of control transformer. The angular position of the two-phase motor is transmitted to the rotor of control transformer through gear train arrangement and it represents the control condition alpha.

Initially, there exist a difference between the synchro generator shaft position and control transformer shaft position. This error is reflected as the voltage across the control transformer. This error voltage is applied to the servo amplifier and then to the control phase of the motor.

With the control voltage, the rotor of the motor rotates in required direction till the error becomes zero. This is how the desired shaft position is ensured in AC servo motors.

Alternatively, modern AC servo drives are embedded controllers like PLCs, microprocessors and microcontrollers to achieve variable frequency and variable voltage in order to drive the motor.

SYNCHRO :-

INTRODUCTION

The term synchro is a generic name for a family of inductive devices which works on the principle of a rotating transformer (Induction motor). The trade names for synchronous are Selsyn, Autosyn and Telesyn. Basically they are electro mechanical devices or electromagnetic transducer which produces an output voltage depending upon angular position of the rotor.

A Synchro system is formed by interconnection of the devices called the Synchro Transmitter and the **synchro control transformer**. They are also called as synchro pair. The synchro pair measures and compares two angular displacements and its output voltage is approximately linear with angular difference of the axis of both the shafts. They can be used in the following two ways.

i. To control the angular position of load from a remote place / long distance.

ii. For automatic correction of changes due to disturbance in the angular position of the load.

SYNCHRO TRANSMITTER

The constructional features, electrical circuit and a schematic symbol of **Synchro Transmitter** are shown in figure-2. The two major parts of **Synchro Transmitters** are stator and rotor.The stators identical to the stator of three phase alternator. It is made of laminated silicon steel and slotted on the inner periphery to accommodate a balance three phase winding. The stator winding is concentric type with the axis of the three coil 120° apart. The stator winding is star connected(Y - connection).

The rotor is of dumb bell construction with a single winding. The ends of the rotor winding are terminated on two slip rings. A single phase AC excitation voltage is applied to the rotor through the slip rings.

Working Principles

When the rotor is excited by AC voltage, the rotor current flows, and a magnetic field is produced. The rotor magnetic field induces an emf in the stator coil by transformer action. The effective voltage induced in any stator coil depends upon the angular position of the coils axis with respect to rotor axis.

Constructional Features of **Synchro Transmitter**

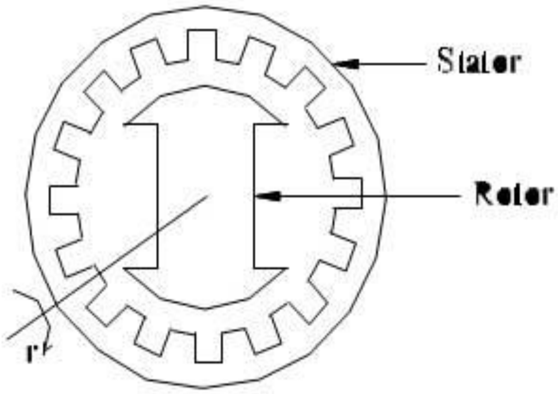


Fig: Constructional Features of Synchro Transmitter

Let er = Instantaneous value of AC voltage applied to rotor.

e ,e ,es1s2 s3 = Instantaneous value of emf induced in stator coils S ,S , S with respect to12 3 neutral respectively.

Er = Maximum value of rotor excitation voltage.

T = Angular frequency of rotor excitation voltage.

K_t = Turns ratio of stator and rotor winding.
K_c = Coupling coefficient.

2 = Angular displacement of rotor with respect to reference.

The instantaneous value of excitation voltage, e = Er sinrTt ---- (1)

Let the rotor rotates in antic lock wise direction. When the rotor rotates by an angle, 2 emfs are induced in stator coils. The frequency of induced emfs is same as that of rotor frequency. The magnitude of induced emfs is proportional to the turn's ratio and coupling coefficient. The turns ratio , K is a constant, but a coupling coefficient, K is a function of rotor angular position.$_t$c
Induced emf in stator coil = K K E since rTt------ (2)

SYNCHRO TRANSMITTER / RECEIVER

Let e be reference vector. With reference to figure 2, when 2 = 0, the flux linkage of coil s i Zero. Hence the flux linkage of coil S is function of cos22 (K = K) Cos c1 2 for coil S). The flux2 linkage of coil S will be maximum after a rotation of 120°in anti-clock wise direction and that3 of S after a rotation of 240°.1

Coupling coefficient, K for coil – S1

Coupling coefficient, K for coil – S2

Coupling coefficient, K for coil – S3

$$e_{S1S2} = e_{S1} - e_{S2} = \sqrt{3}\, KE_r \sin(\theta + 240°) \sin\omega t$$

$$e_{S2S3} = e_{S2} - e_{S3} = \sqrt{3}\, KE_r \sin(\theta + 120°) \sin\omega t$$

$$e_{S3S1} = e_{S3} - e_{S1} = \sqrt{3}\, KE_r \sin\theta \sin\omega t$$

$$e_{S1S2} = e_{S1} - e_{S2} = KE_r \cos(\theta - 240°) \sin\omega t - K E_r \cos\theta \sin\omega t$$

$$= KE_r [\cos\theta \cos 240° + \sin\theta \sin 240° - \cos\theta] \sin\omega t$$

$$= KE_r \left[\cos\theta (-0.5) + \sin\theta \left(-\frac{\sqrt{3}}{2}\right) - \cos\theta\right] \sin\omega t$$

$$= \sqrt{3}\, KE_r \left[\sin\theta \left(-\frac{1}{2}\right) + \cos\theta \left(-\frac{\sqrt{3}}{2}\right)\right] \sin\omega t$$

$$= \sqrt{3}\, KE_r [\sin\theta \cos 240° + \cos\theta \sin 240°] \sin\omega t$$

$$= \sqrt{3}\, KE_r \sin(\theta + 240°) \sin\omega t$$

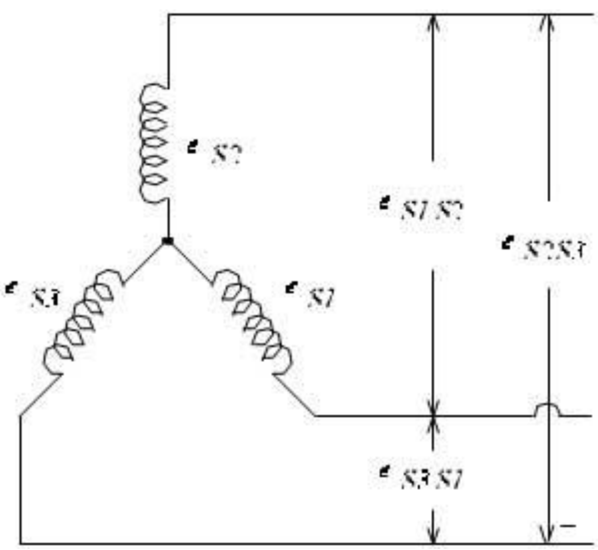


Fig:Induced emf in stator coils

When 2 = 0, from equation 3 we can say that maximum emf is induced in coil S. But from2 equation 8, it is observed that the coil - to coil voltage ES3S1 is zero. This position of the rotor is defined as the electrical zero of the transmitter

$$e_{S2S3} = e_{S2} - e_{S3} = K\,E_r \cos\theta \sin\omega t - KE_r \cos(\theta - 120^\circ) \sin\omega t$$

$$e_{S3S1} = e_{S2} - e_{S3} = KE_r \cos(\theta - 120^\circ) \sin\omega t - K\,E_r \cos(\theta - 240^\circ) \sin \omega t$$

$$= KE_r \left[\cos\theta - \cos\theta \cos 120^\circ - Sin\ \theta\ Sin\ 120^\circ \right] \sin\omega t$$

$$= \sqrt{3}\ KE_r \left[\sin\theta \cos 120^\circ + \cos\theta\ Sin\ 120^\circ \right] \sin\omega t$$

$$= \sqrt{3}\ KE_r \left[\sin\theta \left(-\frac{1}{2}\right) + \cos\theta \left(\frac{\sqrt{3}}{2}\right) \right] \sin\omega t$$

$$= \sqrt{3}\ KE_r \sin(\theta + 120^\circ) \sin\omega t$$

$$= KE_r \left[\cos\theta \cos 120^\circ + Sin\ \theta\ Sin 120^\circ - \cos\theta \cos 240^\circ - Sin\ \theta\ Sin 240^\circ \right] \sin\omega t$$

$$= KE_r \left[\cos\theta\,(-0.5) + \sin\theta \left(\frac{\sqrt{3}}{2}\right) - \cos\theta\,(-0.5) - \sin\theta \left(-\frac{\sqrt{3}}{2}\right) \right] \sin\omega t$$

$$= KE_r \left[\cos\theta - \cos\theta\,(-0.5) - \sin\theta \left(\frac{\sqrt{3}}{2}\right) \right] \sin\omega t$$

$$= \sqrt{3}\ KE_r \sin\theta \sin\omega t$$

angular position of its rotor shaft and the output is a set of three stator coil-to-coil voltages. By measuring and identifying the set of voltages at the stator terminals, it is possible to identify the angular position of the rotor. [A device called synchro / digital converter is available to measure the stator voltages and to calculate the angular measure and then display the direction and angle of rotation of the rotor].

SYNCHRO CONTROL TRANSFORMER

Construction

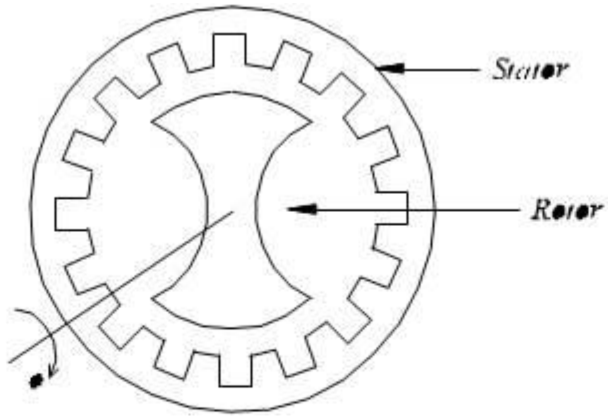


Fig:Constructional Features

The constructional features of **synchro control transformer** are similar to that of **Synchro Transmitter**, except the shape of rotor. The rotor of the control transformer is made cylindrical so that the air gap is practically uniform. This feature of the control transformer minimizes the changes in the rotor impedance with the rotation of the shaft. The constructional features, electrical circuit and a schematic symbol of control transformer are shown in figure 4.

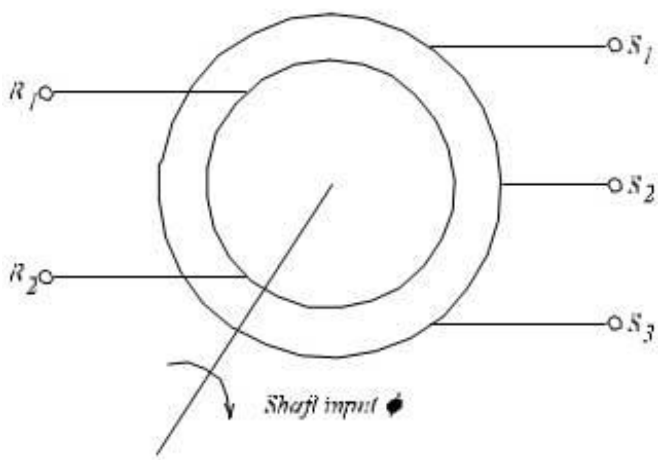


Fig:Schematic Symbol of **synchro control transformer**

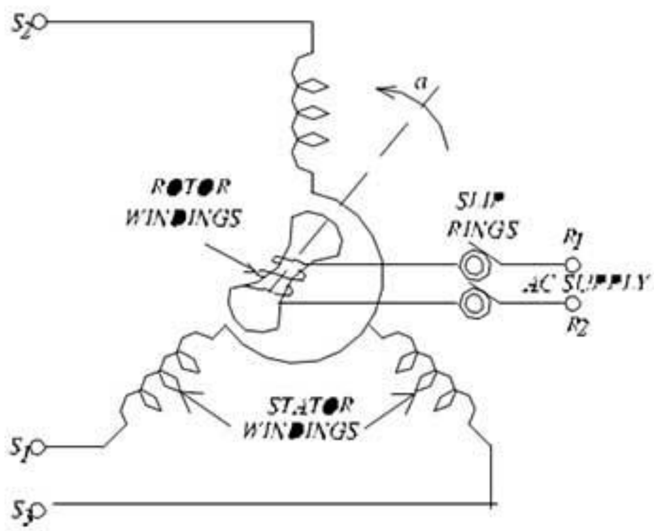


Fig: Electrical Circuit of **synchro control transformer**

Working

The generated emf of the **Synchro Transmitter** is applied as input to the stator coils of control transformer. The rotor shaft is connected to the load whose position has to be maintained at the desired value. Depending on the current position of the rotor and the applied emf on the stator, an emf is induced on the rotor winding. This emf can be measured and used to drive a motor so that the position of the load is corrected.