UNIT-3

SIGNAL TRANSMISSION THROUGH LINEAR SYSTEMS

System: A system is defined as set of rules that associates an o/p time function to every i/p time function.

(or)

A system is an interconnection of elements which produces expected o/p for available i/p.

i/p excitation $f(t)$ —— $\boxed{\text{system}}$ —— o/p Response $r(t)$

$$f(t) \longrightarrow r(t)$$

→ System is an mathematical operator which maps i/p into o/p

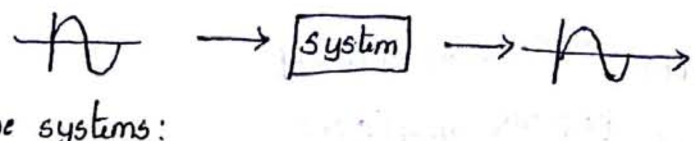
classification of system
↓
- continuous time system
- Discrete time system.

continuous time system ↓

1. Static & Dynamic systems
2. Linear & Non-Linear
3. Time invariant & Time variant
4. Linear TIV & LTIV
5. Stable system
6. Casual & non-causal systems.

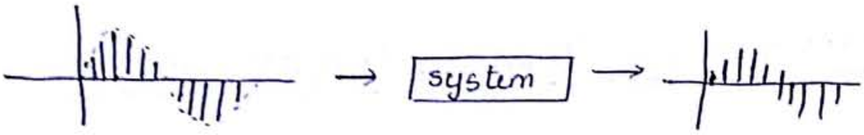
(i) continuous time systems

→ A continuous time system operates on a continuous time i/p signal to produce a continuous time o/p signal



(ii) Discrete time systems:

A discrete time system operates on a discrete time i/p signal to produce a discrete time o/p signal.



classifications :

(i) static and Dynamic Systems:

→ A static system or system is said to be static if its o/p at any instant depends only on present values of i/p.

Ex: $y(t) = ax(t)$ (ii) $y(t) = a^{r} x(t)$

at $t=0$ $y(0) = ax(0)$ at $t=0$ $y(0) = a^{r} x(0)$

at $t=1$ $y(1) = ax(1)$ at $t=1$ $y(1) = a^{r} x(1)$

→ A system is said to be dynamic if its o/p depends on present & past values of i/p.

Ex: $y(t) = x(t-1) + x(t-2) + x(t)$

at $t=2$

$$y(2) = x(2-1) + x(2-2) + x(2) = \underbrace{x(1) + x(0)}_{\text{past}} + \underbrace{x(2)}_{\text{present}}$$

(ii) Linear and Non Linear Systems :

→ A system is said to be linear if its satisfies the superposition principle.

→ It states that the response of the system to a weighted sum of signals be equal to the corresponding weighted sum of o/p's of the system to each of the individual i/p signal.

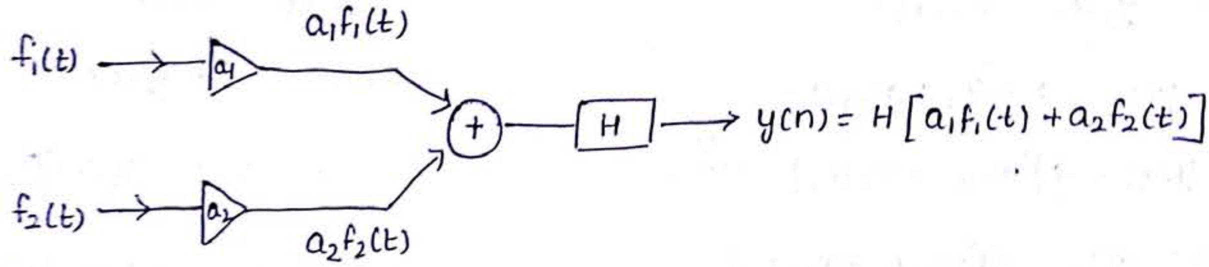
$$H[a_1 f_1(t) + a_2 f_2(t)] = a_1 H[f_1(t)] + a_2 H[f_2(t)]$$

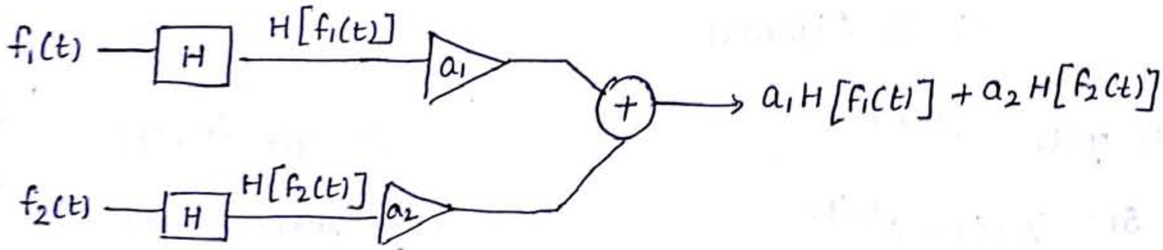
where a_1, a_2 are weighted constants.

$a_1 f_1(t) \xrightarrow{\text{Response}} a_1 H[f_1(t)]$

$a_2 f_2(t) \xrightarrow{\text{Response}} a_2 H[f_2(t)]$

$$H[a_1 f_1(t) + a_2 f_2(t)] \longrightarrow a_1 H[f_1(t)] + a_2 H[f_2(t)]$$





Block diagram.

→ Any system which does not obey the above principle is called as non-linear systems.

check for Linearity:

Procedure:

1. Apply different i/p's separately and get the o/p.
2. Apply different i/p's simultaneously and get the output.
3. If both outputs are same it is linear otherwise non-linear.

Ex:

(i) $y(t) = 4 \sin t \, x(t)$

step 1: $y_1(t) = 4 \sin t \, x_1(t)$

$y_2(t) = 4 \sin t \, x_2(t)$

$y_1(t) + y_2(t) = 4 \sin t \, [x_1(t) + x_2(t)]$

step 2: $y(t) = 4 \sin t \, [x_1(t) + x_2(t)]$

$S_1 = S_2$

(2) $y(t) = ax(t)$

Sol $S_1: y_1(t) = a\,x_1(t)$

$y_2(t) = a\,x_2(t)$

$y(t) = a\,x_1(t) + a\,x_2(t)$

$y(t) = a[x_1(t) + x_2(t)]$

$S2: \; y(t) = a[x_1(t) + x_2(t)]$

$S_1 = S_2$ (Linear)

③ $y(t) = x^2(t)$

Sol $S1: y_1(t) = x_1^2(t)$

$y_2(t) = x_2^2(t)$

$y(t) = x_1^2(t) + x_2^2(t)$

$S2: \; y(t) = [x_1(t) + x_2(t)]^2 \rightarrow$

$S_1 \neq S_2$ (Non-Linear)

(4) $y(t) = e^{x(t)}$

Sol $S1: \; y_1(t) = e^{x_1(t)}$

$y_2(t) = e^{x_2(t)}$

$y(t) = e^{x_1(t)} + e^{x_2(t)}$

$y(t) \doteq e^{[x_1(t) + x_2(t)]}$

$S2: \; e^{x_1(t)} \cdot e^{x_2(t)}$

$S_1 \neq S_2$ (Non-Linear)

⑤ $y(t) = t\,x(t)$

Sol $S1: \; y_1(t) = t\,x_1(t)$

$y_2(t) = t\,x_2(t)$

$y(t) = t[x_1(t) + x_2(t)]$

$S2: \; y(t) = t[x_1(t) + x_2(t)]$

$S_1 = S_2$ (Linear)

⑪ $y(t) = x(t^2)$

Sol $S1: y_1(t) = x_1(t^2)$

$y_2(t) = x_2(t^2)$

$y(t) = x_1(t^2) + x_2(t^2)$

$S2: y(t) = x_1(t^2) + x_2(t^2)$

$S_1 = S_2$ (Linear)

⑥ $y(t) = x(t - t_0)$

Sol $S1: \; y_1(t) = x_1(t - t_0)$

$y_2(t) = x_2(t - t_0)$

$y(t) = x_1(t - t_0) + x_2(t - t_0)$

$S2: \; y(t) = x_1(t - t_0) + x_2(t - t_0)$

$S_1 = S_2$ (Linear

⑦ $y(t) = 3x(t+3)$

Sol $S1: \; y_1(t) = 3x_1(t+3)$

$y_2(t) = 3x_2(t+3)$

$y(t) = 3[x_1(t+3) + x_2(t+3)]$

$S2: \; 3[x_1(t+3) + x_2(t+3)]$

$S_1 = S_2$ (Linear)

(8) $y(t) = Ax(t) + B$

Sol $y_1(t) = Ax_1(t) + B$

$y_2(t) = Ax_2(t) + B$

$y(t) = A[x_1(t) + x_2(t)] + 2B$

$y(t) = A[x_1(t) + x_2(t)] + B$

$S_1 \neq S_2$ = Nonlinear

⑧ $y(t) = x(t+1)e^{-t}$

Sol $S1: y_1(t) = x_1(t+1)e^{-t}$; $y_2(t) = x_2(t+1)e^{-t}$

$y(t) = e^{-t}[x_1(t+1) + x_2(t+1)]$

$S2: \; y(t) = e^{-t}[x_1(t+1) + x_2(t+1)]$

$S_1 = S_2$ (Linear)

⑫ $y(t) = 4x(t) + 2\dfrac{dx(t)}{dt} \rightarrow$ Linear.

⑨ $y(t) = \cos[x(t)]$

Sol $S1: y_1(t) = \cos[x_1(t)]$; $y_2(t) = \cos[x_2(t)]$

$y(t) = \cos[x_1(t)] + \cos[x_2(t)]$

$S2: \; \cos[x_1(t) + x_2(t)]$

$S_1 \neq S_2$ (Non-Linear)

⑩ $y(t) = k\Delta x(t)$ where $\Delta x(t) = [x(t+1) - x(t)]$

Sol $S1: \; y_1(t) = k[x_1(t+1) - x_1(t)]$; $y_2(t) = k\Delta x_2(t)$

$y(t) = [x_1(t+1) - x_1(t) + x_2(t+1) - x_2(t)].k$

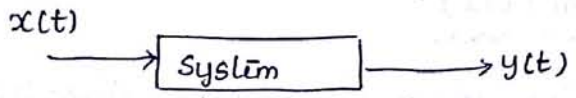
$S2: \; y(t) = k[x_1(t+1) - x_1(t) + x_2(t+1) - x_2(t)]$

$S_1 = S_2$ (Linear)

Time Variant And Time Invariant Systems

→ A system is said to be time invariant if the system does not depend on time i.e system delay is not function of time.

Ex:



$x(t) \longrightarrow y(t)$

$x(t-t_0) \longrightarrow y(t-t_0)$

→ A time shift t_0 in the input results in the same amount of time shift in the o/p but the waveshape does not change.

i.e the i/p and/ to o/p characteristics does not change with time.

→ Any system which doesnot obey the above principle is called as time varying system.

→ An electrical system is said to be time invariant if its component values (R, L, C) does not change with time.

Check for time Invariant:

1. Shift the i/p only and get the o/p.
2. Shift the entire system and get the o/p.
3. If both steps are identical for o/p then it is time invariant system.

Ex!

(1) $y(t) = 4x(t)$

Sol $S_1: y(t) = 4x(t-1)$

$S_2: y(t-1) = 4x(t-1)$ $\Big] \rightarrow S_1 = S_2$ (TIV)

(2) $y(t) = 4t\,x(t)$

Sol $S_1: y(t) = 4t\,x(t-1)$

$S_2: y(t-1) = 4(t-1)(x(t-1))$ $\Big] \rightarrow S_1 \neq S_2$ (TV)

(3) $y(t) = a x(t)$

Sol $S_1: y(t) = a x(t-1)$

$S_2\; y(t-1) = a x(t-1)$ $\Big] \rightarrow S_1 = S_2$ (TIV)

(4) $y(t) = a x(t) + b$

Sol $S_1: y(t) = a x(t-1) + b$

$S_2: y(t-1) = a x(t-1) + b$ $\Big]$ $S_1 = S_2$ (TIV)

(5) $y(t) = 5t\,[x(t)]^2$

Sol $S_1: y(t) = 5t\,[x(t-1)]^2$

$S_2: y(t-1) = 5(t-1)[x(t-1)]^2$ $\Big] \rightarrow S_1 \neq S_2$ (TV)

(6) $y(t) = x(t+1)e^{-t}$

Sol $S_1: y(t) = x(t+1-1)\,e^{-t} = x(t)e^{-t}$

$S_2: y(t-1) = x(t+1-1)e^{-(t-1)}$

$= x(t)\cdot e^{-t}\,e^{-1}$ → constant

$S_1 = S_2$ (TIV)

1 ⑦ $y(t) = x(t+3)$

Sol $S_1: y(t) = x(t+3-1)$

$S_2: y(t-1) = x(t+3-1) = x(t+2)$] $\rightarrow S_1 = S_2$ (TIV)

⑧ $y(t) = x^2(t)$

Sol $S_1: y(t) = x^2(t-1)$

$S_2: y(t-1) = x^2(t-1)$

$\therefore S_1 = S_2$ (TIV)

⑨ $y(t) = e^{x(t)}$

Sol $S_1: y(t) = e^{x(t-1)}$

$S_2: y(t-1) = e^{x(t-1)}$

$S_1 = S_2$ (TIV)

Linear Time Invariant System (LTI):

→ Any system which obeys the linearity and time invariant property is called as LTI system.

Linear Time Variant System (LTV):

→ Any system which obeys the linearity and does not obey time invariant property is called LTV system.

Ex: $y(t) = a x(t)$

Linearity : $y_1(t) = a x_1(t)$; $y_2(t) = a x_2(t)$

$y(t) = a x_1(t) + a x_2(t)$

$y(t) = a[x_1(t) + x_2(t)]$

$S_2: y(t) = a[x_1(t) + x_2(t)]$.

$\therefore S_1 = S_2$.

T.I :

$y(t) = a x(t)$

$S_1: y(t) = a x(t-1)$

$S_2: y(t-1) = a x(t-1)$] $\rightarrow S_1 = S_2$ (TIV)

$\therefore$ It is a linear time invariant system (LTI)

||ly :

(2) $y(t) = t x(t) \rightarrow$ LTI

(3) $y(t) = a x(t) + b \rightarrow$ NLTI

(4) $y(t) = a x^2(t) \rightarrow$ NLTI

(5) $y(t) = e^{x(t)} \rightarrow$ NLTI

(6) $y(t) = x(t - t_0) \rightarrow$ LTI

Stable System:

→ System is absolutely integrable

$$\int_{-\infty}^{\infty} |f(t)|\, dt < \infty$$

Causal And Non Causal Systems:

→ A system is said to be causal if o/p $y(t_0)$ depends only on the values of i/p $x(t)$ at $t < t_0$ {present, i/p, past i/ps, past o/ps} { $x(t)=0$, for $t<0$; noncausal $t \leq 0$, or $t \leq 0$ and $t>0$ — $x(t) \neq 0$ for $t<0$ }

Ex: $y(t) = 4x(t-1)$

$y(2) = 4x(2-1) \Rightarrow 4x(2)$

$y(t) = 4x(t-1) + x(t)$

$y(2) = 4x(1) + x(2)$

→ A system is said to be non-causal if the o/p depends on future values of i/p i.e future i/ps & o/ps.

Ex: $y(t) = 4x(t+1)$

$y(2) = 4x(3)$

Examples whether it is causal & Non causal:

(i) $y(t) = K[x(t+1) - x(t)]$

$y(0) = K[x(1) - x(0)]$ → Noncausal

(2) $y(t) = 3x(t+3)$

$y(0) = 3x(3)$ → Non causal

(3) $y(t) = (t+3)x(t-3)$

$y(0) = (0+3)x(0-3)$

$= 3x(-3)$ → causal

(4) $y(t) = x(t) + 3x(t+4)$

when $t=0$, $y(0) = x(0) + 3x(4)$

when $t=1$, $y(1) = x(1) + 3x(5)$

So here response at $t=0$, $y(0)$ depends on the present i/p & future i/p

hence system is noncausal.

(5) $y(t) = x(t^2)$

$t=-1$, $y(-1) = x(1)$ → future

$t=0$, $y(0) = x(0)$ → present

$t=1$, $y(1) = x(1)$ → present

$t=2$, $y(2) = x(4)$ → future

Noncausal.

Except at $t=0$, $t=1$, the response of any value of t depends on future i/p.

(6) $y(t) = x(2t)$ → Noncausal

(7) $y(t) = x(t) - x(t-1)$ → causal

(8) $y(t) = x(t) + \int_0^t x(\lambda)\, d\lambda$

$= x(t) + z(\lambda)]_0^t \Rightarrow$ Causal.

At $t=0$, $t=1$, $t=2$

Impulse Response:

The response of a system for an impulse i/p is called a impulse response of the system and it is denoted by $h(t)$

$$\delta(t) \longrightarrow \boxed{\text{system}} \longrightarrow h(t)$$

$$\delta(t) \rightarrow h(t)$$

→ Every system is characterised by its impulse response.

$$\underset{f(t)}{i/p} \longrightarrow \boxed{h(t)} \longrightarrow \underset{r(t)}{o/p}$$

Response of a System for an arbitary i/p:

Response of $\delta(t) \rightarrow h(t)$

$$\delta(t-t_0) \rightarrow h(t-t_0)$$

$$\delta(t)+\delta(t-t_0) = h(t)+h(t-t_0)$$

The response of a system for a given i/p $f(t)$ is determined by using superposition principle.

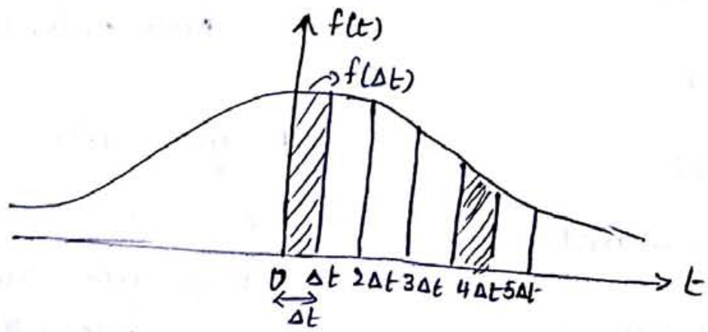
step 1: Resolve the i/p function interms of impulse functions.

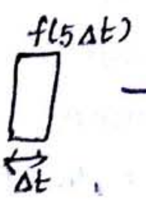
step 2: Determine individually the response of LTI system for impulse function.

step 3: Find the sum of individual responses which will become the overall response $r(t)$.

Representation of a function f(t) in terms of an impulse function:

Here the function $f(t)$ is a impulse train function.





→ area $f(5\Delta t) \times \Delta t$

$$f(t) = \sum_{n=-\infty}^{\infty} f(n\Delta t)\,\Delta t \cdot \delta(t-n\Delta t)$$

$$f(t) = \underset{\Delta t \to 0}{Lt} \sum_{n=-\infty}^{\infty} f(n\Delta t)\, \Delta t\, \delta(t-n\Delta t)$$

The rectangular of width Δt & height $f(n\Delta t)$ and area under the rectangles is $\Delta t \cdot f(n\Delta t)$ and this n^{th} element approached a delta function of strength $f(n\Delta t)\,\Delta t$ located at $t = n\Delta t$. and this delta function is represented as $f(n\Delta t)\,\Delta t\, \delta(t-n\Delta t)$

$$\boxed{f(t) = \underset{\Delta t \to 0}{Lt} \sum_{n=-\infty}^{\infty} f(n\Delta t) \cdot \Delta t \cdot \delta(t-n\Delta t)}$$

As $\Delta t \to 0$, the n^{th} element may be considered.

2) Determination of $r(t)$ for the input $f(t)$:

Let $h(t)$ be the impulse response of the system.

$$\delta(t) \longrightarrow \boxed{\text{system}} \longrightarrow h(t)$$

then $\delta(t) \longrightarrow h(t)$

$\delta(t-n\Delta t) \longrightarrow h(t-A\Delta t)$

$f(n\Delta t)\,\delta(t-n\Delta t) \longrightarrow f(n\Delta t) \cdot h(t-n\Delta t)$

$f(n\Delta t) \cdot \Delta t\, \delta(t-n\Delta t) \longrightarrow f(n\Delta t) \cdot \Delta t\, h(t-n\Delta t)$

$$\underset{\Delta t \to 0}{Lt} \sum_{n=-\infty}^{\infty} f(n\Delta t) \cdot \Delta t\, \delta(t-n\Delta t) \longrightarrow \underset{\Delta t \to 0}{Lt} \sum_{n=-\infty}^{\infty} f(n\Delta t) \cdot \Delta t\, h(t-n\Delta t).$$

$$f(t) \longrightarrow \boxed{\text{system}} \longrightarrow r(t)$$

$$r(t) = \underset{\Delta t \to 0}{Lt} \sum_{n=-\infty}^{\infty} f(n\Delta t) \cdot \Delta t\, h(t-n\Delta t)$$

$\Delta t \to 0$ means summation becomes integration.

$$r(t) = \int_{-\infty}^{\infty} f(\tau) \cdot h(t-\tau)\, d\tau$$

$$r(t) = f(t) \otimes h(t)$$

$$f(t) \longrightarrow \boxed{h(t)} \longrightarrow r(t) \to f(t) \otimes h(t).$$

i.e if the response of a system is known for unit impulse fn. then response to any other function f(t) can be obtained from the above eqn.

→ An unit impulse function is called as a Test function and it is used to characterise a system.

$$r(t) = f(t) \otimes h(t)$$

In frequency domain $r(t) \xleftrightarrow{FT} R(W)$

$$f(t) \xleftrightarrow{FT} F(W)$$

$$h(t) \xleftrightarrow{FT} H(W)$$

Using convolution property

$$f(t) \otimes h(t) = F(\omega) \cdot H(\omega)$$

$$R(\omega) = F(\omega) \cdot H(\omega)$$

$$\boxed{H(\omega) = \frac{R(\omega)}{F(\omega)}}$$

→ When $F(\omega) = 1$; i.e i/p is unit impulse $H(\omega) = R(\omega)$

→ Transfer function $H(\omega)$ of a system is defined as the transform of the response of a system where the i/p is unit impulse function.

$$H(\omega) = \underbrace{|H(\omega)|}_{\text{Amplitude response of the system}} e^{\overbrace{j\theta(\omega)}^{\text{phase response of the system}}}$$

$$\boxed{\begin{aligned} Ln[H(\omega)] &= Ln[|H(\omega)|] + j\theta(\omega) \\ &= \underbrace{L(\omega)}_{\text{Gain of the system}} + \underbrace{j\theta(\omega)}_{\text{phase shift introduced by system}} \end{aligned}}$$

Note: An impulse function contains all frequencies in equal amount so we can use it as a test function.

$$\underbrace{H(\omega)}_{F.T[h(t)]} = \frac{R(\omega)}{F(\omega)} \rightarrow \text{Transfer fn of LTI system.}$$

$$h(t) = I.F.T[H(\omega)]$$

FILTER CHARACTERISTICS OF LINEAR SYSTEMS :

IDEAL LOW PASS FILTERS :

→ It transmits all the signals below certain frequency 'B' Hz without any distortion.

→ The range of frequencies from 0Hz to 'B' Hz is called passband of lowpass filter.

→ The frequency 'B' Hz is called cut-off frequency of the ideal lowpass filter.

→ The transfer function of ideal low pass filter can be written as

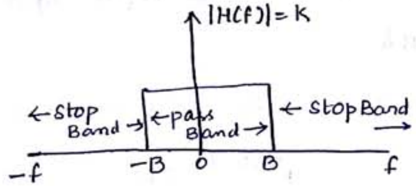
$$H(f) = K e^{-j2\pi f t_0} \quad ; \; -B \leq f < B$$
$$= 0 \quad ; \; |f| > B.$$

K = amplitude is assumed to be unity.

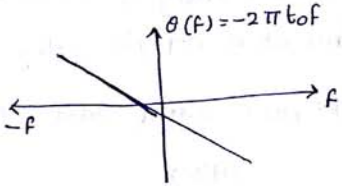
→ By K=1 in above eqn

$$H(f) = e^{-j2\pi f t_0} \quad ; \; -B \leq f < B$$
$$= 0 \quad ; \; |f| > B$$

→ By inverse fourier transform, h(t) can be obtained for ideal LPF



a) magnitude response.



(b) phase response

$$h(t) = \int_{-B}^{B} e^{-j2\pi f t_0} \cdot e^{j2\pi f t} \, df$$

$$= \int_{-B}^{B} \left[e^{j2\pi f (t-t_0)} \right] df = \frac{1}{j2\pi(t-t_0)} \left[e^{j2\pi f(t-t_0)} \right]_{-B}^{B}$$

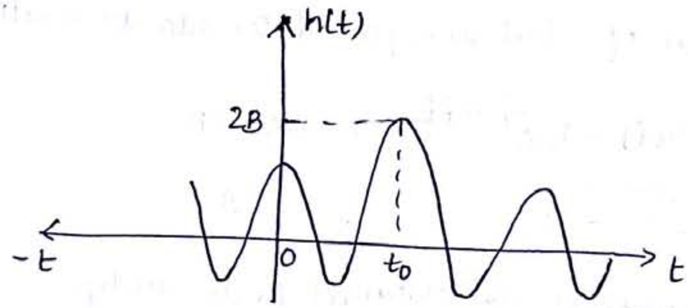
$$= \frac{1}{j2\pi(t-t_0)} \left[e^{j2\pi B(t-t_0)} - e^{-j2\pi B(t-t_0)} \right]$$

$$= \frac{1}{\pi(t-t_0)}\left[\frac{e^{j2\pi B(t-t_0)} - e^{-j2\pi B(t-t_0)}}{2j}\right]$$

$$= \frac{1}{\pi(t-t_0)} \sin\left[2\pi B(t-t_0)\right]$$

$$h(t) = 2B\left(\frac{\sin[2\pi B(t-t_0)]}{2\pi B(t-t_0)}\right) = 2B\,\mathrm{sinc}\left[2B(t-t_0)\right]$$

Response



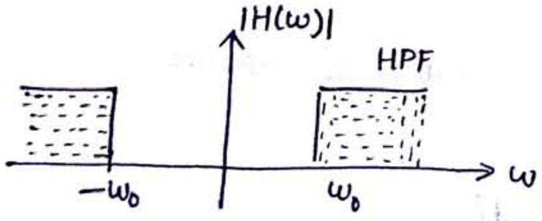
→ Figure shows that impulse response exists for negative values of 't'. But actually unit impulse is applied at t=0 always.

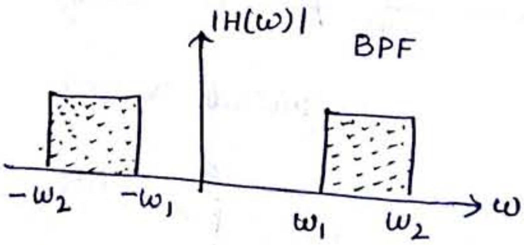
→ Practically it is impossible to implement such a system.

OTHER IDEAL FILTERS SUCH AS HPF, BPF etc.,

→ In realizability of ideal LPF its response begins before input is applied and hence it is not physically realizable.

→ $||^{ly}$ HPF, BPF ideal have frequency response as shown in figure





→ These have sharp transition in frequency response.

→ All ideal filters are physically not realizable since their impulse response is non-causal.

INTRODUCTION FOR FILTER CHARACTERISTICS:

$$f(t) \longrightarrow \boxed{h(t)} \longrightarrow r(t)$$

$$F(\omega) \qquad H(\omega) \qquad R(\omega)$$

$$R(\omega) = F(\omega).H(\omega)$$

→ The spectrum of o/p is $F(\omega).H(\omega)$ i.e the system acts as a kind of filter to various frequency components.

→ Some frequency components are boosted in strength and some are attenuated and some remain unaffected.

→ $||^{ly}$ each freq. component undergoes a different amount of phase shift i.e the modification is carried out according to $H(\omega)$.

↳ acts as waiting fn for two different frequencies.

DISTORTIONLESS TRANSMISSION THROUGH SYSTEM:

→ It means output signal is an exact replica of the i/p signal.

$$i/p \longrightarrow \boxed{\text{System}} \longrightarrow o/p$$

→ The difference between i/p and o/p of such system is that

1. Amplitude of the o/p signal may increase or decrease by some factor w.r. to i/p.
2. The o/p sgl may be delayed in time w.r. to i/p sgl because of system delay.

→ o/p sgl $y(t)$ can be written in terms of i/p $x(t)$ as

$$y(t) = k\,x(t-t_0)$$

k ↓ constant Represents change in amplitude

t_0 ↳ time delay in transmission of signal through a system.

By taking fourier transform

$$Y(f) = F[y(t)] = F\{k\,x(t-t_0)\}$$

From time shifting property of FT

$$Y(f) = k\,X(f)\,e^{-j2\pi f t_0}$$

Transfer fn $H(f) = \dfrac{Y(f)}{X(f)}$

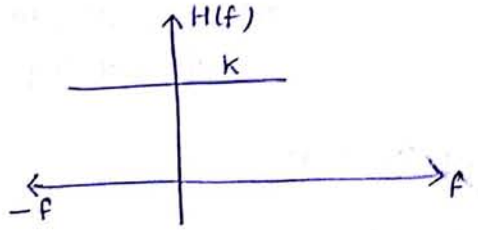
$$\therefore \quad H(f) = \frac{Y(f)}{X(f)} = k \cdot e^{-j2\pi f t_0}.$$

Magnitude of transfer fn independent of frequency.

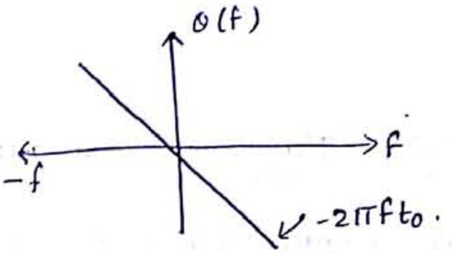
→ Transfer function has constant amplitude at all frequencies. The phase shift is

$$\theta(f) = -2\pi f t_0$$
$$= (-2\pi t_0) f.$$

→ phase shift is linearly proportional to frequency.



(a) Amplitude Spectrum



(b) phase spectrum passing through origin

→ By considering simple example

Let there be signal in time domain as

$$x(t) = \cos(2\pi f t)$$

Let the o/p sgl be same in amplitude but shifted in time by t_0 sec

$$y(t) = \cos[2\pi f(t - t_0)]$$

$$\therefore \; y(t) = \cos(2\pi f t - 2\pi f t_0) = \cos(2\pi f t - \theta(f))$$

$\therefore$ phase shift of $y(t)$ is

$$\theta(f) = -2\pi f t_0.$$

which is proportional to frequency 'f'.

Two types:

(i) Amplitude distortion

(ii) phase distortion.

AMPLITUDE DISTORTION

→ This distortion occurs when $|H(\omega)|$ is not constant over frequency band of interest and the frequency components present in i/p sgl are transmitted with different gain and attenuation.

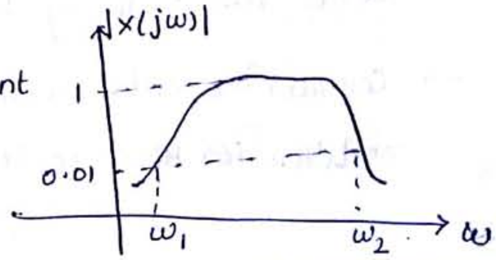
PHASE DISTORTION:

→ This distortion occurs when phase of $H(\omega)$ is not linearly changing with time and different frequency components in i/p are subjected to different time delays during transmission.

SIGNAL BANDWIDTH: The band of frequencies that contains most of signal energy is called B.W of signal denoted by fm.

→ It is the range of significant signal frequencies which are present in the signal.

→ observe in the waveform $x(t)$ has significant frequencies from ω_1 to ω_2.

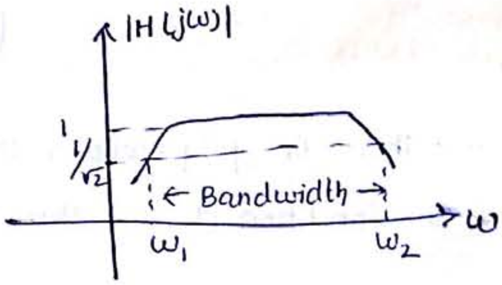


→ The B.W of this signal is $\omega_2 - \omega_1$

→ All the physically obtained signals have limited bandwidth.

SYSTEM BANDWIDTH

→ The B.W of a system is defined as range of frequencies over which $|H(\omega)|$ remains within $1/\sqrt{2}$ times of its mid-band value. for distortionless transmission the system must have infinite B.W but physical system are limited to finite BW.



→ So a system with finite BW can provide distortionless transmission for a band limited signal if $|H(\omega)|$ remains constant over BW of the signal.

→ The range of frequencies for which magnitude $|H(j\Omega)|$ of the systems remains within $\frac{1}{\sqrt{2}}$ of its maximum value

CASUAL SYSTEMS:

→ A system is said to be causal if $h(t)=0$ if $t<0$

$h(t-t_0)=0$; $t<t_0$

i.e if i/p is zero for $t<t_0$, then o/p is also zero for $t<t_0$.

→ Any system which does not obey the above rule is non-causal system.

→ If two i/p to a causal system are equal upto some time 't_0' then corresponding o/p must be equal upto that time instant.

POLY-WIENER CRITERION

→ This gives the condition for causality in frequency domain (or) in other words the frequency domain equivalent of causal system i.e $H(\omega)$.

→ Consider a system with transfer function $H(\omega)$, the necessary and sufficient condition for $H(\omega)$ to be transfer function of causal fn is

$$\int_{-\infty}^{\infty} \frac{|\ln(H(j\omega))|}{1+\omega^2} d\omega < \infty \rightarrow ①$$

provided $|H(j\omega)|$ is square integral.

$$\int_{-\infty}^{\infty} |H(j\omega)|^2 d\omega < \infty \rightarrow ②$$

→ This is poly-wiener criteria. If condition ② is not satisfied then the condition ① is neither necessary nor sufficient.

PHYSICAL REALIZABILITY:

→ A system is said to be physically realizable if it obeys the causal condition.

i.e $h(t)=0$ for $t<0$.

Ex: $H(\omega) = \frac{1}{1+j\omega}$

$h(t) = e^{-t} u(t)$

$= 0$ for $t<0$

So the above system for transfer fn is realizable in freq. domain.

$$\int_{-\infty}^{\infty} \frac{|\ln|H(\omega)||}{H(\omega)^2} d\omega < \infty$$

→ The frequency domain statements can be interpreted as $|H(\omega)|$ of a physically realizable system may be zero for some discrete frequency but it can never be zero for a finite band of frequencies.

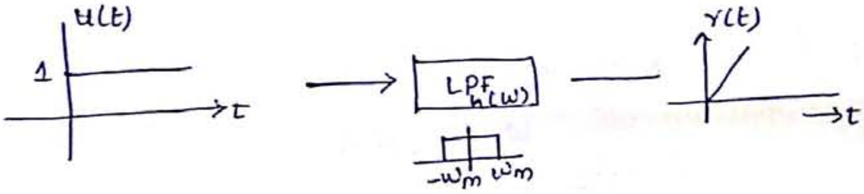
→ $H(\omega)$ for a realisable system cannot decay faster than a function of exponential order.

Ex: A system with T.F $e^{-\omega}$ is realisable where as $e^{-\omega^2}$ is not as it decay faster

RELATIONSHIP BETWEEN RISE TIME AND BANDWIDTH:

→ If a unit step fn $u(t)$ is applied to an ideal LPF, the o/p will show a gradual rise instead of a sharp rise in the i/p.

→ The rise time (t_r) is the time required by the response to reach its final value from initial value.



Transfer function of ideal low pass filter is

$$H(\omega) = |H(\omega)|\, e^{j\theta(\omega)}$$

$$= G(\omega)\, e^{-j\omega t_0}$$

↓ Rectangular pulse with magnitude K.
for $-B < f \leq B$ i.e $-\omega_m \leq \omega \leq \omega_m$ where $\omega_m = 2\pi B$.

and $\theta(\omega) = -2\pi f t_0 = -\omega t_0$.

→ Fourier transform of unit step fn $u(t)$

$$FT\{u(t)\} \Rightarrow U(\omega) = \pi\delta(\omega) + \frac{1}{j\omega}$$

→ Fourier transform of response $R(\omega)$, input and $H(\omega)$ related as

$$R(\omega) = \left[\pi\delta(\omega) + \frac{1}{j\omega}\right] H(\omega) = \pi\delta(\omega).H(\omega) + \frac{1}{j\omega} H(\omega).$$

$\delta(\omega)$ exists only for $\omega=0$ and $H(\omega)\big|_{\omega=0}=1$

$$R(\omega)=\pi\delta(\omega)+\frac{1}{j\omega}H(\omega).$$

By taking IFT for above eqn

$$r(t)=IFt[R(\omega)]=IFT\left\{\pi\delta(\omega)+\frac{1}{j\omega}H(\omega)\right\}$$

$$=IFT\left\{\pi\delta(\omega)+\frac{1}{j\omega}G(\omega)e^{-j\omega t_0}\right\} \quad (\because H(\omega)=G(\omega)e^{-j\omega t_0})$$

Inverse fourier transform of $\pi\delta(\omega)$ is $\frac{1}{2}$.

$$\left\{\begin{array}{l}\because 1\longrightarrow 2\pi\delta(\omega)\\ 1/2\longleftarrow \pi\delta(\omega)\end{array}\right\}$$

$$r(t)=\frac{1}{2}+IFT\left\{\frac{1}{j\omega}G(\omega)e^{-j\omega t_0}\right\}$$

$$=\frac{1}{2}+\frac{1}{2\pi}\int_{-\infty}^{\infty}\frac{1}{j\omega}G(\omega)e^{-j\omega t_0}\cdot e^{j\omega t}d\omega$$

we know $G(\omega)=1$ for $-\omega_m\leq\omega\leq\omega_m$

$$=\frac{1}{2}+\frac{1}{2\pi}\int_{-\omega_m}^{\omega_m}\frac{e^{j\omega(t-t_0)}}{j\omega}d\omega$$

$$=\frac{1}{2}+\frac{1}{2\pi}\int_{-\omega_m}^{\omega_m}\frac{\cos\omega(t-t_0)+j\sin\omega(t-t_0)}{j\omega}d\omega$$

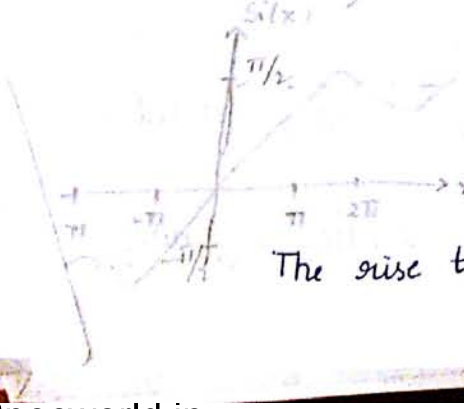
$$=\frac{1}{2}+\frac{1}{2\pi}\underbrace{\int_{-\omega_m}^{\omega_m}\frac{\cos\omega(t-t_0)}{j\omega}d\omega}_{=\text{zero for its odd term}}+\frac{1}{2\pi}\underbrace{\int_{-\omega_m}^{\omega_m}\frac{\sin\omega(t-t_0)}{\omega}d\omega}_{\text{even}}.$$

$Si(x)$ is an odd fn

$Si(-x)=-Si(x)$

(ii) $Si(0)=0$

(iii) $Si(\infty)=\frac{\pi}{2}$; $Si(-\infty)=-\frac{\pi}{2}$



$$r(t)=\frac{1}{2}+\frac{1}{2\pi}\times 2\int_{0}^{\omega_m}\frac{\sin\omega(t-t_0)}{\omega}d\omega=\frac{1}{2}+\frac{1}{\pi}\int_{0}^{\omega_m}\frac{\sin\omega(t-t_0)}{\omega}d\omega$$

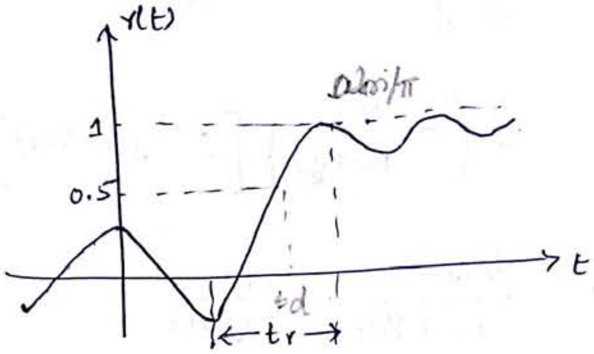
$$=\frac{1}{2}+\frac{1}{\pi}\left[Si\ \omega(t-t_0)\right]_0^{\omega_m}$$

$$=\frac{1}{2}+\frac{1}{\pi}Si\ \omega_m(t-t_0)\longrightarrow \text{sine integral}$$

$$\rightarrow \frac{dr(t)}{dt}\Big|_{t=t_0}=\frac{1}{\pi}\cos[\omega_m(t-t_0)].\ \omega_m$$

The rise time is given as $t_r=\underbrace{\frac{2\pi}{\omega_m}}_{\text{cut off frequency of LPF}}=\frac{1}{B}$

$$\frac{1}{t_r}=\frac{\omega_m}{\pi}\Rightarrow t_r=\frac{\omega_m}{\pi}$$



$w_c \to \infty$

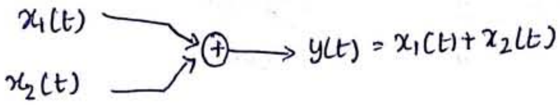
$y(t) = 1$

$w_c \to -\infty$

$y(t) = 0$

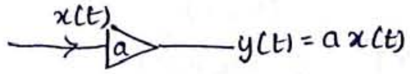
Note : { Elements of block diagram)

① Adder :



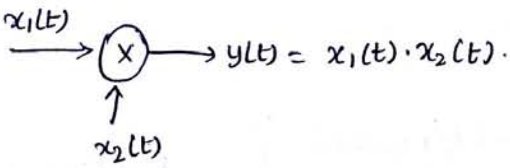
which performs the addition of two signal sequences to form sum

② constant multiplier :



It represents applying a scale factor on i/p $x(t)$.

③ Signal multiplier :



The multiplication of two signal to form product sequence.

PROBLEMS :

① The impulse response of continuous time system is given as

$$h(t) = \frac{1}{RC}\, e^{-t/RC} \cdot u(t)$$

Determine the frequency response & plot the magnitude phase plots .

Sol Take FT

$$H(\omega) = \int_{-\infty}^{\infty} h(t) \cdot e^{-j\omega t}\, dt$$

$$= \int_{-\infty}^{\infty} \frac{1}{RC} \cdot e^{-t/RC} \cdot u(t)\, e^{-j\omega t}\, dt$$

$$= \frac{1}{RC} \int_{0}^{\infty} e^{-t/RC} \cdot e^{-j\omega t}\, dt \quad \left(\because u(t) = \overset{\text{step}}{1} \text{ for } t \geq 0, \; 0 \text{ otherwise} \right)$$

$$= \frac{1}{RC} \int_0^{\infty} e^{-t\left(j\omega + \frac{1}{RC}\right)}\, dt$$

$$= \frac{1}{RC}\left(-\frac{1}{j\omega + \frac{1}{RC}}\right)\left[e^{-t\left(j\omega + 1/RC\right)}\right]_0^{\infty}$$

$$H(\omega) = \frac{1/RC}{j\omega + 1/RC} = \frac{1}{1 + j\omega RC}.$$

Magnitude & phase

$$H(\omega) = \frac{1}{1 + j\omega RC} \times \frac{1 - j\omega RC}{1 - j\omega RC} = \frac{1 - j\omega RC}{1 + (\omega RC)^2}$$

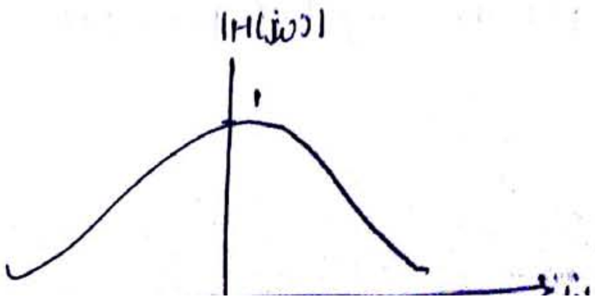
$$= \frac{1}{1 + (\omega RC)^2} + j\, \frac{-\omega RC}{1 + (\omega RC)^2}$$

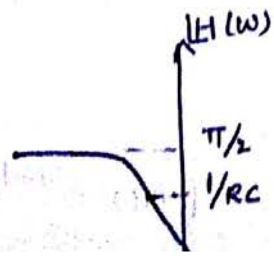
$$|H(\omega)| = \left\{ \frac{1}{\left[1 + (\omega RC)^2\right]^2} + \frac{(\omega RC)^2}{\left[1 + (\omega RC)^2\right]^2} \right\}^{1/2}$$

$$= \frac{1}{\sqrt{1 + (\omega RC)^2}}$$

$$\angle H(\omega) = \tan^{-1}\left\{ \frac{(-\omega RC)/1 + (\omega RC)^2}{1 + \left[1 + (\omega RC)^2\right]} \right\} = -\tan^{-1}(\omega RC)$$

If $RC = 1$, $|H(\omega)| = \frac{1}{\sqrt{1 + \omega^2}}$; $\angle H(\omega) = -\tan^{-1}(\omega)$.





2. For the system shown find the T.T & impulse response of the system.

$$f(t) = \begin{cases} e^{-at} & t>0 \\ 0 & \text{elsewhere} \end{cases} \quad ; \quad Y(\omega) = \frac{1}{\alpha + j\omega}$$

Sol

$$H(\omega) = \frac{R(\omega)}{F(\omega)}$$

$R(\omega) = Y(\omega)$

$F(\omega) \rightarrow \boxed{} \rightarrow Y(\omega)$

$$F(t) = e^{-at}$$

$$F(\omega) = \frac{1}{a+j\omega} \quad ; \quad Y(\omega) = \frac{1}{\alpha + j\omega}$$

$$H(\omega) = \frac{1/\alpha+j\omega}{1/a+j\omega} = \frac{a+j\omega}{\alpha+j\omega}$$

$$F^{-1}\left[\frac{a+j\omega}{\alpha+j\omega}\right] \Rightarrow \frac{a+\alpha-\alpha+j\omega}{\alpha+j\omega} = \frac{a-\alpha}{\alpha+j\omega} + \frac{\alpha+j\omega}{\alpha+j\omega}$$

$$= \frac{a-\alpha}{\alpha+j\omega} + 1$$

$$\boxed{h(t) = (a-\alpha)\, e^{-\alpha t} u(t) + \delta(t)}$$

3. The linear system impulse response is $[e^{-2t}+e^{-3t}]u(t)$ find the excitation to produce an o/p of $t \cdot e^{-2t} u(t)$?

Sol

$$h(t) = [e^{-2t} + e^{-3t}]u(t)$$

$F(\omega) \rightarrow \boxed{H(\omega)} \rightarrow R(\omega)$

$$r(t) = t \cdot e^{-2t} u(t)$$

$$H(\omega) = \frac{R(\omega)}{F(\omega)}$$

$$F(\omega) = \frac{R(\omega)}{H(\omega)}$$

$$r(t) = t \cdot e^{-2t} u(t) \xleftrightarrow{F.T} \frac{1}{(2+j\omega)^2} \qquad \left(\because t \cdot e^{-at} u(t) \longleftrightarrow \frac{1}{(a+j\omega)^2}\right)$$

$$R(\omega) = \frac{1}{(2+j\omega)^2}$$

$$h(t) = e^{-2t}u(t) + e^{-3t}u(t)$$

$$H(\omega) = \frac{1}{2+j\omega} + \frac{1}{3+j\omega}$$

$$R(\omega) = \frac{1/(2+j\omega)^{2}}{\frac{3+j\omega+2+j\omega}{(2+j\omega)(3+j\omega)}} = \frac{1}{2+j\omega} \times \frac{3+j\omega}{5+2j\omega}$$

$$\frac{3+j\omega}{(2+j\omega)(5+2j\omega)} = \frac{A}{2+j\omega} + \frac{B}{5+2j\omega}$$

$$3+j\omega = A(5+2j\omega) + B(2+j\omega)$$

put $3+j\omega = 5A+2B+j\omega(2A+B)$

put $j\omega = 0$; put $j\omega(-2)$

$(3 = 5A+2B) \times 1$

$(1 = 2A+B) \times 2$

$A = 1,\ B = -1$

$$R(\omega) = \frac{1}{2+j\omega} - \frac{1}{5+2j\omega} = \frac{1}{2+j\omega} - \frac{1}{2[5/2+j\omega]}$$

$$\boxed{r(t) = e^{-2t}u(t) - \frac{1}{2}e^{-5/2t}u(t)}$$

DIFFERENTIAL EQUATION:

→ To obtain frequency response & impulse response.

$$\sum_{k=0}^{N} a_k \frac{d^k}{dt^k} y(t) = \sum_{k=0}^{M} b_k \frac{d^k}{dt^k} x(t).$$

differentiation property of FT is

$$\frac{d}{dt}x(t) \xleftrightarrow{FT} j\omega X(\omega).$$

$$\sum_{K=0}^{N} a_K (j\omega)^K Y(\omega) = \sum_{K=0}^{M} b_K (j\omega)^K X(j\omega)$$

$$H(\omega) = \frac{Y(\omega)}{X(\omega)} = \frac{\sum_{K=0}^{M} b_K (j\omega)^K}{\sum_{K=0}^{N} a_K (j\omega)^K}.$$

system transfer fn.

PROBLEMS:

(1) The differential equation of system is given as $\frac{d^2y(t)}{dt^2} + 5\frac{dy(t)}{dt} + 6y(t) = -\frac{dx(t)}{dt}$

Determine the frequency response & impulse response.

Sol

$$\frac{d^2y(t)}{dt^2} + 5\frac{dy(t)}{dt} + 6y(t) = -\frac{dx(t)}{dt}$$

Taking F.T

$$(j\omega)^2 Y(\omega) + 5(j\omega)Y(\omega) + 6Y(\omega) = -j\omega X(\omega)$$

$$Y(\omega)\left[(j\omega)^2 + 5j\omega + 6\right] = -j\omega X(\omega)$$

$$H(\omega) = \frac{Y(\omega)}{X(\omega)} = \frac{-j\omega}{(j\omega)^2 + 5j\omega + 6}$$

$$H(\omega) = \frac{-j\omega}{(j\omega+2)(j\omega+3)} = \frac{A}{j\omega+2} + \frac{B}{j\omega+3}$$

$$= \frac{2}{j\omega+2} - \frac{3}{j\omega+3}$$

$$h(t) = \left[2 \cdot e^{-2t} - 3e^{-3t}\right] u(t)$$

impulse response of the system.

$$\left\{ \because e^{-at}u(t) \xleftrightarrow{FT} \frac{1}{a+j\omega} \right\}$$

② The input voltage to the RC circuit is given by $x(t) = te^{-t/RC}u(t)$ and impulse response of this circuit is given by $h(t) = \frac{1}{RC}e^{-t/RC}u(t)$. Find output $y(t)$

Sol

output $y(t) = x(t) * h(t)$

In frequency domain

$$Y(\omega) = X(\omega)H(\omega)$$

and $H(\omega) = F[h(t)]$

$$H(\omega) = F\left\{\frac{1}{RC}e^{-t/RC}.u(t)\right\}$$

$$= \frac{1}{RC} \cdot \frac{1}{\frac{1}{RC}+j\omega} = \frac{1}{1+j\omega RC}$$

$$X(\omega) = F\left[t.e^{-t/RC}u(t)\right]$$

$$= \int_{-\infty}^{\infty} t.e^{-t/RC}\,e^{-j\omega t}\,dt = \frac{1}{\left(\frac{1}{RC}+j\omega\right)^2} = \frac{(RC)^2}{(1+j\omega RC)^2}$$

$$\left[\because te^{-at}u(t) \xleftrightarrow{FT} \left(\frac{1}{a+j\omega}\right)^2\right]$$

$$Y(\omega) = X(\omega).H(\omega)$$

$$= \frac{(RC)^2}{(1+j\omega RC)^2} \cdot \frac{1}{(1+j\omega RC)} = \frac{(RC)^2}{(1+j\omega RC)^3}$$

$$y(t) = F^{-1}\{Y(\omega)\} = F^{-1}\left\{\frac{(RC)^2}{(1+j\omega RC)^3}\right\} = F^{-1}\left\{\frac{\cancel{(RC)^2}}{\cancel{(RC)^3}\left(\frac{1}{RC}+\right.}\right.$$

$$\boxed{y(t) = \frac{1}{RC} \cdot \frac{t^2.e^{-t/RC}}{2}u(t)}$$

1) $h(t) = e^{-5|t|}$

For stability $\int_{-\infty}^{\infty} |h(t)|\, dt < \infty$

$$\therefore \int_{-\infty}^{\infty} |h(t)|\, dt = \int_{-\infty}^{\infty} |e^{-5|t|}|\, dt = \int_{-\infty}^{\infty} e^{-5|t|}\, dt$$

$$= \int_{-\infty}^{0} e^{5t}\, dt + \int_{0}^{\infty} e^{-5t}\, dt = \left[\frac{e^{5t}}{5}\right]_{-\infty}^{0} + \left[\frac{e^{-5t}}{-5}\right]_{0}^{\infty}$$

$= \frac{2}{5}$ = constant / so system is stable.

2) $h(t) = e^{4t} u(t)$

$$= \int_{-\infty}^{\infty} |e^{4t} u(t)|\, dt = \int_{-\infty}^{\infty} e^{4t} u(t)\, dt$$

$$= \int_{0}^{\infty} e^{4t}\, dt = \frac{e^{\infty}}{4} - \frac{e^{0}}{4} = \infty - \frac{1}{4} = \infty$$ (Unstant.

3) $h(t) = e^{-4t} u(t)$ (stable)

4) $h(t) = t \cos t\, u(t)$ unstable.

$$\int_{0}^{\infty} t \cos t\, dt$$

5) $h(t) = e^{-t} \sin t\, u(t)$ (stable.

$$= \int_{0}^{\infty} e^{-t} \sin t\, dt$$

→ The system produces the o/p of $y(t) = e^{-t}u(t)$ for an input of $x(t) = e^{-2t}u(t)$. Determine the impulse response and frequency response of the system.

Sol

$$y(t) = e^{-t}u(t)$$
$$x(t) = e^{-2t}u(t)$$

Consider standard fourier transform pair $e^{-at}u(t) \xleftrightarrow{FT} \frac{1}{a+j\omega}$

$$Y(\omega) = \frac{1}{1+j\omega}; \quad X(\omega) = \frac{1}{2+j\omega}.$$

From equation

$$H(\omega) = \frac{Y(\omega)}{X(\omega)} = \frac{1/1+j\omega}{1/2+j\omega} = \frac{2+j\omega}{1+j\omega}.$$

Multiply ~~and divide~~ the numerator & denominator by $1-j\omega$

$$H(\omega) = \frac{2+j\omega}{1+j\omega} \times \frac{1-j\omega}{1-j\omega} = \frac{(2+j\omega)(1-j\omega)}{(1)^2-(j\omega)^2} = \frac{2-2j\omega+j\omega+\omega^2}{1+\omega^2}$$

$$= \frac{2+\omega^2-j\omega}{1+\omega^2} \Rightarrow \frac{2+\omega^2}{1+\omega^2} + j\frac{-\omega}{1+\omega^2}.$$

Magnitude $|H(\omega)| = \left\{\sqrt{\left[\frac{2+\omega^2}{1+\omega^2}\right]^2 + \left[\frac{\omega^2}{1+\omega^2}\right]^2}\right]$

$$|H(\omega)| = \sqrt{\frac{4+\omega^2}{1+\omega^2}} \quad ; \quad \angle H(\omega) = \tan^{-1}\left(\frac{\frac{-\omega}{\cancel{1+\omega^2}}}{\frac{2+\omega^2}{\cancel{1+\omega^2}}}\right)$$

$$= -\tan^{-1}\left(\frac{\omega}{2+\omega^2}\right\}$$

$$\therefore H(\omega) = \frac{2+j\omega}{1+j\omega} \Rightarrow \frac{1+j\omega+1}{1+j\omega} \Rightarrow 1+\frac{1}{1+j\omega}.$$

Inverse fourier transform

$$\boxed{h(t) = IFT\{H(\omega)\} = \delta(t) + e^{-t}u(t)}$$

impulse response.

(1) The transfer function of LPF is given by

$$H(\omega) = \begin{cases} (1+K\cos\omega T)e^{-j\omega T} & ; |\omega| < 2\pi B \\ 0 & ; |\omega| > 2\pi B. \end{cases}$$

Determine the output y(t) when a pulse x(t) bandlimited in B is applied at the input.

Sol)

$$Y(\omega) = X(\omega)H(\omega)$$

$$= X(\omega)[1+K\cos\omega T]e^{-j\omega \tau}$$

$$= X(\omega)e^{-j\omega \tau} + K X(\omega)\cos\omega T e^{-j\omega \tau}$$

we know

$$x(t-\tau) + x(t+\tau) \longleftrightarrow 2X(\omega)\cos\omega T$$

$$x(t-T) \longleftrightarrow X(\omega)e^{-j\omega \tau}$$

$$y(t) = F^{-1}[Y(\omega)]$$

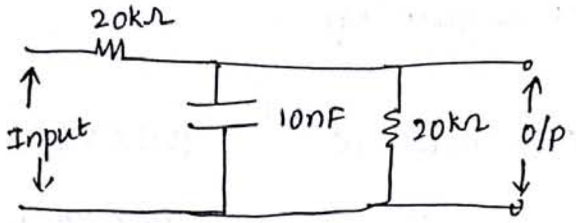
$$= F^{-1}[X(\omega)e^{-j\omega \tau} + K X(\omega)e^{-j\omega \tau}\cos\omega T]$$

$$= x(t-T) + \frac{k}{2}[x(t-\tau-T) + x(t-T+T)]$$

$$y(t) = x(t) + \frac{k}{2}[x(t-T) + x(t+T)]$$

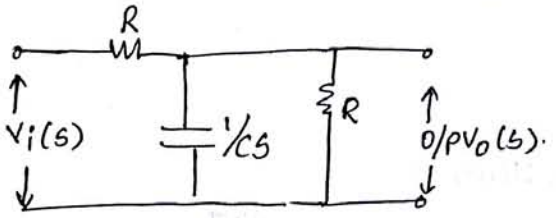
delayed by τ.

2) Determine the maximum bandwidth of signals that can be transmitted through low pass RC filter as shown in figure , if over this bandwidth, the gain variation is to be 10% and the phase variation is to be within 7% of ideal characteristics.



Sol

RC network transformed into s-domain representation.



$$H(s) = \frac{V_o(s)}{V_i(s)}$$

$$= \frac{[R \parallel (1/Cs)]}{([R \parallel (1/Cs)] + R]}$$

$$= \frac{(R/Cs) / [R + 1/Cs]}{[R/Cs] / [(R + 1/Cs)] + R}$$

$$= \frac{R/(1+sCR)}{[R/(1+sCR)] + R} = \frac{R}{R + R(1+sCR)} = \frac{\cancel{R}}{\cancel{R}(1+1+sCR)}$$

$$H(s) = \frac{1}{2+sCR}.$$

But given $R = 20k\Omega$

$C = 10nF.$

$$H(s) = \frac{1}{2+s(10\times10^{-9}\times20\times10^{3})} = \frac{1}{2+s(2\times10^{-4})} = \frac{1}{2+(2s/10^4)} = \frac{10^4}{2\times10^4+2s}$$

$$\therefore H(s) = \frac{5000}{s+10000}$$

put $s = j\omega$

$$H(w) = \frac{5000}{jw+10000}$$

$$|H(\omega)| = \frac{5000}{\sqrt{\omega^2+10000}}$$

$$\phi(w) = -\tan^{-1}\left(\frac{w}{10000}\right).$$

At $w=0$, $|H(\omega)|_{\omega=0} = \frac{5000}{10000} = 0.5.$

But there is 10% variation in gain over bandwidth B.

$$H(w) = 0.5 - 0.5 \times 10\% = 0.45$$

$$|H(\omega)| = \frac{5000}{\sqrt{B^2+10^8}}$$

$$B^2+10^8 = \left(\frac{5000}{0.45}\right)^2 \Rightarrow B^2 = 23.46 \times 10^6$$

$$B = 4.84\text{ KHZ}$$

But $B = 2\pi f$

$$f = \frac{B}{2\pi} = \frac{4.84 \times 10^3}{2\pi} = 770.8\text{ HZ}$$

phase at frequency, $f = 770.8$ HZ

$$\phi(\omega) = -\tan^{-1}\left(\frac{4.84}{10}\right) = -25.83\%.$$

(2) There are several possible ways of estimating an essential bandwidth of non-bandlimited signal. For a low pass signal, for example, the essential b.w may