# UNIT-I

## ECONOMIC OPERATION OF POWER SYSTEMS

Planning operation and control of interconnected power systems presents a variety of challenging problems. An important problem in this area is the economic operation of the system, which means, that every step in planning, scheduling and operation of the system, unit-wise, plant wise and inter connection-wise must be optimal, leading to absolute economy. In this, the transmission losses too play an important role. In this unit on economic operation of power system, both thermal and hydro system will be dealt with using suitable analytical models that result in meaningful savings.

### 1.1 Characteristics of Steam Plants :

In analyzing the economics of operation of thermal systems, modelling of input output characteristics assumes significance. For this purpose a single unit comprising of boiler, turbine, and generator may be considered. The unit has to supply power to the local needs for the auxiliaries in the station. This later component may be around 2-5%. The station auxiliaries includes boiler feed pumps, condenser circulating water pumps, fans etc. The total input to the unit could be either K-cal/hr in terms of heat supplied or Rs/hr in terms of the cost of the fuel such as coal, gas or fossil fuel of any other form. The net output of the unit that is supplied into the system at the generator bus will be in KW or MW. Scheduling is the process of allocation of generation among various operating generator units. Economic scheduling is the cost effective mode of generation allocation among the various units. This can also be termed as optimal operation. The analytical solution proposed in general for optimal operation depends on incremental cost concept.

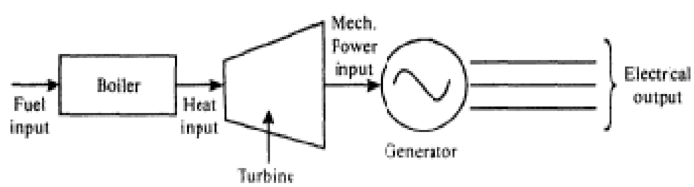


Fig. Boiler turbine generator unit

### 1.2 Input Output Curves:

As has been already stated the input output characteristics for any thermal unit or units that comprise the plant can be obtained from the operating data. The input can be in kilo calories per hour and the output may be in kilowatts or preferably in mega watts. Typical characteristic is shown in below Fig, for the unit shown in above Fig.

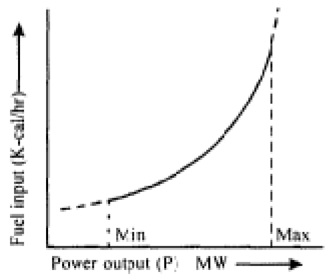


Fig. Thermal unit input-output characteristic

**1.3 The Incremental Heat Rate Characteristic:**

From the input output characteristic the incremental heat rate characteristic can be obtained which is the ratio of the differentials.

$$\text{Incremental fuel rate or heat rate} = \frac{d(in\,put)}{d(output)}$$

By calculating the slope of the characteristic in input-output curve at every point, the incremental fuel rate characteristic can be plotted as shown in below. This characteristic infact tells about the thermal efficiency of the unit under consideration that can be used for comparison with other units in performance.

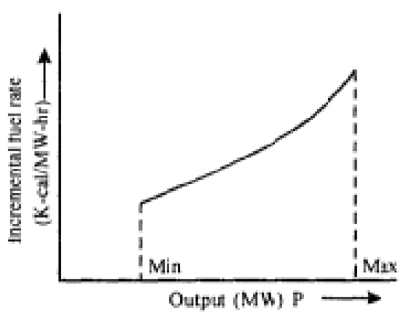


Fig. Incremental fuel rate characteristic for thermal unit

**1.4 The Incremental Fuel Cost Characteristic:**

The incremental fuel rate in K-caI/K-whr can be multiplied by cost-of the fuel in terms of Rs per K-cal. In any case the ordinates are in Rs/Kw-hr. or Rs/MW-hr. The calorific value of the fuel is required in these calculations. This characteristic is shown in below Fig.

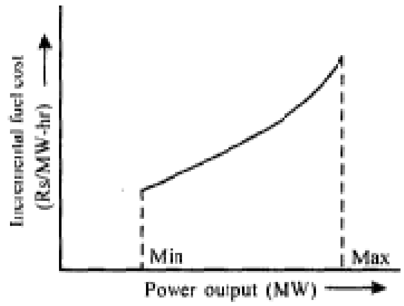


Fig. Incremental fuel cost characteristic for thermal unit

**1.5 -Heat Rate Characteristic :**

Some times the unit net heat rate characteristic is also considered important. To obtain this characteristic the net heat rate in k.caI/K-whr is plotted against the power output (below Fig). The thermal efficiency of the unit is influenced by factors like steam condition, reheat stages, condenser pressure and the steam cycle used. The efficiency of the units in practice is around 30%.

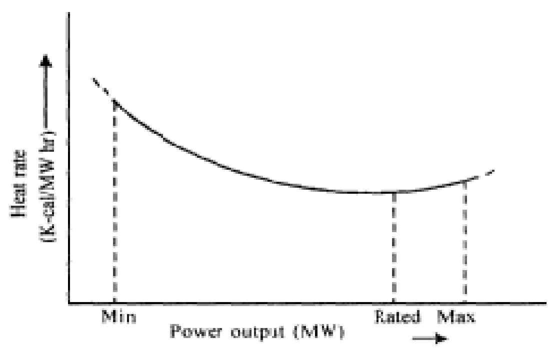


Fig. Heat rate characteristic

**1.6 Incremental Production Cost Characteristics:**

The production cost of the power generated actually depends on several items such as fuel cost, labour charges, cost of items such as oil, water and other supplies needed and also the cost of maintenance. It is well known that in thermal generation the fuel cost is by far the largest cost head and is directly related to the power generated. Even the other charges, that is the additional running expenses too are, more or less, related to the amount of generation. Thus, it is a simple practical proposition to assume that, all the additional costs as a fixed percentage of the incremental fuel cost. The sum of incremental fuel cost and other incremental running expenses is called incremental production cost. The ordinates of incremental fuel cost characteristics will also represent incremental production cost to some other scale. An explicit mathematical relationship involving all the factors involved in power generation to the total power generated is in fact a very difficult task. In general, the input-output data is fitted into a quadratic characteristic even though it is also possible to fit into any polynomial curve for ease of mathematical manipulation.

Large turbine-generator units may have several steam admission valves which are opened in a sequence to meet the increasing steam demand. The input output characteristic for such a unit with two valves may show discontinuity as shown in below Fig.

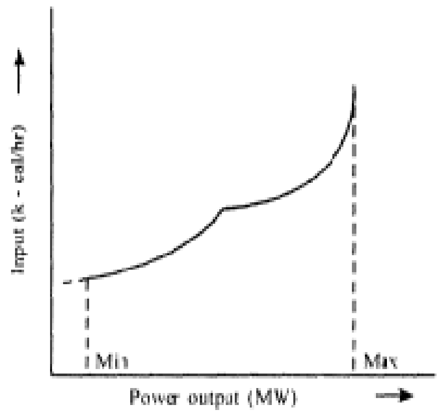


Fig. Input-output characteristic for multiple admission valves (for two valves)

**1.7 Incremental Production Cost Characteristic:**

The incremental water rate characteristic can be converted into incremental production cost characteristic by multiplying the incremental water rate characteristic by water rate or cost of water in rupees per cubic meter $C_w$. The incremental production cost characteristic is shown in below Fig.

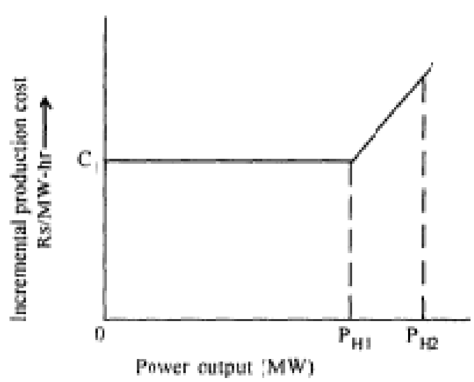


Fig. Incremental production cost representation

**1.8 Generating Costs at Thermal Plants:**

Consider any ith plant among n thermal plants supplying active power. CI is the cost per unit of PI in the neighborhood of PI then the generating cost is

$$F_i = C_i P_i$$

The total cost of operating a system with N  generating sets can be represented by

$$F = \sum_{i=1}^{N_g} C_i P_i$$

If the system operates over a time period T then the total expenditure involved will be

$$F_T = \int_0^T \sum_{i=1}^{N_g} C_i P_i(t) dt$$

Steam plants with partial admission nozzle governing give better performance at partial loads since the cost coefficient increases with increasing megawatt loading. However, such units cannot be shut down, frequently, because of the complexities of steam chest. But, steam plants with throttle governing are more suitable for periodic shut down due to their simpler steam chest. Such units are more suitable for rapid starting and loading. From minimum to maximum permissible limits of operation, the cost coefficients of the units are substantially constant.

**1.9 Analytical Form for Input-Output Characteristics of Thermal Units:**

It is already pointed out that the input-output characteristic may be fitted into a polynomial of the form

$$\text{Cost } F = A + BP + CP^2 + \ldots\ldots\ldots.$$

where A, B, C ...... are constants

If a quadratic representation is made then denoting the cost

$$F = \tfrac{1}{2} a P^2 + bP + C$$

will lead to an incremental cost characteristic of the form a linear relationship around any operating point P.

$$\frac{dF}{dP} = a\ P + b$$

**1.10 Constraints in Operation:**

The power system has to satisfy several constraints while in operation. These may be broadly divided into two types. The first of these arises out of the necessity for the system to satisfy load balance and are called equality constraints. At any bus i. P s, and Os, correspond to the scheduled generation. P D, and O[), are the load demands then the following equations must be satisfied at the bus i :

$$P_{Si} - P_{Di} - P_i = G_i = 0$$
$$Q_{Si} - Q_{Di} - Q_i = H_i = 0$$

P, and Q, are given by

$$P_i = \sum_{j=1}^{n} V_i V_j Y_{ij} \, Cos(\delta_i - \delta_j - \theta_{ij})$$

$$Q_i = \sum_{j=1}^{n} V_i V_j Y_{ij} \, Sin(\delta_i - \delta_j - \theta_{ij}), i = 1, 2....., n$$

with usual notation and G, and H, are the residuals at bus i which should become zero at the point of solution.

In addition, a number of other constrains due to physical and operational limitations of the units and components will arise in economic scheduling. These are in the form of inequality constraints.

Each generator in operation will have a minimum and maximum permissible output and the production must be constrained to ensure that

$$P_i^{min} \leq P_i \leq P_i^{max},\ i = 1, 2\ ....,\ ng$$

Similarly limits may also have to be considered over the range of reactive power capabilities of the generator units requiring that

$$Q_i^{min} \leq Q_i \leq Q_i^{max},\ i = 1, 2\ ....,\ nq$$

where nq is the total number of reactive sources in the system.

Further, the constraint

$$P_i^2 + Q_i^2 \leq (S_i^{rated})^2$$

must be satisfied, where S is the MVA capacity of the generating unit for limiting stator heating. Dynamic limits may also have to be considered when fast changes in generation are envisaged for picking up or for shedding down of loads. These limits put additional constraints of the form.

$$\left|\frac{dP_i(t)}{dt}\right|^{min} \leq \left|\frac{dP_i(t)}{dt}\right| \leq \left|\frac{dP_i(t)}{dt}\right|^{max}$$

The maximum and minimum operating conditions for a group of generators within a power station may be different from the respective sum of the maximum and minimum operating levels of turbines that are supplied by a single boiler. The extremes ofboiler operating conditions will determine these limits. Thus. groups of generators from individllal boiler units may have to be subjected to additional constraints of the nature

$$P_{kg}^{min} \leq \underset{i \in G_i}{P_{ki}} \leq P_{kg}^{max},\ k = 1, 2....GR$$

where GR is the total number of generator groups. the outputs of which are to be separately limited.

Spare capacity is required to account for the errors in load prediction, sudden and fast changes in load demand and the inadvertent loss of scheduled generation. Thus. the total generation G available at any time must be in excess of the total anticipated load demand and system losses by an amount not less than a specified minimum spare capacity P sp'

$$G \geq \sum_{i=1}^{ng} P_i + P_{SP}$$

In a similar manner, constrains may be required to be associated with groups of generators, where all plants are not equally operationally suitable for taking up additional load. If TG is the total number of groups then

$$G_k \geq \sum_{i \in G} P_{ki} + P_{SG}$$

where k = I, 2 ..... , TG.

The summation is over the set G over which group constraints are applied.

..... (3.19)

Thermal considerations may require that the transmission lines be subjected to branch transfer constraints of the type.

$$-S_i^{max} \leq S_{bi} \leq S_i^{max}, i = 1, 2, \ldots\ldots, nb$$

where nb is the number of branches and S bl is the branch transfer MVA.

In addition, constraints are to be imposed for bus voltage magnitudes and for phase displacements between them for maintaining voltage profile and for limiting overloading respectively.

Thus we have

$$V_{ij}^{min} \le V_{ij} \le V_{ij}^{max}, i = 1, 2......, n$$

$$\delta_{ij}^{min} \le \delta_{ij} \le \delta_{ij}^{max}, i = 1, 2......, n$$

$$\text{where} \quad j = 1, 2|.....m \quad \text{and} \quad j \ne i$$

where n is the total number of nodes and m is the number of nodes neighbouring each node with interconnecting branches.

In case transformer tap positions are to be included for optimization, then the tap positions T) must lie within the range available, i.e.

$$T_i^{min} \le T_i \le T_i^{max}$$

Sometimes, phase shifting transformers are made available in the system. If such equipment exists then constraints of the type

$$PS_i^{min} \le PS_i \le PS_i^{max}$$

must be reckoned where PSI is the phase shift obtained from the ith phase shifting transformer.

If power system security is also required to be considered in the formulation for economic operation then power flows between certain important buses may also have to be considered for the final solution. It may be mentioned that consideration of each and every possible branch for outage will not be a feasible proposition.

Equal Incremental Cost Method: Transmission Losses Neglected

**1.11 Method of Lagrange multipliers:**

It is but natural that for a given load to be allocated between several generating units, the most efficient unit identified by incremental cost of production should be the one to get priority. When this is applied repeatedly to all the units the load allocation will become complete when all of them, that are involved in operation, are all working at the same incremental cost of production.

The above can be proved mathematically as follows:

Consider ng generating units supplying PI' Pc' ............ Png active powers to supply a total load demand Po'

The objective function for minimization is the total input to the system in rupees per hour.

$$F(P_1, P_2, \ldots\ldots, P_{ng}) = F(P) = \int_{i=1}^{ng} C_i(P_i)$$

where C,(P) is the generation cost for the ith unit and ng is the total number of generating units.

The equality constraint is given by

$$G(P_1, P_2, \ldots, P_{ng}) = G(P) = P_D - \sum_{i=1}^{ng} P_i = 0$$

i.e., total supply = total demand neglecting losses and reserve. Using the method of Lagrange multipliers for equality constraints. the Lagrange function is defined as

$$L(P, \lambda) = F(P) + \lambda G(P)$$

Where A is a Langrange multiplier.

The necessary conditions are given by

$$\frac{\partial L}{\partial P_i} = 0 \text{, and}$$

$$\frac{\partial L}{\partial \lambda} = 0$$

since $$L = \sum_{i=1}^{ng} C_i(P_i) + \lambda\left(P_D - \sum_{i=1}^{ng} P_i\right)$$

we obtain $$\frac{\partial L}{\partial P_i} = \frac{\partial}{\partial P_i} C_i(P_i) + \lambda(-1) = 0$$

and $$\frac{\partial L}{\partial \lambda} = P_D - \sum_{i=1}^{ng} P_i = 0$$

The later equation is the load demand constraint only; while the former gives

$$\frac{\partial C_1(P_1)}{\partial P_1} = \frac{\partial C_2(P_2)}{\partial P_2} = \dots\dots\dots = \frac{\partial C_{ng}(P_{ng})}{\partial P_{ng}} = \lambda$$

The above equation states that at the optimum all the generating stations operate at the same incremental cost for optimum economy and their incremental production cost is equal to the Lagrange multiplier A at the optimum.

$$(P_1 + P_2 + \dots\dots\dots + P_{ng}) = P_D$$

The principle underlying the mathematical treatment is that load should be taken up always at the lowest incremental cost. It must be ensured that the generations so determined are within their capacities. Otherwise, the generation has to be kept constant at the capacity limit for that unit and eliminated from further optimum calculations.

It can be seen that in this method at the optimum the incremental cost of production is also the incremental cost of the power received.

**1.11 Transmission Loss Formula - B. Coefficients:**

An expression for the transmission loss was derived by Kirchamayer with several assumptions made. In his derivation he used Kron's tensorial transformation. However a simplified procedure for the derivation of the B-coefficients will be presented here.

Consider a power system supplying nlloads. Let the load currents be ill' iL2, .•....• iLnl.

These loads are supplied by ng generators. Let the generator currents be igl • ig2,····· .. igng.

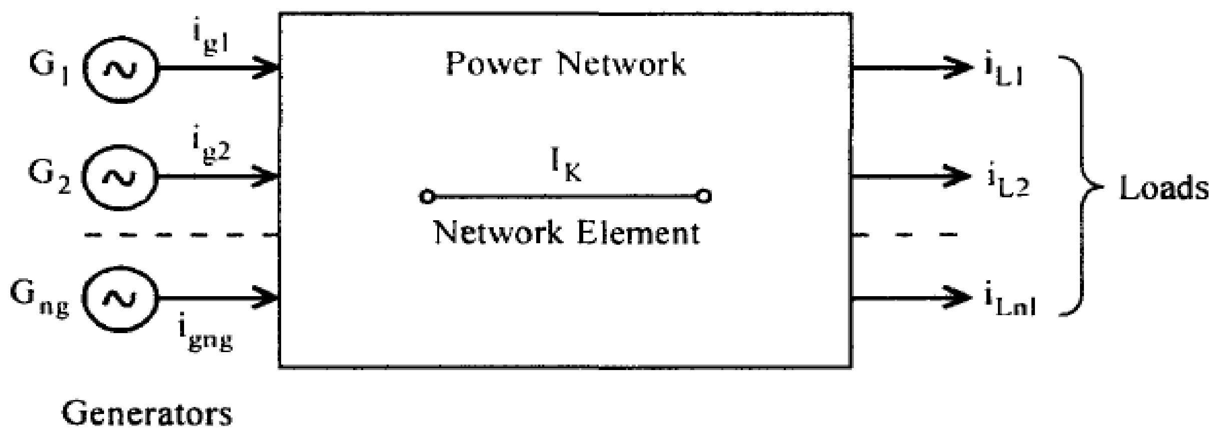


Fig. Power system with generator and load currents

Consider a network element K (an inter connected line in the system) carrying current IK. Let generator I alone supply the entire load current IL where

$$I_L = i_{L1} + i_{L2} + \dots\dots\dots + i_{Lnl}$$

$$= \sum_{j=1}^{nl} i_{Lj}$$

under this condition let the current in K be iKI

In a similar manner if each of the ng generators operating alone also supply the total load current IL while the rest of the generators are disconnected the current carried by the network element K changes from iKI to iK2 ' iK3 to iKng .

Let the ratio of iK I to iL be dK 1

$$d_{K1} = \frac{i_{K1}}{I_L}$$

Also

$$d_{K2} = \frac{i_{K2}}{I_L}$$

and so on.

Now, if all the generators are connected to the power system simultaneously to supply the same load, by the principle of super position.

$$I_K = d_{K1} i_{g1} + d_{K2} i_{g2} + \dots\dots\dots + \dots\dots\dots\dots\dots + d_{Kng} i_{gng}$$

Let the individual load currents remain a constant complex ratio of the total load current Iv It is assumed that (X/R) ratio for all the line elements or branches in the network remains the same. The factors dK1 will then be real and not complex.

The individual generator currents may have phase angles 81, 82, .•.•• 8ng with respect to a reference axis. The generator currents can be expressed as :

$$i_{g1} = |i_{g1}| \cos\delta_1 + |i_{g1}| \sin\delta_1$$

$$i_{g2} = |i_{g2}| \cos\delta_2 + |i_{g2}| \sin\delta_2$$

$$\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots$$

$$i_{gng} = |i_{gng}| \cos\delta_{ng} + |i_{gng}| \sin\delta_{ng}$$

For simplicity to derive the formula

Let ng = 3 so that eqn.  becomes

$$I_K = d_{k1}\, i_{g1} + d_{k2}\, i_{g2} + d_{k3}\, i_{g3}$$

$$|I_K|^2 = \left(d_{k1}|i_{g1}|\cos\delta_1 + d_{k2}|i_{g2}|\cos\delta_2 + d_{k3}|i_{g3}|\cos\delta_3\right)^2$$

$$+ \left(d_{k1}|i_{g1}|\sin\delta_1 + d_{k2}|i_{g2}|\sin\delta_2 + d_{k3}|i_{g3}|\sin\delta_3\right)^2$$

$$= d_{k1}^2|i_{g1}|^2 + d_{k2}^2|i_{g2}|^2 + d_{k3}^2|i_{g3}|^2$$

$$+ 2d_{k1}d_{k2}|i_{g1}||i_{g2}|\cos\delta_1\cos\delta_2$$

$$+ 2d_{k2}d_{k3}|i_{g2}||i_{g3}|\cos\delta_2\cos\delta_3$$

$$+ 2d_{k3}d_{k1}|i_{g3}||i_{g1}|\cos\delta_3\cos\delta_1$$

$$+ 2d_{k1}d_{k2}|i_{g1}||i_{g2}|\sin\delta_1\sin\delta_2$$

$$+ 2d_{k2}d_{k3}|i_{g2}||i_{g3}|\sin\delta_2\sin\delta_3$$

$$+ 2d_{k3}d_{k1}|i_{g3}||i_{g1}|\sin\delta_3\sin\delta_1$$

$$= d_{k1}^2|i_{g1}|^2 + d_{k2}^2|i_{g2}|^2 + d_{k3}^2|i_{g3}|$$

$$+ 2d_{k1}d_{k2}|i_{g1}||i_{g2}|\cos(\delta_1 - \delta_2)$$

$$+ 2d_{k2}d_{k3}|i_{g2}||i_{g3}|\cos(\delta_2 - \delta_3)$$

$$+ 2d_{k3}d_{k1}|i_{g3}||i_{g1}|\cos(\delta_3 - \delta_1)$$

Eliminating currents in terms of powers supplied by the generators

$$i_{g1} = \frac{P_1}{\sqrt{3}|V_1|\text{Cos}\phi_1}$$

$$i_{g2} = \frac{P_2}{\sqrt{3}|V_2|\text{Cos}\phi_2} \qquad \text{and}$$

$$i_{g3} = \frac{P_3}{\sqrt{3}|V_3|\text{Cos}\phi_3}$$

where PI' P2 and P3 are the active power supplied by the generators 1,2 and 3 at voltages IV II, IV 21 and IV 31 and power factors at the generator buses being COS ~I' COS ~2 and Cos <1>3 respectively.

$$|I_K|^2 = \frac{d_{k1}^2 P_1^2}{\left(\sqrt{3}|V_1|\cos\phi_1\right)^2} + \frac{d_{k2}^2 P_2^2}{\left(\sqrt{3}|V_2|\cos\phi_2\right)^2} + \frac{d_{k3}^2 P_3^2}{\left(\sqrt{3}|V_3|\cos\phi_3\right)^2}$$

$$+ \frac{2d_{k1}d_{k2}P_1P_2\cos(\delta_1 - \delta_2)}{3|V_1||V_2|\cos\phi_1\cos\phi_2} + \frac{2d_{k2}d_{k3}P_2P_3\cos(\delta_2 - \delta_3)}{3|V_2||V_3|\cos\phi_2\cos\phi_3}$$

$$+ \frac{2d_{k3}d_{k1}P_3P_1\cos(\delta_3 - \delta_1)}{3|V_3||V_1|\cos\phi_3\cos\phi_1}$$

The power losses in the network comprising of nb network elements or branches PLOSS is given by

$$P_L = \sum_{k=1}^{nb} 3|i_k|^2 R_K$$

where RK is the resistance of the element K.

$$P_L = \frac{P_1^2 \sum_{k=1}^{ng} d_{k1}^2 R_K}{|V_1|^2 (\cos\phi_1)^2} + \frac{P_2^2 \sum_{k=1}^{ng} d_{k2}^2 R_K}{|V_2|^2 (\cos\phi_2)^2} + \frac{P_3^2 \sum_{k=1}^{ng} d_{k3}^2 R_K}{|V_3|^2 (\cos\phi_3)^2}$$

$$+ \frac{2P_1 P_2 \sum_{k=1}^{ng} d_{k1} d_{k2} R_K \cos(\delta_1 - \delta_2)}{|V_1||V_2| \cos\phi_1 \cos\phi_2}$$

$$+ \frac{2P_2 P_3 \sum_{k=1}^{ng} d_{k2} d_{k3} R_K \cos(\delta_2 - \delta_3)}{|V_2||V_3| \cos\phi_2 \cos\phi_3}$$

$$+ \frac{2P_3 P_1 \sum_{k=1}^{ng} d_{k3} d_{K1} R_K \cos(\delta_3 - \delta_1)}{|V_3||V_1| \cos\phi_3 \cos\phi_1}$$

Let

$$B_{11} = \frac{1}{|V_1|^2(\cos\phi_1)^2}\sum d_{k1}^2 R_K$$

$$B_{22} = \frac{1}{|V_2|^2(\cos\phi_2)^2}\sum d_{k2}^2 R_K$$

$$B_{33} = \frac{1}{|V_3|^2(\cos\phi_3)^2}\sum d_{k3}^2 R_K$$

$$B_{12} = \frac{\cos(\delta_1 - \delta_2)\sum_{k=1}^{ng} d_{k1} d_{k2} R_K}{|V_1||V_2|(\cos\phi_1)(\cos\phi_2)}$$

$$B_{23} = \frac{\cos(\delta_2 - \delta_3)\sum_{k=1}^{ng} d_{k2} d_{k3} R_K}{|V_2||V_3|(\cos\phi_2)(\cos\phi_3)}$$

$$B_{31} = \frac{\cos(\delta_3 - \delta_1)\sum_{k=1}^{ng} d_{k3} d_{k1} R_K}{|V_1||V_3|(\cos\phi_3)(\cos\phi_1)}$$

$$P_{LOSS} = P_1^2 B_{11} + P_2^2 B_{22} + P_3^2 B_{33}$$

$$+ 2P_1 P_2 B_{12} + 2P_2 P_3 B_{23} + 2P_3 P_1 B_{31} + \ldots\ldots$$

$$= \sum_{m=1}^{3} \sum_{m=1}^{3} P_m B_{mn} P_n$$

In general the formula for Bmn coefficients can be expressed a~

$$B_{mn} = \frac{\cos(\delta_m - \delta_n)}{|V_m||V_n|(\cos\phi_m)(\cos\phi_n)} \sum_k d_{km} d_{kn} R_k$$

In the matrix form, the loss formula is expressed for an n generator system as :

$$\begin{bmatrix} P_1 & P_2 & \dots & P_n \end{bmatrix} \begin{bmatrix} B_{11} & B_{12} & \dots & B_{1n} \\ B_{21} & B_{22} & \dots & B_{2n} \\ \dots & \dots & \dots & \dots \\ B_{n1} & B_{n2} & \dots & B_{nn} \end{bmatrix} \begin{bmatrix} P_1 \\ P_2 \\ \dots \\ P_n \end{bmatrix}$$

The coefficients can be considered constant, if in addition to the assumptions already

made we further assume that the generator voltages V I' V 2' .••. etc remain constant in magnitude

and generator bus power factors cos <PI' cos <P2 ....... also remains constant.

**1.12 Active Power Scheduling:**

Economic scheduling of thermal plants considering effect of transmission losses:

The objective function is

$$F(P) = F(P_1, P_2, \dots\dots\dots P_{ng})$$

$$= \sum_{i=1}^{ng} C_i(P_i)$$

which has to be minimized over a given period of time. As only active power is scheduled the

equality constraint G(P) is given by

$$G(P) = P_D - P_L - \sum_{i=1}^{ng} P_i = 0$$

must be satisfied at every generator bus where PI is the generation at bus i ; P D' the total load demand and PL the total transmission loss in all the lines. It is desired to minimize eqn. subject to the constraint eqn.

The Lagrange function L is formed as

$$L(P, \lambda) = F(P) + 1 \ \lambda\left[P_D + P_L - \sum_{i=1}^{ng} P_i\right]$$

Applying the necessary conditions for the minimum of L

$$\frac{\partial L}{\partial P_i}(P, \lambda) = \frac{\partial}{\partial P_i} F(P_i) + \frac{\partial}{\partial P_i} \lambda\left[P_D + P_L - \sum_{i=1}^{ng} P_i\right] = 0$$

i.e.,

$$\frac{\partial}{\partial P_i} C[P1 \ldots\ldots., Pi,..Png] + \frac{\partial}{\partial P_i} \lambda\left[P_D + P_L - \sum_{i=1}^{ng} P_i\right] = 0$$

$\therefore$

$$\frac{\partial C_i(P_i)}{\partial P_i} + \lambda \frac{\partial P_L}{\partial P_i} - \lambda = 0$$

**Further,**

$$\frac{\partial C_i(P_i)}{\partial P_i} = \lambda\left[1 - \frac{\partial L}{\partial P_i}\right] \ ; \ i = 1,2\ldots\ldots.ng$$

**Also, it can be expressed as**

$$\frac{\partial F_i}{\partial P_i} + \lambda \frac{\partial P_L}{\partial P_i} = \lambda$$

The sum of the incremental production cost of power at any plant i and the incremental transmission losses incurred due to generation Piat bus i charged at the rate of $\lambda$ must be constant for all generators and equal to λ. This constant λ is equal to the incremental cost of the received power.

the loss formula is derived as

$$P_L = \sum_i \sum_j P_i B_{ij} P_j$$

$$= B_{11} P_1^2 + B_{22} P_2^2 + ..... + B_{ngng} P_{ng}^2 + 2B_{12} P_1 P_2 +$$

$$2B_{13} P_1 P_3 + .... + 2B_{1ng} P_{ng}^2 + .......$$

Differentiating

$$\frac{\partial P_L}{\partial P_i} = \sum_j 2B_{ij} P_j$$

Also

$$\frac{\partial C_i(P_i)}{\partial P_i} = \frac{dC_i(P_i)}{dP_i} \quad i = 1, 2, ....... ng$$

$$\frac{dC_i(P_i)}{dP_i} + \sum_{j=1}^{ng} 2B_{ij} P_j = \lambda \quad j = 1, 2, ....., ng$$

If the incremental costs are represented by a linear relationship following a quadratic input-output characteristic.

Then, denoting

$$\frac{dC_i(P_i)}{dP_i} = a_i P_i + b_i$$

$$a_i P_i + b_i + \lambda \sum_{j=1}^{ng} 2B_{ij} P_j = \lambda$$

$$\sum_{j=1}^{ng} 2B_{ij}P_j = 2B_{ii}P_j + \sum_{\substack{j=1 \\ j\neq i}}^{ng} 2B_{ij}P_j$$

$$a_iP_i + b_i + \lambda 2B_{ii}P_i + \lambda \sum B_{ij}P_j = \lambda$$

$$a_iP_i + b_i + \lambda 2B_{ii}P_i + \lambda \sum_{\substack{i=1 \\ j\neq i}}^{ng} B_{ij}P_j = \lambda$$

solving for Pi

There are ng equations to be solved for ng powers (P) for which Gauss's or

Gauss - Seidel method is well suited. Knowing ai' bl

, and BIJ coefficients for any assumed

value of A, PI values may converge to a solution, giving the generator scheduled powers. It is very important that a suitable value is assumed for A, so that a quick convergence of the equations is obtained.

**1.13 Penalty Factor:**

$$\frac{dC_i(P_i)}{dP_i} + \lambda \frac{\partial P_L}{\partial P_i} = \lambda$$

It can be rewritten as

$$\frac{\partial C_i(P_i)}{\partial P_i} = \lambda \left[ 1 - \frac{\partial P_L}{\partial P_i} \right]$$

in other words

$$\frac{\partial C_i(P_i)}{\partial P_i} \left[ \frac{1}{1 - \frac{\partial P_L}{\partial P_i}} \right] = \lambda$$

When transmission losses are included, the incremental production cost at each plant i must be multiplied by a factor $\left(\frac{1}{1-\frac{\partial P_L}{\partial P_i}}\right)$ which then will be equal to the incremental cost of power delivered. Hence, the factor $\left(\frac{1}{1-\frac{\partial P_L}{\partial P_i}}\right)$ is called penalty factor :

Since $\frac{\partial P_L}{\partial P_i}$ is much less than unity some times it is approximated by $1+\frac{\partial P_L}{\partial P_i}$ so that

$$\frac{\partial C_i(P_i)}{\partial P_i}\left(1+\frac{\partial P_L}{\partial P_i}\right)=\lambda$$

The term $1+\frac{\partial P_L}{\partial P_i}$ is called approximate penalty factor.

**1.14 Evaluation of λ for Computation:**

$$P_i = \frac{1-\frac{b_i}{\lambda}-\sum_{\substack{j=1\\ j\neq i}}^{ng} 2B_{ij}P_j}{\frac{ai}{\lambda}+2B_{ii}P_j};i=1,2,.....,ng$$

solution to P1 values depends upon the value of λ chosen. The value of λ determines a set of generations for a particular received load.

It is established that for scheduling purposes it is necessary to start the computation with two different values of λ. As the arbitrarity assigned generations are improved from iteration to iteration, a new value of λ may be computed for each new iteration using the following algorithm for a specified total received load power PR.

$$\lambda^{(i)} = \lambda^{(i-1)} + \left(P_R^d - P_R^{(i-1)}\right)\left(\frac{\lambda^{(i-1)} - \lambda^{(i-2)}}{P_R^{(i-1)} - P_R^{(i-2)}}\right)$$

where $P_R^{(i-1)}$ = received power with $\lambda^{(i-1)}$

$P_R^{(i-2)}$ = received power with $\lambda^{(i-2)}$

$\lambda^{(i)}, \lambda^{(i-1)}, \lambda^{(i-2)}$ are the values of $\lambda$ during iterations (i), (i – 1) and (i – 2)

$P_R^d$ = the desired total power to be received by the loads.

$$= \sum_{i=1}^{ng} P_i - P_L$$

When two values of PR calculated successively during the iterations converge to a single value with reasonable accuracy, the latest value of P R calculated will also converge to PR.