Unit 1

Electromechanical Energy Conversion Principles

Introduction

For energy conversion between electrical and mechanical forms, electromechanical devices are developed. In general, electromechanical energy conversion devices can be divided into three categories:

(1) Transducers (for measurement and control)

These devices transform the signals of different forms. Examples are microphones, pickups, and speakers.

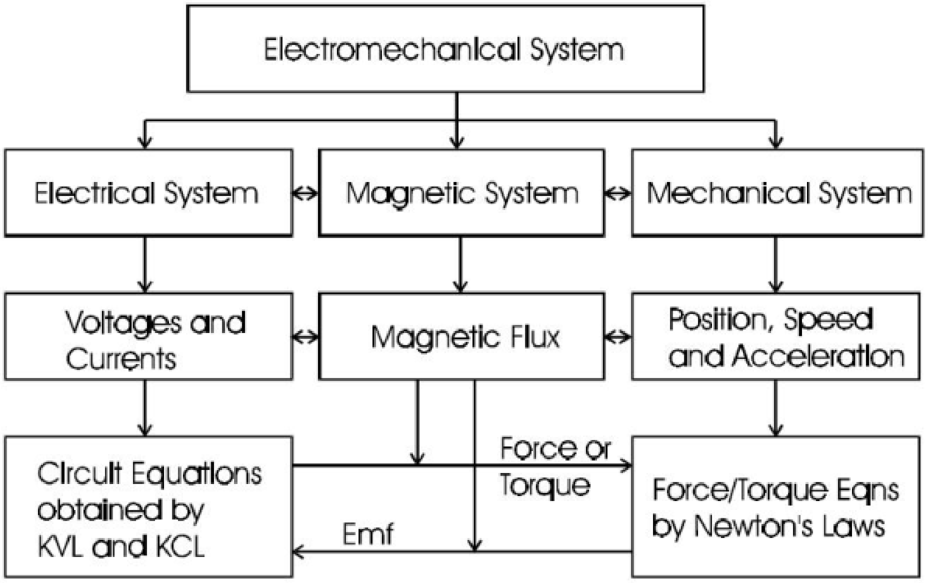
(2) Force producing devices (linear motion devices)

These type of devices produce forces mostly for linear motion drives, such as relays, solenoids (linear actuators), and electromagnets.

(3) Continuous energy conversion equipment

These devices operate in rotating mode. A device would be known as a generator if it converts mechanical energy into electrical energy, or as a motor if it does the other way around (from electrical to mechanical).

Concept of Electromechanical System Modeling



Concept map of electromechanical system modeling

- Electromechanical system coverts electrical energy into mechanical energy through magnetic media and vice-versa as shown in fig.
- Measurements in electrical system are voltage and currents.
- Measurements in mechanical system are Force, torque, angular speed and position & angle.
- Measurements in Magnetic system are mmf and flux.
- Analysis by KVL and KCL in Electrical system.
- Analysis by Newtons laws in mechanical system.

Principle of Energy Conversion

The principle of energy conversion is based on the principle of conservation of energy. According to this principle "Energy can neither be created nor destroyed".

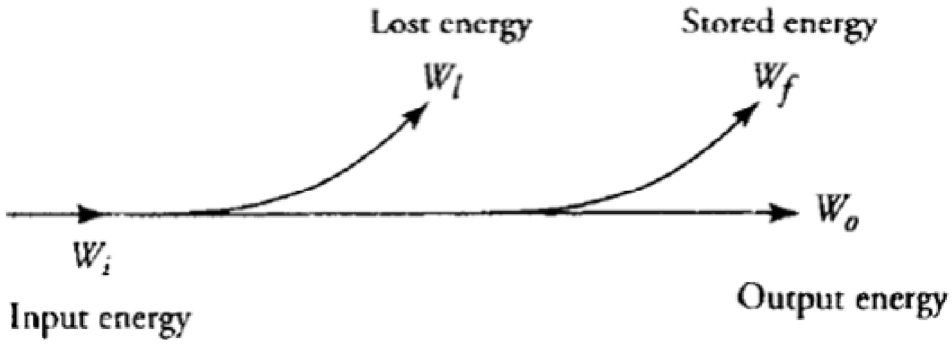
In energy conversion device, out of the total input energy, some energy is converted into the required output, some energy is stored and the rest is dissipated.

It can be written as

Energy input = Energy output + Energy stored + Energy dissipated ---- 1

This is as shown in energy flow diagram in fig.

Energy Flow Diagram



Energy Flow Diagram

Induced *emf* in Electromechanical Systems

A conductor of length l placed in a uniform magnetic field of flux density **B**. When the conductor moves at a speed **v**, the induced *emf* in the conductor can be determined by

$$\mathbf{e} = l\mathbf{v} \times \mathbf{B}$$

The direction of the *emf* can be determined by the "right hand rule" for cross products.

In a coil of N turns, the induced *emf* can be calculated by

$$e = -\frac{d\lambda}{dt}$$

Where λ is the flux linkage of the coil and the minus sign indicates that the induced current opposes the variation of the field. It makes no difference whether the variation of the flux linkage is a result of the field variation or coil movement.

In practice, it would convenient if we treat the *emf* as a voltage. The above express can then be rewritten as

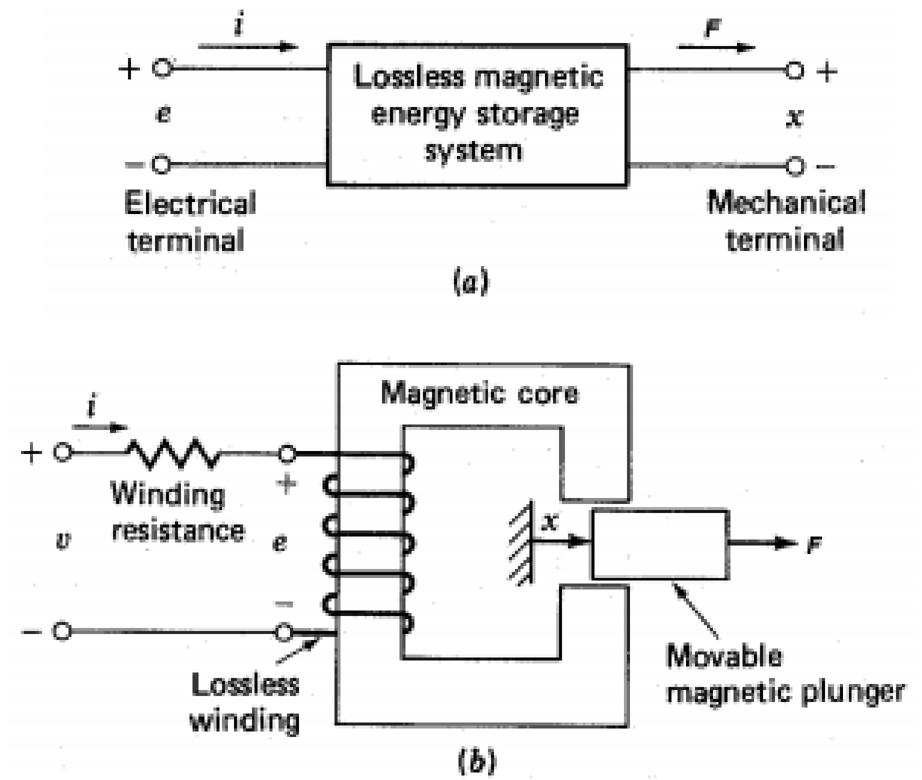
$$e = \frac{d\lambda}{dt} = L\frac{di}{dt} + i\frac{dL}{dx}\frac{dx}{dt}$$

if the system is magnetically linear, i.e. the self inductance is independent of the current. It should be noted that the self ***inductance is a function of the displacement*** *x* since there is a moving part in the system.

Singly Excited System:

Consider a singly excited linear actuator as shown below. The winding resistance is R. At a certain time instant *t*, the terminal voltage, v is applied to the excitation winding, the excitation winding current *i*, the position of the movable plunger *x*, and the force acting on the plunger **F** with the reference direction chosen in the positive direction of the *x* axis, as shown in the diagram. After a time interval *dt*, notice that the plunger has moved for a distance *dx* under the action of the force **F**. The mechanical done by the force acting on the plunger during this time interval is thus

$$dw_m = Fdx$$



Singly Excited system energy conversion

The amount of electrical energy that has been transferred into the magnetic field and converted into the mechanical work during this time interval can be calculated by subtracting the power loss dissipated in the winding resistance from the total power fed into the excitation winding as

$$dw_e = dw_f + dw_m = vidt - Ri^2dt$$

Since,

$$e = \frac{d\lambda}{dt} = v - Ri$$

So,

$$dw_f = dw_e - dw_m = eidt - Fdx = id\lambda - Fdx$$

We can also write,

$$e = \frac{d\lambda}{dt} = v - Ri$$

$$dw_f(\lambda, x) = \frac{dw_f(\lambda, x)}{d\lambda} d\lambda + \frac{dw_f(\lambda, x)}{dx} dx$$

the energy stored in a magnetic field can be expressed as

$$w_f(\lambda, x) = \int_0^\lambda i((\lambda, x) d\lambda$$

For a magnetically linear (with a constant permeability or a straight line magnetization curve such that the inductance of the coil is independent of the excitation current) system, the above expression becomes

$$W_f(\lambda, x) = \frac{1}{2} \frac{\lambda^2}{L(x)}$$

and the force acting on the plunger is then

$$F = -\frac{\partial W_f(\lambda, x)}{\partial x} = \frac{1}{2}\left[\frac{\lambda}{L(x)}\right]^2 \frac{dL(x)}{dx} = \frac{1}{2} i^2 \frac{dL(x)}{dx}$$

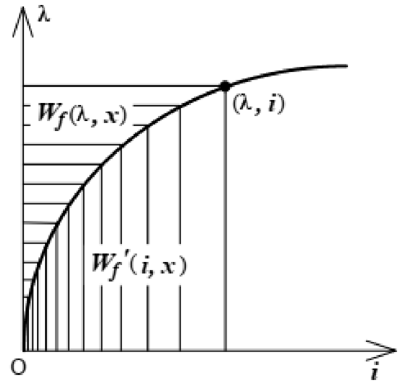
In the diagram below, it is shown that the magnetic energy is equivalent to the area above the magnetization or λ-i curve.

Mathematically, we define the area underneath the magnetization curve as the ***co-energy*** (which does not exist physically), i.e.

$$W_f'(i, x) = i\lambda - W_f(\lambda, x)$$

we can obtain

$$\begin{aligned} dW_f'(i, x) &= \lambda di + id\lambda - dW_f(\lambda, x) \\ &= \lambda di + Fdx \\ &= \frac{\partial W_f'(i, x)}{\partial i} di + \frac{\partial W_f'(i, x)}{\partial x} dx \end{aligned}$$



Energy and coenergy

Therefore,

$$\lambda = \frac{\partial W_f'(i, x)}{\partial i} \qquad \text{and} \qquad F = \frac{\partial W_f'(i, x)}{\partial x}$$

From the above diagram, the co-energy or the area underneath the magnetization curve can be calculated by

$$W_f'(i, x) = \int_0^i \lambda(i, x) di$$

For a magnetically linear system, the above expression becomes

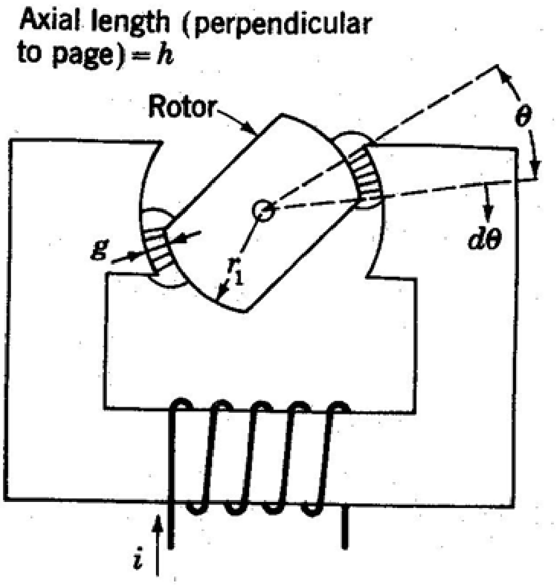
$$W_f'(i,x) = \frac{1}{2} i^2 L(x)$$

and the force acting on the plunger is then

$$F = \frac{\partial W_f'(i,x)}{\partial x} = \frac{1}{2} i^2 \frac{dL(x)}{dx}$$

Singly Excited Rotating Actuator

The singly excited linear actuator mentioned above becomes a singly excited rotating actuator if the linearly movable plunger is replaced by a rotor, as illustrated in the diagram on the right hand side. Through a derivation similar to that for a singly excited linear actuator, one can readily obtain that the torque acting on the rotor can be expressed as the negative partial derivative of the energy stored in the magnetic field against the angular displacement or as the positive partial derivative of the coenergy against the angular displacement, as Summarized in the following table.



A singly excited rotating actuator

Table: Torque in a singly excited rotating actuator

Energy	**Coenergy**
In general,	
$dW_f = i d\lambda - T d\theta$	$dW_f' = \lambda di + T d\theta$
$W_f(\lambda,\theta) = \int_0^\lambda i(\lambda,\theta) d\lambda$	$W_f'(i,\theta) = \int_0^i \lambda(i,\theta) di$
$i = \frac{\partial W_f(\lambda,\theta)}{\partial \lambda}$	$\lambda = \frac{\partial W_f'(i,\theta)}{\partial i}$
$T = -\frac{\partial W_f(\lambda,\theta)}{\partial \theta}$	$T = \frac{\partial W_f'(i,\theta)}{\partial \theta}$
If the permeability is a constant,	
$W_f(\lambda,\theta) = \frac{1}{2} \frac{\lambda^2}{L(\theta)}$	$W_f'(i,\theta) = \frac{1}{2} i^2 L(\theta)$
$T = \frac{1}{2}\left[\frac{\lambda}{L(\theta)}\right]^2 \frac{dL(\theta)}{d\theta} = \frac{1}{2} i^2 \frac{dL(\theta)}{d\theta}$	$T = \frac{1}{2} i^2 \frac{dL(\theta)}{d\theta}$

Multiply Excited Magnetic Field Systems or Doubly Excited Rotating Actuator

Multiply Excited Magnetic Field Systems – Singly-excited devices discussed earlier, are generally employed for motion through a limited distance or rotation through a prescribed angle. Electro-mechanical transducers have the special requirement of producing an electrical signal proportional to forces or velocities or producing force proportional to electrical signal (current or voltage). Such transducers require two excitations one excitation establishes a magnetic field of specified strength while the other excitation produces the desired signal (electrical or mechanical). Also continuous energy conversion devices motors and generators require multiple excitations.

Figure 4.15 shows a magnetic field system with two electrical excitations one on stator and the other on rotor. The system can be described in either of the two sets of three independent variables; (λ_1, λ_2, Θ) or (i_1, i_2, Θ). In terms of the first set

$$T_f = -\frac{\partial W_f(\lambda_1, \lambda_2, \theta)}{\partial \theta} \tag{4.62}$$

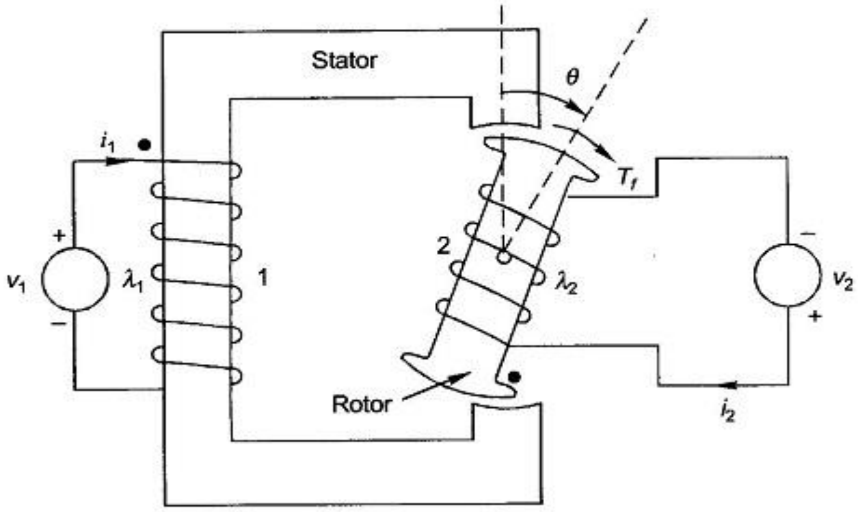


Fig. 4.15

Where the field energy is given by

$$W_f(\lambda_1, \lambda_2, \theta) = \int_0^{\lambda_1} i_1\, d\lambda_1 + \int_0^{\lambda_2} i_2 d\lambda_2 \tag{4.63}$$

Analogous to Eq. (4.28)

$$i_1 = \frac{\partial W_f(\lambda_1, \lambda_2, \theta)}{\partial \lambda_1}$$

$$i_2 = \frac{\partial W_f(\lambda_1, \lambda_2, \theta)}{\partial \lambda_2}$$

Assuming linearity

$$\lambda_1 = L_{11}i_1 + L_{12}i_2 \tag{4.64a}$$

$$\lambda_2 = L_{21}i_1 + L_{22}i_2;\ (L_{12} = L_{21}) \tag{4.64b}$$

Solving for i_1 and i_2 in terms of λ_1, λ_2 and substituting in Eq. (4.63) gives upon integration*

$$W_f(\lambda_1, \lambda_2, \theta) = \frac{1}{2}\ \beta_{11}\lambda_1^2 + \beta_{12}\lambda_1\lambda_2 + \frac{1}{2}\ \beta_{22}\lambda_2^2 \tag{4.65}$$

where
$$\beta_{11} = L_{22}/(L_{11}L_{22} - L_{12}^2)$$

$$\beta_{22} = L_{11}/(L_{11}L_{22} - L_{12}^2)$$

$$\beta_{12} = \beta_{21} = -L_{12}/(L_{11}L_{22} - L_{12}^2)$$

The self- and mutual-inductance of the two exciting coils are functions of angle θ.

If currents are used to describe the system state

$$T_f = \frac{\partial W_f'(i_1, i_2, \theta)}{\partial\theta} \tag{4.66}$$

where the coenergy is given by

$$W_f'(i_1, i_2, \theta) = \int_0^{i_1} \lambda_1\, di_1 + \int_0^{i_2} \lambda_2\, di_2 \tag{4.67}$$

In the linear case

$$W'_f(i_1, i_2, \theta) = \frac{1}{2} L_{11} i_i^2 + L_{12} i_1 i_2 + \frac{1}{2} L_{22} i_2^2$$

where inductances are functions of angle θ.

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* $$i_1 = \beta_{11}\lambda_1 + \beta_{12}\lambda_2$$

$$i_2 = \beta_{21}\lambda_1 + \beta_{22}\lambda_2;\ \beta_{21} = \beta_{12}$$

$$W_f(\lambda_1, \lambda_2, \theta) = \int_0^{\lambda_1} (\beta_{11}\lambda_1 + \beta_{12}\lambda_2)\, d\lambda_1 + \int_0^{\lambda_2} (\beta_{12}\lambda_1 + \beta_{22}\lambda_2)\, d\lambda_2$$

$$= \beta_{11} \int_0^{\lambda_1} \lambda_1 d\lambda_1 + \beta_{12} \left[\int_0^{\lambda_1} \lambda_2 d\lambda_1 + \int_0^{\lambda_2} \lambda_1 d\lambda_2\right] + \beta_{22} \int_0^{\lambda_2} \lambda_2 d\lambda_2$$

$$= \beta_{11} \int_0^{\lambda_1} \lambda_1 d\lambda_1 + \beta_{12} \int_0^{\lambda_1, \lambda_2} d(\lambda_1\lambda_2) + \beta_{22} \int_0^{\lambda_2} \lambda_2 d\lambda_2$$

$$= \frac{1}{2}\ \beta_{11}\lambda_1^2 + \beta_{12}\lambda_1\lambda_2 + \frac{1}{2}\ \beta_{22}\lambda_2^2$$

EMF & Torque Equations in rotating machines

EMF Equation

Consider a D.C generator whose field coil is excited to produce a flux density distribution along the air gap and the armature is driven by a prime mover at constant speed as shown in figure 37.1.

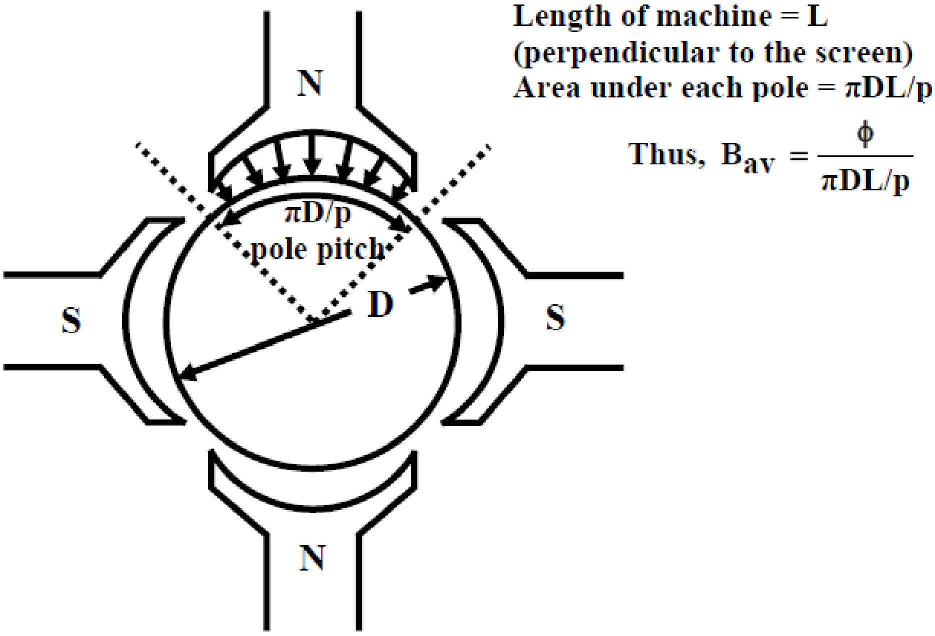

Figure 37.1: Pole pitch & area on armature surface per pole.

Let us assume a ***p*** polar d.c generator is driven (by a prime mover) at ***n*** rps. The excitation of the stator field is such that it produces a flux **φ** Wb per pole. Also let ***z*** is the total number of armature conductors and '***a'*** is the number of parallel paths in the armature circuit.

Total average voltage across the brushes is calculated on the basis of average flux density $\boldsymbol{B_{av}}$

$$\text{Average flux density } B_{av} = \frac{\phi}{\left(\frac{\pi D}{p}\right)L}$$

$$= \frac{\phi\, p}{\pi\, DL}$$

$$\text{Induced voltage in a single conductor} = B_{av}Lv$$

$$\text{Number of conductors present in each parallel path} = \frac{z}{a}$$

$$\text{If } v \text{ is the tangential velocity then, } v = \pi Dn$$

$$\text{Therefore, total voltage appearing across the brushes} = \frac{z}{a}B_{av}Lv$$

$$= \frac{z}{a}\frac{\phi p}{\pi\, DL}L\pi\, Dn$$

$$\text{Thus voltage induced across the armature, } E_A = \frac{pz}{a}\phi\, n \qquad (37.1)$$

We thus see that across the armature a voltage will be generated so long there exists some flux per pole and the machine runs with some speed. Therefore irrespective of the fact that the machine is operating as generator or as motor, armature has an induced voltage in it governed essentially by the above derived equation. This emf is called *back emf* for motor operation.

Torque equation

Whenever armature carries current in presence of flux, conductor experiences force which gives rise to the electromagnetic torque. In this section we shall derive an expression for the electromagnetic torque T_e developed in a d.c machine. Obviously T_e will be developed both in motor and generator mode of operation. It may be noted that the direction of conductor currents reverses as we move from one pole to the other. This ensures unidirectional torque to be produced. The derivation of the torque expression is shown below.

$$\text{Let, } I_a = \text{Armature current}$$

$$\text{Average flux density } B_{av} = \frac{\phi p}{\pi DL}$$

$$\text{Then, } \frac{I_a}{a} = \text{Current flowing through each conductor.}$$

$$\text{Force on a single conductor} = B_{av}\frac{I_a}{a}L$$

$$\text{Torque on a single conductor} = B_{av}\frac{I_a}{a}L\frac{D}{2}$$

$$\text{Total electromagnetic torque developed, } T_e = zB_{av}\frac{I_a}{a}L\frac{D}{2}$$

$$\text{Putting the value of } B_{av}\text{, we get } T_e = \frac{pz}{2\pi a}\phi I_a \qquad (37.2)$$

Thus we see that the above equation is once again applicable both for motor and generator mode of operation. The direction of the electromagnetic torque, T_e will be along the direction of rotation in case of motor operation and opposite to the direction of rotation in case of generator operation. When the machine runs steadily at a constant rpm then $T_e = T_{load}$ and $T_e = T_{pm}$, respectively for motor and generator mode.