NBKRIST

ELECTROMECHANICAL ENERGY CONVERSION – III
LECTURE NOTES

UNIT-1

DEPARTMENT OF ELECTRICAL AND ELECTRONICS ENGINEERING

Alternator Synchronous Generator

***Definition**: The definition of alternator is hidden in the name of this machine itself. An alternator is such a machine which converts mechanical energy from a prime mover to AC electric power at specific voltage and current. It is also known as synchronous generator.*

Alternator.

An alternator consists of armature winding and field magnet, but the difference between the alternator and DC generator is that in the DC generator armature rotates and the field system is stationary. This arrangement is the alternator is just reverse of it there the armature is stationary called as stator and field system is rotating called as Rotor.

For generating EMF, three things are essential:

1) Magnetic field
2) System of conductors
3) Relative motion between those two.

The conductors are mounted on the stators and the field poles are mounted on the Rotor core Relative motion between the stator conductors and the field is brought about rotating the field system.

The rotor is coupled mechanically to a suitable prime mover. When the prime mover runs, the rotor core also rotates and the field flux is cut by the stationary stator conductors and EMF's are induced in them.

If a load is connected across the stator terminals electric power would be delivered to it.

Advantages of Stationary Armature

- The generated power can be easily taken from the stator.
- There is no possibility of the armature conductors flying off, when the machine runs at high speed since they are housed in the stator slots.
- There is no difficulty in insulating the armature (stationary) winding for very High voltages, i.e, as high as 30 kV or more.
- Two slip rings are required for the supply of DC energy required for rotor field excitation. Since exciting current is to be supplied at low voltage, there is no difficulty in insulating them.
- Rotating field is competitively light and can run with high speeds.

Differences:-

STATIONARY FIELD SYSTEM	ROTATING FIELD SYSTEM
4 slip rings are required.	2 slip rings are required.
Heavy armature current passes through slip rings.	Very low field current passes through slip rings.
More sparking at slip rings.	No sparking at slip rings.
Armature supply is taken through slip rings.	Armature supply is taken through fixed connections.
Capacity is limited to 30KVA.	It can be designed to any capacity.
Voltage is limited to 440 V.	Voltage is up to 33KV is generated.

Low efficiency.	High efficiency.
More maintenance.	Less maintenance.

Working Principle of Alternator

The working principle of an alternator or AC generator is similar to the basic working principle of a DC generator.

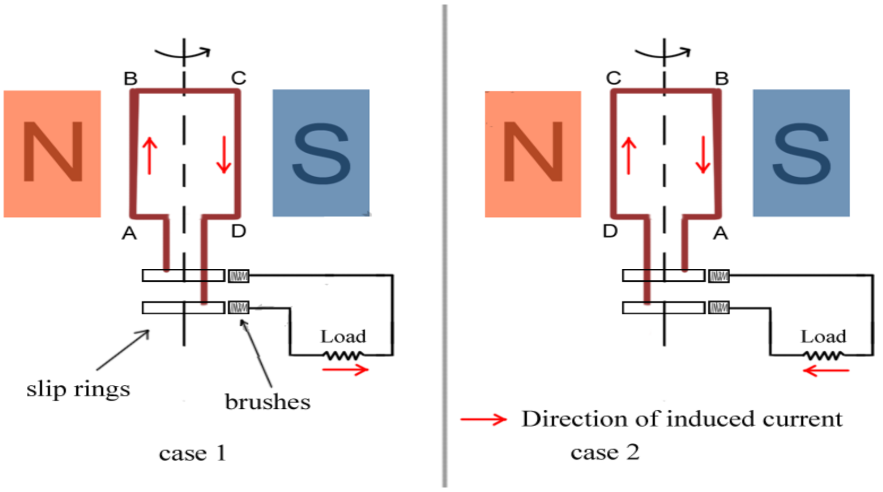


Fig. 1.1

Above fig.1.1 helps you understanding **how an alternator or AC generator works**. According to the Faraday's law of electromagnetic induction, whenever a conductor moves in a magnetic field EMF gets induced across the conductor. If the close path is provided to the conductor, induced emf causes current to flow in the circuit.

Now, see the above fig.1.1. Let the conductor coil ABCD is placed in a magnetic field. The direction of magnetic flux will be form N pole to S pole. The coil is connected to slip rings, and the load is connected through brushes resting on the slip rings.

Now, consider the case 1 from above figure. The coil is rotating clockwise, in this case the direction of induced current can be given by Fleming's right hand rule, and it will be along A-B-C-D.

As the coil is rotating clockwise, after half of the time period, the position of the coil will be as in second case of above fig.1. In this case, the direction of the induced current according to Fleming's right hand rule will be along D-C-B-A. It shows that, the direction of the current changes after half of the time period that means we get an alternating current.

Here, we have considered that the magnetic field is stationary and conductors (armature) are rotating. But generally in practical construction of alternator, armature conductors are stationary and field magnets rotate between them. The rotor of an alternator or a synchronous generator is mechanically coupled to the shaft or the turbine blades, which on being made to rotate at synchronous speed N_s under some mechanical force results in magnetic flux cutting of the stationary armature conductors housed on the stator. As a direct consequence of this flux cutting an induced emf and current starts to flow through the armature conductors which first flow in one direction for the first half cycle and then in the other direction for the second half cycle for each winding with a definite time lag of 120° due to the space displaced arrangement of 120° between them as shown in the fig. 1.2 below. This particular phenomena result in 3φ power flow out of the alternator which is then transmitted to the distribution stations for domestic and industrial uses.

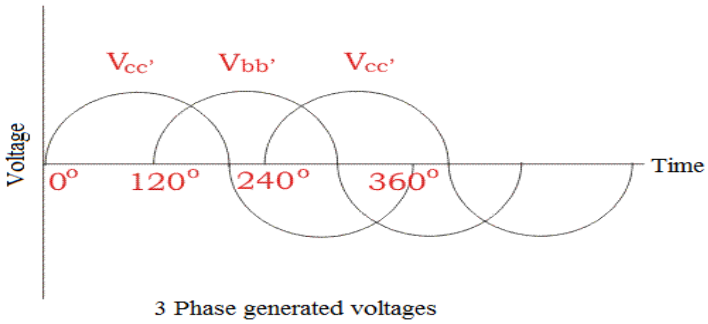


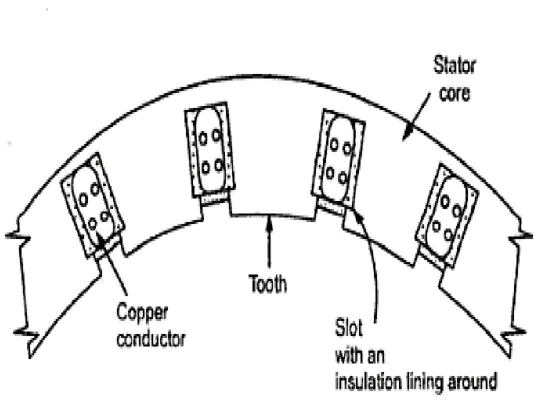
Fig. 1.2

Construction:-

An alternator consists of mainly two parts

1. Stator
2. Rotor

1. <u>Stator:-</u>



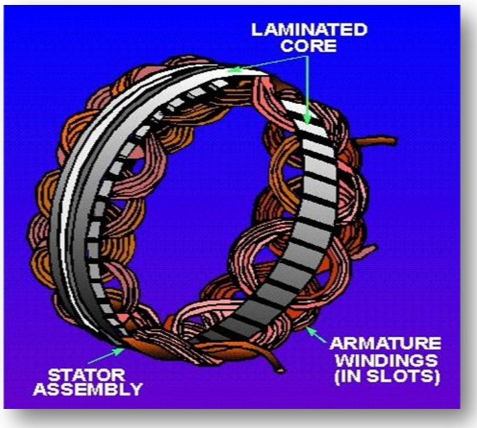


Fig.1.3

The armature core is supported by the stator frame and is built up of laminations of special magnetic iron or steel iron alloy the core is laminated to minimize the loss due to Eddy currents.

The laminations are stamped out in complete rings or segments. The laminations are insulated from each other and have space between them for allowing the cooling air to pass through.

The inner periphery of the stator is slotted and copper conductors which are joined to one another constituting armature winding housed in these slots as shown in fig. 1.3.. The other ends of the winding are brought out are connected to fixed terminal from which the generator power can be taken out.

2. Rotor:-

Depending upon the type of application, these are classified into two types

1) Salient-pole or projecting pole type
2) Non salient-pole or round rotor or cylindrical rotor

Salient-pole or projecting pole type: The fig.1.4 shows the Salient-pole or projecting pole type rotor.

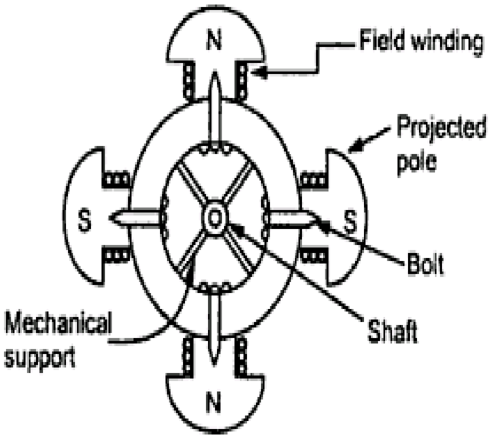
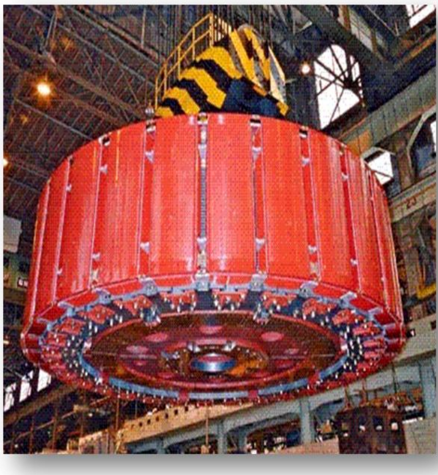


Fig.1.4



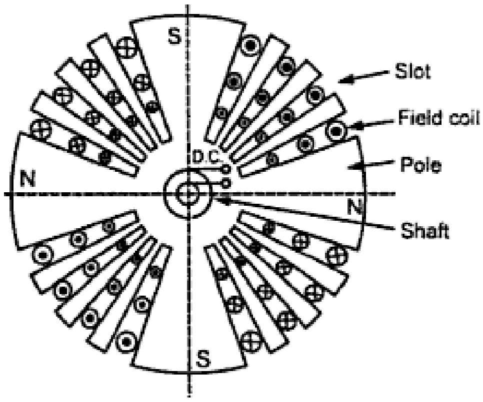
Actual Salient Rotor

- It is used for and medium speed alternators used in hydro and diesel power generating station.
- The poles are made of laminated sheets and fixed to the rotor by dove tail joint.
- Short circuited damper bars are placed in the slots provided on the pole surfaces.
- These are used to prevent hunting and to provide starting torque in synchronous motors.
- The field coils are placed on the poles as shown in the figure 2.

Key features:-

- It has non-uniform air gap.
- The diameter of the rotor is more than of the cylindrical rotor.
- The no. of holes is higher than that of the non salient-pole rotor
- Axial length is less.
- The prime mover speed is less and is driven in hydal turbines
- These generators are used in hydro electric stations so these are called as hydro generators.

Non-Salient pole type (or) cylindrical type (or) Round rotor: The fig.1.5 shows the Non salient-pole or round rotor or cylindrical rotor.



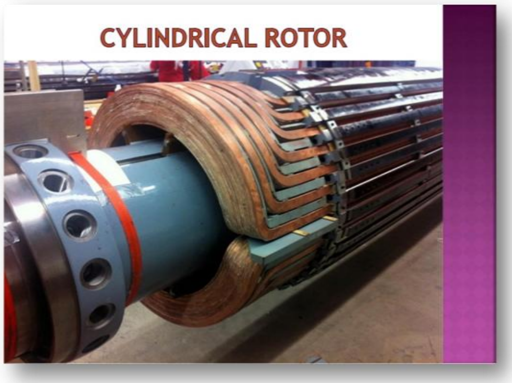


Fig.1.5

- Slots are provided in between the poles and these slots are placed with field winding Conductors

Features :-

- No. of poles are less when compared to salient pole type.
- Diameter is less
- Axial length is more
- Air gap is uniform
- Prime mover speed is more and is driven in thermal turbines.
- These are used in thermal stations so, these are called as turbo Generators.

<u>Frequency of the induced EMF:-</u>

Consider an alternate whose rotor is driver at a constant speed N rpm.
Let "p" be no. of poles and f is the frequency of the generated voltage
No. of cycles of the induced EMF per sec

= No. of cycles per revolutions × no. of revolutions\sec

= P /2 × N/60 = PN ×120. Frequency f = PN/120 Hz.

For a given alternator, the no. of poles is fixed.

Hence in order to generate power at a specified frequency, the machine is to be run at a definite speed which is termed as synchronous speed.

<u>Armature Winding</u>

Armature Winding is the windings, in which voltage is induced. The **Field Winding** is the winding in which the main field flux is produced when the current through the winding is passed.

Some of the **basic terms** related to the Armature Winding are defined as follows:

- **Turn**: A turn consists of two conductors connected to one end by an end connector as shown in Fig.1.6 (a).
- **Coil**: A coil is formed by connecting several turns in the series as shown in Fig.1.6 (b).
- **Winding**: A winding is formed by connecting several coils in series as shown in Fig.1.6 (c).

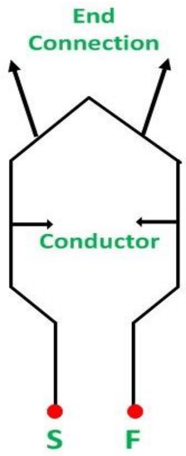


Fig.1.6 (a)

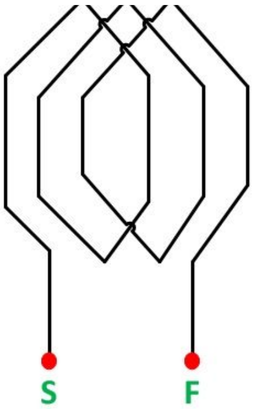


Fig.1.6 (b)

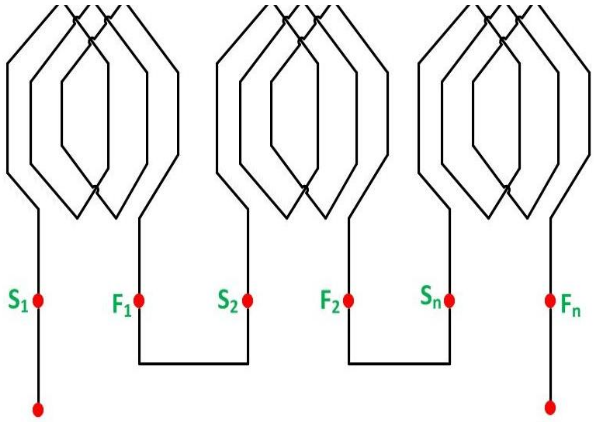


Fig.1.6 (c)

The beginning of the turn or coil is identified by the symbol **(S)** meaning **Start**, and the end of the turn or coil is represented by the symbol **(F)** meaning **Finish**.

The concept of electrical degree is very important in the study of the machine. For a (P) pole machine, the electrical degree is defined as given below.

$$\theta_{ed} \triangleq \frac{P}{2}\theta_{md} \;\ldots\ldots\ldots (1)$$

Where,

$\boldsymbol{\Theta_{md}}$ is the mechanical degrees or an angular measure in space.

$\boldsymbol{\Theta_{ed}}$ is the electrical degrees or an angular measure in cycles.

The advantage of this notation is that the expressions written in terms of electrical angles apply to the machine any number of poles.

The angular distance between the centers of two adjacent poles on a machine is known as **pole pitch** or **pole span**.

$$\text{One pole pitch} = 180^\circ_{ed} = \frac{360^\circ_{md}}{P} \;\ldots\ldots (2)$$

The pole pitch is always 180 degrees electrical regardless of the number of poles in a machine.

The two sides of a coil are placed in two slots on the stator surface. The distance between the two sides of a coil is called the **coil-pitch**. If the coil pitch is one pole pitch, it is called the **Full Pitch Coil**. If the coil pitch is less than one pole pitch, the coil is called the **Short Pitch** or **Fractional Pitch coil**.

EMF Equation of an alternator:

Consider the following

Φ = flux per pole in wb

P = Number of poles

N_S = Synchronous speed in rpm

f = frequency of induced emf in Hz

Z = total number of stator conductors

Z_{ph} = conductors per phase connected in series

T_{ph} = Number of turns per phase

Assuming concentrated winding, considering one conductor placed in a slot

According to Faradays Law electromagnetic induction,

The average value of emf induced per conductor in one revolution

$e_{avg} = d\Phi /dt$ = Change of Flux in one revolution/ Time taken for one revolution

Change of Flux in one revolution = $P\Phi$

Time taken for one revolution = $60/N_S$ seconds

Hence $e_{avg} = (P\Phi) / (60/Ns) = (P\Phi) Ns / 60$

We know f = PNs /120

Hence PNs /60 = 2f

Hence eavg = 2Φf volts

Hence average emf per turn = 2 x 2Φf volts = 4Φf volts

If there are T_{ph}, number of turns per phase connected in series, then average emf induced in Tph turns is

$E_{ph, avg} = T_{ph}$ x eavg = $4\Phi f T_{ph}$ volts

Hence RMS value of emf induced, E = 1.11 x $E_{ph, avg}$ = 1.11 x $4f\Phi T_{ph}$ volts

= $4.44f\Phi$Tph volts

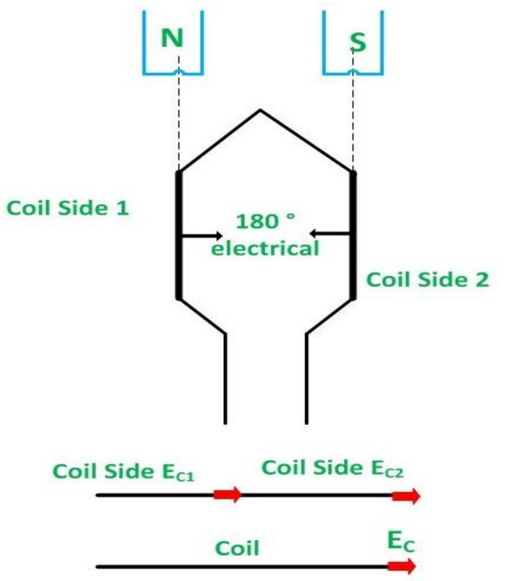
This is the general emf equation for the machine having concentrated and full pitched winding.

In practice, alternators will have short pitched winding and hence coil span will not be 180^0, but one or two slots short than the full pitch.

Coil Span Factor

The **Coil Span Factor** or **Pitch Factor** K_C is defined as the ratio of the voltage generated in the short pitch coil to the voltage generated in the full pitch coil. The distance between he two sides of a coil is called the **Coil Span** or **Coil Pitch Factor**. It is also known as **Chording Factor**

The angular distance between the central line of one pole to the central line of the next pole is called **Pole Pitch**. A pole pitch is always 180 electrical degrees, regardless of the number of poles on the machine. A coil having a span equal to 180^0 electrical is called a **full pitch coi**l as shown in the Fig. 1.7(a).



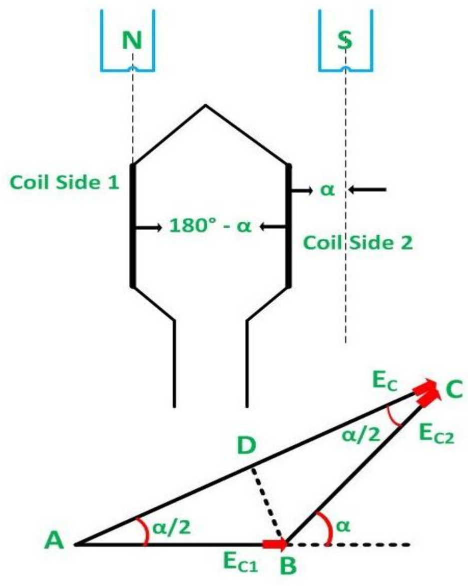


Fig. 1.7(a) Fig.1.7(b)

A Coil having a span less than 180^0 electrical is called a **short pitch coil** or fractional pitch coil. It is also called a Chorded coil. The short pitch coil factor is shown in the Fig. 1.7(b)

A stator winding using fractional pitch coil is called a chorded winding. If the span of the coil is reduced by an angle α electrical degrees, the coil span will be (180 – α) electrical degrees.

In case of a full pitch coil, the distance between the two sides of the coil is exactly equal to the pole pitch of 180^0 electrical. As a result, the voltage in a full pitch coil is such that the voltage of each side of the coil is in phase.

Let E_{C1} and E_{C2} be the voltages generated in the coil sides, and E_C is the resultant coil voltage.

Then the equation is given as shown below.

$$E_C = E_{C1} + E_{C2}$$

$$|E_{C1}| = |E_{C2}| = E_1 \quad \text{(Say)}$$

Since E_{C1} and E_{C2} are in phase, the resultant coil voltage E_C is equal to their arithmetic sum. Therefore,

$$E_C = E_{C1} + E_{C2} = 2E_1$$

If the coil span of a single coil is less than the pole pitch of 180^0 electrical, the voltages generated on each coil side are not in phase. The resultant coil voltage E_C is equal to the phasor sum of E_{C1} and E_{C2}

If the coil span is reduced by an angle α electrical degrees, the coil span is (180 – α) electrical degrees. The voltage generated E_{C1} and E_{C2} in the two coil sides will be out of phase with respect to each other by an angle α electrical degrees. The phasor sum of E_{C1} and E_{C2} is E_C, which is equal to AC as shown in the phasor diagram Fig. 1.7(b).

The coil span factor is represented as

$$K_C = \frac{\text{Actual Voltage Generated in the Coil}}{\text{Voltage Generated in the coil of span } 180° \text{ electrical}}$$

$$K_C = \frac{\text{Phasor sum of the voltages of two coil sides}}{\text{Arithmetic sum of the voltages of two coil sides}}$$

$$K_C = \frac{AC}{2AB} = \frac{2AD}{2AB} = \cos\frac{\alpha}{2}$$

$$K_C = \cos\frac{\alpha}{2}$$

For full pitch coil, the value of α will be 0^0, cos α/2 = 1 and $\mathbf{K_C = 1}$.

For a short pitch coil $\boldsymbol{K_C < 1}$.

Advantages of Short Pitch Coil or Chording

- It shortens the ends of the winding and, therefore, there is a saving in the conductor's material.
- It Reduces the effects of distorting harmonics and thus the waveform of the generated voltage is improved and making it a sine wave.

Distribution Factor

The **Distribution Factor** or the **Breadth Factor** is defined as the ratio of the actual voltage obtained to the possible voltage if all the coils of a polar group were concentrated in a single

$$K_d = \frac{\text{Phasor Sum of the coil volatges per phase}}{\text{Arithmetic sum of coil voltages per phase}} \quad \ldots\ldots\ldots(1)$$

In a concentrated winding, each phase of a coil is concentrated in a single slot. The individual coil voltages induced are in phase with each other. These voltages must be added arithmetically. In order to determine the induced voltage per phase, a given coil voltage is multiplied by the number of series connected coils per phase. In actual practice, in each phase, coils are not concentrated in a single slot. They are distributed in a number of slots in space to form a polar group under each pole.

The voltages induced in coil sides are not in phase, but they differ by an angle β which is known as the angular displacement of the slots. The phasor sum of the individual coil voltages is equal to the total voltage induced in any phase of the coil.

Let,

M = slots per pole per phase

$$m = \frac{\text{slots}}{\text{poles x phases}} \quad \ldots\ldots(2)$$

β = angular displacement between adjacent slots in electrical degrees

$$\beta = \frac{180°}{\text{slots/pole}} = \frac{180° \text{ x poles}}{\text{slots}} \quad \ldots\ldots(3)$$

Thus, one phase of the winding consists of coils arranged in m consecutive slots. Voltages E_{C1}, E_{C2}, E_{C3}….. are the individual coil voltages. Each coil voltage E_C will be out of phase with the next coil voltages by the slot pitch β.

The Fig. 1.8 shows the voltage polygon of the induced voltages in the four coils of a group (m = 4)

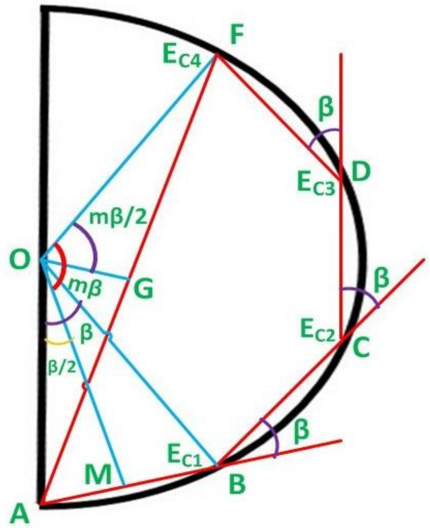


Fig. 1.8

The voltages E_{C1}, E_{C2}, E_{C3} and E_{C4} are represented by the phasors AB, BC, CD and DF respectively. Each of these phasors is a chord of a circle with the center O and subtends an angle β at the point O. The phasor sum AF, represents the resultant winding voltage, subtends at an angle mβ at the center.

The arithmetic sum of the individual coil voltage is given as

$$mE_C = mAB = m(2AM)$$

$$= 2mOA \text{ Sin } AOM = 2m \text{ OA Sin } \beta/2$$

The phasor sum of the individual coil voltages is given as

$$= AF = 2AG = 2 \text{ OA Sin AOG} = 2 \text{ OA Sin } \frac{m\beta}{2}$$

Therefore, from the equation (1) shown above, we know that,

$$K_d = \frac{\text{Phasor Sum of the coil volatges per phase}}{\text{Arithmetic sum of coil voltages per phase}} = \frac{2 \text{ OA Sin m } \beta/2}{2 \text{ OA m Sin } \beta/2} \quad \text{or}$$

$$K_d = \frac{\text{Sin m } \beta/2}{\text{m Sin } \beta/2} \quad \dots\dots (4)$$

The distribution factor K_d for a given number of phases is dependent only on the number of distributed slots under a given pole. It is independent of the type of the winding, lap or wave or the number of turns per coil, etc. the distribution factor decreases as the number of slots per pole increases.

$\mathbf{E_{ph} = 4.44\, K_p K_d\, f T_{ph}}$ vlolts

Hence the line Voltage $\mathbf{E_L = \sqrt{3} \text{ x phase voltage} = \sqrt{3}\, E_{ph}}$

Armature Reaction

The effect of the armature flux on the main flux affecting its value and the distribution is called armature reaction.

The effect of the armature flux not only depends on the magnitude of the current flowing through the armature winding but also depends on the nature of the power factor of the load connected to the alternator.

Now we will study the effect of nature of the load power factor on the armature reaction.

Unity Power Factor Load

Consider a purely resistive load connected to the alternator, having unity power factor. As induced e.m.f. E_{ph} drives a current of I_{aph} and load power factor is unity, E_{ph} and I_{ph} are in phase with each other.

If Φ_f is the main flux produced by the field winding responsible for producing E_{ph} then E_{ph} lags Φ_f by 90^o .

Now current through armature I_a, produces the armature flux say Φ_a. So flux Φ_a and I_a are always in the same direction.

This relation between Φ_f , Φ_a, E_{ph} and I_{aph} can be shown in the phasor diagram. (See Fig. 1.9)

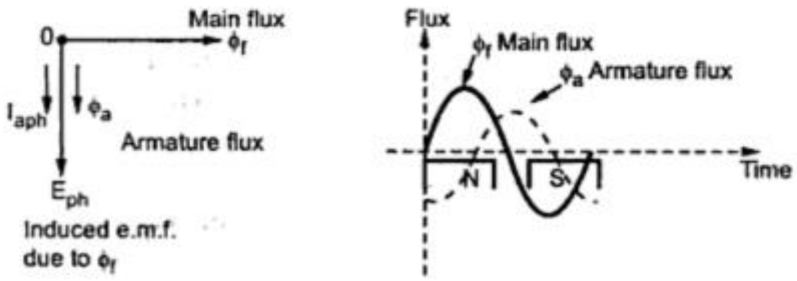


Fig. 1.9 Armature reaction for unity power factor

It can be seen from the phasor diagram that there exists a phase difference of 90^0 between the armature flux and the main flux. The waveforms for the two fluxes are also shown in the Fig. 1.9. From the waveforms it can be seen that the two fluxes oppose each other on the left half of each pole while assist each other on the right half of each pole. Hence average flux in the air gap remains constant but its distribution gets distorted.

Note: Hence such distorting effect of armature reaction under unity p.f. condition of the load is called cross magnetizing effect of armature reaction.

Due to such distortion of the flux, there is small drop in the terminal voltage of the alternator.

Zero Lagging Power Factor Load

Consider a purely inductive load connected to the alternator having zero lagging power factor. This indicates that I_{aph} driven by E_{ph} lags E_{ph} by 90^o which is the power factor angle Φ.

Induced e.m.f. E_{ph} lags main flux Φ_f by 90^o while Φ_a is in the same direction as that of I_a. So the phasor diagram and the waveforms are shown in the Fig. 1.10.

It can be seen from the phasor diagram that the armature flux and the main flux are exactly in opposite direction to each other.

Note : So armature flux tries to cancel the main flux. Such an effect of armature reaction is called demagnetizing effect of the armature reaction.

As this effect causes reduction in the main flux, the terminal voltage drops. This drop in the terminal voltage is more than the drop corresponding to the unity p.f. load.

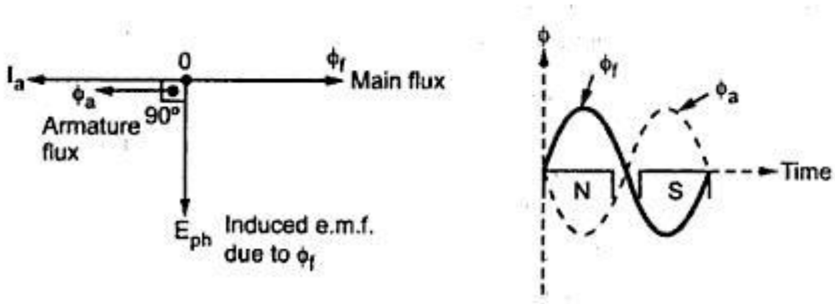


Fig. 1.10 Armature reaction for zero lagging p.f. load

Zero Leading Power Factor Load

Consider a purely capacitive load connected to the alternator having zero leading power factor. This means that armature current I_{aph} driven by E_{ph}, leads E_{ph} by 90^o, which is the power factor angle Φ.

Induced e.m.f. E_{ph} lags Φ_f by 90^o while I_{aph} and Φ_a are always in the same direction. The phasor diagram and the waveforms are shown in the Fig. 1.11.

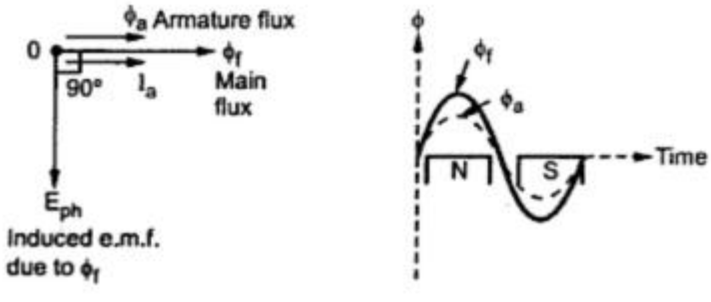


Fig. 1.11 Armature reaction for zero leading p.f. load

It can be seen from the phasor diagram and waveforms shown in the Fig. 1.10, the armature flux and the main field flux are in the same direction i.e. they are helping each other. This results into the addition in main flux.

Note : Such an effect of armature reaction due to which armature flux assists field flux is called magnetising effect of the armature reaction.

As this effect adds the flux to the main flux, greater e.m.f. gets induced in the armature. Hence there is increase in the terminal voltage for leading power factor loads.

For intermediate power factor loads i.e. between zero lagging and zero leading the armature reaction is partly cross magnetising and partly demagnetising for lagging power factor loads or partly magnetising for leading power factor loads.

Armature Reaction Reactance (X_{ar})

In all the conditions of the load power factors, there is change in the terminal voltage due to the armature reaction. Mainly the practical loads are inductive in nature, due to demagnetising effect of armature reaction, there is reduction in the terminal voltage. Now this drop in the voltage due to the interaction of armature and main flux. This drop is not across any physical element.

But to quantify the voltage drop due to the armature reaction, armature winding is assumed to have a fictitious reactance. This fictitious reactance of the armature is called armature reaction reactance denoted as X_{ar} Ω/ph. And the drop due to armature reaction can be accounted as the voltage drop across this reactance as $I_{ar} X_{ar}$.

Note : The value of this reactance changes as the load power factor changes, as armature reaction depends on the load power factor.

Armature Leakage Reactance

When armature carries a current, it produces its own flux. Some part of this flux completes its path through the air around the conductors itself. Such a flux is called leakage flux. This is shown in the Fig. 1.12.

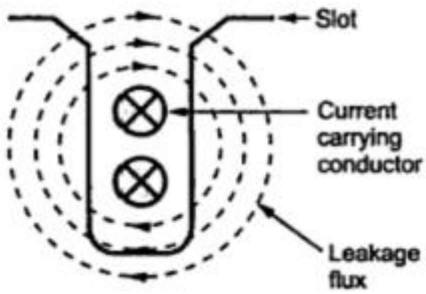


Fig. 1.12 Armature leakage flux

Note : This leakage flux makes the armature winding inductive in nature. So winding possesses a leakage reacatnce, in addition to the resistance.

So if 'L' is the leakage inductance of the armature winding per phase, then leakage reactance per phase is given by $X_L = 2 \pi f L$ Ω/ph. The value of leakage reactance is much higher than the armature resistance. Similar to the d.c. machines, the value of armature resistance is very very small.

Equivalent Circuit of an Alternator

From the previous discussion it is clear that in all there are three important parameters of armature winding namely armature resistance R_a, leakage reactance X_L and armature reaction reactance X_{ar}. If E_{ph} is induced e.m.f. per phase on no load condition then on load it changes to E' due to armature reaction as shown in the equivalent circuit Fig.1.13. As current flows through the armature, there are two voltage drops across R_a and X_L as I_a R_a and respectively. Hence finally terminal voltage V_t is less than E' by the amount equal to the drops across R_a and X_L.

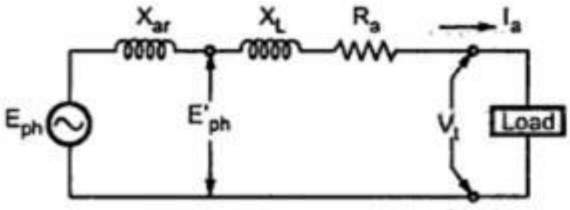


Fig. 1.13 Equivalent circuit

In practice, the leakage reactance X_L and the armature reaction reactance X_{ar} are combined to get synchronous reactance X_s.

Hence the equivalent circuit of an alternator gets modified as shown in the Fig. 1.14.

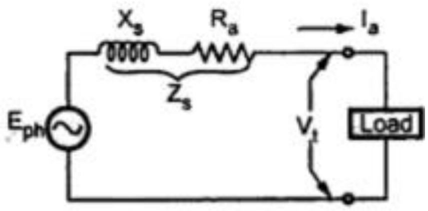


Fig. 1.14 Equivalent circuit of an alternator

Thus in the equivalent circuit shown,

E_{ph} = induced e.m.f. per phase on no load

V_{tph} = terminal voltage per phase on load

I_{aph} = armature resistance per phase

Z_s = synchronous impedance per phase

and $\quad \overline{E}_{ph} = \overline{V}_{tph} + \overline{I_a Z_s}$ ... (Phasor sum)

Phasor Diagram of a Loaded Alternator

$$\overline{E}_{ph} = \overline{V}_{ph} + \overline{I_a R_a} + \overline{I_a X_s}$$

The above voltage equation is to be realised using phasor diagrams for various load power factor conditions

Lagging Power Factor Load

The power factor of the load is cosΦ lagging so I_a lags V_{ph} by angle Φ. The Phasor diagram can be drawn as shown in the Fig. 1.16.

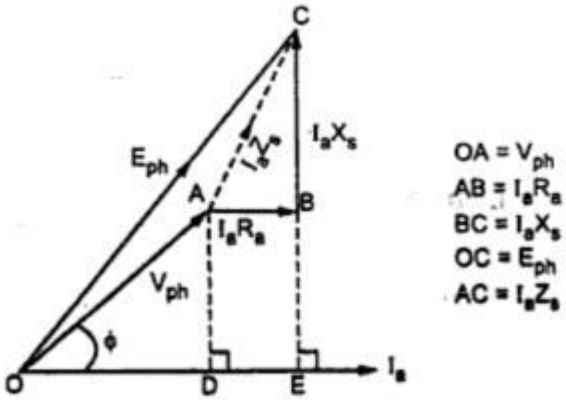


Fig. 1.16

To derive the relationship between E_{ph} and V_{ph}, the perpendiculars are drawn on the current phasor from points A and B. These intersect current phasor at points D and E respectively.

∴ $(E_{ph})^2 = (OD + DE)^2 + (BE - BC)^2$

∴ $(E_{ph})^2 = (V_{ph}\cos\Phi + I_a R_a)^2 + (V_{ph}\sin\Phi - I_a X_s)^2$

∴ $$E_{ph} = \sqrt{(V_{ph}\cos\phi + I_a R_a)^2 + (V_{ph}\sin\phi + I_a X_s)^2}$$

It can be observed that the sign of the $I_a X_s$ is negative as against its positive sign for lagging p.f. load. This is because X_s consists of X_{ar} i.e. armature reaction reactance. Armature reaction is demagnetising for lagging while magnetising for leading power factor loads. So sign of $I_a X_s$ is opposite for lagging and leading p.f. conditions.

Unity Power Factor Load

The power factor of the load is unity i.e. cosΦ = 1. So Φ = 0, which means V_{ph} is in phase with I_a. So phasor diagram can be drawn as shown in the Fig. 1.17.

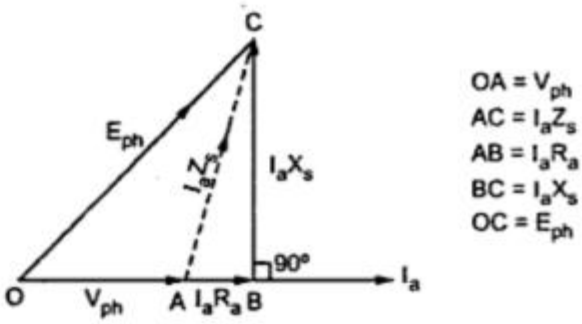


Fig. 1.17 Phasor diagram for unity p.f. load

Consider ΔOBC, for which we can write,

$(OC)^2 = (OB)^2 + (BC)^2$

∴ $(E_{ph})^2 = (OA + AB)^2 + (BC)^2$

∴ $(E_{ph})^2 = (V_{ph} + I_a R_a)^2 + (I_a Xs)^2$

∴ $$E_{ph} = \sqrt{(V_{ph} + I_a R_a)^2 + (I_a X_s)^2}$$

As cosΦ = 1, so sinΦ = 0 hence does not appear in the equation.

Note : The phasor diagrams can be drawn by considering voltage V_{ph} as a reference phasor. But to derive the relationship, current phasor selected as a reference makes the derivation much more simplified. Hence current is selected as a reference phasor.

It is clear from the phasor diagram that V_{ph} is less than E_{ph} for lagging and unity p.f. conditions due to demagnetising and cross magnetising effects of armature reaction. While V_{ph} is more than E_{ph} for leading p.f. condition due to the magnetising effect of armature reaction.

Thus in general for any power factor condition,

$(E_{ph})^2 = (V_{ph} \cos + I_a R_a)^2 + (V_{ph} \sin I_a X_s)^2$

+ sign for lagging p.f. loads

- sign for leading p.f. loads

and V_{ph} = per phase rated terminal voltage

I_a = per phase full load armature current