NBKRIST

ELECTROMECHANICAL ENERGY CONVERSION – III LECTURE NOTES

UNIT-3

DEPARTMENT OF ELECTRICAL AND ELECTRONICS ENGINEERING

Salient pole alternators and Blondel's Two reaction Theory

In round rotor or non salient pole alternators the air gap is uniform throughout and hence the effect of mmf will be same whether it acts along the pole axis or the inter polar axis. Hence reactance of the stator is same throughout and hence it is called synchronous reactance.

But in case salient pole machines the air gap is non uniform and it is smaller along pole axis and is larger along the inter polar axis. These axes are called direct axis or d-axis and quadrature axis or q-axis.

Hence the effect of mmf when acting along direct axis will be different than that when it is acting along quadrature axis. Hence the reactance of the stator cannot be same when the mmf is acting along d - axis and q- axis.

As the length of the air gap is small along direct axis reluctance of the magnetic circuit is less and the air gap along the q – axis is larger and hence the along the quadrature axis will be comparatively higher. Hence along d-axis more flux is produced than q-axis. Therefore the reactance due to armature reaction will be different along d-axis and q-axis.

These reactances are

 X_{ad} = direct axis reactance; X_{aq} = quadrature axis reactance

Hence the effect of armature reaction in the case of a salient pole synchronous machine can be taken as two components - one acting along the direct axis (coinciding with the main field pole axis) and the other acting along the quadrature axis (inter-polar region or magnetic neutral axis).

The direct-axis component \mathbf{F}_{ad} acts over a magnetic circuit identical with that of the main field system and produces a comparable effect while the quadrature-axis component \mathbf{F}_{aq} acts along the inter polar axis, resulting in an altogether smaller effect and, in addition, a flux distribution totally different from that of \mathbf{F}_{ad} or the main field m.m.f. This explains why the application of cylindrical-rotor theory to salient-pole machines for predicting the performance gives results not conforming to the performance obtained from an actual test.

Blondel's two-reaction theory considers the effects of the direct and quadrature -axis components of the armature reaction separately. Neglecting saturation, their different effects are considered by assigning to each an appropriate value of armature-reaction "reactance," respectively X_{ad} and X_{aq} . The effects of armature resistance and true leakage reactance (X_L) may be treated separately, or may be added to the armature reaction coefficients. Thus the combined reactance values can be expressed as : $X_{sd} = X_{ad} + X_L$ and $X_{sq} = X_{aq} + X_L$ for the direct- and cross-reaction axes respectively.

In a salient-pole machine, X_{aq} , the quadrature-axis reactance is smaller than X_{ad} , the direct-axis reactance, since the flux produced by a given current component in that axis is smaller as the reluctance of the magnetic path consists mostly of the inter polar spaces. It is essential to clearly note the difference between the direct and quadrature -axis components I_{ad} and I_{aq} of the armature current I_a , and the active and reactive components I_{aa} and I_{ar} . Although both pairs are represented by phasors in phase quadrature, the former are related to the induced emf E_t while the latter are referred to the terminal voltage V. These phasors are clearly indicated with reference to the phasor diagram of a (salient pole) synchronous generator supplying a lagging power factor (pf) load, shown in Fatigue 3.1



Figure: 3.1. Phasor diagram of salient pole alternator

$$I_{aq} = I_a \cos(\delta + \phi); I_{ad} = I_a \sin(\delta + \phi); \text{ and } I_a = \sqrt{[(I_{aq})^2 + (I_{ad})^2]}$$

$$I_{aa} = I_a \cos \varphi$$
; $I_{ar} = I_a \sin \varphi$; and $I_a = \sqrt{[(I_{aa})^2 + (I_{ar})^2]}$

Where δ = torque or power angle and φ = the p.f. angle of the load.

The phasor diagram shows figure 3.1. the two reactance voltage components $I_{aq}*X_{sq}$ and $I_{ad}*X_{sd}$ which are in quadrature with their respective components of the armature current. The resistance drop I_a*R_a is added in phase with I_a although we could take it as $I_{aq}*R_a$ and $I_{ad}*R_a$ separately, which is unnecessary as $I_a = I_{ad} + jI_{aq}$.

Power output of a Salient Pole Synchronous Machine

Neglecting the armature winding resistance, the power output of the generator is given by:

$$P = V*I_a*\cos \varphi$$

This can be expressed in terms of δ , by noting that

$$\begin{split} I_{a}\cos\phi &= I_{aq}\cos\delta + I_{ad}\sin\delta\\ V\cos\delta &= E_{o} - I_{ad} * X_{sd} \text{ and } V\sin\delta &= I_{aq} * X_{sq} \end{split}$$

Substituting the above expressions for power we get

$$P = V \left[(V \sin \delta / X_{sd}) * \cos \delta + (E_o - V \cos \delta) / X_{sd} * \sin \delta \right]$$

On simplification we get

$$P = (V * E_o/X_{sd}) \sin \delta + V^2 * (X_{sd} - X_{sq})/(2 * X_{sq} * X_{sq}) * \sin 2 \delta$$

The above expression for power can also be written as

$$P = (E_o * V * \sin \delta / X_d) + V^2 * (X_d - X_q) * \sin 2 \delta / (2 * X_q * X_q)$$

The above expression for power consists of two terms first is called electromagnetic power and the second is called reluctance power.

It is clear from the above expression that the power is a little more than that for a cylindrical rotor synchronous machine, as the first term alone represents the power for a cylindrical rotor synchronous machine. A term in $(\sin 2\delta)$ is added into the power – angle characteristic of a non-salient pole synchronous machine. This also shows that it is possible to generate an emf even if the excitation E_0 is zero. However this magnitude is quite less compared with that obtained with a finite E_0 . Likewise it can be shown that the machine develops a torque - called the reluctance torque - as this torque is developed due to the variation of the reluctance in the magnetic circuit even if the excitation E_0 is zero.

Determination of X_d and X_q by slip test:

The direct and quadrature axis reactances X_d and X_q can be of a synchronous machine can be experimentally determined by a simple test known as slip test. Basic circuit diagram for conducting this test is shown in figure 3.2. Here the armature terminals are supplied with a subnormal voltage of rated frequency with field circuit left open. The generator is driven by a prime mover at a slip speed which is slightly more or less than the synchronous speed. This is equivalent to the condition in which the armature mmf remains stationary and rotor rotates at a slip speed with respect to the armature mmf. As the rotor poles slip through the armature mmf the armature mmf will be in line with direct axis and quadrature axis alternately. When it is in line with the direct axis the armature mmf directly acts on the magnetic circuit and at this instant the voltage applied divided by armature current gives the direct axis synchronous reactance. When the armature mmf coincides with the quadrature axis then the voltage impressed divided by armature current gives the direct axis then the voltage impressed divided by armature current gives the quadrature axis then the voltage impressed divided by armature current gives the quadrature axis then the voltage impressed divided by armature current gives the quadrature axis then the voltage impressed divided by armature current gives the quadrature axis be the top the science $X_d > X_q$ the pointers of the ammeter reading the armature current will oscillate from a minimum to a maximum. Similarly the terminal voltage will also oscillate between the minimum and maximum as in figure 3.3.



Figure: 3.2. Slip test

: Xd = Maximum voltage / minimum current

Xq = Minimum voltage / maximum current.

$$X_{d} = \frac{\text{Maximum voltage}}{\text{Minimum current}} = \frac{(V_{t}) \text{ line (at minimum } I_{a})}{\sqrt{3} I_{a} (\text{min})}$$
$$X_{q} = \frac{\text{Minimum voltage}}{\text{Maximum current}} = \frac{(V_{t}) \text{ line (at maximum } I_{a})}{\sqrt{3} I_{a} (\text{max})}$$

and

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Derivation of Various Power Conditions in Alternators and Synchronous Motors

In order to derive various conditions for power in both and synchronous motors, let us consider the general problem of power flow through inductive impedance. The circuit diagram shown in figure 3.4 consists of voltage source E_1 , voltage source E_1 and load which consists of one resistor in series with an inductor. Now if we assume that the voltage source E_1 is greater than the voltage source E_2 then the voltage equation for this circuit is given by the equation,



Fig. 3.4 (a) circuit diagram (b) Phasor digram

Where, Z is R + jX as shown in the above circuit diagram. From the above expression we write the expression of current as

$$I = \frac{E_1}{Z} - \frac{E_2}{Z}$$

Phasor Diagram for above Circuit (fig. 3.4(a)) will discuss here the simplest way of drawing the phasor diagram for the above circuit.

Here, θ_z to represent angle between the voltage E_1 and current E_1/Z or voltage E_2 and current E_2/Z ,

I to represent current in the above circuit

and δ to represent angle between E_1 and E_2 .

The phasor diagram for the above circuit:

In order to draw the Phasor diagram first draw the E_1 voltage and the current E_1/Z , mark angle E_1 and E_1/Z as θ_z .

Similarly draw phasor E_2 and E_2/Z , such that the angle between E_1 and E_2 should be δ . Complete the phasor diagram by drawing the voltage drops IX and IR as shown above.

Derivation of expression for power supplied by the source E₁.

Let the power supplied by the source E_1 be P_1 . We define power as the product of the voltage and current, using this we can write from the phasor diagram as $P_1=E_1*$ (component of current I in phase with the voltage source E_1). Component of current in phase with the voltage source E_1 is

$$\frac{E_1}{Z}cos\theta_z - \frac{E_2}{Z}sin(\theta_z + \delta)]$$

On substituting this expression in the above equation we have

$$P_1 = E_1 \times \left[\frac{E_1}{Z}\cos\theta_z - \frac{E_2}{Z}\sin(\theta_z + \delta)\right]$$

$$\Rightarrow P_1 = E_1 \times \frac{E_1}{Z} cos\theta_z - E_1 \times \frac{E_2}{Z} sin(\theta_z + \delta)$$

From the phasor diagram, we have $\theta_z=90^\circ - \alpha_z$. On substituting the value of the angle θ_z in the above expression we have

$$P_1 = E_1 \times \frac{E_1}{Z} - E_1 \times \frac{E_2}{Z} sin(\delta - \alpha_z) \cdots \cdots \cdots (1)$$

This is the required expression for the power supplied by the source E_1 . Let the power supplied by the source E_2 be P_2 . We define power as the product of the voltage and current, using this we can write from the phasor diagram as

 $P_2 = E_2 \times (component \ of \ current \ I \ in \ phase \ with \ the \ voltage \ source \ E_2)$

Component of current in phase with the voltage source E₂ is

$$\frac{E_1}{Z}cos(\theta_z - \delta) - \frac{E_2}{Z}sin\theta_z$$

On substituting this expression in the above equation we have

$$P_2 = E_2 \times \left[\frac{E_1}{Z}\cos(\theta_z - \delta) - \frac{E_2}{Z}\sin\theta_z\right]$$
$$\Rightarrow P_2 = E_2 \times \frac{E_1}{Z}\cos(\theta_z - \delta) - E_2 \times \frac{E_2}{Z}\sin\theta_z$$

From the phasor diagram, we have $\theta_z=90^\circ$ - α_z . On substituting the value of the angle θ_z in the above expression we have

$$P_2 = -E_2 \times \frac{E_2}{Z} + E_1 \frac{E_2}{Z} \sin(\delta + \alpha_z) \cdots \cdots (2)$$

This is the required expression for the power supplied by the source E_2 .

Now let us derive various equations for the power flow the cylindrical rotor alternator. In order to derive various power equation for an alternator let us substitute voltage source E_1 equal to the excitation voltage (E_f), voltage source E_2 equals to the terminal voltage (V_t), inductive impedance of the above circuit equals to synchronous impedance (Z_s) and $Z_s=r_a+jX_s$. After replacing all these, we will have power input by the source E_1 is equal to power input to the generator (P_{ig}). So,

$$P_{ig} = P_1 = E_f \times r_a \times \frac{E_f}{Z_s} - E_f \times \frac{V_t}{Z_s} sin(\delta - \alpha_z)$$

Similarly we have output of the generator

$$P_{og} = P_2 = -V_t \times r_a \times \frac{V_t}{Z_s} - E_f \times \frac{V_t}{Z_s} sin(\delta + \alpha_z)$$

Similarly, we have output of the generator One important result can be derived from these equations. The difference of the power input to the generator and the power output to the generator gives ohmic losses in the generator. So in order to prove above statement let us subtract output power from the input power to the generator:

$$P_{ig} - P_{og} = \left[E_f \times r_a \times \frac{E_f}{Z_s} - E_f \times \frac{V_t}{Z_s} sin(\delta - \alpha_z) \right] \\ - \left[-V_t \times r_a \times \frac{V_t}{Z_s} - E_f \times \frac{V_t}{Z_s} sin(\delta + \alpha_z) \right]$$

On expanding the expression, we have

$$P_{ig} - P_{og} = \frac{r_a}{Z_s^2} \times (E_f^2 + V_t^2 - 2 \times E_f V_t \cos\delta)$$

From the phasor diagram we have

$$E_f^2 + V_t^2 - 2E_f V_t \cos\delta = I_a^2 \times r_a$$

So substituting the value, from this equation we have

$$P_{ig} - P_{og} = I_a^2 \times r_a$$

Usually we neglect the value of armature resistance, due to this αz becomes zero and Z_s becomes equal to X_s . Hence we have,

$$P_{ig} = P_{og} = \frac{E_f \times V_t}{Xs} sin\delta$$

Now we are in state to derive the expression for maximum power output conditions for the generator. In order to derive the maximum power output conditions we will first differentiate the expression of the power output equation of the generator that we have already derived above, after this we equate the equation with zero. On equating with zero we will get the angle relationship between alpha and delta at maximum power out conditions. Mathematically we have

$$\frac{dP_{og}}{d\delta} = \theta = \frac{E_f \times V_t}{Z_s} cos(\delta + \alpha_z)$$
$$\mathbf{L}\delta + \alpha_z = 90^o \Rightarrow \delta = 90^o - \alpha_z$$

This is the required condition for the maximum power output, at maximum power output we have load angle is equal to the impedance angle.

On substituting the above relation in output power relation we have maximum power output equals to

$$\frac{E_f \times V_t}{Z_s} - \left(\frac{V_t}{Z_s}\right)^2 \times r_a$$

Which is the required expression for the maximum power output of the generator. Here we are interested in drawing the phasor diagrams for the maximum power output in case of generator. Given below is the phasor diagram for the generator in case of maximum power output. All the symbols have their usual meanings in the phasor diagram.



Similarly, we can derive the expression for maximum input to the generator. In order to derive the maximum power input conditions, we will first differentiate the expression of the power input equation of the generator that we have already derived above, after this we equate the equation with zero. On equating with zero we will get the angle relationship between alpha and delta at maximum power out conditions. Mathematically, we have

$$\frac{dP_{ig}}{d\delta} = \theta = \frac{E_f \times V_t}{Z_s} cos(\delta - \alpha_z)$$
$$\Delta - \alpha_z = 90^o \Rightarrow \delta = 180^o - \theta_z$$

This is the required condition for the maximum power input, at maximum power input we have load angle is equal to the 180 degree minus impedance angle. On substituting the above relation in input power relation we have maximum power input equals to

$$\frac{E_f \times V_t}{Z_s} + \left(\frac{E_f}{Z_s}\right)^2 \times r_a$$

Which is the required expression for the maximum power input for the generator? Here, we are interested in drawing the phasor diagrams for the maximum power input in case of generator. Given below is the phasor diagram for the generator in case of maximum power input. All the symbols have their usual meanings in the phasor diagram. From the phasor diagram of maximum power input we can derive various conditions for the power factor and these conditions are written below:

(a) When $(E_f \cos \delta - I_a r_a \cos \theta)$ is less than terminal voltage then the power factor will be leading.

(b) When $(E_f \cos \delta - I_a r_a \cos \theta)$ is equal to terminal voltage then the power factor will be unity.

(c) When $(E_f \cos \delta - I_a r_a \cos \theta)$ is greater than terminal voltage then the power factor will be lagging.

Let us now derive the expression for Reactive power flow in case of synchronous generator. We can derive expression for the reactive power at the output terminal of the generator as

$$Q_{og} = E_2 \times (component \ of \ current \ in \ phase \ quadrature \ lagging \ with \ E_2)$$

$$\Rightarrow Q_{og} = V_t \times \left[\frac{E_f}{Z_s} sin(\theta_z - \delta) - \frac{V_t}{Z_s} sin\theta_z\right]$$

Further we can write this equation as

$$Q_{og} = \frac{V_t \times E_f}{Z_s} cos(\delta + \alpha_z) - \left(\frac{V_t}{Z_s}\right)^2 \times X_s$$

From the above equation in generating mode if we have armature resistance equals to zero then the above equation will reduce to

$$Q_{og} = \frac{V_t}{X_s} \left[E_f \cos\delta - V_t \right]$$

From the above equation we can derive various conditions for the power factor and reactive power, these conditions are written below:

(a) When $E_f cos \delta$ is less than terminal voltage then the power factor will be leading and reactive power is negative at the output terminal.

(b) When $E_f \cos \delta is$ equal to terminal voltage then the power factor will be unity and the reactive power is zero at the output terminal of the generator.

(c) When $E_f \cos \delta$ is greater than terminal voltage then the power factor will be lagging and reactive power is positive.

Now let us derive various equations for the power flow the cylindrical rotor synchronous motor. In order to derive various power equation for a synchronous motor let us substitute voltage source E_1 equal to the excitation voltage (V_t), voltage source E_2 equals to the terminal voltage (E_f), inductive impedance of the above circuit equals to synchronous impedance (Z_s) and $Z_s = r_a + jX_s$. After replacing all these, we will have power input by the source E_1 is equal to power input to the generator (P_{ig}). So,

$$P_{im} = P_1 = V_t \times r_a \times \frac{V_t}{Z_s} + E_f \times \frac{V_t}{Z_s} sin(\delta - \alpha_z)$$

Similarly, we have output of the synchronous motor

$$P_{om} = P_2 = -E_f \times r_a \times \frac{E_f}{Z_s} + E_f \times \frac{V_t}{Zs} sin(\delta + \alpha_z)$$

One important result can be derived from these equations. The difference of the power input to the synchronous motor and the power output to the synchronous motor gives ohmic losses in the generator. So in order to prove above statement let us subtract output power from the input power to the synchronous motor:

$$P_{im} - P_{om} = \left[E_f \times r_a \frac{E_f}{Z_s} - E_f \times \frac{V_t}{Z_s} sin(\delta - \alpha_z) \right]$$
$$- \left[-V_t \times r_a \frac{V_t}{Z_s} + E_f \times \frac{V_t}{Z_s} sin(\delta + \alpha_z) \right]$$

On expanding the expression, we have

$$P_{im} - P_{om} = \frac{r_a}{Z_s^2} \times \left[E_f^2 + V_t^2 - 2E_f V_t \cos\delta\right]$$

From the phasor diagram we have

$$E_f^2 + V_t^2 - 2E_f V_t \cos\delta = I_a^2 \times r_a$$

So substituting the value, from this equation we have,

$$P_{im} - P_{om} = I_a^2 \times r_a$$

Usually we neglect the value of armature resistance, due to this α_z becomes zero and Z_s becomes equal to X_s . Hence we have

$$P_{im} = P_{om} = \frac{E_f \times V_t}{X_s} sin\delta$$

Now we are in state to derive the expression for maximum power output conditions for the synchronous motor. In order to derive the maximum power output conditions, we will first differentiate the expression of the power output equation of the synchronous motor that we have already derived above, after this we equate the equation with zero. On equating with zero we will get the angle relationship between alpha and delta at maximum power out conditions. Mathematically we have

$$\frac{dP_{om}}{d\delta} = 0 = \frac{E_f \times V_t}{Z_s} cos(\delta + \alpha_z)$$
$$\mathbf{L}\delta + \alpha_z = 90^o \Rightarrow \delta = 90^o - \alpha_z$$

This is the required condition for the maximum power output, at maximum power output we have load angle is equal to the impedance angle. On substituting the above relation in output power relation we have maximum power output equals to

$$\frac{E_f \times V_t}{Zs} - \frac{E_f}{Z_s^2} \times r_a$$

Which is the required expression for the maximum power output for the synchronous motor.

Here we are interested in drawing the phasor diagrams for the maximum power output in case of synchronous motor. Given below is the phasor diagram for the synchronous motor in case of maximum power output. All the symbols have their usual meanings in the phasor diagram.



Similarly, we can derive the expression for maximum input to the motor. In order to derive the maximum power input conditions, we will first differentiate the expression of the power input equation of the generator that we have already derived above, after this we equate the equation with zero. On equating with zero we will get the angle relationship between alpha and delta at maximum power out conditions. Mathematically, we have

$$\frac{dP_{im}}{d\delta} = 0 = \frac{E_f \times V_t}{Z_s} cos(\delta - \alpha_z)$$
$$\Delta - \alpha_z = 90^o \Rightarrow \delta = 180^o - \theta_z$$

This is the required condition for the maximum power input, at maximum power input we have load angle is equal to the 180 degree minus impedance angle. On substituting the above relation in input power relation we have maximum power input equals to

$$\frac{E_f \times V_t}{Z_s} + \left(\frac{V_t}{Z_s}\right)^2 \times r_a$$

Which is the required expression for the maximum power input for the synchronous motor.

Here we are interested in drawing the phasor diagrams for the maximum power input in case of synchronous motor. Given below is the phasor diagram for the synchronous motor in case of maximum power input. All the symbols have their usual meanings in the phasor diagram.

From the phasor diagram of maximum power input we can derive various conditions for the power factor and these conditions are written below:

(a) When $(E_f \cos \delta + I_a r_a \cos \theta)$ is less than terminal voltage then the power factor will belagging. (b) When $(E_f \cos \delta + I_a r_a \cos \theta)$ is equal to terminal voltage then the power factor will be unity. (c) When $(E_f \cos \delta + I_a r_a \cos \theta)$ is greater than terminal voltage then the power factor will be leading.

Let us now derive the expression for Reactive power flow in case of synchronous motor. We can derive expression for the reactive power at the input terminal of the synchronous motor

$$Q_{im} = E_2 \times (component \ of \ current \ in \ phase \ quadrature \ lagging \ with \ E_2)$$
$$Q_{im} = -V_t \times \left[\frac{E_f}{Z_s} cos(\alpha_z - \delta) + \frac{V_t}{Z_s^2} X_s\right]$$

Further we can write this equation as

$$Q_{im} = -\frac{V_t \times E_f}{Z_s} \cos(\alpha_z - \delta) - \left(\frac{V_t}{Z_s}\right)^2 X_s$$

From the above equation in motoring mode if we have armature resistance equals to zero then the above equation will reduce to

$$Q_{im} = -\frac{V_t}{Xs} E_f cos\delta - \frac{V_t^2}{X_s}$$

From the above equation we can derive various conditions for the power factor and reactive power, these conditions are written below:

(a) When $E_f \cos \delta$ is less than terminal voltage then the power factor will be lagging and reactive power is positive at the input terminal.

(b) When $E_f \cos \delta$ is equal to terminal voltage then the power factor will be unity and the reactive power is zero at the input terminal of the synchronous motor.

(c) When $E_f cos \delta$ is greater than terminal voltage then the power factor will be leading and reactive power is negative.

Note on Power Angle Characteristics of Cylindrical Rotor Synchronous Machine and Two Reaction Model of Salient Pole Machine.

Power angle characteristics of cylindrical rotor synchronous machine:

Let us consider a synchronous machine connected to the infinite bus having a voltage of (V_t) also resistance R_a of machine neglected and only (X_s) is considered.



Fig 3.7: Single line diagram



Synchronous machine operation (generating/motoring mode)

Fig 3.8: Phasor diagrams

By solving the phasor diagrams. We can calculate;

$$P_i = \frac{EV \sin \delta}{X_s}$$

This equation is known as phase angle equation of synchronous machine having cylindrical rotor.



Fig 3.9: Graph showing maximum power



Torque- δ angle characteristics of salient pole machine

Fig 3.10: Generating & motoring mode

The electrical power output for the salient pole machine is;

$$Pe = \frac{Eb Vt sin\delta}{Xd} + \frac{Vt * Vt}{2} \left(\frac{1}{Xq} - \frac{1}{Xd}\right)$$

Which gives the total power developed due to salient pole machine.

Power Angle Characteristics of Synchronous Generator

We have seen previously,

$$P_i = \frac{EV\sin\delta}{X_s}$$

The relation between P_i and δ is known as power angle characteristics of the machine. It is shown in the Fig. 3.11.



Fig. 3.11 Power Angle Characteristics

The maximum power occurs at $\delta = 90^{\circ}$. Beyond this point the machine falls out of step and loses synchronism. The machine can be taken up to P_{i max} only by gradually increasing the load. This is known as the steady state stability limit of the machine. The is normally operated at δ much less than 90°.