UNIT-III POWER SYSTEM TRANSIENTS

POWER SYSTEM TRANSIENTS

Introduction-Circuit closing transients - Recovery transient due to removal of a short circuit-Travelling waves on transmission line -Surge impedance and wave velocity-Specification of travelling waves-Reflections and refractions of waves - Different types of terminations-Forked line-Successive reflections - Bewley’s Lattice diagram-Attenuation and distortion.

INTRODUCTION

- A transient occurs in the power system when the network changes from one steady state into another.
- The majority of power system transients is, however, the result of a switching action.
- Load break switches and disconnectors switch off and switch on parts of the network under load and no-load conditions.
- Fuses and circuit breakers interrupt higher currents and clear short-circuit currents flowing in faulted parts of the system.
- The time period when transient voltage and current oscillations occur is in the range of microseconds to milliseconds.

TRANSIENTS

- The transient is a pulse which is of very short duration but of very high intensity.

- The good example of transient is the lightening occurring in the sky in the period of Monsoon , which is nothing but the heavy discharge of millions of volts and millions of amperes .

- This wave travels along the length of three line at a certain velocity.

- This may cause damage to the equipments connected to the system.

- The other examples of power system transient are, opening of the circuit breaker , ground fault on line or in equipment etc. Insulation of the equipment gets punctured due this high voltage.

OVERVOLTAGE

- An overvoltage is an increase in the r.m.s. ac voltage greater than 110 % at the power frequency for a duration longer than 1 min.
- It is a result of load switching.

    Eg : Switching OFF a Large load

    Energizing a capacitor bank

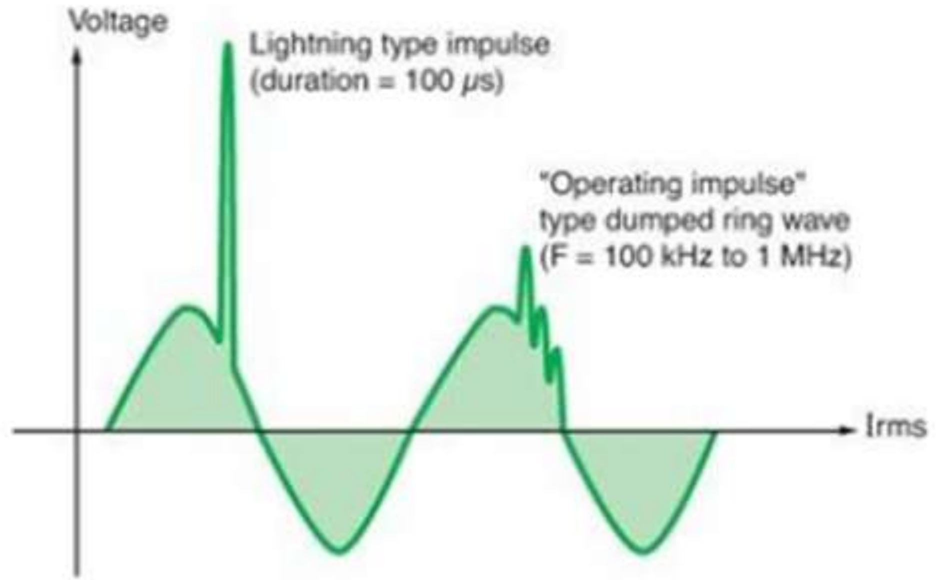
- Switchings in a power system occur frequently.
- A variety of switchings are performed for routine operations or automatically by control and protection systems.
- Typical switchings are as follows:
  - Lines (transmission or distribution)
  - Cables
  - Shunt/series capacitors
  - Shunt reactors
  - Transformers
  - Generators/motors

OVER VOLTAGES

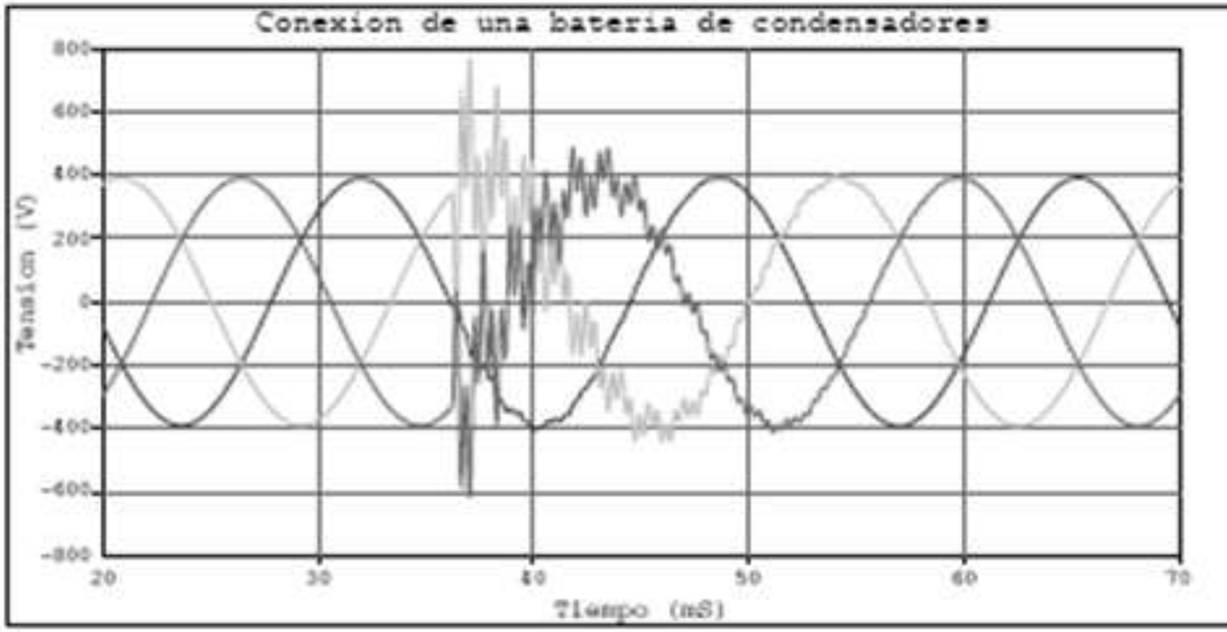
- The causes of power system over voltages are numerous and the waveforms are complex.
- It is customary to classify the transients on the basis of frequency content of the waveforms.
- In this sense, the following three broad categories are defined:
  - Power frequency overvoltages
  - Switching overvoltages
  - Lightning overvoltages

SWITCHING TRANSIENTS

- OVER VOLTAGES



Transients



- **Causes**
  - shortcircuits
  - switching operations
  - lightning strokes
- **Effects**
  - overcurrents
  - equipment aging and breakdown

VOLTAGE SURGE OR TRANSIENT VOLTAGE

Definition :-

- "**Increase in voltage for very short time in power system is called the voltage surge**."

- This voltage surge comes only for few micro seconds. But due to it there i much increase in the voltage level of the system. This may cause damage the equipments connected to the system.

- There are different causes of occurring voltage surge. External causes include lightning in the sky where as internal causes include switching su or fault.

- Flashover occurs over the insulator due to the voltage surge and the equipments like generator, transformer may get damaged.

CAUSES OF SYSTEM TRANSIENTS

- The causes of power system transients can be divided in to two categor
  - (a) External causes
  - (b) Internal causes.

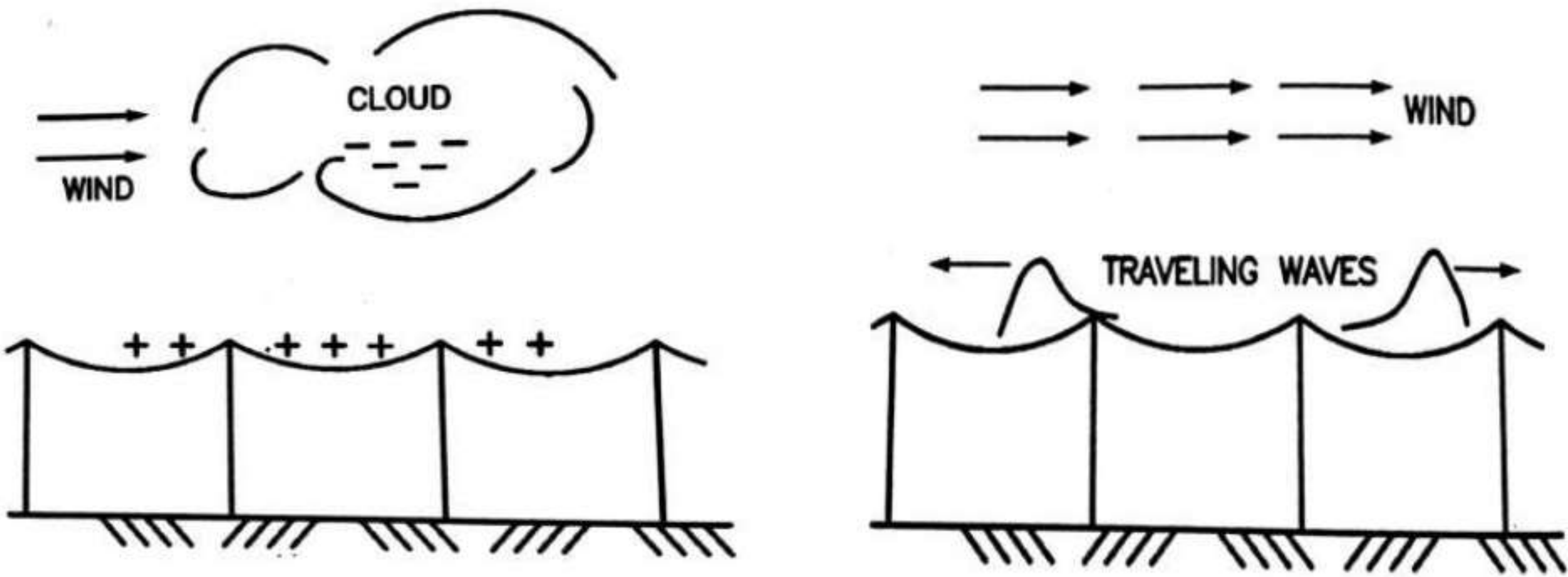
- **Due to external cause**

- High voltage can be produced in the line by the travelling voltage wave produced due to the static charges in addition to the direct stroke.

- Suppose there is a cloud with negative charges over the line as shown i figure this cloud will induce positive charges on the line.

- Suppose the strong wind blows so the cloud will be shifted away from t line so the positive charges on the line will not remain bound and will be free and can travel on the two sides of the line.

- The magnitude of this travelling wave is of 10 KV to 15 KV and it has the steep wave front. Characteristic of this wave is as shown i figure.



Due to internal causes

- When there is sudden change in the circuit conditions of the power system, oscillations are produced due to the inductance and the capacitance of the circuit and there is increase in the system voltage.

- This voltage is almost double the system voltage.

- Thus the over voltage due to the internal cause is very less than that produced due to the lightening is not necessary to provide additional protection for the over voltage caused due to the internal causes if the insulation of the equipment is designed properly.

- **Internal causes occurred due to following reason**

A. Switching Surges

B. Arcing Ground

C. Resonance

A. Switching Surges

- When switching operation is done in power system with or without load, the voltage produced as a result of this is called the switching surge.

B. Arcing Ground

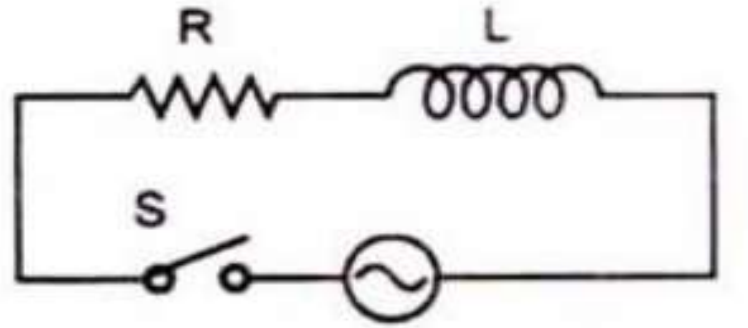
- When neutral is not earthed in three-phase line and if line to ground fault occurs, the phenomenon of arcing ground occurs. Due to this, oscillations of three to four times the normal magnitude are produced.

C. Resonance

- When the inductive reactance of the line becomes equal to its capacitive reactance the net impedance of the line becomes the minimum and it is equal to the resistance of the line.
- Series resonance occurs at this time.
- If the distortion occurs in the waveform of the e. m. f. the harmonics are produced. So the value of Xu and Xc may become equal at the fifth or multiple harmonics and may result in resonance.
- Over voltages are produced due to the resonance.

CIRCUIT CLOSING TRANSIENT

Consider an R-L series circuit, which is connected, at the instant t = 0, to a source of alternating voltage $v = V_{max} \sin(\omega t/ + \alpha)$ where α is the phase displacement between the voltage v and the reference wave which passes through zero at the time t = 0.



$$v = V_{max} \sin(\omega t + \alpha)$$

Fig. ***R-L AC Series Circuit***

The equation relating the applied voltage and the current in the given circuit is given as –

$$v = iR + L\, di/dt$$

or $L\, di/dt + i\, R = V_{max} \sin(\omega t + \alpha)$ .. .(1)

The complete solution of the above equation consists of two parts which are called the particular integral and complementary function. The particular integral is the solution corresponding to the steady-state conditions, namely –

$$i = \frac{V_{max}}{\sqrt{R^2 + \omega^2 L^2}} \sin(\omega t + \alpha - \phi) = \frac{V_{max}}{Z} \sin(\omega t + \alpha - \phi)$$

Where $\sqrt{R^2 + (\omega L)^2}$ = Z, the circuit impedance, ϕ is the phase angle between the current and voltage determined by $\phi = \mathrm{Tan}^{-1}\, \omega L/R$.

The complete solution, which is the sum of the general and particular solution, is

$$i(t) = i_c(t) + i_p(t)$$

$$= A\, e^{-(R/L)t} + \frac{E_{max}}{\sqrt{R^2 + \omega^2 L^2}} \sin\left[\omega t + \varphi - \tan^{-1}\left(\frac{\omega L}{R}\right)\right] \qquad (2)$$

Complementary function is, therefore, necessary to represent the initial conditions.

Therefore, substituting initial conditions to calculate A

at $t = 0$, i= 0

$$A + \frac{V_{\max}}{\sqrt{R^2 + \omega^2 L^2}} \sin\left[\varphi - \tan^{-1}\left(\frac{\omega L}{R}\right)\right] = 0$$

$$A = -\frac{V_{\max}}{\sqrt{R^2 + \omega^2 L^2}} \sin\left[\varphi - \tan^{-1}\left(\frac{\omega L}{R}\right)\right.$$

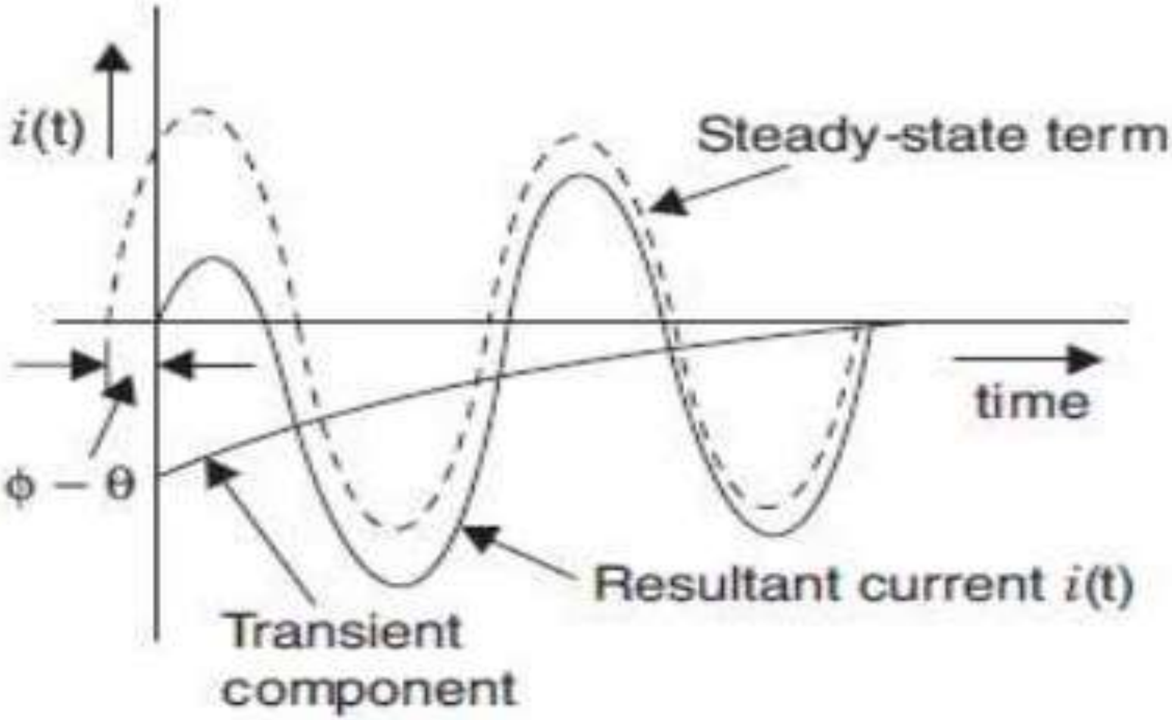
This gives us the value for A ; hence, the complete expression for the current becomes

$$i(t) = e^{-(R/L)t}\left\{\frac{-V_{max}}{\sqrt{R^2+\omega^2L^2}}\sin\left[\varphi - \tan^{-1}\left(\frac{\omega L}{R}\right)\right]\right\}$$

$$+\frac{V_{max}}{\sqrt{R^2+\omega^2L^2}}\sin\left[\omega t + \varphi - \tan^{-1}\left(\frac{\omega L}{R}\right)\right] \qquad (3)$$

The first part of Equation (3) contains the term $\exp[-(R/L)t]$ and damps out. This is called the *DC component.*

The second component is a.c component with an amplitude $\frac{V_{max}}{\sqrt{R^2+\omega^2L^2}}$ and lagging the voltage by an angle $\tan^{-1}\left(\frac{\omega L}{R}\right)$



i(t)
Steady-state term
time
φ − θ
Resultant current i(t)
Transient
component

- The first term in equ3 is transient part of current which vanishes theoretically after infinite time. Practically vanishes very quickly after two or three cycles.
- The transient decay depends on time constant of RL circuit.
- The second term is the steady state sinusoidal variation.
- It can be seen that the transient component will be zero in case the switching of the voltage wave is done when $\phi = \theta$

• If $\phi - \theta = \pm \frac{\pi}{2}$,the transient term will have its max value and the first peak of resulting will be twice the peak value of sinusoidal steady state component.

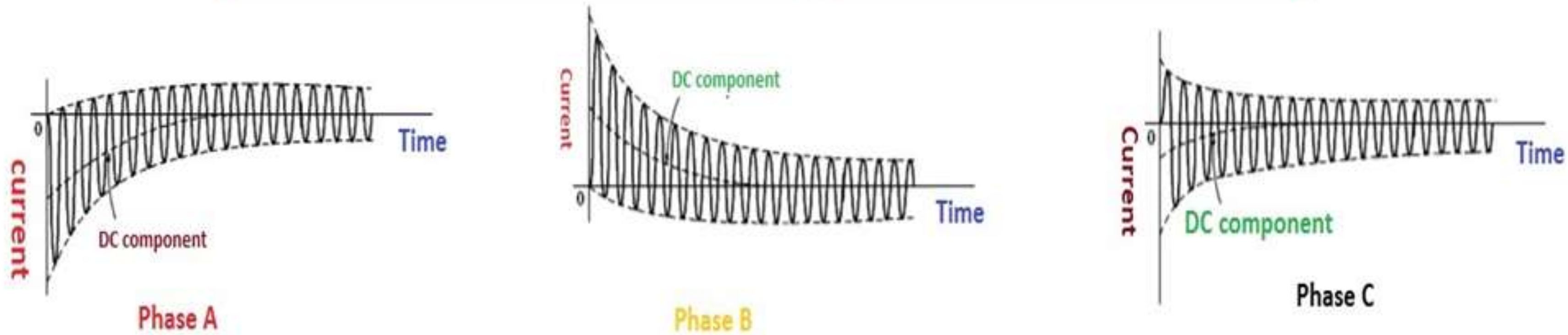
Sudden symmetrical Short Circuit of Alternator

- The study of three phase alternator is of almost same as RL circuit previously discussed.
- However, there is one important difference, that is, in case of an R-L series circuit, reactance X (ωL) is a constant quantity where as in case of the synchronous generator the reactance is not a constant one but is a function of time.
- Consider a 3phase alternator running at synchronous speed and with field excited by a constant DC voltage.

- Whenever a 3phase s.c occurs at the terminals of an alternator, the current in alternator circuit increases suddenly to a large value(about 10 to 18 times of full load current) during the first quarter cycle
- Since the resistance of circuit is small as compared with reactance, the current is highly lagging & p.f is approximately Zero.
- Due to this sudden switching there are two components of currents
- i) a.c component
- ii)d.c component

- Since the voltages of 3phases are 120 degrees out of phase from each other, the s.c occurs at different points on the voltage wave of each phase.
- Therefore the d.c component will be different in each phase as shown in figure

Short-Circuit Transients in Synchronous Generators



The total fault currents as a function of time during a three-phase fault at the terminals of a synchronous generator

➢**If the d.c component subtracted, the oscillogram of the armature current has a typical wave shape shown in fig.**

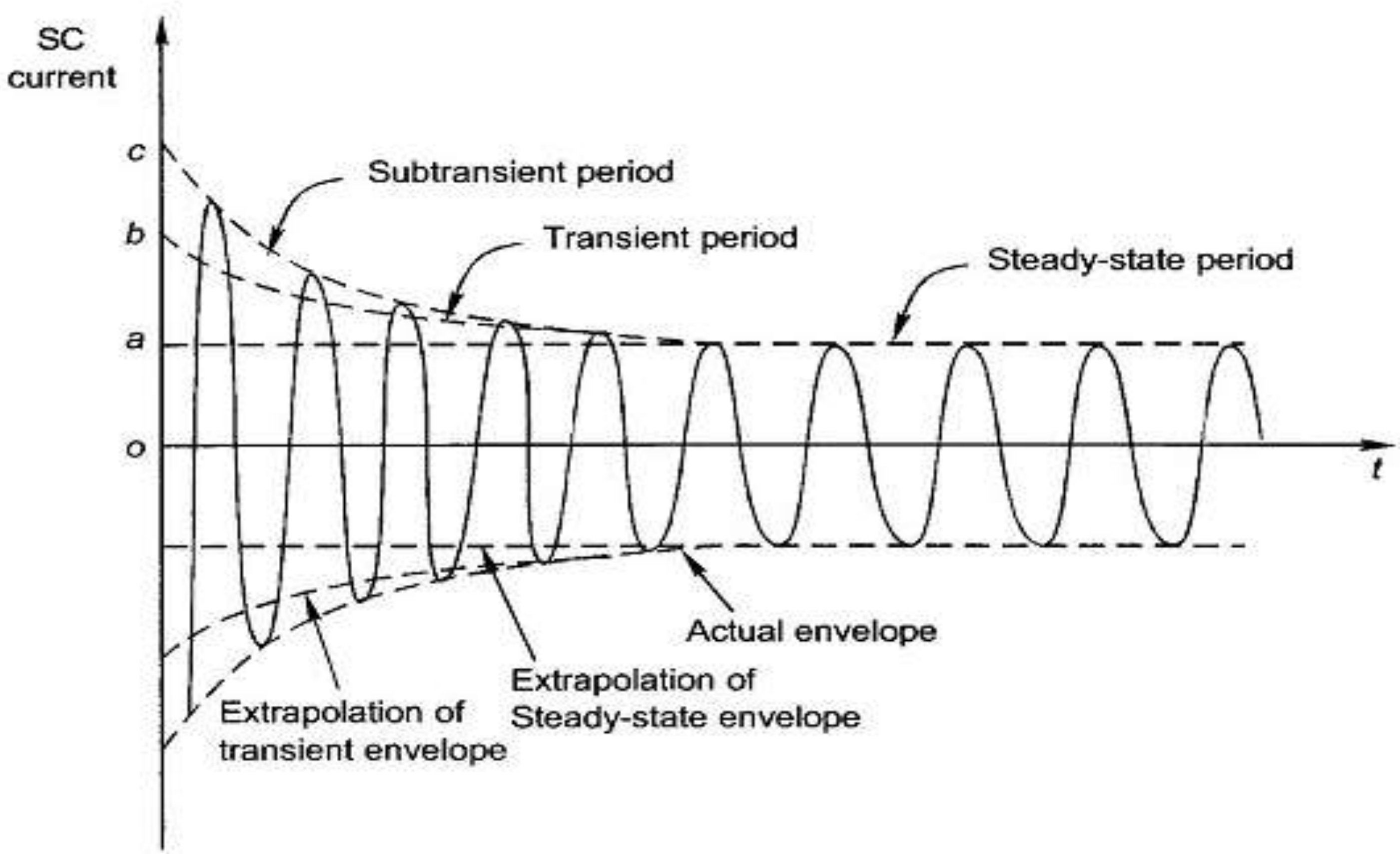


Fig. *Symmetrical short circuit current in synchronous generator*

- Fig shows the complete waveform of the symmetrical short-circuit current in a synchronous machine.
- The wave may be divided into three distinct time periods
- 1.sub-stransient lasting for only about 2cycles during which the current decrement is very rapid
- 2. Transient period lasting for about 30 cycles or so
- 3.Steady-state period
- These are indicated on the current envelope.

- Extrapolation of the subtransient, transient and steady-state current envelopes identifies the ordinates Oa, Ob, and Oc on the current coordinate.
- The rms value of initial current(i.e at the instant of s.c) is known as sub-transient current.
- The corresponding reactance is called the direct axis sub transient reactance.

- The extrapolation of the transient envelope cuts the y-axis at point b.
- The RMS value of current, represented by intercept ob, i.e 0.707(ob) in amperes.
- Corresponding reactance is transient reactance
- Similarly the rms value of the steady state current represented by intercept oc, i.e 0.707 (oc) in ampere and corresponding reactance is direct axis synchronous reactance

- The decaying envelope is clearly indicative of the fact that the equivalent d-axis reactance offered by the machine continuously increases as time progresses and finally settles to the steady value X_d .
- The machine presents three different reactances, during the short circuit, as defined below:

Subtransient reactance, $X''_d = \frac{E_f}{Oc/\sqrt{2}} = \frac{E_f}{I''}$ -- 1

Transient reactance, $X'_d = \frac{E_f}{Ob/\sqrt{2}} = \frac{E_f}{I'}$ -- 2

Steady-state reactance, $X_d = \frac{E_f}{Oa/\sqrt{2}} = \frac{E_f}{I}$ -- 3

where E_f = excitation emf (open-circuit voltage) (rms phase value)

I'' = subtransient SC current (rms)

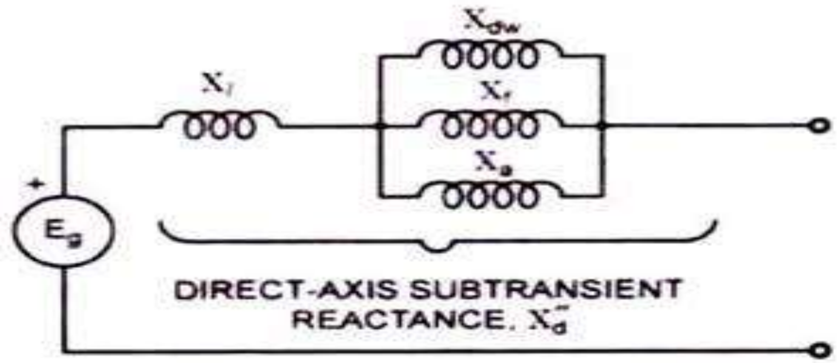
I' = transient SC current (rms)

I = steady SC current (rms)

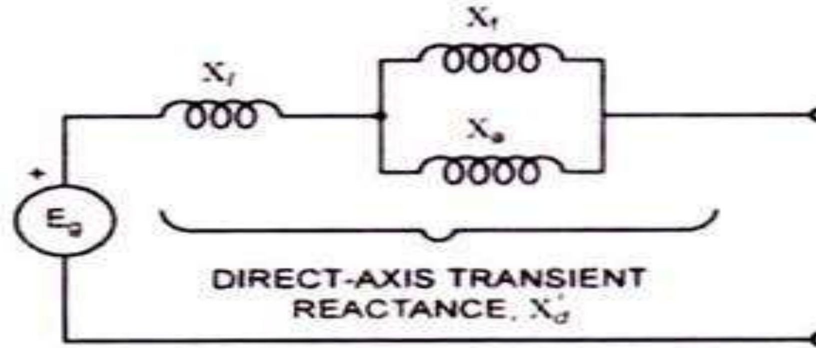
Obviously

$$X''_d < X'_d < X_d \qquad \text{--- 4}$$

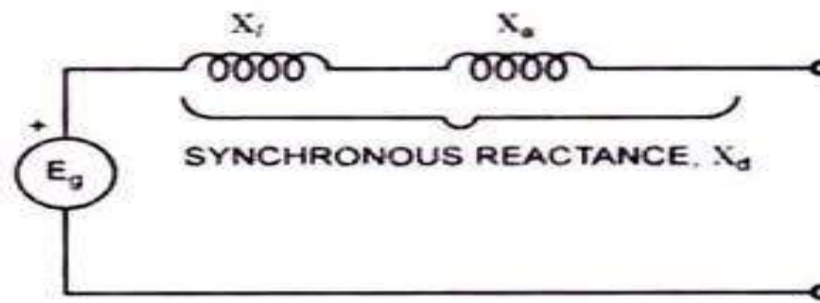
In fact as stated earlier, X''_d almost equals the leakage reactance of the machine.



(a) Approximate Circuit Model During Subtransient Period of Short Circuit



(b) Approximate Circuit Model During Transient Period of Short Circuit

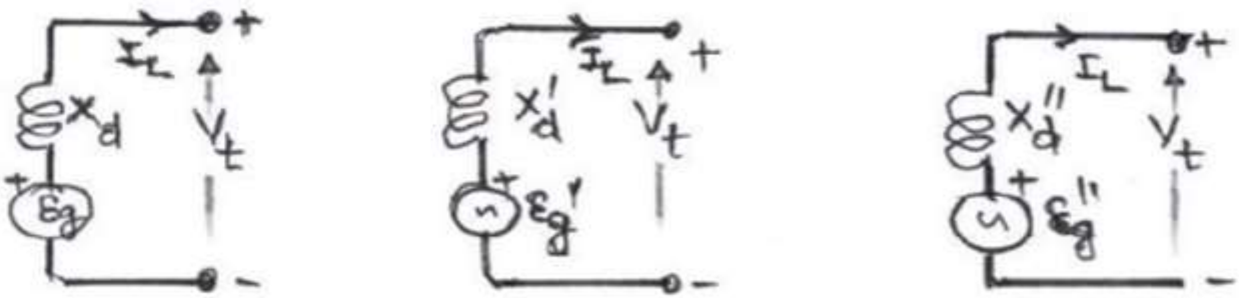


(c) Steady-State Short-Circuit Model of a Synchronous Machine

Fig. 4.4

short circuit of a loaded machine

Figure gives the circuit model of a synchronous generator operating under steady state conditions supplying a load current I_l to the bus at a terminal voltage V_t. E_g is the induced emf under the loaded conditions and X_d is the direct axis synchronous reactance of the generator.



Circuit model for computing **steady state** *current*

Circuit model for computing transient current

Circuit model for computing subtransient current

Figure Circuit models for a fault on a loaded machine.

Also shown in figure , are the circuit models to be used for short circuit current calculations when a fault occurs at the terminals of the generator, for sub-transient current and transient current values. The induced emf values used in these models are given by the expressions as under:

$E_g = V_t + j\, I_L X_d$ = Voltage behind syn. reactance

$E_g' = V_t + j\, I_L X_d'$ = Voltage behind transient reactance

$E_g'' = V_t + j\, I_L X_d''$ = Voltage behind subtr. Reactance

The subtransient and transient currents during short circuit are given by

$$I'' = \frac{E''_g}{X''_d}$$

$$I' = \frac{E'_g}{X'_d}$$

Of course the steady-state short circuit current is given by

$$I = \frac{E_g}{X_d}$$

where E_g is the excitation emf and Xd, the steady-state d-axis reactance.

Restriking voltage after removal of short circuit

- The opening of a circuit breaker under faulty condition results in a transient known as recovery voltage or restriking voltage transients.
- The transient has an important effect on behaviour and rating of circuit interruption and protective devices.

Circuit Breaker

- A circuit breaker is a piece of equipment which can
- (i) Make or break a circuit either manually or by remote control under normal conditions.
- (ii) Break a circuit automatically under fault conditions
- (iii) Make a circuit either manually or by remote control under fault conditions
- A circuit breaker essentially consists of fixed and moving contacts, called Electrodes.
- Under normal operating conditions, these contacts remain closed and will not open automatically until and unless the system becomes faulty.

- When the contacts of a CB are separated under fault conditions, an arc is struck between them.
- The current is thus able to continue until the discharge ceases.
- Therefore the main problem in a CB is to extininguish the arc within the shortest possible time .

- **Restriking voltage:**
- It is the transient voltage that appears across the breaking contacts at the instant of arc extinction.
- This voltage tries to restrike the arc
- i.e if dielectric strength rise is greater than the rise of restriking voltage then the arc will not restrike.
- In other words it is the transient voltage that exists during the arcing time(natural frequency kHz).

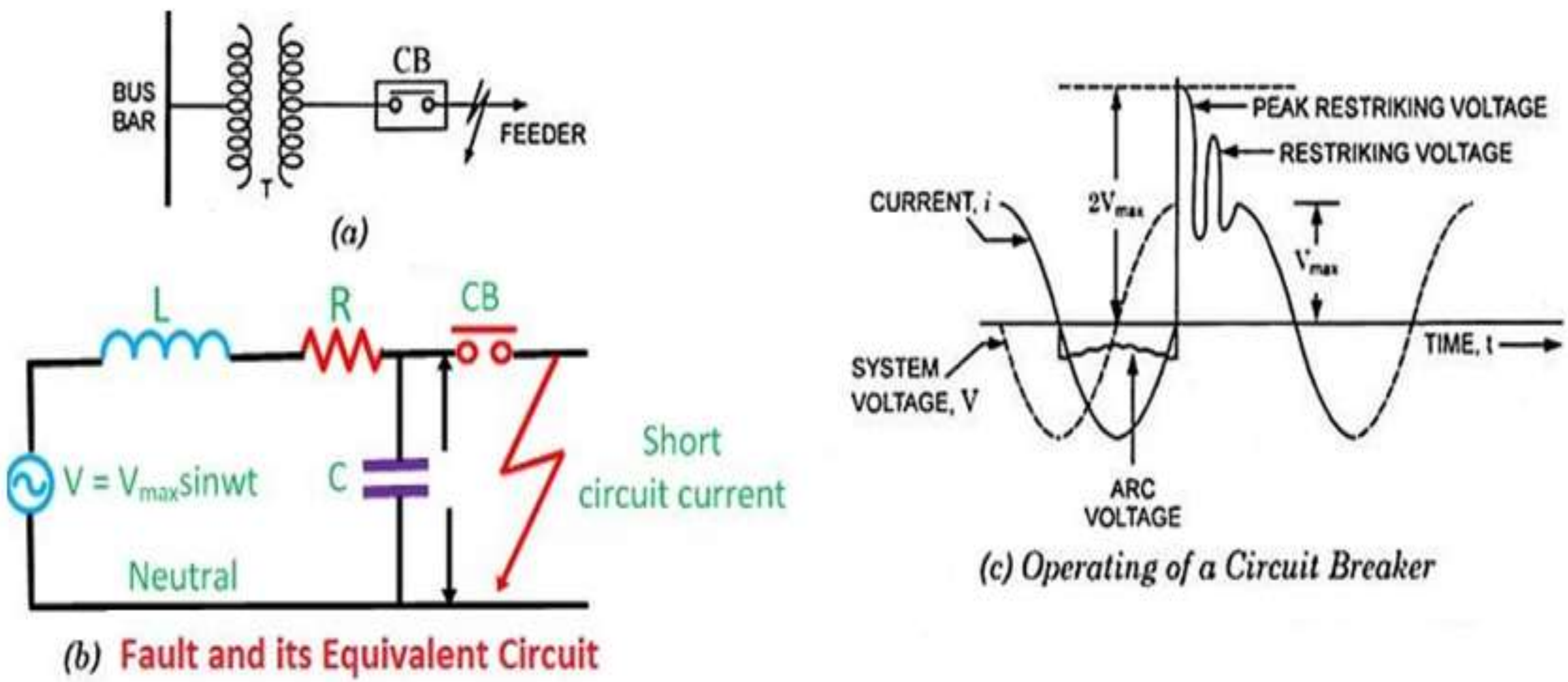
Recovery voltage

- It is defined as the voltage that appears across the breaker contact after the complete removal of transient oscillations and final arc extinction.
- In other words it is the RMS voltage after final arc extinction (normal frequency 50 or 60 Hz).
- Both Restriking and recovery voltages appear between circuit breaker poles.

Calculation of Restriking Voltage

- Let’s take the some assumption, at the maximum restriking voltage after the removal of fault by breaker contact opening to Calculate the Restriking Voltage,
- 1.Current interruption is assumed to be taking place at natural current zero.
- 2.Assumed the whole is system is lossless.
- 3.The fault does not involve any arcing. Example, the fault is solid one.
- 4.Effect of corona and saturation is ignored.

- Let us consider a simple circuit, having a circuit breaker CB, as shown in Fig.1 (a) and that a S.C occurs on the feeder close to the bus-bars.
- The equivalent circuit is shown in Fig.1 (b).
- Let L be the inductance & R be the resistance per phase of the system up to the fault point and C be the capacitance to ground of busbar, the bushings ect is lumped.



(a)

(b) Fault and its Equivalent Circuit

(c) Operating of a Circuit Breaker

Fig. 1

- Consider the opening of a circuit breaker under fault conditions shown in Fig.1 (b).
- Before current interruption, the capacitance C is short circuited by the fault and the S.C current through the breaker is limited by R and L of the system.
- If R is negligible compared to L, the short-circuit current i will lag behind the system voltage v by 90°, as shown in Fig. 1 (c).

- The voltage across CB contact will be equal to the multiplication of fault current and the system impedance as seen from the CB contacts with voltage source shorted.
- Let us assume this voltage to be v(t).
- For making calculations easy, consider every parameter in Laplace domain.
- Therefore,
- I(f) = Fault Current
  V(s) = Supply Voltage
  Z(s) = System Impedance as seen from CB contacts

- Hence,
  $V(s) = I(f)Z(s)$ ………….(1)
- But source voltage is V_mSinωt, hence source voltage at natural current zero i.e. when breaker contact open will be V_m.(When voltage is passing through max value V_m)
- Therefore $V(s) = V_m/s$ and impedance is mainly offered by inductance, this means $Z(s) = Ls$

- Thus from (1),
- $V_m/s = I(f)Z(s)$
  $I(f) = V_m / Ls^2$ ………..(2)

- Now let us find the Zo(s).
- Therefore,

- Now the impedance between the C.B. contacts after the short circuiting the voltage source will be the impedance of parallel combination of L and C i.e.,

$$Z_0(s) = \frac{Ls \,.\, 1/Cs}{Ls + 1/Cs} = \frac{Ls}{LCs^2 + 1} = \frac{s/C}{s^2 + 1/LC}$$

- Now from (2),
- V(s) = Voltage across CB contact immediately after opening
  = Restriking Voltage
  = Transient Recovery Voltage
  = I(f)Zo(s)
  $= [(V_m/s)(1/sL)(s / C)] / [s^2 + 1/LC]$
- $V(s) = V_m [1/s - s / (s^2 + 1/LC)]$

- $V(s) = V_m [1/s - s / (s^2 + 1/LC)]$

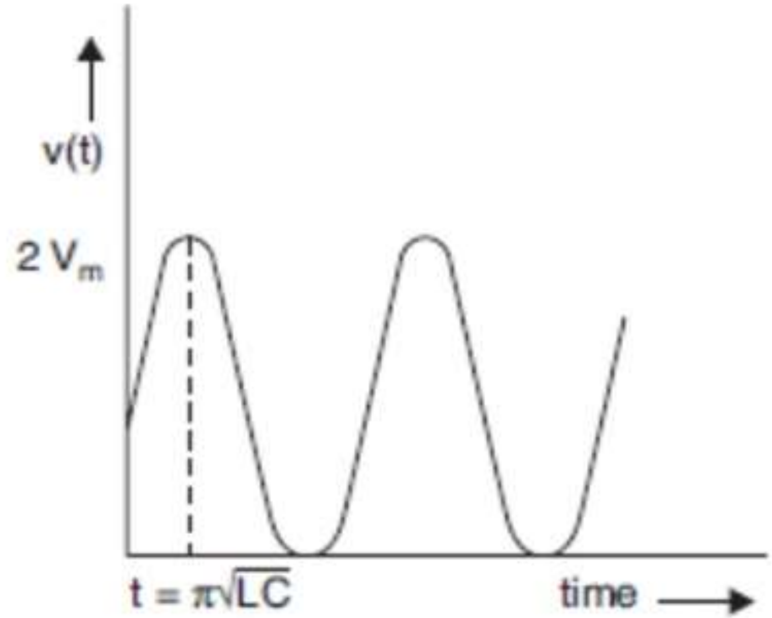
Taking inverse laplace transform, we get

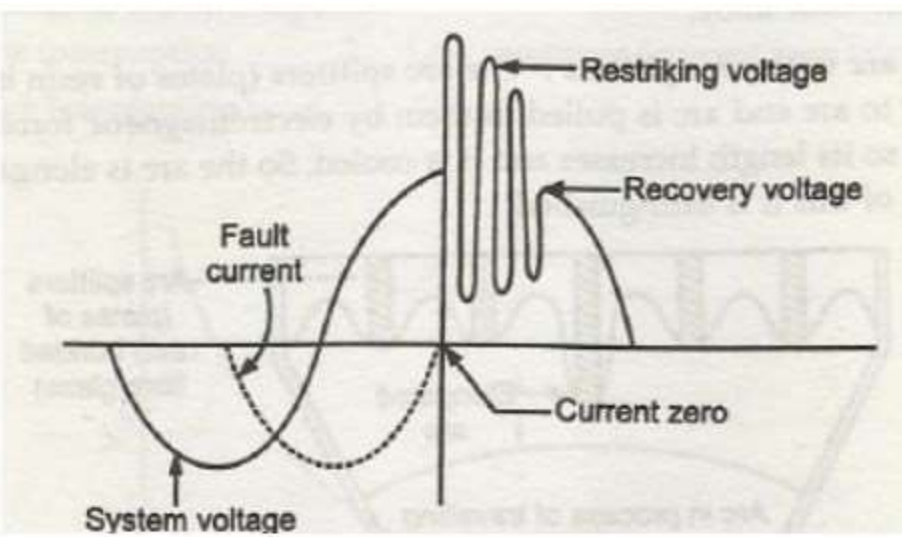
- **$v(t) = V_m[1 - \cos\omega_0 t]$** ....(3)
- where $\omega_0 = 1 / \sqrt{LC}$ and hence $f_0 = (1/2\pi\sqrt{LC})$ where f_0 is the natural frequency of oscillation.

$$\omega_0 t = \pi$$

$$\frac{1}{\sqrt{LC}} t = \pi$$

$$t = \pi\sqrt{LC}$$





- The maximum value of TRV = $2V_m$ when $\omega_0 t = \pi$
- $t = \pi / \omega_0$

  $= \pi / (1 / \sqrt{LC})$

  $= \pi \sqrt{LC}$
- Equ. 3 is the restriking or transient recovery voltage.

Characteristic of Restriking Voltage

- The important characteristic of restriking voltage which affects the performance of the circuit breaker is as follows –
- **Amplitude Factor:** It is defined as the ratio of the peak of transient voltage to the peak system frequency voltage.
- **The rate of Rising of Restriking Voltage:** It is defined as the slope of the steepness tangent of the restriking voltage curve. It is expressed in kV/μs.
- RRRV is directly proportional to the natural frequency. The expression for the restriking voltage is expressed as

$$RRRV_{max} = \frac{V_{max}}{\sqrt{LC}}$$

- The transient voltage vanishes rapidly due to the damping effect of system resistance, and the normal frequency system voltage is established.
- This voltage across the breakers contact is called recovery voltage.

Travelling Wave

- **Definition:** Travelling wave is a temporary wave that creates a disturbance and moves along the transmission line at a constant speed.
- Such type of wave occurs for a short duration (for a few microseconds) but cause a much disturbance in the line.

- The transient wave is set up in the transmission line mainly due to switching, faults and lightning.

- These waves helps in designing the insulators, protective equipment, the insulation of the terminal equipment, and overall insulation coordination.

Travelling Wave on Transmission Line

- To understand the travelling wave phenomenon over transmission line consider a transmission line
- The line is assumed to be lossless line.
- Considered a long transmission line having a distributed parameter inductance (L) and capacitance (C).
- The long transmission line has been represented by a large number of L and C pi sections as shown in fig b.

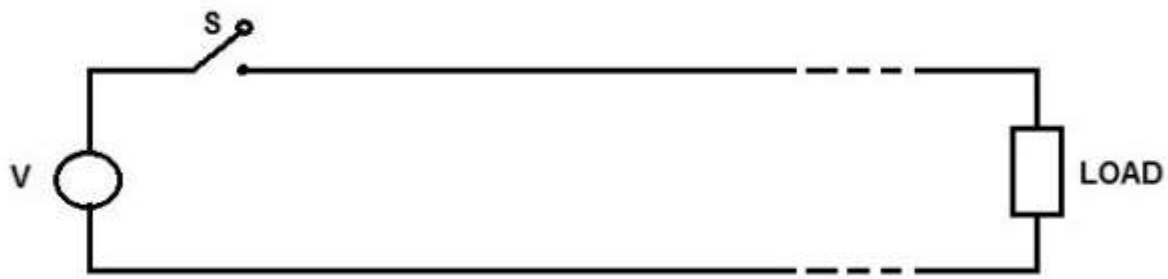


Fig a Long transmission line

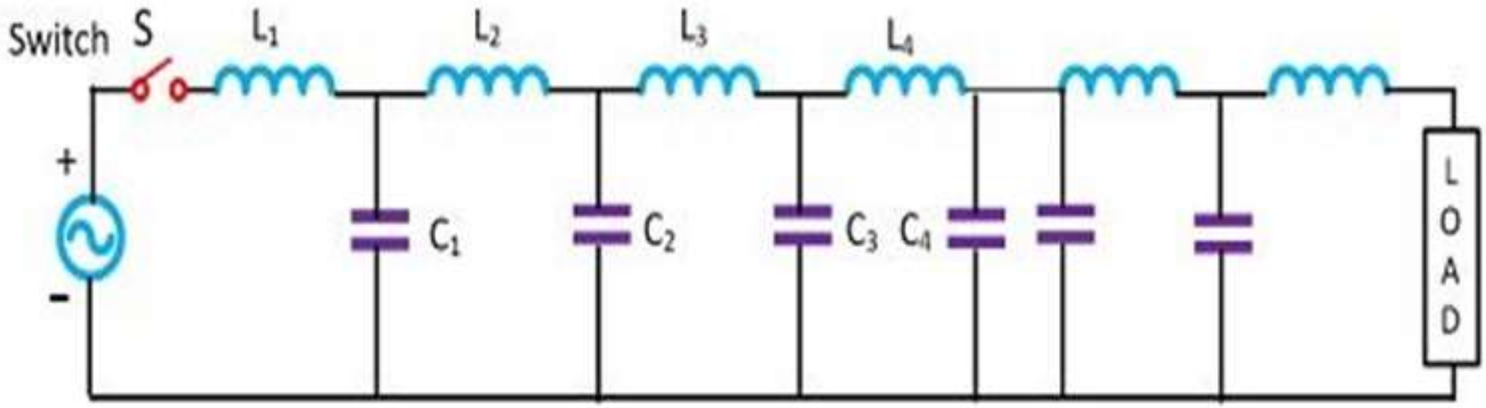


Fig b Equivalent Pi-Section of a Long Transmission Line

- When a transmission line is suddenly connected to a voltage source by the closing of a switch S, the voltage does not appear instantaneously at the other end.
- When the switch is closed the L_1 act as an open circuit and C_1 act as a short circuit.
- At the same instant, the voltage at the next section cannot be charged because the voltage across the capacitor C_1 is zero.

- So unless the capacitor C_1 is charged to some value the charging of the C_2 through L_2 is not possible which will obviously take some time.
- The same argument applies to the third section, fourth section, and so on.
- The voltage at the successive sections builds up gradually.
- This gradual build up of voltage over the transmission conductor can be regarded as a voltage wave is travelling from one end to the other end .
- The gradual charging of the capacitances is due to associate current wave.

- The current wave, which is accompanied by voltage wave steps up a magnetic field in the surrounding space.
- At junctions and terminations, these waves undergo reflection and refraction.
- The total energy of the resultant wave cannot exceed the energy of the incident wave.

- Now it is desired to find out expression for the relation between the voltage and current waves travelling over the transmission lines and their velocity propagation.
- Suppose that the wave after time t has travelled through a distance x.
- Since we have assumed lossless lines, what ever is the value of V and I at the start, remain same throughout the travel.
- Consider a distance dx which is travelled by the waves in time dt.
- The electrostatic flux is associated with the voltage wave and electromagnetic flux with current wave.

- Now the electrostatic flux which is equal to the charge between the conductors of the line up to a distance x is given by,

$$q = VCx$$

Now current is given by

$$I = \frac{dq}{dt} = VC\frac{dx}{dt} = VCv \quad \text{..................(1)}$$

Where, v=velocity of the travelling wave

But dx/dt = velocity of travelling wave = v (say)

- Now electromagnetic flux linkages created around the conductor due to the current is,

$$\psi = ILx$$

So voltage is given by,

$$V = IL\frac{dx}{dt} = ILv \qquad \text{.......................(2)}$$

- Dividing eq (2) by eq (1),

$$\frac{V}{I} = \frac{ILv}{VCv} = \frac{I}{V} \cdot \frac{L}{C}$$

$$\therefore \qquad \frac{V}{I} = \sqrt{\frac{L}{C}} = Z_n$$

- Zn is called surge impedance will remain constant for a given transmission line.
- This value will not change due to change in length of line. The value of surge impedance for a typical transmission line is around 400 Ohm and that for a cable is around 40 ohm.

- Now Multiplying eq (1) and (2)

$$VI = VCv \cdot ILv = VILCv^2$$

$$v^2 = \frac{1}{LC}$$

$$v = \frac{1}{\sqrt{LC}}$$

The velocity of travelling wave is constant.

For overhead line the values of L and C are given as

$$L = 2 \times 10^{-7} \ln\frac{d}{r} \; {}^{H}/_{meter}$$

$$C = \frac{2\pi\epsilon}{\ln\frac{d}{r}} \; {}^{F}/_{meter}$$

$$\therefore \quad v = \frac{1}{\left(2\times10^{-7}\ln\frac{d}{r}\cdot\frac{2\pi\varepsilon}{\ln\frac{d}{r}}\right)^{1/2}} = 3\times 10^{-8} \quad {}^{meter}/_{second}$$

- So it means velocity of travelling wave in T.L. is equal to the velocity of light.

UNIT-III POWER SYSTEM TRANSIENTS

POWER SYSTEM TRANSIENTS

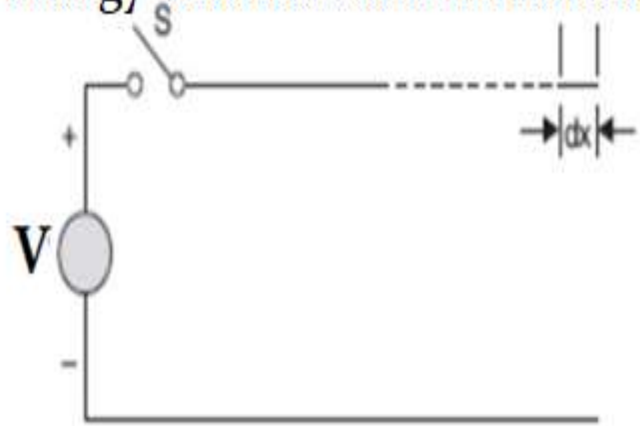
Introduction-Circuit closing transients - Recovery transient due to removal of a short circuit-Travelling waves on transmission line -Surge impedance and wave velocity-Specification of travelling waves-Reflections and refractions of waves - Different types of terminations-Forked line-Successive reflections - Bewley's Lattice diagram-Attenuation and distortion.

Travelling Wave With Open End Line

- Let us consider a T.L. open circuited at the receiving end and a steady voltage **v** is suddenly applied at the sending end.
- When switch S is closed on an unenergised line of length L, the voltage V and current I= V/Z travel towards open end on line at the velocity v
- Necessarily, at the open end no current can flow

- Equating electrostatic and electromagnetic energy in dx, since the current at the open end is zero, the electromagnetic energy vanishes and is transformed into electrostatic energy.

As a result change in voltage be then

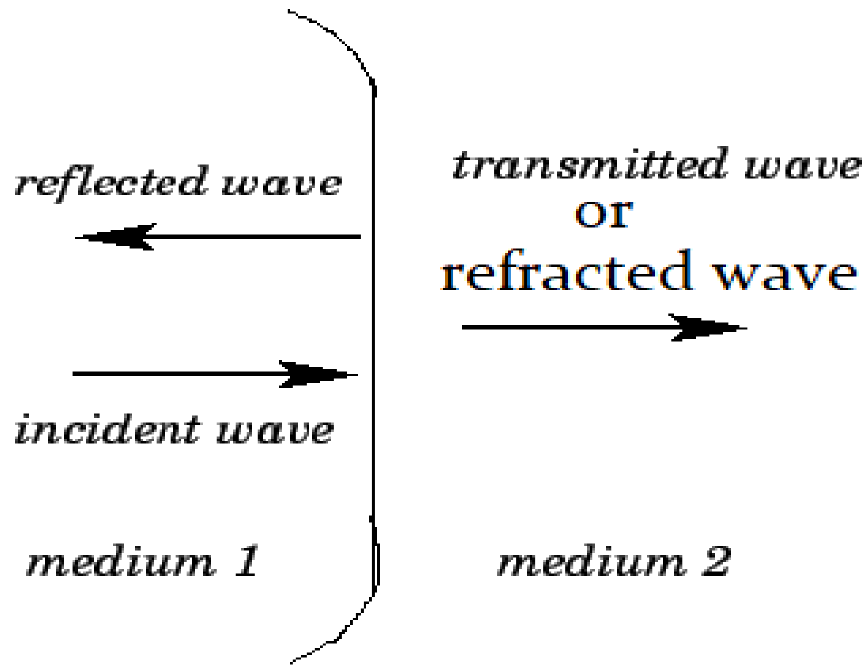


$$\frac{1}{2}Cdxe^2 = \frac{1}{2}LdxI^2$$

$$e = \sqrt{\frac{L}{C}}I = Z_n I = \mathbf{V}$$

This means that voltage at the open end is raised by V volts. So total voltage at the open end when wave reaches at the end,

$$V+V=2V \quad \cdots\cdots\cdots\cdots \quad (3)$$



Figure

Transmitted Wave= Incident Wave + Reflected Wave

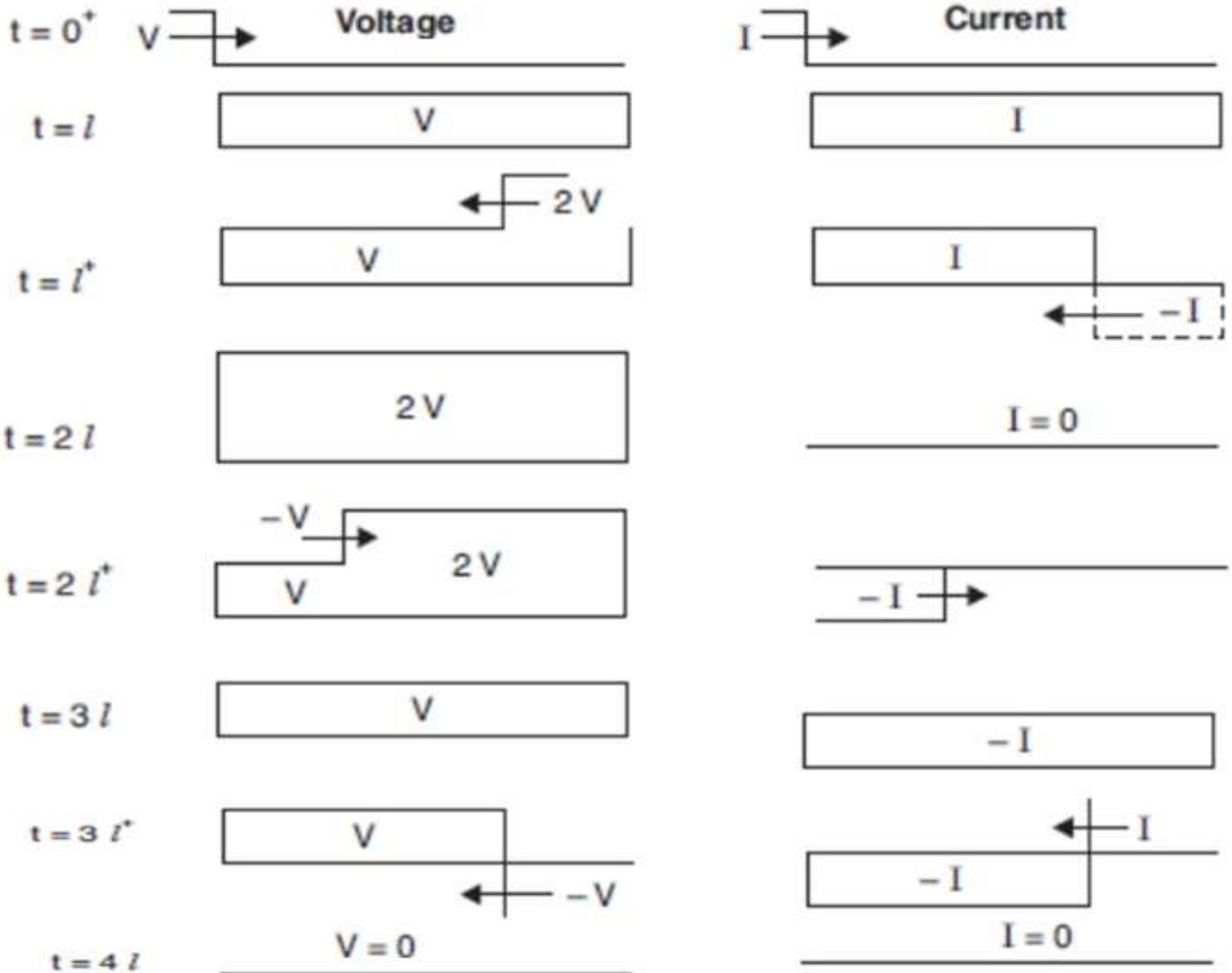
- The wave that starts travelling over the line when the switch S is closed, can be considered as the **incident wave** and after the wave reaches the open end the rise in potential V can be considered due to a wave which is **reflected** at the open end and actual voltage at the open end can be considered as **refracted wave or transmitted wave** and so, Transmitted Wave= Incident Wave + Reflected Wave

- From eq(3) it is cleared that for voltage, a travelling wave is reflected back with **+ve** sign.
- For a open end line a travelling wave is reflected back with +sign and coefficient of reflection as unity
- Now let us see about current wave.
- Here, as incident current wave I reaches the open end the current at the open end is zero, this means that for a open end line , a current wave of I magnitude travels back over T.L. so far an open end line. a current wave is reflected with **–ve** sign and coefficient of reflection is unity.

- After the voltage and current waves are reflected back from open end, they reach the source end, the voltage over the line becomes 2V and the current is zero.
- The voltage at the source end can not be more than source voltage V .
- So a voltage wave of –V and a current wave of

–I is reflected back into the line.

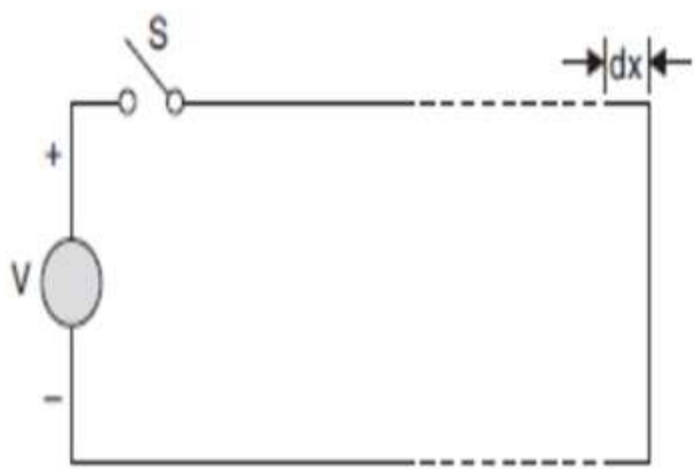
- It can be seen that the wave have travelled through a distance of 4l when l is length of line, they would have wiped out both voltage and current waves, leaving the line momentarily in its original state.
- The above cycle repeats itself as shown in figure.



t = 0+
V
Voltage
I
Current
t = l
V
I
2 V
t = l+
V
I
−I
t = 2 l
2 V
I = 0
−V
t = 2 l+
2 V
V
−I
t = 3 l
V
−I
t = 3 l+
V
I
−I
−V
t = 4 l
V = 0
I = 0

SHORT CIRCUITED LINE

- Here when switch S is closed, a voltage wave of magnitude of V and current of magnitude I starts travelling towards the S.C. end. Consider dx element where electrostatic energy is $\frac{1}{2}CdxV^2$ and electromagnetic energy is $\frac{1}{2}LdxI^2$.



- Equating electrostatic and electromagnetic energy in dx, since the voltage at the s.c end is zero, the electrostatic energy vanishes and is transformed into electromagnetic energy.

  As a result change in current be then

$$\frac{1}{2} C dx\, v^2 = \frac{1}{2} L dx\, i^2$$

$$V = iZ$$

$$i = \frac{V}{Z} = I$$

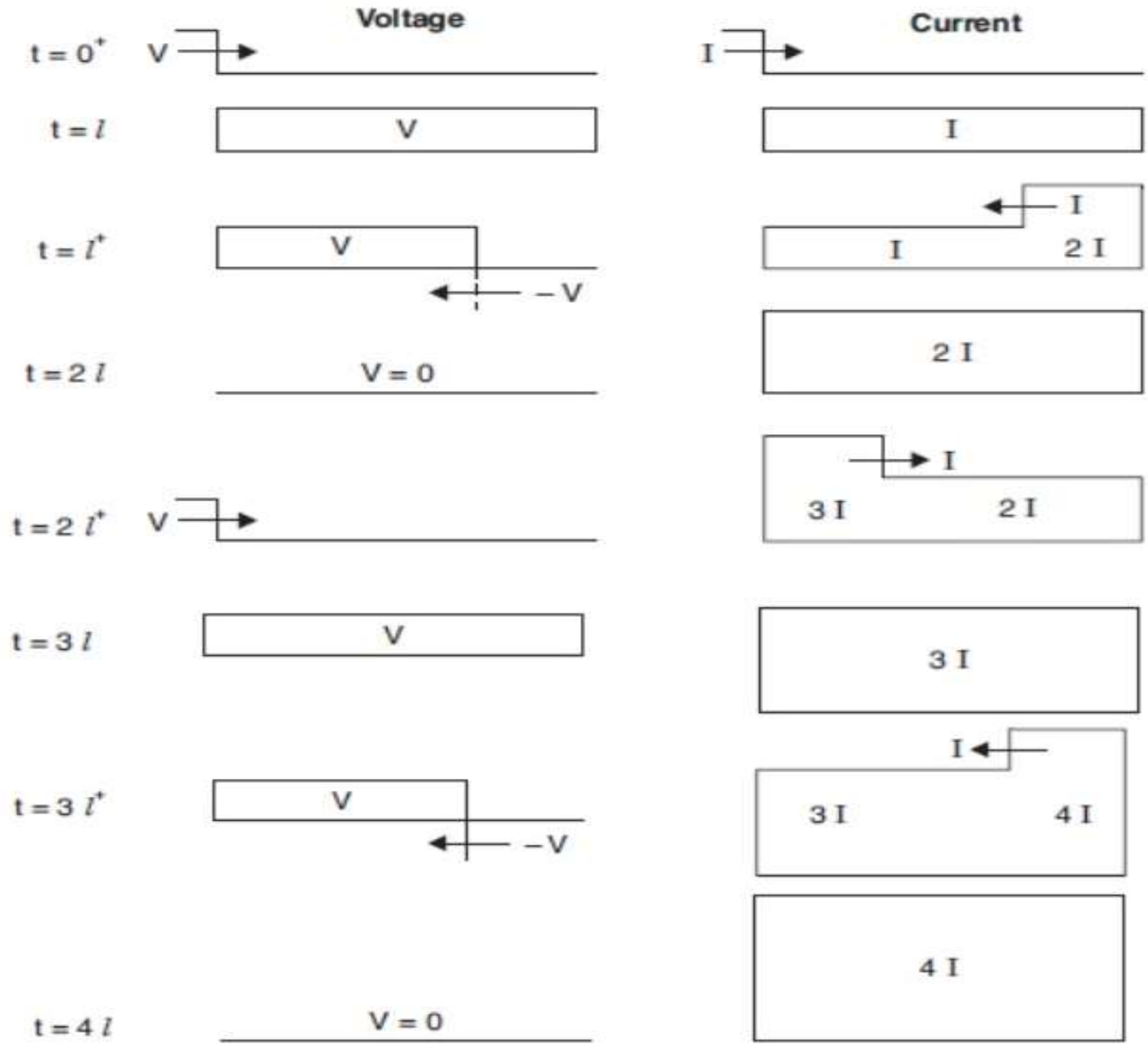
- This means the increase in current is I Amperes. As a result the the total current at the shorted end, when the current wave reaches the end is (I+I) = 2I Amperes .

This could be considered due to a reflected current wave of magnitude I amperes. Therefore for a short circuited end the current wave reflected back with +sign and coefficient of reflection as unity.

- Since the voltage at the shorted end is zero, a voltage wave of –V have been reflected back into the line.
- It is seen from the fig. that the voltage wave periodically reduces to zero after it has travelled through a distance of twice the length of line.
- Whereas after each reflection at either end the current is build up by an amount I.

- Theoretically the reflection will be infinite and the current will reach infinite value.
- But practically in an actual system the current will be limited by resistance of line and final value of current will be I = V/R
- Where R is the resistance of line

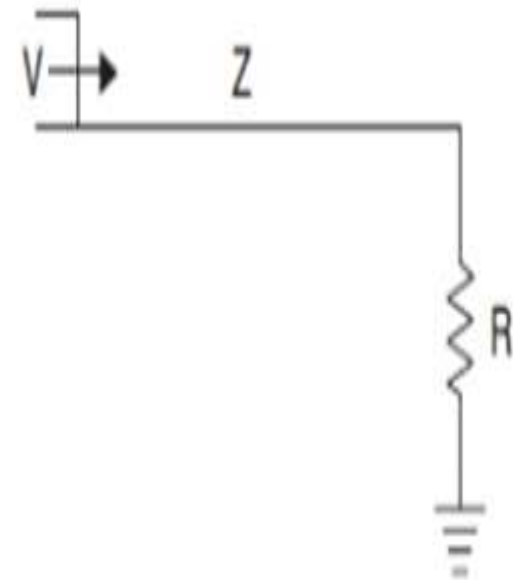
- Variation of voltage and current over line is shown in fig.



Voltage
Current
t = 0⁺
V
I
t = l
V
I
I
t = l⁺
V
I
2 I
–V
t = 2 l
V = 0
2 I
I
t = 2 l⁺
V
3 I
2 I
t = 3 l
V
3 I
I
t = 3 l⁺
V
3 I
4 I
–V
4 I
t = 4 l
V = 0

LINE TERMINATED THROUGH A RESISTANCE

- Consider a lossless T.L. which has a surge impedance of Z terminated through a resistance R.



Let V & I - INCIDENT WAVES

V' & I' -REFLECTED WAVES

V" &I" -REFRACTED OR TRANSMITTED WAVES

LINE TERMINATED THROUGH A RESISTANCE

Whatever the value of terminating impedance , whether it is open or short circuited, one of two voltage or current wave is reflected back with -ve sign. Therefore

$$I' = -\frac{V'}{Z}$$

Where V' and I' are reflected voltage and current wave

- But Refracted or transmitted wave =(Incident Wave + Reflected Wave)

Let V'' & I'' be the refracted voltage and current waves into the Resistance R,

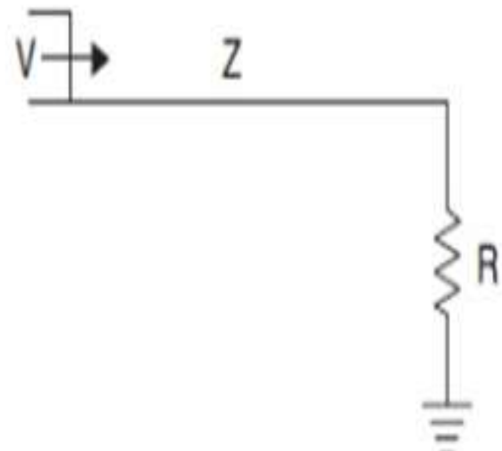
When incident waves V and I reach resistance R.

The following relations holds good

$$I = \frac{V}{Z}$$

$$I' = -\frac{V'}{Z}$$

$$I'' = \frac{V''}{R}$$



So $I'' = I + I'$

$$\frac{V''}{R} = \frac{V}{Z} - \frac{V'}{Z} = \frac{V}{Z} - \frac{V'' - V}{Z} = \frac{2V}{Z} - \frac{V''}{Z}$$

$$V'' = \frac{2R}{Z+R} V \qquad \text{———} \quad (1)$$

$$I'' = \frac{V''}{R} = \frac{2V}{R+Z} = \frac{V}{Z} \frac{2Z}{R+Z}$$

$$= I \frac{2Z}{R+Z} \qquad \text{———} \quad (2)$$

substituting V'' interms of $V+V'$ in above equ

$$\frac{V''}{R} = \frac{V}{Z} - \frac{V'}{Z}$$

$$\frac{V+V'}{R} = \frac{V}{Z} - \frac{V'}{Z}$$

$$V' = V.\ \frac{R-Z}{R+Z} \qquad (3)$$

$$I' = -\frac{V'}{Z} = -\frac{V}{Z}\ \frac{R-Z}{R+Z} = -I\ \frac{R-Z}{R+Z} \qquad (4)$$

From above equations

Coefficient of refraction for current waves $= \dfrac{2Z}{R+Z}$

Coefficient of refraction for voltage waves $= \dfrac{2R}{Z+R}$

Coefficient of reflection for current waves $= -\dfrac{R-Z}{R+Z}$

Coefficient of reflection for voltage waves $= \dfrac{R-Z}{R+Z}$

EXTREMITIES

OPEN CIRCUIT(R=∞)	**SHORT CIRCUIT**(R=0)
coefficient of reflected current wave= **-1**	coefficient of reflected current wave= **1**
coefficient of reflected voltage wave= **1**	coefficient of reflected voltage wave= **0**
coefficient of refracted current wave= **0**	coefficient of refracted current wave= **2**
coefficient of refracted voltage wave= **2**	coefficient of refracted voltage wave= **0**

- **In case of R=Z**
- **Coefficient of reflection for current and voltage wave is zero**
- **Coefficient of refraction for current and voltage wave is 1**
- Therefore when a transmission line is terminated through a resistance equal to its surge impedance , the wave does not suffer reflection such lines
- Such lines are said to be infinite length lines and also the load corresponding to this is known as surge impedance loading or natural loading.

Line connected to cable

- A wave travelling over the line entering the cable as shown in Fig., looks into different impedance and therefore, it suffers reflection and refraction at the junction.
- The refracted voltage is given by

$$V'' = V \frac{2Z_2}{Z_1 + Z_2}$$

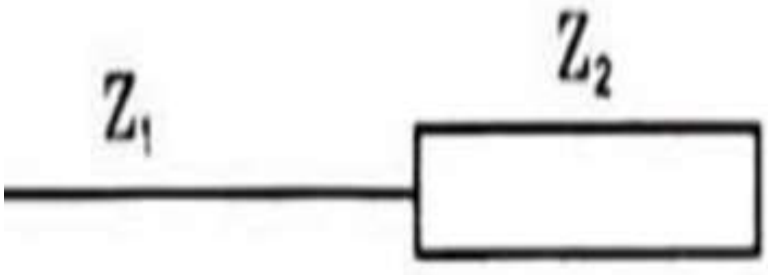


Fig Line connected to a cable

The surge impedances of the overhead line and cable are approximately 400 Ω and 40 Ω respectively. With these values it can be seen that the voltage entering the cable will be –

$$V'' = V \frac{2Z_2}{Z_1 + Z_2} = V.\frac{2 \times 40}{40 + 400} = \frac{2}{11}V$$

or roughly 20% of the incident voltage.

This is the reason that an overhead line is terminated near a station by connecting the station equipment to the overhead line through a short length of underground cable. Besides the reduction in magnitude of the voltage wave, it also reduces the steepness of the wave. It is because of capacitance of the cable.

Reflection and Refraction at a T-Junction

- A voltage wave V is travelling over the line with surge impedance Z1 as shown in fig1
- When it reaches the junction, it looks a change in impedance.
- Therefore suffers reflection and refraction. Let V_2'', I_2'' & V_3'', I_3'' be the voltages and currents in lines having surge impedances Z2 and Z3 respectively.
- Since Z2 and Z3 form a parallel path as far as surge wave is concerned, $V_2'' = V_3'' = V''$

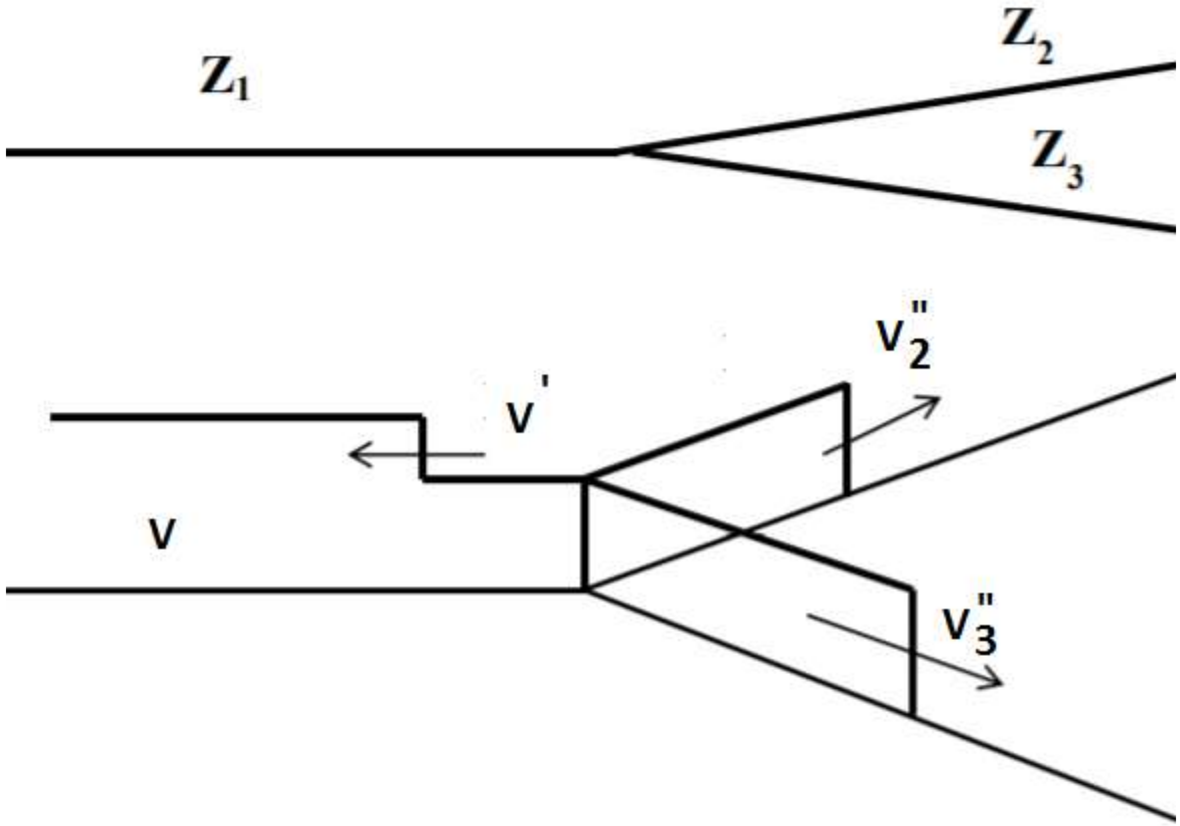


Fig1 A bifurcated line

Let V, I be the incident voltage and current

V', I' be the reflected voltage and current

V_2'', I_2'' be the transmitted voltage and current along Z_2

V_3'', I_3'' be the transmitted voltage and current along Z_3

Since $V_2'' = V_3'' = V''$

Following relations holds good

$V + V' = V''$

$$I = \frac{V}{Z_1}, \quad I' = -\frac{V'}{Z_1}$$

Then $I_2'' = \dfrac{V''}{Z_2}$ and $I_3'' = \dfrac{V''}{Z_3}$

$$\& \quad I + I' = I_2'' + I_3'' \quad \text{———} \quad 1$$

$$\frac{V}{Z_1} - \frac{V'}{Z_1} = \frac{V''}{Z_2} + \frac{V''}{Z_3}$$

substituting V' = V"- V

$$\text{Also } \frac{V}{Z_1} - \left(\frac{V'' - V}{Z_1}\right) = \frac{V''}{Z_2} + \frac{V''}{Z_3}$$

$$\frac{2V}{Z_1} = V''\left(\frac{1}{Z_1} + \frac{1}{Z_2} + \frac{1}{Z_3}\right)$$

$$V'' = \frac{\frac{2V}{Z_1}}{\frac{1}{Z_1} + \frac{1}{Z_2} + \frac{1}{Z_3}} \quad \text{———} \quad 2$$

The solution of which is

$$V' = \frac{V\left(\frac{1}{Z_1} - \frac{1}{Z_2} - \frac{1}{Z_3}\right)}{\frac{1}{Z_1} + \frac{1}{Z_2} + \frac{1}{Z_3}}$$

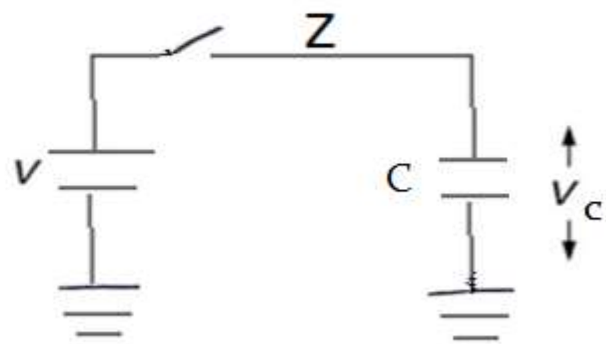
Knowing V, all the other quantities can be calculated. If we put $Z_3 = \infty$ in the above expression, we have

$$V'' = \frac{2\,V Z_2}{Z_1 + Z_2}$$

The case becoming that of a simple junction of two lines having different characteristics.

Line terminated through a capacitor

- Consider a D.C surge of infinite length, travels over the line of surge impedance Z and is incident on the capacitor as shown in fig



Line terminated by capacitance

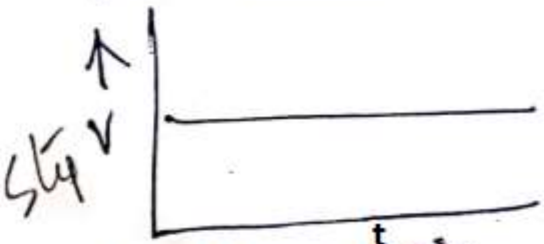
Refracted Voltage $= v''(s) = \dfrac{2 \cdot 1/cs}{Z + 1/cs} \cdot \dfrac{v}{s}$

$$= \frac{2v}{s} \cdot \frac{1}{Zcs+1}$$

$$= \frac{2v}{s} \cdot \frac{1/Zc}{s + 1/Zc} = 2v\left[\frac{1}{s} - \frac{1}{s+1/Zc}\right]$$

Voltage across capa. $= v''(t) = 2v\left[1 - e^{-t/Zc}\right]$

The variation of voltage is shown in fig



Figa Incident voltage

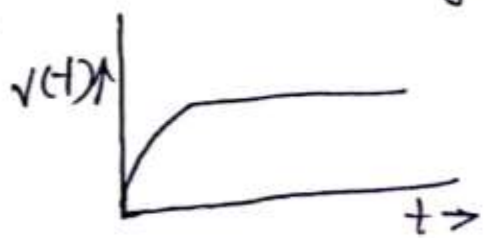
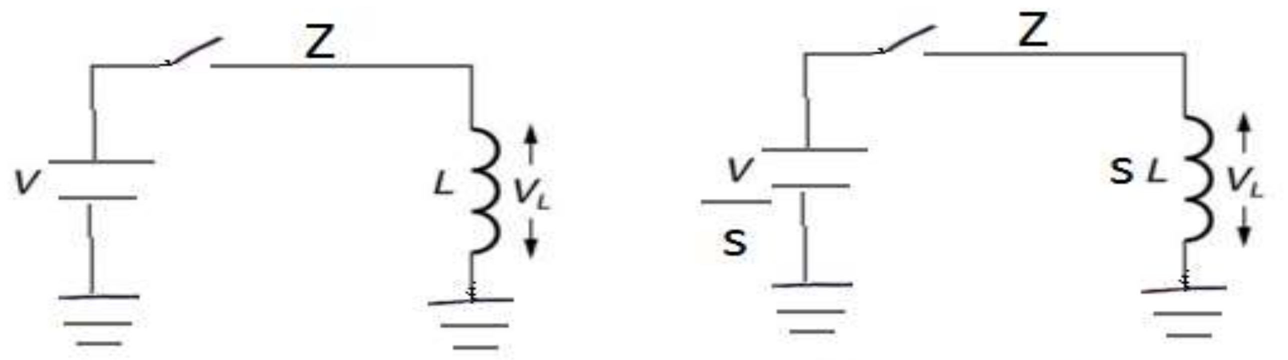


Fig b voltage across capacitor

LINE TERMINATED THROUGH A INDUCTANCE



Line terminated by inductance

Transformed network

Refracted voltage= $V''(S) = \frac{V}{S} \frac{2\,.SL}{SL+Z}$

$= 2VL\,.\frac{1/L}{S+Z/L}$

take ILT

$V''(t) = 2V\,e^{-zt/L}$

1) 3-ϕ transmission line has conductors 1.5 cm in dia. spaced 1m apart in equilatual formation. The resistance and leakance are negligible. calculate (i) natural impedance of line.
ii) the line current if a voltage wave of 11 kv travels along the line iii) the rate of energy absorption, the rate of reflection and the state and the form of reflection if the line is terminated thro' a star connected load of 1000Ω
iv) the value of terminating resistance for no reflection and v) the amount of reflected and transmitted power if the line is connected to a cable extension with inductance and capacitance per phase per cm of 0.5×10^{-6} H and 1×10^{-6} μF resp.

Sol Inductance per unit length $= 2\times10^{-7} \ln \frac{d}{r}$ H/m

$= 2\times10^{-7} \ln \frac{100}{0.75}$

$= 2\times10^{-7} \ln 133.3$

$= 2\times10^{-7} \times 4.89$

$= 9.78\times10^{-7}$ H/m

Capacitance per phase per unit length $= \frac{2\pi\epsilon}{\ln \frac{d}{r}}$

$= \frac{2\pi \times 10^{-9}}{36\pi \ln \frac{d}{r}} = 1.136\times10^{-11}$

$\therefore$ Natural impedance $= \sqrt{\frac{L}{C}} = \sqrt{\frac{9.78\times10^{-7}}{1.136\times10^{-11}}} = 294\ \Omega$

ii) line current $= \frac{11\times10^3}{\sqrt{3}\times294} = 21.6\,A$

iii) ∵ the terminating resistance is of higher value as compared to value of surge impedance of line, reflection is with a +ve sign.

$\left(\frac{E\,Z_2 - Z_1}{Z_1+Z_2}\right)$ → terminating surge imp.

voltage across terminating resistance $E'' = \frac{2Z_2E}{Z_1+Z_2}$

where Z_1 = line surge impedance, Z_2 = terminating impedance, E = incident voltage

$$E'' = 2\times\frac{11000}{\sqrt{3}}\,\frac{1000}{1294} = 9.8\ kV$$

∴ The rate of Power consumption = $\frac{3E''^2}{R}$

$= \frac{3 \times 9.8 \times 9.8}{1000} \times 1000$ kW

$= 288$ kW

Reflected voltage $E' = \frac{Z_2 - Z_1}{Z_2 + Z_1} E = \frac{1000 - 294}{1294} \times \frac{11}{\sqrt{3}}$

$= 3.465$ kV

∴ The rate of reflected energy = $\frac{3 \times 3.466^2}{294} \times 1000$ kW

294 → surge impedance

$= 121.8$ kW

iv) In order that the incident wave when reaches terminating resistance, does not suffer reflection, the terminating resistance should be equal to the surge impedance of the line i.e. 294 Ω

v) The surge impedance of cable $= \sqrt{\frac{L}{C}} = \sqrt{\frac{0.5 \times 10^{-8}}{10^{-12}}} = 70.7\ \Omega$

$\frac{2R}{R+Z}$

The refracted voltage $= \frac{2 \times 70.7}{294 + 70.7} \times \frac{11}{\sqrt{3}} = 2.46\ kV$

$\frac{R-Z}{R+Z}$

The reflected voltage $= \frac{70.7 - 294}{364.7} \times \frac{11}{\sqrt{3}} = -3.9\ kV$

∴ Refracted and reflected Powers are

$\frac{3 \times 2.46^2}{70.7} \times 1000 = 256\ kW$ $\quad$ $\frac{3 \times 3.9^2}{294} \times 1000 = 155\ kW$

2. A cable has a conductor of radius 0.75 cm and a sheath of inner radius 2.5 cm. Find (i) the inductance per meter length (ii) capacitance per meter length (iii) surge impedance and (iv) velocity of propagation, if the permittivity of insulation is 4.

Solution:

Radius of conductor, $r = 0.75$ cm

Inner radius of sheath, $R = 2.5$ cm

Permittivity of insulation, $\varepsilon_r = 4$

(i) The inductance per metre length of the cable is,

$$L = 2\times10^{-7}\ln\frac{R}{r}\ \text{H/m}$$

$$= 2\times10^{-7}\times\ln\frac{2.5}{0.75}$$

$$= 2.41\times10^{-7}\ \text{H/m}$$

(ii) The capacitance per metre length of the cable is,

$$C = \frac{2\pi\varepsilon_0\varepsilon_r}{\ln\frac{R}{r}} \text{ F/m}$$

$$= \frac{4\times10^{-9}}{18\times\ln\frac{2.5}{0.75}}$$

$$= 0.1846\times10^{-9} \text{ F/m}$$

(iii) Surge impedance of the cable is,

$$Z_c = \sqrt{\frac{L}{C}}$$

$$= \sqrt{\frac{2.41\times10^{-7}}{0.1846\times10^{-9}}}$$

$$= 36.13\ \Omega$$

(iv) The velocity of wave propagation is,

$$\upsilon = \frac{1}{\sqrt{LC}}$$

$$= \frac{1}{\sqrt{2.41\times10^{-7}\times0.1846\times10^{-9}}}$$

$$= 1.5\times10^{8}\ \text{m/s}$$

Example 3 *A transmission line of surge impedance 250 Ω and is connected to a cable of surge impedance 50 Ω at the other end, if a surge of 400 kV travels along the line to the junction point, find the voltage build at the junction.*

☺***Solution:*** $z_1 = 250\ \Omega$, $z_2 = 50\ \Omega$, $V = 400$ kV

$$V_1' = \frac{z_2 - z_1}{z_1 + z_2} \times V = \frac{250 - 50}{50 + 250} \times 400 = 266.67 \text{ kV}$$

$$\text{Voltage build at the junction} = V + V_1'$$

$$= 400 + 266.67 = 666.67 \text{ kV}$$

Example 4 ***An underground cable of inductance 0.150 mH/km and of capacitance 0.2 μF/km is connected to an overhead line having an inductance of 1.2 mH/km and capacitance of 0.006 μF/km. Calculate the transmitted and reflected voltage and current waves at the junction, if a surge of 200 kV travels to the junction, (1) along the cable and (2) along the overhead line.***

Solution:

$$\text{Surge voltage, } V = 200 \text{ kV}$$

$$L_{cable} = 0.150 \text{ mH/km}$$

$$C_{cable} = 0.2 \text{ μf/km}$$

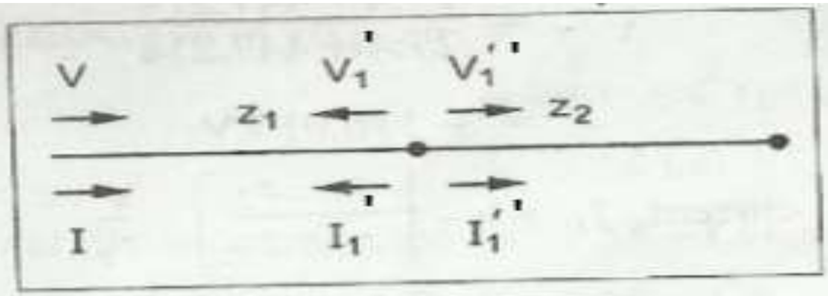
$$\text{Surge impedance of cable, } z_1 = \sqrt{\frac{L}{C}} = \sqrt{\frac{0.15 \times 10^{-3}}{0.2 \times 10^{-6}}}$$

$$= 27.4$$

$$L_{line} = 1.2 \text{ mH/km}$$

$$C_{line} = 0.006 \text{ μf/km}$$

$$\text{Surge impedance of line, } z_2 = \sqrt{\frac{L}{C}} = \sqrt{\frac{1.2 \times 10^{-3}}{0.006 \times 10^{-6}}} = 447.214$$



Case (i): Along the cable:

$$\text{Reflected voltage, } V_1' = \frac{z_2 - z_1}{z_1 + z_2} \cdot V$$

$$= \frac{447.214 - 27.4}{27.4 + 447.214} \times 200$$

$$= 176.91 \text{ kV}$$

$$\text{Reflected current, } I_1' = -\left[\frac{z_2 - z_1}{z_1 + z_2}\right] \cdot \frac{V}{z_1}$$

$$= -\left[\frac{447.214 - 27.4}{27.4 + 447.214}\right] \times \frac{200}{27.4}$$

$$= -6.45 \text{ kA}$$

$$\text{Transmitted voltage, } V_1'' = \frac{2 z_2}{z_1 + z_2} \cdot V = \frac{2 \times 447.214}{27.4 + 447.214} \times 200$$

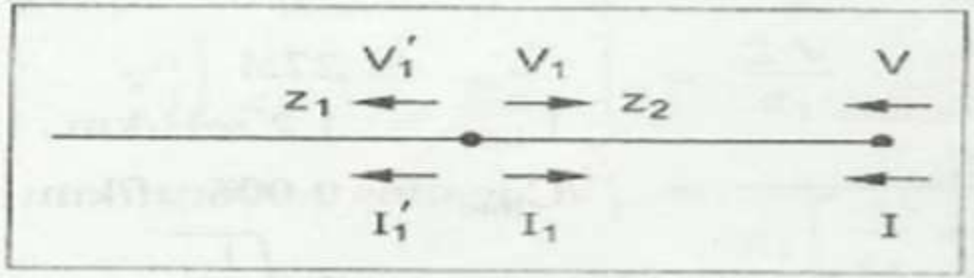
$$= 376.91 \text{ kV}$$

$$\text{Transmitted current, } I_1' = \frac{2 z_1}{z_1 + z_2} \times \frac{V}{z_1}$$

$$= \frac{2 \times 27.4}{27.4 + 447.214} \times \frac{200}{27.4}$$

$$= 0.84 \text{ kA}$$

Case (ii): Along the line



$$\text{Reflected voltage, } V_1' = \frac{z_1 - z_2}{z_1 + z_2} \cdot V$$

$$= \frac{27.4 - 447.214}{27.4 + 447.214} \times 200$$

$$= -176.91 \text{ kV}$$

$$\text{Reflected current, } I_1' = -\left[\frac{z_1 - z_2}{z_1 + z_2}\right] \cdot \frac{V}{z_2}$$

$$= -\left[\frac{27.4 - 447.214}{27.4 + 447.214}\right] \times \frac{200}{447.214}$$

$$= -0.395 \text{ kA}$$

$$\text{Transmitted voltage, } V_1'' = \frac{2 z_1}{z_1 + z_2} \cdot V$$

$$= \frac{2 \times 27.4}{27.4 + 447.214} \times 200$$

$$= 23.09 \text{ kV}$$

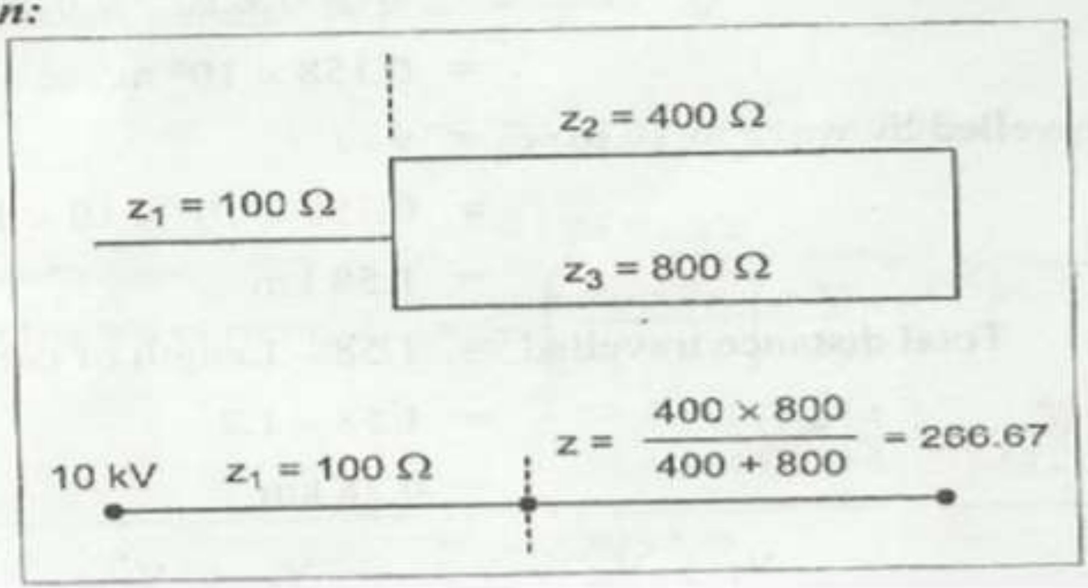
$$\text{Transmitted current, } I_1'' = \frac{2 z_2}{z_1 + z_2} \times \frac{V}{z_2}$$

$$= \frac{2 \times 200}{27.4 + 447.214}$$

$$= 0.84 \text{ kA}$$

Example 5 *A cable with a surge impedance of 100 Ω is terminated in two parallel connected overhead lines having surge impedances 400 Ω and 800 Ω respectively. If a step voltage wave of 10 kV travels along the cable, find the voltages and currents in the cable and overhead lines immediately after the travelling wave reaches the junction of cable and overhead lines.*

☺***Solution:***



$$V_1'' = \frac{2z}{z_1 + z} \cdot V$$

$$= \frac{2 \times 266.67}{100 + 266.67} \times 10 = 14.55 \text{ kV}$$

$$V_1' = \frac{z - z_1}{z_1 + z} \times V$$

$$= \frac{266.67 - 100}{100 + 266.67} \times 10 = 4.55 \text{ kV}$$

$$I_2' = \frac{V_1'}{z_2} = \frac{14.55 \times 10^3}{400} = 36.375 \text{ Amp}$$

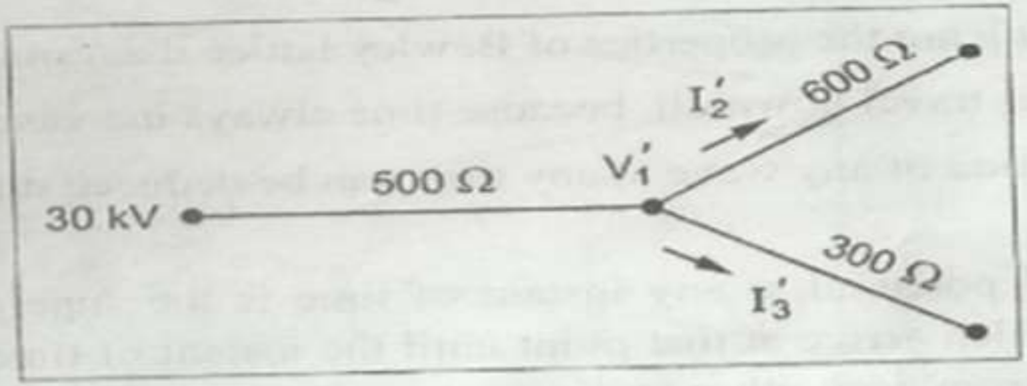
$$I_3' = \frac{V_1'}{z_3} = \frac{14.55 \times 10^3}{800} = 18.188 \text{ Amp}$$

Reflected current, $I_1' = \frac{-(z - z_1)}{z + z_1} \times \frac{V}{z_1}$

$$= \frac{-(266.67 - 100)}{(266.67 + 100)} \times \frac{10 \times 10^3}{100}$$

$$= -45.5 \text{ Amp}$$

Example 6 *A surge of 30 kV traveling in a line of $z_C = 500\ \Omega$, arrives at a junction with two lines having a surge impedances as 600 Ω and 300 Ω respectively. Determine the surge voltage and currents transmitted into each branch line.*



Solution: Refracted or transmitted voltage at the junction V_1''

$$= \frac{2V}{z_1\left[\frac{1}{z_1} + \frac{1}{z_2} + \frac{1}{z_3}\right]}$$

$$= \frac{2 \times 30}{500\left[\frac{1}{500} + \frac{1}{600} + \frac{1}{300}\right]} = 17.14 \text{ kV}$$

Solution: Refracted or transmitted voltage at the junction V_1''

$$= \frac{2V}{z_1\left[\frac{1}{z_1}+\frac{1}{z_2}+\frac{1}{z_3}\right]}$$

$$= \frac{2 \times 30}{500\left[\frac{1}{500}+\frac{1}{600}+\frac{1}{300}\right]} = 17.14 \text{ kV}$$

Transmitted current through line 600 Ω

$$= \frac{V_1''}{600} = \frac{17.14}{600} = 0.028 \text{ kA} = 28 \text{ Amp}$$

Transmitted current through line 300 Ω

$$= \frac{V_1''}{300} = \frac{17.14}{300} = 0.057 \text{ kA} = 57 \text{ Amp}$$

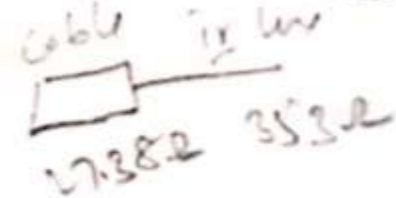
EX 7. A surge of 15 kV magnitude travels along a cable towards its junction with an overhead line. The inductance and capacitance of cable and overhead line are 0.3 mH, 0.4 μF and 1.5 mH, 0.012 μF per km. Find the voltage rise at the junction due to surge.

Sol: ∵ the surge travels from cable towards the overhead line and hence there will be +ve voltage reflection at the junction.

Natural impedance of cable $= \sqrt{\frac{0.3\times10^{-3}}{0.4\times10^{-6}}} = 27.38\,\Omega$

Natural " line $= \sqrt{\dfrac{1.5\times10^{-3}}{0.012\times10^{-6}}} = 353\,\Omega = Z_2$

Voltage rise at the junction is the voltage transmitted into overhead line as voltage is zero before surge reaching junction.

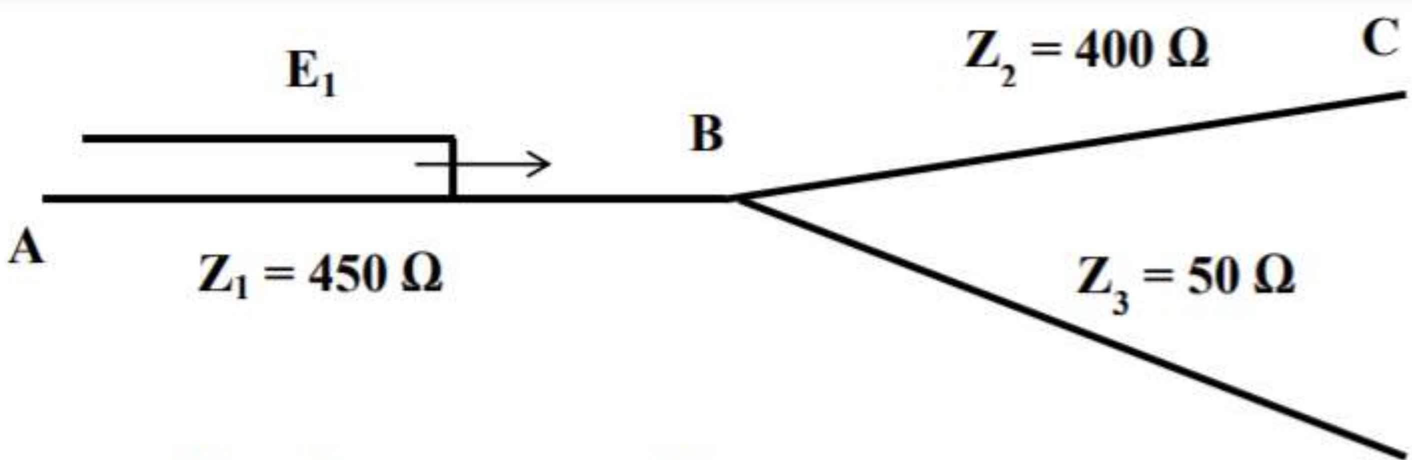


$$E'' = \frac{2\times353\times15}{353+27} = 27.87\ \text{kV}$$

$$\frac{2Z_2}{Z_1+Z_2}\times V$$

Example 8

An overhead transmission line having a surge impedance of 450 ohms runs between two substations A and B; at B it branches into two lines C and D, of surge impedances 400 and 50 ohms respectively. If a travelling wave of vertical front and magnitude 25 kV travels along the line AB, calculate the magnitude of the voltage and current waves which enter the branches at C and D.



Incident voltage = E_1 =25000 V

Incident current = $I_1 = E_1/Z_1$ = 25000/450 = 55.6 A

Transmitted voltage along BC and BD

$$E'' = \frac{\frac{2E_1}{Z_1}}{\frac{1}{Z_1} + \frac{1}{Z_2} + \frac{1}{Z_3}}$$

$$E'' = \frac{\frac{2\times 25}{450}}{\frac{1}{450}+\frac{1}{400}+\frac{1}{50}} = 4.5\ kV$$

Transmitted current along BC:

$I_2 = E''/Z_2 = 4500/400 = 11.25$ A

Transmitted current along BD:

$I_3 = E''/Z_3 = 4500/50 = 90$ A

Thus, the current reflected back into line AB = 90 + 11.25 – 55.6 = 45.65 A

1) A surge of 100KV travelling in a line of natural impedance 600Ω arrives at a junction with two lines of impedances 800Ω and 200Ω respectively. Find the surge voltages and currents transmitted into each branch line

Sol $Z_1 = 600\Omega$ $Z_2 = 800\Omega$ $Z_3 = 200\Omega$

surge magnitude = 100KV

$$E'' = \frac{2E/Z_1}{\frac{1}{Z_1}+\frac{1}{Z_2}+\frac{1}{Z_3}} = \frac{2\times 100/600}{\frac{1}{600}+\frac{1}{800}+\frac{1}{200}} = 42.04 \text{ kV}$$

Sol $Z_1 = 600\,\Omega$ $Z_2 = 800\,\Omega$ $Z_3 = 200\,\Omega$

Surge magnitude = 100 kV

$$E'' = \frac{2E/Z_1}{\frac{1}{Z_1}+\frac{1}{Z_2}+\frac{1}{Z_3}} = \frac{2\times 100/600}{\frac{1}{600}+\frac{1}{800}+\frac{1}{200}} = 42.04\text{ kV}$$

Transmitted current in line $Z_2 = \frac{42.04\times 1000}{800} = 52.55\text{A}$

" " $Z_3 = \frac{42.04\times 1000}{200} = 210.2\text{ A}$

10 A Surge of 200 kV travelling on a line of surge impedance 400 Ω (600 Ω) reaches a junction of line with two branch lines of surge impedances 500 Ω (800 Ω) and 300 Ω (700 Ω) respectively. Find the surge voltage and current transmitted into each branch line. Also find the reflected surge voltage and current.

$Z_1 = 400\,\Omega$

$Z_2 = 500\,\Omega$

$Z_3 = 300\,\Omega$

$E = 200\,kV$ (The surge mag.)

$e_t'' = E'' = \dfrac{2E/Z_1}{\frac{1}{Z_1} + \frac{1}{Z_2} + \frac{1}{Z_3}} = 128\,kV$

$e_t = 127.66\,kV = e_i - e_t$

transmi. current in line 2: $i_{t1} = \dfrac{e_t}{Z_1} = \dfrac{127.66}{500} = 0.255\,kA$

$i_{t2} = \dfrac{e_t}{Z_2} = \dfrac{127.66}{300} = 0.425\,kA$

$e_f = e_t'' + e_3''$

$e_r = e_t - e_f = 127.66 - 200 = -72.34\,kV$ (incident)

$i = i_t'' + i_3''$

$i_r = i_{t1} + i_{t2} - i_f = 0.255 + 0.425 - \dfrac{200}{400}$

$= 0.18\,kA$

11) A Step wave of 100 kV travels on a line having a surge impedance of 40 Ω. The line is terminated by an inductance of 4000 μH. Find the voltage across the inductance and the reflected voltage wave.

Voltage across inductance $= 2V e^{-tZ/L}$

$= 2 \times 100 \times e^{-t \frac{400}{4000}}$

$= 200 e^{-0.1t}$ kV

$V' = V'' - V$

$= (2V e^{-tZ/L} - V)$

Bewley Lattice Diagram

https://www.youtube.com/watch?v=rpBJf9-FxdE

- In order to keep track of the multiplicity of successive reflections of discontinuities in the system, Bewley has devised a time-space diagram.
- Which shows at a glance, the motion and direction of motion of every incident, reflected, and transmitted wave on the system at every instant of time.

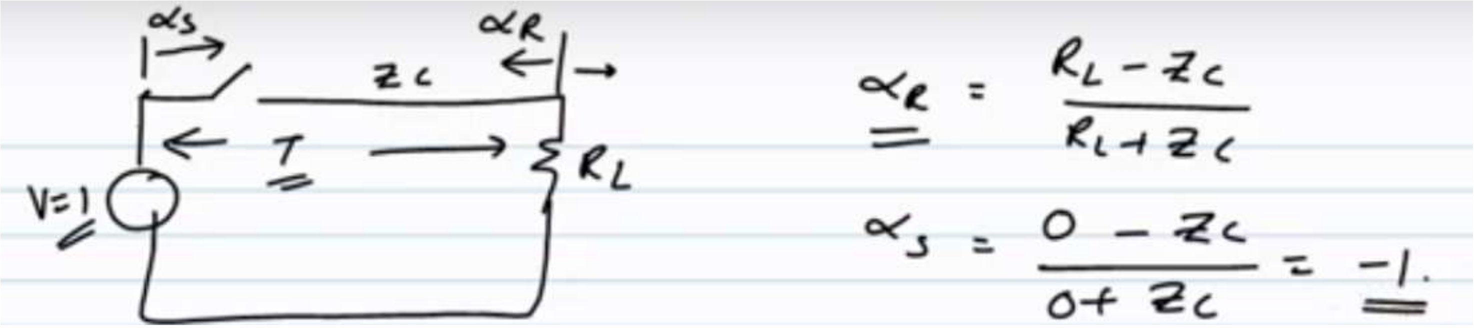
properties of Bewley lattice diagram

- (1)All waves travel downhill, because time always increases.
- (2)The position of any wave at any time can be deduced directly from the diagram.
- (3)The total potential at any point, at any instant of time is the superimposed of all the waves which have arrived at that point up until that instant of time.
- (4)The history of the wave is easily traced.
- (5)Attenuation is included.

Bewley Lattice Diagram

- In lattice diagram two axis are established, a horizontal one scaled in distance along the system, and a vertical one scaled in time.
- Line showing the passage of surge are drawn such that there is a slope gives the time corresponding to distance travelled.
- At each point of change in impedance, the reflected and transmitted waves are obtained by multiplying the incident wave magnitude by proper reflection and refraction coefficient.

- Lattice diagram for current may also be drawn.
- The reflection coefficient for current is always negative of reflection coefficient of voltage.
- Consider a simple system shown in figure
- A generator of unit voltage is switched on to a loss less line of characteristic impedance Zc with load resistance R_L at the receiving end.



- It is assumed that generator has zero impedance such that a unit voltage wave is continuously feed to the line after switching instant.

The reflection coefficient at receiving end is

$$\alpha_r = \frac{R_L - Z_c}{R_L + Z_c}$$

The reflection coefficient at sending end is

$$\alpha_s = \frac{0 - Z_c}{0 + Z_c} = -1.$$

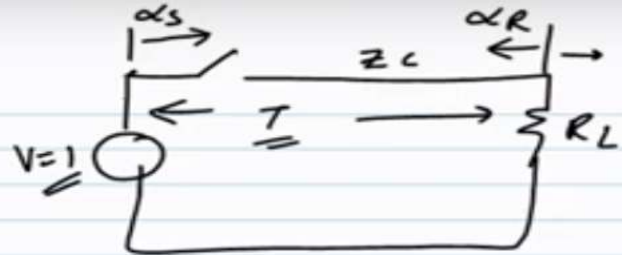
- The generator acts like a S.C with the assumption of zero impedance.
- Let the time of travel of surge from one end to other be T.
- Immediately upon switching a unit step voltage surge travels down the line towards the receiving end, this fact is diagrammatically recorded by a line sloping downwards(Left to right) as shown in fig.

- When surge reaches the line end, a surge of amplitude α_R is originated in the process of reflection, which then travels the generator end at t=2T represented by a sloping line(right to left).
- The reflection of generator end causes an outward surge of strength $-\alpha_R$ this process continues indefinitely.
- It is easily observed from the diagram that at the receiving end the increment of voltage at each refraction is sum of incident and reflected wave.

- After infinite reflections the voltage becomes unity, the receiving end voltage is plotted against time and shown in figure
- The resulting voltage at various instants are

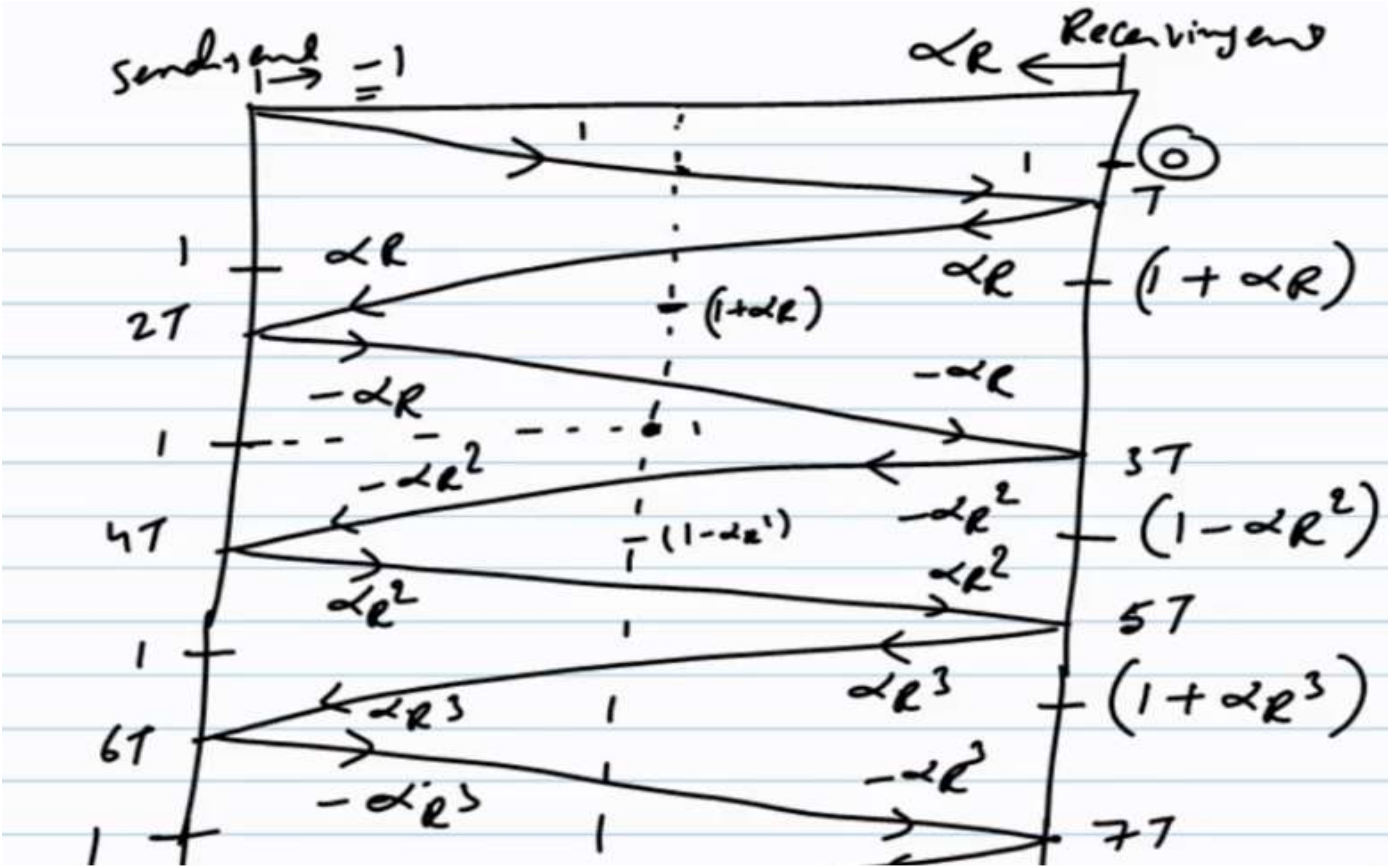
$$V_R = (1+\alpha_R) - (\alpha_R + \alpha_R^2) + (\alpha_R^2 + \alpha_R^3) - (\alpha_R^3 + \alpha_R^4) + \text{so on}$$

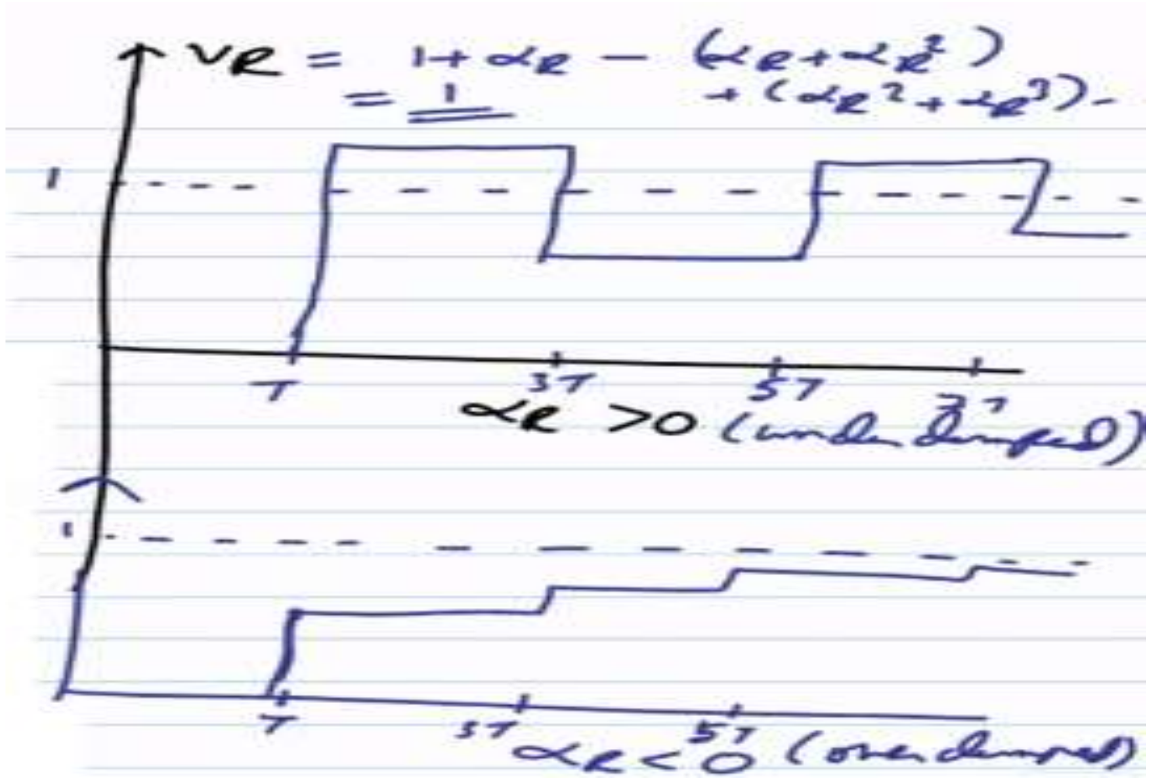
$$= 1$$



$$\alpha_R = \frac{R_L - Z_c}{R_L + Z_c}$$

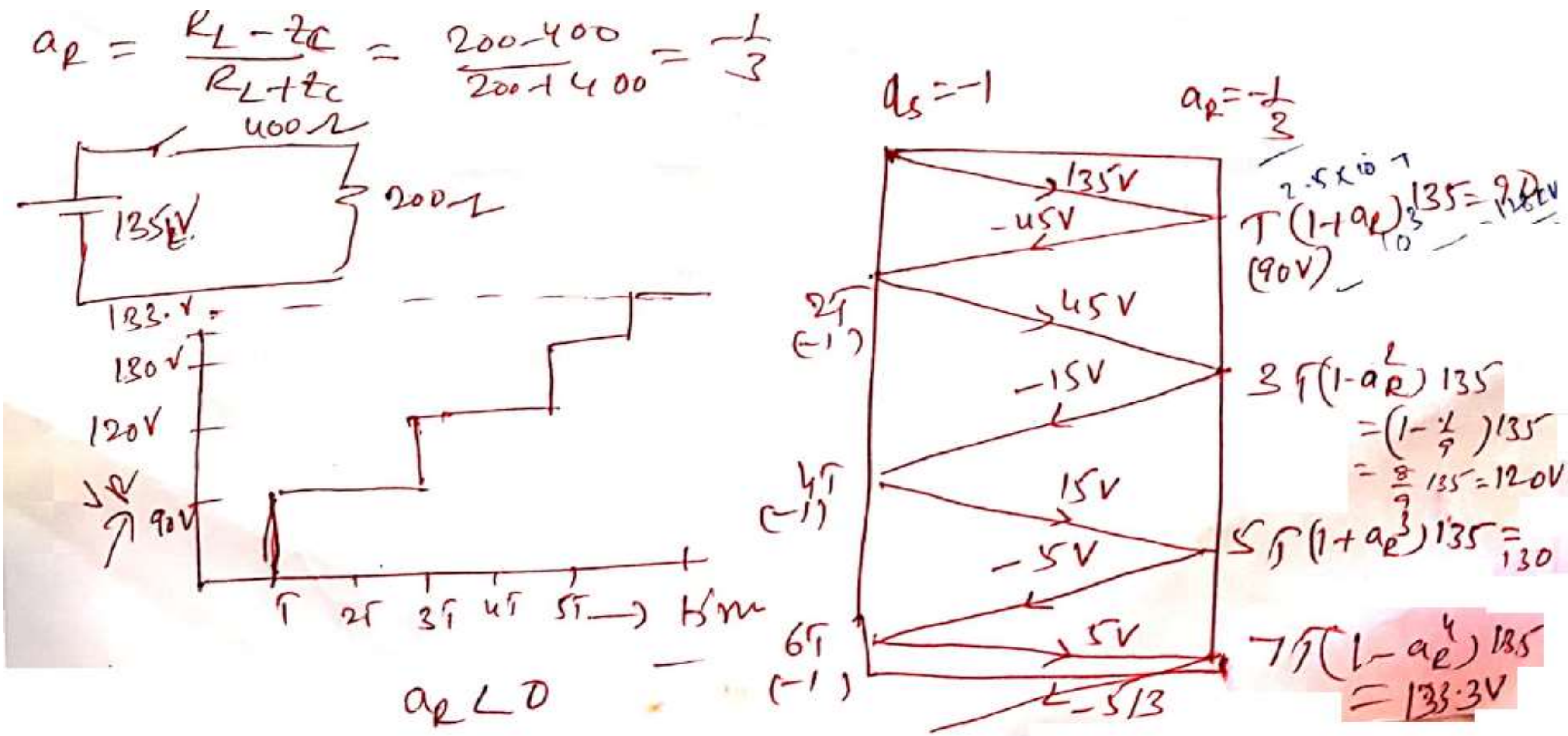
$$\alpha_S = \frac{0 - Z_c}{0 + Z_c} = -1.$$





$v_R = 1 + \alpha_R - (\alpha_R + \alpha_R^2) + (\alpha_R^2 + \alpha_R^3) -$
$= 1$
1
T
3T
5T
7T
$\alpha_R > 0$ (underdamped)
1
T
3T
5T
$\alpha_R < 0$ (overdamped)

- Ex.1 A line of source impedance 400 ohms charged through a battery of constant voltage of 135V. The line is 300m long and terminated by a resistance of 200 ohms. Plot the reflection lattice and voltage across terminating resistance.



Example ***A long transmission line is energized by a unit step voltage 1.0 V at the sending end and is open circuited at the receiving end. Construct the Bewley lattice diagram and obtain the value of the voltage at the receiving end after a long time. Take the attenuation factor $\alpha = 0.9$.***

Solution: For the open circuited at the receiving end,

Let the time of travel be 1 unit.

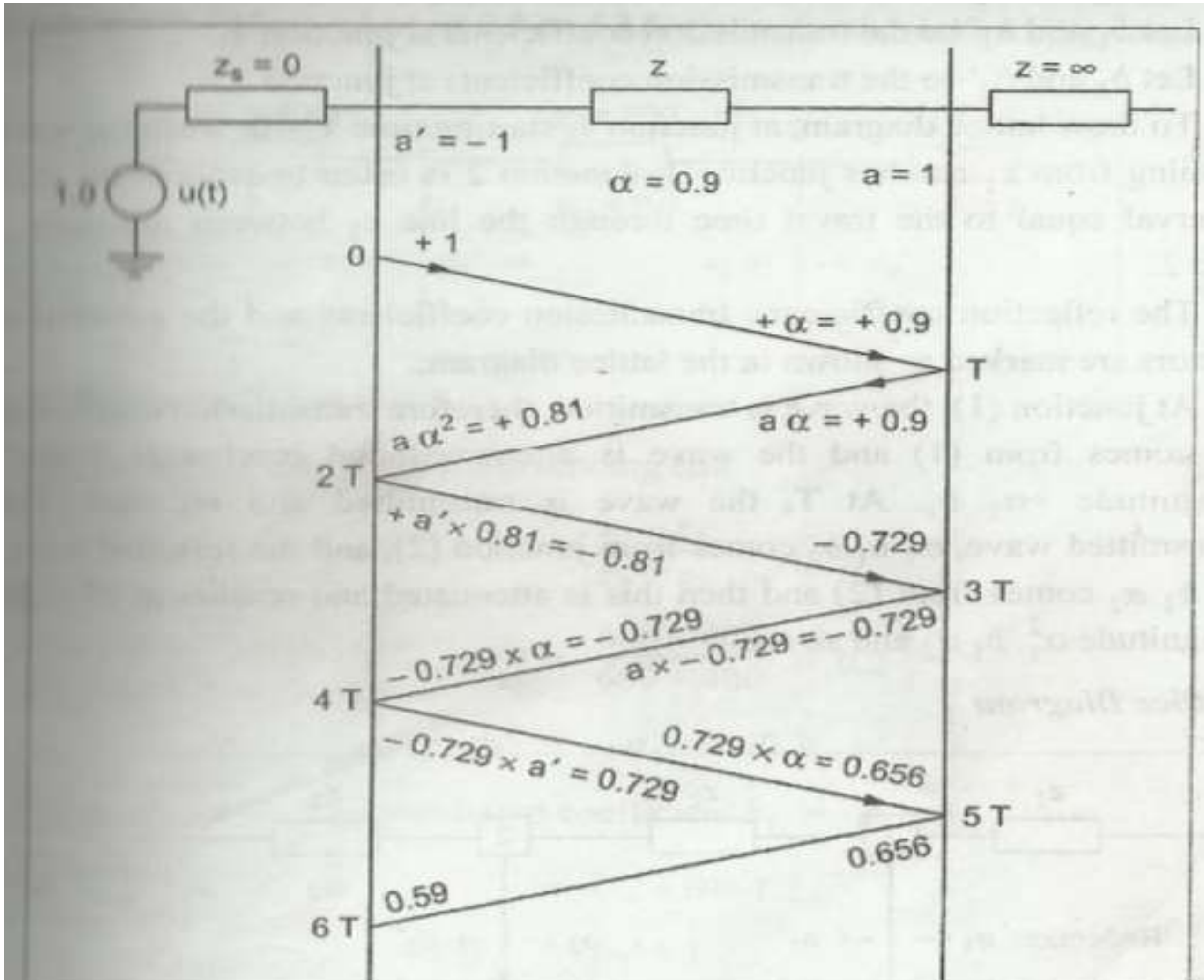
At the receiving end,

$$\text{Reflection coefficient, } a = \frac{(\infty - z)}{\infty + z} = 1.0$$

At the sending end,

$$\text{Reflection coefficient, } a' = \frac{0 - z}{0 + z} = -1.0$$

(Source impedance is zero.)



$Z_s = 0$
Z
$Z = \infty$
1.0
u(t)
a′ = − 1
α = 0.9
a = 1
0
+ 1
+ α = + 0.9
T
a α² = + 0.81
a α = + 0.9
2 T
+ a′ × 0.81 = − 0.81
− 0.729
3 T
− 0.729 × α = − 0.729
a × − 0.729 = − 0.729
4 T
− 0.729 × a′ = 0.729
0.729 × α = 0.656
5 T
0.656
0.59
6 T

Attenuation and Distortion of Travelling Waves:

- As a Travelling Waves moves along a line, it suffers both attenuation and distortion.
- The decrease in the magnitude of the wave as it propagates along the line is called **attenuation**.
- The elongation or change of wave shape that occurs is called distortion.
- Sometimes, the steepness of the wave is reduced by distortion. Also, the current and voltage wave shapes become dissimilar even though they may be the same initially.

- Attenuation is caused due to the energy loss in the line and Distortion of Travelling Waves is caused due to the inductance and capacitance of the line.
- The energy loss in the system are due to line resistance, leakage conductance and corona.
- At low voltages the losses due to line resistance are important , but at high voltages losses due to corona are very much greater than these due to line resistance.

As the wave travel along the line thus undergo three changes

- 1.The crest of wave is decreased in magnitude or attenuated.
- 2.The wave changes the shape i.e gets elongated , its irregularities are smoothed out and steepness is reduced.
- 3.Voltage and current waves ceases to be similar.

- All the above changes occurs simultaneously, the last two changes together known as distortion.
- Attenuation of waves due to corona is more (pronounced) for positive waves than for negative waves because of greater loss due to corona for positive waves.
- The effect on short waves is more than that on long waves.