UNIT – VI

- **System modeling:** Representation of transmission lines-circuit representation of synchronous machine-two winding and three winding transformers-Per unit representation and advantages - single line diagram representation-impedance and reactance diagrams-changing the base of per unit quantities.

Per Unit System

- For the analysis of electrical machines or electrical machine system, different values are required, thus, per unit system provides the value for voltage, current, power, impendence, and admittance.
- The **Per Unit System** also makes the calculation easier as all the values are taken in the same unit. The per-unit system is mainly used in the circuit where variation in voltage occurs.

- **Definition:** The per-unit value of any quantity is defined as the ratio of actual value in any unit to the base or reference value in the same unit.
- Any quantity is converted into per unit quantity by dividing the numeral value by the chosen base value of the same dimension. The per-unit value is dimensionless.

$$\text{Per Unit Value} = \frac{\text{Actual value in any unit}}{\text{Base or reference value in the same unit}}$$

Advantages of Per Unit system(PU)

- 1.Manufacturers usually specify the impedance value of equipment in pu of the equipment rating.
- 2.The PU impedance of m/c’s of the same type and widely different rating usually lie within narrow range
- 3.PU data representation yields important information about relative magnitudes
- 4.In a transformer if the base values are selected properly, the pu impedance is same on both sides of Transformer.

Dis advantages of PU systems

1. For transmission lines, its value of impedance and admittances in physical units (eg: ohms/km) that are of same magnitude regardless of voltage level or MVA rating
2. Equivalent circuits of the components are modified, making them somewhat more abstract. Sometimes phase shifts that are clearly presented in the unscaled circuits varnish in per-unit system

Selection bases

where S_B, V_B, I_B and Z_B represent the base power, base voltage, base current and base impedance expressed in volt-amperes, volts, amperes and ohms respectively. Thus from above four quantities , if any two of the four quantities are specified, the remaining two may be determined without any problem. base kVA and base kV are usually specified.

For a common representation, base kVA and base voltage are to be chosen. Then the base current and base impedance can be expressed as follows :

Let kVA_B and kV_B be the base kVA and base kV, then

$$\text{Base current, } I_B = \frac{\text{Base kVA } (kVA_B)}{\text{Base kV } (kV_B)} \text{ amperes} \quad \ldots 1$$

$$\text{Base impedance, } Z_B = \frac{\text{Base voltage in volts, } V_B}{\text{Base current in amperes, } I_B} \text{ ohms}$$

$$= \frac{V_B \times V_B}{I_B \times V_B}$$

$$= \frac{V_B^2}{V_B\ I_B}$$

$$= \frac{\left(\frac{V_B}{1{,}000}\right) \times \left(\frac{V_B}{1{,}000}\right) \times 1{,}000}{\frac{V_B\ I_B}{1{,}000}}$$

$$= \frac{(kV_B)^2 \times 1{,}000}{kVA_B}$$

$$= \frac{(kV_B)^2}{MVA_B} \text{ ohms} \qquad \ldots(2\ )$$

Per unit current, $I_{pu} = \frac{\text{Actual current}}{\text{Base current}}$

$$= \frac{\text{Actual current}}{kVA_B} \times kV_B \quad ...(3)$$

Per unit impedance,

$$Z_{pu} = \frac{\text{Actual impedance}}{\text{Base impedance}}$$

$$= \text{Actual impedance} \times \frac{kVA_B}{(kV_B)^2 \times 1,000} \quad ...(4)$$

$$Z_{pu} = \frac{\text{Actual Impedence x } KVA_B}{(KV_B)^2 \text{ x } 1000}$$

In a **3-phase system** rather than obtaining the per unit values using per phase quantities, the per unit values can be obtained directly by using three-phase base quantities (total kVA, line-to-line voltage and line currents). Let kVA_B be the base kVA (total output of three phases) and kV_B be the base line-to-line voltage in kV.

Assuming star connection (equivalent star can always be found)

$$\text{Base current, } I_B = \frac{kVA_B}{\sqrt{3}\, kV_B} \text{ amperes}$$

Base impedance,

$$Z_B = \frac{kV_B \times 1{,}000}{\sqrt{3} \times I_B}$$

$$= \frac{kV_B}{\sqrt{3}} \times \frac{\sqrt{3}\, kV_B}{kVA_B} \times 1{,}000$$

$$= \frac{(kV_B)^2 \times 1{,}000}{kVA_B} \text{ ohms} \qquad \ldots(5)$$

$$= \frac{(kV_B)^2}{MVA_B} \text{ ohms}$$

Per unit impedance,

$$Z_{pu} = \frac{\text{Actual impedance}}{\text{Base impedance}}$$

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$$Z_{pu} = \frac{\text{Actual impedance}}{\text{Base impedance}}$$

$$= \frac{\text{Actual impedance} \times \text{kVA}_B}{(\text{kV}_B)^2 \times 1{,}000} \qquad \ldots(6)$$

$$= \frac{\text{Actual impedance} \times \text{MVA}_B}{(\text{kV}_B)^2} \qquad \ldots(7)$$

It should be noted that the same expression is obtained for a single-phase and three-phase systems. In power system problems the data are given in terms of 3-phase kVA, line-to-line kV, the machine kVA and machine phase-to-phase (or line-to-line) kV rating. Further the direct-axis synchronous reactance is also known as positive sequence reactance of the machine.

CHANGE OF BASE VALUE

Normally the per unit impedance of various components corresponding to its own rating voltage and kVA are given and since we choose one common base kVA and base kV for the whole system, therefore, it becomes imperative to determine the per unit impedance of the various equipments corresponding to the common base kV and base kVA.

If the individual quantities are $Z_{pu\ old}$, kVA_{old}, kV_{old} and the common base quantities are $Z_{pu\ new}$, kVA_{new} and kV_{new}, then making use of Eq. (6), according to which per unit impedance is directly proportional to the base kVA and inversely proportional to the square of the base kV, we have

Per unit impedance to new base,

$$Z_{pu\ new} = Z_{pu\ old} \times \frac{kVA_{new}}{kVA_{old}} \times \frac{(kV_{old})^2}{(kV_{new})^2} \qquad ...(8)$$

This is very important relation used in power system analysis.

PER UNIT IMPEDANCE OF TRANSFORMERS

(a) Two winding transformer :-

In circuits containing transformers the base volt-amp (or KVA) is the same on the two sides. However, the voltage base on the two sides must have the same ratio as the turn ratio. This selection makes the p.u impedance equal on the two sides.

Consider a single phase transformer having V_1 and V_2 as the primary and secondary voltages and I_1 and I_2 as primary and secondary currents. Let impedance as referred to primary be Z_1, then

$$K = \frac{V_2}{V_1} = \frac{I_1}{I_2}$$

$$\text{Base Volt ampere} = V_1 I_1 = V_2 I_2$$

Base Voltage = V_1 for primary and V_2 for secondary

Base impedence for primary $= \frac{V_1}{I_1}$

P.U impedence as referred to primary $= \frac{\text{actual impedence}}{\text{Base impedence}}$

$$= \frac{Z_1}{V_1/I_1} = \frac{I_1 Z_1}{V_1} \rightarrow (1)$$

Actual impedence as referred to secondary $= Z_1 K^2$

$$= Z_1 \left(\frac{V_2}{V_1}\right)^2$$

Base Impedence for Secondary $= \frac{V_2}{I_2}$

Per unit impedance as referred to secondary $= \dfrac{Z_1\left(\dfrac{V_2}{V_1}\right)^2}{V_2/I_2}$

$$\Rightarrow \frac{Z_1 V_2 I_2}{(V_1)^2} \Rightarrow \frac{Z_1 V_1 I_1}{(V_1)^2}$$

$$= \frac{I_1 Z_1}{V_1} \longrightarrow (2)$$

= p.u impedance as referred to primary.

from equations (1) and (2) the p.u value of Impedance will be same either referred to primary or secondary side.

REPRESENTATION OF POWER SYSTEM COMPONENTS

For the purpose of the power system studies such as transient stability studies, load flow studies or short-circuit studies, there is a need for the development of mathematical model of the power system network.

For the development of fairly accurate models of the power system network, it is necessary that model reflects correctly the terminal behaviour of each component of the network for the purpose of study for which the model has been developed.

The different components of power system are : synchronous machines, transmission network, transformers, distribution network including static loads, composite loads and dynamic loads such as induction machines etc.

Now, the representation of different components (synchronous machines, transformers and transmission lines) of the power system will be discussed in brief here.

Representation of Synchronous Machines

The synchronous machine is the most important component of a power system. It converts mechanical power into electrical form and supplies it into the power system network or, in the case of a motor, it draws electrical power from the network and converts it into the mechanical form.

Most simplified model *i.e.*, representation of a synchronous generator for the purpose of transient stability studies is a constant voltage source behind proper reactance. The voltage source may be subtransient, transient or steady-state voltages and the reactance may be the corresponding reactances.

However, for understanding this model, let us consider a synchronous generator operating at no load before occurrence of a 3-phase short circuit at its terminals.

The current flowing in the synchronous generator just after occurrence of the three-phase short circuit at its terminals is similar to the current that flows in an R-L circuit upon which sudden an ac voltage is applied

Hence the current will have both ac (*i.e.*, steady-state) component as well as the dc (*i.e.*, transient) component which decays exponentially with time constant τ equal to L/R. However, if the dc component is neglected the oscillograph of the ac component of the current that flows in the synchronous generator just after the fault occurs, will have the waveshape as shown in Fig. 1

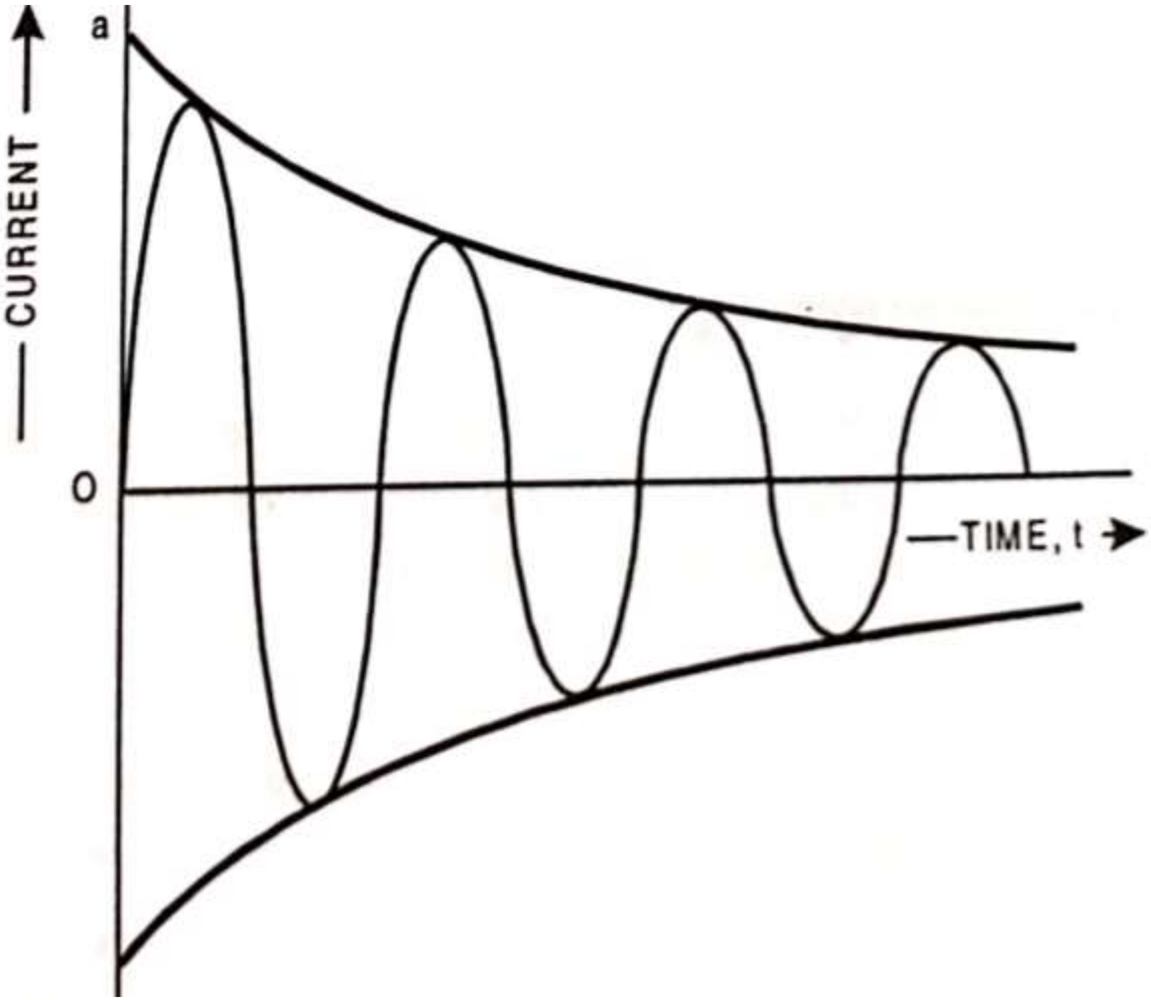
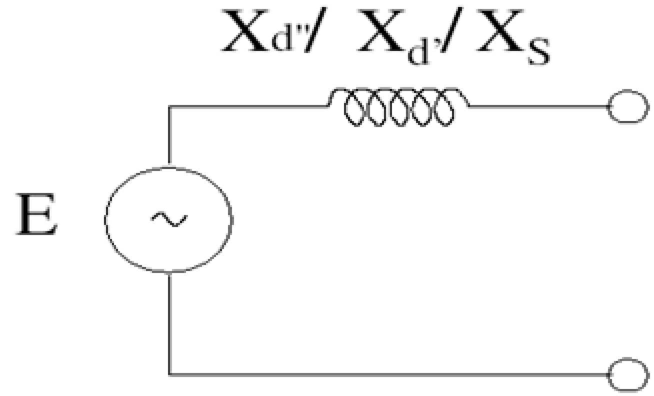


Fig. 1. ***Oscillogram of The Current That Flows in The Unloaded Synchronous Generator Just After a 3-Phase Fault at Its Terminals Excluding DC Component***

- Representation of Rotating Machines.



Representation of Transformer

As we know the transformer is a static machine which is used to transform electric power from one circuit to another without change in frequency. It may raise or lower the voltage in a circuit

but with a corresponding decrease or increase in current. Thus transformer plays a vital role in power systems.

Practically every transformer is provided with taps for ratio control, and thus it is also used for controlling secondary voltage level.

Most of the transformers are provided with *off-load tap-changers* while some of the transformers are provided with *on-load tap-changers*.

A power transformer with turn ratio $K = \frac{E_2}{E_1} = \frac{N_2}{N_1}$ can be represented as illustrated in Fig. 2.

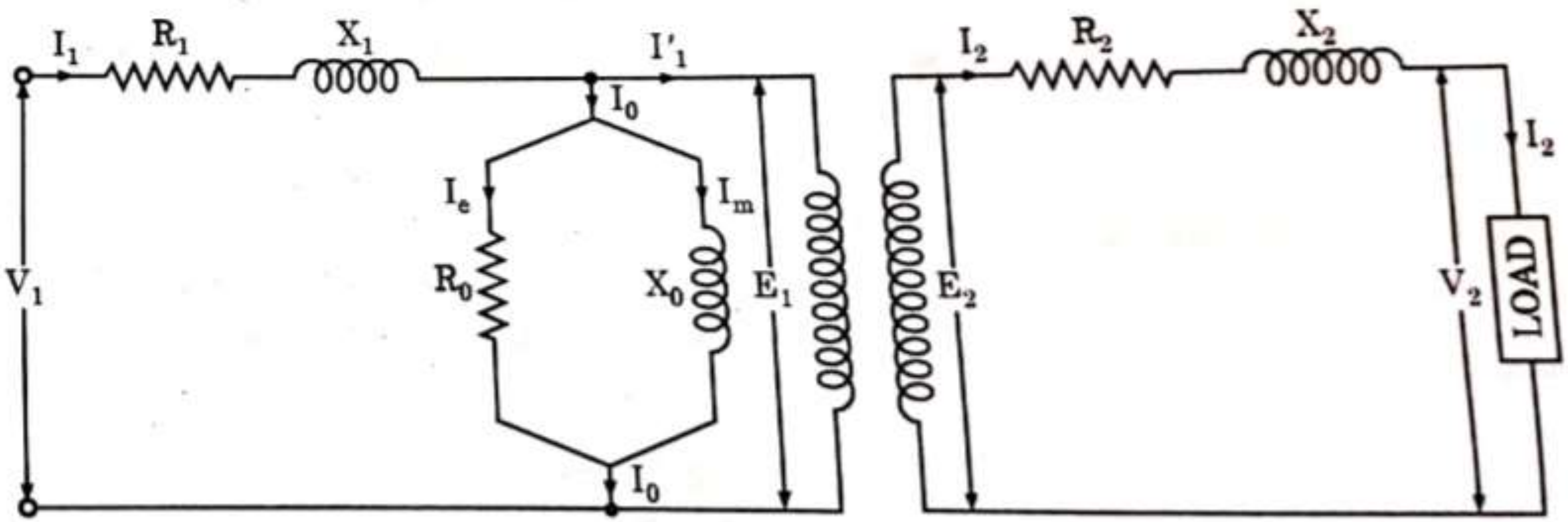


Fig. 2 ***Equivalent Circuit of a Transformer***

The induced emf in primary winding E_1 is primary applied voltage V_1 less primary voltage drop. This voltage causes iron loss current I_e and magnetising current I_m and we can, therefore, represent these two components of no-load current by the current drawn by a non-inductive resistance R_0 and pure reactance X_0 having the voltage E_1 or (V_1—primary voltage drop) applied across them, as shown in Fig. 2.2.

$$\text{Secondary current, } I_2 = \frac{\mathbf{I}_1'}{K} = \frac{\mathbf{I}_1 - \mathbf{I}_0}{K}$$

Terminal voltage V_2 across load is induced emf E_2 in secondary winding less voltage drop in secondary winding.

The equivalent circuit can be simplified by transferring the voltage, current and impedance to the primary side. After transferring the secondary voltage, current and impedance to primary side equivalent circuit is reduced to that shown in Fig. 3.

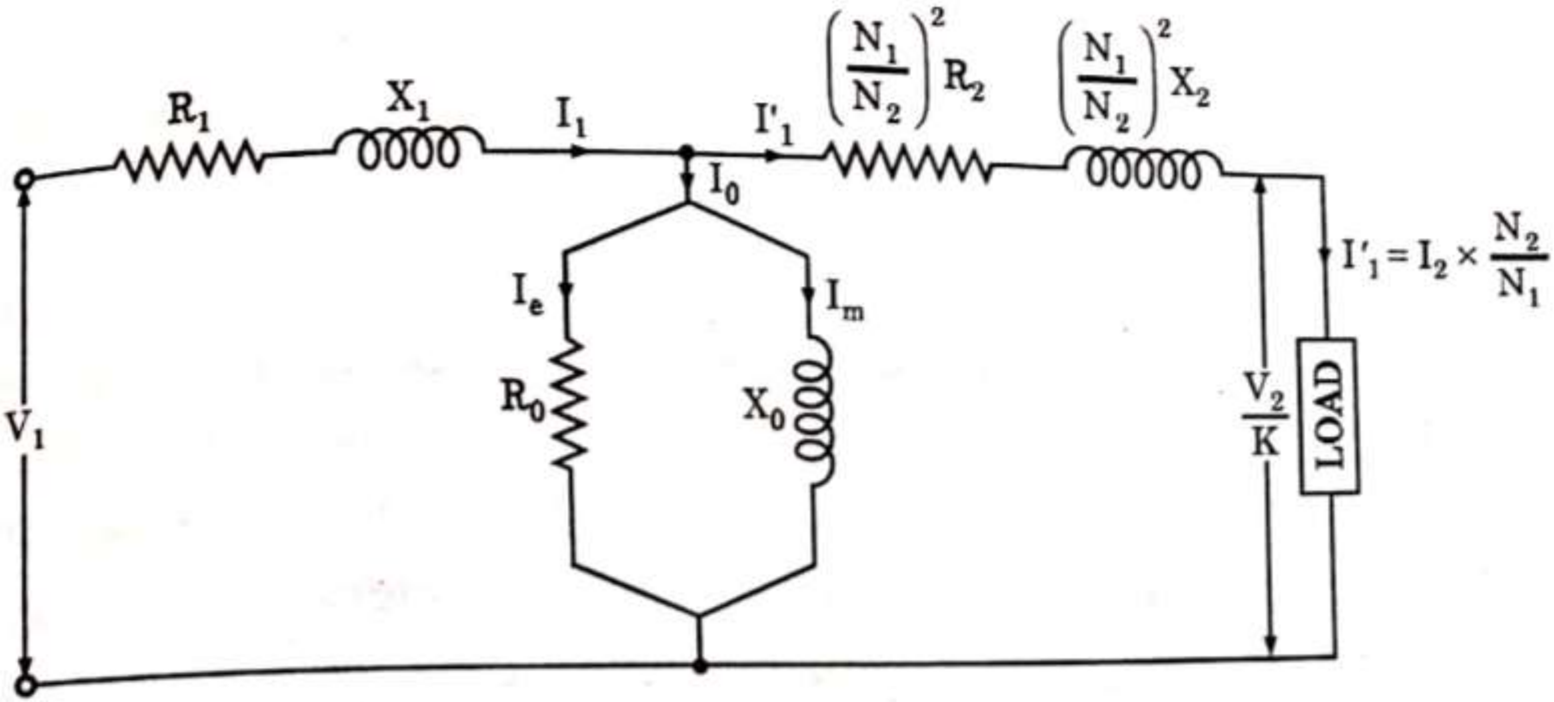


Fig. 3. ***Equivalent Circuit of a Transformer With All Secondary Impedances Transferred To Primary Side***

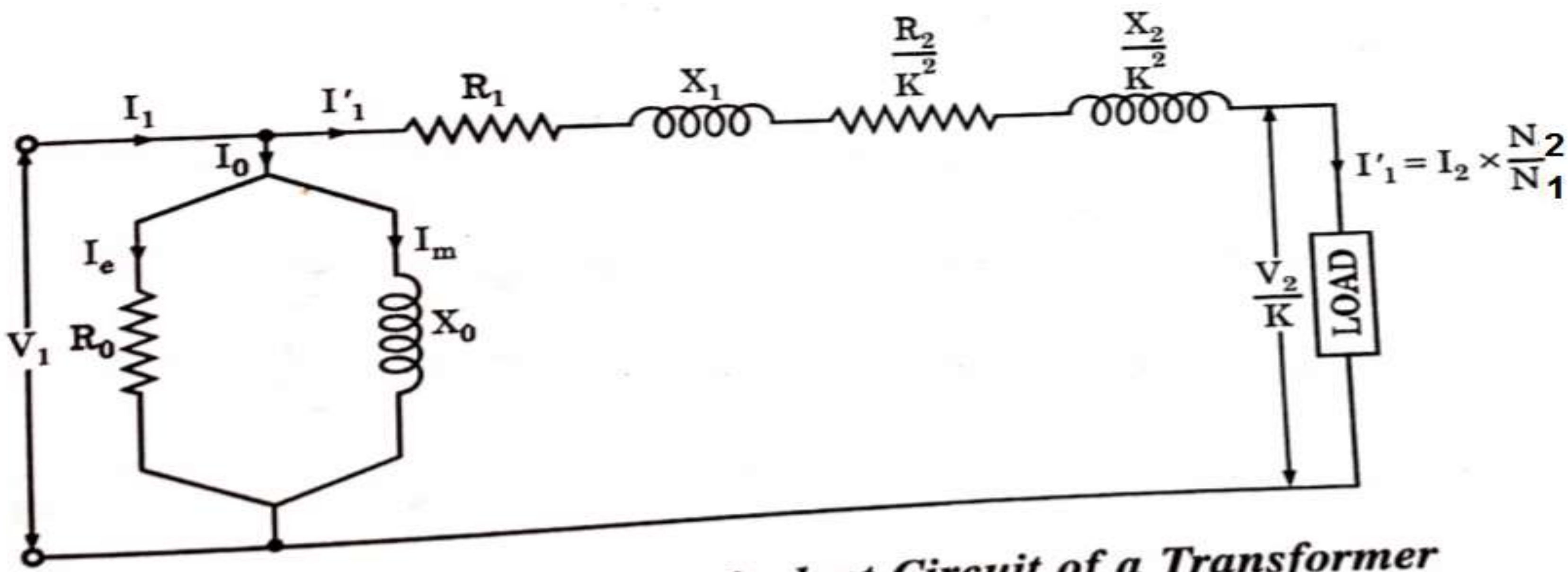


Fig. 4. ***Approximate Equivalent Circuit of a Transformer***

The equivalent circuit diagram can further be simplified by transferring the resistance R_0 and reactance X_0 towards left end, as shown in Fig. 4. The error introduced by doing so is very small and can be neglected.

No-load current I_0 is hardly 3 to 5 per cent of the full-load rated current, the parallel branch consisting of R_0 and X_0 can be omitted

The equivalent circuit referred to primary side (neglecting no-load current I_0) is shown in Fig. 5.

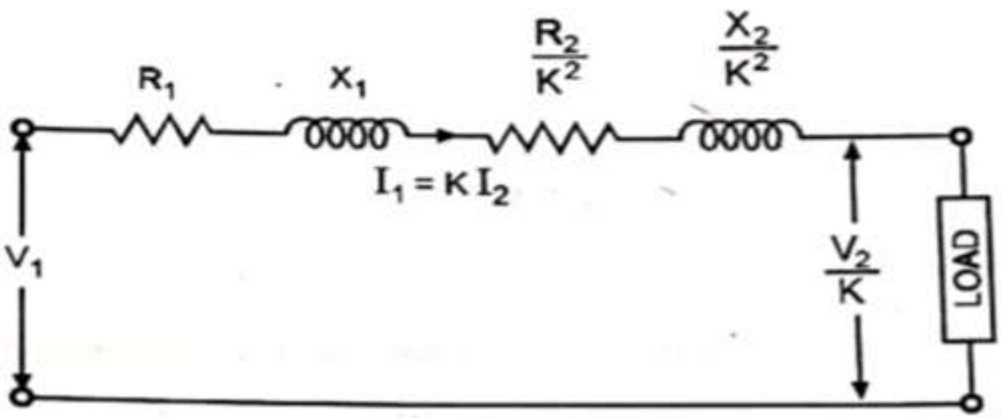


Fig. 5. ***Equivalent Circuit of a Transformer Referred to Primary***

Three Winding Transformer:

Three Winding Transformer may be built with a third winding, called the tertiary, in addition to the primary and secondary. Various purposes the use of a tertiary winding are given below:

1. To supply the substation auxiliaries at a voltage different from those of the primary and secondary windings.
2. Static capacitors or synchronous condensers may be connected to the tertiary winding for reactive power injection into the system for voltage control.

3. It limits voltage imbalance when the load is unbalanced. It also permits the third harmonic current to flow thereby reducing third-harmonic voltages.
4. Three windings may be used for interconnecting three transmission lines at different voltages.
5. Tertiary can serve the purpose of measuring voltage of an HV testing transformer.

Equivalent Circuit:

- The equivalent circuit of a Three Winding Transformers can be represented by the single-phase equivalent circuit of Fig.
- wherein each winding is represented by its equivalent resistance and reactance.
- All the values are reduced to a common rating base and respective voltage bases.
- The subscripts 1, 2 and 3 indicate the primary, secondary and tertiary respectively.

3winding transformer

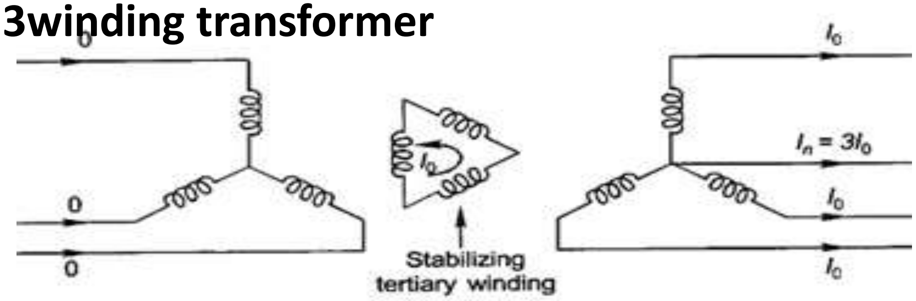
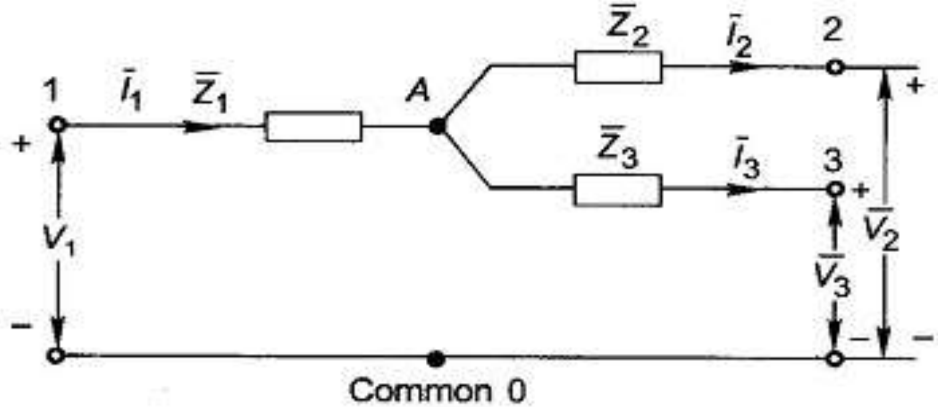


Fig.

Equivalent Circuit of 3winding transformer



- The impedance of above Fig. can be readily obtained from three simple short-circuit tests. If Z_{12} indicates the SC impedance of windings 1 and 2 with winding 3 open, then from the equivalent circuit,

$$\bar{Z}_{12} = \bar{Z}_1 + \bar{Z}_2 \qquad (1)$$

Similarly

$$\bar{Z}_{23} = \bar{Z}_2 + \bar{Z}_3 \qquad (2)$$

$$\bar{Z}_{13} = \bar{Z}_1 + \bar{Z}_3 \qquad (3)$$

where $\bar{Z}_{23}$ = SC impedance of windings 2 and 3 with winding 1 open.

$\bar{Z}_{13}$ = SC impedance of windings 1 and 3 with winding 2 open.

- All the impedances are referred to a common base.
- Solving Eq. (1) to Eq. (3) yields

$$\bar{Z}_1 = \frac{1}{2}(\bar{Z}_{12} + \bar{Z}_{13} - \bar{Z}_{23}) \quad (4)$$

$$\bar{Z}_2 = \frac{1}{2}(\bar{Z}_{23} + \bar{Z}_{12} - \bar{Z}_{13}) \quad (5)$$

$$\bar{Z}_3 = \frac{1}{2}(Z_{13} + Z_{23} - Z_{12}) \quad (6)$$

To express these impedances in p.u , the base values are selected as under

- 1.kVA base is the same for all the three windings.
- 2.The related line to line kV is same for each of the 3 windings, is selected as base kV for the winding

Representation of Transmission Lines

3-phase network consisting of transmission system and also the distribution system are assumed to be symmetrical or balanced.

The performance of a transmission line is governed by its four parameters—series resistance R, and inductance L, shunt capacitance C and the shunt conductance G.

The resistance R is due to the fact that every conductor offers opposition to the flow of current. The inductance L is due to the fact that the current carrying conductor is surrounded by the magnetic lines of force.

The capacitance of the line is due to the fact that the conductor carrying current forms a capacitor with the earth which is always at lower potential than the conductor and the air between them forms a dielectric medium.

The shunt conductance G is mainly due to flow of leakage currents over the surface of the insulators especially during bad weather.

However, the conductance is normally neglected in the case of transmission line calculations since leakage at normal frequency are negligible.

REPRESENTATION OF POWER SYSTEM NETWORK

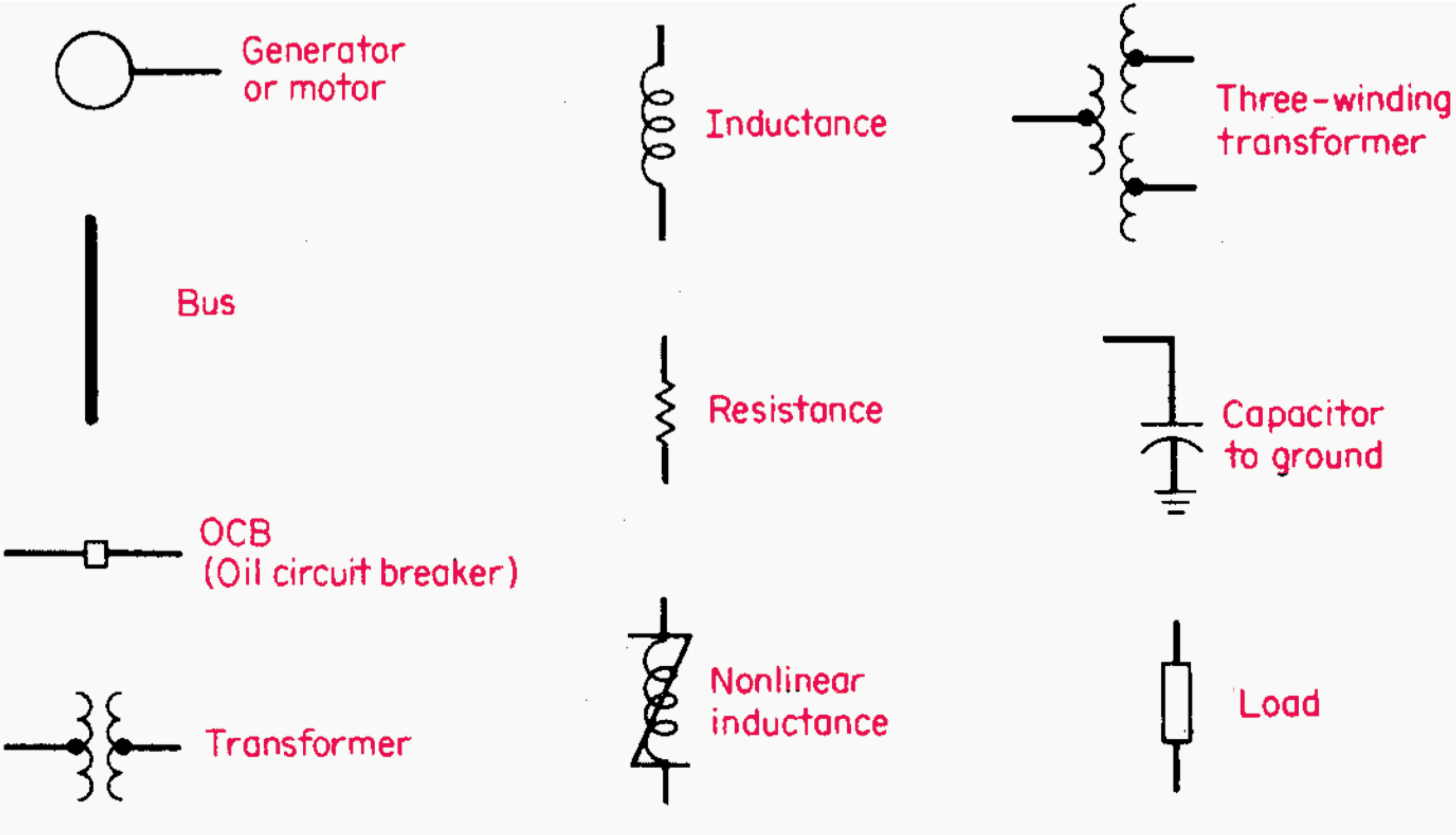
- A balanced three phase network is generally solved as a single phase or per phase equivalent circuit.
- A power system is represented by one of three phases and the neutral.
- The components such as generators, transformers and loads are indicated by **standard symbols**.

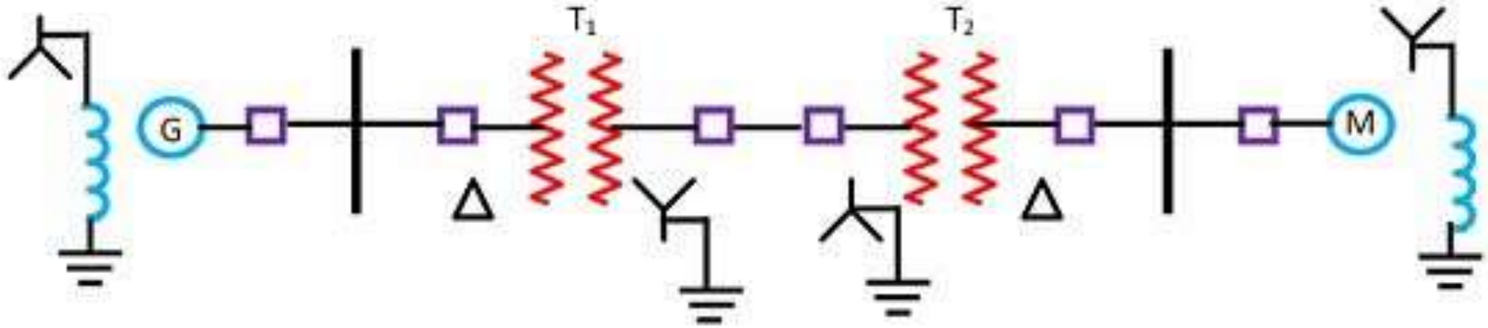
Single Line Diagram of Power System

- Single line diagram is the representation of a power system using the simple symbol for each component.
- The single line diagram of a power system is the network which shows the main connections and arrangement of the system components along with their data (such as output rating, voltage,
  - Such a simplified diagram of an electrical system is called **single-line or one-line diagram**.

- Generator and transformer connections, star, delta and neutral earthing are indicated by symbols drawn by the side of the representation of these elements.

Figure 1 – Common power symbols used in single line diagrams





Single Line Representation of a Typical Power System

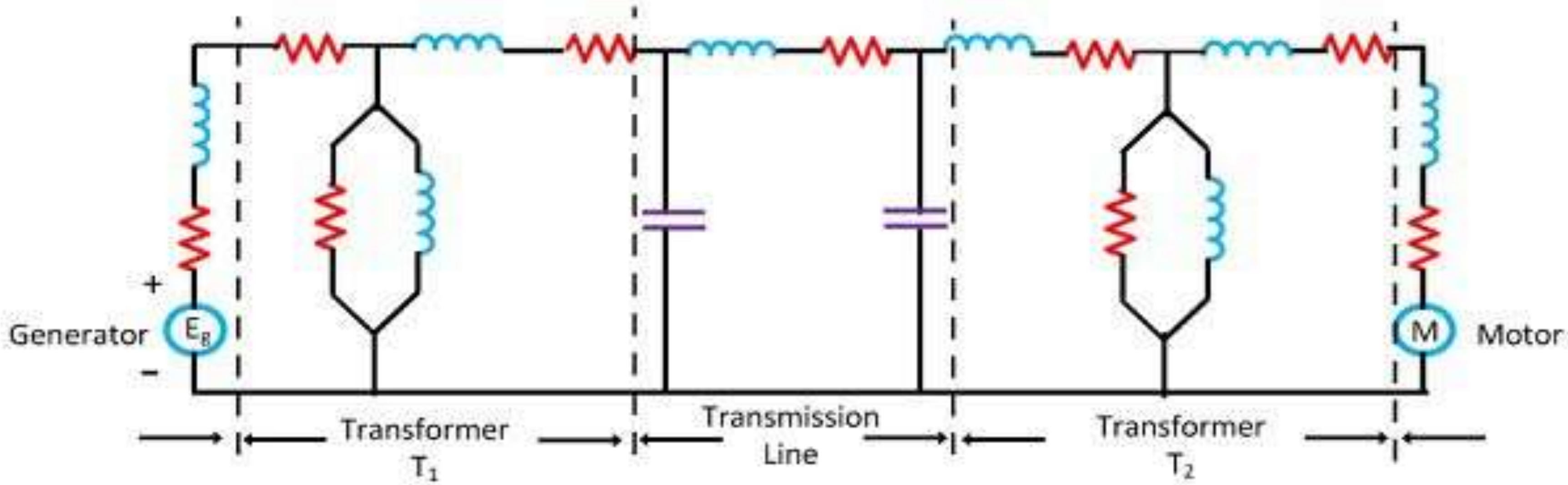
Impedance or reactance Diagram for the Power System:

- In impedance or reactance diagram, each component is represented by its equivalent circuit.
- This equivalent circuit of power system is used to analyse the performance of a system under load or to analyse the condition of the system under fault.

- The diagram shown below is the balanced 3-phase diagram.
- The impedance diagram on single phase basis for use under balanced operating conditions can be easily drawn from the single line diagram.
- The generators are represented as voltage source with series impedance .

- single phase transformer equivalent are shown as ideal transformer with transformer impedances indicated on appropriate side.
- A short transmission line is represented by series impedance, medium and long lines by nominal pie circuit.
- A very long lines represented by equivalent pie circuit

- The static load is represented by a resistive and inductive reactance in series.
- Motors are represented by equivalent circuits.
- Neutral earthing impedance do not appear in the diagram as the balanced condition is assumed.

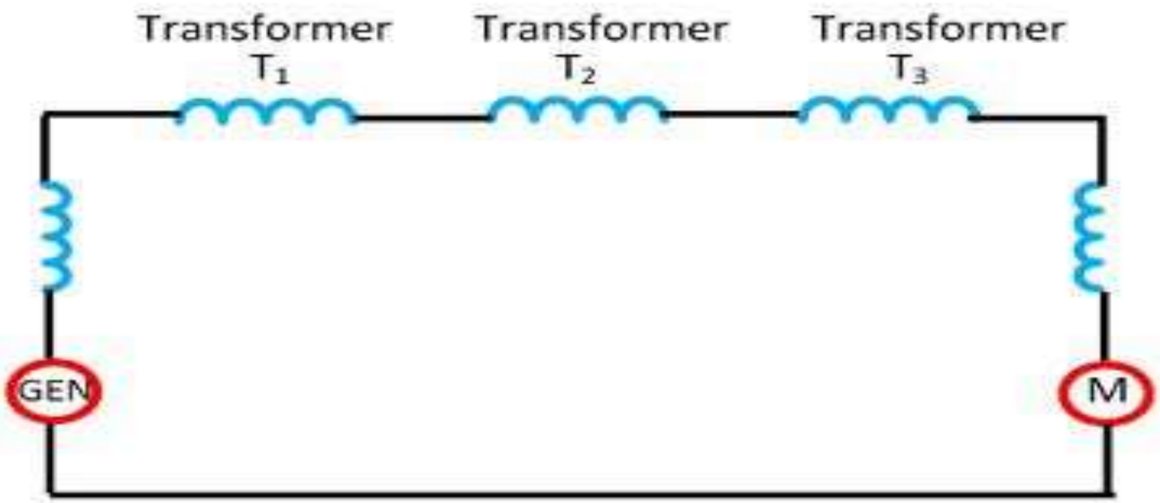


Impedance Diagram For The Power System

- The impedance diagram can further be simplified by making certain assumptions and reduced to simplified reactance.
- Reactance diagram is drawn by neglecting the effective resistance of generator armature, transformer winding resistance, transmission line resistance line charging and the magnetising circuit of transformers.
- Reactance diagram of the power system is shown below.

Reactance Diagram for the Power System

- The reactance diagram gives an accurate result for many power system studies, such as short-circuit studies, etc.
- The winding resistance, including the line resistance, are quite small in comparison with leakage reactance and shunt path which includes line charging and transformer magnetising circuit provide a very high parallel impedance with fault.



Reactance Diagram For Power System

- It is considered that if the resistance is less than one-third of the reactance, and resistance is ignored, then the error introduced will be not more than 5 %.
- If the resistance and reactance ignored errors up to 12% may be introduced. The errors mean their calculation gives a higher value than the actual value.

PROCEDURE TO FORM REACTANCE DIAGRAM FROM SINGLE LINE DIAGRAM

- 1.Select a base power kVAb or MVAb
- 2. Select a base voltage kVb. Consider the system to be divided into no.of sections by the transformers. calculate the kV bases of other sections in the ratio of transformation.
- i.e The voltage conversion is achieved by means of transformer kVb on LT side= kVb on HT side x LT voltage rating/HT voltage rating
- 3. calculate the p.u values of reactance in each section and connect them up as per the topology of single line diagram
- The result is the single phase p.u reactance diagram

Example .1. Base voltage = 1,100 volts. Base kVA = 10^6. What is base impedance ?

Solution : Base kVA $= 10^6$

$$\text{Base kV} = \frac{1{,}100}{1{,}000} = 1.1 \text{ kV}$$

$$\text{Base current, } I_B = \frac{kVA_B}{kV_B} = \frac{10^6}{1.1}$$

$$\text{Base impedance, } Z_B = \frac{kV_B \times 1{,}000}{I_B} = \frac{1.1 \times 1{,}000 \times 1.1}{10^6}$$

$$= 0.00121\ \Omega \ \textbf{Ans.}$$

Base impedance can also be determined directly by substituting the values of base kVA and base kV and we get

$$Z_B = \frac{(kV_B)^2 \times 1{,}000}{kVA_B} = \frac{(1.1)^2 \times 1{,}000}{10^6} = 0.00121\ \Omega \ \textbf{Ans.}$$

Same as abo

Example 2. **If the resistance in ohms is 5 Ω, find the per unit value. Given base kVA = 10 and base kV = 11.**

Solution : Resistance, $R = 5\ \Omega$

$$\text{Base resistance, } R_B = \frac{(kV_B)^2 \times 1{,}000}{kVA_B} = \frac{(11)^2 \times 1{,}000}{10}$$

$$= 12{,}100\ \Omega$$

$$\text{Per unit resistance, } R_{pu} = \frac{R}{R_B} = \frac{5}{12{,}100} = 0.000413 \text{ pu} \quad \textbf{Ans.}$$

Example 3. A single-phase transformer is rated as 2.5 kVA, 11/0.4 kV. If the leakage reactance is 0.96 Ω when referred to low-voltage side, then determine its leakage reactance in per unit.

Solution : Base voltage, $kV_B = 0.4$ kV

Base kVA, $kVA_B = 2.5$ kVA

Low-voltage side base impedance,

$$Z_{BLV} = \frac{(kV_B)^2 \times 1{,}000}{kVA_B} = \frac{(0.4)^2 \times 1{,}000}{2.5} = 64\ \Omega$$

Leakage reactance in per unit,

$$Z_{pu} = \frac{\text{Actual reactance}}{\text{Base impedance}} = \frac{0.96}{64} = 0.015 \text{ pu}$$ **Ans.**

Example A 100 MVA, 33 kV, 3-phase generator has a subtransient reactance of 15%. The generator is connected to the motors through a transmission line and transformer as shown in Fig. 2.20. The motors have rated inputs of 30 MVA, 20 MVA and 50 MVA at 30 kV with 20% subtransient reactance. The 3-ϕ transformers are rated at 110 MVA 32 kV Δ/110 kV Y with leakage reactance 8%. The line has a reactance of 50 Ω. Selecting the generator rating as the base quantities in the generator circuit, determine the base quantities in other parts of the system and evaluate the corresponding pu values.

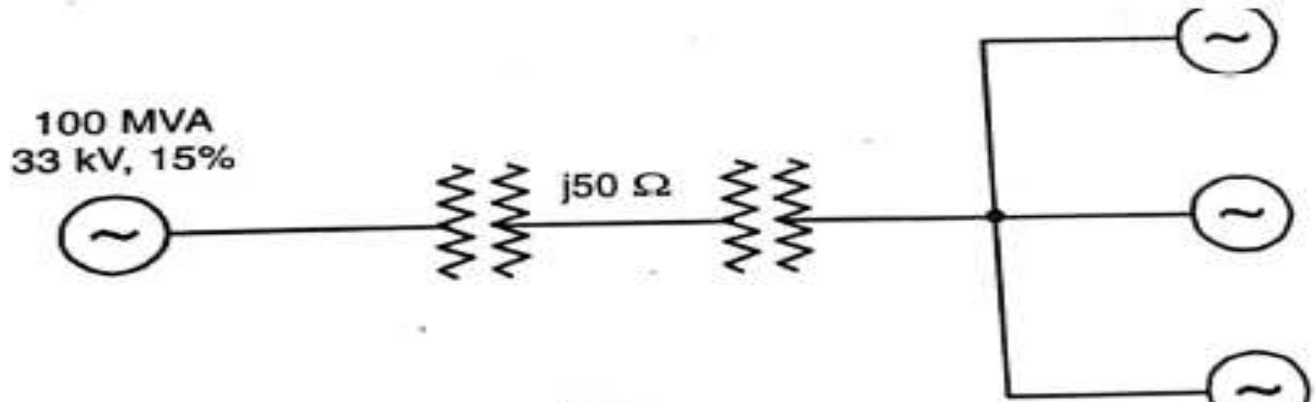


Fig.

Sol) Assuming the base value as 100MVA

Base kV = 33 kV in gn. cct.

Then P.U reactance of gen will be 15% = 0.15 P.U

The base value of voltage in the line will be

= Base kV of T_1 on HV side

= Base kV of T_1 on LV side $\times \frac{HV}{LV\,side}$ = $33 \times \frac{110}{32}$ = 113.43kV

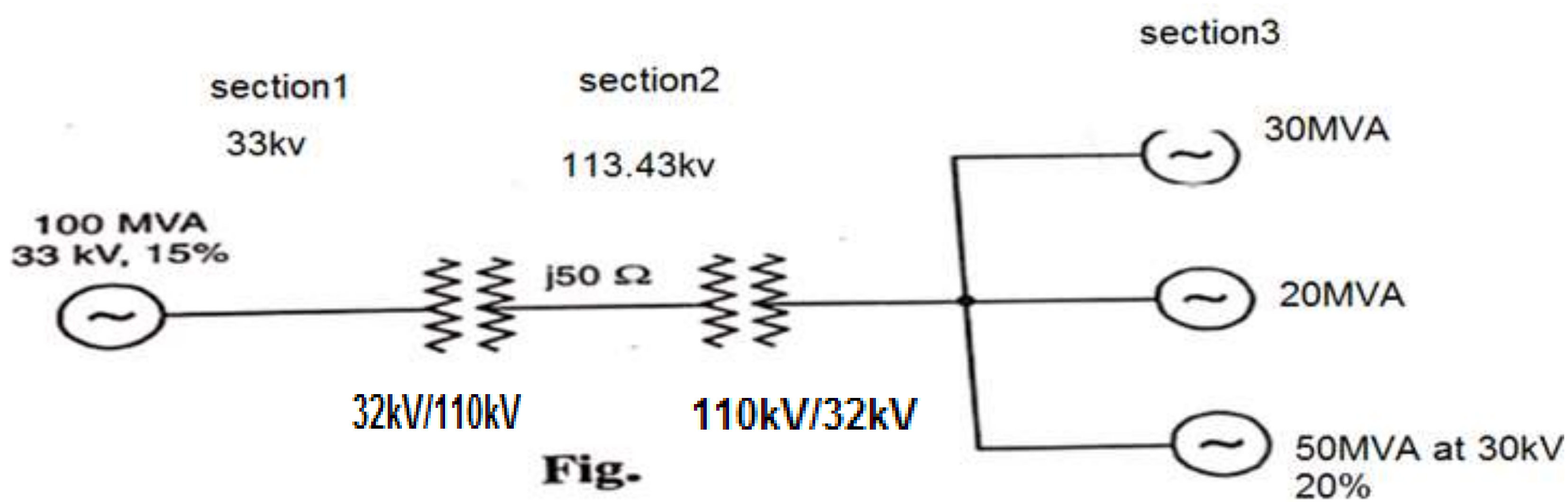


Fig.

The base value of voltage in the motor circuit

$$= \text{Base kV of } T_2 \text{ on LV side}$$

$$= \text{Base kV of } T_2 \text{ on HV side} * \frac{\text{LV}}{\text{HV side}}$$

$$= 113.43 * \frac{32}{110} = 33 \text{ kV}$$

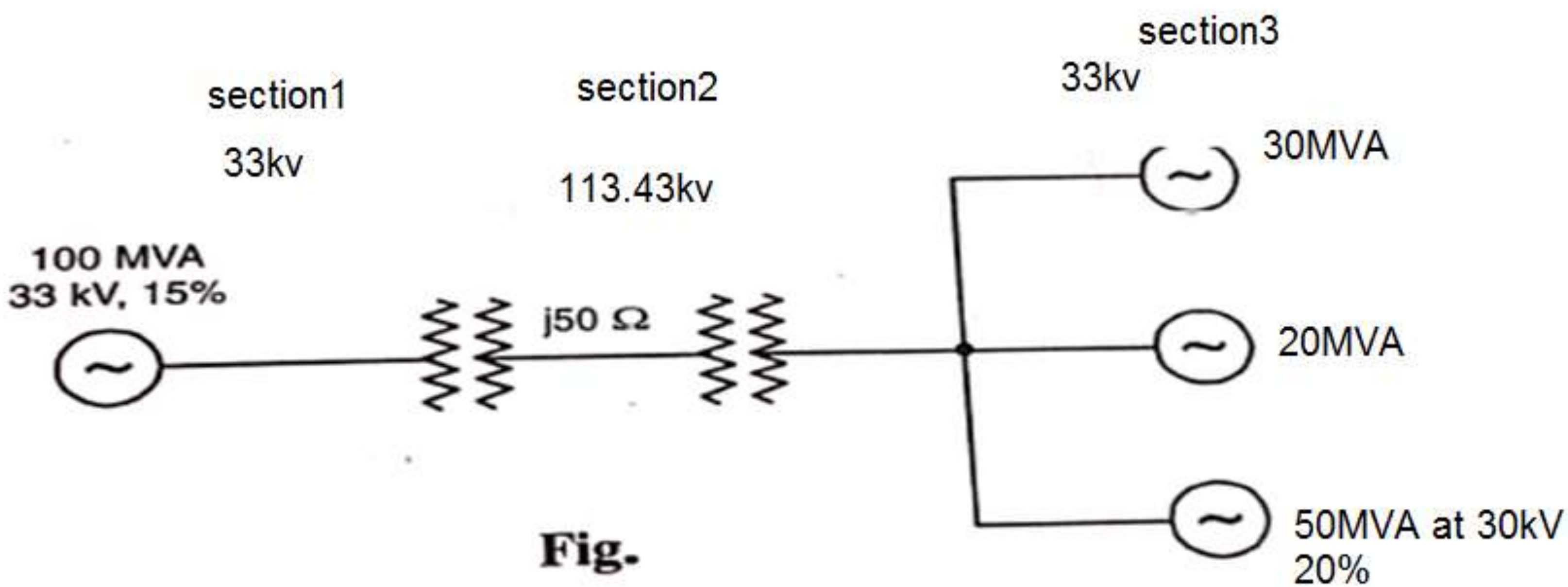


Fig.

$$Z_{pu\ new} = Z_{pu\ old} \times \frac{kVA_{new}}{kVA_{old}} \times \frac{(kV_{old})^2}{(kV_{new})^2}$$

The reactance of the transformer given is 8% corresponding to 110MVA, 32 kV. (w.r.t. primary)

∴ corresponding to 100MVA & 33 kV

T_1 the PU reactance $= 0.08 * \frac{100_{new}}{110_{old}} * \left(\frac{32_{old}}{33_{new}}\right)^2 = 0.06838$ PU

The P.U reactance of line $= \frac{Z_{act}}{Z_{base}} = \frac{50 \times 100}{113.43^2} = 0.3886$ P.U

T_2 same → T_1

T_2 PU reactance on HV side $= 0.08 * \frac{100_{new}}{110_{old}} * \left(\frac{110_{old}}{113.43}\right)^2 = 0.06838$ PU

The p.u reactance of motor 1 $= 0.2 * \frac{100}{30} \times \left(\frac{30}{33}\right)^2 = 0.55$ p.u

" motor 2 $= 0.2 * \frac{100}{20} \times \left(\frac{30}{33}\right)^2 = 0.826$ p.u

" motor 3 $= 0.2 * \frac{100}{50} \times \left(\frac{30}{33}\right)^2 = 0.3305$ p.u

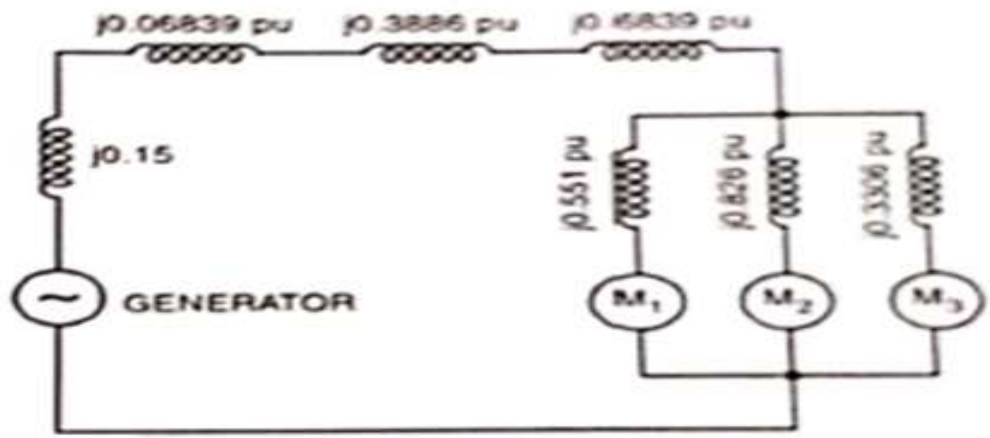
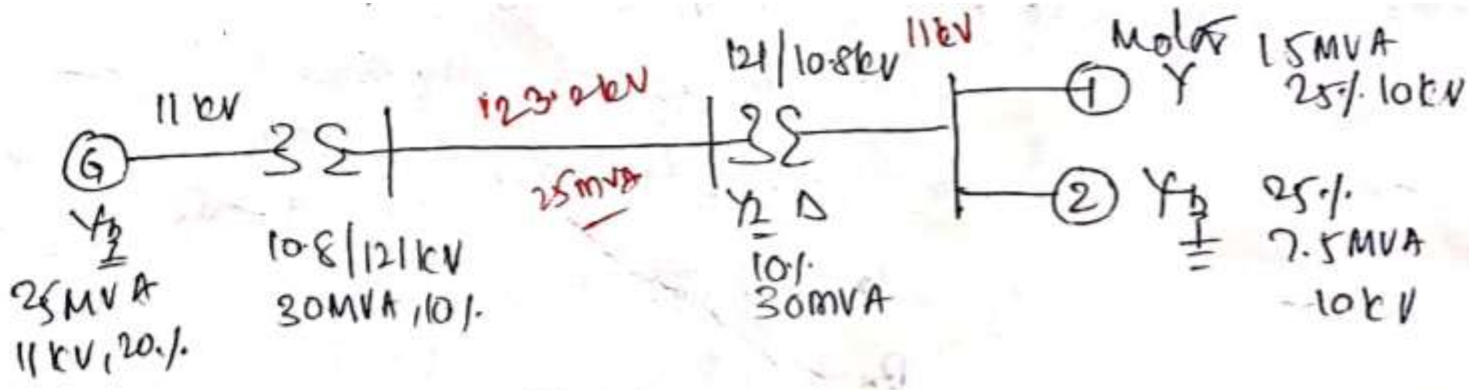


Fig. ***PU Reactance Diagram of The System***

- Ex2: A 25MVA,11kV Three phase generator has a sub transient reactance of 20%. The generator supplies two motors over a transmission line with transformers at both ends as shown in one line diagram of figure. The motors have rated inputs of 15 and 7.5 MVA, both 10kV with 25% sub transient reactance. The three phase transformers are both rated 30MVA, 10.8/121kV, connection delta-star with leakage reactance of 10% each. The series reactance of the line is 100 ohms.



let Base MVA = 25 MVA } Gen. ckt. requires a 25 MVA base in all other ckts & the foll. voltage bases

Base kV = 11 kV

Voltage base of Gen = 11 kV

Voltage base of Gen = 11 kV

T.L voltage base = Base kV of T_1 on HV side

= Base kV of T_1 on LV side * $\frac{HV}{LV}$ = $11 * \frac{121}{10.8}$ = 123.2 kV

Motor voltage base = base kV of T_2 on LV side

= base kV of T_2 on HV side * $\frac{LV}{HV}$

= $123.2 * \frac{10.8}{121}$ = 11 kV

The reactances of tr's, Tr lines and motors are converted to P.U values on appropriate bases as follows. ← Ref. Pri.

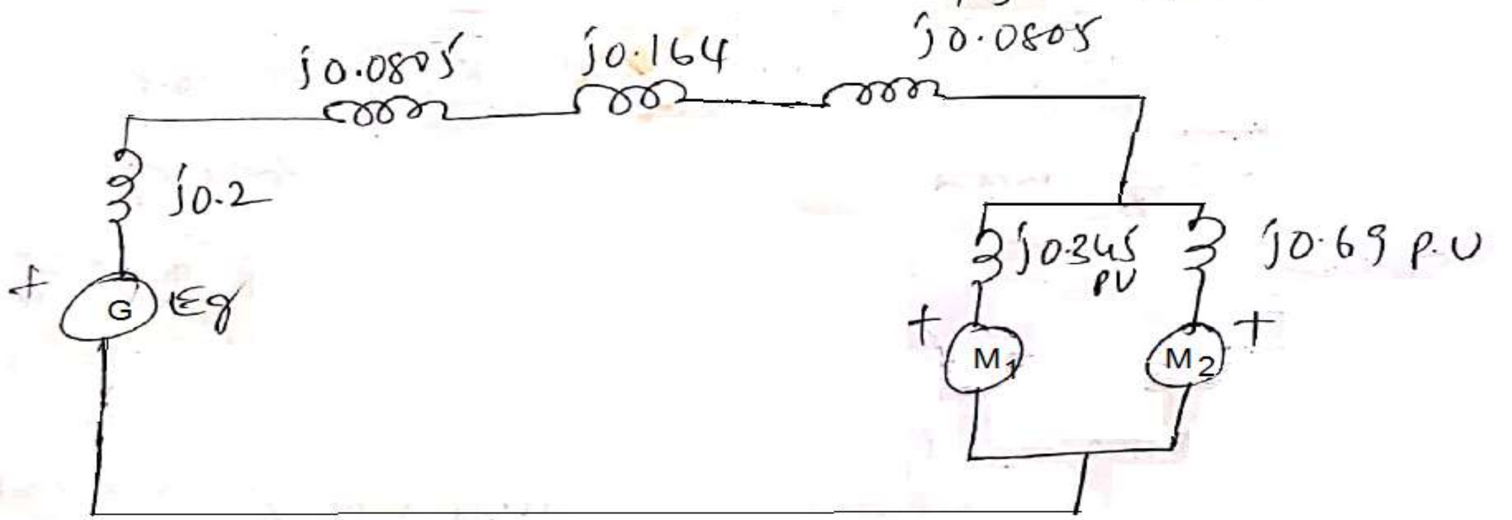
$$\text{Transformer reactance} = Z_{pu\,old} * \frac{MVA_{new}}{MVA_{old}} \times \left(\frac{kV_{old}}{kV_{new}}\right)^2$$

$$T_1 \quad = 0.1 * \frac{25}{30} * \left(\frac{10.8}{11}\right)^2 = 0.0805\ PU$$

$$\text{Line reactance in P.U} = \frac{Z_{act}}{Z_{base}} = \frac{Z_{act}}{\left(\frac{(kV)^2}{MVA}\right)} = \frac{100 \times 25}{123.2^2} = 0.164\ P.U$$

$$\text{Reactance of motor 1} = 0.25 * \frac{25}{15} \times \left(\frac{10}{11}\right)^2 = 0.345\ P.U$$

$$\text{〃 motor 2} = 0.25 * \frac{25}{7.5} * \left(\frac{10}{11}\right)^2 = 0.69\ P.U$$



- Ex3: A 30MVA,11kV Three phase generator has a sub transient reactance of 15%. The generator supplies one motor over a transmission line with transformers at both ends The motors have rated inputs of 15 MVA, 10kV with 20% sub transient reactance. The three phase transformers are both rated 25MVA, 11/120kV, connection star-delta with leakage reactance of 5% each. The series reactance of the line is 50 ohms.

Example . Draw the pu impedance diagram for the power system shown in Fig. Neglect resistance and use a base of 100 MVA, 220 kV in 50 Ω line. The rating of the generator, motor and transformers are :

Generator	:	40 MVA, 25 kV, X'' = 20 %
Motor	:	50 MVA, 11 kV, X'' = 30 %
Y-Y Transformer	:	40 MVA, 33Y-220 Y kV, X = 15 %
Y-Δ Transformer	:	30 MVA, 11 Δ-220 Y kV, X = 15 %

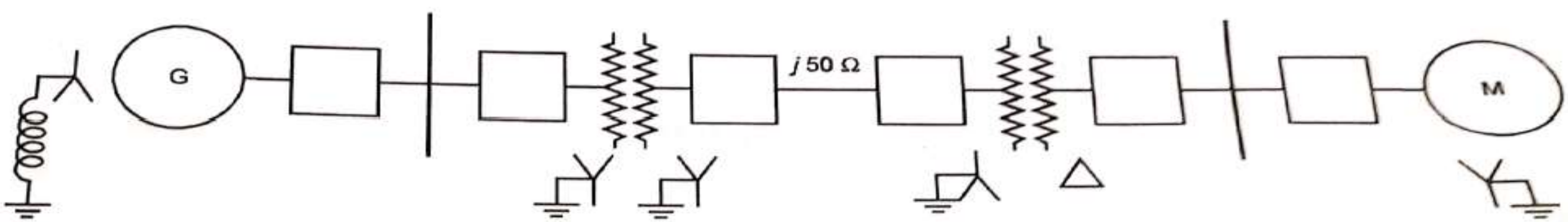


Fig.

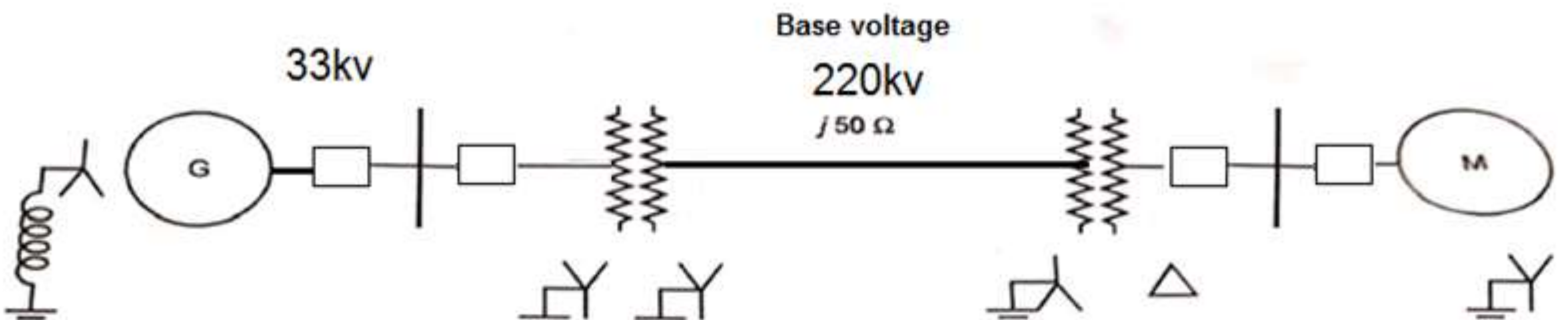
Solution : Base kVA for the complete circuit = 100 MVA

Base kV in 50 Ω line = 220 kV

Base kV in generator circuit

$$= 220 \times \frac{1}{\text{Transformation ratio of Y - Y transformer}}$$

$$= 220 \times \frac{33}{220} = 33 \text{ kV}$$

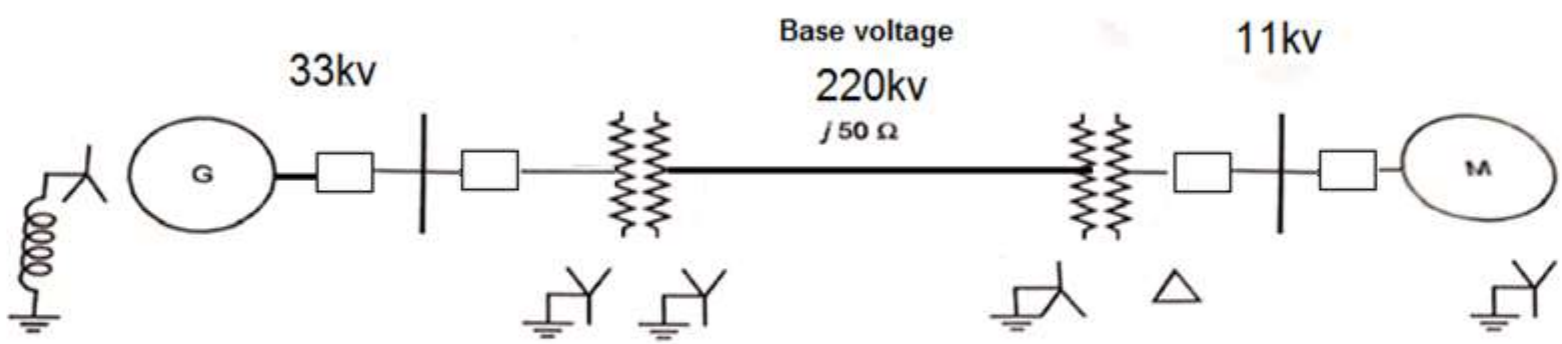


Fig

Base kV in motor circuit

$$= 220 \times \text{transformation ratio of Y–}\Delta \text{ transformer}$$

$$= 220 \times \frac{11}{220} = 11 \text{ kV}$$



Fig

Per unit impedance of line

$$Z_{pu} = \frac{\text{Actual impedance}}{\text{Base impedance}}$$

$$= \frac{\text{Actual impedance} \times MVA_B}{(kV_B)^2}$$

$$Z_{pu\ new} = Z_{pu\ old} \times \frac{kVA_{new}}{kVA_{old}} \times \frac{(kV_{old})^2}{(kV_{new})^2}$$

Per unit reactance of the generator

$$= \frac{20}{100} \times \frac{100{,}000}{40{,}000}\left(\frac{25}{33}\right)^2 = j\,0.287 \text{ pu}$$

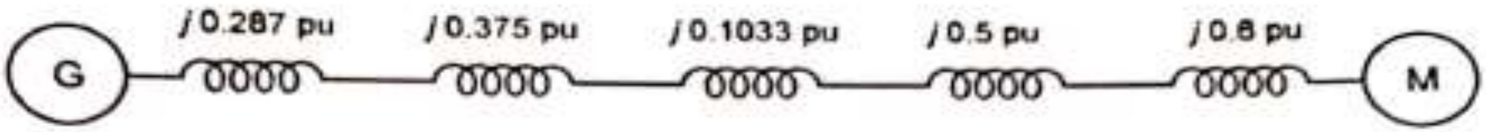


Fig. ***Reactance Diagram***

Per unit reactance of the motor

$$= \frac{30}{100} \times \frac{100{,}000}{50{,}000} \times \left(\frac{11}{11}\right)^2 = j\,0.6 \text{ pu}$$

Per unit reactance of Y–Y transformer

$$= \frac{15}{100} \times \frac{100{,}000}{40{,}000} \times \left(\frac{33}{33}\right)^2 = j\,0.375 \text{ pu}$$

Per unit reactance of Y–Δ transformer

$$= \frac{15}{100} \times \frac{100{,}000}{30{,}000} \times \left(\frac{11}{11}\right)^2 = j\,0.5 \text{ pu}$$

The reactance diagram for the system is shown in Fig.

Example A simple power system is shown in Fig. Redraw this system where the per unit impedances are represented on a common 5,000 VA base and common system base voltage of 250 V.

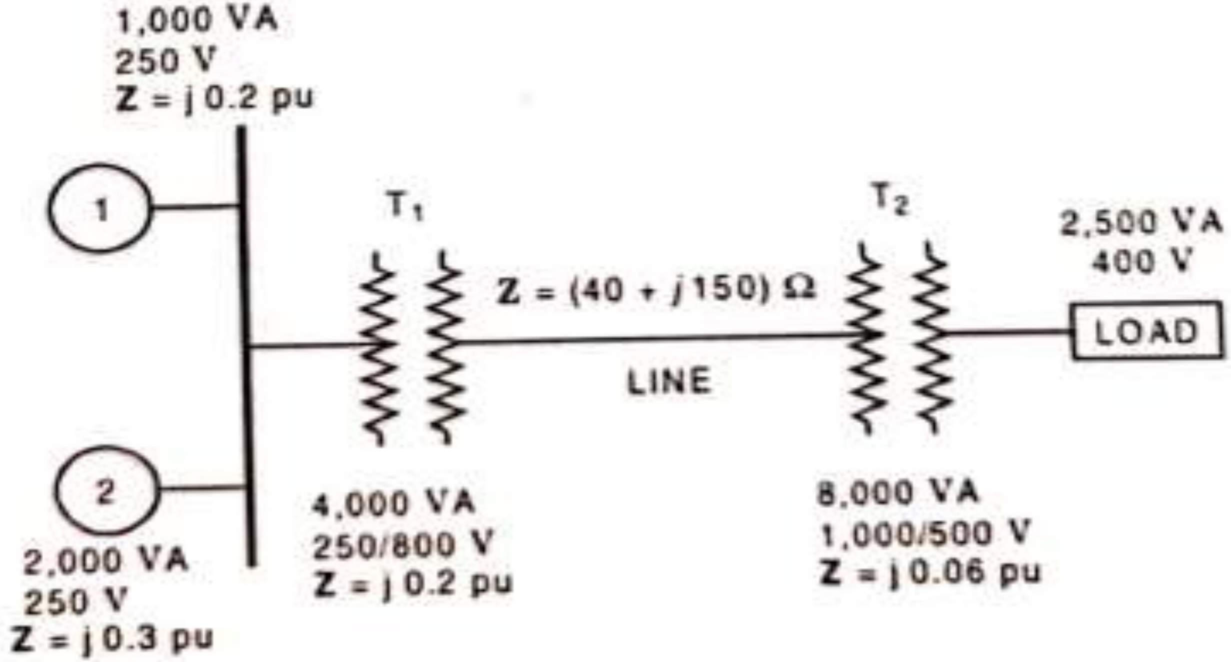


Fig.

Solution : Base kVA for the complete circuit $= \dfrac{5{,}000}{1{,}000} = 5$ kVA

Base voltage in generator circuits

$$= \frac{250}{1{,}000} = 0.25 \text{ kV}$$

Per unit reactance of machine 1,

$$X_{pu\,M_1} = j0.2 \times \left(\frac{0.25}{0.25}\right)^2 \times \left(\frac{5.0}{1.0}\right) = j\,1.0 \text{ pu}$$

Per unit reactance of machine 2,

$$X_{pu\,M_2} = j0.3 \times \left(\frac{0.25}{0.25}\right)^2 \times \frac{5}{2} = j\,0.75 \text{ pu}$$

Per unit reactance of transformer T_1,

$$X_{pu\,T_1} = j0.2 \times \left(\frac{0.25}{0.25}\right)^2 \times \left(\frac{5}{4}\right) = j\,0.25 \text{ pu}$$

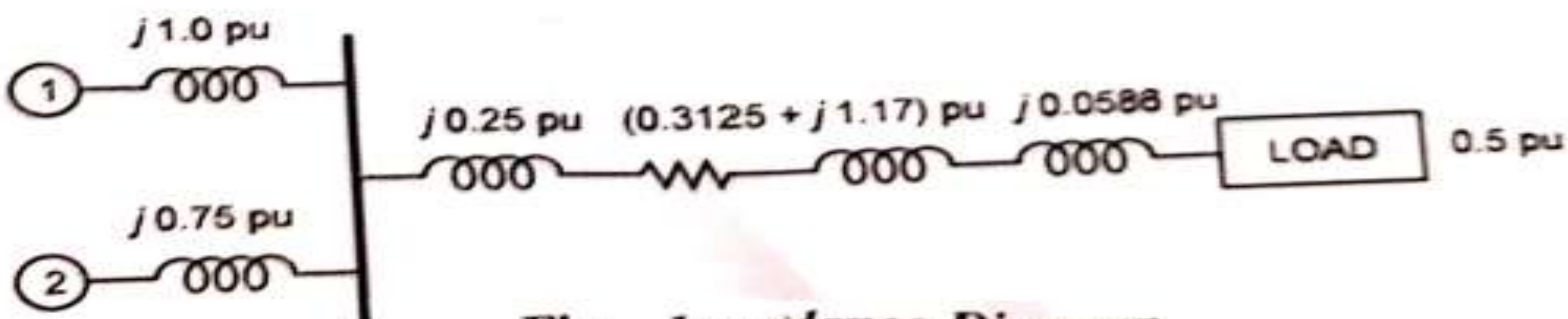


Fig. ***Impedance Diagram***

Example A 100 MVA, 33 kV, 3-phase generator has a subtransient reactance of 15%. The generator is connected to the motors through a transmission line and transformer as shown in Fig. 2.20. The motors have rated inputs of 30 MVA, 20 MVA and 50 MVA at 30 kV with 20% subtransient reactance. The 3-ϕ transformers are rated at 110 MVA 32 kV Δ/110 kV Y with leakage reactance 8%. The line has a reactance of 50 Ω. Selecting the generator rating as the base quantities in the generator circuit, determine the base quantities in other parts of the system and evaluate the corresponding pu values.

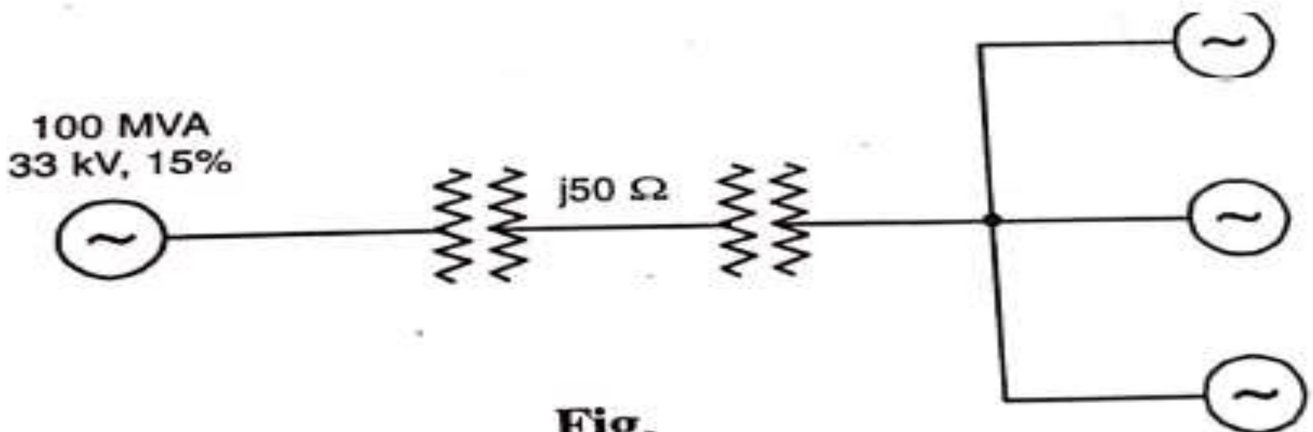


Fig.

Solution : Taking generator's rating as values, Base MVA for the whole circuit,

$$MVA_B = 100 \text{ MVA}$$

Base kV in generator circuit, $kV_B = 33$ kV

PU reactance of the generator $= j0.15$ pu

The base value of the line,

$$kV_{BL} = 33 \times \frac{110}{32} = 113.4375 \text{ kV} \quad \textbf{Ans.}$$

The base value in the motor circuit,

$$kV_{BM} = 113.4375 \times \frac{32}{110} = 33 \text{ kV} \quad \textbf{Ans.}$$

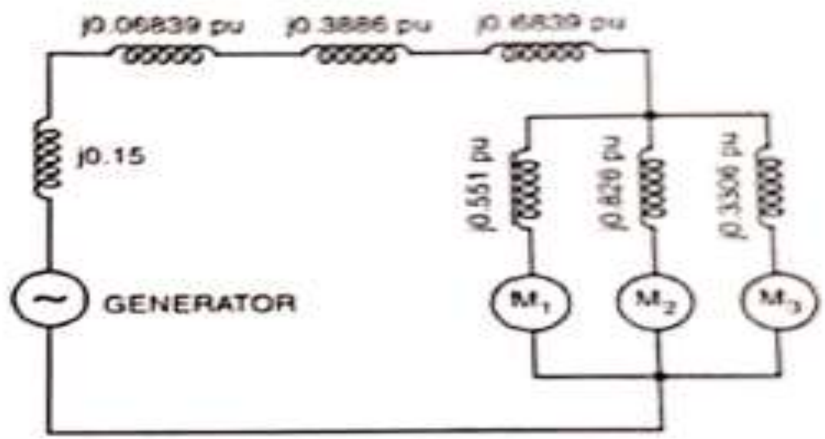


Fig. *PU Reactance Diagram of The System*

PU reactance of transformers $= 0.08 \times \frac{100}{110} \times \left(\frac{32}{33}\right)^2$

$= 0.06839$ pu **Ans.**

PU impedance of line $= j50 \times \frac{100}{(113.4375)^2} = 0.3886$ pu **Ans.**

PU reactance of motor 1 $= 0.2 \times \frac{100}{30} \times \left(\frac{30}{33}\right)^2 = 0.551$ pu **Ans.**

PU reactance of motor 2 $= 0.2 \times \frac{100}{20} \times \left(\frac{30}{33}\right)^2 = 0.826$ pu **Ans.**

PU reactance of motor 3 $= 0.2 \times \frac{100}{50} \times \left(\frac{30}{33}\right)^2 = 0.3306$ pu **Ans.**

The pu reactance diagram for the system is shown in Fig.

Example A 300 MVA, 20 kV, 3-phase generator has a subtransient reactance of 20%. The generator supplies a number of synchronous motors over a 64 km transmission line having transformers at both ends as shown in one line diagram of Fig. The motors all rated 13.2 kV, are represented by just two equivalent motors. The neutral of one motor M_1 is grounded through reactance. The neutral of the second motor M_2 is not grounded. Rated inputs to the motors are 200 MVA and 100 MVA for M_1 and M_2 respectively. For both motors $X'' = 20\%$. The 3-phase transformers T_1 is rated 350 MVA, 230/20 kV with leakage reactance of 10%. Transformer T_2 is composed of three single-phase transformers each rated 127/13.2 kV, 100 MVA with leakage reactance of 10%. Series reactance of transmission line is 0.5 Ω/km. Draw the reactance diagram with all reactances marked in per unit. Select the generator rating as base in generator circuit.

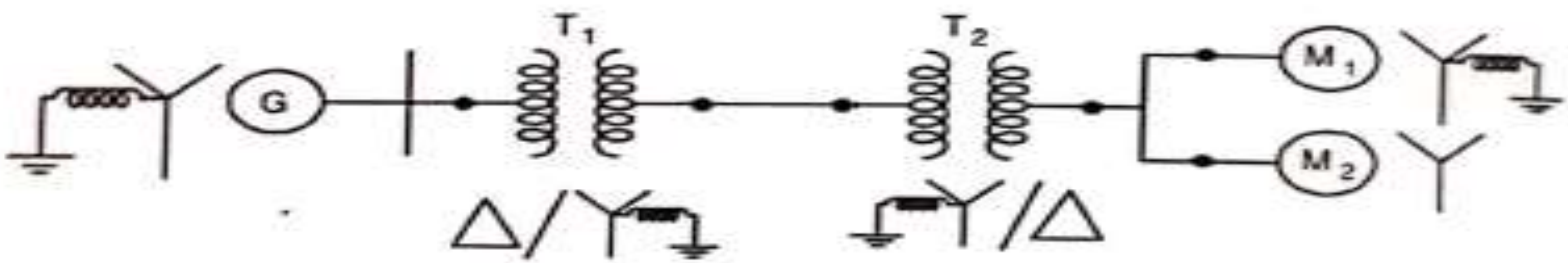


Fig.

Solution : Base MVA, $MVA_B = 300$ MVA

$$\text{Base voltage for generator} = 20 \text{ kV}$$

$$\text{Base voltage for line} = 20 \times \frac{230}{20} = 230 \text{ kV}$$

Line to line voltage ratio of transformer T_2

$$= \sqrt{3} \times \frac{127}{13.2} = 16.664$$

$$\text{Base voltage for motors} = \frac{230}{16.664} = 13.8 \text{ kV}$$

$$\text{PU reactance of generator} = 20\% = j0.2 \text{ pu}$$

$$\text{PU reactance of transformer } T_1 = \frac{10}{100} \times \frac{300}{350} \times \left(\frac{230}{230}\right)^2$$

$$= j0.0857 \text{ pu}$$

The reactance of transformer T_2 is 10% on $\sqrt{3} \times 127$: 13.2 kV 3×100 *i.e.*, 300 MVA base. For 300 MVA, 230 : 13.8

So pu reactance of transformer T_2

$$= \frac{10}{100} \times \left(\frac{13.2}{13.8}\right)^2 \times \frac{300}{300} = j0.0915 \text{ pu}$$

$$\text{Reactance of line} = 64 \times 0.5 = 32\ \Omega$$

$$\text{Base reactance of line} = \frac{(\text{Base kV})^2}{\text{Base MVA}} = \frac{(230)^2}{300} = 176.333\ \Omega$$

$$\text{Per unit reactance of line} = \frac{32}{176.333} = j0.1815 \text{ pu}$$

$$\text{Per unit reactance of motor } M_1 = \frac{20}{100} \times \frac{300}{200} \times \left(\frac{13.2}{13.8}\right)^2 = j0.2745 \text{ pu}$$

$$\text{Per unit reactance of motor } M_2 = \frac{20}{100} \times \frac{300}{100} \times \left(\frac{13.2}{13.8}\right)^2 = j0.549 \text{ pu}$$

Reactance diagram is shown in Fig. 2.25. E_g is the emf of generator and E_1 and E_2 are the emfs of motors.

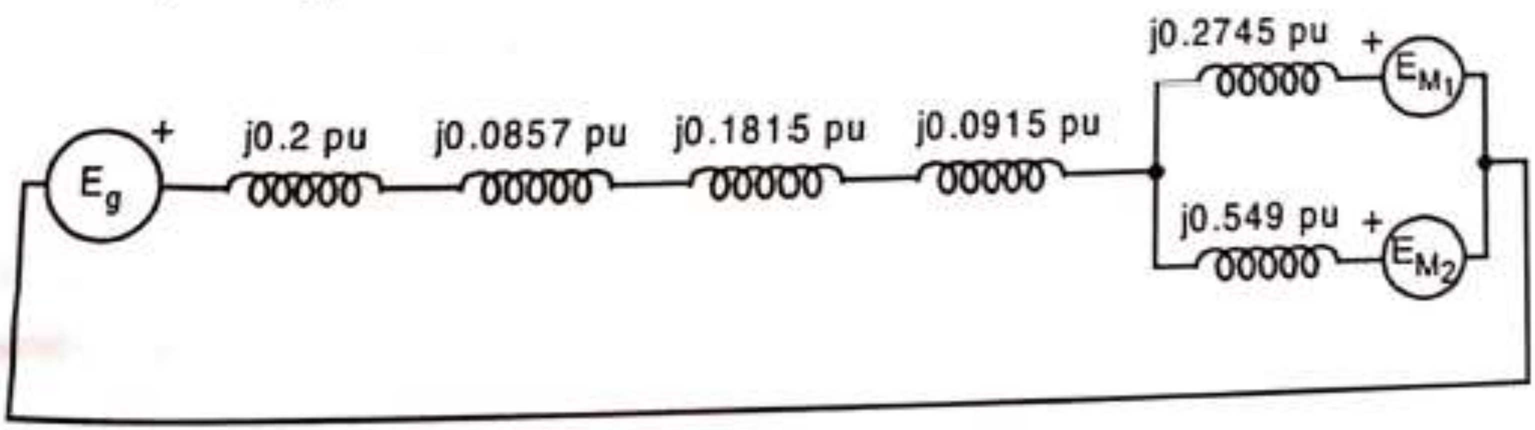
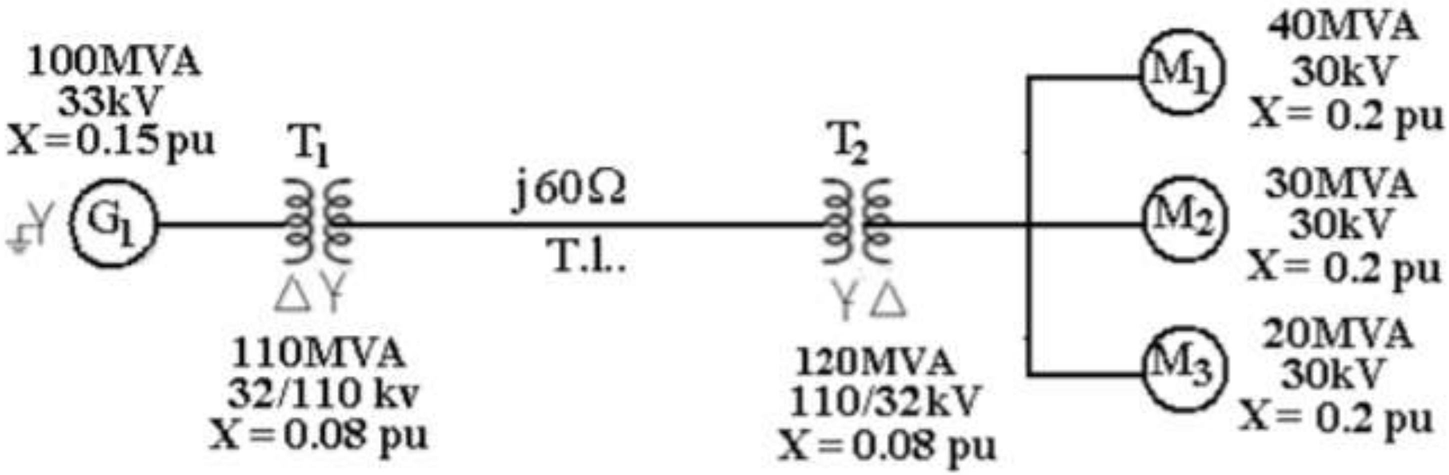


Fig. Reactance Diagram

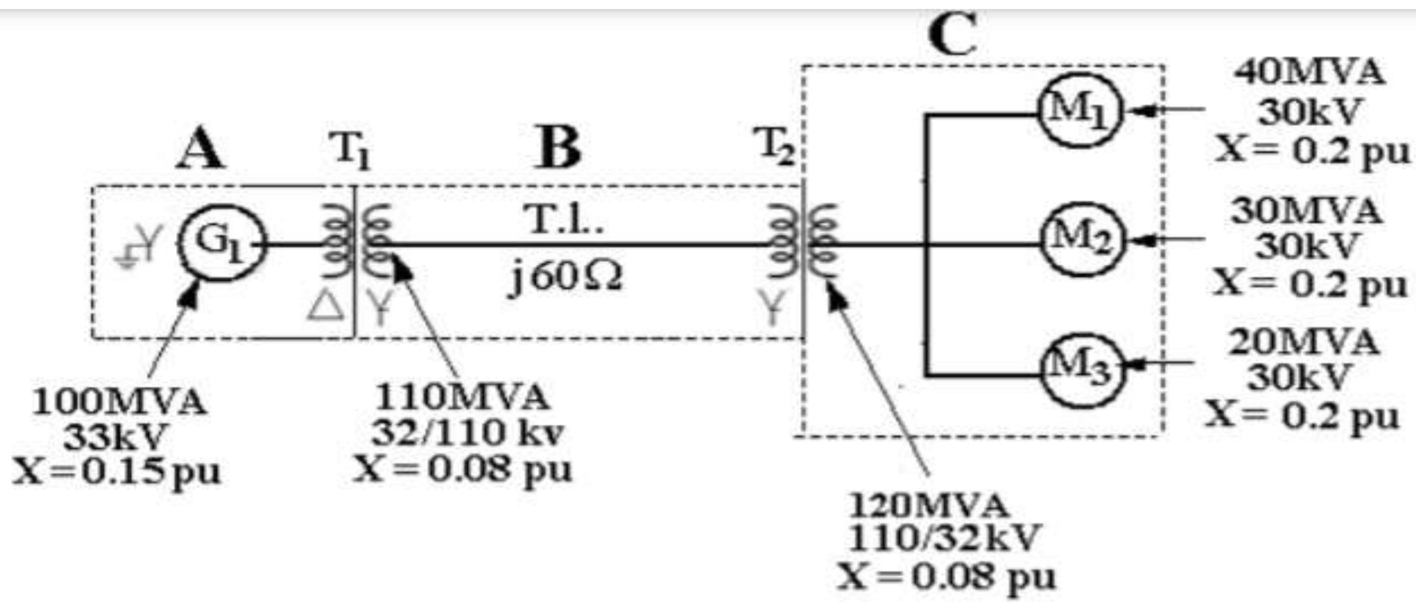
- Ex. A 90MVA ,11kV,three phase generator has a reactance of 25%. The generator supplies two motors through transmission line. The transformer T1 is three phase transformer, 100MVA, 10kV/132kV, 6% reactance. The transformer T2 is composed of 3 single phase units each rated at 30MVA, 66/10kV with 5% reactance. The motors are rated at 50MVA & 40MVA both 10kV ,20% reactance. Taking generator ratings as base, draw reactance diagram and indicate reactance in p.u.

Ex. A 100MVA, 33kV three-phase synchronous generator has a reactance of 15%. This generator is connected to a group of motors through a transmission line and two transformers as shown in figure. Draw the per-unit reactance diagram for this system.



Solution:

Select as a base 100MVA and 33kV in G_1 circuit



- For circuit A (G_1 circuit):

$MVA_b = 100, \quad kV_b = 33$

$$Z_{pu,\,new} = Z_{pu,\,old}\left[\frac{(MVA_b)_{new}}{(MVA_b)_{old}}\right]\left[\frac{(kV_b)_{old}}{(kV_b)_{new}}\right]^2$$

$$\boxed{Z_{G1} = j0.15\,pu}$$

$$Z_{T1} = j0.08 \times \left(\frac{100}{110}\right)\left(\frac{32}{33}\right)^2 = j0.0684\,\text{pu}$$

- For circuit B:

$$MVA_b = 100, \quad kV_b = 33 \times \left(\frac{110}{32}\right) = 113.44$$

$$Z_b = \frac{(kV_b)^2}{MVA_b} = \frac{(113.44)^2}{100} = 128.68\,\Omega$$

$$Z_{T.L.} = \frac{j60}{128.68} = j0.466\,\text{pu}$$

$$Z_{T2} = j0.08 \times \left(\frac{100}{120}\right)\left(\frac{110}{113.44}\right)^2 = j0.0627\,\text{pu}$$

- For circuit C:

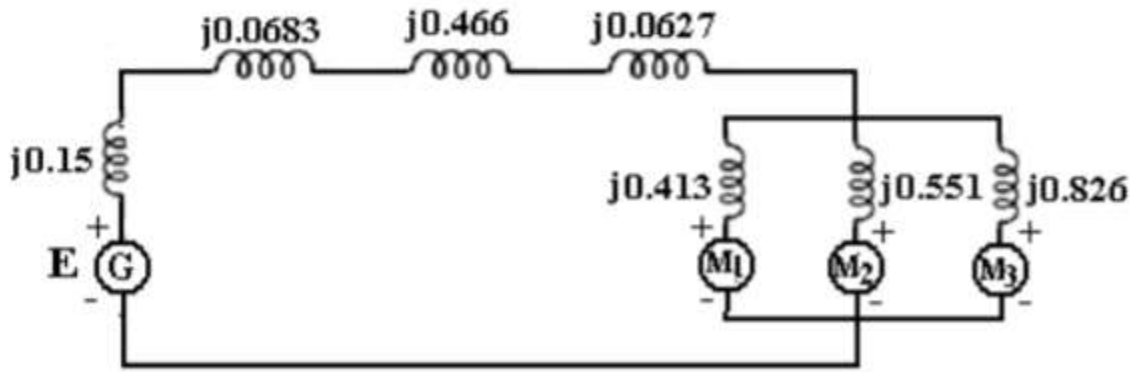
$$MVA_b = 100, \quad kV_b = 113.44 \times \frac{32}{110} = 33$$

$$Z_{M1} = j0.2 \times \left(\frac{100}{40}\right)\left(\frac{30}{33}\right)^2 = j0.413\,\text{pu}$$

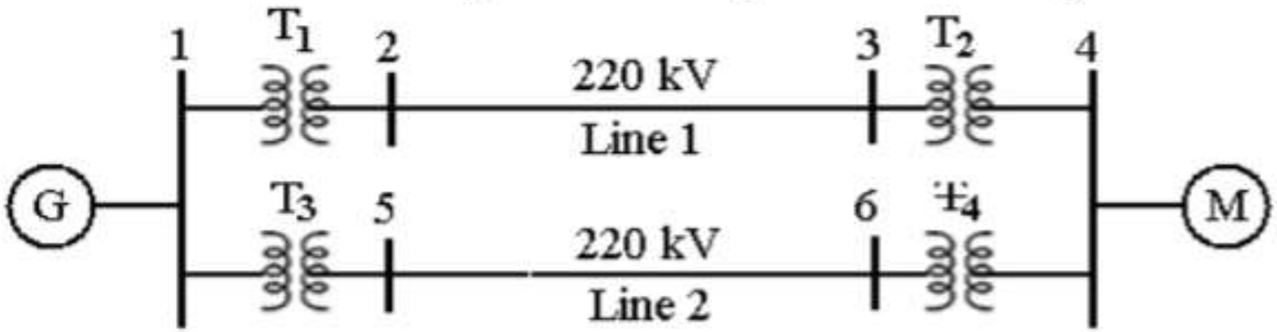
$$Z_{M2} = j0.2 \times \left(\frac{100}{30}\right)\left(\frac{30}{33}\right)^2 = j0.551\,\text{pu}$$

$$Z_{M3} = j0.2 \times \left(\frac{100}{20}\right)\left(\frac{30}{33}\right)^2 = j0.826\,\text{pu}$$

and the per-unit reactance diagram is as shown in following figure:



(Ex) The single-line diagram of a 3-phase power system is shown in figure. Choose as a common base the values 100MVA and 13.8kV in the generator circuit. Draw the per-unit diagram for this system.



Data of the system components are:

Generator: 90MVA, 13.8kV. X_G=18%

Transformer T_1: 50MVA, 13.8/220kV. X_{T1}=10%

Transformer T_2: 50MVA, 220/11kV. X_{T2}=10%

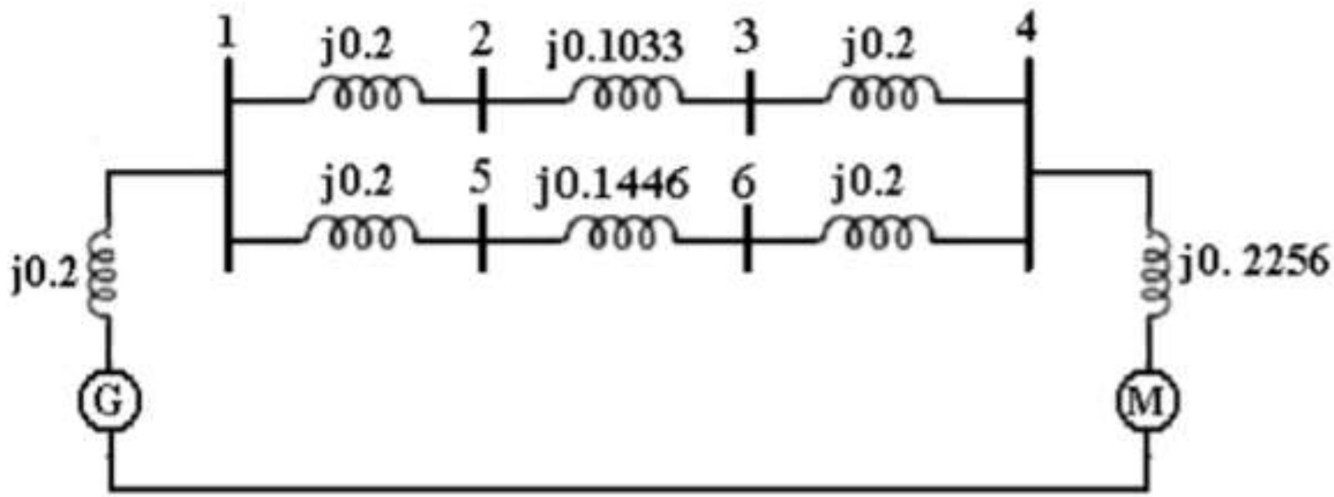
Transformer T_3: 50MVA, 13.8/132kV. X_{T3}=10%

Transformer T_4: 50MVA, 132/11kV. X_{T4}=10%

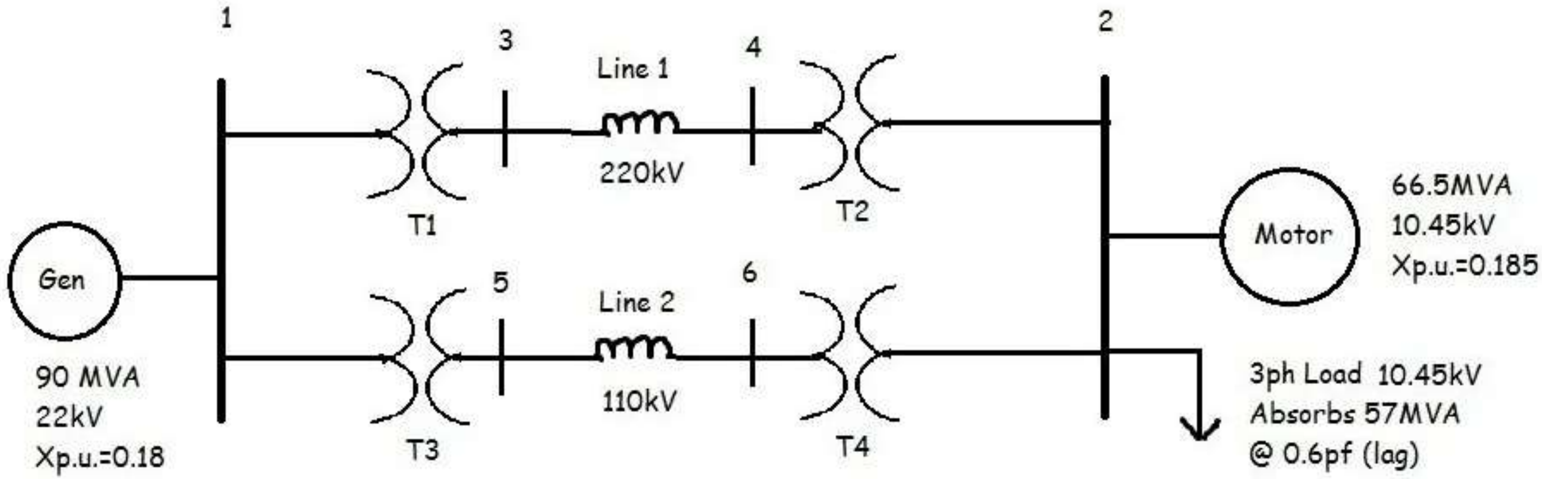
Motor M: 80MVA, 10.45kV. X_M=20%

Transmission lines: $X_{line1} = 50\Omega$, $X_{line12} = 70\Omega$

Solution:



Example:



T1:	50MVA	22/220kV	Xp.u.= 0.10
T2:	40MVA	220/11kV	Xp.u.= 0.06
T3:	40MVA	22/110kV	Xp.u.= 0.064
T4:	40MVA	110/11kV	Xp.u.=0.08

Line 1: 48.4Ohms (total)

Line 2: 65.43Ohms (total)

Figure 1: Oneline Diagram Of A Power System

Now that you have carefully examined the system and its parameters, the equivalent impedance diagram for the above system would look something like the following.

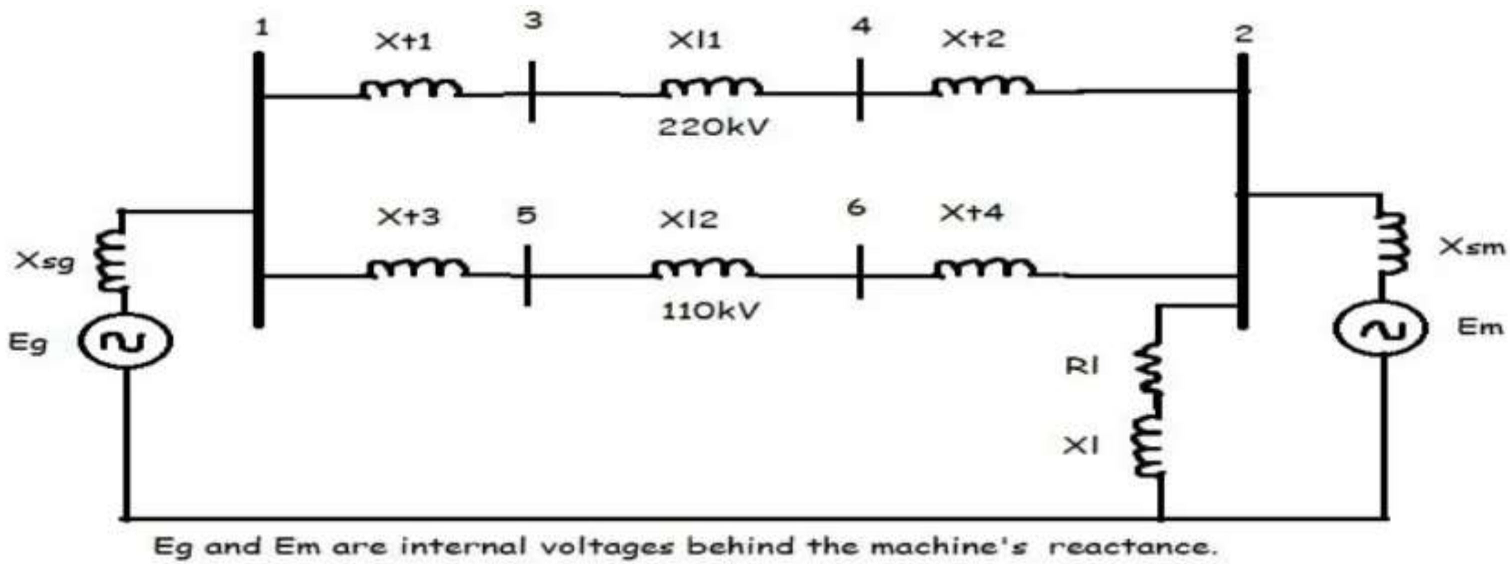


Figure 2: Impedance Diagram Of A Power System

Resistive impedance for most components have been ignored. Rotating machines have been replaced with a voltage source behind their internal reactance. Capacitive effects between lines and to ground are ignored as well.

To obtain the new normalized per unit impedances, first we need to figure out the base values (Sbase, Vbase, Zbase) in the power system. Following steps will lead you through the process.

Step 1: Assume a system base

Assume a system wide S_{base} of 100MVA. This is a random assumption and chosen to make calculations easy when calculating the per unit impedances.

So, S_{base} = 100MVA

Step 2: Identify the voltage base

Voltage base in the system is determined by the transformer. For example, with a 22/220kV voltage rating of T1 transformer, the V_{base} on the primary side of T1 is 22kV while the secondary side is 220kV. It does not matter what the voltage rating of the other components are that are encompassed by the V_{base} zone.

See figure below for the voltage bases in the system.

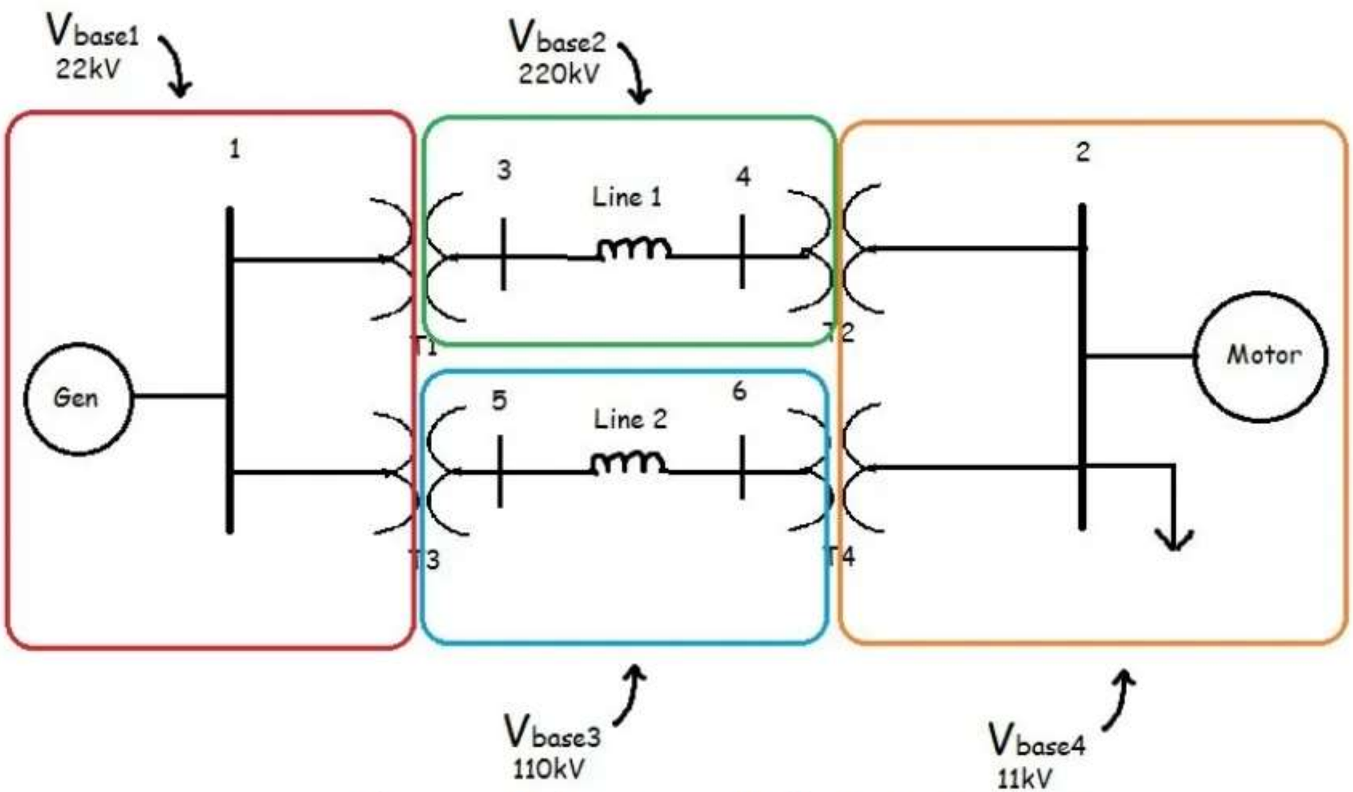


Figure 3: Voltage Base In The Power System

For 3-phase load: $Z_{base} = \frac{(11)^2}{100}$ = 1.21 Ohms

Step 4: Calculate the per unit impedance

The per unit impedance is calculated using the following formulas:

$$Z_{p.u.} = \frac{Z_{actual}}{Z_{base}} \quad .....(2)$$

$$Z_{p.u._{new}} = Z_{p.u._{old}} \left(\frac{S_{base_{new}}}{S_{base_{old}}}\right)\left(\frac{V_{base_{old}}}{V_{base_{new}}}\right)^2 \quad .....(3)$$

The voltage ratio in equation (3) is not equivalent to the transformers voltage ratio. It is the ratio of the transformer's voltage rating on the primary or secondary side to the

For T-line 2 using equation (2): $X_{l2_{p.u.}} = \frac{65.43}{121}$ = 0.5 pu

For 3-Phase load:

Power Factor: $\cos^{-1}(0.6) = \angle 53.13$

Thus, $S_{3\phi}(load) = 57\angle 53.13$

$Z_{act} = \frac{(V_{rated})^2}{S^*} = \frac{10.45^2}{57\angle -53.13}$

= 1.1495+j1.53267 Ohms

For generator, the new per unit reactance using equation (3)

$$X_{sg} = 0.18(\frac{100}{90})(\frac{22}{22})^2$$

= 0.2 pu

For transformer T1: $X_{t1} = 0.1(\frac{100}{50})(\frac{22}{22})^2$ = 0.2 pu

For transformer T2: $X_{t2} = 0.06(\frac{100}{40})(\frac{220}{220})^2$ = 0.15 pu

For transformer T3: $X_{t3} = 0.064(\frac{100}{40})(\frac{22}{22})^2$ = 0.16 pu

For transformer T4: $X_{t4} = 0.08(\frac{100}{40})(\frac{110}{110})^2$ = 0.2 pu

E-REFERENCES

- https://www.slideshare.net/PRABHAHARAN429/power-system-analysis?next_slideshow=1
- https://www.eee.hku.hk/~work1104/3-Per%20Unit%20System.pdf
- https://slideplayer.com/slide/10377772/

- https://portal.zu.edu.ly/upload/file-1588897089069.pdf
- https://www.academia.edu/5738318/Lecture_15_Power_system_Representation_and_Equations
- https://www.bharathuniv.ac.in/colleges1/downloads/courseware_eee/Notes/sem7/SEM%20VII%20BEE701%20power%20system%20analysis.pdf