NBKRIST

POWER ELECTRONICS LECTURE NOTES

UNIT-2

DEPARTMENT OF ELECTRICAL AND ELECTRONICS ENGINEERING

PHASE CONTROLLED RECTIFIERS

Line Commutated AC to DC converters

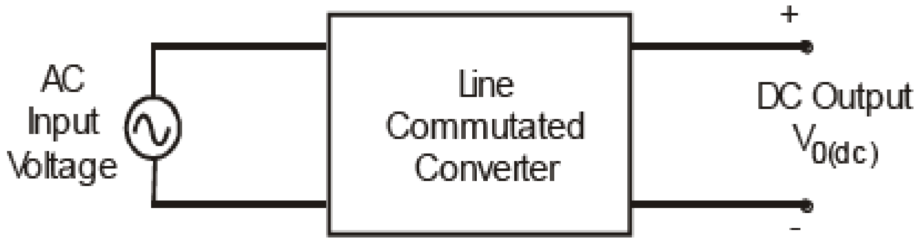


Fig.2.1 line commutated converter block diagram

The Fig.2.1 shows the block diagram of line commutated converter. It has

- Type of input: Fixed voltage, fixed frequency ac power supply.
- Type of output: Variable dc output voltage
- Type of commutation: Natural / AC line commutation

Applications of Phase Controlled Rectifiers

- DC motor control in steel mills, paper and textile mills employing dc motor drives.
- AC fed traction system using dc traction motor.
- Electro-chemical and electro-metallurgical processes.
- Magnet power supplies.
- Portable hand tool drives

Classification of Phase Controlled Rectifiers

- Single Phase Controlled Rectifiers.
- Three Phase Controlled Rectifiers

Different types of Single Phase Controlled Rectifiers.

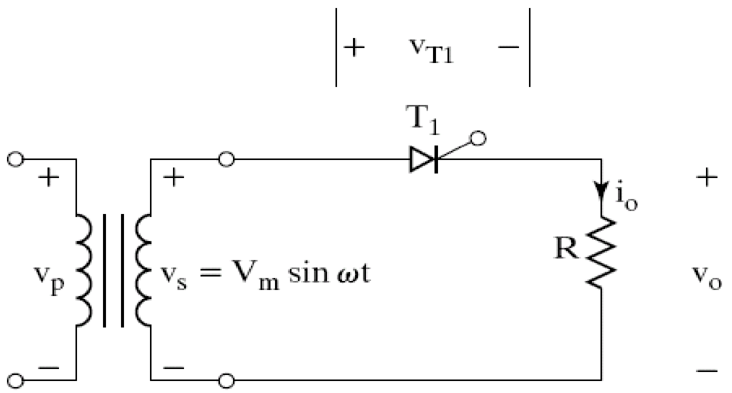
- Half wave controlled rectifiers.
- Full wave controlled rectifiers using a center tapped transformer.
- Full wave bridge circuit.
- Semi converter.

Different Types of Three Phase Controlled Rectifiers

- Half wave controlled rectifiers.
- Full wave controlled rectifiers.
- Semi converter (half controlled bridge converter).
- Full converter (fully controlled bridge converter).

Principle of Phase Controlled Rectifier Operation

The basic principle of operation of a phase controlled rectifier circuit is explained with reference to a single phase half wave phase controlled rectifier circuit with a resistive load as shown in the figure.2.2



$R = R_L =$ Load Resistance

Fig.2.2: Single Phase Half-Wave Thyristor Converter with a Resistive Load

A single phase half wave thyristor converter which is used for ac-dc power conversion is shown in the above figure. The input ac supply is obtained from a main supply transformer to provide the desired ac supply voltage to the thyristor converter depending on the output dc voltage required. V_P represents the primary input ac supply voltage. V_S represent the secondary ac supply voltage which is the output of the transformer secondary.

During the positive half cycle of input supply when the upper end of the transformer secondary is at a positive potential with respect to the lower end, the thyristor anode is positive with respect to its cathode and the thyristor is in a forward biased state. The thyristor is triggered at a delay angle of $\omega t = \alpha$, by applying a suitable gate trigger pulse to the gate lead of thyristor. When the thyristor is triggered at a delay angle of $\omega t = \alpha$, the thyristor conducts and assuming an ideal thyristor, the thyristor behaves as a closed switch and the input supply voltage appears across the load when the thyristor conducts from $\omega t = \alpha$ to π radians. Output voltage $v_O = v_S$, when the thyristor conducts from $\omega t = \alpha$ to π.

For a purely resistive load, the load current i_O (output current) that flows when the thyristor T_1 is on, is given by the expression

$$i_O = \frac{v_O}{R_L}, \text{ for } \alpha \le \omega t \le \pi$$

Note that when the thyristor conducts (T_1 is on) during $\omega t = \alpha$ to π, the thyristor current i_{T1}, the load current i_O through R_L and the source current i_S flowing through the transformer secondary winding are all one and the same.

Hence we can write

$$i_S = i_{T1} = i_O = \frac{v_O}{R} = \frac{V_m \sin \omega t}{R} \;\; ; \text{ for } \alpha \le \omega t \le \pi$$

When the supply voltage falls to zero at $\omega t = \pi$, the thyristor and the load current also falls to zero at $\omega t = \pi$. Thus the thyristor naturally turns off when the current flowing through it falls to zero at $\omega t = \pi$.

During the negative half cycle of input supply when the supply voltage reverses and becomes negative during $\omega t = \pi$ to 2π radians, the anode of thyristor is at a negative potential with respect to its cathode and as a result the thyristor is reverse biased and hence it remains cut-off (in the reverse blocking mode). The thyristor cannot conduct during its reverse biased state between $\omega t = \pi$ to 2π. An ideal thyristor under reverse biased condition behaves as an open switch and hence the load current and load voltage are zero during $\omega t = \pi$ to 2π. The maximum or peak reverse voltage that appears across the thyristor anode and cathode terminals is V_m.

The trigger angle α (delay angle or the phase angle α) is measured from the beginning of each positive half cycle to the time instant when the gate trigger pulse is applied. The thyristor conduction angle is from α to π, hence the conduction angle $\delta = (\pi - \alpha)$. The maximum conduction angle is π radians (180^0) when the trigger angle $\alpha = 0$.

The waveforms shows in the fig.2.3 the input ac supply voltage across the secondary winding of the transformer which is represented as v_S, the output voltage across the load, the output (load) current, and the thyristor voltage waveform that appears across the anode and cathode terminals.

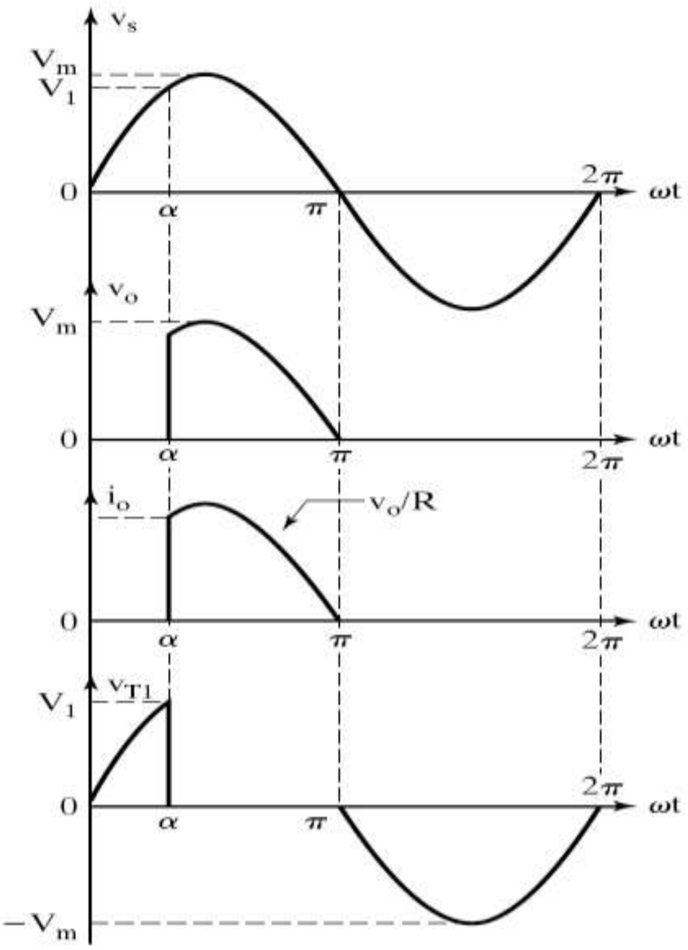


Fig.2.3: Waveforms of single phase half-wave controlled rectifier with resistive load

Equations

$v_s = V_m \sin \omega t =$ The ac supply voltage across the transformer secondary.

$V_m =$ Max. (Peak) value of input ac supply voltage across transformer secondary.

$V_S = \frac{V_m}{\sqrt{2}} =$ RMS value of input ac supply voltage across transformer secondary.

$v_O = v_L =$ The output voltage across the load; $i_O = i_L =$ output (load) current.

When the thyristor is triggered at $\omega t = \alpha$ (an ideal thyristor behaves as a closed switch) and hence the output voltage follows the input supply voltage.

$v_O = v_L = V_m \sin \omega t$; for $\omega t = \alpha$ to π, when the thyristor is on.

$i_O = i_L = \frac{v_O}{R}$ = Load current for $\omega t = \alpha$ to π, when the thyristor is on.

3.4.1 An Expression for the Average (Dc) Output Voltage across the Load

If V_m is the peak input supply voltage, the average output voltage V_{dc} can be found from

$$V_{O(dc)} = V_{dc} = \frac{1}{2\pi}\int_{\alpha}^{\pi} v_O.d(\omega t) \quad => \quad V_{O(dc)} = V_{dc} = \frac{1}{2\pi}\int_{\alpha}^{\pi} V_m \sin \omega t.d(\omega t)$$

$$V_{O(dc)} = \frac{1}{2\pi}\int_{\alpha}^{\pi} V_m \sin \omega t.d(\omega t) \quad => V_{O(dc)} = \frac{V_m}{2\pi}\int_{\alpha}^{\pi} \sin \omega t.d(\omega t)$$

$$V_{O(dc)} = \frac{V_m}{2\pi}\left[-\cos \omega t \Big/_{\alpha}^{\pi}\right] \quad => V_{O(dc)} = \frac{V_m}{2\pi}\left[-\cos \pi + \cos \alpha\right]; \quad \cos \pi = -1$$

$$V_{O(dc)} = \frac{V_m}{2\pi}\left[1 + \cos \alpha\right] \quad ; \quad V_m = \sqrt{2}V_S$$

The maximum average (dc) output voltage is obtained when $\alpha = 0$ and the maximum dc output voltage $V_{dc(\max)} = V_{dm} = \frac{V_m}{\pi}$.

The average dc output voltage can be varied by varying the trigger angle α from 0 to a maximum of $180^0 \left(\pi \text{ radians}\right)$.

We can plot the control characteristic, which is a plot of dc output voltage versus the trigger angle α by using the equation for $V_{O(dc)}$.

Control Characteristic of Single Phase Half Wave Phase Controlled Rectifier with Resistive Load

The average dc output voltage is given by the expression

$$V_{O(dc)} = \frac{V_m}{2\pi}\left[1 + \cos \alpha\right]$$

The fig.2.4 shows the control characteristics i.e graph between DC output voltage and trigger angle α

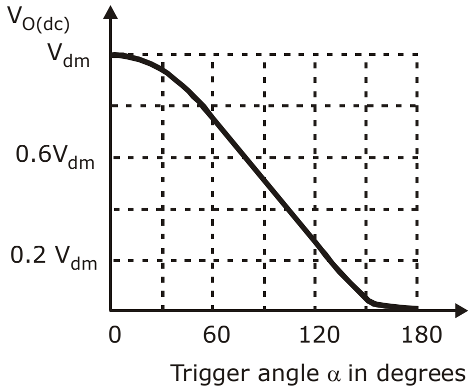


Fig.2.4: Control characteristic

Normalizing the dc output voltage with respect to V_{dm}, the normalized output voltage

$$V_{dcn} = \frac{V_{O(dc)}}{V_{dc(\max)}} = \frac{V_{dc}}{V_{dm}} \quad => \quad V_{dcn} = V_n = \frac{V_{dc}}{V_{dm}} = \frac{\frac{V_m}{2\pi}\left(1+\cos\alpha\right)}{\frac{V_m}{\pi}}$$

$$V_n = \frac{V_{dc}}{V_{dm}} = \frac{1}{2}\left(1+\cos\alpha\right) = V_{dcn}$$

An Expression for the RMS Value of Output Voltage of a Single Phase Half Wave Controlled Rectifier with Resistive Load

The rms output voltage is given by

$$V_{O(RMS)} = \left[\frac{1}{2\pi}\int_0^{2\pi} v_O^2 . d\left(\omega t\right)\right]$$

Output voltage $v_O = V_m \sin\omega t$; for $\omega t = \alpha$ to π

$$V_{O(RMS)} = \left[\frac{1}{2\pi}\int_\alpha^{\pi} V_m^2 \sin^2 \omega t . d\left(\omega t\right)\right]^{\frac{1}{2}}$$

$$V_{O(RMS)} = \left[\frac{1}{2\pi}\int_\alpha^{\pi} V_m^2 \frac{\left(1-\cos 2\omega t\right)}{2} . d\left(\omega t\right)\right]^{\frac{1}{2}} \quad => V_{O(RMS)} = \left[\frac{V_m^2}{4\pi}\int_\alpha^{\pi}\left(1-\cos 2\omega t\right). d\left(\omega t\right)\right]^{\frac{1}{2}}$$

$$V_{O(RMS)} = \left[\frac{V_m^2}{4\pi}\left\{\int_\alpha^{\pi} d\left(\omega t\right) - \int_\alpha^{\pi} \cos 2\omega t . d\left(\omega t\right)\right\}\right]^{\frac{1}{2}} \quad =>$$

$$V_{O(RMS)} = \frac{V_m}{2}\left[\frac{1}{\pi}\left\{\left(\omega t\right)\Big/_{\alpha}^{\pi} - \left(\frac{\sin 2\omega t}{2}\right)\Big/_{\alpha}^{\pi}\right\}\right]^{\frac{1}{2}}$$

$$V_{O(RMS)} = \frac{V_m}{2}\left[\frac{1}{\pi}\left(\left(\pi-\alpha\right) - \frac{\left(\sin 2\pi - \sin 2\alpha\right)}{2}\right)\right]^{\frac{1}{2}} \; ; \; \sin 2\pi = 0$$

Hence we get,

$$V_{O(RMS)} = \frac{V_m}{2}\left[\frac{1}{\pi}\left((\pi-\alpha)+\frac{\sin 2\alpha}{2}\right)\right]^{\frac{1}{2}} \quad => \quad V_{O(RMS)} = \frac{V_m}{2\sqrt{\pi}}\left((\pi-\alpha)+\frac{\sin 2\alpha}{2}\right)^{\frac{1}{2}}$$

Single Phase Half Wave Controlled Rectifier with an RL Load

A single phase half wave controlled rectifier circuit with an RL load using a thyristor T_1 (T_1 is an SCR) is shown in the figure.2.5 below.

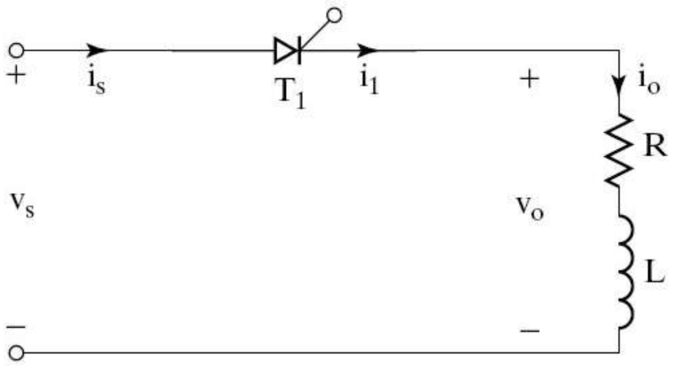


Figure2.5: Single Phase Half Wave Controlled Rectifier with RL-Load

The thyristor T_1 is forward biased during the positive half cycle of input supply. Let us assume that T_1 is triggered at $\omega t = \alpha$, by applying a suitable gate trigger pulse to T_1 during the positive half cycle of input supply. The output voltage across the load follows the input supply voltage when T_1 is ON. The load current i_O flows through the thyristor T_1 and through the load. This load current pulse flowing through T_1 can be considered as the positive current pulse. Due to the inductance in the load, the load current i_O flowing through T_1 would not fall to zero at $\omega t = \pi$, when the input supply voltage starts to become negative. A phase shift appears between the load voltage and the load current waveforms, due to the load inductance.

The thyristor T_1 will continue to conduct the load current until all the inductive energy stored in the load inductor L is completely utilized and the load current through T_1 falls to zero at $\omega t = \beta$, where β is referred to as the Extinction angle, (the value of ωt) at which the load current falls to zero. The extinction angle β is measured from the point of the beginning of the positive half cycle of input supply to the point where the load current falls to zero.

The thyristor T_1 conducts from $\omega t = \alpha$ to β. The conduction angle of T_1 is $\delta = (\beta - \alpha)$, which depends on the delay angle α and the load impedance angle ϕ. The waveforms of the input supply voltage, the gate trigger pulse of T_1, the thyristor current, the load current and the load voltage waveforms appear as shown in the figure. 2.6.

From β to 2π, the thyristor remains cut-off as it is reverse biased and behaves as an open switch. The thyristor current and the load current are zero and the output voltage also remains at zero during the non conduction time interval between β to 2π. In the next cycle the thyristor is triggered again at a phase angle of $\left(2\pi+\alpha\right)$, and the same operation repeats.

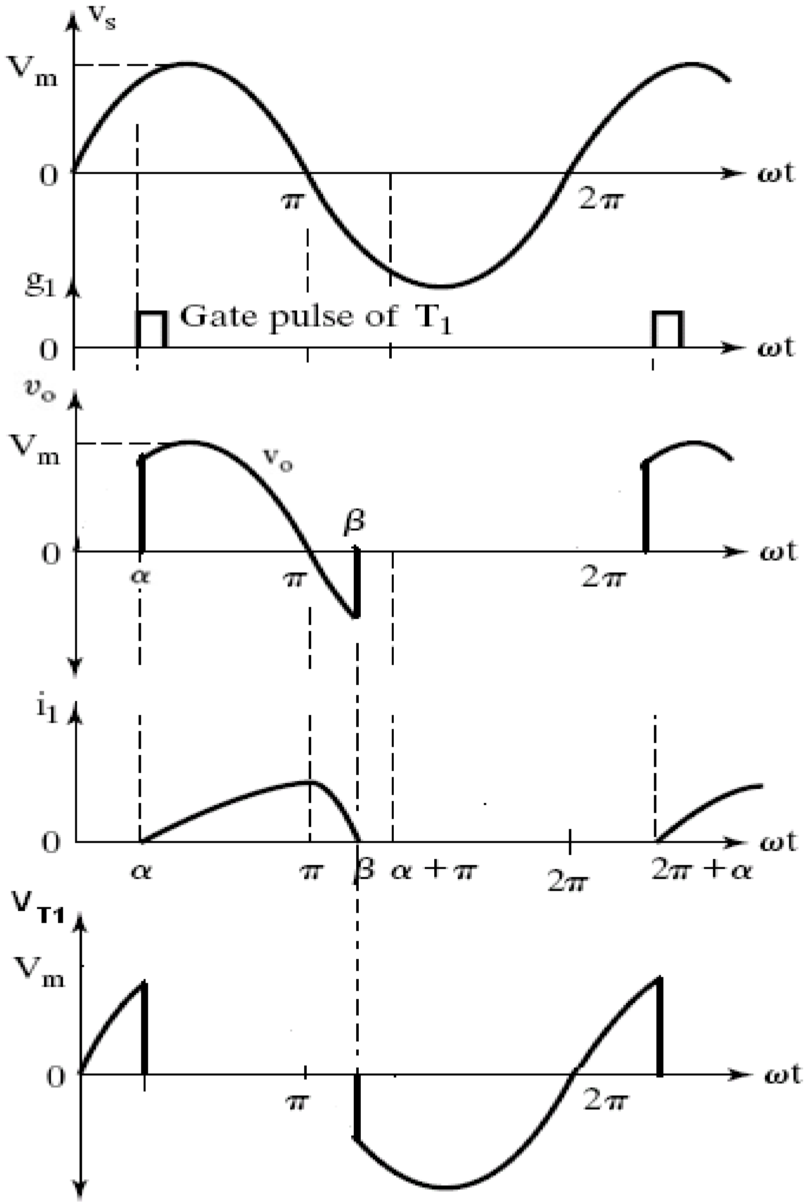


Fig.2.6: Input voltage, Output (load) voltage, Thyristor current (Load current) and Thyristor voltage waveforms of a single phase half wave controlled rectifier with RL load

An Expression for the Output (Inductive Load) Current, During $\omega t=\alpha$ to β When Thyristor T_1 Conducts

Considering sinusoidal input supply voltage we can write the expression for the supply voltage as

$v_S = V_m \sin \omega t$ = instantaneous value of the input supply voltage.

Let us assume that the thyristor T_1 is triggered by applying the gating signal to T_1 at $\omega t = \alpha$. The load current which flows through the thyristor T_1 during $\omega t = \alpha$ to β can be found from the equation

$$L\left(\frac{di_O}{dt}\right) + Ri_O = V_m \sin \omega t \quad ;$$

The solution of the above differential equation gives the general expression for the output load current which is of the form

$$i_O = \frac{V_m}{Z}\sin\left(\omega t - \phi\right) + A_1 e^{\frac{-t}{\tau}} \quad ;$$

Where $V_m = \sqrt{2}V_S$ = maximum or peak value of input supply voltage.

$Z = \sqrt{R^2 + \left(\omega L\right)^2}$ = Load impedance.

$\phi = \tan^{-1}\left(\frac{\omega L}{R}\right)$ = Load impedance angle (power factor angle of load).

$\tau = \frac{L}{R}$ = Load circuit time constant.

Therefore the general expression for the output load current is given by the equation

$$i_O = \frac{V_m}{Z}\sin\left(\omega t - \phi\right) + A_1 e^{\frac{-R}{L}t} \quad ;$$

The value of the constant A_1 can be determined from the initial condition. i.e. initial value of load current $i_O = 0$, at $\omega t = \alpha$. Hence from the equation for i_O equating i_O to zero and substituting $\omega t = \alpha$, we get

$$i_O = 0 = \frac{V_m}{Z}\sin\left(\alpha - \phi\right) + A_1 e^{\frac{-R}{L}t}$$

Therefore $\quad A_1 e^{\frac{-R}{L}t} = \frac{-V_m}{Z}\sin\left(\alpha - \phi\right) \quad => \quad A_1 = \frac{1}{e^{\frac{-R}{L}t}}\left[\frac{-V_m}{Z}\sin\left(\alpha - \phi\right)\right]$

$$A_1 = e^{\frac{+R}{L}t}\left[\frac{-V_m}{Z}\sin\left(\alpha - \phi\right)\right] \quad => \quad A_1 = e^{\frac{R(\omega t)}{\omega L}}\left[\frac{-V_m}{Z}\sin\left(\alpha - \phi\right)\right]$$

By substituting $\omega t = \alpha$, we get the value of constant A_1 as $\quad A_1 = e^{\frac{R(\alpha)}{\omega L}}\left[\frac{-V_m}{Z}\sin\left(\alpha - \phi\right)\right]$

Substituting the value of constant A_1 from the above equation into the expression for i_O, we obtain $i_O = \frac{V_m}{Z}\sin\left(\omega t - \phi\right) + e^{\frac{-R}{L}t} e^{\frac{R(\alpha)}{\omega L}}\left[\frac{-V_m}{Z}\sin\left(\alpha - \phi\right)\right]$;

$$i_O = \frac{V_m}{Z}\sin\left(\omega t - \phi\right) + e^{\frac{-R(\omega t)}{\omega L}} e^{\frac{R(\alpha)}{\omega L}}\left[\frac{-V_m}{Z}\sin\left(\alpha - \phi\right)\right]$$

$$i_O = \frac{V_m}{Z}\sin\left(\omega t - \phi\right) + e^{\frac{-R}{\omega L}(\omega t - \alpha)}\left[\frac{-V_m}{Z}\sin\left(\alpha - \phi\right)\right]$$

Therefore we obtain the final expression for the inductive load current of a single phase half wave controlled rectifier with RL load as

$$i_O = \frac{V_m}{Z}\left[\sin(\omega t - \phi) - \sin(\alpha - \phi)e^{\frac{-R}{\omega L}(\omega t - \alpha)}\right] \quad ; \quad \text{Where } \alpha \le \omega t \le \beta .$$

The above expression also represents the thyristor current i_{T1}, during the conduction time interval of thyristor T_1 from $\omega t = \alpha$ to β.

To calculate extinction angle β

The extinction angle β, which is the value of ωt at which the load current i_O falls to zero and T_1 is turned off can be estimated by using the condition that $i_O = 0$, at $\omega t = \beta$

By using the above expression for the output load current, we can write

$$i_O = 0 = \frac{V_m}{Z}\left[\sin(\beta - \phi) - \sin(\alpha - \phi)e^{\frac{-R}{\omega L}(\beta - \alpha)}\right] \quad \text{As } \frac{V_m}{Z} \ne 0, \text{ we can write}$$

$$\left[\sin(\beta - \phi) - \sin(\alpha - \phi)e^{\frac{-R}{\omega L}(\beta - \alpha)}\right] = 0$$

Therefore we obtain the expression

$$\sin(\beta - \phi) = \sin(\alpha - \phi)e^{\frac{-R}{\omega L}(\beta - \alpha)}$$

The extinction angle β can be determined from this transcendental equation by using the iterative method of solution (trial and error method). After β is calculated, we can determine the thyristor conduction angle $\delta = (\beta - \alpha)$.

β is the extinction angle which depends upon the load inductance value. Conduction angle δ increases as α is decreased for a specific value of β.

Conduction angle $\delta = (\beta - \alpha)$; for a purely resistive load or for an RL load when the load inductance L is negligible the extinction angle $\beta = \pi$ and the conduction angle $\delta = (\pi - \alpha)$

Equations

$v_s = V_m \sin \omega t =$ Input supply voltage

$v_O = v_L = V_m \sin \omega t =$ Output load voltage for $\omega t = \alpha$ to β,

When the thyristor T_1 conducts (T_1 is on).

Expression for the load current (thyristor current): for $\omega t = \alpha$ to β

$$i_O = \frac{V_m}{Z}\left[\sin(\omega t - \phi) - \sin(\alpha - \phi)e^{\frac{-R}{\omega L}(\omega t - \alpha)}\right] \quad ; \quad \text{Where } \alpha \le \omega t \le \beta .$$

Extinction angle β can be calculated using the equation

$$\sin(\beta - \phi) = \sin(\alpha - \phi)e^{\frac{-R}{\omega L}(\beta - \alpha)}$$

To Derive an Expression for Average (Dc) Load Voltage

$$V_{O(dc)} = V_L = \frac{1}{2\pi}\int_0^{2\pi} v_O.d(\omega t)$$

$$V_{O(dc)} = V_L = \frac{1}{2\pi}\left[\int_0^{\alpha} v_O.d(\omega t) + \int_{\alpha}^{\beta} v_O.d(\omega t) + \int_{\beta}^{2\pi} v_O.d(\omega t)\right] ;$$

$$v_O = 0 \text{ for } \omega t = 0 \text{ to } \alpha \text{ \& for } \omega t = \beta \text{ to } 2\pi \;;$$

$$\therefore \quad V_{O(dc)} = V_L = \frac{1}{2\pi}\left[\int_{\alpha}^{\beta} v_O.d(\omega t)\right]; v_O = V_m \sin\omega t \text{ for } \omega t = \alpha \text{ to } \beta$$

$$V_{O(dc)} = V_L = \frac{1}{2\pi}\left[\int_{\alpha}^{\beta} V_m \sin\omega t.d(\omega t)\right]$$

$$V_{O(dc)} = V_L = \frac{V_m}{2\pi}\left[-\cos\omega t \Big/_{\alpha}^{\beta}\right] = \frac{V_m}{2\pi}(\cos\alpha - \cos\beta)$$

$$\therefore \quad V_{O(dc)} = V_L = \frac{V_m}{2\pi}(\cos\alpha - \cos\beta)$$

Note: During the period $\omega t = \pi$ to β, we can see from the output load voltage waveform that the instantaneous output voltage is negative and this reduces the average or the dc output voltage when compared to a purely resistive load.

Average DC Load Current

$$I_{O(dc)} = I_{L(Avg)} = \frac{V_{O(dc)}}{R_L} = \frac{V_m}{2\pi R_L}(\cos\alpha - \cos\beta)$$

Single Phase Half Wave Controlled Rectifier with RL Load and Free Wheeling Diode

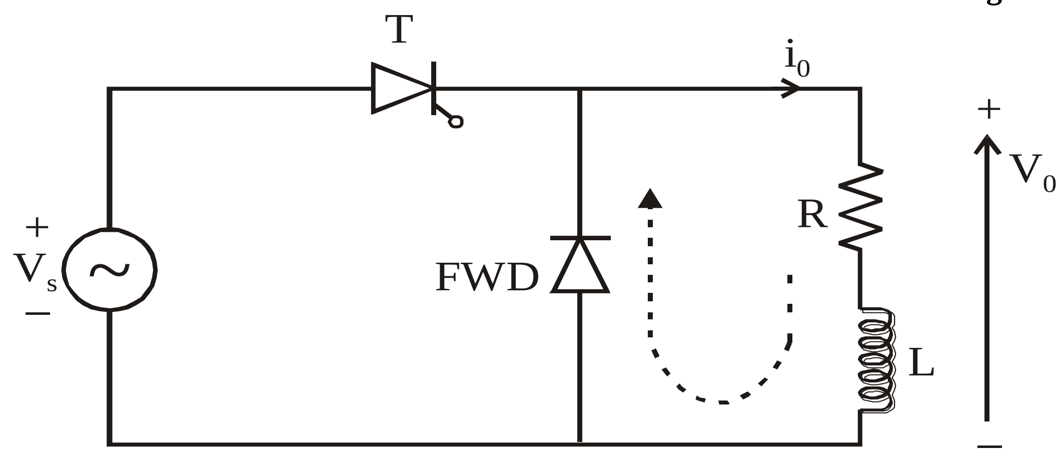


Fig.2.7: Single Phase Half Wave Controlled Rectifier with RL Load and Free Wheeling Diode

With a RL load it was observed that the average output voltage reduces. This disadvantage can be overcome by connecting a diode across the load as shown in fig.2.7. The diode is called as a ***Free Wheeling Diode (FWD)***. The waveforms are shown below as in fig.2.8.

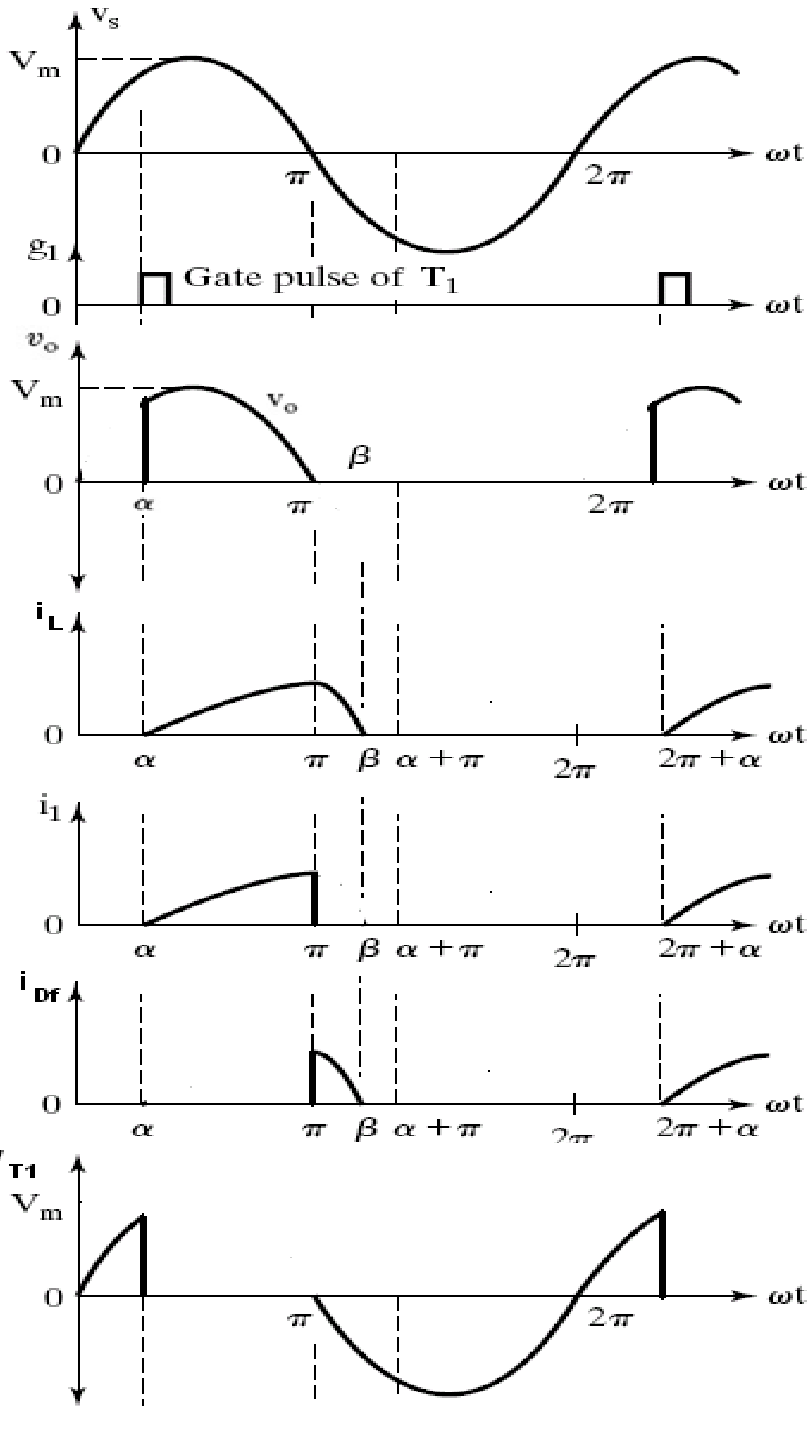


Fig.2.8: Input voltage, Output (load) voltage, Load current, Thyristor current, FWD Current and Thyristor voltage waveforms of a single phase half wave controlled Rectifier with RL load and freewheeling diode (FWD)

At $\omega t = \pi$, the source voltage v_S falls to zero and as v_S becomes negative, the freewheeling diode is forward biased. The stored energy in the inductance maintains the load current flow through R, L, and the FWD. Also, as soon as the FWD is forward biased, at $\omega t = \pi$, the SCR becomes reverse biased, the current through it becomes zero and the SCR turns off. During the period $\omega t = \pi$ to β, the load current flows through FWD (freewheeling load current) and decreases exponentially towards zero at $\omega t = \beta$.

Also during this freewheeling time period the load is shorted by the conducting FWD and the load voltage is almost zero, if the forward voltage drop across the conducting FWD is neglected. Thus there is no negative region in the load voltage wave form. This improves the average output voltage.

The average output voltage $V_{dc} = \frac{V_m}{2\pi}\left[1+\cos\alpha\right]$, which is the same as that of a purely resistive load.

Disadvantages of Single Phase Half Wave Controlled Rectifiers

Single phase half wave controlled rectifier gives

- Low dc output voltage.
- Low dc output power and lower efficiency.
- Higher ripple voltage & ripple current.
- Higher ripple factor.
- Low transformer utilization factor.
- The input supply current waveform has a dc component which can result in dc saturation of the transformer core.

Single phase half wave controlled rectifiers are rarely used in practice as they give low dc output and low dc output power. They are only of theoretical interest.

The above disadvantages of a single phase half wave controlled rectifier can be overcome by using a full wave controlled rectifier circuit. Most of the practical converter circuits use full wave controlled rectifiers.

Single Phase Full Wave Controlled Rectifier Using A Center Tapped Transformer with R-Load

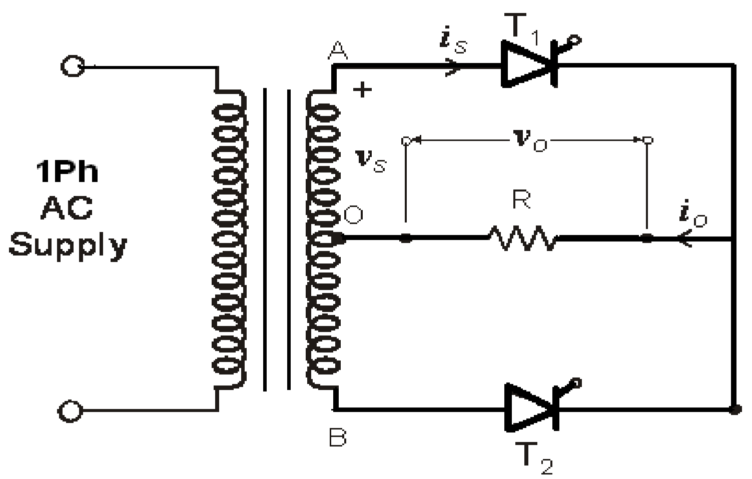


Fig.2.9: Single Phase Full Wave Controlled Rectifier Using A Center Tapped Transformer with R Load

v_S = Supply Voltage across the upper half of the transformer secondary winding

$v_S = v_{AO} = V_m \sin \omega t$

$v_{BO} = -v_{AO} = -V_m \sin \omega t =$ Supply voltage across the lower half of the transformer secondary winding.

The Single Phase Full Wave Controlled Rectifier Using A Center Tapped Transformer with R-Load as shown in fig.2.9 .This type of full wave controlled rectifier requires a center tapped transformer and two thyristors T_1 and T_2. The input supply is fed through the mains supply transformer, the primary side of the transformer is connected to the ac line voltage which is available (normally the primary supply voltage is 230V RMS ac supply voltage at 50Hz supply frequency in India). The secondary side of the transformer has three lines and the center point of the transformer (center line) is used as the reference point to measure the input and output voltages.

The upper half of the secondary winding and the thyristor T_1 along with the R-load act as a half wave controlled rectifier, the lower half of the secondary winding and the thyristor T_2 with the common R-load act as the second half wave controlled rectifier so as to produce a full wave load voltage waveform.

During the positive half cycle of input supply, when the upper line of the secondary winding is at a positive potential with respect to the center point 'O' the thyristor T_1 is forward biased and it is triggered at a delay angle of α. The load current flows through the thyristor T_1, through the load and through the upper part of the secondary winding, during the period α to π. The output voltage across the load follows the input supply voltage that appears across the upper part of the secondary winding from $\omega t = \alpha$ to π. The load current through the thyristor T_1 decreases and drops to zero at $\omega t = \pi$, and the thyristor T_1 naturally turns off at $\omega t = \pi$.

Similarly T_2 will triggered and source voltage across lower part of secondary winding is connected across load and load current flows during the period $\pi + \alpha$ to 2π. The load current through the thyristor T_2 decreases and drops to zero at $\omega t = 2\pi$, and the thyristor T_2 naturally turns off at $\omega t = 2\pi$. The related wave forms are as shown in fig.2.10.

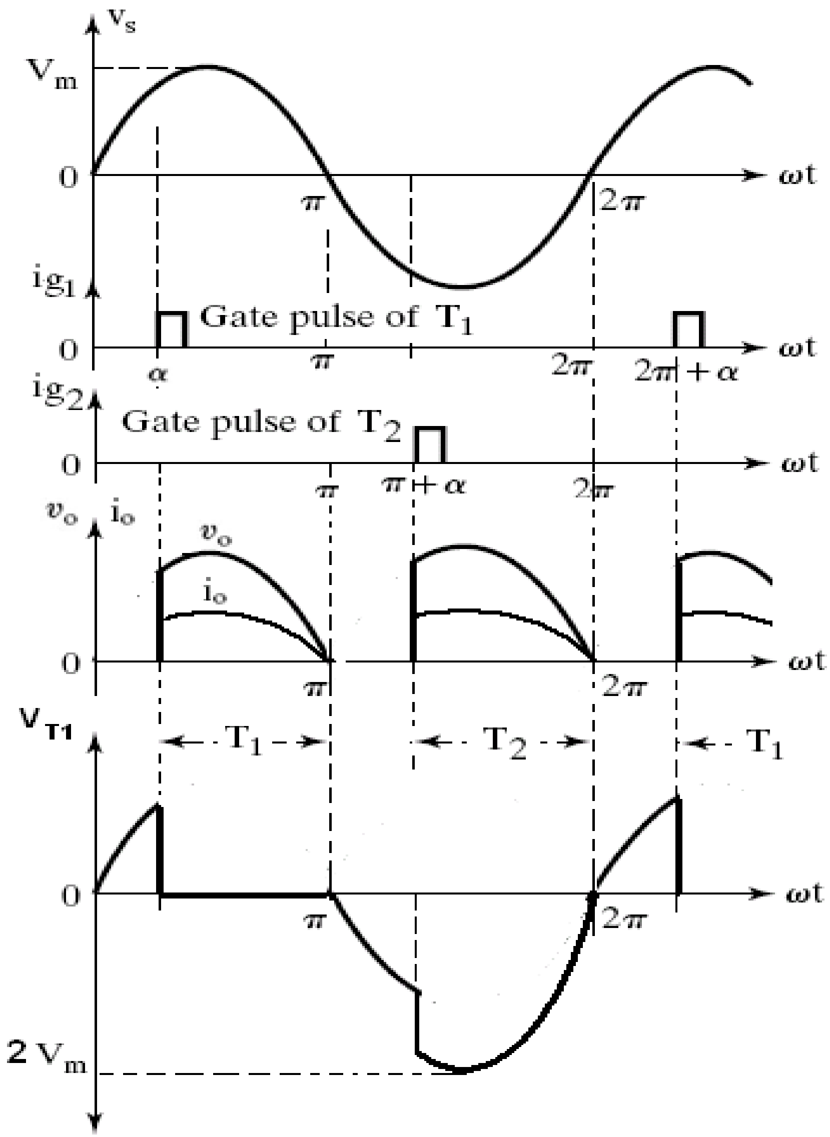


Fig.2.10: Input voltage, Output (load) voltage, Load current and Thyristor voltage Waveforms of a Single Phase Full Wave Controlled Rectifier Using A Center Tapped Transformer with R-Load

Single Phase Full Wave Controlled Rectifier Using A Center Tapped Transformer with RL Load

Fig.2.11 shows the circuit of Single Phase Full Wave Controlled Rectifier Using Center Tapped Transformer with RL-Load. It is operated in two modes of operations.

1) Mode 1: Discontinuous Load Current Operation.
2) Mode 2: Continuous Load Current Operation.

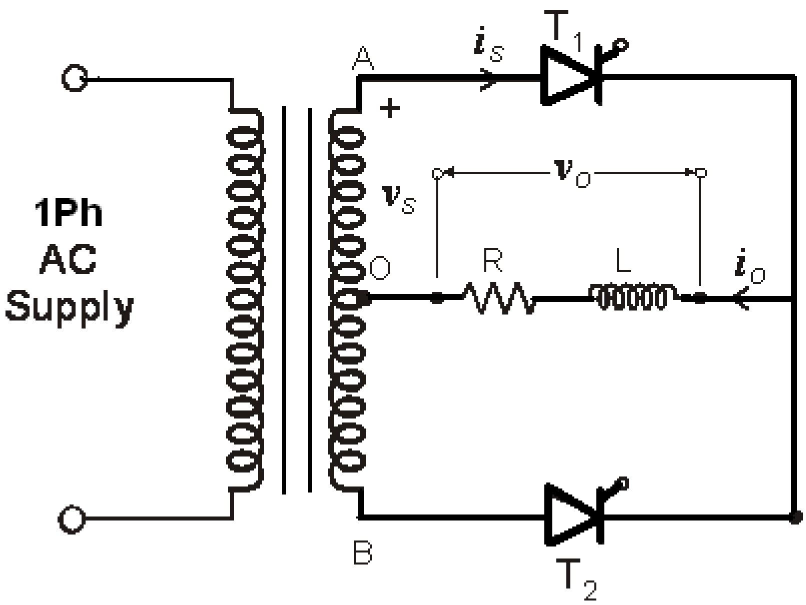


Fig.2.11: Single Phase Full Wave Controlled Rectifier Using A Center Tapped Transformer with R- Load

Discontinuous Load Current Operation (for low value of load inductance)

Generally the load current is discontinuous when the load is purely resistive or when the RL load has a low value of inductance.

During the positive half cycle of input supply, when the upper line of the secondary winding is at a positive potential with respect to the center point ‘O’ the thyristor T_1 is forward biased and it is triggered at a delay angle of α. The load current flows through the thyristor T_1, through the load and through the upper part of the secondary winding, during the period α to β, when the thyristor T_1 conducts.

The output voltage across the load follows the input supply voltage that appears across the upper part of the secondary winding from $\omega t = \alpha$ to β. The load current through the thyristor T_1 decreases and drops to zero at $\omega t = \beta$, where $\beta > \pi$ for RL type of load and the thyristor T_1 naturally turns off at $\omega t = \beta$.

During the negative half cycle of the input supply the voltage at the supply line ‘A’ becomes negative whereas the voltage at line ‘B’ (at the lower side of the secondary winding) becomes positive with respect to the center point ‘O’. The thyristor T_2 is forward biased during the negative half cycle and it is triggered at a delay angle of $(\pi + \alpha)$. The current flows through the thyristor T_2, through the load, and through the lower part of the secondary winding when T_2 conducts during the negative half cycle the load is connected to the lower half of the secondary winding when T_2 conducts.

The load current falls to zero at $\omega t = \beta = \pi$, when the input supply voltage falls to zero at $\omega t = \pi$. For low values of load inductance the load current would be discontinuous and the extinction angle $\beta > \pi$ but $\beta < (\pi + \alpha)$.

For large values of load inductance the load current would be continuous and does not fall to zero. The thyristor T_1 conducts from α to $(\pi+\alpha)$, until the next thyristor T_2 is triggered. When T_2 is triggered at $\omega t = (\pi+\alpha)$, the thyristor T_1 will be reverse biased and hence T_1 turns off.

The related wave forms of Single phase full wave midpoint converter fed to RL load for discontinuous current as shown in fig.2.12.

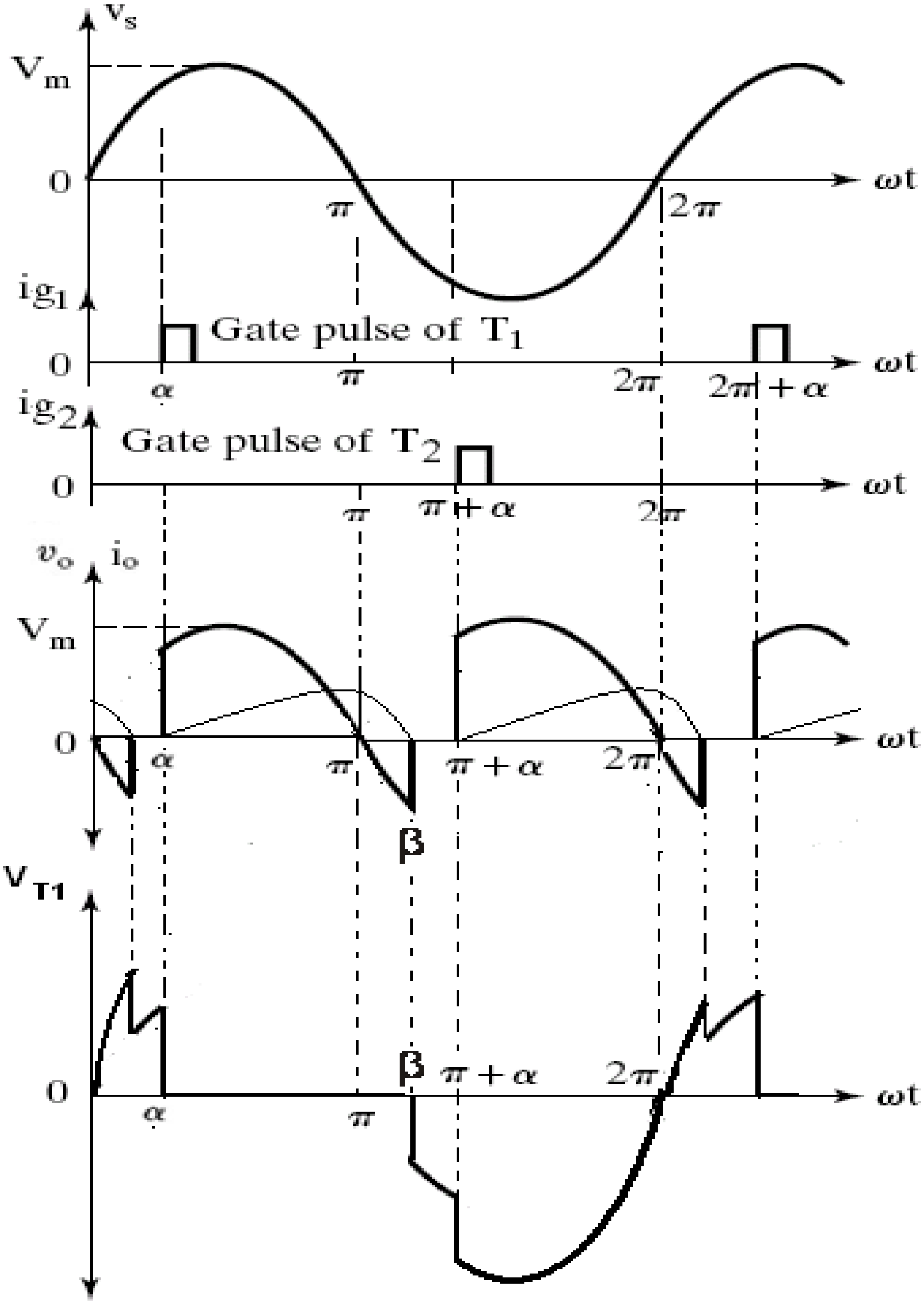


Fig.2.12: Waveforms for Discontinuous Load Current Operation without FWD

DC Output voltage of a single phase full wave controlled rectifier with RL load for discontinues conduction.

The average or dc output voltage :

$$V_{O(dc)} = V_{dc} = \frac{2}{2\pi}\int_{\omega t=\alpha}^{\beta} v_O.d(\omega t)$$

$$V_{O(dc)} = V_{dc} = \frac{1}{\pi}\int_{\omega t=\alpha}^{\beta} v_O.d(\omega t) \quad => \quad V_{O(dc)} = V_{dc} = \frac{1}{\pi}\left[\int_{\alpha}^{\beta} V_m \sin\omega t.d(\omega t)\right]$$

$$V_{O(dc)} = V_{dc} = \frac{V_m}{\pi}\left[-\cos\omega t \Big/_{\alpha}^{\beta}\right] \quad => \quad V_{O(dc)} = V_{dc} = \frac{V_m}{\pi}(\cos\alpha - \cos\beta)$$

Therefore $V_{O(dc)} = \frac{V_m}{\pi}(\cos\alpha - \cos\beta)$, for discontinuous load current operation, $\pi < \beta < (\pi + \alpha)$.

Single Phase Full Wave Controlled Rectifier Using A Center Tapped Transformer with R Load and Freewheeling Diode (FWD)

The Fig.2.13 shows the Single Phase Full Wave Controlled Rectifier Using A Center Tapped Transformer with R Load and Freewheeling Diode.

When T_1 is triggered at $\omega t = \alpha$, during the positive half cycle of the input supply the FWD is reverse biased during the time period $\omega t = \alpha$ to π. FWD remains reverse biased and cut-off from $\omega t = \alpha$ to π. The load current flows through the conducting thyristor T_1, through the RL load and through upper half of the transformer secondary winding during the time period α to π.

At $\omega t = \pi$, when the input supply voltage across the upper half of the secondary winding reverses and becomes negative the FWD turns-on. The load current continues to flow through the FWD from $\omega t = \pi$ to β. The related wave forms are as shown in fig.2.14.

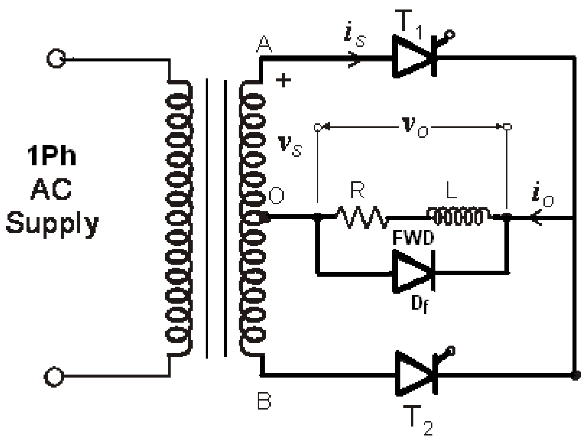


Fig2.13: Single Phase Full Wave Controlled Rectifier Using A Center Tapped Transformer with R Load and Freewheeling Diode (FWD)

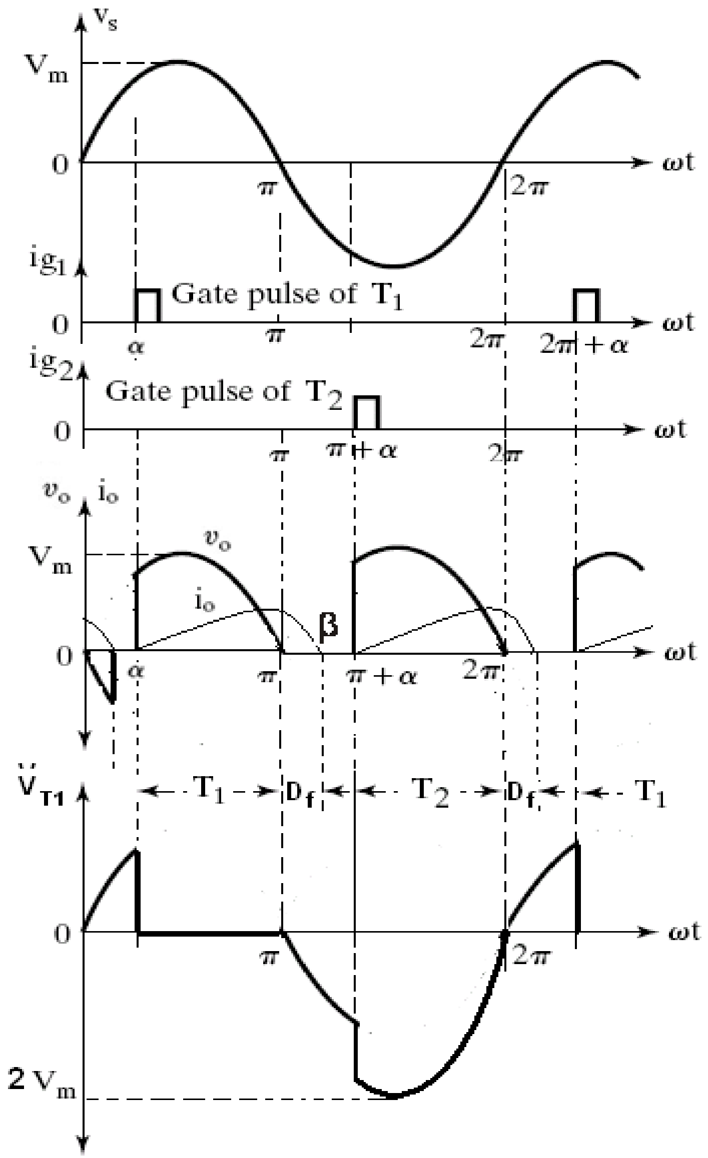


Fig2.14: Waveform for Discontinuous Load Current Operation with FWD

Expression for the Dc Output Voltage of a Single Phase Full Wave Controlled Rectifier with RL Load and FWD

Output Voltage: $$V_{O(dc)} = V_{dc} = \frac{1}{\pi}\int_{\alpha}^{\pi} V_m \sin\omega t.d(\omega t)$$

$$V_{O(dc)} = V_{dc} = \frac{V_m}{\pi}\left[-\cos\omega t \Big/_{\alpha}^{\pi}\right]$$

$$V_{O(dc)} = V_{dc} = \frac{V_m}{\pi}\left[-\cos\pi + \cos\alpha\right] \quad ; \quad \cos\pi = -1$$

Therefore $$V_{O(dc)} = V_{dc} = \frac{V_m}{\pi}\left(1 + \cos\alpha\right)$$

The DC output voltage V_{dc} is same as the DC output voltage of a single phase full wave controlled rectifier with resistive load. Note that the dc output voltage of a single phase full wave controlled rectifier is two times the dc output voltage of a half wave controlled rectifier.

Control characteristics of a single phase full wave controlled rectifier with R load or RL load with FWD

The control characteristic shown in fig.2.15 can be obtained by plotting the dc output voltage V_{dc} versus the trigger angle α .

The average or dc output voltage of a single phase full wave controlled rectifier circuit with R load or RL load with FWD is

$$V_{O(dc)} = V_{dc} = \frac{V_m}{\pi}\left(1+\cos\alpha\right)$$

Normalizing the dc output voltage with respect to its maximum value, we can write the normalized dc output voltage as

$$V_{dcn} = V_n = \frac{V_{dc}}{V_{dc(\max)}} = \frac{V_{dc}}{V_{dm}}$$

$$V_{dcn} = V_n = \frac{\frac{V_m}{\pi}\left(1+\cos\alpha\right)}{\left(\frac{2V_m}{\pi}\right)} = \frac{1}{2}\left(1+\cos\alpha\right)$$

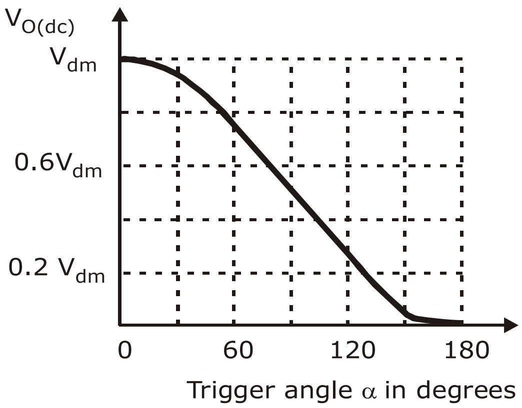


Fig2.15: Control characteristic of a single phase full wave controlled rectifier with R load or RL load with FWD

Continuous Load Current Operation (Without Fwd)

For large values of load inductance the load current flows continuously without decreasing and falling to zero and there is always a load current flowing at any point of time. This type of operation is referred to as continuous current operation. The waveforms for continuous current operation are as shown in fig.2.16.

In the case of continuous current operation the thyristor T_1 which is triggered at a delay angle of α , conducts from $\omega t = \alpha$ to $\left(\pi+\alpha\right)$. Output voltage follows the input supply voltage across the upper half of the transformer secondary winding $v_O = v_{AO} = V_m \sin\omega t$.

The next thyristor T_2 is triggered at $\omega t = (\pi + \alpha)$, during the negative half cycle input supply. As soon as T_2 is triggered at $\omega t = (\pi + \alpha)$, the thyristor T_1 will be reverse biasing and T_1 turns off due to natural commutation (ac line commutation). The load current flows through the thyristor T_2 from $\omega t = (\pi + \alpha)$ to $(2\pi + \alpha)$. Output voltage across the load follows the input supply voltage across the lower half of the transformer secondary winding $v_O = v_{BO} = -V_m \sin \omega t$.

Each thyristor conducts for π radians (180^0) in the case of continuous current operation

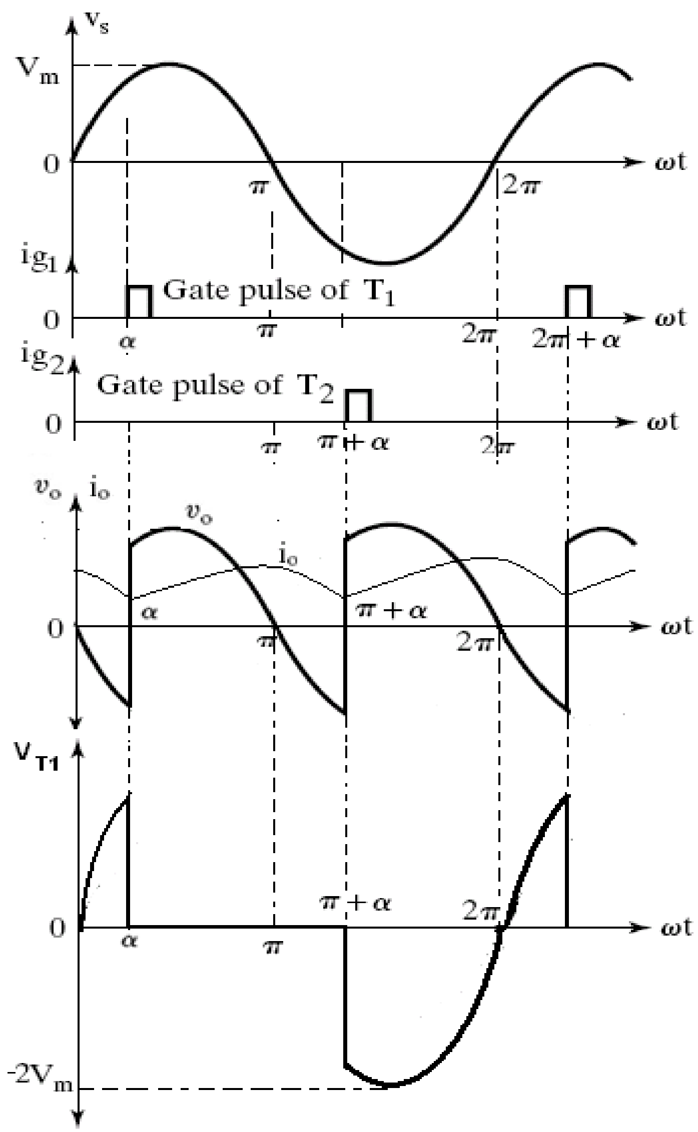


Fig.2.16: Load voltage and load current waveform of a single phase full wave controlled rectifier with RL load & without FWD for continuous load current operation

The Average DC Output Voltage of Single Phase Full Wave Controlled Rectifier with Continuous Load Current Operation.

$$V_{O(dc)} = V_{dc} = \frac{1}{\pi} \int_{\omega t=\alpha}^{(\pi+\alpha)} v_O . d(\omega t)$$

$$V_{O(dc)} = V_{dc} = \frac{1}{\pi} \left[\int_{\alpha}^{(\pi+\alpha)} V_m \sin \omega t . d(\omega t) \right]$$

$$V_{O(dc)} = V_{dc} = \frac{V_m}{\pi} \left[-\cos \omega t \Big/_{\alpha}^{(\pi+\alpha)} \right]$$

$$V_{O(dc)} = V_{dc} = \frac{V_m}{\pi} \left[\cos \alpha - \cos(\pi + \alpha) \right] \quad ; \qquad \cos(\pi + \alpha) = -\cos \alpha$$

$$V_{O(dc)} = V_{dc} = \frac{V_m}{\pi} \left[\cos \alpha + \cos \alpha \right]$$

$$\therefore \qquad V_{O(dc)} = V_{dc} = \frac{2V_m}{\pi} \cos \alpha$$

The above equation can be plotted to obtain the control characteristic of a single phase full wave controlled rectifier with RL load assuming continuous load current operation as shown i9n fig.2.17.

Normalizing the dc output voltage with respect to its maximum value, the normalized dc output voltage is given by

$$V_{dcn} = V_n = \frac{V_{dc}}{V_{dc(\max)}} = \frac{\frac{2V_m}{\pi}(\cos \alpha)}{\frac{2V_m}{\pi}} = \cos \alpha$$

Therefore $\qquad V_{dcn} = V_n = \cos \alpha$

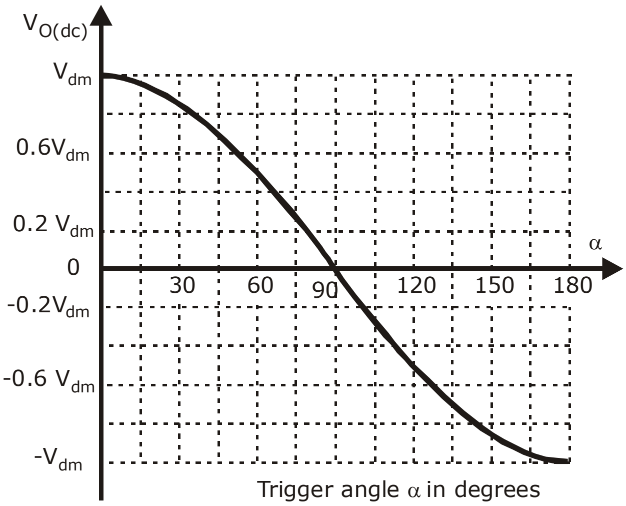


Fig.2.17: Control Characteristic

We notice from the control characteristic that by varying the trigger angle α we can vary the output dc voltage across the load. Thus it is possible to control the dc output voltage by changing the trigger angle α. For trigger angle α in the range of 0 to 90 degrees $\left(i.e.,\ 0 \le \alpha \le 90^0\right)$, V_{dc} is positive and the circuit operates as a controlled rectifier to convert ac supply voltage into dc output power which is fed to the load.

For trigger angle $\alpha > 90^0, \cos\alpha$ becomes negative and as a result the average dc output voltage V_{dc} becomes negative, but the load current flows in the same positive direction. Hence the output power becomes negative. This means that the power flows from the load circuit to the input ac source. This is referred to as *line commutated inverter operation*. During the inverter mode operation for $\alpha > 90^0$ the load energy can be fed back from the load circuit to the input ac source.

RMS Output Voltage

The rms value of the output voltage is calculated by using the equation

$$V_{O(RMS)} = \left[\frac{2}{2\pi}\int_{\alpha}^{(\pi+\alpha)} v_O^2 .d\left(\omega t\right)\right]^{\frac{1}{2}} \quad => \quad V_{O(RMS)} = \left[\frac{1}{\pi}\int_{\alpha}^{(\pi+\alpha)} V_m^2 \sin^2\omega t.d\left(\omega t\right)\right]^{\frac{1}{2}}$$

$$V_{O(RMS)} = \left[\frac{V_m^2}{\pi}\int_{\alpha}^{(\pi+\alpha)} \sin^2\omega t.d\left(\omega t\right)\right]^{\frac{1}{2}} \quad => \quad V_{O(RMS)} = \left[\frac{V_m^2}{\pi}\int_{\alpha}^{(\pi+\alpha)} \frac{\left(1-\cos 2\omega t\right)}{2}.d\left(\omega t\right)\right]^{\frac{1}{2}}$$

$$V_{O(RMS)} = V_m\left[\frac{1}{2\pi}\left\{\int_{\alpha}^{(\pi+\alpha)} d\left(\omega t\right) - \int_{\alpha}^{(\pi+\alpha)} \cos 2\omega t.d\left(\omega t\right)\right\}\right]^{\frac{1}{2}}$$

$$V_{O(RMS)} = V_m\left[\frac{1}{2\pi}\left\{\left(\omega t\right)\Big/_{\alpha}^{(\pi+\alpha)} - \left(\frac{\sin 2\omega t}{2}\right)\Big/_{\alpha}^{(\pi+\alpha)}\right\}\right]^{\frac{1}{2}}$$

$$V_{O(RMS)} = V_m\left[\frac{1}{2\pi}\left\{\left(\pi+\alpha-\alpha\right) - \left(\frac{\sin 2\left(\pi+\alpha\right) - \sin 2\alpha}{2}\right)\right\}\right]^{\frac{1}{2}}$$

$$V_{O(RMS)} = V_m\left[\frac{1}{2\pi}\left\{\left(\pi\right) - \left(\frac{\sin 2\pi \times \cos 2\alpha + \cos 2\pi \times \sin 2\alpha - \sin 2\alpha}{2}\right)\right\}\right]^{\frac{1}{2}}$$

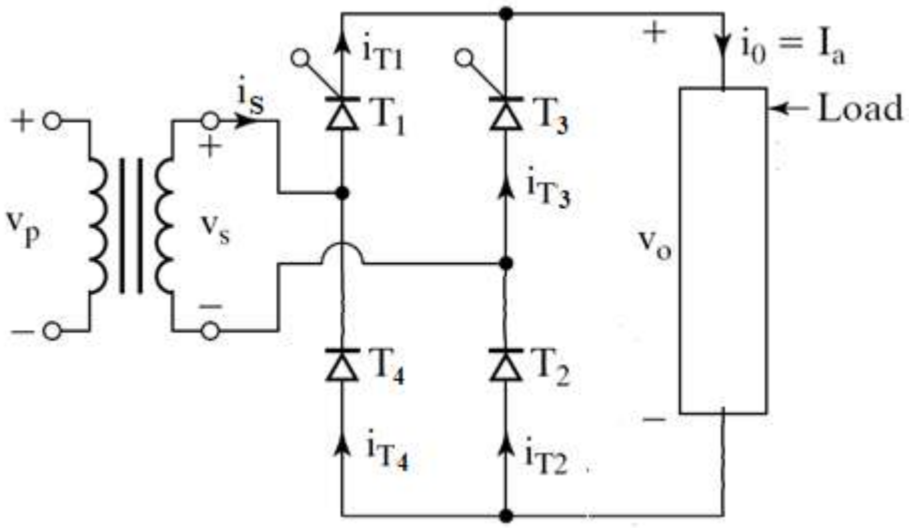
$$V_{O(RMS)} = V_m\left[\frac{1}{2\pi}\left\{\left(\pi\right) - \left(\frac{0 + \sin 2\alpha - \sin 2\alpha}{2}\right)\right\}\right]^{\frac{1}{2}}$$

$$V_{O(RMS)} = V_m\left[\frac{1}{2\pi}\left(\pi\right)\right]^{\frac{1}{2}} = \frac{V_m}{\sqrt{2}}$$

Therefore

$V_{O(RMS)} = \frac{V_m}{\sqrt{2}}$; The rms output voltage is same as the input rms supply voltage.

Single Phase Fully Controlled Bridge Converter



Note: For +ve pulse of source T_1T_2, pair and for -ve pulse of source T_3T_4, pair will conduct. The operation, wave forms and expressions is similar as 1-Ph mid-point controlled rectifier

Single Phase Semi-converters

(a) With R-Load

Single phase semi-converter circuit is a full wave half controlled bridge converter which uses two thyristors and two diodes connected in the form of a full wave bridge configuration.

The two thyristors are controlled power switches which are turned on one after the other by applying suitable gating signals (gate trigger pulses). The two diodes are uncontrolled power switches which turn-on and conduct one after the other as and when they are forward biased.

The circuit diagram of a single phase semi-converter (half controlled bridge converter) with R- load is as shown in the fig.2.18.

During the positive half cycle of input ac supply voltage, when the transformer secondary output line 'A' is positive with respect to the line 'B' the thyristor T_1 and the diode D_1 are both forward biased. The thyristor T_1 is triggered at $\omega t = \alpha$; $(0 \le \alpha \le \pi)$ by applying an appropriate gate trigger signal to the gate of T_1. The current in the circuit flows through the secondary line 'A', through T_1 , through the load in the downward direction, through diode D_1 back to the secondary line 'B'.

T_1 And D_1 conduct together from $\omega t = \alpha$ to π and the load is connected to the input ac supply. The output load voltage follows the input supply voltage (the secondary output voltage of the transformer) during the period $\omega t = \alpha$ to π .

During the negative half cycle of input supply voltage the secondary line 'A' becomes negative with respect to line 'B'. The thyristor T_2 and the diode D_2 are both forward biased. T_2 is triggered at $\omega t = (\pi + \alpha)$, during the negative half cycle. The load current continues to flow through T_2 and D_2 during the period $\omega t = (\pi + \alpha)$ to 2π

The related wave forms of a single phase semi-converter (half controlled bridge converter) with R- load is as shown in the fig.2.19.

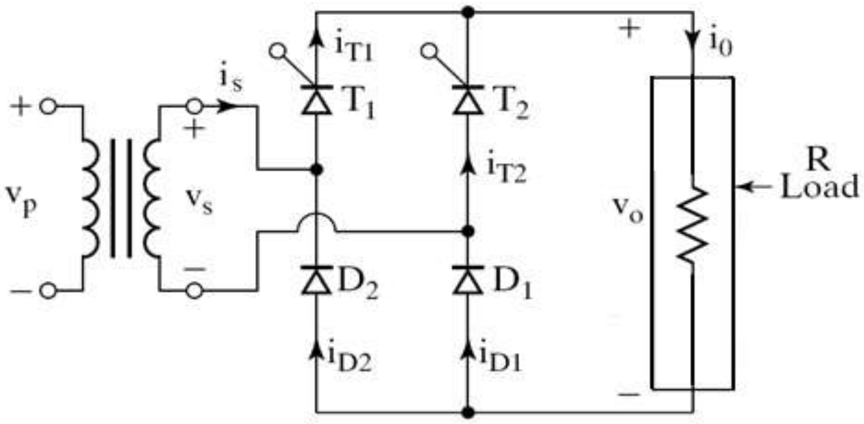


Fig.2.18: Single Phase Semi-converter with R-Load

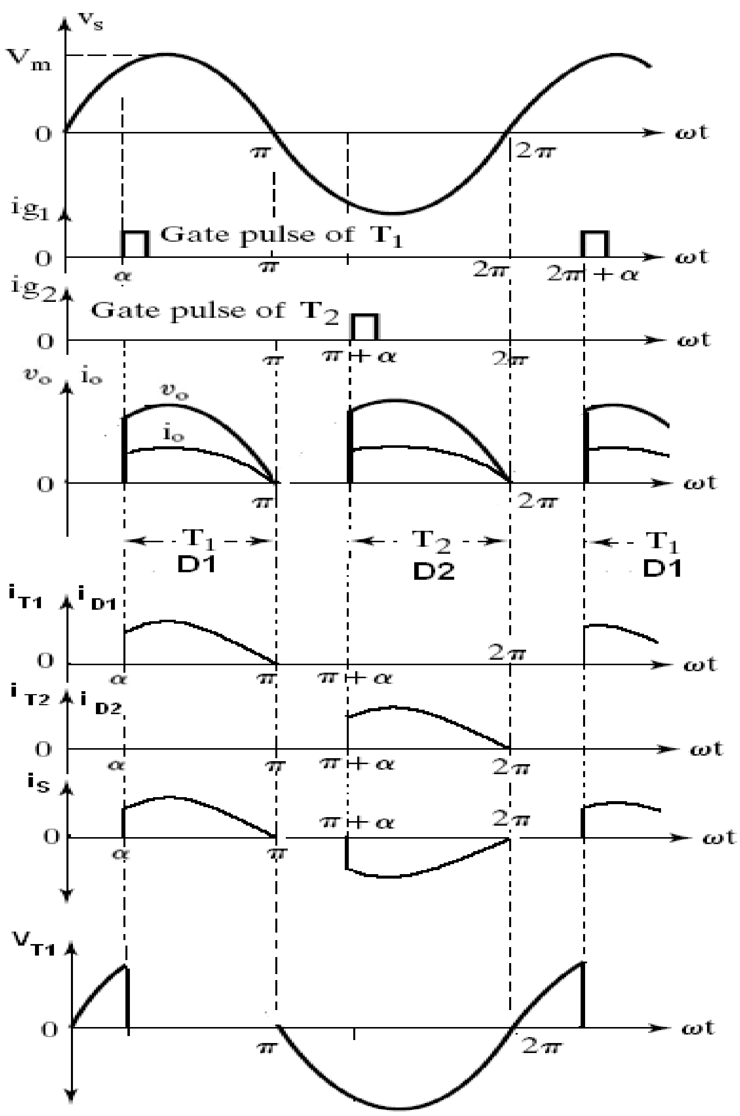


Fig.2.19: Related wave forms of a single phase semi-converter with R- load

(b) With RL-Load

The circuit diagram of a single phase semi-converter with RL- Load is as shown in the Fig.2.20. The related wave forms of a single phase semi-converter with RL- load for continuous load current and discontinuous load current are as shown in the fig.2.21 and fig.2.22 respectively.

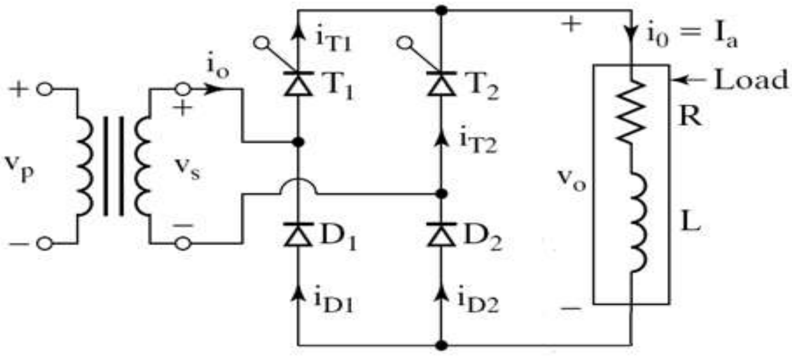


Fig.2.20: Single Phase Semi-converter with RL-Load

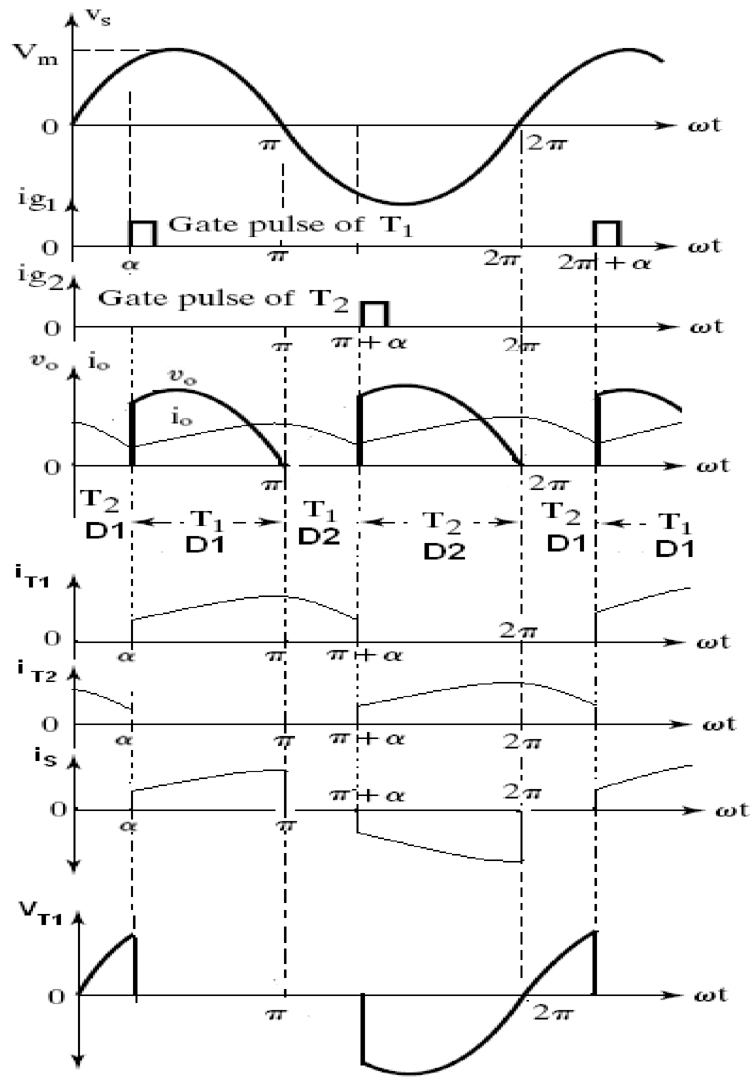


Fig.2.21: Related wave forms of a 1-Φ semi-converter with RL- load (For continuous load current)

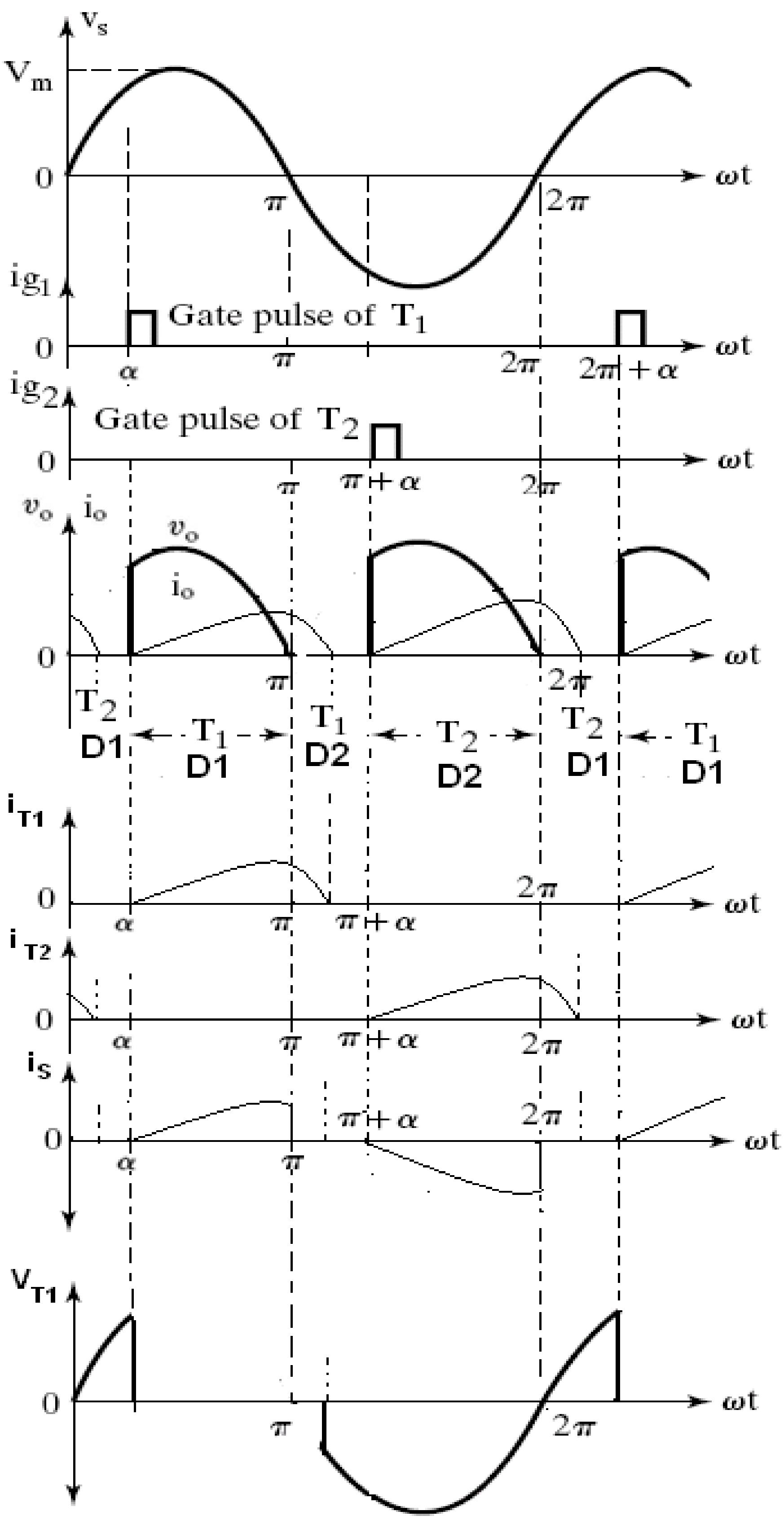


Fig.2.22: wave forms of a 1-Φ semi-converter with RL- load (For discontinuous load current)

Average or DC output voltage of a single phase semi-converter

The average output voltage can be found from

$$V_{dc} = \frac{2}{2\pi}\int_{\alpha}^{\pi} V_m \sin\omega t.d(\omega t) \quad => \quad V_{dc} = \frac{2V_m}{2\pi}\left[-\cos\omega t\right]_{\alpha}^{\pi}$$

$$V_{dc} = \frac{V_m}{\pi}\left[-\cos\pi + \cos\alpha\right] \;;\; \cos\pi = -1$$

Therefore $$V_{dc} = \frac{V_m}{\pi}\left[1+\cos\alpha\right]$$

V_{dc} Can be varied from $\frac{2V_m}{\pi}$ to 0 by varying α from 0 to π .

RMS output voltage of a single phase semi-converter

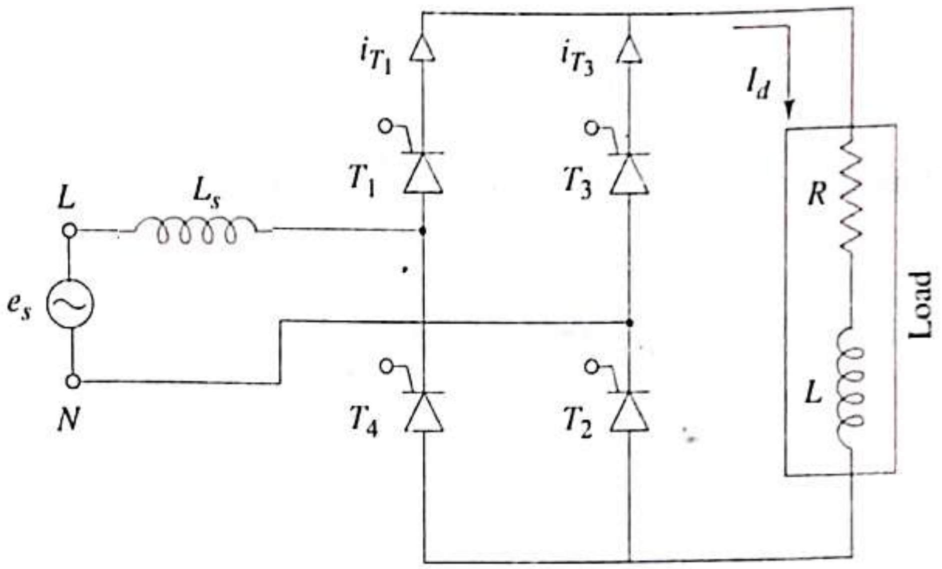
The rms output voltage is found from

$$V_{O(RMS)} = \left[\frac{2}{2\pi}\int_{\alpha}^{\pi} V_m^2 \sin^2\omega t.d(\omega t)\right]^{\frac{1}{2}}$$

$$V_{O(RMS)} = \frac{V_m}{\sqrt{2}}\left[\frac{1}{\pi}\left(\pi - \alpha + \frac{\sin 2\alpha}{2}\right)\right]^{\frac{1}{2}}$$

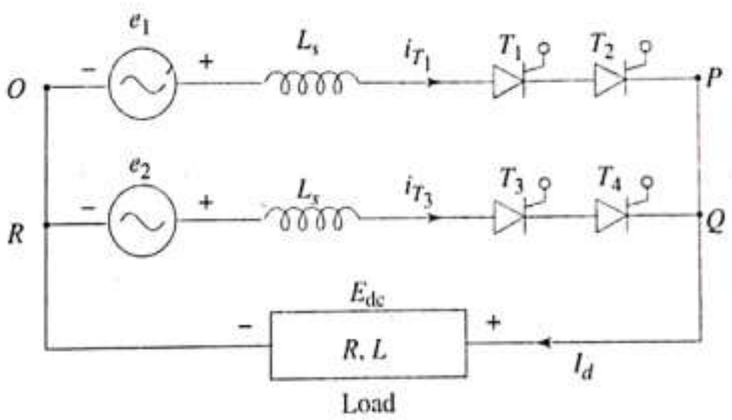
Source Inductance effect on 1-Φ full Bridge controlled -converter

The single phase full bridge converter with source inductance is as shown in below figure.

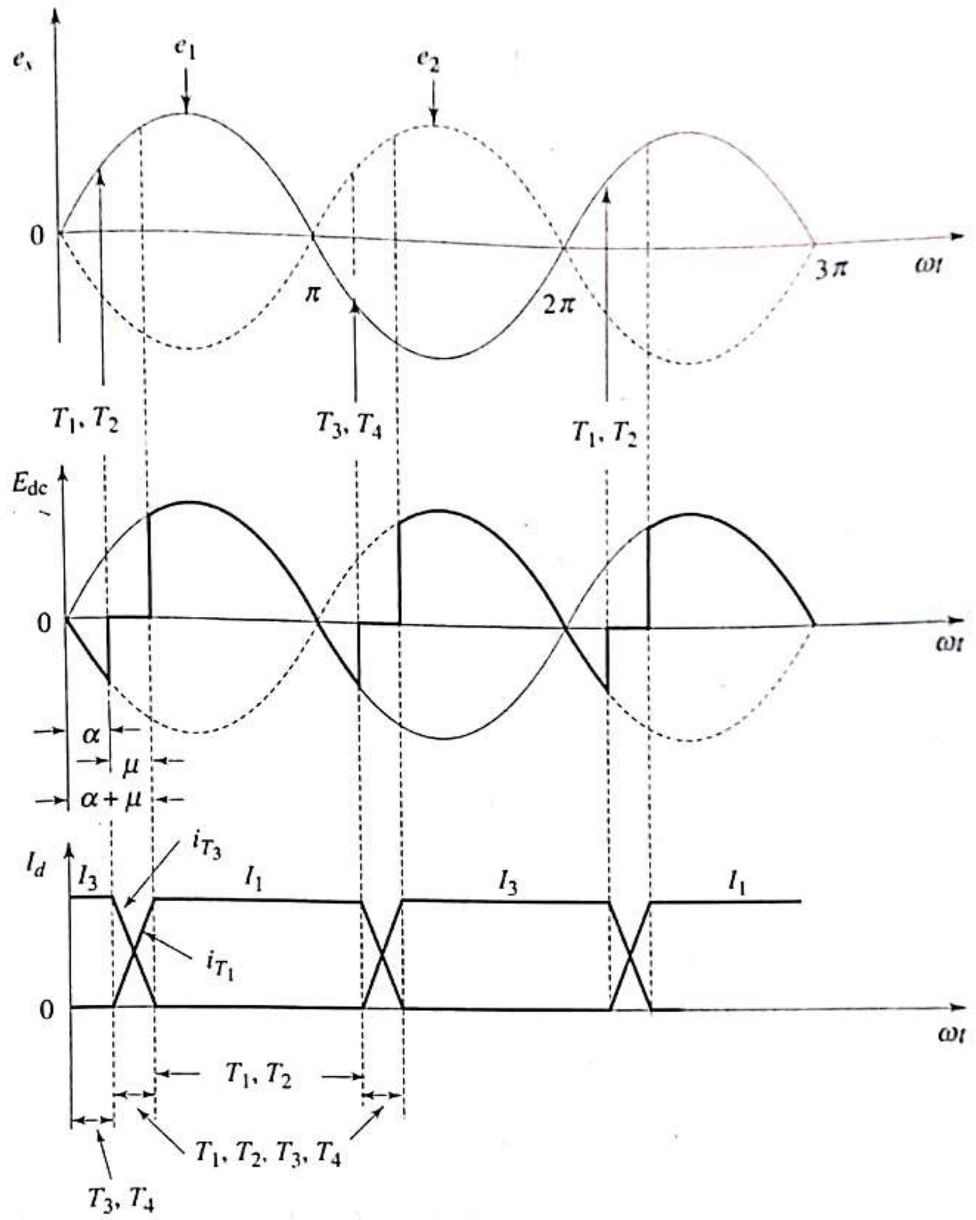


Single-phase fully-controlled converter with source inductance L_s

the equivalent circuit is as shown in below figure.



Equivalent circuit



Voltage and current-waveforms with L_s

Form the KVL equations of the equivalent circuit

$$e_1 - L_s \frac{d_{iT_1}}{dt} = e_2 - L_s \frac{d_{i_3}}{dt}$$

$$e_1 - e_2 = L_S \left(\frac{d_{iT_1}}{dt} - \frac{d_{i_3}}{dt} \right) \quad \text{-------------- (1)}$$

From wave forms

$$e_1 = E_m \sin \omega t$$

and

$$e_2 = - E_m \cdot \sin \omega t.$$

Then, the equation(1) becomes

$$L_s \left(\frac{d_{iT_1}}{dt} - \frac{d_{i_3}}{dt} \right) = 2E_m \times \sin \omega t \quad \text{--------------(2)}$$

The load current is

$$i_{T_1} + i_{T_3} = I_d \quad \text{--------------(3)}$$

Differentiating with respect to ‘t’ and we get

$$\frac{d_{iT_1}}{dt} = - \frac{d_{iT_3}}{dt} \quad \text{--------------(4)}$$

Substitute in equation (2)

$$L_s \left(2 \frac{d_{iT_1}}{dt} \right) = 2\, E_m \cdot \sin \omega t$$

$$\therefore \quad \frac{d_{iT_1}}{dt} = \frac{E_m}{L_s} \sin \omega t \quad \text{----------- (5)}$$

If the overlap angle is ‘µ’, then the current through T_1T_2 pair builds up from zero to I_d during this interval.

at $\omega t = \alpha, i_{T_1} = 0$ and

at $\omega t = (\alpha + \mu), i_{T_1} = I_d$

Therefore the equation (5) can be write as

$$\int_0^{I_d} d_{iT_1} = \frac{E_m}{L_s} \int_{\alpha/\omega}^{(\alpha+\mu)/\omega} \sin \omega t \, d(\omega t)$$

$$\therefore \quad I_d = \frac{E_m}{\omega L_s} [\cos \alpha - \cos(\alpha + \mu)] \quad \text{---------(6)}$$

From waveforms the average output voltage is given by

$$E_{dc} = \frac{E_m}{\pi} \int_{\alpha+\mu}^{\pi+\alpha} \sin \omega t \, d(\omega t)$$

$$= \frac{E_m}{\pi} [-\cos \omega t]_{\alpha+\mu}^{\pi+\alpha}$$

$$= \frac{E_m}{\pi} [\cos(\alpha+\mu) - \cos(\alpha+\pi)]$$

$$= \frac{E_m}{\pi} [\cos \alpha + \cos(\alpha+\mu)] \quad \text{-----------(7)}$$

Now, from equation (6)

$$\cos(\alpha+\mu) = \cos \alpha - \frac{\omega L_s}{E_m} I_d$$

Substituting this in equation (7) and we get

$$E_{dc} = \frac{E_m}{\pi} \left[\cos \alpha + \cos \alpha - \frac{\omega L_s}{E_m} I_d \right]$$

$$E_{dc} = \frac{2E_m}{\pi} \cos \alpha - \frac{\omega L_s}{\pi} I_d \quad \text{-----------(8)}$$

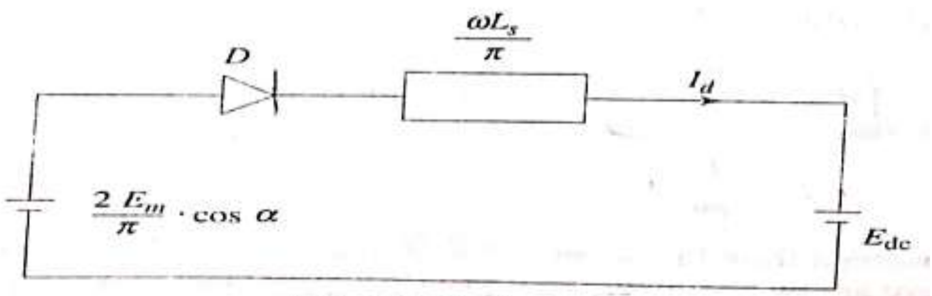
Also from equation (6), we get

$$\cos \alpha = \frac{\omega L_s}{E_m} I_d + \cos(\alpha+\mu)$$

Substitute this in equation (7) and we get

$$E_{dc} = \frac{2E_m}{\pi} \cos(\alpha+\mu) + \frac{\omega L_s}{\pi} I_d \quad \text{-----------(9)}$$

With the help of equation (8), a DC equivalent circuit of two pulse single phase controlled rectifier is as shown in figure below



D.C. equivalent circuit