6. UNSYMMETRICAL FAULT ANALYSIS

6.1 INTRODUCTION

The majority of faults on the power system are of unsymmetrical nature; the most common type being a short-circuit from one line to ground. When such a fault occurs, it gives rise to unsymmetrical currents *i.e.* the magnitude of fault currents in the three lines are different having unequal phase displacement. The method of symmetrical components is used to determine the currents and voltages on the occurrence of an unsymmetrical fault. In this chapter, we shall focus our attention on the analysis of unsymmetrical faults.

6.2 UNSYMMETRICAL FAULTS ON 3-PHASE SYSTEM

The faults on the power system which give rise to unsymmetrical fault currents (i.e. unequal fault currents in the lines with unequal phase displacement) are known as **unsymmetrical faults.**

On the occurrence of an unsymmetrical fault, the currents in the three lines become unequal and so there is a phase displacement among them. There are three ways in which unsymmetrical faults may occur in a power system (see Fig. 6.1).

(*i*) Single line-to-ground fault (L — G)

(*ii*) Line-to-line fault (*L* — *L*)

(*iii*) Double line-to-ground fault (*L* — *L* — *G*)

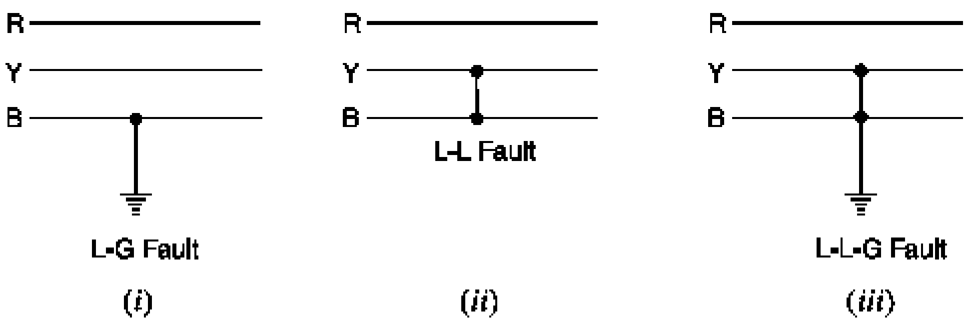


Fig.(6.1)

The solution of unsymmetrical fault problems can be obtained by using the following two methods

(*a*) Kirchhoff's laws

(*b*) Symmetrical components method.

The latter method is preferred because of the following reasons

(*i*) It is a simple method and gives more generality to be given to fault performance studies.

(*ii*) It provides a useful tool for the protection engineers, particularly in connection with tracing out of fault currents.

6.3 SYMMETRICAL COMPONENT METHOD

The analysis of unsymmetrical poly phase network by the method of symmetrical components was introduced by Dr. C.L. Fortescue in 1918, an American scientist. According to Fortescue theorem any unbalanced system of 3-phase currents ,voltages or other sinusoidal quantities can be resolved into three balanced system of vectors, which are called symmetrical components (i.e in general an unbalanced system of n related vectors can be resolved into n system of balanced vectors called symmetrical components of original vector.)

The symmetrical components of a 3-phase system are classified as

1. **Positive sequence components:** The positive sequence components consists of three vectors equal in magnitude, displaced from each other by 120^0 in phase, and having the same phase sequence as the original vectors.
2. **Negative sequence components:** The negative sequence components consists of three vectors equal in magnitude, displaced from each other by 120^0 in phase, and having the phase sequence opposite to that of the original vectors.
3. **Zero sequence components:** The zero sequence components consists of three vectors equal in magnitude and with zero phase displacement from each other.

The vector diagram of positive, negative, zero sequence components are shown in the following fig.6.2.

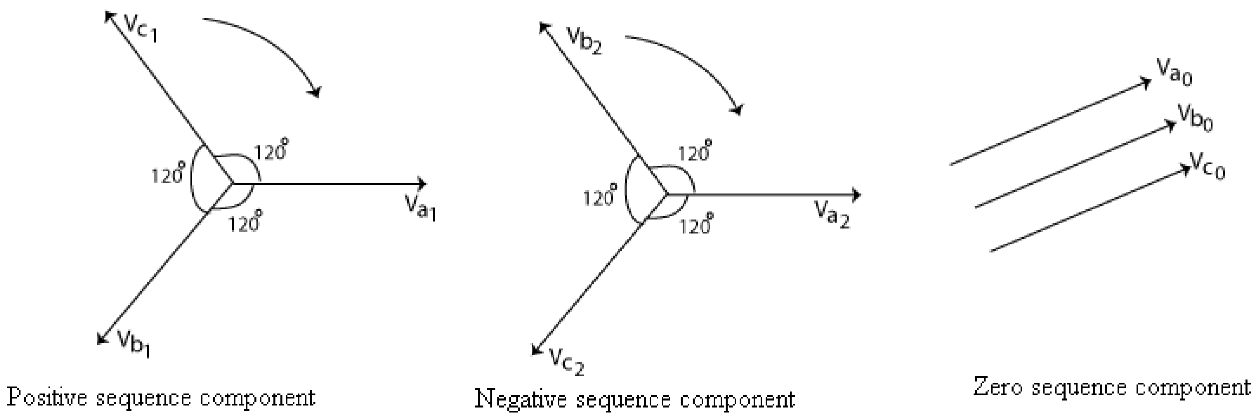


Fig.(6.1)

6.4 SYMMETRICAL COMPONENT TRANSFORMATION

The operator "a" is defined as

$a=1\angle 120 = \cos 120 + j\sin 120 = -0.5+j0.866$

$a^2 = 1\angle -120 = \cos 120 - j\sin 120 = -0.5 - j0.866$

$a^3 = 1\angle 360 = 1, \quad 1 + a + a^2 = 0$

The symmetrical components for voltages are derived as follows

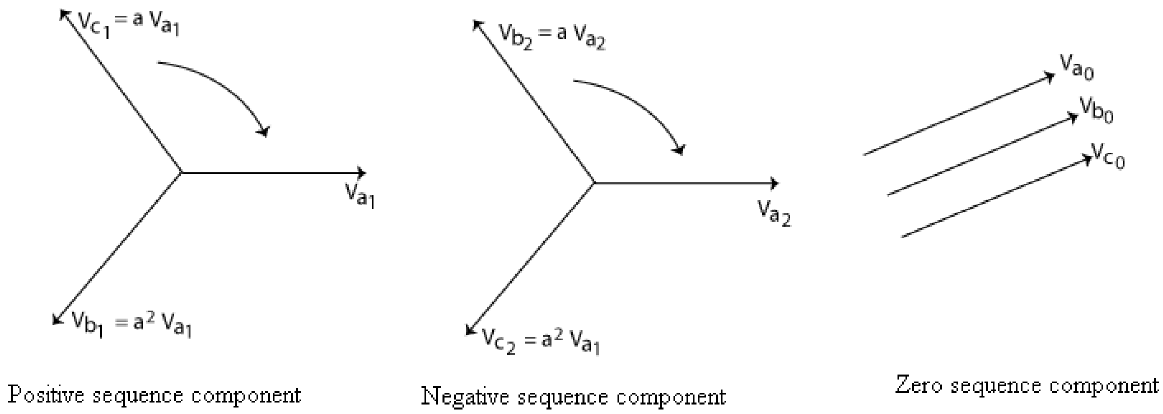


Fig.(6.2)

Each of the original unbalanced vector is the sum of its positive, negative and zero sequence components. Therefore the original unbalanced 3 phase voltage vectors can be expressed in terms of their symmetrical components as given below

$V_a = V_{a0} + V_{a1} + V_{a2}$

$V_b = V_{b0} + V_{b1} + V_{b2} = V_{a0} + a^2 V_{a1} + aV_{a2}$

$V_c = V_{c0} + V_{c1} + V_{c2} = V_{a0} + aV_{a1} + a^2 V_{a2}$

$$\begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} . \begin{bmatrix} V_{a0} \\ V_{a1} \\ V_{a2} \end{bmatrix} \quad \text{--- (6.1)}$$

$$\Rightarrow V_{abc} = AV_{012} \quad \text{--- (6.2)}$$

$$\text{where } A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix}$$

$\Rightarrow A^{-1} V_{abc} = A^{-1} A V_{012}$

$\Rightarrow A^{-1} V_{abc} = I\, V_{012}$

$\Rightarrow V_{012} = A^{-1} V_{abc}$

$$|A| = \begin{vmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{vmatrix} = (a^4 - a^2) - (a^2 - a) + (a - a^2) = 3(a - a^2)$$

$$\text{adj A}= \begin{bmatrix} a-a^2 & a-a^2 & a-a^2 \\ a-a^2 & a(a-a^2) & a^2(a-a^2) \\ a-a^2 & a^2(a-a^2) & a(a-a^2) \end{bmatrix}^T = \begin{bmatrix} a-a^2 & a-a^2 & a-a^2 \\ a-a^2 & a(a-a^2) & a^2(a-a^2) \\ a-a^2 & a^2(a-a^2) & a(a-a^2) \end{bmatrix}$$

$$A^{-1} = \frac{AdjA}{|A|} = \frac{1}{3(a-a^2)} \begin{bmatrix} a-a^2 & a-a^2 & a-a^2 \\ a-a^2 & a(a-a^2) & a^2(a-a^2) \\ a-a^2 & a^2(a-a^2) & a(a-a^2) \end{bmatrix}$$

$$= \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} = \frac{1}{3} A^*$$

We know that

$$V_{012} = A^{-1} V_{abc} \qquad \text{--- (6.3)}$$

$$\begin{bmatrix} V_{a0} \\ V_{a1} \\ V_{a2} \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} = \frac{1}{3} \begin{bmatrix} V_a + V_b + V_c \\ V_a + aV_b + a^2V_c \\ V_a + a^2V_b + aV_c \end{bmatrix}$$

$$\left. \begin{aligned} V_{a0} &= \frac{1}{3}(V_a + V_b + V_c) \\ V_{a1} &= \frac{1}{3}(V_a + aV_b + a^2V_c) \\ V_{a2} &= \frac{1}{3}(V_a + a^2V_b + aV_c) \end{aligned} \right\} \qquad \text{--- (6.4)}$$

Similarly, the symmetrical components for currents

$$\left. \begin{aligned} I_{a0} &= \frac{1}{3}(I_a + I_b + I_c) \\ I_{a1} &= \frac{1}{3}(I_a + aI_b + a^2I_c) \\ I_{a2} &= \frac{1}{3}(I_a + a^2I_b + aI_c) \end{aligned} \right\} \qquad \text{--- (6.5)}$$

The above equation can be written in matrix form as given below

$$\begin{bmatrix} V_{a0} \\ V_{a1} \\ V_{a2} \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix}$$

$$\begin{bmatrix} I_{a0} \\ I_{a1} \\ I_{a2} \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix}$$

Certain observations can be made regarding a 3 phase system with neutral as shown in the following figure:

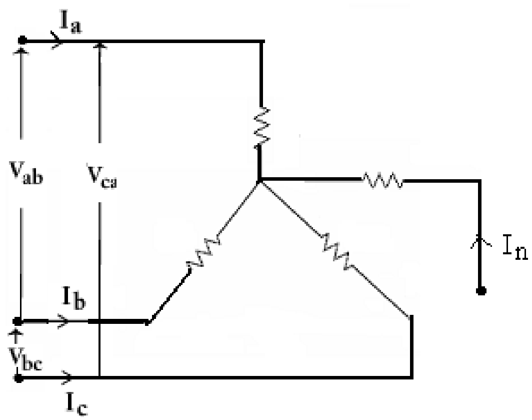


Fig.(6.3)

The sum of the three line voltages will always be zero. Therefore, the zero sequence component of line voltage is always zero.

i.e $V_{ab0} = \frac{1}{3}(V_{ab} + V_{bc} + V_{ca}) = 0$ --- (6.6)

On the other hand the sum of the phase voltages (line to neutral) may not be zero so that their zero sequence component V_{a0} may exist.

Since the sum of the three line currents equals the current in the neutral wire , we have

$$I_{a0} = \frac{1}{3}(I_a + I_b + I_c) = \frac{1}{3}(I_n)$$

i.e the current in the neutral is three times the zero sequence line current.

If the neutral connection is severed

$$I_{a0} = \frac{1}{3}(I_n) = 0$$

i.e in the absence of a neutral connection the zero sequence line current is always zero.

we know that

$V_{abc} = Z_{abc} I_{abc}$

From eq.(6.2) $V_{abc}=AV_{012}$ & $I_{abc}=AI_{012}$

$AV_{012}= Z_{abc} A I_{012}$

$A^{-1}AV_{012}=A^{-1}Z_{abc} AI_{012}$

$I V_{012}= (A^{-1}Z_{abc}A) I_{012}$

$V_{012}= (A^{-1}Z_{abc} A) I_{012}$

$V_{012}=Z_{012} I_{012}$

$Z_{012}=A^{-1} Z_{abc} A= \frac{1}{3} A^{*}Z_{abc} A$ --- (6.7)

6.5 POWER INVARIANCE

In a 1-Φ system volt amperes S is given by

$S_{1\Phi} = P + jQ = VI^{*}$

In a 3- Φ system volt amperes S is given by

$S_{3\Phi} = S_{abc} = V_a I_a^{*} +V_b I_b^{*} +V_c I_c^{*}$

$$= \begin{bmatrix} V_a & V_b & V_c \end{bmatrix} \begin{bmatrix} I_a^* \\ I_b^* \\ I_c^* \end{bmatrix} = \begin{bmatrix} V_a & V_b & V_c \end{bmatrix} \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix}^*$$

$$= \begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix}^T . \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix}^* = V_{abc}^{T} . I_{abc}^{*}$$

we know that

$V_{abc}= A V_{012}$

$I_{abc}= A I_{012}$

There fore

$S_{abc}= [AV_{012}]^{T} [A I_{012}]^{*}$

$= V_{012}^{T} A^{T} A^{*} I_{012}^{*}$

$= V_{012}^{T} 3I I_{012}^{*}= 3 V_{012} I_{012}^{*} = 3 S_{012}$

$$=3\times\begin{bmatrix} V_{a0} & V_{a1} & V_{a2} \end{bmatrix} \begin{bmatrix} I_a^* \\ I_b^* \\ I_c^* \end{bmatrix} =3\times [V_{ao} I_{ao}^{*} V_{a1} I_{a1}^{*} V_{a2} I_{a2}^{*}]$$

i.e $S_{abc} = 3V_{ao} I_{ao}^{*} + 3V_{a1} I_{a1}^{*} + 3V_{a2} I_{a2}^{*}$ ---.(6.8)

The above eqn.(6.8) shows that the total complex power in the unbalanced system is equal to the sum of the complex power of three symmetrical components. Hence we can say that the symmetrical component transformation is power invariant.

Problem -1 : The voltage across a 3-Φ unbalanced loads are $V_a= 300 \angle 20^0$,$V_b= 360 \angle 90^0$ and $V_c = 500\angle -140^0$ V. Determine the symmetrical components of voltages. Take phase sequence as ABC

Solution: Given that

$V_a = 300\angle 20^0 = 281.91 + j 102.6$

$V_b= 360\angle 90^0 = 0+ j 36$

$V_c = 500\angle -140^0 = 383.02 - j\,321.89$

$aV_b = 1\angle 120 \times 360\angle 90 = 360\angle 210 = -311.77 - j\,180$

$a^2V_b = 1\angle 240 \times 360\angle 90 = 360\angle 330 = 311.77 - j\,180$

$a\,V_c = 1\angle 120 \times 500\angle -140 = 500\angle -20 = 470 - j\,171$

$a^2\,V_c = 1\angle 240 \times 500\angle -140 = 500\angle 100 = -86.86 + j\,472.4$

The symmetrical components of phase-A are given by

$$V_{ao} = \frac{1}{3}[V_a + V_b + V_c]$$

$$= \frac{1}{3}\,[281.91 + j\,102.61 + j360 - 33 - j\,321.4]$$

$$= -33.7 + j\,47.07 = 57.89\ \angle 126V$$

$$V_{a1} = \frac{1}{3}\,[\,V_a + aV_b + a^2\,V_c\,]$$

$$= \frac{1}{3}\,[281.91 + j\,102.6 - 311.77 - j\,180 - 86.82 + j\,492.4]$$

$$= -38.89 + j\,138.34 = 143.\,7\ \angle 106\ V$$

$$V_{a2} = \frac{1}{3}\,[V_a + a^2\,V_b + a\,V_c]$$

$$= \frac{1}{3}\,[281.91 + j\,102.61 + 311.77 - j\,180 + 470 - j\,171.01]$$

$$= 354.57 - j\,82.8 = 364.05\ \angle -13V$$

We know that $V_{ao} = V_{bo} = V_{co}$

The zero sequence components are

$V_{ao} = V_{bo} = V_{co} = 57.89\ \angle 126\ V$

The positive sequence components are

$V_{a1} = 143.7\angle 106\ V$

$V_{b1} = a^2\,V_{a1} = 1\angle 240 \times 143.7\ \angle 106 = 143.7\ \angle 346V$

$V_{c1} = a\,V_{a1} = 1\angle 120 \times 143.7\ \angle 106 = 143.7\ \angle 226V$

The negative sequence components are

$V_{a2} = 364.05\angle -13\ V$

$V_{b2} = a\,V_{a2} = 1\angle 120 \times 364.05\angle -13 = 364.05\angle 107V$

$V_{c2} = a^2\,V_{a2} = 1\angle 240 \times 364.05\ \angle -13 = 364.05\ \angle 227V$

Problem-2: The symmetrical components of phase A voltage in a 3-Φ unbalanced system are $V_{ao}=10\angle 180V$, $V_{a1} = 50 \angle 0V$, $V_{a2} = 20\angle 90$ V. Determine the phase voltages V_a, V_b and V_c.

Solution: Given that

$V_{ao}=10\angle 180V$, $V_{a1} = 50 \angle 0V$, $V_{a2} = 20\angle 90$ V

The phase voltages are given by

$V_a = V_{a0}+ V_{a1}+ V_{a2}$

$=10\angle 180 +50\angle 0 +20\angle 90$

$= -10+50+j20$

$= 40+j20$

$V_b = V_{a0}+ a^2 V_{a1}+ aV_{a2}$

$=10\angle 180 +1\angle -120\times 50 \angle 0 +1\angle 120\times 20\angle 90$

$= -10+50\angle -120+20\angle 210$

$= -10 -25-j43.3-17.3-j10$

$= -52.3-j53$

$V_c = V_{a0}+ aV_{a1}+ a^2 V_{a2}$

$=10\angle 180 +1\angle 120\times 50 \angle 0 +1\angle -120\times 20\angle 90$

$= -10 +50\angle 120 +20\angle -30$

$= -10 -25+j43.3+17.3-j10$

$= -17.7 +j33.3$

Problem-3: One conductor of a three phase line is open. The current flowing through the line A is 10A. Assuming line B is open, find symmetrical components of line currents?

Solution: Let us assume, I_a as reference phasor

$I_a = 10 \angle 0$ then $I_c = 10 \angle 180$, $I_b = 0$

Symmetrical components of line-A currents are

$$I_{a0} = \frac{1}{3}\left(I_a + I_b + I_c\right) = 0A$$

$$I_{a1} = \frac{1}{3}\left(I_a + aI_b + a^2 I_c\right) = \frac{1}{3}(10+0+1\angle -120\times 10\angle 180)$$

$$= \frac{1}{3}(10+10\angle 60) = 5.77\angle 30A$$

$$I_{a2} = \frac{1}{3}\left(I_a + a^2 I_b + aI_c\right) = \frac{1}{3}(10+0+1\angle 120\times 10\angle 180)$$

$$= \frac{1}{3}(10 + 10\angle 300 = 5.77\angle -30A$$

Symmetrical components of line-B currents are

$I_{b0} = I_{a0} = 0$ A

$I_{b1} = a^2 I_{a1} = 1\angle -120 \times 5.77\angle 30 = 5.77\angle -90$

$I_{b2} = aI_{a1} = 1\angle 120 \times 5.77\angle 30 = 5.77\angle 150$

Symmetrical components of line-C currents are

$I_{c0} = I_{a0} = 0$ A

$I_{c1} = aI_{a1} = 1\angle 120 \times 5.77\angle 30 = 5.77\angle 150$

$I_{b2} = a^2 I_{a1} = 1\angle -120 \times 5.77\angle 30 = 5.77\angle -90$

6.6 SEQUENCE IMPEDANCES

The sequence impedances are the impedances offered by the circuit elements (or power system components) to the flow of sequence currents. The sequence impedances of an equipment or a component of a power system are the positive, negative and zero sequence impedances. They are defined as follows.

The positive sequence impedance of an equipment is the impedance offered by the equipment to the flow of positive sequence currents. Similarly the negative and zero sequence impedance of an equipment is the impedance offered by the equipment to the flow of corresponding sequence currents. Let us represent positive, negative and zero sequence impedances respectively by Z_0 , Z_1 ,Z_2, Z_0

6.7 SEQUENCE NETWORK EQUATIONS

The sequence network equations will be derived for an unloaded alternator with neutral solidly grounded (as shown in fig.6.4) and by assuming that the system is balanced. When an unsymmetrical fault occurs on the generator terminals, unbalanced currents I_a, I_b and I_c will flow as shown in the figure. These currents can be resolved into their symmetrical components by drawing the sequence network of the generator.

Let V_a, V_b, V_c be the generated voltages and V_{a0}, V_{a1}, V_{a2} be the zero, positive and negative sequence voltages of phase 'a' respectively.

$V_b = a^2 V_a$

$V_c = a V_a$

$$V_{a0} = \frac{1}{3}(V_a + V_b + V_c) = \frac{1}{3}(V_a + a^2 V_a + a V_a) = \frac{V_a}{3}(1 + a^2 + a) = 0$$

i.e $V_{a0} = 0$

$V_{a1} = \frac{1}{3}(V_a + a^2 V_c + a V_b) = \frac{1}{3}(V_a + a^3 V_a + a^3 V_a) = \frac{V_a}{3}(1 + 1 + 1) = V_a$

i.e $V_{a1} = V_a$ and

$V_{a2} = \frac{1}{3}(V_a + a^2 V_b + a V_c) = \frac{1}{3}(V_a + a^4 V_a + a^2 V_a) = \frac{V_a}{3}(1 + a + a^2) = 0$

i.e $V_{a2} = 0$

From these relations it is observed that a symmetrically designed generator generates only positive sequence voltage. The zero and negative sequence generated voltages are zero.

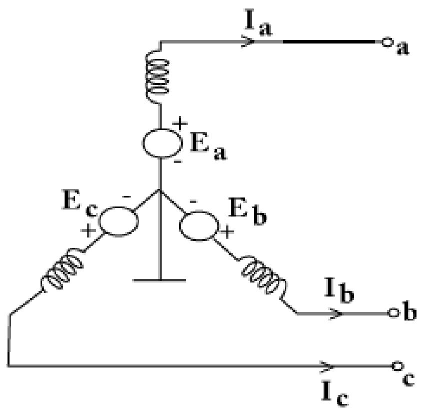


Fig.(6.4): An unloaded generator

There fore, the three sequence network equations are

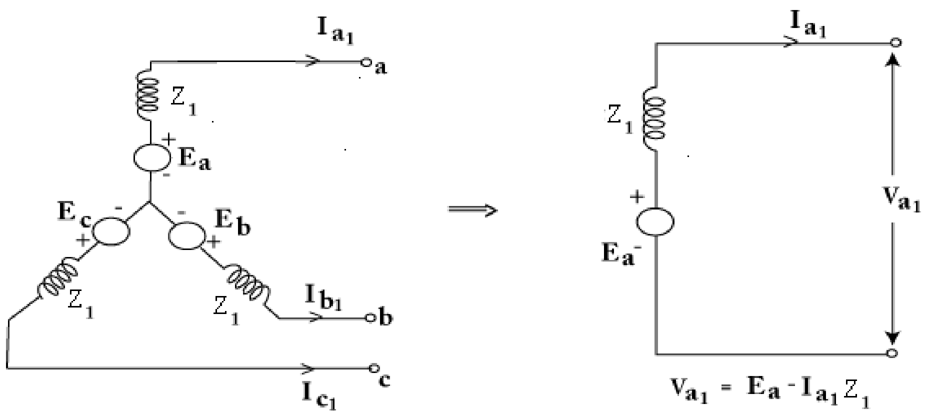
$V_{a1} = E_a - I_{a1} Z_1$

$V_{a2} = -I_{a2} Z_2$

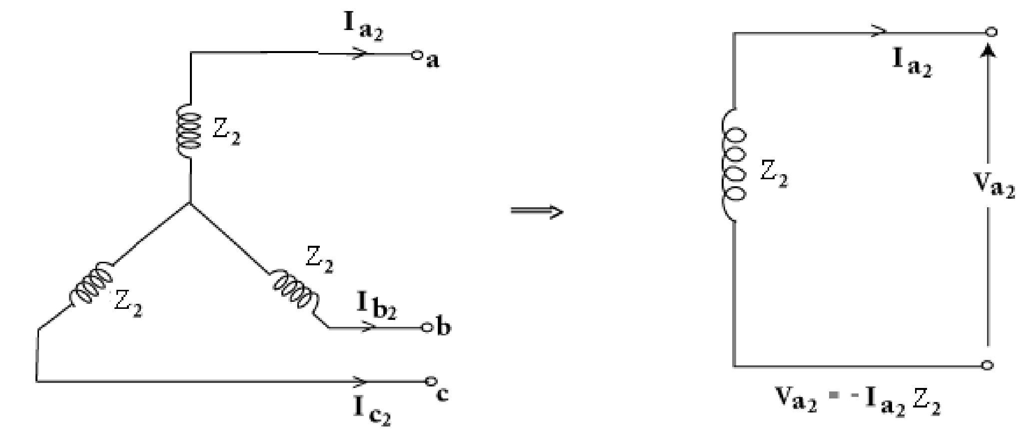
$V_{a0} = -I_{a0} Z_0$

The sequence network equations can be written in matrix form as given below

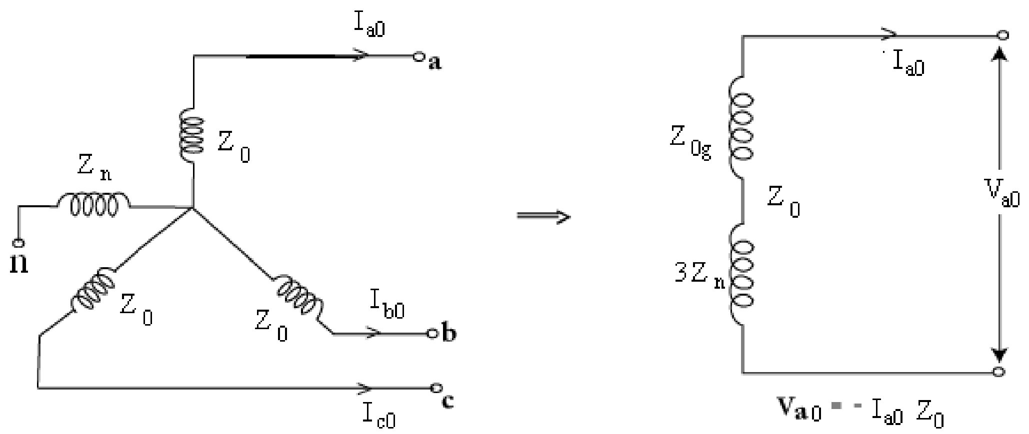
$$\begin{bmatrix} V_{a0} \\ V_{a1} \\ V_{a2} \end{bmatrix} = \begin{bmatrix} 0 \\ E_a \\ 0 \end{bmatrix} - \begin{bmatrix} Z_0 & 0 & 0 \\ 0 & Z_1 & 0 \\ 0 & 0 & Z_2 \end{bmatrix} \begin{bmatrix} I_{a0} \\ I_{a1} \\ I_{a2} \end{bmatrix} \quad \text{--- (6.9)}$$



(a) Positive sequence network



(b) Negative sequence network



(c) Zero sequence network

Fig.(6.5): Sequence networks of an un-loaded alternator

6.9 SEQUENCE NETWORKS

The 1-ϕ equivalent circuit (impedance or reactance diagram) formed by using the impedances of any one sequence only is called the sequence network for that particular sequence. The impedance or reactance diagram formed by using positive sequence impedance is called positive sequence network. Similarly the impedance diagrams formed by using negative and zero sequence are called negative sequence network and zero sequence network respectively . The positive sequence network consists of an emf in series with positive sequence impendence of generator . The negative and zero sequence networks will not have any sources but includes their respective sequence impedances.

6.10 ZERO SEQUENCE NETWORKS OF TRANSFORMERS

Before considering the zero sequence network of various types of transformer connections, three important observations are made.

i) If magnetizing currents is neglected, transformer primary would carry current only if there is a current flow on the secondary side.

ii) Zero sequence currents can flow in the legs of a star connection only if the star point is grounded, which provides the necessary returns path for zero sequence currents.

iii) No zero sequence current can flow in the lines connected to a delta connection, as no return path is available for these currents. Zero sequence currents can, however flow in the legs of a delta; such currents are caused by the present of zero sequence voltage in the delta connection.

Note: *When the neutral of star connection is grounded through reactance Z_n, then $3Z_n$ should be added to zero sequence impedance of transformer to get the total zero sequence impedan ce*

The zero sequence circuits of 3 ϕ transformers require special attention because of the possibility of various combinations. The general circuit for any combinations is given in the fig.6.20.

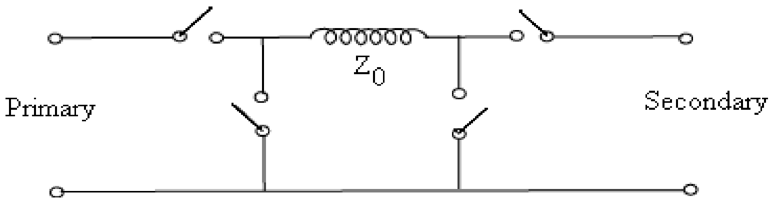
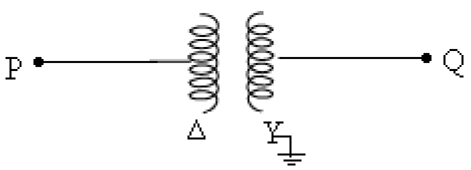
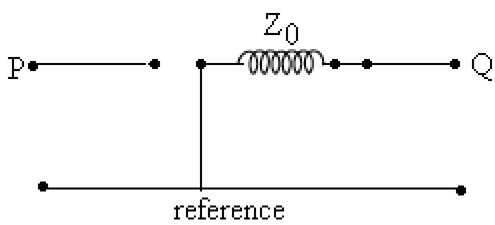


Fig.(6.6)

Here Z_0 is the zero sequence impedance of the winding of the transformer. There are two series and shunt switches. If we observe the location of the switches, one series and one shunt switch are for both sides separately. The series switch of a particular side is closed if it is star grounded and the shunt switch is closed if that side is delta connected, otherwise they are left open.

Example: Say the T/F is Δ/Y connected with star grounded. Since the primary is delta connected, the shunt switch of primary side is closed and the series switch is left open. The secondary is star grounded, therefore, the series switch is closed and the shunt switch is left open.





The zero sequence networks for all transformer connections are shown in fig.6.7

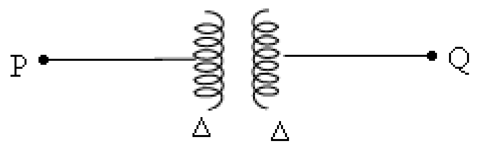
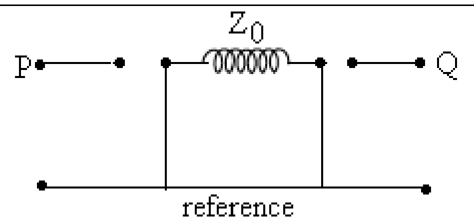
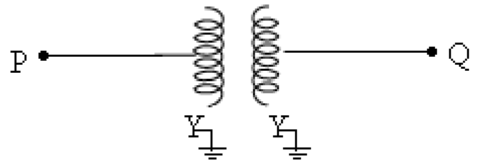
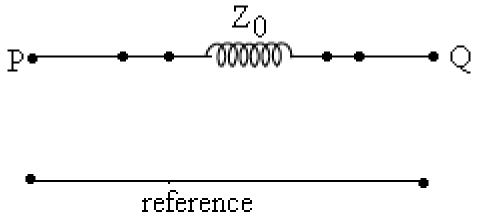
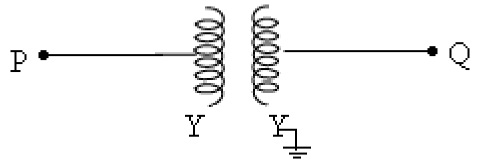
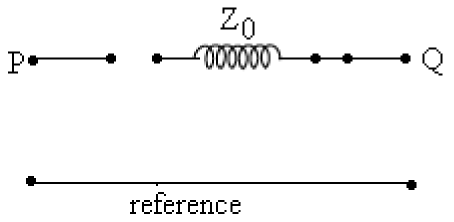
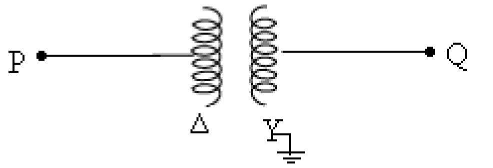
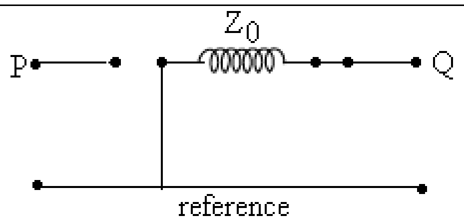
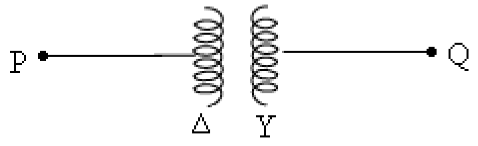
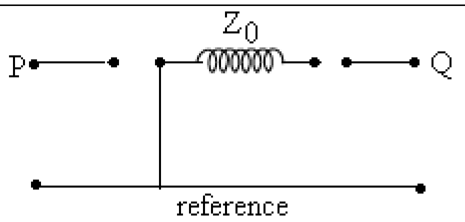
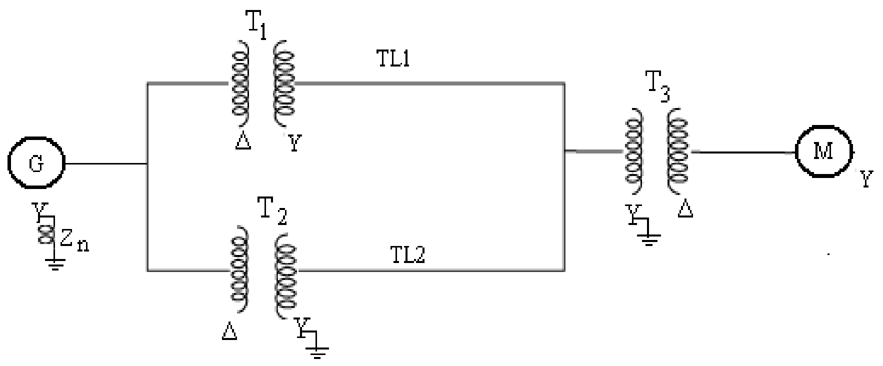
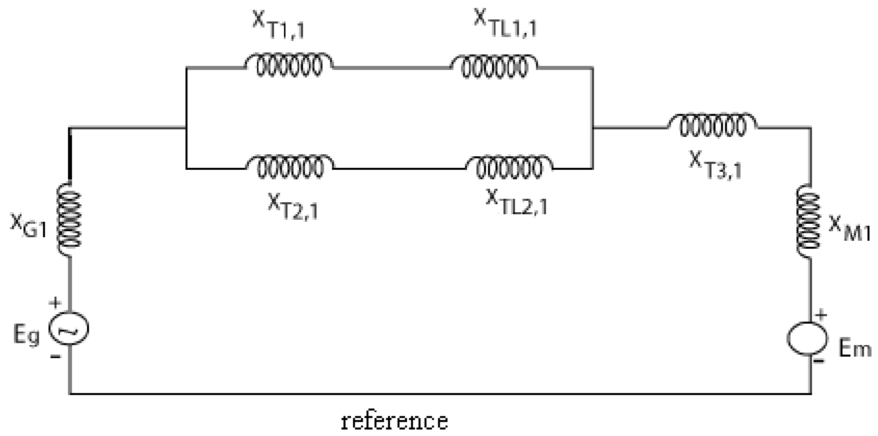
S.No	Winding symbol	Zero sequence equivalent circuit
1.		
2.		
3.		
4.		
5.		

Fig.(6.7)

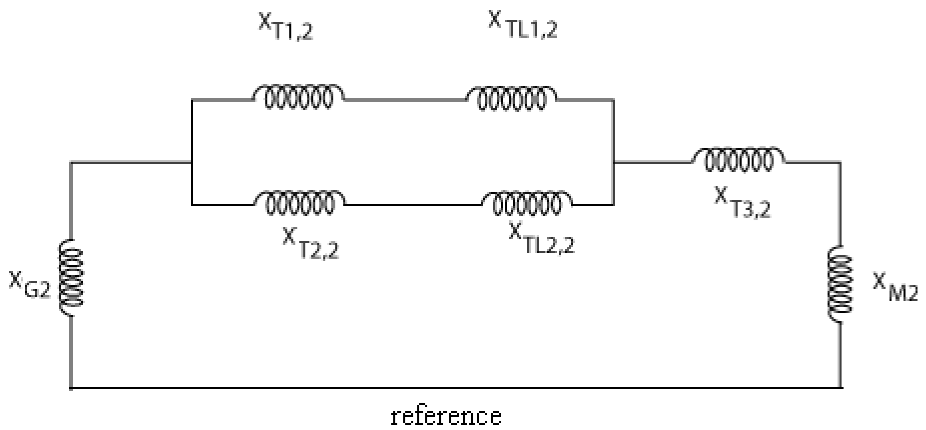
Problem-4: Draw the positive , negative and zero sequence networks of the power system shown in the figure?



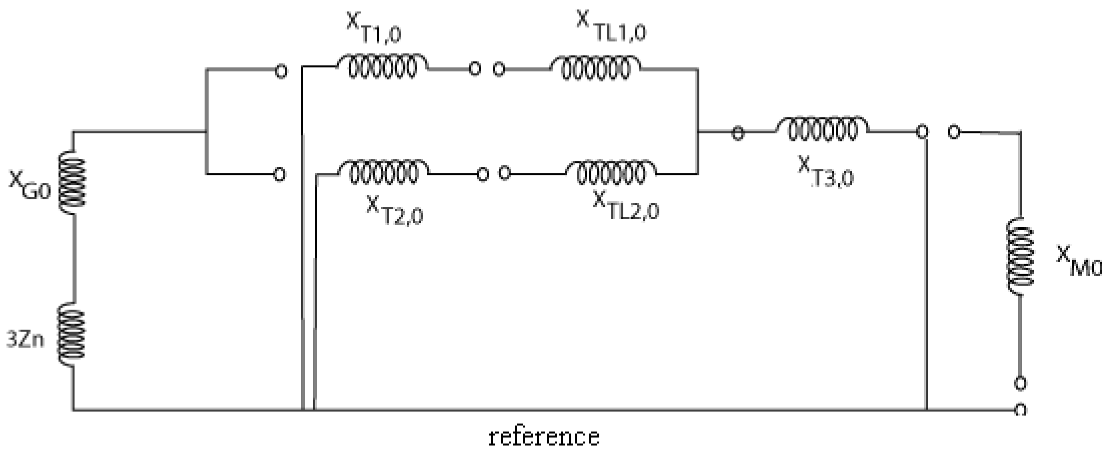
Solution: The positive sequence network is



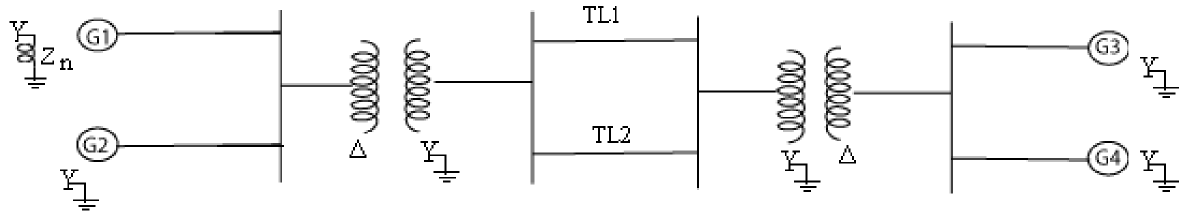
The negative sequence network is



The zero sequence network is



Problem-5: Draw the positive , negative and zero sequence networks of the power system shown in the figure?



The system data is

Generator G_1 : $X_1 = X_2 = 1.0,\ X_0 = 0.3,\ X_n = 0.2\,pu$

Generator G_2 : $X_1 = X_2 = 1.0,\ X_0 = 0.3$

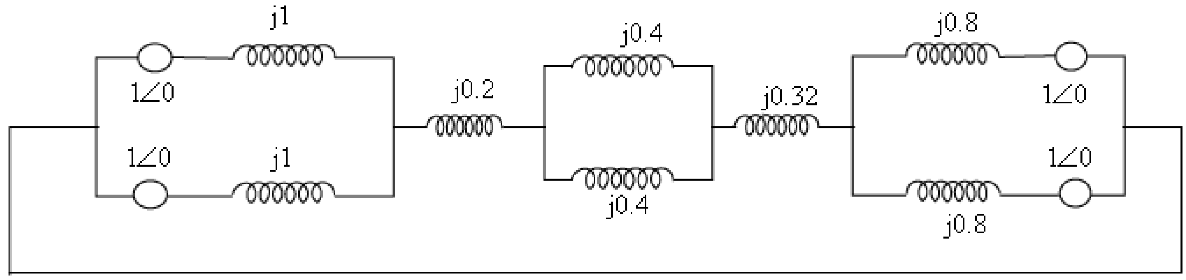
Generator G_3, G_4: $X_1 = X_2 = 0.8,\ X_0 = 0.2$

T/F T_1 : $X_1 = X_2 = X_0 = 0.2$

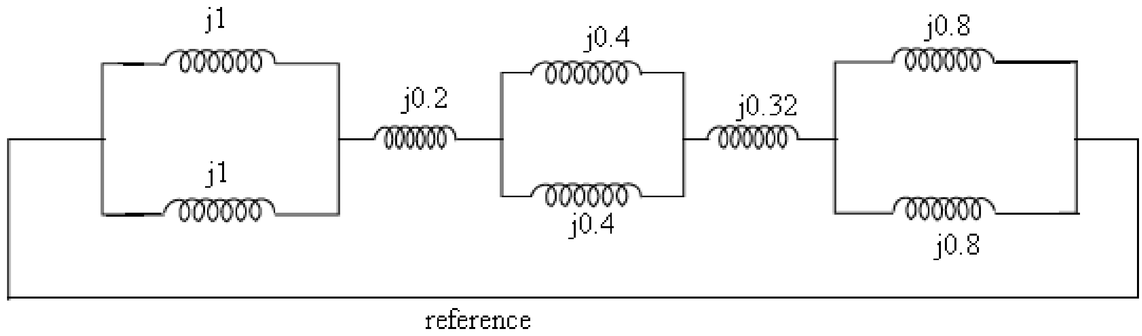
T/F T_2 : $X_1 = X_2 = X_0 = 0.32$

Lines (each) : $X_1 = X_2 = 0.4,\ X_0 = 0.5$

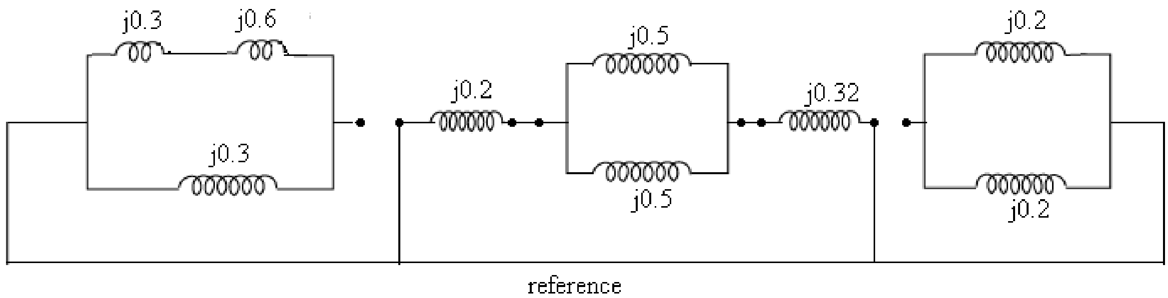
Solution: The positive sequence network is



The negative network is



The zero sequence network is



Problem-6: The following figure shows a power system network. Draw zero sequence network for this system.

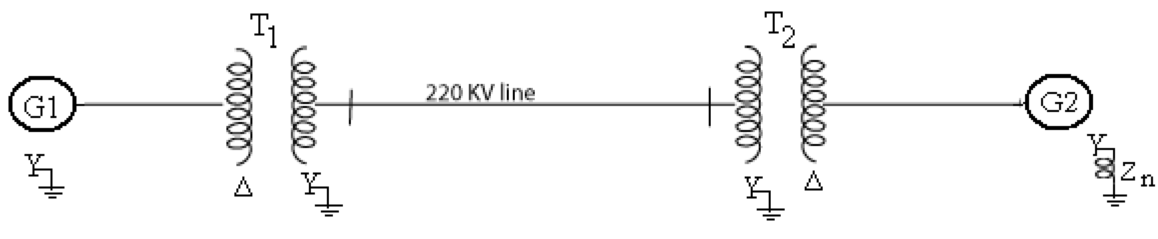
Generator G_1 : 50 MVA , 11kV, $X_0 = 0.08\ pu$

Transformer T_1 : 50 MVA, 11/220 kV, $X_0 = 0.1\ pu$

Generator G_2 : 30 MVA , 11kV, $X_0 = 0.01 pu, Z_n = j3\Omega$

Transformer T_2 : 30 MVA, 11/220 kV, $X_0 = 0.09 pu$

Zero sequence reactance of line is 555.6 Ω



Solution: Base MVA = 50MVA

Base voltage = 11kV for LT side and 220 kV for HT side of transformer T_1

Base impedance of line= $\dfrac{220 \times 220}{50} = 968\,\Omega$

pu impedance of line = $\dfrac{j555.6}{968} = j0.574$

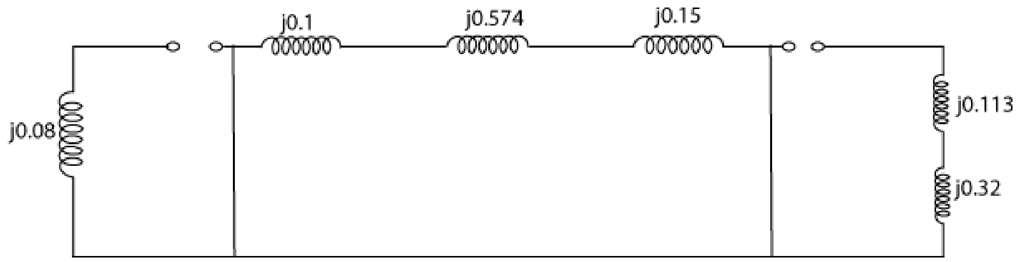
pu impedance of transformer, T_2= $j0.09 \times \frac{50}{30} = j0.15$

pu impedance of G_2 = $j0.07 \times \frac{50}{30} = j0.117$

Base impedance for generator = $j0.07 \times \frac{11 \times 11}{50} = 2.42\,\Omega$

pu impedance of neutral rector = $\frac{j3}{2.42} = j1.24$

The zero sequence network of the given power system network is



6.11 UNSYMMETRICAL FAULT CALCULATIONS

6.11.1 Line to Ground Fault (L-G)

Case (a): Without fault impedance

Let us assume an L-G fault occurs on phase-a.

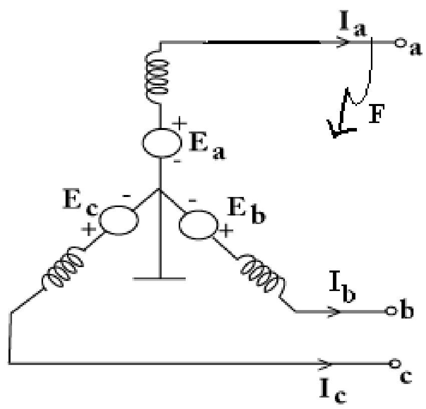


Fig.(6.8)

The boundary conditions are

$V_a = 0$; $I_b = 0$; $I_c = 0$ --- (6.10)

The fault current is

$$I_f = I_a \qquad \text{--- (6.11)}$$

The symmetrical components of currents are

$$\begin{bmatrix} I_{ao} \\ I_{a1} \\ I_{a2} \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix}\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} \qquad \text{--- (6.12)}$$

From eqn (6.10) & (6.12)

$$\begin{bmatrix} I_{a0} \\ I_{a1} \\ I_{a2} \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix}\begin{bmatrix} I_a \\ 0 \\ 0 \end{bmatrix} = \frac{1}{3}\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix}$$

$$I_{a0}=I_{a1}=I_{a2}= \frac{I_a}{3} \qquad \text{--- (6.13)}$$

The sequence network equations are given by

$$\begin{aligned} V_{a0} &= -I_{a0}Z_0 \\ V_{a1} &= E_a - I_{a1}Z_1 \\ V_{a2} &= -I_{a2}Z_2 \end{aligned} \qquad \text{--- (6.14)}$$

We know that from eqn (6.10)

$V_a=0$

$V_{a0}+V_{a1}+V_{a2}=0$

$-I_{a0}Z_0+E_a-I_{a1}Z_1-I_{a2}Z_2=0$

$I_{a1}(Z_0+Z_1+Z_2) = E_a$

$$I_{a1} = \frac{E_a}{Z_0+Z_1+Z_2} \qquad \text{--- (6.15)}$$

The fault current is

$I_f = I_a = I_{a0}+I_{a1}+I_{a2}=3I_{a1}$

$$I_f = I_a = 3I_{a1}= \frac{3E_a}{Z_0+Z_1+Z_2} \qquad \text{--- (6.16)}$$

In the case of line to ground fault, the neutral current is

$$I_n=I_a=3I_{a1} \qquad \text{--- (6.17)}$$

Using eqn (6.15), the equivalent circuit of generator during L-G fault is drawn as shown in the figure. Here, the positive, negative and zero sequence currents of the generator are connected in series. If the neutral of the generator is not grounded, the zero sequence network is open circuited and Z_0 is infinite. Under these conditions, I_{a0} is zero and so I_{a1} and I_{a2} must be zero. Therefore , no path exists for the flow of current in the fault unless the generator neutral is grounded.

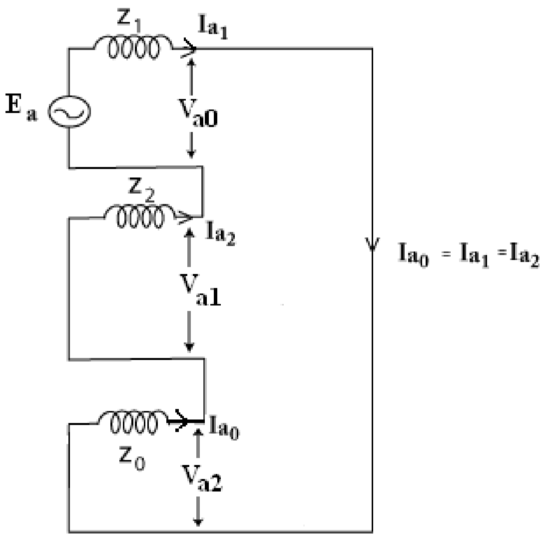


Fig.(6.9): The equivalent circuit of generator during L-G fault

Case (b): With fault impedance (Z_f)

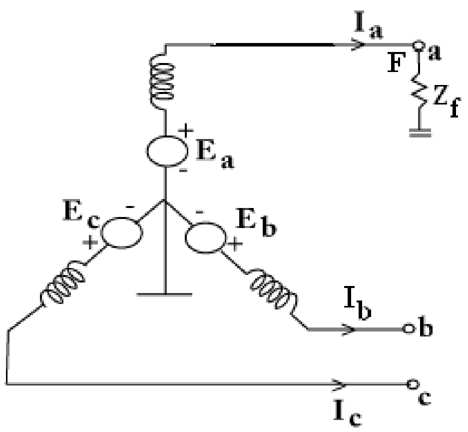


Fig.(6.10)

The boundary conditions are

$V_a = I_a Z_f$; $I_b=0$; $I_c=0$ --- (6.18)

The fault current is

$I_f = I_a$ --- (6.19)

The symmetrical components of the currents are

$$\begin{bmatrix} I_{a0} \\ I_{a1} \\ I_{a2} \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix}\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix}\begin{bmatrix} I_a \\ 0 \\ 0 \end{bmatrix}$$

$\Rightarrow I_{a0} = I_{a1} = I_{a2} = \frac{I_a}{3}$ --- (6.20)

The sequence network equations are given by

$$\left.\begin{aligned} V_{a0} &= -I_{a0}Z_0 \\ V_{a1} &= E_a - I_{a1}Z_1 \\ V_{a2} &= -I_{a2}Z_2 \end{aligned}\right\} \quad \text{--- (6.21)}$$

We know that from eqn (6.18)

$V_a = I_a Z_f$

$\Rightarrow V_{a0}+V_{a1}+V_{a2} = I_a Z_f$

$\Rightarrow -I_{a0}Z_0+E_a-I_{a1}Z_1-I_{a2}Z_2 = I_a Z_f$

$\Rightarrow -I_{a1}Z_0+E_a-I_{a1} Z_1-I_{a1}Z_2 = 3 I_{a1} Z_f$

$\Rightarrow I_{a1} (Z_0+Z_1+Z_2+3Z_f) = E_a$

$\Rightarrow I_{a1} = \frac{E_a}{Z_0 + Z_1 + Z_2 + 3Z_f}$ --- (6.22)

The fault current is

$I_f = I_a = I_{a0}+I_{a1}+I_{a2} = 3I_{a1}$

i.e $I_f = I_a = 3I_{a1} = \frac{3E_a}{Z_0 + Z_1 + Z_2 + 3Z_f}$ --- (6.23)

Using eqn.(6.22), the sequence network diagram can be drawn as shown in the fig.6.11.

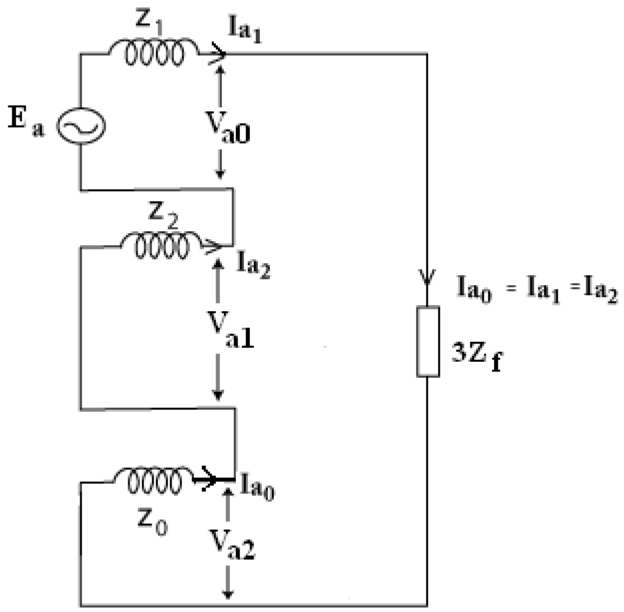


Fig.(6.11): The equivalent circuit of generator during L-G fault with fault impedance

6.11.2 Line to line Fault (L-G)

Case (a): Without fault impedance

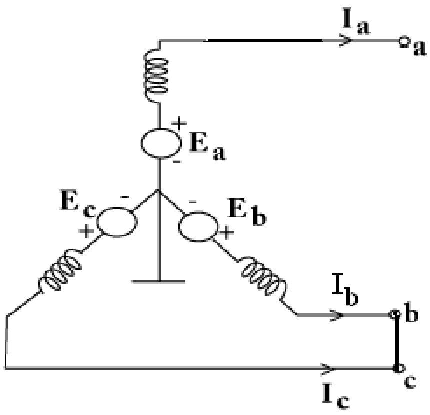


Fig.(6.12)

The boundary conditions are

$$\left.\begin{aligned} &V_b = V_c, \quad I_a = 0 \\ &I_b + I_c = 0 \Rightarrow I_b = -I_c \end{aligned}\right\} \qquad \text{--- (6.24)}$$

The symmetrical components of voltages are

$$\begin{bmatrix} V_{a0} \\ V_{a1} \\ V_{a2} \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix}\begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} \qquad \text{--- (6.25)}$$

From eqns. (6.24) & (6.25)

$$\begin{bmatrix} V_{a0} \\ V_{a1} \\ V_{a2} \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix}\begin{bmatrix} V_a \\ V_b \\ V_b \end{bmatrix}$$

$$\Rightarrow V_{a0} = \frac{1}{3}\left(V_a + V_b + V_c\right)$$

$$V_{a1} = \frac{1}{3}\left[V_a + \left(a + a^2\right)V_b\right] = \frac{1}{3}\left[V_a - V_b\right]$$

$$V_{a2} = \frac{1}{3}\left[V_a + \left(a + a^2\right)V_b\right] = \frac{1}{3}\left[V_a - V_b\right]$$

$$\Rightarrow V_{a1} = V_{a2} \qquad \text{--- (6.26)}$$

The symmetrical components of currents are

$$\begin{bmatrix} I_{a0} \\ I_{a1} \\ I_{a2} \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix}\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix}\begin{bmatrix} 0 \\ I_b \\ -I_b \end{bmatrix}$$

$$\Rightarrow I_a = 0$$

$$I_{a1} = \frac{I_b}{3}(a - a^2)$$

$$I_{a2} = \frac{I_b}{3}(a^2 - a) = -I_{a1}$$

$$\Rightarrow I_{a0} = 0, I_{a1} = -I_{a2} \qquad \text{--- (6.27)}$$

From eqn.(6.26)

$V_{a1}=V_{a2}$

$$E_a - I_{a1}Z_1 = -I_{a2}Z_2$$
$$E_a = I_{a1}Z_1 - I_{a2}Z_2 = I_{a1}\left(Z_1 + Z_2\right)$$

$$\Rightarrow I_{a1} = \frac{E_a}{Z_1 + Z_2} \qquad \text{--- (6.28)}$$

The fault current is given by

$$I_f = I_b = I_{a0} + a^2 I_{a1} + aI_{a2}$$
$$= \left(a^2 - a\right)I_{a1} = \left(-\frac{1}{2} - j\frac{\sqrt{3}}{2} + \frac{1}{2} - j\frac{\sqrt{3}}{2}\right)I_{a1}$$

$$\Rightarrow I_f = I_b = -j\sqrt{3}I_{a1} = \frac{-j\sqrt{3}E_a}{Z_1 + Z_2} \qquad \text{--- (6.29)}$$

Using eqn.(6.28), the equivalent circuit of a generator during L-L fault is shown in the fig.6.13

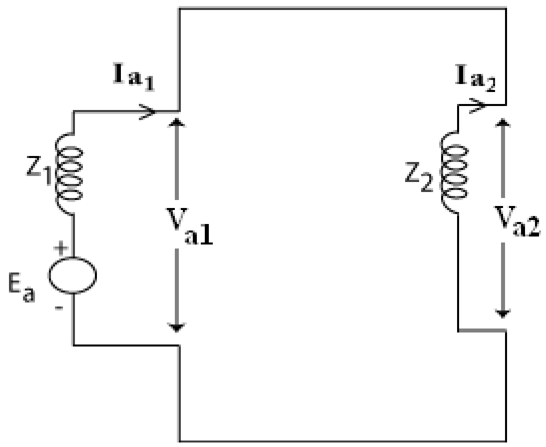


Fig.(6.13): The equivalent circuit of generator during L-L fault without fault impedance

Case (b): With fault impedance

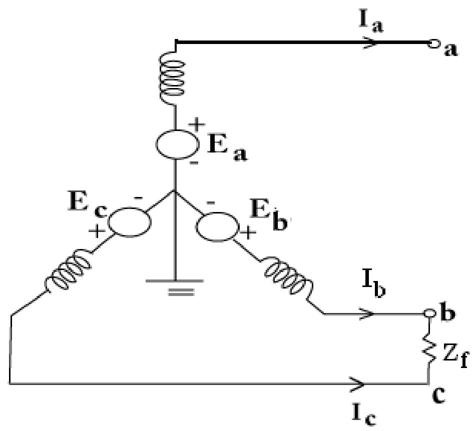


Fig.(6.14)

The boundary conditions are

$$\left.\begin{array}{l} V_b = V_c + I_b Z_f, \quad I_a = 0 \\ I_b + I_c = 0 \Rightarrow I_b = -I_c \end{array}\right\} \qquad \text{--- (6.30)}$$

The symmetrical components of currents are

$$\begin{bmatrix} I_{a0} \\ I_{a1} \\ I_{a2} \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix}\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix}\begin{bmatrix} 0 \\ I_b \\ -I_b \end{bmatrix}$$

$$\Rightarrow I_a = 0$$

$$I_{a1} = \frac{I_b}{3}(a - a^2)$$

$$I_{a2} = \frac{I_b}{3}(a^2 - a) = -I_{a1}$$

$$\Rightarrow I_{a0} = 0, I_{a1} = -I_{a2} \qquad \text{--- (6.32)}$$

From the eqn. (6.30)

$$V_b = V_c + I_b Z_f$$

$$\Rightarrow V_{a0} + a^2 V_{a1} + a V_{a2} = V_{a0} + a V_{a1} + a^2 V_{a2} + I_b Z_f$$

$$\Rightarrow \left(a^2 - a\right)V_{a1} + \left(a - a^2\right)V_{a2} = I_b Z_f$$

$$\Rightarrow \left(a^2 - a\right)\left(V_{a1} - V_{a2}\right) = \left[I_{a0} + a^2 I_{a1} + a I_{a2}\right]Z_f \qquad \text{--- (6.33)}$$

From eqns. (6.32) & (6.33)

$$\left(a^2 - a\right)\left(V_{a1} - V_{a2}\right) = \left(a^2 - a\right)I_{a1} Z_f$$

$$\Rightarrow V_{a1} - V_{a2} = I_{a1} Z_f \qquad \text{--- (6.34)}$$

$$E_a - I_{a1} Z_1 + I_{a2} Z_2 = I_{a1} Z_f \;\left(\because \textit{From sequence network quations}\right)$$

$$I_{a1}Z_1 - I_{a2}Z_2 + I_{a1}Z_f = E_a$$

$$I_{a1}\left(Z_1 + Z_2 + Z_f\right) = E_a \ (\because I_{a2} = -I_{a1})$$

$$\Rightarrow I_{a1} = \frac{E_a}{Z_1 + Z_2 + Z_f} \qquad \text{--- (6.35)}$$

The fault current is given by

$$I_f = I_b = I_{a0} + a^2 I_{a1} + aI_{a2}$$

$$= \left(a^2 - a\right) I_{a1} = \left(-\frac{1}{2} - j\frac{\sqrt{3}}{2} + \frac{1}{2} - j\frac{\sqrt{3}}{2}\right) I_{a1}$$

$$\Rightarrow I_f = I_b = -j\sqrt{3} I_{a1} = \frac{-j\sqrt{3}E_a}{Z_1 + Z_2} \qquad \text{--- (6.36)}$$

The sequence network diagram for a line to line fault through an impedance Z_f is shown in the following figure.

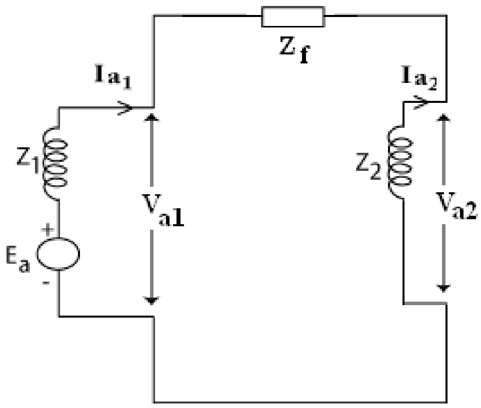


Fig.(6.15): The equivalent circuit of generator during L-L fault with fault impedance

6.11.3 Double line to ground fault (L-L-G fault)

Case (a): Without fault impedance

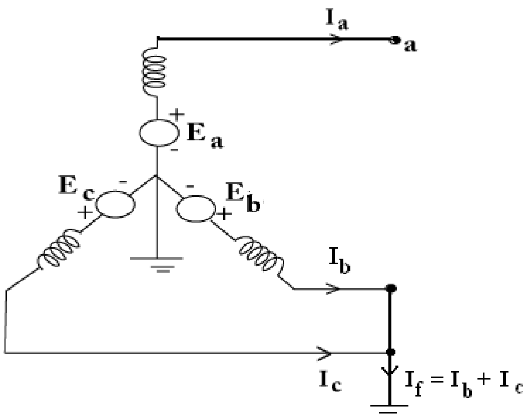


Fig.(6.16)

The boundary conditions are given by

$$V_c = 0, V_b = 0, I_a = 0 \qquad \text{--- (6.37)}$$

The fault current is

$$I_f = I_b + I_c$$

The sequence components of the voltage are

$$\begin{bmatrix} V_{a0} \\ V_{a1} \\ V_{a2} \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix}\begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix}\begin{bmatrix} V_a \\ 0 \\ 0 \end{bmatrix}$$

$$\Rightarrow V_{a0} = V_{a1} = V_{a2} = \frac{V_a}{3} \qquad \text{--- (6.38)}$$

$$V_{a0} = V_{a1}$$

$$\Rightarrow -I_{a0}Z_0 = E_a - I_{a1}Z_1$$

$$\Rightarrow I_{a0} = \frac{I_{a1}Z_1 - E_a}{Z_0} \qquad \text{--- (6.39)}$$

$$V_{a0} = V_{a1}$$

$$E_a - I_{a1}Z_1 = -I_{a2}Z_2$$

$$\Rightarrow I_{a2} = \frac{I_{a1}Z_1 - E_a}{Z_2} \qquad \text{--- (6.40)}$$

From eqn. (6.37), we know that

$$I_a = 0$$

$$I_{a0} + I_{a1} + I_{a2} = 0$$

$$\frac{I_{a1}Z_1 - E_a}{Z_0} + I_{a1} + \frac{I_{a1}Z_1 - E_a}{Z_2} = 0$$

$$\Rightarrow I_{a1}\frac{Z_1}{Z_0} + I_{a1} + I_{a1}\frac{Z_1}{Z_2} = \frac{E_a}{Z_0} + \frac{E_a}{Z_2}$$

$$\Rightarrow I_{a1}\frac{\left(Z_1Z_2 + Z_0Z_1 + Z_0Z_2\right)}{Z_0Z_2} = I_a \times \frac{\left(Z_0 + Z_2\right)}{Z_0Z_2}$$

$$\Rightarrow I_{a1} = \frac{E_a\left(Z_0 + Z_2\right)}{Z_1\left(Z_0 + Z_2\right) + Z_0Z_2}$$

$$\Rightarrow I_{a1} = \frac{E_a}{Z_1 + \dfrac{Z_0 Z_2}{Z_0 + Z_2}} \qquad \text{--- (6.39)}$$

The fault current is given by

$$I_f = I_b + I_c$$

$$= I_{a0} + a^2 I_{a1} + aI_{a2} + I_{a0} + aI_{a1} + a^2 I_{a2}$$

$$= 2I_{a0} + \left(a^2 + a\right) I_{a1} + \left(a^2 + a\right) I_{a2}$$

$$= 2I_{a0} + \left(a^2 + a\right)\left(I_{a1} + I_{a2}\right)$$

$$= 2I_{a0} - \left(I_{a1} + I_{a2}\right) = 3I_{a0} \qquad \left(\because I_a = I_a + I_{a1} + I_{a2} = 0\right) \qquad \text{--- (6.40)}$$

The sequence network diagram by using eqn. (6.39) can be drawn as shown in the fig.6.17

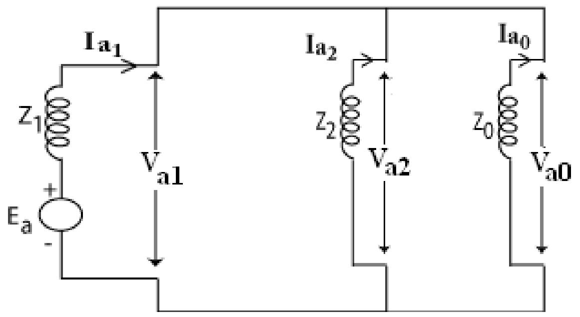


Fig.(6.17): The equivalent circuit of generator during L-L-G fault without fault impedance

Case (b): With fault impedance (Z_f)

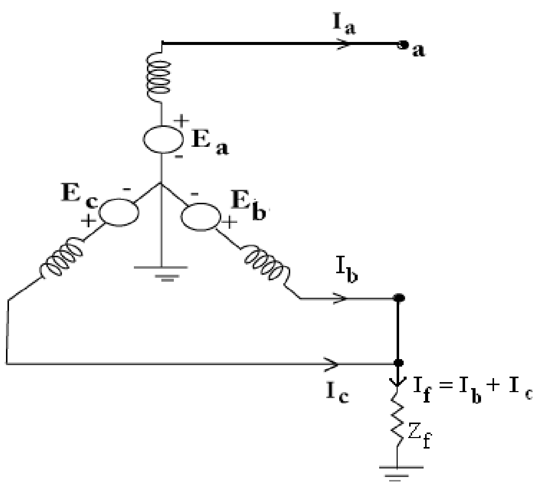


Fig.(6.18)

The boundary conditions are

$$\left.\begin{aligned} I_a &= 0 \\ V_b = V_c &= \left(I_b + I_c\right) Z_f = 3I_{a0} Z_f \end{aligned}\right\} \qquad \text{--- (6.41)}$$

From eqn. (6.41)

$$V_b = V_c$$

$$\Rightarrow V_{a0} + a^2 V_{a1} + aV_{a2} = V_{a0} + aV_{a1} + a^2 V_{a2}$$

$$\Rightarrow \left(a^2 - a\right)V_{a1} = \left(a^2 - a\right)V_{a2}$$

$$\Rightarrow V_{a1} = V_{a2} \qquad \text{--- (6.42)}$$

Again from eqn. (6.41)

$$V_b = 3I_{a0}Z_f$$

$$\Rightarrow V_{a0} + a^2 V_{a1} + aV_{a2} = 3I_{a0}Z_f$$

$$\Rightarrow V_{a0} + \left(a^2 + a\right)V_{a1} = 3I_{a0}Z_f$$

$$\Rightarrow V_{a0} - V_{a1} = 3I_{a0}Z_f$$

$$\Rightarrow V_{a1} = V_{a0} - 3I_{a0}Z_f \qquad \text{--- (6.43)}$$

From sequence network equations

$$E_a - I_{a1}Z_1 = -I_{a0}Z_0 - 3I_{a0}Z_f$$

$$\Rightarrow I_{a0} = -\frac{\left(E_a - I_{a1}Z_1\right)}{Z_0 + 3Z_f}$$

From eqn. (6.42)

$$V_{a1} = V_{a2}$$

$$\Rightarrow E_a - I_{a1}Z_1 = -I_{a2}Z_2$$

$$\Rightarrow I_{a2} = \frac{I_{a1}Z - E_a}{Z_2} \qquad \text{--- (6.44)}$$

From eqn. (6.41)

$$I_a = 0$$

$$I_{a0} + I_{a1} + I_{a2} = 0$$

$$\frac{I_{a1}Z_1 - E_a}{Z_0 + 3Z_f} + I_{a1} + \frac{I_{a1}Z_1 - E_a}{Z_2} = 0$$

$$\Rightarrow \frac{I_{a1}Z_1}{Z_0 + 3Z_f} + I_{a1} + \frac{I_{a1}Z_1}{Z_2} = \frac{E_a}{Z_0 + 3Z_f} + \frac{E_a}{Z_2}$$

$$\Rightarrow I_{a1}\left(\frac{Z_1}{Z_0 + 3Z_f} + 1 + \frac{Z_1}{Z_2}\right) = E_a\left(\frac{1}{Z_0 + 3Z_f} + \frac{1}{Z_z}\right)$$

$$\Rightarrow I_{a1}\left(\frac{Z_1Z_2+\left(Z_0+3Z_f\right)Z_2+Z_1\left(Z_0+3Z_f\right)}{\left(Z_0+3Z_f\right)Z_2}\right)=\frac{E_a\left(Z_2+Z_0+3Z_f\right)}{\left(Z_2\right)\left(Z_0+3Z_f\right)}$$

$$\Rightarrow I_{a1}=\frac{E_a\left(Z_2+Z_0+3Z_f\right)}{Z_1Z_2+\left(Z_0+3Z_f\right)Z_2+Z_1\left(Z_0+3Z_f\right)}$$

$$=\frac{E_a}{\frac{Z_1\left(Z_2+Z_0+3Z_f\right)}{Z_2+Z_0+3Z_f}+\frac{Z_2\left(Z_0+3Z_f\right)}{Z_2+Z_0+3Z_f}}$$

$$\Rightarrow I_{a1}=\frac{E_a}{Z_1+\frac{Z_2\left(Z_0+3Z_f\right)}{Z_2+Z_0+3Z_f}} \qquad \text{--- (6.45)}$$

The fault current is given by

$$I_f=I_b+I_c=3I_{a0} \qquad \text{--- (6.46)}$$

The sequence network diagram, by using eqn.(6.45) can be drawn as shown in the fig.(6.19)

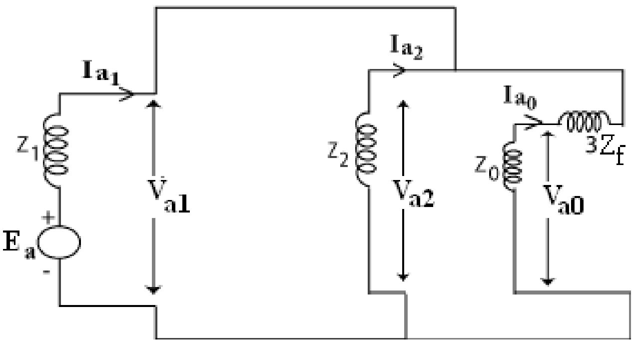


Fig.(6.17): The equivalent circuit of generator during L-L-G fault with fault impedance

6.11.4 3-phase Fault

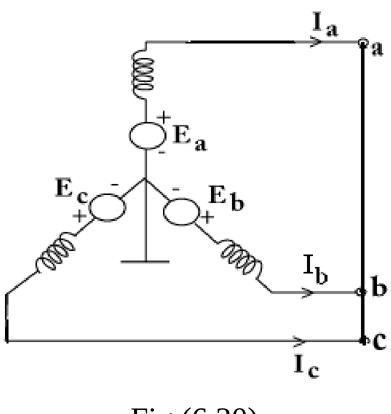


Fig.(6.20)

The boundary conditions are

$$I_a+I_b+I_c=0$$

$V_a = V_b = V_c$

Since $|I_a| = |I_b| = |I_c|$ and if I_a is taken as reference

$\therefore I_a = I_a\angle 0,\ I_b = a^2 I_a,\ I_c = a\ I_a$

The symmetrical components of the currents are

$$\begin{bmatrix} I_{a0} \\ I_{a1} \\ I_{a2} \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix}\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix}\begin{bmatrix} 0 \\ a^2 I_a \\ a I_a \end{bmatrix}$$

$\Rightarrow I_a = 0,\ I_{a1} = I_a,\ I_{a2} = 0$ --- (6.47)

From the above equation, it is clear that for a 3ϕ fault zero as well as negative sequence components of current are absent and the positive sequence component of current is equal to the phase current.

The sequence components for the voltages are

$$\begin{bmatrix} V_{a0} \\ V_{a1} \\ V_{a2} \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix}\begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix}\begin{bmatrix} V_a \\ V_a \\ V_a \end{bmatrix}$$

$\Rightarrow V_{a0} = V_a,\ V_{a1} = V_{a2} = 0$ --- (6.48)

Since $V_{a1} = 0$

$$E_a - I_{a1} Z_1 = 0$$

$$\Rightarrow I_{a1} = \frac{E_a}{Z_1} \quad \text{--- (6.49)}$$

The sequence network diagram, from eqn.(6.49) can be drown as shown in the fig.6.19

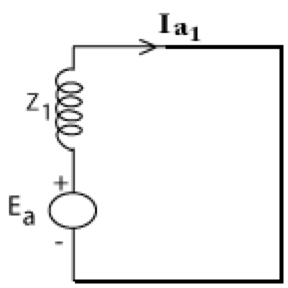


Fig.(6.21)

Problem-7: A 25 MVA,13.2 kV alternator with solidly grounded neutral has a sub-transient reactance of 0.25Ω. The negative and zero sequence reactance's are 0.35 Ω and 0.1 Ω respectively. Determine the fault current and line to line voltages at the fault

i) when a L-G fault occurs at the terminals of the alternator

ii) when a L-L fault occurs at the terminals of the alternator

iii) when a L-L-G fault occurs at the terminals of the alternator

Solution:

Let the line to neutral voltage at the fault point before the fault be 1+j0

i.e. E_a = 1+j0

Z_0 = 0·1 pu , Z_1 = 0·25pu , Z_2=0·35pu

Z= Z_0 + Z_1+Z_2 = j0.1+j0.25+j0.35= j0·7

i) For L-G fault

$$I_{a1} = \frac{E_a}{Z_0 + Z_1 + Z_2} = \frac{1 + j0}{j0.7} = -j1.428$$

$$\therefore I_{a0} = I_{a1} = I_{a2} = -j1.428$$

The p.u fault current

$$I_f = I_a = I_{a0} + I_{a1} + I_{a2}$$

$$= 3\,I_{a1} = 3 \times -j1.428 = -j4.285$$

Let the base quantities be 25 MVA, 13.2 kV

Base current = $\frac{25 \times 10^3}{\sqrt{3} \times 13.2} = 1093A$

The fault current in Amps

$$I_f = I_a = 1093 \times 4.285 = 4685A$$

To determine the line voltages, we first find out the sequence components of voltages.

$$V_{a1} = E_a - I_{a1}Z_1 = 1 - (-j1.428) \times j0{\cdot}25 = 1 - 0.357 = 0.643$$

$$V_{a2} = -I_{a2}Z_2 = j1.428 \times j0.35 = -0.5$$

$$V_{a0} = -I_{a0}Z_0 = j1.428 \times j0.1 = -0.1428$$

$$V_a = V_{a0} + V_{a1} + V_{a2} = 0.643 - 0.5 - 0.1428 = 0$$

$$V_b = V_{a0} + a^2V_{a1} + aV_{a2}$$

$$= -0.1428 + 1\angle -120 \times 0.643\angle 0 + 1\angle 120 \times -0.5\angle 0$$

$$= -0.2143 - j0.9898$$

$$V_c = V_{a0} + aV_{a1} + a^2V_{a2}$$

$$= -0.1428 + 1\angle 120 \times 0.643\angle 0 + 1\angle -120 \times (-0.5\angle 0)$$

$= 0.2143 + j0.9898$

Now

$V_{ab} = V_a - V_b = 0 + 0.2143 - j0.9896 = 1.0127\angle 77.78$

$V_{bc} = V_b - V_c = -0.2143 - j0.9896 - (-0.2143 + j0.9898) = -j2 \times 0.9898 = 1.9796\angle -90$

$V_{ca} = V_c - V_a = -0.2143 + j0.9896 = 1.0127\angle 102.2$

The line to line voltage are

$$V_{ab} = 1.0127 \times \frac{13.2}{\sqrt{3}} = 7.71\, kV$$

$$V_{bc} = 1.9796 \times \frac{13.2}{\sqrt{3}} = 15.08 kV$$

$$V_{ca} = 1.0127 \times \frac{13.2}{\sqrt{3}} = 7.71 kV$$

ii) For L-L fault

$$I_{a1} = \frac{E_a}{Z_1 + Z_2} = \frac{1}{j0.25 + j0.35} = -j1.667$$

$I_{a2} = -I_{a1} = j1.667,\ I_{a0} = 0$

Fault current $I_f = I_b = -I_c$

$$= I_{a0} + a^2 I_{a1} + aI_{a2}$$

$$= -j\sqrt{3} I_{a1} = -j\sqrt{3} \times (-j1.667) = -2.8872 pu$$

The base current is 1093A.

$\therefore$ Fault current $I_f = 2.8872 \times 1093 = 3155.7 A$

To find out the to line to line voltages we find out the sequence components of the voltages

$V_{a1} = E_a - I_{a1} Z_1 = 1 - (-j1.667)(j0.25) = 1 - 0.4167 = 0.5833$

$V_{a2} = -I_{a2}\ Z_2 = -j1.667 \times j0.35 = 0.5834$

$V_{a0} = I_{a0} \times Z_0 = 0$

$V_a = V_{a0} + V_{a1} + V_{a2} = 0 + 0.5833 + 0.5833 = 1.166 pu$

$V_b = V_{a0} + a^2 V_{a1} + aV_{a2}$

$= 0 + 1\angle -120 \times 0.5833\angle 0 + 1\angle 120 \times 0.5833\angle 0 =$ -0.5833

$V_b = V_c = 0.5833$

The line voltages

$V_{ab} = V_a - V_b = 1.1666 + 0.5833 = 1.75$

$V_{bc} = V_b - V_c = 0$

$V_{ca} = V_c - V_a = -0.5833 - 1.1666 = -1.7499$

$\therefore$ The line to line voltage are

$$V_{ab} = 1.75 \times \frac{13.2}{\sqrt{3}} = 13.33kV$$

$$V_{bc} = 0 \times \frac{13.2}{\sqrt{3}} = 0$$

$$V_{ca} = \frac{13.2}{\sqrt{3}} \times 1.75 = 13.33kV$$

iii) L-L-G fault

$$I_{a1} = \frac{E_a}{Z_1 + \dfrac{Z_0\ Z_2}{Z_0 + Z_2}} = \frac{1 + j0}{j0.25 + \dfrac{j0.1 \times j0.35}{j0.45}} = \frac{1 + j0}{j0.25 + j0.0778} = -j3.0506\ pu$$

To find out I_{a2} and I_{a0} we should first find V_{a1} and from that we can find $I_{a2} = I_{a0}$

$V_{a1} = E_a - I_{a1}\ Z_1$ = 1+j3.0506 $\times j0.25$ = 1-0.7626 = 0.2374

$\therefore V_{a0} = V_{a1} = V_{a2} = 0.2374$

$$I_{a2} = \frac{-V_{a2}}{Z_2} = \frac{-0.2374}{j0.35} = j0.678$$

$$I_{a0} = \frac{-V_{a0}}{Z_0} = \frac{-0.2374}{j0.1} = j2.374$$

Fault current $I_f = I_b + I_c = 3I_{a0}$

$= 3 \times j2.374 = j7.122\ pu$

Since the base current is 1093A, the fault current in amps is

$$I_f = 7.122 \times \frac{13.2}{\sqrt{3}} = 7784.3A$$

$V_a = V_{a0} + V_{a1} + V_{a2} = 3V_{a1} = 3 \times 0.2374 = 0.7122$

$V_b = 0$; $V_c = 0$

$V_{ab} = V_a - V_b = 0.7122$

$V_{bc} = V_b - V_c = 0$

$V_{ca} = V_c - V_a = -0.7122$

The line to line voltage are

$$V_{ab} = 0.7122 \times \frac{13.2}{\sqrt{3}} = 5.42\, kV$$

$$V_{bc} = 0 \times \frac{13.2}{\sqrt{3}} = 0$$

$$V_{ca} = 0.7122 \times \frac{13.2}{\sqrt{3}} = 5.42\, kV$$

Problem-8: A 50MVA, 11kV, 3- ϕ alternator was subjected to different types of faults. The fault currents were

3-ϕ fault ———————— 1870A

L-L fault ———————— 2590A

L-G fault ———————— 4130A

The alternator neutral is solidly grounded. Find the per unit values of the three sequence reactance of the alternator.

Solution:

For 3-ϕ fault:

$$\text{Fault current } (I_f) = I_a = \frac{\text{Line to neutral voltage i.e } E_a}{X_1}$$

$$\Rightarrow 1870 = \frac{11000/\sqrt{3}}{X_1}$$

$$\Rightarrow X_1 = 3.396\,\Omega$$

For L-L fault:

$$\text{Fault current}, I_f = \frac{\sqrt{3}E_a}{X_1 + X_2}$$

$$2590 = \frac{\sqrt{3} \times 11000/\sqrt{3}}{X_1 + X_2}$$

$$\Rightarrow X_1 + X_2 = \frac{11000}{2590} = 4.247$$

$$\Rightarrow X_2 = 4.247 - 3.396 = 0.851\,\Omega$$

For L-G fault:

$$\text{Fault current, } I_f = \frac{3E_a}{X_1 + X_2 + X_0}$$

$$4130 = \frac{3 \times 11000/\sqrt{3}}{X_1 + X_2 + X_0}$$

$$\Rightarrow X_1 + X_2 + X_0 = \frac{\sqrt{3} \times 11000}{4130} = 4.613$$

$$X_0 = 4.613 - 4.247 = 0.366\,\Omega$$

Base impedance $= \frac{11 \times 11 \times 10^3 \times 10^3}{50 \times 10^6} = 2.42\,\Omega$

$$\therefore X_1 = \frac{3.396}{2.42} = 1.4\,pu$$

$$X_2 = \frac{0.851}{2.42} = 0.35\ pu$$

$$X_0 = \frac{0.366}{2.42} = 0.15\,pu$$

Problem-9: A 3-ϕ, 37.5 M VA, 33kV alternator having $X_1 = 0.4pu$, $X_2 = 0.2\ pu$ and $X_0 = 0.1pu$, based on its rating, is connected to a 33kV overhead line having X_1= 6.5 Ω, X_2 = 7.3 Ω and X_0 = 10.5 Ω per phase. An LG fault occurs at the remote end of the line. The alternator neutral is solidly grounded. Calculate the fault current.

Solution: Base MVA = 37.5 MVA

Base voltage = 33 kV

Base impedance = $\frac{33 \times 1000 \times 33 \times 1000}{37.5 \times 10^6} = 29.04\,\Omega$

Total $X_1 = j0.4 + \frac{j6.5}{29.04} = j0.6238\,\Omega$

Total $X_2 = j0.2 + \frac{j7.3}{29.04} = j0.4513\Omega$

Total $X_0 = j0.10 + \frac{j10.5}{29.04} = j0.4616\Omega$

Fault current = $\frac{3 \times 1\angle 0}{j0.6238 + j0.4513 + j0.4616} = -j1.9522\ pu$

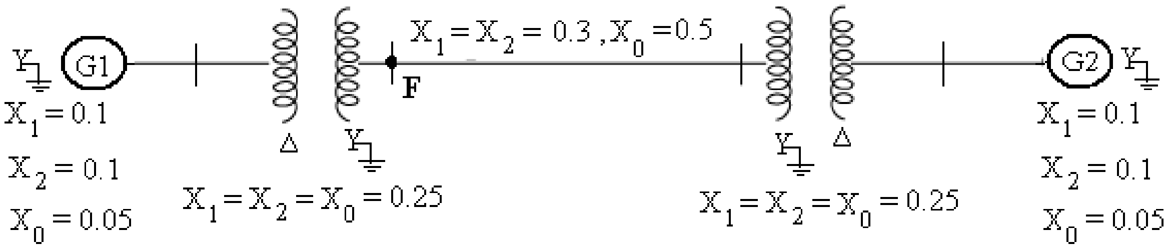
Base current = $\frac{37.5 \times 10^6}{\sqrt{3} \times 33 \times 10^3} = 656.08A$

Fault current $= 1.9522 \times 656.08 = 1280.8A$

ADDITIONAL SOLVED PROBLEMS

Problem-1: For the system network as shown in figure , if the fault occurs at point F, find the fault current current in the following cases

i) L-G fault
ii) L-L fault
iii) L-L-G fault



[JNTU, Regular, November-2009]

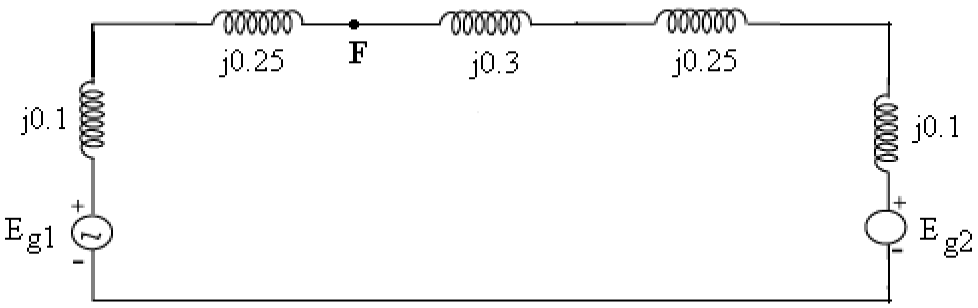
Solution: Given that

For Generators, X_1=0.1, X_2=0.1, X_0=0.05

. For Transformers, X_1=0.25, X_2=0.25, X_0=0.25

For Transmission line, X_1=0.3, X_2=0.3, X_0=0.5

The Positive sequence network diagram is

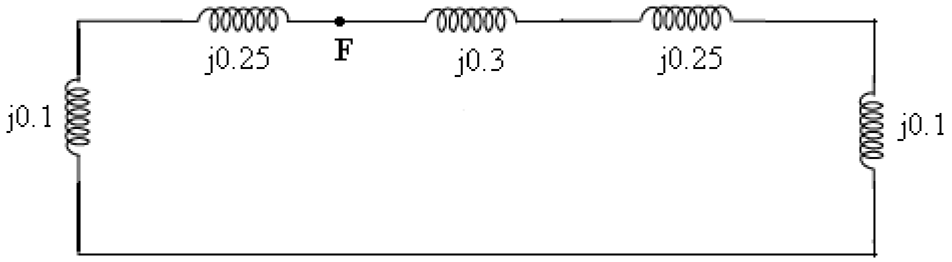


Equivalent positive sequence impedance is

$$Z_1 = (X_{g1,1} + X_{T1,1}) // (X_{L1} + X_{g2,1} + X_{T2,1})$$

$$= (j0.1 + j0.25) // (j0.3 + j0.1 + j0.25) = j0.2275$$

The negative sequence network diagram

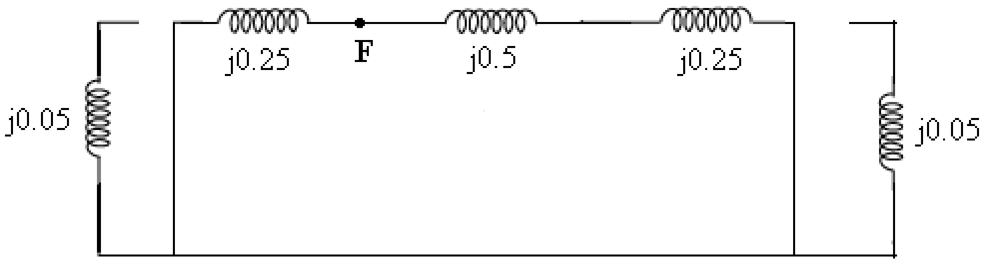


Equivalent negative sequence impedance is,

$$Z_2 = (X_{g1,2} + X_{T1,2}) // (X_{L2} + X_{g2,2} + X_{T2,2})$$

$$= (j0.1 + j0.25) // (j0.3 + j0.1 + j0.25) = j0.2275$$

The zero sequence network diagram is



Equivalent zero sequence impedance is,

$$Z_0 = (j0.25)//(j0.5 + j0.25) = j0.185$$

i) Fault current for an L-G fault

$$I_f = \frac{3\times E_a}{Z_0 + Z_1 + Z_2} = \frac{3\times 1}{j0.185 + j0.2275 + j0.2275} = -j4.669$$

Magnitude of fault current I_f = 4.669 p.u

ii) Fault current for an L-L fault

$$I_f = \frac{-j\sqrt{3}\times E_a}{Z_1 + Z_2} = \frac{-j\sqrt{3}\times 1}{j0.2275 + j0.2275} = -3.8$$

Magnitude of fault current I_f = 3.8 p.u

iii) Fault current for an L-L-G fault

$I_f = 3I_{a0}$

$$I_{a1} = \frac{E_a}{Z_1 + \frac{Z_0 Z_2}{Z_0 + Z_2}} = \frac{1}{j0.2275 + \frac{j0.185\times j0.2275}{j0.185 + j0.2275}} = -j3.483$$

$V_{a1} = E_{a1} - I_{a1}Z_1 = 1 - j3.483\times j0.2275 = 1.7924$

In case of LLG fault, $V_{a1} = V_{a2} = V_{a0}$

$V_{a1} = V_{a0} \Rightarrow -I_{a0} Z_0 = V_{a1} = 1.7924$

$$\Rightarrow I_{a0} = \frac{-1.7924}{Z_0} = \frac{-1.7924}{j0.185} = j9.688$$

There fore, fault current, $I_f = 3I_{a0} = j29.064$