3. LOAD FLOW ANALYSIS

3.1. INTRODUCTION

Successful operation of electrical power systems requires that:

- Generation must supply the demand (load) plus the losses,
- Bus voltage magnitudes must remain close to rated values,
- Generators must operate within specified real and reactive power limits,
- Transmission lines and transformers should not be overloaded for long periods.

Therefore it is important that voltages and power flows in an electrical system can be determined for a given set of loading and operating conditions. This is known as the load flow problem. The study of various methods of solution to a complex power system network is referred to as load flow study. The solution provides the voltages at various buses, power flowing in various lines and line losses. The main information obtained from a load flow study are the magnitude and phase angle of the voltage at each bus and the real and reactive power flowing in each line. The load flow solution also gives the initial conditions of the system when the transient behaviour of the system is to be studied. The load flow study of a power system is essential to decide the best operation of existing system, for planning the future expansion of the system and for designing a new power system.

A load flow study of a power system generally requires the following steps

i. Representation of the system by single line diagram
ii. Determine the impedance (admittance) diagram using the information in single line diagram
iii. Formulation of network equations
iv. Solution of network equations

3.2. BUS CLASSIFICATION

In a power system the buses are meeting points of various components. The generators will feed energy to buses and loads will draw energy from buses. In the network of a power system the buses becomes nodes and so a voltage can be specified for each bus. Therefore each bus in a power system, is associated with four quantities and they are real power, reactive power, magnitude of voltage and phase angle of voltage. In a load flow problem two quantities (out of four) are specified for each bus and the remaining two quantities are obtained by solving the load flow equations. The buses of a power system can be classified into following three types based on the quantities being specified for the buses

i) Load bus or PQ bus

The bus is called load bus, when real and reactive components of power are specified for the bus. The load flow equations can be solved to find the magnitude and phase of bus voltage. In a load bus the voltage is allowed to vary within permissible limits, for example $\pm 5\%$.

ii) Generator bus or voltage controlled bus or PV bus

The bus is called generator bus, when real power and magnitude of bus voltage are specified for the bus. The load flow equations can be solved to find the reactive power and phase angle of bus voltage. Usually for generator buses, reactive power limits will be specified.

iii) **Slack bus or swing bus or reference bus**

The bus is called slack bus if the magnitude and phase angle of bus voltage are specified for the bus. The slack bus is the reference bus for load flow solution and usually one of the generator buses is selected as the slack bus.

The following table gives the summary of the above classifications.

Bus type	Quantities specified	Quantities to be obtained
Load bus	P,Q	$\|V\|, \delta$
Generator bus	P,$\|V\|$	Q, δ
Slack bus	$\|V\|, \delta$	P,Q

3.3. NECESSITY OF SLACK BUS

Basically the power system has only two types of buses and they are load and generator buses. In these buses only power injected by generators and power drawn by loads are specified, but the power loss in transmission lines are not accounted.

In a power system the total power generated will be equal to sum of power consumed by loads and losses

$$\text{i.e} \begin{pmatrix} \textit{Sum of complex} \\ \textit{power of generators} \end{pmatrix} = \begin{pmatrix} \textit{Sum of complex} \\ \textit{power of loads} \end{pmatrix} + \begin{pmatrix} \textit{Total power loss in} \\ \textit{transmission lines} \end{pmatrix}$$

or

$$\begin{pmatrix} \textit{Total power loss in} \\ \textit{transmission lines} \end{pmatrix} = \begin{pmatrix} \textit{Sum of complex} \\ \textit{power of generators} \end{pmatrix} - \begin{pmatrix} \textit{Sum of complex} \\ \textit{power of loads} \end{pmatrix}$$

The transmission line losses can be estimated only if the real and reactive power of all buses is known. The power in the buses will be known only after solving the load flow equations. For these reasons, the real and reactive power of one of the generator bus is not specified and this bus is called slack bus. It is assumed that the slack bus generates the real and reactive power required for transmission line losses. Hence for a slack bus, the magnitude and phase of bus voltage are specified and real and reactive powers are obtained through the load flow solution.

3.4. DATA FOR LOAD FLOW STUDIES

Irrespective of the method for the solution, the data required is common for any load flow. These are presented below. All data is normally in p.u.

i) System Data: This should give information on

- Number of buses n
- Number of PV buses
- Number of load buses
- Number of loads
- Slack bus number
- Voltage magnitude of slack bus
- Reactive power limits for the generator bus
- Number of transmission lines
- Number of transformers
- Number of shunt elements
- Base MVA
- Tolerance limit
- Maximum permissible number of iterations

ii) Generator bus Data: For every generator bus p the data required is

- Bus number
- Active power generation, P_{Gp}
- Reactive power limits
- Voltage magnitude, $V_{p,spec}$.

iii) Load Data: For all loads, the data required is

- Bus number
- Active power demand, P_{Dp}
- Reactive power demand Q_{Dp}

iv) Transmission line Data: For every transmission line connected between buses p and q, the data required is

- Starting bus number, p
- Ending bus number, q
- Resistance and reactance of the line
- Half line charging admittance

v) Transformer data: For every transformer connected between buses p and q, the data required is

- Starting bus number, p
- Ending bus number, q
- Resistance and reactance of the transformer
- Off nominal turns ratio, a

vi) Shunt element data: The data needed for shunt element is

- Bus number where element is connected
- Shunt admittance (G_{sh}+jB_{sh})

3.5. FORMULATION OF LOAD FLOW EQUATIONS USING Y_{BUS} MATRIX (STATIC LOAD F LOW EQUATION)

The load flow equations can be formed using either the mesh or node basis equations of a power system. However, from the view point of computer time and memory, the nodal admittance formulation using the nodal voltages as the independent variables is the most economic. As far as power system networks are concerned, the major advantages of the nodal approach may be listed as follows:

- Data preparation is easy.
- The number of variables and equations is usually less than with the mesh method for power networks.
- Parallel branches do not increase the number of variables or equations.
- Node voltages are available directly from the solution, and branch currents are easily calculated.
- Off-nominal transformer taps can easily be represented.

The load flow equations, using nodal admittance matrix formulation for a three bus system as shown in fig.(3.1), are developed first and then they are generalized for n-bus system.

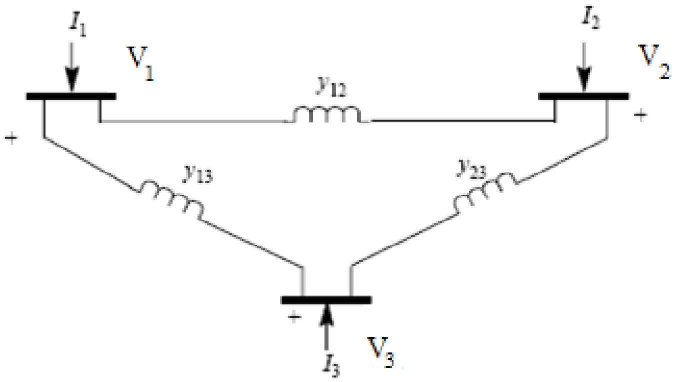


Fig.(3.1): 3-bus system

By applying KCL at node 1

$I_1 = (V_1-V_2)y_{12}+(V_1-V_3)y_{13}$

$= V_1(y_{12}+y_{13}) - V_2y_{23} - V_3y_{13}$

$\Rightarrow$ $I_1 = V_1Y_{11} + V_2Y_{12} + V_3Y_{13}$ --- (3.1)

Where $y_{12} = \frac{1}{z_{12}}$, $y_{23} = \frac{1}{z_{23}}$, $y_{31} = \frac{1}{z_{31}}$

Here $Y_{11} = (y_{12}+y_{13})$ is shunt charging admittance at bus 1.

$Y_{12} = -y_{12}$ is the mutual admittance between the buses 1 and 2

$Y_{13} = -y_{13}$ is the mutual admittance between the buses 1 and 3

Similarly nodal current equations for the other nodes can be written as follows:

$I_2 = V_1Y_{21} + V_2Y_{22} + V_3Y_{23}$ --- (3.2)

$I_3 = V_1Y_{31} + V_2Y_{32} + V_3Y_{33}$ --- (3.3)

These equations can be written in matrix form as follows

$$\begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} & Y_{13} \\ Y_{21} & Y_{22} & Y_{23} \\ Y_{31} & Y_{32} & Y_{33} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} \quad \text{--- (3.4)}$$

In general the above equation can be written in matrix notation as

$I = Y\,V$ --- (3.5)

The elements Y_{11} , Y_{22}, Y_{33} forming the diagonal terms are called self admittances. The self admittance of a node 'n' is equal to the sum of admittances of all the elements connected to node 'n'. In general the diagonal element Y_{pp} of the bus admittance matrix is equal to the sum of admittances of all the elements connected to bus p.

i.e. $Y_{pp} = y_{p1}+y_{p2}+\text{-----------}+y_{pn}$

The elements Y_{12} , Y_{13}, Y_{21} , Y_{23} , Y_{31} , Y_{32} forming the off-diagonal terms are called mutual admittances.

$$Y_{12} = Y_{21} = -y_{12}\ ,\ \ Y_{23} = Y_{32} = -y_{23}\ ,\ \ Y_{13} = Y_{31} = -y_{13}$$

It is to be noted that all mutual admittance terms have a negative sign. In general, the off-diagonal term of the bus admittance matrix is equal to the negative of admittance connected between nodes ‘p’ and ‘q’

i.e $Y_{pq} = -y_{pq}$

In compact form, the eq.(3.4) can be written as

$$I_p = \sum_{q=1}^{3} Y_{pq} V_q\ ,\quad p=1,2,----n \qquad \text{--- (3.6)}$$

From this we can write nodal current equation for an ‘n’ bus system where each node is connected to all other nodes.

$$I_p = \sum_{q=1}^{n} Y_{pq} V_q\ ,\ p=1,2,----n \qquad \text{--- (3.7)}$$

$$= Y_{pp} V_p + \sum_{\substack{q=1 \\ q \neq p}}^{n} Y_{pq} V_q$$

$$\Rightarrow V_p = \frac{I_p}{Y_{pp}} - \frac{1}{Y_{pp}} \sum_{\substack{q=1 \\ q \neq p}}^{n} Y_{pq} V_q \qquad \text{--- (3.8)}$$

I_p has been substituted by the real and reactive powers, because normally in a power system these quantities are specified.

Now, we know that

$$V_p^* I_p = P_p - jQ_p$$

$$\Rightarrow I_p = \frac{P_p - jQ_p}{V_p^*} \qquad \text{--- (3.9)}$$

From equations (3.8) & (3.9)

$$V_p = \frac{1}{Y_{pp}} \left[\frac{P_p - jQ_p}{V_p^*} - \frac{1}{Y_{pp}} \sum_{\substack{q=1 \\ q \neq p}}^{n} Y_{pq} V_q \right],\ p = 1,2----n \qquad \text{--- (3.10)}$$

If the power system elements have mutual coupling, the bus admittance matrix cannot be found directly by inspection of the single line diagram. In presence of mutual coupling between power system elements the inspection method fails. In such a case Y_{bus} can be formed from graph theory approach. However, the mutual coupling between power system elements exist only in case of transmission lines running in parallel for a long distance. But this coupling is also weak. Therefore, for all practical purposes the mutual coupling can be ignored and Y_{bus} is formed by inspection method.

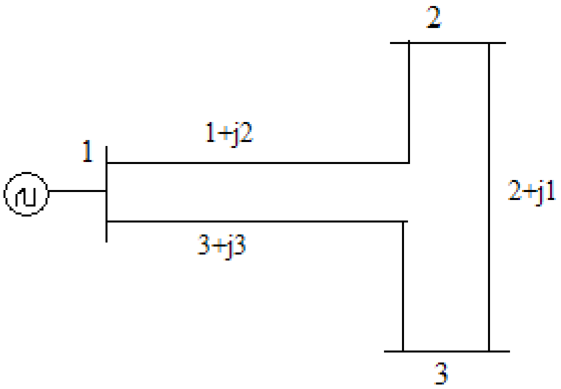
Properties of Y_{bus} matrix

The nodal admittance matrix in (3.4) or (3.5) has a well-defined structure, which makes it easy to construct. Its properties are as follows:

- Square of order $n \times n$.
- Symmetrical, since $Y_{pq} = Y_{qp}$
- Complex.
- Each off-diagonal element Y_{pq} is the negative of the branch admittance between nodes p and q, and is frequently of value zero.
- Each diagonal element Y_{pp} is the sum of the admittances of the branches which terminate at node p including branches to ground.
- Very few non-zero mutual admittances exist in practical networks. Therefore matrix Y is generally highly sparse.

Note: Why P+jQ= VI^*. Let $V = V\angle\delta$ and $I = I\angle -\theta$. If We write the expression for power as S=VI, we get $VI\angle\delta-\theta$. But the power factor angle should be $\delta+\theta$.From the phasor diagram the phase angle between V and I is $\delta+\theta$. So we have to the expression for power as $S=VI^* = V^*I$.

Problem-1: Determine the nodal admittance matrix for the power system represented by the single line diagram as shown in the fig.



Solution:

$Y_{11} = y_{12}+y_{13} = 1+j2+3+j3 = 4+j5$

$Y_{12} = -1- j2$

$Y_{13} = -3- j3$

$Y_{22} = y_{21}+y_{23} = 1+j2+2+j1 = 3+j3$

$Y_{23} = -2- j1= Y_{32}$

$Y_{33} = y_{31}+y_{32} = 3+j3+2+j1 = 5+j4$

The nodal admittance matrix (Y_{BUS}) is

$$Y_{BUS} = \begin{bmatrix} Y_{11} & Y_{12} & Y_{13} \\ Y_{21} & Y_{22} & Y_{23} \\ Y_{31} & Y_{32} & Y_{33} \end{bmatrix} = \begin{bmatrix} 4+j5 & -1-j2 & -3-j3 \\ -1-j2 & 3+j3 & -2-j1 \\ -3-j3 & -2-j1 & 5+j4 \end{bmatrix}$$

3.6. ITERATIVE METHODS OF LOAD FLOW SOLUTION

Iterative methods can be used to solve the load flow equations which are non-linear.

The iterative methods are:

i. Gauss- Seidal method
   a. Without PV bus
   b. With PV bus
ii. Newton Rapshon method
iii. Decoupled load flow method
iv. Fast-Decoupled load flow method

The static load flow equations are of such complexity that it is not possible to obtain exact analytical solution. We must use some approximate techniques that will give a sufficiently accurate numerical solution.

The solution of the load flow problem is obtained in the following manner

1. Draw the single line diagram and write bus admittance matrix.
2. Identify the buses and branches by numbers.
3. Write the power flow equations for the given network in suitable form.
4. An initial solution is guessed for the given power system network.
5. This solution is used in conjunction with static load flow equations to compute a new and better second estimation.
6. The second estimation is then used for finding the third estimation and so on.
7. The iterations are continued till the desired convergence is reached.
8. Calculate the desired quantities at the various buses.

3.6.1 GAUSS-SEIDAL (GS) METHOD

Case-1: Gauss-Seidal (GS) method without PV bus

- Gauss-Seidal method without PV bus, is an iterative method can be chosen first because of its inherent simplicity. We shall apply this method to solve our static load flow equations of general n-bus system. Presently we shall consider the case, when voltage control buses or PV buses are not present. This means that we have n-1 load buses or PQ buses, the remaining one bus is the slack bus.

Thus in this method our unknown variables are

$V_p = |V_p| e^{j\delta p}$, p = 2,3……..n

Which are n-1 complex unknown variables $V_2, V_3, V_4 \ldots\ldots V_n$ and $S_1 = P_1 + jQ_1$

Where the complex power S_1 at the slack bus can only be computed if the unknown $|V_p|$ and δ_p at the (n-1) load buses are computed first.

- The current entering at the p^{th} bus of an n-bus system is given by

$$I_p = \sum_{q=1}^{n} Y_{pq} V_q \text{ , p=1,2,----n}$$

$$= Y_{pp} V_P + \sum_{\substack{q=1 \\ q \neq p}}^{n} Y_{pq} V_q$$

$$\Rightarrow V_p = \frac{I_p}{Y_{pp}} - \frac{1}{Y_{pp}} \sum_{\substack{q=1 \\ q \neq p}}^{n} Y_{pq} V_q \quad \text{--- (3.11)}$$

Now, we know that for p^{th} bus

$$V_p^* I_p = P_p - jQ_p$$

$$\Rightarrow I_p = \frac{P_p - jQ_p}{V_p^*} \quad \text{--- (3.12)}$$

By substituting I_p from eqn.(12) in eqn.(11) , we have

$$V_p = \frac{1}{Y_{pp}} \left[\frac{P_p - jQ_p}{V_P^*} - \sum_{\substack{q=1 \\ q \neq p}}^{n} Y_{pq} V_q \right] \quad \text{--- (3.13)}$$

For Gauss-Seidal iterative method without PV bus, we can write the above equation as

$$V_p^{K+1} = \frac{1}{Y_{pp}} \left[\frac{P_p - jQ_p}{(V_p^*)^K} - \sum_{\substack{q=1 \\ q \neq p}}^{n} Y_{pq} V_q^K \right] \quad \text{--- (3.14)}$$

Algorithm when PV buses are absent

1. Read the system data and formulate Y_{BUS} for the given power system network.
2. Assume a flat voltage profile (1+j0) for all node voltages except the slack bus. Let slack bus voltage be (a+j0) and it is not modified in any iteration.
3. Assume a suitable value of convergence criterion $\in$. If the absolute value of the maximum change in voltage between any two consecutive iterations is less than a pre-

specified tolerance $\in$, the convergence is achieved and the iterative procedure is terminated.

4. Set iteration count k= 0
5. Set bus count p= 1
6. Check for the slack bus. If it is a slack bus then go to step (8), since voltage at the slack bus is fixed both in magnitude and phase, it does not vary during iterative procedure. If it is not a slack bus then go to next step.
7. Calculate bus voltage V_p^{k+1} using equation $V_p^{K+1} = \frac{1}{Y_{pp}}\left[\frac{P_p - jQ_p}{(V_p^*)^K} - \sum_{\substack{q=1 \\ q \neq p}}^{n} Y_{pq} V_q^K\right]$ and the difference in the bus voltage using $\Delta V_p^k = V_p^{k+1} - V_p^k$
8. Advance the bus count by 1 to evaluate other values of V_p^{k+1} and ΔV_p^k Check all the buses have been taken into account or not. If yes, go to the next step, otherwise go back to step (6).
9. Determine the largest absolute value of change in voltage $|\Delta V|_{max}$
10. If $|\Delta V|_{max}$ is less than the pre specified tolerance ϵ , then evaluate line flows and print the voltages and line flows. If not, advance the iteration count k= k+1 and go back to step (5).

Case-2: Gauss -Seidal (GS) method including PV buses

- The GS method is an iterative algorithm for solving a set of non-linear load flow equations. The non-linear load flow equations are given by eqn.(3.13) can be represented for convenience as follows

$$V_p = \frac{1}{Y_{pp}}\left[\frac{P_p - jQ_p}{V_P^*} - \sum_{q=1}^{p-1} Y_{pq} V_q - \sum_{q=p+1}^{n} Y_{pq} V_q\right] \text{ where p=1,2,3------n,} \qquad \text{--- (3.15)}$$

  The variables obtained from the above equation are node voltages V_1, V_2, ------V_n.
- In the GS method an initial value of voltages are assumed and they are denoted as V_1^0, V_2^0-------, V_n^0. On substituting these initial values in the above equation and by taking p=1, the revised value of bus 1 voltage V_1^1 is computed. The revised value of bus voltage V_1^1 is replaced for initial value V_1^0 and the revised bus 2 voltage V_2^1 is computed. Now replace the value of V_1^1 for V_1^0 and V_2^1 for V_2^0 and again calculate the voltage for bus 3 and so on.

The process of computing all the bus voltages as explained above is called one iteration. The iterative procedure is repeated till the bus voltages converges within prescribed accuracy.

- Based on the above discussion the load flow eqn.(3.15) can be written in modified form as given below

$$V_p^{K+1} = \frac{1}{Y_{pp}}\left[\frac{P_p - jQ_p}{(V_p^*)^K} - \sum_{q=1}^{p-1} Y_{pq} V_q^{K+1} - \sum_{q=p+1}^{n} Y_{pq} V_q^K\right] \quad \text{--- (3.16)}$$

Where $V_p^k = k^{th}$ iteration value of bus voltage V_p

$V_p^{k+1} = (k+1)^{th}$ iteration value of bus voltage V_p

In eqn.(3.16), to compute the $(k+1)^{th}$ iteration value of bus-p voltage, the $(k+1)^{th}$ iteration values of voltages are used for all buses less than p and k^{th} iteration values of voltages are used for all buses greater than or equal to p.

- The eqn.(3.16) is applicable for load bus, since in load bus, changes in both magnitude and phase of voltages are allowed. But in generator bus the magnitude of voltage remains constant and so the eqn.(3.16) is used to calculate the phase angle of voltage.
- In the load flow analysis one of the bus is taken as a slack bus or reference bus and so its voltage will not change. Therefore in each iteration the slack bus voltage remains constant and it is not modified.
- For a generator bus, the reactive power is not specified. Therefore in order to calculate the phase of bus voltage of a generator bus using eqn.(3.16), we have to estimate first the reactive power, from the bus voltages and admittance matrix as shown below

$$\frac{P_p - jQ_p}{V_P^*} = \sum_{q=1}^{p-1} Y_{pq} V_q + \sum_{q=p}^{n} Y_{pq} V_q$$

$$P_p - jQ_p = V_P^*\left[\sum_{q=1}^{p-1} Y_{pq} V_q + \sum_{q=p}^{n} Y_{pq} V_q\right] \quad \text{--- (3.17)}$$

From the above eqn.(3.17), the equation for complex power in bus-p during $(k+1)^{th}$ iteration can be obtained as given below.

$$P_p^{k+1} - jQ_p^{k+1} = (V_P^k)^*\left[\sum_{q=1}^{p-1} Y_{pq} V_q^{k+1} + \sum_{q=p}^{n} Y_{pq} V_q^k\right] \quad \text{--- (3.18)}$$

The reactive power of bus-p during $(k+1)^{th}$ iteration is given by imaginary part of eqn.(18). So the reactive power of bus-p during $(k+1)^{th}$ iteration is given by

$$Q_p^{k+1} = (-1)I..P.of\left\{(V_p^k)^*\left[\sum_{q=1}^{p-1} Y_{pq}V_q^{k+1} + \sum_{q=p}^{n} Y_{pq}V_q^k\right]\right\} \quad \text{--- (3.19)}$$

- Also for a generator bus a lower and upper limits for reactive power will be specified. In each iteration, the reactive power of generator bus is calculated using eqn.(3.19) and then checked with specified limits. If it violates the specified limits then the reactive power of the bus is equated to the limit violated and it is treated as load bus. If it does not violate the limits then the bus is treated as generator bus.

Computation of Slack bus power and Line flows

The slack bus power can be calculated after the voltages have converged. The eqn.(3.17) can be used to calculate the slack bus power. Here, bus-p is slack bus.

$$P_p - jQ_p = V_P^*\left[\sum_{q=1}^{n} Y_{pq}V_q^k\right]$$

Consider a line connecting between buses p and q as shown in fig. (3.2).Usually the transmission line is connected to buses using transformer at its ends. The π-equivalent model of a transmission line with transformer at its both ends is as shown in fig. (3.2).

Fig.(3.2)

From fig.(3.2)

$$I_{pq} = (V_p - V_q)y_{pq} + V_p y_{pq0}$$

$$I_{qp} = (V_q - V_p)y_{pq} + V_q y_{pq0}$$

Complex power injected by bus-p in the line pq is

$$S_{pq} = V_p^* I_{pq} = V_p^*\left[(V_p - V_q)y_{pq} + V_p y_{pq0}\right]$$

Complex power injected by bus-q in the line pq is

$$S_{qp} = V_q^* I_{pq} = V_q^*\left[(V_q - V_p)y_{pq} + V_q y_{pq0}\right]$$

The complex power loss in the line pq is given by

$S = S_{pq} + S_{qp}$

Algorithm when PV buses are present

1) Read the system data and formulate Y_{BUS} for the given power system network.
2) Assume a flat voltage profile (1+j0) for all the bus voltages except the slack bus. Let slack bus voltage be (a+j0) and it is not modified in any iteration.
3) Assume a suitable value of ϵ called convergence criterion. Here ϵ is a specified change in the bus voltage that is used to compare the actual change in bus voltage between k^{th} and $(k+1)^{th}$ iteration.
4) Set iteration count k= 0
5) Set bus count p=1.
6) Check for slack bus. If it is a slack bus then go to step (13), otherwise go to next step.
7) Check for generator bus. If it is a generator bus go to next step, otherwise go to step (9)
8) Replace the value of the voltage magnitude of generator bus in that iteration by the specified value. Keep the phase angle same as in that iteration. Calculate Q for generator bus.

- The reactive power of the generator bus can be calculated by using the following equation

$$Q_{p,cal}^{k+1} = (-1) I.P.of \left\{ (V_P^k)^* \left[\sum_{q=1}^{p-1} Y_{pq} V_q^{k+1} + \sum_{q=p}^{n} Y_{pq} V_q^k \right] \right\}$$

- The calculated reactive power may be within specified limits or it may violate the limits. If the calculated reactive power violates the specified limit for the reactive power then treat this bus as the load bus. The magnitude of the reactive power at this bus will correspond to the limit it has violated

i.e. if $Q_{p,cal}^{k+1} < Q_{p,\min}$ then $Q_p = Q_{p,\min}$

or if $Q_{p,cal}^{k+1} > Q_{p,\max}$ then $Q_p = Q_{p,\max}$

Since the bus is treated as load bus, take actual value of V_p^k for $(k+1)^{th}$ iteration

i.e. $|V_p^k|$ need not be replaced by $|V_p|_{sep}$ when the generator bus is treated as load bus. Go to step (10).

9) For generator bus the magnitude of voltage does not change and so for all iterations the magnitude of bus voltage is the specified value only. The phase of the bus voltage can be calculated as shown below.

$$V_{p,temp}^{K+1} = \frac{1}{Y_{pp}}\left[\frac{P_p - jQ_p}{(V_P^k)^*} - \sum_{q=1}^{p-1} Y_{pq}V_q^{K+1} - \sum_{q=p}^{n} Y_{pq}V_q^{K}\right]$$

$$\delta_p^{k+1} = \tan^{-1}\left[\frac{I.P.\ of\ \ V_{p,temp}^{k+1}}{R.P.\ of\ \ V_{p,temp}^{k+1}}\right]$$

Now the (k+1)th iteration voltage of the generator bus is given by

$$V_p^{k+1} = \ |V_p\ |_{spe}\ \delta_p^{k+1}$$

Where $|V_p\ |_{spe}$ is magnitude of specified voltage.

After calculating V_p^{k+1} for generator bus go to step (12)

10) For the load bus the (k+1)th iteration value of load bus-p voltage, V_p^{k+1} can be calculated with the following equation.

$$V_p^{K+1} = \frac{1}{Y_{pp}}\left[\frac{P_p - jQ_p}{(V_P^k)^*} - \sum_{q=1}^{p-1} Y_{pq}V_q^{K+1} - \sum_{q=p+1}^{n} Y_{pq}V_q^{K}\right]$$

11) An acceleration factor ∝ can be used for faster convergence. If acceleration factor is specified then modify the (k+1)th iteration value of bus-p voltage using the following equation.

$$V_{p,acc}^{k+1} = V_p^k + \alpha(V_p^{k+1} - V_p^k)$$

Then set $V_p^{k+1} = V_{p,acc}^{k+1}$

12) Calculate the change in bus-p voltage, using the relation

$$\Delta V_p^k \ = \ V_p^{k+1} - V_p^k$$

Advance the bus count by 1 to evaluate other values of V_p^{k+1} and ΔV_p^k

13) Check all the buses have been taken into account or not. If yes, go to the next step, Otherwise go back to step (6).
14) Determine the largest absolute value of change in voltage $|\Delta V|_{max}$
15) If $|\Delta V|_{max}$ is less than the pre specified tolerance €, then evaluate line flows and print the bus voltages and line flows. If not, advance the iteration count K= K+1 and go back to step (5).

Important Note

- For load bus the active and reactive powers are considered as negative, when generation of active power(P_G) and reactive power(Q_G) are not specified for the given power system network. When both generation and demand of load bus are given then the active power is $P=P_G-P_D$ and reactive power is $Q=Q_G-Q_D$.

- For generator bus the active and reactive powers are always considered as positive.
- In a particular iteration if the calculated reactive power for the generator bus violates the given limits, then in that iteration that bus is taken as load bus. But the signs of active and reactive powers will remains positive, even if the bus is treated as load bus.
- In buses having generators and loads connected to it, either the net power will be specified or the generator and load power will be individually specified.

3.6.1.1 Acceleration factor (α)

- In the GS method, a large number of iterations are required to arrive at the specified convergence. The rate of convergence can be increased by the use of acceleration factor to the solution obtained after each iteration. The acceleration factor is a multiplier that enhances correction between the values of voltage in two successive iterations.
- If the acceleration factor(α) is specified then modify the $(k+1)^{th}$ iteration value of the bus-p voltage using the following equation

$$V_{p,acc}^{k+1} = V_p^k + \alpha(V_p^{k+1} - V_p^k)$$

Then set $V_p^{k+1} = V_{p,acc}^{k+1}$

- The choice of a specific value of acceleration n factor depends upon the system parameters. The optimum value of $\propto$ is 1.6

Problem-2: Given the simultaneous equations

$$x_1+x_2=4, \quad 2x_1+x_2=5$$

Using initial values $x_1^0=2$ and $x_2^0= 3$, write down the values for x_1^1 and x_2^1 using GS method.

Solution: From the given equation, we can get

$$x_1=4-x_2, \quad x_2=5-2x_1$$

By using GS method

$x_1^1 = 4 – x_2^0 =4 – 3= 1$

$x_2^1 = 5 – 2x_1^1=5 – 2x_2 = 5-2 = 3$

Problem 3: The system data for load flow solution are given in the following tables. Determine the voltages at various buses at the end of the first iteration by using GS method. Take α=1.6

Table1:Line admittances	
Bus code	Admittance
1-2	2-j8
1-3	1-j4
2-3	0.666-j2.664
2-4	1-j4
3-4	2-j8

Table-2: Bus specifications				
Bus code	P	Q	V	Remarks
1	-	-	1.06	Slack bus
2	0.5	0.2	-	PQ bus
3	0.4	0.3	-	PQ bus
4	0.3	0.1	-	PQ bus

Solution :

From the table-1,the admittances of various branches are calculated as follows

y_{12}= 2-j8, y_{13}=1-j4, y_{23}= 0.666-j2.664, y_{24}=1-j4, y_{34}= 2-j8

Y_{11}= y_{12} + y_{13}= 2-j8 +1-j4 = 3-j12

Y_{22}= y_{12}+ y_{23} + y_{24}= 2- j8 +0.666- j2.664+ 1- j4=3.666- j 14.664

Y_{33}= y_{31}+y_{32}+ y_{34}= 1- j4 +0.666-j2.664 +2-j8 = 3.666-j14.664

Y_{44}= y_{42}+y_{43}= 1- j4 +2- j8 =3-j12

Y_{12}=Y_{21}= -y_{12} =-2+j8

Y_{13}=Y_{31} = -y_{13} = -1+j4

Y_{14}= Y_{41}=0

Y_{23}=Y_{32}= -y_{23}= -0.666+j2.664

Y_{24}= Y_{42} = -y_{24} = -1+j4

The bus admittance matrix of the given power system is

$$Y_{BUS} = \begin{bmatrix} Y_{11} & Y_{12} & Y_{13} & Y_{14} \\ Y_{21} & Y_{22} & Y_{23} & Y_{24} \\ Y_{31} & Y_{32} & Y_{33} & Y_{34} \\ Y_{41} & Y_{42} & Y_{43} & Y_{44} \end{bmatrix}$$

$$= \begin{bmatrix} 3-j12 & -2+j8 & -1+j4 & 0 \\ -2+j8 & 3.666-j14.664 & -0.666+j2.664 & -1+j4 \\ -1+j4 & -0.666+j2.664 & 3.666-j14.664 & -2+j8 \\ 0 & -1+j4 & -2+j8 & 3-j12 \end{bmatrix}$$

The initial values of the bus voltages are considered as 1p.u except the slack bus.

$V_2^0 = V_3^0 = V_4^0 = 1+j0$

The bus 1 is slack bus and so its voltage remains at the specified value for all iterations.

i.e $V_1^0 = V_1^1 = V_1^k = 1.06+j0.0$

Since the buses are PQ buses the specified real and reactive powers are considered as load powers. Therefore negative sign is attached to the specified powers. For first iteration k=0, the system has four buses and p will take values from 1 to 4. Here all the buses are load buses except bus1.

The calculations of bus voltages for first iterations are shown below.

$V_1^0 = V_1^1 = 1.06 + j0$ (sl ack bus)

$$V_p^{K+1} = \frac{1}{Y_{pp}}\left[\frac{P_p - jQ_p}{(V_P^k)^*} - \sum_{q=1}^{p-1} Y_{pq}V_q^{K+1} - \sum_{q=p+1}^{n} Y_{pq}V_q^K\right]$$

$$V_2^1 = \frac{1}{Y_{22}}\left[\frac{P_2 - jQ_2}{1-j0} - Y_{21}V_1^1 - Y_{23}V_3^0 - Y_{24}V_4^0\right]$$

$$= \frac{1}{3.666 - j14.664}\left[\frac{-0.5 + j0.2}{1-j0} - (-2+j8)1.06 - (-0.666 + j2.664)(1+j0) - (-1+j4)(1+j0)\right]$$

$$= \frac{-0.5 + j0.2 + 2.12 - j8.48 + 0.666 - j2.664 + 1 - j4}{3.666 - j14.664}$$

$$= \frac{3.286 - j14.944}{3.666 - j14.664}$$

= (1.0119-j0.029) p.u

$$V_{2,acc}^1 = V_2^0 + \alpha(V_2^1 - V_2^0)$$

=1+1.6(1.0119-j0.029-1)

= (1.019-j0.0464) pu

Now $V_2^1 = V_{2,acc}^1$ = (1.019-j0.0464) pu

$$V_3^1 = \frac{1}{Y_{33}}\left[\frac{P_3 - jQ_3}{1-j0} - Y_{31}V_1^1 - Y_{32}V_2^1 - Y_{34}V_4^0\right]$$

$$= \frac{1}{3.666 - j14.664}\left[\frac{-0.4 + j0.3}{1-j0} - (-1+j4)1.06 - (-0.666 + j2.664)(1.019 - j0.0464) - (-2+j8)(1+j0)\right]$$

$$= \frac{-0.4 + j0.3 + 1.06 - j4.24 + 0.555 - j2.755 + 2 - j8}{3.666 - j14.664}$$

$$= \frac{3.215 - j14.6855}{3.666 - j14.664}$$

= (0.9942-j0.0293) pu

$$V_{3,acc}^{1} = V_3^0 + \alpha(V_3^1 - V_3^0)$$

= 1+1.6(0.9942-j0.0293-1)

= (0.9907-j0.0469) pu

Now $V_3^1 = V_{3,acc}^1 =$ 0.9907-j0.0469

$$V_4^1 = \frac{1}{Y_{44}}\left[\frac{P_4 - jQ_4}{1 - j0} - Y_{41}V_1^1 - Y_{42}V_2^1 - Y_{43}V_3^1\right]$$

$$= \frac{1}{3 - j12}\left[\frac{-0.3 + j0.1}{1 - j0} - 0 \times 1.06 - (-1 + j4)(1.019 - j0.0464) - (-2 + j8)(0.9907 - j0.0469)\right]$$

$$= \frac{2.1396 - j12.0418}{3 - j12}$$

= (0.9864-j0.0683) pu

$$V_{4,acc}^{1} = V_4^0 + \alpha(V_4^1 - V_4^0)$$

= 1+1.6(0.9864-j0.0683-1)

= (0.9762-j0.1093) pu

Now $V_4^1 = V_{4,acc}^1$ = (0.9762-j0.1093)pu

The bus voltages at the end of first iteration are

V_1^1 = (1.06+j0) p.u

V_2^1 = (1.019-j0.0464) p.u

V_3^1 = (0.9907-j0.0469) p.u

V_4^1 =(0.9762-j0.1093) p.u

Problem-4: The system data for load flow solution are given in the following tables. Determine the voltages at the end of first iteration by GS method. Take α=1 and bus specifications are given in the following table.

Table1:Line admittances	
Bus code	Admittance
1-2	1-j5
1-3	1.2-j4
2-3	0.5-j4
2-4	1.1-j2
3-4	1.2-j3

Table-2: Bus specifications				
Bus code	P	Q	V	Remarks
1	-	-	1.06	Slack bus
2	0.5	$0.12 \le Q_2 \le 0.5$	1.04	PV bus
3	0.4	0.3	-	PQ bus
4	0.2	0.1	-	PQ bus

$$\begin{bmatrix} 2.2-j9 & -1+j5 & -1.2+j4 & 0 \\ -1+j5 & 2.6-j11 & -0.5+j4 & -1.1+j2 \\ -1.2+j4 & -0.5+j4 & 2.9-j11 & -1.2+j3 \\ 0 & -1.1+j2 & -1.2+j3 & 2.3-j5 \end{bmatrix}$$

- In the given system bus-1 is slack, bus-2 is generator bus and bus-3 , bus-4 are load buses. The initial voltages of load buses are assumed as (1+j0) pu. For slack and generator buses the specified voltages are used as initial values.
  $V_1^0 = V_1^1 = \ldots\ldots\ldots\ldots.. = V_1^k = 1.06$ (slack bus)
  $V_2^0 = 1.04+j0$ (generator bus) [initial phase is assumed as '0']
  $V_3^0 = (1+j0)$ pu (load bus)
  $V_4^0 = (1+j0)$ pu (load bus)
- For the generator bus the specified powers are considered as positive powers but for load buses the specified powers are considered as –ve powers.
- For first iteration, k=0, in each iteration the slack bus voltage need not be recalculated. In each iteration the reactive power for generator bus as to be calculated and checked for violation of the specified limits. If the limits are violated then it is treated as load bus.
- The calculation of bus voltage for first iteration is shown below.
  $V_1^1 = V_1^0 = (1.06+j0)$ pu ($\because$ bus 2 is slack bus)
- The bus-2 is a generator bus and to calculate its reactive power Q_2

$$Q_{p,cal}^{k+1} = (-1) I.P.of \left\{ (V_P^k)^* \left[\sum_{q=1}^{p-1} Y_{pq} V_q^{k+1} + \sum_{q=p}^{n} Y_{pq} V_q^k \right] \right\}$$

Here p=2, k=0, n=4

$$\therefore Q_{2,cal}{}^{1} = -I.Pof\left\{\left(V_2{}^0\right)^*\left[Y_{21}V_1^1 + Y_{22}V_2{}^0 + Y_{23}V_3{}^0 + Y_{24}V_4{}^0\right]\right\}$$

Note: Here $\left|V_2{}^0\right|$ is same as $|V_2|_{\text{spec}}$ and so $V_2{}^0$ is used for calculation as such if it is not same then we have to replace $\left|V_2{}^0\right|$ with $|V_2|_{\text{spec.}}$

$$\therefore\ Q_{2,cal}{}^{1} = -I.P\ of\left\{1.04\left[(-1+j5)1.06+(2.6-j11)1.04+(-0.5+j4)1+(-1.1+j2)1\right]\right\}$$

$$= -I.P\ of\left[0.0458 - j0.145\right] \quad = 0.145 \text{ pu}$$

- The specified range for Q_2 is $0.12 \le Q_2 \le 0.5$. The calculated value of Q_2 is within this range and so the reactive limit is not violated. Therefore the bus can be treated as generator bus.

  Now $P_2 = 0.5, Q_2 = 0.145, V_2{}^0 = 1.04 + j0$

- Since the bus-2 is treated as generator bus, then $\left|V_2{}^1\right| = |V_2|_{spec}$ and phase of $V_2{}^1$ is given by the phase of $V_2{}^1{}_{temp}$.

$$V_{p,temp}{}^{k+1} = \frac{1}{Y_{pp}}\left[\frac{P_p - jQ_p}{\left(V_p{}^k\right)^*} - \sum_{q=1}^{p-1} Y_{pq}Vq^{k+1} - \sum_{q=p+1}^{n} Y_{pq}V_q{}^k\right]$$

$$V_{2,temp}^1 = \frac{1}{Y_{22}}\left[\frac{P_2 - jQ_2}{(V_2^0)^*} - Y_{21}V_1^1 - Y_{23}V_3^0 - Y_{24}V_4^0\right]$$

$$= \frac{1}{2.6 - j11}\left[\begin{matrix}\dfrac{0.5 - j0.145}{1.04 - j0} - (-1+j5)(1.06+j0) - (-0.5+j4)(1+j0) \\ -(-1.1+j2)(1+j0)\end{matrix}\right]$$

$$= \frac{3.1408 - j11.44}{2.6 - j11}$$

$$= \frac{11.8633\angle -74.65^0}{11.3031\angle -76.7^0}$$

$$= 1.0496\angle 2.05^0$$

$$\therefore\ V_2{}^1{}_{temp} = 1.0496\angle 2.05^0\ pu$$

$$\therefore\ \delta_2{}^1 = \angle V_2{}^1{}_{temp} = 2.05^0$$

$$\therefore V_2{}^1 = |V_2|_{spec}\angle\delta_2{}^1 = 1.04\angle 2.05\,pu = 1.0393 + j0.0372\,pu$$

- The bus-3 and bus-4 are load buses. The voltages of load bus are calculated using the following equation

$$V_p^{\,k+1} = \frac{1}{Y_{pp}}\left[\frac{P_p - jQ_p}{\left(V_p^{\,k}\right)^*} - \sum_{q=1}^{p-1} Y_{pq}V_q^{\,k+1} - \sum_{q=p+1}^{n} Y_{pq}V_q^{\,k}\right]$$

$$V_3^1 = \frac{1}{Y_{33}}\left[\frac{P_3 - jQ_3}{\left(V_3^0\right)^*} - Y_{31}V_1^1 - Y_{32}V_2^1 - Y_{34}V_4^0\right]$$

$$= \frac{1}{2.9 - j11}\left[\begin{array}{l}\frac{-0.4 + j0.3}{1 - j0} - (-1.2 + j4)1.06 - (-0.5 + j4)(1.0489 + j0.0375) \\ -(1.2 + j3)(1 + j0)\end{array}\right]$$

$$= \frac{2.7405 - j11.0786}{2.9 - j11}$$

$$= \frac{11.4125\angle -76.11^0}{11.3759\angle -75.23^0}$$

$$= 1.0032\angle -0.88^0$$
$$= (1.0031 - j0.01545)pu$$

$$V_4^1 = \frac{1}{Y_{44}}\left[\frac{P_4 - jQ_4}{\left(V_4^0\right)^*} - Y_{41}V_1^1 - Y_{42}V_2^1 - Y_{43}V_3^1\right]$$

$$= \frac{1}{2.3 - j5}\left[\begin{array}{l}\frac{-0.2 + j0.1}{1 - j0} - (0 \times 1.06) - (-1.1 + j2)(1.0393 + j0.0372) \\ -(-1.2 + j3)(1.0031 - j0.01545)\end{array}\right]$$

$$= \frac{2.1751 - j4.9655}{2.3 - j5}$$

$$= \frac{5.4210\angle -66.34^0}{5.5036\angle -65.3^0}$$

$$= 0.9850\angle -1.04^0 \ pu$$
$$= (0.9848 - j0.0178)pu$$

$\therefore$ The bus voltages at the end of the first iteration are

$$V_1^1 = 1.06 + j0 = 1.06\angle 0^0 \text{ pu}$$

$$V_2^1 = 1.0393 + j0.0371 = 1.04\angle 2.05^0 \text{ pu}$$

$$V_3^1 = 1.0031 - j0.01545 = 1.0032\angle -0.88^0 \text{ pu}$$

$$V_4^1 = 0.9848 - j0.0179 = 0.9850\angle -1.04^0 \text{ pu.}$$

Problem-5: In the problem-4, the reactive power constraints on generator bus-2 changed to $0.2 \le Q_2 \le 0.5$. With the other data in the previous problem remaining same, find the voltages of all the buses at the end of the first iteration by GS method?

Solution: The formation of bus impedance matrix and calculation of $Q^1_{2,cal}$ are same in the above problem. The $Q^1_{2,cal}$ corresponding to initial value V_2^0 = 1.4+j0 is 0.145p.u. This value of Q_2 violates the lower limit of the specified range for Q_2. Therefore the reactive power generation for bus-2 is fixed at lower limit i.e 0.2 and bus-2 is treated as load bus for first iteration. Now V_2^0 = 1.0+j0, similar to other load buses for first iteration. But P and Q are considered as positive for bus-2 and P and Q are negative for other load buses.

The bus voltages are obtained as follows

$V_1^1 = V_1^0$ = 1.06+j0 (slack bus)

$$V_p^{K+1} = \frac{1}{Y_{pp}}\left[\frac{P_p - jQ_p}{(V_P^k)^*} - \sum_{q=1}^{p-1} Y_{pq}V_q^{K+1} - \sum_{q=p+1}^{n} Y_{pq}V_q^K\right]$$

$$V_2^1 = \frac{1}{Y_{22}}\left[\frac{P_2 - jQ_2}{(V_2^0)^*} - Y_{21}V_1^1 - Y_{23}V_3^0 - Y_{24}V_4^0\right]$$

$$= \frac{1}{2.6 - j11}\left[\begin{array}{l}\frac{0.5 - j0.2}{1 - j0} - (-1 + j5)(1.06 + j0) - (-0.5 + j4)(1 + j0) \\ -(-1.1 + j2)(1 + j0)\end{array}\right]$$

$$= \frac{3.16 - j11.5}{2.6 - j11}$$

$$= \frac{11.9263\angle -74.65^0}{11.3031\angle -76.7^0} = 1.0551\angle 2.05^0$$

= 1.0544+j0.0379 p.u

$$V_3^1 = \frac{1}{Y_{33}}\left[\frac{P_3 - jQ_3}{\left(V_3^0\right)^*} - Y_{31}V_1^1 - Y_{32}V_2^1 - Y_{34}V_4^0\right]$$

$$= \frac{1}{2.9 - j11}\left[\begin{array}{l}\frac{-0.4 + j0.3}{1 - j0} - (-1.2 + j4)1.06 - (-0.5 + j4)(1.0544 + j0.0379) \\ -(-1.2 + j3)(1 + j0)\end{array}\right]$$

$$= \frac{2.7508 - j8.4387}{2.9 - j11}$$

$$= \frac{8.8757\angle -71.95^0}{11.3759\angle -75.23^0}$$

$$= 0.7802\angle 3.28^0$$
$$= (0.7789 + j0.0446)pu$$

$$V_4^1 = \frac{1}{Y_{44}}\left[\frac{P_4 - jQ_4}{(V_4^0)^*} - Y_{41}V_1^1 - Y_{42}V_2^1 - Y_{43}V_3^1\right]$$

$$= \frac{1}{2.3 - j5}\left[\begin{array}{l}\frac{-0.2 + j0.1}{1 - j0} - (0 \times 1.06) - (-1.1 + j2)(1.0544 + j0.0379) \\ -(-1.2 + j3)(0.7789 - j0.0446)\end{array}\right]$$

$$= \frac{2.1041 - j4.2503}{2.3 - j5}$$

$$= \frac{4.7426\angle -63.66^0}{5.5036\angle -65.3^0}$$

$$= 0.8617\angle 1.64^0\ pu$$
$$= (0.8613 + j0.0247)pu$$

$\therefore$ The bus voltages at the end of the first iteration are

$$V_1^1 = 1.06 + j0 = 1.06\angle 0^0 \text{ pu}$$

$$V_2^1 = 1.0544 + j0.0379 = 1.055\angle 2.05^0 \text{ pu}$$

$$V_3^1 = 0.7789 + j0.0446 = 0.7802\angle 3.28^0 \text{ pu}$$

$$V_4^1 = 0.8614 + j0.0247 = 0.8617\angle 1.64^0 \text{ pu.}$$

Problem-6: In the problem-4, the reactive power constraints on generator bus-2 changed to $0.01 \leq Q_2 \leq 0.12$. With the other data in the previous problem remaining same, find the voltages of all the buses at the end of the first iteration by GS method?

Solution: The formation of bus impedance matrix and calculation of $Q^1_{2,cal}$ are same in the above problem. The $Q^1_{2,cal}$ corresponding to initial value V_2^0 = 1.4+j0 is 0.145p.u. This value of Q_2 violates the upper limit of the specified range for Q_2. Therefore the reactive power generation for bus-2 is fixed at upper limit i.e 0.12 and bus-2 is treated as load bus for first iteration. Now V_2^0 = 1.0+j0, similar to other load buses for first iteration. But P and Q are considered as positive for bus-2 and P and Q are negative for other load buses.

The bus voltages are obtained as follows

$V_1^1 = V_1^0$ = 1.06+j0 (slack bus)

$$V_p^{K+1} = \frac{1}{Y_{pp}}\left[\frac{P_p - jQ_p}{(V_P^k)^*} - \sum_{q=1}^{p-1} Y_{pq}V_q^{K+1} - \sum_{q=p+1}^{n} Y_{pq}V_q^K\right]$$

$$V_2^1 = \frac{1}{Y_{22}}\left[\frac{P_2 - jQ_2}{(V_2^0)^*} - Y_{21}V_1^1 - Y_{23}V_3^0 - Y_{24}V_4^0\right]$$

$$= \frac{1}{2.6 - j11}\left[\begin{matrix}\frac{0.5 - j0.12}{1 - j0} - (-1 + j5)(1.06 + j0) - (-0.5 + j4)(1 + j0) \\ -(-1.1 + j2)(1 + j0)\end{matrix}\right]$$

$$= \frac{3.16 - j11.42}{2.6 - j11}$$

$$= \frac{11.8491\angle -74.53^0}{11.3031\angle -76.7^0} = 1.0551\angle 2.17^0$$

= 1.0475+j0.0397 p.u

$$V_3^1 = \frac{1}{Y_{33}}\left[\frac{P_3 - jQ_3}{\left(V_3^0\right)^*} - Y_{31}V_1^1 - Y_{32}V_2^1 - Y_{34}V_4^0\right]$$

$$= \frac{1}{2.9 - j11}\left[\begin{matrix}\frac{-0.4 + j0.3}{1 - j0} - (-1.2 + j4)1.06 - (-0.5 + j4)(1.0475 + j0.0379) \\ -(-1.2 + j3)(1 + j0)\end{matrix}\right]$$

$$= \frac{2.7546 - j8.4102}{2.9 - j11}$$

$$= \frac{8.8498\angle -71.85^0}{11.3759\angle -75.23^0}$$

$$= 0.7779\angle 3.38^0$$

$$= (0.7766 + j0.0457)p.u$$

$$V_4^1 = \frac{1}{Y_{44}}\left[\frac{P_4 - jQ_4}{\left(V_4^0\right)^*} - Y_{41}V_1^1 - Y_{42}V_2^1 - Y_{43}V_3^1\right]$$

$$= \frac{1}{2.3 - j5}\left[\begin{matrix}\frac{-0.2 + j0.1}{1 - j0} - (0 \times 1.06) - (-1.1 + j2)(1.0475 + j0.0397) \\ -(-1.2 + j3)(0.7766 - j0.0457)\end{matrix}\right]$$

$$= \frac{2.1007 - j4.2263}{2.3 - j5}$$

$$= \frac{4.7196\angle -63.56^0}{5.5036\angle -65.3^0}$$

$$= 0.8575\angle 1.74^0 \; pu$$

$$= (0.8571 - j0.0259)p.u$$

$\therefore$ The bus voltages at the end of the first iteration are

$V_1^1 = 1.06 + j0 = 1.06\angle 0^0$ pu

$V_2^1 = 1.0475 + j0.0397 = 1.0483\angle 2.17^0$ pu

$V_3^1 = 0.7766 + j0.0457 = 0.7779\angle 3.37^0$ pu

$V_4^1 = 0.8571 + j0.0259 = 0.8575\angle 1.74^0$ pu.

3.6.1.2 Advantages and disadvantages of GS method

Advantages of GS method

i) Calculations are simple and so programming task is lesser.
ii) The memory requirement is less.
iii) Use full for small size systems.

Disadvantages of GS method

i) Requires large number of iterations to reach convergence.
ii) Not suitable for large systems.
iii) Convergence time increases with the size of the system.

3.6.2 NEWTON-RAPSHON METHOD

3.6.2.1 Development of load flow equations

The NR method of load flow analysis is an iterative method which approximates the set of non-linear simultaneous load flow equations to a set of linear simultaneous equations using Taylor's series expansion and the terms are limited to first order approximation. The equations for NR method are derived as follows

Case 1: In rectangular form

- We know that for an n-bus system

$$P_p - jQ_p = V_p^* I_p = V_p^* \sum_{q=1}^{n} Y_{pq} V_q$$

Let $V_p = e_p + jf_p$,

$V_q = e_q + jf_q$

$Y_{pq} = G_{pq} - jB_{pq}$

Where e_p & f_p are real and imaginary parts of V_p respectively.

e_q & f_q are real and imaginary parts of V_q respectively.

G_{pq} & B_{pq} are conductance and susceptance of admittance Y_{pq}

$$P_p - jQ_p = V_p^* \sum_{q=1}^{n} Y_{pq} V_q$$

$$= (e_p - jf_p) \sum_{q=1}^{n} [G_{pq} - jB_{pq})(e_q + jf_q)$$

$$= (e_p - jf_p) \sum_{q=1}^{n} [(G_{pq} e_q + f_q B_{pq}) + j(f_q G_{pq} - e_q B_{pq})]$$

$$\Rightarrow P_p - jQ_p = \sum_{q=1}^{n} e_p (e_q G_{pq} + f_q B_{pq}) + f_p (f_q G_{pq} - e_q B_{pq}) -$$

$$j \sum_{q=1}^{n} f_p (e_q G_{pq} + f_q B_{pq}) - e_p (f_q G_{pq} - e_q B_{pq})$$

By comparing real and imaginary parts on both sides

$$P_p = \sum_{q=1}^{n} [e_p (e_q G_{pq} + f_q B_{pq}) + f_p (f_q G_{pq} - e_q B_{pq})] \quad \text{--- (3.20)}$$

$$Q_p = \sum_{q=1}^{n} [f_p (e_q G_{pq} + f_q B_{pq}) - e_p (f_q G_{pq} - e_q B_{pq})] \quad \text{--- (3.21)}$$

$$|V_p|^2 = e_p^2 + f_p^2 \quad \text{--- (3.22)}$$

- The above three set of equations are the load flow equations and it can be seen that they are non-linear equations in terms of the real and imaginary components of nodal voltages.
- The voltages of a slack bus will be a known quantity in a power system and so it need not be solved. For load buses P_p and Q_p will be specified and we have to solve V_p. For a generator bus P_p and $|V_p|$ will be specified and we have to solve Q_p and phase angle of V_p i.e. δ_p.

Case 2: In polar form

- We can also formulate the load flow problem using NR method in polar coordinates.
- Say for any buses p and q we have

$$V_p = |V_p| e^{j\delta_p} \quad V_p^* = |V_p| e^{-j\delta_p}$$

$$V_q = |V_q| e^{j\delta_q}, \quad Y_{pq} = |Y_{pq}| e^{-j\theta_{pq}}$$

For any bus 'p' we have

$$P_p - jQ_p = V_p^* \sum_{q=1}^{n} Y_{pq} V_q$$

$$= \left|V_p\right| e^{-j\delta_p} \sum_{q=1}^{n} \left|Y_{pq}\right| e^{-j\theta_{pq}} \left|V_q\right| e^{j\delta_q}$$

$$= \sum_{q=1}^{n} \left|V_p V_q Y_{pq}\right| e^{-j(\theta_{pq}+\delta_p-\delta_q)}$$

By comparing real and imaginary parts on both sides

$$P_p = \sum_{q=1}^{n} \left|V_p Y_{pq} V_q\right| \cos(\theta_{pq} + \delta_p - \delta_q)$$

$$= \left|V_p^2 Y_{pp}\right| \cos\theta_{pp} + \sum_{\substack{q=1 \\ q\neq p}}^{n} \left|V_p Y_{pq} V_q\right| \cos(\theta_{pq} + \delta_p - \delta_q) \qquad \text{--- (3.23)}$$

$$Q_p = \sum_{q=1}^{n} \left|V_p Y_{pq} V_q\right| \sin(\theta_{pq} + \delta_p - \delta_q)$$

$$= \left|V_p^2 Y_{pp}\right| \sin\theta_{pp} + \sum_{\substack{q=1 \\ q\neq p}}^{n} \left|V_p Y_{pq} V_q\right| \sin(\theta_{pq} + \delta_p - \delta_q) \qquad \text{--- (3.24)}$$

The above two equations are the load flow equations in polar form.

3.6.2.2 Mathematical background for N-R method

- Let (x_1, x_2,------------x_n) be a set of unknown variables and (y_1,y_2,-----------y_n) be set of specified quantities. Now the specified quantities can be expressed as a non-linear function of unknown variables as shown below.

$$\left.\begin{aligned} y_1 &= f_1(x_1, x_2, \text{--------------} x_n) \\ y_2 &= f_2(x_1, x_2, \text{--------------} x_n) \\ &\vdots \\ &\vdots \\ y_n &= f_n(x_1, x_2, \text{--------------} x_n) \end{aligned}\right\} \qquad \text{--- (3.25)}$$

- Let us assume an approximate initial solution $x_1^0, x_2^0, -------- x_n^0$ for the above equations. The prefix zero refers to zeroth iteration in the processing of solving the above non-linear equations. Let $\Delta x_1^0, \Delta x_2^0, -------- \Delta x_n^0$ are the corrections required for x_1^0 , x_2^0 ________ x_n^0 respectively for the next better estimation.

- Now the non-linear equations can be expressed as shown below, i.e. they can be expressed as functions of modified variables $x_1^0+\Delta x_1^0, x_2^0+\Delta x_2^0, ------ x_n^0+\Delta x_n^0$

$$\left.\begin{aligned} y_1 &= f_1(x_1^0+\Delta x_1^0, x_2^0+\Delta x_2^0, --------- x_n^0+\Delta x_n^0) \\ y_2 &= f_2(x_1^0+\Delta x_1^0, x_2^0+\Delta x_2^0, --------- x_n^0+\Delta x_n^0) \\ &\vdots \\ &\vdots \\ y_n &= f_n(x_1^0+\Delta x_1^0, x_2^0+\Delta x_2^0, --------- x_n^0+\Delta x_n^0) \end{aligned}\right\} \quad \text{--- (3.26)}$$

The above equations are linearized about the initial guess using Tailor's expansion.

- The linearized equations with second order and higher order derivatives neglected are given below.

$$\left.\begin{aligned} y_1 &= f_1(x_1^0, x_2^0, \text{-------} x_n^0) + \Delta x_1^0\left(\frac{\partial f_1}{\partial x_1}\right)^0 + \Delta x_2^0\left(\frac{\partial f_1}{\partial x_2}\right)^0 + --- + \Delta x_n^0\left(\frac{\partial f_1}{\partial x_n}\right)^0 \\ y_2 &= f_2(x_1^0, x_2^0, \text{-------} x_n^0) + \Delta x_1^0\left(\frac{\partial f_2}{\partial x_1}\right)^0 + \Delta x_2^0\left(\frac{\partial f_2}{\partial x_2}\right)^0 + ---- + \Delta x_n^0\left(\frac{\partial f_2}{\partial x_n}\right)^0 \\ &\vdots \\ &\vdots \\ y_n &= f_n(x_1^0, x_2^0, \text{--------} x_n^0) + \Delta x_1^0\left(\frac{\partial f_n}{\partial x_1}\right)^0 + \Delta x_2^0\left(\frac{\partial f_n}{\partial x_2}\right)^0 + ---- + \Delta x_n^0\left(\frac{\partial f_n}{\partial x_n}\right)^0 \end{aligned}\right\} \quad \text{--- (3.27)}$$

Note: Tailor's series expansion for any function f(x) is given by

$$f(x^0+\Delta x^0) = f(x^0) + \Delta x^0\left(\frac{\partial f}{\partial x}\right)^0 + \frac{(\Delta x^0)^2}{2!}\left(\frac{\partial^2 f}{\partial x^2}\right)^0 + ---------$$

- The above equations can be written as

$$\left.\begin{aligned} y_1 - f_1(x_1^0, x_2^0, --- x_n^0) &= \Delta x_1^0\left(\frac{\partial f_1}{\partial x_1}\right)^0 + \Delta x_2^0\left(\frac{\partial f_1}{\partial x_2}\right)^0 + ---- + \Delta x_n^0\left(\frac{\partial f_1}{\partial x_n}\right)^0 \\ y_2 - f_2(x_1^0, x_2^0, --- x_n^0) &= \Delta x_1^0\left(\frac{\partial f_2}{\partial x_1}\right)^0 + \Delta x_2^0\left(\frac{\partial f_2}{\partial x_2}\right)^0 + ---- + \Delta x_n^0\left(\frac{\partial f_2}{\partial x_n}\right)^0 \\ &\vdots \\ &\vdots \\ y_n - f_n(x_1^0, x_2^0, --- x_n^0) &= \Delta x_1^0\left(\frac{\partial f_n}{\partial x_1}\right)^0 + \Delta x_2^0\left(\frac{\partial f_n}{\partial x_2}\right)^0 + ---- + \Delta x_n^0\left(\frac{\partial f_n}{\partial x_n}\right)^0 \end{aligned}\right\} \quad \text{---(3.28)}$$

Let $y_1 - f_1(x_1^0, x_2^0, ------x_n^0) = \Delta y_1$

$y_2 - f_2(x_1^0, x_2^0, ------x_n^0) = \Delta y_2$

$\vdots \qquad \vdots$

$\vdots \qquad \vdots$

$y_n - f_1(x_1^0, x_2^0, ------x_n^0) = \Delta y_n$

- Now the above equations can be written as

$$\left.\begin{aligned}
\Delta y_1 &= \Delta x_1^0\left(\frac{\partial f_1}{\partial x_1}\right)^0 + \Delta x_2^0\left(\frac{\partial f_1}{\partial x_2}\right)^0 + ------- + \Delta x_n^0\left(\frac{\partial f_1}{\partial x_n}\right)^0 \\
\Delta y_2 &= \Delta x_1^0\left(\frac{\partial f_2}{\partial x_1}\right)^0 + \Delta x_2^0\left(\frac{\partial f_2}{\partial x_2}\right)^0 + ------- + \Delta x_n^0\left(\frac{\partial f_2}{\partial x_n}\right)^0 \\
&\vdots \qquad \vdots \\
&\vdots \qquad \vdots \\
\Delta y_n &= \Delta x_1^0\left(\frac{\partial f_n}{\partial x_1}\right)^0 + \Delta x_2^0\left(\frac{\partial f_n}{\partial x_2}\right)^0 + ------- + \Delta x_n^0\left(\frac{\partial f_n}{\partial x_n}\right)^0
\end{aligned}\right\} \quad \text{--- (3.29)}$$

- The above equations can be written in matrix form

$$\begin{bmatrix} \Delta y_1 \\ \Delta y_2 \\ \vdots \\ \vdots \\ \Delta y_3 \end{bmatrix} = \begin{bmatrix} \left(\frac{\partial f_1}{\partial x_1}\right)^0 & \left(\frac{\partial f_1}{\partial x_2}\right)^0 & \cdots & \cdots & \left(\frac{\partial f_1}{\partial x_n}\right)^0 \\ \left(\frac{\partial f_2}{\partial x_1}\right)^0 & \left(\frac{\partial f_2}{\partial x_2}\right)^0 & \cdots & \cdots & \left(\frac{\partial f_2}{\partial x_n}\right)^0 \\ \vdots & \vdots & & & \\ \vdots & \vdots & & & \\ \left(\frac{\partial f_n}{\partial x_1}\right)^0 & \left(\frac{\partial f_n}{\partial x_2}\right)^0 & \cdots & \cdots & \left(\frac{\partial f_n}{\partial x_n}\right)^0 \end{bmatrix} \begin{bmatrix} \Delta x_1^0 \\ \Delta x_2^0 \\ \vdots \\ \vdots \\ \Delta x_n^0 \end{bmatrix} \quad \text{--- (3.30)}$$

$$B = JC \quad \text{--- (3.31)}$$

Where $B = \begin{bmatrix} \Delta y_1 \\ \Delta y_2 \\ \vdots \\ \vdots \\ \Delta y_3 \end{bmatrix}$; $J = \begin{bmatrix} \left(\frac{\partial f_1}{\partial x_1}\right)^0 & \left(\frac{\partial f_1}{\partial x_2}\right)^0 & \cdots & \cdots & \left(\frac{\partial f_1}{\partial x_n}\right)^0 \\ \left(\frac{\partial f_2}{\partial x_1}\right)^0 & \left(\frac{\partial f_2}{\partial x_2}\right)^0 & \cdots & \cdots & \left(\frac{\partial f_2}{\partial x_n}\right)^0 \\ \vdots & \vdots & & & \\ \vdots & \vdots & & & \\ \left(\frac{\partial f_n}{\partial x_1}\right)^0 & \left(\frac{\partial f_n}{\partial x_2}\right)^0 & \cdots & \cdots & \left(\frac{\partial f_n}{\partial x_n}\right)^0 \end{bmatrix}$; $C = \begin{bmatrix} \Delta x_1^0 \\ \Delta x_2^0 \\ \vdots \\ \vdots \\ \Delta x_n^0 \end{bmatrix}$

- Here J is the first derivative matrix and it is called Jacobian matrix. The elements of Jacobian matrix are obtained by evaluating the first derivatives at the assumed solution. The B matrix is called residual column vector. The elements of B are the difference between the specified quantities and calculated quantities at the assumed solution. With the elements of 'J' and 'B' are known, the elements of matrix 'C' are obtained by solving matrix eqn. (3.31)
- The solution of matrix eqn.(3.31) gives $\Delta x_1^0, \Delta x_2^0, --------- \Delta x_n^0$. The next better estimation is obtained as follows.

$$\left.\begin{aligned} x_1^1 &= x_1^0 + \Delta x_1^0 \\ x_2^1 &= x_2^0 + \Delta x_2^0 \\ &\vdots \\ &\vdots \\ x_n^1 &= x_n^0 + \Delta x_n^0 \end{aligned}\right\} \quad \text{--- (3.32)}$$

With the new solution given by eqn. (3.32) the process is repeated to find next solution.

- The iterative process is terminated if any one of the following condition is satisfied.

(i) The largest (magnitude of the) element in the B matrix is less than a pre-specified value.

(ii)The largest (magnitude of the) element in the C matrix is less than pre-specified value.

3.6.2.3 Applying NR method to load flow problem

Consider a power system with n-buses. The bus-1 is usually selected as slack bus. The other buses (i.e bus-2 to bus-n) can be either generator bus or load bus. The specified quantities for load buses are P_p and Q_p and for generator buses are P_p and $|V_p|$.

Case (i) : When the power system has all the (n-1) buses are load buses.

In this case, bus-1 is slack bus and bus-2 to bus-n are load buses. Let P_2,P_3-------- P_n be the specified real powers and Q_2,Q_3, ---------Q_n be the specified reactive powers of (n-1) load buses. The unknown variables are real part of voltages e_2,e_3, --------e_n and imaginary part of bus voltages f_2,f_3, --------f_n. Now the matrix equation B=JC for this power system problem will be in the form shown below.

$$\begin{bmatrix} \Delta P_2 \\ \vdots \\ \vdots \\ \Delta P_n \\ \Delta Q_2 \\ \vdots \\ \vdots \\ \Delta Q_n \end{bmatrix} = \begin{bmatrix} \frac{\partial P_2}{\partial e_2} & \frac{\partial P_2}{\partial e_3} & \cdots & \cdots & \frac{\partial P_2}{\partial e_n} & \frac{\partial P_2}{\partial f_2} & \frac{\partial P_2}{\partial f_3} & \cdots & \cdots & \frac{\partial P_2}{\partial f_n} \\ \vdots & \vdots & \cdots & \cdots & \vdots & \vdots & \vdots & \cdots & \cdots & \vdots \\ \frac{\partial P_n}{\partial e_2} & \frac{\partial P_n}{\partial e_3} & \cdots & \cdots & \frac{\partial P_n}{\partial e_n} & \frac{\partial P_n}{\partial f_2} & \frac{\partial P_n}{\partial f_3} & \cdots & \cdots & \frac{\partial P_n}{\partial f_n} \\ \frac{\partial Q_2}{\partial e_2} & \frac{\partial Q_2}{\partial e_3} & \cdots & \cdots & \frac{\partial Q_2}{\partial e_n} & \frac{\partial Q_2}{\partial f_2} & \frac{\partial Q_2}{\partial f_3} & \cdots & \cdots & \frac{\partial Q_2}{\partial f_n} \\ \vdots & \vdots & \cdots & \cdots & \vdots & \vdots & \vdots & \cdots & \cdots & \vdots \\ \frac{\partial Q_n}{\partial e_2} & \frac{\partial Q_n}{\partial e_3} & \cdots & \cdots & \frac{\partial Q_n}{\partial e_n} & \frac{\partial Q_n}{\partial f_2} & \vdots & \cdots & \cdots & \frac{\partial Q_n}{\partial f_n} \end{bmatrix} \begin{bmatrix} \Delta e_2 \\ \vdots \\ \vdots \\ \Delta e_n \\ \Delta f_2 \\ \vdots \\ \vdots \\ \Delta f_n \end{bmatrix} \quad \text{--- (3.33)}$$

2(n-1)x2(n-1)

$$\Rightarrow \quad \begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} = \begin{bmatrix} J_1 & J_2 \\ J_3 & J_4 \end{bmatrix} \begin{bmatrix} \Delta e \\ \Delta f \end{bmatrix} \quad \text{--- (3.34)}$$

Case (ii) : When the power system has both load and generator buses

- In this case also bus-1 is slack bus and buses 2 to m be load buses and buses (m+1) to n are generator buses. Let P_2, P_3,----------P_n be the specified real power of (n-1) buses. Let Q_2, Q_3, -------Q_m be the specified reactive powers of load buses. Let $|V_{m+1}|$, $|V_{m+2}|$,-------- $|V_n|$ be the specified magnitude of voltages of generator buses. The unknown variables are real part of bus voltages e_2,e_3,----e_n and imaginary part of bus voltages f_2,f_3, --------f_n . Now the matrix equation B = JC for this case will be in the following form

$$\begin{bmatrix} \Delta P_2 \\ \vdots \\ \vdots \\ \Delta P_n \\ \Delta Q_2 \\ \vdots \\ \vdots \\ \Delta Q_m \\ |\Delta V_{m+1}|^2 \\ \vdots \\ \vdots \\ |\Delta V_n|^2 \end{bmatrix} = \begin{bmatrix} \frac{\partial P_2}{\partial e_2} & \frac{\partial P_2}{\partial e_3} & \cdots & \cdots & \frac{\partial P_2}{\partial e_n} & \frac{\partial P_2}{\partial f_2} & \frac{\partial P_2}{\partial f_3} & \cdots & \cdots & \frac{\partial P_2}{\partial f_n} \\ \vdots & \vdots & \cdots & \cdots & \vdots & \vdots & \vdots & \cdots & \cdots & \vdots \\ \frac{\partial P_n}{\partial e_2} & \frac{\partial P_n}{\partial e_3} & \cdots & \cdots & \frac{\partial P_n}{\partial e_n} & \frac{\partial P_n}{\partial f_2} & \frac{\partial P_n}{\partial f_3} & \cdots & \cdots & \frac{\partial P_n}{\partial f_n} \\ \frac{\partial Q_2}{\partial e_2} & \frac{\partial Q_2}{\partial e_3} & \cdots & \cdots & \frac{\partial Q_2}{\partial e_n} & \frac{\partial Q_2}{\partial f_2} & \frac{\partial Q_2}{\partial f_3} & \cdots & \cdots & \frac{\partial Q_2}{\partial f_n} \\ \vdots & \vdots & \cdots & \cdots & \vdots & \vdots & \vdots & \cdots & \cdots & \vdots \\ \frac{\partial Q_m}{\partial e_2} & \frac{\partial Q_m}{\partial e_3} & \cdots & \cdots & \frac{\partial Q_m}{\partial e_n} & \frac{\partial Q_m}{\partial f_2} & \frac{\partial Q_m}{\partial f_3} & \cdots & \cdots & \frac{\partial Q_m}{\partial f_n} \\ \frac{\partial |V_{m+1}|^2}{\partial e_2} & \frac{\partial |V_{m+1}|^2}{\partial e_3} & \cdots & \cdots & \frac{\partial |V_{m+1}|^2}{\partial e_n} & \frac{\partial |V_{m+1}|^2}{\partial f_2} & \frac{\partial |V_{m+1}|^2}{\partial f_3} & \cdots & \cdots & \frac{\partial |V_{m+1}|^2}{\partial f_n} \\ \vdots & \vdots & \cdots & \cdots & \vdots & \vdots & \vdots & \cdots & \cdots & \vdots \\ \frac{\partial |V_n|^2}{\partial e_2} & \frac{\partial |V_n|^2}{\partial e_3} & \cdots & \cdots & \frac{\partial |V_n|^2}{\partial e_n} & \frac{\partial |V_n|^2}{\partial f_2} & \frac{\partial |V_n|^2}{\partial f_3} & \cdots & \cdots & \frac{\partial |V_n|^2}{\partial f_n} \end{bmatrix} \begin{bmatrix} \Delta e_2 \\ \vdots \\ \vdots \\ \Delta e_n \\ \Delta f_2 \\ \vdots \\ \vdots \\ \Delta f_m \\ \Delta f_{m+1} \\ \vdots \\ \vdots \\ \Delta f_n \end{bmatrix}$$

--- (3.35)

$$\Rightarrow \quad \begin{bmatrix} \Delta P \\ \Delta Q \\ |\Delta V|^2 \end{bmatrix} = \begin{bmatrix} J_1 & J_2 \\ J_3 & J_4 \\ J_5 & J_6 \end{bmatrix} \begin{bmatrix} \Delta e \\ \Delta f \end{bmatrix} \quad \text{--- (3.36)}$$

Computing the elements of Jacobian matrix

The elements of Jacobian matrix (J) can be derived from the load flow equations as given below.

Case 1: NR method in Rectangular form

The load flow equations can be written in rectangular form as given below

$$P_P = \sum_{q=1}^{n}\left[e_p\left(e_q G_{pq} + f_q B_{pq}\right) + f_p\left(f_q G_{pq} - e_q B_{pq}\right)\right]$$

$$= e_p\left(e_p G_{pp} + f_p B_{pp}\right) + f_p\left(f_p G_{pp} - e_p B_{pp}\right) + \sum_{\substack{q=1 \\ q\neq p}}^{n}\left[e_p\left(e_q G_{pq} + f_q B_{pq}\right) + f_p\left(f_q G_{pq} - e_q B_{pq}\right)\right]$$

$$Q_P = \sum_{q=1}^{n}\left[f_p\left(e_q G_{pq} + f_q B_{pq}\right) - e_p\left(f_q G_{pq} - e_q B_{pq}\right)\right]$$

$$= f_p\left(e_p G_{pp} + f_p B_{pp}\right) - e_p\left(f_p G_{pp} - e_p B_{pp}\right) + \sum_{\substack{q=1 \\ q\neq p}}^{n}\left[f_p\left(e_q G_{pq} + f_q B_{pq}\right) - e_p\left(f_q G_{pq} - e_q B_{pq}\right)\right]$$

$$\left|V_P\right|^2 = e_p{}^2 + f_p{}^2$$

J_1 : Off-diagonal elements are

$$\frac{\partial P_p}{\partial e_q} = \sum_{q=1}^{n}\left(e_p G_{pq} - f_p B_{pq}\right), \quad q \neq p$$

$$= \left(e_p G_{pq} - f_p B_{pq}\right) \qquad \text{--- (3.37)}$$

Diagonal elements are

$$\frac{\partial P_p}{\partial e_P} = 2e_p G_{pp} + \cancel{f_p B_{pp}} - \cancel{f_p B_{pp}} + \sum_{\substack{q=1 \\ q\neq p}}^{n}\left(e_q G_{pq} + f_q B_{pq}\right)$$

$$= 2e_p G_{pp} + \sum_{\substack{q=1 \\ q\neq p}}^{n}\left(e_q G_{pq} + f_q B_{pq}\right) \qquad \text{--- (3.38)}$$

J_2 : Off-diagonal elements are

$$\frac{\partial P_p}{\partial f_q} = \sum_{q=1}^{n}\left(e_p B_{pq} + f_p G_{pq}\right), \quad q \neq p$$

$$= \left(e_p B_{pq} + f_p G_{pq}\right) \qquad \text{--- (3.39)}$$

Diagonal elements are

$$\frac{\partial P_p}{\partial f_p} = 2f_pG_{pp} + \cancel{e_pB_{pp}} - \cancel{e_pB_{pp}} + \sum_{\substack{q=1\\q\neq p}}^{n}\left(f_qG_{pq} - e_qB_{pq}\right)$$

$$= 2f_pG_{pp} + \sum_{\substack{q=1\\q\neq p}}^{n}\left(f_qG_{pq} - e_qB_{pq}\right) \qquad \text{--- (3.40)}$$

J_3 : Off-diagonal elements are

$$\frac{\partial Q_p}{\partial e_q} = \left(e_pB_{pq} + f_pG_{pq}\right),\ \ q \neq p \qquad \text{--- (3.41)}$$

Diagonal elements are

$$\frac{\partial Q_p}{\partial e_p} = 2e_pB_{pp} - \sum_{\substack{q=1\\q\neq p}}^{n}\left(f_qG_{pq} - e_qB_{pq}\right) \qquad \text{--- (3.42)}$$

J_4 : Off-diagonal elements are

$$\frac{\partial Q_p}{\partial f_q} = \left(-e_pG_{pq} + f_pB_{pq}\right),\ \ q \neq p \qquad \text{--- (3.43)}$$

Diagonal elements are

$$\frac{\partial Q_p}{\partial f_p} = 2f_pB_{pp} + \sum_{\substack{q=1\\q\neq p}}^{n}\left(e_qG_{pq} + f_qB_{pq}\right) \qquad \text{--- (3.44)}$$

J_5 : Off-diagonal elements are

$$\frac{\partial \left|V_p\right|^2}{\partial e_q} = 0\ ,\ q \neq p \qquad \text{--- (3.45)}$$

Diagonal elements are

$$\frac{\partial \left|V_p\right|^2}{\partial e_p} = 2e_p \qquad \text{--- (3.46)}$$

J_6 : Off-diagonal elements are

$$\frac{\partial \left|V_p\right|^2}{\partial f_q} = 0\ ,\ q \neq p \qquad \text{---(3.47)}$$

diagonal elements are

$$\frac{\partial \left|V_p\right|^2}{\partial f_p} = 2f_p \qquad \text{--- (3.48)}$$

- The elements of the matrices are obtained by partially differentiating the load flow equations w.r.t. a unknown variable and then evaluating the first derivatives using the solution of previous iteration. For first iteration the initial assumed values are e_p^0 and f_p^0 for p = 2,3, -----,n

- The elements of the residual column matrix 'B' is the difference between the specified value of the quantity and the calculated value of the quantity using the solution of previous iteration.
- Let $P_{p,spec}$, $Q_{p,spec.}$ and $|V_p|_{spec}$ be the specified quantities at the bus-P. For the initial solution the value of P_p^0 ,Q_p^0 ,$|V|_p^0$ can be calculated using the load flow equations.

  Now for the first iteration

  ΔP_p = $P_{p,spec}$ - P_p^0

  ΔQ_p = $Q_{p \cdot spec}$- Q_p^0

  $$\left|\Delta V_p\right|^2 = \left|V_p\right|_{spec}^2 - \left|V_p{}^0\right|^2$$

- After calculating the elements of Jacobian matrix 'J' and residual column vector 'B' the elements of increment voltage vector 'C' can be calculated by using any standard technique.

  Now the next better solution will be

  $$e_p^1 = e_p^0 + \Delta e_p^0$$

  $$f_p^1 = f_p^0 + \Delta f_p^0$$

These values of voltages will be used in the next iteration.

- The process will be repeated and in general the new better estimation for bus voltages will be

  $$e_p^{K+1} = e_p^K + \Delta e_p^K$$

  $$f_p^{K+1} = f_p^K + \Delta f_p^K$$

  The process is repeated till the magnitude of the largest element in the residual column vector 'B' is less than a pre-specified value.

Algorithm for NR method in rectangular form

1) Read the system data and formulate Y_{BUS} for the given power system network.
2) Assume a flat voltage profile (1+j0) for all nodal voltages except the slack bus. Let slack bus voltage be (a+j0) and it is not modified in any iteration.
3) Assume a suitable value of ε called convergence criterion i.e. if the largest of the absolute value of the residues exceeds ε, the process is repeated, otherwise terminated.

4) Set iteration count k= 0

5) Set bus count p=1.

6) Check for slack bus. If it is a slack bus **go to step (11)**, otherwise go to next step.

7) Calculate the real and reactive power of bus-P using the following equations

$$P_p^k = \sum_{q=1}^{n} \left[e_p^k \left(e_q^k G_{pq} + f_q^k B_{pq} \right) + f_p^k \left(f_q^k G_{pq} - e_q^k B_{pq} \right) \right]$$

$$Q_p^k = \sum_{q=1}^{n} \left[f_p^k \left(e_q^k G_{pq} + f_q^k B_{pq} \right) - e_p^k \left(f_q^k G_{pq} - e_q^k B_{pq} \right) \right]$$

8) Calculate the change in active power

$$\Delta P_p^k = P_{p,spec} - P_p^k$$

9) Check for generator bus. If yes, compare the calculated reactive power, Q_p^k with the limits. The calculated reactive power may be within specified limits or it may violate the given limits. If the calculated reactive power violates the specified limit, then fix the reactive power generation to the corresponding violated limit and treat this bus as load bus and go to the next step.

i.e if $Q_p^k < Q_{p,\min}$ then $Q_{p,spec} = Q_{p,\min}$

or if $Q_p^k > Q_{p,\max}$ then $Q_{p,spec} = Q_{p,\max}$

If the reactive power limit is not violated then evaluate the voltage residue

$$\left|\Delta V_P^K\right|^2 = \left|V_{P,spec}\right|^2 - \left|V_P^K\right|^2$$

And **go to step (11).**

10) Calculate the change in reactive power for load bus (or for the generator bus treated as load bus)

Change in reactive power $\Delta Q_P^K = Q_{P,Spec} - Q_P^K$

11) Advance the bus count by 1 i.e p=p+1 and check if all the buses have been taken into account or not. If yes, go to the next step, Otherwise **go back to step (6).**

12) Determine the largest of the absolute value of the residue.

13) If the largest of the absolute value of the residue is less than ε, **go to step (18).**

14) Evaluate the elements for Jacobian matrix.

15) Calculate voltage increments Δe_k^p *and* Δf_p^k by using matrix inverse technique.

16) The new bus voltages can be calculated as follows

$e_p^{K+1} = e_p^K + \Delta e_p^K$; p=1,2,----,n. except slack bus

$f_p^{K+1} = f_p^K + \Delta f_p^K$; p=1,2,----,n. except slack bus

$$\left|V_P^{K+1}\right| = \sqrt{(e_P^{K+1})^2 + (f_P^{K+1})^2}$$

$$\delta_p^{K+1} = \tan^{-1}\left(\frac{f_P^{K+1}}{e_p^{K+1}}\right)$$

$$\therefore V_P^{K+1} = \left|V_P^{K+1}\right|\angle\delta_P^{K+1}$$

17) Advance iteration count i.e k=k+1 and **go back to step (5).**

18) Evaluate the line flows and slack bus power.

Case 2: NR method in Polar form

The load flow equations can be written in polar form as given below

$$P_p = \sum_{q=1}^{n} \left|V_p Y_{pq} V_q\right| \cos(\theta_{pq} + \delta_p - \delta_q)$$

$$= \left|V_p^2 Y_{pp}\right| \cos\theta_{pp} + \sum_{\substack{q=1 \\ q \neq p}}^{n} \left|V_p Y_{pq} V_q\right| \cos(\theta_{pq} + \delta_p - \delta_q) \quad \text{--- (3.49)}$$

$$Q_p = \sum_{q=1}^{n} \left|V_p Y_{pq} V_q\right| \sin(\theta_{pq} + \delta_p - \delta_q)$$

$$= \left|V_p^2 Y_{pp}\right| \sin\theta_{pp} + \sum_{\substack{q=1 \\ q \neq p}}^{n} \left|V_p Y_{pq} V_q\right| \sin(\theta_{pq} + \delta_p - \delta_q) \quad \text{--- (3.50)}$$

Now the linear equation in the polar form becomes

$$\begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} = \begin{bmatrix} J_1 & J_2 \\ J_3 & J_4 \end{bmatrix} \begin{bmatrix} \Delta\delta \\ \Delta|V| \end{bmatrix} \quad \text{--- (3.51)}$$

Where J_1,J_2,J_3,J_4 are the elements of Jacobian matrix, which can be calculated in the following manner

J_1 : off-diagonal elements are

$$\frac{\partial P_p}{\partial \delta_q} = \left|V_p Y_{pq} V_q\right| \sin(\theta_{pq} + \delta_p - \delta_q),\ q \neq p \quad \text{--- (3.52)}$$

Diagonal elements are

$$\frac{\partial P_p}{\partial \delta_p} = -\sum_{\substack{q=1 \\ q \neq p}}^{n} \left|V_p Y_{pq} V_q\right| \sin(\theta_{pq} + \delta_p - \delta_q) \quad \text{--- (3.53)}$$

J_2 : off-diagonal elements are

$$\frac{\partial P_p}{\partial |V_q|} = \sum_{\substack{q=1 \\ q \neq p}}^{n} |V_p Y_{pq}| \cos(\theta_{pq} + \delta_p - \delta_q),\ q \neq p \quad \text{--- (3.54)}$$

diagonal elements are

$$\frac{\partial P_p}{\partial |V_p|} = 2|V_p Y_{pp}| \cos\theta_{pp} + \sum_{\substack{q=1 \\ q \neq p}}^{n} |V_q Y_{pq}| \cos(\theta_{pq} + \delta_p - \delta_q) \quad \text{--- (3.55)}$$

J_3 : off-diagonal elements are

$$\frac{\partial Q_p}{\partial \delta_q} = - |V_p Y_{pq} V_q| \cos(\theta_{pq} + \delta_p - \delta_q),\ q \neq p \quad \text{--- (3.56)}$$

diagonal elements are

$$\frac{\partial Q_p}{\partial \delta_p} = \sum_{\substack{q=1 \\ q \neq p}}^{n} |V_p Y_{pq} V_q| \cos(\theta_{pq} + \delta_p - \delta_q) \quad \text{--- (3.57)}$$

J_4 : off-diagonal elements are

$$\frac{\partial Q_p}{\partial |V_q|} = \sum_{\substack{q=1 \\ q \neq p}}^{n} |V_p Y_{pq}| \sin(\theta_{pq} + \delta_p - \delta_q), \quad \text{--- (3.58)}$$

diagonal elements are

$$\frac{\partial Q_p}{\partial |V_p|} = 2|V_p Y_{pp}| \sin\theta_{pp} + \sum_{\substack{q=1 \\ q \neq p}}^{n} |V_q Y_{pq}| \sin(\theta_{pq} + \delta_p - \delta_q) \quad \text{--- (3.59)}$$

Algorithm for NR method in polar form

1) Read the system data and formulate Y_{BUS} for the given power system network.
2) Assume a flat voltage profile (1+j0) for all nodal voltages except the slack bus. Let slack bus voltage be (a+j0) and it is not modified in any iteration.
3) Assume a suitable value of ϵ called convergence criterion i.e. if the largest of the absolute value of the residues exceeds ϵ , the process is repeated, otherwise terminated.
4) Set iteration count k= 0
5) Set bus count p=1.
6) Check for slack bus. If it is a slack bus **go to step (11)**, otherwise go to next step.
7) Calculate the real and reactive power of bus-p using the following equations

$$P_p^{\ k} = \sum_{q=1}^{n} |V_p^{\ k} V_q^{\ k} Y_{pq}| \cos(\theta_{pq} + \delta_p^{\ k} - \delta_q^{\ k})$$

$$Q_p{}^k = \sum_{q=1}^{n} \left|V_p{}^k V_q{}^k Y_{pq}\right| \sin(\theta_{pq} + \delta_p{}^k - \delta_q{}^k)$$

8) Calculate the change in active power

$$\Delta P_p^k = P_{p,spec} - P_p^k$$

9) Check for generator bus. If yes, compare the calculated reactive power, Q_p with the given limits. The calculated reactive power may be within the specified limits or it may violate given the limits. If the calculated reactive power violates the specified limit, then fix the reactive power generation to the corresponding violated limit and treat this bus as load bus and go to the next step.

i.e if $Q_p^k < Q_{p,\min}$ then $Q_{p,spec} = Q_{p,\min}$

or if $Q_p^k > Q_{p,\max}$ then $Q_{p,spec} = Q_{p,\max}$

If the reactive power limit is not violated treat this bus as generator bus.

10) Calculate the change in reactive power for load bus or generator bus (or for the generator bus treated as load bus)

Change in reactive power $\Delta Q_p^k = Q_{p,Spec} - Q_p^k$

11) Advance the bus count by 1 i.e p = p+1 and check if all the buses have been taken into account or not. If yes, go to the next step, Otherwise **go back to step (6).**

12) Determine the largest of the absolute value of the residue.

13) If the largest of the absolute value of the residue is less than ϵ , **go to step (18).**

14) Evaluate the elements for Jacobian matrix.

15) Calculate phase angle and voltage increments $\Delta\delta$ and $\Delta|V|$ by using matrix inverse technique.

16) The new bus voltages can be calculated as follows

$$\delta_p{}^{k+1} = \delta_p{}^k + \Delta\delta_p{}^k$$

$$|V_p|^{k+1} = |V_p|^k + \Delta|V_p|^k$$

$$\therefore V_p^{k+1} = \left|V_p^{k+1}\right| \angle \delta_p^{k+1}$$

17) Advance iteration count i.e k=k+1 and **go back to step (5).**

18) Evaluate the line flows and slack bus power.

Symmetry property in Jacobian matrix

- Using the rectangular coordinates (see equations3.37, 3.39, 3.41, 3.43,) a careful examination for the off-diagonal elements of the sub-matrices [J_1], [J_2], [J_3] and [J_4] would reveal certain interesting properties.

$$[J_4]_{pq} = -[J_1]_{pq} \qquad \text{---(3.60)}$$

And $$[J_2]_{pq} = [J_3]_{pq} \qquad \text{---(3.61)}$$

- This property reduces the computational efforts considerably as it is enough only to compute the off-diagonal elements of any two sub-matrices.
- It may be noted that we do not see the symmetry property in the Jacobian, if polar coordinates are used. However, if we replace Δ|v| by $\frac{\Delta |V|}{|V|}$ in the eqn.(3.51), we have.

$$\begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} = \begin{bmatrix} J_1 J_2 \\ J_3 J_4 \end{bmatrix} \begin{bmatrix} \Delta\delta \\ \frac{\Delta |V|}{|V|} \end{bmatrix} \qquad \text{---(3.62)}$$

- Now the expressions for off-diagonal terms

$$[J_1]_{pq} = \frac{\partial P_p}{\partial \delta_q} = \left|V_p Y_{pq} V_q\right| \sin(\theta_{pq} + \delta_p - \delta_q),\ q \neq p \qquad \text{--- (3.63)}$$

$$[J_4]_{pq} = \frac{\partial Q_p}{\partial |V_q|} \left|V_q\right| = \left|V_p V_q Y_{pq}\right| \sin(\theta_{pq} + \delta_p - \delta_q),\ q \neq p \qquad \text{--- (3.64)}$$

$$[J_2]_{pq} = \frac{\partial P_p}{\partial |V_q|} \left|V_q\right| = \left|V_p V_q Y_{pq}\right| \cos(\theta_{pq} + \delta_p - \delta_q),\ q \neq p \qquad \text{--- (3.65)}$$

$$[J_3]_{pq} = \frac{\partial Q_p}{\partial \delta_q} = - \left|V_p Y_{pq} V_q\right| \cos(\theta_{pq} + \delta_p - \delta_q),\ q \neq p \qquad \text{--- (3.66)}$$

- From the above equations (3.63), (3.64), (3.65) and (3.66), it is seen

$$[J_1]_{pq} = [J_4]_{pq} \qquad \text{--- (3.67)}$$

$$[J_2]_{pq} = -[J_3]_{pq} \qquad \text{--- (3.68)}$$

Thus with slight modification in equation (3.51), we get the symmetry property in the Jacobian, which is observed in the case of expressing Jacobian in rectangular coordinates. The elements of Jacobian (J) are calculated with the latest voltage estimate and calculated power. However, the procedure (i.e algorithm) here, is the same as that of the rectangular coordinates. The formulation in the polar coordinates takes less computational effort and also requires less memory space.

3.6.2.4. Advantages and Disadvantage of NR method

Advantages:

1. The NR method is faster, more reliable and the results are accurate.
2. Requires less number of iterations for convergence.
3. The number of iterations are independent of the size of the system (i.e. no. of buses)
4. Suitable for large size systems

Disadvantages:

1. The programming logic is more complex than GS method.
2. The memory requirement is more.
3. No. of calculations per iteration are higher than GS method

3.6.2.5 COMPARISON OF GS AND NR METHOD

1. For GS method the variables are expressed in rectangular coordinates where as in NR method, they are expressed in polar coordinates. If rectangular coordinates are used for NR method then memory requirement will be more.

2. The no. of mathematical operations per iteration will be lesser in GS method than NR method. Hence computation time per iteration is less in GS method.

3. The GS method has linear convergence characteristics where as the NR method has quadratic convergence characteristics. Hence NR method converges faster than GS method.

4. In GS method no. of iteration increases with no. of buses but in NR method the no. of iterations remains constant and it does not depend on the size of the system.

5.In GS method convergence is affected by the choice of slack bus and the presence of series capacitors but the NR method is less sensitive to these factors.

6. The NR method needs only 3 to 5 iterations to reach an acceptable solution for a large system. But GS method requires large no. of iteration (30 or more) for same level of accuracy.

Problem-7: The load flow data for a sample power system are given below. The voltage magnitude of bus 2 is to be maintained at 1.04 p.u. The maximum and minimum reactive power limits of the generator at bus 2 are 0.35 and 0.0 p.u respectively. Determine the set of load flow equations at the end of first iteration by using NR method.

Impedance for sample system

Bus code	Impedance	Line changing admittance

1-2	0.08+j0.24	0.0
1-3	0.02+j0.06	0.0
2-3	0.06+j0.18	0.0

Schedule of generation and loads

Bus code	Assumed voltages	Generation		Load	
		MW	MVAR	MW	MVAR
1	1.06+j0.0	0	0	0	0
2	1.0+j0.0	0.2	0.0	0.0	0.0
3	1.0+j0.0	0	0	0.6	0.25

Solution: From the given impedance table

$$y_{12} = \frac{1}{z_{12}} = \frac{1}{0.08 + j0.24} = 1.25\text{-}j3.75$$

$$y_{13} = \frac{1}{z_{32}} = \frac{1}{0.02 + j0.06} = 5\text{-}j15$$

$$y_{23} = \frac{1}{z_{23}} = \frac{1}{0.06 + j0.18} \quad 1.667\text{-}j5$$

$$Y_{BUS} = \begin{bmatrix} 6.25 - j18.75 & -1.25 + j3.75 & -5 + j15 \\ -1.25 + j3.75 & 2.916 - j8.75 & -1.666 + j5 \\ -5 + j15 & -1.666 + j5 & 6.666 - j20 \end{bmatrix}$$

From the nodal admittance matrix and assumed voltage solution

$G_{11} = 6.25$ $\quad$ $B_{11} = 18.75$ $\quad$ $e_1 = 1.06,\ f_1 = 0.0$

$G_{12} = -1.25 = G_{21}$ $\quad$ $B_{12} = -3.75 = B_{21}$ $\quad$ $e_2 = 1.0,\ f_2 = 0.0$

$G_{13} = -5 = G_{31}$ $\quad$ $B_{13} = -15 = B_{31}$ $\quad$ $e_3 = 1.0,\ f_3 = 0.0$

$G_{22} = 2.916$ $\quad$ $B_{22} = 8.75$

$G_{23} = -1.666 = G_{32}$ $\quad$ $B_{23} = -5$

$G_{33} = 6.66$ $\quad$ $B_{33} = 20$

$$P_P = \sum_{q=1}^{n}\left[e_p\left(e_q G_{pq} + f_q B_{pq}\right) + f_p\left(f_q G_{pq} - e_q B_{pq}\right)\right]$$

$$P_2 = \sum_{q=1}^{n}\left[e_2\left(e_q G_{2q} + f_q B_{2q}\right) + f_2\left(f_q G_{2q} - e_q B_{2q}\right)\right]$$

$$= e_2 (e_1G_{21} + f_1B_{21}) + f_2 (f_1G_{21} - e_1B_{21}) + e_2 (e_2G_{22} + f_2B_{22})$$

$$+ f_2 (f_2G_{22} - e_2B_{22}) + e_2 (e_3G_{23} + f_3B_{23}) + f_2 (f_3G_{23} - e_3B_{23})$$

$= 1(1.06x\text{-}1.25 + 0x\text{-}3.5) + 0 – 1.6x\text{-}3.75 + 1(1x2.916) + 0+ 1x1x\text{-}1.666 + 0$

$= -1.325 + 2.916 + - 1.666$

$= -0.075$

Similarly $P_3 = -0.3$

$$Q_P = \sum_{q=1}^{n}\left[f_p\left(e_q G_{pq} + f_q B_{pq}\right) - e_p\left(f_q G_{pq} - e_q B_{pq}\right)\right]$$

$Q_2 = -0.225$

$Q_3 = -0.9$

$\Delta P_2 = P_{2\ specified} – P_{2\ calculated}$ i.e $P_2^0 = 0.2 -(-0.075) = 0.275$

$\Delta P_3 = -0.6 –(-0.3) = -0.3$

Since the lower limit on Q_2 is 0.0 and the value of Q_2 as calculated above violates this limit, bus-2 is treated as a load bus

$Q_{2,\ spec} = 0.0$

$\Delta Q_2 = 0.0 - (-0.225) = 0.225$

$\Delta Q_3 = - 0.25 - (-0.9) = 0.65$

Diagonal elements:

$$\frac{\partial P_p}{\partial e_p} = 2e_p G_{PP} + \sum_{\substack{q=1 \\ q\neq P}}^{n} (e_q G_{pq} + f_q B_{pq})$$

$$\frac{\partial P_2}{\partial e_2} = 2e_2 G_{22} + \sum_{\substack{q=1 \\ q\neq 2}}^{3} (e_q G_{2q} + f_q B_{2q})$$

$= 2e_2G_{22} + e_1G_{21} + f_1B_{21} + e_3G_{23} + f_3B_{23}$

$= 2(1) (2.916) +1.06(-1.25) +0(-3.75) +1(-1.666) +0(-5)$

$= 2.848$

$$\frac{\partial P_3}{\partial e_3} = 2e_3G_{33} + e_1G_{31} + f1B31 + e_2G_{32} + f_2B_{32}$$

$= 2(1) (6.666) +1.06(-5) +0(-15) +1(-1.666) +0(-5)$

$= 6.3666$

$$\frac{\partial P_p}{\partial f_p} = 2f_p G_{PP} + \sum_{\substack{q=1 \\ q\neq P}}^{n} (f_q G_{pq} - e_q B_{pq})$$

$$\frac{\partial P_2}{\partial f_2} = 2f_2 G_{22} + \sum_{\substack{q=1 \\ q\neq 2}}^{3} (f_q G_{2q} - e_q B_{2q})$$

$=2f_2G_{22} + f_1G_{21} – e_1B_{21}+f_3G_{23} - e_3B_{23}$

$= 2(0) (2.916)-1.06(-3.75)-1(-5)$

$= 8.975$

$$\frac{\partial P_3}{\partial f_3} = 20.9$$

Off-diagonal elements:

$$\frac{\partial P_p}{\partial e_q} = e_p G_{pq} - f_p B_{pq}$$

$$\frac{\partial P_2}{\partial e_3} = e_2 G_{23} - f_2 B_{23} = -1.666$$

$$\frac{\partial P_3}{\partial e_2} = e_3 G_{32} - f_3 B_{32} = -1.666$$

$$\frac{\partial P_p}{\partial f_q} = e_p B_{pq} + f_p G_{pq}$$

$$\frac{\partial P_2}{\partial f_3} = e_2 B_{23} + f_2 G_{23} = -5.0$$

$$\frac{\partial P_3}{\partial f_2} = e_3 B_{32} + f_3 G_{32} = -5.0$$

Similarly we find out the derivatives of the reactive power

Diagonal elements:

$$\frac{\partial Q_p}{\partial e_p} = 2e_p B_{PP} - \sum_{\substack{q=1 \\ q\neq P}}^{n} (f_q G_{pq} - e_q B_{pq})$$

$$\frac{\partial Q_2}{\partial e_2} = 2e_2 B_{22} - f_1 G_{21} + e_1 B_{21} - f_3 G_{23} + e_3 B_{23}$$

$= 2(2)(8.75)+1.06(-3.75)+1(-5) = 8.525$

$$\frac{\partial Q_3}{\partial e_3} = 19.1$$

$$\frac{\partial Q_p}{\partial f_p} = 2f_p B_{PP} + \sum_{\substack{q=1 \\ q\neq P}}^{n} (e_q G_{pq} + f_q B_{pq})$$

$$\frac{\partial Q_2}{\partial f_2} = -2.991, \qquad \frac{\partial Q_3}{\partial f_3} = -6.966$$

Off-diagonal elements:

$$\frac{\partial Q_p}{\partial e_q} = e_p B_{pq} + f_p G_{pq}$$

$$\frac{\partial Q_2}{\partial e_3} = e_2B_{23} + f_3G_{23} = 1(-5) + 0 = -5$$

$$\frac{\partial Q_3}{\partial e_2} = e_3B_{32} + f_2G_{32} = 1(-5) + 0 = -5$$

$$\frac{\partial Q_p}{\partial f_q} = -e_pG_{pq} + f_pB_{pq}$$

$$\frac{\partial Q_2}{\partial f_3} = -e_2G_{23} + f_2B_{23} = -1(-1.666) + 0 = 1.666$$

$$\frac{\partial Q_3}{\partial f_2} = -e_3G_{32} + f_3B_{32} = -1(-1.666) + 0$$

$\therefore$ The set of linear equations at the end of first iteration are

$$\begin{bmatrix} 0.275 \\ -0.3 \\ 0.225 \\ 0.65 \end{bmatrix} = \begin{bmatrix} 2.846 & -1.666 & 8.975 & -5 \\ -1.666 & 6.366 & -5.0 & 20.9 \\ 3.525 & -5 & -2.991 & 1.666 \\ -5 & 19.1 & 1.666 & -6.966 \end{bmatrix} \begin{bmatrix} \Delta e_2 \\ \Delta e_3 \\ \Delta f_2 \\ \Delta f_3 \end{bmatrix}$$

Problem-8: In case the reactive power constraints at bus-2 in the previous problem is $-0.3 \leq Q_2 \leq 0.3$ Determine the equations at the end of first iteration.

Solution:

Since $Q_2 = 0.225$ and the lower limit is -0.3, therefore the bus-2 behaves like a generator bus

$\Delta P_2 = 0.2 - (-0.075) = 0.275$

$\Delta P_3 = -0.6 - (-0.3) = -0.3$

$\Delta Q_3 = -0.25 - (-0.9) = 0.65$

Since bus-2 behave like a generator bus therefore

$$\Delta|V_2|^2 = |V_2|^2 - |V_{2cal}|^2$$
$$= 1.04^2 - 1.0^2 = 0.0816$$

The Jacobian elements corresponding to rows ΔP_2, ΔP_3, ΔQ_3 remains same as in previous problem, those of Q_2 will be change and they are calculated as follows

$$\frac{\partial |V_2|^2}{\partial e_2} = 2e_2 = 2$$

$$\frac{\partial |V_2|^2}{\partial f_2} = 2f_2 = 0$$

$$\frac{\partial |V_2|^2}{\partial e_3} = 0$$

$$\frac{\partial |V_2|^2}{\partial f_3} = 0$$

The set of equations will be as given below

$$\begin{bmatrix} 0.275 \\ -0.3 \\ 0.65 \\ 0.0816 \end{bmatrix} = \begin{bmatrix} 2.846 & -1.666 & 8.975 & -5 \\ -1.666 & 6.366 & -5.0 & 20.9 \\ -5.0 & 19.1 & 1.666 & -6.966 \\ 2.0 & 0.0 & 0.0 & 0.0 \end{bmatrix} \begin{bmatrix} \Delta e_2 \\ \Delta e_3 \\ \Delta f_2 \\ \Delta f_3 \end{bmatrix}$$

3.6.3 DECOUPLED LOAD FLOW (DLF) METHOD

In NR method, the inverse of the Jacobian has to be computed at every iteration. When solving large interconnected power system, alternative solution methods are possible, taking into account certain observations made of practical systems. These are

- . The real power changes (ΔP) are less sensitive to changes in voltage magnitude and are mainly sensitive to change in bus voltage angles. In other words the coupling between active power 'P' and the bus voltage magnitude |V| is relatively weak
- The reactive power changes (ΔQ) are less sensitive to change in bus voltage angles and are mainly sensitive to change in voltage magnitude. In other words the coupling between reactive power (Q) and bus voltage phase angle (δ) is also weak.

Thus the weak coupling is utilized in the development of decoupled load flow method in which 'P' is decoupled from ΔV and 'Q' is decoupled from Δδ.

- With these assumptions the equation

$$\begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} = \begin{bmatrix} J_1 & J_2 \\ J_3 & J_4 \end{bmatrix} \begin{bmatrix} \Delta \delta \\ \Delta |V| \end{bmatrix} \qquad \text{--- (3.69)}$$

is reduced to the following form

$$\begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} = \begin{bmatrix} J_1 & 0 \\ 0 & J_4 \end{bmatrix} \begin{bmatrix} \Delta \delta \\ \Delta |V| \end{bmatrix} \qquad \text{--- (3.70)}$$

Therefore $\Delta P = [J_1]\,[\Delta\delta]$ --- (3.71)

$\Delta Q = [J_4]\,[\Delta|V|]$ --- (3.72)

The load flow equations of NR method in polar form are

$$P_p = |V_p^2\ Y_{pp}|\cos\theta_{pp} + \sum_{\substack{q=1 \\ q\neq p}}^{n} |V_p V_q Y_{pq}|\cos(\theta_{pq} + \delta_p - \delta_q)$$

$$Q_p = |V_p^2 \ Y_{pp}| \sin\theta_{pp} + \sum_{\substack{q=1 \\ q \neq p}}^{n} |V_p V_q Y_{pq}| \sin(\theta_{pq} + \delta_p - \delta_q)$$

J_1: Off-diagonal elements are

$$\frac{\partial P_p}{\partial \delta_q} = |V_p V_q Y_{pq}| \sin(\theta_{pq} + \delta_p - \delta_q), \quad q \neq p \qquad \text{--- (3.73)}$$

Diagonal elements are

$$\frac{\partial P_p}{\partial \delta_p} = -\sum_{\substack{q=1 \\ q \neq p}}^{n} |V_p V_q Y_{pq}| \sin(\theta_{pq} + \delta_p - \delta_q), \qquad \text{--- (3.74)}$$

J_4: Off-diagonal elements are

$$\frac{\partial Q_p}{\partial |V_q|} = |V_p Y_{pq}| \sin(\theta_{pq} + \delta_p - \delta_q), \quad q \neq p \qquad \text{--- (3.75)}$$

Diagonal elements are

$$\frac{\partial Q_p}{\partial |V_p|} = 2|V_p Y_{pp}| \sin\theta_{pp} + \sum_{\substack{q=1 \\ q \neq p}}^{n} |V_q Y_{pq}| \sin(\theta_{pq} + \delta_p - \delta_q), \qquad \text{--- (3.76)}$$

- Equations (3.71) and (3.72) can be constructed and solved simultaneous with each other at each iteration, updating the $[J_1]$ and $[J_4]$ matrices in each iteration using the equations (3.73), (3.74), (3.75) and (3.76).
- A better approach is to conduct each iteration by first solving equation (3.71) for $\Delta\delta$, and use the updated 'δ' in constructing and then solving equation (3.72) for $\Delta|V|$. This will result in faster convergence then in the simultaneous mode.
- The main advantage of the decoupled load flow (DLF) method as compared to the NR method is its reduced memory requirements in storing the Jacobian elements. However, the time required per iteration of the DLF method is practically the same as that of NR method. In DLF method more no. of iterations are required for convergence because of the approximations made in it.

3.6.4 FAST DECOUPLED LOAD FLOW (FLDF) METHOD

- This FDLF method is an extension of NR method formulated in polar coordinates with certain approximations which results a faster algorithm for load flow solution. In this method both the speed as well as the sparsity are exploited (make good use)
- The load flow equations for NR method in polar form can be written as

$$P_p = |V_p^2 Y_{pp}| \cos\theta_{pp} + \sum_{\substack{q=1 \\ q\neq p}}^{n} |V_p V_q Y_{pq}| \cos(\theta_{pq} + \delta_p - \delta_q) \quad \text{--- (3.77)}$$

$$Q_p = |V_p^2 Y_{pp}| \sin\theta_{pp} + \sum_{\substack{q=1 \\ q\neq p}}^{n} |V_p V_q Y_{pq}| \sin(\theta_{pq} + \delta_p - \delta_q) \quad \text{--- (3.78)}$$

These equations after linearization can be written in matrix form as

$$\begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} = \begin{bmatrix} H & N \\ M & L \end{bmatrix} \begin{bmatrix} \Delta\delta \\ \dfrac{\Delta|V|}{|V|} \end{bmatrix} \quad \text{--- (3.79)}$$

Where H, N, M and L are the elements (viz J_1, J_2, J_3, J_4) of the Jacobian matrix.

- The first assumption under decoupled load flow method is that the real power changes (ΔP) are less sensitive to change in voltage magnitude and mainly sensitive to changes in phase angle. Similarly, the reactive power changes (ΔQ) are less sensitive to changes in phase angle but mainly sensitive to change in voltage magnitude. With these assumptions, the equation (3.79) is reduced to

$$\begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} = \begin{bmatrix} H & 0 \\ 0 & L \end{bmatrix} \begin{bmatrix} \Delta\delta \\ \dfrac{\Delta|V|}{|V|} \end{bmatrix} \quad \text{--- (3.80)}$$

Equation (3.80) is decoupled equation which can be expressed as

$$[\Delta P] = [H][\Delta\delta] \quad \text{--- (3.81)}$$

$$[\Delta Q] = [L]\left[\frac{\Delta|V|}{|V|}\right] \quad \text{--- (3.82)}$$

Using equations (3.77) and (3.78) the elements of the Jacobian matrices H and L are obtained as follows:

- **Off-diagonal elements of H**

$$H_{pq} = \frac{\partial P_p}{\partial \delta_q} = |V_p V_q Y_{pq}| \sin(\theta_{pq} + \delta_p - \delta_q)$$

$$= |V_p V_q Y_{pq}| [\sin\theta_{pq} \cos(\delta_p - \delta_q) + \cos\theta_{pq} \sin(\delta_p - \delta_q)]$$

$$= |V_p V_q| [|Y_{pq}| \sin\theta_{pq} \cos(\delta_p - \delta_q) + |Y_{pq}| \cos\theta_{pq} \sin(\delta_p - \delta_q)]$$

$$= |V_p V_q| [-B_{pq} \cos(\delta_p - \delta_q) + G_{pq} \sin(\delta_p - \delta_q)]$$

$$H_{pq} = |V_p V_q| [-B_{pq} \cos(\delta_p - \delta_q) + G_{pq} \sin(\delta_p - \delta_q)] \quad \text{--- (3.83)}$$

- **Off-diagonal elements of L**

$$L_{pq} = \frac{\partial Q_p}{\partial V_q}|V_q| = |V_p V_q Y_{pq}| \sin(\theta_{pq} + \delta_p - \delta_q)$$

$$= |V_p V_q| [|Y_{pq}| \sin\theta_{pq} \cos(\delta_p - \delta_q) + |Y_{pq}| \cos\theta_{pq} \sin(\delta_p - \delta_q)]$$

$$= |V_p V_q| [-B_{pq} \cos(\delta_p - \delta_q) + G_{pq} \sin(\delta_p - \delta_q)]$$

$$L_{pq} = |V_p V_q| [-B_{pq} \cos(\delta_p - \delta_q) + G_{pq} \sin(\delta_p - \delta_q)] \quad \text{--- (3.84)}$$

From equations (3.83) and (3.84)

$$H_{pq} = L_{pq} = |V_p V_q| [-B_{pq} \cos(\delta_p - \delta_q) + G_{pq} \sin(\delta_p - \delta_q)]$$

- **The diagonal elements of H**

$$H_{pp} = \frac{\partial P_p}{\partial \delta_p} = -\sum_{\substack{q=1 \\ q \neq p}}^{n} |V_p V_q Y_{pq}| \sin(\theta_{pq} + \delta_p - \delta_q)$$

$$= -Q_p + V_p{}^2 Y_{pp} \sin\theta_{pp} \qquad \text{(From eqn. (3.78))}$$

$$= -Q_p + V_p{}^2 B_{pp} \quad \text{--- (3.85)}$$

- **The diagonal elements of L:**

$$L_{pp} = \frac{\partial Q_p}{\partial V_p}|V_p| = 2|V_p{}^2 Y_{pp}| \sin\theta_{pp} + \sum_{\substack{q=1 \\ q \neq p}}^{n} |V_p V_q Y_{pq}| \sin(\theta_{pq} + \delta_p - \delta_q)$$

$$= 2|V_p{}^2 Y_{pp}| \sin\theta_{pp} + Q_p - |V_p{}^2 Y_{pp}| \sin\theta_{pp}$$

$$= Q_p + |V_p{}^2 Y_{pp}| \sin\theta_{pp}$$

$$L_{pp} = Q_p - |V_p{}^2| B_{pp} \quad \text{--- (3.86)}$$

- In the case of fast decoupled load flow methods following approximations are further made for evaluating Jacobian elements
  i) $B_{pq} >> G_{pq}$(since, the X/R ratio of transmission lines is high in well designed system
  ii) The voltage angle difference $(\delta_p - \delta_q)$ between two buses in the system is very small. This means that $\cos(\delta_p - \delta_q) \approx 1$ and $\sin(\delta_p - \delta_q)$=0
  iii) $Q_p \leq B_{pp} V_p{}^2$
- With these assumption the Jacobian elements now become

$$L_{pq} = H_{pq} = -|V_p||V_q| B_{pq} \qquad \text{for } q \neq p$$

$$L_{pp} = H_{pp} = -B_{pp}|V_p|^2$$

- With these Jacobian elements, equations (3.81) and (3.82) becomes

$$\Delta P = [|V_p||V_q|B^{|}_{pq}][\Delta\delta_q] \quad \text{--- (3.87)}$$

$$\Delta Q = [|V_p||V_q|B^{||}_{pq}]\frac{\Delta|V_q|}{|V_q|} \quad \text{--- (3.88)}$$

Where $B^{|}_{pq}$ and $B^{||}_{pq}$ are the elements of $[-B_{pq}]$ matrix.

- Further decoupling and logical simplification of the FDLF algorithm is achieved by
  1. Omitting effect of phase shifting transformers.
  2. Setting off-nominal turns ratio of transformers to 1.0
  3. In forming $[B^{|}]$, omitting the effect of shunt reactors and capacitors which mainly effect reactive power.
  4. Ignoring series resistance of lines in forming the $Y_{bus.}$
  5. Dividing each of the equations (3.87) and (3.88) by $|V_p|$ and setting $|V_q| = 1$ p.u, we get

$$\left[\frac{\Delta P}{|V|}\right] = [B^1]\times[\Delta\delta] \quad \text{--- (3.89)}$$

$$\left[\frac{\Delta Q}{|V|}\right] = [B^{11}]\times[\Delta|V|] \quad \text{--- (3.90)}$$

Here both $[B^{|}]$ and $[B^{||}]$ are real and sparse and have structures of H and L respectively. Since, they contain only network admittances, they are constant and need to be triangularised only once at the beginning of the iteration. This algorithm, which results in a very fast solution of $\Delta\delta$ and ΔV, is known as fast decoupled load flow formulation of load flow studies.

Algorithm for FDLF method

1) Read the system data and formulate Y_{BUS} for the given power system network.
2) Assume a flat voltage profile (1+j0) for all nodal voltages except the slack bus. Let slack bus voltage be (a+j0) and it is not modified in any iteration.
3) Assume a suitable value of ϵ called convergence criterion.
4) Set iteration count K= 0.
5) Set bus count p=1.
6) Check for slack bus. If it is a slack bus goes to step (11), other wise go to next step.
7) Calculate active power and reactive power by using the following formula

$$P_p = P_P = \sum_{q=1}^{n} \left|V_p Y_{pq} V_q\right| \cos(\theta_{pq} + \delta_p - \delta_q)$$

$$Q_P = \sum_{q=1}^{n} \left|V_p Y_{pq} V_q\right| \sin(\theta_{pq} + \delta_p - \delta_q)$$

8) Calculate the mismatches (i.e changes) in active power ΔP^k and reactive powers ΔQ^k. If the mismatches are within the desirable tolerance then stop the iteration process.

9) Normalize the mismatches by dividing each entry by its respective bus voltage magnitude.

$$\Delta P^k = \frac{\Delta P_2^k}{V_2^k} \qquad \Delta Q_p^k = \frac{\Delta Q_2^k}{V_2^k}$$

$$= \frac{\Delta P_3^k}{V_3^k} \qquad = \frac{\Delta Q_3^K}{V_3^k}$$

$$- \qquad -$$

$$- \qquad -$$

$$- \qquad -$$

$$- \qquad -$$

$$= \frac{\Delta P_n^k}{V_n^k} \qquad = \frac{\Delta Q_n^k}{V_n^k}$$

10) Solve the following equations for the correction factors ΔV^k and $\Delta\delta^k$ by using the constant matrices B^I and B^{II} which are extracted from the bus admittance matrix.

$$\left[\frac{\Delta P}{|V|}\right] = \left[B^1\right] \times \left[\Delta\delta\right]$$

$$\left[\frac{\Delta Q}{|V|}\right] = \left[B^{11}\right] \times \left[\Delta|V|\right]$$

11) The new bus voltages can be calculated as follows

$$\delta_p^{k+1} = \delta_p^k + \Delta\, \delta_p^k$$

$$V_p^{k+1} = V_p^k + \Delta V_p^k$$

$$\therefore V_P^{K+1} = \left|V_P^{K+1}\right| \angle \delta_P^{K+1}$$

12) Check if all the buses are taken into account or not. If yes go to next step otherwise increase bus count by 1 i.e set p=p+1 and go back to step (6).

13) Advance iteration count i.e k=k+1 and go back to step (5).

14) Evaluate the line flows and slack bus power.

3.7. COMPARISON OF DIFFERENT LOAD FLOW METHODS

S.No	GS Method	NR Method	FDLF Method
1	Rectangular coordinates are preferred for solution	Polar coordinates are preferred for solution	Polar coordinates are preferred for solution
2	More no.of iteration(i.e 30 or more) are required to get the acceptable solution	Less no.of iteration(i.e 3 to 5) are required to get the acceptable solution	Less no.of iteration(i.e 2 to 5) are required to get the acceptable solution
3	The computation time per iteration will be less due to less no. of mathematical operation	The computation time per iteration is more i.e 8 times than GS method	The computation time per iteration is more i.e 2/3 times than GS method and 5 times than NR method
4	Acceleration factor is used to get fast convergence	No such factor is used	No such factor is used
5	The number of iterations increases as the size of the system increases.	The number of iterations independent of the size of the system.	The number of iterations independent of the size of the system
6	Less computer memory is required	More computer memory is required	Memory requirement is intermediate of GS and NR method
7	High computation cost	Less computation cost	Less computation cost
8	Suitable for small size of systems	Suitable for large size of systems	Suitable for large size of systems
9	Convergence is effected on the selection of slack bus	Convergence is independent on the selection of slack bus	Convergence is independent on the selection of slack bus
10	Convergence is uncertain	Convergence is certain	Convergence is certain

3.8. DC LOAD FLOW METHOD

- In certain power system studies (e.g. reliability studies) a very large no. of load flow runs may be needed. Therefore, a very fast (and not necessarily accurate, due to the linear approximation involved) method can be used for such studies.
- The method of calculating the real power flows by solving first for the bus angles is known as the dc load flow method, in contrast with the exact non 0linear solution, which is known as the ac solution.
- Assume that bus p is connected to bus q over an impedance of Z_{pq}. Therefore, the active power flow can be expressed as

$$P_{pq} = \frac{V_p V_q}{Z_{pq}} \sin(\delta_p - \delta_q) \quad \text{--- (3.91)}$$

Where $V_p = |V_p| \angle \delta_p \qquad V_q = |V_q| \angle \delta_q$

- The following simplifying approximations are made

$$X_{pq} \cong Z_{pq} \text{ Since } X_{pq} \gg R_{pq}$$

$$|V_p| = 1pu$$

$$|V_q| = 1pu$$

$$\sin(\delta_p - \delta_q) = \delta_p - \delta_q$$

- Now, the active load flow eqn. (3.91) can be expressed as

$$P_{pq} = \frac{\delta_p - \delta_q}{X_{pq}} = B_{pq}(\delta_p - \delta_q) \quad \text{--- (3.92)}$$

In matrix form

$$[P] = [B][\delta] \quad \text{--- (3.93)}$$

$$[\delta] = [B]^{-1}[P] \quad \text{--- (3.94)}$$

$$[\delta] = [X][P] \quad \text{--- (3.95)}$$

Where [B] matrix is an (n-1) × (n-1) matrix dimensionally for an n-bus system. The diagonal and off-diagonal elements of the [B] matrix can be found by adding the series susceptances of the branches connected to bus and by setting then equal to negated series susceptance of branch pq, respectively.

- The linear equation (3.93) can be solved for by using matrix techniques.
- It is possible with the dc load flow method to carry out the thousands of load – flow runs that are required for comprehensive contingency analysis on large scale systems.

- In summary, the choice of a load –flow method is a matter of choice between speed and accuracy. For a given degree of accuracy, the speed depends on the size, complexity, and configuration of the power system and on the numerical approach chosen.

ADDITIONAL SOLVED PROBLEMS

Problem-1: The load flow data for the system shown in figure is given below in the following tables

Bus-Code	Impedance (Z_{pq})
1-2	j0.05pu
1-3	j0.1pu
2-3	j0.05pu

Table (1)

Bus Code	Assumed Bus Voltage	Generation		Load	
		MW	MVar	MW	MVar
1	1.03 + j0 pu	0	0	0	0
2	1.0 + j0 pu	50	-	20	10
3	1.0 + j0 pu	0	0	20	20

Table(2)

The Voltage magnitude at bus-2 to be held at 1.0p.u.The maximum and minimum reactive power limits at bus-2 are 50 and -10 MVars respectively .With bus-1 as the slack bus, use GS method and Y_{bus} matrix to obtain a load flow solution up to one iteration?

[JNTU, Regular, Nov - 2004]

Solution: From the table (1)

$$y_{12} = \frac{1}{Z_{12}} = \frac{1}{j0.05} = -j20$$

$$y_{13} = \frac{1}{Z_{13}} = \frac{1}{j0.1} = -j10$$

$$y_{23} = \frac{1}{Z_{23}} = \frac{1}{j0.05} = -j20$$

$$Y_{bus} = \begin{bmatrix} Y_{11} & Y_{12} & Y_{13} \\ Y_{21} & Y_{22} & Y_{23} \\ Y_{31} & Y_{32} & Y_{33} \end{bmatrix}$$

$Y_{11} = y_{12} + y_{13} = -j20-j10 = -j30$

$Y_{22} = y_{12} + y_{23} = -j20-j20 = -j40$

$Y_{33} = y_{31} + y_{32} = -j10-j20 = -j30$

$Y_{12} = Y_{21} = -y_{12} = j20$

$Y_{23} = Y_{32} = -y_{23} = j20$

$Y_{31} = Y_{13} = -y_{13} = j10$

Nodal admittance Matrix $Y_{bus} = \begin{bmatrix} -j30 & j20 & j10 \\ j20 & -j40 & j20 \\ j10 & j20 & -j30 \end{bmatrix}$

The data in table-2 is to be converted into per unit value

$$\text{Per unit value} = \frac{actual\ value}{base\ value}$$

Let the base value =50 MVA , so the data in table-2 is changed accordingly

i.e $P_{G2} = 50\ MW = \frac{50}{50} = 1.0$ p.u

$P_{D2} = 20\ MW = \frac{20}{50} = 0.4$ p.u

$Q_{D2} = 10\ MVar = \frac{10}{50} = 0.2$ p.u

$P_{D3} = 20\ MW = \frac{20}{50} = 0.4$ p.u

$Q_{D3} = 20\ MVar = \frac{20}{50} = 0.4$ p.u

Assume flat voltage profile for all the buses except slack bus i.e

$V_2^0 = 1\ p.u, \qquad V_3^0 = 1\ p.u, \ |V_2|_{spec} = 1.0\ p.u$

Since bus-1 is slack bus, its voltage remains constant at the specified value for all the iterations.

$$V_1^0 = V_1^1 = V_1^2 = ... = V_1^k = (1.03 + j0.0)\,p.u$$

For generator bus ,the reactive power limits are

$Q_{2min} = \frac{-10}{50}$ = -0.2 p.u

$Q_{2max} = \frac{50}{50}$ = 1.0 p.u

The bus-2 is a generator bus and so calculate its reactive power, Q_2

$$Q_{p,cal}^{k+1} = (-1)I.P.of\left\{(V_P^k)^*\left[\sum_{q=1}^{p-1} Y_{pq}V_q^{k+1} + \sum_{q=p}^{n} Y_{pq}V_q^k\right]\right\}$$

Here p = 2 , k=0 ,n=3

$$Q_{2,cal}^1 = (-1)I.P.of\left\{(V_2^0)^*\left[Y_{21}V_1^1 + Y_{22}V_2^0 + Y_{23}V_3^0\right]\right\}$$

$$= (-1)I.P.of\left\{(1)\left[(j20)(1.03) + (-j40)(1) + (j20)(1)\right]\right\}$$

= -0.6 p.u

The specified range for Q_2 is -0.2<Q<1.0. The calculated value of Q_2 violates the lower limit of the specified range for Q_2 . Therefore the reactive power generation for bus-2 is fixed at -0.2 (lower limit) and the bus-2 is treated as load bus for 1st iteration. Now $V_2^0 = 1 + j0$, similar to other load buses for first iteration .But P and Q are considered as positive for bus-2 and P and Q are negative for other load buses.

$$V_p^{K+1} = \frac{1}{Y_{pp}}\left[\frac{P_p - jQ_p}{(V_P^k)^*} - \sum_{q=1}^{p-1} Y_{pq}V_q^{K+1} - \sum_{q=p+1}^{n} Y_{pq}V_q^K\right]$$

$$V_2^1 = \frac{1}{Y_{22}}\left[\frac{P_2 - jQ_2}{(V_2^0)} - Y_{21}V_1^1 - Y_{23}V_3^0\right]$$

$$= \frac{1}{-j40}\left[\frac{0.6 + j0.2}{1} - (j20)(1.03) - (j20)(1)\right]$$

$$= \frac{-(0.6 - j40.4)}{j40} = \frac{40.4\angle 90.9}{40\angle 90} = 1.01\angle 0.9\,p.u$$

Voltage at bus-3

$$V_3^1 = \frac{1}{Y_{33}}\left[\frac{P_3 - jQ_3}{(V_3^0)} - Y_{31}V_1^1 - Y_{32}V_2^1\right]$$

$$= \frac{1}{-j30}\left[\frac{-0.4 + j0.4}{1} - (j10)(1.03) - (j20)(1.01\angle 0.9)\right]$$

$$= \frac{(-0.083 - j30.1)}{-j30} = \frac{30.1\angle 89.842}{30\angle 90} = 1.003\angle -0.158\, p.u$$

The voltages at the end of first iteration are

$V_1^1 = 1.03\angle 0\, p.u$ $\qquad$ $V_2^1 = 1.01\angle 0.9\, p.u$

$V_3^1 = 1.003\angle -0.158\, p.u$

Problem-2: The load flow data for the power system as shown in the figure is given in the following tables.

Bus-Code	Impedance (Z_{pq})
1-2	0.08+j0.24pu
1-3	0.02+j0.06pu
2-3	0.06+j0.18pu

Table (1)

Bus Code	Assumed Bus Voltage	Generation		Load	
		MW	MVar	MW	MVar
1	1.05 + j0 pu	0	0	0	0
2	1.0 + j0 pu	20	0	50	20
3	1.0 + j0 pu	0	0	60	25

Table(2)

The voltage magnitude at bus-2 is to be maintained at 1.03 p.u .The maximum and minimum reactive power limits of the generator at bus-2 are 35 and 0 Mvars respectively. With bus1 as slack bus, obtain voltage at bus-3 using GS method after first iteration (assume base Mva = 50) [**JNTU , Supplementary, Feb-2007]**

Solution: From the table (1)

$$y_{12} = \frac{1}{Z_{12}} = \frac{1}{0.08 + j0.24} = 1.25 - j3.75$$

$$y_{13} = \frac{1}{Z_{13}} = \frac{1}{0.02 + j0.06} = 5 - j15$$

$$y_{23} = \frac{1}{Z_{23}} = \frac{1}{0.06 + j0.18} = 1.667\text{-}j5$$

Nodal admittance matrix

$$Y_{bus} = \begin{bmatrix} Y_{11} & Y_{12} & Y_{13} \\ Y_{21} & Y_{22} & Y_{23} \\ Y_{31} & Y_{32} & Y_{33} \end{bmatrix}$$

Nodal admittance Matrix $Y_{bus} = \begin{bmatrix} 6.25 - j18.75 & -1.25 + j3.75 & -5 + j15 \\ -1.25 + j3.75 & 2.916 - j8.75 & -1.666 + j5 \\ -5 + j15 & -1.666 + j5 & 6.666 - j20 \end{bmatrix}$

The data in table-2 is to be converted into per unit value

$$\text{Per unit value} = \frac{actualvalue}{basevalue}$$

Let the base value =50 MVA. So the data in table-2 is changed accordingly

i.e P_{G2} = 20 MW = $\frac{20}{50}$ =0.4 p.u

Q_{G2} = 0, Q_{G3} = 0, P_{G3} = 0

P_{D2} = 50 MW = $\frac{50}{50}$ =1.0p.u

Q_{D2} = 20 MVar = $\frac{20}{50}$ =0.4 p.u

P_{D3} = 60 MW = $\frac{60}{50}$ =1.2 p.u

Q_{D3} = 25 MVar = $\frac{25}{50}$ =0.2 p.u

Assume flat voltage profile for all the buses except slack bus i.e

$V_2^0 = 1.03 p.u$, $V_3^0 = 1 p.u$, $|V_2|_{spec} = 1.03 p.u$

Since bus-1 is slack bus, its voltage remains constant at the specified value for all the iterations.

$$V_1^0 = V_1^1 = V_1^2 = ... = V_1^k = (1.05 + j0.0) p.u$$

For generator bus ,the reactive power limits are

Q_{2min}= $\frac{0}{50}$ = 0 p.u

Q_{2max}= $\frac{35}{50}$ = 0.7 p.u

The bus-2 is a generator bus and so calculate its reactive power, Q_2

$$Q_{p,cal}^{k+1} = (-1)I.P.of \left\{ (V_P^k)^* \left[\sum_{q=1}^{p-1} Y_{pq} V_q^{k+1} + \sum_{q=p}^{n} Y_{pq} V_q^k \right] \right\}$$

Here p = 2 , k=0 ,n=3

$$Q_{2,cal}^{1} = (-1)I.P.of \left\{ (V_2^0)^* \left[Y_{21}V_1^1 + Y_{22}V_2^0 + Y_{23}V_3^0 \right] \right\}$$

$$= (-1)I.P.of \left\{ (1.03)\left[(-1.25 + j3.75)(1.05) + (2.916 - j8.75)(1.03) + (-1.666 + j5)(1) \right] \right\}$$

= I.P of (0.2544 – j0.07725)

=0.07725 pu

The specified range for Q_2 is $0 \leq Q_2 \leq 0.7$p.u. The calculated value of Q_2 is lies with in the given reactive power limits and so this bus can be treated as generator bus.

Now P_2= -0.6, Q_2 = 0.07725, $V_2^0 = 1.03 + j0$, since the bus-2 is treated as generator bus, the $|V_2^1| = |V_2|_{spec}$ and phase of V_2^1 is given by the phase of V_2^1 temp.

$$V_{p,temp}^{K+1} = \frac{1}{Y_{pp}} \left[\frac{P_p - jQ_p}{(V_P^k)^*} - \sum_{q=1}^{p-1} Y_{pq} V_q^{K+1} - \sum_{q=p+1}^{n} Y_{pq} V_q^K \right]$$

$$V_{2,temp}^{1} = \frac{1}{Y_{22}} \left[\frac{P_2 - jQ_2}{(V_2^0)^*} - Y_{21}V_1^1 - Y_{23}V_3^0 \right]$$

$$= \frac{1}{2.916 - j8.75} \left[\frac{-0.6 - j0.077}{1.03} - (-1.25 + j3.75)(1.05) - (-1.666 + j5)(1) \right]$$

$$= \frac{1}{2.916 - j8.75} \left[2.38 - j8.99 \right]$$

$$= \frac{9.29\angle -75.17}{9.20\angle -71.56} = 1.009\angle -3.61$$

$$V_2^1 = 1.03\angle -3.61 p.u$$

Voltage at bus-3

$$V_3^1 = \frac{1}{Y_{33}} \left[\frac{P_3 - jQ_3}{(V_3^0)^*} - Y_{31}V_1^1 - Y_{32}V_2^1 \right]$$

$$= \frac{1}{6.666 - j20} \left[\frac{-1.2 + j0.5}{1} - (-5.02 + j15.06)(1.05) - (-1.66 + j5)(1.03\angle -3.61) \right]$$

$$= \frac{(5.451 - j20.543)}{6.666 - j20} = \frac{21.25\angle -75.13}{21.12\angle -71.56} = 1.006\angle -3.57 p.u$$

Problem-3: The data for 2-bus system is given below S_{G1} = unknown , S_{D1} = unknown $V_1=1.0\angle 0$ p.u S_1 = To be determined , $S_{G2} = 0.25 + j\ Q_{G2}$ p.u ,S_{D2} = 1+j0.5p.u The two buses are connected by a transmission line p.u reactance of 0.5 p.u . Find Q_2 and $\angle V_2$. Neglect shunt susceptance of the tie line .Assume $|V_2| = 1.0$. Perform Two iterations Using GS Method. **[JNTU ,Supplementary , Feb-2008]**

Fig.(1)

Solution: From the given data, the single line diagram can be drawn as

$Z_{12} = j0.5$

$$y_{12} = \frac{1}{Z_{12}} = -j2$$

$P_2 = 0.25 - 1 = -0.75$, $V_1 = 1.0$ p.u

Let us assume the bus-1 as slack bus , so its voltage remains constant throughout all the iterations

$$V_1^0 = V_1^1 = V_1^2 = \ldots\ldots\ldots\ldots\ldots = V_1^k = (1 + j0.0)\,p.u$$

$$V_2^0 = 1.0\,p.u$$

The bus admittance matrix

$$Y_{bus} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} = \begin{bmatrix} -j2 & j2 \\ j2 & -j2 \end{bmatrix}$$

$$Q_{p,cal}^{k+1} = (-1) I.P.of \left\{ (V_P^k)^* \left[\sum_{q=1}^{p-1} Y_{pq} V_q^{k+1} + \sum_{q=p}^{n} Y_{pq} V_q^k \right] \right\}$$

Here p = 2 , k=0 ,n=2

$$Q_{2,cal}^1 = (-1) I.P.of \left\{ (V_2^0)^* \left[Y_{21} V_1^1 + Y_{22} V_2^0 \right] \right\} = (-1) I.P.of \left\{ (1) \left[(j2)(1) + (-j2)(1) \right] \right\}$$

$$= (-1) I.P.of\,(j2 - j2) \ =0$$

Q_2=0 p.u

$$V_p^{K+1} = \frac{1}{Y_{pp}}\left[\frac{P_p - jQ_p}{(V_P^k)^*} - \sum_{q=1}^{p-1} Y_{pq}V_q^{K+1} - \sum_{q=p+1}^{n} Y_{pq}V_q^K\right]$$

$$V_2^1 = \frac{1}{Y_{22}}\left[\frac{P_2 - jQ_2}{(V_2^0)^*} - Y_{21}V_1^1\right]$$

$$= \frac{-1}{j2}\left[\frac{-0.75 - j0}{1} - j2\right]$$

$$= \frac{0.75 + j2}{j2} \quad = 1 - j0.375$$

$$V_2^1 = 1.068\angle -20.56\,p.u$$

$$\delta_2^1 = -20.56$$

$$V_2^1 = |V_2|\angle\delta_2^1 = 1\angle -20.56\,p.u = 0.936 - j0.35$$

$$V_2^2 = \frac{1}{Y_{22}}\left[\frac{P_2 - jQ_2}{(V_2^1)^*} - Y_{21}V_1^2\right]$$

$$= \frac{1}{-j2}\left[\frac{-0.75 - j0}{0.936 + j0.35} - j2\times 1\right]$$

$$= \frac{1}{j2}\left[2.674 - j0.2628 + j2\right]$$

= 0.868-j1.337 = 1.594 $\angle$ -57

$$\delta_2^2 = -57$$

$$V_2^2 = |V_2|\angle\delta_2^2 = 1\angle -57\,p.u$$