UNIT 5

SINGLE PHASE TRANSFORMERS:

Types of transformers, constructional details, principle of operation, EMF equation, phasor diagram, losses and efficiency, regulation, all day efficiency, effect of variations of frequency and supply voltage on iron losses,

Auto transformers- equivalent circuit, comparison with two winding transformers.

INTRODUCTION

- The transformer can **change the magnitude of alternating voltage or current from one value to another.**
- This useful property of transformer is mainly responsible for the widespread use of alternating currents rather than direct currents i.e**., electric power is generated, transmitted and distributed in the form of alternating current.**
- Transformers have **no moving parts**, rugged and durable in construction, thus requiring very little attention.
- They also have a very high efficiency—as high as **99%.**

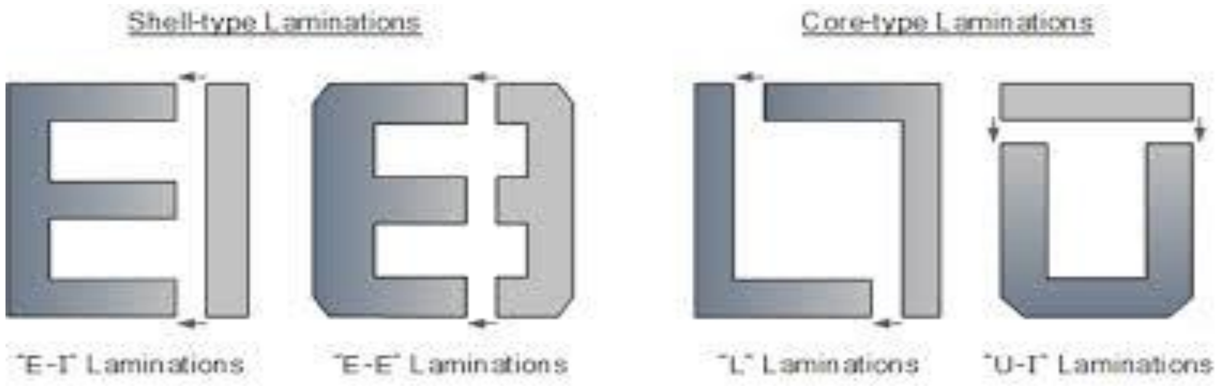
TRANSFORMER

A transformer is a static device that transfers AC electrical power from one circuit to the another circuit without change in frequency, but the voltage level and current level are usually changed.

A transformer is a static piece of equipment used either for raising or lowering the voltage of an a.c. supply with a corresponding decrease or increase in current.

CONSTRUCTION OF A TRANSFORMER

We usually design a power transformer so that it approaches the characteristics of an ideal transformer. To achieve this, following design features are incorporated:



(i) **The core is made of silicon steel** which has low hysteresis loss and high permeability. Further, **core is laminated in order to reduce eddy current loss.** These features considerably reduce the iron losses under the no-load current.

(ii) Instead of placing primary on one limb and secondary on the other, it is a usual practice to **wind one-half of each winding on one limb.** This ensures tight coupling between the two windings. Consequently, **leakage flux is considerably reduced.**

(iii) The winding resistances R_1 and R_2 are minimized to reduce I^2R loss and resulting rise in temperature and to ensure high efficiency.

TYPES OF TRANSFORMERS

Depending upon the manner in which the primary and secondary are wound on the core, transformers are of two types viz., (i) core-type transformer and (ii) shell-type transformer.

(i) **Core-type transformer**.

In a core-type transformer, half of the primary winding and half of the secondary winding are placed round each limb as shown in Fig. (5.1). This reduces the leakage flux. It is a usual practice to place the low-voltage winding below the high-voltage winding for mechanical considerations.

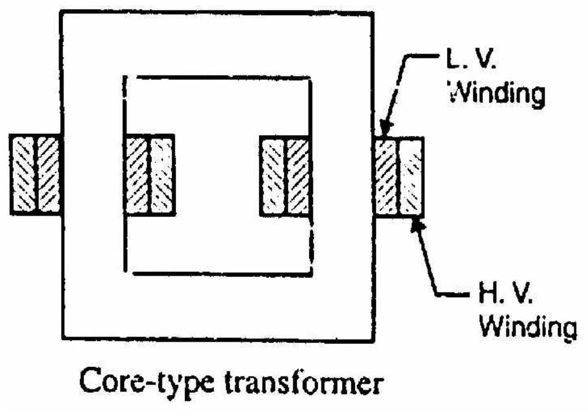


Fig.5.1

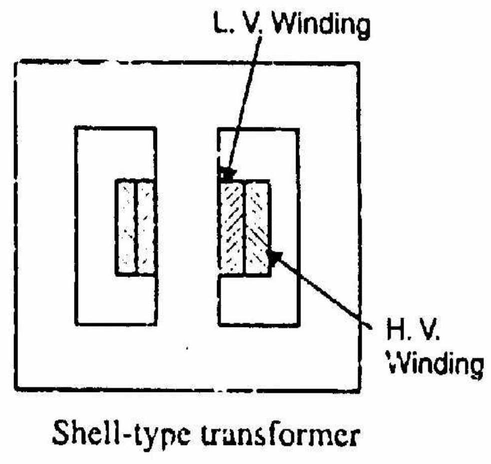
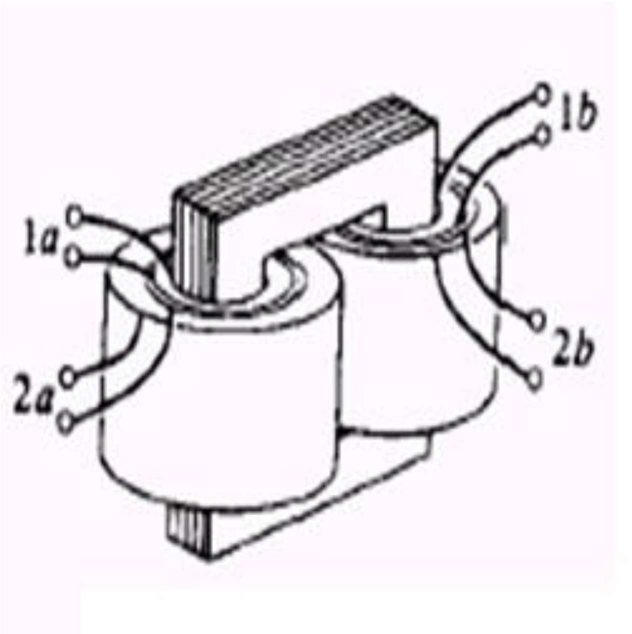
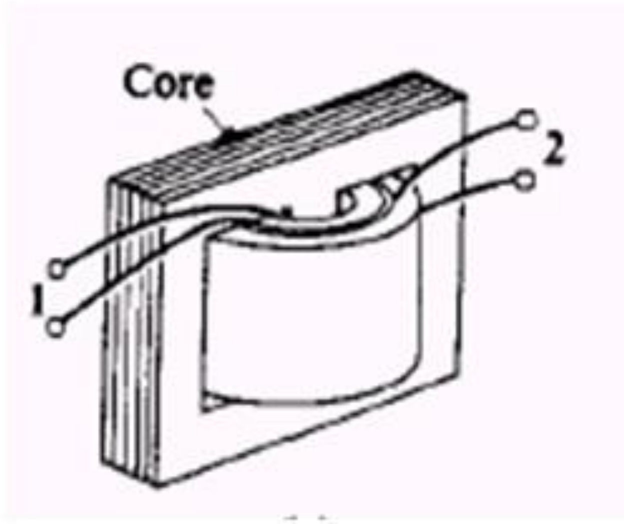


Fig.5.2

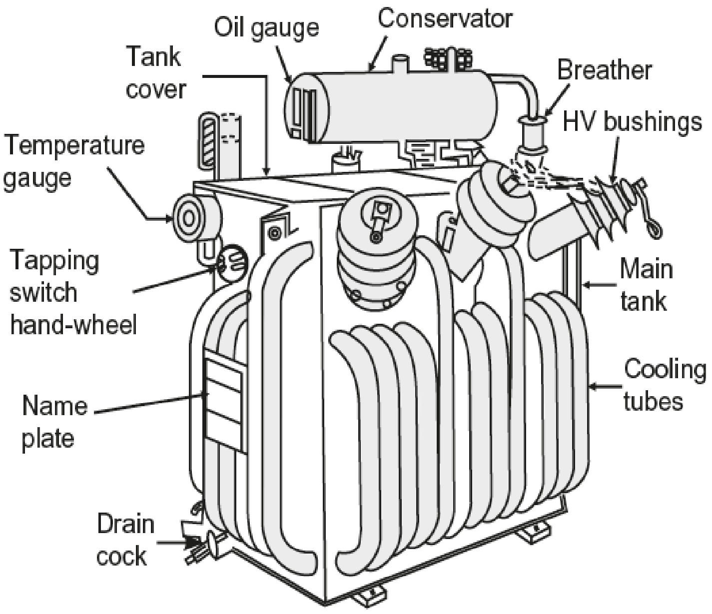




(ii) **Shell-type transformer**.

This method of construction involves the use of a double magnetic circuit. **Both the windings are placed round the central limb** (See Fig. 5.2), the other two limbs acting simply as a low-reluctance flux path. The choice of type (whether core or shell) will not greatly affect the efficiency of the transformer.

The core type is generally more suitable for **high voltage and small output** while the shell-type is generally more suitable for **low voltage and high output.**



500 kVA, 11/0.4 kV
(a)

(b)

TRANSFORMER

A transformer is a static device that transfers AC electrical power from one circuit to the another circuit without change in frequency, but the voltage level and current level are usually changed. A transformer is a static piece of equipment used either for raising or lowering the voltage of an a.c. supply with a corresponding decrease or increase in current.

It essentially consists of two windings, the primary and secondary, wound on a common laminated magnetic core as shown in Fig. 5.3. The winding connected to the a.c. source is called primary winding (or primary) and the one connected to load is called secondary winding (or secondary).

The alternating voltage V_1 whose magnitude is to be changed is applied to the primary. Depending upon the number of turns of the primary (N_1) and secondary (N_2), an alternating e.m.f. E_2 is induced in the secondary. This induced e.m.f. E_2 in the secondary causes a secondary current I_2. Consequently, terminal voltage V_2 will appear across the load. If $V_2 > V_1$, it is called a step up-transformer. On the other hand, if $V_2 < V_1$, it is called a step-down transformer.

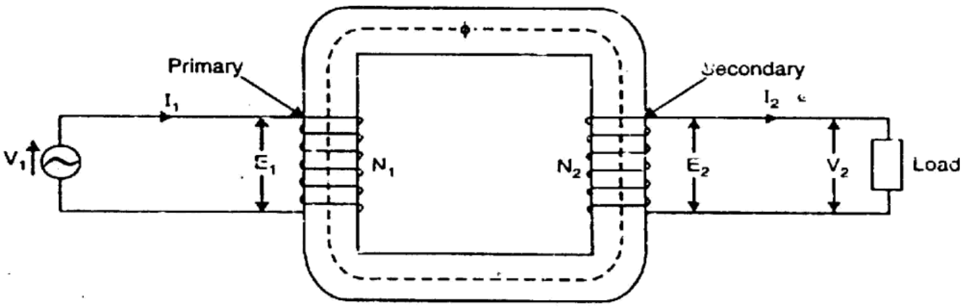


Fig.5.3

WORKING PRINCIPLE

When an alternating voltage V_1 is applied to the primary, an alternating flux is set up in the core. This alternating flux links both the windings and induces e.m.f.s E_1 and E_2 in them according to Faraday's laws of electromagnetic induction. The e.m.f. E_1 is termed as primary e.m.f (self induced emf) and e.m.f. E_2 is termed as secondary e.m.f (mutually induced emf).

$$E_1 = -N_1 \frac{d\phi}{dt}$$

and

$$E_2 = -N_2 \frac{d\phi}{dt}$$

$$\frac{E_2}{E_1} = \frac{N_2}{N_1}$$

Note that magnitudes of E_2 and E_1 depend upon the number of turns on the secondary and primary respectively. **If $N_2 > N_1$, then $E_2 > E_1$**

(or $V_2 > V_1$) and we get a step-up transformer.

On the other hand, **if $N_2 < N_1$, then $E_2 < E_1$ (or $V_2 < V_1$) and we get a step-down transformer.**

If load is connected across the secondary winding, the secondary e.m.f. E_2 will cause a current I_2 to flow through the load. Thus, a transformer enables us to transfer a.c. power from one circuit to another with a change in voltage level.

The following points may be noted carefully:

(i) The transformer action is based on the **laws of electromagnetic induction.**

(ii) There is no electrical connection between the primary and secondary. **The a.c. power is transferred from primary to secondary through magnetic flux.**

(iii) There is no change in frequency i.e., **output power has the same frequency as the input power.**

(iv) The losses that occur in a transformer are:

(a) **core losses** —eddy current and hysteresis losses

(b) **copper losses** —in the resistance of the windings

In practice, these losses are very small so that output power is nearly equal to the input primary power. In other words, a **transformer has very high efficiency.**

E.M.F. EQUATION OF A TRANSFORMER

Consider that an alternating voltage V_1 of frequency f is applied to the primary .The sinusoidal flux ϕ produced by the primary can be represented as:

$$\phi = \phi_m \sin \omega t$$

The instantaneous e.m.f. e_l induced in the primary is

$$e_l = -N_l \frac{d\phi}{dt} = -N_l \frac{d}{dt}(\phi_m \sin \omega t)$$
$$= -\omega N_l \phi_m \cos\omega t = -2\pi f N_l \phi_m \cos\omega t$$
$$= 2\pi f N_l \phi_m \sin(\omega t - 90^0)$$

It is clear from the above equation that maximum value of induced e.m.f. in the primary is $E_{ml} = 2\pi f N_l \phi_m$

The r.m.s value E_l of the primary e.m.f. is

$$E_l = \frac{E_{ml}}{\sqrt{2}} = \frac{2\pi f N_l \phi_m}{\sqrt{2}}$$

or $E_l = 4.44\, f N_l \phi_m$

$$E_2 = 4.44\, f N_2 \phi_m$$

*similarly

In an ideal transformer, E_l=V_l and V_2.

Note. It is clear from exp. (i) above that e.m.f. E_1 induced in the primary lags behind the flux ϕ by 90°. Likewise, e.m.f. E_2 induced in the secondary lags behind flux ϕ by 90°.

VOLTAGE TRANSFORMATION RATIO (K)

From the above emf equations

$$\frac{E_2}{E_1} = \frac{N_2}{N_1} = k$$

The constant K is called *voltage transformation ratio.* Thus if K = 5 (i.e. N_2/N_1 = 5), then E_2=5E_1

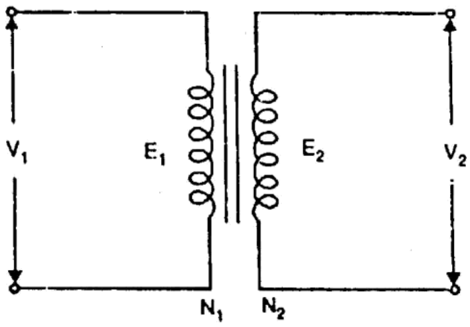


Fig.5.4

For an ideal transformer:

(i) E_l=V_l and E_2 = V_2 as there is no voltage drop in the windings.

$$\therefore \frac{E_2}{E_1} = \frac{V_2}{V_1} = \frac{N_2}{N_1} = k$$

(ii) There are no losses. Therefore, volt-amperes input to the primary are equal to the output volt-amperes i.e $V_1 I_1 = V_2 I_2$

Or $I_2/I_1 = V_1/V_2 = 1/k$.

Hence, currents are in the inverse ratio of voltage transformation ratio. This simply means that if **we raise the voltage, there is a corresponding decrease of current.**

THEORY OF AN IDEAL TRANSFORMER (on no load)

An ideal transformer is one that has

(i) **no winding resistance**

(ii) **no leakage flux** i.e., the same flux links both the windings

(iii) **no iron losses** (i.e., eddy current and hysteresis losses) in the core

Although ideal transformer cannot be physically realized, yet its study provides a very powerful tool in the analysis of a practical transformer. In fact, practical

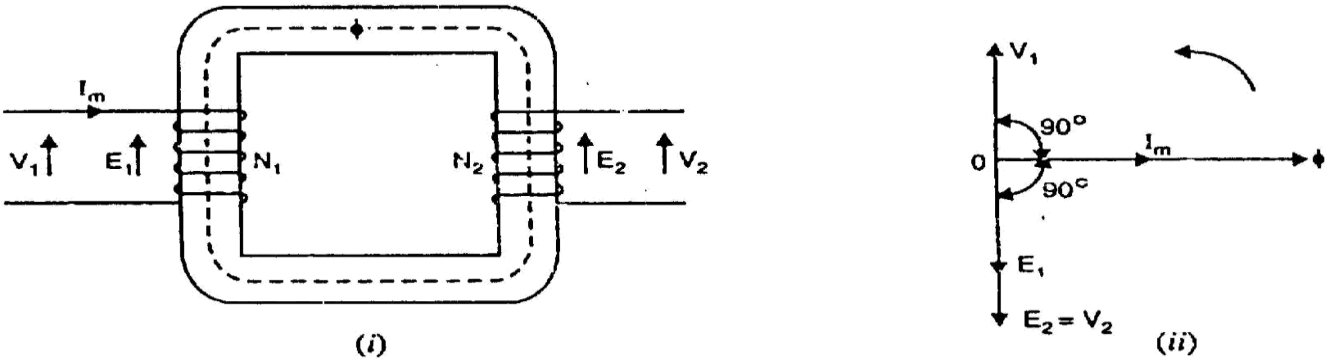


Fig.5.5

transformers have properties that approach very close to an ideal transformer.

Consider an ideal transformer on no load i.e., secondary is open-circuited as shown in Fig. (5.5 (i)). Under such conditions, the primary is simply a coil of pure inductance. When an alternating voltage V_1 is applied to the primary, it draws a small magnetizing current I_m which lags behind the applied voltage by90°.

This alternating current I_m produces an alternating flux ϕ which is proportional to and in phase with it. The alternating flux ϕ links both the windings and induces e.m.f. E_1 in the primary and e.m.f. E_2 in the secondary. The primary e.m.f. E_1 is, at every instant, equal to and in opposition to V_1 (Lenz's law). Both e.m.f.s E_1 and E_2 lag behind flux ϕ by 90°. However, their magnitudes depend upon the number of primary and secondary turns.

Fig. (5.5 (ii)) shows the phasor diagram of an ideal transformer on no load. Since flux ϕ is common to both the windings, it has been taken as the reference phasor. The primary e.m.f. E_1 and secondary e.m.f. E_2 lag behind the flux ϕ by 90. Note that E_1 and E_2 are in phase. But E_1 is equal to V_1 and 180° out of phase with it.

PRACTICAL TRANSFORMER

A practical transformer differs from the ideal transformer in many respects. The practical transformer has (i) iron losses (ii) winding resistances and (iii) magnetic leakage, giving rise to leakage reactances.

(i) **Iron losses**. Since the iron core is subjected to alternating flux, there occurs eddy current and hysteresis loss in it. These two losses together are known as iron losses or core losses. The iron losses depend upon the supply frequency, maximum flux density in the core, volume of the core etc. It may be noted that magnitude of iron losses is quite small in a practical transformer.

(ii) **Winding resistances**. Since the windings consist of copper conductors, it immediately follows that both primary and secondary will have winding resistance. The primary resistance R_1 and secondary resistance R_2 act in series with the respective windings as shown in Fig. (5.6). When current flows through the windings, there will be power loss as well as a loss in voltage due to IR drop. This will affect the power factor and E_1 will be less than V_1 while V_2 will be less than E_2.

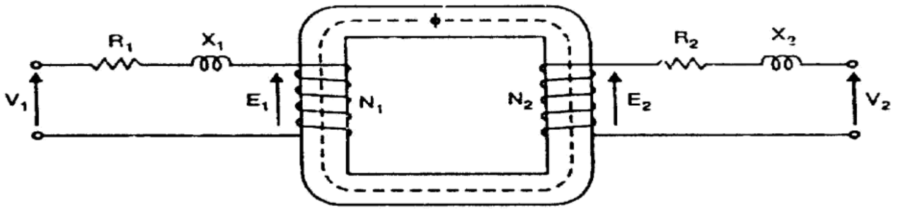


Fig.5.6

(iii) **Leakage reactances**. Both primary and secondary currents produce flux.

The flux ϕ which links both the windings is the useful flux and is called mutual flux. However, primary current would produce some flux ϕ which would not link the secondary winding (See Fig. 5.7).

Similarly, secondary current would produce some flux ϕ_2 that would not link the primary winding. The flux such as ϕ_1 or ϕ_2 which links only one winding is called leakage flux. The leakage flux paths are mainly through the air. The effect of these leakage fluxes would be the same as though inductive reactance were connected in series with each winding of transformer that had no leakage flux as shown in Fig. (5.6).

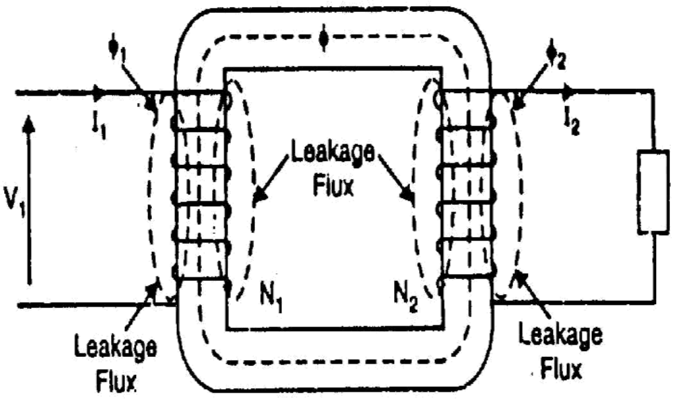


Fig.5.7

In other words, the effect of primary leakage flux ϕ_1 is to introduce an inductive reactance X_1 in series with the primary winding as shown in Fig. (5.6). Similarly, the secondary leakage flux ϕ_2 introduces an inductive reactance X_2 in series with the secondary winding. There will be no power loss due to leakage reactance. However, the presence of leakage reactance in the windings changes the power factor as well as there is voltage loss due to IX drop.

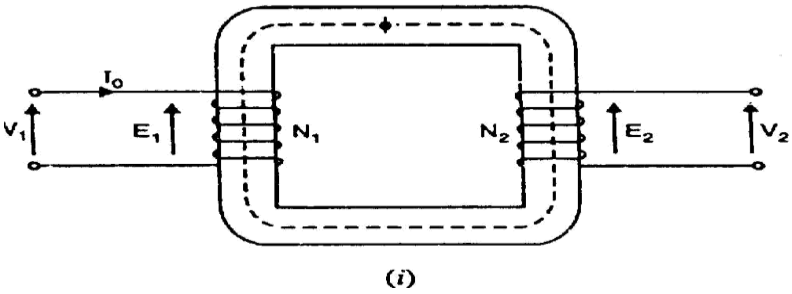
Note. Although leakage flux in a transformer is quite small (about 5% of ϕ) compared to the mutual flux ϕ , yet it cannot be ignored. It is because leakage flux paths are through air of high reluctance and hence require considerable e.m.f. It may be noted that energy is conveyed from the primary winding to the secondary winding by mutual flux ϕ which links both the windings.

PRACTICAL TRANSFORMER ON NO LOAD

Consider a practical transformer on no load i.e., secondary is open-circuit as shown in Fig. (5.8 (i)). The primary will draw a small current I_0 to supply

(i) the iron losses and (ii) a very small amount of copper loss in the primary. Hence the primary no load current I_0 is not 90° behind the applied voltage V_1 but lags it by an angle $\phi_0 < 90°$ as shown in the phasor diagram in Fig. (5.8 (ii)).

No load input power, $W_0 = V_1 I_0 \cos\phi_0$



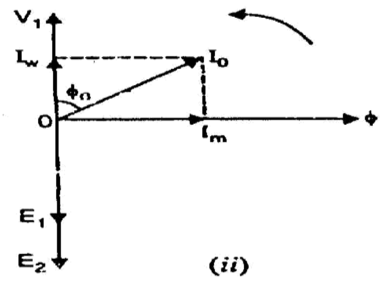


Fig.5.8

As seen from the phasor diagram in Fig. (5.8 (ii)), the no-load primary current I_0 can be resolved into two rectangular components viz.

(i) The component I_w in phase with the applied voltage V_1. This is known as **active or working or iron loss component and supplies the iron loss and a very small primary copper loss.**

$$I_W = I_0 \cos\phi_0$$

(ii) The component I_m lagging behind V_1 by 90° and is known as magnetizing component. It is this component which **produces the mutual flux** ϕ in the core.

$$I_m = I_0 \sin\phi_0$$

Clearly, I_0 is phasor sum of I_m and I_W,

$$I_0 = \sqrt{(I_m^2 + I_w^2)}$$

No load p.f., $\cos\phi_0 = I_W / I_0$

It is emphasized here that no load primary copper loss (i.e. $I_0^2 R_1$) is very small and may be neglected. Therefore, the no load primary input power is practically equal to the iron loss in the transformer i.e.

No load input power, W_0= Iron loss

Note. At no load, there is no current in the secondary so that $V_2=E_2$. On the primary side, the drops in R_1 and X_1 due to I_0 are also very small because of the smallness of I_0. Hence, we can say that at no load, $V_1=E_1$.

IDEAL TRANSFORMER ON LOAD

Let us connect a load Z_L across the secondary of an ideal transformer as shown in Fig. (5.9 (i)). **The secondary e.m.f. E_2 will cause a current I_2 to flow through the load.**

$$I_2 = \frac{E_2}{Z_L} = \frac{V_2}{Z_L}$$

The angle at which I_2 leads or lags V_2 (or E_2) depends upon the resistance and reactance of the load. In the present case, we have considered inductive load so that current I_2 lags behind V_2 (or E_2) by Φ_2.

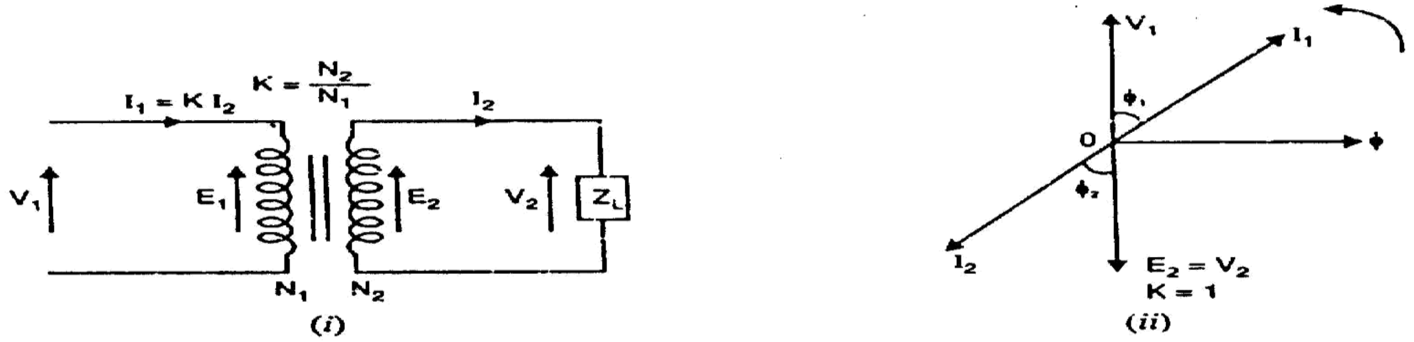


Fig.5.9

The **Secondary current I_2 sets up an m.m.f. N_2I_2 which produces a flux in the opposite direction to the flux** ϕ originally set up in the primary by the magnetizing current. This will change the flux in the core from the original value. However, the flux in the core should not change from the original value

In order to fulfill this condition, **the primary must develop an m.m.f. which exactly counterbalances the secondary m.m.f. N_2I_2.** Hence a primary current I_1 must flow such that:

$N_1 I_1 = N_2 I_2$

Or $I_1 = (N_2 / N_1) I_2 = K I_2$

Thus when a transformer is loaded and carries a secondary current I_2, then a current I_1, (= $K I_2$) must flow in the primary to maintain the m.m.f. balance. In other words, the primary must draw enough current to neutralize the demagnetizing effect of secondary current so that mutual flux ϕ remains constant.

Thus as the secondary current increases, the primary current I_1 (= KI_2) increases in unison and keeps the mutual flux ϕ constant. The power input, therefore, automatically increases with the output. For example if K = 2 and I_2 =2A, then primary will draw a current $I_1 = KI_2 = 2 \times 2 = 4A$. If secondary current is increased to 4A, then primary current will become $I_1 = KI_2 = 2 \times 4 = 8A$.

Phasor diagram: Fig. (5.9 (ii)) shows the phasor diagram of an ideal transformer on load. Note that in drawing the phasor diagram, the value of K has been assumed unity so that primary phasors are equal to secondary phasors. The secondary current I_2 lags behind V_2 (or E_2) by ϕ_2. It causes a primary current

$I_1 = KI_2 = 1 \times I_2$ which is in anti phase with it.

$$\phi_1 = \phi_2$$

Or $\cos\phi_1 = \cos\phi_2$

Thus, power factor on the primary side is equal to the power factor on the secondary side.

(ii) Since there are no losses in an ideal transformer, input primary power is equal to the secondary output power i.e.

PRACTICAL TRANSFORMER ON LOAD

We shall consider two cases (i) when such a transformer is assumed to have no winding resistance and leakage flux (ii) when the transformer has winding resistance and leakage flux.

(i) Transformer with resistance and leakage reactance

Fig. (5.10) shows a practical transformer having winding resistances and leakage reactances. These are the actual conditions that exist in a transformer. There is voltage drop in R_1 and X_1 so that primary e.m.f. E_1 is less than the applied voltage V_1.

Similarly, there is voltage drop in R_2 and X_2 so that secondary terminal voltage V_2 is less than the secondary e.m.f. E_2. Let us take the usual case of inductive load which causes the secondary current I_2 to lag behind the secondary voltage V_2 by ϕ_2. The total primary current I_1 must meet two requirements viz.

(a) It must supply the no-load current I_0 to meet the iron losses in the transformer and to provide flux in the core.

(b) It must supply a current I'_2 to counteract the demagnetizing effect of secondary current I_2. The magnitude of I'_2 will be such that:

$$N_1 I'_2 = N_2 I_2$$

or $I'_2 = N_2 / N_1 \times I_2 = K I_2$

The total primary current I_1 will be the phasor sum of I'_2 and I_0 i.e.,

$$I_1 = I'_2 + I_0$$

Where $I'_2 = K I_2$

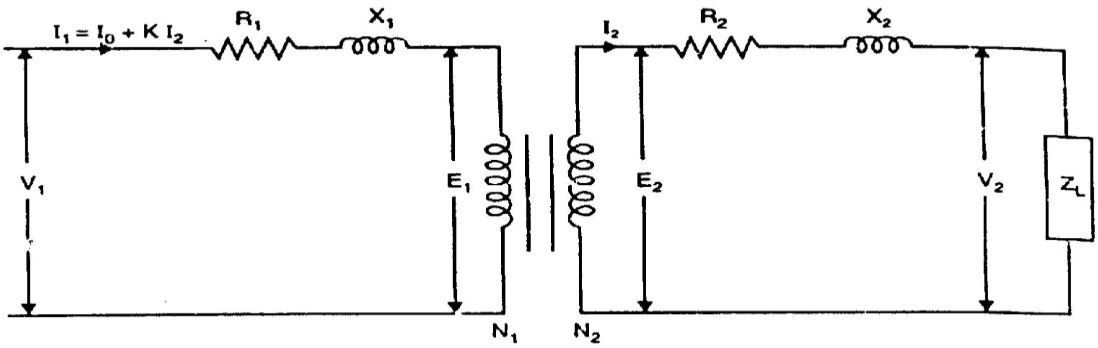


Fig.5.10

$$V_1 = E_1 + I_1(R_1 + jX_1) \text{ where } I_1 = I_0 + (-KI_2)$$

$= E_1 + I_1 Z_1$

$$V_2 = E_2 - I_2(R_2 + jX_2)$$

$= E_2 - I_2 Z_2$

Phasor diagram. Fig. (5.11) shows the phasor diagram of a practical transformer for the usual case of inductive load. Both E_1 and E_2 lag the mutual flux ϕ by 90°. The current I'_2 represents the primary current to neutralize the demagnetizing effect of secondary current I_2.

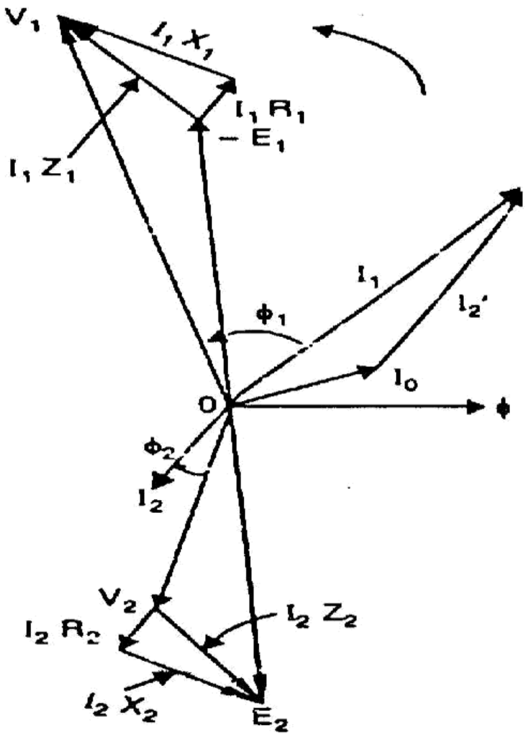


Fig.5.11

Now $I'_2 = K I_2$ and is opposite to I_2. Also I_0 is the no-load current of the transformer. The phasor sum of I'_2 and I_0 gives the total primary current I_1.

Note that counter e.m.f. that opposes the applied voltage V_1 is $- E_1$. Therefore, if we add I_1R_1 (in phase with I_1) and $I_1 X_1$ (90° ahead of I_1) to $- E_1$, we get the applied primary voltage V_1.

The phasor E_2 represents the induced e.m.f. in the secondary by the mutual flux ϕ. The secondary terminal voltage V_2 will be what is left over after subtracting $I_2 R_2$ and $I_2 X_2$ from E_2.

Load power factor = $\cos \phi_2$

Primary power factor = $\cos \phi_1$

Input power to transformer, $P_1 = V_1 I_1 \cos \phi_1$

Output power of transformer, $P_2 = V_2 I_2 \cos \phi_2$

Note: The reader may draw the phasor diagram of a loaded transformer for (i)unity p.f. and (ii) leading p.f. as an exercise.

SHIFTING IMPEDANCES IN A TRANSFORMER

fig. (5.12) shows a transformer where resistances and reactances are shown external to the windings. The resistance and reactance of one winding can be transferred to the other by appropriately using the factor K^2. This makes the analysis of the transformer a simple affair because then we have to work in one

Winding only..

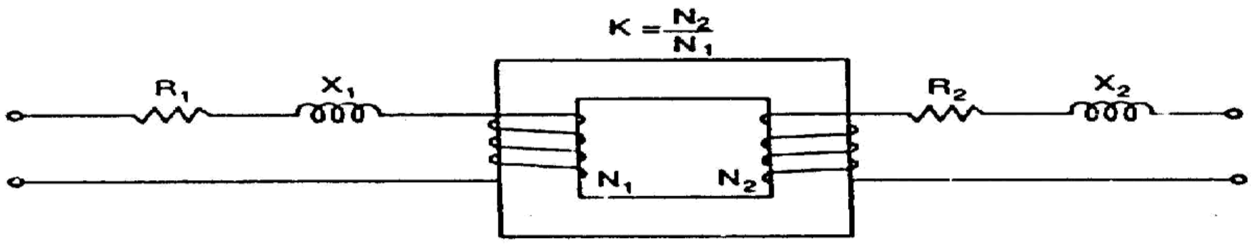


Fig.5.12

(i) Referred to primary: When secondary resistance or reactance is transferred to the primary, it is divided by K^2. It is then called equivalent secondary resistance or reactance referred to primary and is denoted by R'_2 or X'_2.

Equivalent resistance of transformer referred to primary is
$R_{01} = R_1 + R_2^1 = R_1 + R_2 / k^2$

Equivalent reactance of transformer referred to primary is

$X_{01} = X_1 + X_2^1 = X_1 + X_2 / k^2$

Equivalent impedance of transformer referred to primary is

$Z_{01} = \sqrt{(R^2{}_{01} + X_{01}^2)}$

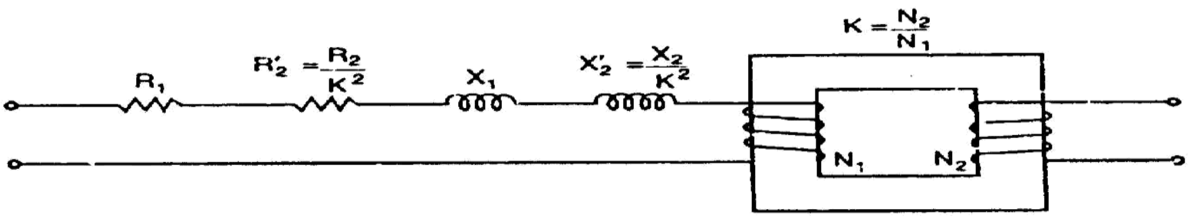


Fig.5.13

Fig. (5.13) shows the resistance and reactance of the secondary referred to the primary. Note that secondary now has no resistance or reactance.

(ii) Referred to secondary

When primary resistance or reactance is transferred to the secondary, it is multiplied by K^2. It is then called equivalent primary resistance or reactance referred to the secondary and is denoted by R'_1 or X'_1.

Equivalent resistance of transformer referred to secondary

$R_{02} = R_2 + R_1' = R_2 + K^2 R_1$

Equivalent reactance of transformer referred to secondary is

$X_{02} = X_2 + X_1' = X_2 + K^2 X_1$

Equivalent impedance of transformer referred to secondary is $Z_{02} = \sqrt{(R_{02}^2 + X_{02}^2)}$ Fig. (5.14) shows the resistance and reactance of the primary referred to the secondary. Note that primary now has no resistance or reactance.

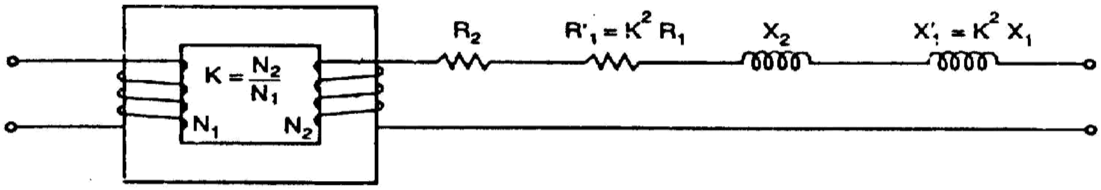


Fig.5.14

EQUIVALENT CIRCUIT OF SINGLE PHASE TRANSFORMER

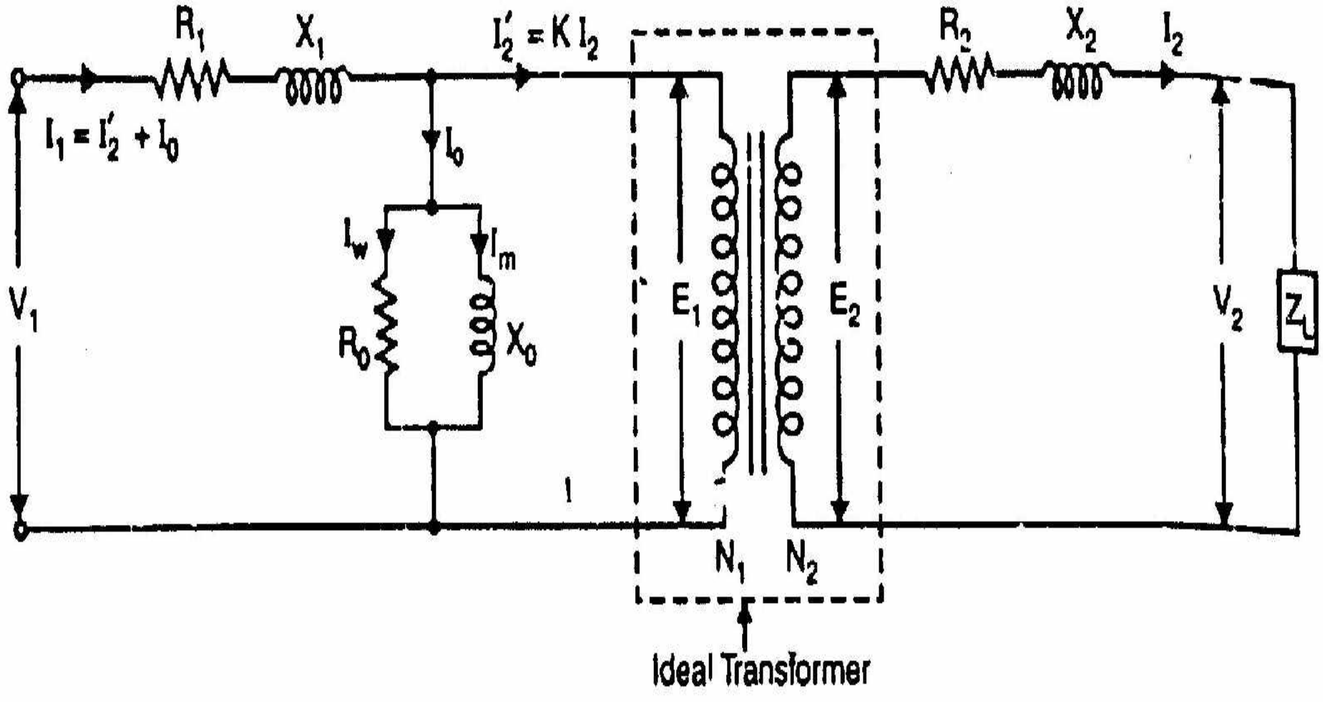


Fig.5.15

Fig. (5.15) shows the exact equivalent circuit of a transformer on load. Here R_1 is the primary winding resistance and R_2 is the secondary winding resistance.

Similarly, X_1 is the leakage reactance of primary winding and X_2 is the leakage reactance of the secondary winding. The parallel circuit R_0 - X_0 is the no-load equivalent circuit of the transformer.

The resistance R_0 represents the core losses (hysteresis and eddy current losses) so that current I_W which supplies the core losses is shown passing through R_0.

The inductive reactance X_0 represents a loss-free coil which passes the magnetizing current I_m. The phasor sum of I_W and I_m is the no-load current I_0 of the transformer.

Note that in the equivalent circuit shown in Fig. (5.15), the imperfections of the transformer have been taken into account by various circuit elements. Therefore, the transformer is now the ideal one. Note that equivalent circuit has created two normal electrical circuits separated only by an ideal transformer whose function is to change values according to the equation:

$$\frac{E_2}{E_1} = \frac{N_2}{N_1} = \frac{I_2'}{I_2}$$

The following points may be noted from the equivalent circuit:

(i) When the transformer is on no-load (i.e., secondary terminals are open- circuited), there is no current in the secondary winding. However, the primary draws a small no-load current I_0. The no-load primary current I_0 is composed of (a) magnetizing current (I_m) to create magnetic flux in the core and (b) the current I_W required to supply the core losses.

(ii) When the secondary circuit of a transformer is closed through some external load Z_L, the voltage E_2 induced in the secondary by mutual flux will produce a secondary current I_2. There will be $I_2 R_2$ and $I_2 X_2$ drops in the secondary winding so that load voltage V_2 will be less than E_2.

$$V_2 = E_2 - I_2(R_2 + JX_2) = E_2 - I_2 Z_2$$

(iii) When the transformer is loaded to carry the secondary current I_2, the primary current consists of two components:

(a) The no-load current I_0 to provide magnetizing current and the current required to supply the core losses.

(b) The primary current I'_2 (= K I_2) required to supply the load connected to the secondary.

Total primary current $I_1 = I_0 + (-KI_2)$

(iv) Since the transformer in Fig. (5.15) in now ideal, the primary induced voltage E_1 can be calculated from the relation:

$$\frac{E_2}{E_1} = \frac{N_2}{N_1}$$

If we add I_1R_1 and I_1X_1 drops to E_1, we get the primary input voltage V_1

$$V_1 = -E_1 + I_1(R_1 + jX_1) = -E_1 + I_1 Z_1$$

$$V_1 = -E_1 + I_1 Z_1$$

SIMPLIFIED EQUIVALENT CIRCUIT OF A LOADED TRANSFORMER

The no-load current I_0 of a transformer is small as compared to the rated primary current. Therefore, voltage drops in R_1 and X_1 due to I_0 are negligible. The equivalent circuit shown in Fig. (5.15)) above can, therefore, be simplified by transferring the shunt circuit R_0 - X_0 to the input terminals as shown in Fig. (5.16). This modification leads to only slight loss of accuracy.

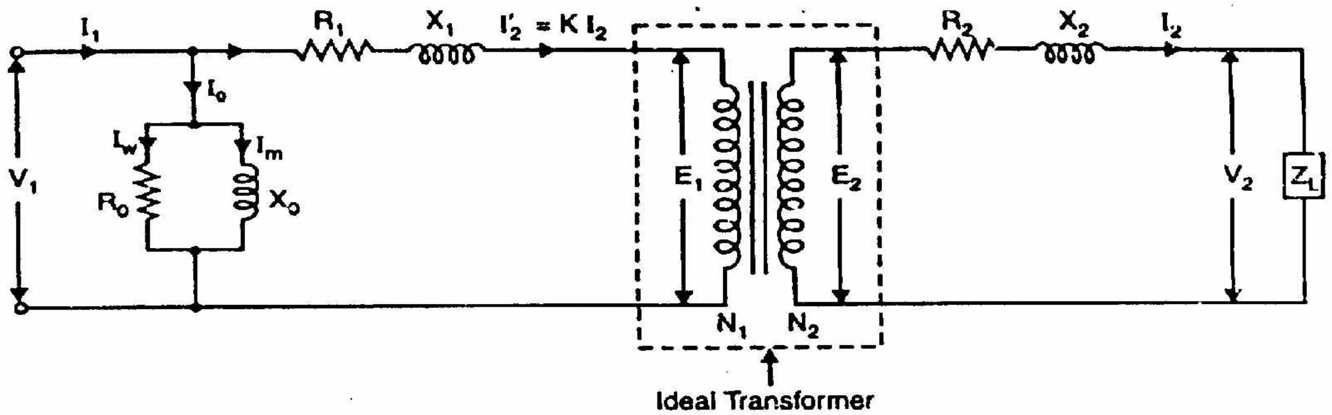


Fig.5.16

i) Equivalent circuit referred to primary

If all the secondary quantities are referred to the primary, we get the equivalent circuit of the transformer referred to the primary as shown in Fig. (2.19 (i)). This further reduces to Fig. (2.19 (ii)).

Note that when secondary quantities are referred to primary, resistances/reactances/impedances are divided by K^2, voltages are divided by K and currents are multiplied by K.

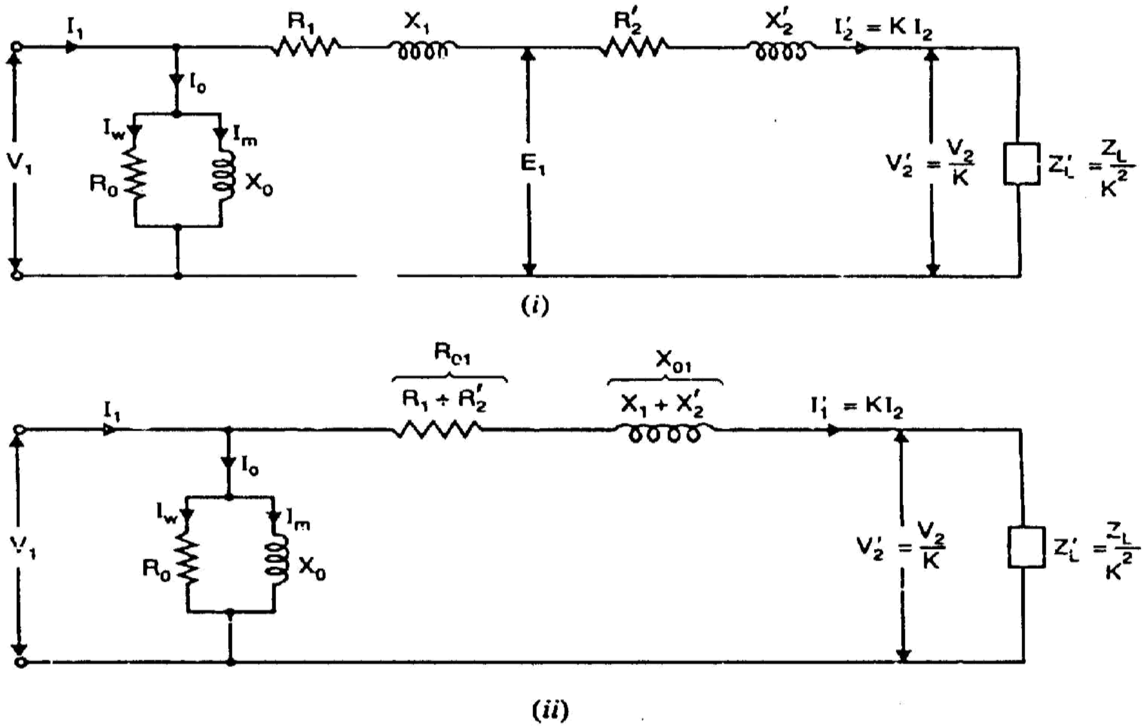


Fig.5.19

$$R_2^{'} = \frac{R_2}{K^2}; X_2^{'} = \frac{X_2}{K^2}; Z_L^{'} = \frac{Z_L}{K^2}; V_2^{'} = \frac{V_2}{K}; I_2^{'} = KI_2$$

$$Z_{01} = R_{01} + jX_{01}$$

$$R_{01} = R_1 + R_2^{'}; X01 = X_1 + X_2^{1}$$

(ii) Equivalent circuit referred to secondary.

If all the primary quantities are referred to secondary, we get the equivalent circuit of the transformer referred to secondary as shown in Fig. (2.20 (i)). This further reduces to Fig. (2.20 (ii)).

Note that when primary quantities are referred to secondary resistances/reactances/impedances are multiplied by K^2, voltages are multiplied by K. and currents are divided by K.

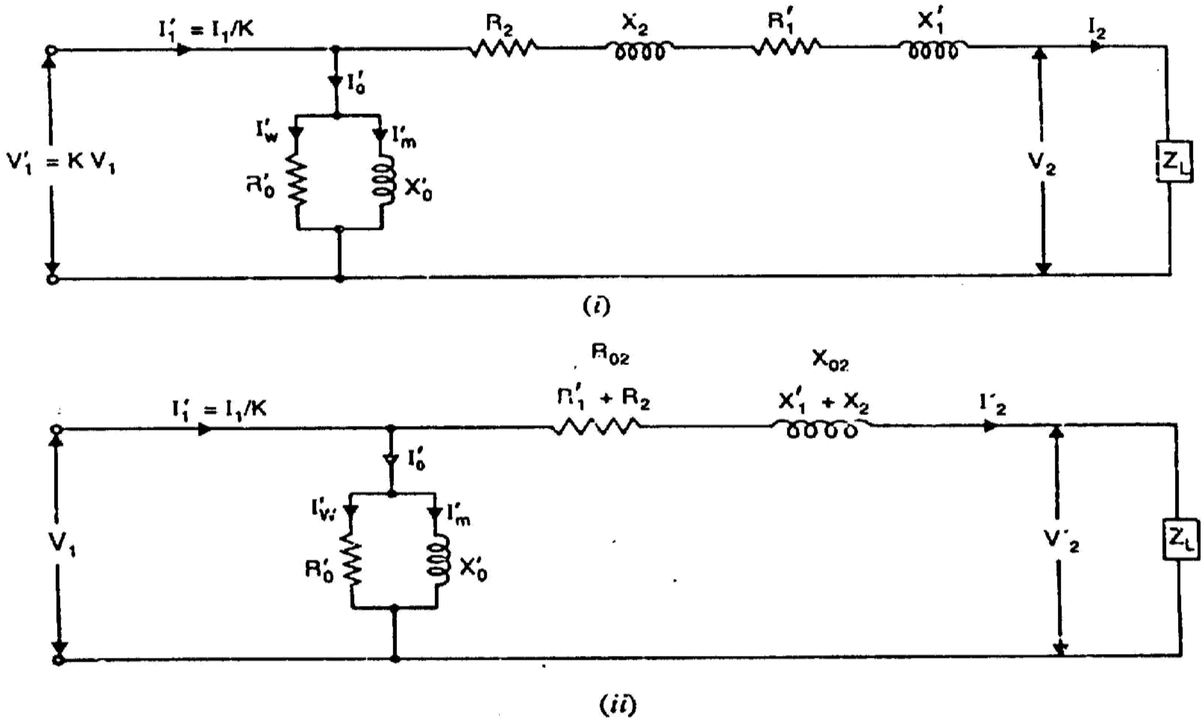


Fig.5.20

APPROXIMATE VOLTAGE DROP IN A TRANSFORMER

The approximate equivalent circuit of transformer referred to secondary is shown in Fig. (2.21). At no-load, the secondary voltage is K V_1. When a load having a lagging p.f. $\cos\phi_2$ is applied, the secondary carries a current I_2 and voltage drops occur in (R_2 + K^2 R_1) and (X_2 + K^2 X_1). Consequently, the secondary voltage falls from K V_1 to V_2. Referring to Fig. (2.21), we have,

$$V_2 = KV_1 - I_2\left[\left(R_2 + K^2R_1\right) + j\left(X_2 + K^2X_1\right)\right]$$

$$= KV_1 - I_2\left(R_{02} + jX_{02}\right)$$

$$= KV_1 - I_2Z_{02}$$

Drop in secondary voltage= $KV_1 - V_2 = I_2Z_{02}$

The phasor diagram is shown in Fig. (2.22). It is clear from the phasor diagram that drop in secondary voltage is AC = $I_2 Z_{02}$. It can be found as follows. With O as

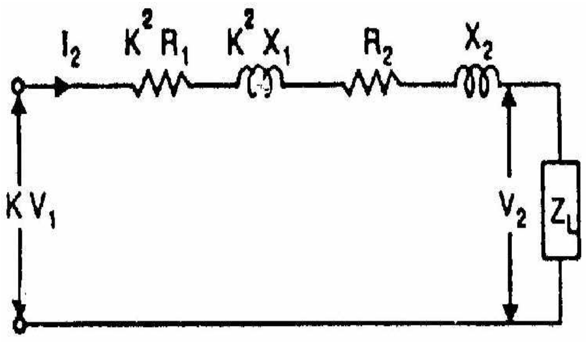


Fig.5.21

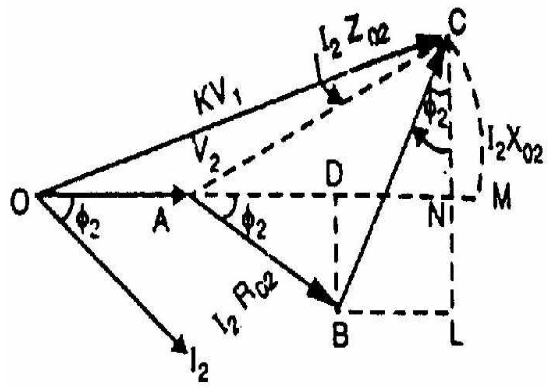


Fig.5.22

centre and OC as radius, draw an arc cutting OA produced at M. Then AC = AM = AN. From B, draw BD perpendicular to OA produced. Draw CN perpendicular to OM and draw BL | | OM.

Approximate drop in secondary voltage=AN=AD+DN

=AD+BL

$= I_2R_{02}\cos\phi_2 + I_2X_{02}\sin\phi_2$

For a load having a leading p.f. cos ϕ_2 we have ,

Approximate voltage drop= $I_2R_{02} * \cos\phi_2 - I_2X_{02}\sin\phi_2$

Note: If the circuit is referred to primary, then it can be easily established that:

Approximate voltage drop= $I_1R_{01} * \cos\phi_1 \pm I_1X_{01}\sin\phi 1$

VOLTAGE REGULATION

The voltage regulation of a transformer is the arithmetic difference (not phasor difference) between the no-load secondary voltage ($_0V_2$) and the secondary voltage V_2 on load expressed as percentage of no-load voltage i.e

% age voltage regulation = $\frac{_0V_2 - V_2}{_0V_2} \times 100$

Where $\quad _0V_2 =$ No-load secondary voltage $=KV_1$

V_2 =Secondary voltage on load

As shown in art **8.15,** $\quad _0V_2 - V_2 = I_2 R_{02} \cos\phi_2 \pm I_2 X_{02}\sin\phi_2$

The +ve sign is for lagging p.f. and -ve sign for leading p.f.

It may be noted that %age voltage regulation of the transformer will be the same whether primary or secondary side is considered.

TRANSFORMER LOSSES

The transformer is a static machine and, therefore, there are no friction or windage losses. The various power losses occurring in a transformer are enumerated below:

1. Iron or Core Losses.

Iron loss is caused by the alternating flux in the core and consists of hysteresis and eddy current losses.

(a) Hysteresis Loss. The core of a transformer is subjected to an alternating magnetizing force and for each cycle of emf a hysteresis loop is traced out. The hysteresis loss per second is given by the equation

Hysteresis loss, $P_h = \eta'(B_{max})^x f v$ joules per second or watt ... (1)

Where f is the supply frequency in Hz, v is the volume of core in cubic metres, η' is the hysteresis coefficient, B_{max} is peak value of flux density in the core and x lies between 1.5 and 2.5 depending upon the material and is often taken as 1.6.

(b) Eddy Current Loss. If the magnetic circuit is made up of iron and if the flux in the circuit is variable, **currents will be induced by induction in the iron circuit itself. All such currents are known as eddy currents.**

Eddy currents result in a loss of power, with consequent beating of the material.

The eddy current loss is given by equation

$P_e = K_e(B_{max})^2 f^2 t^2 v$ watts or joules per second ...(2)

From the above expression for eddy current loss in a thin sheet it is obvious that eddy current loss varies (i) as the square of maximum flux density (ii) as the square of the frequency and (iii) as the square of thickness of laminations.

The hysteresis and eddy current losses depend upon the maximum flux density in the core and supply frequency. Since it has been determined that the mutual flux varies somewhat with the load (its variation being 1 to 3% from no load to full load), the core losses will vary somewhat with the load and its power factor. It may be emphasized here that core losses are assumed to remain constant from no load to full load, the variations in losses from no load to full load being very small and negligible.

These losses are determined from the open-circuit test.

The input to the transformer with rated voltage applied to the primary and secondary open circuited is equal to the core loss.

These losses are minimized by **using steel of high silicon** content for the core and by using **very thin laminations (0.3 mm to 0.5 mm**) insulated from each other either by insulating varnish or by layer of papers.

2. Copper or Ohmic Losses. These losses occur due to ohmic resistance of the transformer windings. If I_1 and $I_2$are the primary and secondary currents respectively and R_1 and R_2 are the respective resistances of primary and secondary windings then copper losses occurring in primary and secondary windings will be $I_1^2R_1$ and $I_2^2R_2$ respectively.

So total copper losses will **be $(I_1^2R_1 + I_2^2R_2)$.** These losses vary as the square of the load current or kVA. For example if the copper losses at full load are P_C then copper losses at one-half or one-third of full load will be $\left(\frac{1}{2}\right)^2 P_C$ or $\left(\frac{1}{3}\right)^2 P_C$ i.e. $\frac{P_C}{4}$ or $\frac{P_C}{9}$ respectively.

TRANSFORMER EFFICIENCY

The rated capacity of a transformer is defined as the **product of rated voltage and full-load (rated) current** on the output side. The power output depends upon the power factor of the load.

The efficiency (η) of a transformer, like that of any other apparatus, is defined as the **ratio of useful power output to the input power**, the two being measured in same units (either in watt or killowatts).

i.e. Transformer efficiency,

$$\eta = \frac{Output}{Input} = \frac{Output}{Output + losses}$$

$$= \frac{Output}{Output + iron\ loss + copper\ loss}$$

$$= 1 - \frac{iron\ loss + copper\ loss}{Output + iron\ loss + copper\ loss}$$

$$Now\ power\ output = V_2 I_2 \cos\emptyset$$

Where V_2 is the secondary terminal voltage on load, I2 is the secondary current at load and cos Φ is the power factor of the load.

Iron Loss, P_i = Hysteresis loss + eddy current loss

Copper loss = $I_1^2R_1 + I_2^2R_2 = I_1^2R_{01} = I_2^2R_{02}$

DETERMINATION OF TRANSFORMER EFFICIENCY.

The ordinary transformer has a very **high efficiency (in the range of 96-99%)**.Hence the transformer efficiency cannot be determined with high precision by direct measurement of output and input, since the losses are of the order of only 1-4%. The difference between the readings of output and input instruments is then so small that an instrument error as low as 0.5% would cause an error of the order of 15% in the losses.

Further, it is inconvenient and costly to have the necessary loading devices of the correct current and voltage ratings and power factor to load the transformer. There is also a wastage of large amount of power (equal to that of power output + losses) and no information is available from such a test about the proportion of copper and iron losses.

The best and accurate method of **determining of efficiency of a transformer would be to compute losses from open-circuit and short-circuit tests and determine efficiency as follows:**

Iron loss, $P_i = W_0$ or P_0 , determined from open-circuit test

Copper loss at full load, $P_c = W_S$ or P_S, determined from short-circuit test

Copper loss at a load x times full load $= I_2^2 R_{02} \quad = x^2 P_C$

Where x is the ratio of load currentI_2 to full-load secondary current.

And transformer efficiency,

$$\eta = \frac{V_2 I_2 \cos\emptyset}{V_2 I_2 \cos\emptyset + P_i + x^2 P_C} \qquad (1)$$

In Eq. (1) the effect of instrument readings is confined to losses only so that overall efficiency obtained from it is far more accurate than that obtained by direct loading. Another great advantage of this method is that it is not necessary for the transformer to be loaded to its full-load rating during testing, and the kW rating of the test plant need be equal to the value of the individual transformer losses.

Efficiency versus Load.

It has been pointed out that with constant voltage the mutual flux of the transformer is practically constant from no load to full load (maximum variation is from 1 to 3%). **The core or iron loss is, therefore, considered constant regardless of load. Copper loss varies as the square of the load current or kVA output.** The variations in copper loss with the increase in load current (or kVA) is shown in Fig. 2.23.

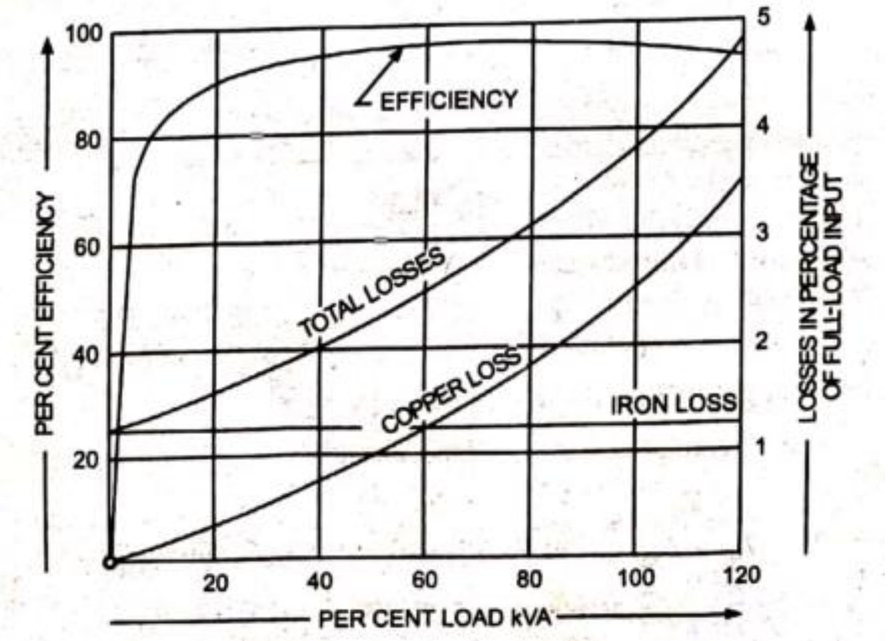


Fig.5.23

From the efficiency-load curve shown in Fig.2.23 it is obvious that the efficiency is very high even at light load, as low as 10 % of rated load. The efficiency is practically constant from about 20% rated load to about 20% overload. At light loads the efficiency is poor because of constant iron loss whereas at high loads the efficiency falls off due to increase in copper loss as the square of load. From Fig. 2.23 it is also obvious that the transformer efficiency is maximum at the point of intersection of copper loss and iron loss curves i.e. when copper loss equals iron loss.

CONDITION FOR MAXIMUM EFFICIENCY.

Transformer efficiency,

$$\eta = \frac{V_2 I_2 \cos\emptyset}{V_2 I_2 \cos\emptyset + P_i + x^2 P_C} = \frac{xP}{xP + P_i + x^2 P_C} \qquad (1)$$

where P is equal to the full load (rated output in volt-amperes or kVA) x load power factor (cos Φ), P_i is the total iron loss, P_C is the full-load copper loss and x is the fraction of full-load kVA at which efficiency is maximum,

Differentiating both sides of Eq. (1.) we get

$$\frac{d\eta}{dx} = \frac{(xP + x^2 P_C + P_i)P - xP(P + 2xP_C)}{(xP + P_i + x^2 P_C)^2} = \frac{P(P_i - x^2 P_C)}{(xP + P_i + x^2 P_C)^2}$$

Efficiency n will be maximum if $\frac{d\eta}{dx} = 0$

Or $$P(P_i - x^2 P_C) = 0$$

$$P_i - x^2 P_C = 0$$

Or $$x^2 P_C = P_i \qquad (2)$$

Hence the efficiency will be maximum when variable loss (copper loss) is equal to constant loss (iron loss).

Output kVA corresponding to maximum efficiency

$$= x\ X\ Rated\ kVA = Rated\ kVA\ X \sqrt{\frac{P_i}{P_C}} \qquad (3)$$

Since from Eq.(2) $x = \sqrt{\frac{P_i}{P_C}}$

Output current I_2 corresponding to maximum efficiency is determined as below:

Copper loss at given load at which efficiency is maximum $= I_2^2 R_{02}$

and since for maximum efficiency to occur it is necessary that copper loss equals P_i ,

$$I_2^2 R_{02} = P_i$$

Or current corresponding to maximum efficiency,

$$I_2 = \sqrt{\frac{P_i}{R_{02}}} \qquad (4)$$

Power transformers employed for bulk power transmission are operated continuously near about full load and are, therefore, designed to have maximum efficiency at full load. On the other hand, the distribution transformers, which supply load varying over the day through a wide range, are designed to have maximum efficiency at about three-fourths the full load.

It is important to appreciate that, since copper loss depends on current and the iron loss depends on voltage, the total loss in the transformer depends on volt-ampere product, and not on the phase angle between voltage and current, i.e. it is independent of the load power factor. The transformers are, therefore, rated in kilovolt amperes (kVA) and not in kilowatts.

ALL DAY EFFICIENCY (OR ENERGY EFFICIENCY)

The transformer efficiency discussed so for is the ordinary, also called commercial efficiency, which is defined as the ratio of power output to power input.

There are certain types of transformers whose performance cannot be judged by ordinary or commercial efficiency. For instance**, distribution transformers are energized for 24 hours, but they deliver very light loads for major portion of the day. Thus iron or core loss occur for the whole day but the copper loss occurs only when the transformer is loaded.**

The performance of such a transformer must be judged by its all-day efficiency, also called the energy efficiency or operational efficiency which is computed on the basis of **energy consumed during the whole day (24 hours).**

The all-day efficiency is defined as **the ratio of energy (kWh) output over 24 hours to the energy input over the same period.**

i.e. All-day efficiency = $\frac{Output\ in\ kWh}{Input\ in\ kWh}$ (5)

Since the distribution transformer does not supply the rated load for the whole day so the all-day efficiency of such a transformer will be lesser than ordinary or commercial efficiency.

For determination of all-day efficiency of a transformer, it is necessary, of course, to know how the load varies from hour to hour during the day.

EFFECTS OF VOLTAGE AND FREQUENCY VARIATIONS

Power transformers are not ordinarily subjected to frequency variation and usually are subject to only modest voltage variations, but it is interesting to consider the effects .

Variations in voltage and/or frequency affects the iron losses (hysteresis and eddy current losses) in a transformer. As long as flux variations are sinusoidal with respect to line, hysteresis loss (P_h) and eddy current loss (P_e) varies according to the following relations

$$P_h \propto f(\emptyset_{max})^x$$

Where x lies between 1.5 and 2.5 depending on the grade of iron used in transformer core and

$$P_e \propto f^2(\emptyset_{max})^2$$

If the transformer is operated with the frequency and voltage changed in the same proportion, the flux density will remain unchanged and apparently the no load current will also remain unaffected.

The transformer can be operated safely at- frequency less than rated one with correspondingly reduced voltage. In this case iron losses will be reduced. But if the transformer is operated with increased voltage and frequency in the same proportion, the core losses may increase to an intolerable level.

Increase in frequency with constant supply voltage will cause-reduction in hysteresis loss and leave the eddy current losses unaffected. Some increase in voltage could, therefore; be tolerated at higher frequencies, but exactly how, much depends on the relative magnitude of the hysteresis and eddy current losses and the grade of iron used in the transformer core.

AUTO-TRANSFORMER

The operating principle and general construction of an auto-transformer is the same as that of conventional two-winding transformer. **The auto-transformer differs from a conventional two-winding transformer in a way in which the primary and secondary are interrelated**.

In a conventional two-winding transformer, the primary and secondary windings are completely insulated from each other but are magnetically linked by a common core. In an **auto-transformer, the two windings, primary and secondary, are connected electrically as well as magnetically**, in fact, a part of the single continuous winding is common to primary and secondary.

The single winding is wound on a laminated silicon steel core and, therefore, both primary and secondary sections of this one winding are on the same magnetic circuit.

The auto-transformers are of two types in construction. In one type of auto-transformers, **there is a continuous winding with taps brought out at convenient points determined by the designed secondary voltages** and in the other type of auto-transformer, there are two or more distinct coils which are electrically connected to form continuous winding. In either case, the same laws governing conventional two-winding transformers apply equally well to auto-transformer.

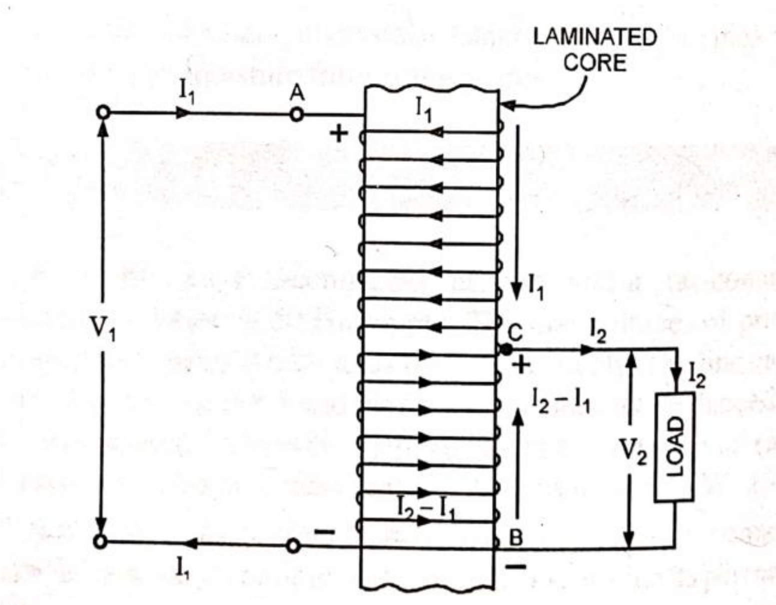


Fig.5.24 Step-Down Auto transformer

In an auto-transformer shown in Fig. 5.24, the primary winding is AB consisting of whole turns between terminals A and B, let it be N_1. Voltage V_1 is applied across primary terminals A and B.

To obtain the secondary winding the winding AB is tapped at C and, therefore, secondary winding consists of turns between terminals B and C, let it be N_2 and secondary voltage is available across terminals B and C. If the internal impedance drop is neglected, then the voltage per turn is $\frac{V_1}{N_1}$ and, therefore, the voltage across BC is $\frac{V_1 N_2}{N_1}$

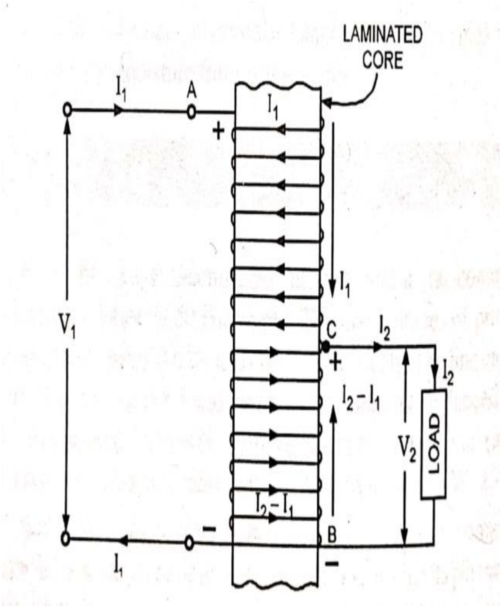
Although diagrammatically the auto-transformer has the appearance of a resistance type potential divider, its operation is quite different, as described below:

- A resistive potential divider cannot step up voltage, whereas an auto-transformer can do.
- In a potential divider the input current is always more than its output current. But it is not so in an auto-transformer. In a step-down auto-transformer, the output (load) current is more than the input current.
- In a potential-divider, almost entire power to the load flows by conduction, whereas in auto-transformer, a part of the power is conducted and the rest is transferred to the load by transformer action.
- Thus in a potential divider, there is a great loss of power in the different resistances whereas it is not so in case of an auto-transformer. Voltage regulation is also poor in case of a potential divider.

Volt-Ampere Relations. Consider the case where V_1 is applied voltage and voltage V_2 is available across the secondary terminals, as illustrated in Fig. 5.24.

Neglecting losses

$$V_1 I_1 \cos \emptyset_1 = V_2 I_2 \cos \emptyset_2$$



If internal leakage impedance drops and losses are negligible, then

$$\cos \emptyset_1 = \cos \emptyset_2$$

And

$$V_1 I_1 = V_2 I_2$$

Or

$$\frac{V_2}{V_1} = \frac{I_1}{I_2} = \frac{N_2}{N_1} = K \qquad (1)$$

Where K is transformation ratio (as in case of two-winding transformers).

Note: Because the exciting current and the leakage reactance are small in case of auto-transformers so it is an accepted practice to assume the input power and output power to be the same.

In an auto-transformer, **only a part of the power input is transferred from the primary to the secondary side by transformer action, while the remainder is transferred directly** from the primary to the secondary side as mentioned above. The relative amounts of power transformed (or inductively transferred) and power conductively transferred depend upon the ratio of transformation, as worked out below;

Under ideal conditions, the mmf due to primary current and secondary current will balance exactly, leaving the mutual flux unchanged by load currents I_2 will, therefore, oppose I_1 in the common section.

The current flowing in section AC of winding is primary current I_1 flowing from A to C, current supplied to load is I_2 and current in section BC will be $(I_2 - I_1)$ flowing from B to C, as illustrated in Fig. 5.24. Neglecting the exciting current $(I_2 - I_1)$ may be determined algebraically.

Power delivered to load = $V_2 I_2$ volt-amperes

Power in winding A C = $E_{AC} I_1 = (V_1 - V_2) I_1$ volt-amperes

Power transformed = Power in winding BC

$=V_2(I_2 - I_1)$

$=V_2 I_2\left(1 - \frac{I_1}{I_2}\right) = V_2 I_2 (1 - K)$ (2)

Since from Eq(1), $\frac{I_1}{I_2} = K$

Ratio of power transformed to total power delivered

$$\frac{V_2 I_2 (1-K)}{V_2 I_2} = (1 - K) \quad (3)$$

Power conducted directly = Power delivered to load - power transferred by transformer action

$$V_2 I_2 - V_2 I_2 (1 - K) = K V_2 I_2 = K \; X \; power \; output \quad (4)$$

Equation (3) may be rewritten as

$$\frac{\text{Inductively transferred power}}{Total\ power} = 1 - K = 1 - \frac{V_2}{V_1} = \frac{V_1 - V_2}{V_1} = \frac{High\ voltage - Low\ voltage}{High\ voltage}$$

EQUIVALENT CIRCUIT. The auto-transformer may be considered as a two winding transformer when the part AC of winding (Fig. 5.24) can be taken as primary with impressed voltage V_{AC} and primary current as I_1 and CB as the secondary with the output voltage V_2 and current I_2-I_1. The current ratio is, therefore, given as

$$\frac{I_{AC}}{I_{BC}} = \frac{I_1}{I_2 - I_1} = \frac{I_1/I_2}{1 - I_1/I_2} = \frac{K}{1 - K}$$

Hence the ratio of currents, in the parts of the winding is $\frac{K}{1-K}$.

The ratio of induced emfs in the parts of the winding CB and AC i.e.

$$\frac{E_{CB}}{E_{AC}} = \frac{N_2}{N_1 - N_2} = \frac{N_2/N_1}{1 - N_2/N_1} = \frac{K}{1-K}$$

The occurrence of the factor $\frac{K}{1-K}$ in the voltage and current, ratios signifies that the auto-transformer with a turn-ratio of K is equivalent in its transformation to the ordinary transformer with a ratio of $\frac{K}{1-K}$.

The approximate equivalent circuit for an auto-transformer is shown Fig. 14.3.

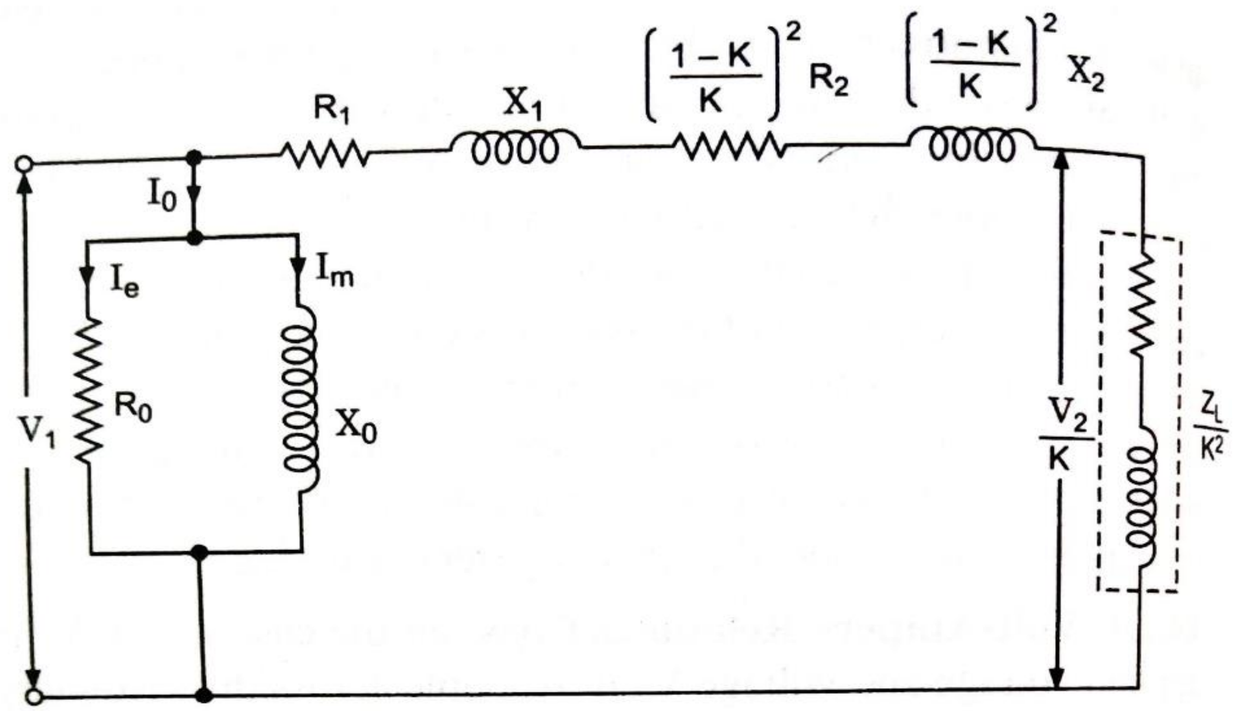


Fig. 5.25 Approximate Equivalent Circuit diagram for an Auto transformer

COMPARISON BETWEEN AUTO-TRANSFORMER AND TWO WINDING TRANSFORMER.

1. VA Ratings. Visualizing that a two-winding transformer is connected as an auto-transformer, let us compare the volt-ampere ratings of the two.

Voltage rating as two-winding transformer

$=(V_1 - V_2)\, I_1 = (I_2 - I_1) V_2$ (5)

When used as an auto-transformer,

Voltage rating as an auto-transformer

$= V_1 I_1 = V_2 I_2$ (6)

$$\frac{\text{Volt} - \text{ampere rating as an auto} - \text{transformer}}{\text{Volt} - \text{ampere rating as two winding transformer}}$$

$$= \frac{V_1 I_1}{(V_1 - V_2)\, I_1} = \frac{V_2 I_2}{V_2 (I_2 - I_1)} = \frac{1}{1-K} \qquad (7)$$

From Eq. (14.7) it is obvious that a two-winding transformer of a given volt-ampere rating when connected as an auto-transformer can handle high volt-amperes. This is because in the auto-transformer connection part of VA is conducted directly from the primary to the secondary.

2. CONDUCTOR MATERIAL REQUIREMENTS(SAVING OF COPPER).

The cross section of conductor (copper or aluminum) is proportional to the current to be carried and length of the conductor in a winding is proportional to the number of turns. Hence the **weight of the conductor material in the winding being proportional to product of cross-sectional area and length of the conductor .ie proportional to the product of number of turns and current to be carried.**

In an ordinary transformer, the total weight of conductor material required is proportional to

$$N_1 I_1 + N_2 I_2 \; or \; 2N_1 I_1$$

because $N_2 I_2 = N_1 I_1$

In an auto-transformer (Fig. 5.24), **The top section AC has (N_1-N_2) turns and carries current I_1 and bottom section BC has turns N_2 and carries current I_2-I_1.**

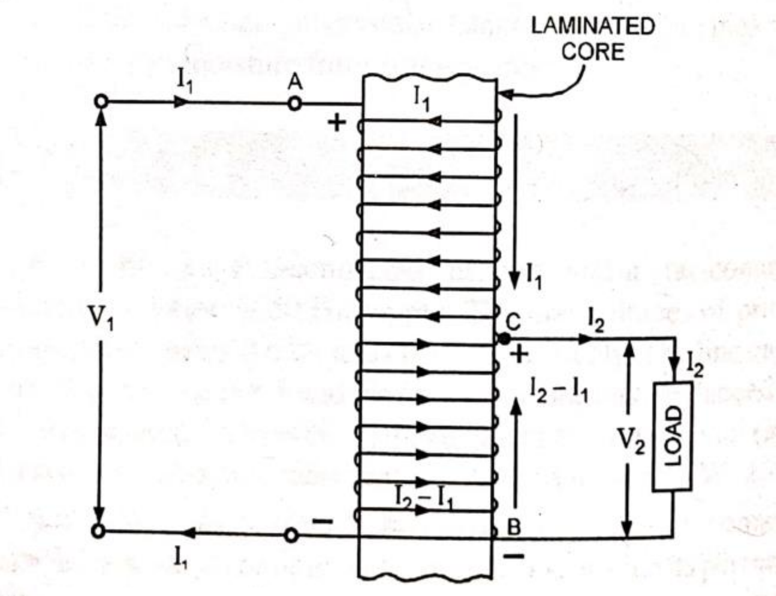


Fig.5.24 Step-Down Auto transformer

So weight of conductor material in an auto-transformer having the same input ($V_1 I_1$), the same output (V_2 I_2) and the same voltage ratio ($\frac{V_2}{V_1}$) as that of the two-winding transformer is proportional to $[(N_1 - N_2)I_1 + N_2(I_2 - I_1)]$

Or $[(N_1 - N_2)I_1 + N_2 I_2 - N_2 I_1]$

Or $[(N_1 - N_2)I_1 + N_1 I_1 - N_2 I_1]$

Or $2(N_1 - N_2)I_1$

So $\frac{\text{Weight of conductor in an auto-transformer}}{\text{Weight of conductor in a two winding transformer}}$

$$= \frac{2(N_1 - N_2)I_1}{2N_1 I_1} = 1 - \frac{N_2}{N_1} = 1 - K$$

Hence saving in conductor material required for an auto-transformer

= K x weight of conductor in a two winding transformer (9)

From Eq. (9) it is obvious that nearer the ratio of transformation is to unity, the greater is the economy in conductor material requirement. If $V_2 = V_1$ i.e. when K is equal to unity the auto-transformer, according to Eq. (8), needs no conductor material, for actually no transformer is required. If voltage is to be transformed from very-high voltage to very low voltage i.e. when K is to be very small, there will be a little saving in the conductor material and in such cases it is preferable to use an ordinary two-winding transformer.

APPLICATIONS.

As said earlier, **auto-transformers are used when transformation ratio K is nearly equal to unity** and where there is no objection to direct electrical connection between primary and secondary. Hence auto-transformers are extensively used for ratio ranging from at 0.4 to 1.0 for the following applications:

(i) To obtain a neutral in a 3-wire a c distribution system in the same way as a balancer set is used in a 3-wire dc distribution system. The connection of an auto-transformer used in this way is shown in Fig. 5.24. If the load on the upper half of system is greater than on the lower half, the winding BC acts as the primary and winding AC acts as secondary to supply the extra load required by upper half of the system. The auto-transformer used in this manner is known as a balance coil. The balance coil is cheaper and more efficient than balancer set.

(ii) Auto-transformers with a **number of tapings are used for starting induction motors and synchronous motors.** When auto-transformers are used for this purpose, these are known as auto-starters.

(iii) A continuously variable auto-transformer finds useful application in **electrical testing laboratory.**

(iv) Auto-transformer have the biggest sphere of usefulness as regulating transformers (the tap point C is variable in such cases).

(v) Auto-transformers are also used as **boosters** to raise the voltage in an ac feeder.

(vi) As **furnace transformers** for getting a convenient supply to suit the furnace winding from normal 230V ac supply.

(vii) Auto-transformers are mainly used for **interconnecting systems** that are operating at roughly the same voltage. The inter connection of EHV system(e.g. 220kV and 132kV) by the auto-transformers results in considerable saving of bulk and cost as compared to the conventional two winding transformers.