CONTROL SYSTEMS

UNIT…1

SYSTEM:-

A system is defined as a number of elements or components are connected in a sequence to perform a specific function (or) a device.

(or)

It is an arrangement of physical components related in such a manner as to form an entire unit.

CONTROL SYSTEMS:-

In a system when the output quantity is controlled by varying the input quantity, the system is called ***control system.*** The output quantity is called controlled variable or response and input quantity is called command signal or excitation.

Examples of control systems…..

CLASSIFICATION OF CONTROL SYSTEMS

Control Systems can be classified as open loop control systems and closed loop control systems based on the **feedback path.**

1. Open –loop control system

2. Closed – loop control system

OPEN LOOP SYSTEM

Any physical system which does not automatically correct the variation in its output, is called an ***open loop system***, or control system in which the output quantity has no effect upon the input quantity are called open-loop control system. This means that the output is not fedback to the input for correction.

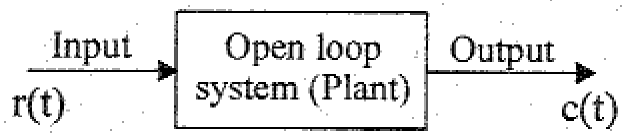


Fig 1.1 : *Open loop system.*

In open loop system the output can be varied by varying the input. But due to external disturbances the system output may change. When the output changes due to disturbances, it is not followed by changes in input to correct the output. In open loop systems the changes in output are corrected by changing the input manually.

Open loop control systems are also known as manual control systems or non feedback systems

Advantages of open loop systems

1. The open loop systems are simple and economical.
2. The open loop systems are easier to construct.
3. Generally the open loop systems are stable.

Disadvantages of open loop systems

1. The open loop systems are inaccurate and unreliable.
2. The changes in the output due to external disturbances are not corrected automatically.

CLOSED LOOP SYSTEM

Control systems in which the output has an effect upon the input quantity in order to maintain the desired output value are called ***closed loop systems.***

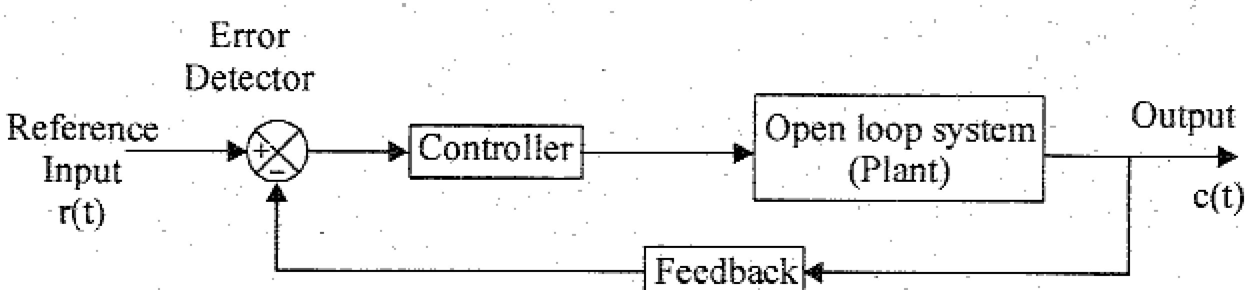


Fig 1.2 : *Closed loop system.*

The open loop system can be modified as closed loop system by providing a feedback. The provision of feedback automatically corrects the changes in output due to disturbances. Hence the closed loop system is also called ***automatic control system.*** The general block diagram of an automatic control system is shown in fig 1.2. It consists of an error detector, a controller, plant (open loop system) and feedback path elements.

The reference signal (or input signal) corresponds to desired output. The feedback path elements samples the output and converts it to a signal of same type as that of reference signal. The feedback signal is proportional to output signal and it is fed to the error detector. The error signal generated by the error detector is the difference between reference signal and feedback signal. The controller modifies and amplifies the error signal to produce better control action. The modified error signal is fed to the plant to correct its output.

Advantages of open loop systems

Advantages of closed loop systems

1. The closed loop systems are accurate.
2. The closed loop systems are accurate even in the presence of non-linearities.
3. The sensitivity of the systems may be made small to make the system more stable.
4. The closed loop systems are less affected by noise.

Disadvantages of closed loop systems

1. The closed loop systems are complex and costly.
2. The feedback in closed loop system may lead to oscillatory response.
3. The feedback reduces the overall gain of the system.
4. Stability is a major problem in closed loop system and more care is needed to design a stable closed loop system.

EXAMPLES OF CONTROL SYSTEMS

EXAMPLE 1 : TEMPERATURE CONTROL SYSTEM

OPEN LOOP SYSTEM

The electric furnace shown in fig 1.3. is an open loop system. The output in the system is the desired temperature. The temperature of the system is raised by heat generated by the heating element. The output temperature depends on the time during which the supply to heater remains ON.

The ON and OFF of the supply is governed by the time setting of the relay. The temperature is measured by a sensor, which gives an analog voltage corresponding to the temperature of the furnace. The analog signal is converted to digital signal by an Analog - to - Digital converter (A/D converter).

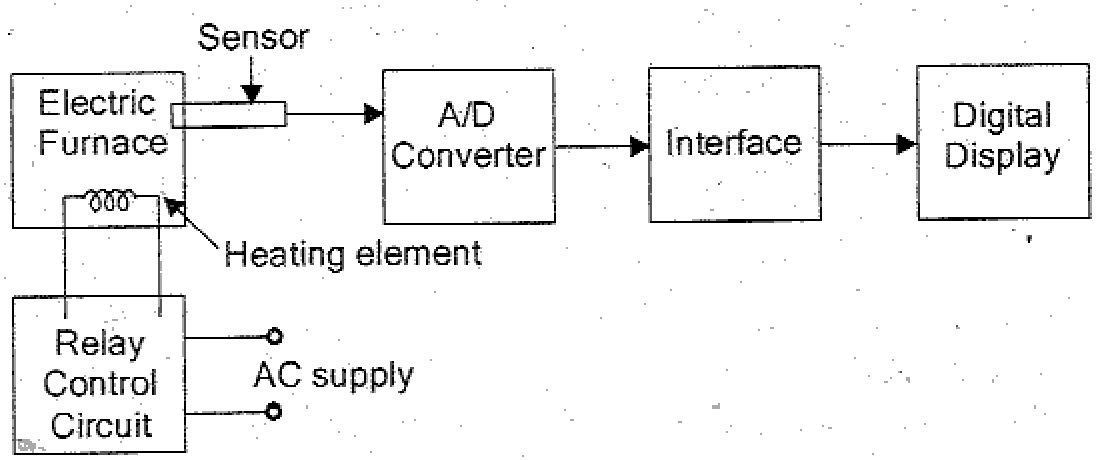


Fig 1.3 : Open loop temperature control system.

The digital signal is given to the digital display device to display the temperature. In this system if there is any change in output temperature then the time setting of the relay is not altered automatically.

CLOSED LOOP SYSTEM

CLOSED LOOP SYSTEM

The electric furnace shown in fig 1.4 is a closed loop system. The output of the system is the desired temperature and it depends on the time during which the supply to heater remains ON.

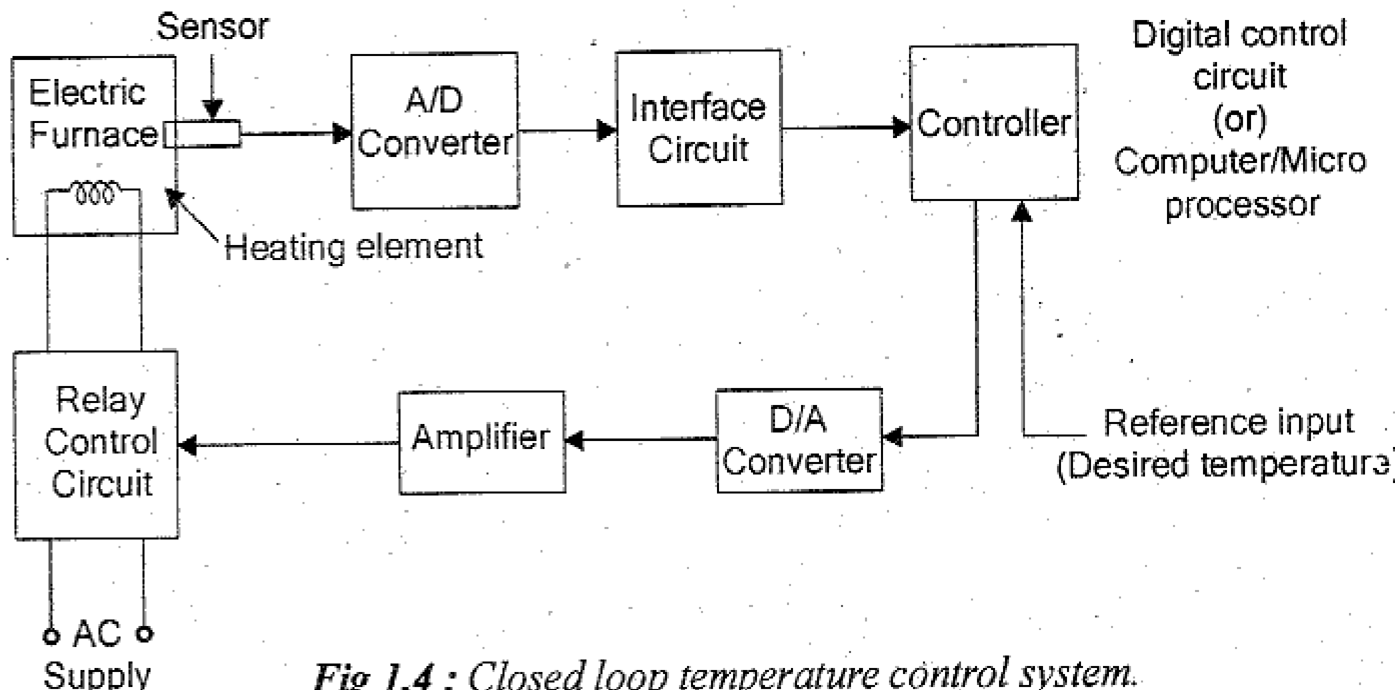


Fig 1.4 : Closed loop temperature control system.

The switching ON and OFF of the relay is controlled by a controller which is a digital system or computer. The desired temperature is input to the system through keyboard or as a signal corresponding to desired temperature via ports. The actual temperature is sensed by sensor and converted to digital signal by the A/D converter. The computer reads the actual temperature and compares with desired temperature. If it finds any difference then it sends signal to switch ON or OFF the relay through D/A converter and amplifier. Thus the system automatically corrects any changes in output. Hence it is a closed loop system.

EXAMPLE 2 : TRAFFIC CONTROL SYSTEM

OPEN LOOP SYSTEM

Traffic control by means of traffic signals operated on a time basis constitutes an open-loop control system. The sequence of control signals are based on a time slot given for each signal. The time slots are decided based on a traffic study. The system will not measure the density of the traffic before giving the signals. Since the time slot does not changes according to traffic density, the system is open loop system.

CLOSED LOOP SYSTEM

Traffic control system can be made as a closed loop system if the time slots of the signals are decided based on the density of traffic. In closed loop traffic control system, the density of the traffic is measured on all the sides and the information is fed to a computer . The timings of the control signals are decided by the computer based on the density of traffic . Since the closed loop system dynamically changes the timings, the flow of vehicles will be better than open loop system.

COMPARISON BETWEEN OPEN AND CLOSED LOOP SYSTEMS.

S.No.	Open Loop system	Closed system
1.	Any change in output has no effect on the input.	Changes in output, effects on the input.
2.	These are not reliable.	These are reliable.
3.	It is easier to build.	It is difficult to build.
4.	Open loop systems are less accurate	They are accurate because of feed back.
5.	Open loop systems are generally more stable	These are less stable
6.	Optimization is not possible	Optimization is possible
7.	Highly sensitive to the disturbances & environmental changes.	Less sensitive to the disturbances & environmental changes.
8.	Open loop systems are known as manual control systems	Closed loop systems are known as automatic control systems

FEED BACK AND ITS EFFECTS

In general closed loop control systems are more commonly called feed back control system. It is the property of a closed loop system which permits the output to be compared with the input of the system so that the appropriate control action may be formed.

The comparison between input and output is carried by feedback elements in closed loop systems

Types of Feedback

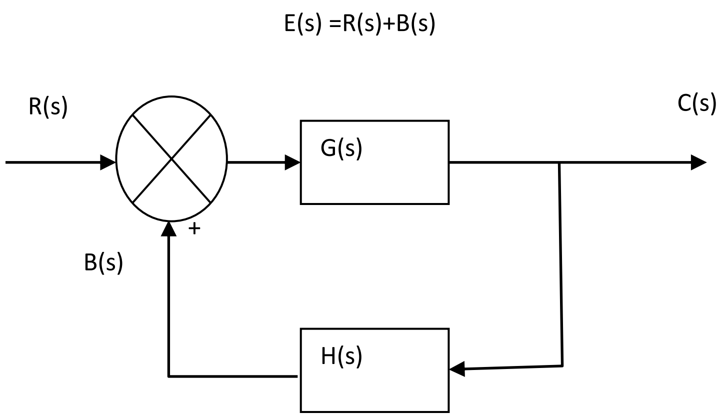
There are two types of feedback –

1) Positive feedback

2)Negative feedback

Positive feed back:

The following figure shows the block diagram of **positive feedback control system.**



The positive feedback adds the reference input(s) and feedback output

$$E(s) = R(s) + B(s) \qquad \text{------------} \ (1)$$

$$C(s) = G(s)\ E(s) \qquad \text{------------} \ (2)$$

$$B(s) = C(s)\ H(s) \qquad \text{------------} \ (3)$$

$$E(s) = R(s) + C(s)\ H(s)$$

$$c(s) = G(s)\left[R(s) + C(s)H(s)\right]$$

$$\Rightarrow\ C(s)[1 - G(s)\ H(s)] = G(s)R(s)$$

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1 - G(s)H(s)} \rightarrow \text{Transfer Function.}$$

Transfer function is defined as, it is the ratio of Laplace transform of the output to the Laplace transform of the input.

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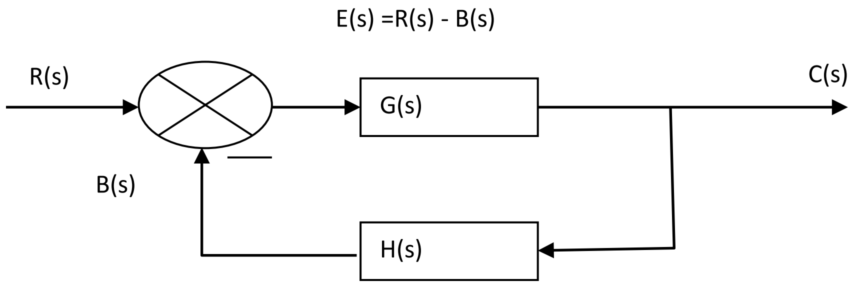
Negative Feedback

In the negative feed back control system, the resulting error signal is the difference between input (or) reference signal and feed back signal

. The following figure shows the block diagram of the **negative feedback control system.**



$$E(s) = R(s) - B(s)$$

$$C(s) = G(s)\, E(s)$$

$$B(s) = C(s) \times H(s)$$

$$E(s) = R(s) - C(s)\, H(s)$$

$$C(s) = G(s) \times \left(R(s) - C(s) \times H(s)\right)$$

$$C(s)\left[1 + G(s)\, H(s)\right] = G(s) R(s)$$

$$\frac{C(s)}{R(s)} = \frac{G(s)}{\left(1 + G(s) H(s)\right)} \rightarrow \text{Transfer Function.}$$

Transfer function is defined as, it is the ratio of Laplace transform of the output to the Laplace transform of the input.

Effects of Feedback

1) Effect of Feedback on Overall Gain

The overall gain of negative feedback closed loop control system is the ratio of 'G' and (1+GH). So, the overall gain may increase or decrease depending on the value of (1+GH).

If the value of (1+GH) is less than 1, then the overall gain increases

If the value of (1+GH) is greater than 1, then the overall gain decreases.

In general, 'G' and 'H' are functions of frequency. So, the feedback will increase the overall gain of the system in one frequency range and decrease in the other frequency range.

2)Effect of Feedback on Stability

A system is said to be stable, if its output is under control. Otherwise, it is said to be unstable.

The overall gain of negative feedback closed loop control system is

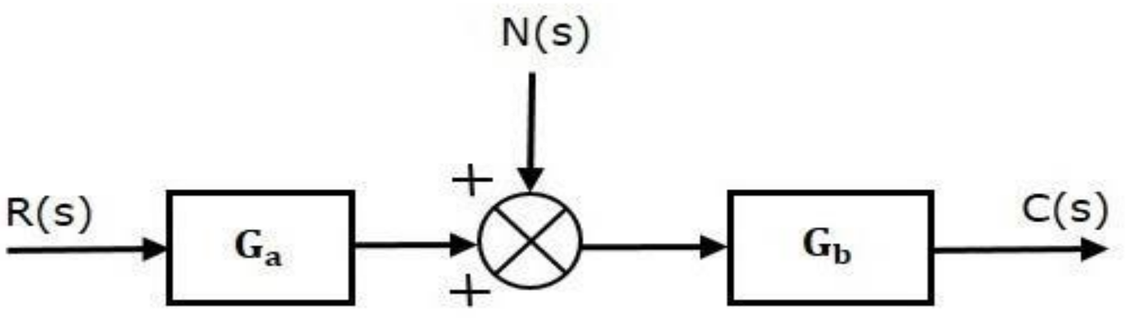
$$\frac{C(s)}{R(s)} = \frac{G(s)}{(1+G(s)H(s))}.$$

if the denominator value is zero (i.e., GH = -1),
then the output of the control system will be infinite. So, the control system becomes unstable.
Therefore, we have to properly choose the feedback in order to make the control system stable.

3)Effect of Feedback on Noise

To know the effect of feedback on noise, let us compare the transfer function relations with and without feedback due to noise signal alone.

Consider an **open loop control system with noise signal as shown below.**



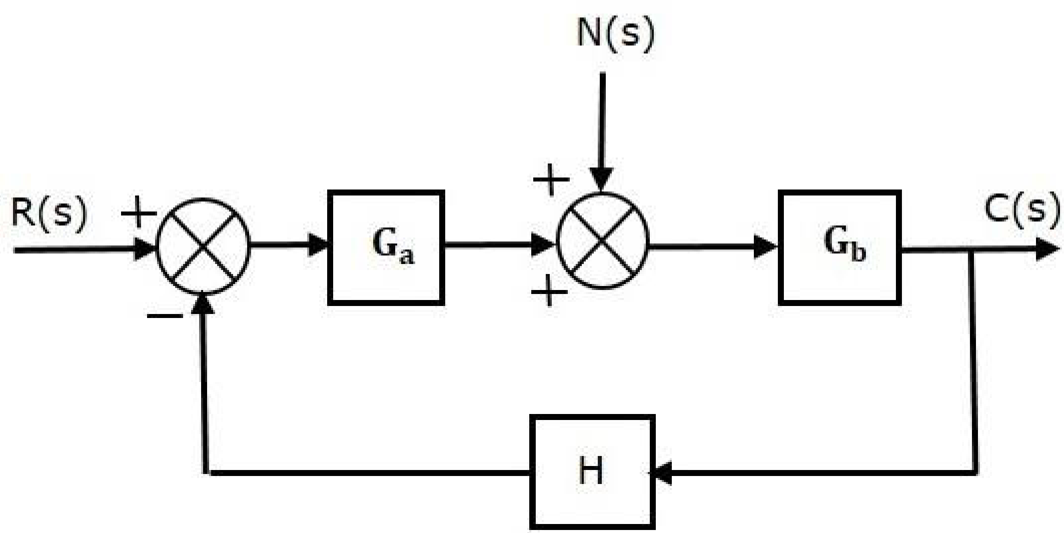
The **open loop transfer function due to noise signal alone is**

transfer function due to noise signal alone is

$$\frac{C(s)}{N(s)} = G_b \quad \text{(Equation 1)}$$

It is obtained by making the other input R(s) equal to zero.

Consider a **closed loop control system with noise signal as shown below.**



The **closed loop transfer function due to noise signal alone is**

$$\frac{C(s)}{N(s)} = \frac{G_b}{1+G_aG_bH} \quad \text{(Equation 2)}$$

It is obtained by making the other input R(s) equal to zero.

Compare Equation 1 and Equation 2, In the closed loop control system, the gain due to noise signal is decreased by a factor of $(1+G_aG_bH)$ provided that the term $(1+G_aG_bH)$ is greater than one.

4) Effect of Feedback on Sensitivity

The parameters of any control system changes with the change in environment conditions. Also these parameters cannot be constant through out the life. These parameter variations affects the performance of the system

for example, the resistance of the winding of a motor changes due to the change in temperature during its operation.

So, a control system should be insensitive to the parameter variations

Sensitivity of the overall gain of negative feedback closed loop control system (T) to the variation in open loop gain (G) is defined as

$$S = \frac{\frac{\partial T}{T}}{\frac{\partial G}{G}} = \frac{\partial T}{T} \times \frac{G}{\partial G}$$

$$= \frac{\partial T}{\partial G} \times \frac{G}{T}$$

$$\frac{\partial T}{\partial G} = \frac{\partial}{\partial G}\left(\frac{G}{1+GH}\right) = \frac{1\cdot(1+GH) - G(H)}{(1+GH)^2}$$

$$= \frac{1}{(1+GH)^2}$$

$$\frac{G}{T} = (1+GH)$$

$$S = \frac{1}{(1+GH)^2} \times (1+GH) = \frac{1}{1+GH}$$

So, we got the **sensitivity of the overall gain of closed loop control system as the** reciprocal of (1+GH). So, Sensitivity may increase or decrease depending on the value of (1+GH).

If the value of (1+GH) is less than 1, then sensitivity increases.

If the value of (1+GH) is greater than 1, then sensitivity decreases.

Transfer function

The ***transfer function*** of a system is defined as the ratio of Laplace transform of output to the Laplace transform of input with zero initial conditions.

$$\text{Transfer function} = \left.\frac{\text{Laplace Transform of output}}{\text{Laplace Transform of input}}\right|_{\text{with zero initial conditions}}$$

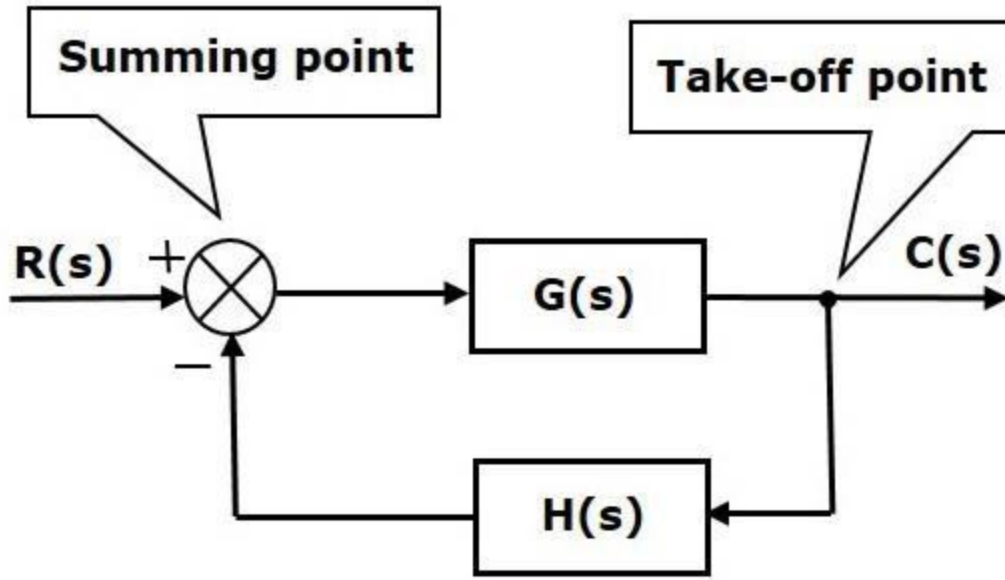
The transfer function can be obtained by taking Laplace transform of the differential equations governing the system with zero initial conditions and rearranging the resulting algebraic equations to get the ratio of output to input.

Block diagrams

Block diagrams consist of a single block or a combination of blocks. These are used to represent the control systems in pictorial form.

Basic Elements of Block Diagram

The basic elements of a block diagram are a block, the summing point and the take-off point. Let us consider the block diagram of a closed loop control system as shown in the following figure to identify these elements.



The above block diagram consists of two blocks having transfer functions G(s) and H(s). It is also having one summing point and one take-off point. Arrows indicate the direction of the flow of signals.

Advantages of Block Diagram Representation

1. Very simple to construct block diagram for a complicated system
2. Function of individual element can be visualized
3. Individual & Overall performance can be studied
4. Over all transfer function can be calculated easily.

Disadvantages of Block Diagram Representation

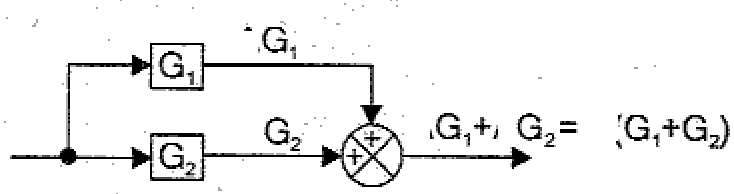
1. No information about the physical construction
2. Source of energy is not shown

<u>BLCOK DIAGRAM REDUCTION</u>

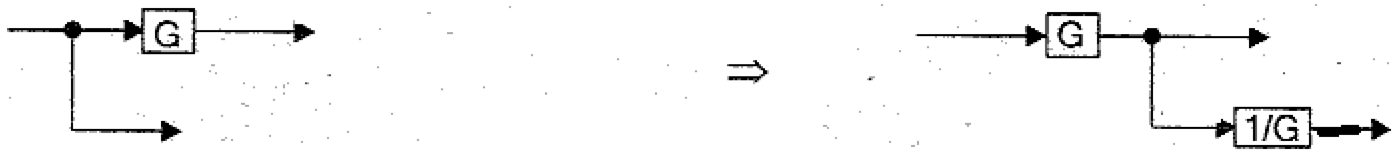
When a number of blocks are connected, the over all transfer function can be obtained by block diagram reduction technique. The following rules are associated with the block reduction technique.

1. Cascaded blocks

<u>Rule-2 :</u> ***Combining Parallel blocks***

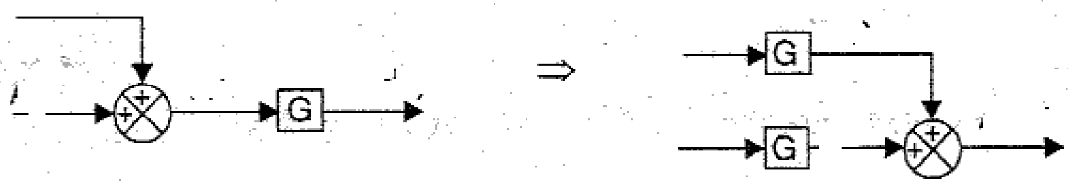


Rule-3 : ***Moving the branch point ahead of the block***



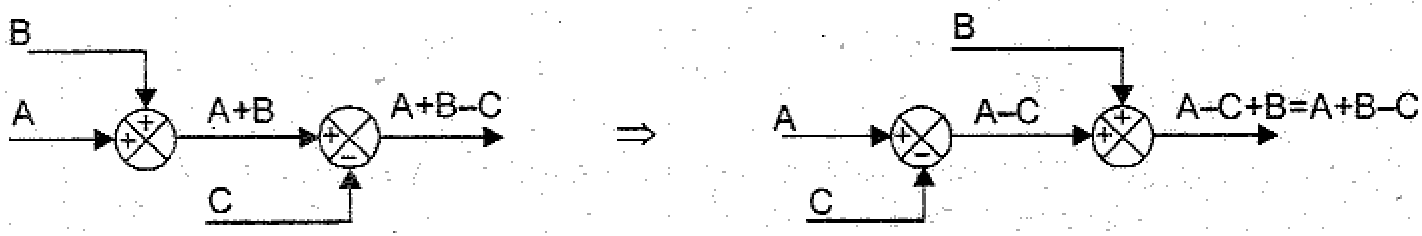
Rule-4 : ***Moving the branch point before the block***

Rule-5 : ***Moving the summing point ahead of the block***

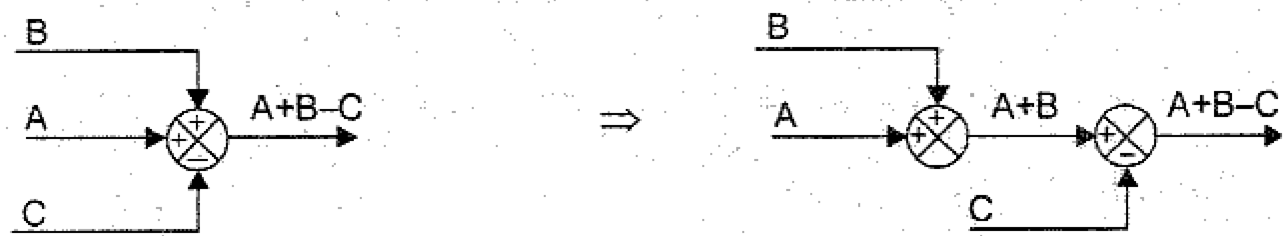


Rule-6 : ***Moving the summing point before the block***

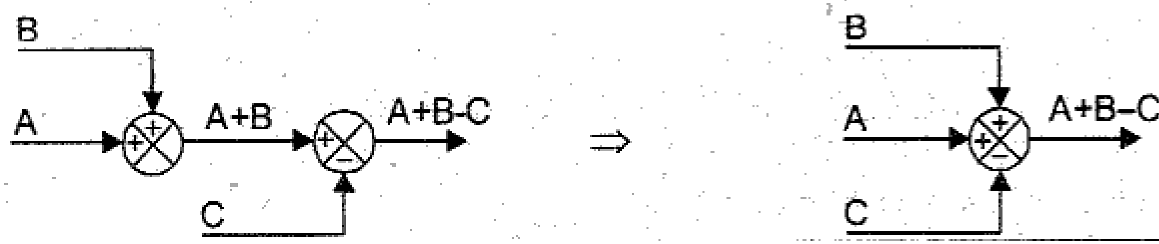
Rule-7 : ***Interchanging summing point***



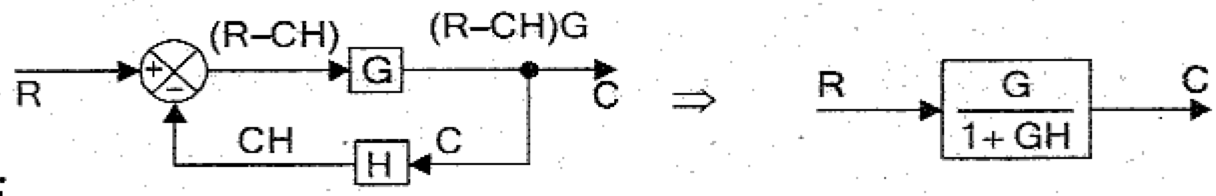
Rule-8 : ***Splitting summing points***



Rule-9 : ***Combining summing points***



Rule-10 : Elimination of (negative) feedback loop

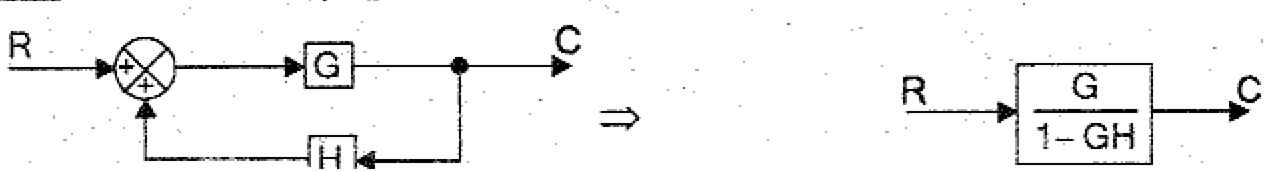


Proof:

$$C = (R - CH)G \Rightarrow C = RG - CHG \Rightarrow C + CHG = RG$$

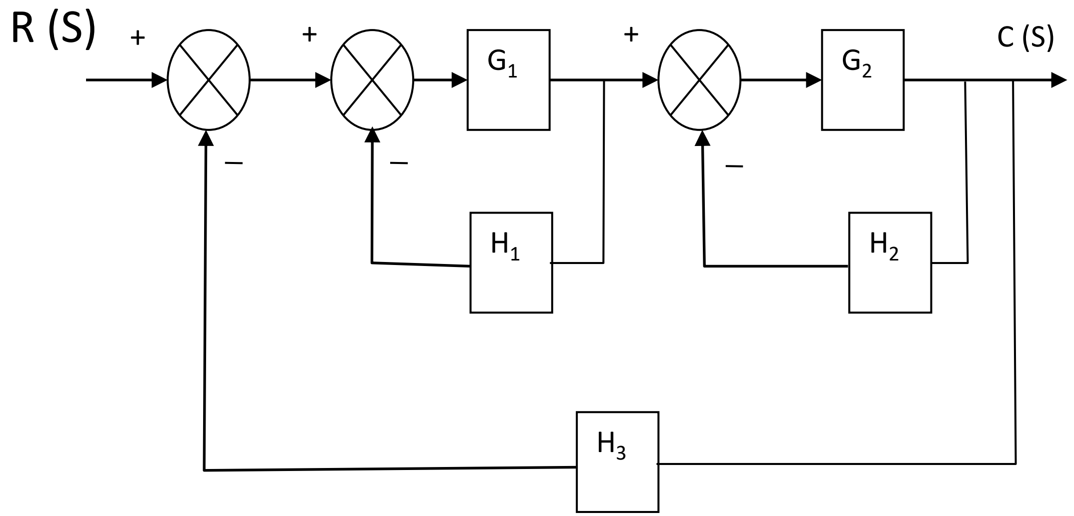
$$\therefore C(1 + HG) = RG \Rightarrow \frac{C}{R} = \frac{G}{1 + GH}$$

Rule-11 : Elimination of (positive) feedback loop

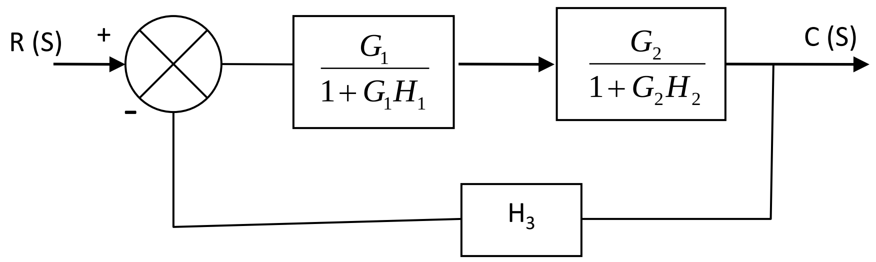


PROBLEMS ON BLOCK DIAGRAM REDUCTION

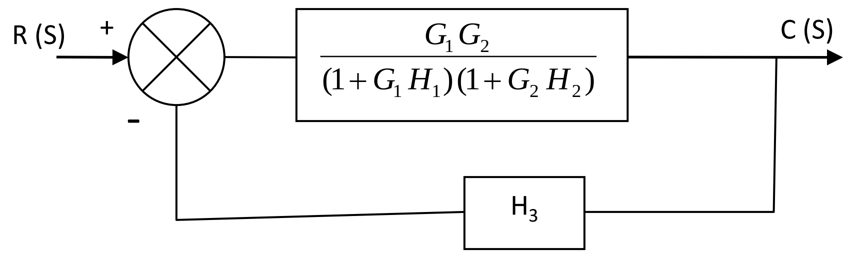
Derive the Transfer Function using block diagram reduction technique



Step 1 : Eliminating the feedback paths

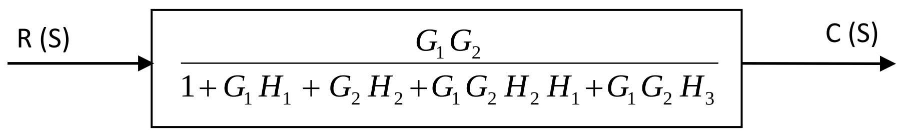


Step 2 : Combining the blocks in cascade



$$\frac{G_1 G_2}{(1+G_1 H_1)(1+G_2 H_2)}$$

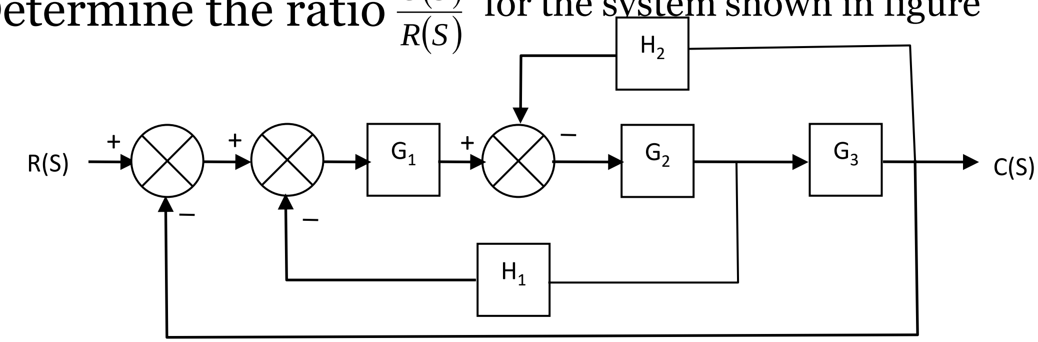
Step 3 : Eliminating the feedback path



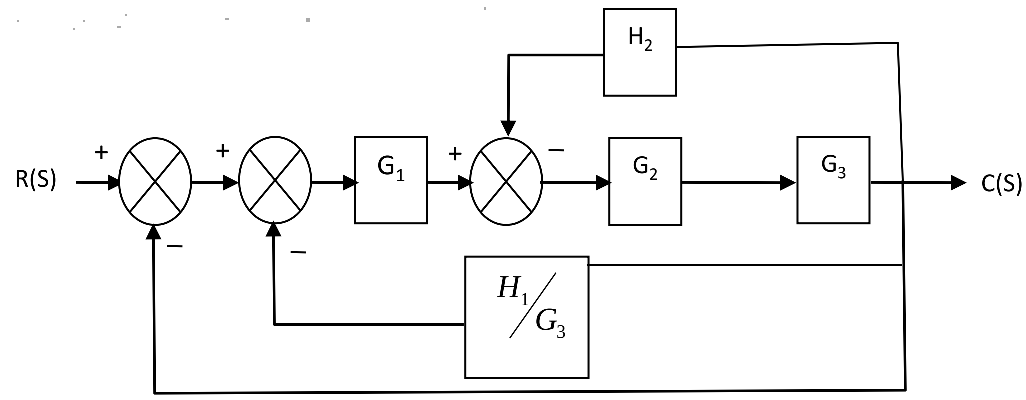
$$\frac{G_1 G_2}{1+G_1 H_1 + G_2 H_2 + G_1 G_2 H_2 H_1 + G_1 G_2 H_3}$$

- The Transfer Function $\dfrac{C(S)}{R(S)} = \dfrac{G_1 G_2}{1+G_1 H_1 + G_2 H_2 + G_1 G_2 H_1 H_2 + G_1 G_2 H_3}$

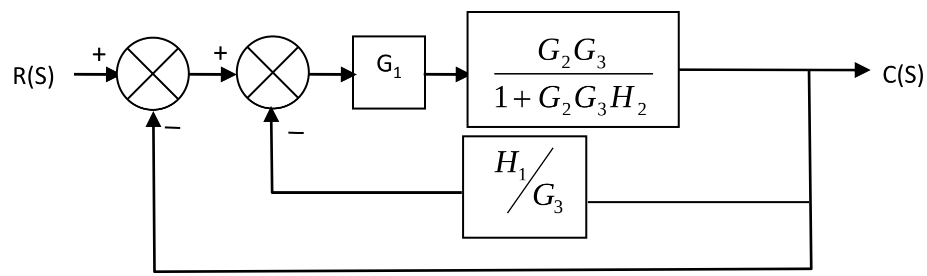
2) Determine the ratio $\dfrac{C(S)}{R(S)}$ for the system shown in figure



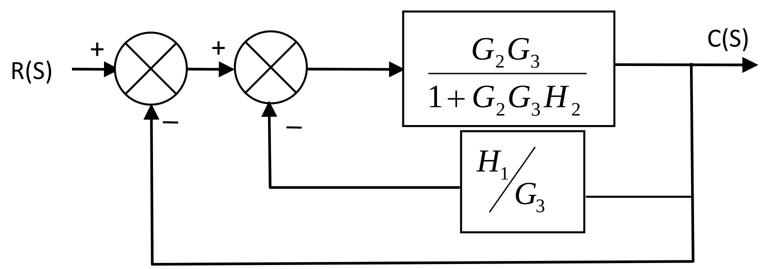
Step 1: Moving the branch point after the block



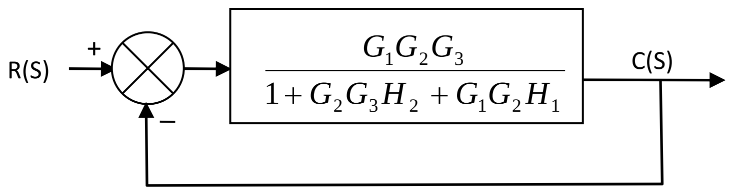
Step 2: Combining the blocks in cascade and eliminating feed back loop



Step 3: combining the cascade blocks



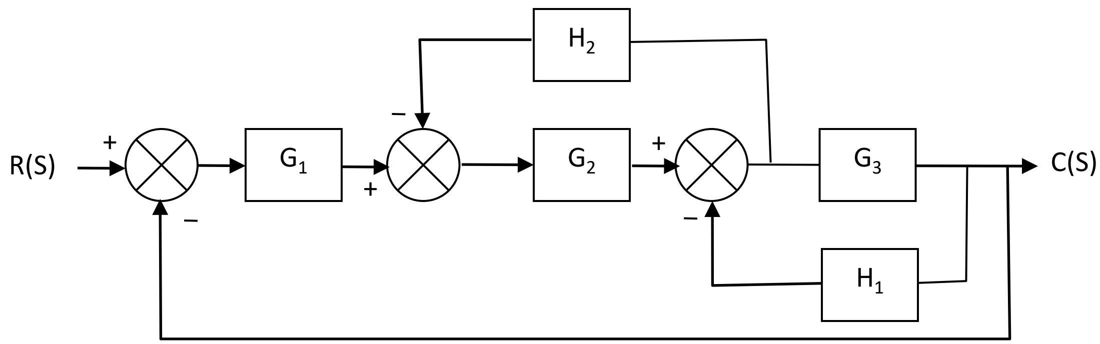
Step 4: Eliminating feed back loop



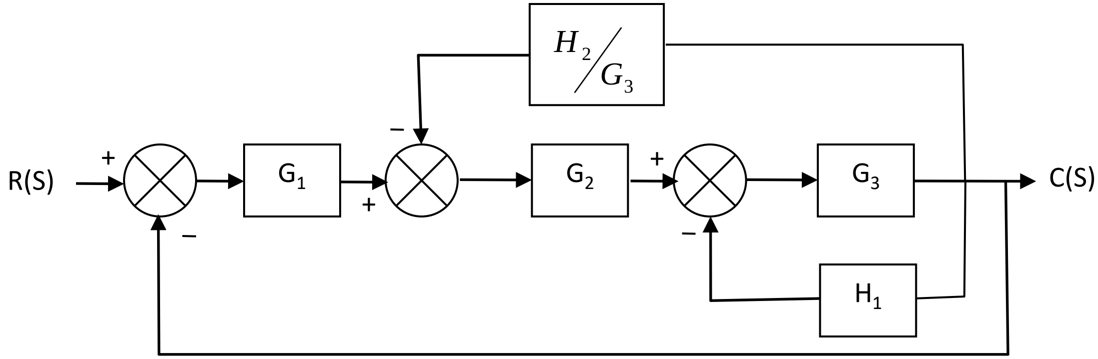
Step 5: Eliminating feed back loop

∴ The Transfer Function $\frac{C(S)}{R(S)} = \frac{G_1G_2G_3}{1+G_2G_3H_2+G_1G_2H_1+G_1G_2G_3}$

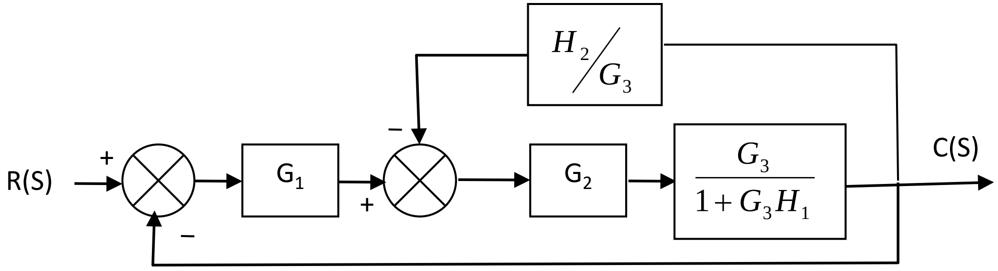
3) Find the ratio $C(S)/R(S)$ of the system shown in fig.



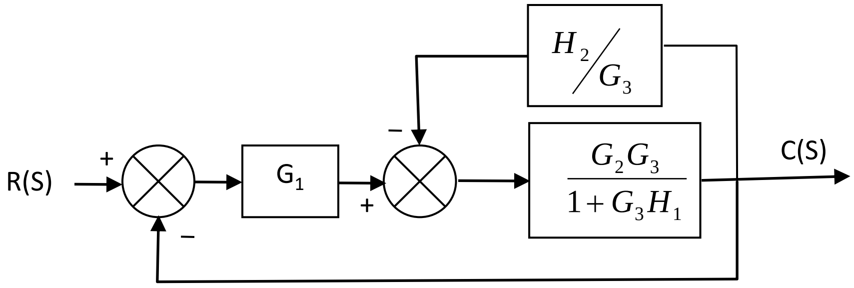
Step 1: Moving the branch point **after** the block



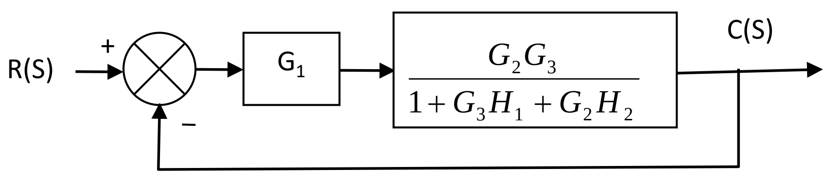
Step 2: Eliminating the feedback **path**



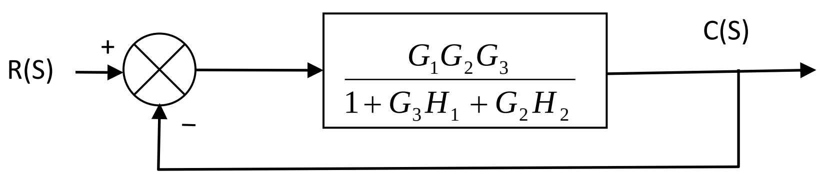
Step 3: combining the cascade blocks



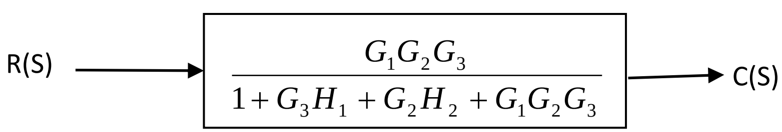
Step 4: Eliminating feed back loop



Step 5: combining the cascade blocks



Step 6: Eliminating feed back loop



∴ Transfer function of the system is

$$\frac{C(S)}{R(S)} = \frac{G_1G_2G_3}{1+G_3H_1+G_2H_2+G_1G_2G_3}$$

4) Reduce the block diagram shown in fig 1 and find C/R.

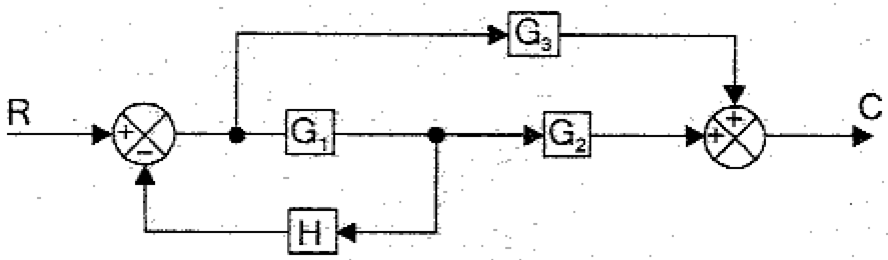
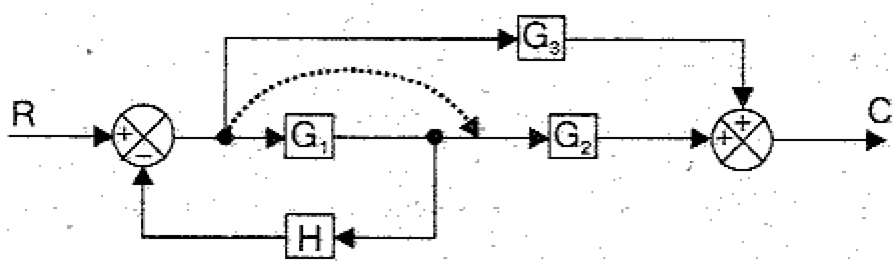


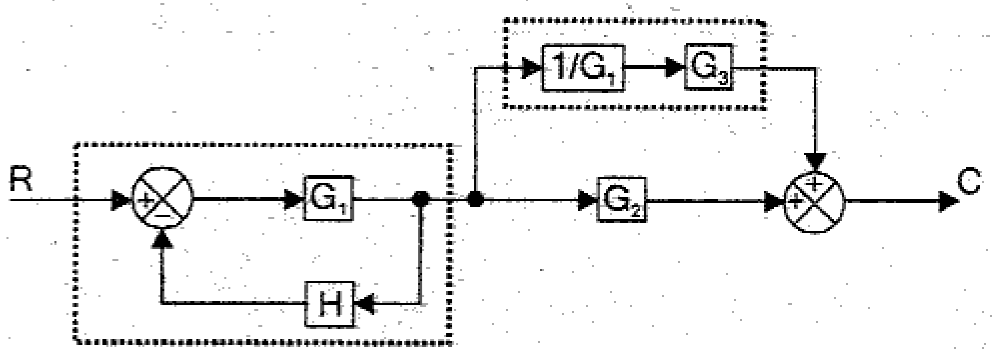
Fig 1.

SOLUTION

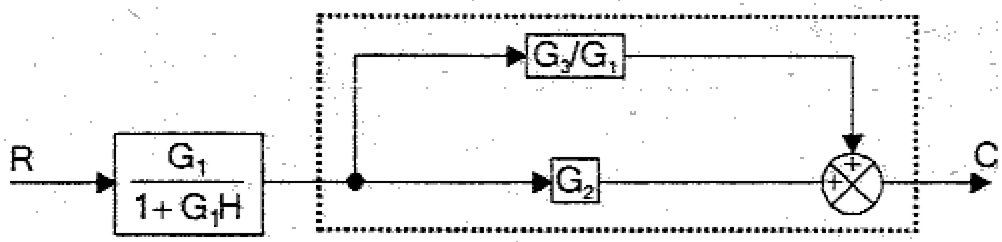
Step 1: Move the branch point after the block.



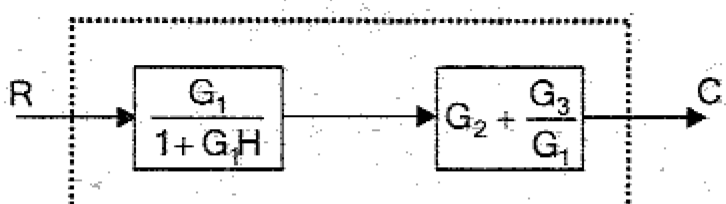
Step 2: Eliminate the feedback path and combining blocks in cascade.



Step 3: Combining parallel blocks



Step 4: Combining blocks in cascade



$$\frac{C}{R} = \left(\frac{G_1}{1+G_1H}\right)\left(G_2 + \frac{G_3}{G_1}\right) = \left(\frac{G_1}{1+G_1H}\right)\left(\frac{G_1G_2+G_3}{G_1}\right) = \frac{G_1G_2+G_3}{1+G_1H}$$

RESULT

The overall transfer function of the system, $\frac{C}{R} = \frac{G_1G_2 + G_3}{1 + G_1H}$

5)

The block diagram of a closed loop system is shown in fig 1. Using the block diagram reduction technique determin[e the clo]sed loop transfer function C(s)/R(s).

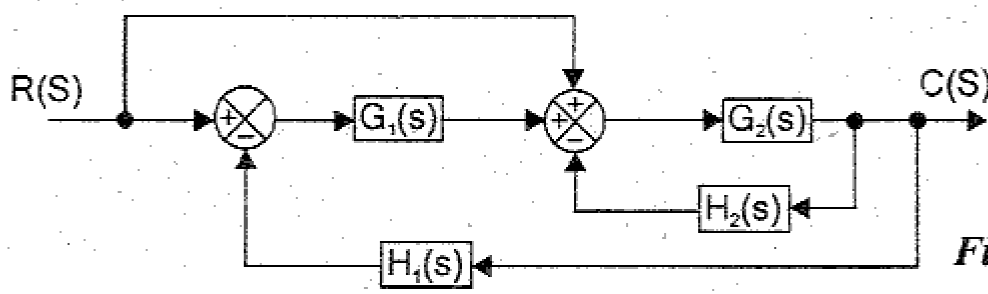
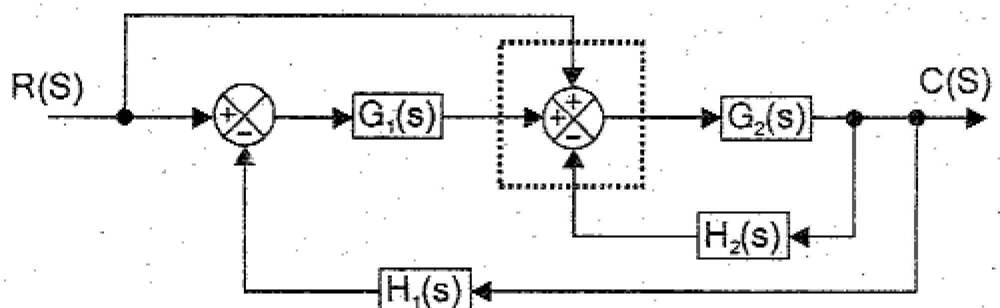


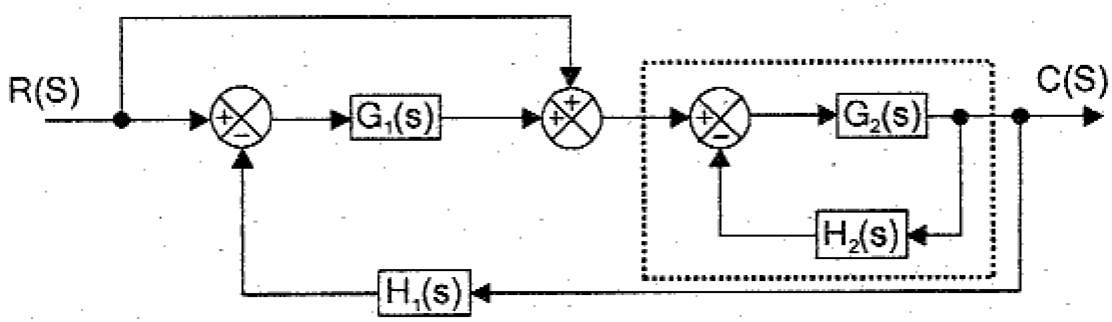
Fig 1.

SOLUTION

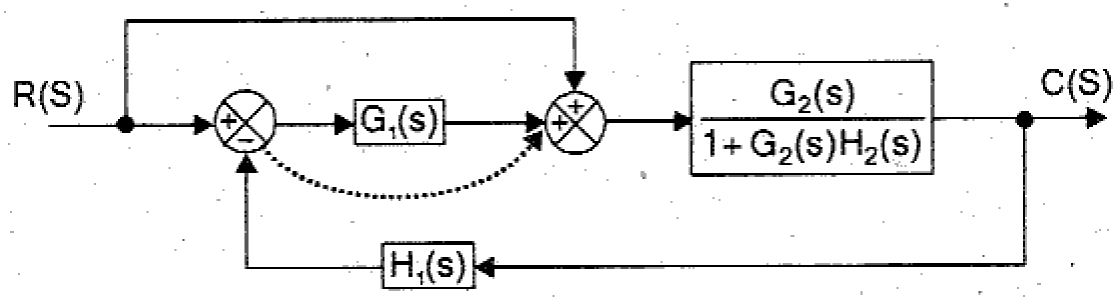
Step 1 : Splitting the summing point.



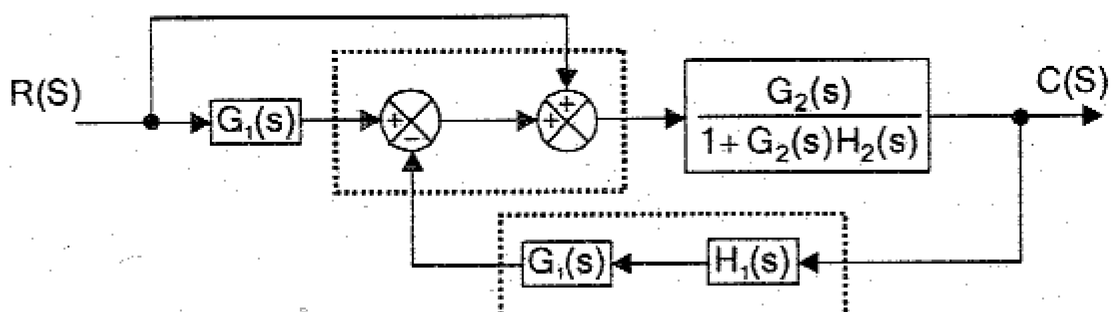
Step 2 : Eliminating the feedback path.



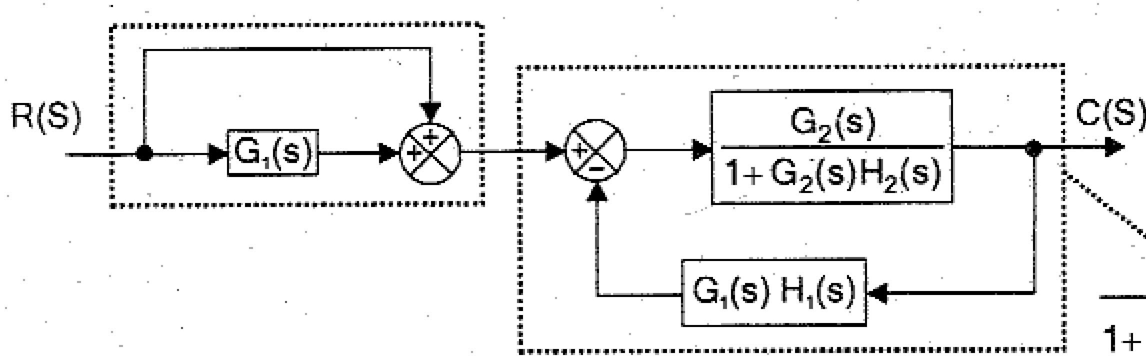
Step 3 : Moving the summing point after the block.



Step 4 : Interchanging the summing points and combining the blocks in cascade

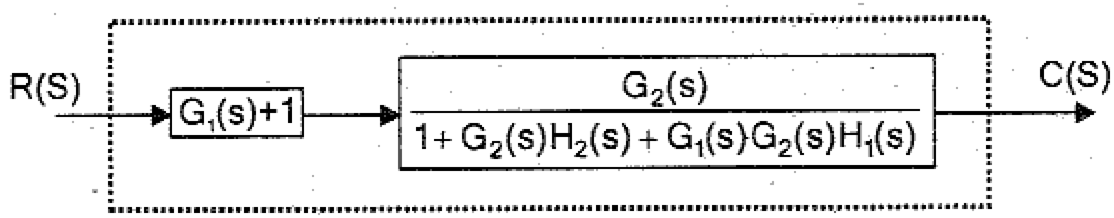


Step 5 : Eliminating the feedback path and feed forward path



$$\frac{\dfrac{G_2(s)}{1+G_2(s)H_2(s)}}{1+\dfrac{G_2(s)}{1+G_2(s)H_2(s)}\,G_1(s)H_1(s)}$$

Step 6 : Combining the blocks in cascade



$$\therefore \frac{C(s)}{R(s)} = \frac{G_2(s)\,[G_1(s)+1]}{1+G_2(s)\,H_2(s)+G_1(s)\,G_2(s)\,H_1(s)}$$

RESULT

The transfer function of the system is,

$$\frac{C(s)}{R(s)} = \frac{G_2(s)\,[G_1(s)+1]}{1+G_2(s)\,H_2(s)+G_1(s)\,G_2(s)\,H_1(s)}$$

Using block diagram reduction technique find closed loop transfer function of the system whose block diagram is shown in fig 1.

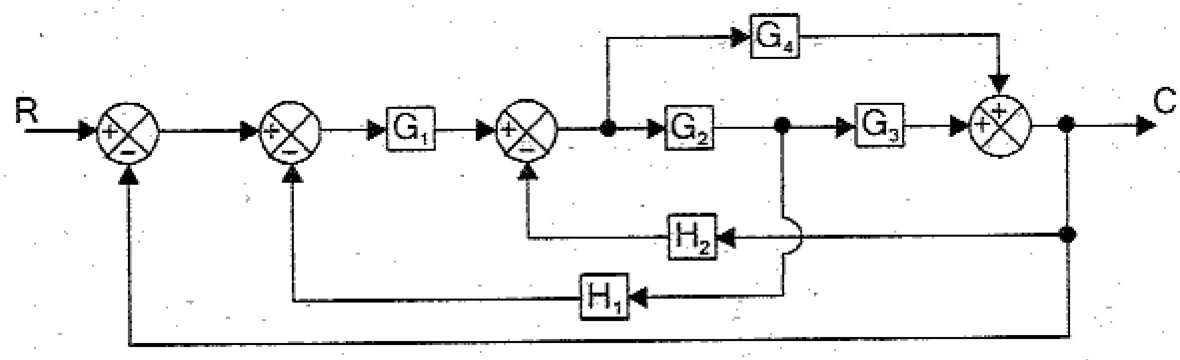
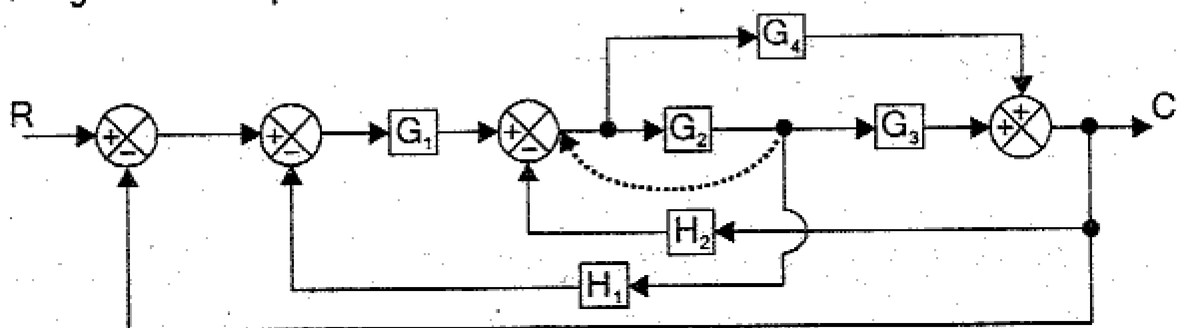


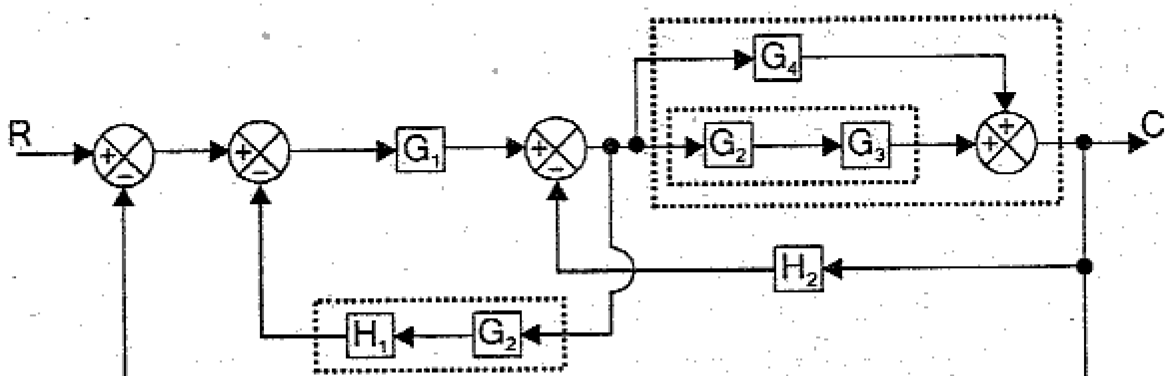
Fig 1.

SOLUTION

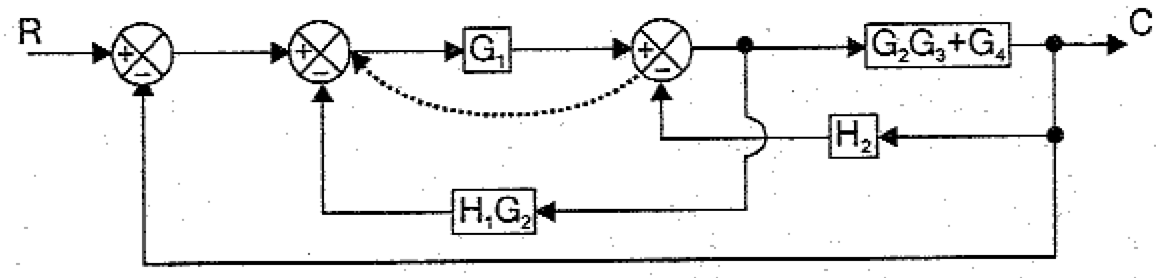
Step 1: Moving the branch point before the block



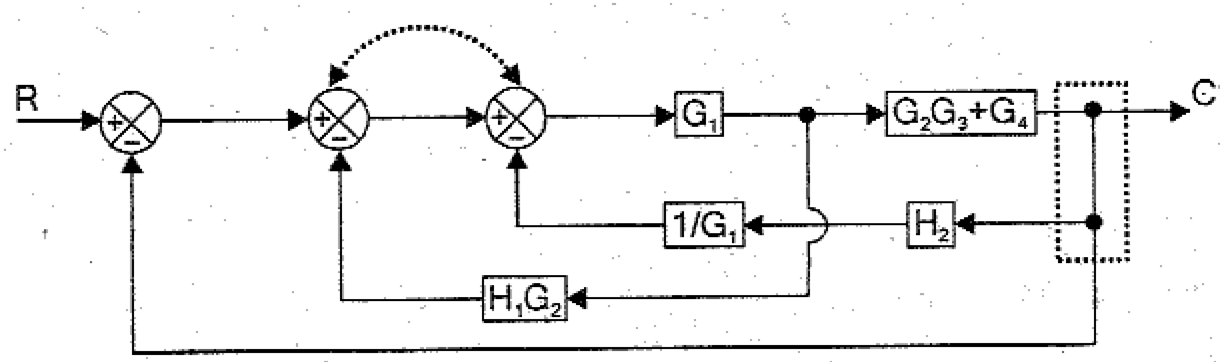
Step 2: Combining the blocks in cascade and eliminating parallel blocks



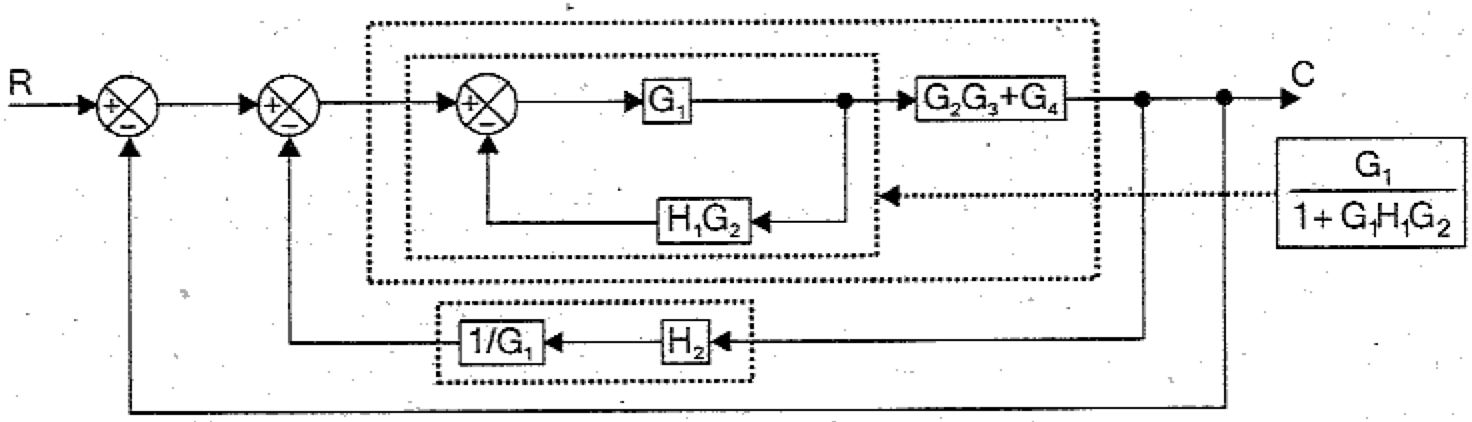
Step 3: Moving summing point before the block.



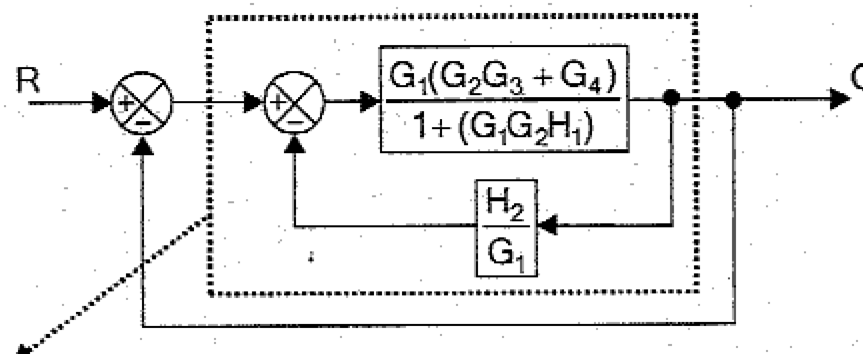
Step 4: Interchanging summing points and modifying branch points.



Step 5: Eliminating the feedback path and combining blocks in cascade

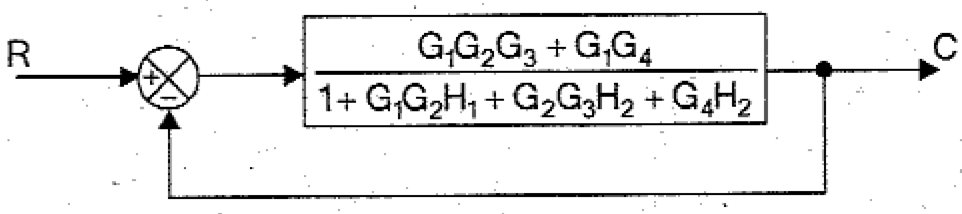


Step 6: Eliminating the feedback path



$$\frac{\dfrac{G_1(G_2G_3+G_4)}{1+G_1G_2H_1}}{1+\dfrac{G_1(G_2G_3+G_4)}{1+G_1G_2H_1}\,\dfrac{H_2}{G_1}} \Rightarrow \frac{\dfrac{G_1G_2G_3+G_1G_4}{1+G_1G_2H_1}}{\dfrac{1+G_1G_2H_1+G_2G_3H_2+G_4H_2}{1+G_1G_2H_1}} \Rightarrow \frac{G_1G_2G_3+G_1G_4}{1+G_1G_2H_1+G_2G_3H_2+G_4H_2}$$

Step 7: Eliminating the feedback path



$$\frac{C}{R} = \frac{\dfrac{G_1G_2G_3 + G_1G_4}{1 + G_1G_2H_1 + G_2G_3H_2 + G_4H_2}}{1 + \dfrac{G_1G_2G_3 + G_1G_4}{1 + G_1G_2H_1 + G_2G_3H_2 + G_4H_2}} = \frac{G_1G_2G_3 + G_1G_4}{1 + G_1G_2H_1 + G_2G_3H_2 + G_4H_2 + G_1G_2G_3 + G_1G_4}$$

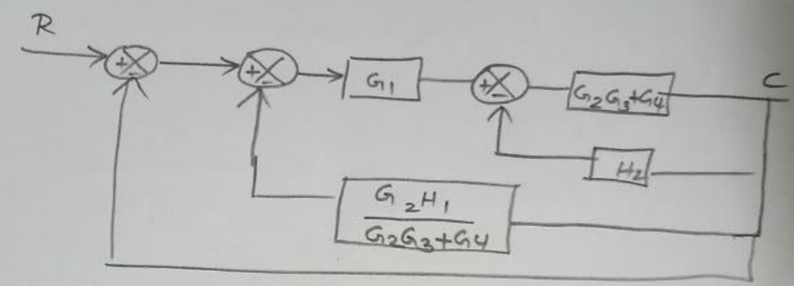
RESULT

The overall transfer function is given by,

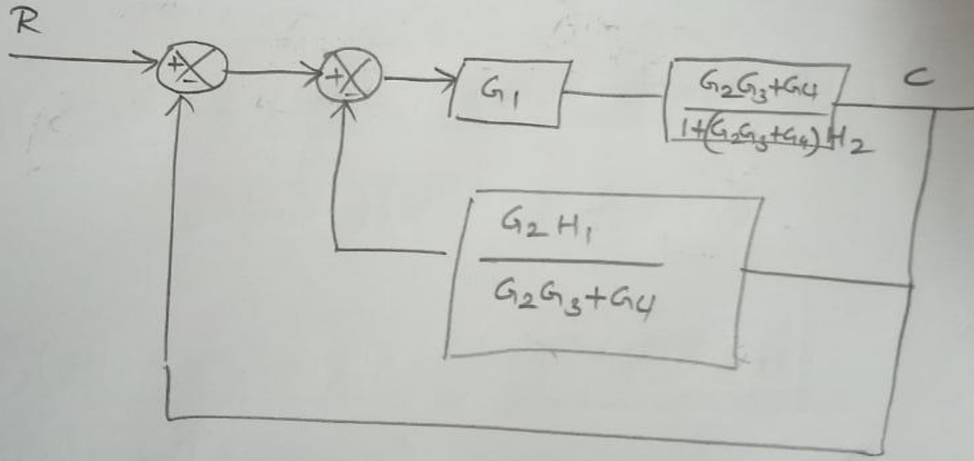
$$\frac{C}{R} = \frac{G_1G_2G_3 + G_1G_4}{1 + G_1G_2H_1 + G_2G_3H_2 + G_4H_2 + G_1G_2G_3 + G_1G_4}$$

From step …3 t can be done as

step- 4 Moving Branch point after the block



step-5:- Eliminating feed back loop



step-6 :- Cascading the blocks and Eliminating feed back loop

$$= \frac{G_1 \left(\frac{G_2G_3 + G_4)}{1 + (G_2G_3H_2 + G_4H_2} \right.}{1 + \frac{G_1 (G_2G_3 + G_4)}{1 + (G_2G_3H_2 + G_4H_2} \times \frac{G_2H_1}{G_2G_3 + G_4}}$$

$$= \frac{G_1 (G_2G_3 + G_4) / 1 + (G_2G_3H_2 + G_4H_2)}{\frac{(1 + G_2G_3H_2 + G_4H_2)(G_2G_3 + G_4) + G_1(G_2G_3 + G_4)(G_2H_1)}{(1 + G_2G_3H_2 + G_4H_2)(G_2G_3 + G_4}}$$

$$= \frac{G_1 (G_2G_3 + G_4}{1 + G_2G_3H_2 + G_4H_2} \times \frac{(1 + G_2G_3H_2 + G_4H_2)(G_2G_3 + G_4)}{(G_2G_3 + G_4)\left[1 + G_2G_3H_2 + G_4H_2) \quad G_1G_2H_1)\right.}$$

$$= \frac{G_1G_2G_3 + G_1G_4}{1 + G_1G_2H_1 + G_2G_3H_2 + G_4H_2}$$

7)

Using block diagram reduction technique find the transfer function C(s)/R(s) for the system shown in fig 1.

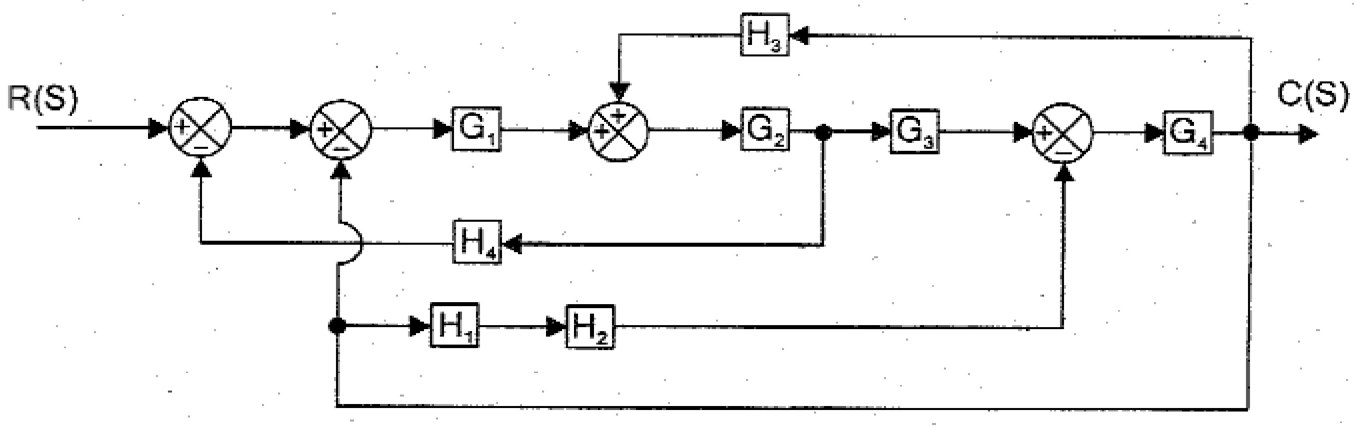
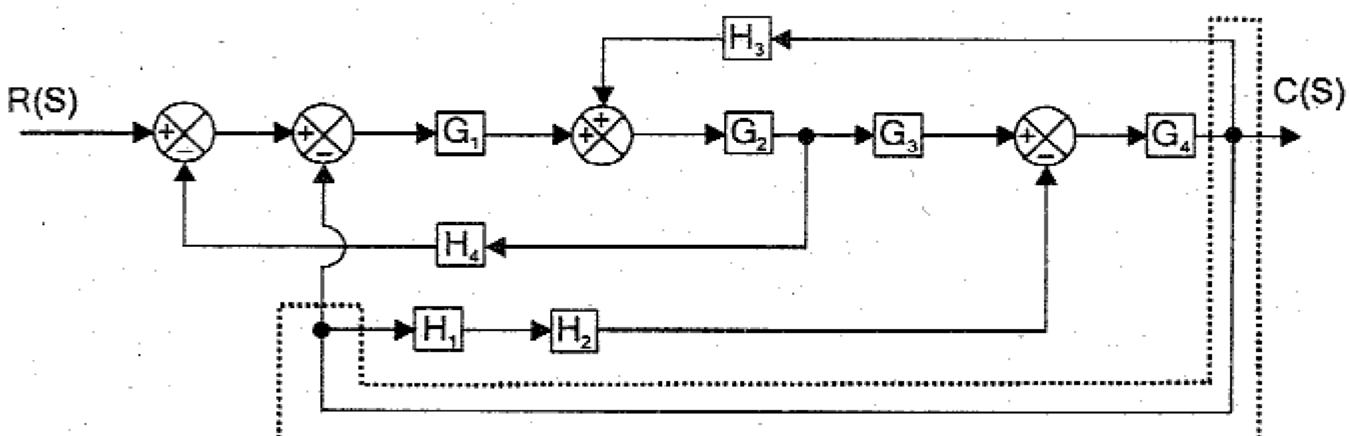


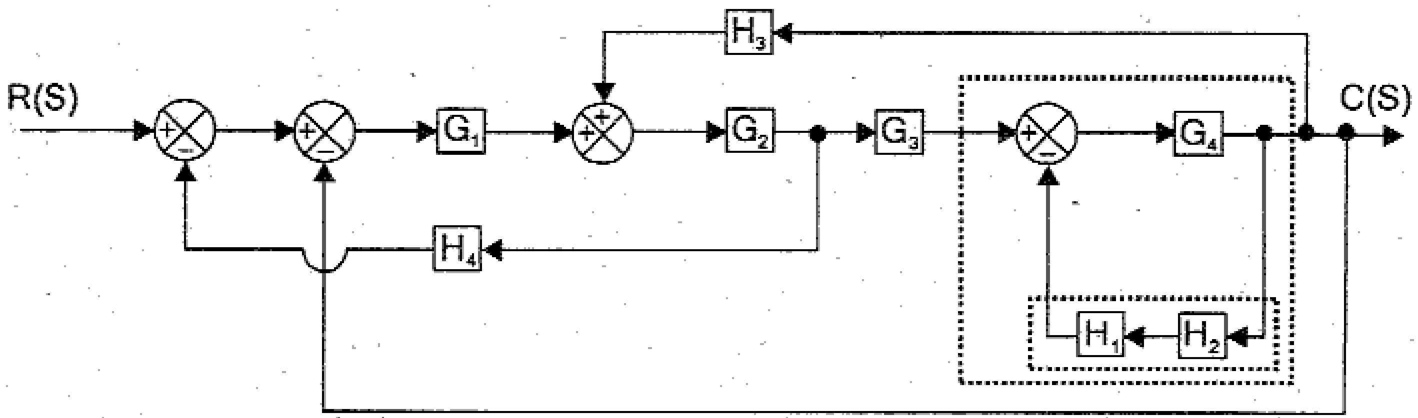
Fig 1.

SOLUTION

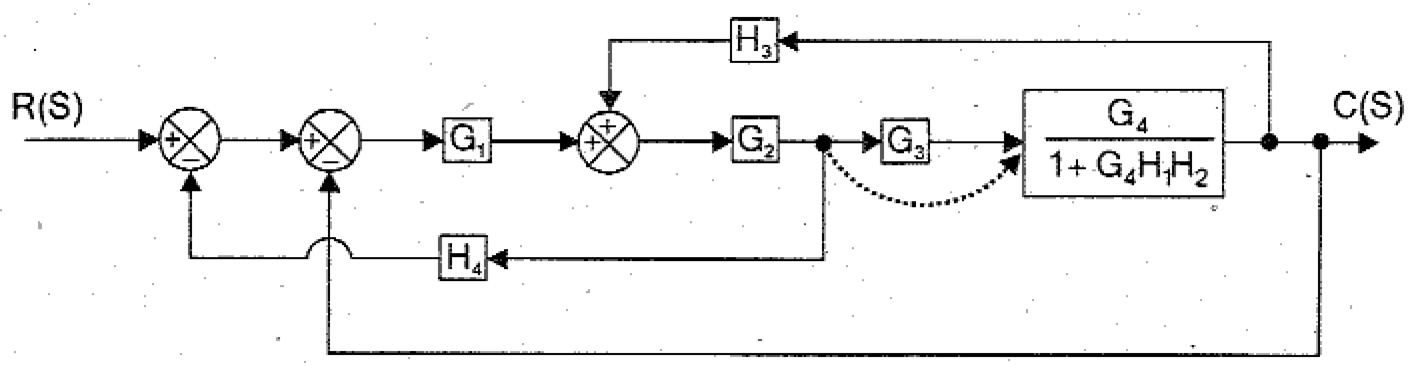
Step 1 : Rearranging the branch points



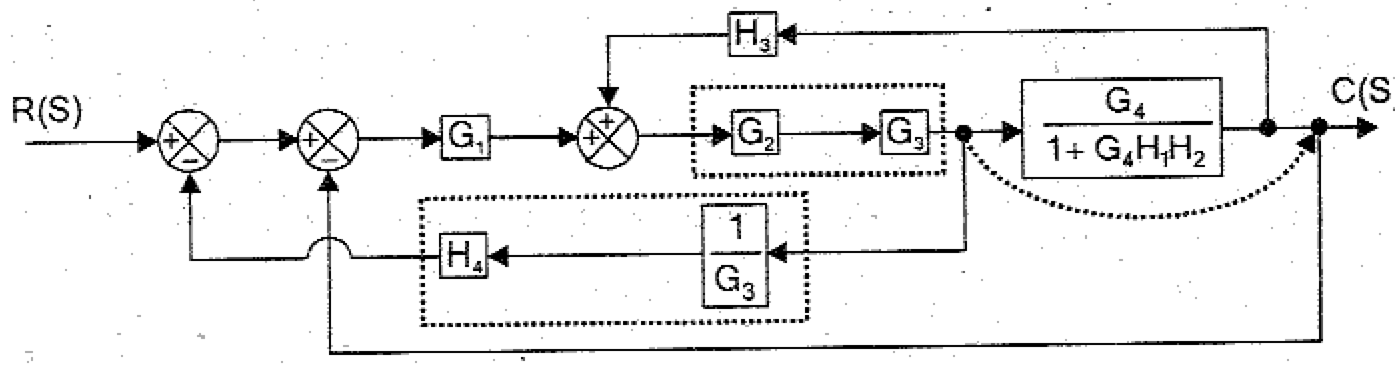
Step 2 : Combining the blocks in cascade and eliminating the feedback path.



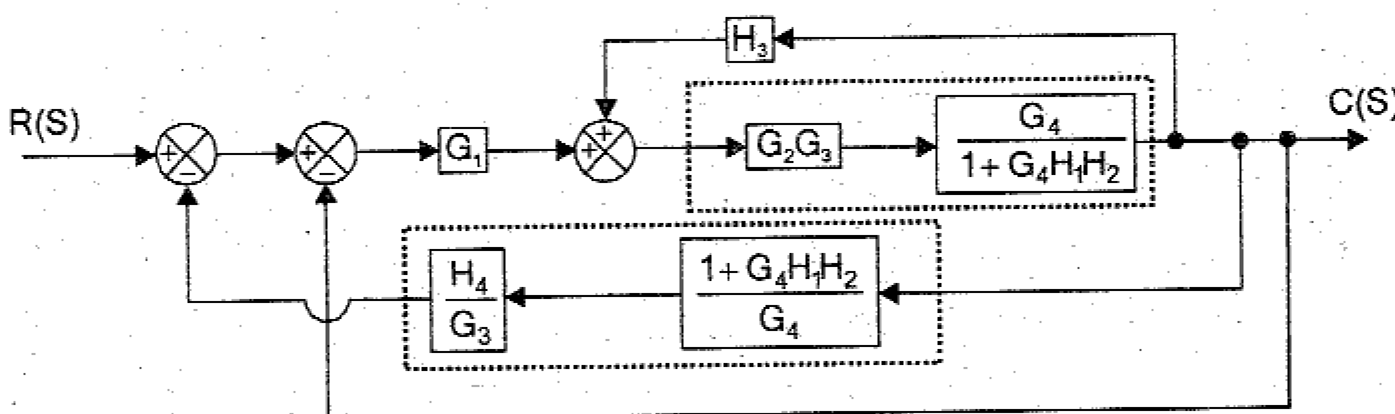
Step 3 : Moving the branch point after the block.



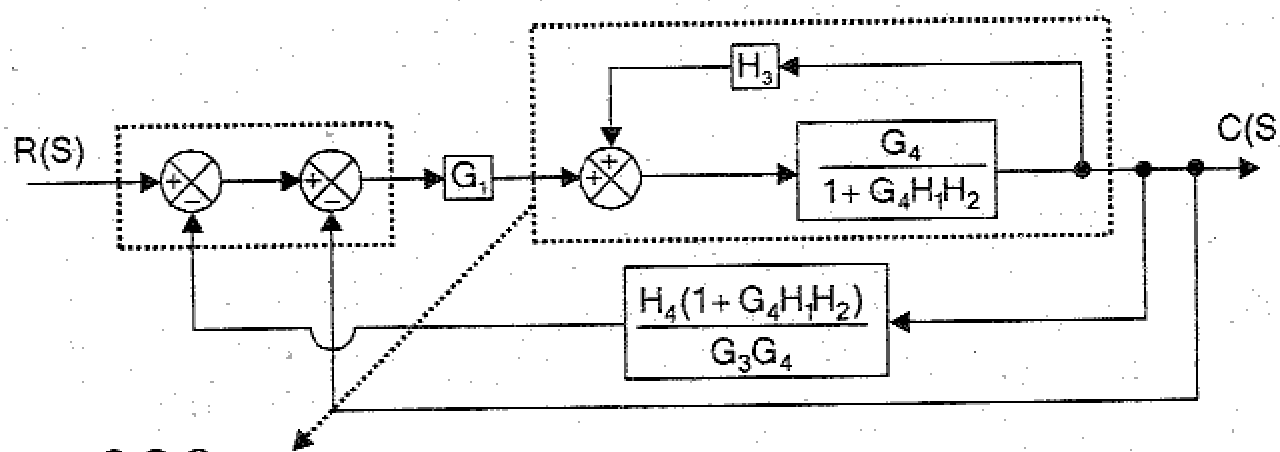
Step 4 : Moving the branch point and combining the blocks in cascade.



Step 5 : Combining the blocks in cascade

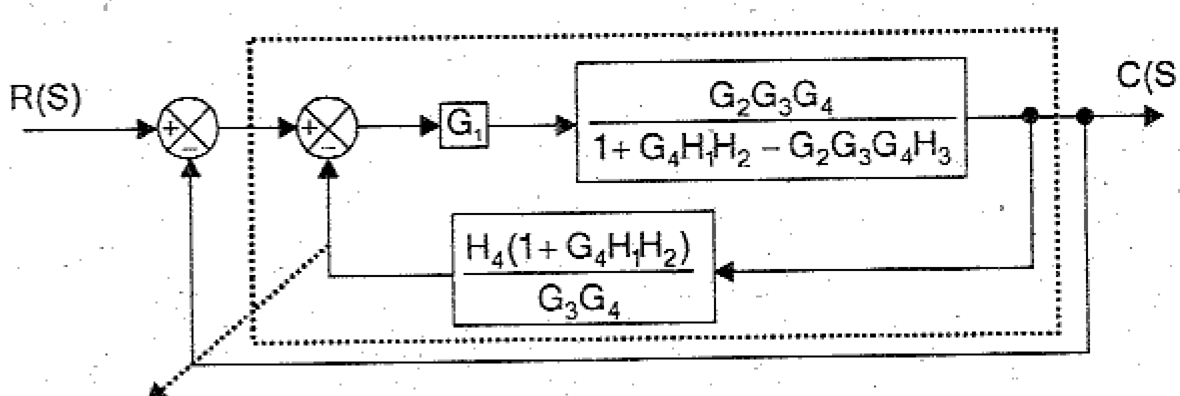


Step 6: Eliminating feedback path and interchanging the summing points.



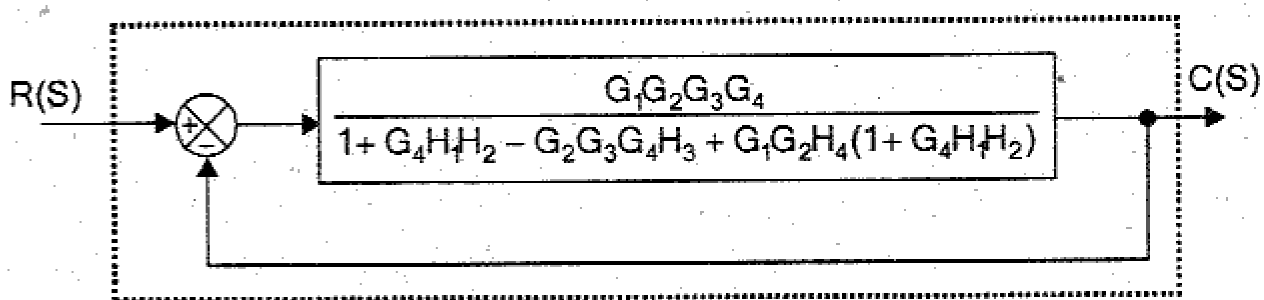
$$\frac{\dfrac{G_2G_3G_4}{1+G_4H_1H_2}}{1-\dfrac{G_2G_3G_4H_3}{1+G_4H_1H_2}} = \frac{G_2G_3G_4}{1+G_4H_1H_2-G_2G_3G_4H_3}$$

Step 7: Combining the blocks in cascade and eliminating the feedback path



$$\frac{\dfrac{G_1G_2G_3G_4}{1+G_4H_1H_2-G_2G_3G_4H_3}}{1+\left(\dfrac{G_1G_2G_3G_4}{1+G_4H_1H_2-G_2G_3G_4H_3}\right)\left(\dfrac{H_4(1+G_4H_1H_2)}{G_3G_4}\right)} = \frac{G_1G_2G_3G_4}{1+G_4H_1H_2-G_2G_3G_4H_3+G_1G_2H_4(1+G_4H_1H_2)}$$

Step 8 : Eliminating the unity feedback path.



$$\therefore \frac{C(s)}{R(s)} = \frac{\dfrac{G_1G_2G_3G_4}{1+G_4H_1H_2 - G_2G_3G_4H_3 + G_1G_2H_4(1+G_4H_1H_2)}}{1+\dfrac{G_1G_2G_3G_4}{1+G_4H_1H_2 - G_2G_3G_4H_3 + G_1G_2H_4(1+G_4H_1H_2)}}$$

$$= \frac{G_1G_2G_3G_4}{1+G_4H_1H_2 - G_2G_3G_4H_3 + G_1G_2H_4(1+G_4H_1H_2) + G_1G_2G_3G_4}$$

$$= \frac{G_1G_2G_3G_4}{1+H_1H_2(G_4 + G_1G_2G_4H_4) + G_1G_2(H_4 + G_3G_4) - G_2G_3G_4H_3}$$

ESULT

The transfer function of the system is,

$$\frac{C(s)}{R(s)} = \frac{G_1G_2G_3G_4}{1+H_1H_2(G_4 + G_1G_2G_4H_4) + G_1G_2(H_4 + G_3G_4) - G_2G_3G_4H_3}$$

For the system represented by the block diagram shown in the fig 1, determine C_1/R_1 and C_2/R_1.

8)

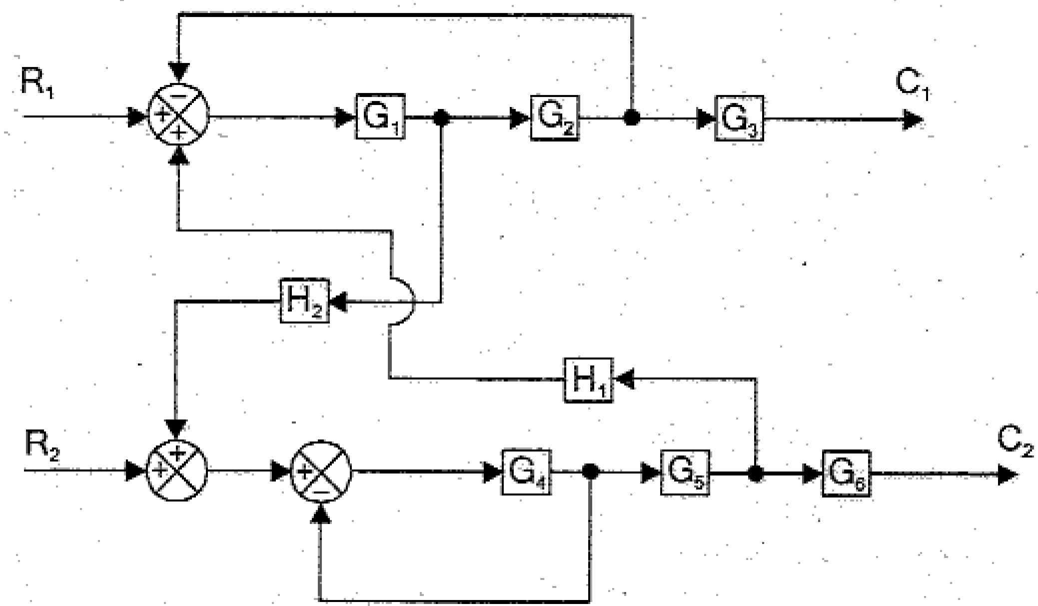


Fig 1

SOLUTION

Case (i) To find $\frac{C_1}{R_1}$

In this case set $R_2 = 0$ and consider only one output C_1. Hence we can remove the summing point which adds R_2 and need not consider G_6, since G_6 is on the open path. The resulting block diagram is shown in fig 2.

Step 1: Eliminating the feedback path

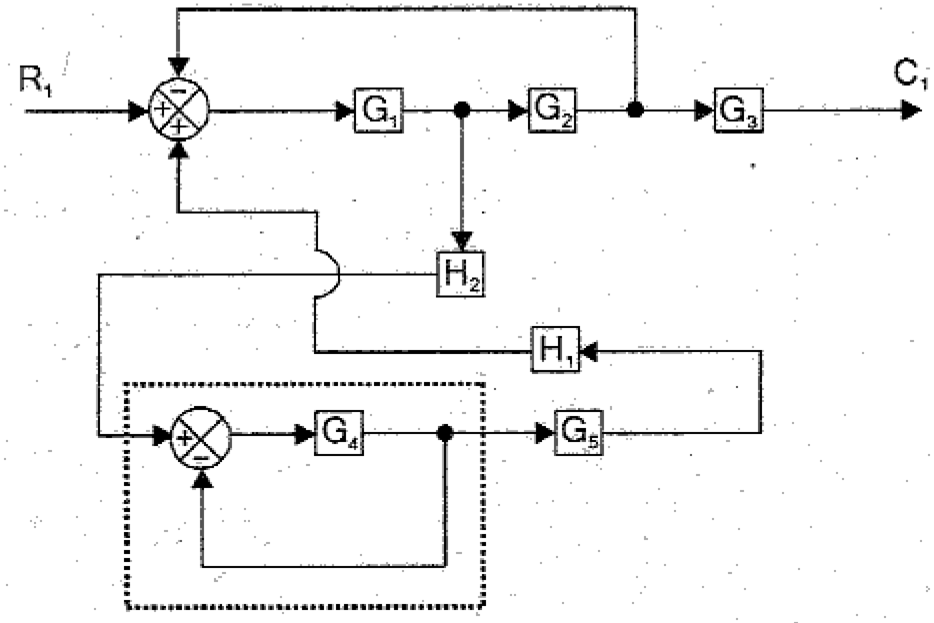
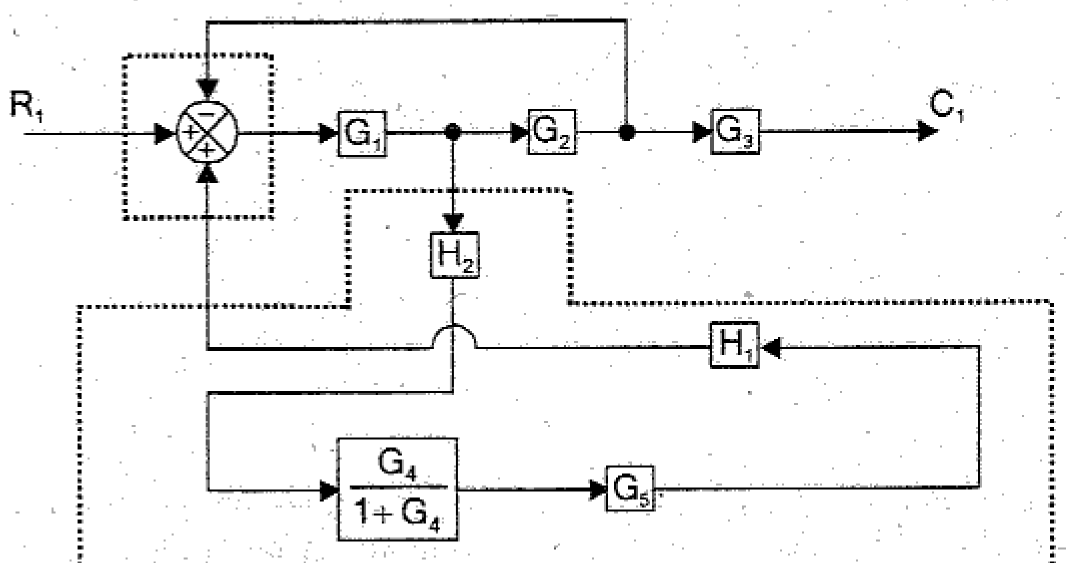
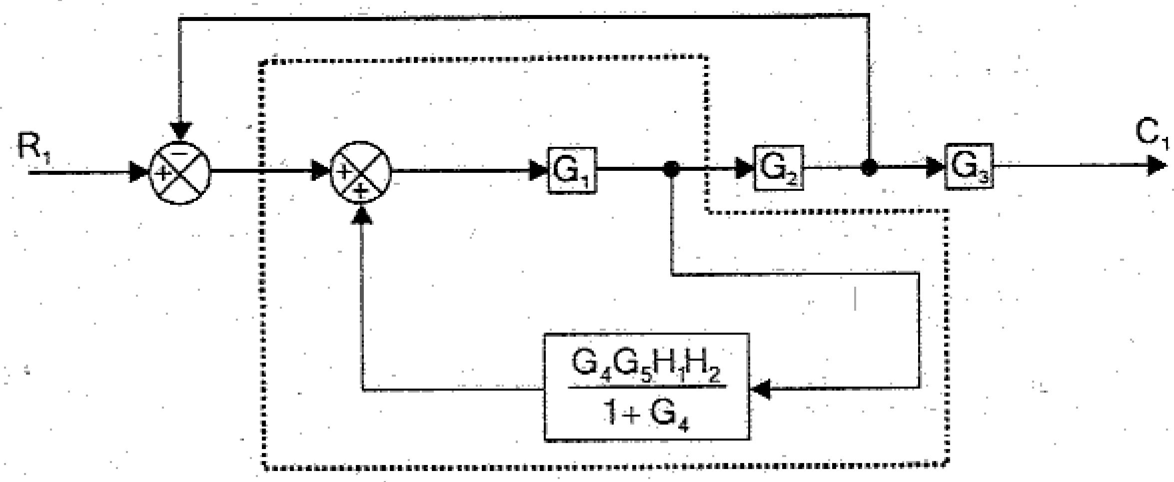


Fig 2.

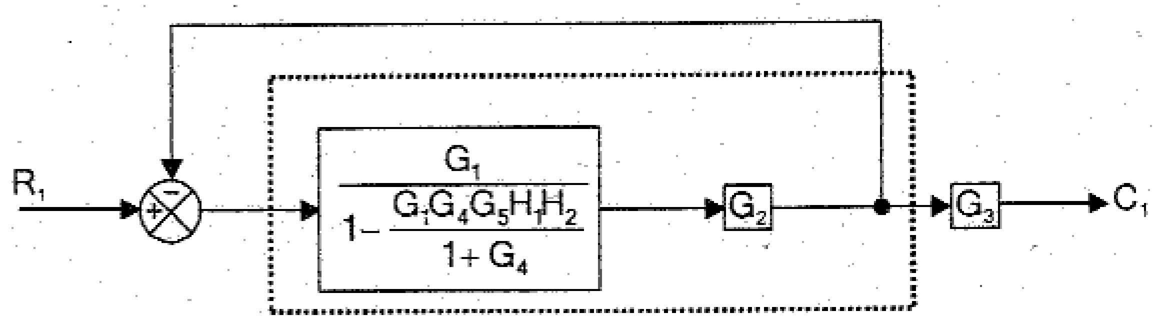
Step 2: Combining the blocks in cascade and splitting the summing point



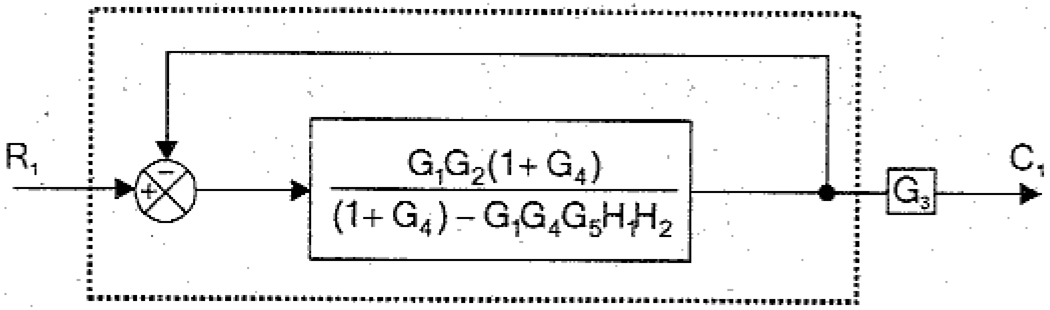
Step 3: Eliminating the feedback path

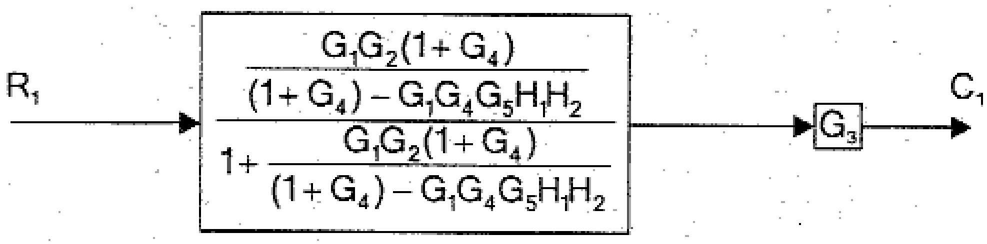


Step 4: Combining the blocks in cascade

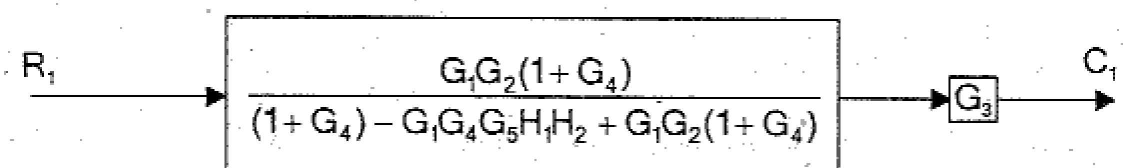


Step 5: Eliminating the feedback path





Step 6: Combining the blocks in cascade



$$\frac{C_1}{R_1} = \frac{G_1G_2G_3(1+G_4)}{(1+G_1G_2)\,(1+G_4)-G_1G_4G_5H_1H_2}$$

Case 2 : To find $\frac{C_2}{R_1}$

In this case set $R_2 = 0$ and consider only one output C_2. Hence we can remove the summing point which adds R_2 and need not consider G_3, since G_3 is on the open path. The resulting block diagram is shown in fig 3.

Step 1: Eliminate the feedback path.

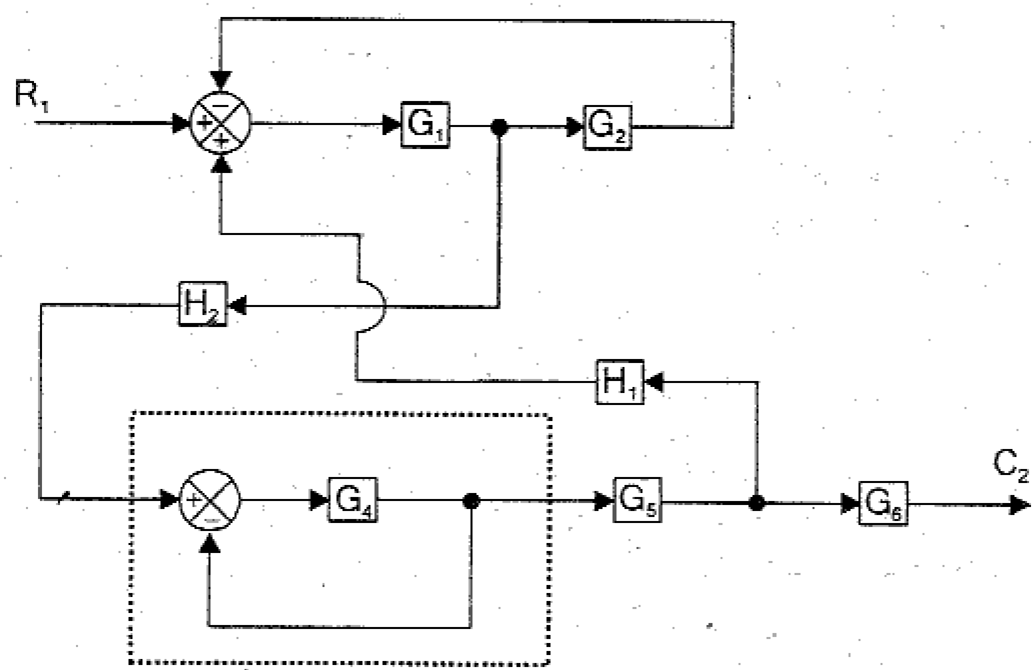
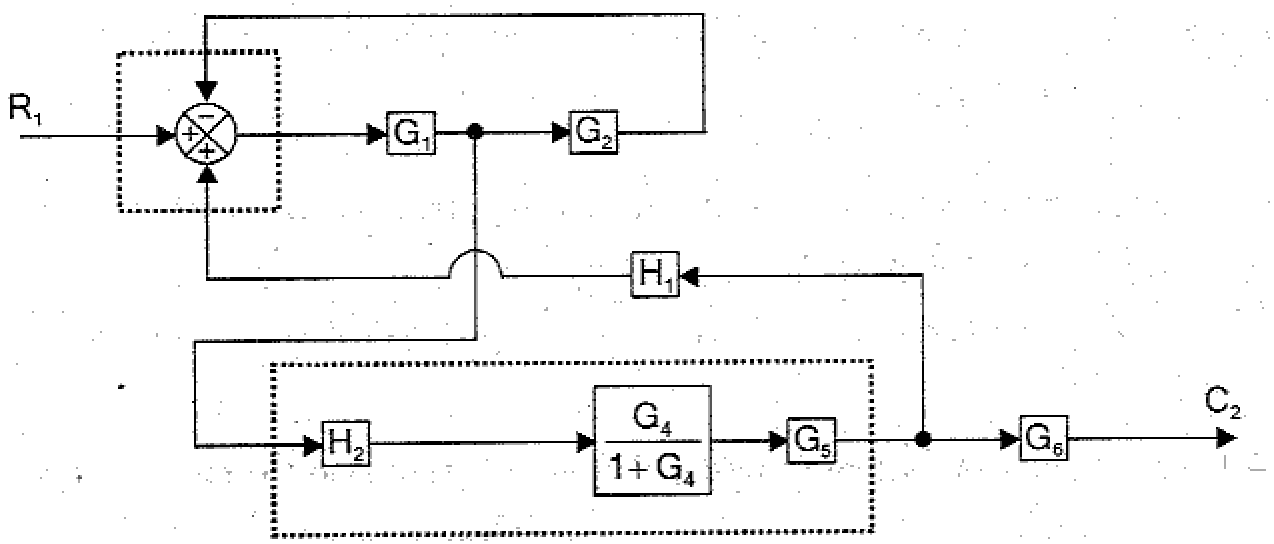
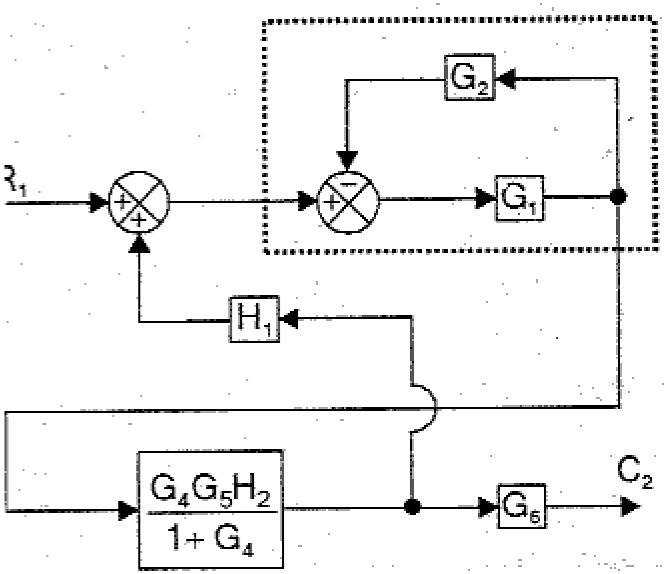


Fig 3.

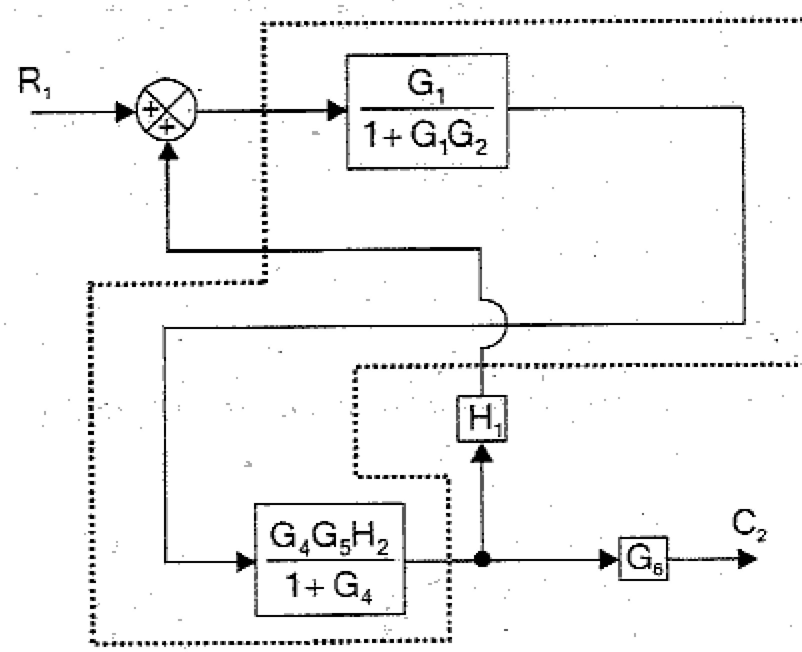
Step 2: Combining blocks in cascade and splitting the summing point



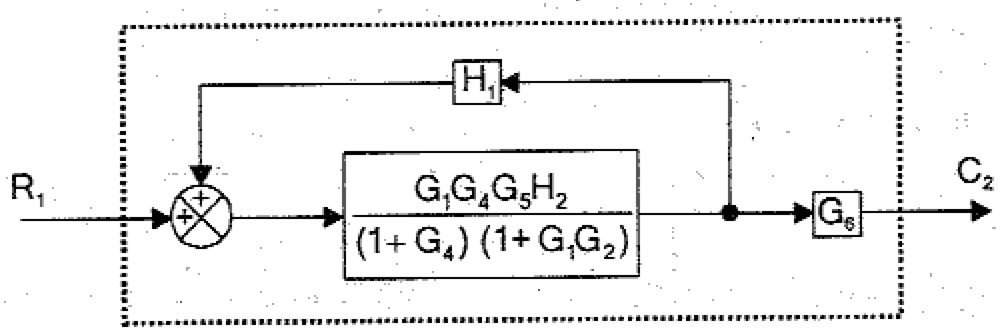
Step 3: Eliminating the feedback path

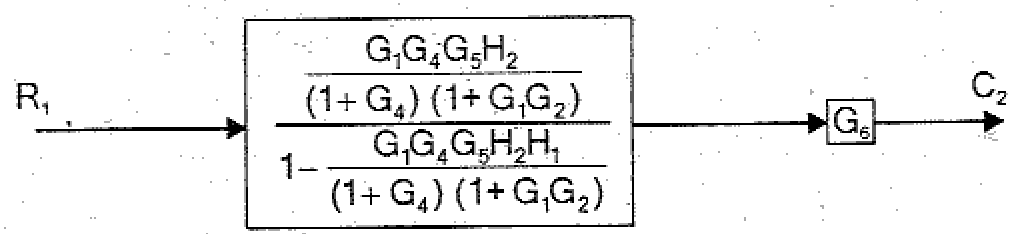


Step 4 : Combining the blocks in cascade

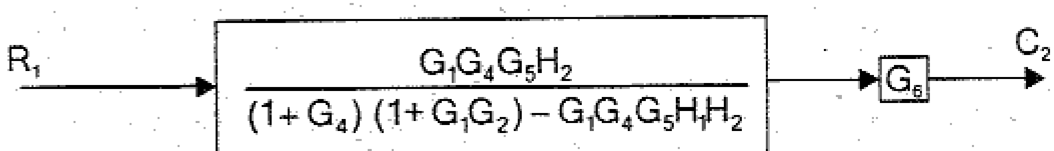


Step 5: Eliminating the feedback path





Step 6: Combining the blocks in cascade



$$\frac{C_2}{R_1} = \frac{G_1G_4G_5G_6H_2}{(1+G_4)\,(1+G_1G_2) - G_1G_4G_5H_1H_2}$$

RESULT

The transfer function of the system when the input and output are R_1 and C_1 is given by,

$$\frac{C_1}{R_1} = \frac{G_1G_2G_3\,(1+G_4)}{(1+G_1G_2)\,(1+G_4) - G_1G_4G_5H_1H_2}$$

The transfer function of the system when the input and output are R_1 and C_2 is given by,

SIGNAL FLOW GRAPH

The signal flow graph is used to represent the control system graphically and it was developed by **S.J. Mason.**

A signal flow graph is a diagram that represents a set of simultaneous linear algebraic equations. By taking Laplace transform, the time domain differential equations governing a control system can be transferred to a set of algebraic equations in s-domain. The signal flow graph of the system can be constructed using these equations.

In a signal flow graph, the signal flows in only one direction. The direction of signal flow is indicated by an arrow placed on the branch and the gain (multiplication factor) is indicated along the branch.

TERMS USED IN SIGNAL FLOW GRAPH

- **Node: A node is a point representing a variable or signal.**
- **Branch: A branch is a directed line segment joining two nodes.**
- **Transmittance: It is the gain between two nodes.**
- **Input node: A node that has only outgoing branche(s). It is also, called as** source and corresponds to independent variable.
- **Output node: A node that has only incoming branches. This is also called as** sink and corresponds to dependent variable.
- **Path: A path is a traversal of connected branches in the direction of branch** arrow.

- **Loop: A loop is a closed path.**
- **Self loop: It is a feedback loop consisting of single branch.**
- **Loop gain: The loop gain is the product of branch transmittances of the loop.**
- **Non touching loops: Loops that do not posses a common node.**
- **Forward path: A** path from source to sink without traversing an node more than once.
- **Feedback path: A path which originates and terminates at the same node.**

Forward path gain: Product of branch transmittances of a forward path.

ROPERTIES OF SIGNAL FLOW GRAPH

The basic properties of signal flow graph are the following :

The algebraic equations which are used to construct signal flow graph must be in the form of cause and effect relationship.

Signal flow graph is applicable to linear systems only.

A node in the signal flow graph represents the variable or signal.

A node adds the signals of all incoming branches and transmits the sum to all outgoing branches.

The signals travel along branches only in the marked direction and when it travels it gets multiplied by the gain or transmittance of the branch.

The signal flow graph of system is not unique.

SIGNAL FLOW GRAPH REDUCTION

The signal flow graph of a system can be reduced either by using the rules of a signal flow graph algebra or by using Mason's gain formula.

The signal flow graph reduction by **rule base** will be time consuming and tedious. **S.J.Mason** has developed a simple procedure to determine the transfer function of the system

He has developed a formula called by his name **Mason's gain formula** which can be directly used to find the transfer function of the system.

MASON'S GAIN FORMULA

The Mason's gain formula is used to determine the transfer function of the system from the signal flow graph of the system.

Let, R(s) = Input to the system

C(s) = Output of the system

Now, Transfer function of the system, $T(s) = \dfrac{C(s)}{R(s)}$ .....(1.34)

Mason's gain formula states the overall gain of the system [transfer function] as follows,

Overall gain, $T = \dfrac{1}{\Delta} \sum_K P_K \Delta_K$ .....(1.35) where, T = $T(s)$ = Transfer function of the system

P_K = Forward path gain of K^{th} forward path

K = Number of forward paths in the signal flow graph

$$\Delta = 1 - (\text{Sum of individual loop gains}) + \begin{pmatrix}\text{Sum of gain products of all possible} \\ \text{combinations of two non - touching loops}\end{pmatrix} - \begin{pmatrix}\text{Sum of gain products of all possible} \\ \text{combinations of three non - touching loops}\end{pmatrix} + \ldots\ldots$$

Δ_K = Δ for that part of the graph which is not touching K^{th} forward path

Find the overall transfer function of the system whose signal flow graph is shown in fig 1.

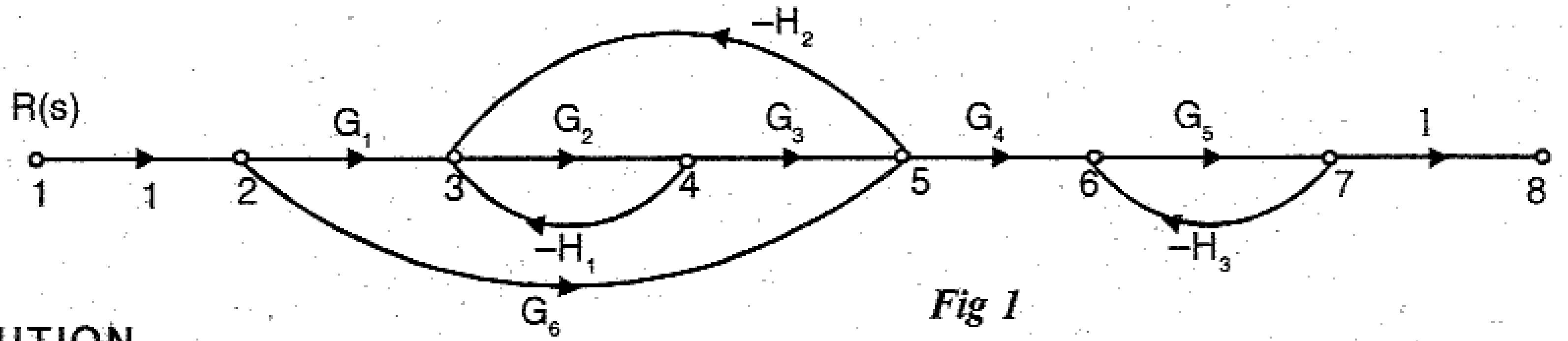


Fig 1

OLUTION

Forward Path Gains

There are two forward paths. $\therefore K = 2$

Let forward path gains be P_1 and P_2.

***Fig 2 :** Forward path-1.*

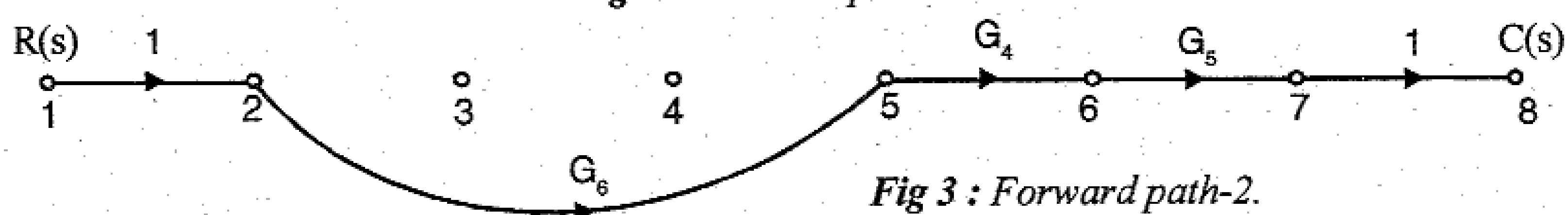


***Fig 3 :** Forward path-2.*

Gain of forward path-1, $P_1 = G_1 G_2 G_3 G_4 G_5$

Gain of forward path-2, $P_2 = G_4 G_5 G_6$

Individual Loop Gain

There are three individual loops. Let individual loop gains be P_{11}, P_{21} and P_{31}.

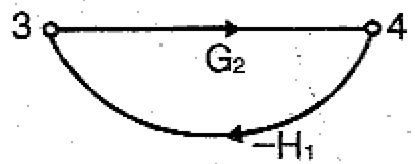


Fig 4 : Loop-1.

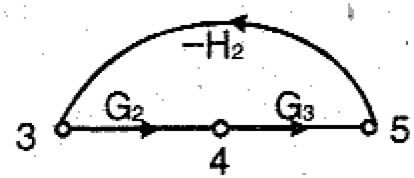


Fig 5 : Loop-2.

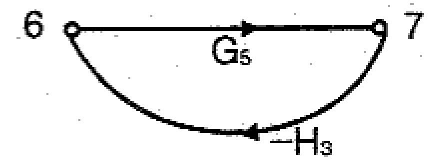


Fig 6 : Loop-3.

Loop gain of individual loop-1, $P_{11} = -G_2H_1$

Loop gain of individual loop-2, $P_{21} = -G_2G_3H_2$

Loop gain of individual loop-3, $P_{31} = -G_5H_3$

Gain Products of Two Non-touching Loops

There are two combinations of two non-touching loops. Let the gain products of two non touching loops be P_{12} and P_{22}.

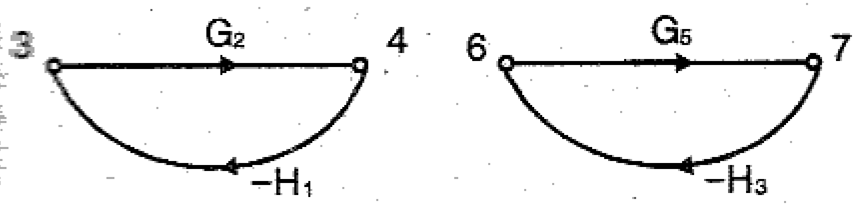


Fig 7 : First combination of 2 non-touching loops.

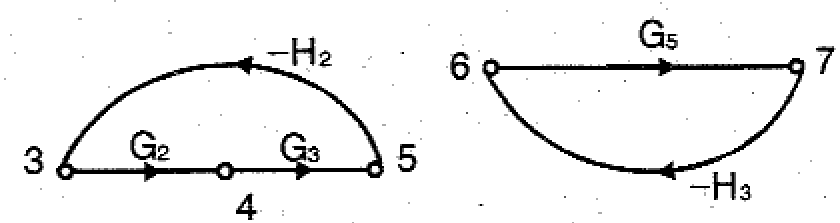


Fig 8 : Second combination of 2 non-touching loops.

Gain product of first combination of two non touching loops $\Big\}$ $P_{12} = P_{11}P_{31} = (-G_2H_1)(-G_5H_3) = G_2G_5H_1H_3$

Gain product of second combination of two non touching loops $\Big\}$ $P_{22} = P_{21}P_{31} = (-G_2G_3H_2)(-G_5H_3) = G_2G_3G_5H_2H_3$

IV. Calculation of Δ and Δ_K

$$\Delta = 1 - (P_{11} + P_{21} + P_{31}) + (P_{12} + P_{22})$$

$$= 1 - (-G_2H_1 - G_2G_3H_2 - G_5H_3) + (G_2G_5H_1H_3 + G_2G_3G_5H_2H_3)$$

$$= 1 + G_2H_1 + G_2G_3H_2 + G_5H_3 + G_2G_5H_1H_3 + G_2G_3G_5H_2H_3$$

$\Delta_1 = 1$, Since there is no part of graph which is not touching with first forward path.

The part of the graph which is non touching with second forward path is shown in fig 9.

Fig 9

$$\Delta_2 = 1 - P_{11} = 1 - (-G_2H_1) = 1 + G_2H_1$$

V. Transfer Function, T

By Mason's gain formula the transfer function, T is given by,

$$T = \frac{1}{\Delta}\sum_K P_K\Delta_K = \frac{1}{\Delta}(P_1\Delta_1 + P_2\Delta_2)$$ (Number of forward paths is 2 and so K = 2)

$$= \frac{G_1G_2G_3G_4G_5 + G_4G_5G_6(1 + G_2H_1)}{1 + G_2H_1 + G_2G_3H_2 + G_5H_3 + G_2G_5H_1H_3 + G_2G_3G_5H_2H_3}$$

$$= \frac{G_1G_2G_3G_4G_5 + G_4G_5G_6 + G_2G_4G_5G_6H_1}{1 + G_2H_1 + G_2G_3H_2 + G_5H_3 + G_2G_5H_1H_3 + G_2G_3G_5H_2H_3}$$

$$= \frac{G_2G_4G_5\left[G_1G_3 + G_6/G_2 + G_6H_1\right]}{1 + G_2H_1 + G_2G_3H_2 + G_5H_3 + G_2G_5H_1H_3 + G_2G_3G_5H_2H_3}$$

Find the overall gain C(s)/R(s) for the signal flow graph shown in fig 1.

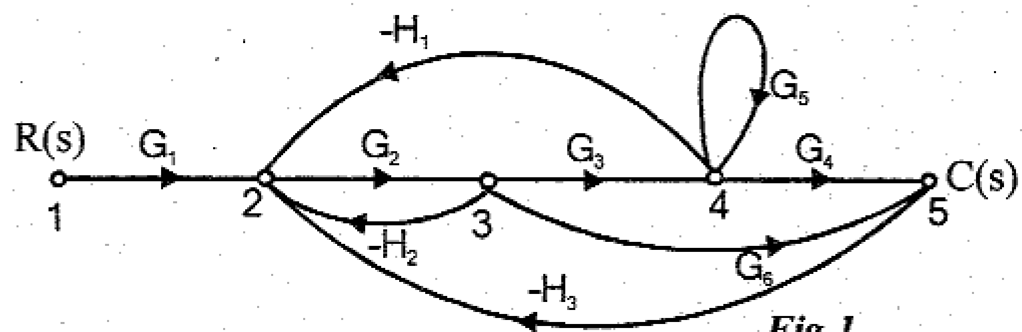


Fig 1

SOLUTION

I. Forward Path Gains

There are two forward paths. ∴ K = 2. Let the forward path gains be P_1 and P_2.

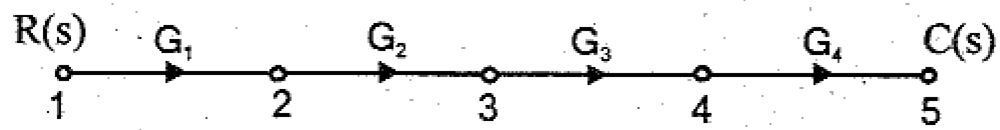


***Fig 2 :** Forward path-1*

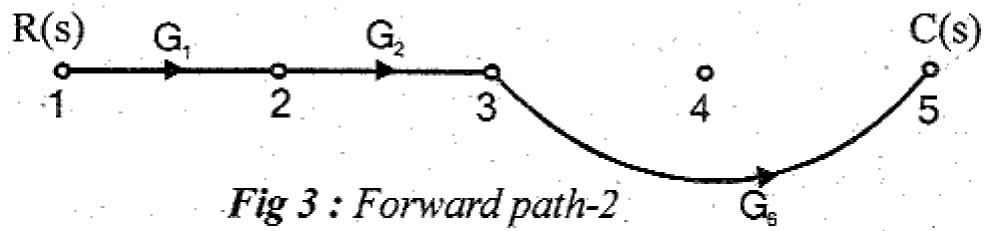


***Fig 3 :** Forward path-2*

Gain of forward path-1, $P_1 = G_1G_2G_3G_4$

Gain of forward path-2, $P_2 = G_1G_2G_6$

Individual Loop Gain

There are five individual loops. Let the individual loop gains be p_{11}, p_{21}, p_{31}, p_{41} and p_{51}.

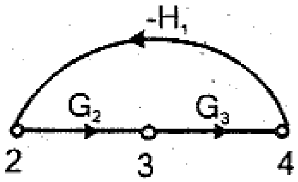


Fig 4 : loop-1

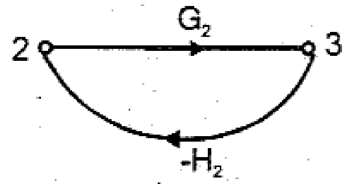


Fig 5 : loop-2

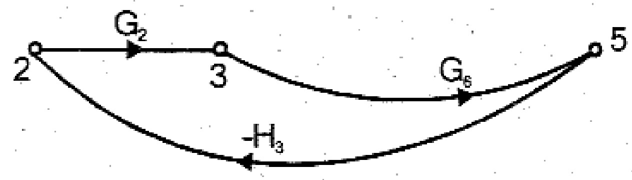


Fig 6 : loop-3

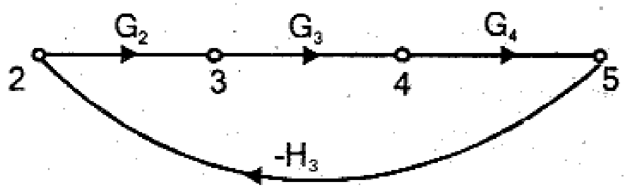


Fig 7 : loop-4

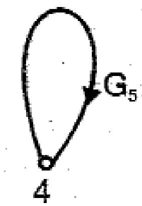


Fig 8 : loop-5

Loop gain of individual loop-1, $P_{11} = -G_2G_3H_1$

Loop gain of individual loop-2, $P_{21} = -H_2G_2$

Loop gain of individual loop-3, $P_{31} = -G_2G_6H_3$

Loop gain of individual loop-4, $P_{41} = -G_2G_3G_4H_3$

Loop gain of individual loop-5, $P_{51} = G_5$

Gain Products of Two Non-touching Loops

There are two combinations of two non-touching loops.

Let the gain products of two non-touching loops be P_{12} and P_{22}.

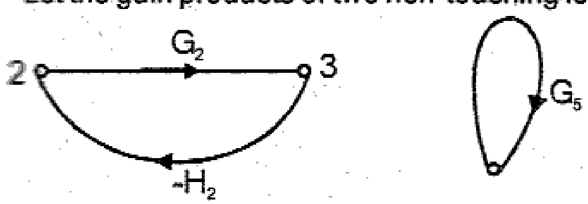


Fig 9 : First combination of two non-touching loops

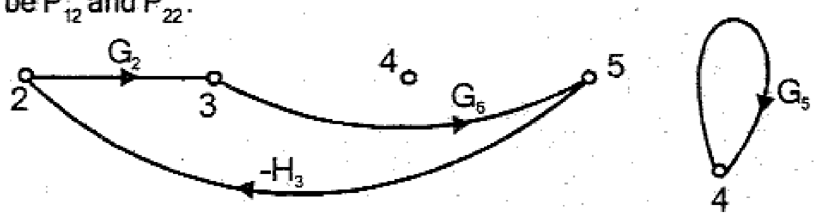


Fig 10 : Second combination of two non-touching loops

Gain product of first combination of two non touching loops $\Big\}$ $P_{12} = P_{21}P_{51} = (-G_2H_2)(G_5) = G_2G_5H_2$

Gain product of second combination of two non touching loops $\Big\}$ $P_{22} = P_{31}P_{51} = (-G_2G_6H_3)(G_5) = -G_2G_5G_6H_3$

Calculation of Δ and Δ_K

$$\begin{aligned}\Delta &= 1-(P_{11}+P_{21}+P_{31}+P_{41}+P_{51})+(P_{12}+P_{22})\\ &= 1-(-G_2G_3H_1 - H_2G_2 - G_2G_3G_4H_3 + G_5 - G_2G_6H_3)\\ &\quad +(-G_2H_2G_5 - G_2G_5G_6H_3)\end{aligned}$$

Since there is no part of graph which is not touching forward path-1, $\Delta_1 = 1$.

The part of graph which is not touching forward path-2 is shown in fig 11.

Fig 11

$\therefore \Delta_2 = 1 - G_5$

Transfer Function, T

By Mason's gain formula the transfer function, T is given by,

$$T = \frac{1}{\Delta}\sum_K P_K\Delta_K \text{ (Number of forward path is 2 and so K = 2)}$$

$$= \frac{1}{\Delta}[P_1\Delta_1 + P_2\Delta_2] = \frac{1}{\Delta}[G_1G_2G_3G_4 \times 1 + G_1G_2G_6(1-G_5)]$$

$$= \frac{G_1G_2G_3G_4 + G_1G_2G_6 - G_1G_2G_5G_6}{1 + G_2G_3H_1 + H_2G_2 + G_2G_3G_4H_3 - G_5 + G_2G_6H_3 - G_2H_2G_5 - G_2G_5G_6H_3}$$

Find the overall gain C(s)/R(s) for the signal flow graph shown in fig 1.

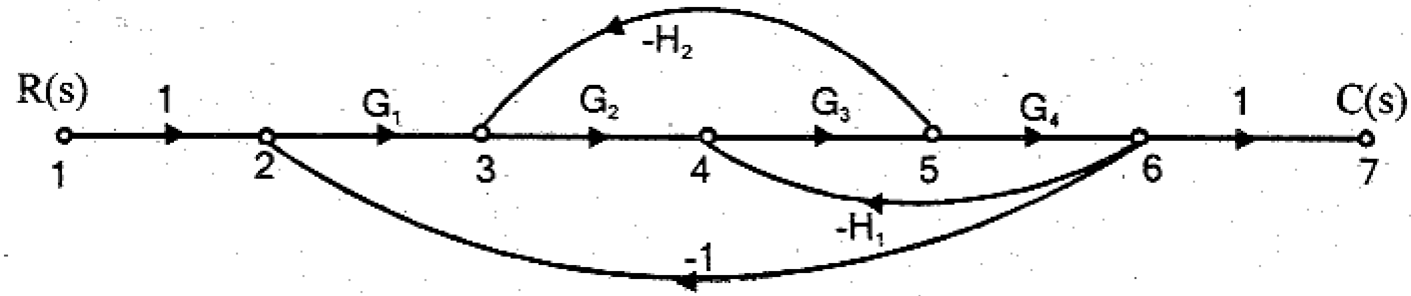


Fig 1

SOLUTION

I. Forward Path Gains

There is only one forward path. $\therefore$ K = 1.

Let the forward path gain be P_1.

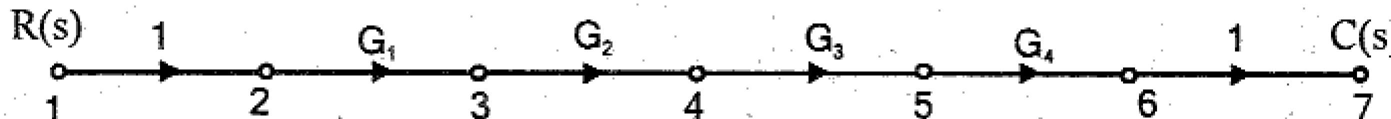


Fig 1 : Forward path-1

Gain of forward path-1, $P_1 = G_1G_2G_3G_4$

II. Individual Loop Gain

There are three individual loops. Let the loop gains be P_{11}, P_{21}, P_{31}.

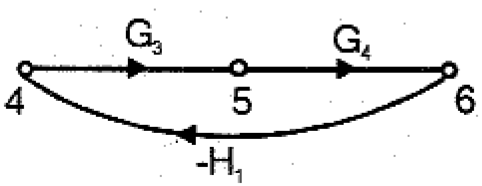


Fig 3 : loop-1

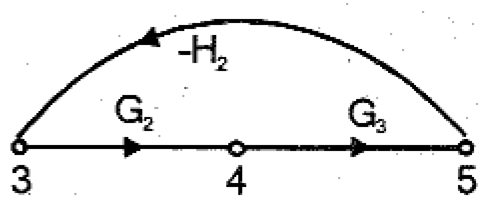


Fig 4 : loop-2

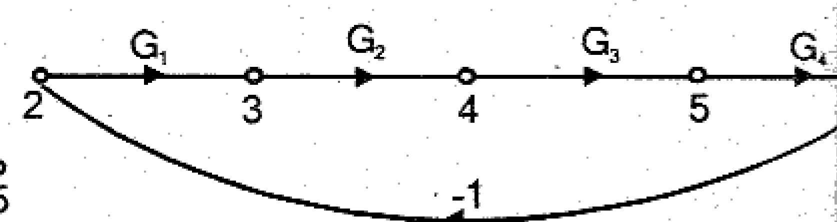


Fig 5 : loop-3

Loop gain of individual loop-1, $P_{11} = -G_3G_4H_1$

Loop gain of individual loop-2, $P_{21} = -G_2G_3H_2$

Loop gain of individual loop-3, $P_{31} = -G_1G_2G_3G_4$

III. Gain Products of Two Non-touching Loops

There are no possible combinations of two non-touching loops, three non-touching loops, etc.

IV. Calculation of Δ and Δ_K

$$\begin{aligned}\Delta &= 1-(P_{11}+P_{21}+P_{31})\\ &= 1-(-G_3G_4H_1 - G_2G_3H_2 - G_1G_2G_3G_4)\\ &= 1+G_3G_4H_1+G_2G_3H_2+G_1G_2G_3G_4\end{aligned}$$

Since no part of the graph is non-touching with forward path-1, $\Delta_1 = 1$.

Transfer Function, T

By Mason's gain formula the transfer function, T is given by,

$$T = \frac{C(s)}{R(s)} = \frac{1}{\Delta}\sum_K P_K\Delta_K = \frac{1}{\Delta}P_1\Delta_1 \text{ (Number of forward path is 1 and so K = 1)}$$

$$= \frac{G_1G_2G_3G_4}{1+G_3G_4H_1+G_2G_3H_2+G_1G_2G_3G_4}$$

The signal flow graph for a feedback control system is shown in fig 1. Determine the closed loop transfer function C(s)/R(s).

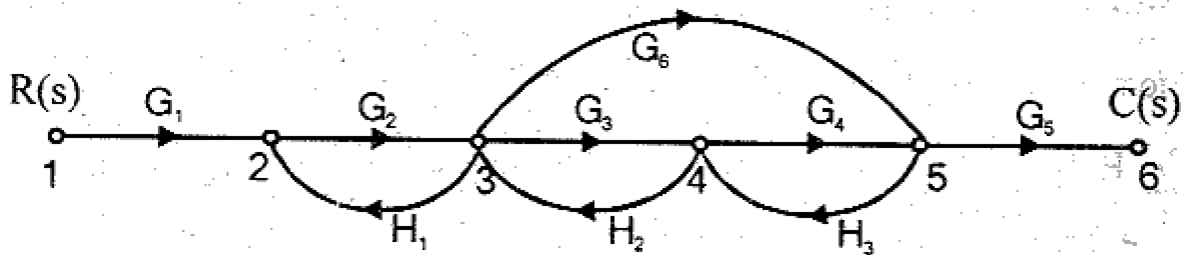


Fig 1

SOLUTION

1 Forward Path Gains

There are two forward paths. $\therefore$ K = 2.

Let forward path gains be P_1 and P_2.

***Fig 2 :** Forward path-1*

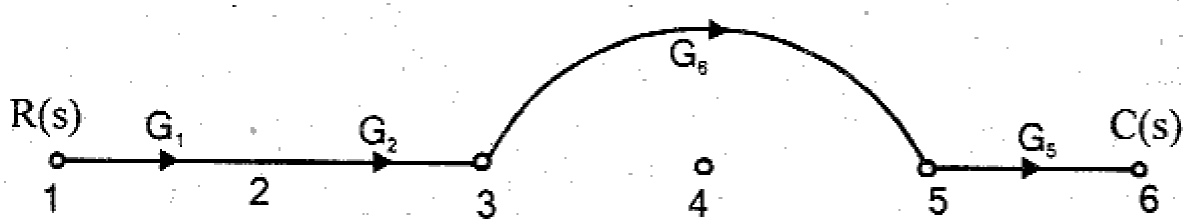


***Fig 3 :** Forward path-2*

Gain of forward path-1, $P_1 = G_1 G_2 G_3 G_4 G_5$

Gain of forward path-2, $P_2 = G_1 G_2 G_6 G_5$

Individual Loop Gain

There are four individual loops. Let individual loop gains be P_{11}, P_{21}, P_{31} and P_{41}.

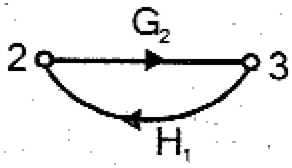


Fig 4 : loop-1

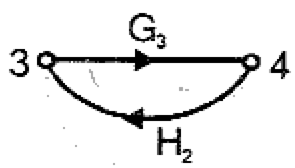


Fig 5 : loop-2

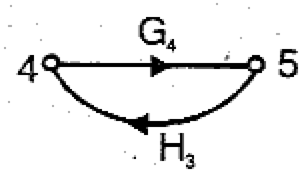


Fig 6 : loop-3

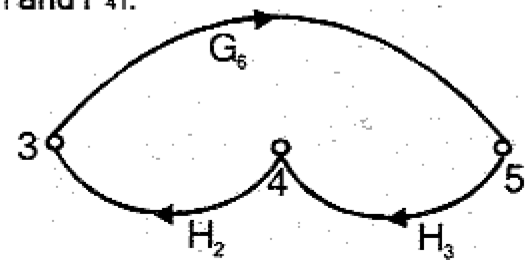


Fig 7 : loop-4

Loop gain of individual loop-1, $P_{11} = G_2H_1$

Loop gain of individual loop-2, $P_{21} = G_3H_2$

Loop gain of individual loop-3, $P_{31} = G_4H_3$

Loop gain of individual loop-4, $P_{41} = G_6H_2H_3$

Gain Products of Two Non-touching Loops

There is only one combination of two non-touching loops. Let the gain ...ucts of two non-touching loops be P_{12}.

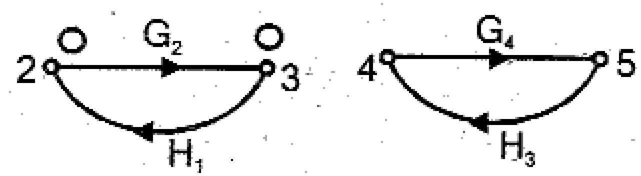


Fig 8 : First combination of two non touching loops

$$\left.\begin{array}{l}\text{Gain product of first combination}\\ \text{of two non-touching loops}\end{array}\right\} P_{12} = (G_2H_1)(G_4H_3)$$

$$= G_2G_4H_1H_3$$

IV. Calculation of Δ and Δ_K

$$\Delta = 1-(P_{11}+P_{21}+P_{31}+P_{41})+P_{12}$$
$$= 1-(G_2H_1+G_3H_2+G_4H_3+G_6H_2H_3)+G_2G_4H_1H_3$$
$$= 1-G_2H_1-G_3H_2-G_4H_3-G_6H_2H_3+G_2G_4H_1H_3$$

Since there is no part of graph which is non-touching with forward path-1 and 2, $\Delta_1 = \Delta_2 = 1$

V. Transfer Function, T

By Mason's gain formula the transfer function, T is given by,

$$T = \frac{1}{\Delta}\sum_{K} P_K \Delta_K = \frac{1}{\Delta}(P_1\Delta_1 + P_2\Delta_2) \quad \text{(Number of forward paths is two and so K = 2)}$$

$$= \frac{G_1G_2G_3G_4G_5 + G_1G_2G_5G_6}{1 - G_2H_1 - G_3H_2 - G_4H_3 - G_6H_2H_3 + G_2G_4H_1H_3}$$

EXAMPLE 1.26

Find the overall gain of the system whose signal flow graph is shown in fig 1.

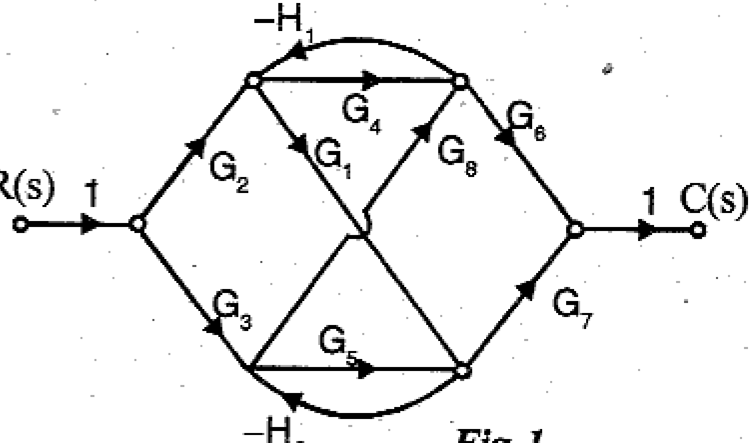


Fig 1

SOLUTION

Let us number the nodes as shown in fig 2.

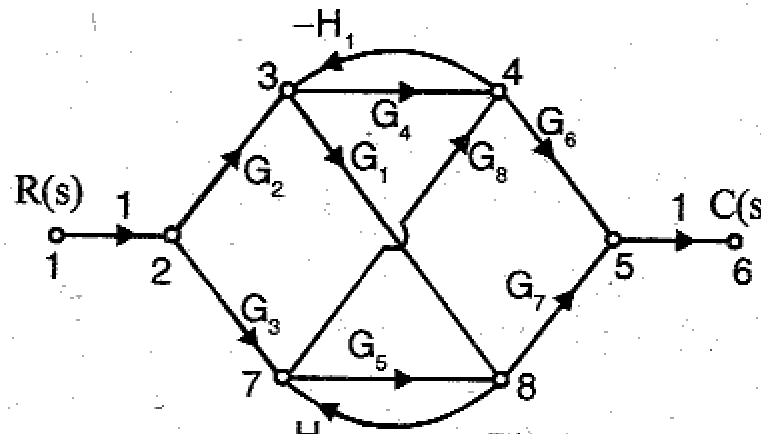


Fig 2

I. Forward Path Gains

There are six forward paths. ∴ K = 6

Let the forward path gains be P_1, P_2, P_3, P_4, P_5 and P_6.

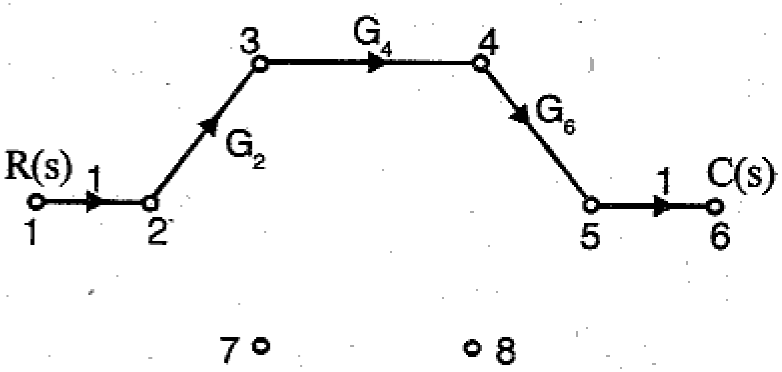


Fig 3 : Forward path-1.

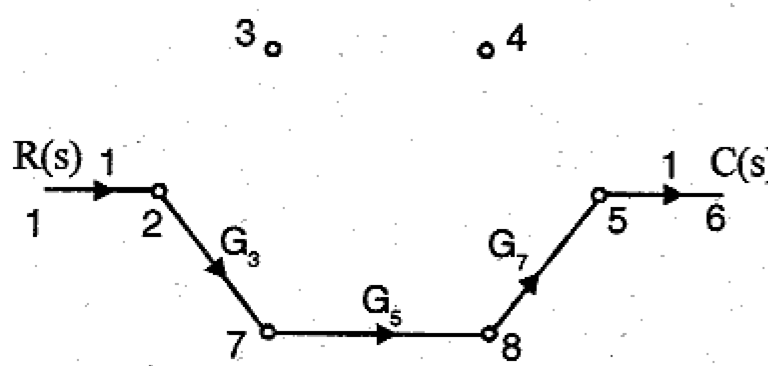


Fig 4 : Forward path-2.

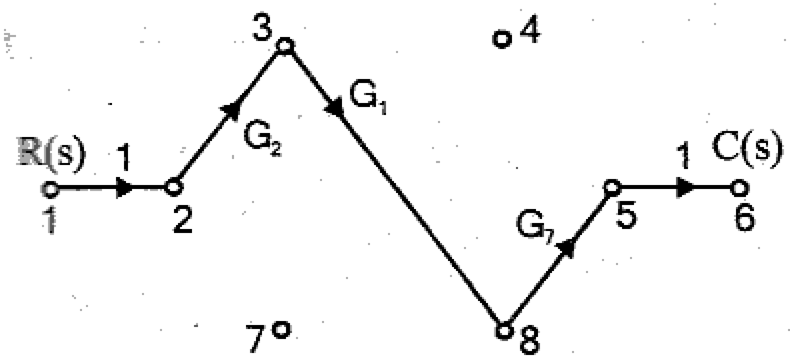


Fig 5 : Forward path-3

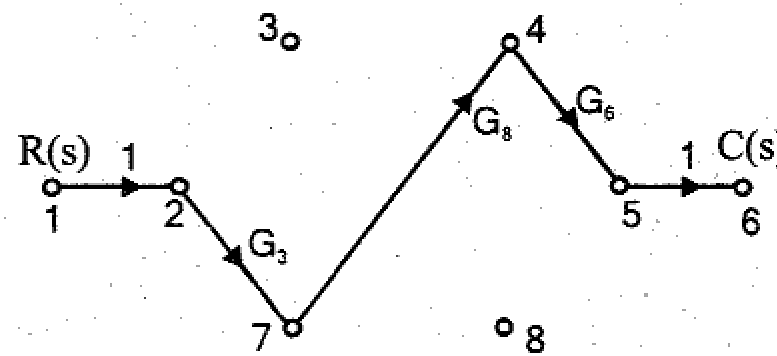


Fig 6 : Forward path-4

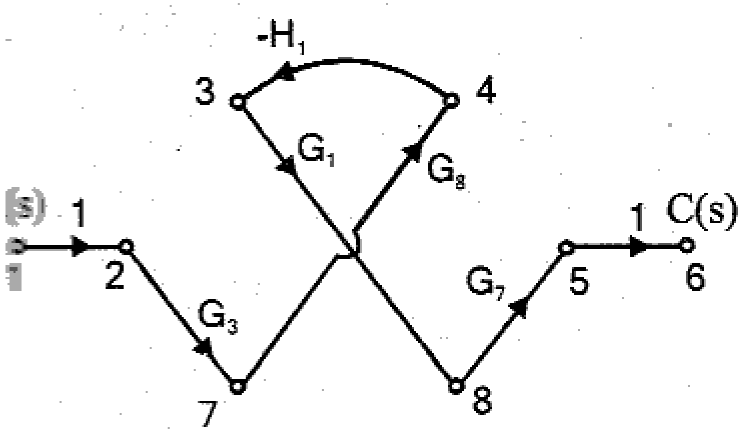


Fig 7 : Forward path-5

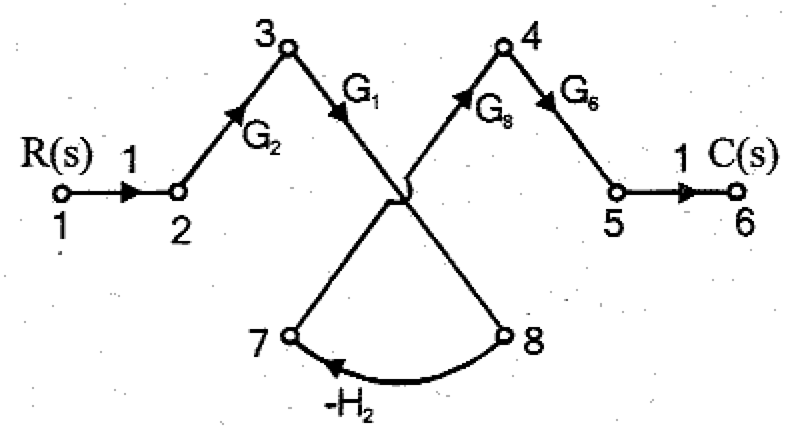


Fig 8 : Forward path-6

Gain of forward path-1, $P_1 = G_2\,G_4\,G_6$

Gain of forward path-2, $P_2 = G_3\,G_5\,G_7$

Gain of forward path-3, $P_3 = G_1\,G_2\,G_7$

Gain of forward path-4, $P_4 = G_3\,G_8\,G_6$

Gain of forward path-5, $P_5 = -G_1G_3G_7G_8H_1$

Gain of forward path-6, $P_6 = -G_1G_2G_6G_8H_2$

Individual Loop Gain

There are three individual loops.

Let individual loop gains be P_{11}, P_{21} and P_{31}.

Fig 9 : Loop-1

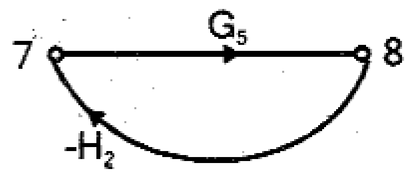


Fig 10 : Loop-2

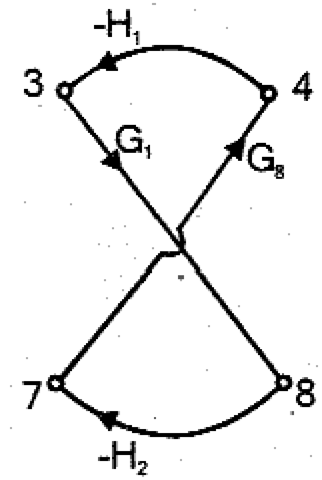


Fig 11 : Loop-3

Loop gain of individual loop-1, $P_{11} = -G_4H_1$

Loop gain of individual loop-2, $P_{21} = -G_5H_2$

Loop gain of individual loop-3, $P_{31} = G_1G_8H_1H_2$

Gain Products of Two Non-touching Loops

There is only one combination of two non-touching ccs. Let gain product of two non-touching loops be P_{12}.

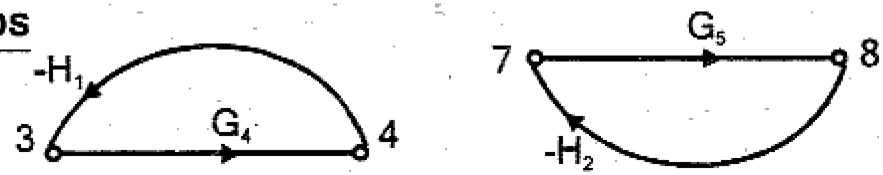


Fig 12 : Combination of 2 non-touching loops

$$\left.\begin{array}{l}\text{Gain product of first combination}\\ \text{of two non-touching loops}\end{array}\right\} P_{12} = P_{11}P_{21} = (-G_4H_1)(-G_5H_2) = G_4G_5H_1H_2$$

Calculation of Δ and Δ_K

$$\Delta = 1 - (P_{11} + P_{21} + P_{31}) + P_{12} = 1 - (-G_4H_1 - G_5H_2 + G_1G_8H_1H_2) + G_4G_5H_1H_2$$

$$= 1 + G_4H_1 + G_5H_2 - G_1G_8H_1H_2 + G_4G_5H_1H_2$$

The part of the graph non-touching forward path - 1 is shown in fig 13.

$$\therefore \Delta_1 = 1 - (-G_5H_2) = 1 + G_5H_2$$

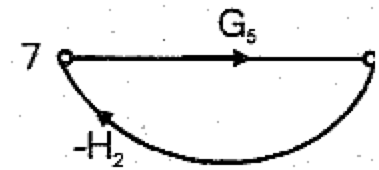


Fig 13

The part of the graph non-touching forward path -2 is shown in fig 14.

$$\therefore \Delta_2 = 1 - (-G_4H_1) = 1 + G_4H_1$$

Fig 14

There is no part of the graph which is non-touching with forward paths 3, 4, 5 and 6.

$$\therefore \Delta_3 = \Delta_4 = \Delta_5 = \Delta_6 = 1$$

V. Transfer Function, T

By Mason's gain formula the transfer function, T is given by,

$$T = \frac{1}{\Delta}\left(\sum_K P_K\,\Delta_K\right) \qquad \text{(Number of forward paths is six and so K = 6)}$$

$$= \frac{1}{\Delta}\left(P_1\Delta_1 + P_2\Delta_2 + P_3\Delta_3 + P_4\Delta_4 + P_5\Delta_5 + P_6\Delta_6\right)$$

$$= \frac{G_2G_4G_6(1+G_5H_2) + G_3G_5G_7(1+G_4H_1) + G_1G_2G_7 + G_3G_6G_8 - G_1G_3G_7G_8H_1 - G_1G_2G_6G_8H_2}{1 + G_4H_1 + G_5H_2 - G_1G_8H_1H_2 + G_4G_5H_1H_2}$$

6) Convert the given block diagram to signal flow graph and determine C(s)/R(s).

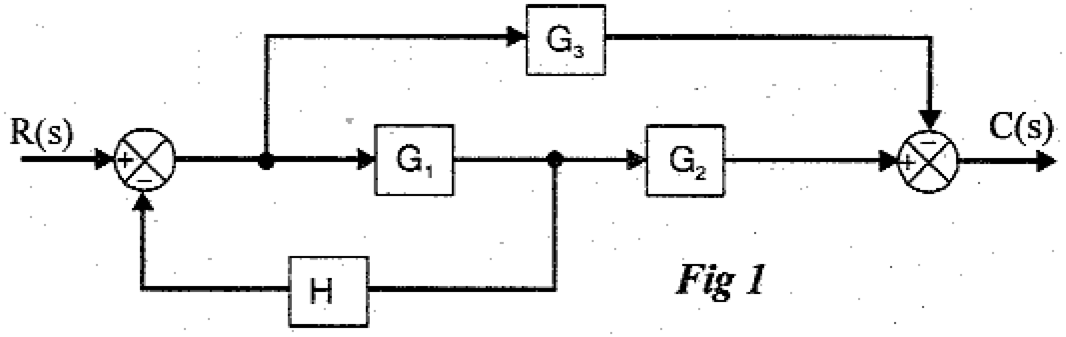


Fig 1

SOLUTION

The nodes are assigned at input, output, at every summing point & branch point as shown in fig 2.

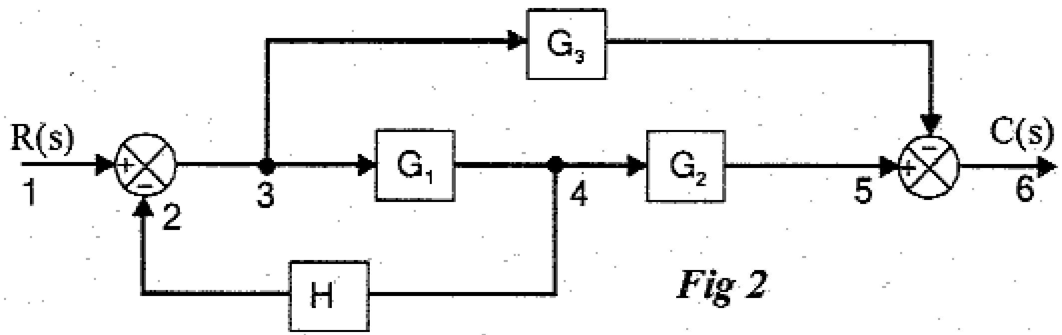


Fig 2

The signal flow graph of the above system is shown in fig 3.

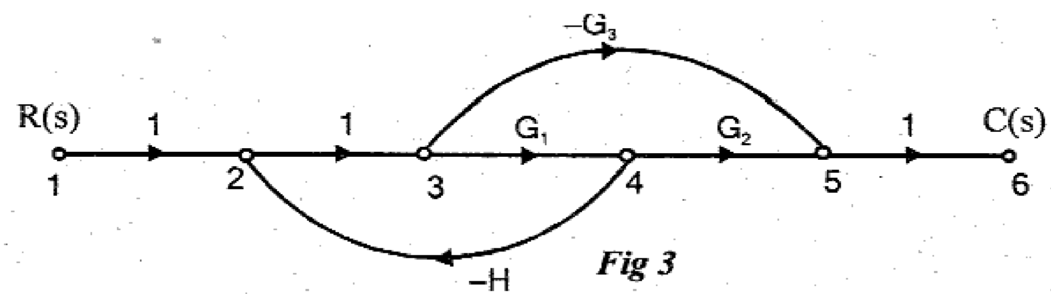


Fig 3

I. Forward Path Gains

There are two forward paths. $\therefore$ K=2

Let the forward path gains be P_1 and P_2.

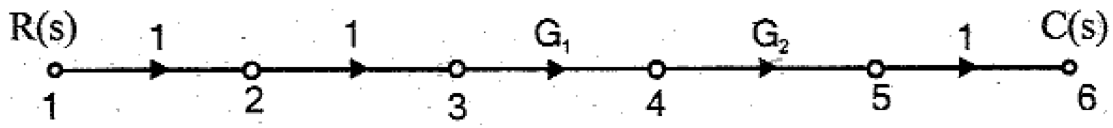


Fig 4 : Forward path-1

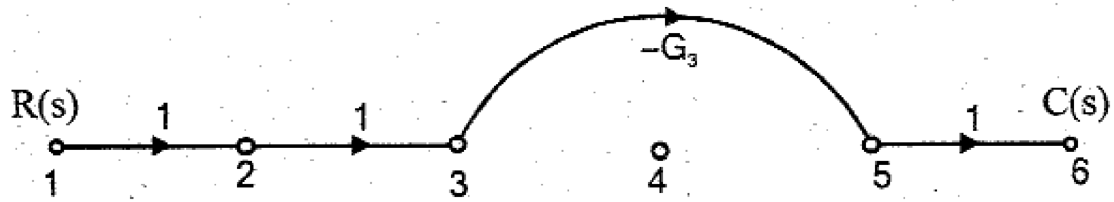


Fig 5 : Forward path-2

Gain of forward path-1, $P_1 = G_1G_2$

Gain of forward path-2, $P_2 = -G_3$

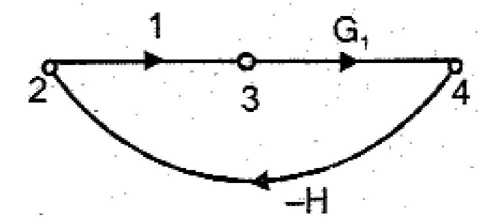


Fig 3 : loop-1

Individual Loop Gain

There is only one individual loop. Let the individual loop gain be P_{11}.

Loop gain of individual loop-I, $P_{11} = -G_1H$.

Gain Products of Two Non-touching Loops

There are no combinations of non-touching Loops.

Calculation of Δ and Δ_K

$$\Delta = 1 - [P_{11}] = 1 + G_1H$$

Since there are no part of the graph which is non-touching with forward path-1 and 2,

$$\Delta_1 = \Delta_2 = 1$$

Transfer Function, T

By Mason's gain formula the transfer function, T is given by,

$$T = \frac{1}{\Delta}\sum_K P_K\Delta_K = \frac{1}{\Delta}[P_1\Delta_1 + P_2\Delta_2] = \frac{G_1G_2 - G_3}{1 + G_1H}$$

7)

Convert the block diagram to signal flow graph and determine the transfer function using Mason's gain formula.

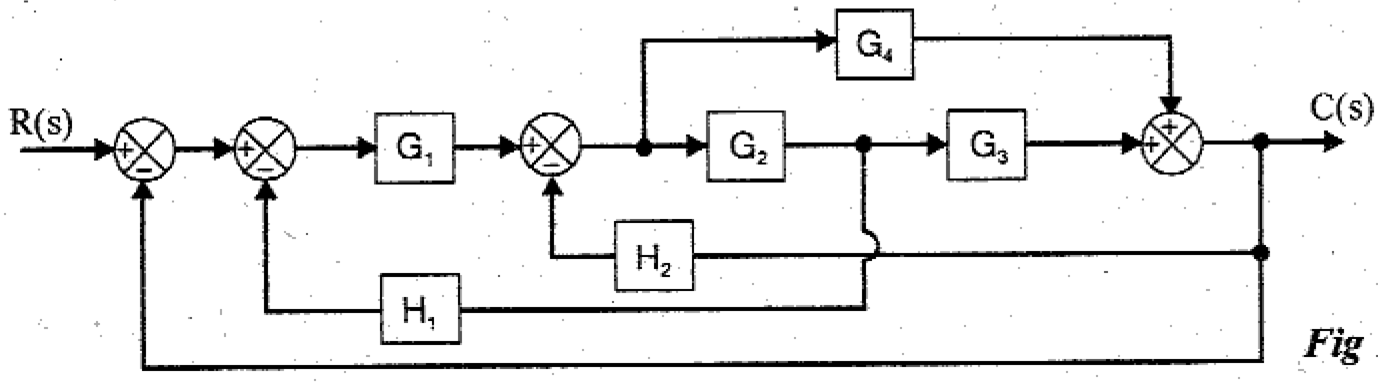


Fig 1

LUTION

The nodes are assigned at input, ouput, at every summing point & branch point as shown in fig 2.

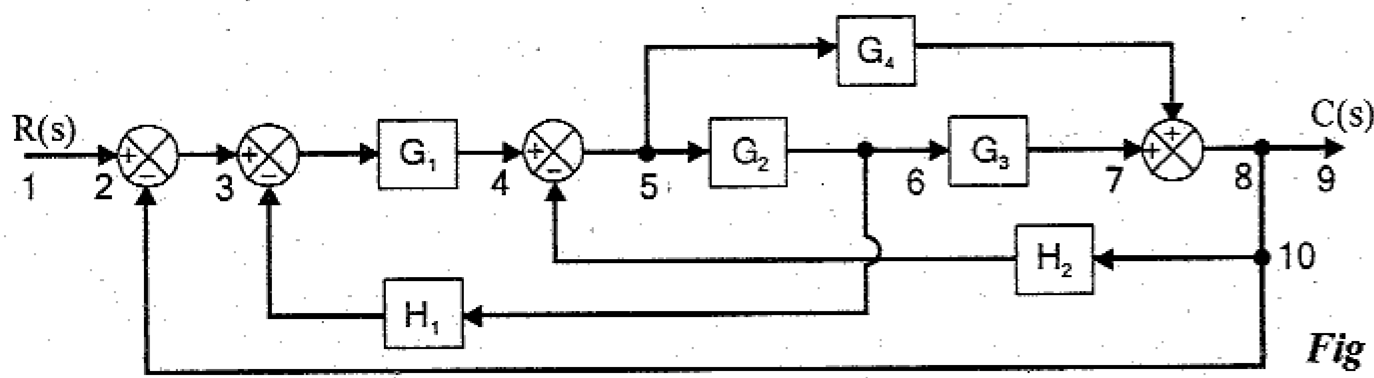


Fig 2

The signal flow graph for the above block diagram is shown in fig 3.

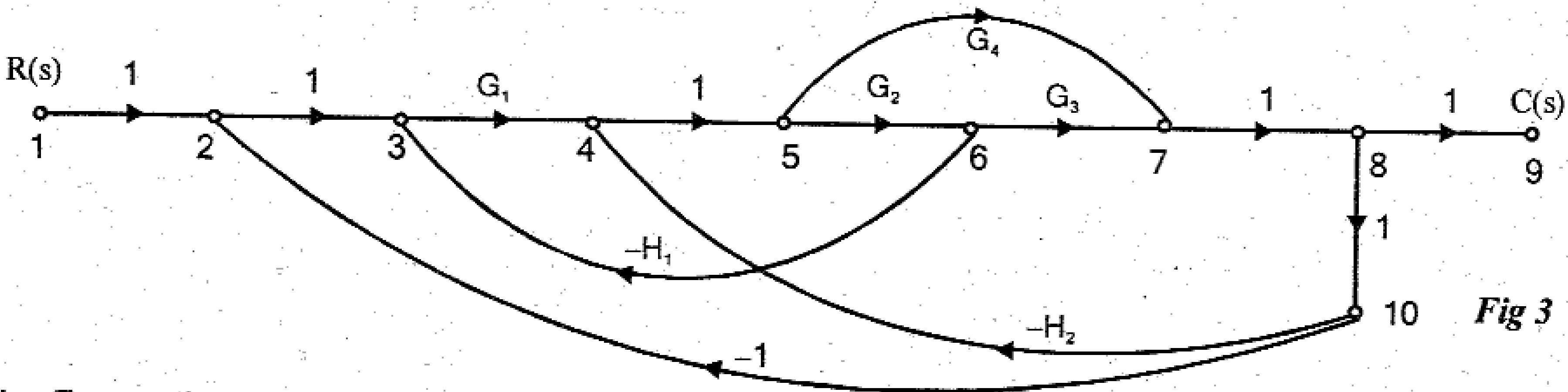


Fig 3

I. Forward Path Gains

There are two forward paths. ∴ K=2.

Let the gain of the forward paths be P_1 and P_2.

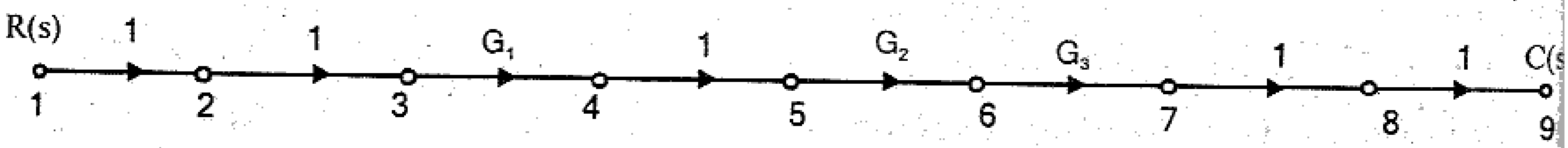


***Fig 4** : Forward path-1*

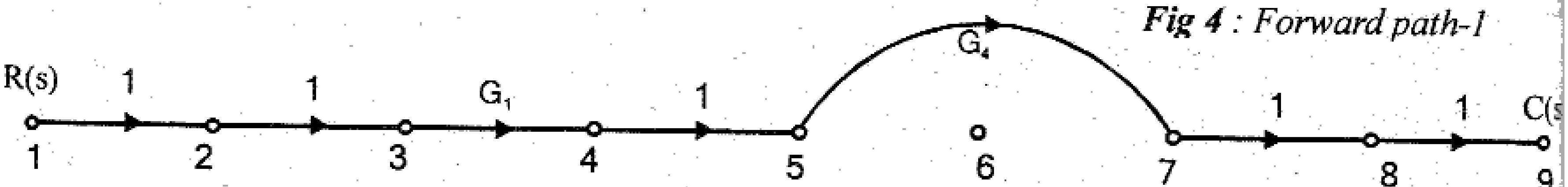


***Fig 5** : Forward path-2*

Gain of forward path-1, $P_1 = G_1G_2G_3$

Gain of forward path-2, $P_2 = G_1G_4$

II. Individual Loop Gain

There are five individual loops. Let the individual loop gain be P_{11}, P_{21}, P_{31}, P_{41} and P_{51}.

Loop gain of individual loop-1, $P_{11} = -G_1G_2G_3$

Loop gain of individual loop-2, $P_{21} = -G_2G_1H_1$

Loop gain of individual loop-3, $P_{31} = -G_2G_3H_2$

Loop gain of individual loop-4, $P_{41} = -G_1G_4$

Loop gain of individual loop-5, $P_{51} = -G_4H_2$

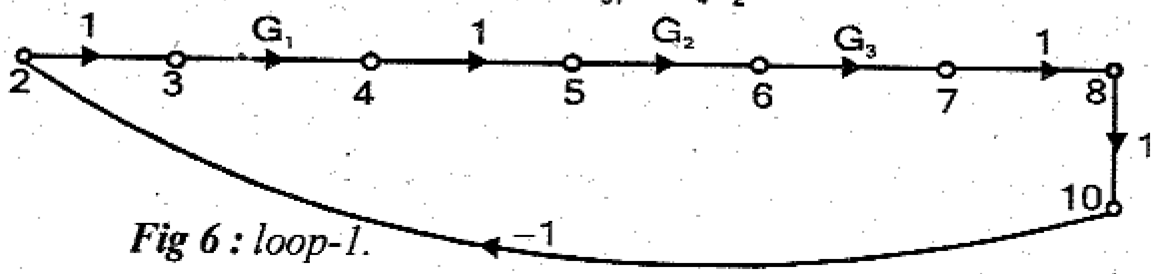


Fig 6 : loop-1.

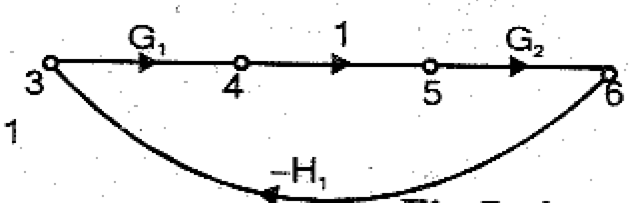


Fig 7 : loop-2.

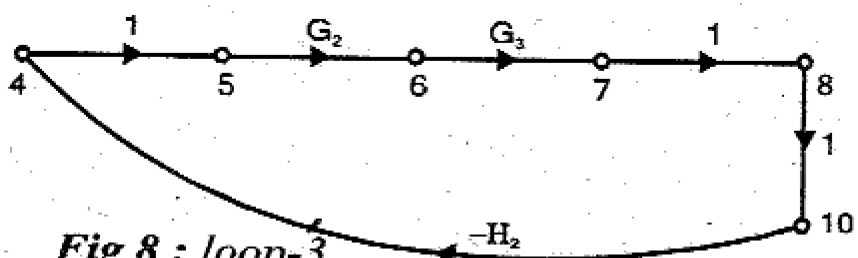


Fig 8 : loop-3.

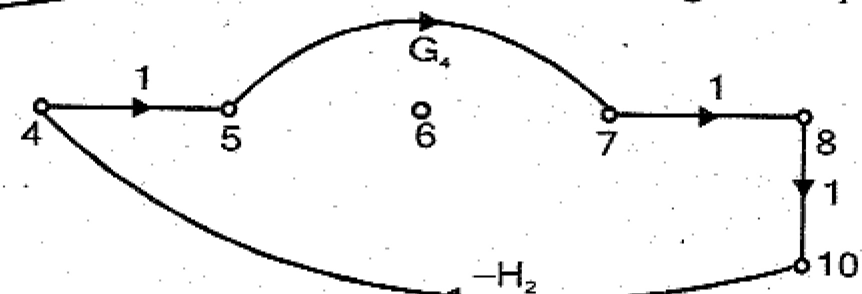


Fig 10 : loop-5.

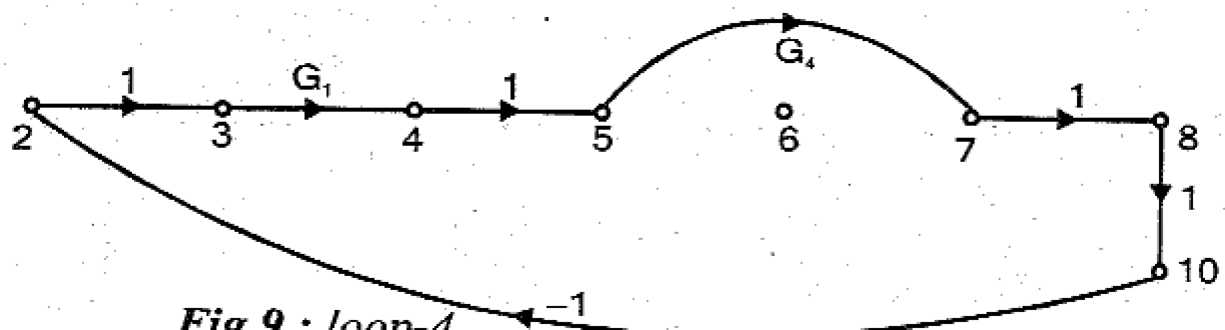


Fig 9 : loop-4.

Gain Products of Two Non-touching Loops

There are no possible combinations of two non-touching loops, three non-touching loops, etc.,.

Calculation of Δ and Δ_K

$$\Delta = 1-[P_{11}+P_{21}+P_{31}+P_{41}+P_{51}] = 1+G_1G_2G_3+G_1G_2H_1+G_2G_3H_2+G_1G_4+G_4H_2$$

Since no part of graph is non touching with forward paths-I and 2, $\Delta_1 = \Delta_2 = 1$.

Transfer Function, T

By Mason's gain formula the transfer function, T is given by,

$$T = \frac{1}{\Delta}\sum_k P_k\Delta_k = \frac{1}{\Delta}[P_1\Delta_1 + P_2\Delta_2]$$

$$= \frac{G_1G_2G_3 + G_1G_4}{1+G_1G_2G_3 + G_1G_2H_1 + G_2G_3H_2 + G_1G_4 + G_4H_2}$$