UNIT – I

Concept of Electric Circuits: Introduction to circuit elements, V-I relationships of R, L and C elements, Ideal and Practical Sources, Kirchhoff's laws, Source Transformation, Network reduction techniques-series, parallel, series-parallel and Star-Delta transformation.

What is an electrical circuit?

An **electric circuit** is a path in which electrons from a voltage or current source flow. The point where those electrons enter an **electrical circuit** is called the "source" of electrons.

What do you mean by electrical network?

An **electrical network** is an interconnection of **electrical** components (e.g. batteries, resistors, inductors, capacitors, switches) or a model of such an interconnection, consisting of **electrical** elements (e.g. voltage sources, current sources, resistances, inductances, capacitances).

How do electrons flow around a circuit?

Current only **flows** when a **circuit** is complete—when there are no gaps in it. In a complete **circuit**, the electrons **flow** from the negative terminal (connection) on the power source, through the connecting wires and components, such as bulbs, and back to the positive terminal.

Current:

An electric current is a flow of electric charge (electrons).

Unit of Electric current is **Ampere**

Electric current is measured using a device called an **ammeter.**

Voltage:

Voltage is the electromotive force or the electrical potential (Charge) difference between two points in a circuit.

Unit of Voltage is **volt**

Voltage is measured using a device called **voltmeter.**

Types of network Elements:

The circuit elements are classified into following categories,

1. Passive and active elements.
2. Unilateral and Bilateral elements.
3. Linear and Non-Linear elements.

Passive and Active Elements

Passive Element: The elements that absorbs or stores energy is called passive element.

` Examples: Resistor (R), Capacitor (C), Inductor (L), Transformer

Active Element: The elements that supply energy to the circuit is called active element.

Examples: Voltage and Current sources, Generators, Transistor.

Unilateral and Bilateral Element

Bilateral Element: Conduction of current in both directions in an element with same magnitude is termed as bilateral element.

Examples: Resistance; Inductance; Capacitance

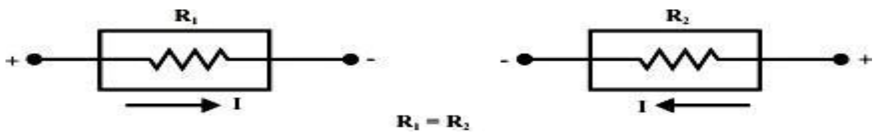


Fig 1.1

Unilateral Element: Conduction of current in one direction in an element is termed as unilateral element

Examples: Diode, Transistor.

Fig 1.2

Linear and Non Linear Elements

Linear Element: The elements that obeys ohm's law and homogeneity principle is called linear element.

Examples: Resistor (R), Capacitor (C), Inductor (L)

Fig 1.3

Non-Linear Element: The elements that does not obey ohm‟s law and homogeneity principle is called Non-Linear element.

Examples: Semiconductors, Diode, Transistor

Types of Sources:

Independent Sources:

Independent sources are those in which generated voltage (V_S) or the generated current (I_S) are not affected by the load connected across the source terminals or across any other element that exists elsewhere in the circuit or external to the source.

Ideal and Practical Voltage Sources:

- An ideal voltage source, which is represented by a model in below fig, is a device that produces a constant voltage across its terminals no matter what current is drawn from it (terminal voltage is independent of load (resistance) connected across the terminals)
- The V- I characteristic of ideal voltage source is a straight line parallel to the x-axis.

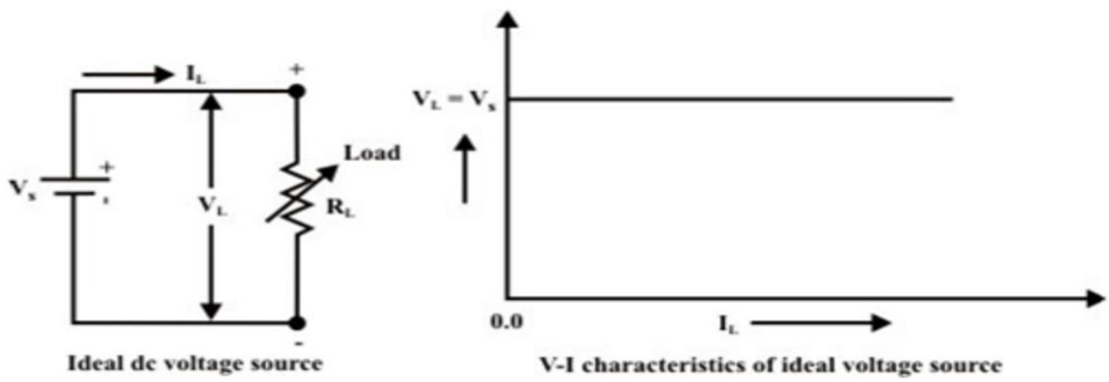


Fig 1.4

- Internal resistance (R_S) of a ideal voltage source is zero.

$$R_S = 0$$

- A practical voltage source, which is represented by a model in below fig, is a device that does not produces a constant voltage.

- The V- I characteristic of a practical voltage source can be described by the following equation

$$V_L = V_S – IR_S$$

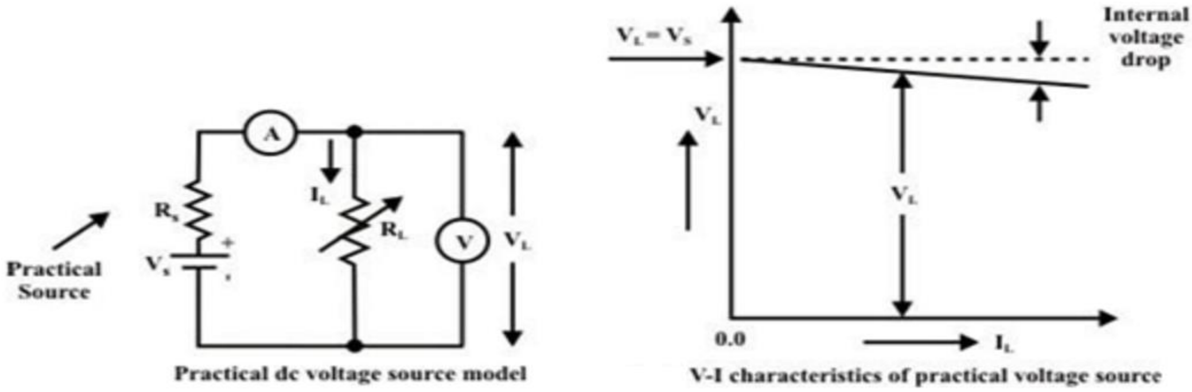


Fig 1.5

<u>Ideal and Practical Current Sources</u>:

- An ideal current source, which is represented by a model in fig is a device that delivers a constant current to any load resistance connected across it, no matter what the terminal voltage is developed across the load (i.e., independent of the voltage across its terminals across the terminals).
- The V- I characteristic of ideal current source is a straight line parallel to the y-axis.

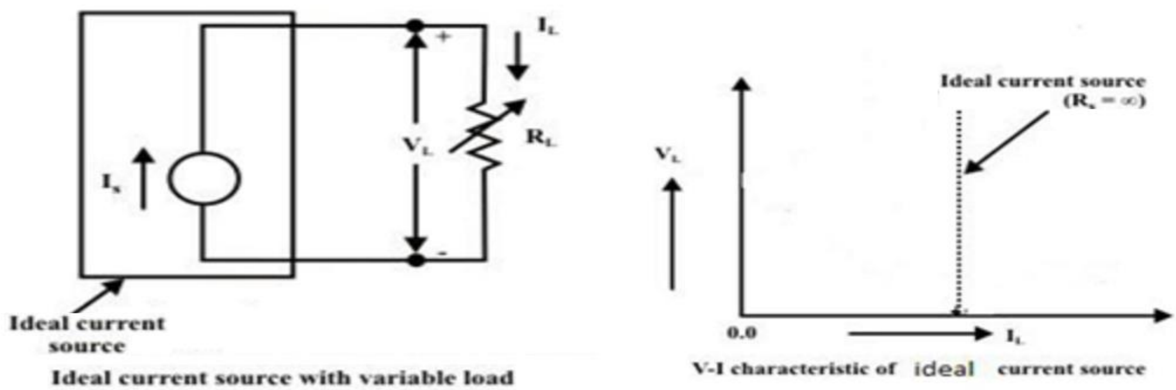


Fig 1.6

- Internal resistance (RS) of a ideal Current source is Infinity.

$$R_S = \infty$$

- A practical voltage source, which is represented by a model in below fig, is a device that does not produces a constant current.

- The V- I characteristic of a practical current source can be described by the following equation

$$I_L = I_S – \frac{V_L}{R_S}$$

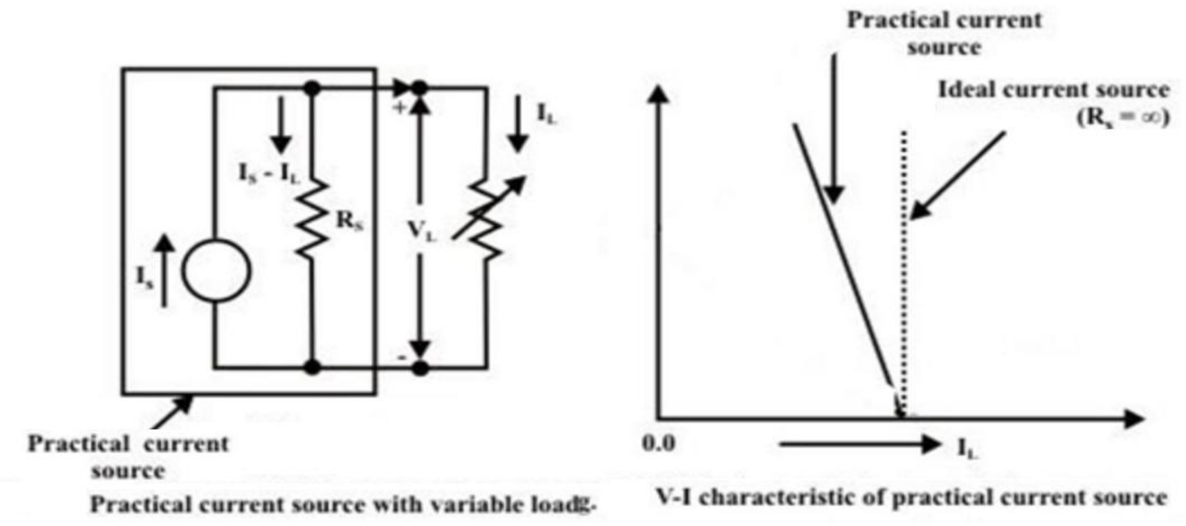


Fig 1.7

DEPENDENT OR CONTROLLED SOURCES:

In some network, in which some of the voltage sources or current sources are controlled by changing of current or voltage elsewhere in the circuit. Such sources are termed as "Dependent or Controlled sources".

There are four types of dependent sources.

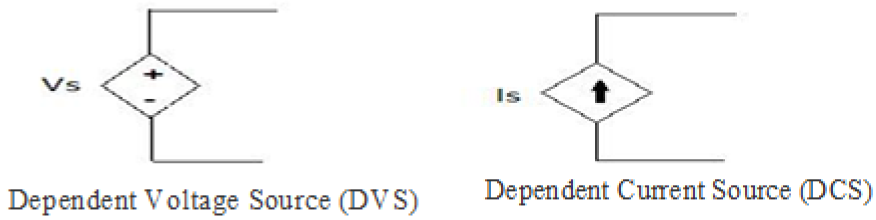


Fig 1.8

- CDVS(Current DVS)
- CDCS(Current DCS)
- VDVS(Voltage DVS)
- VDCS(Voltage DCS)

Resistor:

Resistance (R): The opposition offered to the flow of electric current flowing through the material is called Resistance.

Unit: Ohm(Ω)

Laws of Resistance:

Electrical resistance (R) of a conductor is

1. directly proportional to its length, l i.e. $R \propto l$,
2. inversely proportional to its area of cross-section, a i.e.

$$R \propto \frac{1}{a}$$

Combining these two laws we get,

$$R \propto \frac{l}{a} \Rightarrow R = \rho\frac{l}{a}$$

Where ρ is a constant depending on the nature of the material of the conductor and is known as it's Specific Resistance or Resistivity.

Specific Resistance or Resistivity:

Specific Resistance or Resistivity is the resistance of a material with unit length and unit cross sectional area.

Unit:

The **unit of resistivity** can be easily determined form its equation

$$R = \rho\frac{l}{a} \Rightarrow \rho = \frac{Ra}{l} \quad \frac{\Omega - m^2}{m} \rightarrow \Omega - m$$

Conductance:

It is the inverse of resistance. $G = \frac{1}{R}$

Unit:

$G = \frac{1}{\Omega} = mho$ (or) siemen

Conductivity:

It is the inverse of resistivity. $\sigma = \frac{1}{\rho}$

Unit:

$\sigma = \frac{1}{\Omega - m} = mho/m$ (or) Siemen/m

Factors affecting the Resistance:

1. Length of the material:

The Resistance "R" is directly proportional with its length: **"L"**

$$\mathbf{R \propto l}$$

As length of the wire increases, resistance also increases.

2. Cross Sectional Area of the material:

The Resistance "R" is inversely proportional with its Cross Sectional Area: **"A"**

$$R \propto \frac{1}{a}$$

As Cross Sectional Area of the wire increases, resistance also decreases.

3. Nature of the material:

The Resistance "R" is dependent on the Nature of the material.

- In Conductors, No of free electrons are very high so resistance of the conductor is very less.
- In Insulators and Semi conductors, No of free electrons are less so resistance of the conductor is very high.

4. Temperature of the conductor:

The Resistance "R" is dependent on the Temperature of the conductor.

$\mathbf{R_2 = R_1 (1 + \alpha * \Delta T)}$

Where 'α' is the temperature coefficient of resistance

- For Conductors **'α' = +ve, as** temperature increases, resistance also increases.
- For Insulators and Semi conductors **'α' = -ve, as** temperature increases, resistance decreases.

Ohm's law:

At a constant temperature the voltage across a conducting material is directly proportional to the current flowing through it.

According to definition

$V \propto I$

$\mathbf{V = IR}$

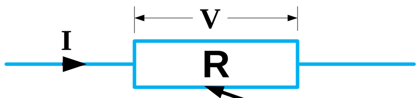
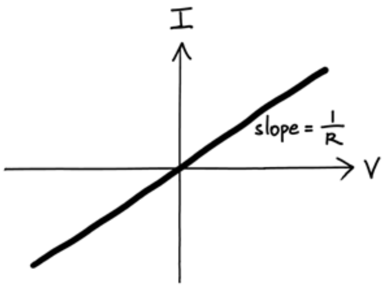


Fig 1.9

Where, V = Voltage across the conductor in volts
I = Current flowing through the conductor in Ampere
R = Proportionality constant (resistance in ohms)

I-V Characteristics of Resister



I-V Characteristic of Resistor

Fig 1.10

Power consumed by Resister, $P=VI=V^2/R=I^2R$
Energy consumed by Resister, $W=VIt = V^2t/R=I^2Rt$

Inductor

The property of the coil of inducing emf due to the changing flux linked with it is known as **inductance of the coil**. Due to this property all electrical coil can be referred as **inductor**.

In other way, an inductor can be defined as an energy storage device which stores energy in form of magnetic field.

Whenever a time-changing current is passed through a coil or wire, the voltage across it is proportional to the rate of change of current through the coil. This proportional relationship may be expressed by the equation is

$$v = L\frac{di}{dt}$$

Where L is the constant of proportionality known as inductance and is measured in Henrys (H).
Remember v and i are both functions of time.

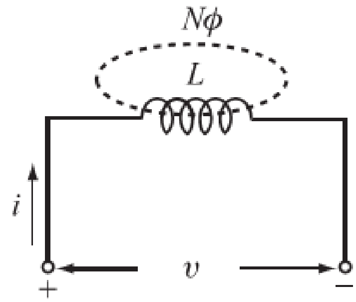


Figure 1.11 Model of the inductor

Let us assume that the coil shown in Fig. 1.6 has N turns and the core material has a high permeability so that the magnetic flux Φis connected within the area A. The changing flux creates an induced voltage in each turn equal to the derivative of the flux Φ, so the total voltage v across N turns is

$$v = N\frac{d\phi}{dt} \quad \text{------}\rightarrow 1$$

Since the total flux NΦis proportional to current in the coil,
We have NΦ = Li -------→ 2
Where L is the constant of proportionality. Substituting equation (2) into equation (1), we get

$$v = L\frac{di}{dt}$$

The power in an inductor is

$$p = vi = L\left(\frac{di}{dt}\right)i$$

The energy stored in the inductor is

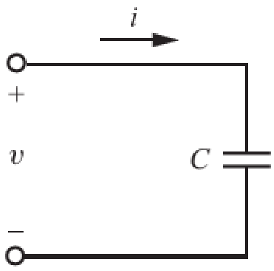
$$w = \int_{-\infty}^{t} p\ d\tau$$

$$= L\int_{i(-\infty)}^{i(t)} i\ di = \frac{1}{2}Li^2 \text{ Joules}$$

Note that when $t = -\infty. i(-\infty) = 0$. Also note that w(t)>=0 for all i(t), so the inductor is a passive element. The inductor does not generate energy, but only stores energy.

Capacitor

A capacitor is a two-terminal element that is a model of a device consisting of two conducting plates separated by a dielectric material. Capacitance is a measure of the ability of a device to store energy in the form of an electric field.



1.12 Circuit symbol for a capacitor

Capacitance is defined as the ratio of the charge stored to the voltage difference between the two conducting plates or wires

$$C = \frac{q}{v}$$

The current through the capacitor is given by

$$i = \frac{dq}{dt} = C\frac{dv}{dt}$$

The energy stored in a capacitor is

$$w = \int_{-\infty}^{t} vi \; d\tau$$

Remember that v and i are both functions of time and could be written as v(t) and i(t).
Since

$$i = C\frac{dv}{dt}$$

We have

$$w = \int_{-\infty}^{t} v \; C\frac{dv}{d\tau} \; d\tau$$

$$= C \int_{v(-\infty)}^{v(t)} v \; dv = \frac{1}{2}Cv^2 \Big|_{v(-\infty)}^{v(t)}$$

Since the capacitor was uncharged at $t = -\infty, \; v(-\infty) = 0.$
Hence

$$w = w(t)$$

$$= \frac{1}{2}Cv^2(t) \text{ Joules}$$

Since q =Cv we may write

$$w(t) = \frac{1}{2C}q^2(t) \text{ Joules}$$

Note that since w(t) >= 0 for all values of v(t), the element is said to be a passive element.

Kirchhoff Laws:

Gustav Kirchhoff (1824-1887), an eminent Germen physicist did a considerable amount of work on the principle of governing behavior of electric circuits. He gave his finding in a set of two laws which together called Kirchhoff's laws. These two laws are

1. Kirchhoff's Current Law (KCL)
2. Kirchhoff's Voltage Law (KVL)

<u>Kirchhoff's Current Law (KCL)</u>

Kirchhoff's Current Law states that the algebraic sum of the current meeting at a node (junction) is equal to zero

i.e., $\sum I = 0$

This law is illustrated below. Five branches are connected to node O which carries currents I_1, I_2, I_3, I_4 and I_5 as shown in figure 1.13. Consider current entering (I_1, I_3 & I_5) to the node as positive and current leaving (I_2 & I_4) from the node as negative.

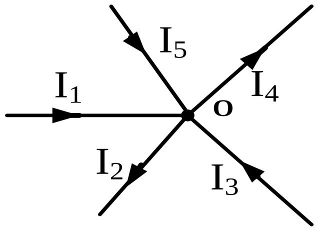


Fig. 1.13 Five branches are connected to node o

From above diagram

$$-I_1 - I_2 + I_3 + I_4 + I_5 = 0$$

or

$$I_1 + I_2 = I_3 + I_4 + I_5$$

i. e., Incoming currents = Outgoing currents

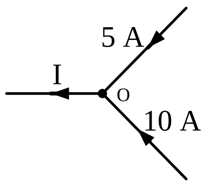
Hence Kirchhoff's first law can be stated as:

The currents flowing towards any junction in an electric circuit is equal to the sum of the currents flowing away from the junction

<u>Examples:</u>

Find out the value of unknown current I from the given networks

Example 1



Where incoming currents are 5A and 10A, outgoing current is I. By applying KCL at node o

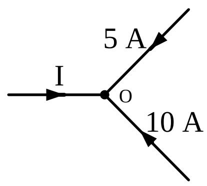
$$I - 5 - 10 = 0$$

or

$$I = 15\ A$$

Unknown current I = 15A

Example 2



Where incoming currents are I, 10A and 5A

By applying KCL at node o

$$-I - 5 - 10 = 0 \times$$

or

$$I = -15\ A$$

Unknown current I = -15A

This minus sign indicates that the actual current direction is opposite.

<u>Kirchhoff's Voltage Law (KVL)</u>

The algebraic sum of the all branch voltages in a loop (or closed path) is equal to zero

or

Kirchhoff's Voltage Law ***states that in a closed circuit, the algebraic sum of all source voltages must be equal to the algebraic sum of all the voltage drops.***

Steps to follow

Step. 1. Mark all the nodes

Step. 2. Mark all branch currents

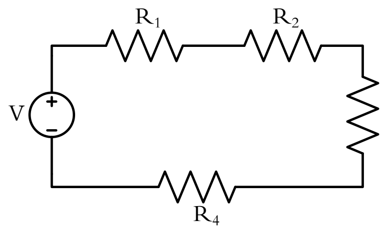
Step. 3. Mark voltage drop across each resistor (mark current entering point as positive and current leaving point as negative).

Step. 4. Depend up on the number of unknowns write KVL equations (At the time of writing equations consider the sign which see first for the voltage drops and voltage sources)

Step. 5. By solving this equations calculate the unknown branch currents and determine the desired responses.

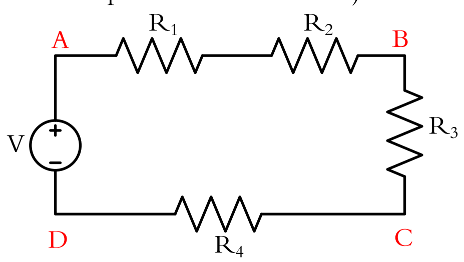
Illustration of Kirchhoff's law

Example: Apply KVL and determine current flowing through each element in the circuit shown below.

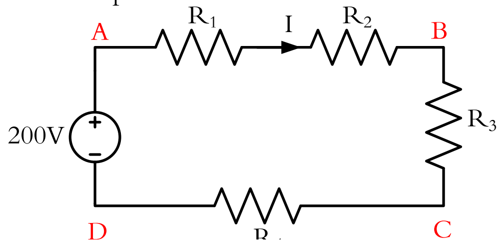


Solution,

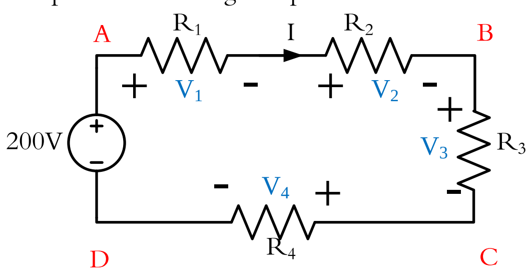
Step 1– Mark all the nodes)



Step 2 – Mark all the branch currents



Step 3 – Mark voltage drop across each resistor



Step 4 – Write KVL equations

From loop – ABCDA

$$V_1 + V_2 + V_3 + V_4 - V = 0 \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots .(1)$$

From ohm's law

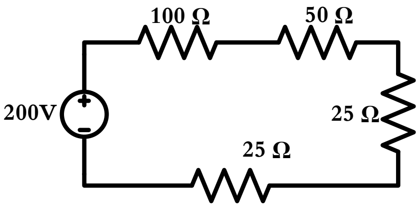
$V_1 = I \times R_1$, $V_2 = I \times R_2$, $V_3 = I \times R_3$ & $V_4 = I \times R_4$, Substitute these values in to equation 1.

$$I \times R_1, + I \times R_2 + I \times R_3 + I \times R_4 - V = 0$$

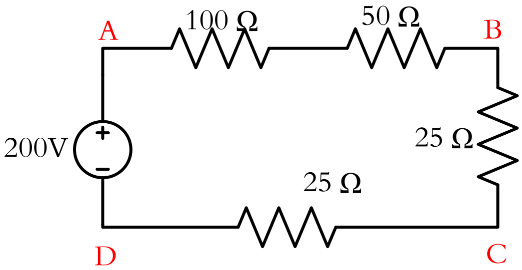
$$I \times (R_1, + R_2 + R_3 + R_4) = V$$

V

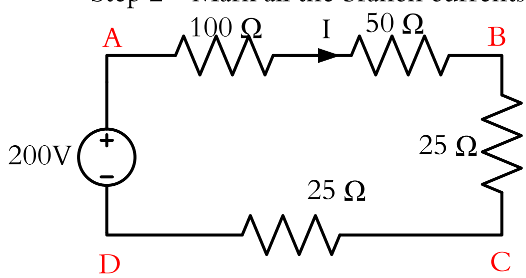
Example:Find out the value of unknown current I from the given networks



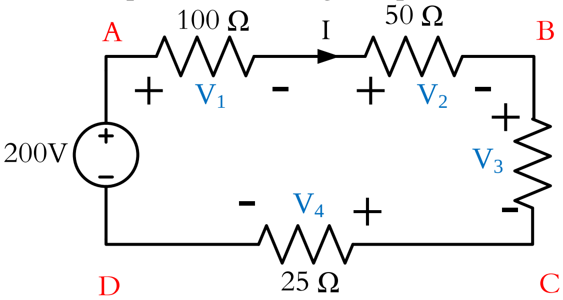
Step 1– Mark all the nodes)



Step 2 – Mark all the branch currents



Step 3 – Mark voltage drop across each resisto:



Step 4 – Write KVL equations

From loop – ABCDA

$V_1 + V_2 + V_3 + V_4 - V = 0 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots.(1)$

From ohm's law

$V_1 = I \times 100,\ V_2 = I \times 50,\ V_3 = I \times 25\ \&\ \ V_4 = I \times 25,$

Substitute these values in to equation 1.

$I \times 100 + I \times 50 + I \times 25 + I \times 25, -200 = 0$

$I \times (100 + 50 + 25 + 25_1) = 200$

$I = \frac{200}{200} = 1A$

Source Transformation

• Not possible to transform ideal current (voltage) sources to ideal voltage (current) sources.

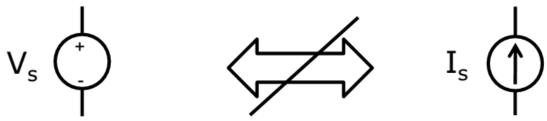


Fig 1.14

• But we can transform Practical current (voltage) sources to Practical voltage (current) sources.

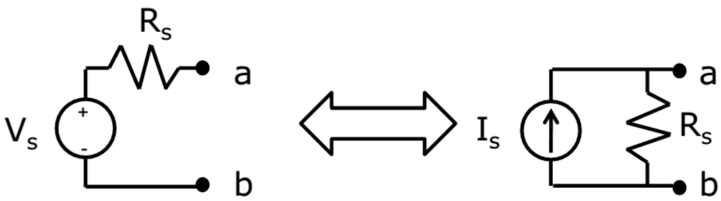


Fig 1.15

Relationships:

$I_S = V_S/R$ **$V_S = I_S.R$**

Resistors in Series

Consider the series combination of N resistors shown in Fig. 1.16 a.We want to simplify the circuit with replacing the N resistors with a single resistor Req so that the remainder of the circuit, in this case only the voltage source, does not realize that any change has been made. The current, voltage, and power of the source must be the same before and after the replacement.

First, apply KVL:

$$\mathbf{v_S = v_1 + v_2 + \cdots + v_N}$$

and then Ohm's law:

$$\mathbf{v_S = R_1 i + R_2 i + \cdots + R_N i = (R_1 + R_2 + \cdots + R_N) i}$$

Now compare this result with the simple equation applying to the equivalent circuit shown in Fig. 1.16 b:

$$\mathbf{v_S = R_{eq} i}$$

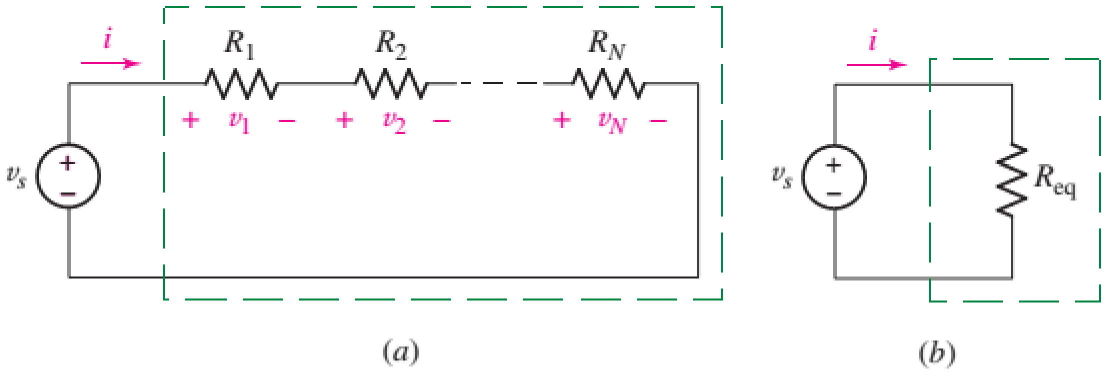


Fig 1.16

Thus, the value of the equivalent resistance for N series resistors is

$$\mathbf{R_{eq} = R_1 + R_2 + \cdots + R_N}$$

Voltage Division

Voltage division is used to express the voltage across one of several series resistors in terms of the voltage across the combination. In Fig. 1.17, the voltage across R2 is found via KVL and Ohm's law:

$$\mathbf{v = v_1 + v_2 = iR_1 + iR_2 = i(R_1 + R_2)}$$

So

$$i = \frac{v}{R_1 + R_2}$$

Thus

$$v_2 = iR_2 = \left(\frac{v}{R_1 + R_2}\right) R_2$$

Or

$$v_2 = \frac{R_2}{R_1 + R_2} v$$

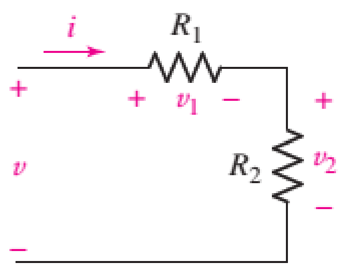


Fig 1.17

and the voltage across R1 is, similarly,

$$v_1 = \frac{R_1}{R_1 + R_2} v$$

If the network of Fig.1.17 is generalized by removing R_2 and replacing it with the series combination of R_2, R_3............RN, then we have the general result for voltage division across a string of N series resistors

$$\boxed{v_k = \frac{R_k}{R_1 + R_2 + \cdots + R_N} v}$$

which allows us to compute the voltage $\mathbf{v_k}$ that appears across an arbitrary resistor R_k of the series.

Resistors in Parallel

Similar simplifications can be applied to parallel circuits. A circuit containing N resistors in parallel, as in Fig. 1.18 a, leads to the KCL equation

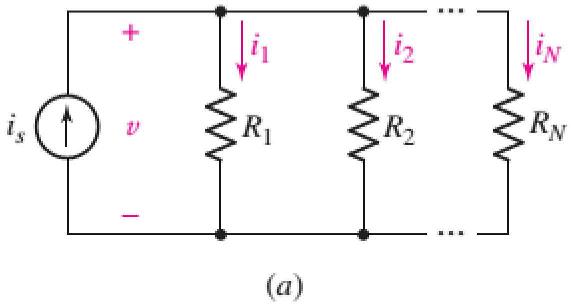
$\mathbf{i_S = i_1 + i_2 + \cdots + i_N}$

$$i_s = \frac{v}{R_1} + \frac{v}{R_2} + \cdots + \frac{v}{R_N}$$
$$= \frac{v}{R_{eq}}$$

or, in terms of conductances, as

$\mathbf{G_{eq} = G_1 + G_2 + \cdots + G_N}$

The simplified (equivalent) circuit is shown in Fig. 1.18 b.



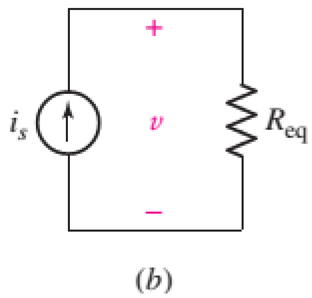


Fig 1.18

Current Division

The dual of voltage division is current division. We are now given a total current supplied to several parallel resistors, as shown in the circuit of Fig. 1.19.

The current flowing through R2 is

$$i_2 = \frac{v}{R_2} = \frac{i(R_1 \| R_2)}{R_2} = \frac{i}{R_2} \frac{R_1 R_2}{R_1 + R_2}$$

Or

$$\boxed{i_2 = i \frac{R_1}{R_1 + R_2}}$$

and, similarly,

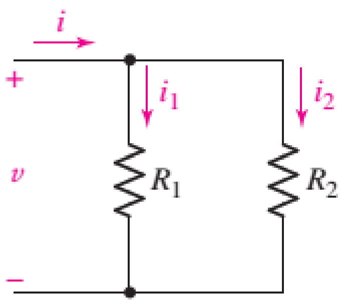


Fig 1.19

$$i_1 = i \frac{R_2}{R_1 + R_2}$$

For a parallel combination of N resistors, the current through resistor R_k is

$$i_k = i \frac{\frac{1}{R_k}}{\frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_N}}$$

Delta-Star(Wye) Conversion

Star connection and delta connection are the two different methods of connecting three basic elements which cannot be further simplified into series or parallel.

The two ways of representation can have equivalent circuits in either form.

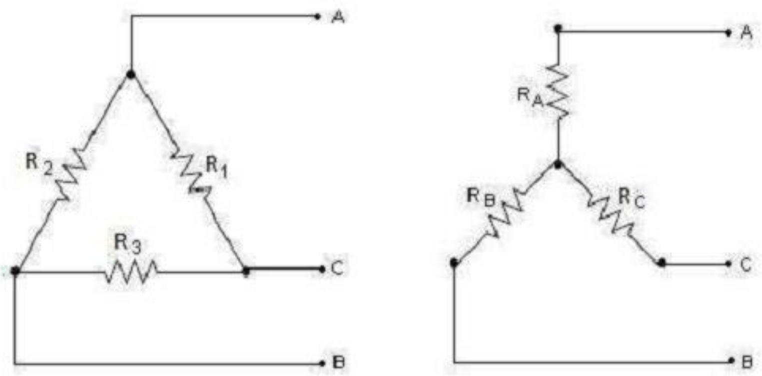


Fig 1.20

Assume some voltage source across the terminals AB.

$$R_{eq} = R_a + R_b$$

$$R_{eq} = R_1(R_2 + R_3)/(R_1 + R_2 + R_3)$$

Therefore $R_a + R_b = R_1(R_2 + R_3)/(R_1 + R_2 + R_3)$…………(1)

Similarly $R_b + R_c = R_3(R_1 + R_2)/(R_1 + R_2 + R_3)$…………(2)

$R_c + R_a = R_2(R_3 + R_1)/(R_1 + R_2 + R_3)$…………..(3)

Subtracting (2) from (1) and adding to (3) ,

$R_a = R_1R_2/(R_1 + R_2 + R_3)$…………….(4)

$R_b = R_1R_3/(R_1 + R_2 + R_3)$…………….(5)

$R_c = R_2R_3/(R_1 + R_2 + R_3)$…………….(6)

A delta connection of R_1, R_2, R_3 can be replaced by an equivalent star connection with the values from equations (4),(5),(6).

Multiply (4)(5) ; (5)(6) ; (4)(6) and then adding the three we get,

` $R_aR_b + R_bR_c + R_cR_a = R_1R_2R_3/(R_1 + R_2 + R_3)$

Dividing LHS by R_a gives R_3, by R_b gives R_2, by R_c gives R_1

$$R_1 = (R_aR_b + R_bR_c + R_cR_a) / R_c$$

$$R_2 = (R_aR_b + R_bR_c + R_cR_a) / R_b$$

$$R_3 = (R_a R_b + R_b R_c + R_c R_a) / R_a$$

UNIT – II

Fundamentals of AC circuits:

R.M.S, Average valves, form factor and crest factor for different periodic wave forms, Sinusoidal Alternating Quantities - Phase and Phase Difference, Complex and Polar Forms of Representations, j-Notation. Concept of Reactance, Impedance, Susceptance and Admittance.

What is Alternating Voltage?

Alternating voltage is the voltage which constantly changes in amplitude, and which reverses direction at regular intervals.

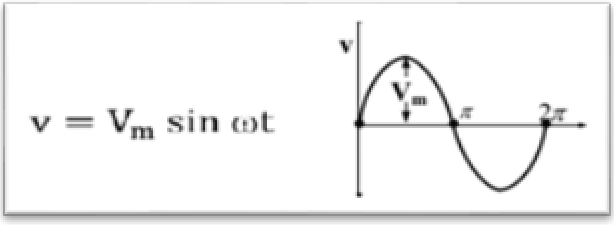


Fig 2.1

What is Alternating Current (A.C.)?

When the current flowing in the circuit varies in magnitude as well as in direction periodically is called as an alternating current..

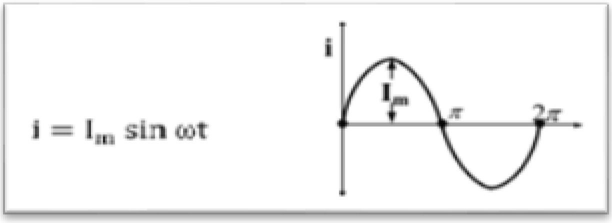


Fig 2.2

Advantages of AC system over DC system

1. AC voltages can be efficiently stepped up/down using transformer
2. AC motors are cheaper and simpler in construction than DC motors
3. Switchgear for AC system is simpler than DC system

Types of Periodic Waveform

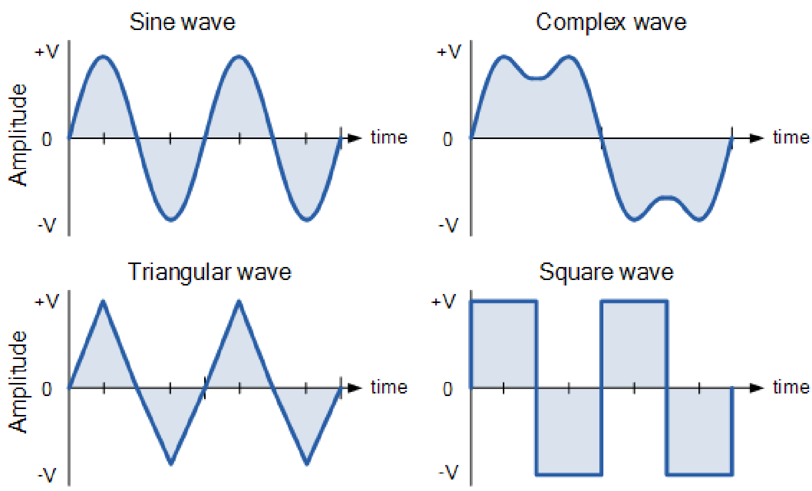


Fig 2.3

<u>Single Phase AC Generator</u>

There are two kinds of sources of electrical power:

(1) Direct current or voltage source (DC source) in which the current and voltage remains constant over time.
(2) Alternating current or voltage source (AC source) in which the current or voltage constantly changes with time. The voltage of the electrical power source that we use in our homes or offices (line voltage) is a sinusoidal signal that goes through a complete cycle 60 times in one second. In this section we will discuss how single phase AC voltage is generated.

Figure 2.4 shows a conductor placed in a magnetic field. A voltage is induced between its terminals (x, y) due to the change in flux linkage when the conductor is rotated in the magnetic field. The change in flux linkage is at its minimum when the conductor is moving parallel to the field and it is at its maximum when the conductor is moving perpendicular to the magnetic field. In a half rotation, the conductor moves from being parallel to the field to being perpendicular to the field and eventually moving back to being parallel to the field. Accordingly, the induced voltage increases from zero to its maximum value and then back to zero at the end of the half rotation.

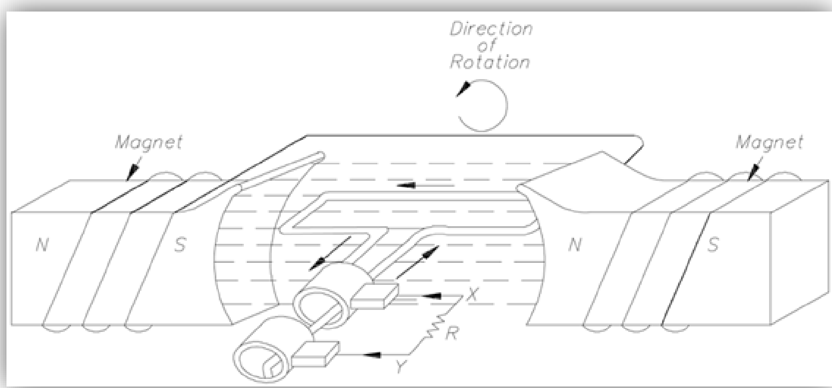


Figure 2.4: A rotating conductor in a magnetic field*.

Fig. 2.5 shows the changes in the induced voltage as the conductor rotates in the magnetic field. During the second half of the rotation the flux linkage changes through the conductor the same way as before; however, it induces the voltage with the opposite polarity because the position of the conductors is now reversed. Due to the shape of the poles and the rotary motion of the conductor the induced voltage turns out to be a sine wave.

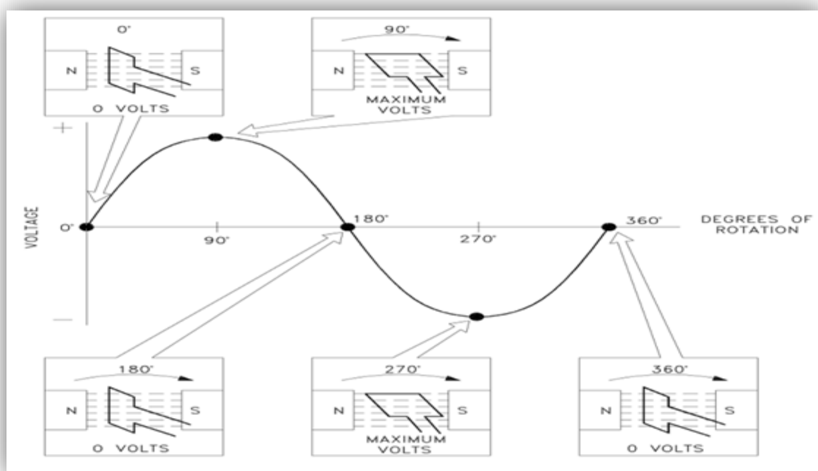
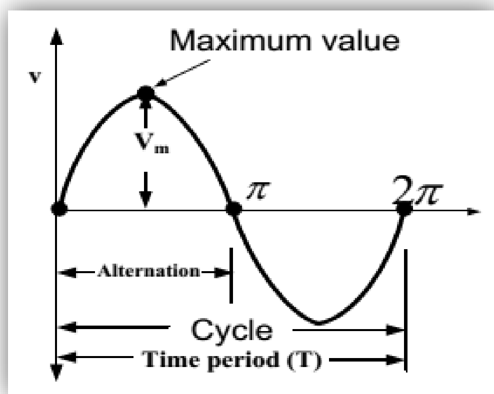


Fig 2.5: Induced voltage vs. rotation of the conductor*.

Properties of Alternating Current

Frequency (f): It is the number of cycles that occur in one second. The unit for frequency is Hz or cycles/sec.

Period (T): It is the Time Taken in seconds to complete one cycle of an alternating quantity.



Wavelength (λ): wavelength is measured in distance per cycle.

$\lambda = c/f$.

Amplitude: The amplitude of a sine wave is the value of that sine wave at its peak. This is the maximum value, positive or negative, that it can attain.

Peak-Peak value: The difference between the peak positive value and the peak negative value is called the peak-to-peak value of the sine wave. Fig 2.6

This value is twice the maximum or peak value of the sine wave.

Instantaneous Value: It is the value of the quantity at any instant.

<u>Average Value</u>

The arithmetical average of an alternating quantity over one cycle is called its average value. Average Value can be determined by Graphical Method or Analytical Method.

From Fig 2.7 average value can calculate by using graphical method as.

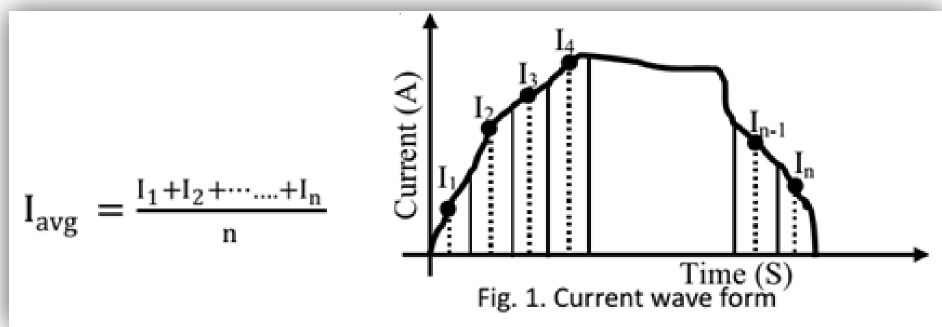


Fig 2.7

From Fig 2.8 average value can calculate by using Analytical method as.

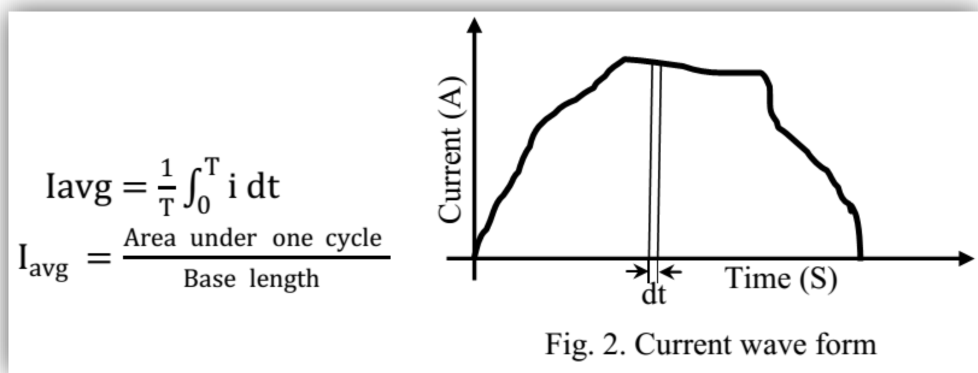


Fig 2.8

RMS Value or Effective Value

The effective or RMS value of an alternating current is the steady current (D.C) which when flowing through a given resistance for a given time produces the same amount of heat as produced by a alternating current when flowing through the same resistance for the same time.

RMS Value can be determined by Graphical Method or Analytical Method.

Graphical Method: This method is best suitable for complicated waveforms, with this method approximate RMS value can calculate very easily.

From fig (1) RMS value can calculate as

$$I = \sqrt{I_1^{\,2} + I_2^{\,2} \ldots\ldots I_n^{\,2}}$$

$$I = \sqrt{\text{mean }(i^2)}$$

From fig (2) RMS value can calculate as.

$$I = \sqrt{\frac{1}{T}\int_0^T i^2\,dt}$$

Where 'I' is the RMS value of alternating current, 'i' is the instantaneous value of current and where T is the time period.

Peak Factor (or) Crest Factor

The peak factor of an alternating quantity is defined as the ratio of its maximum value to the RMS value.

$$\text{Peak factor} = \frac{\text{Maximum value}}{\text{RMS value}}$$

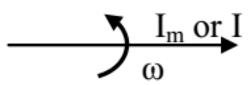
Form Factor

The form factor of an alternating quantity is defined as the ratio of RMS value to the average value.

$$\text{Form factor} = \frac{\text{RMS value}}{\text{average value}}$$

Phasor

A phasor is a line of definite length rotating in anti-clock wise direction at a constant angular velocity (ω). Length of this phasor is the maximum or RMS value of the alternating quantity.



RMS and Average Value for Sinusoidal alternating quantity

Maximum Value: Maximum value of the given wave form is $I_{max} = I_m$

RMS Value

The given wave form is a symmetrical wave form with a time period 2π.

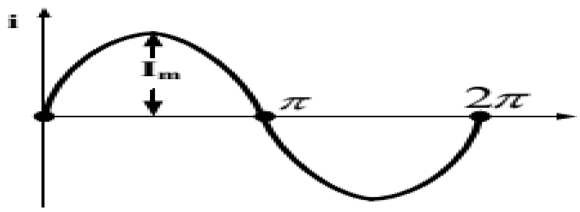


Fig 2.9

$$I = \sqrt{\frac{1}{T}\int_0^T i^2 . dt}$$

$$I^2 = \frac{1}{2\pi}\int_0^{2\pi} (I_m \sin \omega t)^2 \; d\,\omega t = \frac{I_m{}^2}{2\pi}\int_0^{2\pi} \left(\frac{1-\cos 2\omega t}{2}\right) d\,\omega t$$

$$I^2 = \frac{I_m{}^2}{4\pi}\left[\int_0^{2\pi} 1 \; d\,\omega t - \int_0^{2\pi} \cos 2\omega t \; d\,\omega t\right]$$

$$I^2 = \frac{I_m{}^2}{4\pi}\left[\int_0^{2\pi} 1. d\omega t - \int_0^{2\pi} \cos 2\omega t. d\omega t\right] = \frac{I_m{}^2}{4\pi}[(2\pi - 0) - 0] = \frac{I_m{}^2}{4\pi} \times 2\pi$$

$$= \frac{I_m{}^2}{2}$$

$$I = \frac{I_m}{\sqrt{2}} = 0.707\, I_m$$

Average Value

The given wave form is a symmetrical wave form, consider only alternation.

$$Iavg = \frac{1}{T/2}\int_0^{T/2} i\, dt$$

$$Iavg = \frac{1}{\pi}\int_0^{\pi} (I_m \sin\omega t)\, d\omega t$$

$$Iavg = \frac{I_m}{\pi} \times \int_0^{\pi} \sin\omega t\, d\omega t = \frac{I_m}{\pi} \times [-\cos\omega t]_0^{\pi} = \frac{I_m}{\pi} \times [\cos\omega t]_{\pi}^{0}$$

$$= \frac{I_m}{\pi} \times [1 + 1]$$

$$Iavg = \frac{2I_m}{\pi} = 0.6366\, I_m$$

$$\text{Form factor} = \frac{\text{RMS value}}{\text{Average value}} = \frac{0.707\, I_m}{0.6366\, I_m} = 1.11$$

$$\text{Peak factor} = \frac{\text{Maximum value}}{\text{RMS value}} = \frac{I_m}{0.707\, I_m} = 1.414$$

Phasor Representation:

An alternating quantity can be represented using

i) Waveform

ii) Equations

iii) Phasor

A sinusoidal alternating quantity can be represented by a rotating line called a Phasor. A phasor in a line of definite length rotating in anticlockwise direction at a constant angular velocity.

The waveform and equation representation of an alternating current is as shown in Fig 2.10. This sinusoidal quantity can also be represented using phasors.

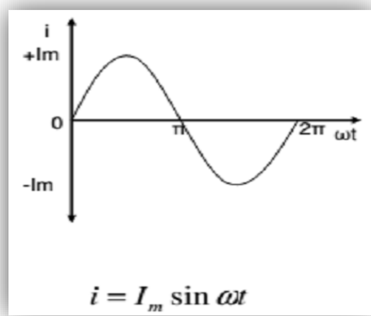


Fig 2.10

In phasor form the above wave is written as $\bar{I} = I_m \angle 0^0$

Draw a line OP of length equal to Im. This line OP rotates in the anticlockwise direction with a uniform angular velocity ω rad/sec and follows the circular trajectory shown in figure 2.11. At any instant, the projection of OP on the y-axis is given by OM=OPsinθ = $I_m$sinωt. Hence the line OP is the phasor representation of the sinusoidal current.

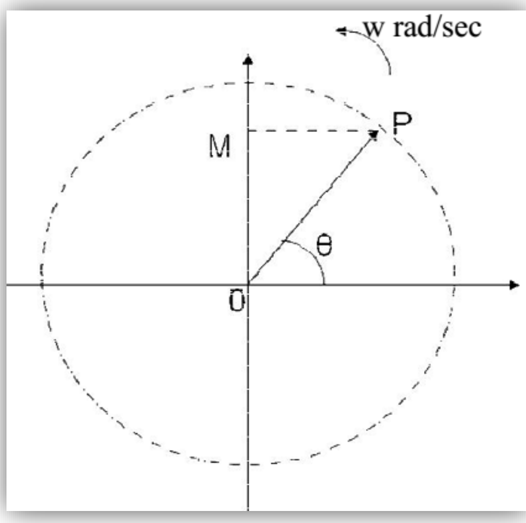


Fig 2.11

Phase

Phase is defined as the fractional part of time period or cycle through which the quantity has advanced from the selected zero position of reference.

Phase of +Em is $\pi/2$rad or T/4 sec

Phase of -Em is $\pi/2$rad or 3T/4 sec

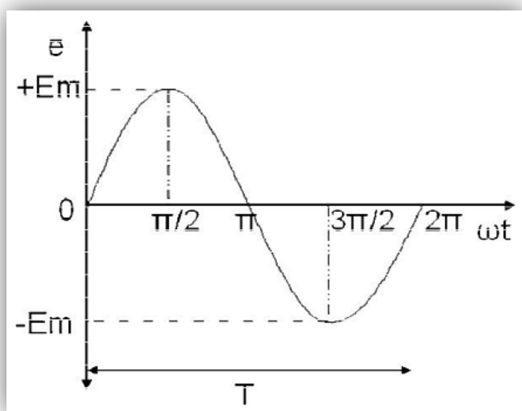


Fig 2.12

Phase Difference

When two alternating quantities of the same frequency have different zero points, they are said to have a phase difference. The angle between the zero points is the angle of phase difference.

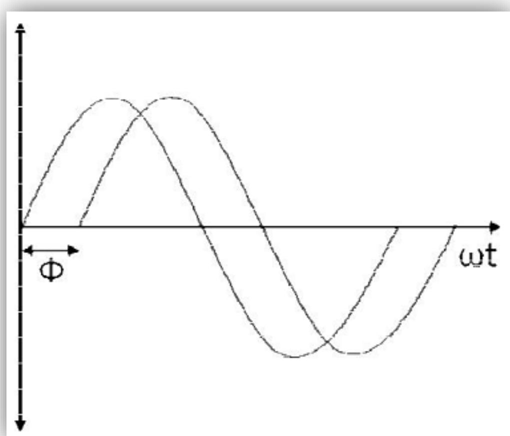


Fig 2.13

In Phase

Two waveforms are said to be in phase, when the phase difference between them is zero.

That is the zero points of both the waveforms are same. The waveform, phasor and equation representation of two sinusoidal quantities which are in phase is as shown. The figure 2.14 shows that the voltage and current are in phase.

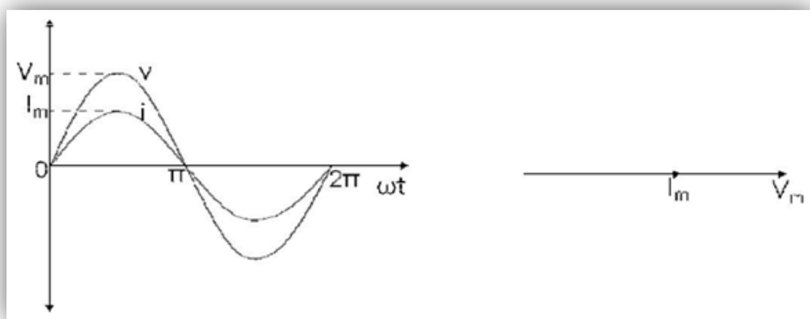
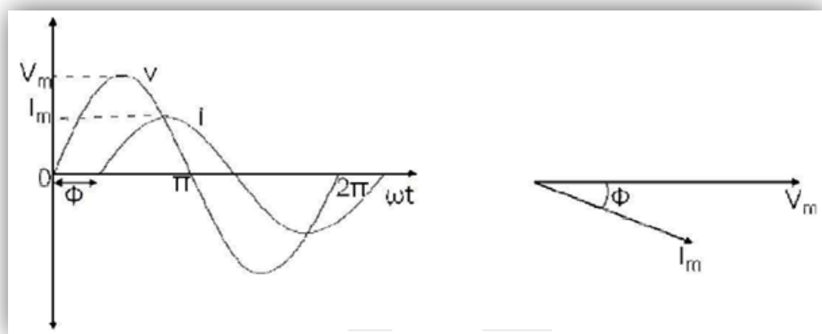


Fig 2.14

$$v = v_m \sin(\omega t)$$
$$i = i_m \sin(\omega t)$$

Lagging

In the figure 2.15, the zero point of the current waveform is after the zero point of the voltage waveform. Hence the current is lagging behind the voltage. The waveform, phasor



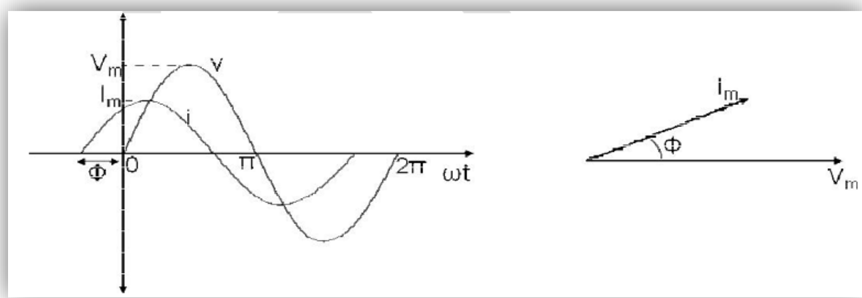
and equation representation is as shown.

Fig 2.15

$$v = v_m \sin(\omega t) => \bar{V} = V_m \angle 0^0$$
$$i = i_m \sin(\omega t - \theta) => \bar{I} = I_m \angle{-\theta^0}$$

Leading

In the figure 2.16, the zero point of the current waveform is before the zero point of the voltage waveform. Hence the current is leading the voltage. The waveform, phasor and



equation representation is as shown.

Fig 2.16

$$v = v_m \sin(\omega t) => \bar{V} = V_m\angle 0^0$$
$$i = i_m \sin(\omega t + \theta) => \bar{I} = I_m\angle \theta^0$$

Complex Numbers in Rectangular and Polar Form

To represent complex numbers (x+jy) geometrically, we use the rectangular coordinate system with the horizontal axis representing the real part and the vertical axis representing the imaginary part of the complex number.

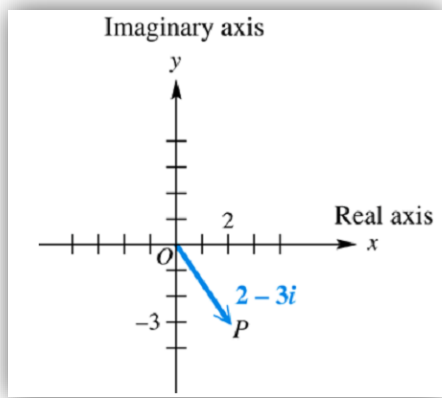
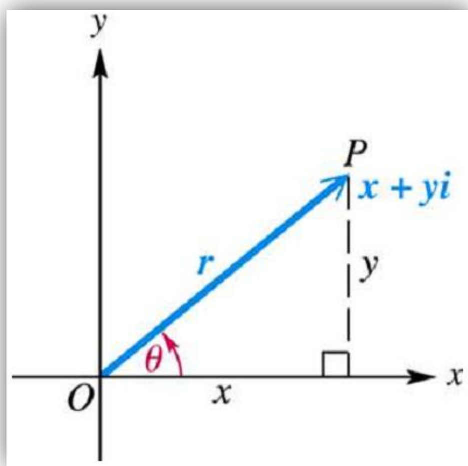


Fig 2.17

We sketch a vector with initial point (0, 0) and terminal point P(x, y). The length r of the vector is the absolute value or modulus of the complex number and the angle Θ with the positive x-axis is the is called the direction angle or argument of x+ jy.

Fig 2.18



Conversions between rectangular and polar form follows the same rules as it does for vectors.

Rectangular to Polar

For a complex number x+yi

$$|x + yi| = r = \sqrt{x^2 + y^2}$$
$$\tan\theta = \frac{y}{x},\ x \neq 0$$

Polar to Rectangular

$x = r\cos\Theta$
y r sin Θ
The polar form $r(\cos\Theta + i\sin\Theta)$ is sometimes abbreviated $r\ \text{cis}\ \Theta$
and written as $r\angle\Theta$ and read as "r at an angle Θ"

Example

Convert $-\sqrt{3} + i$ to polar form.

Solution

$x = -\sqrt{3}$ and $y = 1$ so that

$$r = \sqrt{\left(-\sqrt{3}\right)^2 + 1^2} = 2$$

and

$$\tan\theta = \frac{1}{-\sqrt{3}}$$

$$\theta = 150°$$

Example

Converting polar to rectangular form is straightforward.

$$\begin{aligned} 4\,\text{cis}\,240° &= 4\cos 240° + i\sin 240° \\ &= 4\left(-\frac{1}{2}\right) + i\left(-\frac{\sqrt{3}}{2}\right) \\ &= -2 - 2i\sqrt{3} \end{aligned}$$

Addition and Subtraction of complex numbers

To add or subtract two complex numbers, you add or subtract the real parts and the imaginary parts.

$(a + bi) + (c + id) = (a + c) + (b + d)i.$

$(a + bi) - (c + id) = (a - c) + (b - d)i.$

Example 1:

$(3 - 5i) + (6 + 7i) = (3 + 6) + (-5 + 7)i = 9 + 2i.$

$(3 - 5i) - (6 + 7i) = (3 - 6) + (-5 - 7)i = -3 - 12i.$

Product and Quotient Theorems

The advantage of polar form is that multiplication and division are easier to accomplish.

Product Theorem

$$(r_1\,\text{cis}\,\theta_1)(r_2\,\text{cis}\,\theta_2) = r_1 r_2\,\text{cis}(\theta_1 + \theta_2)$$

Quotient Theorem

$$\frac{(r_1\,\text{cis}\,\theta_1)}{(r_2\,\text{cis}\,\theta_2)} = \frac{r_1}{r_2}\,\text{cis}(\theta_1 - \theta_2)$$

The advantage of using polar form will become even more pronounced when we calculate powers.

Example

Find $(2\,\text{cis}\,45°)(3\,\text{cis}\,135°)$ and convert the answer to rectangular form.

Solution

$$\begin{aligned}(2\,\text{cis}\,45°)(3\,\text{cis}\,135°) &= 2 \cdot 3\,\text{cis}(45° + 135°)\\ &= 6\,\text{cis}\,180°\end{aligned}$$

In rectangular form, this answer is –6.

Example

Find $\dfrac{10\,\text{cis}(-60°)}{5\,\text{cis}(150°)}$ and convert the answer to rectangular form.

Solution

$$\begin{aligned}\frac{10\,\text{cis}(-60°)}{5\,\text{cis}(150°)} &= \frac{10}{5}\,\text{cis}(-60° - 150°)\\ &= 2\,\text{cis}(-210°)\end{aligned}$$

Converting the polar result gives

$$\begin{aligned}2\,\text{cis}(-210°) &= 2(\cos(-210°) + i\sin(-210°))\\ &= 2(\cos(210°) - i\sin(210°))\\ &= 2\left(-\frac{\sqrt{3}}{2} - i\left(-\frac{1}{2}\right)\right)\\ &= -\sqrt{3} + i\end{aligned}$$

IMPEDANCE :

"Impedance is the total resistance/opposition offered by the circuit elements to the flow of alternating or direct current!"

OR

"The impedance of a circuit is the ratio of the phasor voltage (V) to the phasor current (I)"

It is denoted by Z.   Z=V/I

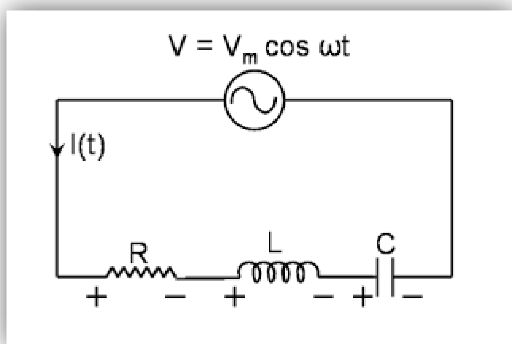


Fig 2.19

As complex quantity, we can write as: Z=R+jX

It is a vector (two-dimensional) quantity consisting of two independent scalar (one-dimensional)phenomena: resistance and reactance !

RESISTANCE:

"Resistance of an element denotes its ability to resists the flow of electric current"

OR

"It is a measure of the extent to which a substance opposes the movement of electrons among its atoms"

It is denoted by R.

The more easily the atoms give up and/or accept electrons, the lower the resistance, which is measured in ohms.

It is observed with alternating current (AC) and also with direct current (DC).

<u>Types of Resistance:</u>

· HIGH RESISTANCE:

Substances with High-resistance are called insulators or dielectrics, and include materials such as polyethylene, mica, and glass.

· LOW RESISTANCE:

Substances with low-resistance are called electrical conductors, and include materials such as copper, silver, and gold.

· INTERMEDIATE RESISTANCE:

Substances with intermediate levels of resistance are called semiconductors, and include materials such as silicon, germanium, and gallium arsenide.

REACTANCE:

"Reactance is a form of opposition that electronic components exhibit to the passage of AC (alternating current) because of capacitance or inductance"

It is denoted by X.

It is expressed in ohms.

It is observed for AC (alternating current), but not for DC (direct current).

<u>TYPES OF REACTANCE:</u>

· INDUCTIVE REACTANCE:

When AC (alternating current) passes through a component that contains reactance, energy might be stored and released in the form of a magnetic field which is known as inductive reactance.

It is denoted by $+jX_L$

· CAPACITIVE REACTANCE:

When AC (alternating current) passes through a component that contains reactance, energy might be stored and released in the form of an electric field which is known as capacitive reactance.

It is denoted by $-jX_C$

EXPLANATION:

Reactance is conventionally multiplied by the positive square root of -1, which is the unit imaginary number called the *j operator*, to express Z as a complex number of the form $R + jX_L$ (when the net reactance is inductive) or $R - jX_C$ (when the net reactance is capacitive).

ADMITTANCE:

"Admittance is the allowance of circuit elements to the flow of alternating current or direct current ".

OR

"It is the inverse of impedance"

It is denoted by Y.

We can write as:

Y=1/Z=I/V

As complex quantity, we can write as:

Y=G+jB

Admittance is a vector quantity comprised of two independent scalars phenomena: conductance and susceptance

CONDUCTANCE:

"Conductance is the ability of an element to conduct electric current."

OR

"It is the inverse of resistance"

It is denoted by G.

G=1/R

The more easily the charge carriers move in response to a given applied electric potential, the higher the conductance, which is expressed in positive real-number (Siemens) or (Mhos).

Conductance is observed with AC and also with direct current DC.

SUSCEPTANCE:

"Susceptance is an expression of the readiness with which an electronic component, circuit, or system releases stored energy as the current and voltage fluctuate"

OR

"It is a reciprocal of reactance"

It is denoted by B.

B=1/X

Susceptance is expressed in imaginary number Siemens.

Susceptance is observed with AC, but not for DC.

<u>TYPES OF SUSCEPTANCE:</u>

- INUDUCTIVE SUSCEPTANCE:

When AC (alternating current) passes through a component that contains susceptance, energy might be stored and released in the form of a magnetic field which is known is inductive susceptance.

It is denoted by $-jB_L$

- CAPACITIVE SUSCEPTANCE:

When AC (alternating current) passes through a component that contains susceptance, energy might be stored and released in the form of an electric field which is known is capacitive susceptance.

It is denoted by $+jB_C$

EXPLANATION:

Admittance is the vector sum of conductance and susceptance. Susceptance is conventionally multiplied by the positive square root of -1, the unit imaginary number called symbolized by j, to express Y as a complex quantity $G - jB_L$(when the net susceptance is inductive) or $G + jB_C$ (when the net susceptance is capacitive).

In parallel circuits, conductance and susceptance add together independently to yield the composite admittance. In series circuits, conductance and susceptance combine in a more complicated manner. In these situations, it is easier to convert conductance to resistance, susceptance to reactance, and then calculate the composite impedance.

<u>Impedance & Admittance:</u>

ELEMENT	IMPEDENCE Z=V/I	ADMITTANCE Y = I/V
R	**ZR**= R	**YR**= 1/R
L	ZL= jwL	**YL**= 1/jwL
C	**ZC**= 1/jwC	**YC**= jwC

UNIT – III

Single Phase AC Circuits: Concept of Active and reactive power, power factor –power triangle. Examples Steady state Analysis of R, L and C elements (in series, parallel and series parallel combinations) –with sinusoidal Excitation - Phasor diagrams-Examples

Power:

In an AC circuit, the various powers can be classified as

1. Real or Active power or Average power.
2. Reactive power
3. Apparent power

Real or active power in an AC circuit is the power that does useful work in the circuit. Reactive power flows in an AC circuit but does not do any useful work. Apparent power is the total power in an AC circuit.

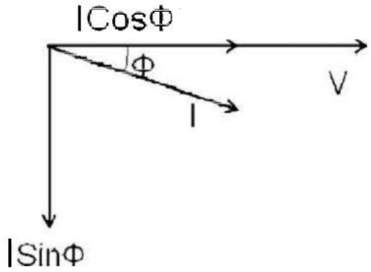


Fig 3.1

Instantaneous Power:

The instantaneous power is product of instantaneous values of current and voltages and it can be derived as follows

$$P = vi$$

$$p = V_m \sin(\omega t + \theta_v) * I_m \sin(\omega t + \theta_i)$$

From trigonometric expression:

$$\cos(A - B) - \cos(A + B) = 2\sin(A)\sin(B)$$

$$p = \frac{V_m I_m}{2}(\cos(\theta_v - \theta_i) - \cos(2\omega t + \theta_v + \theta_i))$$

$$p = \frac{V_m I_m}{2}\cos(\theta_v - \theta_i) - \frac{V_m I_m}{2}\cos(2\omega t)$$

The instantaneous power consists of two terms. The first term is called as the constant power term and the second term is called as the fluctuating power term.

Average Power:

From instantaneous power we can find average power over one cycle as following.

$$P = \frac{1}{2\pi}\int_0^{2\pi}(\frac{V_m I_m}{2}\cos(\theta_v - \theta_i) - \frac{V_m I_m}{2}\cos(2\omega t + \theta_v + \theta_i))d(\omega t)$$

$$P = \frac{1}{2\pi}\left(\frac{V_m I_m}{2}\cos(\theta_v - \theta_i) * (2\pi - 0)\right) - \frac{1}{2\pi}\int_0^{2\pi} - \frac{V_m I_m}{2}\cos(2\omega t)\, d(\omega t)$$

$$P = \frac{V_m I_m}{2}\cos(\theta_v - \theta_i) = \frac{V_m}{\sqrt{2}} * \frac{I_m}{\sqrt{2}} = V_{RMS} * I_{RMS}\cos(\theta_v - \theta_i)$$

As seen above the average power is the product of the RMS voltage and the RMS current.

Real Power:

The power due to the active component of current is called as the active power or real power. It is denoted by P.

$$P = V * Icos(\emptyset) = I^2 Rcos(\emptyset)$$

Real power is the power that does useful power. It is the power that is consumed by the resistance. The unit for real power is Watt (W).

Reactive Power:

The power due to the reactive component of current is called as the reactive power. It is denoted by Q.

$$Q = V * Isin(\emptyset) = I^2 X_L \sin(\emptyset)$$

Reactive power does not do any useful work. It is the circulating power in the L and C components. The unit for reactive power is Volt Amperes Reactive (VAR).

Apparent Power:

The apparent power is the total power in the circuit. It is denoted by S.

$$S = VI = I^2 Z$$

$$S = \sqrt{P^2 + Q^2}$$

The unit for apparent power is Volt Amperes (VA).

Power Triangle:

From the impedance triangle, another triangle called the power triangle can be derived as shown.

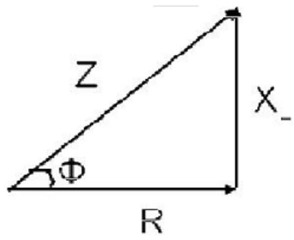


Fig.3.2 a

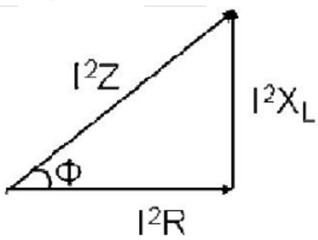


Fig.3.2b

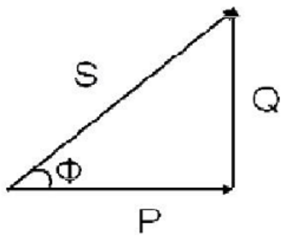


Fig3.2c

The power triangle is right angled triangle with P and Q as two sides and S as the hypotenuse. The angle between the base and hypotenuse is Φ. The power triangle enables us to calculate the following things.

$$\text{Apparent Power } S = \sqrt{P^2 + Q^2}$$

$$Power\ factor = \cos(\emptyset) = \frac{P}{Q} = \frac{Real\ power}{Apparent\ power}$$

The power Factor in an AC circuit can be calculated by any one of the following Methods

$$= Cosine\ of\ angle\ between\ V\ and\ I$$

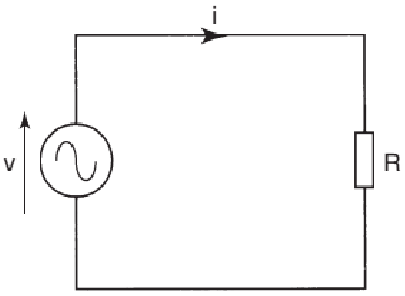
$$= \frac{Resistance}{Impedance} = \frac{R}{Z}$$

$$= \frac{Real\ power}{Apparent\ power}$$

Single phase circuits with Sinusoidal AC excitation

Pure Resistance

Consider a perfect (pure) resistor, connected to an a.c. supply, as shown in Fig. 3.3. The current flowing at any instant is directly proportional to the instantaneous applied voltage, and inversely proportional to the resistance value. The voltage is varying sinusoidally, and the resistance is a constant value. Thus the current flow will also be sinusoidal, and will be in phase with the applied voltage. This can be written as follows



$$i = \frac{v}{R} \text{ amp}$$

$$\text{but, } v = V_m \sin \omega t \text{ volt}$$

$$\text{therefore, } i = \frac{V_m}{R} \sin \omega t \text{ amp}$$

$$\text{but, } i = I_m \sin \omega t \text{ amp}$$

Fig.3.3

Hence,

$$I_m = \frac{V_m}{R} \text{ amp, or } I = \frac{V}{R} \text{ amp}$$

The relevant waveform and phasor diagrams are shown in Figs. 3.4a. and 3.4b. respectively.

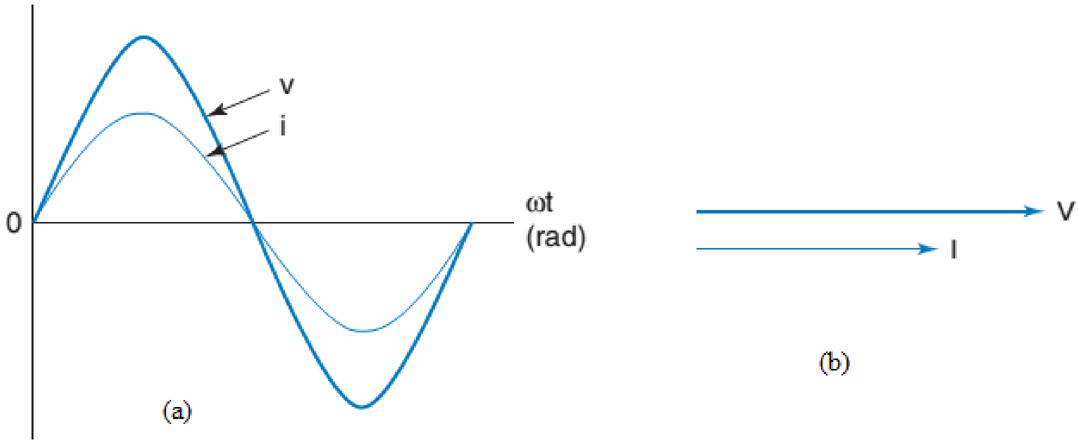


Fig.3.4

The instantaneous power (p) is given by the product of the instantaneous values of voltage and current. Thus p = vi. The waveform diagram is shown in Fig.3.4a. From this diagram, it is obvious that the power reaches its maximum and minimum values at the same time as both voltage and current. Therefore

$$P_m = V_m I_m$$

$$\text{hence, } P = VI = I^2R = \frac{V^2}{R} \text{ watt}$$

Note: When calculating the power, the r.m.s.values must be used.

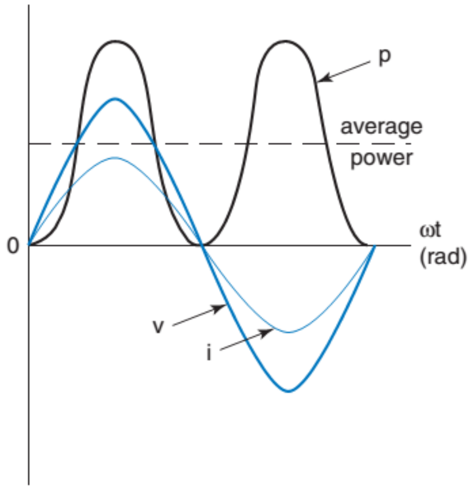


Fig.3.5

From these results, we can conclude that a pure resistor, in an a.c. circuit, behaves in exactly the same way as in the equivalent d.c. circuit.

Pure Inductance

Consider a pure inductor, connected to an a.c. supply, as shown in Fig.3.6.

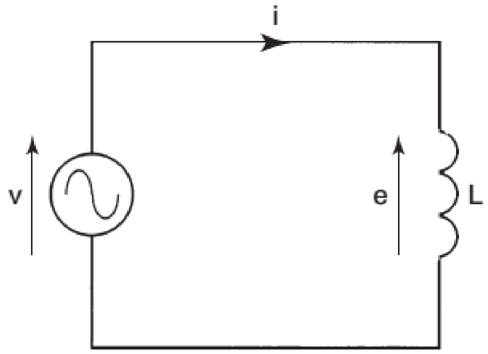


Fig.3.6

An alternating current will now flow through the circuit. Since the current is continuously changing, then a back emf, e will be induced across the inductor. In this case, e will be exactly equal and opposite to the applied voltage, v. the equation for this back emf is

$$e = -L\frac{\mathrm{d}i}{\mathrm{d}t}\,\text{volt}$$

E will have its maximum values when the rate of change of current, d i/d t, is at its maximum values. These maximum rates of change occur as the current waveform passes through the zero position. The related waveforms are shown in Fig. 3.7 a . From this waveform diagram, it may be seen that the applied voltage, V leads the circuit current, I, by π /2 rad, or 90°. The corresponding phasor diagram is shown in Fig. 3.7 b.

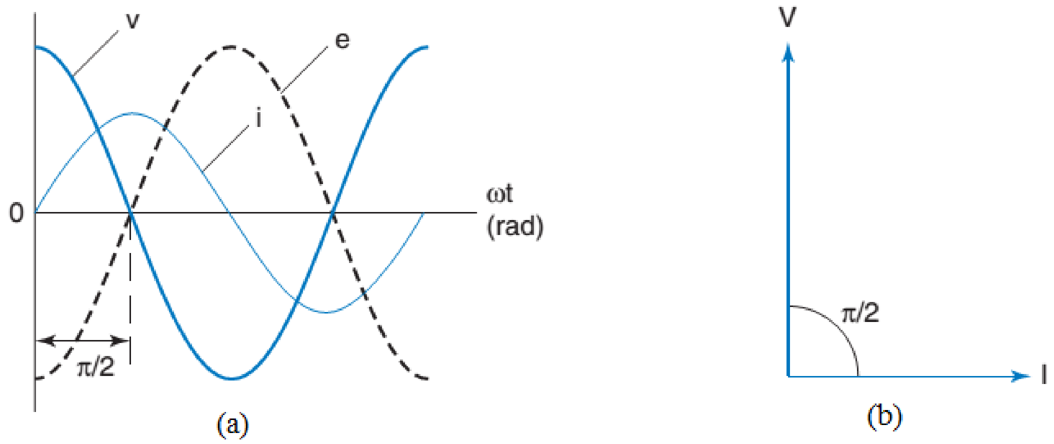


Fig.3.7

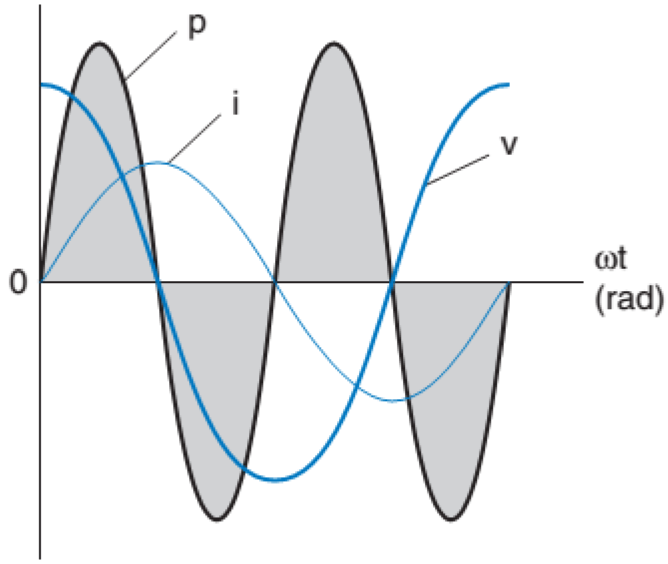


Fig.3.7 c

Inductive Reactance

Inductive reactance is defined as the opposition offered to the flow of a.c., by a perfect inductor. It is measured in ohms, and the quantity symbol is X L

$$e = -L\frac{di}{dt} \text{ volt, and } e = -v$$

$$\text{therefore, } v = L\frac{di}{dt} \text{ volt}$$

Now, $i = I_m \sin \omega t$ amp, so, $v = L\dfrac{d}{dt}(I_m \sin \omega t)$

therefore, $v = \omega L I_m \cos \omega t$

at time $t = 0$, $v = V_m$; and $\cos \omega t = 1$

hence, $V_m = \omega L I_m$; and dividing by I_m

$$\frac{V_m}{I_m} = \frac{V}{I} = \omega L \text{ ohm}$$

so, inductive reactance is:

$$X_L = \omega L = 2\pi f L \text{ ohm}$$

Pure Capacitance

Consider a perfect capacitor, connected to an a.c. supply, as shown in Fig.3.8a. The charge on the capacitor is directly proportional to the p.d. across it. Thus, when the voltage is at its maximum, so too will be the charge, and so on. The waveform for the capacitor charge will therefore be in phase with the voltage. Current is the rate of change of charge. This means that when the rate of change of charge is a maximum, then the current will be at a maximum, and so on.. The resulting waveforms are shown in Fig.3.8b. It may therefore be seen that the current now leads the voltage by π/2 rad, or 90°.

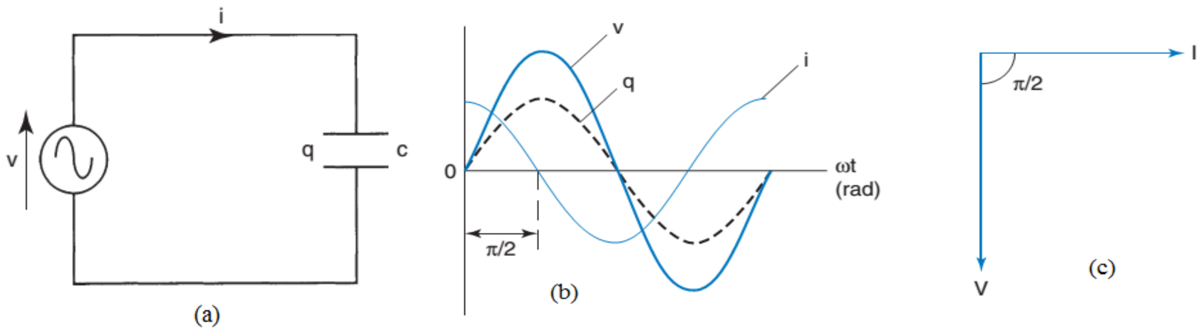


Fig.3.8

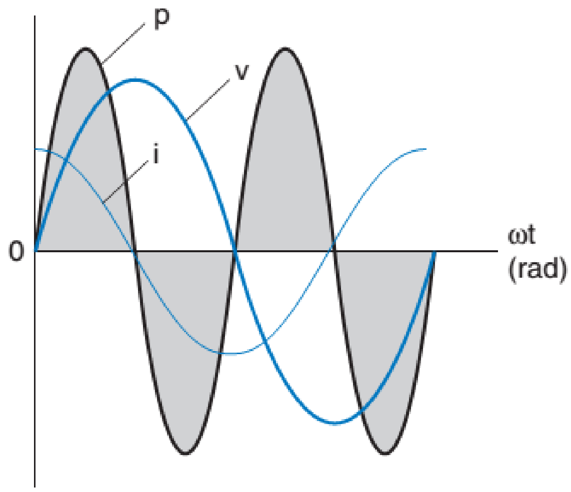


Fig.3.8d

Capacitive Reactance(X_C)

It is defined as the opposition offered to the flow of a.c. through a perfect capacitor

$$q = vC \text{ coulomb}; \quad \text{and } i = \frac{dq}{dt} \text{ amp}$$

therefore, $i = C\dfrac{dV}{dt}$

and since $v = V_m \sin\omega t$ volt, then

$$i = C\frac{d}{dt}(V_m \sin\omega t)$$

$$= \omega C V_m \cos\omega t$$

when time $t = 0$, $I = I_m$; and $\cos\omega t = 1$

$$\text{therefore, } I_m = \omega C V_m$$

$$\text{and } \frac{V_m}{I_m} = \frac{V}{I} = \frac{1}{\omega C} \text{ ohm}$$

capacity reactance,

$$X_c = \frac{1}{\omega C} = \frac{1}{2\pi f C} \text{ ohm}$$

Impedance

This is the total opposition, offered to the flow of a.c. current, by a circuit that contains both resistance and reactance. It is measured in ohms, and has the quantity symbol Z.

$$\text{Thus, } Z = \frac{V}{I} \text{ ohm}$$

Where V is the circuit applied voltage, and I is the resulting circuit current.

Inductance and Resistance in Series

A pure resistor and a pure inductor are shown connected in series in Fig....... The circuit current, I, will produce the p.d. V R across the resistor, due to its resistance, R. Similarly, the p.d. V_L results from the inductor's opposition, the inductive reactance, X_L. Thus, the only circuit quantity that is common, to both the resistor and the inductor, is the circuit current, I. For this reason, the current is chosen as the reference phasor.

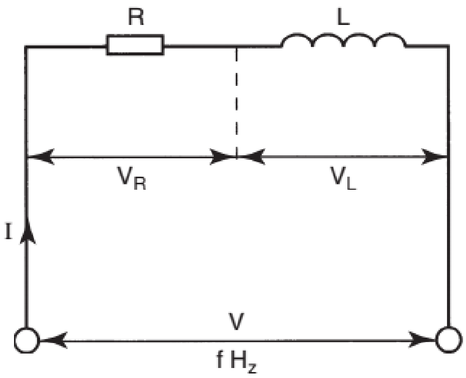


Fig.3.9

The p.d. across the resistor will be in phase with the current through it (Φ = 0). The p.d. across the inductor will lead the current by 90° (Φ = 90°). The total applied voltage, V, will be the phasor sum of V_R and V_L. This last statement may be considered as the ' a.c. Version' of Kirchhoff's voltage law. In other words, the term ' phasor sum ' has replaced the term 'algebraic sum ' , as used in d.c. circuits. The resulting phasor diagram is shown in Fig.3.10. The angle Φ, shown on this diagram, is the angle between the circuit applied voltage, V,and the circuit current, I.It is therefore known as the circuit phase angle.

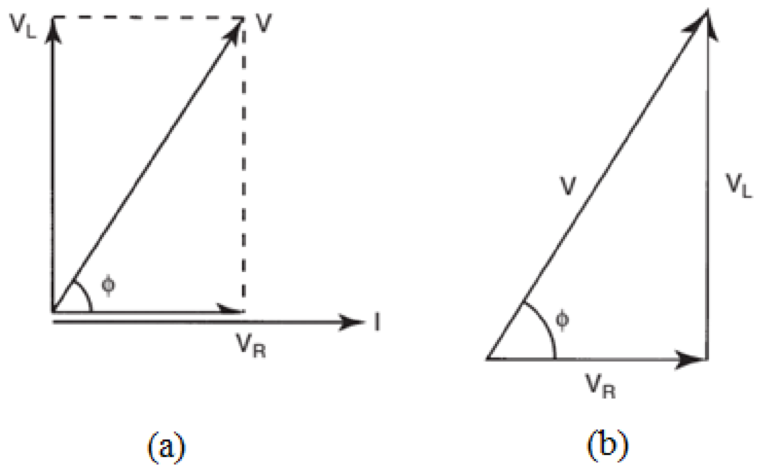


Fig.3.10

The applied voltage Vis the phasor sum of the circuit p.d.s. These p.d.s form horizontal and vertical components.

$$V^2 = V_R^2 + V_L^2$$

Now, $V_R = IR$; $V_L = IX_L$; and $V = IZ$ volt

and substituting these into equation

we have: $$(IZ)^2 = (IR)^2 + (IX_L)^2$$

and dividing through by I^2 we have:

$$Z^2 = R^2 + X_L^2$$

therefore, $Z = \sqrt{R^2 + X_L^2}$ ohm

From the last equation, it may be seen that Z, R and X_L also form a right-angled triangle. This is known as the impedance triangle, and is shown in Fig.3.11.

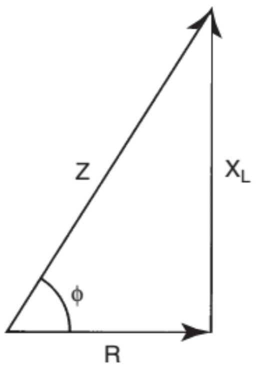


Fig.3.11

From both the voltage and impedance triangles, the following expressions for the circuit phase angle, Φ, are obtained:

$$\cos\phi = \frac{R}{Z} = \frac{V_R}{V}$$
$$\text{or, } \sin\phi = \frac{X_L}{Z} = \frac{V_L}{V}$$
$$\text{or, } \tan\phi = \frac{X_L}{R} = \frac{V_L}{V_R}$$

Resistance and Capacitance in Series

Figure 3.12 shows a pure capacitor and resistor connected in series, across an a.c. supply. Again, being a series circuit, the circuit current is common to both components. Each will have a p.d. developed. In this case however, the p.d. across the capacitor will lag the current by 90°.

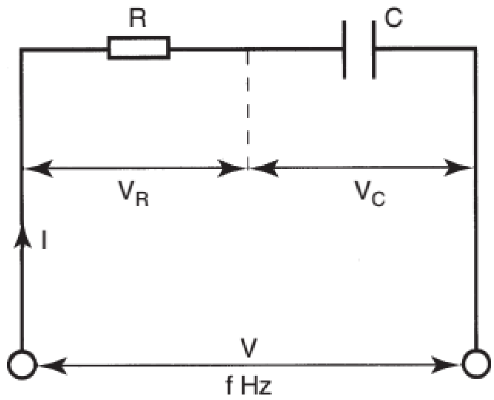


Fig.3.12

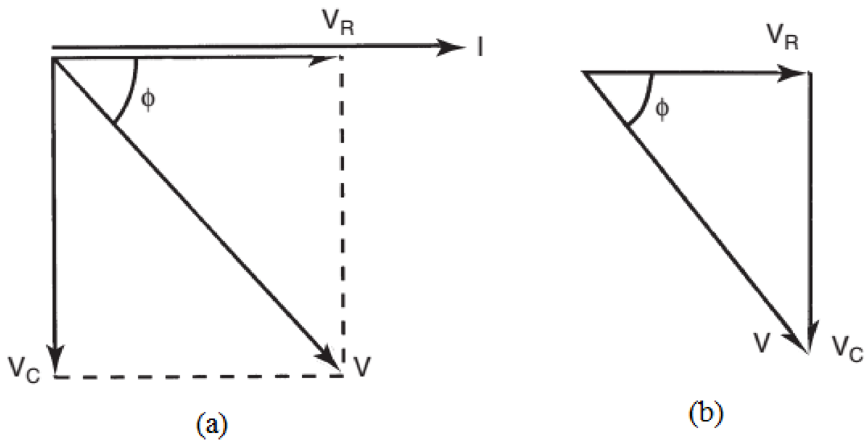


Fig.3.13

The voltage and impedance triangles of RC circuit are shown in Figs.3.13. and .3.14.

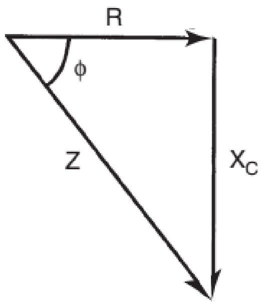


Fig.3.14

The following equations result:

$$Z = \sqrt{R^2 + X_C^2} \text{ ohm}$$

$$\phi = \cos^{-1} \frac{R}{Z}$$

$$\sin \phi = \frac{X_C}{Z} = \frac{V_C}{V}$$

$$\text{and } \tan \phi = \frac{X_C}{R} = \frac{V_C}{V_R}$$

Resistance, Inductance and Capacitance in Series

These three elements, connected in series, are shown in Fig. 3.15. Of the three p.d.s, V_R will be in phase with the current I, V_L will lead I by 90°, and V_C will lag I by 90°. The associated phasor diagram is shown in Fig.3.16.

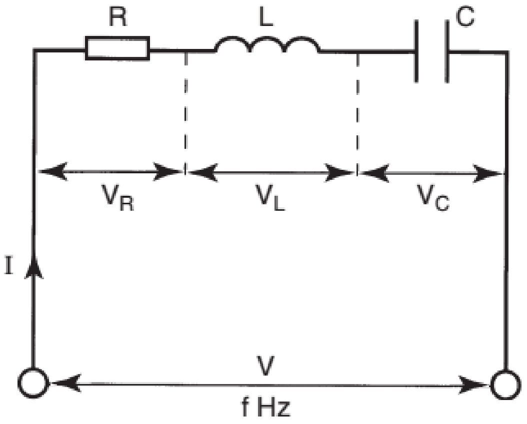


Fig.3.15

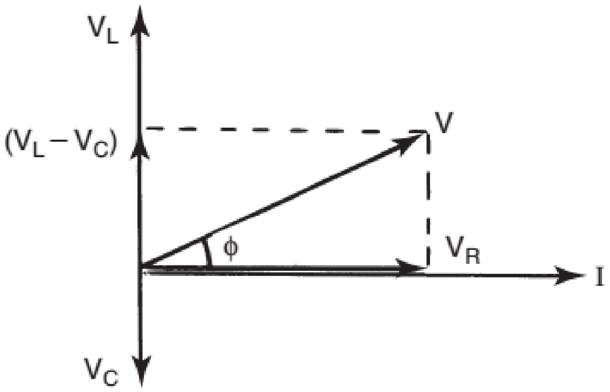


Fig.3.16

The applied voltage V is the phasor sum of the circuit p.d.s. These p.d.s form horizontal and vertical components.

$$V^2 = V_R^2 + (V_L - V_C)^2$$

$$\text{but, } V = IZ,\ V_R = IR,\ V_L = IX_L \text{ and } V_C = IX_C \text{ volt}$$

$$\text{therefore, } (IZ)^2 = (IR)^2 + (IX_L - IX_C)^2$$

$$\text{hence, } Z^2 = R^2 + (X_L - X_C)^2$$

$$\text{and, } Z = \sqrt{R^2 + (X_L - X_C)^2} \text{ ohm}$$

The associated impedance triangle is shown in Fig.3.17. Note that if X_C X_L, then the circuit phase angle φ will be lagging, instead of leading as shown.

$$\cos\phi = \frac{R}{Z}; \qquad \sin\phi = \frac{X_L - X_C}{Z};$$

$$\text{and } \tan\phi = \frac{X_L - X_C}{R}$$

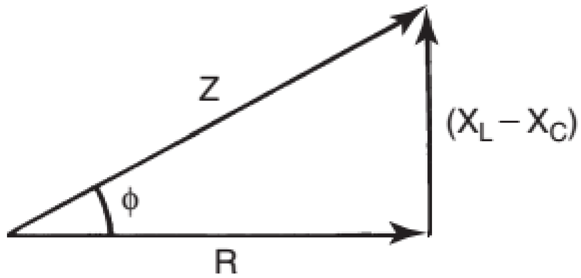


Fig.3.17

NBKRIST

BASIC ELECTRICAL SCIENCES
LECTURE NOTES

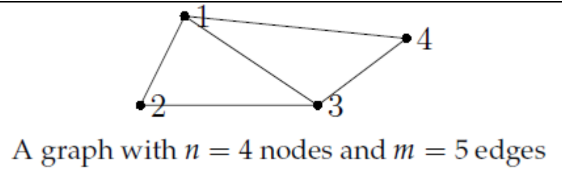
UNIT-4

I.PRABHAKAR REDDY
DEPARTMENT OF ELECTRICAL AND ELECTRONICS ENGINEERING

NETWORK TOPOLOGY

When all the elements in a network are replaces by lines with circles or dots at both ends, configuration is called the graph of the network.

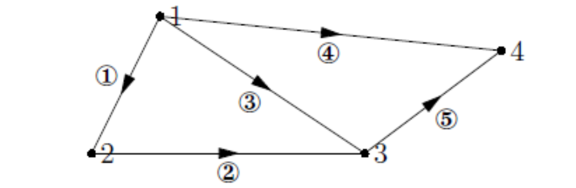
Graph: A graph is a collection of nodes joined by edges (line segments), the fig. 4.1 shows a small graph.



A graph with $n = 4$ nodes and $m = 5$ edges

Fig. 4.1

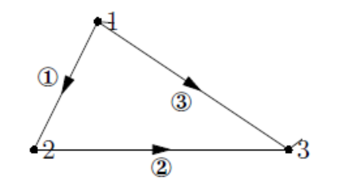
Directed (or Oriented) graph:- A graph is said to be directed (or oriented) when all the nodes and branches are numbered or direction assigned to the branches by arrow.



The graph of Fig 4.1 with a direction on each edge.

Fig. 4.2

Sub graph:- A graph G_S said to be sub-graph of a graph G if every node of G_S is a node of G and every branch of G_S is also a branch of G.



Sub Graph of the Fig.4.2

Fig. 4.3

Connected Graph:- When at least one path along branches between every pair of a graph exits , it is called a connected graph

A. Terminology used in network graph:-

(i) Path:-A sequence of branches traversed in going from one node to another is called a path.

(ii) Node:-A node point is defined as an end point of a line segment and exits at the junction between two branches or at the end of an isolated branch.

(iii) Degree of a node: - It is the no. of branches incident to it. Example for Degree of node as shown in fig. 4.4

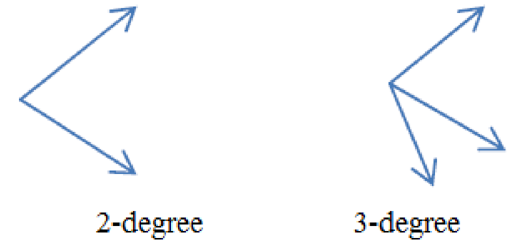


Fig. 4.4

(iv) Tree: - It is a connected sub graph with no closed loops. It has only one path between any pair of nodes. A Graph has many trees. Some of the trees of graph shown in fig. 4.1.

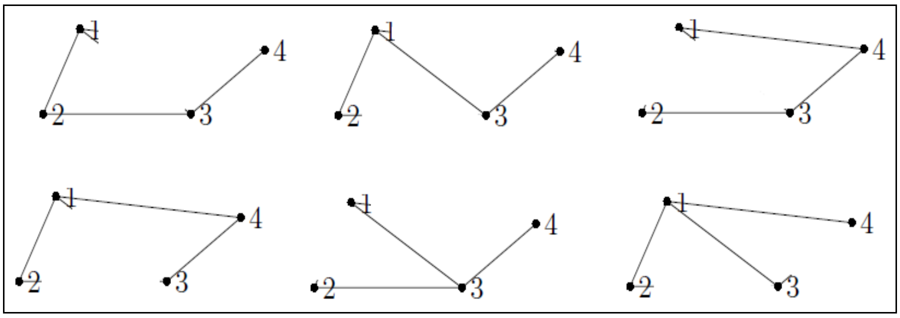


Fig. 4.5 some of the Trees of Graph shown in fig. 4.1

Properties of a Tree-

(i) It consists of all the nodes of the graph.

(ii) If the graph has N nodes, then the tree has (N-1) branch.

(iii) There will be no closed path in a tree.

(iv) There can be many possible different trees for a given graph depending on the no. of nodes and branches.

(v) Co-Tree: collection of branches that are not part of the tree.

(vi) Tree branch (Twig):- It is the branch of a tree. It is also named as twig.

(vii) Tree link (or chord):-It is the branch of a co-tree.

(viii) Loop:- This is a closed path in a graph.

Relation between twigs and links-

Let N=no. of nodes
L= total no. of links
B= total no. of branches
No. of twigs= N-1
Then, L= B-(N-1)

Incidence matrix (Node-Incidence Matrix):- Any oriented graph can be described completely in a compact matrix form. Here we specify the orientation of each branch in the graph and the nodes at which this branch is incident. This matrix is called incident matrix. When one row is completely deleted from the matrix the remaining matrix is called a **reduced incidence matrix.**

FORMATION OF INCIDENCE MATRIX:-

- This matrix shows which branch is incident to which node.
- Each row of the matrix being representing the corresponding node of the graph.
- Each column corresponds to a branch.
- If a graph contain N- nodes and B branches then the size of the incidence matrix [A] will be NXB.

A. Procedure:-

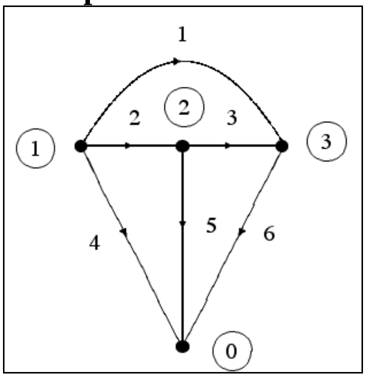
(i) If the branch j is incident at the node I and oriented away from the node, a_{ij} =1. In other words, when a_{ij} =1, branch j leaves away node i.

(ii) If branch j is incident at node j and is oriented towards node i, a_{ij} =-1. In other words j enters node i.

(iii) If branch j is not incident at node i. a_{ij} =0.

The complete set of incidence matrix is called augmented incidence matrix.

Example:

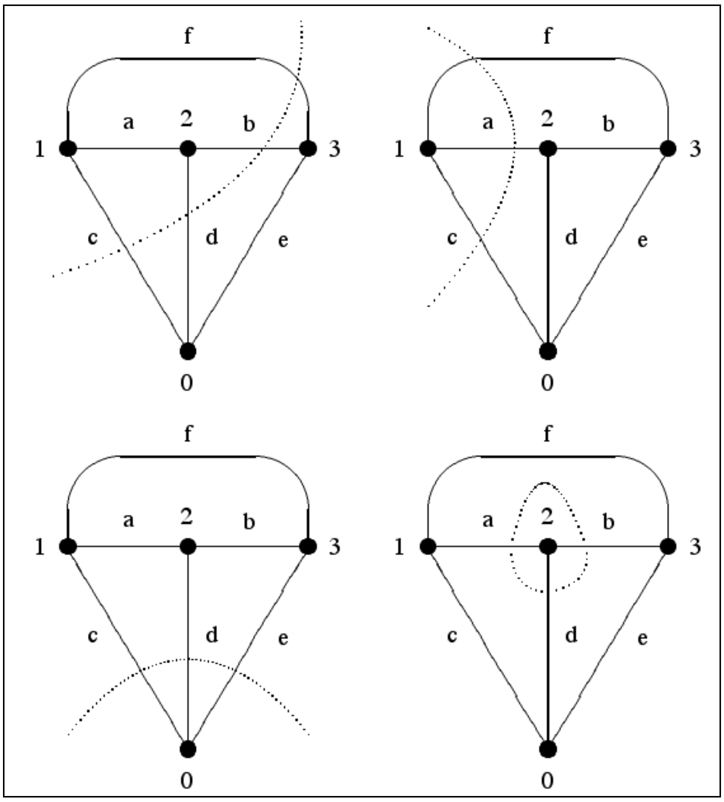


$$A = \begin{array}{c} \\ 0 \\ 1 \\ 2 \\ 3 \end{array} \begin{array}{c} \begin{array}{cccccc} 1 & 2 & 3 & 4 & 5 & 6 \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & -1 & -1 & -1 \\ 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & -1 & 1 & 0 & 1 & 0 \\ -1 & 0 & -1 & 0 & 0 & 1 \end{bmatrix} \end{array}$$

Isomorphism:- It is the property between two graphs so that both have got same incidence matrix.

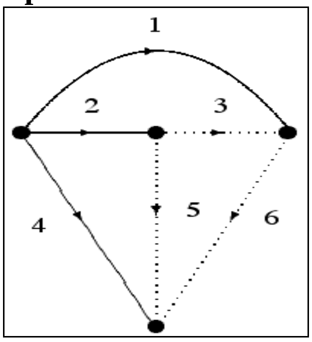
Cut-Set:- It is that set of elements or branches of a graph that separated two parts of a graph(network). If any branch of the cut-set is not removed, the network remains connected.

Example: cut-Sets: i) {f, b, d, c} ii) {f, a, c} iii) {c, d, e} and iv) {a, b, d}



Basic cutest: it is a cut set containing only one tree branch.

Example:



So, there are *t* basic cutsets in a graph.

In this example, the basic cutsets are

{ **1**, 3, 6 }
{ **2**, 3, 5 }
{ **4**, 5, 6 }

tree branches

The importance of basic cutsets is the formulation of **independent** KCL equations:

$$\begin{aligned} I_1 + I_3 - I_6 &= 0 \\ I_2 - I_3 - I_5 &= 0 \\ I_4 + I_5 + I_6 &= 0 \end{aligned}$$

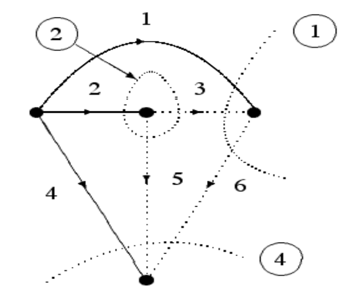
Basic cutset matrix (Q-matrix):

- The Q-matrix describes the way the basic cutset is chosen.
- Each column corresponds to a branch (b columns).
- Each row corresponds to a basic cutset (t rows).

Construction

For each row:

1. Put a "+1" in the entry corresponding to the cutset tree branch.
2. Put a "0" in the entry corresponding to other tree branches.
3. Put a "+1" or "–1" in the entry corresponding to each cutset co-tree branch; "+" if it is consistent with the tree branch direction and "–" otherwise.

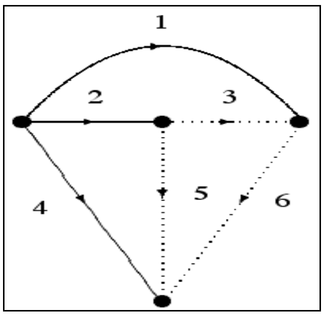


$$Q = \begin{array}{c} \\ 1 \\ 2 \\ 4 \end{array} \begin{array}{c} \begin{array}{cccccc} 1 & 2 & 4 & 3 & 5 & 6 \end{array} \\ \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & -1 \\ 0 & 1 & 0 & -1 & -1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 \end{array}\right] \end{array}$$

$\boxed{Q = [\, \mathbf{I} \mid Q_1 \,]}$

Basic loop: It is a loop containing only one co-tree branch.

Example:



So, there are *t* basic **loops** in a graph.

In this example, the basic **loops** are

{ 1, 2, **3** }
{ 2, 4, **5** }
{ 1, 4, **6** }

co-tree branches

The importance of basic loops is the formulation of **independent** KVL equations:

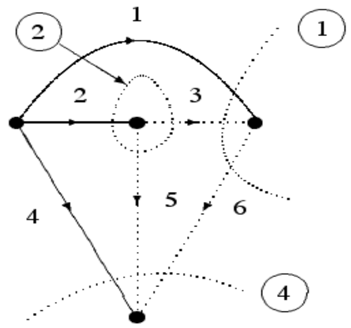
$$\begin{aligned} -V_1 + V_2 + V_3 &= 0 \\ V_2 + V_5 - V_4 &= 0 \\ V_1 - V_4 - V_6 &= 0 \end{aligned}$$

Basic loop matrix (B-matrix):

- The B-matrix describes the way the basic loop is chosen.
- Each column corresponds to a branch (b columns).
- Each row corresponds to a basic loop (b–t rows).

Construction

1. For each row: Put a "+1" in the entry corresponding to the loop co-tree branch.
2. Put a "0" in the entry corresponding to other co-tree branches.
3. Put a "+1" or "–1" in the entry corresponding to each loop tree branch; "+" if it is consistent with the co-tree branch direction and "–" otherwise.



$$B = \begin{array}{c} \\ 3 \\ 5 \\ 6 \end{array} \begin{array}{c} \begin{array}{cccccc} 1 & 2 & 4 & 3 & 5 & 6 \end{array} \\ \left[\begin{array}{ccc|ccc} -1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & -1 & 0 & 1 & 0 \\ 1 & 0 & -1 & 0 & 0 & 1 \end{array}\right] \end{array}$$

$$B = [\ B_1 \mid \mathbf{I}\ ]$$

Relationship between Q and B

$$Q = \begin{array}{c} \\ 1 \\ 2 \\ 4 \end{array} \begin{array}{c} \begin{array}{cccccc} 1 & 2 & 4 & 3 & 5 & 6 \end{array} \\ \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & -1 \\ 0 & 1 & 0 & -1 & -1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 \end{array}\right] \end{array} \qquad B = \begin{array}{c} \\ 3 \\ 5 \\ 6 \end{array} \begin{array}{c} \begin{array}{cccccc} 1 & 2 & 4 & 3 & 5 & 6 \end{array} \\ \left[\begin{array}{ccc|ccc} -1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & -1 & 0 & 1 & 0 \\ 1 & 0 & -1 & 0 & 0 & 1 \end{array}\right] \end{array}$$

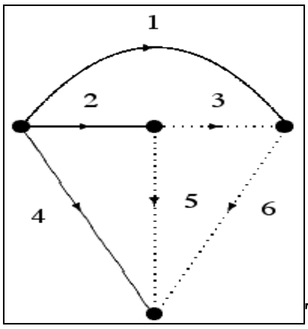
$$Q = [\ \mathbf{1} \mid Q_1\ ] \qquad B = [\ B_1 \mid \mathbf{1}\ ]$$

It is always true that $Q_1 = -B_1^T$ or $B_1 = -Q_1^T$

Thus, once we have Q, we know B, and vice versa.

Tie-Set:- It is a set of branches forming a loop through which link current flows.

Example:



Tie-Sets :{ 1, 2, 3}, {2, 4, 5}, {1, 4, 6}

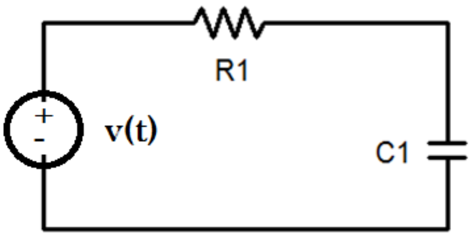
Duality and Dual net works

Duals: Two circuits are said to be duals of one another if they are described by the same characterizing equations with the dual pairs interchanged.

(Or) Two networks are said to be duals, if mesh equations characterize one of them has the same mathematical form as the nodal equations that characterize the other.

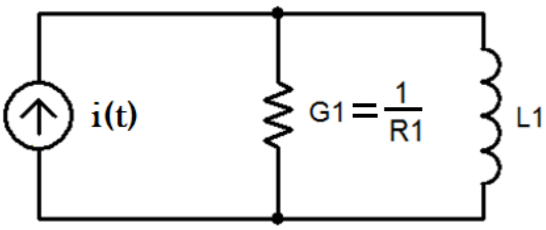
Principle of duality: Identical behavior patterns observed between voltage and currents, between two independent circuits illustrate the principle of duality.

Example:



Mesh → KVL → $-V + iR_1 + \frac{1}{C_1}\int i\,dt = 0$

$$V = IR_1 + \frac{1}{C}\int i\,dt$$



Nodal → KCL → $-I + VG_1 + \frac{1}{L_1}\int v\,dt$

$$I = VG_1 + \frac{1}{L_1}\int v\,dt$$

Dual Pairs	
Resistance (R)	Conductance (G)
Inductance (L)	Capacitance (C)
Voltage (v)	Current (i)
Voltage Source v(t) Vsinωt	Current Source i(t) Icosωt
Node	Mesh/Loop
Series Path	Parallel Path
Open Circuit	Short Circuit
KVL	KCL
Thévenin	Norton
Switch in series(getting closed)	Switch in parallel (getting opened)

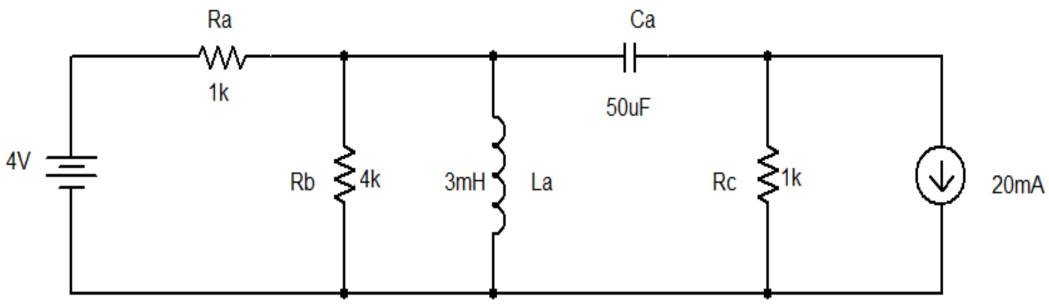
Procedure to Construct Dual Circuits

1. Place a node at the center of each mesh of the circuit.
2. Place a reference node (ground) outside of the circuit.
3. Draw lines between nodes such that each line crosses an element.
4. Replace the element by its dual pair.

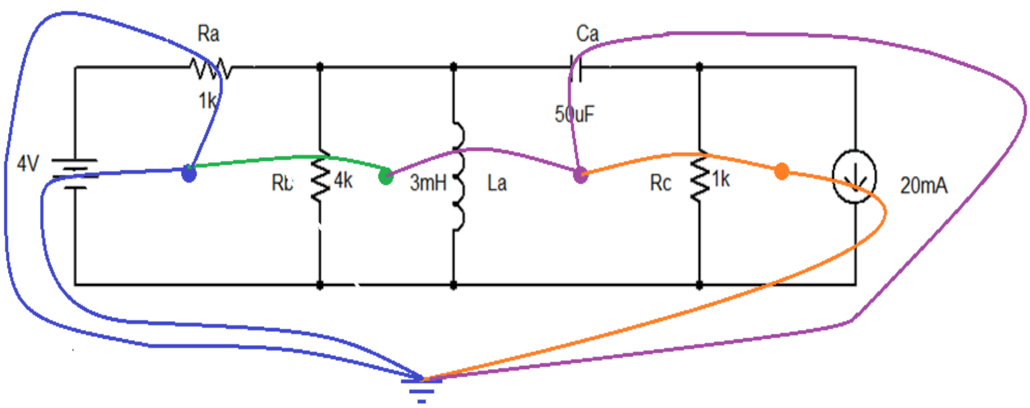
5. Determine the polarity of the voltage source and direction of the current source.

A voltage source that produces a positive mesh current has as its dual a current source that forces current to flow from the reference ground to the node associated with that mesh.

Example:



Follow the first three steps



Follow the 4th step replace the elements by its dual pairs

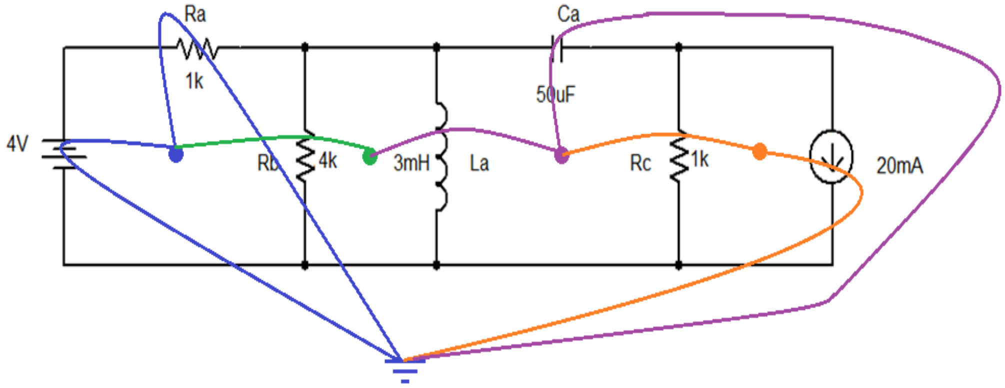
Component in Original circuit	**Its Dual**
Voltage source (4 V)	Current source (4 A)
Resistor Ra (1 kW)	Conductor R1 (1/1kW = 1 mW)
Resistor Rb (4 kW)	Conductor R2 (1/4kW = 0.25 mW)
Resistor Rc (4 kW)	Conductor R3 (1/1kW = 1 mW)
Inductor La (3 mH)	Capacitor C1 (3 mF)
Capacitor Ca (50 mF)	Inductor L1 (50 mH)
Current Source (20 mA)	Voltage source (20 mV)

Follow the 5th step

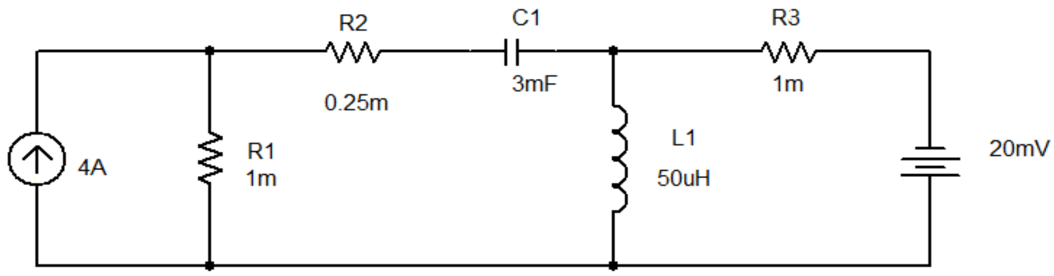
- In the original circuit:
  - The voltage source forces current to flow towards Ra.

    - Its dual should force current to flow from the reference ground to the node that is shared by the current source and R1, the dual of Ra.
- The current source causes current to flow from the node where Rc is connected towards the other meshes.
    - Its dual should cause current to flow from the node between it and R3 to distributed node (reference) of the rest of the circuit.

Draw Dual Circuit



Its dual circuit is



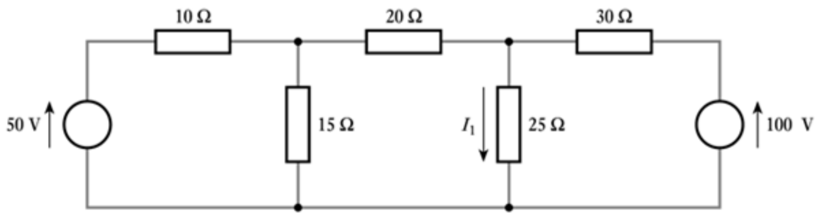
Nodal Analysis:

The aim of nodal analysis is to determine the voltage at each node relative to the reference node (or ground). Once you have done this you can easily work out anything else you need.

The procedure for analyzing a circuit with the node method is based on the following steps.

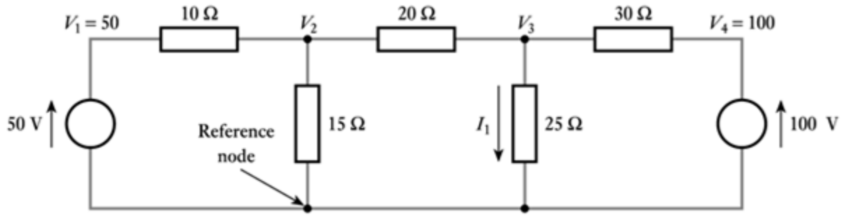
1. Clearly label all circuit parameters and distinguish the unknown parameters from the known.
2. Identify all nodes of the circuit.
3. Select a node as the reference node also called the ground and assign to it a potential of 0Volts. all other voltages in the circuit are measured with respect to the reference node.
   There are a few general guidelines that for the selection of the reference node.
   i) A useful reference node is one which has the largest number of elements connected to it.
   ii) A useful reference node is one which is connected to the maximum number of voltage sources.
4. Label the voltages at all other nodes.
5. Assign and label polarities.
6. Apply KCL at each node and express the branch currents in terms of the node voltages.
7. Solve the resulting simultaneous equations for the node voltages.
8. Now that the node voltages are known, the branch currents may be obtained from Ohm's law.

Example: Determine the current *I*1 in the following circuit



Sol.

1. Label all circuit parameters and distinguish the unknown parameters from the known.
2. Identify all nodes of the circuit and label node voltages.
3. Select a node as the reference node.



4. Apply KCL at node 2(V_2) and 3(V_3)

$$\frac{50 - V_2}{10} + \frac{V_3 - V_2}{20} + \frac{0 - V_2}{15} = 0 \quad \text{---}\rightarrow 1$$

$$\frac{V_2 - V_3}{20} + \frac{100 - V_3}{30} + \frac{0 - V_3}{25} = 0 \quad \text{-}\rightarrow 2$$

Solving the above two equations gives

$V2 = 32.34$ V

$V3 = 40.14$ V

and the required current is given by

$$I_1 = \frac{V_3}{25\,\Omega} = \frac{40.14\text{ V}}{25\,\Omega} = 1.6\text{ A}$$

<u>Nodal analysis with floating voltage sources (or) The Supernode</u>.

If a voltage source is not connected to the reference node it is called a floating voltage source.

In the circuit of Figure 6 the voltage source V_y is not connected to the reference node and thus it is a floating voltage source.

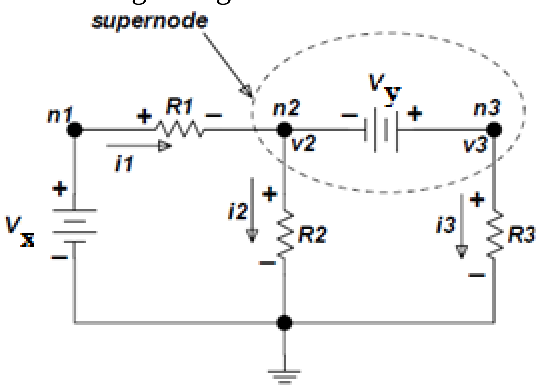


Figure 6. Circuit with a supernode.

The part of the circuit enclosed by the dotted ellipse is called a supernode. Kirchhoff's current law may be applied to a supernode in the same way that it is applied to any other regular node.

In our example application of KCL at the supernode gives

i1=i2+i3-------------1

In term of the node voltages Equation (4.22) becomes:

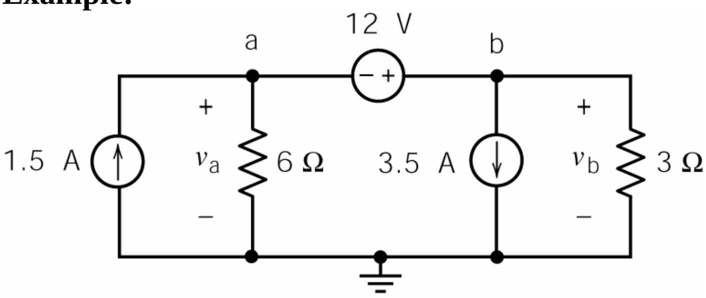
$$\frac{V_x - v2}{R1} = \frac{v2}{R2} + \frac{v3}{R3} \quad \text{--------2}$$

The relationship between node voltages ***v2*** and ***v3*** is the constraint that is needed in order to completely define the problem. The constraint is provided by the voltage source. ***Vy***

Vy=V3-V2-----------3

From 2 and 3 equations determine the node voltages ***v2*** and ***v3***

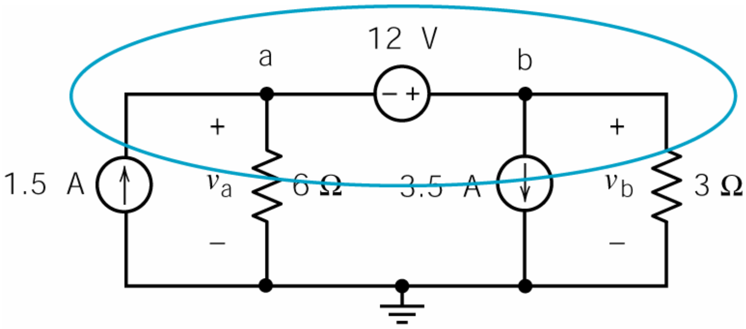
Example:



Determine the values of the node voltages, v_a and $v_{b,}$ for this circuit.

Sol:

Identify the supernode corresponding to the voltage source(Shown below in vowel shape).



Apply KCL to the supernode to get

$$1.5 = \frac{v_a}{6} + 3.5 + \frac{v_b}{3} \quad \Rightarrow \quad -2.0 = \frac{v_a}{6} + \frac{v_b}{3} \text{ -----------1}$$

The voltage source voltage is related to the node voltages at by

$$v_b - v_a = 12 \quad \Rightarrow \quad v_b = v_a + 12 \text{ ----------------2}$$

By solving the 1 and 2 equations

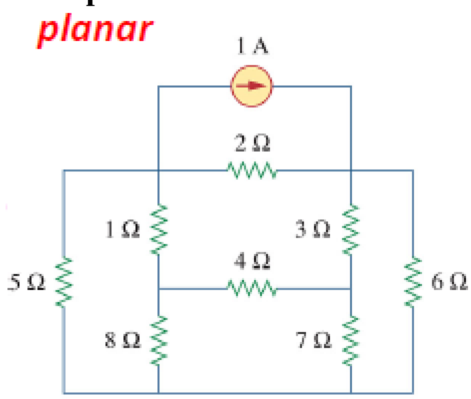
$$v_a = -12 \text{ V} \quad \text{and} \quad v_b = 0 \text{ V}$$

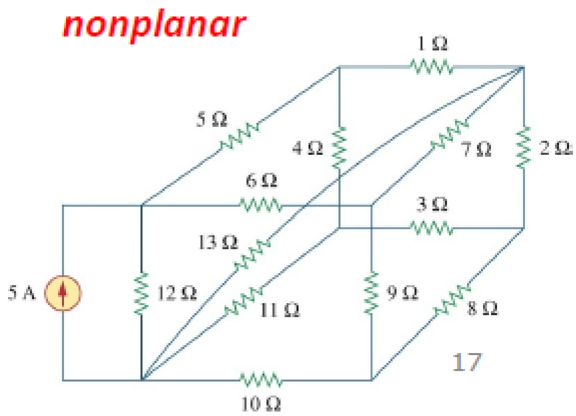
<u>Mesh Analysis</u>

The aim of this method is to find branch currents and Mesh analysis is only applicable to a circuit that is ***planar***.

- A ***planar*** circuit is one that can be drawn in a plane with no branches crossing one another.

Example:





<u>The procedure for obtaining the solution with mesh analysis as follow the steps are given below.</u>

1. Clearly label all circuit parameters and distinguish the unknown parameters from the known.
2. Identify all meshes of the circuit.
   - A **mesh** is defined as a loop which does not contain any other loops.
3. Assign mesh currents and label polarities.
4. Apply KVL at each mesh and express the voltages in terms of the mesh currents.
5. Solve the resulting simultaneous equations for the mesh currents.
6. Now that the mesh currents are known, the voltages may be obtained from Ohm's law**.**

Example:

Our circuit example has three loops but only two meshes as shown on Figure 9. Note that we have assigned a ground potential to a certain part of the circuit.

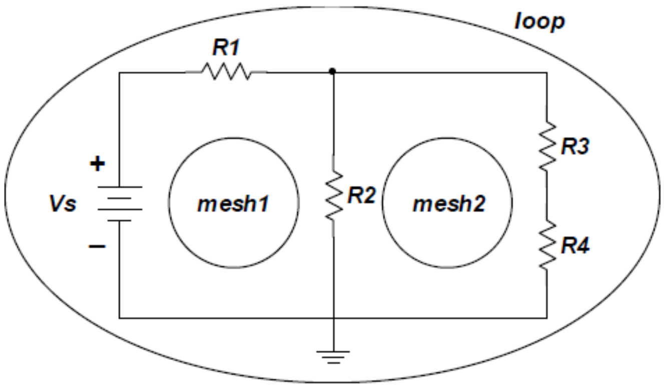


Figure 9. Identification of the meshes

Identify the meshes, they are mesh1 and mesh2 and assign the mesh currents, define current direction and voltage polarities.

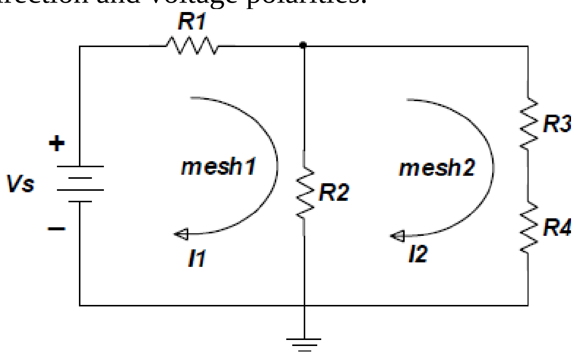
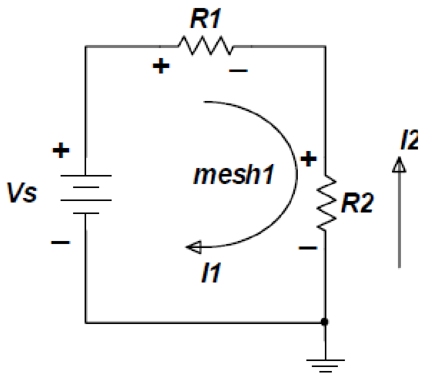


Figure 10. labeling mesh current direction



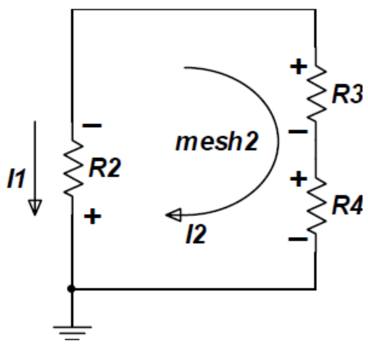


Figure 11. Sub-circuit for mesh1

Figure 12. Sub-circuit for mesh2

Apply KVL to mesh1 and mesh2 write the KVL equations as follows

$I1R1 + (I1 - I2)R2 - Vs = 0$ --------1

$I2(R3 + R4) + (I2 - I1)R2 = 0$ -----2

Rewrite the equations 1 and 2

$$\boxed{\begin{array}{l} I1(R1 + R2) - I2R2 = Vs \\ \\ -I1R2 + I2(R2 + R3 + R4) = 0 \end{array}}$$ --------3

Solve the equation for the mesh currents ***I1*** and ***I2***

NBKRIST

BASIC ELECTRICAL SCIENCES
LECTURE NOTES

UNIT-5

I.PRABHAKAR REDDY
DEPARTMENT OF ELECTRICAL AND ELECTRONICS ENGINEERING

Magnetically Coupled Circuits

The circuits we have considered so far may be regarded as *conductively coupled*, because one loop affects the neighboring loop through current conduction. When two loops with or without contacts between them affect each other through the magnetic field generated by one of them, they are said to be *magnetically coupled*.

The transformer is an electrical device designed on the basis of the concept of magnetic coupling. It uses magnetically coupled coils to transfer energy from one circuit to another. Transformers are key circuit elements.

Self Inductance

Self inductance is the ration between the induced Electro Motive Force (EMF) across a coil to the rate of change of current through this coil. **Self inductance** is related term to self induction phenomenon. Because of self induction self inductance generates. Self-inductance or Co-efficient of Self-induction is denoted as L. Its unit is Henry (H). First we have to know what self induction is. Self induction is the phenomenon by which in a coil a change in electric current produces an induced Electro Motive Force across this coil itself. This induced Electro Motive Force (ε) across this coil is proportional to the current changing rate. The higher the rate of change in current, the higher the value of EMF.

We can write that,

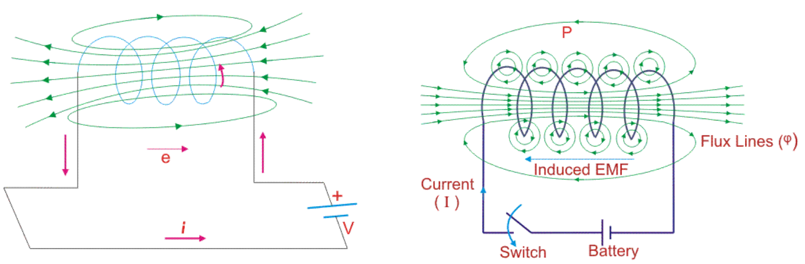
$$\varepsilon \propto \frac{dI}{dt},\ or,\ \varepsilon = L\frac{dI}{dt}$$

$$L = \frac{\varepsilon}{\frac{dI}{dt}} = self\ inductance\ \ or\ \ co-efficient\ of\ self\ induction$$

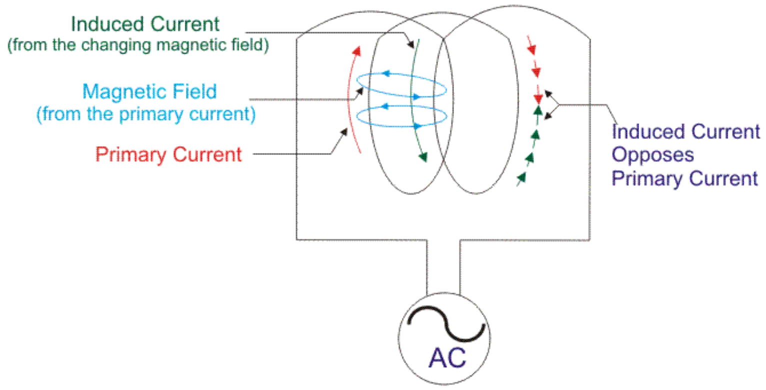
$$\varepsilon = -L\frac{dI}{dt}$$

But the actual equation is This Induced EMF across this coil is always opposite to the direction of the rate of change of current as per Lenz's Law.

When current (I) flows through a coil some electric flux produces inside the coil in the direction of the current flowing. At that moment of self induction phenomenon, the induced EMF generates to oppose this rate of change of current in that coil. So their values are same but sign differs. Look at the figure below.

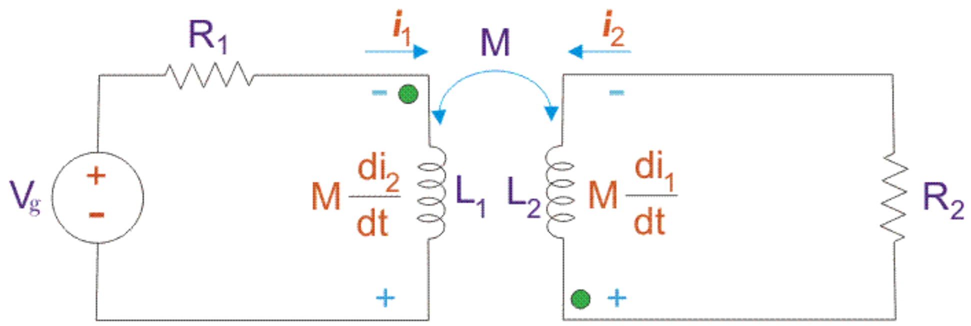


Take a closer look at a coil that is carrying current. The magnetic field forms concentric loops that surround the wire and join to form larger loops that surround the coil. When the current increases in one loop the expanding magnetic field will cut across some or all of the neighboring loops of wire, inducing a voltage in these loops.



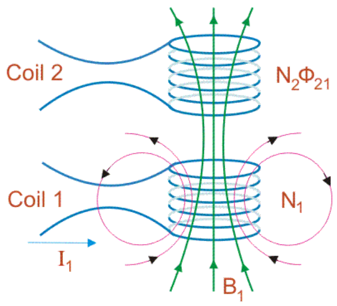
Mutual Inductance

Mutual Inductance is the ratio between induced Electro Motive Force across a coil to the rate of change of current of another adjacent coil in such a way that two coils are in possibility of flux linkage.Mutual induction is a phenomenon when a coil gets induced in EMF across it due to rate of change current in adjacent coil in such a way that the flux of one coil current gets linkage of another coil. Mutual inductance is denoted as (M), it is called co-efficient of Mutual Induction between two coils.



Mutual inductance for two coils gives the same value when they are in mutual induction with each other. Induction in one coil due to its own rate of change of current is called self inductance (L), but due to rate of change of current of adjacent coil it gives **mutual inductance** (M).

From the above figure, first coil carries current i_1 and its self inductance is L_1. Along with its self inductance it has to face mutual induction due to rate of change of current i_2 in the second coil. Same case happens in the second coil also. Dot convention is used to mark the polarity of the mutual induction. Suppose two coils are placed nearby.



Coil 1 carries I_1 current having N_1 number of turn. Now the flux density created by the coil 1 is B_1. Coil 2 with N_2 number of turn gets linked with this flux from coil 1. So flux linkage in coil 2 is N_2 . φ_{21} [φ_{21} is called leakage flux in coil 2 due to coil 1].

Consider φ_{21} is also changing with respect to time, so an EMF appears across coil 2. This EMF is called mutually induced EMF.

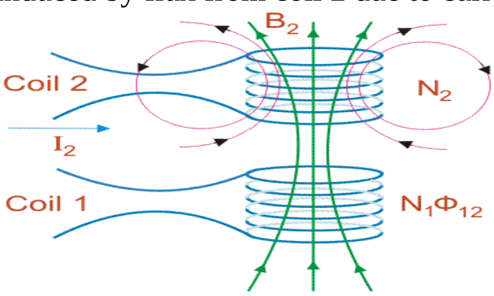
$$\varepsilon_2 = -N_2 . \frac{d\varphi_{21}}{dt} \ volt.$$

$$Again, \ \ \varepsilon_2 = -M_{21} . \frac{dl_1}{dt} \ volt.$$

Now it can be written from these equations,

$$M_{21} = \frac{\varphi_{21} N_2}{I_1}$$

Again, coil 1 gets induced by flux from coil 2 due to current I2 in the coil 2.



In same manner it can be written that for coil 1.

$$M_{21} = \frac{\varphi_{21} N_2}{I_1}$$

However, using the reciprocity theorem which combines Ampere's law and the Biot-Savart law, one may show that the constants are equal. i.e. $M_{12} = M_{21} = M$. M is the mutual inductance for both coil in Henry. **The value of mutual inductance is a function of the self-inductances** Suppose two coils are place nearby such that they are in mutual induction. L_1

and L_2 are co-efficient of self-induction of them. M is the mutual inductance.

$$Now,\ \ L_1 = \frac{N_1^2 . \mu_0 . A_1}{l_1}\ and\ L_2 = \frac{N_2^2 . \mu_0 . A_2}{l_2}, M\ \alpha\ L_1\ and\ M\ \alpha\ L_2$$

$$So,\ M^2\ \alpha\ L_1 . L_2$$

$$Or,\ M^2 = \text{ƙ}.\ L_1 . L_2$$

$$Or,\ M = \text{ƙ}\sqrt{M_1 . M_2}$$

Here, ƙ is called co-efficient of coupling and it is defined as the ratio of mutual inductance actually present between the two coils to the maximum possible value. If the flux due to first coil completely links with second coil, then ƙ = 1, then two coils are tightly coupled. Again if no linkage at all then ƙ = 0 and hence two coils are magnetically isolated. Merits and demerits of mutual inductance: Due to mutual inductance, transformer establishes its operating principle. But due to mutual inductance, in any circuit having inductors, has to face extra voltage drop.

Dot Convention

The self-inductance of a circuit is intimately associated with the magnetic field linking the circuit. The self-inductance emf may be thought of as the emf induced in the circuit by a magnetic field produced by the circuit current.

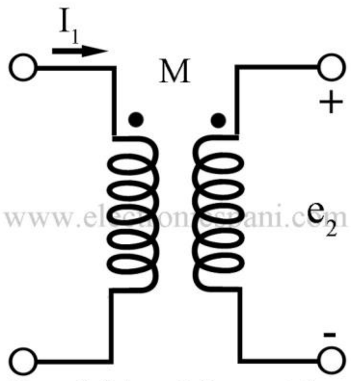
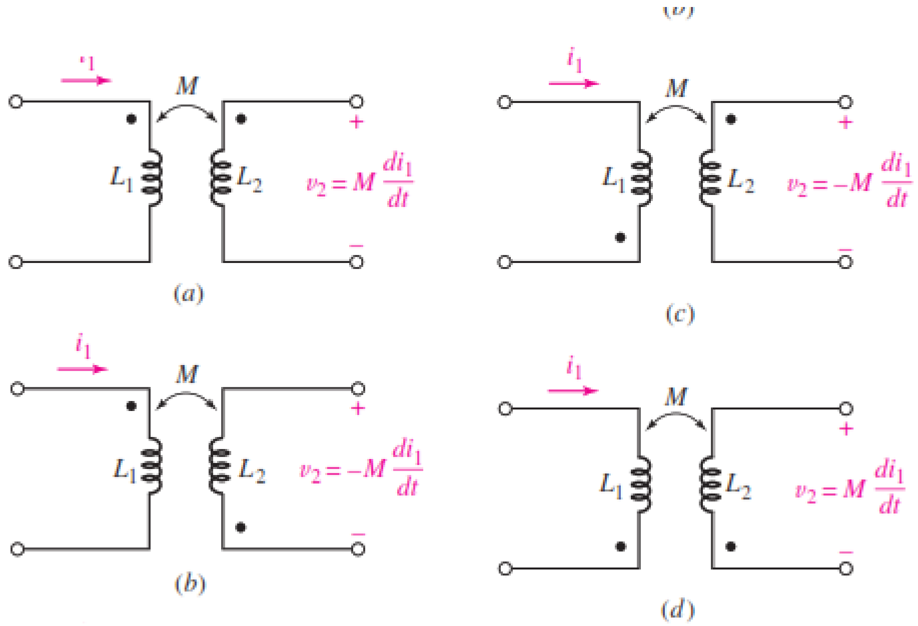


Figure 2: Schematic Representation of Mutual Inductance

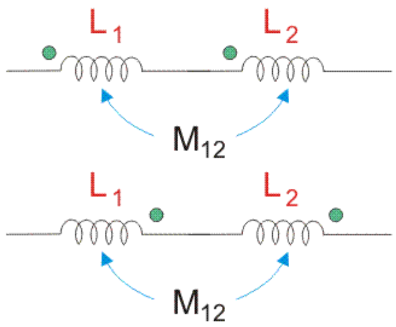
Fig .5.2

Since a magnetic field exists in the region around the current that develops it, there is also may a possibility that an emf be induced in the other circuits linked by the field. Two circuits linked by the same magnetic field are said to be *coupled* to each other. The circuit element used to represent *magnetic coupling* is shown in Figure 5.2 and is called *mutual inductance*. It is represented by symbol M and is measured in henrys. The volt-ampere relationship is one which gives the induced emf in one circuit by a current in another and is given as

$$e_2 = M\frac{di_1}{dt}$$



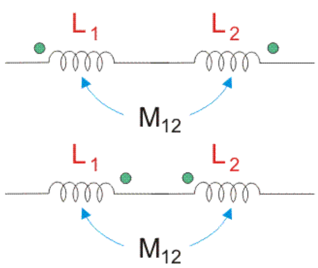
How to find out L_{eq} in a circuit having mutual inductance with dot convention Suppose two coils are in series with same place dot.



Mutual inductance between them is positive.

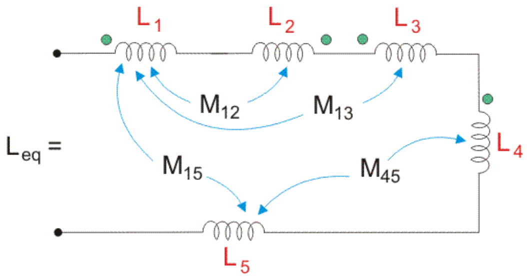
$$So,\quad L_{eq} = L_1 + L_2 + 2M_{12}$$

Suppose two coils are in series with opposite place dot.



$$So, \quad L_{eq} = L_1 + L_2 - 2M_{12}$$

When a few numbers of inductors are in series with mutual inductances.



$$L_{eq} = L_1 + L_2 + L_3 + L_4 + L_5 - 2M_{12} + 2M_{13} + 2M_{15} - 2M_{23} + 2M_{45}$$

Inductor in parallel

Let us consider two coils of inductance L1 and L_2 connected in parallel as shown in figure 8

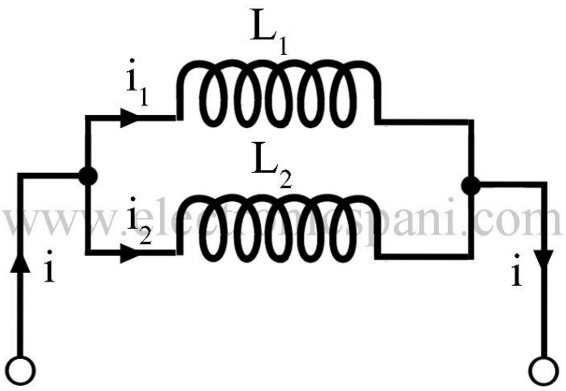


Figure 8: Inductor in Parallel

The supply circuit divides into two components i_1 and i_2 following through the coils.

i.e. $I = i_1 + i_2$

or $\frac{di}{dt} = \frac{di_1}{dt} + \frac{di_2}{dt}$

Self-induced emf in coil A, $e_{LA} = -L_1 \frac{di_1}{dt}$

Mutually induced emf in coil A due to change of current in coil B,

$$e_{MA} = -M\frac{di_2}{dt}$$

Where M is the mutual coefficient of inductance

Resultant emf induced in coil A,

$$= e_{LA} + e_{MA} = -(L_1\frac{di_1}{dt} + M\frac{di_2}{dt})$$

Similarly, resultant emf induced in coil B

$$= -(L_2\frac{di_2}{dt} + M\frac{di_1}{dt})$$

As both coils are connected in parallel, therefore, resultant emf induced in both of the coils must be equal

$$L_1\frac{di_1}{dt} + M\frac{di_2}{dt} = L_2\frac{di_2}{dt} + M\frac{di_1}{dt}$$

Or $(L_1 - M)\frac{di_1}{dt} = (L_2 - M)\frac{di_2}{dt}$

Or $\frac{di_1}{dt} = [\frac{L_2 - M}{L_1 - M}]\frac{di_2}{dt}$ ……(11)

Or $\frac{di_1}{dt} + \frac{di_2}{dt} = [\frac{L_2 - M}{L_1 - M} + 1]\frac{di_2}{dt}$

$$\frac{di}{dt} = (\frac{L_1 + L_2 - 2M}{L_1 - M})\frac{di_2}{dt} \quad ……(12)$$

If L is the equivalent inductance of the combination then induced emf

$$= -L\frac{di}{dt}$$

Since induced emf in parallel combination = Induced emf in either of the coils.

$$L\frac{di}{dt} = L_1\frac{di_1}{dt} + M\frac{di_2}{dt}$$

Or

$$L = \frac{L_1\frac{di_1}{dt} + M\frac{di_2}{dt}}{\frac{di}{dt}}$$

Substituting $\frac{di_1}{dt} = [\frac{L_2 - M}{L_1 - M}]\frac{di_2}{dt}$ from equation (11) and $\frac{di}{dt} = [\frac{L_1 + L_2 - 2M}{L_1 - M}]\frac{di_2}{dt}$ from equation (12) we get

$$L = \frac{L_1[\frac{L_2 - M}{L_1 - M}]\frac{di_2}{dt} + M\frac{di_2}{dt}}{[\frac{L_1 + L_2 - 2M}{L_1 - M}]\frac{di_2}{dt}}$$

Or $$L = \frac{L_1L_2 - L_1M + L_1M - M^2}{L_1 + L_2 - 2M}$$

Or $$L = \frac{L_1L_2 - M^2}{L_1 + L_2 - 2M}$$

When mutual flux helps the individual flux

Or $$L = \frac{L_1L_2 - M^2}{L_1 + L_2 + 2M}$$

When mutual flux opposes the individual flux.

NBKRIST

BASIC ELECTRICAL SCIENCES
LECTURE NOTES

UNIT-6

I.PRABHAKAR REDDY
DEPARTMENT OF ELECTRICAL AND ELECTRONICS ENGINEERING

Resonance & Locus Diagram

8.1 SERIES RESONANCE

In many electrical circuits, resonance is a very important phenomenon. The study of resonance is very useful, particularly in the area of communications. For example, the ability of a radio receiver to select a certain frequency, transmitted by a station and to eliminate frequencies from other stations is based on the principle of resonance. In a series RLC circuit, the current lags behind, or leads the applied voltage depending upon the values of X_L and X_C. X_L causes the total current to lag behind the applied voltage, while X_C causes the total current to lead the applied voltage. When $X_L > X_C$, the circuit is predominantly inductive, and when $X_C > X_L$, the circuit is predominantly capacitive. However, if one of the parameters of the series RLC circuit is varied in such a way that the current in the circuit is in phase with the applied voltage, then the circuit is said to be in resonance.

Consider the series RLC circuit shown in Fig. 8.1.

The total impedance for the series RLC circuit is

$$Z = R + j(X_L - X_C) = R + j\left(\omega L - \frac{1}{\omega C}\right)$$

$$Z = R + j(X_L - X_C) = R + j\left(\omega L - \frac{1}{\omega C}\right)$$

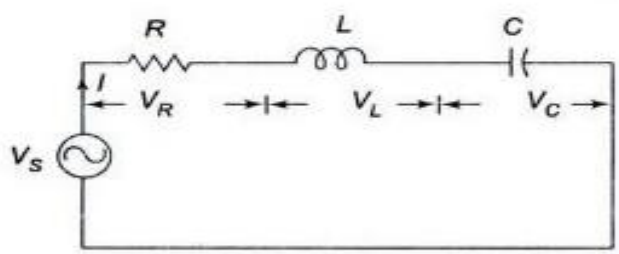


Fig. 8.1

It is clear from the circuit that the current $I = V_S/Z$.

The circuit is said to be in resonance if the current is in phase with the applied voltage. In a series RLC circuit, series resonance occurs when $X_L = X_C$. The frequency at which the resonance occurs is called the *resonant frequency*.

Since $X_L = X_C$, the impedance in a series RLC circuit is purely resistive. At the resonant frequency, f_r, the voltages across capacitance and inductance are equal in magnitude. Since they are 180° out of phase with each other, they cancel each other and, hence zero voltage appears across the LC combination.

At resonance

$$X_L = X_C \quad \text{i.e.} \quad \omega L = \frac{1}{\omega C}$$

$$2\pi f_r L = \frac{1}{2\pi f_r C}$$

$$f_r^2 = \frac{1}{4\pi^2 LC}$$

$$\therefore \quad f_r = \frac{1}{2\pi\sqrt{LC}}$$

In a series RLC circuit, resonance may be produced by varying the frequency, keeping L and C constant; otherwise, resonance may be produced by varying either L or C for a fixed frequency.

Example 8.1 *For the circuit shown in Fig. 8.2, determine the value of capacitive reactance and impedance at resonance.*

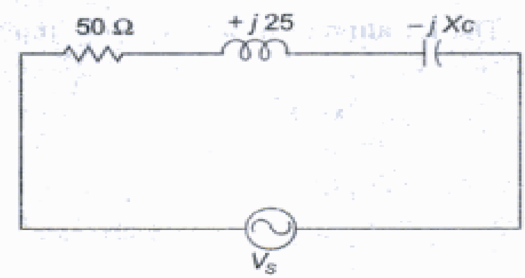


Fig. 8.2

Solution At resonance

$$X_L = X_C$$

Since $X_L = 25\ \Omega$

$$X_C = 25\ \Omega \qquad \therefore \frac{1}{\omega C} = 25$$

The value of impedance at resonance is

$$Z = R$$

$$\therefore \quad Z = 50\ \Omega$$

8.2 IMPEDANCE AND PHASE ANGLE OF A SERIES RESONANT CIRCUIT

The impedance of a series RLC circuit is

$$|Z| = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$$

The variation of X_C and X_L with frequency is shown in Fig. 8.4.

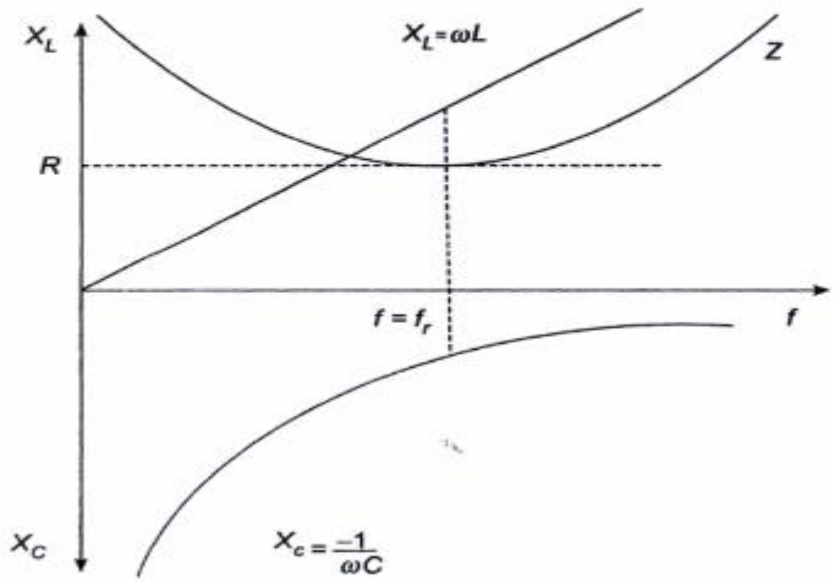


Fig. 8.4

At zero frequency, both X_C and Z are infinitely large, and X_L is zero because at zero frequency the capacitor acts as an open circuit and the inductor acts as a short circuit. As the frequency increases, X_C decreases and X_L increases. Since X_C is larger than X_L, at frequencies below the resonant frequency f_r, Z decreases along with X_C. At resonant frequency f_r, $X_C = X_L$, and $Z = R$. At frequencies above the resonant frequency f_r, X_L is larger than X_C, causing Z to increase. The phase angle as a function of frequency is shown in Fig. 8.5.

At a frequency below the resonant frequency, current leads the source voltage because the capacitive reactance is greater than the inductive reactance. The phase angle decreases as the frequency approaches the resonant value, and is 0° at resonance. At frequencies above resonance, the current lags behind the source voltage, because the inductive reactance is greater than capacitive reactance. As the frequency goes higher, the phase angle approaches 90°.

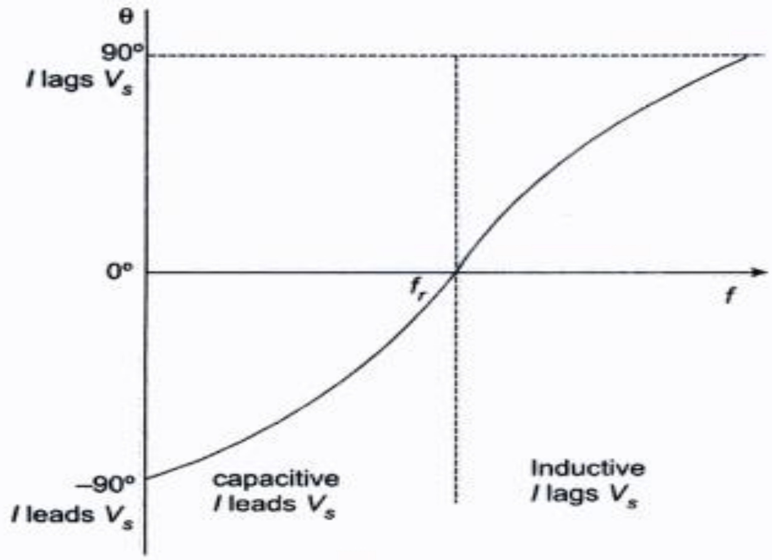


Fig. 8.5

Example 8.3 For the circuit shown in Fig. 8.6, determine the impedance at resonant frequency, 10 Hz above resonant frequency, and 10 Hz below resonant frequency.

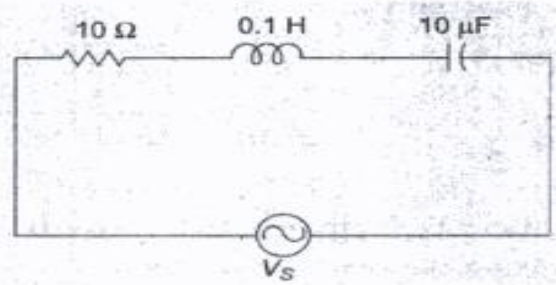


Fig. 8.6

Solution Resonant frequency

$$f_r = \frac{1}{2\pi\sqrt{LC}}$$

$$= \frac{1}{2\pi\sqrt{0.1\times10\times10^{-6}}} = 159.2 \text{ Hz}$$

At 10 Hz below f_r = 159.2 − 10 = 149.2 Hz

At 10 Hz above f_r = 159.2 + 10 = 169.2 Hz

Impedance at resonance is equal to R

$$\therefore \quad Z = 10\ \Omega$$

Capacitive reactance at 149.2 Hz is

$$X_{C_1} = \frac{1}{\omega_1 C} = \frac{1}{2\pi\times149.2\times10^{-6}\times10}$$

$$\therefore \quad X_{C_1} = 106.6\ \Omega$$

Capacitive reactance at 169.2 Hz is

$$X_{C_2} = \frac{1}{\omega_2 C} = \frac{1}{2\pi \times 169.2 \times 10 \times 10^{-6}}$$

$$\therefore \quad X_{C_2} = 94.06\ \Omega$$

Inductive reactance at 149.2 Hz is

$$X_{L_1} = \omega_2 L = 2\pi \times 149.2 \times 0.1 = 93.75\ \Omega$$

Inductive reactance at 169.2 Hz is

$$X_{L_2} = \omega_2 L = 2\pi \times 169.2 \times 0.1 = 106.31\ \Omega$$

Impedance at 149.2 Hz is

$$|Z| = \sqrt{R^2 + (X_{L_1} - X_{C_1})^2}$$

$$= \sqrt{(10)^2 + (93.75 - 106.6)^2}$$

$$= 16.28\ \Omega$$

Here X_{C_1} is greater than X_{L_1}, so Z is capacitive.
Impedance at 169.2 Hz is

$$|Z| = \sqrt{R^2 + (X_{L_2} - X_{C_2})^2}$$

$$= \sqrt{(10)^2 + (106.31 - 94.06)^2}$$

$$= 15.81\ \Omega$$

Here X_{L_1} is greater than X_{C_1}, so Z is inductive.

8.3 VOLTAGES AND CURRENTS IN A SERIES RESONANT CIRCUIT

The variation of impedance and current with frequency is shown in Fig. 8.7.

At resonant frequency, the capacitive reactance is equal to inductive reactance, and hence the impedance is minimum. Because of minimum impedance, maximum current flows through the circuit. The current variation with frequency is plotted.

The voltage drop across resistance, inductance and capacitance also varies with frequency. At $f = 0$, the capacitor acts as an open circuit and blocks current. The complete source voltage appears across the capacitor. As the frequency increases, X_C decreases and X_L increases, causing total reactance $X_C - X_L$ to decrease. As a result, the impedance decreases and the current increases. As the current increases, V_R also increases, and both V_C and V_L increase.

When the frequency reaches its resonant value f_r, the impedance is equal to R, and hence, the current reaches its maximum value, and V_R is at its maximum value.

As the frequency is increased above resonance, X_L continues to increase and X_C continues to decrease, causing the total reactance, $X_L - X_C$ to increase. As a result there is an increase in impedance and a decrease in current. As the current decreases, V_R also decreases, and both V_C and V_L decrease. As the frequency becomes very high, the current approaches zero, both V_R and V_C approach zero, and V_L approaches V_s.

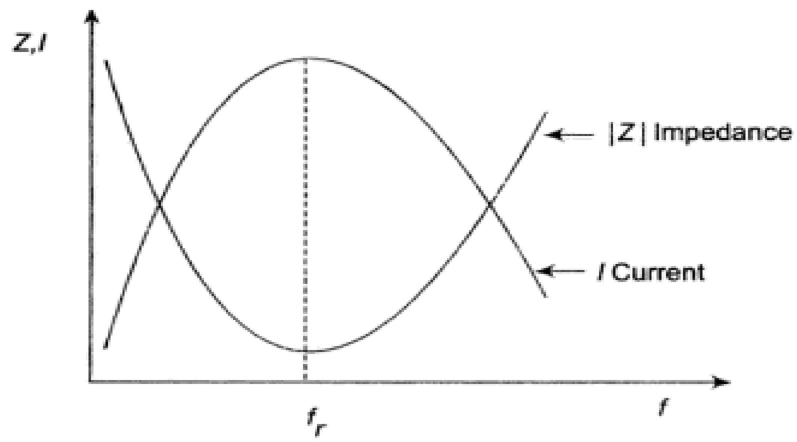


Fig. 8.7

The response of different voltages with frequency is shown in Fig. 8.8.

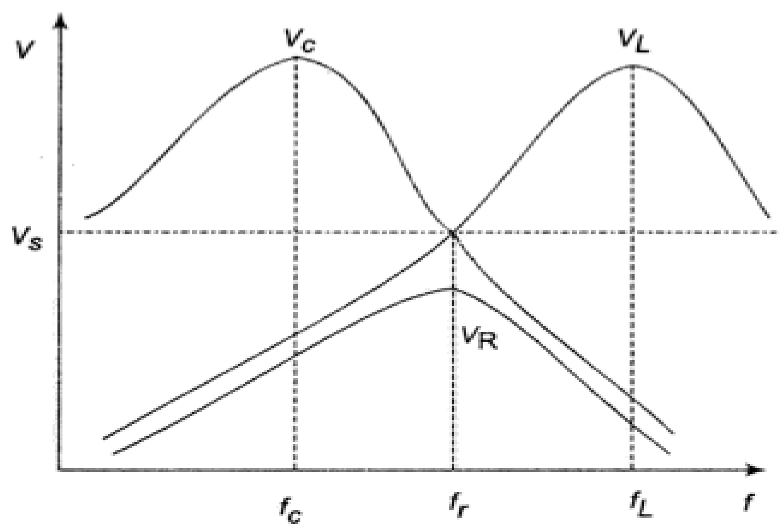


Fig. 8.8

The drop across the resistance reaches its maximum when $f = f_r$. The maximum voltage across the capacitor occurs at $f = f_c$. Similarly, the maximum voltage across the inductor occurs at $f = f_L$.

The voltage drop across the inductor is

$$V_L = IX_L$$

$$\text{where} \quad I = \frac{V}{Z}$$

$$\therefore \quad V_L = \frac{\omega LV}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}$$

To obtain the condition for maximum voltage across the inductor, we have to take the derivative of the above equation with respect to frequency, and make it equal to zero.

$$\therefore \quad \frac{dV_L}{d\omega} = 0$$

If we solve for ω, we obtain the value of ω when V_L is maximum.

$$\frac{dV_L}{d\omega} = \frac{d}{d\omega}\left\{\omega L V\left[R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2\right]^{-1/2}\right\}$$

$$\frac{LV\left(R^2 + \omega^2 L^2 - \frac{2L}{C} + \frac{1}{\omega^2 C^2}\right)^{-1/2} - \frac{\omega L V}{2}\left(R^2 + \omega^2 L^2 - \frac{2L}{C} + \frac{1}{\omega^2 C^2}\right)\left(2\omega L^2 - \frac{2}{\omega^3 C^2}\right)}{R^2 + \omega^2 L^2 - \frac{2L}{C} + \frac{1}{\omega^2 C^2}} = 0$$

From this

$$R^2 - \frac{2L}{C} + 2/\omega^2 C^2 = 0$$

$$\therefore \quad \omega L = \sqrt{\frac{2}{2LC - R^2 C^2}} = \frac{1}{\sqrt{LC}}\sqrt{\frac{2}{2 - \frac{R^2 C}{L}}}$$

$$f_L = \frac{1}{2\pi\sqrt{LC}}\sqrt{\frac{1}{1 - \frac{R^2 C}{2L}}}$$

Similarly, the voltage across the capacitor is

$$V_C = IX_C = \frac{1}{\omega C}$$

$$\therefore \quad V_C = \frac{V}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}} \times \frac{1}{\omega C}$$

To get maximum value $\frac{dV_C}{d\omega} = 0$

If we solve for ω, we obtain the value of ω when V_C is maximum.

$$\frac{dV_C}{d\omega} = \omega C \frac{1}{2}\left[R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2\right]^{-1/2}\left[2\left(\omega L - \frac{1}{\omega C}\right)\left(L + \frac{1}{\omega^2 C}\right)\right] + \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}\, C = 0$$

From this

$$\omega_C^2 = \frac{1}{LC} - \frac{R^2}{2L}$$

$$\omega_C = \sqrt{\frac{1}{LC} - \frac{R^2}{2L}}$$

$$\therefore \quad f_C = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{2L}}$$

The maximum voltage across the capacitor occurs below the resonant frequency; and the maximum voltage across the inductor occurs above the resonant frequency.

Example 8.4 *A series circuit with $R = 10\,\Omega$, $L = 0.1$ H and $C = 50\,\mu$F has an applied voltage $V = 50\angle 0°$ with a variable frequency. Find the resonant frequency, the value of frequency at which maximum voltage occurs across the inductor and the value of frequency at which maximum voltage occurs across the capacitor.*

Solution The frequency at which maximum voltage occurs across the inductor is

$$f_L = \frac{1}{2\pi\sqrt{LC}}\sqrt{\frac{1}{\left(1 - \dfrac{R^2C}{2L}\right)}}$$

$$= \frac{1}{2\pi\sqrt{0.1\times 50\times 10^{-6}}}\sqrt{\frac{1}{1 - \left(\dfrac{(10)^2\times 50\times 10^{-6}}{2\times 0.1}\right)}}$$

$$= 72.08 \text{ Hz}$$

Similarly $$f_C = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{2L}}$$

$$= \frac{1}{2\pi}\sqrt{\frac{1}{0.1\times 50\times 10^{-6}} - \frac{(10)^2}{2\times 0.1}}$$

$$= \frac{1}{2\pi}\sqrt{200000 - 500}$$

$$= 71.08 \text{ Hz}$$

Resonant frequency $$f_r = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{0.1\times 50\times 10^{-6}}} = 71.18 \text{ Hz}$$

It is clear that the maximum voltage across the capacitor occurs below the resonant frequency and the maximum inductor voltage occurs above the resonant frequency.

8.4 BANDWIDTH OF AN RLC CIRCUIT

The bandwidth of any system is the range of frequencies for which the current or output voltage is equal to 70.7% of its value at the resonant frequency, and it is denoted by BW. Figure 8.9 shows the response of a series RLC circuit.

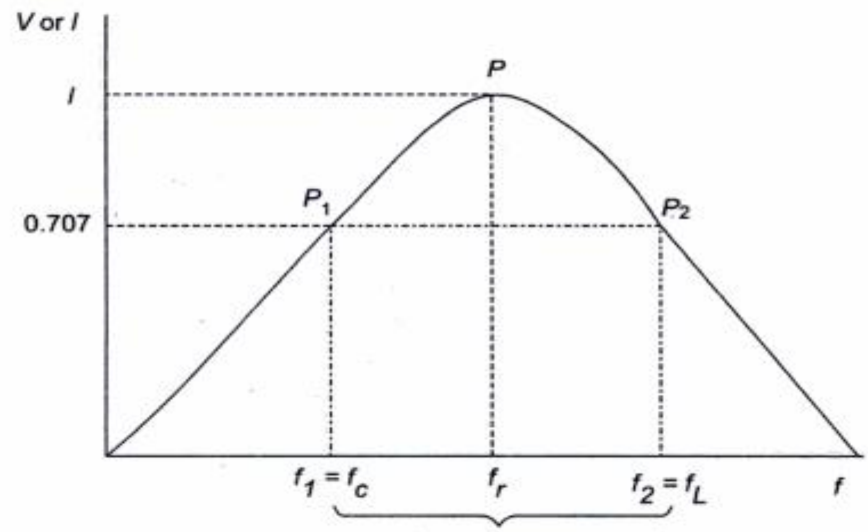


Fig. 8.9

Here the frequency f_1 is the frequency at which the current is 0.707 times the current at resonant value, and it is called the *lower cut-off frequency*. The frequency f_2 is the frequency at which the current is 0.707 times the current at resonant value (i.e. maximum value), and is called the *upper cut-off frequency*. The bandwidth, or BW, is defined as the frequency difference between f_2 and f_1.

$\therefore \quad BW = f_2 - f_1$

The unit of BW is hertz (Hz).

If the current at P_1 is $0.707 I_{max}$, the impedance of the circuit at this point is $\sqrt{2}R$, and hence

$$\frac{1}{\omega_1 C} - \omega_1 L = R \tag{8.1}$$

Similarly, $$\omega_2 L - \frac{1}{\omega_2 C} = R \tag{8.2}$$

If we equate both the above equations, we get

$$\frac{1}{\omega_1 C} - \omega_1 L = \omega_2 L - \frac{1}{\omega_2 C}$$

$$L(\omega_1 + \omega_2) = \frac{1}{C}\left(\frac{\omega_1 + \omega_2}{\omega_1 \omega_2}\right) \tag{8.3}$$

From Eq. 8.3, we get

$$\omega_1\omega_2 = \frac{1}{LC}$$

we have $\omega_r^2 = \dfrac{1}{LC}$

$$\therefore \quad \omega^2{}_r = \omega_1\omega_2 \tag{8.4}$$

If we add Eqs 8.1 and 8.2, we get

$$\frac{1}{\omega_1 C} - \omega_1 L + \omega_2 L - \frac{1}{\omega_2 C} = 2R$$

$$(\omega_2 - \omega_1)L + \frac{1}{C}\left(\frac{\omega_2 - \omega_1}{\omega_1\omega_2}\right) = 2R \tag{8.5}$$

Since $C = \dfrac{1}{\omega_r^2 L}$

and $\omega_1\omega_2 = \omega^2{}_r$

$$(\omega_2 - \omega_1)L + \frac{\omega_r^2 L(\omega_2 - \omega_1)}{\omega_r^2} = 2R \tag{8.6}$$

From Eq. 8.6, we have

$$\omega_2 - \omega_1 = \frac{R}{L} \tag{8.7}$$

$$\therefore \quad f_2 - f_1 = \frac{R}{2\pi L} \tag{8.8}$$

or $BW = \dfrac{R}{2\pi L}$

From Eq. 8.8, we have

$$f_2 - f_1 = \frac{R}{2\pi L}$$

$$\therefore \quad f_r - f_1 = \frac{R}{4\pi L}$$

$$f_2 - f_r = \frac{R}{4\pi L}$$

The lower frequency limit $\quad f_1 = f_r - \dfrac{R}{4\pi L} \quad$ (8.9)

The upper frequency limit $\quad f_2 = f_r + \dfrac{R}{4\pi L} \quad$ (8.10)

If we divide the equation on both sides by f_r, we get

$$\frac{f_2 - f_1}{f_r} = \frac{R}{2\pi f_r L} \tag{8.11}$$

Here an important property of a coil is defined. It is the ratio of the reactance of the coil to its resistance. This ratio is defined as the Q of the coil. Q is known as a *figure of merit*, it is also called *quality factor* and is an indication of the quality of a coil.

$$Q = \frac{X_L}{R} = \frac{2\pi f_r L}{R} \tag{8.12}$$

If we substitute Eq. 8.11 in Eq. 8.12, we get

$$\frac{f_2 - f_1}{f_r} = \frac{1}{Q} \tag{8.13}$$

The upper and lower cut-off frequencies are sometimes called the *half-power frequencies*. At these frequencies the power from the source is half of the power delivered at the resonant frequency.

At resonant frequency, the power is

$$P_{\max} = I^2_{\max} R$$

At frequency f_1, the power is $P_1 = \left(\frac{I_{\max}}{\sqrt{2}}\right)^2 R = \frac{I^2_{\max} R}{2}$

Similarly, at frequency f_2, the power is

$$P_2 = \left(\frac{I_{\max}}{\sqrt{2}}\right)^2 R$$

$$= \frac{I^2_{\max} R}{2}$$

The response curve in Fig. 8.9 is also called the *selectivity curve* of the circuit. Selectivity indicates how well a resonant circuit responds to a certain frequency and eliminates all other frequencies. The narrower the bandwidth, the greater the selectivity.

Example 8.5 *Determine the quality factor of a coil for the series circuit consisting of $R = 10\ \Omega$, $L = 0.1$ H and $C = 10\ \mu F$.*

Solution Quality factor $Q = \frac{f_r}{BW}$

$$f_r = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{0.1 \times 10 \times 10^{-6}}} = 159.2 \text{ Hz}$$

At lower half power frequency, $X_C > X_L$

$$\frac{1}{2\pi f_1 C} - 2\pi f_1 L = R$$

From which $f_1 = \frac{-R + \sqrt{R^2 + 4L/C}}{4\pi L}$

At upper half power frequency $X_L > X_C$

$$2\pi f_2 L - \frac{1}{2\pi f_2 C} = R$$

From which $f_2 = \dfrac{+R + \sqrt{R^2 + 4L/C}}{4\pi L}$

Bandwidth $BW = f_2 - f_1 = \dfrac{R}{2\pi L}$

Hence $Q_0 = \dfrac{f_r}{BW} = \dfrac{2\pi f_r L}{R} = \dfrac{2\times\pi\times 159.2\times 0.1}{10}$

$$Q_0 = \frac{f_r}{BW} = 10$$

8.5 THE QUALITY FACTOR (Q) AND ITS EFFECT ON BANDWIDTH

The quality factor, Q, is the ratio of the reactive power in the inductor or capacitor to the true power in the resistance in series with the coil or capacitor.

The quality factor

$$Q = 2\pi \times \frac{\text{maximum energy stored}}{\text{energy dissipated per cycle}}$$

In an inductor, the max energy stored is given by $\dfrac{LI^2}{2}$

$$\text{Energy dissipated per cycle} = \left(\frac{I}{\sqrt{2}}\right)^2 R \times T = \frac{I^2 RT}{2}$$

$$\therefore \text{ Quality factor of the coil } Q = 2\pi \times \frac{\frac{1}{2}LI^2}{\frac{I^2 R}{2}\times\frac{1}{f}}$$

$$= \frac{2\pi f L}{R} = \frac{\omega L}{2}$$

Similarly, in a capacitor, the max energy stored is given by $\dfrac{CV^2}{2}$

The energy dissipated per cycle $= (I/\sqrt{2})^2 \; R \times T$

The quality factor of the capacitance circuit

$$Q = \frac{2\pi \frac{1}{2} C \left(\frac{1}{\omega C}\right)^2}{\frac{I^2}{2} R \times \frac{1}{f}} = \frac{1}{\omega CR}$$

In series circuits, the quality factor $Q = \dfrac{\omega L}{R} = \dfrac{1}{\omega CR}$

We have already discussed the relation between bandwidth and quality factor, which is $Q = \dfrac{f_r}{BW}$.

A higher value of circuit Q results in a smaller bandwidth. A lower value of Q causes a larger bandwidth.

Example 8.6 For the circuit shown in Fig. 8.10, determine the value of Q at resonance and bandwidth of the circuit.

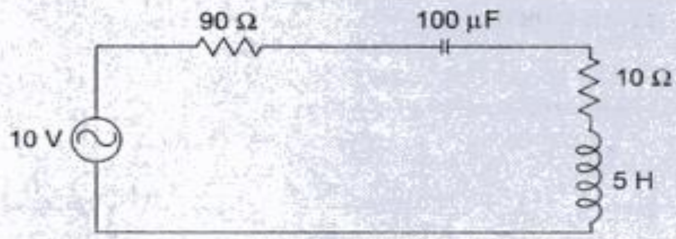


Fig. 8.10

Solution The resonant frequency,

$$f_r = \frac{1}{2\pi\sqrt{LC}}$$

$$= \frac{1}{2\pi\sqrt{5\times100\times10^{-6}}}$$

$$= 7.12 \text{ Hz}$$

Quality factor $Q = X_L/R = 2\pi f_r L/R$

$$= \frac{2\pi\times7.12\times5}{100} = 2.24$$

Bandwidth of the circuit is $BW = \frac{f_r}{Q} = \frac{7.12}{2.24} = 3.178$ Hz

8.6 MAGNIFICATION IN RESONANCE

If we assume that the voltage applied to the series RLC circuit is V, and the current at resonance is I, then the voltage across L is $V_L = IX_L = (V/R)\,\omega_r L$

Similarly, the voltage across C

$$V_C = IX_C = \frac{V}{R\omega_r C}$$

Since $Q = 1/\omega_r CR = \omega_r L/R$

where ω_r is the frequency at resonance.

Therefore $V_L = VQ$

$V_C = VQ$

The ratio of voltage across either L or C to the voltage applied at resonance can be defined as magnification.

$\therefore$ Magnification $= Q = V_L/V$ or V_C/V

8.7 PARALLEL RESONANCE

Basically, parallel resonance occurs when $X_C = X_L$. The frequency at which resonance occurs is called the *resonant* frequency. When $X_C = X_L$, the two branch currents are equal in magnitude and 180° out of phase with each other. Therefore, the two currents cancel each other out, and the total current is zero. Consider the circuit shown in Fig. 8.11. The condition for resonance occurs when $X_L = X_C$.

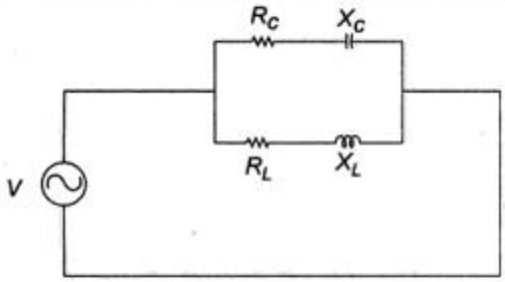


Fig. 8.11

In Fig. 8.11, the total admittance

$$Y = \frac{1}{R_L + j\omega L} + \frac{1}{R_C - (j/\omega C)}$$

$$= \frac{R_L - j\omega L}{R_L^2 + \omega^2 L^2} + \frac{R_C + (j/\omega C)}{R_C^2 + \dfrac{1}{\omega^2 C^2}}$$

$$= \frac{R_L}{R_L^2 + \omega^2 L^2} + \frac{R_C}{R_C^2 + \dfrac{1}{\omega^2 C^2}} + j\left\{\left[\frac{1/\omega C}{R_C^2 + \dfrac{1}{\omega^2 C^2}}\right] - \left[\frac{\omega L}{R_L^2 + \omega^2 L^2}\right]\right\} \quad (8.14)$$

At resonance the susceptance part becomes zero

$$\therefore \quad \frac{\omega_r L}{R_L^2 + \omega_r^2 L^2} = \frac{\dfrac{1}{\omega_r C}}{R_C^2 + \dfrac{1}{\omega_r^2 C^2}} \quad (8.15)$$

$$\omega_r L\left[R_C^2 + \frac{1}{\omega_r^2 C^2}\right] = \frac{1}{\omega_r C}\left[R_L^2 + \omega_r^2 L^2\right]$$

$$\omega_r^2\left[R_C^2 + \frac{1}{\omega_r^2 C^2}\right] = \frac{1}{LC}\left[R_L^2 + \omega_r^2 L^2\right]$$

$$\omega_r^2 R_C^2 - \frac{\omega_r^2 L}{C} = \frac{1}{LC} R_L^2 - \frac{1}{C^2}$$

$$\omega_r^2\left[R_C^2 - \frac{L}{C}\right] = \frac{1}{LC}\left[R_L^2 - \frac{L}{C}\right]$$

$$\omega_r = \frac{1}{\sqrt{LC}}\sqrt{\frac{R_L^2 - (L/C)}{R_C^2 - (L/C)}} \quad (8.16)$$

The condition for resonant frequency is given by Eq. 8.16. As a special case, if $R_L = R_C$, then Eq. 8.16 becomes

$$\omega_r = \frac{1}{\sqrt{LC}}$$

Therefore $$f_r = \frac{1}{2\pi\sqrt{LC}}$$

Example 8.7 Find the resonant frequency in the ideal parallel LC circuit shown in Fig. 8.12.

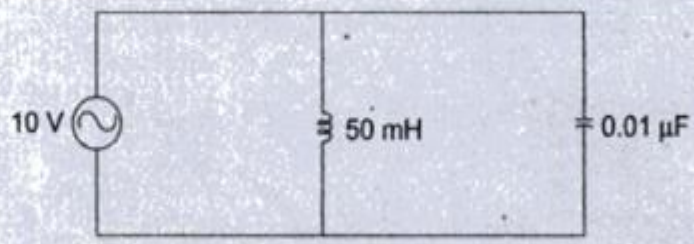


Fig. 8.12

Solution $$f_r = \frac{1}{2\pi\sqrt{LC}}$$

$$= \frac{1}{2\pi\sqrt{50\times10^{-3}\times0.01\times10^{-6}}}$$

$$= 7117.6 \text{ Hz}$$

8.8 RESONANT FREQUENCY FOR A TANK CIRCUIT

The parallel resonant circuit is generally called a tank circuit because of the fact that the circuit stores energy in the magnetic field of the coil and in the electric field of the capacitor. The stored energy is transferred back and forth between the capacitor and coil and vice-versa. The tank circuit is shown in Fig. 8.13. The circuit is said to be in resonant condition when the susceptance part of admittance is zero.

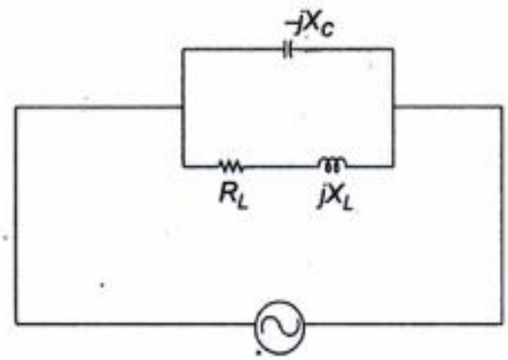


Fig. 8.13

The total admittance is

$$Y = \frac{1}{R_L + jX_L} + \frac{1}{-jX_C} \tag{8.17}$$

Simplifying Eq. 8.17, we have

$$Y = \frac{R_L - jX_L}{R_L^2 + X_L^2} + \frac{j}{X_C}$$

$$= \frac{R_L}{R_L^2 + X_L^2} + j\left[\frac{1}{X_C} - \frac{X_L}{R_L^2 + X_L^2}\right]$$

To satisfy the condition for resonance, the susceptance part is zero.

$$\therefore \quad \frac{1}{X_C} = \frac{X_L}{R_L^2 + X_L^2} \tag{8.18}$$

$$\omega C = \frac{\omega L}{R_L^2 + \omega^2 L^2} \tag{8.19}$$

From Eq. 8.19, we get

$$R_L^2 + \omega^2 L^2 = \frac{L}{C}$$

$$\omega^2 L^2 = \frac{L}{C} - R_L^2$$

$$\omega^2 = \frac{1}{LC} - \frac{R_L^2}{L^2}$$

$$\therefore \quad \omega = \sqrt{\frac{1}{LC} - \frac{R_L^2}{L^2}} \tag{8.20}$$

The resonant frequency for the tank circuit is

$$f_r = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R_L^2}{L^2}} \tag{8.21}$$

Example 8.8 *For the tank circuit shown in Fig. 8.14, find the resonant frequency.*

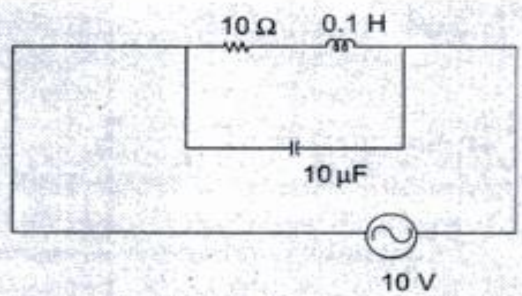


Fig. 8.14

Solution The resonant frequency

$$f_r = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R_L^2}{L^2}}$$

$$= \frac{1}{2\pi}\sqrt{\frac{1}{0.1 \times 10 \times 10^{-6}} - \frac{(10)^2}{(0.1)^2}}$$

$$= \frac{1}{2\pi}\sqrt{(10)^6 - (10)^2} = \frac{1}{2\pi}(994.98) = 158.35 \text{ Hz}$$

8.9 VARIATION OF IMPEDANCE WITH FREQUENCY

The impedance of a parallel resonant circuit is maximum at the resonant frequency and decreases at lower and higher frequencies as shown in Fig. 8.15.

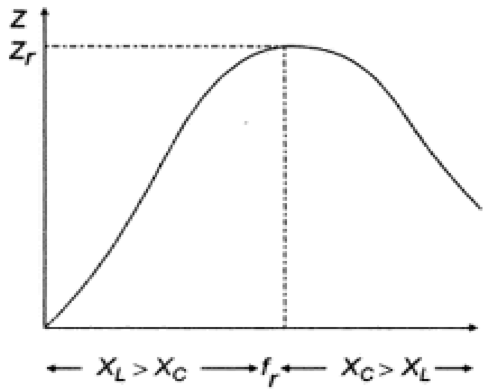


Fig. 8.15

At very low frequencies, X_L is very small and X_C is very large, so the total impedance is essentially inductive. As the frequency increases, the impedance also increases, and the inductive reactance dominates until the resonant frequency is reached. At this point $X_L = X_C$ and the impedance is at its maximum. As the frequency goes above resonance, capacitive reactance dominates and the impedance decreases.

8.10 Q FACTOR OF PARALLEL RESONANCE

Consider the parallel RLC circuit shown in Fig. 8.16.

In the circuit shown, the condition for resonance occurs when the susceptance part is zero.

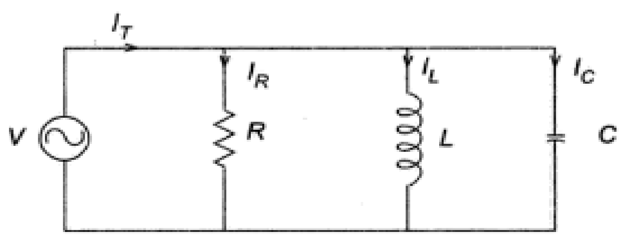


Fig. 8.16

$$\text{Admittance} \quad Y = G + jB \tag{8.22}$$

$$= \frac{1}{R} + j\omega C + \frac{1}{j\omega L}$$

$$= \frac{1}{R} + j\left(\omega C - \frac{1}{\omega L}\right) \tag{8.23}$$

The frequency at which resonance occurs is

$$\omega_r C - \frac{1}{\omega_r L} = 0 \tag{8.24}$$

$$\omega_r = \frac{1}{\sqrt{LC}} \tag{8.25}$$

The voltage and current variation with frequency is shown in Fig. 8.17. At resonant frequency, the current is minimum.

The bandwidth, $BW = f_2 - f_1$

For parallel circuit, to obtain the lower half power frequency,

$$\omega_1 C - \frac{1}{\omega_1 L} = -\frac{1}{R} \tag{8.26}$$

From Eq. 8.26, we have

$$\omega_1^2 + \frac{\omega_1}{RC} - \frac{1}{LC} = 0 \tag{8.27}$$

If we simplify Eq. 8.27, we get

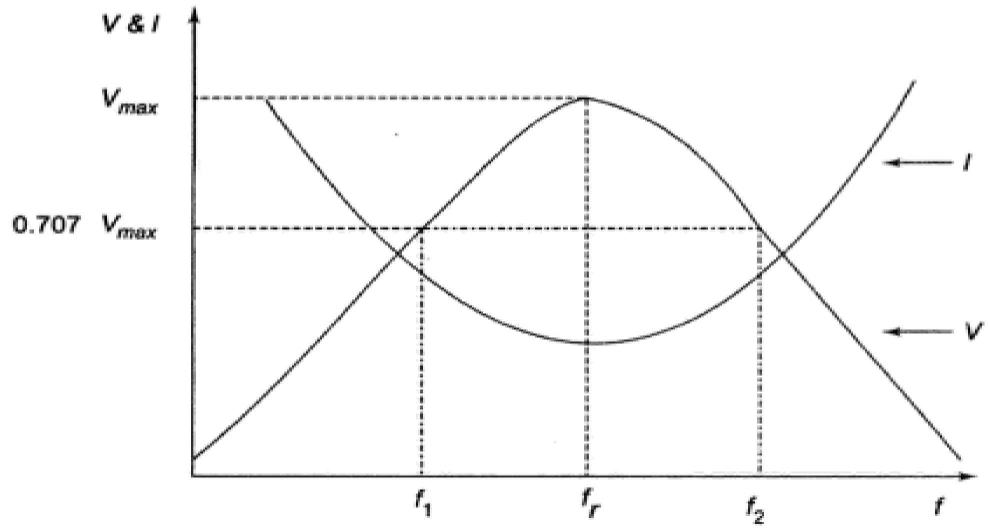


Fig. 8.17

$$\omega_1 = \frac{-1}{2RC} + \sqrt{\left(\frac{1}{2RC}\right)^2 + \frac{1}{LC}} \tag{8.28}$$

Similarly, to obtain the upper half power frequency

$$\omega_2 C - \frac{1}{\omega_2 L} = \frac{1}{R} \tag{8.29}$$

From Eq. 8.29, we have

$$\omega_2 = \frac{1}{2RC} + \sqrt{\left(\frac{1}{2RC}\right)^2 + \frac{1}{LC}} \tag{8.30}$$

Bandwidth $BW = \omega_2 - \omega_1 = \dfrac{1}{RC}$

The quality factor is defined as $Q_r = \dfrac{\omega_r}{\omega_2 - \omega_1}$

$$Q_r = \frac{\omega_r}{1/RC} = \omega_r RC$$

In other words,

$$Q = 2\pi \times \frac{\text{maximum energy stored}}{\text{Energy dissipated /cycle}}$$

In the case of an inductor,

The maximum energy stored $= \dfrac{1}{2} LI^2$

Energy dissipated per cycle $= \left(\dfrac{I}{\sqrt{2}}\right)^2 \times R \times T$

The quality factor $Q = 2\pi \times \dfrac{1/2(LI^2)}{\dfrac{I^2}{2} R \times \dfrac{1}{f}}$

$$\therefore \quad Q = 2\pi \times \frac{\frac{1}{2} L \left(\frac{V}{\omega L} \right)^2 R}{\frac{V^2}{2} \times \frac{1}{f}}$$

$$= \frac{2\pi f LR}{\omega^2 L^2} = \frac{R}{\omega L}$$

For a capacitor, maximum energy stored = 1/2 (CV^2)

Energy dissipated per cycle $= P \times T = \dfrac{V^2}{2 \times R} \times \dfrac{1}{f}$

The quality factor $Q = 2\pi \times \dfrac{1/2(CV^2)}{\dfrac{V^2}{2R} \times \dfrac{1}{f}}$

$$= 2\pi fCR = \omega CR$$

8.11 MAGNIFICATION

Current magnification occurs in a parallel resonant circuit. The voltage applied to the parallel circuit, $V = IR$

Since $I_L = \dfrac{V}{\omega_r L} = \dfrac{IR}{\omega_r L} = IQ_r$

For the capacitor, $I_C = \dfrac{V}{1/\omega_r C} = IR\omega_r C = IQ_r$

Therefore, the quality factor $Q_r = I_L/I$ or I_C/I

8.12 REACTANCE CURVES IN PARALLEL RESONANCE

The effect of variation of frequency on the reactance of the parallel circuit is shown in Fig. 8.18.

The effect of inductive susceptance,

$$B_L = \frac{-1}{2\pi f L}$$

Inductive susceptance is inversely proportional to the frequency or ω. Hence it is represented by a rectangular hyperbola, MN. It is drawn in fourth quadrant, since B_L is negative. Capacitive susceptance, $B_C = 2\pi fC$. It is directly proportional to the frequency f or ω. Hence it is represented by OP, passing through the origin. Net susceptance $B = B_C - B_L$. It is represented by the curve JK, which is a hyperbola. At point ω_r, the total

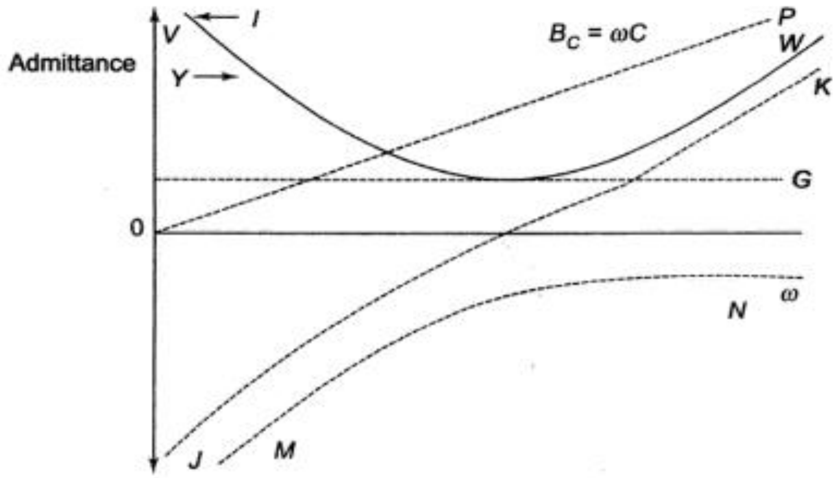


Fig. 8.18

susceptance is zero, and resonance takes place. The variation of the admittance Y and the current I is represented by curve VW. The current will be minimum at resonant frequency.

8.13 LOCUS DIAGRAMS

A phasor diagram may be drawn and is expanded to develop a curve; known as a locus. Locus diagrams are useful in determining the behaviour or response of an RLC circuit when one of its parameters is varied while the frequency and voltage kept constant. The magnitude and phase of the current vector in the circuit depends upon the values of R, L, and C and frequency at the fixed source voltage. The path traced by the terminus of the current vector when the parameters R, L or C are varied while f and v are kept constant is called the current locus.

The term circle diagram identifies locus plots that are either circular or semi-circular loci of the terminus (the tip of the arrow) of a current phasor or voltage phasor. Circle diagrams are often employed as aids in analysing the operating characteristics of circuits like equivalent circuit of transmission lines and some types of AC machines.

Locus diagrams can be also drawn for reactance, impedance, susceptance and admittance when frequency is variable. Loci of these parameters furnish important information for use in circuit analysis. Such plots are particularly useful in the design of electric wave filters.

Series Circuits

To discuss the basis of representing a series circuit by means of a circle diagram consider the circuit shown in Fig. 8.19 (a). The analytical procedure is greatly simplified by assuming that inductance elements have no resistance and that capacitors have no leakage current.

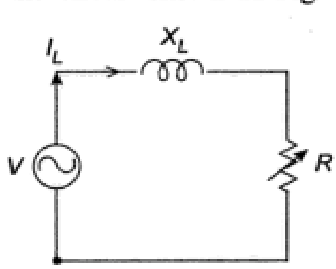


Fig. 8.19 (a)

The circuit under consideration has constant reactance but variable resistance. The applied voltage will be assumed with constant rms voltage V. The power factor angle is designated by θ. If $R = 0$, I_L is obviously equal to $\dfrac{V}{X_L}$ and has maximum value. Also I lags V by 90°. This is shown in Fig. 8.19 (b).

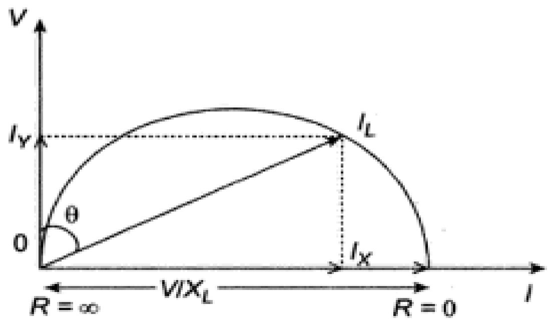


Fig. 8.19 (b)

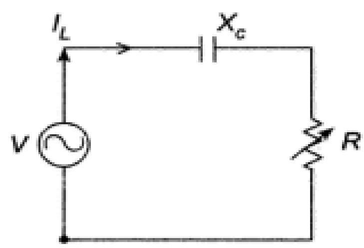


Fig. 8.19 (c)

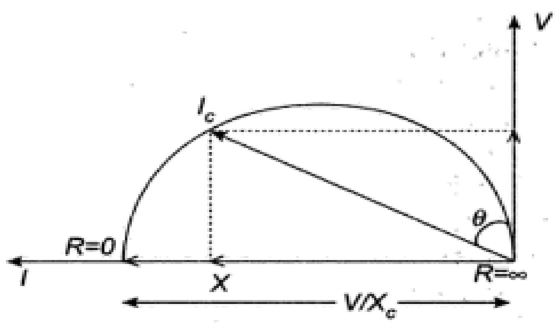


Fig. 8.19 (d)

If R is increased from zero value, the magnitude of I becomes less than $\frac{V}{X_L}$ and θ becomes less than 90° and finally when the limit is reached, i.e. when R equals to infinity, I equals to zero and θ equals to zero. It is observed that the tip of the current vector represents a semicircle as indicated in Fig. 8.19 (b).

In general

$$I_L = \frac{V}{Z}$$

$$\sin\theta = \frac{X}{Z}$$

or $$Z = \frac{X_L}{\sin\theta}$$

$$I = \frac{V}{X_L}\sin\theta \qquad (8.31)$$

For constant V and X, Eq. 8.31 is the polar equation of a circle with diameter $\frac{V}{X_L}$. Figure 8.19 (b) shows the plot of Eq. 8.31 with respect to V as reference.

The active component of the current I_L in Fig. 8.19 (b) is $OI_L \cos\theta$ which is proportional to the power consumed in the RL circuit. In a similar way we can draw the loci of current if the inductive reactance is replaced by a capacitive reactance as shown in Fig. 8.19 (c). The current semicircle for the RC circuit with variable R will be on the left-hand side of the voltage vector OV with diameter $\frac{V}{X_L}$ as shown in Fig. 8.19 (d). The current vector OI_C leads V by θ°. The active component of the current I_CX in Fig. 8.19 (d) is $OI_C \cos\theta$ which is proportional to the power consumed in the RC circuit.

(a) Circle Equations for an RL Circuit

Fixed reactance and variable resistance. The X-co-ordinate and Y-co-ordinate of I_L in Fig. 8.19 (b) respectively are $I_X = I_L \sin\theta$; $I_y = I_L \cos\theta$.

$$\text{Where} \quad I_L = \frac{V}{Z};\ \sin\theta = \frac{X_L}{Z};\ \cos\theta = \frac{R}{Z};\ Z = \sqrt{R^2 + X_L^2}$$

$$\therefore \quad I_X = \frac{V}{Z}\cdot\frac{X_L}{Z} = V\cdot\frac{X_L}{Z^2} \qquad (8.32)$$

$$I_Y = \frac{V}{Z}\cdot\frac{R}{Z} = V\cdot\frac{R}{Z^2} \qquad (8.33)$$

Squaring and adding Eqs 8.32 and 8.33, we obtain

$$I_X^2 + I_Y^2 = \frac{V^2}{R^2 + X_L^2} \tag{8.34}$$

From Eq. 8.32

$$Z^2 = R^2 + X_L^2 = V \cdot \frac{X_L}{I_X}$$

$\therefore$ Equation 8.34 can be written as $\quad I_X^2 + I_Y^2 = \frac{V}{X_L} \cdot I_X$

or $\quad I_X^2 + I_Y^2 - \frac{V}{X_L} \cdot I_X = 0$

Adding $\left(\frac{V}{2X_L}\right)^2$ to both sides the above equation can be written as

$$\left(I_X - \frac{V}{2X_L}\right)^2 + I_Y^2 = \left(\frac{V}{2X_L}\right)^2 \tag{8.35}$$

Equation 8.35 represents a circle whose radius is $\frac{V}{2X_L}$ and the co-ordinates of the centre are $\frac{V}{2X_L}, 0$.

In a similar way we can prove that for a series RC circuit as in Fig. 8.19 (c), with variable R, the locus of the tip of the current vector is a semi-circle and is given by

$$\left(I_X + \frac{V}{2X_C}\right)^2 + I_Y^2 = \frac{V^2}{4X_C^2} \tag{8.36}$$

The centre has co-ordinates of $-\frac{V}{2X_L}$, 0 and radius as $\frac{V}{2X_L}$.

(b) Fixed Resistance, Variable Reactance

Consider the series RL circuit with constant resistance R but variable reactance X_L as shown in Fig. 8.20 (a).

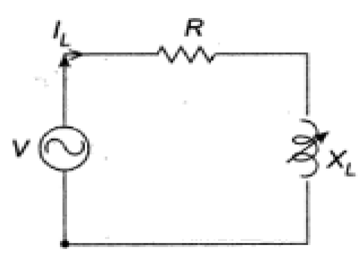


Fig. 8.20 (a)

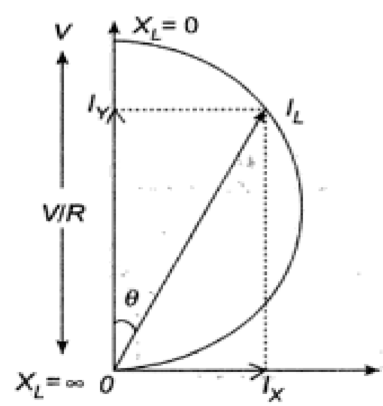


Fig. 8.20 (b)

When $X_L = 0$; I_L assumes maximum value of $\frac{V}{R}$ and $\theta = 0$, the power factor of the circuit becomes unity; as the value X_L is increased from zero, I_L is reduced and finally when X_L is α, current becomes zero and θ will be lagging behind the voltage by 90° as shown in Fig. 8.20 (b). The current vector describes a semicircle with diameter $\frac{V}{R}$ and lies in the right-hand side of voltage vector OV. The active component of the current $OI_L \cos\theta$ is proportional to the power consumed in the RL circuit.

Equation of Circle

Consider Eq. 8.34 $\quad I_X^2 + I_Y^2 = \dfrac{V^2}{R^2 + X_L^2}$

From Eq. 8.33 $\quad Z^2 = R^2 + X_L^2 = \dfrac{VR}{I_Y} \qquad (8.37)$

Substituting Eq. 8.37 in Eq. 8.34

$$I_X^2 + I_Y^2 = \frac{V}{R} I_Y \qquad (8.38)$$

$$I_X^2 + I_Y^2 - \frac{V}{R} I_Y = 0$$

Adding $\left(\dfrac{V}{2R}\right)^2$ to both sides the above equation can be written as

$$I_X^2 + \left(I_Y - \frac{V}{2R}\right)^2 = \left(\frac{V}{2R}\right)^2 \qquad (8.39)$$

Equation 8.39 represents a circle whose radius is $\frac{V}{2R}$ and the co-ordinates of the centre are 0; $\frac{V}{2R}$.

Let the inductive reactance in Fig. 8.20 (a) be replaced by a capacitive reactance as shown in Fig. 8.21 (a).

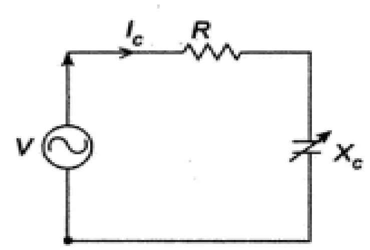


Fig. 8.21 (a)

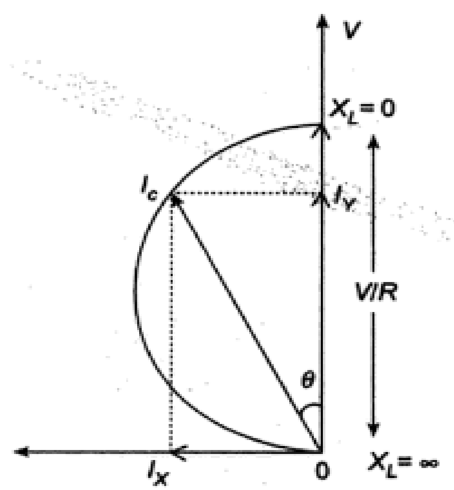


Fig. 8.21 (b)

The current semicircle of a RC circuit with variable X_C will be on the left-hand side of the voltage vector OV with diameter $\frac{V}{R}$. The current vector OI_C leads V by θ°. As before, it may be proved that the equation of the circle shown in Fig. 8.21 (b) is

$$I_X^2 + \left(I_Y - \frac{V}{2R}\right)^2 = \left(\frac{V}{2R}\right)^2$$

Example 8.9 *For the circuit shown in Fig. 8.22 (a) plot the locus of the current, mark the range of I for maximum and minimum values of R, and the maximum power consumed in the circuit. Assume X_L = 25 Ω and R = 50 Ω. The voltage is 200 V; 50 Hz.*

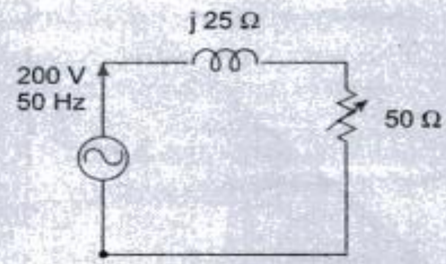


Fig. 8.22 (a)

Solution Maximum value of current

$$I_{max} = \frac{200}{25} = 8\text{ A}; \theta = 90°$$

Minimum value of current

$$I_{min} = \frac{200}{\sqrt{(50)^2 + (25)^2}} = 3.777\text{ A}; \theta = 27.76°$$

The locus of the current is shown in Fig. 8.22 (b).

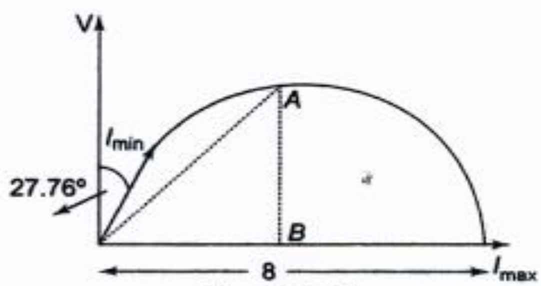


Fig. 8.22 (b)

Power consumed in the circuit is proportional to I cos θ for constant V. The maximum ordinate possible in the semicircle (AB in Fig. 8.22 (b)) represents the maximum power consumed in the circuit. This is possible when θ = 45°, under the condition power factor cos θ = cos 45° = $\frac{1}{\sqrt{2}}$.

Hence, the maximum power consumed in the circuit $= V \times AB = V \times \frac{I_{max}}{L}$

$$I_{max} = \frac{V}{X_L} = 84\text{ A}$$

$$P_{max} = \frac{V^2}{2X_L} = \frac{(200)^2}{2 \times 25} = 800\text{ W}$$

Example 8.13 Draw the locus of I_2 and I for the parallel circuit shown in Fig. 8.29 (a).

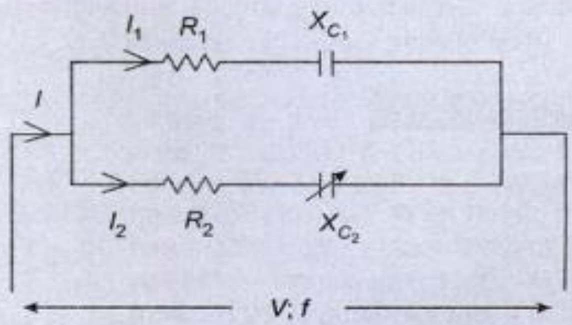


Fig. 8.29 (a)

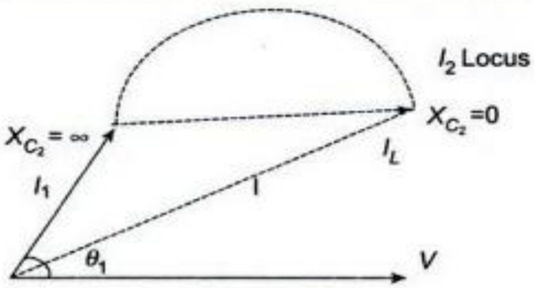


Fig. 8.29 (b)

Solution I_1 leads the voltage by a fixed angle θ_1 given by $\tan^{-1}\dfrac{X_C}{R_1}$

I_2 varies according to the value of X_{C_2}

I_2 is maximum when $X_{C_2} = 0$ and is in phase with V

I_2 is zero when $X_{C_2} = \infty$ as shown in Fig. 8.29 (b).