Three Phase Circuits

Advantages of three phase system

The three phase system has more advantages than a single phase system both from utility point of view and from consumer point of view. Some of the advantages are

1. The amount of conductor material needed to transfer same amount of power is lesser for three phase system than single phase system – thus 3-Ø is economical.
2. Both domestic and industrial power can be provided from a same source.
3. For a given size of frame, the 3-Ø generator produces more output than a 1- Ø.
4. The power in a 1- Ø is pulsating, so the torque produced is also pulsating. Although the power supplied by each phase is pulsating, the total 3- Ø power supplied to a 3- Ø circuit is constant at every instant of time. Because of this , 3- Ø motors have uniform torque.
5. As 3- Ø induction motors are self starting while 1- Ø motors are not, 3- Ø are advantageous.
6. Voltage regulation of a 3- Ø system is better than 1- Ø.

   We can conclude that the operating characteristics of 3- Ø are superior to 1- Ø and also the control equipment to 3- Ø are smaller, cheaper, lighter in weight and more efficient. Therefore the study of 3- Ø circuits has great importance.

Generation of 3- Ø voltages

Three phase voltages can be generated in a stationary armature with a rotating field structure, or in a rotating armature with a stationary field as shown in the figure. Generation of 3-phase voltages is also based on Faraday's laws of electromagnetic induction. Here the coil where the voltage is induced is stationary whereas the magnetic field of the constant magnitude is made to rotate.

In 3- Ø generator the three phase voltages are generated in three separate but identical sets of windings or coils that are displaced by 120^o electrical degrees in the armature. So that the voltages generated in them are 120^o apart in time phase. This arrangement is as shown in the figure. Here RR^l constitutes one coil (R-phase) ; YY^l constitutes one coil (Y-phase) ; BB^l constitutes one coil (B-phase). The field magnets are assumed in clockwise direction.

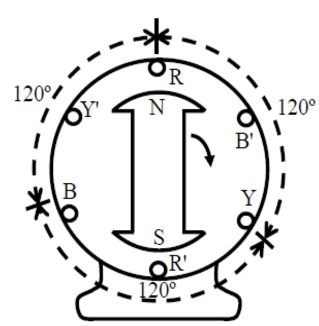
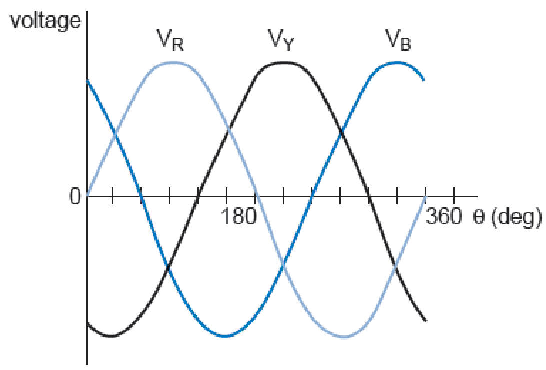
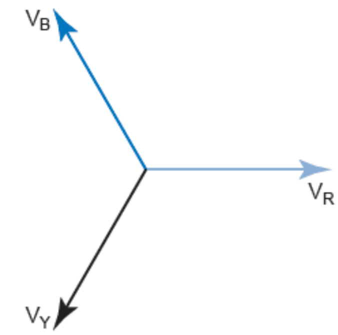


Fig. 2.1 Schematic diagram of three windings of stator
for the generation of three phase balanced voltage (2 pole rotor)

Fig. 2.1 shows a two-pole rotor, the field winding of which is fed from a d.c. source to create magnetic flux for the poles. The poles are so shaped that these produce sinusoidal flux in space. The stator has a balanced three-phase winding with the axis of each phase displaced by 120°. Now, if the rotor is driven by a prime mover in the clockwise direction as shown, at synchronous speed, the voltage induced in the coil will be v x Bl and the direction of induced emf will be given as per Flemings right hand rule. Since the three generated voltages are sinewaves of the same frequency, mutually out-of-phase by 120°, then they may be represented both on a waveform diagram using the same angular or time axis, and as phasors. The corresponding waveform and phasor diagram is as shown in the figure.



3-Ø waveform



phasor diagram

From the phasor diagram that the phasor sum of the three e.m.fs, V_R, V_Y, and V_B is zero. That is

$$V_R+V_Y+V_B = 0$$

Since, E_R has been taken as the reference phasor, the instantaneous values of the three e.m.fs are given as

V_R =Emsin ωt

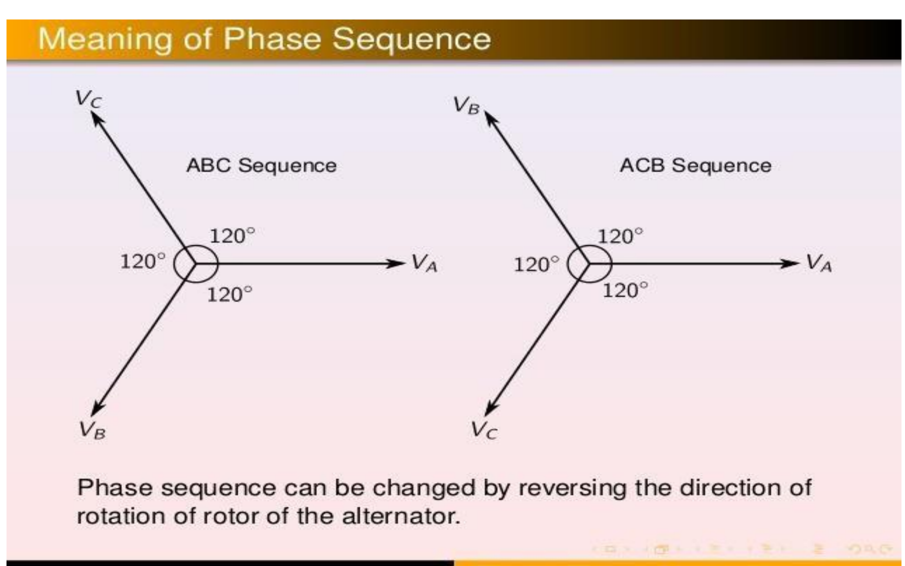
V_Y =Emsin(ωt-120^0) and

V_B = Em sin (ωt - 240°) = Em sin (ωt + 120°)

Phase sequence

This three-phase supply consists of three phases, generally represented as R, Y and B or A, B and C. The order in which the three phase voltages attain their positive peak values is known as the phase sequence. Conventionally the three phases are designated as red-R, yellow-Y and blue-B phases.

The phase sequence is said to be ABC if A attains its peak or maximum value first with respect to the reference as shown in the counter clockwise direction followed by B phase 120° later and C phase 240° later than the A phase. The phase sequence is said to be ACB if A is followed by C phase120° later and B phase 240° later than the A phase. By convention ABC is considered as positive while the sequence ACB as negative.



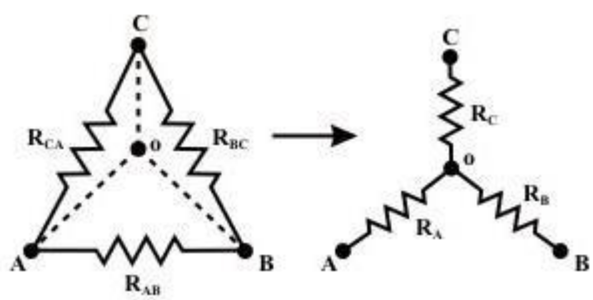
The phase sequence of the voltages applied to a load is determined by the order in which the 3 phase lines are connected. This phase sequence of a three-phase power plays a critical role in controlling the direction of rotation of the three-phase-electrical motors. If this sequence is altered, then the direction of the motor gets altered. So, it is important to keep the phase in sequence or to maintain the proper phase sequence.

The phase sequence can be reversed by interchanging any one pair of lines without causing any change in the supply sequence. Reversal of sequence results in reversal of the direction of rotation in case of induction motor.

Star-Delta Transformation

Delta (Δ)- Star(Y) conversion

Before taking up the examples, the formula for Delta(Δ)-Star(*Y*) conversion and also Star-Delta conversion, using impedances as needed, instead of resistance as elements, which is given in the figure , are presented. The formulas for delta-star conversion, using resistances are,



Let us consider the network shown in figure and assumed the resistances (R_{AB} , R_{BC} , R_{CA}) in Δ network are known. Our problem is to find the values of in Wye (*Y*) network (R_A, R_B , R_C) that will produce the same resistance when measured between similar pairs of terminals. We can write the equivalence resistance between any two terminals in the following form.

Between A & C terminals:

$$R_A + R_C = \frac{R_{CA}\left(R_{AB} + R_{BC}\right)}{R_{AB} + R_{BC} + R_{CA}}$$

Between C & B terminals:

$$R_C + R_B = \frac{R_{BA}\left(R_{AB} + R_{CA}\right)}{R_{AB} + R_{BC} + R_{CA}}$$

Between B & A terminals:

$$R_B + R_A = \frac{R_{AB}\left(R_{CA} + R_{BC}\right)}{R_{AB} + R_{BC} + R_{CA}}$$

By combining above three equations, one can write an expression as given below.

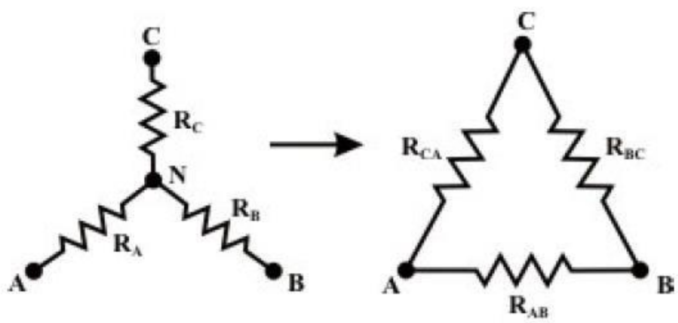
$$R_A + R_B + R_C = \frac{R_{AB}R_{BC} + R_{BC}R_{CA} + R_{CA}R_{AB}}{R_{AB} + R_{BC} + R_{CA}}$$

Subtracting equations (2), (1), and (3) from (4) equations, we can write the express for unknown resistances of Wye (Y) network as

$$R_A = \frac{R_{AB} R_{CA}}{R_{AB} + R_{BC} + R_{CA}}$$
$$R_B = \frac{R_{AB} R_{BC}}{R_{AB} + R_{BC} + R_{CA}}$$
$$R_C = \frac{R_{BC} R_{CA}}{R_{AB} + R_{BC} + R_{CA}}$$

Star or Wye to Delta

To convert a **Wye (**Y) to a **Delta (**Δ), the relationships must be obtained in terms of the **Wye** (R_A, R_B, R_C) resistances for Δ (R_{AB}, R_{BC}, R_{CA}) in network.



Considering the Y connected network, we can write the current expression through resistor R_A as

$$I_A = \frac{(V_A - V_N)}{R_A} \quad \text{(for } Y \text{ network)} \qquad (1)$$

Applying KCL at 'N' for Y connected network (assume N has having higher potential than the terminal ABC terminals) we have

$$\frac{(V_A - V_N)}{R_A} + \frac{(V_B - V_N)}{R_B} + \frac{(V_C - V_N)}{R_C} = 0 \quad \Rightarrow V_N\left(\frac{1}{R_A} + \frac{1}{R_B} + \frac{1}{R_C}\right) = \left(\frac{V_A}{R_A} + \frac{V_B}{R_B} + \frac{V_C}{R_C}\right)$$

$$\text{or,} \quad \Rightarrow V_N = \frac{\left(\frac{V_A}{R_A} + \frac{V_B}{R_B} + \frac{V_C}{R_C}\right)}{\left(\frac{1}{R_A} + \frac{1}{R_B} + \frac{1}{R_C}\right)}$$

For - Δ network

Current entering at terminal A = Current leaving the terminal 'A'

$$I_A = \frac{V_{AB}}{R_{AB}} + \frac{V_{AC}}{R_{AC}} \quad \text{(for } \Delta \text{ network)} \qquad (2)$$

From equations (1) and (2)

$$\frac{(V_A - V_N)}{R_A} = \frac{V_{AB}}{R_{AB}} + \frac{V_{AC}}{R_{AC}}$$

Using the expression V_N in the above equation, we get

$$\frac{\left(V_A - \frac{\left(\frac{V_A}{R_A} + \frac{V_B}{R_B} + \frac{V_C}{R_C} \right)}{\left(\frac{1}{R_A} + \frac{1}{R_B} + \frac{1}{R_C} \right)} \right)}{R_A} = \frac{V_{AB}}{R_{AB}} + \frac{V_{AC}}{R_{AC}} \quad \Rightarrow \quad \frac{\left(\frac{\left(\frac{V_A - V_B}{R_B} + \frac{V_A - V_C}{R_C} \right)}{\left(\frac{1}{R_A} + \frac{1}{R_B} + \frac{1}{R_C} \right)} \right)}{R_A} = \frac{V_{AB}}{R_{AB}} + \frac{V_{AC}}{R_{AC}}$$

$$\text{or} \quad \frac{\left(\frac{\left(\frac{V_{AB}}{R_B} + \frac{V_{AC}}{R_C} \right)}{\left(\frac{1}{R_A} + \frac{1}{R_B} + \frac{1}{R_C} \right)} \right)}{R_A} = \frac{V_{AB}}{R_{AB}} + \frac{V_{AC}}{R_{AC}}$$

Equating the coefficients of V_{AB} and V_{AC} and in both sides of above eq, we obtained the following relationship

$$\frac{1}{R_{AB}} = \frac{1}{R_A R_B \left(\frac{1}{R_A} + \frac{1}{R_B} + \frac{1}{R_C} \right)} \Rightarrow R_{AB} = R_A + R_B + \frac{R_A R_B}{R_C}$$

$$\frac{1}{R_{AC}} = \frac{1}{R_A R_C \left(\frac{1}{R_A} + \frac{1}{R_B} + \frac{1}{R_C} \right)} \Rightarrow R_{AC} = R_A + R_C + \frac{R_A R_C}{R_B}$$

Similarly, I_B for both the networks are given by

$$I_B = \frac{(V_B - V_N)}{R_B} \quad \text{(for } Y \text{ network)}$$

$$I_B = \frac{V_{BC}}{R_{BC}} + \frac{V_{BA}}{R_{BA}} \quad \text{(for } \Delta \text{ network)}$$

Equating the above two equations and using the value of V_N we get the final expression as

$$\frac{\left(\frac{\left(\frac{V_{BC}}{R_C} + \frac{V_{BA}}{R_A} \right)}{\left(\frac{1}{R_A} + \frac{1}{R_B} + \frac{1}{R_C} \right)} \right)}{R_B} = \frac{V_{BC}}{R_{BC}} + \frac{V_{BA}}{R_{BA}}$$

Equating the coefficient of V_{BC} in both sides of the above equations we obtain the following relation

$$\frac{1}{R_{BC}} = \frac{1}{R_B R_C \left(\frac{1}{R_A} + \frac{1}{R_B} + \frac{1}{R_C} \right)} \Rightarrow R_{BC} = R_B + R_C + \frac{R_B R_C}{R_A}$$

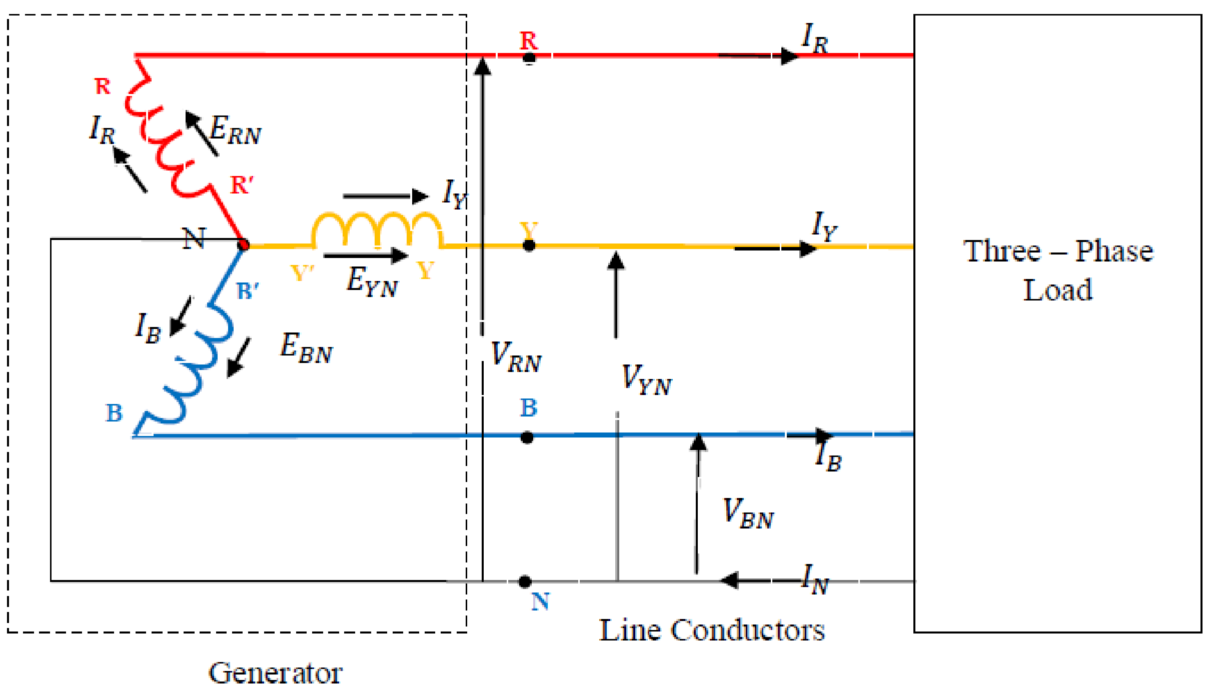
VOLTAGES AND CURRENTS RELATIONS IN 3 PHASE SYSTEMS

In a three-phase system, there are two sets of voltages: *(i)* line voltages, and *(ii)* phase voltages. Similarly, there are two sets of currents: *(i)* line currents, and *(ii)* phase currents. We shall now determine the relations between these two sets of voltage and two sets of currents in both the star-connected system as well as delta connected system.

(1) Star Connected System

Assume the e.m.f in each phase to be positive when acting from the neutral point outwards, as shown in figure. The rms values of the e.m.fs generated in the three phases are ERN, EYN and EBN. In practice, it is the voltage between two lines or between a line conductor and the neutral point that is measured. Due to the impedance voltage-drop in the windings, this potential difference (Pd) is different from the corresponding e.m.f generated in the winding, except when the generator is on open circuit. Hence, in general it is preferable to work with the potential difference, V, rather than the e.m.f, E.

In a three-phase system, there are two sets of voltages we are interested in. One is the set of phase voltages, and the other is the set of line voltages. In Fig.10, VRN is the rms value of the voltage drop from R to N. That is, this is the phase voltage of phase *R*. Thus, V_{RN}, V_{YN} and V_{BN} denote the set of three phase voltages. The term 'line voltage' is used to denote the voltage between two lines. Thus, V_{RY} represents line voltage between the lines R and between the lines R and Y. The other line voltages are V_{YB} and V_{BR}.



A star connected generator supplying a three phase load

To determine the relation between phase voltages and line voltages, we analyze the phasor diagram, shown in Figure. Note that a phasor diagram by itself is meaningless. It is essential to relate the quantities in the phasor diagram to a circuit diagram and to indicate the directions in which the voltage and current is assumed to be positive. The phasor diagram is drawn in terms of effective (or rms) values and it shows voltages (which can be measured). In Fig, V_{RN} represents rms value of the voltage of phase R line with respect to the neutral line N.

By applying Kirchhoff's voltage law, we can get the magnitude and phase angle of the line voltage VRY (which is the voltage drop from R via N to Y, and can be represented by an unambiguous symbol V_{RNY}):

$V_{RY} = V_{RN} + V_{NY}$

This equation simply states that the voltage drop existing from *R* to *Y* is equal to the voltage drop from *R* to *N* plus the voltage drop from *N* to *Y*. The above equation can be written as

$V_{RY} = V_{RN} + V_{NY}$

$= V_{RN} – V_{YN} = V_{RN} + (-V_{YN})$

This shows that to determine V_{RY}, first we reverse the phasor V_{YN} to get - V_{YN} and then add the phasors V_{RN} and $-V_{YN}$, as shown in Figure.

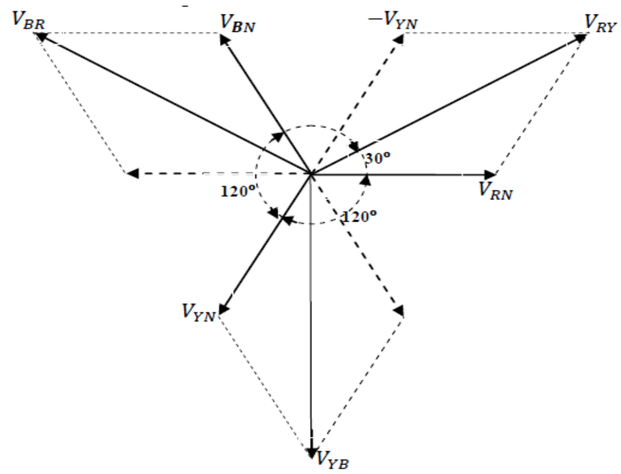


Fig.1 Phasor diagram of star connected system

Analytical Analysis:

In a balanced system, each phase voltage has the same magnitude. So, we can write

$$|V_{RN}|= |V_{YN}|= |V_{BN}|= V_{ph}$$

The three phasors representing the set of phase voltages can be written as

$V_{RN} = V_{Ph}\angle 0°$; $V_{YN} = V_{Ph}\angle -120°$; $V_{BN} = V_{Ph}\angle 120°$;

We have

$V_{RY} = V_{RN} - V_{YN} = V_{Ph}\angle 0° - V_{Ph}\angle -120°$

$= Vph – Vph (\cos 120° - j \sin 120°)$

$= Vph - Vph (- 0.5 - j\, 0.866) = Vph(1.5 + j\, 0.866)$

Thus, the magnitude of V_{RY} is given as

$V_{RY}=Vph \sqrt{(1.5^2 + 0.866^2)} = \sqrt{3}\ Vph$

And the phase angle of V_{RY} with respect to the reference phasor V_{RN} is given as

$\theta = \tan^{-1}\left(\frac{0.866}{1.5}\right) = 30°$

Hence the phasor V_{RY} can be written as $V_{RY} = \sqrt{3}\ Vph\angle 30°$

Similarly $V_{YB} = \sqrt{3}\ Vph\angle -90°$ and $V_{BR} = \sqrt{3}\ Vph\angle 150°$

Thus, we can say that the magnitude of the line voltage V_L for star connection is given as

$$|V_L| = \sqrt{3}\, V_{ph}$$

Geometrical Analysis *:*

Because of the symmetry in Fig, it is evident that the line voltages are equal and are spaced 120° apart. Further, since sides of all the parallelograms are of equal length, the diagonals bisect one another at right angles. Also, they bisect the angle of their respective parallelograms.

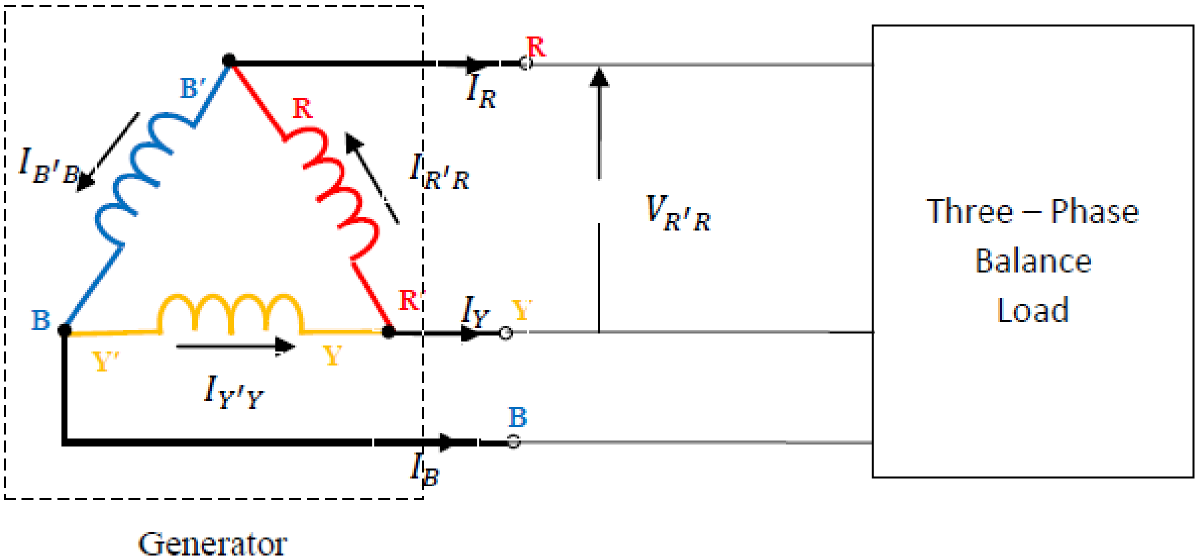
Since the angle between V_{RN} and $-V_{YN}$ is 60°, we have

$V_{RY}= 2(V_{RN}COS30°)$ or $VL =2\ Vph(0.866) = \sqrt{3}Vph$

In a star connection, any current that flows out of the line terminal *R* must be the same as that which flows due to the phase source voltage appearing between terminals Rand N. Therefore, for star-connection, we have IL = Iph

(2) Delta-Connected System

Let $I_{R'R}$, $I_{Y'Y}$, $I_{B'B}$, be the rms values of the phase currents in the three windings of the generator. Their assumed positive directions are indicated by arrows in Fig. Since the load is assumed balanced, these currents are equal in magnitude and differ in phase by 120°, as shown in the phasor diagram.



A delta-connected generator supplying power to a 3 phase load

We can write $|I_{R'R}| = |I_{Y'Y}| = |I_{B'B}| = I_{ph}$

The three phasors representing the set of phase currents can be written as

$I_{R'R} = I_{Ph}\angle 0°$; $I_{Y'Y} = I_{Ph}\angle -120°$; $I_{B'B} = I_{Ph}\angle 120°$;

From Fig. 12, it can be seen that the phase current $I_{R'R}$ flows towards the line conductor R, whereas the phase current $I_{B'B}$ flows away from it. Applying KCL at the terminal R, we can write

$I_R = I_{R'R} - I_{B'B}$

$I_R = I_{R'R} - I_{B'B} = I_{Ph}\angle 0° - I_{Ph}\angle 120°$

$= I_{ph} - I_{ph}(\cos 120° - j \sin 120°)$

$= I_{ph} - I_{ph}(-0.5 + j\,0.866) = V_{ph}(1.5 - j\,0.866)$

Thus, the magnitude of I_R is given as

$I_R = I_{ph}\sqrt{(1.5^2 + 0.866^2)} = \sqrt{3}\, I_{ph}$

And the phase angle of V_{RY} with respect to the reference phasor $I_{R'R}$ is given as

$\theta = \tan^{-1}\left(\frac{-0.866}{1.5}\right) = -30°$

Hence the phasor I_R can be written as $I_R = \sqrt{3}\, I_{ph}\angle -30°$

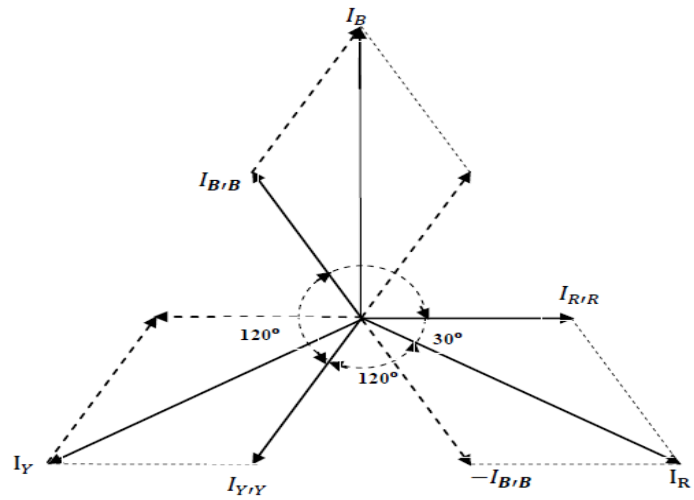
Similarly $I_Y = \sqrt{3}\, I_{ph}\angle -150°$ and $I_B = \sqrt{3}\, I_{ph}\angle 90°$

Thus, we can say that the magnitude of the line current I_L for delta connection is given as

$$|I_L| = \sqrt{3}\, I_{ph}$$

Above vector addition of $I_{R'R}$ and $-I_{B'B}$ is shown in the phasor diagram of Fig. From the symmetrical geometry of the diagram, it is evident that the line currents are equal in magnitude and differ in phase by 120°. From Fig. it is obvious that the line voltage V_{RY} is same as the phase voltage $V_{RR'}$. Hence for a delta connected system, we have

$$|V_L| = V_{ph}$$



Phasor diagram of Delta connected system

Important Points about Three-Phase Systems

Following important points should be noted while dealing with three-phase systems:

(i) For a three-phase system, unless otherwise mentioned, it is normal practice to specify the values of the line voltages and line currents.

(ii) The current in any phase can be determined by dividing the phase voltage by its impedance.

(iii) The power factor of Z is the same as the cosine of the phase difference between phase voltage V_{ph} and phase current Iph.

POWER IN THREE-PHASE SYSTEM WITH A BALANCED LOAD

Consider one phase only. For this load, the voltage is V_{ph} and the current is I_{ph}. The average active power consumed by this load is given by

$P1 = V_{ph} \times I_{ph} \times \cos\Phi$

where Φ is the phase angle of the load.

As the load is balanced, the power in other two phase circuits will also be the same. Hence, the total power consumed is

$P = 3P_1 = 3V_{ph} \times I_{ph} \times \cos\Phi$

Above expression for the total power is in terms of phase voltage and phase current. However, it is a normal practice to mention line voltage and line current in a three-phase system. Hence, the expression for the total power in terms of V_L and I_L:

For a ***star-connected system***, we have $V_L = \sqrt{3}\ V_{ph}$ and $I_L = I_{ph}$. Hence,

$P = 3(V_L/\sqrt{3})\ I_L \cos\Phi = \sqrt{3}\ V_L I_L \cos\Phi$

For a ***delta-connected system***, we have $V_L = V_{ph}$ and $I_L = \sqrt{3}\ I_{ph}$ Hence,

$P = 3\ V_L\ (I_L/\sqrt{3}) \cos\Phi = \sqrt{3}\ V_L I_L \cos\Phi$

Thus, it follows that, for *any balanced load* (connected in either Y or A), the total power is given as

$P = \sqrt{3}\ V_L I_L \cos\Phi$

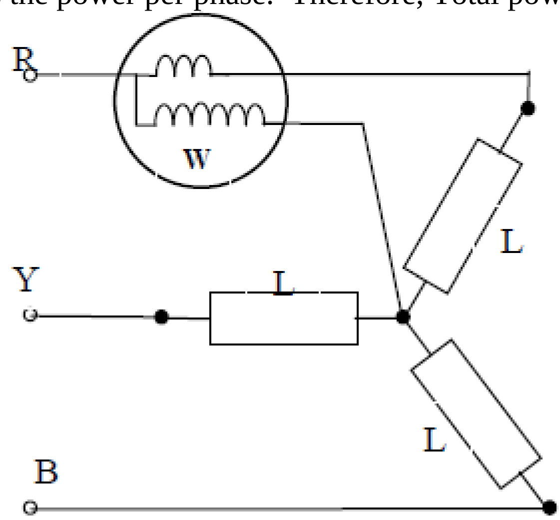
MEASUREMENT OF POWER

The method of measurement of total power in three-phase depends upon the type of system and that of the load. There exist the following methods:

(i) Three-Wattmeter Method: This is the simplest and straight forward method. One wattmeter is inserted in each of the phases. The power consumed by the load is the algebraic sum of the three wattmeter readings.

(ii) One-Wattmeter Method This can be used to Determine the total power consumed by a star connected balanced load, with neutral point accessible. The current coil of the wattmeter is connected in one line and the potential coil is connected between that line and the neutral point, as shown in Fig.

The reading of the wattmeter gives the power per phase. Therefore, Total power = 3 x wattmeter reading

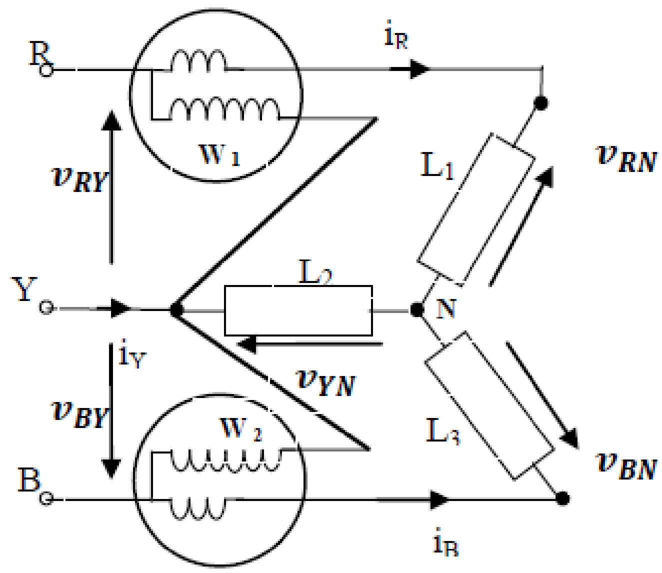


Star connected balanced load

(iii) Two-Wattmeter Method This can be used for any balanced or unbalanced load, star or delta connected. Details of this method are explained below.

Power Measurement by Two-Wattmeter Method

Suppose that the three loads L1, L2 and L3 are connected in star, as shown in Fig. The current coils (CC) of the two wattmeters W1 and W2 are connected in any two lines, say, the R and B lines. The potential coils (PC) of the wattmeters are connected between these lines and the third line. The sum of the wattmeter readings gives the average value of the total power absorbed by the three phases.



Star connected unbalanced load

Proof Let V_{RN}, V_{YN} and V_{BN} be the instantaneous values of the voltages across the loads, with the positive direction marked by arrows in the diagram. Let I_R,I_Y and I_B be the corresponding instantaneous values of the line (and phase) currents.

Total instantaneous power = $V_{RN}\ I_R + V_{YN}\ I_Y + V_{BN}\ I_B$

Since the current through the current coil of W1 is I_R ,and the potential across its potential coil is $V_{RN} - V_{YN}$, we have the instantaneous power measured by W1 = $I_R\,(V_{RN} - V_{YN})$

Similarly, the instantaneous power measured by W2 = $I_B\,(V_{BN} - V_{YN})$

Hence, the sum of the instantaneous powers of W1 and W2 is

P1 + P2 = $I_R\ (V_{RN} - V_{YN}) + I_B\ (vBN - V_{YN}) = I_R\,V_{RN} + I_B\ V_{BN} - V_{YN}\ (I_R + I_B)$

From KCL, the algebraic sum of the instantaneous currents at *N* is zero, i.e.,

$I_R + I_Y + I_B = 0$

$I_Y = -\,(I_R + I_B)$

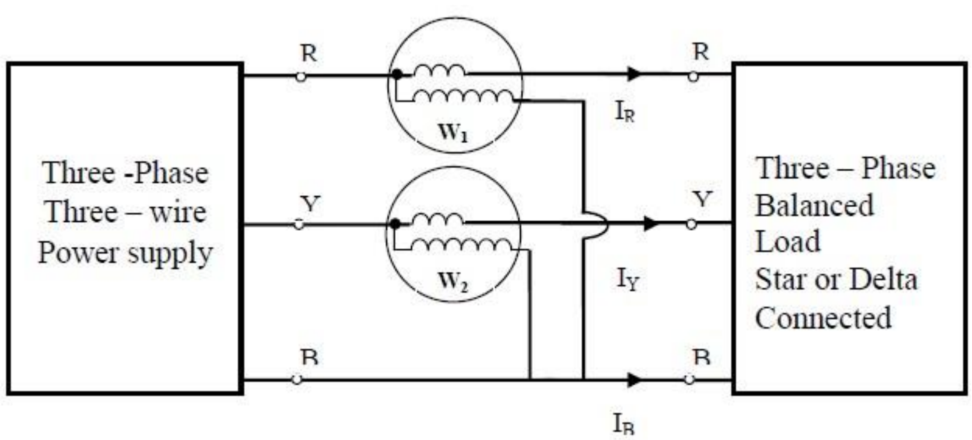
$$P = I_R\,V_{RN} + I_B\ V_{BN} + V_{YN}\ I_Y = \text{total instantaneous power}$$

The power measured by each wattmeter varies from instant to instant, but due to the inertia of the moving system the pointer stays at the average value of the power.

Since the above proof does not assume a balanced load or a sinusoidal waveform, it follows that *the sum of the two wattmeter readings gives the total power under all conditions.* The above proof was derived for a star-connected load. One could derive the same conclusion for a delta-connected load.

Power Factor Measurement by Two-Wattmeter Method

Consider a balanced three-phase inductive load at a power factor Cos Φ (lagging), connected to a 3-wire, 3-phase system, as shown in Fig. The phase sequence is R Y B. The current coils of the two wattmeters W_1 and W_2 are connected in the line conductors Rand Y; respectively. Their potential coils are connected between the corresponding line conductor and the third line conductor B.



Connection of Two Wattmeters

Let I_R, I_Y and I_B be the three line currents, and V_{RN}, V_{YN} and V_{BN} be the three phase-voltages·. Since the load is balanced, the three line currents and the three line voltages will have same magnitude, i.e.,

$I_R = I_Y = I_B$ = Iph and $V_{RN} = V_{YN} = V_{BN} = V_{ph}$

Each line current lags by angle Φ its corresponding voltage as shown in the phasor diagram of Fig .

Since $V_{RB} = V_{RN} - V_{BN}$, and $V_{YB} = V_{YN} - V_{BN}$, we can determine the line voltages V_{RB} and V_{YB} by phasor method. It is seen from Fig 1. that the line voltage V_{RB} lags the phase voltage V_{RN} by 30° and V_{YB} leads V_{YN} by 30°. Thus, the phase angle between the line voltage V_{RB} and the line current I_R is (30° - Φ).

Similarly, the phase angle between the line voltage V_{YB} and the line current I_Y is (30° + Φ).
Therefore, the readings of the two wattmeters are

$$P_1 = V_{RB} I_R \cos(30° - \Phi) = V_L I_L \cos(30° - \Phi)$$

and

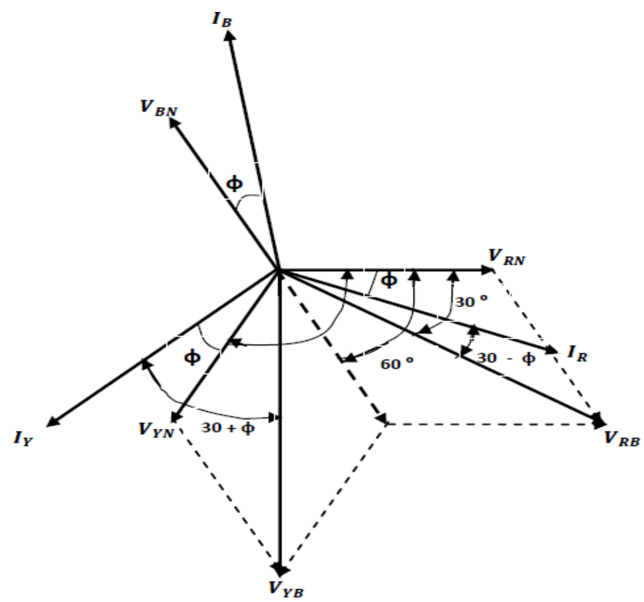
$$P_2 = V_{YB} I_Y \cos(30° + \Phi) = V_L I_L \cos(30° + \Phi)$$

Adding P_1 and P_2

$$P_1 + P_2 = V_L I_L \cos(30° - \Phi) + V_L I_L \cos(30° + \Phi)$$
$$= V_L I_L \{\cos(30° - \Phi) + \cos(30° + \Phi)\}$$
$$= V_L I_L 2\cos 30° \cos \Phi$$

Subtracting P_1 and P_2

$$P_1 - P_2 = V_L I_L \cos(30° - \Phi) - V_L I_L \cos(30° + \Phi)$$
$$= V_L I_L \{\sin(30° - \Phi) + \sin(30° + \Phi)\}$$
$$= V_L I_L 2\sin 30° \sin \Phi$$



Dividing (P_1 - P_2) by (P_1 + P_2), we get

$$\frac{(P_1 - P_2)}{(P_1 + P_2)} = \frac{2 \sin 30^\circ \sin \Phi}{2 \cos 30^\circ \cos \Phi} = \tan 30^\circ \tan \Phi$$

Simplifying, $\tan \Phi = \left\{\sqrt{3}\frac{(P_1 - P_2)}{(P_1 + P_2)}\right\}$ and

$$\Phi = \tan^{-1} \sqrt{3}\frac{(P_1 - P_2)}{(P_1 + P_2)}$$

We can calculate the phase angle Φ from the above relation, and then determine the power factor Cos Φ.

We get the power factor $\cos \Phi = \left\{\tan^{-1}\left(\sqrt{3}\frac{(P_1 - P_2)}{(P_1 + P_2)}\right)\right\}$

1 If the load p.f. > 0.5 (i.e. $\varphi < 60°$); both meters will give a positive reading.

2 If the load p.f.= 0.5 (i.e. $\varphi = 60°$); $W1$ indicates the total power and $W2$ indicates zero.

3 If the load p.f <0.5 (i.e. $\varphi > 60°$); $W2$ attempts to indicate a negative reading. In this case, the connections to the voltage coil of $W2$ need to be reversed, and the resulting reading recorded as a negative value. Under these circumstances, the total load power will be $P = P1 - P2$.

4 The load power factor may be determined from the two wattmeter readings from the equation

$$\phi = \tan^{-1}\sqrt{3}\left(\frac{P_2 - P_1}{P_2 + P_1}\right)$$

hence, power factor, cos φ can be determined

PROBLEMS AND SOLUTION ON THREE PHASE CIRCUITS

1. A 3 phase 230 V supply is given to balanced load which is delta connected. Impedance in each phase of the load is 8+j6,Determine the phase current and the total power consumed

Solution:

Phase voltage = 230 V

balanced delta connected load

Impedance in each phase of the load = 8 + j6 = $10\angle 36.86^0$ ohm

phase current = phase voltage / impedance

= 230 / $10\angle 36.86^0$ = $23\angle -36.86^0$ Amp.

Total power consumed = 3 Vph Iph cos(Vph, Iph)

= 3 x 230 x 23 x cos(36.86) = 12696 w or 12.696kW

2. A balanced 3-phasestar connected load of 150kW takes a leading current of 100 A with a line voltage of 1100 V, 50Hz. Find the circuit constants of the load per phase.

Solution:

Load power = 150kW, Line current = 100 A,

Line voltage = 1100 V, 50Hz.

Power = $\sqrt{3}\ V_L\ I_L \cos\Phi$

150 x 103 = $\sqrt{3}$ x 1100 x 100 cosΦ;

cosΦ = 0.7873; impedance angle, $\Phi = -38.06^0$

Impedance = phase voltage / phase current = $(1100/\sqrt{3})/100$

= 635.08/100 = $6.35\angle -38.06^0$ ohm = (5 – j 3.914) ohm

Resistance, R = 5 ohm, capacitive reactance, XC = 3.914 ohm

Capacitance C = 1/(ω Xc) = 813.67 μF.

3. **Three identical coils each having a resistance of 10 and a reactance of 10 are connected in delta, across 400 V, 3-phase supply. Find the line current and the reading on the two Wattmeters connected to measure the power**.

Solution:

Coil resistance = 10 ohm

coil reactance = 10 ohm. Delta connection.

Supply voltage = 400 V.

Impedance = 10 +j 10 = $14.14\angle 45^0$ ohm.

Phase current = phase voltage / impedance

= 400 / 14,14 $\angle 45^0$ = $28.28\angle -45^0$ Amp.

Line current = $\sqrt{3}$ x phase current = 48.98 A

Wattmeter reading W1 = $V_L I_L \cos(30 - \angle(Vph, Iph))$

= 400 x 28.28 x cos(30 - 45) = 10926.55 watt. = 10.926kW

Wattmeter reading W2 = $V_L I_L \cos(30 + \angle(Vph, Iph))$

= 400 x 28.28 x cos(30 + 45) = 2927.76 w = 2.927kW.

4. Three similar choking coils each having resistance 10 Ω and reactance 10 Ω are connected in star across a 440 V, 3 phase supply. Find the line current and reading of each of two wattmeters connected to measure Power.

Solution:

Coil resistance = 10 ohm

coil reactance = 10 ohm. Star connection.

Supply voltage = 440 V.

Impedance = 10 +j 10 = 14.14 $\angle 45^0$ ohm.

Phase current = phase voltage / impedance

= $(440/\sqrt{3})$ / 14,14 $\angle 45^0$ = 8.983 $\angle -45^0$ Amp.

Line current = phase current = 8.983 A

Wattmeter reading W1 = $V_L I_L \cos(30 - \angle(V_{ph}, I_{ph}))$

= 440 x 8.983 x cos(30 - 45) = 3817.84 watt. = 3.82kW

Wattmeter reading W2 = $V_L I_L \cos(30 + \angle(V_{ph}, I_{ph}))$

= 440 x 8.983 x cos(30 + 45) = 1022.65 W = 1.02265 kW

5. **Three similar impedances are connected in delta across a 3 phase supply. The two wattmeters connected to measure the input power indicate 12 KW. Calculate Power input and Power factor of the load**.

Solution:

Delta connection.

The two wattmeters connected to measure the input power indicating 12 KW. (both are reading equal)

Wattmeter reading $W_1 = V_L I_L \cos(30 - \Phi)$ = 12kW

Wattmeter reading $W_2 = V_L I_L \cos(30 + \Phi)$ = 12kW

Power input = 12 + 12 = 24kW.

We have $W_1 = W_2$

$V_L I_L \cos(30 - \Phi) = V_L I_L \cos(30 + \Phi)$; $\Phi = 0^0$

Power factor of the load = cos(0) = 1, unity power factor.

OR

Wattmeter reading $W_1 = V_L I_L \cos(30 - \Phi)$ = 12kW

Wattmeter reading $W_2 = V_L I_L \cos(30 + \Phi)$ = 12kW

power factor angle $\Phi = \tan^{-1}\{\sqrt{3}\ (W_A - W_B)/(W_A + W_B)\}$

with $W_A = W_B$ = 12kW, $\Phi = 0^0$

Therefore the power factor Cos Φ = 1

6. **The power flowing in a 3 Phase,3 Wire balanced load system is measured by two wattmeter method. The reading in Wattmeter A is 750 Watts and Wattmeter B is 1500 Watts. What is the power factor of the system?**

Solution:

Wattmeter A = 750 Watts

Wattmeter B = 1500 Watts.

power factor angle

$\Phi = \tan^{-1}\{\sqrt{3}\ (W_A - W_B)/(W_A + W_B)\}$

$= \tan^{-1}\{\sqrt{3}(750 - 1500)/(750+1500)\} = -30^0$

power factor of the system = $\cos(-30^0)$

= 0.866 lagging.

7. **Three similar coils each having resistance of 10 Ohm and reactance of 8 Ohm are connected in star across a 400 V, 3 Phase supply, Determine the i)Line current ii)Total Power and iii)Reading of each of two wattmeters connected to measure the power**.

Solution:

Coil impedance = 10 + j 8 Ohm

Star connected.

Supply : 400 V, 3 Phase supply,

i) Line current = phase voltage / impedance

= $(400/\sqrt{3})/(10 + j\ 8)$ = 230.94 /12.8 $\angle 38.66$

= 18.042 $\angle -38.66$ Amp

Ii) Total Power = $\sqrt{3}$ x 400 x 18.042 x cos(38,66) = 10 kW

iii) Reading of each of two wattmeters:

$W_1 = V_L I_L \cos(30 – \Phi) = 400 \times 18.042 \times \cos(68.66) = 2.626kW$

8. **Three similar impedances are connected in delta across a 3 Phase supply. The two Wattmeters connected to measure the input power indicate 12 KW and 7KW calculate: Power input and Power factor of the load.**

Solution:

Wattmeter A = 12kW = 12000 Watts

Wattmeter B = 7kW = 7000 Watts.

power factor angle

$\Phi = \tan^{-1}\{\sqrt{3}(W_A – W_B)/(W_A + W_B)\}$

$= \tan^{-1}\{\sqrt{3}(12 – 7)/(12 + 7)\} = 24.5^0$

power factor of the system = cos 24.5

= 0.9099 lagging.

9. The input power to a three phase motor was measured by two wattmeter method. The readings were 5kW and – 1.7kW, and the line voltage was 400 V. Calculate: (a) the total power; (b) the power factor; and (iii) the line current.

Solution:

Wattmeter A = 5 kW = 5000 Watts

Wattmeter B = - 1.7 kW = 1700 Watts.

Total Power = WA + WB = 5 – 1.7 = 3.3 kW = 3300 W

power factor angle

$\Phi = \tan^{-1}\{\sqrt{3}(W_A – W_B)/(W_A + W_B)\}$

$= \tan^{-1}\{\sqrt{3}(5 – (- 1.7))/(5 + (- 1.7))\} = 74.12^0$

power factor of the system = cos 74.12 = 0.2735 lagging

Line current = Total power /$\{\sqrt{3} \times VL \times \cos \Phi\}$ = 3300 / $\{\sqrt{3} \times 400 \times 0.2735\}$ = 17.415 A

10. **Three similar coils, connected in star, takes a total power of 1.5kW, at a power factor of 0.2, from a three phase, 400V, 50Hz supply. Calculate: (a) the resistance and inductance of each coil, and (b) the line currents.**

Solution:

Input = 1.5 kW = 1500 W

= √3 x line voltage x line current x power factor.

Line current = 1500 / [√3 x 400 x 0.2] = 10.8253 A

Power factor angle = $\cos^{-1}(0.2)$ = 78.46

Impedance per phase = Phase voltage / phase current = (400/√3)/(10.8253∟-78.46)

= 21.33∟78.46 Ohm

= 4.267 + j 20.898

Resistance of the coil = 4.267 Ohms

Inductive reactance = 20.898 = ω L = 2π f L, where L is the inductance of the coil.

Inductance L = 20.898 / 314 = 0.0665 Henry.