UNIT IV

LOSSES AND EFFICIENCY OF DC MACHINE:

Various losses in DC machine and efficiency, condition for maximum efficiency- numerical problems.

Testing of DC machines: Brake test - Swinburne's test - Hopkinson's test – Field's test - Retardation test - Separation of iron and friction Losses- numerical problems.

LOSSES IN A D.C. MACHINE

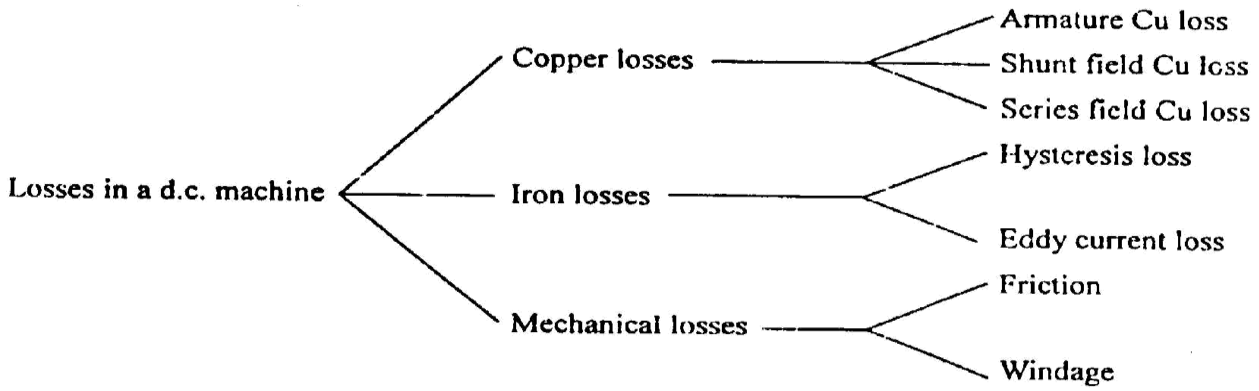


Fig 4.1

The losses in a d.c. machine (generator or motor) may be divided into three classes viz (i) copper losses (ii) iron or core losses and (iii) mechanical losses. All these losses appear as heat and thus raise the temperature of the machine. They also lower the efficiency of the machine.

1. Copper losses

These losses occur due to currents in the various windings of the machine.

$$Armature copper loss = I_a^2 R_a$$

$$Shunt field copper loss = I_{sh}^2 R_{sh}$$

$$Series field copper loss = I_{se}^2 R_{se}$$

2. Iron or Core losses

These losses occur in the armature of a d.c. machine and are due to the rotation of armature in the magnetic field of the poles. They are of two types viz., (i) hysteresis loss (ii) eddy current loss.

(i) Hysteresis loss

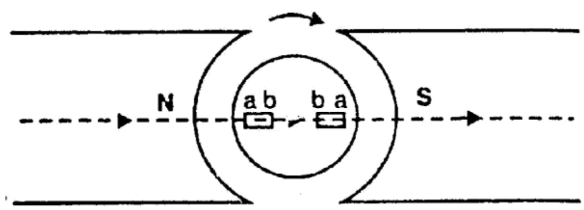


Fig 4.2

Hysteresis loss occurs in the armature of the d.c. machine since any given part of the armature is subjected to magnetic field reversals as it passes under successive poles. Fig. (4.2) shows an armature rotating in two-pole machine. Consider a small piece ab of the armature. When the piece ab is under N-pole, the magnetic lines pass from a to b. Half a revolution later, the same piece of iron is under S-pole and magnetic lines pass from b to a so that magnetism in the iron is reversed. In order to reverse continuously the molecular magnets in the armature core, some amount of power has to be spent which is called hysteresis loss. It is given by Steinmetz formula. This formula is

Hysteresis Loss, $P_h = \eta B_{max}^{16} fV watts$

where B_{max} = Maximum flux density in armature

f = Frequency of magnetic reversals

= NP/120 where N is in r.p.m.

V = Volume of armature in m^3

η= Steinmetz hysteresis co-efficient

In order to reduce this loss in a d.c. machine, armature core is made of such materials which have a low value of Steinmetz hysteresis co-efficient e.g., silicon steel.

(ii) Eddy current loss

In addition to the voltages induced in the armature conductors, there are also voltages induced in the armature core. These voltages produce circulating currents in the armature core as shown in Fig. (4.3). These are called eddy currents and power

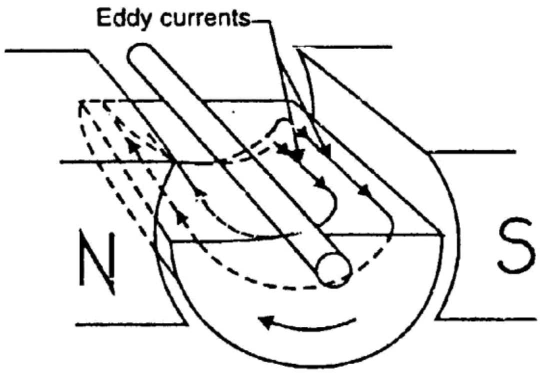


Fig 4.3

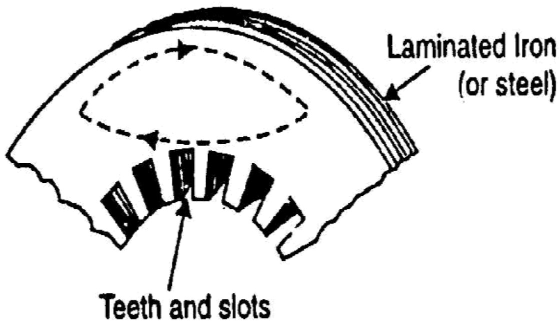


Fig 4.4

loss due to their flow is called eddy current loss. The eddy current loss appears as heat which raises the temperature of the machine and lowers its efficiency.

If a continuous solid iron core is used, the resistance to eddy current path will be small due to large cross-sectional area of the core. Consequently, the magnitude of eddy current and hence eddy current loss will be large. The magnitude of eddy current can be reduced by making core resistance as high as practical. Thecore resistance can be greatly increased by constructing the core of thin, round iron sheets called laminations [See Fig. 4 . 4].

The laminations are insulated from each other with a coating of varnish. The insulating coating has a high resistance, so very little current flows from one lamination to the other. Also, because each lamination is very thin, the resistance to current flowing through the width of a lamination is also quite large. Thus laminating a core increases the core resistance which decreases the eddy current and hence the eddy current loss.

Eddy current loss, $P_e = K_e B^2 \max f^2 t^2 V watts$

where K_e = Constant depending upon the electrical resistance of core and system of units used

B_{max} = Maximum flux density in Wb/m2

f = Frequency of magnetic reversals in Hz

t = Thickness of lamination in m

V = Volume of core in m^3

It may be noted that eddy current loss depends upon the square of lamination thickness. For this reason, lamination thickness should be kept as small as possible.

3. Mechanical losses: These losses are due to friction and windage.

(i) friction loss e.g., bearing friction, brush friction etc.

(ii) windage loss i.e., air friction of rotating armature.

These losses depend upon the speed of the machine. But for a given speed, they are practically constant.

Note. Iron losses and mechanical losses together are called rotational losses.

Constant and Variable Losses

The losses in a d.c. generator (or d.c. motor) may be sub-divided into (i) constant losses (ii) variable losses.

(i) Constant losses

Those losses in a d.c. generator which remain constant at all loads are known as constant losses. The constant losses in a d.c. generator are:

(a) iron losses (b) mechanical losses (c) shunt field losses

(ii) Variable losses

Those losses in a d.c. generator which vary with load are called variable losses. The variable losses in a d.c. generator are:

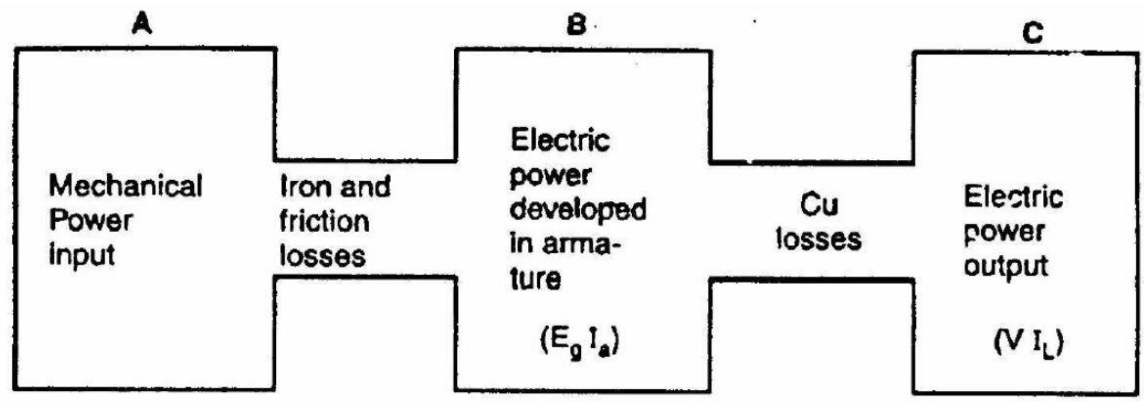
(a) Copper loss in armature winding ($(I_a^2 R_a)$

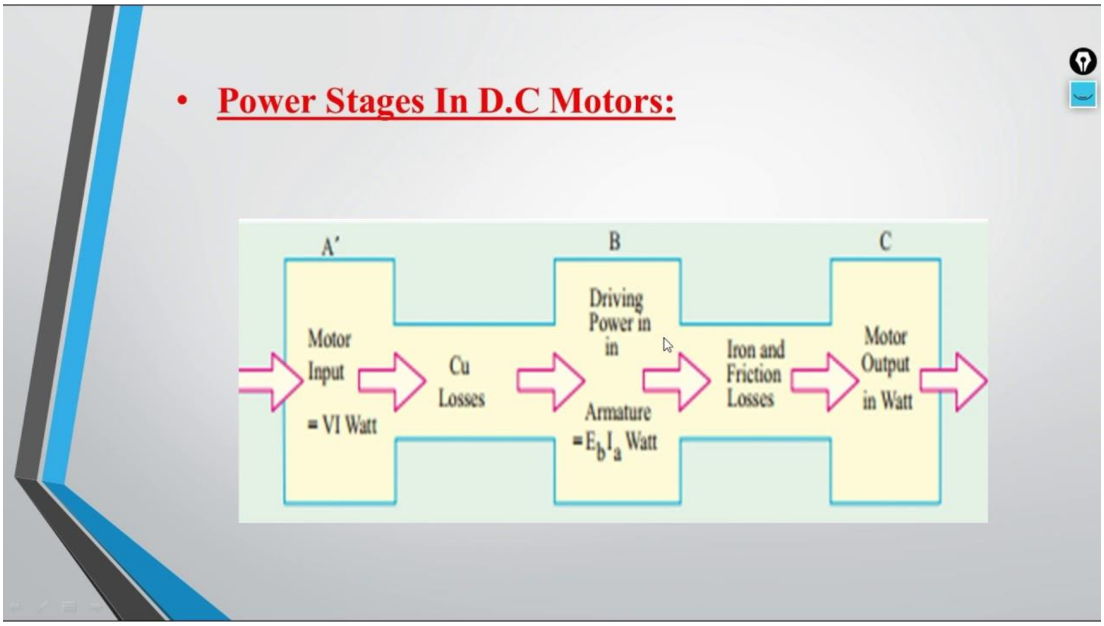
(b) Copper loss in series field winding $(I_{se}^2 R_{se})$

Total losses = Constant losses + Variable losses

Note. Field Cu loss is constant for shunt and compound generators.

Power Stages in DC Generator





EFFICIENCY OF DC MACHINES

The various efficiencies of DC machines are expressed as follows:

1. Generator:

i) Mechanical efficiency,

$$\eta_m = \frac{Electrical\ Power\ delveloped\ by\ the\ armature}{\text{Total mechanical Power input}} = \frac{E_g I_a}{BHP\ of\ Prime\ mover\ X\ 735.5}$$

ii) Electrical efficiency

$$\eta_e = \frac{Useful\ electrical\ Power\ output}{\text{Electrical Power developed}} = \frac{VI_L}{E_g I_a}$$

iii) Commercial or Overall efficiency

$$\eta_G = \eta_m \eta_e = \frac{Useful\ electrical\ Power\ output}{\text{Total mechanical Power input}} = \frac{VI_L}{BHP\ of\ Prime\ mover\ X\ 735.5}$$

2. Motor

i) Electrical efficiency

$$\eta_e = \frac{Mechanical\ Power\ delveloped}{\text{Total Eelectrical Power input}} = \frac{E_b I_a}{VI_L}$$

ii) Mechanical efficiency,

$$\eta_e = \frac{Useful\ Mechanical\ Power\ output}{\text{Mechanical Power developed}} = \frac{BHP\ of\ motor\ X\ 735.5}{E_b I_a}$$

iii) Commercial or Overall efficiency

$$\eta_M = \eta_m \eta_e = \frac{Useful\ Mechanical\ Power\ output}{\text{Total Eelectrical Power input}} = \frac{\text{Total Eelectrical Power input}}{VI_L}$$

The overall efficiency of dc generator and motor can also be expressed as follows:

Overall efficiency of generator,

$$\eta_G = \frac{Useful\ Power\ output}{\text{Total Power input}}$$

$$= \frac{Useful\ Power\ output}{Useful\ Power\ output + total\ losses}$$

$$= \frac{VI_L}{VI_L + total\ losses}$$

Overall efficiency of motor,

$$\eta_M = \frac{Useful\ Power\ output}{\text{Total Power input}}$$

$$= \frac{Total\ Power\ input - \ total\ losses}{Total\ Power\ input}$$

$$= \frac{VI_L - \ total\ losses}{VI_L}$$

CONDITION FOR MAXIMUM EFFICIENCY

Condition for maximum efficiency of a dc generator or of a dc motor is the same and for a dc generator it is derived as follows

Generator output = VI_L watts

Where V is the Terminal voltage and I_L is the output current or load current

Generator input = output + total losses

$$= \quad VI_L + I_a^2 R_a + P_C$$

$$= \quad VI_L + (I_L + I_{sh})^2 R_a + P_C$$

$$= \quad VI_L + I_L^2 R_a + P_C$$

Since, Shunt field current I_{sh} is very small in comparison with load current I_L

Where Ra is the total resistance of armature circuit (including brush contact resistance and resistance of series, interpole and compensating windings, if any).

Generator efficiency,

$$\eta = \frac{Output}{\text{Input}} = \frac{VI_L}{VI_L + I_L^2 R_a + P_C}$$

$$= \frac{1}{1 + \frac{I_L R_a}{V} + \frac{P_C}{VI_L}}$$

Efficiency will be maximum when denominator will be minimum i.e. when

$$\frac{d}{dI_L}\left(\frac{I_L R_a}{V} + \frac{P_C}{VI_L}\right) = 0$$

$$\left(\frac{R_a}{V} - \frac{P_C}{VI_L^2}\right) = 0$$

$$I_L^2 R_a = P_C$$

Hence efficiency will be maximum when variable losses are equal to constant losses. The load current corresponding to maximum efficiency is given by the relation.
$I_L = \sqrt{(P_C/R_a)}$
Variation of efficiency with load is illustrated in which indicates that the efficiency increases with the increase in load current, attains maximum value when the load current equals the value given by $I_L = \sqrt{(P_C/R_a)}$ and then starts decreasing. The machines are usually designed to give maximum efficiency at or near the rated output of the machines.

TESTING OF D.C. MACHINES

INTRODUCTION

There are several tests that are conducted upon a d.c. machine (generator or motor) to judge its performance. One important test is performed to measure the efficiency of a d.c. machine. The efficiency of a d.c. machine depends upon its losses. The smaller the losses, the greater is the efficiency of the machine and vice-versa. The consideration of losses in a d.c. machine is important for two principal reasons. First, losses determine the efficiency of the machine and appreciably influence its operating cost. Secondly, losses determine the heating of the machine and hence the power output that may be obtained without undue deterioration of the insulation.

In this chapter, we shall focus our attention on the various methods for the determination of the efficiency of a d.c. machine.

EFFICIENCY OF A D.C. MACHINE

The power that a d.c. machine receives is called the input and the power it gives out is called the output. Therefore, the efficiency of a d.c. machine, like that of any energy-transferring device, is given by;

Efficiency=output/input ----(1)

Output=input-Losses

and input=output + Losses

Therefore, the efficiency of a.D.C. machine can also be expressed in the following forms:

Efficiency= (input – losses) / input ----- (2)

Efficiency= Output / (output + losses ----- (3)

The most obvious method of determining the efficiency of a d.c. machine is to directly load it and measure the input power and output power. Then we can use Eq.(i) to determine the efficiency of the machine. This method suffers from three main drawbacks.

First, this method requires the application of load on the machine. Secondly, for machines of large rating, the loads of the required sizes may not be available. Thirdly, even 'if it is possible to provide such loads, large power will be dissipated, making it an expensive method.

The most common method of measuring the efficiency of a d.c. machine is to determine its losses (instead of measuring the input and output on load). We can then use Eq.(2) or Eq.(3) to determine the efficiency of the machine.

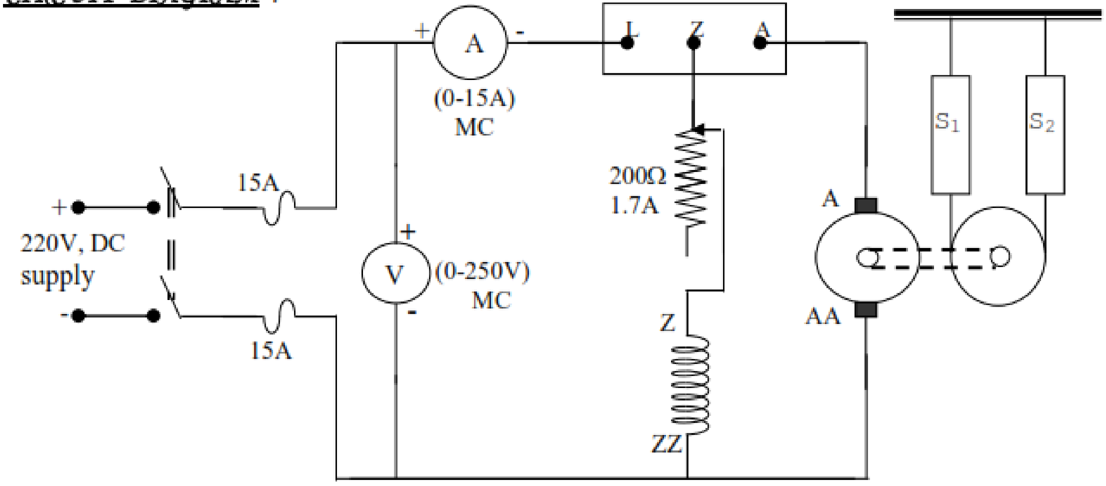
This method has the obvious advantage of convenience and economy.

EFFICIENCY BY DIRECT LOADING

In this method, the d.c. machine is loaded and output and input are measured to find the efficiency. For this purpose, two simple methods can be used.

(I) BRAKE TEST

CIRCUIT DIAGRAM :



In this method, a brake is applied to a water-cooled pulley mounted on the motor shaft as shown in Fig. (4.5). One end of the rope is fixed to the floor via a spring balance S or W1 and a known mass w or w2 is suspended at the other end. The difference of weight W1 and spring balance reading W2 gives the effective force on the pulley.

The mechanical load is adjusted, by varying suspended weights, to cause a flow of full load current through the motor circuit.

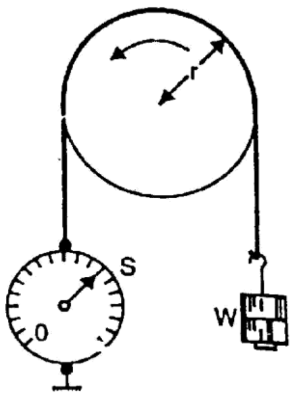


Fig 4.5

If the spring balance reading is (S) kg- and the suspended mass has a weight of W kg, then,
Net pull on the rope = (W-S) kg-Wt
Or (W1-W2) kg.
= (W-S) x 9.81 newtons

If r is the radius of the pulley in metres, then the shaft torque T_{sh} developed by the motor is

$$T_{sh} = (W1 - w2) \times 9.81 \times r \quad \text{N-m}$$

If the speed of the pulley is N r.p.m., then,

$$Outputt\ power = \frac{2\pi N T_{sh}}{60} = \frac{2\pi N \times (W1 - W2) \times 9.81 \times r}{60}\ watts$$

Let V=Supply voltage in volts

I=Current taken by the motor in amperes

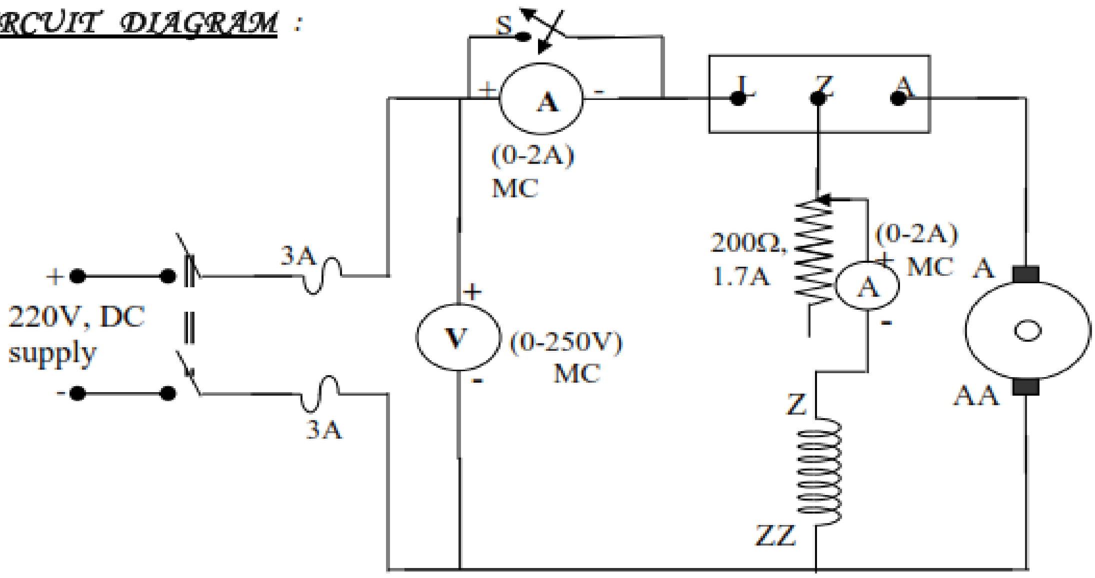
Therefore, Input to motor=V I Watts

$$\text{Efficiency} = \frac{2\pi N(W1 - w2) \times r \times 9.81}{60 \times VI}$$

Because of several disadvantages, direct loading method is used only for determining the efficiency of small machines.

SWINBURNE'S METHOD FOR DETERMINING EFFICIENCY

CIRCUIT DIAGRAM :



In this method, the d.c. machine (generator or motor) is run as a motor at no-load and losses of the machine are determined. Once the losses of the machine are known, its efficiency at any desired load can be determined in advance. It may be noted that this method is applicable to those machines in which flux is practically constant at all loads e.g., shunt and compound machines. Let us see how the efficiency of a d.c. shunt machine (generator or motor) is determined by this method. The test insists of two steps:

(i) Determination of hot resistances of windings

The armature resistance and shunt field resistance are measured using a battery, voltmeter and ammeter. Since these resistances are measured when the machine is cold, they must be converted to values corresponding to the temperature at which the machine would work on full-load. Generally, these values are measured for a temperature rise of 40°C above the room temperature. Let the hot resistances of armature and shunt field be R_a and R_{sh} respectively.

(ii) Determination of constant losses

The machine is run as a motor on no-load with supply voltage adjusted to the rated voltage i.e. voltage stamped on the nameplate. The speed of the motor is adjusted to the rated speed with the help of field regulator R as shown in above Figure.

CALCULATIONS :

Let V=Supply Voltage

I_0=No load current read by Ammeter A_1

I_{sh}= Shunt field current read by Ammeter A_2

No-load Armature Current, $I_{a0}=I_0-I_{sh}$

No-load input power to motor $=VI_0$

No-load input power to Armature $=VI_{a0}=V(I_0-I_{sh})$

Since output of the motor is zero, the no-load input power to the armature supplies (a) iron losses in the core (b) friction loss (c) windage loss (d) armature Cu loss $I_{a0}^2 R_a \, or \, (I_0 - I_{sh})^2 R_a$

Constant losses, Wc = No load Input- No load armature *cu* losses

$$W_C = VI_0 - (I_0 - I_{sh})^2 R_a$$

Since constant losses are known, the efficiency of the machine at any other load can be determined. Suppose it is desired to determine the efficiency of the machine at load current $I_L = I$. Then,

Armature current, $I_a=I-I_{sh}$ … if the machine is motoring

$I_a=I+I_{sh}$ … if the machine is generating

Efficiency when running as a motor:

Input to the motor = V I

Armature current = $I - I_{sh}$

Armature *cu* loss = $(I - I_{sh})^2 * R_a$

Constant losses = Wc

Total losses = Constant losses + Amature *cu* losses

= $W_c + (I - I_{sh})^2 * R_a$

Output = Input - Losses = $(V * I) - (W_c + (I - I_{sh})^2 * R_a)$

% Efficiency = $\frac{\text{Output}}{\text{Input}} = \frac{\text{Input - Losses}}{\text{Input}}$

$$= \frac{VI - (I - I_{sh})^2 R_a - W_C}{VI}$$

Efficiency when running as a generator:

Output of the generator = V I

Armature current = $I+I_{sh}$

Armature *cu* loss = $(I + I_{sh})^2 * R_a$

Constant losses = W_C

Total losses = Constant losses + Amature *cu* losses

$$= W_c + (I + I_{sh})^2 * R_a$$

Input $= \text{Output} + \text{Losses} = (V * I) + (W_c + (I + I_{sh})^2 * R_a)$

% Efficiency $= \dfrac{\text{Output}}{\text{Input}} = \dfrac{\text{Output}}{\text{Output} + \text{Losses}}$

$$= \frac{VI}{VI + (I + I_{sh})^2 R_a + W_C}$$

Advantages of Swinburne's test

(i) The power required to carry out the test is small because it is a no-load test. Therefore, this method is quite **economical.**

(ii) The efficiency can be determined at any load because constant losses are known.

(iii) This test is **very convenient.**

Disadvantages of Swinburne's test

(i) It does not take into account the **stray load losses that occur when the machine is loaded.**

(ii) This test does not enable us to check the performance of the machine on full-load. For example, it does not indicate whether commutation on full load is satisfactory and whether the temperature rise is within the specified limits.

(iii) **This test does not give quite accurate efficiency of the machine**. It is because iron losses under actual load are greater than those measured. This is mainly due to armature reaction distorting the field.

REGENERATIVE OR HOPKINSON'S TEST

This test is also known as back-to-back test. It requires two machines, which must be identical. These machines are mechanically coupled and are so adjusted

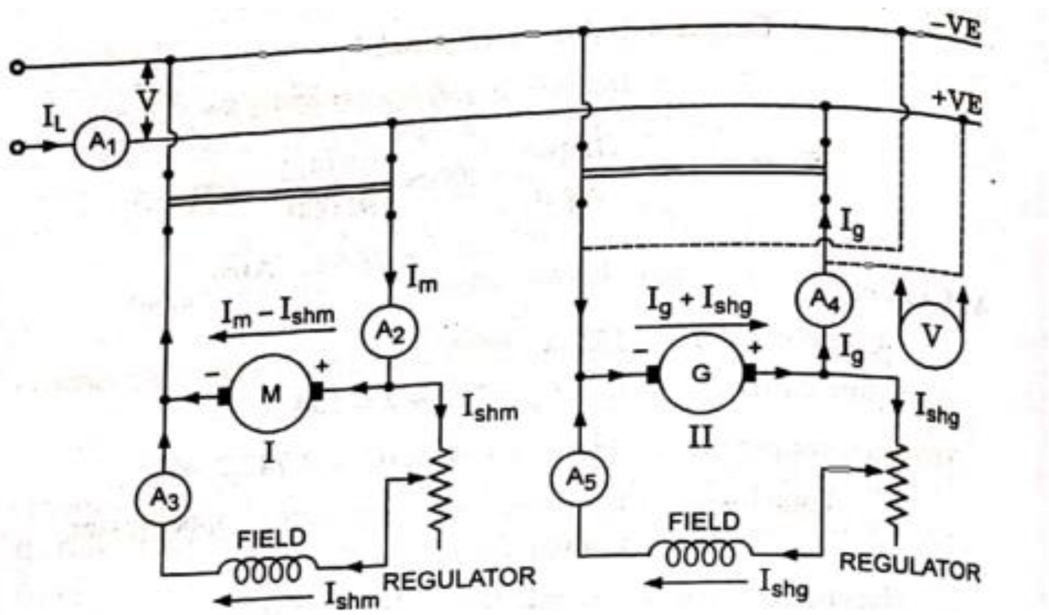


Fig. 4.6 Hopkinson's Test

electrically that one of them acts as a motor and the other as a generator. The motor supplies the mechanical power, which is utilized to drive the generator while generator develops electrical power to be utilized in driving the motor in addition to electrical power drawn from the supply mains. The connection diagram for this test is shown in Fig 4.6.

Thus two identical machines, may be of any size, can be tested under full-load conditions. Under test the power drawn from the supply mains will be that required to overcome the internal losses of the two machines. This method is economical when the large machines are to be tested on full load for longer duration.

The machine acting as a motor, is run up to full speed by means of a starting resistance, the main switch of the generator keeping open. The second machine will, therefore, generate and its voltage is adjusted equal to 1 or 2 V more than bus-bar voltage and of the same polarity. The adjustment is made by means of a paralleling voltmeter V and switched on to the supply.

The machine with smaller excitation acts as a motor and that with larger excitation acts as a generator because back emf of the motor is less than generated emf of generator. Hence to load the machines (both motor and generator) the excitation of the motor is decreased which decreases its back emf so causing the motor to take large current which in turn produces more torque.

During this test the supply voltage is maintained constant at the rated value and the test should be carried out at rated speed.

The armature circuit resistances (i.e. armature winding including brush contact, interpole and compensating windings If any) of the two machines are measured at the end of the experiment when the windings have attained their final temperature by passing full-load current through them at standstill and measuring voltage drops across them, keeping field circuit open.

Let ammeters A_1, A_2, A_3, A_4 and A_5 connected in the circuit shown in Fig. 4.6, measure the line current I_L, motor current I_m, motor shunt field current I_{shm}, generator current I_g and generator shunt field current I_{shg} respectively.

According to Kirchhoffs first law
Motor current, $I_m = I_L + I_g$
Motor input = VI_m
And Generator output = VI_g where V is the Bus bar voltage

If η_m and η_g are the efficiencies of motor and generator respectively,
Then motor output=$VI_m\eta_m$
And generator input=VI_g/η_g
But motor output= Generator input
Or $VI_m\eta_m = VI_g/\eta_g$
Or $\eta_m\eta_g = I_g/I_m$

Assuming armature, field and stray power losses in both of the machines equal then $\eta_m=\eta_g$

and $\eta_m=\eta_g=\sqrt{\left(\frac{I_g}{I_m}\right)}=\sqrt{\left(\frac{I_g}{I_g+I_L}\right)} = \sqrt{\left(\frac{I_m-I_L}{I_m}\right)}$

This assumption will not cause any error in case of large machines due to very slight difference in their armature and excitation current, but in case of

small machines, the difference between the armature currents and shunt field currents is large and it is not good to make above assumption. To obtain accurate results, armature and shunt field losses are determined separately and stray losses are assumed to be equal in both machines.
Current drawn from supply mains, $I_L = I_m - I_g$
Motor intake current $= I_m$
Motor shunt field current $= I_{shm}$
and motor armature current, $I_{am} = (I_m - I_{shm})$
Generator output current $= I_g$
Generator shunt field current $= I_{shg}$
and generator armature current, $I_{ag} = (I_g + I_{shg})$
Bus-bar voltage = V
Total losses of both machines = Power drawn from supply VI_L
Armature copper loss in motor $= (I_{am} - I_{shm})^2 R_{am}$,

R_{am} being motor armature resistance.
Armature copper loss in generator $= (I_g + I_{Shg})^2 R_{ag}$
R_{ag} being generator armature resistance.

Shunt field copper loss in motor $= V I_{shm}$
Shunt field copper loss in generator $= V I_{shg}$
Total copper losses $= (I_m - I_{shm})^2 R_{am} + (I_g + I_{shg})^2 R_{ag} + V I_{shm} + V I_{shg}$
Stray power losses of both machines, $P_s = VI_L - (I_m - I_{shm})^2 R_{am} + (I_g + I_{Shg})^2 R_{ag} + V (I_{shm} + I_{shg})]$
Stray power losses of each machine $= (P_s/2)$

Efficiency of Motor

Motor intake $= VI_m$
Total losses in motor = Armature loss + field loss + stray powerloss
$= (I_m - I_{shm})^2 R_{am} + V I_{shm} + (P_s/2)$
Motor output $= VI_m - [(I_m - I_{shm})^2 R_{am} + VI_{shm} + (P_s/2)]$
Motor efficiency, $\eta_m =$ (Output / Input) X 100

Efficiency of Generator.

Generator output $= V I_g$
Total losses in generator = Armature loss + field loss +stray loss
$= (I_g + I_{shg})^2 R_{ag} + V I_{shg} + (P_s/2)$
Generator input $= V I_g + (I_g + I_{shg})^2 R_{ag} + V I_{shg} + (P_s/2)$
Generator efficiency, $\eta_g =$ (Output / Input) X 100

Hence from the above data determined from the test and relations, the efficiency of motor and generator can be determined.

Advantages of Hopkinson's Test.

(i) It is economical since power required is small as compared to the full-load power of the two machines.

(ii) Since the machines can be tested under full-load conditions for long duration, therefore, the performance of the machines regarding commutation and temperature rise etc. can be conveniently studied.

(iii) The efficiency is being determined under load conditions so that the stray load loss is being taken into account.

Disadvantage.

The main disadvantage of this test is the necessity of two practically identical machines to be available.

TESTING OF DC SERIES MACHINES (FIELD TEST)

Small series machines can be tested by brake test similar to shunt machines, but the large series machines cannot be tested by Swinburne's test in the same way as shunt machines, because series motors cannot be run on no load due to dangerous high speed. In view of this, the field test is quite suitable for Dc series machines. At the same time, there is no difficulty in having two similar dc series machines, because series motors are used for traction purposes and are, therefore, usually available in pairs. However, the accuracy of this test depends on accuracy with which the motor input and generator output are measured.

In this test two similar dc series machines are required. These two machines are mechanically coupled together and their fields are connected in series, as illustrated in Fig. 4.7 in order to make iron losses of both the machines equal. One of the machines operates as a motor and drives the other machine operating as a separately-excited generator. Load resistance R_L is directly connected to the armature without any switch.

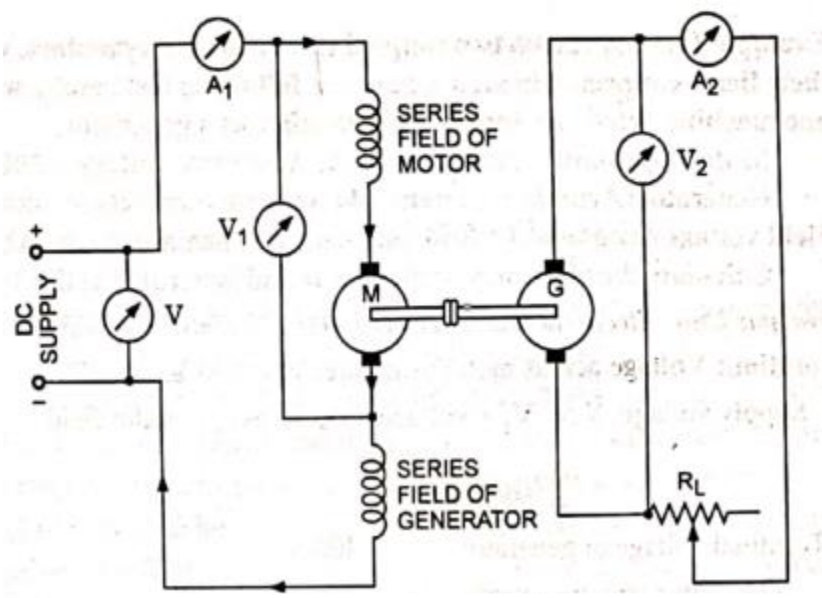


Fig. 4.7 Field Test on Series Machines

The motor M is started in the usual manner and the output of the generator G is dissipated in the variable resistor load. The voltage V_1 across the motor terminals is kept equal to its rated value. Obviously, the supply voltage V will be equal to V_1 plus the voltage drop across the field winding of the generator. The hot resistances of the various windings are measured by voltmeter-ammeter or other suitable method. Load resistance R_L is varied till the ammeter A_l shows the full-load current of the motor. Now the readings of various measuring instruments are noted.

Let supply voltage = Reading of voltmeter V = V volts

Motor input current = Reading of ammeter A_1 = I_1

Terminal voltage of generator = Reading of voltmeter V_2 = V_2

Load current of generator = Reading of ammeter A_2 = I_2

Armature resistance of each machine = R_a

Series field resistance of each machine = R_{se}

Input to the whole set = VI_1

Output = $V_2 I_2$

Total losses of the set, $P_T = VI_1 - V_2 I_2$

Series field and armature copper losses of motor $= I_1^2 (R_a + R_{se})$

Series field and armature copper losses of generator $= I_1^2 R_{se} + I_2^2 R_a$

Total copper losses of the set, $P_c = I_1^2 (R_a + 2R_{se}) + I_2^2 R_a$

Stray power losses for the set = $P_T - P_c$

Stray power losses per machine, Ps = $\frac{P_T - P_c}{2}$

Motor efficiency

Motor input = $V_1 I_1$

Motor losses $= I_1^2 (R_a + R_{se}) + P_s$

Motor output = $V_1 I_1 - I_1^2 (R_a + R_{se}) - Ps$

Motor efficiency $\eta_m = \frac{V_1 I_1 - I_1^2 (\text{Ra} + \text{Rse}) - \text{Ps}}{V_1 I_1}$

Generator Efficiency

Generator output = $V_2 I_2$

Generator losses $= I_2^2 R_a + I_1^2 R_{se} + P_s$

Generator input = $V_2 I_2 + I_2^2 R_a + I_1^2 R_{se} + P_s$

Generator efficiency, $\eta_g = \frac{V_2 I_2}{V_2 I_2 + I_2^2 \text{Ra} + I_1^2 \text{Rse} + \text{Ps}}$

Disadvantage:

1. A relatively small error in the measurement of the input to motor or output of generator may cause a relatively large error in the computed efficiency.
2. Total power supplied to the set is wasted.

RETARDATION TEST

This test is also known as running down test. It is used for finding out the stray losses of shunt wound dc machines.

In this method of testing de machines, machine under test is speeded up slightly above its normal speed and supply to the armature is cut off. Consequently the armature slows down and its kinetic energy is utilized to meet the rotational losses (stray losses).

Kinetic energy of the armature = $\frac{1}{2} J\omega^2$

where J is the moment of inertia of the armature and ω is the angular speed at this instant.

Rotational losses, P_s = Rate of change of kinetic energy

$$= \frac{d}{dt}\left(\frac{1}{2}J\omega^2\right) \qquad = \left(J\omega\frac{d\omega}{dt}\right)$$

Since = $\frac{2\pi N}{60}$, where N is the speed of the armature in rpm at that instant.

Rotational losses, Ps = J $\left(\frac{2\pi N}{60}\right)\frac{d}{dt}\left(\frac{2\pi N}{60}\right)$

$$= \left(\frac{2\pi}{60}\right)^2 JN\frac{dN}{dT}$$

Hence to determine stray losses, the values of moment of inertia of armature (J) and rate of change of speed $\frac{dN}{dT}$ must be known.

The value of J and $\frac{dN}{dT}$ are determined as follows.

(i) Determination of $\frac{dN}{dT}$.

The machine under test is connected, as shown in Fig. 4.8. The voltmeter V connected across the armature shows the instantaneous back emf of the motor. Since back emf of the motor is directly proportional to speed ($E_b \propto N$), the voltmeter can be suitably calibrated to indicate speed. When the supply to armature is cut off, the speed of the motor decreases.

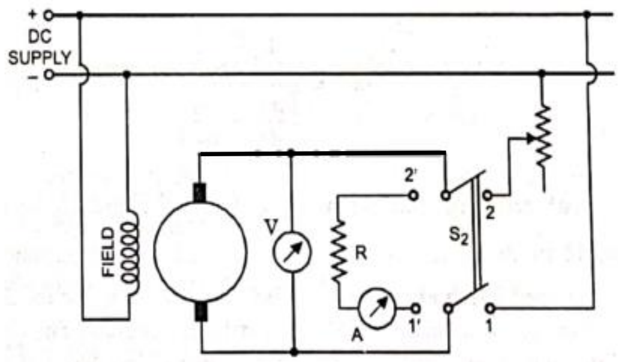


Fig. 4.8

The speed or readings of voltmeter are noted at different intervals of time and a curve is drawn between speed and time, as shown in Fig. 4.9. From any point P lying on speed time curve, tangent is drawn meeting the X-axis and Y-axis at points A and B respectively, then

$$\frac{dN}{dT} = \frac{OB\ in\ rpm}{OA\ in\ Seconds}$$

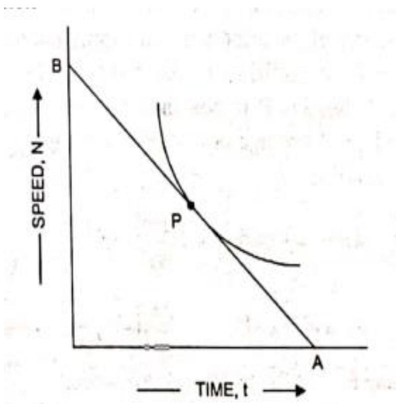


Fig. 4.9

(ii) Determination of moment of inertia

First speed time curve (slowing down curve) is plotted with armature alone as discussed above. Then flywheel of known Moment of inertia (J_1) is keyed to the shaft and speed-time curve is plotted again. From the above two curves, the value of $\left(\frac{dN}{dt_1}\right)$ and $\left(\frac{dN}{dt_2}\right)$ are determined. Since the moment of inertia in second case is more and losses in both cases are almost same, the value of $\left(\frac{dN}{dt_1}\right)$ will be more than the value of $\left(\frac{dN}{dt_2}\right)$.

Hence, In first case, rotational losses,

$$P_s = \left(\frac{2\pi}{60}\right)^2 JN\left(\frac{dN}{dt_1}\right) \qquad (11.17)$$

In second case, rotational losses,

$$P_s = \left(\frac{2\pi}{60}\right)^2 (J + J_1)N\left(\frac{dN}{dt_2}\right) \qquad (11.18)$$

Comparing expressions (11.17) and (11.18) we have

$$\left(\frac{2\pi}{60}\right)^2 (J + J_1)N\left(\frac{dN}{dt_2}\right) = \left(\frac{2\pi}{60}\right)^2 JN\left(\frac{dN}{dt_1}\right)$$

Or $(J + J_1)\left(\frac{dN}{dt_2}\right) = J\left(\frac{dN}{dt_1}\right)$

Or $J\left[\left(\frac{dN}{dt_1}\right) - \left(\frac{dN}{dt_2}\right)\right] = J_1\left(\frac{dN}{dt_2}\right)$

Or $J = \frac{J_1\left(\frac{dN}{dt_2}\right)}{\left[\left(\frac{dN}{dt_1}\right)-\left(\frac{dN}{dt_2}\right)\right]}$ (11.19)

Substituting the values of J_1, $\left(\frac{dN}{dt_1}\right)$ *and* $\left(\frac{dN}{dt_2}\right)$ in Equation (11.19) the value of the inertia of the armature J can be determined.

Second Method:

In this method, first time is noted for the machine to slow down by 5%. Let it be t_1 seconds. Then an additional load known as retarding torque, mechanical or electrical (preferably electrical) is applied and again the time taken by the machine to slow down by 5% in noted, say t_2 seconds. In Fig. 4.8 the additional load (electrical) is applied by throwing the double throw switch S2 over terminals 1', 2' just after disconnecting the armature from the supply mains. Thus a non-inductive resistance R will be in the motor circuit and now kinetic energy of the armature will supply the power to the load resistance R in addition to meet the rotational losses. The additional losses say P' in non-inductive resistance will be equal to the product of average of ammeter reading, I_{av} and average of voltmeter reading, V.

Now rotational loss, $P_s = \left(\frac{2\pi}{60}\right)^2 JN\left(\frac{dN}{dt_1}\right)$ (11.20)

And $P_s + P_s' = \left(\frac{2\pi}{60}\right)^2 JN\left(\frac{dN}{dt_2}\right)$ (11.21)

Dividing Equation (11.21) by Equation (11.20) we get

$$\frac{P_s + P_s'}{P_s} = \frac{\left(\frac{dN}{dt_2}\right)}{\left(\frac{dN}{dt_1}\right)}$$

Or

$$P_s\left[\left(\frac{dN}{dt_2}\right) - \left(\frac{dN}{dt_1}\right)\right] = P_s'\left(\frac{dN}{dt_1}\right)$$

Or

$$P_s = \frac{P_s'\left(\frac{dN}{dt_1}\right)}{\left[\left(\frac{dN}{dt_2}\right)-\left(\frac{dN}{dt_1}\right)\right]} \qquad (11.22)$$

If the change of speed in both cases is kept same and time taken is noted, let it be t_1 and t_2 seconds respectively, then

$$P_s = \frac{P_s' t_2}{t_1 - t_2}$$