UNIT-I

DC & AC CIRCUITS

Circuit: Circuit is the combination of Network elements.

Electrical Circuit: Electrical circuit is a inter connection of Energy sources, active elements and passive elements like Resistor (R), Inductor(L) and capacitor(C).

classification of Network elements:

- Network element
  - Active elements
    - Voltage source
    - Current Source
  - Passive elements
    - Resistor (R)
    - Inductor(L)
    - capacitor (C)

Active elements:

Active elements are the energy sources which may be either voltage source (or) current source

Active elements delivers energy to the circuit

Ex: Voltage source, Current Source

passive elements:

Elements which are not the energy Source is called passive elements.

passive elements neither absorbs the energy or stores the energy.

Ex: Resistor, Inductor, capacitor

Resistor absorbs the energy
Inductor stores the energy in magnetic field
capacitor stores the energy in electrical field

Another classification:

Network elements

- Linear (or) Non-Linear
- Unilateral (or) Bilateral
- Lumped (or) Distributed
- Time Variant (or invariant

Linear & Non-linear Elements:

Linear elements is one which is governed by the linear relationship between exitation (input) and response (output) otherwise it is called Non-linear elements.

Resistor, Air core inductor are linear elements

Diodes, Semiconductor (transistor), Iron core inductor are non-linear elements.

Unilateral & Bilateral Elements:

unilateral elements offers low impedance for the flow of current in one direction and offers high impedance for the flow of current in opposite direction

Eg: Diode (semi conductor devices) transistor

Bilateral elements offers same impedance for the flow of current in either or both directions

Lumped & Distributed Elements:

Lumped parameter is one whose size is very small as compared to the wavelength corresponding to the normal frequency of operation. Otherwise, it is called distributed elements.

Lumped → Resistor, inductor (L), Capacitor (C)

Distributed → Long transmission line

Time Varient and Time invarient:

If the parameter of a network are changing with respect to the time is called time varient. Otherwise time invarient.

Voltage:

The potential difference is measured by the work required to transfer a unit positive charge from one point to another point.

In electrical terminology potential difference called Voltage

$$\boxed{V = \frac{W}{Q} = \frac{dw}{dq}}$$

where W = work done in Joules

Q = charge in columbs

Units for Voltage is "Volt" (or) V

One Volt is the potential difference between the two points, when one Joule of work required to transfer one columb of charge from one point to another point.

voltage is measured with the help of a voltmeter and which is denoted by 'v'.

Current:

Current is the rate of transfer of charge

$$\boxed{i = \frac{Q}{t} = \frac{dQ}{dt}}$$

unit for current is Ampere (or) Columb/sec

current direction is opposite to electron direction i.e, current always flows from higher potential to lower potential.

Power:

power is the rate of doing work (or) power is the rate of transfer of energy.

$$\boxed{P = \frac{w}{t} = \frac{dw}{dt}}$$

As we know, $V = \frac{w}{Q} \Rightarrow w = VQ$

$$P = \frac{VQ}{t} = \frac{d(VQ)}{dt} = V \cdot \frac{dQ}{dt} \qquad \left[\frac{dQ}{dt} = i\right]$$

$$\boxed{P = Vi}$$

units for power J/sec (or) watts

Energy:

Energy is defined as capacity to do work (or) Energy is stored in a work.

→ Energy is available in various forms i.e, electrical energy, Mechanical energy, chemical energy, potential energy & Kinetic energy etc..

$$\boxed{E = Pt} \Rightarrow \text{power} \times \text{time}$$

$$dw = p\,dt$$

$$w = \int_0^t P\,dt$$

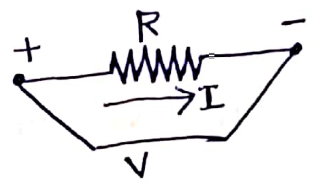
units is joules

Ohm's law:

This law gives relationship between the potential difference (V), the current (I) and the resistance (R) of a d.c circuit. Dr. ohm is 1827 discovered a law called Ohm's law.

"Ohms law states that, the current flowing through the electric circuit is directly propotional to the potential difference across the circuit and inversely propotional to the resistance of the circuit, provided temperature remains constant.

Mathematically, $\boxed{I \propto \frac{V}{R}}$



Note: ohm's law is applicable only for bilateral elements.

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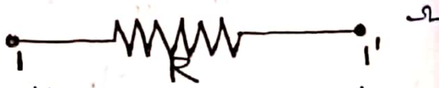
Limitations of ohm's law:

1. It is not applicable to the nonlinear devices such as diodes, Zener diodes, Voltage regulators etc.
2. It does not hold good for non-metallic conductors such as silicon carbide. The law for such conductors is $V = K I^m$

where K, m are constants

Resistance: (R)

Resistance is a circuit element which opposes the flow of current, in doing so the electrical energy is converted in to heat energy. The units of Resistance is ohm Ω the Symbol of Resistor is given by 

According to the ohm's law the voltage drop across the conductor for given length and cross sectional area is proportional to current through conductors.

$$I \propto V$$

$$I = \frac{V}{R}$$

$$V = IR \longrightarrow (1)$$

$$I = \frac{V}{R} = VG \longrightarrow (2)$$

$$\because R = \frac{1}{G}$$

The reciprocal of Resistance is called conductance and the unit of conductance is given by Mho($\mho$).

From the eq (2) the current through the resistor is $I = \frac{V}{R}$

According to physical direction dimensions

$$R = \frac{\rho l}{a}$$

where ρ = Specific resistance of the material

l = length of the conductor

a = Area of cross-section

Power consumed by the Resistor:

$$P = VI$$
$$= (IR) I \quad [\text{from eq (1)}]$$
$$P = I^2 R$$

(or)

$$P = VI$$
$$= V\left(\frac{V}{R}\right)$$
$$P = \frac{V^2}{R}$$

$$\boxed{P = VI = I^2 R = V^2/R}$$

Energy consumed by the Resistor:

$$E = Pt$$
$$= I^2 Rt = \frac{V^2}{R} t$$

$$\boxed{E = Pt = I^2 Rt = V^2/R\, t}$$

Inductance: (L)

An inductance is the element in which energy is stored in the form of electromagnetic field. The inductance is denoted as 'L' and is measured in henries (H).

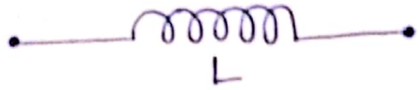
A wire of certain length when twisted into a coil becomes a basic inductor.

For an inductance, the voltage across it is proportional to the rate of change of current passing through it.

$$V \propto \frac{di}{dt} \rightarrow (1)$$

The constant of proportionality in the eq(1) is L

$$\boxed{V = L\frac{di}{dt}} \longrightarrow (2)$$

The symbol for inductance : 

The current through the inductor is given by

$$(2) \Rightarrow \quad di = \frac{V}{L}\,dt$$

$$\int_0^t di = \int_0^t \frac{V}{L}\,dt$$

$$i(t) - i(0) = \frac{1}{L}\int_0^t V\,dt$$

$$\boxed{i(t) = i(0) + \frac{1}{L}\int_0^t V\,dt} \longrightarrow (3)$$

Where $i(0)$ is the intial current through the inducto

The power in the inductor is given by,

$$P = Vi$$

$$P = L\frac{di}{dt}\cdot i \quad (\text{from eq}(2))$$

$$\boxed{P = Li\frac{di}{dt}} \longrightarrow (4)$$

The energy stored in the inductor in the form of an electromagnetic field is,

$$W = \int_0^t P\cdot dt$$

$$W = \int_0^t Li \frac{di}{dt}\, dt \quad \text{(from eq (4))}$$

$$= \int_0^t Li \cdot di = L\int_0^t i \cdot di$$

$$\boxed{W = \frac{1}{2} L i^2 \text{ Joules}}$$

From the above discussion we conclude the following points:

1. If the current passing through the inductor is DC, there is no induced Emf in the inductor.

$$V = L\frac{di}{dt}$$

$$V = L\frac{d}{dt}(\text{const})$$

$$V = 0 \quad \text{(Short circuit)}$$

2. The current through the inductor cannot change instantaneously.

$$V = L\frac{di}{dt} \qquad dt = 0$$

$$V = L\frac{di}{0}$$

Which is not possible

3. A pure Inductor never disipates the energy.

Capacitance: (C)

An element in which energy is stored in the form of an electrostatic field is known as capacitance. It is made up of two conducting plates separated by a dielectric material. It is denoted as 'c' and measured in farads (F).

The charge on a capacitor is proportional to applied voltage

$$q \propto V$$

$$q = CV \longrightarrow (1)$$

$$\boxed{C = \frac{q}{V}} \quad \text{(c is the capacitor of the circuit)}$$

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According to the physical dimensions for parallel plate capacitor

$$\boxed{C = \frac{A \varepsilon_0 \varepsilon_r}{d}}$$

where
A = Surface area of the plate
ε_0 = permitivity of free space $= 8.85 \times 10^{-12}$ F/M
ε_r = permitivity of medium
d = distance of separation

The current through capacitor is given by

$$i = \frac{dq}{dt}$$

$$i = \frac{d}{dt}(CV) \quad \text{from eq(1)}$$

$$\boxed{i = C\frac{dv}{dt}} \rightarrow (2)$$

The voltage drop across capacitor is given by

$$i = C\frac{dv}{dt}$$

$$dV = \frac{i}{C}dt$$

$$\int_0^t dV = \int_0^t \frac{i}{C}dt$$

$$V(t) - V(0) = \frac{1}{C}\int_0^t i.dt$$

$$\boxed{Vt = V(0) + \frac{1}{C}\int_0^t i.dt}$$

Where V(0) is the intial voltage

The power in the capacitor is given by,

$$P = Vi$$

$$\boxed{P = VC\frac{dV}{dt}} \rightarrow (3) \quad \left(i = C\frac{dV}{dt}\right) \text{from eq (2)}$$

The energy stored in the capacitor is given by,

$$P = \frac{dw}{dt}$$

$$dw = P\,dt$$

$$w = \int_0^t P\,dt$$

$$= \int_0^t vc\,\frac{dv}{\cancel{dt}}\cdot \cancel{dt} \quad \text{(from eq (3))}$$

$$= \int_0^t vc\,dv = c\int_0^t v\,dt = c\frac{v^2}{2}$$

$$\boxed{w = \frac{1}{2}cv^2}$$

From the above discussion we conclude the following

→ If the applied voltage is DC the current through the capacitor is zero i.e capacitor acts as open circuit to DC

$$i = c\frac{dv}{dt}$$

$$i = c\frac{d}{dt}(\text{const})$$

→ Voltage across the capacitor does not change instantaneously

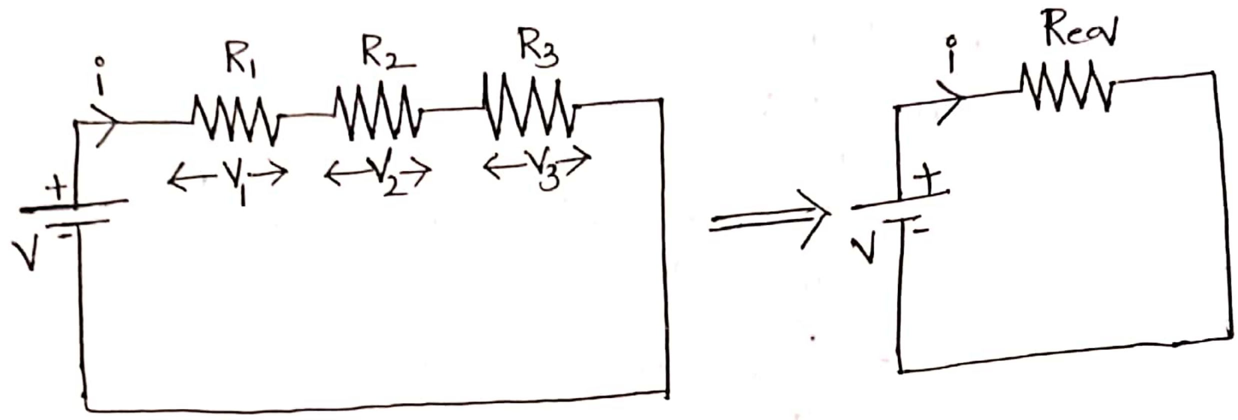
→ A pure capacitors never disipates / absorbs the energy

Voltage - Current Relationship for passive elements:

The behaviour of the three elements can be summarized as

Element	unit	Voltage	Current	power	Energy
R	Ω ohms	$V = iR$	$i = \frac{V}{R}$	$P = i^2R$ $P = V^2/R$	$E = i^2Rt$ $E = V^2/R \cdot t$
L	H henry	$V = L\frac{di}{dt}$	$i(t) = i(0) + \frac{1}{L}\int_0^t V\cdot dt$	$P = Li\frac{di}{dt}$	$E = \frac{1}{2}Li^2$
C	F farad	$V(t) = V(0) + \frac{1}{C}\int_0^t i\cdot dt$	$i = c\frac{dv}{dt}$	$P = cv\frac{dv}{dt}$	$E = \frac{1}{2}cv^2$

Resistors connected in Series:



Consider the circuit, where R_1, R_2, R_3 are connected in Series. This combination is connected across a Source voltage 'V'.

Let, V_1 is the voltage drop across Resistor R_1

V_2 is the voltage drop across Resistor R_2

V_3 is the voltage drop across Resistor R_3

Then, $V = V_1 + V_2 + V_3 \longrightarrow (1)$

According to ohm's law $V_1 = iR_1$, $V_2 = iR_2$, $V_3 = iR_3$ and current through all of them is same i.e i

$$(1) \Rightarrow V = iR_1 + iR_2 + iR_3$$

$$= i(R_1 + R_2 + R_3)$$

$$V = i R_{eq}$$

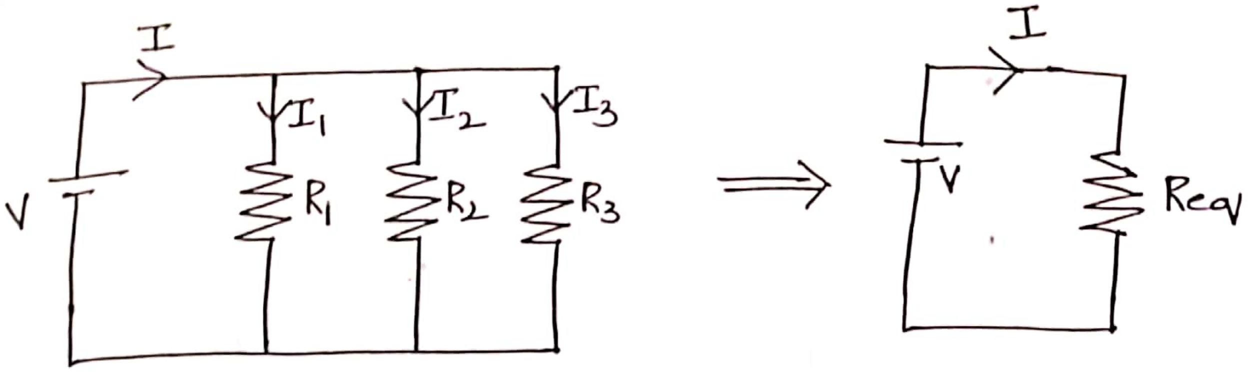
Where R_{eq} = Equivalent resistance of the circuit

$$\boxed{R_{eq} = R_1 + R_2 + R_3}$$

If n resistors are connected in series

then, $\boxed{R = R_1 + R_2 + R_3 - - - - R_n}$

Resistors Connected in parallel :



Consider the circuit, R_1, R_2, R_3 are connected in parallel. This combination is connected across a source voltage 'V'. The voltage across the two ends of each resistance R_1, R_2, R_3 is same and equal to the supply voltage V, and current is dividing at principle node.

$$V_1 = V_2 = V$$

$$I = I_1 + I_2$$

From ohm's law, we know that $I = \frac{V}{R}$

$$\frac{V}{R} = \frac{V_1}{R_1} + \frac{V_2}{R_2}$$

$$\frac{V}{R} = \frac{V}{R_1} + \frac{V}{R_2}$$

$$\frac{\cancel{V}}{R} = \cancel{V}\left[\frac{1}{R_1} + \frac{1}{R_2}\right]$$

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2}$$

$$\boxed{R_{eq} = \frac{R_1 R_2}{R_1 + R_2}}$$

Suppose three elements are connected in parallel

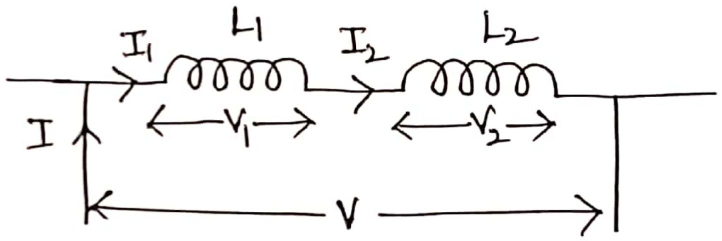
then $\Rightarrow \frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$

$$\boxed{R_{eq} = \frac{R_1 R_2 R_3}{R_1 R_2 + R_2 R_3 + R_3 R_1}}$$

If 'n' resistors are connected in parallel connection then equivalent resistance is given by,

$$\boxed{\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + ----- + \frac{1}{R_n}}$$

Series & parallel connection of inductors:



If two inductors L_1 and L_2 are connected in series as shown in the fig,

For series connection $I = I_1 = I_2$ and $V = V_1 + V_2$

The voltage across the inductor is $V = L\frac{dI}{dt}$

$$V = V_1 + V_2$$

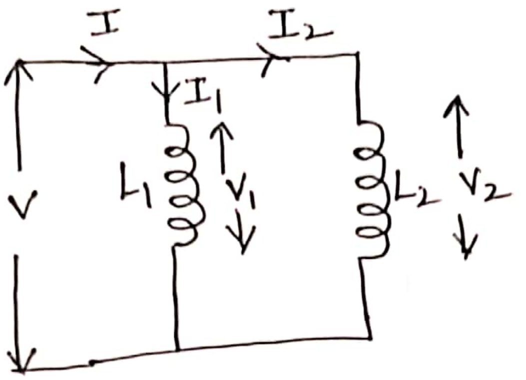
$$L\frac{dI}{dt} = L_1\frac{dI_1}{dt} + L_2\frac{dI_2}{dt}$$

$$L\frac{\cancel{dI}}{\cancel{dt}} = (L_1 + L_2)\frac{\cancel{dI}}{\cancel{dt}}$$

$$\boxed{L_{eq} = L_1 + L_2}$$

Similarly 'n' number of inductors are connected in series $L_{eq} = L_1 + L_2 + L_3 + \cdots + L_n$

→ If L_1 and L_2 Inductors are connected in parallel



For parallel connection of inductors

$V_1 = V_2 = V$

$I_1 + I_2 = I$

The current through the inductor is $I = \frac{1}{L}\int v\,dt$

$$\frac{1}{L_{eq}}\int V dt = \frac{1}{L_1}\int V_1 dt + \frac{1}{L_2}\int V_2 dt$$

$$\frac{1}{L_{eq}}\int V \cancel{dt} = \left(\frac{1}{L_1} + \frac{1}{L_2}\right)\int V \cancel{dt}$$

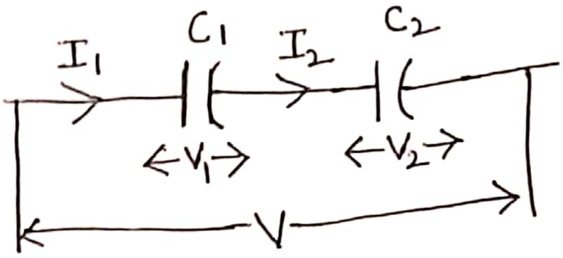
$$\frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2}$$

$$\boxed{L_{eq} = \frac{L_1 L_2}{L_1 + L_2}}$$

If 'n' number of inductors are connected in parallel

$$\frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2} + \cdots + \frac{1}{L_n}$$

Series & parallel connection of capacitor:



If two capacitors C_1 and C_2 are connected in series as shown in the fig

For series connection, $I = I_1 = I_2$

$$V = V_1 + V_2$$

$$C = \frac{Q}{V} \Rightarrow V = \frac{Q}{C}$$

$$\frac{Q}{C_{eq}} = \frac{Q_1}{C_1} + \frac{Q_2}{C_2}$$

$Q \propto i$

$$\frac{\cancel{Q}}{C_{eq}} = \cancel{Q}\left(\frac{1}{C_1} + \frac{1}{C_2}\right)$$

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2}$$

$$\boxed{C_{eq} = \frac{C_1 C_2}{C_1 + C_2}}$$

If 'n' capacitors are connected in series connection then, $\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \cdots + \frac{1}{C_n}$

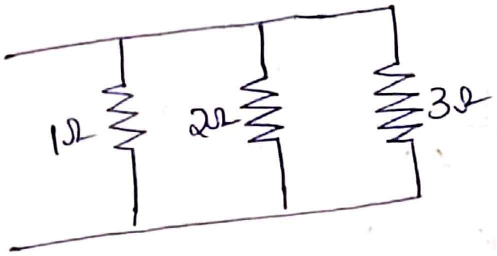
Similarly If the two capacitors are connected in parallel then the equivalent value of capacitor is

$$\boxed{C_{eq} = C_1 + C_2}$$

Problems: Find the Req

1.

2.



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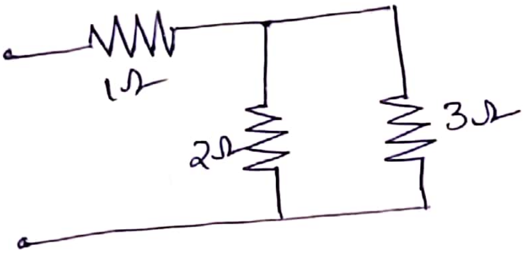

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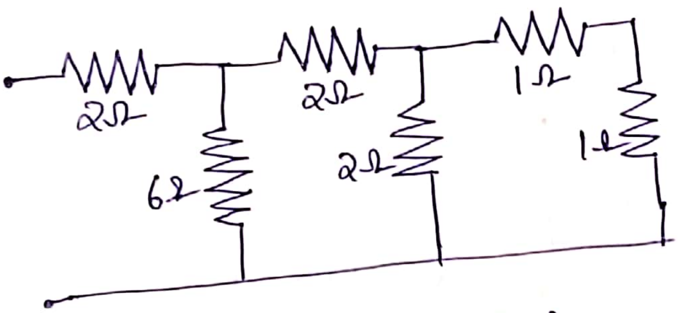

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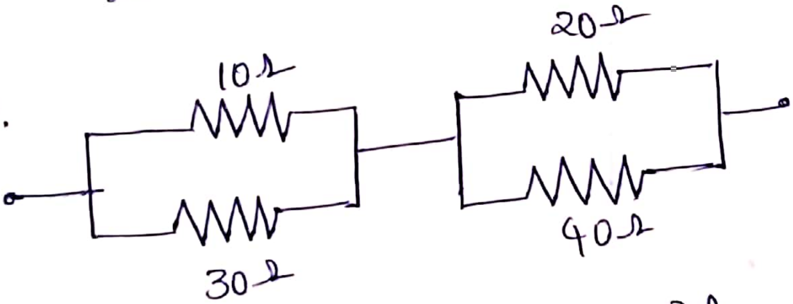

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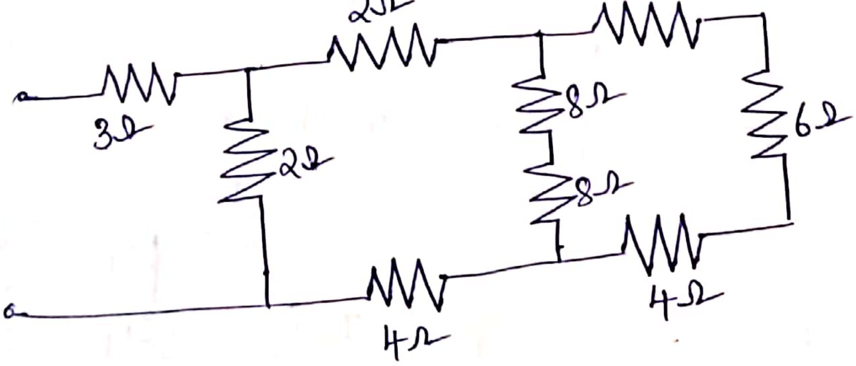

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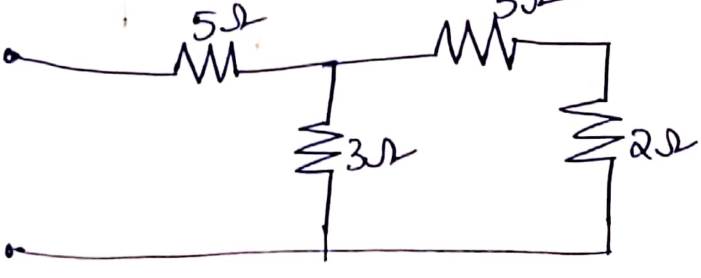
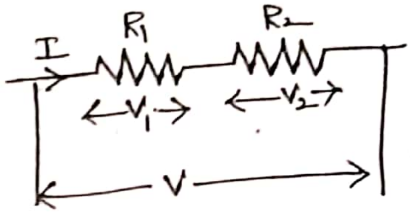


Voltage division Rule:

Consider a simple electrical circuit consists of a voltage source 'V' in volts, two resistive elements R_1 & R_2 are connected in series. Therefore equivalent resistance is

$$R_{eq} = R_1 + R_2$$



From the ohm's law $I \propto V$

$$I = \frac{V}{R} = \frac{V}{R_{eq}} = \frac{V}{R_1 + R_1}$$

The voltage drop across the resistor R_1 is given by

$$V_1 = I R_1 = \frac{V}{R_1 + R_2} \cdot R_1$$

$$\boxed{V_1 = V \cdot \frac{R_1}{R_1 + R_2}}$$

$V \rightarrow$ is total voltage

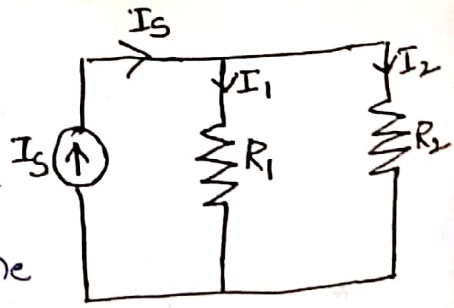
The voltage drop across the resistor R_2 is given by

$$\boxed{V_2 = V \cdot \frac{R_2}{R_1 + R_2}}$$

Current division Rule:

consider a simple electrical circuit consists of a current a current source in parallel with resistive elements R_1 & R_2. are connected i

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} \Rightarrow R_{eq} = \frac{R_1 R_2}{R_1 + R_2}$$



In a parallel circuit the voltage is same and current is dividing at the node.

$$V_1 = V_2 = V$$

$$I_s = I_1 + I_2$$

From the ohms law $V = IR$, $V = I_s R_{eq}$

$$V = I_s\left(\frac{R_1 R_2}{R_1 + R_2}\right)$$

The current entering to the resistor R_1 i.e $I_1 = \frac{V}{R_1}$

$$I_1 = \frac{I_s\left(\frac{\cancel{R_1} R_2}{R_1 + R_2}\right)}{\cancel{R_1}}$$

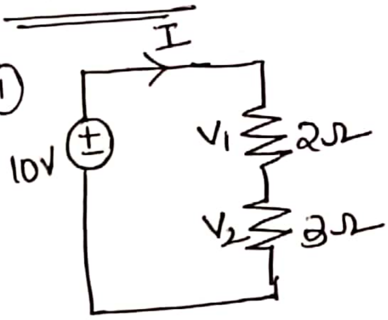
$$\boxed{I_1 = I_s\left(\frac{R_2}{R_1 + R_2}\right)}$$

The current entering to the resistor R_2 i.e $I_2 = \frac{V}{R_2}$

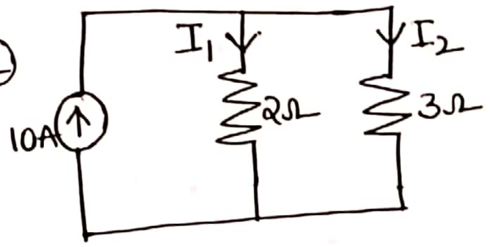
$$I_2 = \frac{I_s\left(\frac{R_1 \cancel{R_2}}{R_1 + R_2}\right)}{\cancel{R_2}}$$

$$\boxed{I_2 = I_s\left(\frac{R_1}{R_1 + R_2}\right)}$$

Problems:

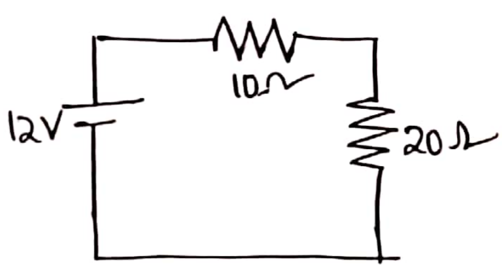
① 

Calculate V_1 & V_2 by voltage division rule

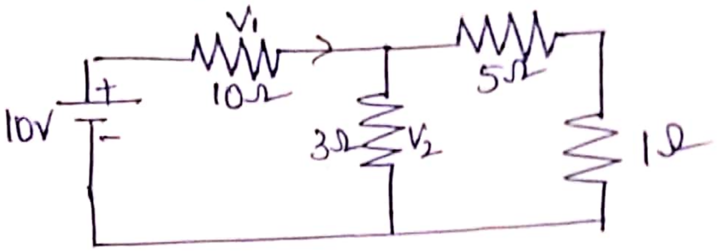
② 

Calculate I_1, I_2 by using current division rule also find voltage drop across the elements.

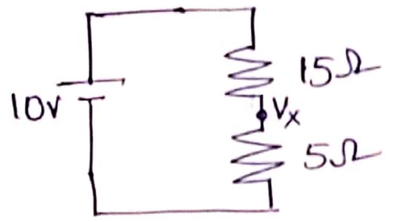
③ Calculate V_1 V_2 in the following circuit



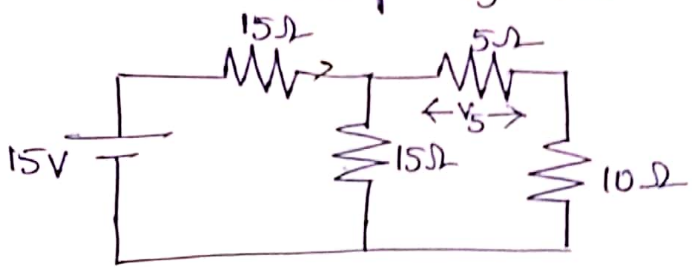
(4) Calculate V_1, V_2 in the following circuit



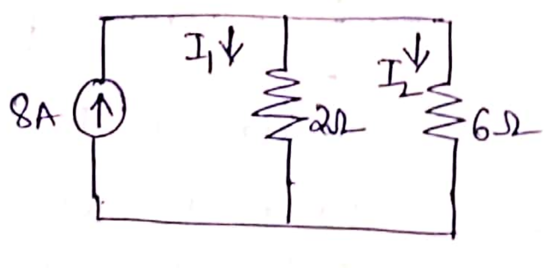
(5) Calculate the voltage V_x



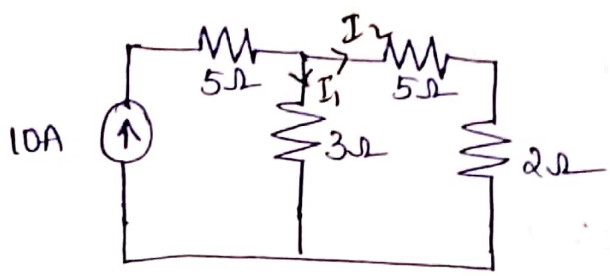
(6) Calculate the voltage V_5 from the given circuit



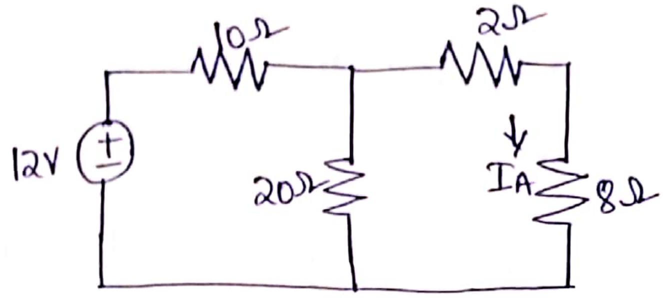
(7) Calculate I_1, I_2 in the following circuit



(8) Calculate I_1, I_2 from the given circuit



(9) Calculate I_A



Kirchoff's laws:

In 1847, a German physicist, Kirchoff, formulate two fundamental laws of electricity. They are

1. Kirchoff's current law (KCL)
2. Kirchoff's voltage law (KVL)

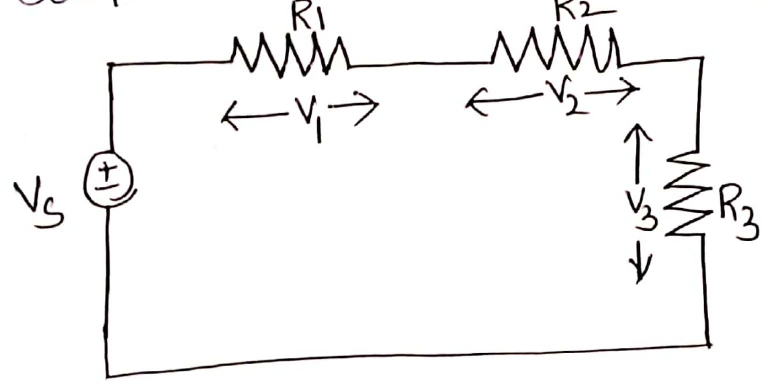
Kirchoff's voltage law:

The KVL is defined for a loop or mesh.

The KVL statement is given by "In any network, the algebraic sum of the voltage drops across the circuit elements of any closed path is always zero".

i.e $\sum V = 0$

consider a simple closed electrical network is having voltage source in series with R_1, R_2, R_3 elements.



Let V_s is the source voltage and $V_1, V_2 \& V_3$ are the voltage drops.

When current entering into negative terminal and leaving from positive terminal the voltage can be treated as positive (voltage rise).

When current entering to positive terminal and leaving from negative terminal the voltage can be taken as negative (voltage drop).

Applying KVL to the network

$$V_S - V_1 - V_2 - V_3 = 0$$

$$\boxed{V_S = V_1 + V_2 + V_3}$$

KVL can also be stated as the sum of voltage rise is equal to the sum of voltage drops.

Kirchoff's current law:

The KCL is defined for a node.

The KCL statement is "The algebraic sum of all the currents meeting at a junction point is alway zero.

i.e $\sum I = 0$

Consider a simple electrical network is having four resistive elements R_1, R_2, R_3 & R_4 with corresponding currents are i_1, i_2, i_3 and i_4

From KCL

$$i_1 + i_2 - i_3 - i_4 = 0$$

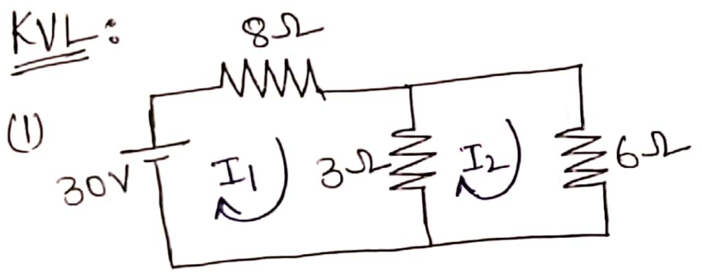
$$\boxed{i_1 + i_2 = i_3 + i_4}$$

From the above expression KCL also stated as "The algebraic sum of currents entering to the node is equal to algebraic sum of currents leaving from the node."

Problems on KVL & KCL:

KVL:

(1)



Calculate the current flowing through 3Ω resistor by KVL

By Applying KVL to the loop 1:

$$30 - 8I_1 - 3(I_1 - I_2) = 0$$

$$30 - 8I_1 - 3I_1 + 3I_2 = 0$$

$$30 - 11I_1 + 3I_2 \Rightarrow 11I_1 - 3I_2 = 30 \rightarrow (1)$$

KVL to the loop 2:

$$-3(I_2 - I_1) - 6I_2 = 0$$

$$-9I_2 + 3I_1 = 0 \rightarrow (2)$$

From eq (1) & (2)

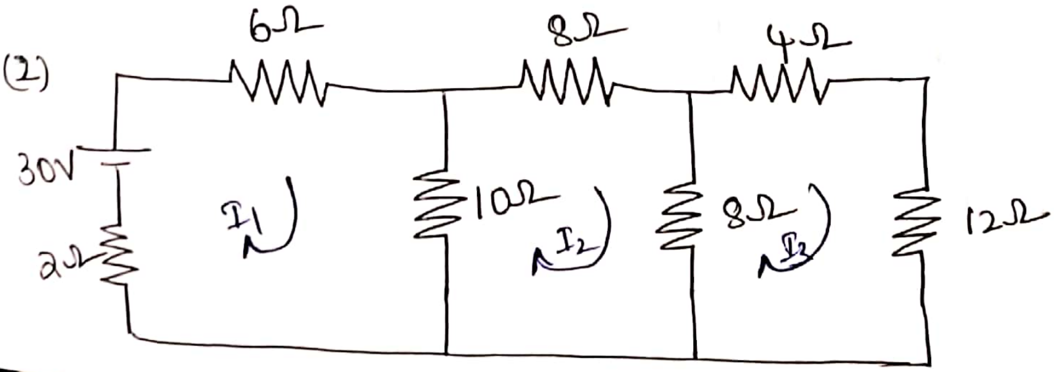
$$I_1 = 3A \quad , I_2 = 1A$$

The current through resistor 3Ω is $I_1 - I_2$

$$I_{3\Omega} = 3 - 1$$

$$= 2A$$

(2)



Calculate the currents by KVL

By Applying KVL to the loop ① is

$$-2I_1 + 30 - 6I_1 - 10(I_1 - I_2) = 0$$

$$-2I_1 - 6I_1 - 10I_1 + 10I_2 + 30 = 0$$

$$-18I_1 + 10I_2 + 30 = 0$$

$$18I_1 - 10I_2 = 30 \longrightarrow ①$$

KVL to the loop ② is

$$-10(I_2 - I_1) - 8I_2 - 8(I_2 - I_3) = 0$$

$$-10I_2 + 10I_1 - 8I_2 - 8I_2 + 8I_3 = 0$$

$$-26I_2 + 10I_1 + 8I_3 = 0 \longrightarrow ②$$

KVL to the loop ③ is

$$-8(I_3 - I_2) - 4I_3 - 12I_3 = 0$$

$$-8I_3 + 8I_2 - 4I_3 - 12I_3 = 0$$

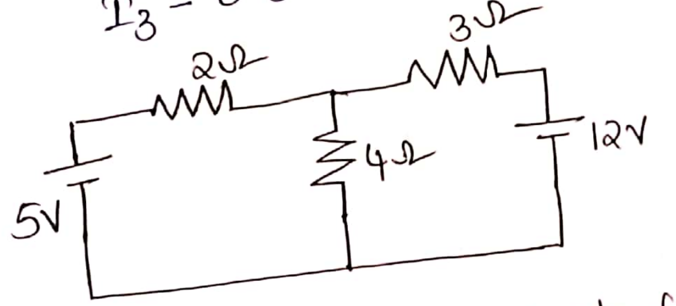
$$-24I_3 + 8I_2 = 0 \longrightarrow ③$$

From eq (1), (2) & (3)

$$I_1 = 2.187A$$

$$I_2 = 0.937A$$

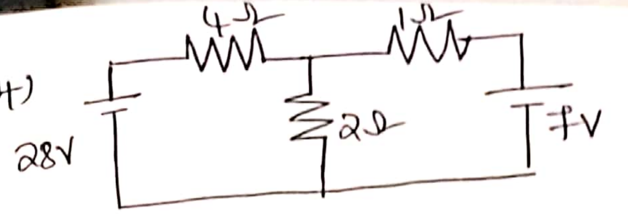
$$I_3 = 0.3125A$$

(3) 

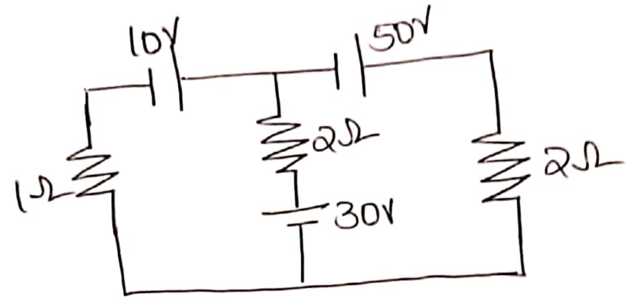
Calculate the current flowing through 4Ω Resistor.

Calculate power loss in each element.

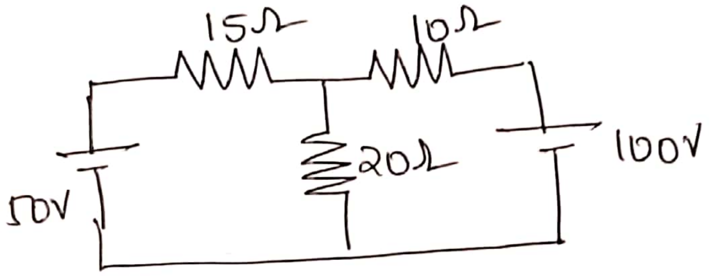
(4)



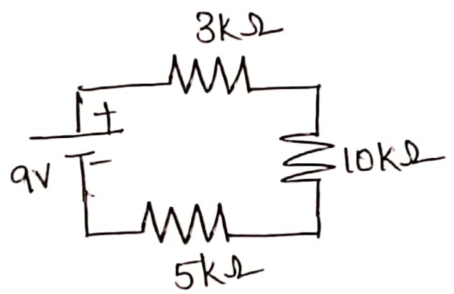
(5)



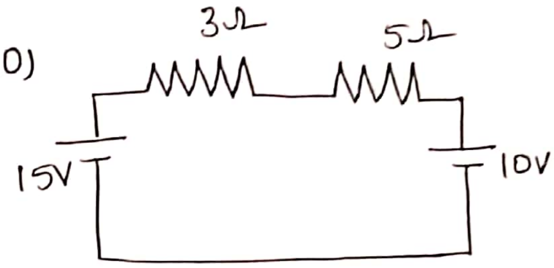
(6)



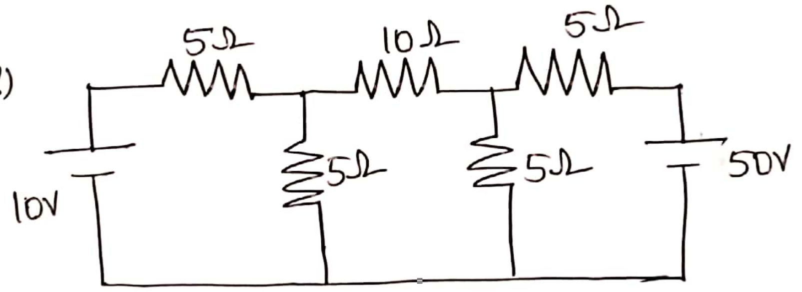
(7)



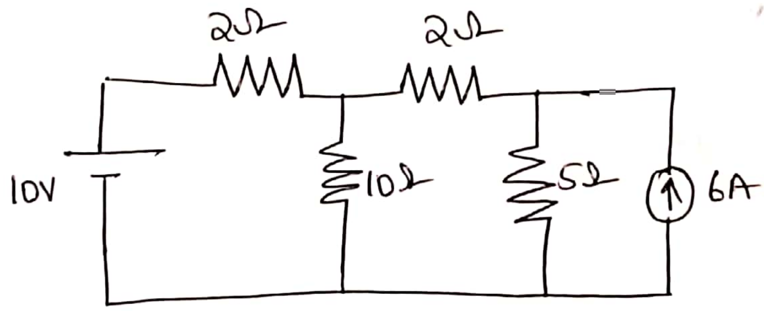
(10)



(8)



(9)



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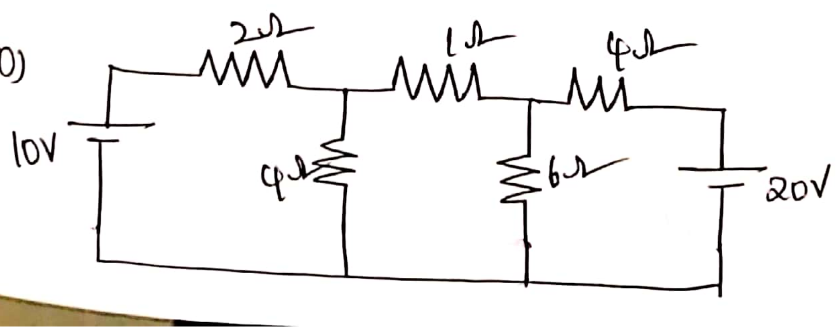

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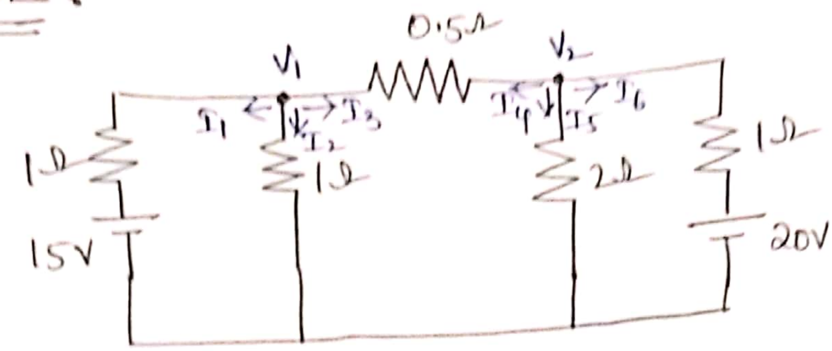
KCL :



Applying KCL to node ①

$$\frac{V_1 - 15}{1} + \frac{V_1}{1} + \frac{V_1 - V_2}{0.5} = 0$$

$$V_1\left(1 + 1 + \frac{1}{0.5}\right) - \frac{V_2}{0.5} = 15$$

$$4V_1 - 2V_2 = 15 \rightarrow ①$$

node ②

$$\frac{V_2 - V_1}{0.5} + \frac{V_2}{2} + \frac{V_2 - 20}{1} = 0$$

$$V_2\left(\frac{1}{0.5} + \frac{1}{2} + 1\right) - \frac{V_1}{0.5} = 20$$

$$3.5V_2 - 2V_1 = 20 \longrightarrow ②$$

From ① & ② $V_1 = 9.25V$, $V_2 = 11V$

$$I_1 = V_1 - 15 = 9.25 - 15 = -5.75A$$

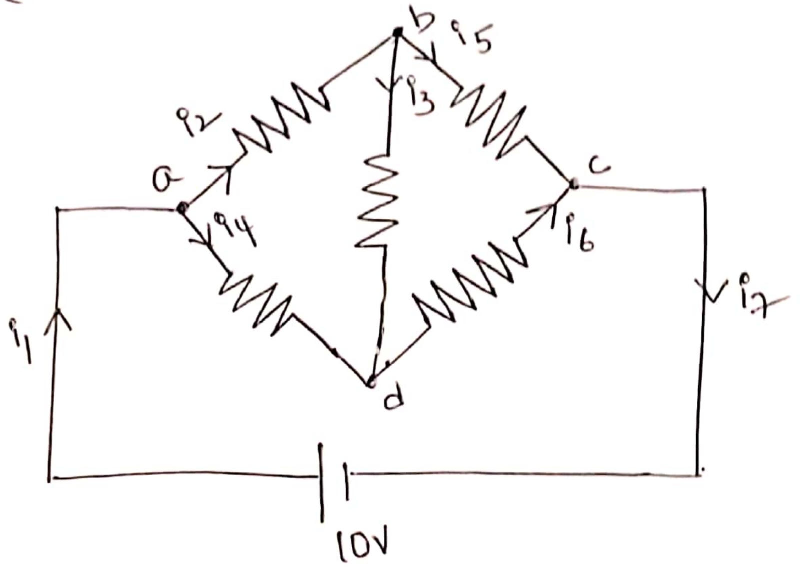
$$I_2 = V_1 = 9.25A$$

$$I_3 = \frac{9.25 - 11}{0.5} = -3.5A$$

$$I_4 = \frac{V}{2} = \frac{11}{2} = 5.5A$$

$$I_5 = V_2 - 20 = 11 - 20 = -9A$$

(2)



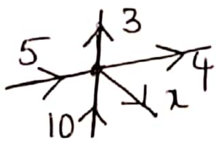
If $i_1 = 10A$
$i_2 = 6A$
$i_5 = 4A$
find i_4, i_3, i_6

At node ⓐ : $i_1 = i_2 - i_4 = 0$
$10 = 6 + i_4$
$i_4 = 4A$

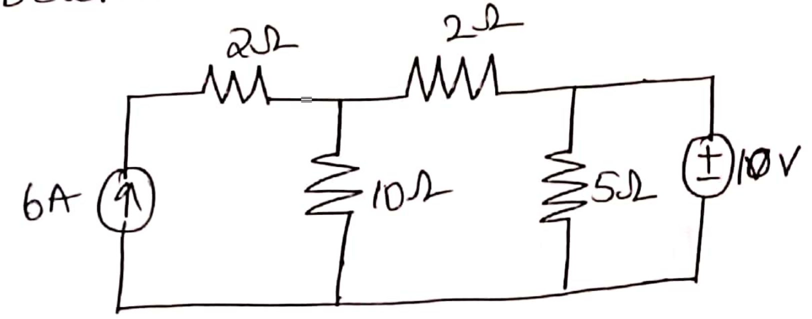
node ⓑ : $i_2 = i_3 + i_5$
$6 = i_3 + 4$
$i_3 = 2A$

node ⓒ : $i_5 + i_6 = i_7$
$4 + i_6 = 10$
$i_6 = 6A$

(3) Determine the unknown current



(4) Determine the current through all branchs



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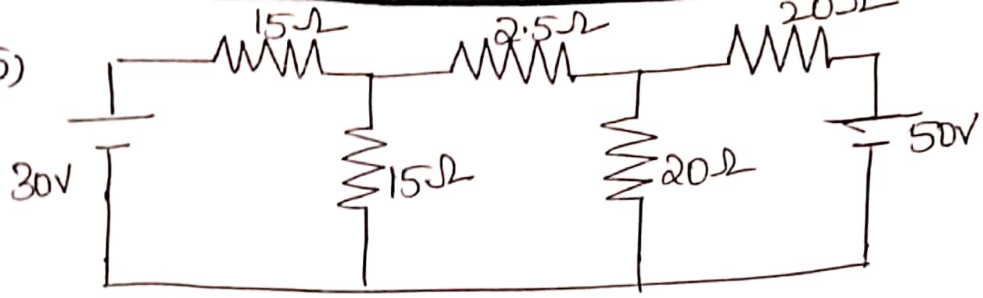
(5)



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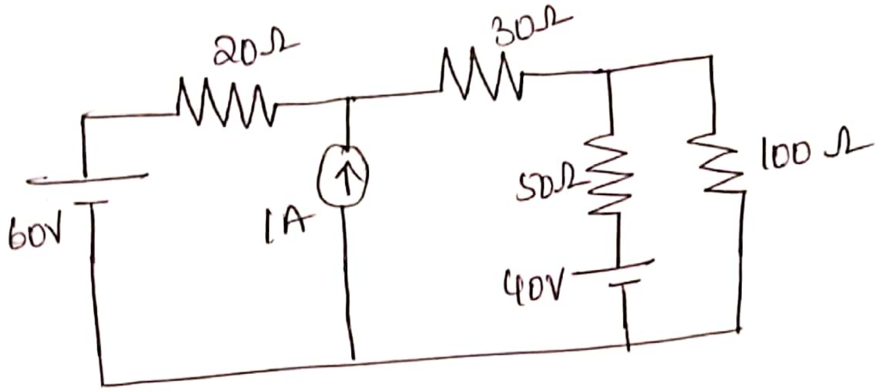
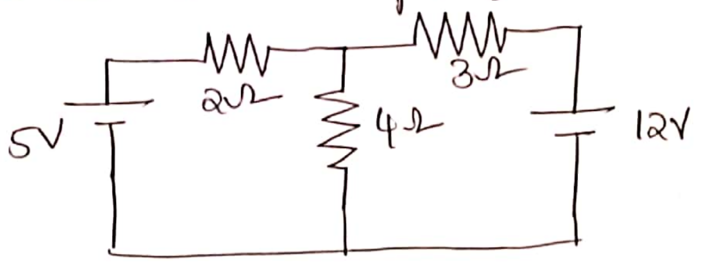


(7) find Current through 4Ω Resistor



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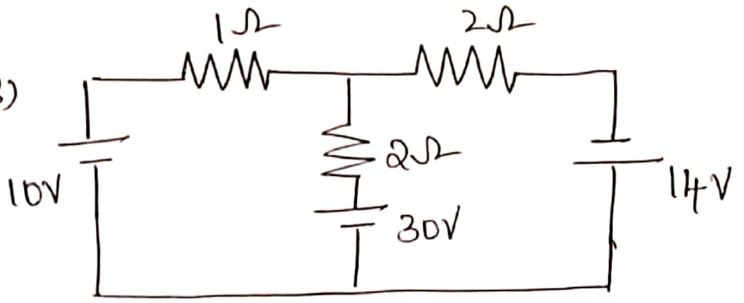

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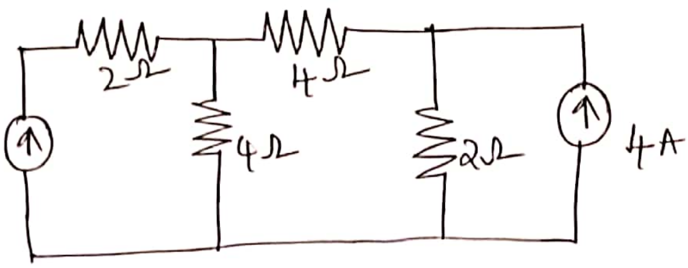
(9)



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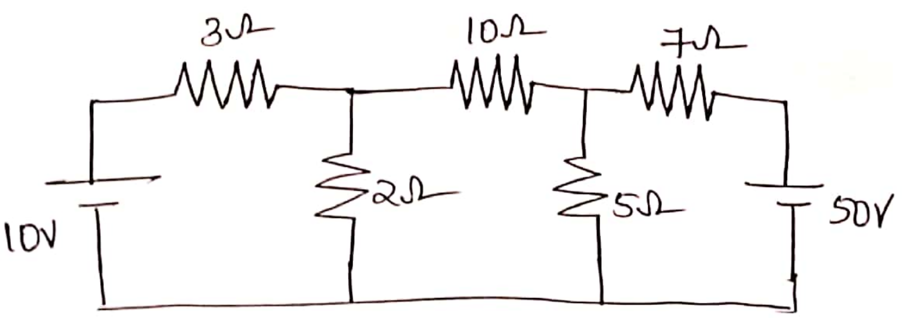

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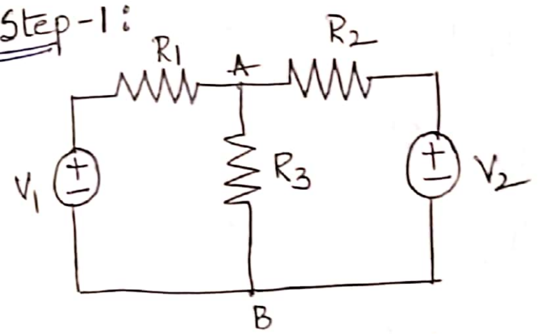
Superposition theorem:

The Superposition theorem states that in any linear network consists of two or more sources, the response in any element is equal to the algebraic sum of the responses produced by each source acting individually

The voltag source in the network must be replaced by short circuit and current source is replaced by open circuit.

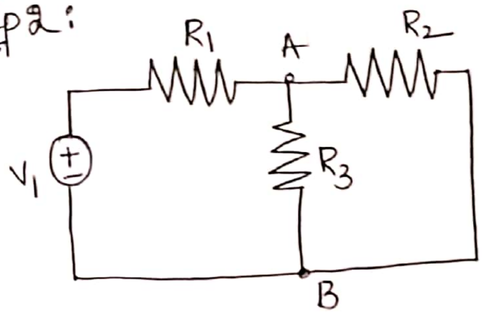
consider a simple network having two voltage sources V_1 & V_2.

Step-1:



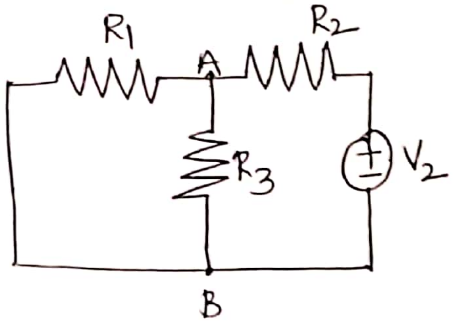
Calculate the current in branch A-B of the network using Superposition theorem. I_{AB}

Step 2:



Now V_1 source is acting independly and V_2 is replaced by short circuit obtain current throug AB branch. (I'_{AB})

step-3:



Now consider V_2 alone and V_1 is replaced by short circuit obtain current throug AB bran (I''_{AB})

$$\boxed{I_{AB} = I'_{AB} + I''_{AB}}$$

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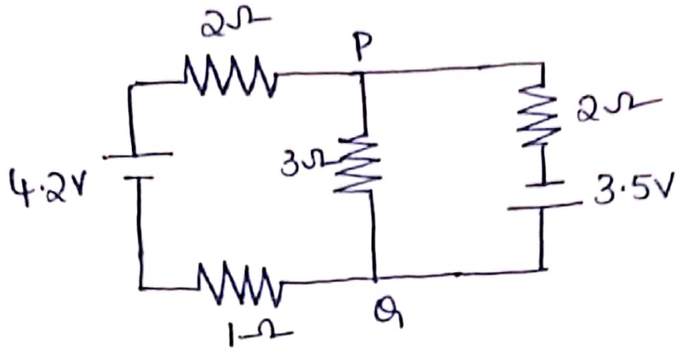
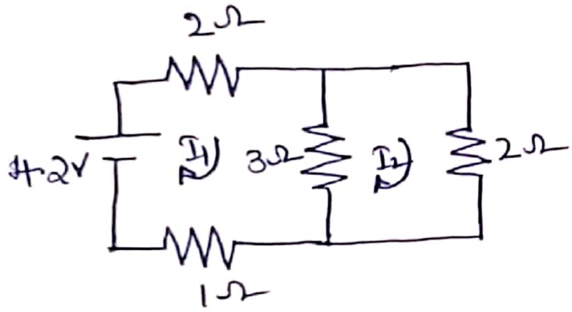
Problems on superposition theorem:

(1) Use the superposition theorem to calculate the current in the branch PQ of the circuit shown



Step 1: consider 4.2V, replace other source by short circuit



$$4.2 - 2I_1 - 3(I_1 - I_2) - I_1 = 0$$

$$4.2 - 6I_1 + 3I_2 = 0 \rightarrow (1)$$

$$-3(I_2 - I_1) - 2I_2 = 0$$

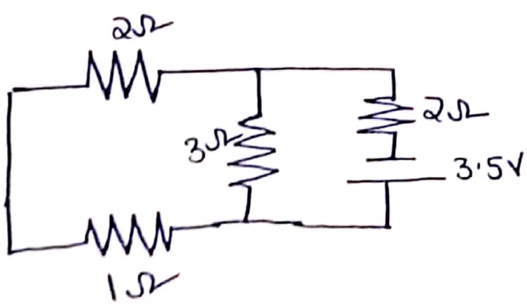
$$3I_1 - 5I_2 = 0 \longrightarrow (2)$$

From eq (1) & (2) we get

$$I_1 = 1A \quad , \quad I_2 = 0.6A$$

$$I'_{PQ} = I_1 - I_2 = 0.4A$$

Step 2: consider 3.5V, replace other source by short circ



$$-I_1 - 2I_1 - 3(I_1 - I_2) = 0$$

$$-6I_1 + 3I_2 = 0 \longrightarrow (1)$$

$$-3(I_2 - I_1) - 2I_2 + 3.5 = 0$$

$$-5I_2 + 3I_1 + 3.5 = 0$$

$$-3I_1 + 5I_2 = 3.5 \longrightarrow (1)$$

From eq (1) & (2) we get

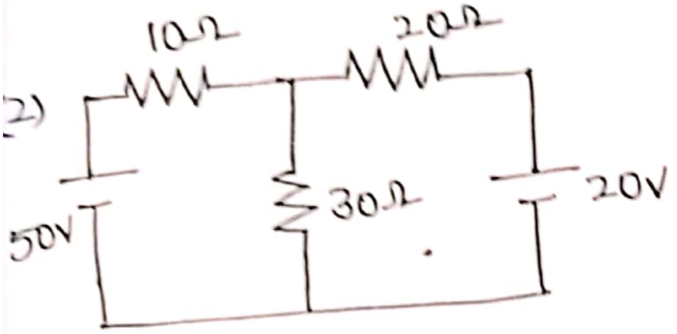
$I_1 = 0.5A \quad I_2 = 1A$

$I''_{PQ} = I_1 - I_2 = -0.5A$

$I_{PQ} = I'_{PQ} + I''_{PQ}$

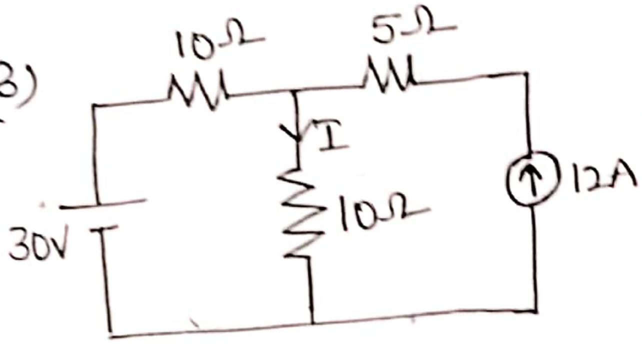
$= 0.4 - 0.5A = \underline{-0.1A}$

2)



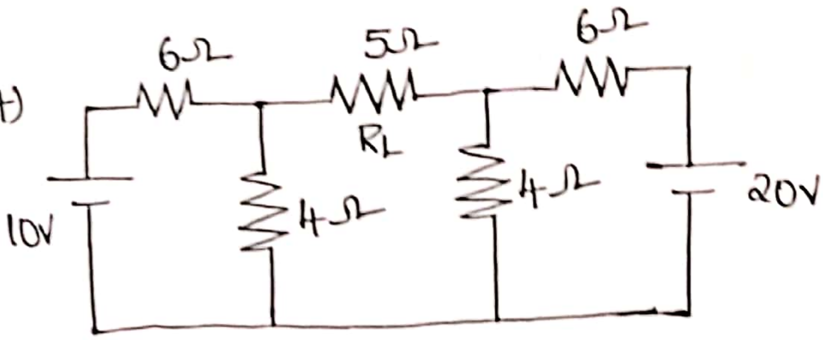
Using superposition theorem find the current flowing through 30Ω Resistor?

(3)



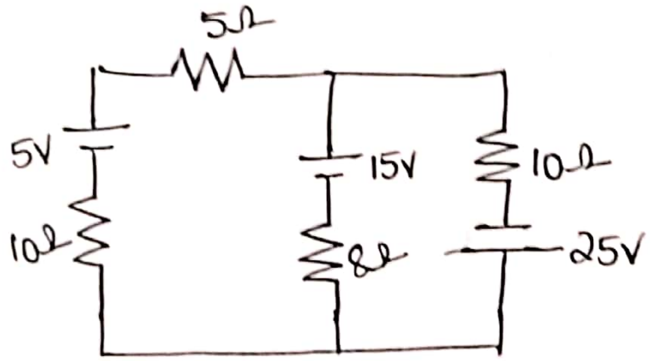
Find the value of 'I' using S.T

(4)



Find I_L using Super-position theorem

(5)



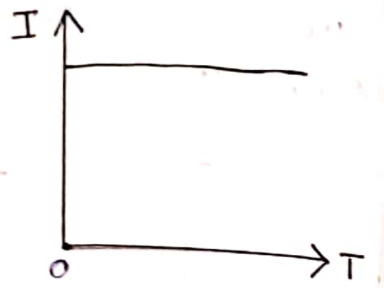
find the current through 8Ω Resistor using S.T ?

AC Circuits

up to now we know about DC supply and DC circuits. But 90% of the Electrical energy used now-a-days is AC in nature.

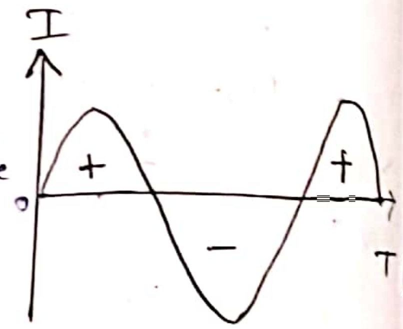
DC Supply:

DC Supply has constant magnitude with respect to time. The Graph shows DC current w.r.t time.



AC Supply:

The Alternating Current is a current which changes periodically both in magnitude and direction



Such change in magnitude and direction is measured interms of cycles. Each cycle of AC consists of two half cycles namely positive cycle and negative cycle. Current increases in magnitude, in one particular direction, attains maximum and starts decreasing, passing through zero, it increases in opposite direction and behaves similarly as shown in the figure.

Advantages of A.C:

1. The voltage in AC system can be raised or lowered with the help of a device called transformer.
2. The AC is easy to generate than dc and easily converted into DC

3. It is cheaper to generate AC than DC

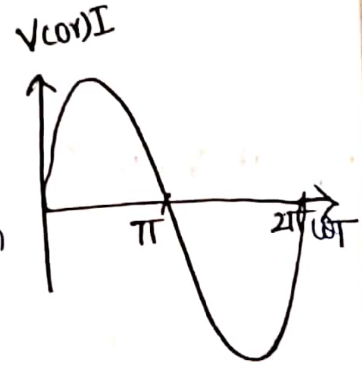
4. AC electrical motors are simple in construction, are cheaper and require less attention from maintenance point of view.

5. The loss of Electrical energy during transmission is lesser for AC

6. The AC generators have higher efficiency than DC.

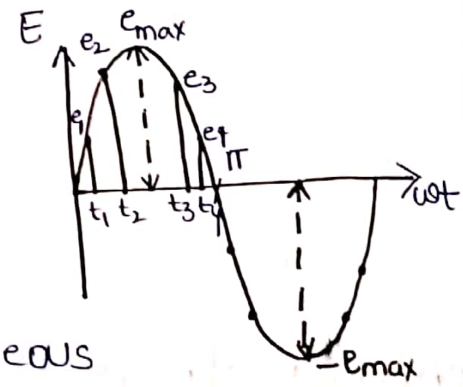
Terminology Related to Alternating Quantity:

(1) Wave form:



The Graph is obtained by plotting the instantaneous values of the current or voltage against a base of time is known as wave form.

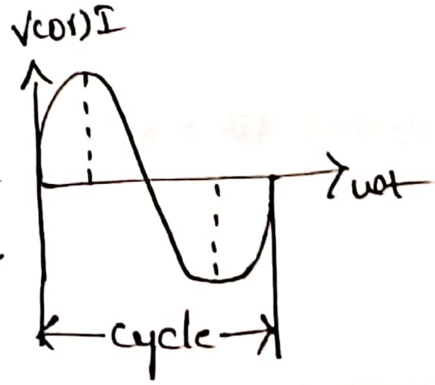
(2) Instantaneous Value:



The value of an alternating quantity at a particular instant is known as instantaneous value.

Ex: e_1, e_2, e_3 & e_4 are instantaneous values of alternating Emf at the instants t_1, t_2, t_3 & t_4 respectively

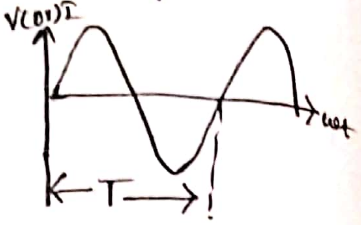
(3) Cycle:



Each repetition of a set of positive and negative instantaneous values of the alternating quantity is called a cycle.

(4) Time period:

The time taken for the waveform to complete one cycle is called time period. It is Denoted by 'T'.

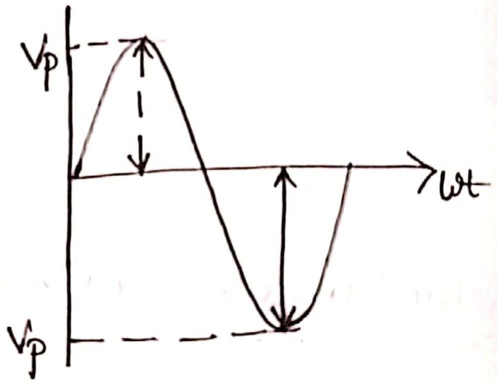


(5) Frequency:

The No. of cycles that the waveform completes in an unit time is known as frequency and denoted by 'f'. It is measured in Hz.

$$f = \frac{1}{T}$$

(6) Peak Value:



The maximum value (+ve or -ve) obtained by the waveform is called peak value.

It is represented by V_p or V_{max}.

* The peak to peak value of sine waveform is the value from +ve peak to -ve peak. denoted by V_{p-p}

(7) Angular frequency:

It is the angle traced per unit time.

It is measured in °/c degrees per second. denoted by ω

$$\omega = 2\pi/T \quad \text{(or)} \quad \frac{360}{T}$$

$$\omega = 2\pi f$$

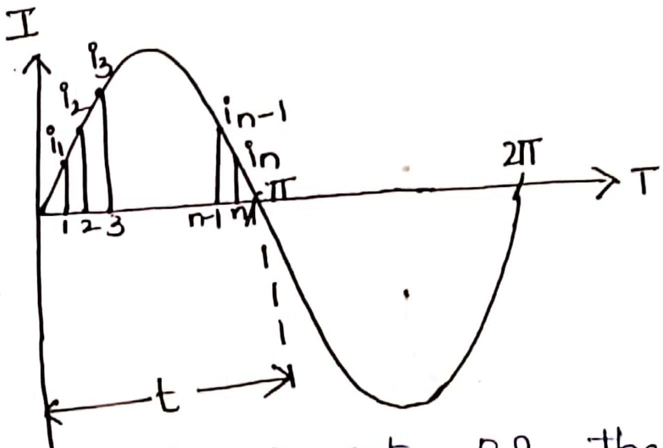
R.M.S Value (or) Effective Value:

The RMS value of an alternating current is given by that steady current (DC) which, when flowing through a given circuit for a given time, produces the same amount of heat as produced by the alternating current (AC) which when flowing through the same circuit for the same time.

RMS value can be obtained by two methods

1. Graphical method
2. Analytical method

1. Graphical method:



Consider sinusoidal varying current.

The RMS value can be obtained by comparing the heat produced.

Let the current passing through the Resistor, $R\Omega$, the heat can be expressed as

Heat produced $= I^2Rt$

The heat produced during 1^{st} interval $= \frac{i_1^2 Rt}{n}$ joules

heat produced during 2^{nd} interval $= \frac{i_2^2 Rt}{n}$ joules

Heat produced during n^{th} interval $= \frac{i_n^2 Rt}{n}$ joules

Total heat produced in 't' sec is

$$= \frac{Rt}{n}\left[i_1^2 + i_2^2 + \cdots + i_n^2\right]$$

$$= Rt\left[\frac{i_1^2 + i_2^2 + \cdots + i_n^2}{n}\right]$$

Now, the heat produced by direct current (DC) 'I' Amp passing through same resistance R for same time 't' is I^2Rt joules.

The Heat produced by both must be equal

$$I^2\cancel{Rt} = \cancel{Rt}\left[\frac{i_1^2 + i_2^2 + \cdots + i_n^2}{n}\right]$$

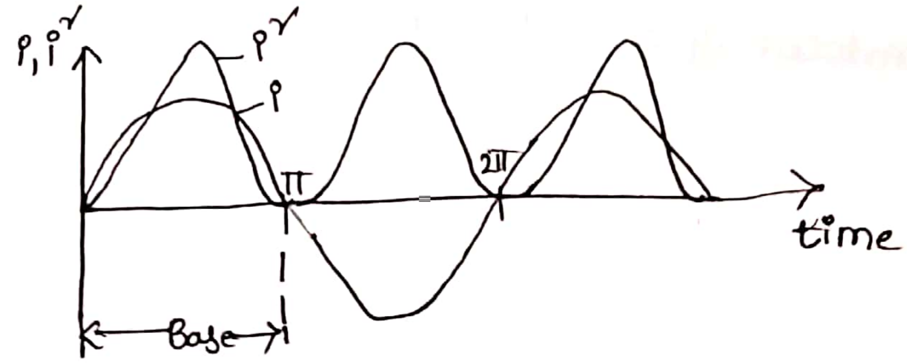
$$I^2 = \frac{i_1^2 + i_2^2 + \cdots + i_n^2}{n}$$

$$\boxed{I = \sqrt{\frac{i_1^2 + i_2^2 + \cdots + i_n^2}{n}} = I_{rms}}$$

This is called RMS value of current

$$\boxed{V_{rms} = \sqrt{\frac{V_1^2 + V_2^2 + \cdots + V_n^2}{n}}}$$

* **Analytical method:**



consider sinusoidal varying AC $i = I_m \sin\theta$

The square of this current is $i^2 = I_m^2 \sin^2\theta$

The Area of the square current over half cycle can be represented as $\int_0^{\pi} i^2 \, d\theta$

The Average value of square of the current over half cycle

$$= \frac{\text{Area of curve over half cycle}}{\text{length of base over half cycle}} = \frac{\int_0^{\pi} i^2 \, d\theta}{\pi}$$

$$= \frac{1}{\pi} \int_0^{\pi} I_m^2 \sin^2\theta \, d\theta \Rightarrow \frac{I_m^2}{\pi} \int_0^{\pi} \sin^2\theta \cdot d\theta$$

$$= \frac{I_m^2}{\pi} \int_0^{\pi} \left(\frac{1-\cos 2\theta}{2} \right) d\theta \Rightarrow \frac{I_m^2}{2\pi} \int_0^{\pi} (1-\cos 2\theta) \cdot d\theta$$

$$= \frac{I_m^2}{2\pi} \left(\theta - \frac{\sin 2\theta}{2} \right)_0^{\pi} = \frac{I_m^2}{2\pi} (0-(-\pi))$$

$$= \frac{I_m^2}{2\cancel{\pi}} \cdot \cancel{\pi}$$

$$I_{avg} = \frac{I_m^2}{2}$$

$$I_{rms} = \sqrt{\text{Avg of Square of current}} = \sqrt{I_{avg}}$$

$$I_{rms} = \sqrt{\frac{I_m^2}{2}} = \frac{I_m}{\sqrt{2}} , \quad V_{rms} = \frac{V_m}{\sqrt{2}}$$

$$\boxed{I_{rms} = 0.707 I_m}$$

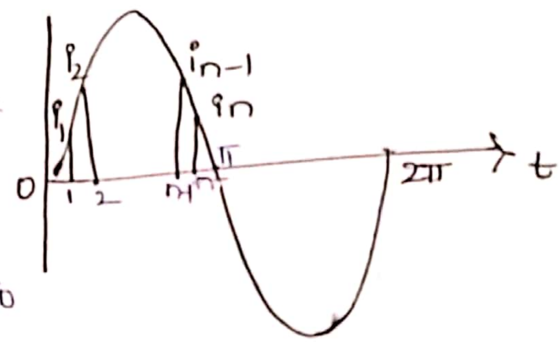
Average Value:

The average value of an alternating quantity is defined as that value which is obtained by averaging all the instantaneous values over a period of half cycle.

(1) Graphical method:

consider 'n' equal intervals of half cycle.

where $i_1, i_2 ---- i_n$ are the instantaneous current for the intervals 1, 2, --- n intervals



the average value is equal to

$$\boxed{\begin{aligned} I_{avg} &= \frac{i_1 + i_2 + --- i_n}{n} \\ V_{avg} &= \frac{V_1 + V_2 + -- + V_n}{n} \end{aligned}}$$

* (2) Analytical method:

Let us consider sinsoidal varying alternative current I and it is represented by $I = I_m \sin\theta$

The average value is = $\dfrac{\text{Area under the current over half cycle}}{\text{length of base over half cycle}}$

$$= \frac{\int_0^\pi I_m \sin\theta \, d\theta}{\pi} \Rightarrow \frac{1}{\pi}\int_0^\pi I_m \sin\theta \cdot d\theta$$

$$= \frac{I_m}{\pi}\int_0^\pi \sin\theta \cdot d\theta = \frac{I_m}{\pi}(-\cos\theta)_0^\pi$$

$$= \frac{-I_m}{\pi}(-1-1)$$

$$= -\frac{I_m}{\pi}(-2)$$

$$I_{avg} = \frac{2I_m}{\pi}$$

$$\boxed{I_{avg} = 0.637\, I_m}$$

The average value of the sinusoidal wave-form is 0.637 times of maximum value

Similarly $\boxed{V_{avg} = 0.637\, V_m}$

Form factor: (K_f)

It is defined as the ratio of rms value to the average value.

$$\text{Form factor, } \boxed{K_f = \frac{\text{RMS Value}}{\text{Average Value}}}$$

$$K_f = \frac{0.707\, I_m}{0.637\, I_m}$$

$\boxed{K_f = 1.11}$ for sinusoidal.

Peak factor : (K_p)

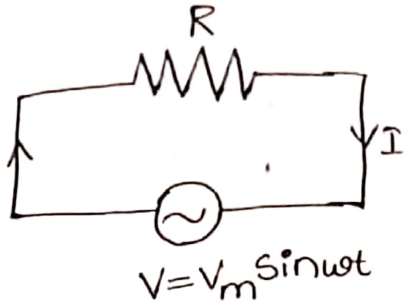
It is defined as the ratio of maximum value to the rms value

$$\text{peak factor, } \boxed{K_p = \frac{\text{Maximum Value}}{\text{RMS Value}}}$$

$$K_p = \frac{I_m}{0.707\, I_m}$$

$\boxed{K_p = 1.414}$ for sinusoidal

AC Response through pure resistance:



Consider a resistor which is excited by sinusoidal voltage

i.e $\boxed{V = V_m \sin\omega t}$

where V_m = maximum voltage

According to ohm's law,

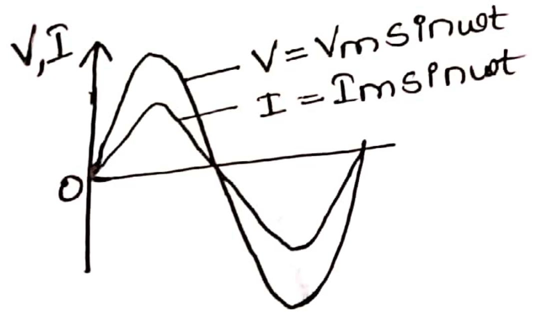
$$I = \frac{V}{R} = \frac{V_m \sin\omega t}{R} = \frac{V_m}{R}\sin\omega t$$

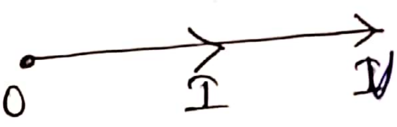
∴ The instantaneous value of current $\boxed{I = I_m \sin\omega t}$

There is No phase difference between the voltage and current.

In pure resistive circuit, the current and the voltage both are in phase with each other.

*The wave form of voltage and current and the corresponding phasor diagram is shown below.





Both are in phase

Power:

$$P = \frac{1}{T}\int_0^T V I \, d\omega t$$

$$P = \frac{1}{T}\int_0^T V_m \sin\omega t \cdot I_m \sin\omega t \cdot d\omega t$$

The total time period is 2π

$$P = \frac{1}{2\pi} \int_0^{2\pi} V_m I_m \sin^2 \omega t \cdot d\omega t$$

$$P = \frac{V_m I_m}{2\pi} \int_0^{2\pi} \sin^2 \omega t \cdot d\omega t$$

$$P = \frac{V_m I_m}{2\pi} \int_0^{2\pi} \left(\frac{1 - \cos 2\omega t}{2} \right) d\omega t \Rightarrow \frac{V_m I_m}{4\pi} \int_0^{2\pi} (1 - \cos 2\omega t) \, d\omega t$$

$$P = \frac{V_m I_m}{4\pi} \left(\omega t - \frac{\sin 2\omega t}{2} \right)_0^{2\pi}$$

$$= \frac{V_m I_m}{4\pi} \cdot 2\pi$$

$$P = \frac{V_m I_m}{2}$$

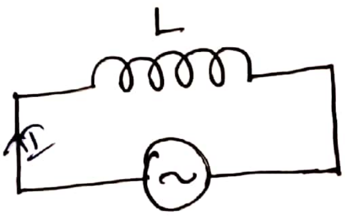
∴ The Average power flowing through resistor is $\frac{V_m I_m}{2}$

$$P = \frac{V_m}{\sqrt{2}} \cdot \frac{I_m}{\sqrt{2}}$$

$$\boxed{P = V_{rms} \cdot I_{rms}} \quad \text{(or)} \quad P = I_{rms}^2 \cdot R$$

AC Response through pure Inductance:

Consider, an Inductor which is excited by AC voltage $V = V_m \sin \omega t$. Then the response in the inductor is,



$V = V_m \sin \omega t$

$$I = \frac{1}{L} \int V \, dt$$

$$= \frac{1}{L} \int V_m \sin \omega t \cdot dt \Rightarrow \frac{V_m}{L} \int \sin \omega t \cdot dt$$

$$I = \frac{V_m}{L} \left(-\frac{\cos \omega t}{\omega} \right)$$

$$I = -\frac{V_m}{\omega L} \cos \omega t$$

$$I = \frac{-V_m}{\omega L} \sin\left(\frac{\pi}{2} - \omega t\right)$$

$$= \frac{V_m}{\omega L} \sin\left(\omega t - \frac{\pi}{2}\right)$$

$$I = \frac{V_m}{X_L} \sin\left(\omega t - \frac{\pi}{2}\right)$$

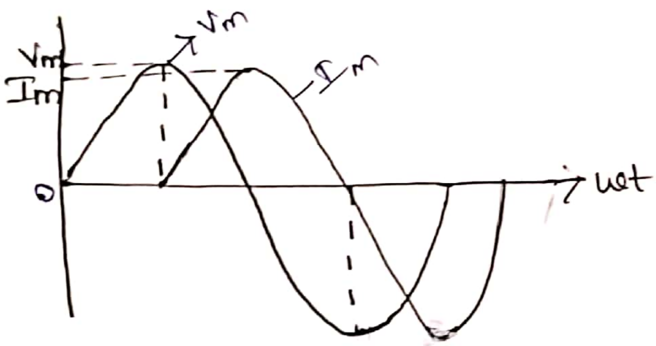
$$I = I_m \sin\left(\omega t - \frac{\pi}{2}\right)$$

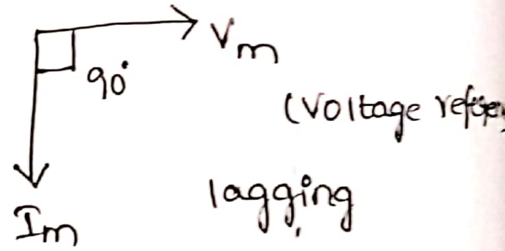
Where X_L = inductive reactance, units ohms (Ω)

The instantaneous value of current is $I = I_m \sin\left(\omega t - \frac{\pi}{2}\right)$

where $I_m = \frac{V_m}{X_L}$

The wave-form and phasor diagram shown below.





Power:

$$P = \frac{1}{T} \int_0^T VI \, d\omega t$$

$$= \frac{1}{2\pi} \int_0^{2\pi} V_m \sin \omega t \cdot I_m \sin\left(\omega t - \frac{\pi}{2}\right) \cdot d\omega t$$

$$= \frac{V_m I_m}{2\pi} \int_0^{2\pi} \sin \omega t \cdot \sin\left(\omega t - \frac{\pi}{2}\right) d\omega t$$

$$= \frac{V_m I_m}{4\pi} \int_0^{2\pi} 2 \sin \omega t \cdot \sin\left(\omega t - \frac{\pi}{2}\right) d\omega t$$

$$= \frac{V_m I_m}{4\pi} \int_0^{2\pi} \cos\left(\omega t - \omega t + \frac{\pi}{2}\right) - \cos\left(\omega t + \omega t - \frac{\pi}{2}\right) d\omega t$$

$$= \frac{V_m I_m}{4\pi} \int_0^{2\pi} \cos\left(\frac{\pi}{2}\right) - \cos\left(2\omega t - \frac{\pi}{2}\right) \cdot d\omega t$$

$$= \frac{V_m I_m}{4\pi} \int_0^{2\pi} 0 - \cos(2\omega t - \tfrac{\pi}{2}) \cdot d\omega t$$

$$= \frac{V_m I_m}{4\pi} \int_0^{2\pi} \cos(\tfrac{\pi}{2} - 2\omega t) \cdot d\omega t$$

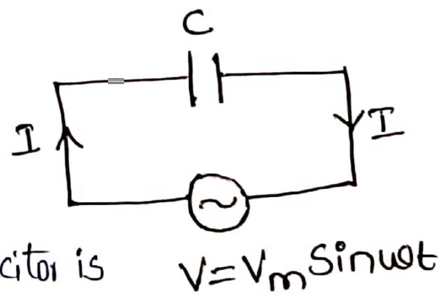
$$= \frac{V_m I_m}{4\pi} \int_0^{2\pi} \sin 2\omega t \cdot d\omega t$$

$$= \frac{V_m I_m}{4\pi} \left(-\frac{\cos 2\omega t}{2} \right)_0^{2\pi}$$

$$P = \frac{-V_m I_m}{8\pi} (\cos 4\pi - \cos 0) = 0$$

The Average power flowing through inductor is zero. Storage element Inductor when it is fully stored the energy and it is returned back to the supply.

AC Response through pure capacitance:



consider, a capacitor which is excited by AC voltage $V = V_m \sin\omega t$

Then the response flowing through capacitor is

$$I = C \cdot \frac{dV}{dt}$$

$$= C \cdot \frac{d}{dt}(V_m \sin\omega t)$$

$$= C \cdot V_m \frac{d}{dt} \sin\omega t \quad = C \cdot V_m \cos\omega t \cdot \omega$$

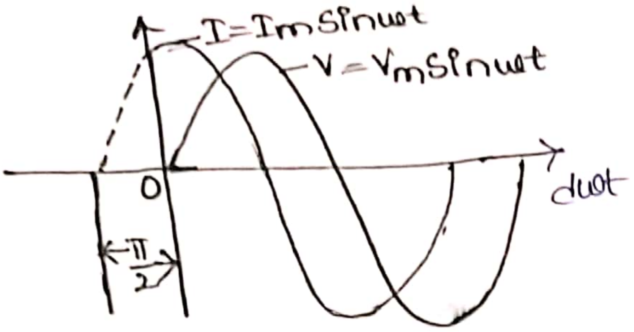
$$= \omega C \cdot V_m \cos\omega t$$

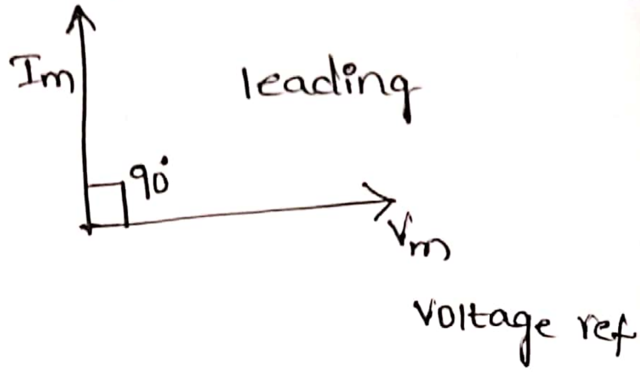
$$I = \frac{V_m}{X_C} \cos\omega t \quad = I_m \sin\omega t \qquad X_C = \frac{1}{\omega C}$$

Where X_C = capacitive reactance, units ohms

$$\boxed{i = I_m \sin(\omega t + \tfrac{\pi}{2})}$$

The wave-form and phasor diagram is shown below





Power:

$$P = \frac{1}{T}\int_0^T V I \, d\omega t$$

$$= \frac{1}{2\pi}\int_0^{2\pi} V_m \sin\omega t \cdot I_m \sin\left(\omega t + \frac{\pi}{2}\right) \cdot d\omega t$$

$$= \frac{V_m I_m}{4\pi}\int_0^{2\pi} 2\sin\omega t \cdot \sin\left(\omega t + \frac{\pi}{2}\right) \cdot d\omega t$$

$$= \frac{V_m I_m}{4\pi}\int_0^{2\pi} \cos\left(\omega t - \omega t - \frac{\pi}{2}\right) - \cos\left(\omega t + \omega t + \frac{\pi}{2}\right) \cdot d\omega t$$

$$= \frac{V_m I_m}{4\pi}\int_0^{2\pi} -\cos\left(2\omega t + \frac{\pi}{2}\right) \cdot d\omega t$$

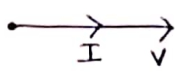
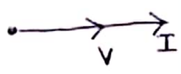
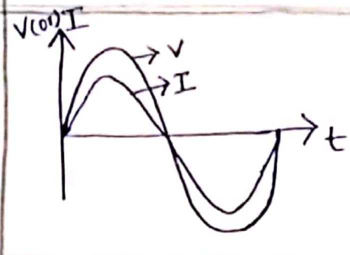
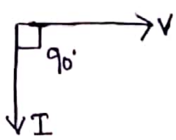
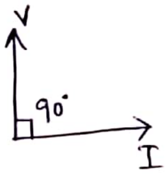
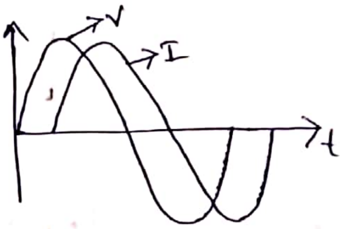
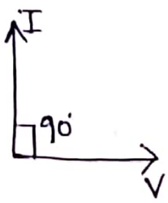
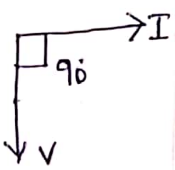
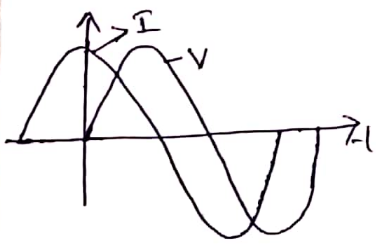
$$= \frac{V_m I_m}{4\pi}\int_0^{2\pi} \sin 2\omega t \cdot d\omega t$$

$$= \frac{V_m I_m}{4\pi}\left(-\frac{\cos 2\omega t}{2}\right)_0^{2\pi}$$

$$P = \frac{-V_m I_m}{8\pi}\left(\cos 4\pi - \cos 0\right)$$

$$P = 0$$

Average power flowing through capacitor is zero.

Element	Voltage	current	Phasor diagram Voltage Ref	 current Ref	waveform
Resistor (R) Ω	$V = V_m \sin\omega t$	$I = I_m \sin\omega t$	 Inphase		
Inductor (L) H	$V = V_m \sin\omega t$	$I = I_m \sin(\omega t - \frac{\pi}{2})$	 lagging		
Capacitor (C) F	$V = V_m \sin\omega t$	$I = I_m \sin(\omega t + \frac{\pi}{2})$	 leading		

Apparent power (S):

It is defined as the product of rms value of voltage (V) & current (I). It is defined by S.

$$\boxed{S = VI} \quad VA$$

unit: volt-amp (VA) or KVA

Real power (or) True power (P):

It is defined as the product of the applied voltage and the active component of the current.

This real component of power measured in watts (W)

$$\boxed{P = VI \cos\phi} \quad watt$$

Reactive power (Q):

It is defined as product of the applied voltage and the reactive component of the current.

It is the imaginary component of power, measured in units volt-amp reactive (VAR) (or) KVAR

$$\boxed{Q = VI \sin\phi} \text{ VAR}$$

Power factor: ($\cos\phi$):

It is defined as factor by which the apparent power must be multiplied in order to obtain true power.

It is the ratio of the true power to apparent power

$$\boxed{\text{power factor} = \frac{\text{True Power}}{\text{Apparent Power}} = \frac{VI\cos\phi}{VI} = \cos\phi}$$

The numerical value of cosine of the phase angle between the applied voltage and current drawn from the supply voltage gives the power factor.

It is also defined as the ratio of resistance to the impedance. $\cos\phi = \frac{R}{Z}$

Importance of powerfactor:

1. For a fixed voltage and power $\cos\phi$ is inversly propotinal to current $\cos\phi \propto \frac{1}{I}$

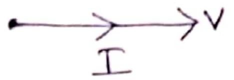
If power factor is more current is less

So that I^2R loss (or) copper loss is less.

2. power factor indicates the amount of power is utilised by the customer.

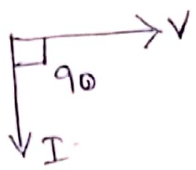
3. power factor values always lies between 0 and 1

4. for Resistor element $\cos\phi$ is 1



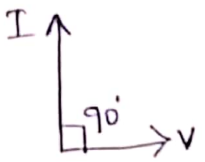
$\phi = 0 \Rightarrow \cos\phi = 1$

5. for Inductor element $\cos\phi = \cos 90° = 0$



$\cos 90° = 0 \quad \therefore \cos\phi = 0$

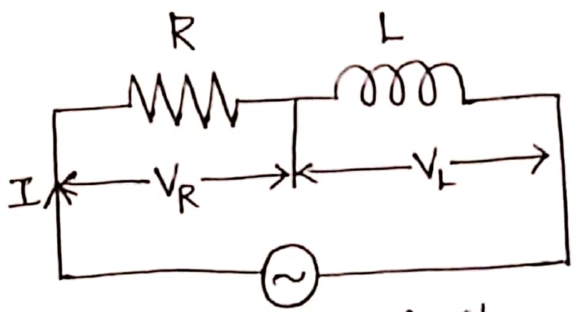
6. for capacitor $\cos\phi = \cos 90° = 0$



$\cos\phi = 0$

AC Response of RL Circuit:

Consider, a circuit consist of Resistor (R) and inductor (L) in Series. When the AC Voltage is applied to it the Circuit drops the current and it has two Voltage drops.



$V = V_m \sin\omega t$

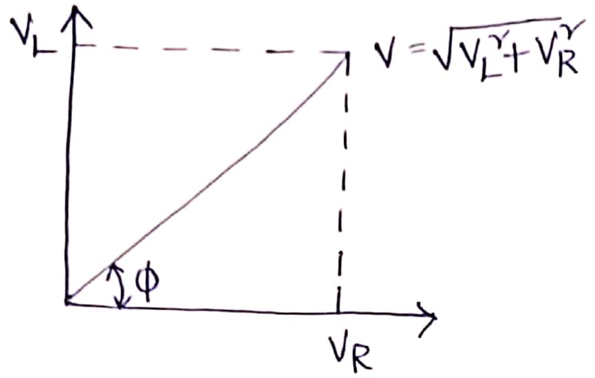
Voltage drop due to resistor $V_R = IR$

Voltage drop due to Inductor $V_L = IX_L$

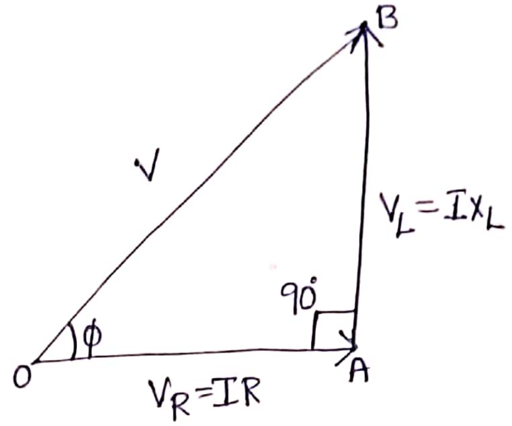
According to Kirchoff voltage law, Supply voltage $\bar{V} = \bar{V}_R + \bar{V}_L$

Scanned with CamScanner

Phasor diagram:



Voltage triangle:



From the voltage triangle we can write,

$$V = \sqrt{(V_R)^2 + (V_L)^2}$$

$$= \sqrt{(IR)^2 + (IX_L)^2}$$

$$= I\sqrt{R^2 + X_L^2}$$

$$V = IZ$$

$$Z = \sqrt{R^2 + X_L^2}$$

where Z is the impedance measured in ohms

Impedance:

Impedance is defined as the opposition of circuit to flow of alternating current. It is denoted by 'Z', unit Ω.

For RL series circuit, it can be observed from phasor diagram that the current lags behind voltage, by an angle ϕ.

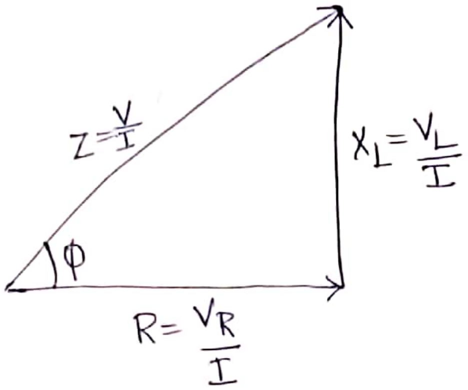
from voltage trangle,

$\tan\phi = \frac{V_L}{V_R} = \frac{X_L}{R}$, $\cos\phi = \frac{V_R}{V} = \frac{R}{Z}$, $\sin\phi = \frac{V_L}{V} = \frac{X_L}{Z}$

$= \frac{IR}{IZ}$

Impedance triangle:

If all the sides of the voltage triangle is divided by current, we get a triangle called impedance triangle.



$R = Z\cos\phi$

$X_L = Z\sin\phi$

$\boxed{Z = R + jX_L\,\Omega}$ → Rectangular form

While in polar form

$Z = |Z| \angle \phi \ \Omega$

$|Z| = \sqrt{R^2 + X_L^2}$, $\phi = \tan^{-1}\left(\frac{X_L}{R}\right)$

Power:

The power is product of instantaneous values of voltage and current.

$$P = V \times I$$

$$= V_m \sin\omega t \times I_m \sin(\omega t - \phi)$$

$$= V_m I_m [\sin\omega t \cdot \sin(\omega t - \phi)]$$

$$= \frac{V_m I_m}{2}[2(\sin\omega t \cdot \sin(\omega t - \phi)]$$

$$= \frac{V_m I_m}{2}\left[\cos(\cancel{\omega t} - \cancel{\omega t} + \phi) - \cos(\omega t + \omega t - \phi)\right]$$

$$= \frac{V_m I_m}{2}[\cos\phi - \cos(2\omega t - \phi)]$$

$$= \frac{V_m I_m}{2}\cos\phi - \frac{V_m I_m}{2}\cos(2\omega t - \phi)$$

The second term is cosine term whose average value over a cycle is zero. Hence average power is

$$P_{avg} = \frac{V_m I_m}{2} \cos\phi$$

$$P_{avg} = \frac{V_m}{\sqrt{2}} \cdot \frac{I_m}{\sqrt{2}} \cos\phi$$

$$\boxed{P = VI \cos\phi} \text{ watts}$$ where V, I are rms value.

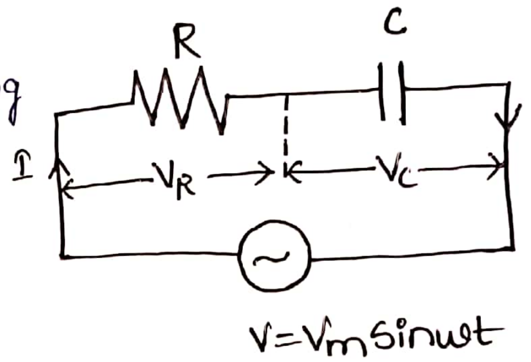
Apparent power: $S = VI$ VA

True power: $P = VI \cos\phi$ watts

Reactive power: $Q = VI \sin\phi$ VAR.

AC Response of RC Circuit:

Consider a circuit consisting of pure Resistance R in Ω and connected in series with a pure capacitor of C in farads



The series combination is connected across ac supply given by $V = V_m \sin\omega t$.

circuit draws a current I, then there are two voltage drops,

Voltage drop across pure resistance $V_R = IR$

Voltage drop across pure capacitance $V_C = IX_C$

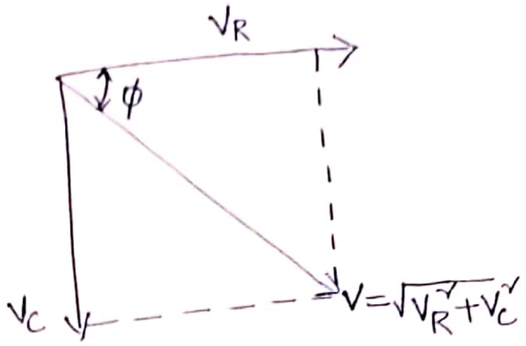
where $X_C = \frac{1}{2\pi fC}$ → capacitive reactance

According to Kirchoff's voltage law total voltage drop is

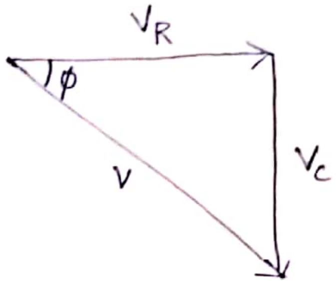
$$V = \overline{V_R} + \overline{V_C}$$

$$V = \overline{IR} + \overline{IX_C}$$

phasor diagram:



Voltage triangle:



From the Voltage triangle

$$V=\sqrt{V_R^2+V_C^2}$$

$$=\sqrt{(IR)^2+(IX_C)^2}=I\sqrt{R^2+X_C^2}$$

$$V=IZ$$

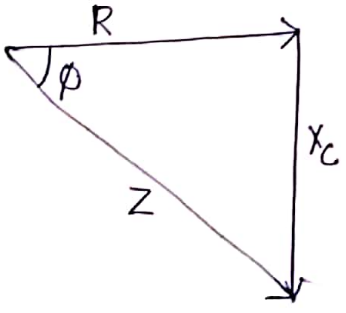
$$Z=\sqrt{R^2+X_C^2}$$

From the phasor diagram In RC series circuit, current leads voltage by angle ϕ (or) supply voltage V laggs current I by angle ϕ.

From voltage triangle

$$\tan\phi=\frac{V_C}{V_R}=\frac{X_C}{R}, \quad \cos\phi=\frac{V_R}{V}=\frac{R}{Z}, \quad \sin\phi=\frac{V_C}{V}=\frac{X_C}{Z}$$

Impedance triangle:



$$R=Z\cos\phi$$

$$X_C=Z\sin\phi$$

The rectangular form of impedance is

$$Z=R-jX_C\ \Omega$$

while polar form, $Z=|Z|\angle-\phi\ \Omega$

$$Z=\sqrt{R^2+X_C^2}, \quad \phi=\tan^{-1}\left(\frac{-X_C}{R}\right)$$

Power :

$$P = VI$$

$$= V_m \sin\omega t \times I_m \sin(\omega t + \phi)$$

$$= V_m I_m \left[\sin\omega t \cdot \sin(\omega t + \phi)\right]$$

$$= \frac{V_m I_m}{2}\left[2\sin\omega t \cdot \sin(\omega t + \phi)\right]$$

$$= \frac{V_m I_m}{2}\left[\cos(\cancel{\omega t} - \cancel{\omega t} - \phi) - \cos(\omega t + \omega t + \phi)\right]$$

$$= \frac{V_m I_m}{2}\left[\cos(-\phi) - \cos(2\omega t + \phi)\right]$$

$$P = \frac{V_m I_m}{2}\left[\cos\phi - \frac{V_m I_m}{2}\cos(2\omega t + \phi)\right]$$

Now, second term is cosine term whose average value over a cycle is zero. Hence average power is

$$P_{avg} = \frac{V_m I_m}{2}\cos\phi = \frac{V_m}{\sqrt{2}} \cdot \frac{I_m}{\sqrt{2}}\cos\phi$$

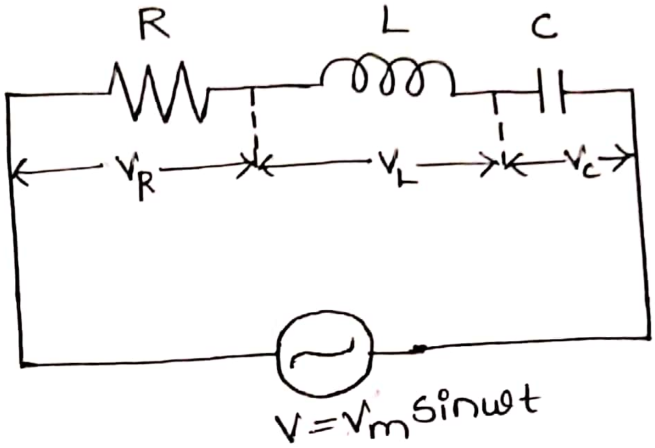
$$P = VI\cos\phi \text{ watts}$$

Apparent power $S = VI$ VA

True power $P = VI\cos\phi$ watts

reactive power $Q = VI\sin\phi$ VAR

AC Response of RLC circuit:



Consider a circuit consisting of resistance (R) Ω pure inductance (L) H and capacitance (C) f connected in series with each other across ac supply.

The AC supply is given by $V = V_m \sin\omega t$.

Voltage drop across resistance R is $V_R = IR$
Voltage drop across inductance L is $V_L = IX_L$
Voltage drop across capacitance C is $V_C = IX_C$
Voltage drop

According to Kirchoff's law we can write

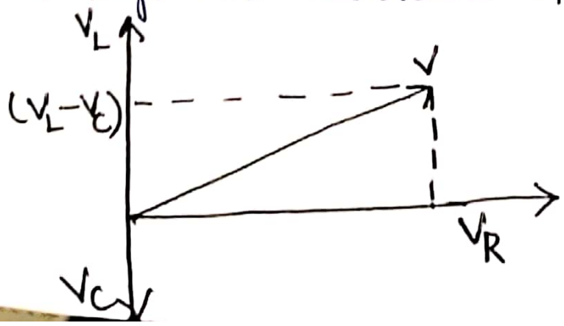
$$V = \overline{V_R} + \overline{V_L} + \overline{V_C}$$

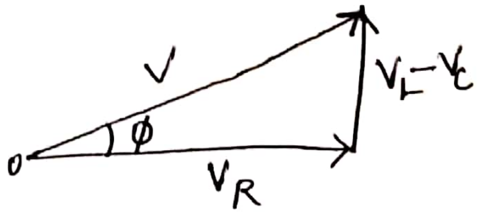
$$V = \overline{IR} + \overline{IX_L} + \overline{IX_C}$$

The phasor diagram depends on the condition of the magnitudes of V_L and V_C ultimately X_L and X_C.
Let us consider different cases.

$X_L > X_C$:

When $X_L > X_C$, i.e V_L is greater than V_C. the current I will lags the resultant of V_L and V_C i.e $(V_L - V_C)$.





From voltage triangle,

$$V = \sqrt{(V_R)^2 + (V_L - V_C)^2}$$

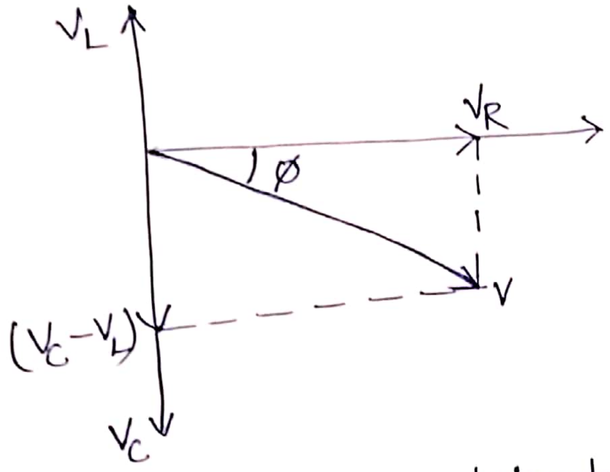
$$V = \sqrt{(IR)^2 + (I(X_L - X_C))^2}$$

$$V = I\sqrt{R^2 + (X_L - X_C)^2}$$

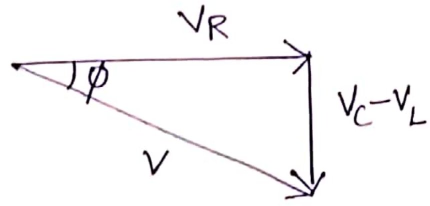
$$V = IZ$$

Where $Z = \sqrt{R^2 + (X_L - X_C)^2}$

(ii) $X_L < X_C$:



Voltage triangle



I leads V

From voltage triangle,

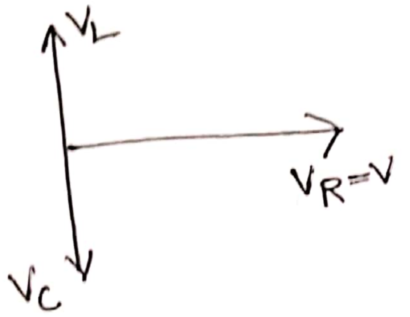
$$V = \sqrt{V_R^2 + (V_C - V_L)^2} = \sqrt{(IR)^2 + (IX_C - IX_L)^2}$$

$$= I\sqrt{R^2 + (X_C - X_L)^2}$$

$$V = IZ$$

where $Z = \sqrt{R^2 + (X_C - X_L)^2}$

(ii) $X_L = X_C$:



$V = V_R$

$V = IR$

$V = IZ$

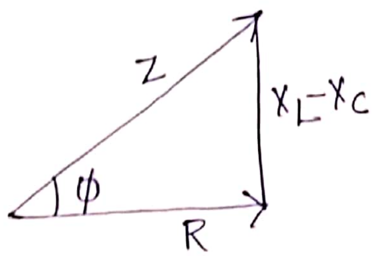
$R = Z$

Impedance triangle

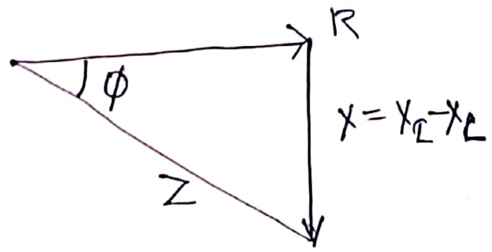
The impedance is expressed as,

$$Z = R + jX$$

for $X_L > X_C \rightarrow \phi$ is positive



for $X_L < X_C \rightarrow \phi$ is negative



power:

Thus, for any condition, $X_L > X_C$ (or) $X_L < X_C$, the power can be expressed as,

$$\boxed{P = VI\cos\phi} \text{ W}$$

Problems:

1. An alternating current, $i = 414 \sin(2\pi \times 50 \times t)$ A, is passed through a series circuit consisting of a resistance of 100Ω and an inductance of 0.31831 H. Find the expressions for the instantaneous values of voltage across (i) the resistance (ii) the inductance

2. Calculate the resistance and inductance or capacitance in series for each of the following impedances. Assume the frequency to be 60 Hz.

   (i) $(12 + j30)\ \Omega$ (ii) $-j60\ \Omega$ (iii) $20 \angle 60^\circ\ \Omega$

3. The wave-forms of the voltage and current of a circuit are given by, $e = 120 \sin(314t)$ and $i = 10 \sin(314t + \frac{\pi}{6})$. Calculate the values of the resistance, capacitance which are connected in series to form the circuit. Also draw wave-forms for current, voltage & phasor diagram. Calculate power.

UNIT-II

DC & AC Machines

→ An Electrical machine is a device which converts mechanical energy in to electrical energy or Vice-Versa.

* An Electrical machine which converts mechanical energy in to an electrical energy is called an Electric Generator.
* An Electrical machine which converts Electrical energy in to an Mechanical energy is called an Electric Motor.

→ Such Electrical machines may be related to an Electrical energy of an alternating type called AC machines or may be related to an Electrical energy of direct type called dc machines.

Magnetic field:

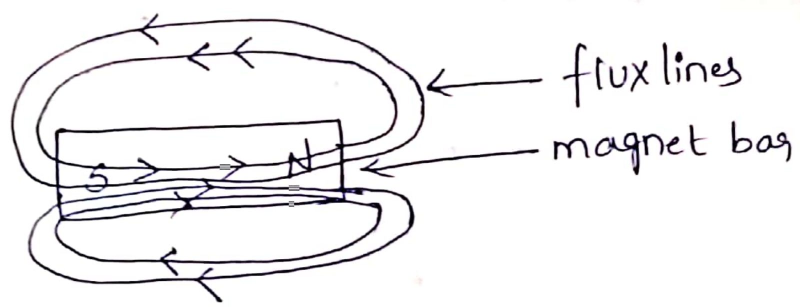
The region around a magnet within which the influence of the magnet can be experienced called its Magnetic field.

Magnetic flux:

The total Number of lines of force existing in a particular magnetic field is called Magnetic flux. denoted by ϕ, measured in a unit weber.

$$1 \text{ weber} = 10^8 \text{ lines of force.}$$

The lines of flux have a fixed direction. these flux lines starts at N-pole and terminate at S-pole, external magnet



Generator:

Generator is a machine which converts mechanical energy to Electrical energy.

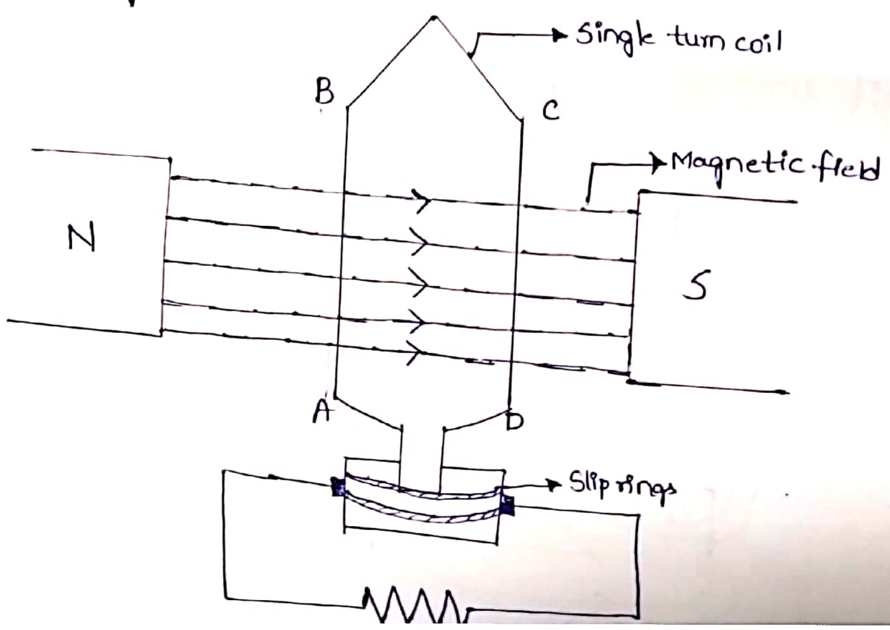
DC Generator works on a principle of "faraday's law of Electromagnetic induction".

The principle states that " Whenever a conductor moves in a magnetic field, an emf gets induced with in the conductor."

In practical generators, the conductors are rotated to cut the magnetic-flux, keeping stationary. To have large voltage as the output the number of conductors are connected together in a specific manner called Armature winding. The necessary magnetic flux produced by field winding.

Working principle:

DC Generator works on a principle of faradays law of Electromagnetic induction. i.e whenever conductor cuts the magnetic field, it produces EMF

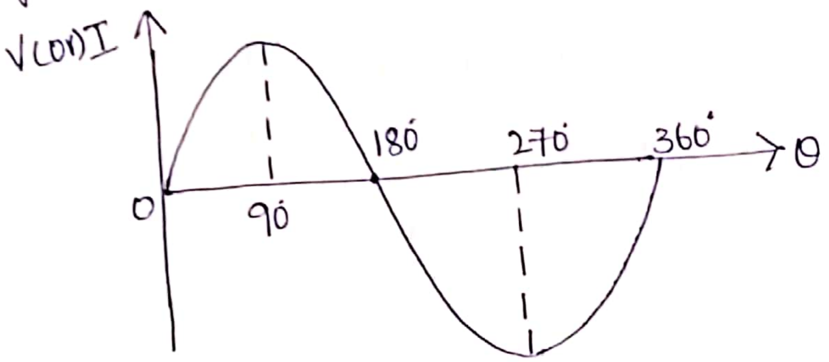


Let us consider a single turn coil (A, B, C, D) is placed in a magnetic field as shown in the figure.
The conductor ends are connected to two slip rings which are mounted on shaft. Two stationary brushes (B_1 & B_2) are mounted on sliprings which collects the current and passes it to external circuit.

Let us take first position of coil i.e $\theta = 0°$

If $\theta = 0°$:
In this position the induced emf is zero, because the coil sides AB and CD are not cutting flux. They are parallel to flux lines.

If $\theta = 0°$ to $90°$:
The coil sides AB and CD are moving with some angle therefore coil cuts flux and EMF will be induced

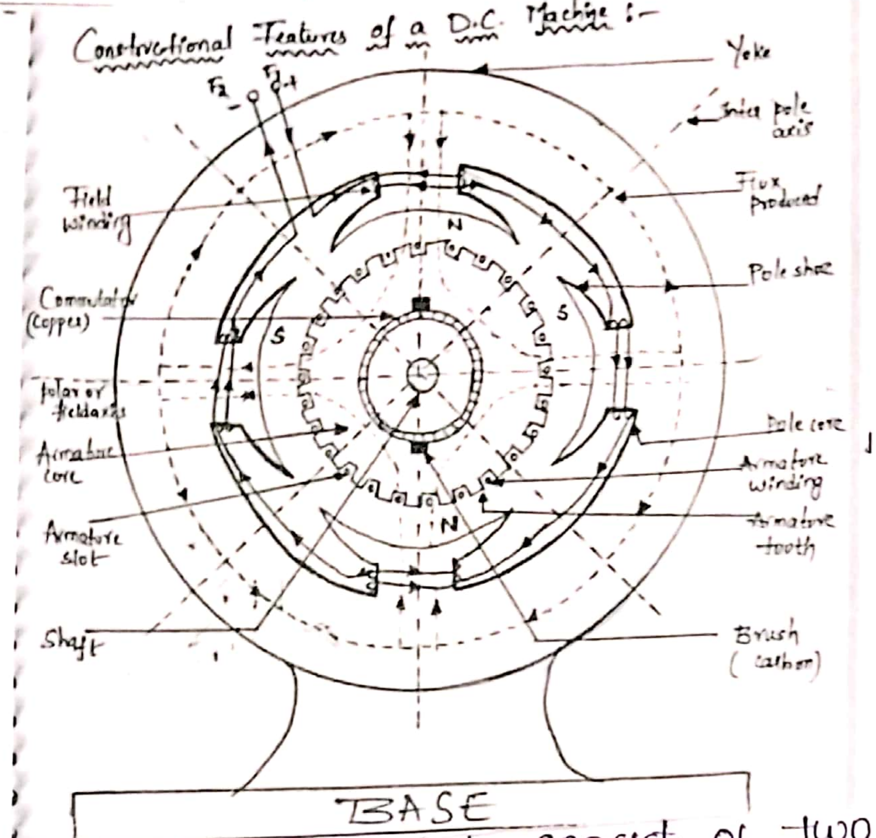


If $\theta = 90°$
The coil sides AB and CD are perpendicular to magnetic flux and therefore the rate of flux cutting is maximum.

If $\theta = 90°$ to $180°$
In this position the flux linking is gradually decreases Hence induced emf gradually decreases to zero

If $\theta = 180°$ to $360°$
In this position the magnitudes of EMF are similar to the first half cycle. The current reverses for every half cycle. Such current is called alternating current.

Constructional features of DC Machine:



A DC Machine mainly consists of two parts such as

1. Stator 2. Rotator

1. <u>Stator</u>: stationary parts of the machine is called as stator. It consists of yoke, pole, field winding, brushes etc

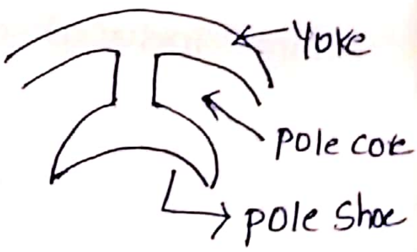
2. <u>Rotor</u>: Ratating parts of the machine is called Rotor. It consist of Armature, commutator, shaft, cooling fan etc.

(A) <u>Yoke</u>: The outer most cyclindrical frame called yoke, used as protecting cover for whole machine. It is made up of cast iron, cast steel.

B) <u>pole system</u>:

It consists of pore core & pole shoe

(i) pole core: pole core is a steel core around which field coils are wounded.



(2) pole shoe: pole shoe enlarges the area of armature core to come across the flux which is necessary to produce large EMF. It is made up of cast Iron (or) steel, laminated construction.

(C) Field winding:

Field winding is wound on the pole core with definite direction. It carries the current and behaves as an electro magnet producing necessary flux. It is made up of copper.

(D) Armature:

It is divides in to (1) Armature core (2) Armature wdg

Armature core:

Armature core is a cylindrical in shape mounted on the shaft. It consists of slots on its periphery. Armature core provides house for armature winding. It is madeup of Iron (or) steel, laminated construction.

Armature winding:

Armature winding is the interconnection of armature conductors placed in the slots of armature core. Generation emf takes place in armature winding.

Armature winding are of two types

(1) Lap winding (A=P) (2) wave winding (A=2

(E) Commutator: the basic nature of EMF Induced in the armature conductors is alterhating type. This needs rectification in DC Generators by using commutator. made up of Copper segments

(F) Brushes : To collect current from commutator and make it available to external circuit. It is made up of soft material like carbon.

(G) Bearings: Ball bearing are usually used, as they are more reliable. for heavy machines roller bearing used

(H) Shaft : used to transfer mechanical power to the machine. made up of mild steel with maximum breaking strength.

EMF Equation of DC Generator:

Let us consider a DC Generator consists of

No. of poles $= P$
Flux per pole $= \phi$
Speed of Armature in RPM $= N$
Total No. of conductors $= Z$
No. of parallel paths $= A$

$A = P$ for lap winding
$A = 2$ for wave winding

Now emf gets induced in the conductor according to faraday's law of electromagnetic induction. Hence average emf induced in each armature conductor is,

$$e = \text{Rate of cutting flux} = \frac{d\phi}{dt}$$

Consider one revolution of conductor. In one revolution the total flux produced by all the poles $\phi \times P$ with $\frac{60}{N}$ seconds & speed of N rpm.

$$e = \frac{\phi P}{\frac{60}{N}} = \frac{\phi PN}{60}$$

This is the emf induced in one conductor. There are total Z conductors with A parallel paths, hence $\frac{Z}{A}$ number of conductors are always in series.

Total emf $E = \frac{\phi PN}{60} \times \frac{Z}{A}$

The emf equation of a DC Generator

$$\boxed{E = \frac{\phi PNZ}{60A}}$$

$E = \frac{\phi NZ}{60}$ for lap type $A = P$

$E = \frac{\phi PNZ}{120}$ for wave type $A = 2$

problems:

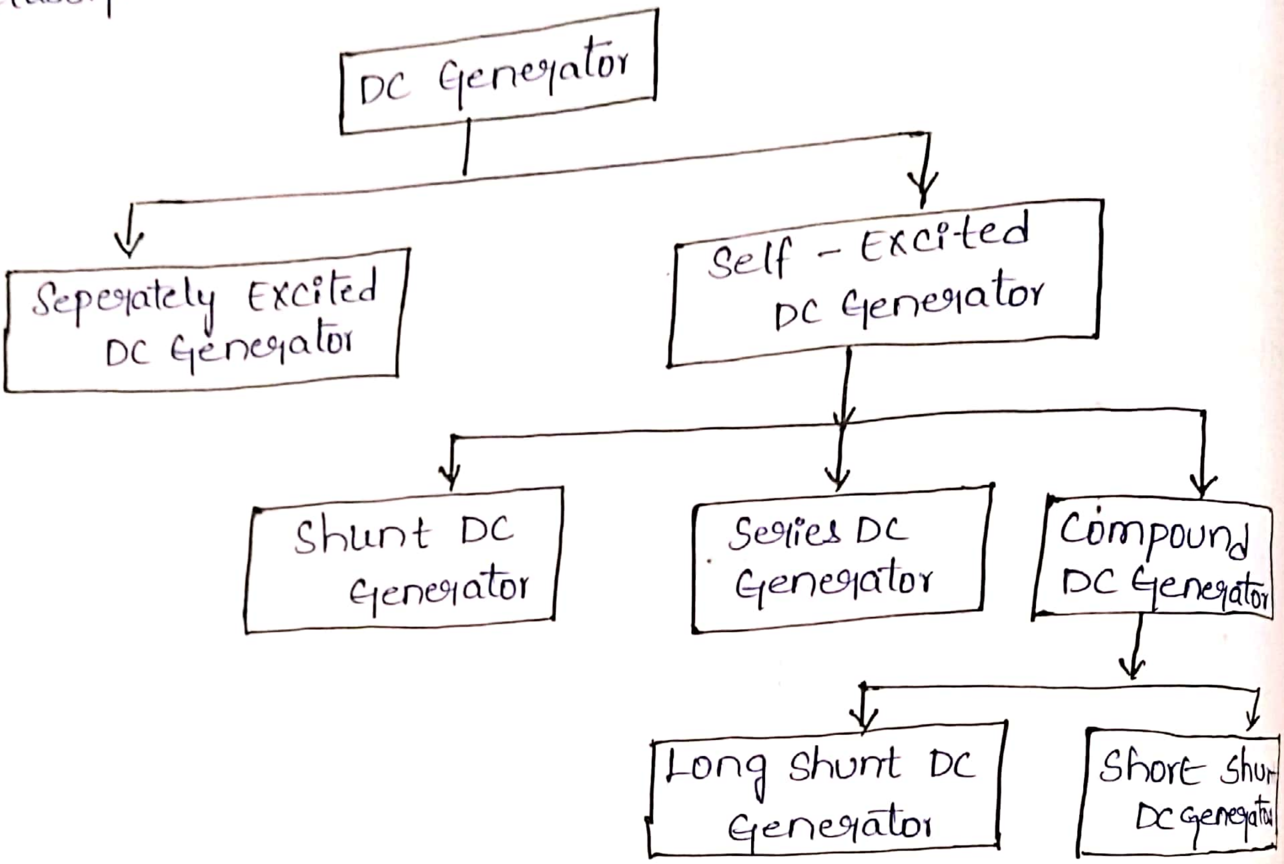
1. A 4-pole generator having lap wound armature winding has 51 slots, each slot containing 20 conductors. what will be the voltage generated in the machine when driven at 1500 rpm. Assuming flux per pole be 7.0mwb?

2. A 8-pole lap wound armature related at 350 rpm is required to generate 260V. The useful flux per pole is about 0.05W. If the armature has 120 slots, calculate the suitable no. of conductors per slot?

3. A 6 pole wave wound generator containing 644 conductors calculate the EMF generated when the flux per pole is 0.06W speed is 250 rpm.

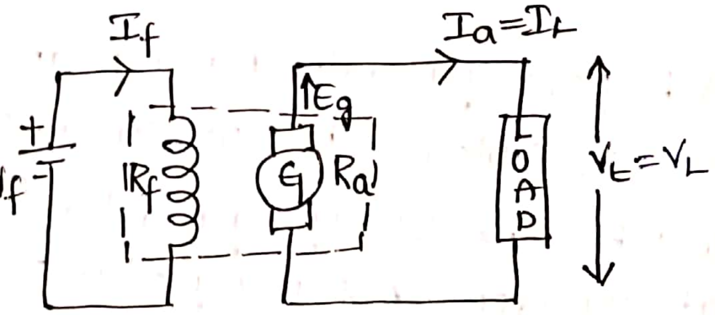
Types of DC Generator:

Based on Excitation DC generators are mainly classified in-to-two-types.



1. Seperately Excited:

When the field winding is Supplied from external Seperate DC Supply then the generator called Seperately Excited generator



The voltage equation for this generator is

$$E_g = V_L + I_a R_a$$

where $I_a = I_L$

Self-Excited generator:

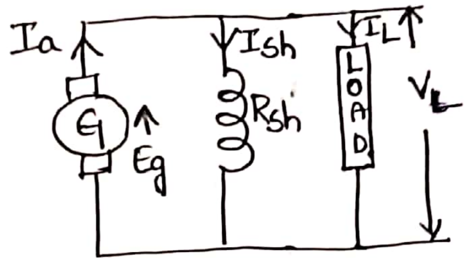
When the field winding is Supplied from the Armature of Generator it Self, then it is called self excited generator

Based on field winding connection to the armature to drive its excitation, this further divided in to following types

1. Shunt Generator
2. Series Generator
3. Compound Generator

1. Shunt Generator :

When the field winding is connected in parallel with armature and the combination of load & Generator called shunt Generator.



The current equation is,

$$\boxed{I_a = I_{sh} + I_L}$$

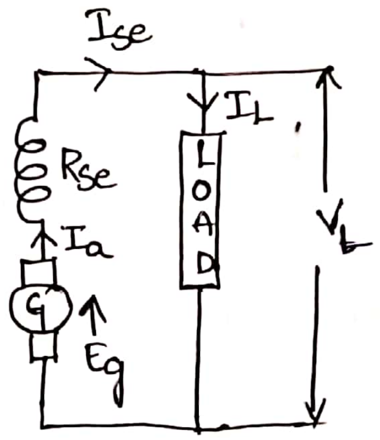
The voltage equation is. $E_g = V_L + I_a R_a + V_b$

If V_b is neglected then

$$\boxed{E_g = V_L + I_a R_a}$$

2. Series Generator :

If the field winding is connected in series with armature winding while supplying the load the generator is called series generator



The current is -

$$\boxed{I_a = I_{se} = I_L}$$

The voltage equation is $E_g = V_L + I_a R_a + I_{se} R_{se} + V_b$

If V_b is neglected then

$$\boxed{E_g = V_L + I_a R_a + I_{se} R_{se}}$$

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3. Compound Generator:

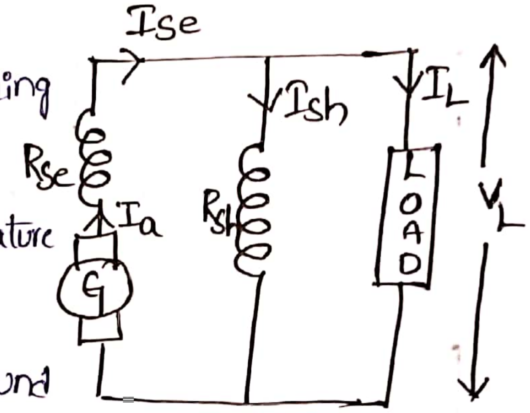
The Generator consists of both the series field winding and shunt field winding then it is called compound generator.

Depending up on the shunt field winding the compound generator is divided in-to two types.

1. Long shunt compound
2. Short shunt compound

(a) Long shunt:

If the shunt field winding is connected in parallel to the series combination of armature and series field winding called long shunt compound Generator



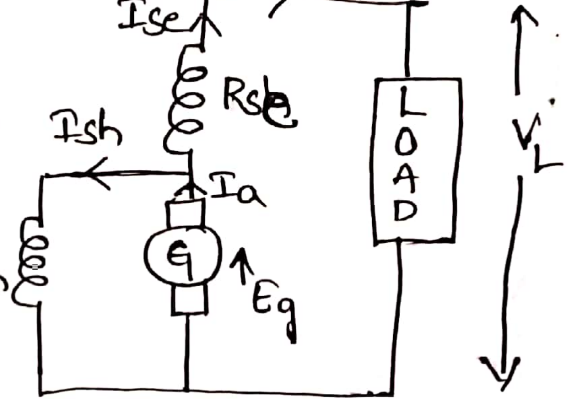
The current equation is,

$$I_a = I_{se}$$
$$I_a = I_{sh} + I_L$$

The voltage equation is, $\boxed{E_g = V_L + I_a R_a + I_{se} R_{se}}$

(b) Short shunt:

If the shunt winding is connected only across the armature terminals called short shunt compound Generator.



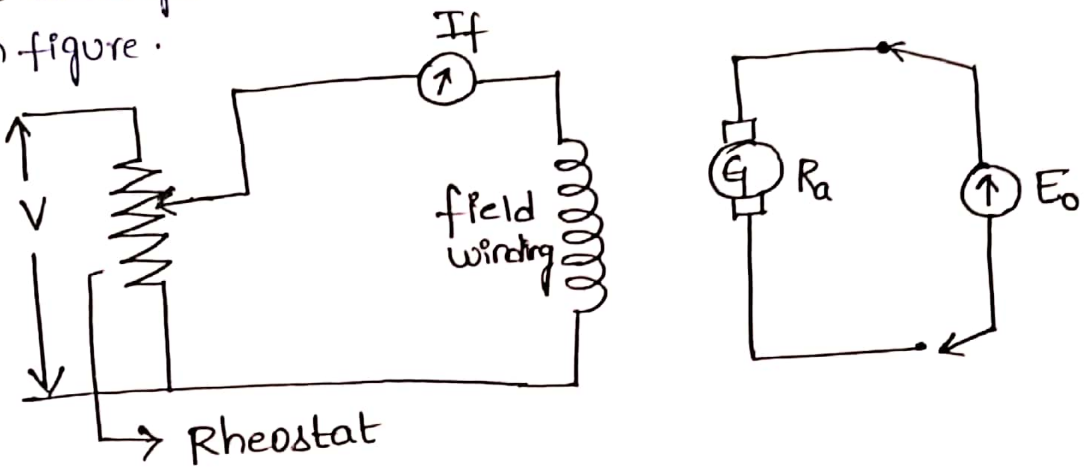
The current equation is

$$\boxed{\begin{array}{c} I_a = I_{se} + I_{sh} \\ I_{se} = I_L \end{array}}$$

The voltage equation is $\boxed{E_g = V_L + I_a R_a + I_{se} R_{se}}$

Open-circuit characteristics of DC shunt Generator:

The arrangement to obtain this characteristics is shown in figure.

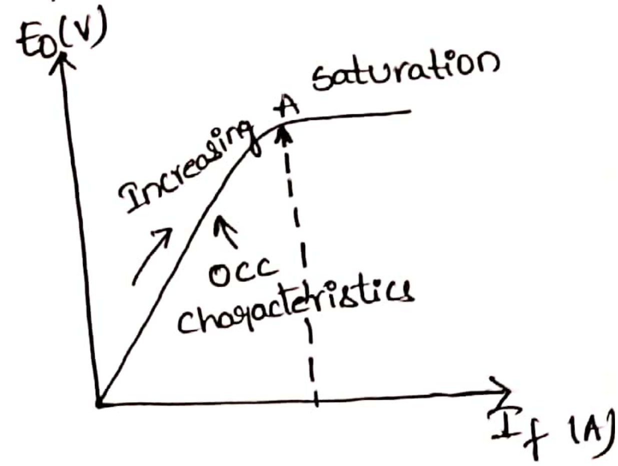


The Rheostat as a potential divider is used to control the field current and the flux. It is varied from zero and is measured on ammeter connected.

$$E_0 = \frac{\phi PNZ}{60A}$$

$E_0 \propto \phi$, $I_f \propto \phi$

$$\boxed{E_0 \propto I_f}$$



As I_f is varied, then ϕ changes and hence induced emf E_0 also varies. It is measured on Voltmeter connected across ~~Am~~ Armature.

No load is connected to machine, hence characteristics are also called "No load characteristics",

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which is the graph of E_0 against field current I_f as shown in the figure. As I_f increases, then flux ϕ increases and E_0 increases. After point A, saturation occurs. when ϕ becomes constant and hence E_0 saturates. (ie due to non linear property of field magnet).