5.1 Analysis for Shear

This section covers the following topics.

- Stress in an Uncracked Beam
- Types of Cracks
- Components of Shear Resistance
- Modes of Failure
- Effect of Prestressing Force

Introduction

The analysis of reinforced concrete and prestressed concrete members for shear is more difficult compared to the analyses for axial load or flexure.

The analysis for axial load and flexure are based on the following principles of mechanics.

1) **Equilibrium** of internal and external forces
2) **Compatibility** of strains in concrete and steel
3) **Constitutive relationships** of materials.

The conventional analysis for shear is based on equilibrium of forces by a simple equation. The compatibility of strains is not considered. The constitutive relationships (relating stress and strain) of the materials, concrete or steel, are not used. The strength of each material corresponds to the ultimate strength. The strength of concrete under shear although based on test results, is empirical in nature.

Shear stresses generate in beams due to bending or twisting. The two types of shear stress are called flexural shear stress and torsional shear stress, respectively. In this section, the analysis for shear refers to flexural shear stress. The torsional shear stress is covered in Section 5.4, Analysis for Torsion.

To understand flexural shear stress, the behaviour of a simply supported beam under uniformly distributed load, without prestressing, will be explained first. The effect of prestressing force will be subsequently introduced. The presentation will be in the following sequence.

1) Stresses in an uncracked (homogenous) beam.
2) Types of cracks generated due to the combination of flexure and shear.
3) Components of shear resistance and the modes of failure.
4) Effect of prestressing force.

5.1.1 Stresses in an Uncracked Beam

The following figure shows the variations of shear and moment along the span of a simply supported beam under a uniformly distributed load. The variations of normal stress and shear stress along the depth of a section of the beam are also shown.

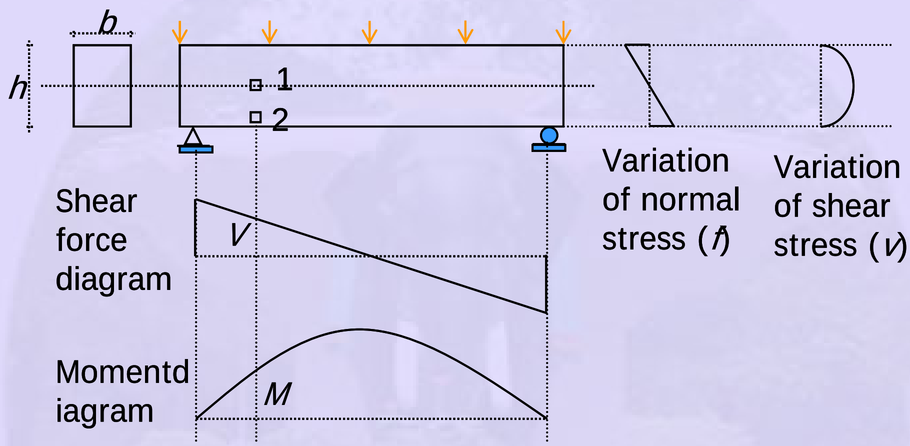

Figure 5-1.1 Variations of forces and stresses in a simply supported beam

Under a general loading, the shear force and the moment vary along the length. The normal stress and the shear stress vary along the length, as well as along the depth. The combination of the normal and shear stresses generate a two-dimensional stress field at a point. At any point in the beam, the state of two-dimensional stresses can be expressed in terms of the principal stresses. The Mohr's circle of stress is helpful to understand the state of stress.

Before cracking, the stress carried by steel is negligible. When the principal tensile stress exceeds the cracking stress, the concrete cracks and there is redistribution of stresses between concrete and steel. For a point on the neutral axis (Element 1), the shear stress is maximum and the normal stress is zero. The principal tensile stress (σ_1) is inclined at 45º to the neutral axis. The following figure shows the state of in-plane stresses.

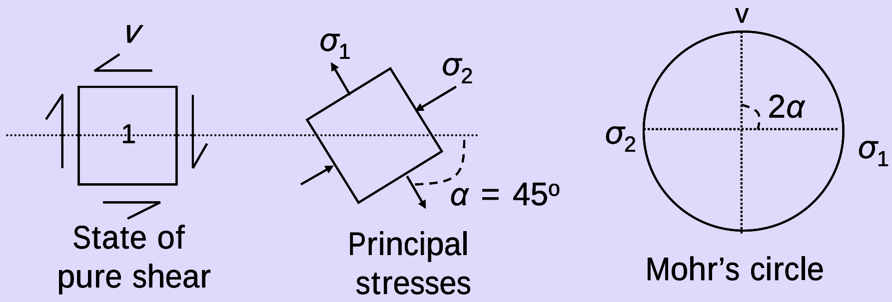

Figure 5-1.2 State of stresses at a point on the neutral axis of a beam

At the level of neutral axis, the normal stress is zero and the shear stress is maximum. An element at that level is under pure shear. A state of pure shear can be conceived as a state of biaxial tensile-compressive stresses. These principle stresses are inclined at 45° with respect to the axis of the beam. It is necessary to study the principle stresses to understand the cracking of concrete. The Mohr's circle is a representation of the state of in-plane stresses on surfaces of various inclinations passing through a point. The horizontal and vertical axes represent the normal and shear stresses, respectively. For a state of pure shear, the centre of the Mohr's circle coincides with the origin of the axes. It is expected that the reader is familiar with these concepts from a course in strength of materials.

Since the shear force is maximum near the supports, cracks due to shear occur near the supports. The cracks are formed around the neutral axis and perpendicular to the principal tensile stress (σ_1). The cracks are thus inclined at 45° to the axis of the beam. The following sketch shows the inclination of the cracks forming at the neutral axis.

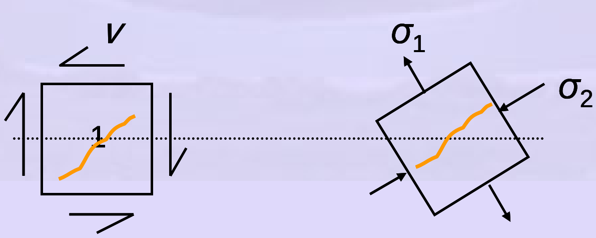

Figure 5-1.3 Inclination of crack at the level of neutral axis

For a point near the bottom edge of the beam (Element 2), the normal stress is maximum and the shear stress is close to zero. The principal tensile stress (σ_1) is almost parallel to the bottom edge. The angle of inclination of σ_1 with respect to the axis of the beam (α) is much smaller than 45°. The following figure shows the state of in-plane stresses.

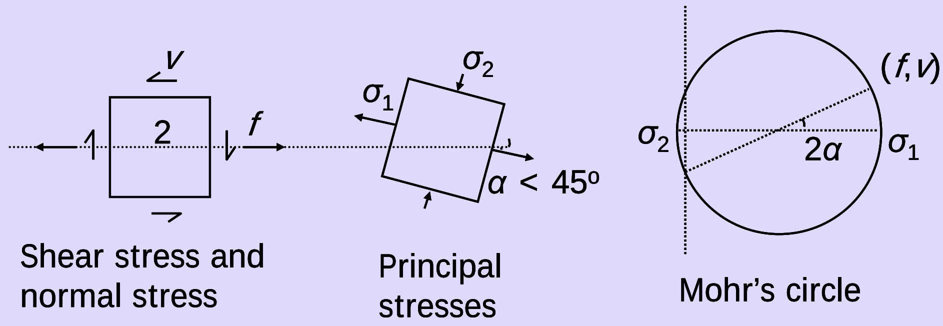

Figure 5-1.4 State of stresses at a point close to the edge under tension

Adjacent to the bottom edge (edge under tension), the tensile stress due to flexure is maximum and the shear stress is zero. The state of stress is nearly uniaxial tensile stress. The principal compressive stress is negligible. The Mohr's circle is shifted towards the axis of principal tensile stress.

Since the moment is maximum at mid span, cracks due to flexure occur near mid span. The cracks are formed at the bottom edge and perpendicular to σ_1. Since σ_1 is parallel to the edge, the cracks are perpendicular to the edge.

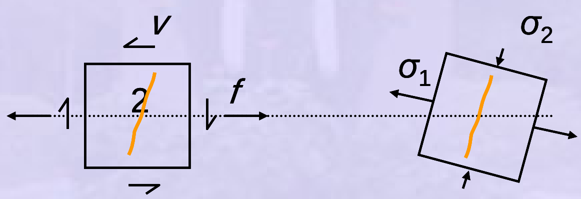

Figure 5-1.5 Inclination of crack close to the edge under tension

The previous concepts can be used to develop the principal stress trajectories. The following figure shows the trajectories for a simply supported beam under a uniformly distributed load. The crack pattern can be predicted from these trajectories.

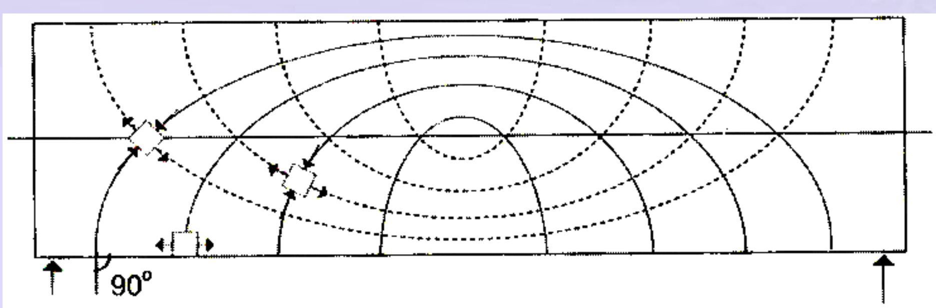

Figure 5-1.6 Principle stress trajectories

(Courtesy: Pillai, S. U., and Menon, D., *Reinforced Concrete Design*)

5.1.2 Types of Cracks

The types and formation of cracks depends on the span-to-depth ratio of the beam and loading. These variables influence the moment and shear along the length of the beam. For a simply supported beam under uniformly distributed load, without prestressing, three types of cracks are identified.

1) **Flexural cracks**: These cracks form at the bottom near the midspan and propagate upwards.
2) **Web shear cracks**: These cracks form near the neutral axis close to the support and propagate inclined to the beam axis.
3) **Flexure shear cracks**: These cracks form at the bottom due to flexure and propagate due to both flexure and shear.

In the following figure, the formation of cracks for a beam with large span-to-depth ratio and uniformly distributed loading is shown.

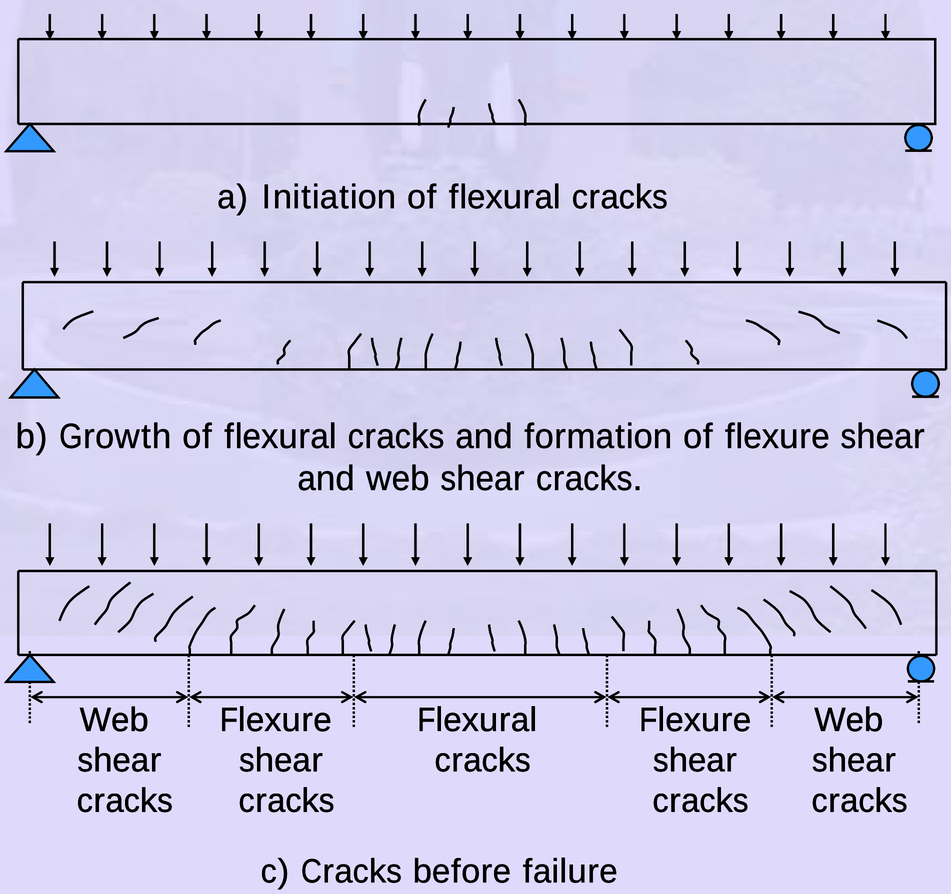

Figure 5-1.7 Formation of cracks in a reinforced concrete beam

5.1.3 Components of Shear Resistance

The components of shear resistance are studied based on the internal forces at a flexure shear crack. The forces are shown in the following figure.

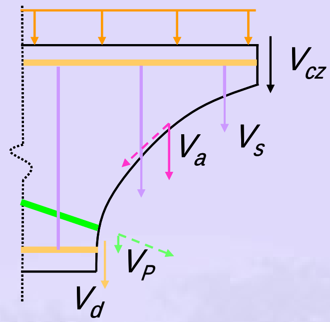

Figure 5-1.8 Internal forces at a flexure shear crack

The notations in the previous figure are as follows.

- V_{cz} = Shear carried by uncracked concrete
- V_a = Shear resistance due to aggregate interlock
- V_d = Shear resistance due to dowel action
- V_s = Shear carried by stirrups
- V_p = Vertical component of prestressing force in inclined tendons

The magnitude and the relative value of each component change with increasing load.

5.1.4 Modes of Failure

For beams with low span-to-depth ratio or inadequate shear reinforcement, the failure can be due to shear. A failure due to shear is sudden as compared to a failure due to flexure. The following five modes of failure due to shear are identified.

1) Diagonal tension failure
2) Shear compression failure
3) Shear tension failure
4) Web crushing failure
5) Arch rib failure

The occurrence of a mode of failure depends on the span-to-depth ratio, loading, cross-section of the beam, amount and anchorage of reinforcement. The modes of failure are explained next (Courtesy: Pillai, S. U., and Menon, D., *Reinforced Concrete Design*).

Diagonal tension failure

In this mode, an inclined crack propagates rapidly due to inadequate shear reinforcement.

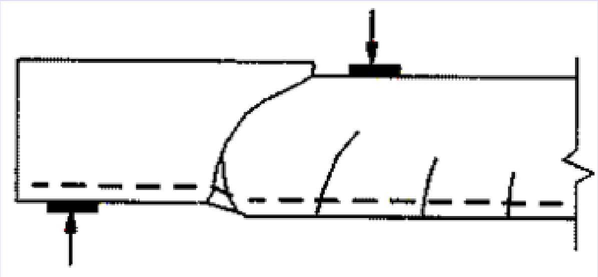

Figure 5-1.9 Diagonal tension failure

Shear compression failure

There is crushing of the concrete near the compression flange above the tip of the inclined crack.

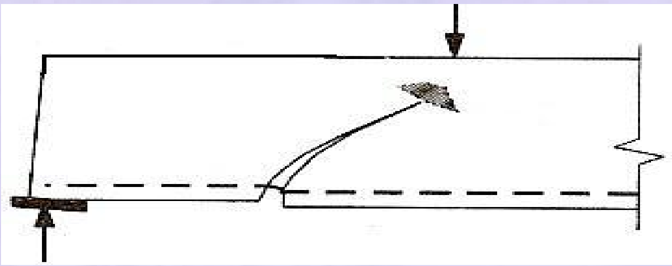

Figure 5-1.10 Shear compression failure

Shear tension failure

Due to inadequate anchorage of the longitudinal bars, the diagonal cracks propagate horizontally along the bars.

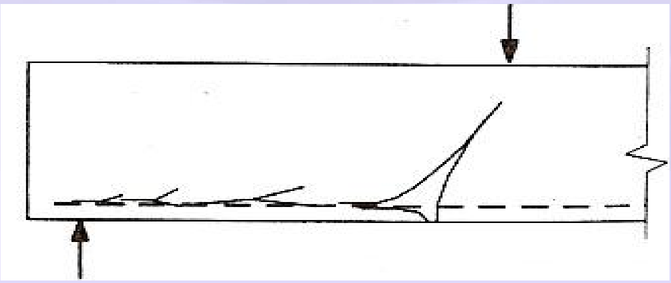

Figure 5-1.11 Shear tension failure

Web crushing failure

The concrete in the web crushes due to inadequate web thickness.

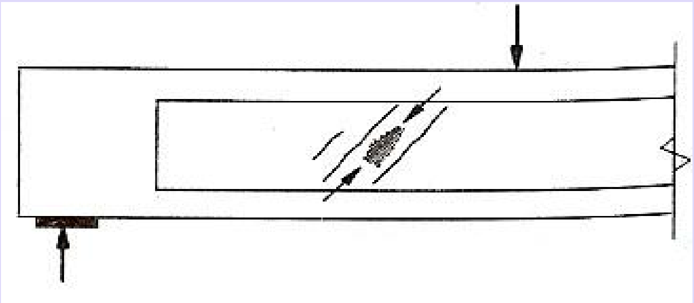

Figure 5-1.12 Web crushing failure

Arch rib failure

For deep beams, the web may buckle and subsequently crush. There can be anchorage failure or failure of the bearing.

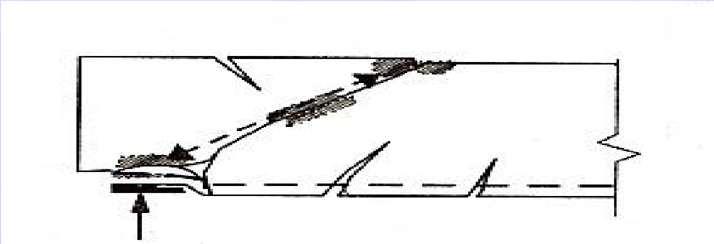

Figure 5-1.13 Arch rib failure

The objective of design for shear is to avoid shear failure. The beam should fail in flexure at its ultimate flexural strength. Hence, each mode of failure is addressed in the design for shear. The design involves not only the design of the stirrups, but also limiting the average shear stress in concrete, providing adequate thickness of the web and adequate development length of the longitudinal bars.

5.1.5 Effect of Prestressing Force

In presence of prestressing force, the flexural cracking occurs at a higher load. For Type 1 and Type 2 sections, there is no flexural crack under service loads. This is evident from the typical moment versus curvature curve for a prestressed section (refer to Section 3.6, Analysis of Member under Flexure (Part V)). In presence of prestressing force, the web shear cracks also generate under higher load.

With increase in the load beyond the cracking load, the cracks generate in a similar sequence. But, the inclinations of the flexure shear and web shear cracks are reduced depending on the amount of prestressing and the profile of the tendon.

The effect of prestressing force is explained for a beam with a concentric effective prestressing force (P_e).

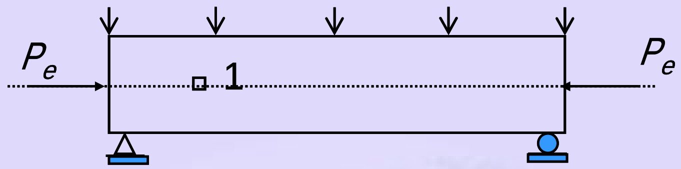

Figure 5-1.14 A simply supported beam under concentric prestress and uniformly distributed loads

For a point at the neutral axis (Element 1), there is normal stress due to the prestressing force ($-f_{pe}$). The principal tensile stress (σ_1) is inclined to the neutral axis at an angle greater than 45º. The following figure shows the state of in-plane stresses.

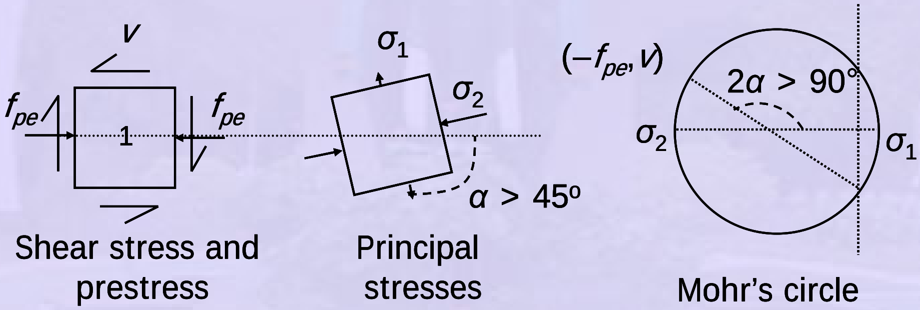

Figure 5-1.15 State of stresses at a point on the neutral axis for a prestressed beam

For a point at the neutral axis (Element 1), there is substantial normal compressive stress due to the prestressing force. With the combination of shear stress, the principal compressive stress (σ_2) is inclined to the neutral axis at an angle much smaller than 45°.

In the following figure, the formation of cracks for a prestressed beam with large span-to-depth ratio and uniformly distributed loading is shown. This figure can be compared with that for a reinforced concrete beam.

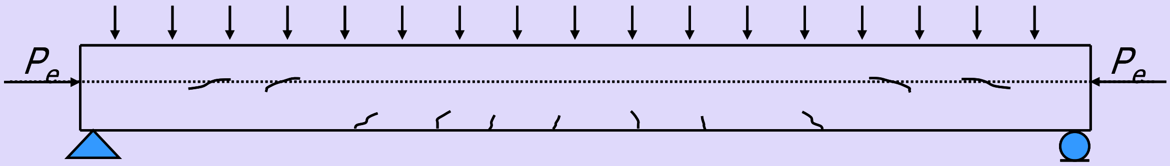

Figure 5-1.16 Formation of cracks in a prestressed beam

After cracking, in presence of prestressing force, the length and crack width of a diagonal crack are low. Thus, the aggregate interlock and zone of concrete under compression are larger as compared to a non-prestressed beam under the same load. Hence, the shear strength of concrete (V_c) increases in presence of prestressing force. This is accounted for in the expression of V_c.