5.4 Analysis for Torsion

This section covers the following topics.

- Stresses in an Uncracked Beam
- Crack Pattern Under Pure Torsion
- Components of Resistance for Pure Torsion
- Modes of Failure
- Effect of Prestressing Force

Introduction

The analysis of reinforced concrete and prestressed concrete members for torsion is more difficult compared to the analyses for axial load or flexure.

The analysis for axial load and flexure are based on the following principles of mechanics.

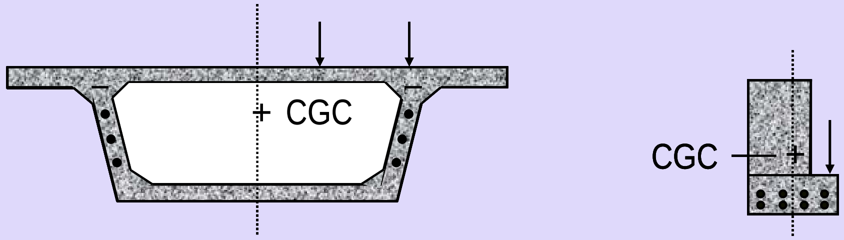
1) **Equilibrium** of internal and external forces
2) **Compatibility** of strains in concrete and steel
3) **Constitutive relationships** of materials.

The conventional analysis of reinforced concrete and prestressed concrete members for torsion is based on equilibrium of forces by simple equations. The compatibility of strains in concrete and steel reinforcement is not considered. The strength of each material, concrete or steel, corresponds to the ultimate strength. The constitutive relationship of each material, relating stress and strain, is not used.

Torsion generated in a member can be classified into two types based on the necessity of analysis and design for torsion.

1) **Equilibrium torsion**: This is generated due to loading eccentric to the centroidal axis. For example, a) in a beam supporting cantilever slab or precast slab or floor joists on one side, b) in a (curved) bridge deck subjected to eccentric live load and c) in an electric pole subjected to loads from wires on one side.

The torsion demand is determined by equilibrium condition only. The member needs to be analysed and designed for torsion. The following figure shows the situations where eccentric loads are acting on the members.



a) Bridge deck b) L-beam

Figure 5-4.1 Examples of members under eccentric load

2) **Compatibility torsion**: This is generated by twisting, to maintain compatibility in deformation with the connected member. This type of torsion generates in a primary beam supporting secondary beams.

In compatibility torsion, the torsion demand is determined by both equilibrium and compatibility conditions. Else, the torsion can be neglected. This implies the primary beam need not be analysed and designed for torsion, if the secondary beams are designed as simply supported.

In this section, the emphasis is laid on equilibrium torsion. To understand the behaviour of a beam under torsion, the presentation will be in the following sequence.

1) Stresses in an uncracked (homogenous) rectangular beam without prestressing due to pure torsion (in absence of flexure), with constant torque along the span.
2) Crack pattern under pure torsion.
3) Components of resistance for pure torsion.
4) Modes of failure under combined torsion and flexure.
5) Effect of prestressing force.

Although pure torsion is absent in structures, understanding the behaviour of a beam under pure torsion helps to analyse a beam under combined torsion, flexure and shear.

5.4.1 Stresses in an Uncracked Beam

The following figure shows a beam of rectangular cross-section under pure torsion. The variations of the torsional shear stress (τ) along radial lines in the cross-section are shown. It can be observed that the maximum shear stress (τ_{max}) occurs at the middle of the longer side. Hence, the subsequent explanation will indicate the stress condition at the middle of the longer side.

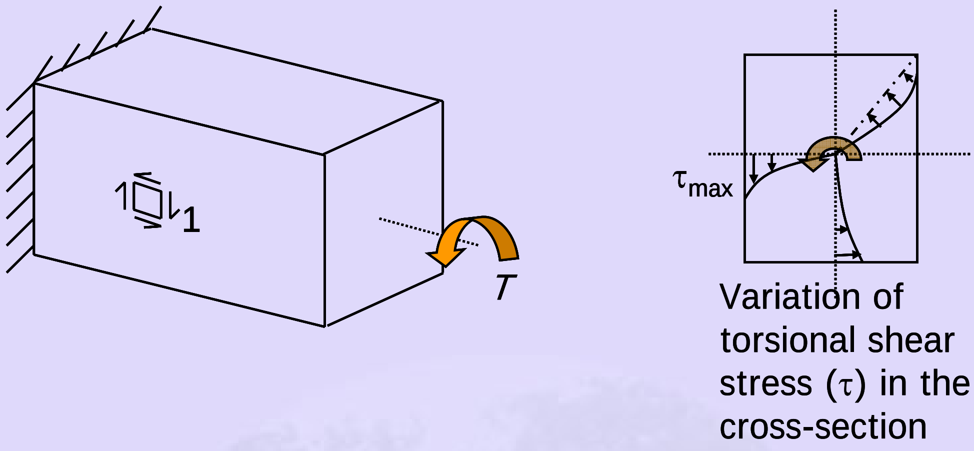


Figure 5-4.2 Beam subjected to pure torsion

At any point in the beam, the state of two-dimensional stresses can be expressed in terms of the principal stresses. The Mohr's circle of stress is helpful to understand the state of stress.

Before cracking, the stress carried by steel is negligible. When the principal tensile stress exceeds the cracking stress, the concrete cracks and there is redistribution of stresses between concrete and steel.

For a point at the middle of the longer side (Element 1), the torsional shear stress is maximum. The principal tensile stress (σ_1) is inclined at 45º to the beam axis.

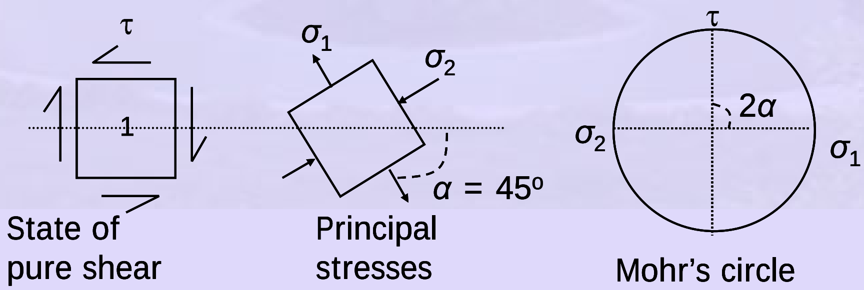
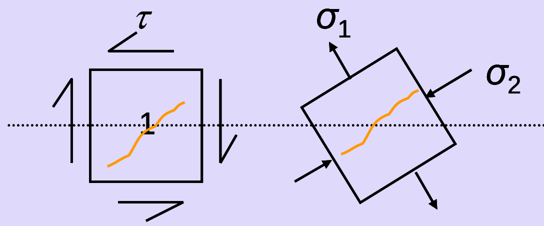


Figure 5-4.3 State of stresses at the side of a beam

Since the torsion is maximum at middle of the longer side, cracks due to torsion occur around that location and perpendicular to σ_1.

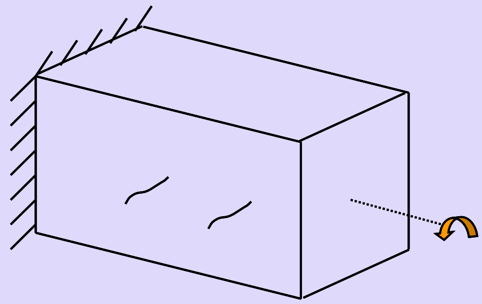


Crack inclination

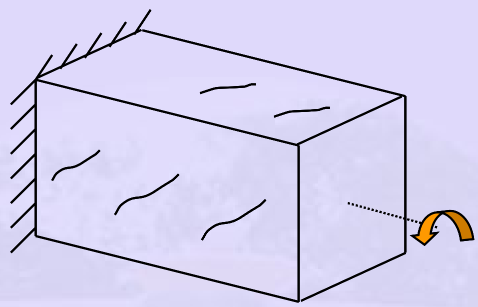
Figure 5-4.4 Inclination of crack at the side of a beam

5.4.2 Crack Pattern under Pure Torsion

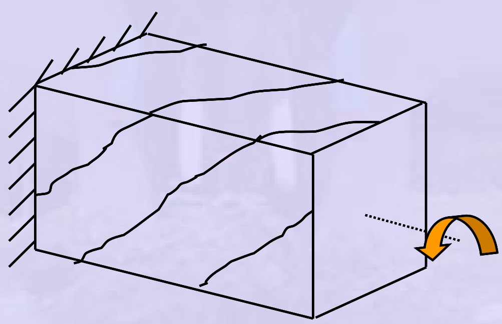
The cracks generated due to pure torsion follow the principal stress trajectories. The first cracks are observed at the middle of the longer side. Next, cracks are observed at the middle of the shorter side. After the cracks connect, they circulate along the periphery of the beam.



a) Initiation of torsional cracks in longer side



b) Initiation of torsional cracks in shorter side



c) Spiral torsional cracks

Figure 5-4.5 Formation of cracks in a beam subjected to pure torsion

In structures, a beam is not subjected to pure torsion. Along with torsion it is also subjected to flexure and shear. Hence, the stress condition and the crack pattern are more complicated than shown before.

5.4.3 Components of Resistance for Pure Torsion

After cracking, the concrete forms struts carrying compression. The reinforcing bars act as ties carrying tension. This forms a space truss. Since the shear stress is larger near the sides, the compression in concrete is predominant in the peripheral zone. This is called the thin-walled tube behaviour. The thickness of the wall is the **shear flow zone**, where the shear flow is assumed to be constant. The portion of concrete inside the shear flow zone can be neglected in calculating the capacity.

The components in vertical and horizontal sections of a beam are shown below.

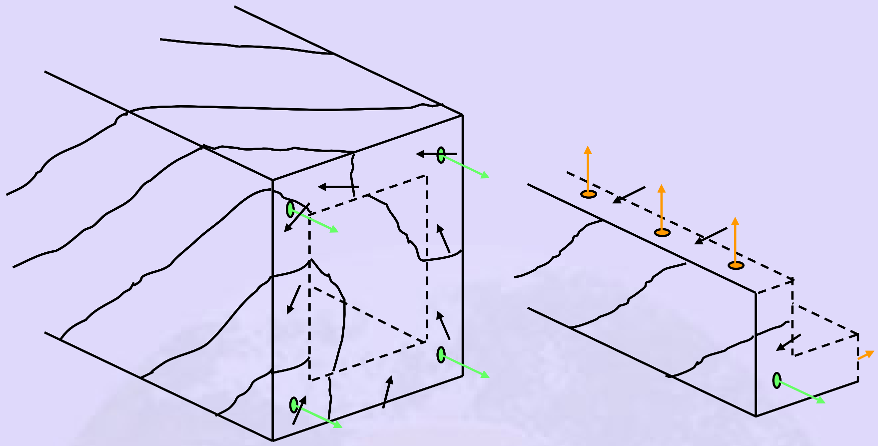


Figure 5-4.6 Internal forces in a beam

The components can be denoted as below.

T_c = torsion resisted by concrete

T_s = torsion resisted by the longitudinal and transverse reinforcing bars.

The magnitude and the relative value of each component change with increasing torque.

5.4.4 Modes of Failure

For a homogenous beam made of brittle material, subjected to pure torsion, the observed plane of failure is not perpendicular to the beam axis, but inclined at an angle. This can be explained by theory of elasticity. A simple example is illustrated by applying torque to a piece of chalk.

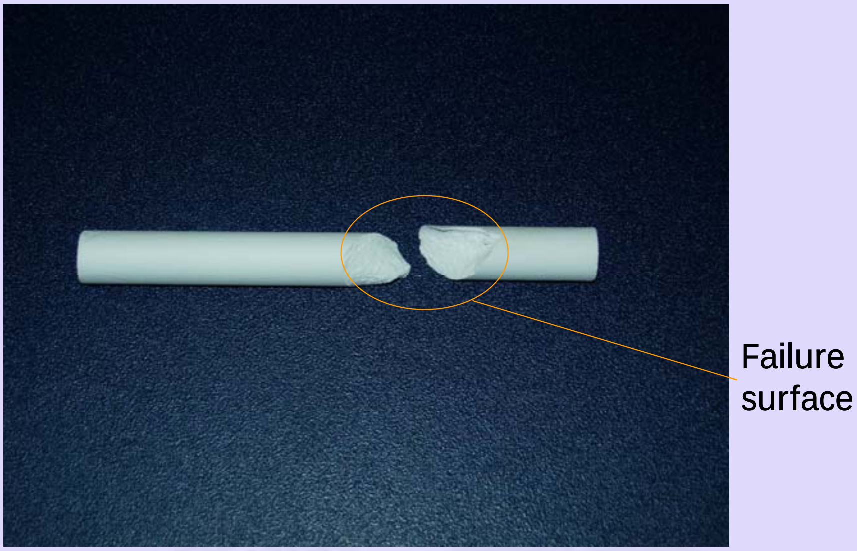


Figure 5-4.7 Failure of a piece of chalk under torque

For a beam of rectangular section, the plane of failure is further influenced by warping. Torsional warping is defined as the differential axial displacement of the points in a section perpendicular to the axis, due to torque.

For a reinforced concrete beam, the length increases after cracking and after yielding of the bars. For a beam subjected to flexure and torsion simultaneously, the modes of failure are explained by the **Skew Bending Theory**. The observed plane of failure is not perpendicular to the beam axis, but inclined at an angle. The curved plane of failure is idealised as a planar surface inclined to the axis of the beam.

The skew bending theory explains that the flexural moment (M_u) and torsional moment (T_u) combine to generate a resultant moment inclined to the axis of the beam. This moment causes compression and tension in a planar surface inclined to the axis of the beam. The following figure shows the resultant moment due to flexural moment and torsion in a beam.

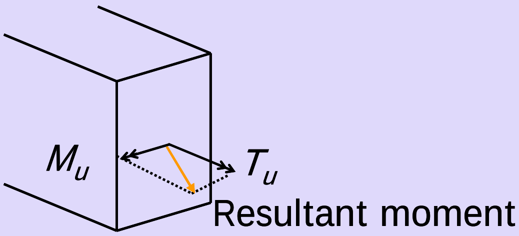


Figure 5-4.8 Beam subjected to flexural moment and torsion

The modes of failure are explained based on the relative magnitudes of the flexural moment (M_u) and torsional moment (T_u) at ultimate. Three discrete modes of failure are defined from a range of failure. The idealised pattern of failure with the plane of failure and the resultant compression (C_u) and tension (T_u) are shown for each mode (Courtesy: Pillai, S. U., and Menon, D., *Reinforced Concrete Design*).

1) **Modified bending failure** (Mode 1): This occurs when the effect of M_u is larger than that of T_u.

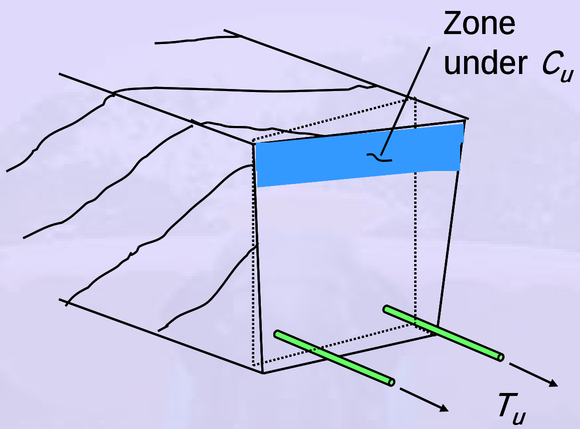


Figure 5-4.9 Idealised pattern for Mode 1 failure

2) **Lateral bending failure** (Mode 2): This is observed in beams with thin webs when the effect of M_u and T_u are comparable.

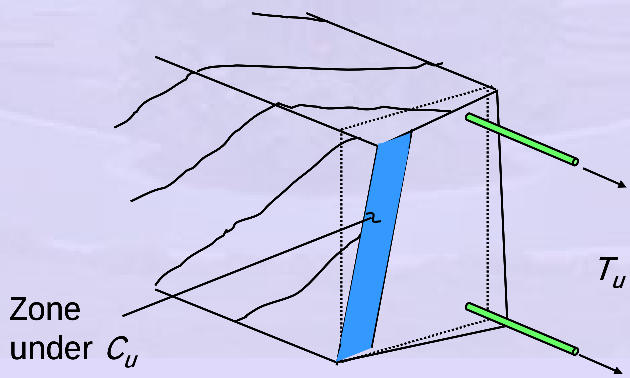


Figure 5-4.10 Idealised pattern for Mode 2 failure

3) **Negative bending failure** (Mode 3): When the effect of T_u is large and the top steel is less, this mode of failure occurs.

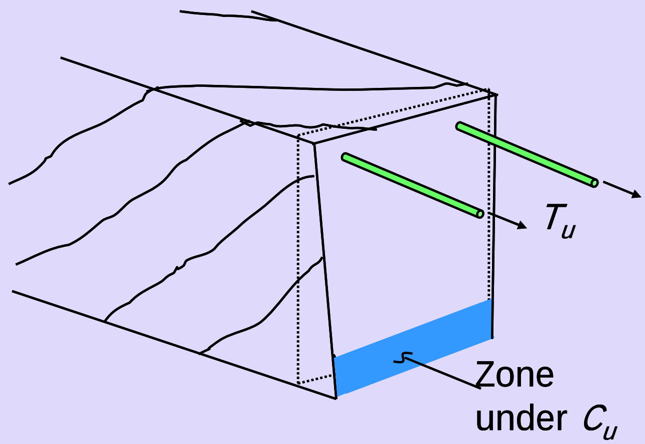


Figure 5-4.11 Idealised pattern for Mode 3 failure

5.4.5 Effect of Prestressing Force

In presence of prestressing force, the cracking occurs at higher load. This is evident from the typical torque versus twist curves for sections under pure torsion. With further increase in load, the crack pattern remains similar but the inclinations of the cracks change with the amount of prestressing. The following figure shows the difference in the torque versus twist curves for a non-prestressed beam and a prestressed beam.

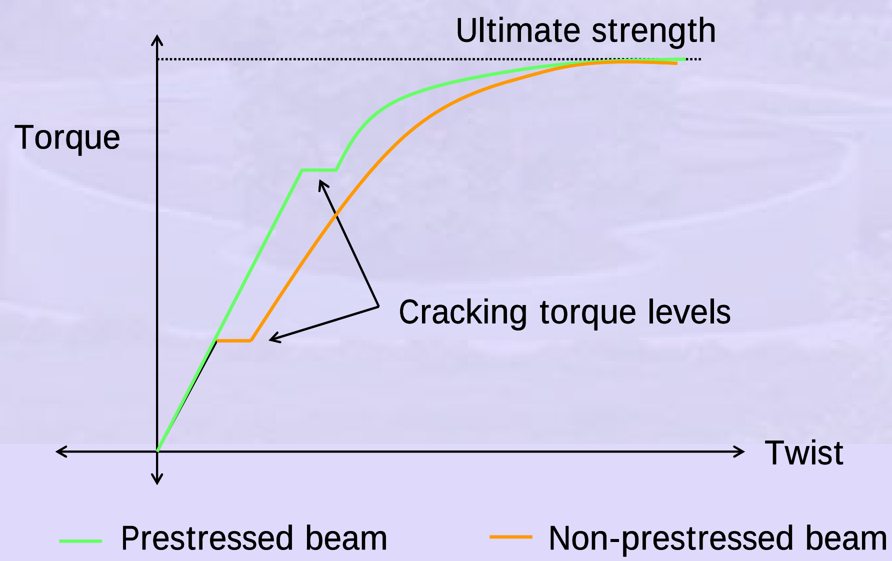


Figure 5-4.12 Torque versus twist curves

The effect of prestressing force is explained for a beam under pure torsion with a concentric prestressing force (P_e). The following figure shows such a beam.

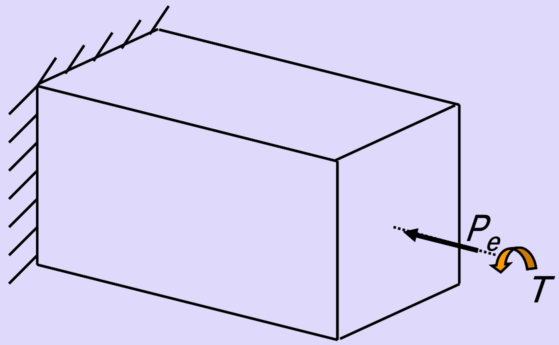


Figure 5-4.13 Beam subjected to pure torsion and prestressing force

For a point at the middle of the longer side (Element 1), there is normal stress due to the prestressing force ($-f_{pe}$). The principal tensile stress (σ_1) is inclined to the neutral axis at an angle greater than 45º.

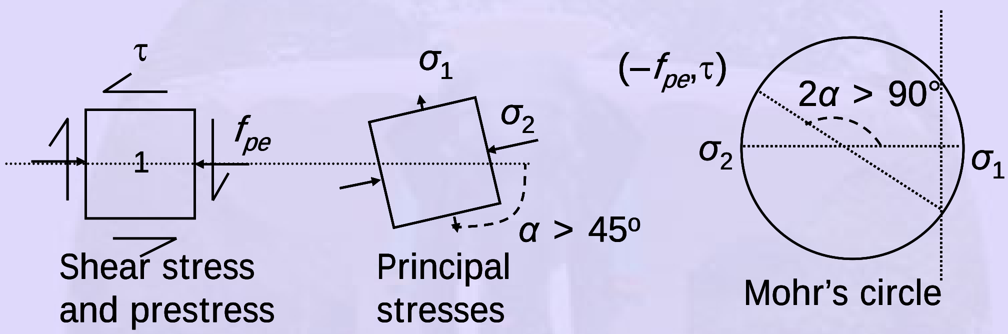


Figure 5-4.14 State of stresses at the side of a prestressed beam

In the following figure, the formation of cracks for a prestressed beam under pure torsion is shown. This figure can be compared with that for a reinforced concrete beam.

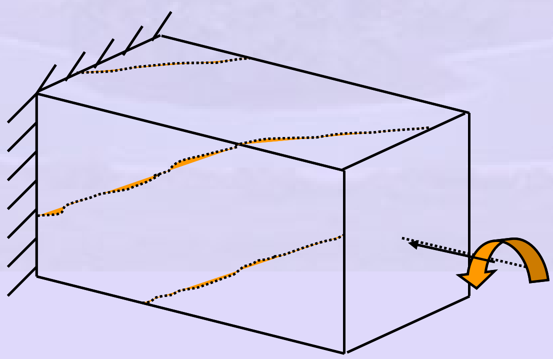


Figure 5-4.15 Formation of cracks in a prestressed beam

In presence of prestressing force, the cracking is at a higher torque. After cracking, the crack width of a spiral crack is low. Thus, the aggregate interlock is larger as compared to a non-prestressed beam under the same torque. Hence, the torsional strength of concrete (T_c) increases in presence of prestressing force. This is accounted for in the expression of T_c.