6.1 Calculation of Deflection

This section covers the following topics.

- Introduction
- Deflection due to Gravity Loads
- Deflection due to Prestressing Force
- Total Deflection
- Limits of Deflection
- Determination of Moment of Inertia
- Limits of Span-to-effective Depth Ratio

6.1.1 Introduction

The deflection of a flexural member is calculated to satisfy a limit state of serviceability. Since a prestressed concrete member is smaller in depth than an equivalent reinforced concrete member, the deflection of a prestressed concrete member tends to be larger.

The total deflection is a resultant of the upward deflection due to prestressing force and downward deflection due to the gravity loads. Only the flexural deformation is considered and any shear deformation is neglected in the calculation of deflection. Shear deformation is included in members such as deep beams and wall type of structures.

The deflection of a member is calculated at least for two cases.

1) Short term deflection **at transfer**
2) Long term deflection under **service loads**

The short term deflection at transfer is due to the prestressing force (before long term losses) and self-weight without the effect of creep and shrinkage of concrete. The long term deflection under service loads is due to the effective prestressing force (after long term losses) and the gravity loads. The permanent components of the gravity loads are considered in the effect of creep. These components are dead load and sustained live load.

6.1.2 Deflection due to Gravity Loads

The methods of calculation of deflection are taught in a course on structural analysis. It is expected that the students are familiar with the methods. The methods include the following.

1) Double integration method
2) Moment-area method
3) Conjugate beam method
4) Principle of virtual work

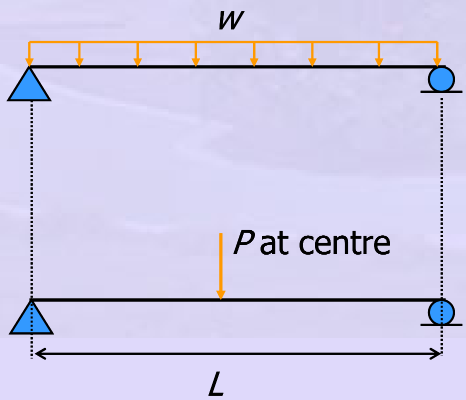
Numerical solution schemes can be developed based on the above methods and executed in a computer. For members with prismatic cross-sections, common support conditions and subjected to conventional loading, the deflections are available in tables of text books on structural analysis.

The expressions of deflection (Δ) for a few cases are provided. Here,

I = moment of inertia

E = modulus of elasticity of concrete

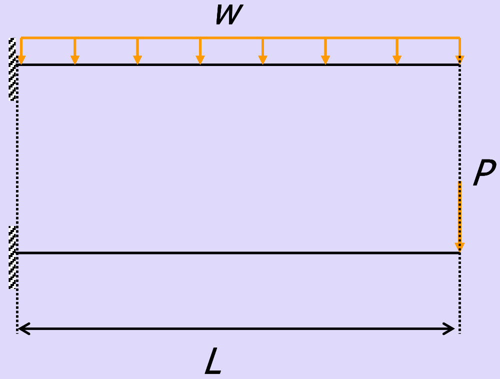
Simply supported beams



$$\Delta = \frac{5}{384}\frac{wL^4}{EI}$$

$$\Delta = \frac{PL^3}{48EI}$$

Cantilever beams



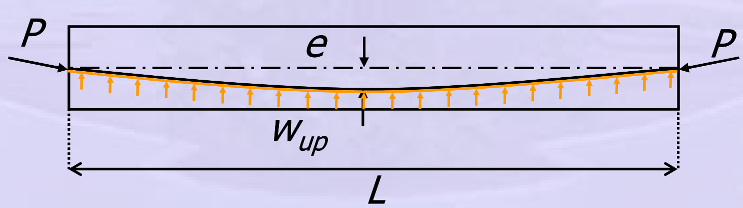
$$\Delta = \frac{wL^4}{8EI}$$

$$\Delta = \frac{PL^3}{3EI}$$

6.1.3 Deflection due to Prestressing Force

The prestressing force causes a deflection only if the CGS is eccentric to the CGC. The deflection due to prestressing force is calculated by the load-balancing method. This method is explained in Section 3.2, Analysis of Member under Flexure (Part I). The upward thrust (represented as w_{up} for curved tendons and W_{up} for bent tendons) and the upward deflection (also called camber and represented as Δ_P) due to the prestressing forces in typical profiles of tendons are reproduced here.

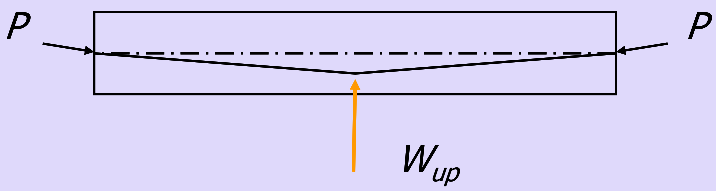
a) For a Parabolic Tendon



$$w_{up} = \frac{8Pe}{L^2}$$

$$\Delta_P = \frac{5}{384}\frac{w_{up}L^4}{EI} \quad \textbf{(6-1.1)}$$

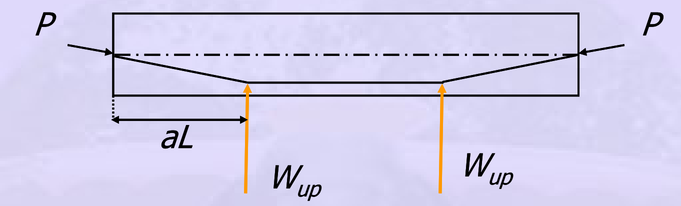
b) For a Singly Harped Tendon



$$W_{up} = \frac{4Pe}{L}$$

$$\Delta_P = \frac{W_{up}L^3}{48EI} \quad \textbf{(6-1.2)}$$

b) For a Doubly Harped Tendon



$$W_{up} = \frac{Pe}{aL}$$

$$\Delta_P = \frac{a\left(3-4a^2\right)W_{up}L^3}{24EI} \quad \textbf{(6-1.3)}$$

6.1.4 Total Deflection

The total deflection is calculated for the following two cases.

1) Short term deflection at transfer
2) Long term deflection under service loads

The short term deflection at transfer (Δ_{st}) is given as follows.

$$\Delta_{st} = -\Delta_{P0} + \Delta_{SW} \quad \textbf{(6-1.4)}$$

Here,

ΔP_0 = magnitude of deflection due to P_0

Δ_{sw} = deflection due to self-weight

P_0 = prestressing force before long term losses.

The long term deflection under service loads is difficult to calculate because the prestressing force and creep strain influence each other. Creep of concrete is defined as the increase in deformation with time under constant load. Due to the creep of concrete, the prestress in the tendon is reduced with time. The ultimate creep strain is found to be proportional to the elastic strain. The ratio of the ultimate creep strain to the elastic strain is called the creep coefficient θ. The values of θ as per **IS:1343 - 1980** are given in Section 1.6, Concrete (Part II).

The following expression of the long term deflection under service loads (Δ_{lt}) is a simplified form, where an average prestressing force is considered to generate the creep strain. The effect of shrinkage on the prestressing force is neglected in the expression.

$$\Delta_{lt} = -\Delta_{Pe} - \left(\frac{\Delta_{P0} + \Delta_{Pe}}{2}\right)\theta + \left(\Delta_{DL} + \Delta_{SL}\right)\left(1+\theta\right) + \Delta_{LL} \quad \textbf{(6-1.5)}$$

The notations in the previous equations are as follows.

- ΔP_0 = magnitude of deflection due to P_0
- ΔP_e = magnitude of deflection due to P_e
- P_e = effective prestressing force after long term losses.
- Δ_{DL} = deflection due to dead load (including self-weight)
- Δ_{SL} = deflection due to sustained live load
- Δ_{LL} = deflection due to additional live load

A more rigorous calculation of total deflection can be done using the **incremental time-step method**. It is a step-by-step procedure, where the change in prestressing force due to creep and shrinkage strains is calculated at the end of each time step. The results at the end of each time step are used for the next time step. This procedure was suggested by the Precast / Prestressed Concrete Institute (PCI) committee and is also called the **General method** (Reference: PCI Committee, "*Recommendations for Estimating Prestress Losses*", PCI Journal, PCI, Vol. 20, No. 4, July-August 1975, pp. 43-75).

In the PCI step-by-step procedure, a minimum of four time steps are considered in the service life of a prestressed member. The following table provides the definitions of the time steps.

Table 6-1.1 Time steps in the step-by-step procedure

Step	Beginning	End
1.	Pre-tension: Anchorage of steel Post-tension: End of curing	Age of prestressing
2.	End of Step 1	30 days after prestressing or when subjected to superimposed load
3.	End of Step 2	1 year of service
4.	End of Step 3	End of service life

The step-by-step procedure can be implemented in a computer program, where the number of time steps can be increased.

6.1.5 Limits of Deflection

Clause 19.3.1 of **IS:1343 - 1980** specifies limits of deflection such that the efficiency of the structural element and the appearance of the finishes or partitions are not adversely affected. The limits of deflection are summarised next.

1) The total deflection due to all loads, including the effects of temperature, creep and shrinkage, should not exceed span / 250.
2) The deflection after erection of partitions or application of finishes, including the effects of temperature, creep and shrinkage, should not exceed span/350 or 20 mm, whichever is less.
3) If finishes are applied, total upward deflection due to prestressing force should not exceed span / 300.

6.1.6 Determination of Moment of Inertia

Type 1 and Type 2 Members

These types of members are designed to be uncracked under service loads. The gross moment of inertia (I_g) can be used to calculate the deflections.

Type 3 Members

This type of members is expected to be cracked under service loads. Strictly, the gross moment of inertia (I_g) cannot be used in the calculations. **IS:1343 - 1980, Clause 22.6.2**, recommends the following.

1) When the permanent load is less than or equal to 25% of the live load, the gross moment of inertia can be used.
2) When the permanent load is greater than 25% of the live load, the span-to-effective depth (*L*/*d*) ratio should be limited to bypass the calculation of deflection.

If the *L*/*d* ratio of a member exceeds the limit, the gross moment of inertia can still be used if the tensile stress under service loads is within the allowable value. This recommendation is suggested because the calculation of gross moment of inertia is simpler as compared to an effective moment of inertia.

6.1.7 Limits of Span-to-Effective Depth Ratio

The calculation of deflection can be bypassed if the span-to-effective depth (*L*/*d*) ratio is within the specified limit.

The limits of *L*/*d* ratios as per **Clause 22.6.2**, **IS:1343 – 1980**, are as follows.

For L ≤ 10 m

For cantilever beams	$L/d \le 7$
For simply supported beams	$L/d \le 20$
For continuous beams	$L/d \le 26$

For L > 10 m

For simply supported beams	$L/d \le (20 \times 10/L)$
For continuous beams	$L/d \le (26 \times 10/L)$

Here, *L* is in metres. Deflection calculations are necessary for cantilevers with $L > 10$ m.