6.2 Calculation of Crack Width

This section covers the following topics.

- Introduction
- Method of Calculation
- Limits of Crack Width

6.2.1 Introduction

The crack width of a flexural member is calculated to satisfy a limit state of serviceability. Among prestressed concrete members, there is cracking under service loads only for Type 3 members. Hence the calculation of crack width is relevant only for Type 3 members. The crack width is calculated for the cracks due to bending which occur at the bottom or top surfaces of a flexural member.

The flexural cracks start from the tension face and propagate perpendicular to the axis of the member. This type of cracks is illustrated in Section 5.1, Analysis for Shear. If these cracks are wide, it leads to corrosion of the reinforcing bars and prestressed tendons. To limit the crack width, Type 3 members have regular reinforcing bars in the tension zone close to the surface, in addition to the prestressed tendons.

The crack width of a flexural crack depends on the following quantities.

1) Amount of prestress
2) Tensile stress in the longitudinal bars
3) Thickness of the concrete cover
4) Diameter and spacing of longitudinal bars
5) Depth of member and location of neutral axis
6) Bond strength
7) Tensile strength of concrete.

6.2.2 Method of Calculation

IS:456 - 2000, Annex F, gives a procedure to determine flexural crack width. The design crack width (W_{cr}) at a selected level on the surface of the section with maximum moment is given as follows.

$$W_{cr} = \frac{3a_{cr}\varepsilon_m}{1+\dfrac{2(a_{cr} - C_{min})}{h - x}} \qquad \textbf{(6-2.1)}$$

The notations in the previous equation are as follows.

a_{cr} = shortest distance from the selected level on the surface to a longitudinal bar

C_{min} = minimum clear cover to the longitudinal bar

h = total depth of the member

x = depth of the neutral axis

ε_m = average strain at the selected level.

The values of C_{min} and h are obtained from the section of the member. The evaluation of the other variables is explained.

Evaluation of a_{cr}

The location of crack width calculation can be at the soffit or the sides of a beam. The value of a_{cr} depends on the selected level. The following sketch shows the values of a_{cr} at a bottom corner (A), at a point in the soffit (B) and at a point at the side (C).

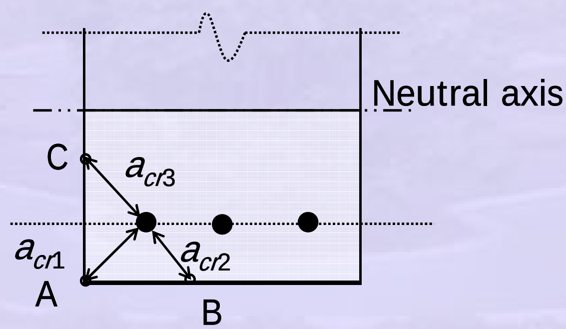

Figure 6-2.1 Cross-section of a beam showing the distances from surface to the nearest longitudinal bar

Usually the crack width is calculated at a point in the soffit, which is equidistant from two longitudinal bars. This point is the location of maximum estimated crack width. The following sketch shows the variables used in computing a_{cr}.

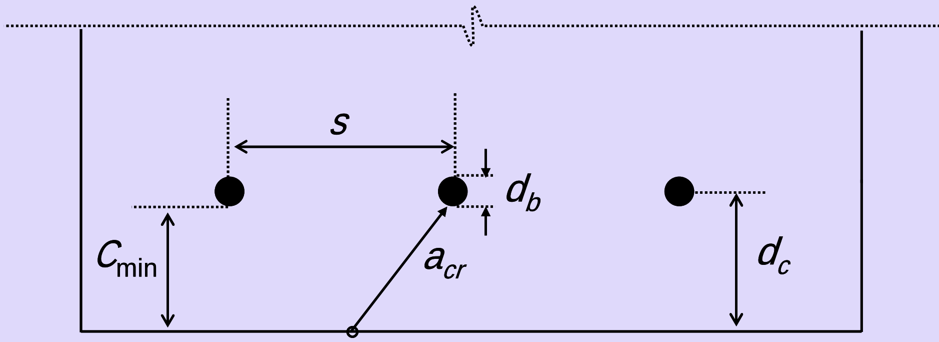

Figure 6-2.2 Cross-section of a beam showing variables for calculation

Using geometry, the value of a_{cr} is obtained from the following equation.

$$a_{cr} = \sqrt{\left(\frac{s}{2}\right)^2 + d_c^{\,2}} - \frac{d_b}{2} \qquad \textbf{(6-2.2)}$$

Here,

d_b = diameter of longitudinal bar

d_c = effective cover = C_{min} + $d_b/2$

s = centre-to-centre spacing of longitudinal bars.

The values of d_b, d_c and s are obtained from the section of the member.

Evaluation of *x* and ε_m

The value of x and ε_m are calculated based on a sectional analysis under service loads. The sectional analysis should consider the tension carried by the uncracked concrete in between two cracks. The stiffening of a member due to the tension carried by the concrete is called the **tension stiffening effect**. The value of ε_m is considered to be an average value of the strain at the selected level over the span. The following sketch illustrates the cracking and the uncracked concrete in a flexural member.

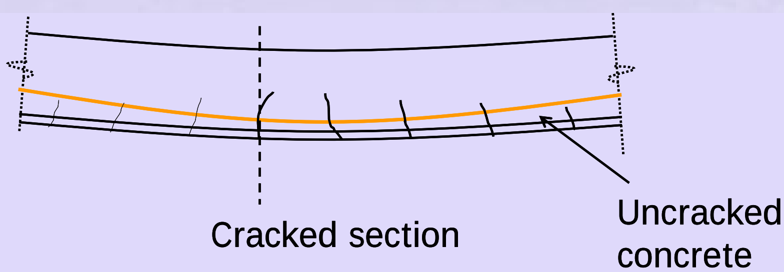

Figure 6-2.3 Elevation of a beam showing the cracking and uncracked concrete

The analysis of a Type 3 member should be based on strain compatibility of concrete and prestressing steel. **IS:456 - 2000** recommends two procedures for the sectional analysis considering the tension stiffening effect.

1) **Rigorous procedure** with explicit calculation of tension carried by the concrete.
2) **Simplified procedure** based on the conventional analysis of a cracked section, neglecting the tension carried by concrete. An approximate estimate of the tension carried by the concrete is subsequently introduced.

Here, the simplified procedure is explained. For a rectangular zone under tension, the simplified procedure gives the following expression of ε_m.

$$\varepsilon_m = \varepsilon_1 - \frac{b(h-x)(a-x)}{3E_sA_s(d-x)} \qquad \textbf{(6-2.3)}$$

For a prestressed member, (E_pA_p + E_sA_s) is substituted in place of E_sA_s. The second term considers the tension carried by the concrete approximately by reducing the strain (ε_1) obtained from the analysis of a cracked section.

In the above expression,

a = distance from the compression face to the level at which crack width is calculated
= h, when the crack width is calculated at the soffit
b = width of the rectangular zone
d = effective depth of the longitudinal reinforcement
A_s = area of non-prestressed reinforcement
A_p = area of prestressing steel.
E_s = modulus of elasticity of non-prestressed steel
E_p = modulus of elasticity of prestressed steel
ε_1 = strain at the selected level based on a cracked sectional analysis
= $\varepsilon_s(a - x)/(d - x)$
ε_s = strain in the longitudinal reinforcement.

The depth of neutral axis (x) can be calculated by a trial and error procedure till the equilibrium equations are satisfied. The following sketch shows the beam cross section, strain profile, stress diagram and force couples under service loads. The contribution of non-prestressed reinforcement is also included.

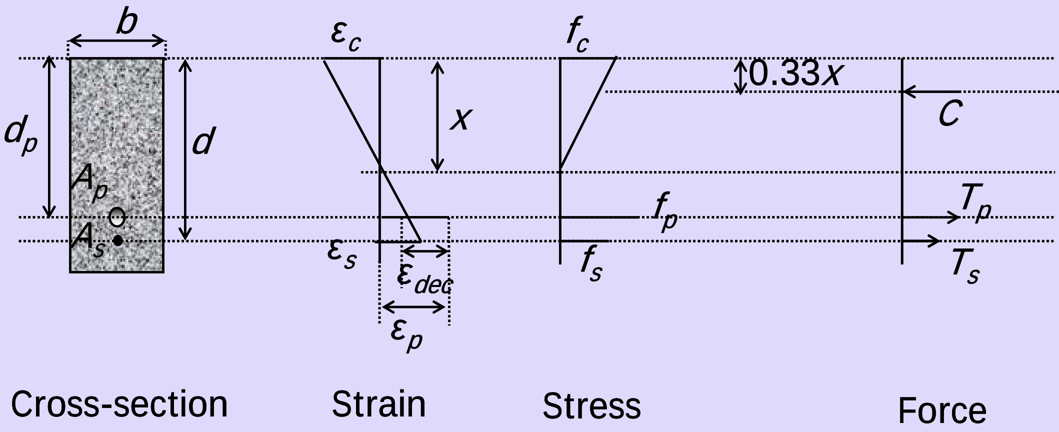

Figure 6-2.4 Sketches for analysis of a rectangular section

The expressions of the forces are as follows.

$$C = 0.5E_c\varepsilon_c xb \qquad \textbf{(6-2.4)}$$

$$T_p = A_pE_p\varepsilon_p \qquad \textbf{(6-2.5)}$$

$$T_s = A_sE_s\varepsilon_s \qquad \textbf{(6-2.6)}$$

Based on the principles of mechanics, the following equations are derived.

1) Equations of equilibrium

The first equation states that the resultant axial force is zero. This means that the compression and the tension in the force couple balance each other.

$$\begin{aligned} &\sum F = 0 \\ &\Rightarrow T_p + T_s = C \\ &\Rightarrow A_pE_p\varepsilon_p + A_sE_s\varepsilon_s = 0.5E_c\varepsilon_c xb \end{aligned} \qquad \textbf{(6-2.7)}$$

The second equation relates the moment under service loads (M) with the internal couple in the force diagram.

$$\begin{aligned} M|_{A_p} &= T_s\left(d - d_p\right) + C\left(d_p - 0.33x\right) \\ &= A_sE_s\varepsilon_s\left(d - d_p\right) + 0.5E_c\varepsilon_c xb\left(d_p - 0.33x\right) \end{aligned} \qquad \textbf{(6-2.8)}$$

The value of M should be equal to the moment due to service loads.

2) Equations of compatibility

The depth of the neutral axis is related to the depth of CGS and the depth of non-prestressed reinforcement by the similarity of the triangles in the strain diagram.

$$\frac{x}{d_p} = \frac{\varepsilon_c}{\varepsilon_c + \varepsilon_p - \varepsilon_{dec}} \quad \text{(6-2.9)}$$

$$\frac{x}{d} = \frac{\varepsilon_c}{\varepsilon_c + \varepsilon_s} \quad \text{(6-2.10)}$$

3) Constitutive relationships

Linear elastic constitutive relationships are used in the earlier expressions of C, T_s and T_p.

The known variables in the analysis are: b, d, A_p, A_s, ε_{dec}, E_c, E_p, E_s, M.

The unknown quantities are: x, ε_c, ε_p, ε_s.

The steps for solving the above equations are given below.

1) Assume ε_c
2) Assume x.
3) Calculate ε_p and ε_s from Eqn. **(6-2.9)** and Eqn. **(6-2.10)**, respectively.
4) Calculate C, T_p and T_s from Eqns. **(6-2.4)**, **(6-2.5)**, **(6-2.6)**, respectively.
5) If Eqn. **(6-2.7)** is not satisfied, change x. If $T_p + T_s < C$, decrease x. If $T_p + T_s > C$, increase x.
6) Calculate M from Eqn. **(6-2.8)**. If the value differs from the given value, change ε_c and repeat from Step 2.

6.2.3 Limits of Crack Width

Clause 19.3.2 of **IS:1343 - 1980** specifies limits of crack width such that the appearance and durability of the structural element are not affected.

The limits of crack width are as follows.

Crack width $\leq$ 0.2 mm for moderate and mild environments

$\leq$ 0.1 mm for severe environment.

The types of environments are explained in **Table 9**, **Appendix A** of **IS:1343 - 1980**.