9.4 Two-way Slabs (Part II)

This section covers the following topics.

- Checking for Shear Capacity
- Spandrel Beams
- Anchorage Devices
- Additional Aspects

9.4.1 Checking for Shear Capacity

The checking of shear capacity of flat plates and flat slabs is of utmost importance. In absence of beams, the shear is resisted by the slab near the slab-column junction.

The shear capacity of a slab should be adequate to resist the shear from two actions.

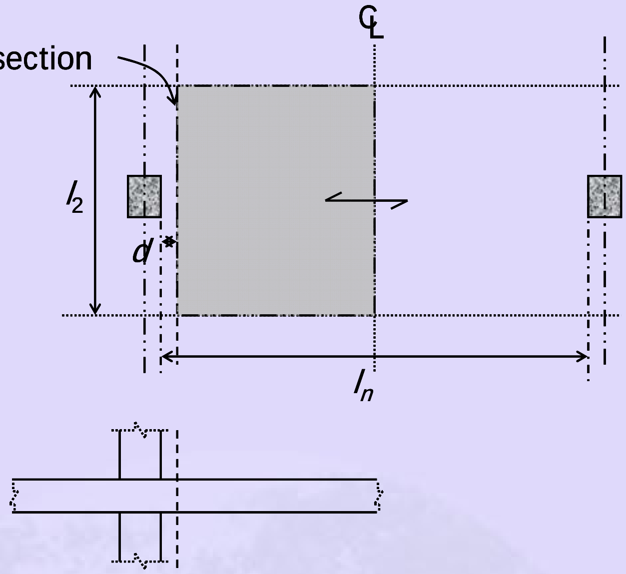
1) One-way (beam) shear
2) Two-way (punching) shear.

One-way shear

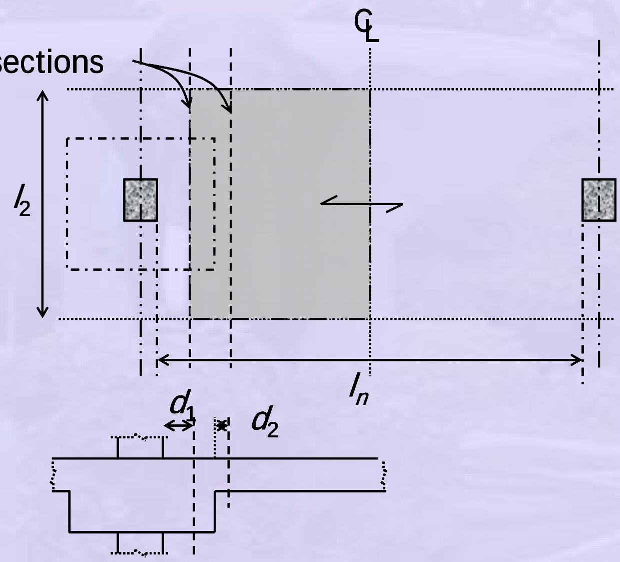
The one-way shear is analogous to that generates in a beam due to flexure. This is checked in a two-way slab for each spanning direction separately.

The critical section for checking the shear capacity is at a distance effective depth '*d*' from the face of the column, across the entire width of the frame. The critical section is transverse to the spanning direction. For gravity loads, the shear demand in the critical section generates from the loads in the tributary area shown in the next figure. For lateral loads, the shear demand is calculated from the analysis of the equivalent frame.

In presence of a drop panel two critical sections need to be checked. The first section is at a distance d_1 from the face of the column, where d_1 is the effective depth of the drop panel. The second section is at a distance d_2 from the face of the drop panel, where d_2 is the effective depth of the slab.



(a) Slab without drop panel



(b) Slab with drop panel

Figure 9-4.1 Critical sections for one-way shear

The calculations for shear can be for unit width of the slab. The shear demand due to gravity loads per unit width is given as follows.

$$V_u = w_u\,(0.5l_n - d) \tag{9-4.1}$$

Here, l_n is the clear span along the spanning direction.

The shear capacity per unit width is given as follows.

$$V_{uR} = V_c \tag{9-4.2}$$

V_c is the shear capacity of uncracked concrete of unit width of slab. The expression of V_c is given in Section 5.2, Design for Shear (Part I).

For adequate shear capacity

$$V_{uR} \geq V_u \qquad \textbf{(9-4.3)}$$

If this is not satisfied, it is preferred to increase the depth of the slab to avoid shear reinforcement along the width of the slab.

Two-way shear

The two-way shear is specific to two-way slabs. If the capacity is inadequate, the slab may fail due to punching around a column. The punching occurs along a conical frustum, whose base is geometrically similar and concentric to the column cross-section. The following figure illustrates the punching shear failure.

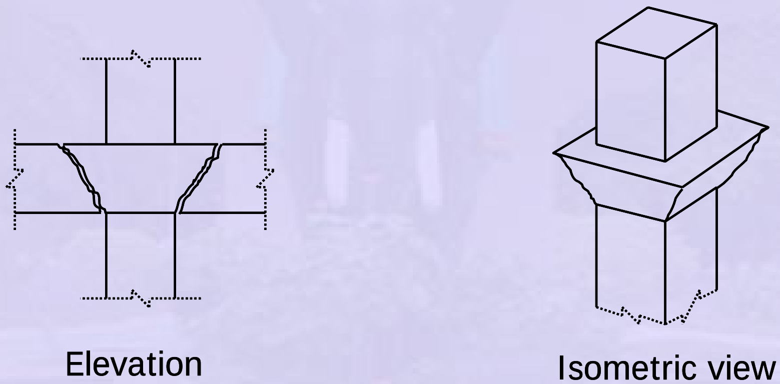
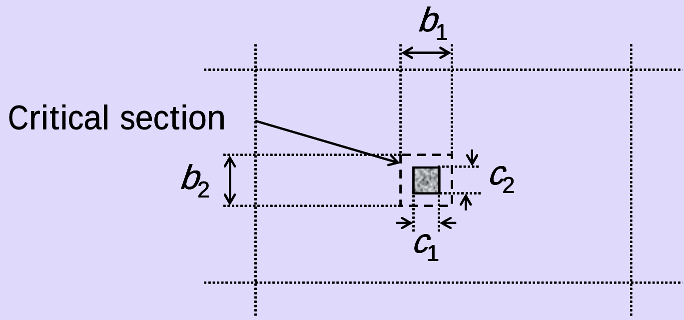
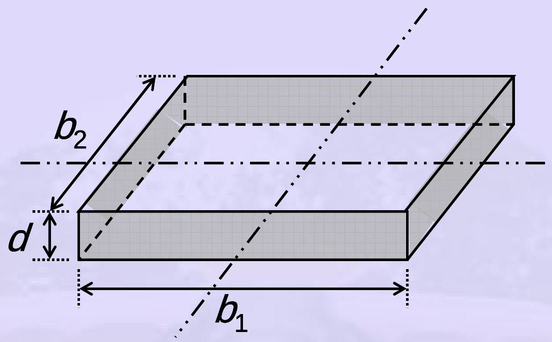


Figure 9-4.2 Punching shear failure

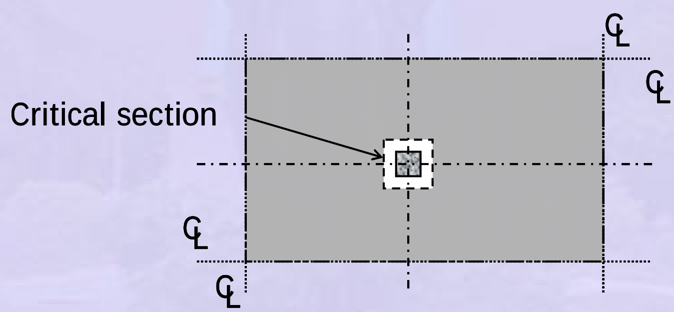
Two-way shear is checked for the two spanning directions simultaneously. The critical section for checking the shear capacity is geometrically similar to the column perimeter and is at a distance $d / 2$ from the face of the column. The depth of the critical section is equal to the average of the effective depths of the slab in the two directions. The sketches below show the critical section. The tributary area of the column is the area within the centre-lines of the spans minus the area within the critical section. It is shown shaded in the third sketch.



(a) Plan



(b) Isometric view



(c) Tributary area for shear due to gravity loads

Figure 9-4.3 Critical section for two-way shear

The lengths of the sides of the critical section along axes 1-1 and 2-2 are denoted as b_1 and b_2, respectively.

$$b_1 = c_1 + d \qquad \textbf{(9-4.4)}$$

$$b_2 = c_2 + d \qquad \textbf{(9-4.5)}$$

Here,

c_1 = dimension of the column or column capital along axis 1-1

c_2 = dimension of the column or column capital along axis 2-2.

For a non-rectangular column, the critical section consists of the slab edges as per **Figure 13, IS:456 - 2000**. For edge and corner columns, the critical section consists of the slab edges as per **Figure 14, IS:456 - 2000.**

The demand in terms of shear stress is given as follows.

$$\tau_v = \frac{V_u}{b_0 d} + \frac{M_{uv}|_{2\text{-}2}\left[{b_1}/{2}\right]}{J|_{2\text{-}2}} + \frac{M_{uv}|_{1\text{-}1}\left[{b_2}/{2}\right]}{J|_{1\text{-}1}} \qquad \textbf{(9-4.6)}$$

Here,

V_u = shear due to gravity loads from the tributary area

M_{uv} = fraction of moment transferred about an axis

b_0 = perimeter of the critical section = $2(b_1 + b_2)$.

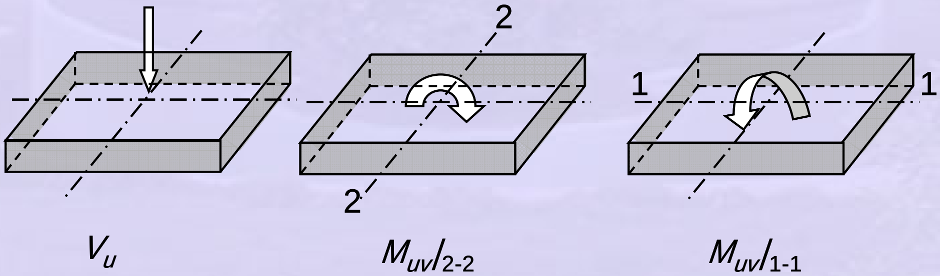
J = polar moment of inertia of the critical section about an axis

The second and third terms are due to transfer of moments from slab to column. The moment about an axis is due to the unbalanced gravity loads for the two sides of the column or due to lateral loads. It is transferred partly by the variation of shear stress in the critical section and the rest by flexure. The fraction transferred by the variation of shear stress about an axis is denoted as M_{uv}.

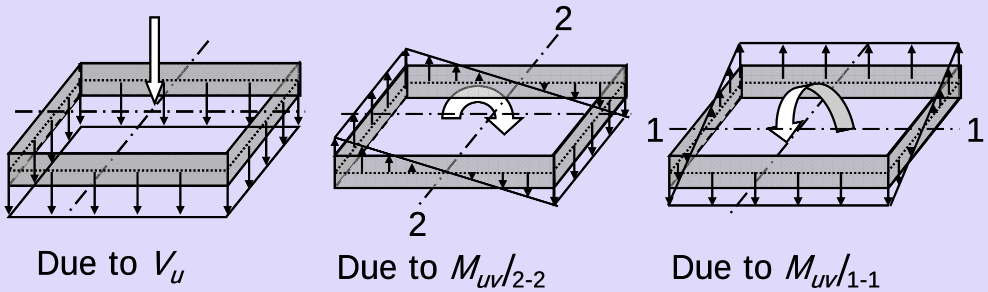
$M_{uv|2\text{-}2}$ = Fraction of moment transferred about axis 2-2

$M_{uv|1\text{-}1}$ = Fraction of moment transferred about axis 1-1

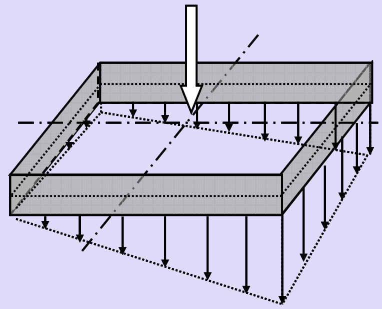
The forces and stresses acting at the critical section are shown below.



(a) Shear and moments acting at the critical section



(b) Stresses acting at the critical section



Due to $V_u + M_{uv}|_{2\text{-}2} + M_{uv}|_{1\text{-}1}$

(c) Resultant shear stress diagram at the critical section

Figure 9-4.4 Forces and stresses at the critical section

The fraction of moment transferred by the variation of shear stress about an axis (M_{uv}), is given in terms of the total moment transferred (M_u) as follows.

$$M_{uv} = (1-\alpha)M_u \tag{9-4.7}$$

The value of M_u due to unbalanced gravity loads is calculated by placing live load on one side of the column only. The value of M_u due to lateral loads is available from the analysis of the equivalent frame. The parameter α is based on the aspect ratio of the critical section.

$$\alpha = \frac{1}{1+\dfrac{2}{3}\sqrt{\dfrac{b_1}{b_2}}} \tag{9-4.8}$$

The polar moments of inertia of the critical section, about the axes are given as follows.

$$J|_{1\text{-}1} = 2\left[\frac{1}{12}b_2d^3 + \frac{1}{12}db_2^{\,3} + b_1d\left(\frac{b_2}{2}\right)^2\right] \tag{9-4.9}$$

$$J|_{2\text{-}2} = 2\left[\frac{1}{12}b_1d^3 + \frac{1}{12}db_1^{\,3} + b_2d\left(\frac{b_1}{2}\right)^2\right] \tag{9-4.10}$$

For adequate shear capacity

$$\tau_v \le k_s\tau_c \tag{9-4.11}$$

The shear stress capacity of concrete for a square column is given as follows.

$$\tau_c = 0.25\sqrt{f_{ck}} \tag{9-4.12}$$

Here, f_{ck} is the characteristic strength of the concrete in the slab. The effect of prestress is neglected. The factor k_s accounts for the reduced shear capacity of non-square columns.

$$k_s = 0.5 + \beta_c \tag{9-4.13}$$

The value of k_s should be less than 1.0. β_c is a parameter based on the aspect ratio of the column cross-section. It is the ratio of the short side to long side of the column or column capital.

If τ_v exceeds $k_s\tau_c$, a drop panel or shear reinforcement needs to be provided at the slab-to-column junction. The shear reinforcement can be in the form of stirrups or I section (shear head) or based on shear studs. The reinforcement based on shear studs reduces congestion for conduits and post-tensioning tendons. If τ_v exceeds $1.5\tau_c$, then the depth of the slab needs to be increased in the form of drop panels.

The stirrups are designed based on the following equation. A_{sv} is the area of the vertical legs of stirrups.

$$A_{sv} = \frac{(\tau_v - 0.5\tau_c)}{0.87f_y} \tag{9-4.14}$$

The stirrups are provided along the perimeter of the critical section. The first row of stirrups should be within a distance of $0.5d$ from the face of the column. They can be continued in outer rows (concentric and geometrically similar to the critical section) at an interval of $0.75d$, till the section with shear stress $\tau_v = 0.5\tau$.

The different types of reinforcement at the slab-to-column junction are shown in the following sketches.

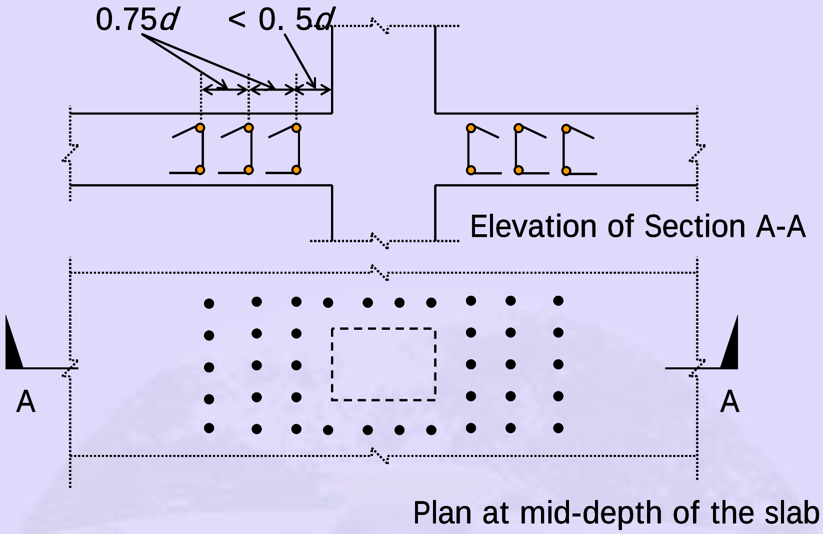
References:

1. Bureau of Indian Standards,
Handbook on Concrete Reinforcement and Detailing
(SP 34 : 1987)

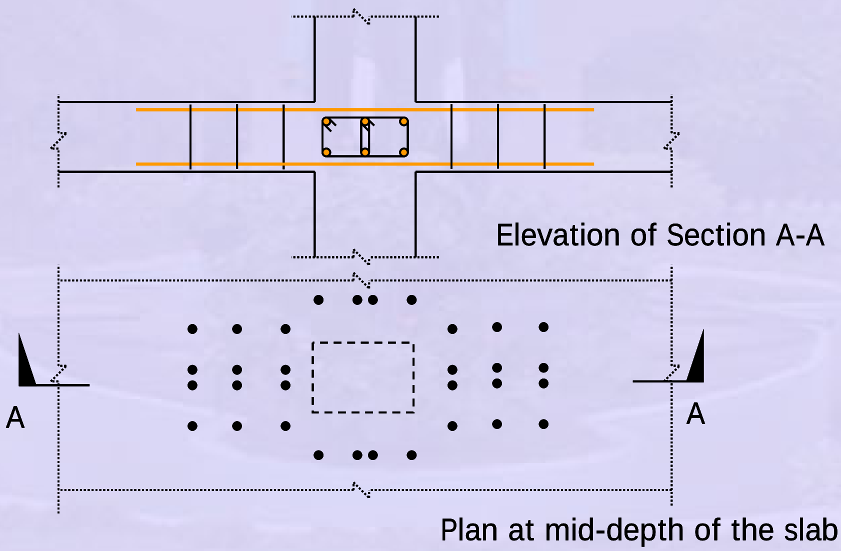
2. Khan, S. and Williams, M.

Post-tensioned Concrete Floors

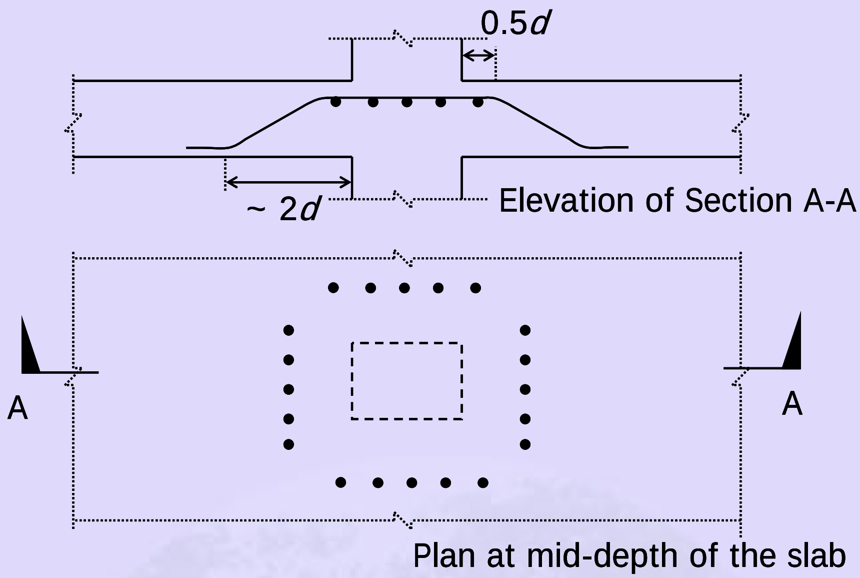
Butterworth-Heinemann Ltd.



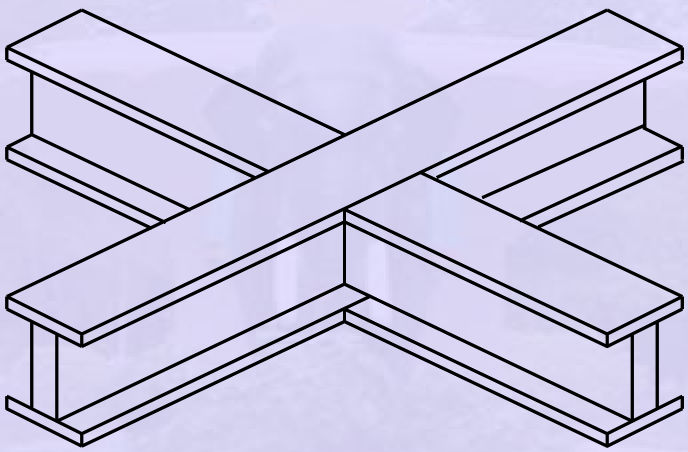
(a) Stirrups in geometrically similar rows



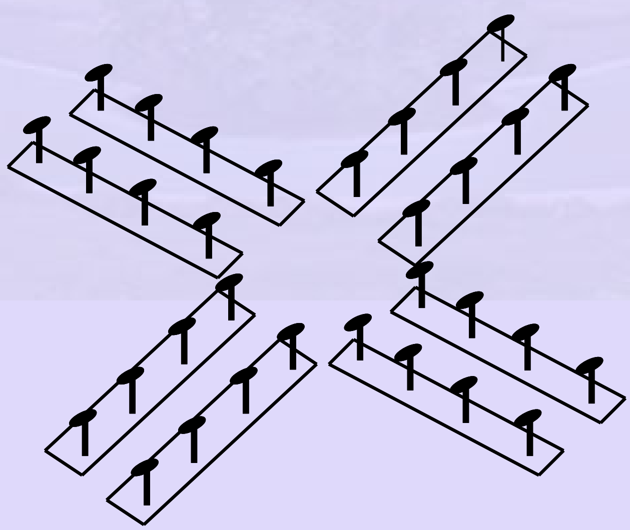
(b) Beam cage stirrups



(c) Beam cage stirrups with bent-up bars



(d) Shear head reinforcement



(e) Shear stud reinforcement

Figure 9-4.5 Shear reinforcement at slab-column junction

The following photo shows the ducts and reinforcement at the slab-column junction in a slab with a drop panel.

Figure 9-3.6 Reinforcement at slab-column junction
(Courtesy: VSL India Pvt. Ltd.)

The residual moment transferred by flexure (M_{uf}), is given in terms of the total moment transferred (M_u) as follows.

$$M_{uf} = \alpha M_u \quad \textbf{(9-4.15)}$$

Additional non-prestressed reinforcement is provided at the top of the slab over a width $c_2 + 3h$ (centred with respect to the column) to transfer M_{uf}.

9.4.2 Spandrel Beams

The flat plates are provided with spandrel beams at the edges. These beams stiffen the edges against rotation. In turn the beams are subjected to torsion.

The maximum torsion is calculated by assuming a uniform torsional loading along the width of the equivalent frame (**ACI 318-02** recommends a triangular distribution). The spandrel beams are provided with closed stirrups to resist the torsion. The design for torsion is given in the Module of "Analysis and Design for Shear and Torsion".

The following figure shows the distribution of the torsional loading on the spandrel beam.

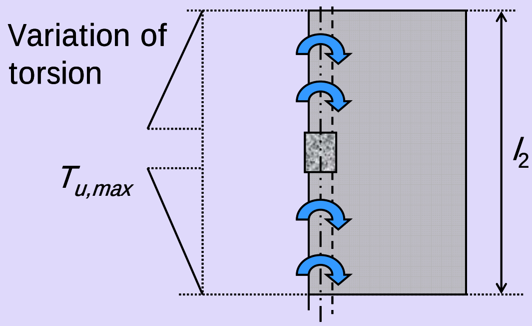


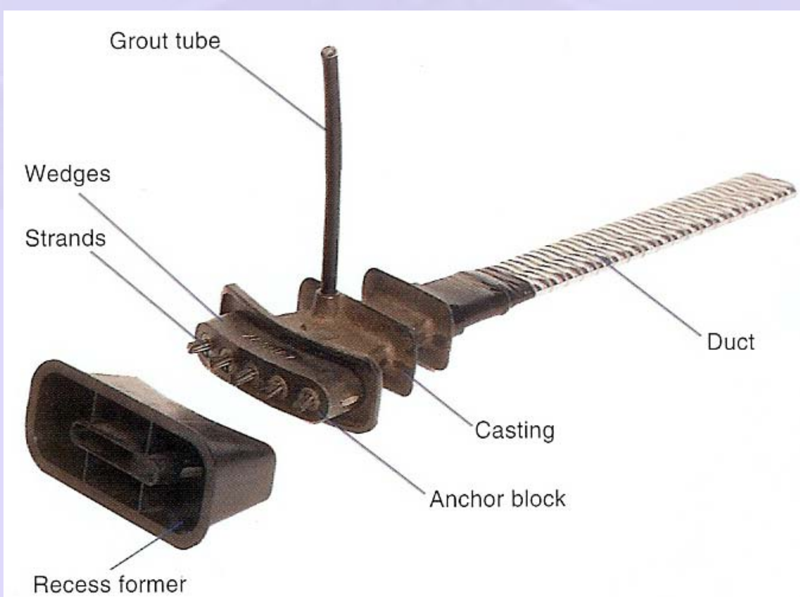
Figure 9-4.7 Torsion in spandrel beams

The maximum torsion ($T_{u,max}$) is given as follows. Here, $M_{u,e}^{-}$ is the moment at the exterior support of the equivalent frame.

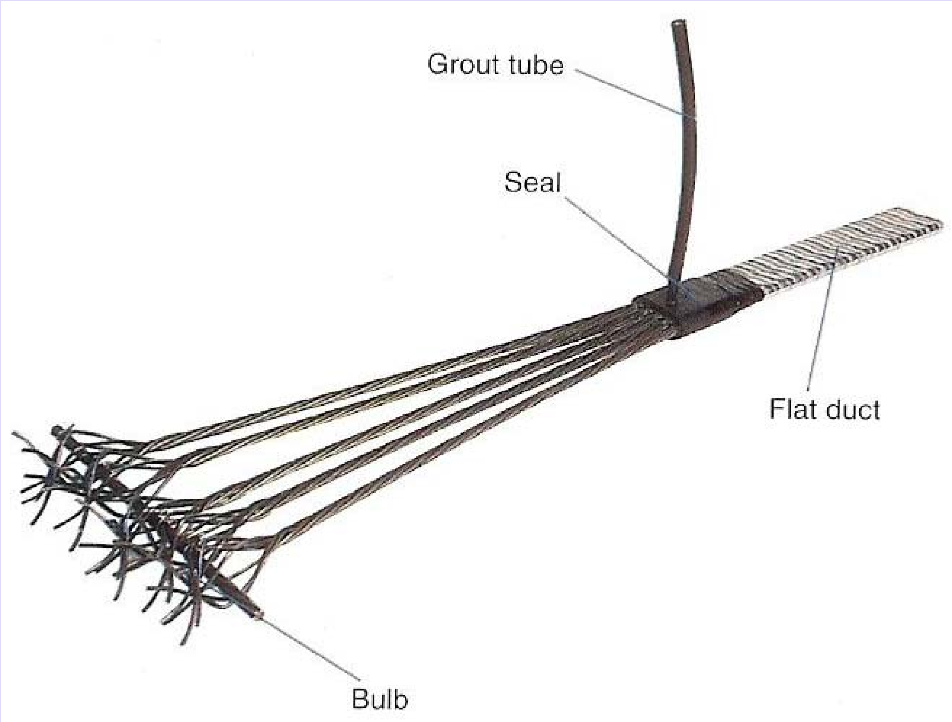
$$\tau_{u,max} = \frac{l_2 - c_2}{2}\frac{M^{-}_{u,e}}{l_2} \quad \textbf{(9-4.16)}$$

9.4.3 Anchorage Devices

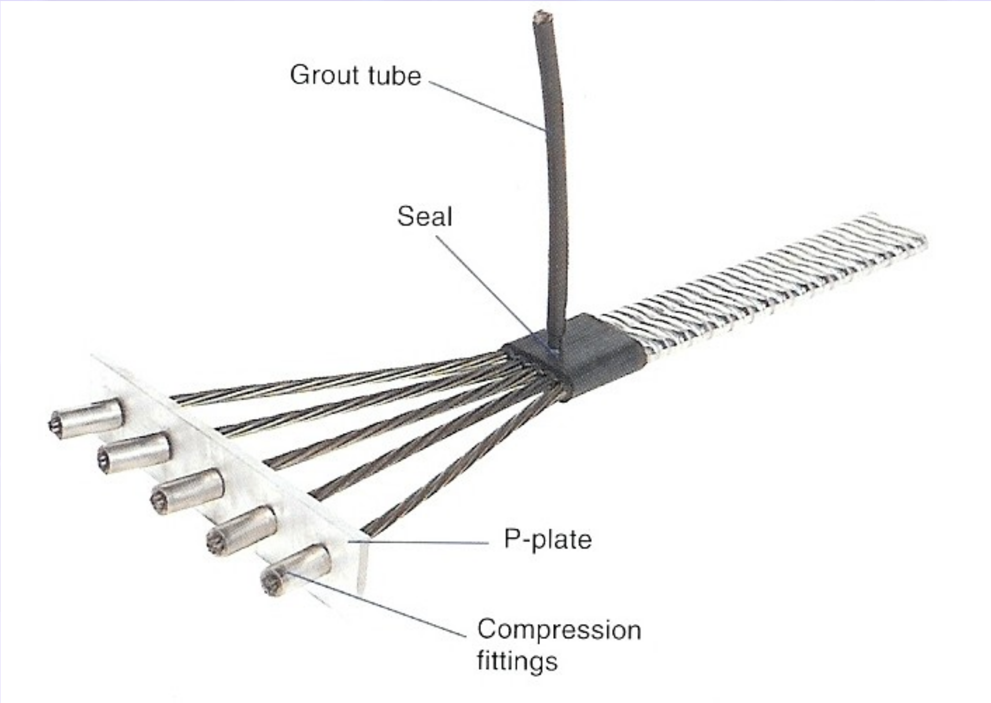
In post-tensioned slabs, the anchorage devices transfer the prestress to the concrete. The device at the stretching end consists of an anchor block and wedges. At the dead end, the wires are looped to provide the anchorage. Bursting links are provided in the end zone to resist transverse tensile stresses in concrete. The following photos show some anchorage devices.



(a) Anchorage device at stretching end



(b) Anchorage device for dead end with bulb

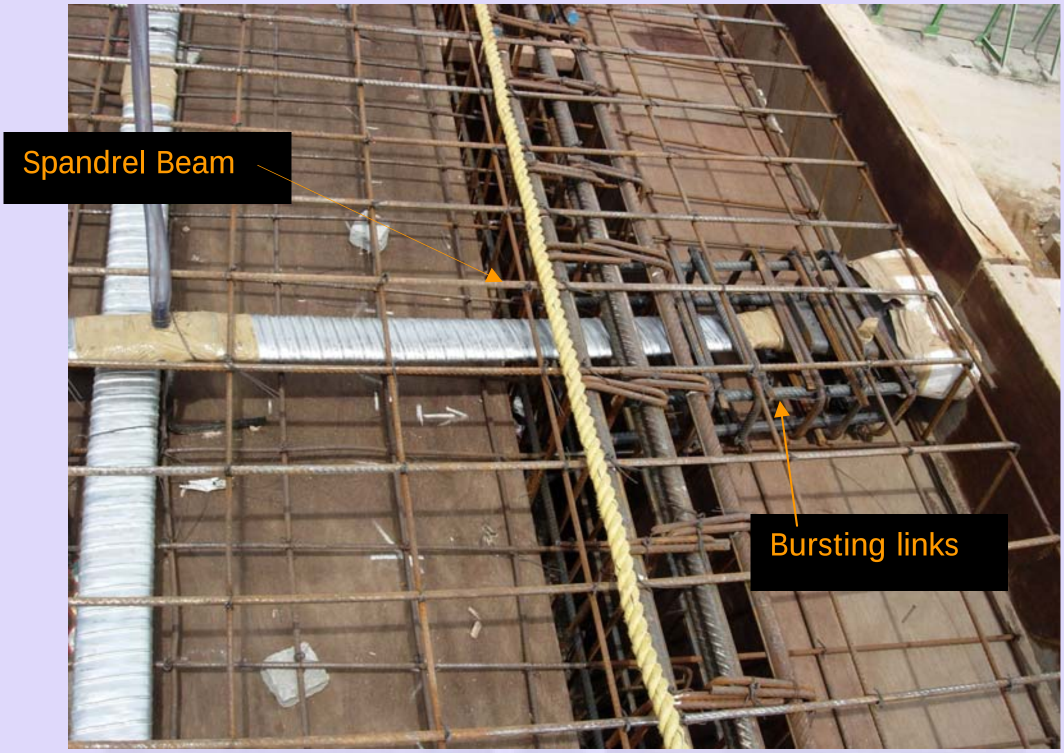


(c) Anchorage device for dead end with plate

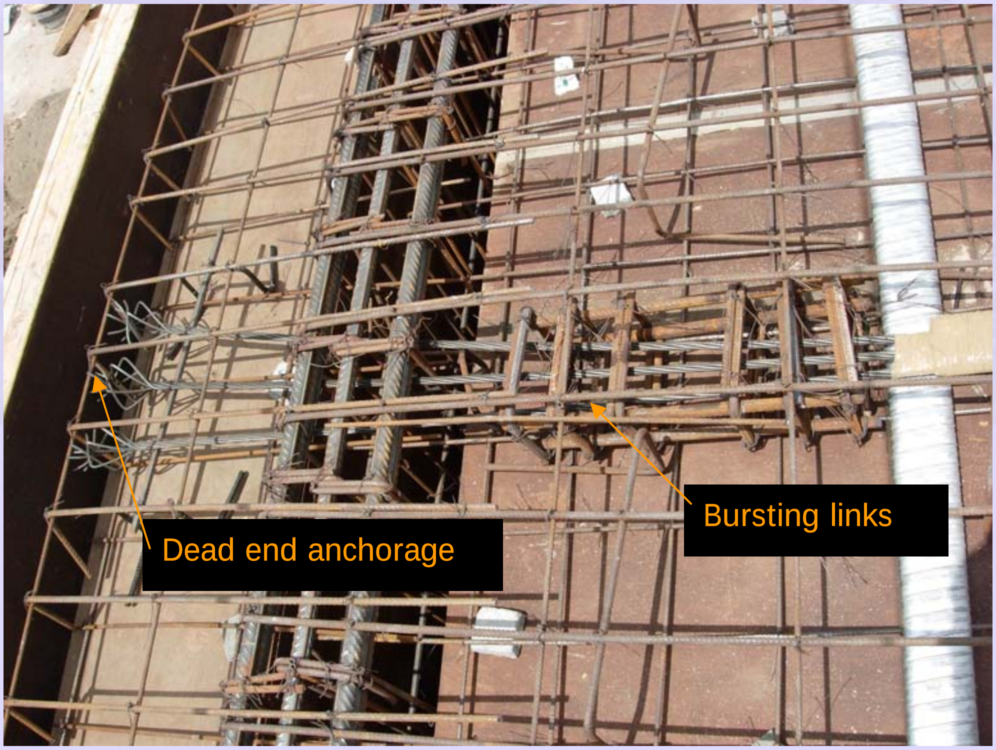
Figure 9-4.8 Anchorage devices for slabs

(Reference: VSL International Ltd.)

The following photos show the anchorage devices, end zone reinforcement, spandrel beam before casting and stretching and anchoring of the tendons after casting of concrete in a slab.



(a) End-zone reinforcement at stretching end



(b) End-zone reinforcement and anchorage at dead end

(c) Stretching of tendons

(d) Anchorage block at stretching end

Figure 9-4.9 End-zone reinforcement and anchoring of tendons in a slab

(Reference: VSL India Pvt. Ltd.)

9.4.4 Additional Aspects

Restraint from vertical elements

Due to the restraint from monolithic columns or walls, the prestressing force in the slab is reduced. Hence, the stiff columns or walls should be located in such a manner that they offer least restraint. Alternatively, sliding joints can be introduced which are made ineffective after post-tensioning of the slab.

Calculation of deflection

The deflection of a two-way slab can be approximately calculated by the equivalent frame method. The deflection at a point is the summation of the deflections of the two orthogonal strips passing through the point.

For an accurate evaluation, the following models can be adopted.

a) Grillage model
b) Finite element model.

Proportioning of drop panels and column capitals

Section 31 of **IS:456 - 2000** provide guidelines for proportioning drop panels and column capitals. A minimum length and a minimum depth (beyond the depth of the slab) of a drop panel are specified. For column capitals it is preferred to have a conical flaring at a subtended angle of 90°. The critical sections are shown in **Figure 12** of the code.