

9.5 Compression Members

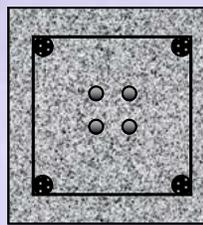
This section covers the following topics.

- Introduction
- Analysis
- Development of Interaction Diagram
- Effect of Prestressing Force

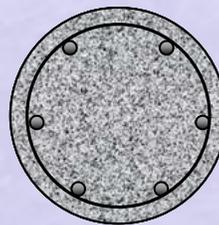
9.5.1 Introduction

Prestressing is meaningful when the concrete in a member is in tension due to the external loads. Hence, for a member subjected to compression with minor bending, prestressing is not necessary. But, when a member is subjected to compression with substantial moment under high lateral loads, prestressing is applied to counteract the tensile stresses. Examples of such members are piles, towers and exterior columns of framed structures.

As the seismic forces are reversible in nature, the prestressing of piles or columns is concentric with the cross-section. Some typical cross sections are shown below.



Partially prestressed column



Prestressed circular and hexagonal piles

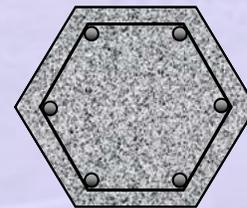


Figure 9-5.1 Examples of prestressed members subjected to compression



Figure 9-5.2 Stacked prestressed piles
(Reference: Industrial Concrete Products Berhad)

Since a prestressed member is under self equilibrium, there is no buckling of the member due to internal prestressing with bonded tendons. In a deflected shape, there is no internal moment due to prestressing.

The justification is explained in the next figure.

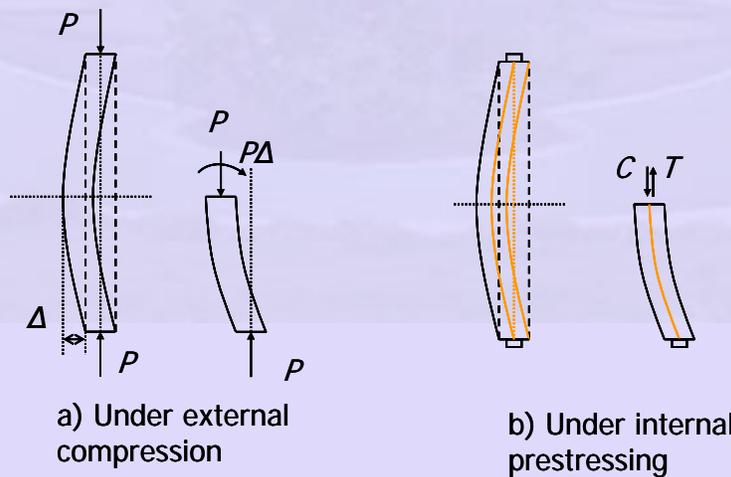


Figure 9-5.3 Internal forces at deflected configuration

In the first free body sketch of the above figure, the external compression P causes an additional moment due to the deflection of the member. The value of the moment at mid-height is $P\Delta$. This is known as the member stability effect, which is one type of $P-\Delta$

effect. If this deflection is not stable, then buckling of the member occurs. In the second free body sketch, there is no moment due to the deflection of the member and the prestressing force, since the compression in concrete (C) and the tension in the tendons (T) balance each other.

When the additional moment due to deflection of the member is negligible, the member is termed as short member. The additional moment needs to be considered when the slenderness ratio (ratio of effective length and a lateral dimension) of the member is high. The member is termed as slender member. In the analysis of a slender member, the additional moment is calculated by an approximate expression or second order analysis. In this module only short members will be considered.

9.5.2 Analysis

Analysis at Transfer

The stress in the section can be calculated as follows.

$$f_c = \frac{P_0}{A} \quad (9-5.1)$$

Here,

A = Area of concrete

P_0 = prestress at transfer after short-term losses.

In this equation, it is assumed that the prestressing force is concentric with the cross-section. For members under compression, a compressive stress is considered to be positive. The permissible prestress and the cross-section area are determined based on the stress to be within the allowable stress at transfer ($f_{cc,all}$).

Analysis at Service Loads

The analysis is analogous to members under flexure. The stresses in the extreme fibres can be calculated as follows.

$$f_c = \frac{P_e}{A} + \frac{N}{A_t} \pm \frac{Mc}{I_t} \quad (9-5.2)$$

In this equation, the external compression for a prestressed member is denoted as N and is concentric with the cross section. The eccentricity is considered in the external moment M .

In the previous equation,

A = area of concrete

A_t = area of the transformed section

c = distance of the extreme fibre from the centroid (CGC)

I_t = moment of inertia of the transformed section

P_e = effective prestress.

The value of f_c should be within the allowable stress under service conditions ($f_{cc,all}$).

Analysis at Ultimate

When the average prestress in a member under axial compression and moment is less than 2.5 N/mm^2 , **Clause 22.2, IS:1343 - 1980**, recommends to analyse the member as a reinforced concrete member, neglecting the effect of prestress. For higher prestress, the analysis of strength is done by the interaction diagrams.

At the ultimate limit state, an interaction diagram relates the axial force capacity (N_{UR}) and the moment capacity (M_{UR}). It represents a failure envelop. Any combination of factored external loads N_u and M_u that fall within the interaction diagram is safe. A typical interaction diagram is shown below. The area shaded inside gives combinations of M_u and N_u that are safe.

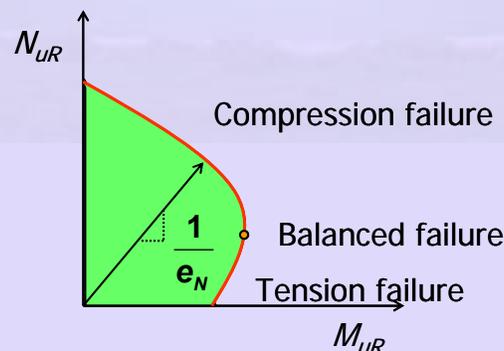


Figure 9-5.4 A typical interaction diagram for compression and bending

The radial line in the previous sketch represents the load path. Usually the external loads increase proportionally. At any load stage, M and N are related as follows.

$$M = N e_N \quad (9-5.3)$$

Here, e_N represents the eccentricity of N which generates the same moment M . The slope of the radial line represents the inverse of the eccentricity ($1/e_N$). At ultimate, the values of M and N (M_u and N_u , respectively) correspond to the values on the interaction diagram. For high values of N as compared to M , that is e_N is small, the concrete in the compression fibre will crush before the steel on the other side yields in tension. This is called the **compression failure**.

For high values of M as compared to N , that is e_N is large, the concrete will crush after the steel yields in tension. This is called the **tension failure**.

The transition of these two cases is referred to as the **balanced failure**, when the crushing of concrete and yielding of steel occur simultaneously. For a prestressed compression member, since the prestressing steel does not have a definite yield point, there is no explicit balanced failure.

9.5.3 Development of Interaction Diagram

An interaction diagram can be developed from the first principles using the non-linear stress-strain curves of concrete under compression and steel under tension. Several sets of N_{uR} and M_{uR} for given values of e_N or x_u are calculated. The distance of neutral axis from the extreme compressive face is denoted as x_u . Partial safety factors for concrete and prestressing steel can be introduced when the interaction diagram is used for design. Here, the procedure is illustrated for a rectangular section with prestressed tendons placed at two opposite faces symmetrically, and without non-prestressed reinforcement.

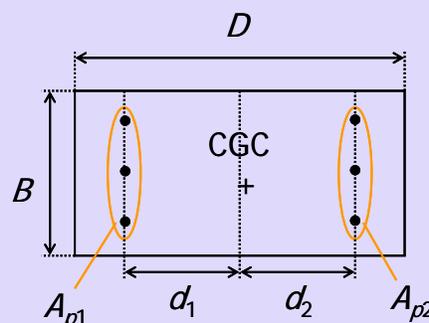


Figure 9-5.5 A rectangular prestressed section

The notations used are as follows.

B = dimension of section transverse to bending

D = dimension of section in the direction of bending

A_{p1} = area of prestressing tendons at the tension face

A_{p2} = area of prestressing tendons at the compression face

d_1, d_2 = distances of centres of A_{p1} and A_{p2} , respectively, from the centroid of the section (CGC).

The strain compatibility equation is necessary to relate the strain in a prestressing tendon with that of the adjacent concrete. Due to a concentric prestress, the concrete at a section undergoes a uniform compressive strain. With time, the strain increases due to the effects of creep and shrinkage. At service, after the long term losses, let the strain be ϵ_{ce} . Also, let the strain in the prestressing steel due to effective prestress be ϵ_{pe} .

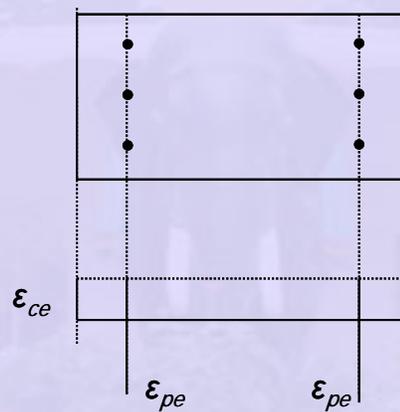


Figure 9-5.6 Strain profile due to effective prestress only

The strain compatibility equation for the prestressed tendons is given below.

$$\epsilon_p = \epsilon_c + \Delta\epsilon_p \quad (9-5.4)$$

$$\text{where, } \Delta\epsilon_p = \epsilon_{pe} - \epsilon_{ce}$$

The strain difference of the strain in a prestressing tendon with that of the adjacent concrete is denoted as $\Delta\epsilon_p$. The design stress-strain curve for concrete under compression is used. This curve is described in Section 1.6, Concrete (Part II). The design stress-strain curve for the prestressed tendon under tension is expressed as $f_p = F(\epsilon_p)$.

The calculation of N_{UR} and M_{UR} for typical cases of e_N or x_U are illustrated. The typical cases are as follows.

- 1) Pure compression ($e_N = 0, x_U = \infty$)

- 2) Full section under varying compression ($0.05D < e_N \leq e_N \mid_{x_u=D}, x_u \geq D$)
- 3) Part of section under tension ($e_N \mid_{x_u=D} < e_N \leq \infty, x_u < D$)
- 4) Pure bending ($e_N = \infty, x_u = x_{u,min}$)

The above cases are illustrated in the following sketches.

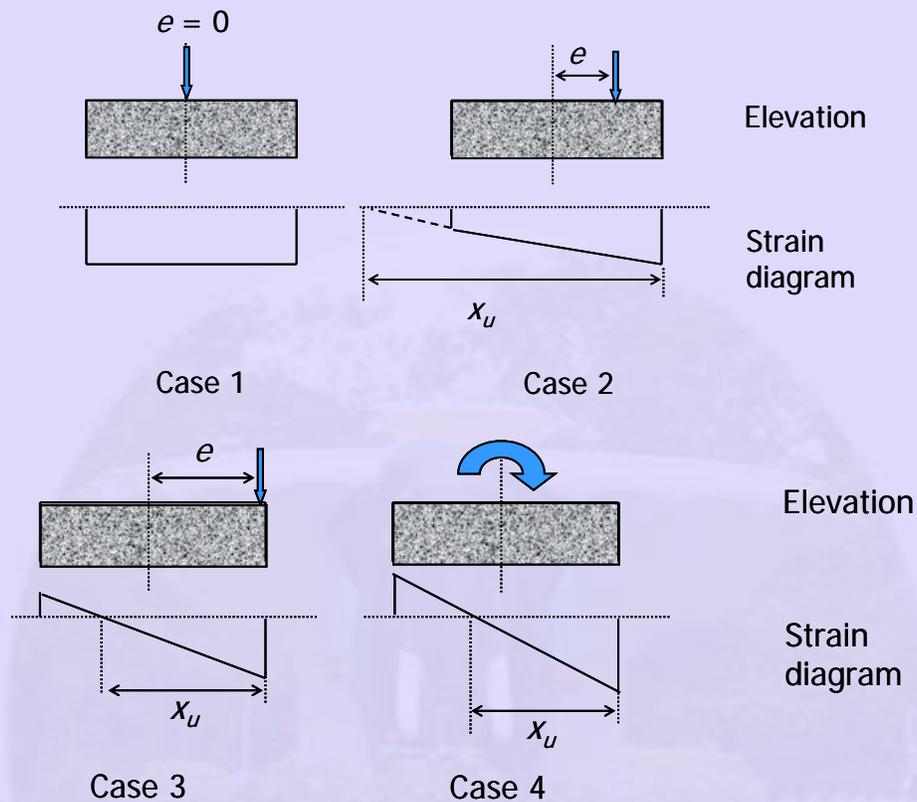


Figure 9-5.7 Typical cases of eccentricity and depth of neutral axis

In addition to the above cases, the case of pure axial tension is also calculated. The straight line between the points of pure bending and pure axial tension provides the interaction between the tensile force capacity and the moment capacity.

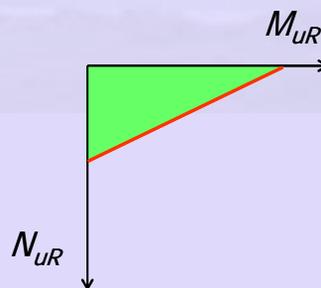


Figure 9-5.8 A typical interaction diagram for tension and bending

1. Pure compression ($e_N = 0, x_u = \infty$).

The following sketches represent the strain and stress profiles across the section and the force diagram.

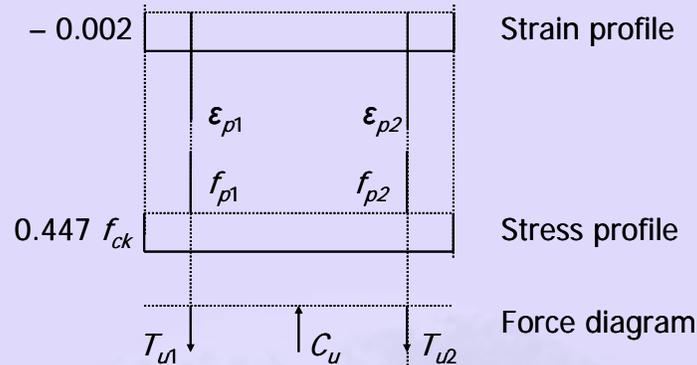


Figure 9-5.9 Sketches for analysis at pure compression

The forces are as follows.

$$C_u = 0.447f_{ck} (A_g - A_p) \quad (9-5.5)$$

$$\begin{aligned} T_{u1} &= T_{u2} = A_{p1} f_{p1} \\ &= A_{p1} E_p (-0.002 + \Delta\epsilon_p) \end{aligned} \quad (9-5.6)$$

The steel is in the elastic range. The total area of prestressing steel is $A_p = A_{p1} + A_{p2}$. The area of the gross-section $A_g = BD$. The moment and axial force capacities are as follows.

$$M_{uR} = 0 \quad (9-5.7)$$

$$\begin{aligned} N_{uR} &= C_u - T_{u1} - T_{u2} \\ &= 0.447f_{ck} (A_g - A_p) - A_p E_p (\epsilon_{pe} - 0.002 - \epsilon_{ce}) \end{aligned} \quad (9-5.8)$$

In design, for simplification the interaction diagram is not used for eccentricities $e_N \leq 0.05D$. To approximate the effect of the corresponding moment, the axial force capacity is reduced by 10%.

$$\therefore N_{uR} = 0.4f_{ck} (A_g - A_p) - 0.9A_p E_p (\epsilon_{pe} - 0.002 + \epsilon_{ce}) \quad (9-5.9)$$

2. Full section under varying compression ($0.05D < e_N \leq e_N |_{x_u = D}, x_u \geq D$)

The following sketches represent the strain and stress profiles across the section and the force diagram.

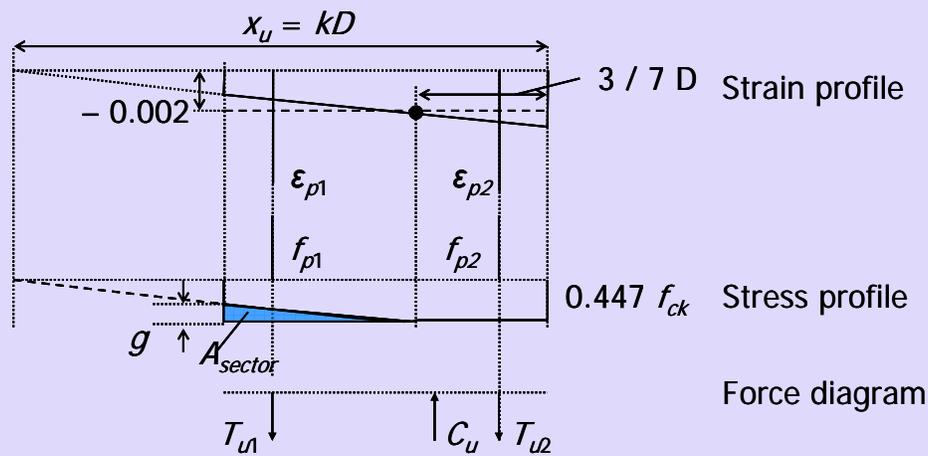


Figure 9-5.10 Sketches for analysis for section under varying compression

The limiting case for full section under compression corresponds to $x_u = D$, when the neutral axis lies at the left edge of the section. The strain diagram pivots about a value of -0.002 at $3/7D$ from the extreme compression face. To calculate C_u , first the reduction of the stress at the edge with lower compression (g) is evaluated. Based on the second order parabolic curve for concrete under compression, the expression of ' g ' is as follows.

$$\begin{aligned}
 g &= 0.447f_{ck} \left(\frac{\left[\frac{4}{7} \right] D}{kD - \left[\frac{3}{7} \right] D} \right)^2 \\
 &= 0.447f_{ck} \left(\frac{4}{7k-3} \right)^2
 \end{aligned} \tag{9-5.10}$$

The area of the complementary sector of the stress block is given as follows.

$$\begin{aligned}
 A_{\text{sector}} &= \frac{1}{3}g \left(\frac{4}{7}D \right) \\
 &= \frac{4}{21}gD
 \end{aligned} \tag{9-5.11}$$

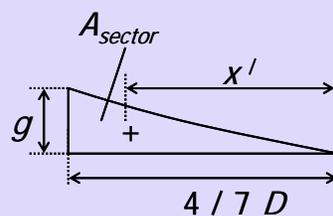


Figure 9-5.11 Complementary area of the stress block

Distance of centroid from apex (x') = $(3/4)(4/7)D = 3/7 D$ The forces are as follows.

$$\begin{aligned}
 C_u &= [0.447f_{ck}D - A_{\text{sector}}]B \\
 &= \left[0.447f_{ck}D - \frac{4}{21}gD\right]B \\
 &= 0.447f_{ck}BD \left[1 - \frac{4}{21} \left(\frac{4}{7k-3}\right)^2\right]
 \end{aligned} \tag{9-5.12}$$

$$\begin{aligned}
 T_{u1} &= A_{p1}f_{p1} \\
 &= A_{p1}E_p \varepsilon_{p1} \\
 &= A_{p1}E_p (\varepsilon_{c1} + \Delta\varepsilon_p) \\
 &= A_{p1}E_p \left(-0.002 \frac{x_u - \left(\frac{D}{2} + d_1\right)}{x_u - \frac{3D}{7}} + \Delta\varepsilon_p \right)
 \end{aligned} \tag{9-5.13}$$

$$\begin{aligned}
 T_{u2} &= A_{p2}f_{p2} \\
 &= A_{p2}E_p \varepsilon_{p2} \\
 &= A_{p2}E_p (\varepsilon_{c2} + \Delta\varepsilon_p) \\
 &= A_{p2}E_p \left(-0.002 \frac{x_u - \left(\frac{D}{2} - d_2\right)}{x_u - \frac{3D}{7}} + \Delta\varepsilon_p \right)
 \end{aligned} \tag{9-5.14}$$

The strains in the concrete at the level of the prestressing steels ε_{c1} and ε_{c2} are determined from the similarity of triangles of the following strain profile.

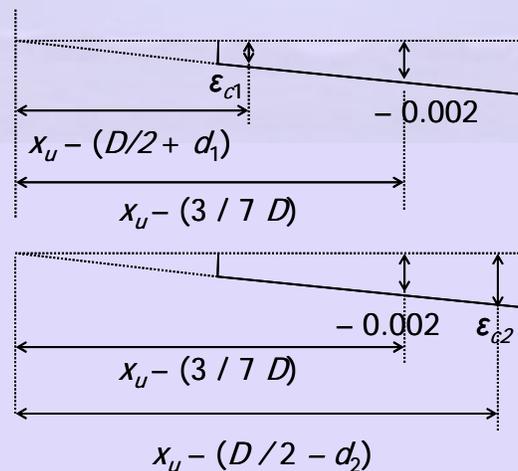


Figure 9-5.12 Strain profile across section

The moment and axial force capacities are as follows.

$$N_{uR} = C_u - T_{u1} - T_{u2} \quad (9-5.15)$$

$$M_{uR} = M_c + M_p \quad (9-5.16)$$

The expressions of M_c and M_p about the centroid are given below. Anticlockwise moments are considered positive. The lever arms of the tensile forces are shown in the following sketch.

$$\begin{aligned} M_c &= 0.447f_{ck}DB \times 0 + A_{\text{sector}}B \left[x' + \frac{3}{7}D - \frac{D}{2} \right] \\ &= \frac{10}{147}gD^2B \end{aligned} \quad (9-5.17)$$

$$M_p = T_{u1}d_1 - T_{u2}d_2 \quad (9-5.18)$$

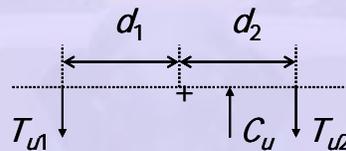


Figure 9-5.13 Force diagram across the section

3. Part of section under tension ($e_N \mid_{x_u=D} < , e_N \leq \infty, x_u < D$)

The following sketches represent the strain and stress profiles across the section and the force diagram.

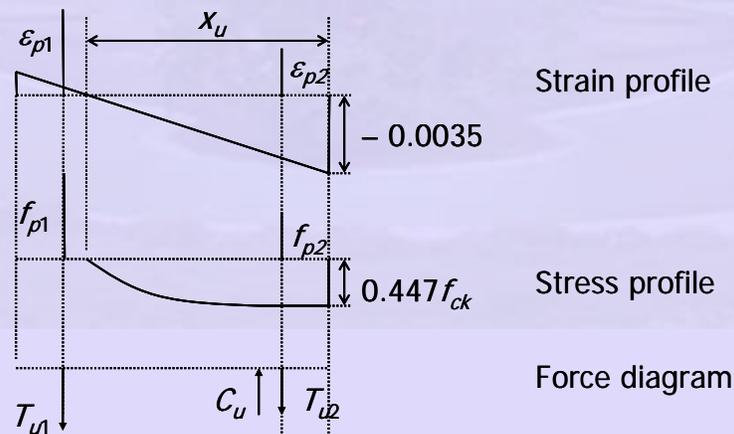


Figure 9-5.14 Sketches for analysis for part of section under tension

The forces are as follows. The compression is the resultant of the stress block whose expression can be derived similar to a reinforced concrete section.

$$C_u = 0.36f_{ck} x_u B$$

$$\begin{aligned}
 T_{u1} &= A_{p1} f_{p1} \\
 &= A_{p1} F(\epsilon_{p1}) \\
 &= A_{p1} F(\epsilon_{c1} + \Delta\epsilon_p) \\
 T_{u2} &= A_{p2} f_{p2} \\
 &= A_{p2} E_p \epsilon_{p2} \\
 &= A_{p2} E_p (\epsilon_{c2} + \Delta\epsilon_p)
 \end{aligned}$$

The strains ϵ_{c1} and ϵ_{c2} are calculated from the similarity of triangles of the following strain diagram.

$$\frac{\epsilon_{c1}}{\frac{D}{2} + d_1 - x_u} = \frac{0.0035}{x_u} \quad (9-5.19)$$

$$\frac{\epsilon_{c2}}{x_u - \left(\frac{D}{2} - d_2\right)} = -\frac{0.0035}{x_u} \quad (9-5.20)$$

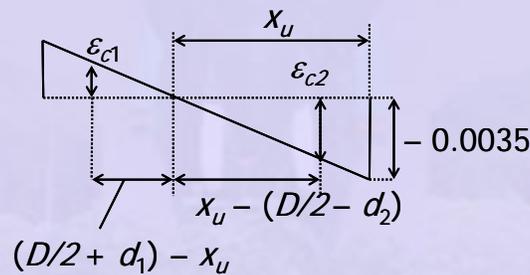


Figure 9-5.15 Strain profile across section

The moment and axial force capacities are as follows.

$$N_{uR} = C_u - T_{u1} - T_{u2} \quad (9-5.21)$$

$$M_{uR} = M_c + M_p \quad (9-5.22)$$

The expressions of M_c and M_p about the centroid are as follows.

$$M_c = 0.36f_{ck} x_u B \left[\left(\frac{D}{2} \right) - 0.42 x_u \right] \quad (9-5.23)$$

$$M_p = T_{u1}d_1 - T_{u2}d_2 \quad (9-5.24)$$

The lever arms of the forces are shown in the following sketch. The location of C_u is similar to that of a reinforced concrete section.

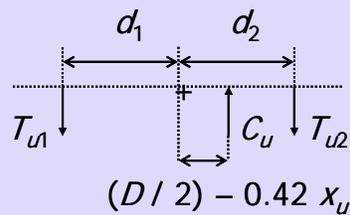


Figure 9-5.16 Force diagram across the section

4. Pure bending ($e_N = \infty, x_u = x_{u,min}$)

The value of x_u is determined by trial and error from the condition that the sum of the forces is zero.

$$C_u - T_{u1} - T_{u2} = 0$$

$$\text{or, } 0.36f_{ck} x_u B - A_{p1} f_{p1} - A_{p2} f_{p2} = 0 \quad (9-5.25)$$

$$\text{or, } x_u = \frac{A_{p1} f_{p1} + A_{p2} E_p \varepsilon_{p2}}{0.36f_{ck} B} \quad (9-5.26)$$

The strains ε_{p1} and ε_{p2} are calculated from the strain compatibility equations. The strain ε_{p2} is within the elastic range, whereas ε_{p1} may be outside the elastic range. The stresses f_{p1} and f_{p2} are calculated accordingly from the stress versus strain relationship of prestressing steel.

The steps for solving x_u are as follows.

- 1) Assume $x_u = 0.15 D$ (say).
- 2) Determine ε_{p1} and ε_{p2} from strain compatibility.
- 3) Determine f_{p1} and f_{p2} from stress versus strain relationship.
- 4) Calculate x_u from Eqn. (9-5.26).
- 5) Compare x_u with the assumed value. Iterate till convergence.

The moment and axial force capacities are as follows.

$$N_{uR} = 0 \quad (9-5.27)$$

$$M_{uR} = M_c + M_p \quad (9-5.28)$$

The expressions of M_c and M_p are same as the previous case.

5. Axial tension

The moment and axial force capacities are as follows. The cracked concrete is neglected in calculating the axial force capacity.

$$N_{uR} = -0.87f_{pk} A_p \quad (9-5.29)$$

$$M_{uR} = 0 \quad (9-5.30)$$

The above sets of N_{uR} and M_{uR} are joined to get the interaction diagram.

Example 9-5.1

Calculate the design interaction diagram for the member given below. The member is prestressed using 8 strands of 10 mm diameter. The strands are stress relieved with the following properties.

Tensile strength (f_{pk}) = 1715 N/mm².

Total area of strands = 8 × 51.6
= 413.0 mm²

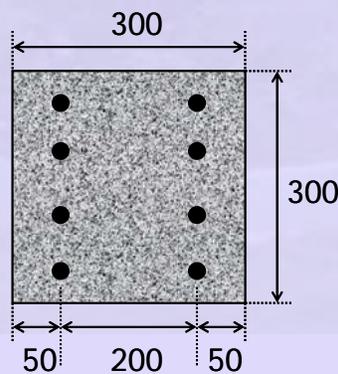
Effective prestress (f_{pe}) = 1034 N/mm²

Modulus (E_p) = 200 kN/mm²

Strain under f_{pe} (ϵ_{pe}) = 0.0042.

Grade of concrete = M40

Strain under f_{pe} (ϵ_{ce}) = -0.0005.



Solution

Calculation of geometric properties and strain compatibility relationship.

$$A_g = 300 \times 300 = 90,000 \text{ mm}^2$$

$$A_{p1} = A_{p2} = 4 \times 51.6 = 206 \text{ mm}^2$$

$$d_1 = d_2 = 100 \text{ mm}$$

$$\Delta \varepsilon_p = 0.0042 + 0.0005 = 0.0047$$

$$\therefore \varepsilon_p = \varepsilon_c + 0.0047$$

1. Pure compression ($e_N = 0$, $x_u = \infty$)

$$M_{uR} = 0 \text{ kNm}$$

$$\begin{aligned} C_u &= 0.447 f_{ck} (A_g - A_p) \\ &= 0.447 \times 40 (90,000 - 413) \\ &= 1601.8 \text{ kN} \end{aligned}$$

$$\begin{aligned} T_{u1} = T_{u2} &= A_{p1} E_p (-0.002 + \Delta \varepsilon_p) \\ &= 206.4 \times 200 \times (0.0047 - 0.002) \\ &= 111.5 \text{ kN} \end{aligned}$$

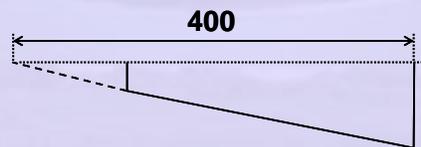
$$\begin{aligned} N_{uR} &= C_u - T_{u1} - T_{u2} \\ &= 1601.8 - 2 \times 111.5 \\ &= 1378.8 \text{ kN} \end{aligned}$$

With 10% reduction, to bypass the use of interaction diagram for eccentricities

$$e_N \leq 0.05 D$$

$$N_{uR} = 1204.9 \text{ kN}$$

2. Full section under compression ($0.05 D < e_N \leq e_N |_{x_u = D}$, $x_u \geq D$)

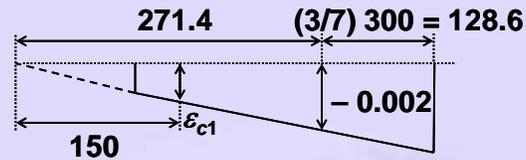


$$\begin{aligned} \text{Select } x_u &= 400 \text{ mm} \\ &= (4 / 3) \times 300 \text{ mm} \end{aligned}$$

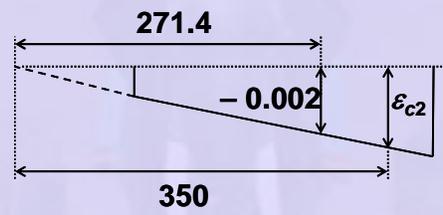
$$\therefore k = 4 / 3$$

$$\begin{aligned} g &= 0.447 \times f_{ck} \left(\frac{4}{7k - 3} \right)^2 \\ &= 0.447 \times 40 \left(\frac{4}{7 \times (4/3) - 3} \right)^2 \\ &= 7.13 \text{ N/mm}^2 \end{aligned}$$

$$\begin{aligned}
 C_u &= 0.447f_{ck}BD \left(1 - \frac{4}{21} \left[\frac{4}{7k-3} \right]^2 \right) \\
 &= 0.447 \times 40 \times 300^2 \left(1 - \frac{4}{21} \left[\frac{4}{7 \times (4/3) - 3} \right]^2 \right) \\
 &= 1486.9 \text{ kN}
 \end{aligned}$$



$$\begin{aligned}
 T_{u1} &= A_{p1}E_p (\epsilon_{c1} + \Delta\epsilon_p) \\
 &= 206.4 \times 200 \left(-0.002 \frac{150}{271.4} + 0.0047 \right) \\
 &= 148.4 \text{ kN}
 \end{aligned}$$



$$\begin{aligned}
 T_{u2} &= A_{p2}E_p (\epsilon_{c2} + \Delta\epsilon_p) \\
 &= 206.4 \times 200 \left(-0.002 \frac{350}{271.4} + 0.0047 \right) \\
 &= 87.5 \text{ kN}
 \end{aligned}$$

$$\begin{aligned}
 N_{uR} &= C_u - T_{u1} - T_{u2} \\
 &= 1486.9 - 148.4 - 87.5 \\
 &= 1251.0 \text{ kN}
 \end{aligned}$$

Limit N_{uR} to 1240.9 kN to bypass the use of interaction diagram for eccentricities $e_N \leq 0.05D$.

$$\begin{aligned}
 M_c &= \frac{10}{147} gD^2B \\
 &= \frac{10}{147} \times 7.13 \times 300^2 \times 300 \\
 &= 13.1 \text{ kNm}
 \end{aligned}$$

$$\begin{aligned}
 M_p &= T_{u1}d_1 - T_{u2}d_2 \\
 &= 148.4 \times 100 - 87.5 \times 100 \\
 &= 6.1 \text{ kNm}
 \end{aligned}$$

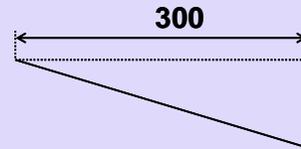
$$\begin{aligned}
 M_{uR} &= M_c + M_p \\
 &= 13.1 + 6.1 \\
 &= 19.2 \text{ kNm}
 \end{aligned}$$

Select $x_u = 300 \text{ mm}$

$$\therefore k = 1$$

By similar calculations,

$$\begin{array}{ll}
 g &= 17.9 \text{ N/mm}^2 & N_{uR} &= 1060.6 \text{ kN} \\
 C_u &= 1304.1 \text{ kN} & M_c &= 32.9 \text{ kNm} \\
 T_{u1} &= 169.9 \text{ kN} & M_p &= 9.6 \text{ kNm} \\
 T_{u2} &= 73.6 \text{ kN} & M_{uR} &= 42.5 \text{ kNm.}
 \end{array}$$



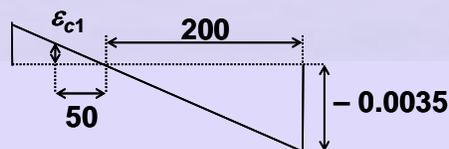
3. Part of section under tension ($e_N \mid_{x_u=D} < e_N \leq \infty, x_u < D$)

Select $x_u = 200 \text{ mm}$.

$$\begin{aligned}
 C_u &= 0.36f_{ck} x_u B \\
 &= 0.36 \times 40 \times 200 \times 300 \\
 &= 864.0 \text{ kN}
 \end{aligned}$$

$$\begin{aligned}
 \epsilon_{c1} &= \frac{0.0035}{200} 50 \\
 &= 0.0009
 \end{aligned}$$

$$\begin{aligned}
 \epsilon_{p1} &= 0.0009 + 0.0047 \\
 &= 0.0056
 \end{aligned}$$



Strain corresponding to elastic limit

$$\begin{aligned}
 \epsilon_{py} &= 0.87 \times 0.8f_{ck} / E_p \\
 &= 0.87 \times 1715 / 200 \times 10^3 \\
 &= 0.0059.
 \end{aligned}$$

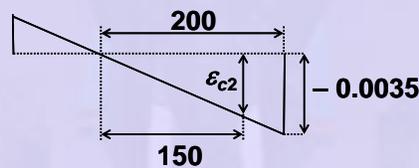
$$\varepsilon_{p1} < \varepsilon_{py}$$

$$\begin{aligned} \therefore f_{p1} &= E_p \varepsilon_{p1} \\ &= 200 \times 10^3 \times 0.0055 \\ &= 1115 \text{ N/mm}^2 \end{aligned}$$

$$\begin{aligned} T_{u1} &= A_{p1} f_{p1} \\ &= 206.4 \times 1115 \\ &= 230.1 \text{ kN} \end{aligned}$$

$$\begin{aligned} \varepsilon_{c2} &= -\frac{0.0035}{200} 150 \\ &= -0.0026 \end{aligned}$$

$$\begin{aligned} \varepsilon_{p2} &= -0.0026 + 0.0047 \\ &= 0.0021 \end{aligned}$$



$$\begin{aligned} f_{p2} &= E_p \varepsilon_{p2} \\ &= 200 \times 10^3 \times 0.0021 \\ &= 416 \text{ N/mm}^2 \end{aligned}$$

$$\begin{aligned} T_{u2} &= A_{p2} f_{p2} \\ &= 206.4 \times 416 \\ &= 85.9 \text{ kN} \end{aligned}$$

$$\begin{aligned} N_{uR} &= C_u - T_{u1} - T_{u2} \\ &= 864 - 230.1 - 85.9 \\ &= 548.0 \text{ kN} \end{aligned}$$

$$\begin{aligned} M_c &= 0.36 f_{ck} x_u B [(D/2) - 0.42x_u] \\ &= 864 (150 - 0.42 \times 200) \\ &= 57.0 \text{ kNm} \end{aligned}$$

$$\begin{aligned} M_p &= T_{u1} d_1 - T_{u2} d_2 \\ &= 230.1 \times 100 - 85.9 \times 100 \\ &= 14.4 \text{ kNm} \end{aligned}$$

$$\begin{aligned} M_{uR} &= M_c + M_p \\ &= 57.0 + 14.4 \\ &= 71.4 \text{ kNm} \end{aligned}$$

4. Pure bending ($e_N = \infty$, $x_u = x_{u,min}$)

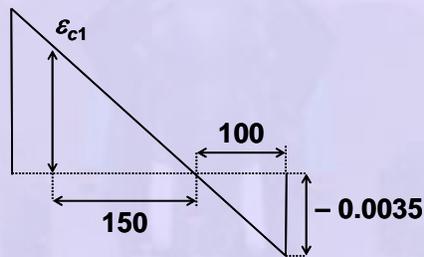
$$N_{uR} = 0.0 \text{ kN}$$

$$\text{Try } x_u = 100 \text{ mm.}$$

$$\begin{aligned} C_u &= 0.36 f_{ck} x_u B \\ &= 0.36 \times 40 \times 100 \times 300 \\ &= 432.0 \text{ kN} \end{aligned}$$

$$\begin{aligned} \epsilon_{c1} &= \frac{0.0035}{100} 150 \\ &= 0.0052 \end{aligned}$$

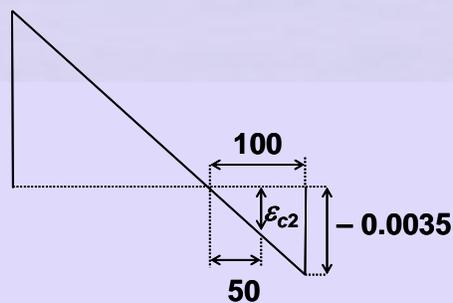
$$\begin{aligned} \epsilon_{p1} &= 0.0052 + 0.0047 \\ &= 0.0099 \end{aligned}$$



From stress-strain curve

$$\begin{aligned} f_{p1} &= 0.87 f_{pk} \\ &= 1492 \text{ N/mm}^2 \end{aligned}$$

$$\begin{aligned} T_{u1} &= A_{p1} f_{p1} \\ &= 206.4 \times 1492 \\ &= 308.0 \text{ kN} \end{aligned}$$



$$\begin{aligned} \epsilon_{c2} &= -\frac{0.0035}{100} 50 \\ &= -0.0017 \end{aligned}$$

$$\begin{aligned}\epsilon_{p2} &= -0.0017 + 0.0047 \\ &= 0.0029\end{aligned}$$

$$\begin{aligned}f_{p2} &= E_p \epsilon_{p2} \\ &= 200 \times 10^3 \times 0.0029 \\ &= 580 \text{ N/mm}^2\end{aligned}$$

$$\begin{aligned}T_{u2} &= A_{p2} f_{p2} \\ &= 206.4 \times 580 \\ &= 120.0 \text{ kN}\end{aligned}$$

$$T_{u1} + T_{u2} = 428.0 \text{ kN}$$

This is close enough to $C_u = 432.0 \text{ kN}$. Hence, the trial value of x_u is satisfactory.

$$\begin{aligned}M_c &= 0.36 f_{ck} x_u B [(D/2) - 0.42x_u] \\ &= 0.36 \times 40 \times 100 \times 300 (150 - 0.42 \times 100) \\ &= 46.6 \text{ kNm}\end{aligned}$$

$$\begin{aligned}M_p &= T_{u1}d_1 - T_{u2}d_2 \\ &= 308.0 \times 100 - 120.0 \times 100 \\ &= 18.8 \text{ kNm}\end{aligned}$$

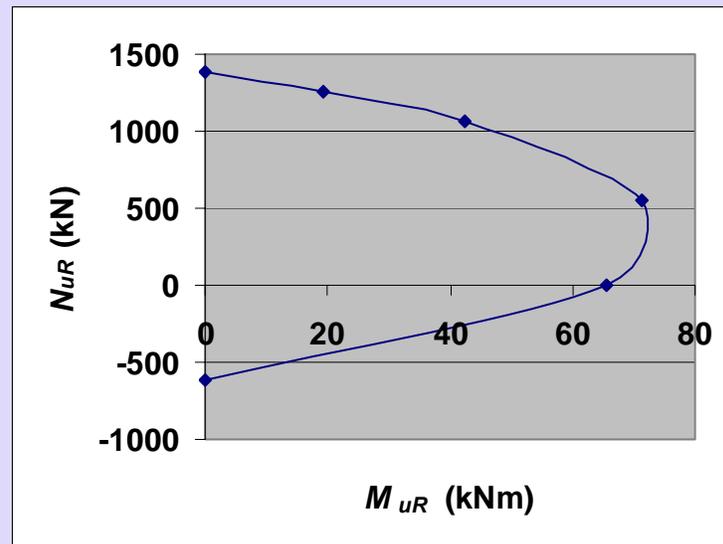
$$\begin{aligned}M_{uR} &= 46.6 + 18.8 \\ &= 65.4 \text{ kNm}\end{aligned}$$

5. Axial tension

$$M_{uR} = 0.0 \text{ kNm}$$

$$\begin{aligned}N_{uR} &= -0.87 f_{pk} A_p \\ &= -0.87 \times 1715 \times 413.0 \\ &= -616.2 \text{ kN}\end{aligned}$$

The above sets of N_{uR} and M_{uR} are joined to get the following interaction diagram. The limit on axial force capacity to consider the effect of eccentricity less than $0.05D$, is not shown.



9.5.4 Effect of Prestressing Force

Along with the interaction curve for the prestressed concrete (PC) section, the interaction curves for two reinforced concrete (RC) sections are plotted. The section denoted as RC 1 has the same moment capacity at zero axial force. The section denoted as RC 2 has the same axial force capacity at zero moment. The gross section of RC 1 is same as that of PC, but the section of RC 2 is smaller.

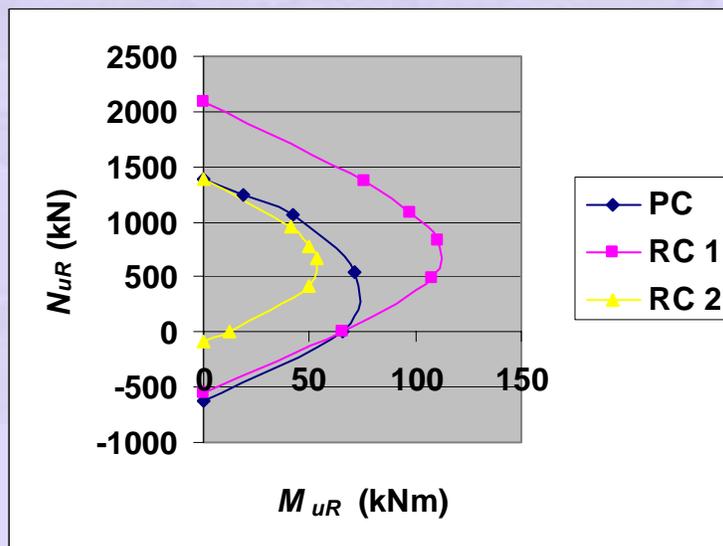


Figure 9-5.17 Interaction diagrams for reinforced and prestressed sections

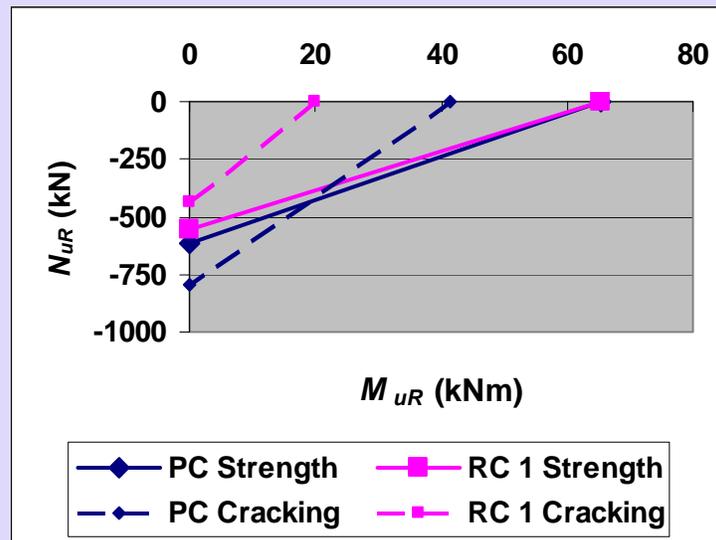


Figure 9-5.18 Interaction of moment and tension for cracking and strength

Comparing the curves for PC and RC 2, it is observed that if the moment demand is small, then a smaller reinforced concrete section is adequate to carry the axial force. Of course with increasing moment, the flexural capacity of the prestressed concrete section is higher. Comparing the curves for PC and RC 1, it is inferred that for two sections with same flexural capacities, the axial load capacity of a prestressed concrete section is less. However if there is tension, the cracking load combination is higher for PC as compared to RC 1.

Thus, prestressing is beneficial for strength when there is occurrence of:

- a) Large moment in addition to compression
- b) Moment along with tension.

Such situations arise in piles or columns subjected to seismic forces. In presence of tension, prestressing is beneficial at service loads due to reduced cracking. Non-prestressed reinforcement may be used for supplemental capacity.