# 9.6 Circular Prestressing

This section covers the following topics.

- Introduction
- General Analysis and Design
- Prestressed Concrete Pipes
- Liquid Storage Tanks
- Ring Beams
- Conclusion

## 9.6.1 Introduction

When the prestressed members are curved, in the direction of prestressing, the prestressing is called circular prestressing. For example, circumferential prestressing in pipes, tanks, silos, containment structures and similar structures is a type of circular prestressing. In these structures, there can be prestressing in the longitudinal direction (parallel to axis) as well. Circular prestressing is also applied in domes, shells and folded plates.

The circumferential prestressing resists the hoop tension generated due to the internal pressure. The prestressing is done by wires or tendons placed spirally, or over sectors of the circumference of the member. The wires or tendons lay outside the concrete core. Hence, the centre of the prestressing steel (CGS) is outside the core concrete section.

The hoop compression generated is considered to be uniform across the thickness of a thin shell. Hence, the pressure line (or C-line) lies at the centre of the core concrete section (CGC). The following sketch shows the internal forces under service conditions. The analysis is done for a slice of unit length along the longitudinal direction (parallel to axis).

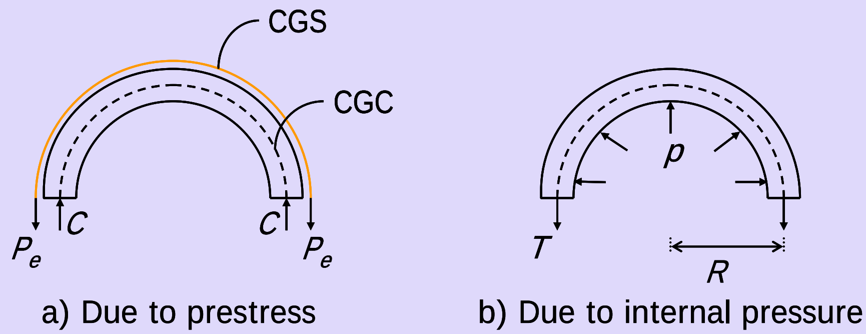



**Figure 9-6.1** Internal forces under service conditions

To reduce the loss of prestress due to friction, the prestressing can be done over sectors of the circumference. Buttresses are used for the anchorage of the tendons. The following sketch shows the buttresses along the circumference.

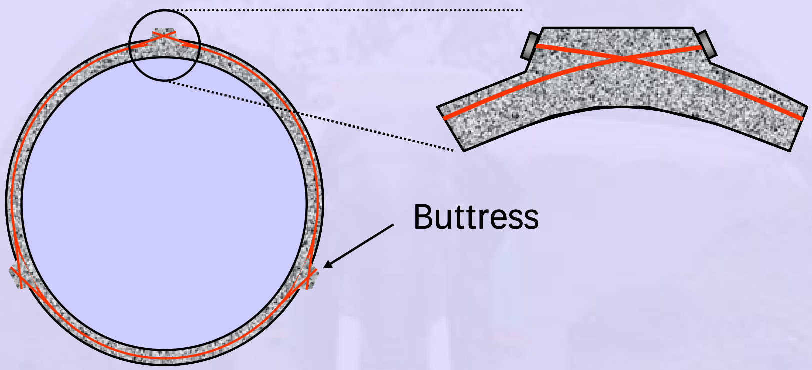



**Figure 9-6.2** Use of buttress in circumferential prestressing

## 9.6.2 General Analysis and Design

**Analysis**

The basics of analysis and design for circumferential prestressing are provided for a general understanding. Specific applications such as pipes, liquid storage tanks and ring beams will be explained later.

**Analysis at Transfer**

The compressive stress can be calculated from the compression $C$. From equilibrium, $C = P_0$, where $P_0$ is the prestress at transfer after short-term losses. The compressive stress ($f_c$) is given as follows

$$f_c = -\frac{P_0}{A} \quad \textbf{(9-6.1)}$$

Here,

$A$ = area of the longitudinal section of the slice.

The permissible prestress is determined based on $f_c$ within the allowable stress at transfer ($f_{cc,all}$).

**Analysis at Service Loads**

The tensile stress due to the internal pressure ($p$) can be calculated from the tension $T$. From equilibrium of half of the slice, $T = pR$ where, $R$ is the radius of the mid-surface of the cylinder. The resultant stress ($f_c$) due to the effective prestress ($P_e$) and internal pressure is given as follows.

$$f_c = -\frac{P_e}{A} + \frac{pR}{A_t} \quad \textbf{(9-6.2)}$$

Here,

$A_t$ = area of the transformed longitudinal section of the slice.

The value of $f_c$ should be compressive and within the allowable stress at service loads ($f_{cc,all}$). In the previous equation, since $P_e = pR$ and $A_t$ is greater than $A$, $f_c$ is always negative. Thus, the concrete will be under compression. To meet the safety standards, a factor of safety can be further introduced.

**Design**

The internal pressure $p$ and the radius $R$ are given variables. It is assumed that the prestressing steel alone carries the hoop tension due to internal pressure, that is $P_e = A_p f_{pe} = pR$.

The steps of design are as follows.

1) Calculate the area of the prestressing steel from the equation $A_p = pR / f_{pe}$.
2) Calculate the prestress at transfer from an estimate of the permissible initial stress $f_{p0}$ and using the equation

$$P_0 = A_p f_{p0}. \quad \textbf{(9-6.3)}$$

3) Calculate the thickness of concrete shell from the following equation.

$$A = P_0 / f_{cc,all} \quad \textbf{(9-6.4)}$$

4) Here, $f_{cc,all}$ is the allowable compressive stress at transfer.

5) Calculate the resultant stress $f_c$ at the service conditions using Eqn. (9-6.2). The value of $f_c$ should be within $f_{cc,all}$, the allowable stress at service conditions.

### 9.6.3 Prestressed Concrete Pipes

Prestressed concrete pipes are suitable when the internal pressure is within 0.5 to 2.0 $N/mm^2$. There are two types of prestressed concrete pipes: cylinder type and the non-cylinder type. A cylinder type pipe has a steel cylinder core, over which the concrete is cast and prestressed. A non-cylinder type of pipe is made of prestressed concrete only. **IS:784 - 2001** (*Prestressed Concrete Pipes (Including Specials) - Specification*) provides guidelines for the design of prestressed concrete pipes with the internal diameter ranging from 200 mm to 2500 mm. The pipes are designed to withstand the combined effect of internal pressure and external loads. The minimum grade of concrete in the core should be M40 for non-cylinder type pipes.

First, the core is cast either by the centrifugal method or by the vertical casting method. In the centrifugal method the mould is subjected to spinning till the concrete is compacted to a uniform thickness throughout the length of the pipe. In the vertical casting method, concrete is poured in layers up to a specified height.

After adequate curing of concrete, first the longitudinal wires are prestressed. Subsequently, the circumferential prestressing is done by the wire wound around the core in a helical form. The wire is wound using a counter weight or a die. Finally a coat of concrete or rich cement mortar is applied over the wire to prevent from corrosion.

For cylinder type pipes, first the steel cylinder is fabricated and tested. Then the concrete is cast around it.

The analysis and design of prestressed concrete pipes consider the stresses due to the different actions. A horizontal layout of the pipe is considered to illustrate them.

**Analysis**

The stresses in the longitudinal direction are due to the following actions.

1. Longitudinal prestressing ($f_{l1}$)
2. Circumferential prestressing ($f_{l2}$)
3. Self weight ($f_{l3}$)
4. Transport and handling ($f_{l4}$)
5. Weight of fluid ($f_{l5}$)
6. Weight of soil above ($f_{l6}$)

**Longitudinal prestressing**

The longitudinal prestressing generates a uniform compression.

$$f_{l1} = -\frac{P_e}{A_{c1}} \qquad \textbf{(9-6.5)}$$

Here,

$P_e$ = effective prestress

$A_{c1}$ = area of concrete in the core.

**Circumferential prestressing**

Due to the Poisson's effect, the circumferential prestressing generates longitudinal tensile stress.

$$f_{l2} = 0.284 \times \frac{P_e}{A_c} \qquad \textbf{(9-6.6)}$$

The above expression estimates the Poisson's effect.

**Self weight**

If the pipe is not continuously supported, then a varying longitudinal stress generates due to the moment due to self weight ($M_{sw}$).

$$f_{l3} = \pm\frac{M_{sw}}{Z_l} \qquad \textbf{(9-6.7)}$$

Here,

$Z_l$ = section modulus about the centroidal axis.

**Transport and handling**

A varying longitudinal stress generates due to the moment during transport and handling ($M_{th}$).

$$f_{l4} = \pm \frac{M_{th}}{Z_l} \quad \textbf{(9-6.8)}$$

**Weight of fluid**

Similar to self weight, the moment due to weight of the fluid inside ($M_f$) generates varying longitudinal stress.

$$f_{l5} = \pm \frac{M_f}{Z_l} \quad \textbf{(9-6.9)}$$

**Weight of soil above**

The weight of soil above for buried pipes is modelled as an equivalent distributed load. The expression of stress ($f_{l6}$) is similar to that for the weight of fluid.

The longitudinal stresses are combined based on the following diagram.

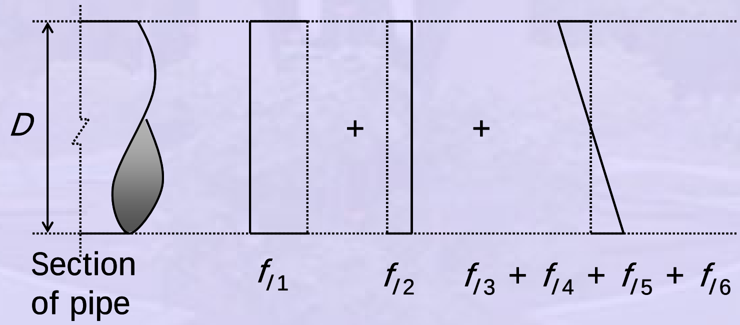



**Figure 9-6.3** Stress profiles across section

The stresses in the circumferential direction are due to the following actions.

1. Circumferential prestressing ($f_{h1}$)
2. Self weight ($f_{h2}$)
3. Weight of fluid ($f_{h3}$)
4. Weight of soil above ($f_{h4}$)
5. Live load ($f_{h5}$)
6. Internal pressure ($f_{h6}$)

**Circumferential prestressing**

The compressive hoop stress ($f_{h1}$) is given as follows.

$$f_{h1} = -\frac{P_s}{A_{c2}}$$
$$= -\frac{P_s}{1 \times t_c} \quad \textbf{(9-6.10)}$$

Here,

$P_s$ = tensile force in spiral wire in unit length of pipe

$A_{c2}$ = area for longitudinal section of unit length

$t_c$ = thickness of the core.

2. to 5. For each of these actions, first the vertical load per unit length (*W*) is calculated. Moment (*M*) and thrust (*T*) develop across the thickness owing to distortion of the section due to *W*, as shown in the following sketch.

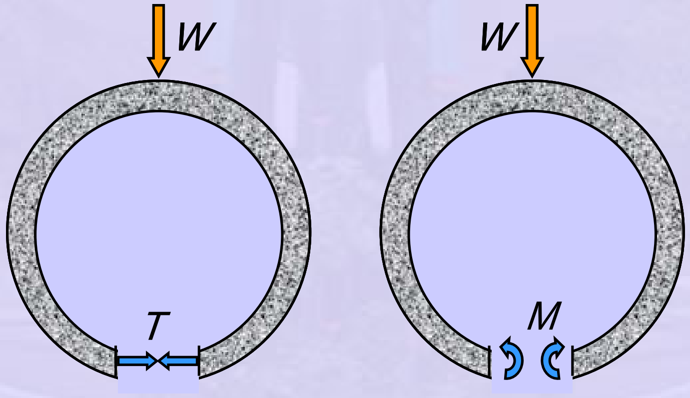



**Figure 9-6.4** Forces due to vertical load

The hoop stress at a point is calculated by the following equation.

$$f_h = \pm\frac{M}{Z_h} + \frac{T}{A} \quad \textbf{(9-6.11)}$$

The expressions of *M* and *T* due to *W* are as follows.

$$M = C_M W R \quad \textbf{(9-6.12)}$$

$$T = C_T W \quad \textbf{(9-6.13)}$$

Here,

$C_M$ = moment coefficient

$C_T$ = thrust coefficient

$W$ = vertical load per unit length

$R$ = mean radius of pipe

$A$ = area of longitudinal section for unit length of pipe

$Z_h$ = section modulus for hoop stress for same length

= $(1/6)t_2 \times 1000$ mm$^3$/m

$t$ = total thickness of core and coat

Values of $C_M$ and $C_T$ are tabulated in **IS:784 - 2001**.

$$f_{h6} = \frac{pR}{A_t} \qquad \textbf{(9-6.14)}$$

The hoop stresses are combined based on the following diagram.

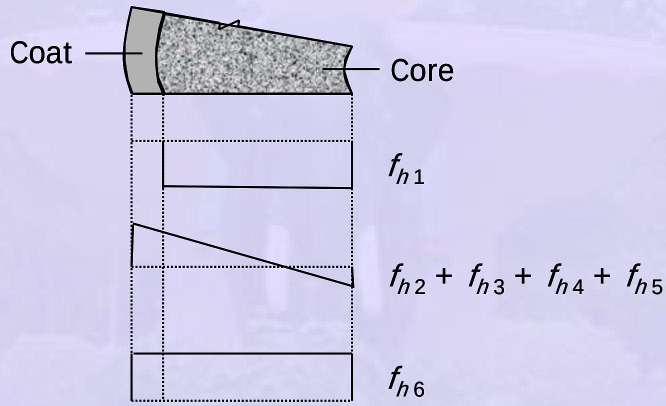



**Figure 9-6.5** Stress profiles across the thickness

## 9.6.4 Liquid Storage Tanks

In the construction of concrete structures for the storage of liquids, the imperviousness of concrete is an important basic requirement. Hence, the design of such construction is based on avoidance of cracking in the concrete. The structures are prestressed to avoid tension in the concrete. In addition, prestressed concrete tanks require low maintenance. The resistance to seismic forces is also satisfactory.

Prestressed concrete tanks are used in water treatment and distribution systems, waste water collection and treatment system and storm water management. Other applications are liquefied natural gas (LNG) containment structures, large industrial process tanks and bulk storage tanks.

The construction of the tanks is in the following sequence. First, the concrete core is cast and cured. The surface is prepared by sand or hydro blasting. Next, the circumferential prestressing is applied by strand wrapping machine. Shotcrete is applied to provide a coat of concrete over the prestressing strands.

A few photographs are provided for illustration.

(a) Circumferential prestressing

(b) Circumferential prestressing (close-up)

(c) Shotcrete operation

**Figure 9-6.6** Construction of a liquid storage tank

(Reference: DYK Incorporated)

**IS:3370 - 1967** (*Code of Practice for Concrete Structures for the Storage of Liquids*) provides guidelines for the analysis and design of liquid storage tanks. The four sections of the code are titled as follows.

Part 1: *General Requirement*

Part 2: *Reinforced Concrete Structures*

Part 3: *Prestressed Concrete Structures*

Part 4: *Design Tables*.

**Analysis**

The analysis of liquid storage tanks can be done by **IS:3370 - 1967, Part 4**, or by the finite element method. The Code provides coefficients for bending moment, shear and hoop tension (for cylindrical tanks), which were developed from the theory of plates and shells. In **Part 4**, both rectangular and cylindrical tanks are covered. Since circular prestressing is applicable to cylindrical tanks, only this type of tank is covered in this module.

The following types of boundary conditions are considered in the analysis of the cylindrical wall.

a) For base: fixed or hinged
b) For top: free or hinged or framed.

The applicability of each boundary condition is explained next.

**For base**

Fixed: When the wall is built continuous with its footing, then the base can be considered to be fixed as the first approximation.

Hinged: If the sub grade is susceptible to settlement, then a hinged base is a conservative assumption. Since the actual rotational restraint from the footing is somewhere in between fixed and hinged, a hinged base can be assumed.

The base can be made sliding with appropriate polyvinyl chloride (PVC) water-stops for liquid tightness.

**For top**

Free: The top of the wall is considered free when there is no restraint in expansion.

Hinged: When the top is connected to the roof slab by dowels for shear transfer, the boundary condition can be considered to be hinged.

Framed: When the top of the wall and the roof slab are made continuous with moment transfer, the top is considered to be framed.

The hydrostatic pressure on the wall increases linearly from the top to the bottom of the liquid of maximum possible depth. If the vapour pressure in the free board is negligible, then the pressure at the top is zero. Else, it is added to the pressure of the liquid throughout the depth. The forces generated in the tank due to circumferential prestress are opposite in nature to that due to hydrostatic pressure. If the tank is built underground, then the earth pressure needs to be considered.

The hoop tension in the wall, generated due to a triangular hydrostatic pressure is given as follows.

$$T = C_T\, w\, H\, R_i \qquad \textbf{(9-6.15)}$$

The bending moment in the vertical direction is given as follows.

$$M = C_M\, w\, H^3 \qquad \textbf{(9-6.16)}$$

The shear at the base is given by the following expression.

$$V = C_V\, w\, H^2 \qquad \textbf{(9-6.17)}$$

In the previous equations, the notations used are as follows.

$C_T$ = coefficient for hoop tension

$C_M$ = coefficient for bending moment

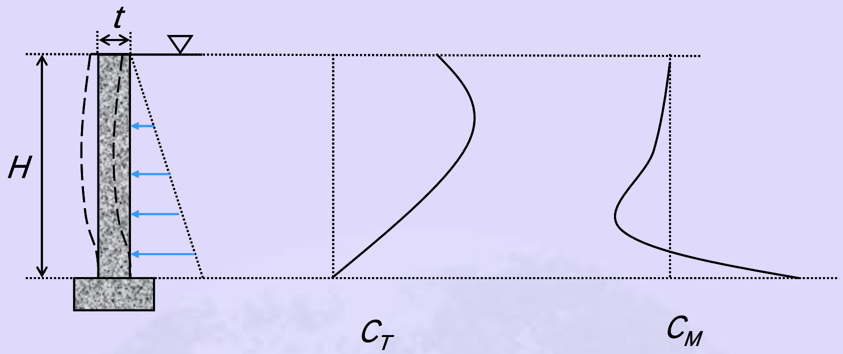
$C_V$ = coefficient for shear

$w$ = unit weight of liquid

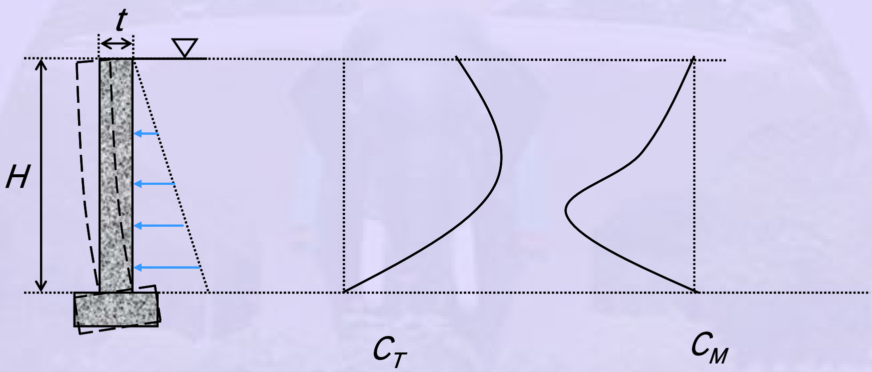
$H$ = height of the liquid

$R_i$ = inner radius of the wall.

The values of the coefficients are tabulated in **IS:3370 - 1967, Part 4**, for various values of $H^2/Dt$, at different depths of the liquid. $D$ and $t$ represent the inner diameter and the thickness of the wall, respectively. The typical variations of $C_T$ and $C_M$ with depth, for two sets of boundary conditions are illustrated.




(a) Fixed base – free top




(b) Hinged base – free top

**Figure 9-6.7** Variations of coefficients for hoop tension and bending moment

The roof can be made of a dome supported at the edges on the cylindrical wall. Else, the roof can be a flat slab supported on columns along with the edges. **IS:3370 - 1967, Part 4**, provides coefficients for the analysis of the floor and roof slabs.

**Design**

**IS:3370 - 1967, Part 3**, provides design requirements for prestressed tanks. A few of them are mentioned.

1) The computed stress in the concrete and steel, during transfer, handling and construction, and under working loads, should be within the permissible values as specified in **IS:1343 - 1980**.
2) The liquid retaining face should be checked against cracking with a load factor of 1.2.

$$\sigma_{CL}/\sigma_{WL} \geq 1.2 \quad \textbf{(9-6.18)}$$

Here,

$\sigma_{CL}$ = stress under cracking load

$\sigma_{WL}$ = stress under working load.

Values of limiting tensile strength of concrete for estimating the cracking load are specified in the Code.

3) The ultimate load at failure should not be less than twice the working load.
4) When the tank is full, there should be compression in the concrete at all points of at least 0.7 N/mm$^2$. When the tank is empty, there should not be tensile stress greater than 1.0 N/mm$^2$. Thus, the tank should be analysed both for the full and empty conditions.
5) There should be provisions to allow for elastic distortion of the structure during prestressing. Any restraint that may lead to the reduction of the prestressing force, should be considered.

**Detailing Requirements**

**IS:3370 - 1967, Part 3**, also provides detailing requirements. The cover requirement is as follows. The minimum cover to the prestressing wires should be 35 mm on the liquid face. For faces away from the liquid, the cover requirements are as per **IS:1343 - 1980**.

Other requirements from **IS:1343 - 1980** are also applicable.

## 9.6.5 Ring Beams

Ring beams support domes in buildings, tanks, silos and nuclear containment structures.

Circular prestressing is applied on a dome by a grid of tendons. The cylindrical wall is prestressed circumferentially and vertically. The ring beam is circumferentially prestressed. The sketches below show schematic representation of the elements and the prestressing tendons.

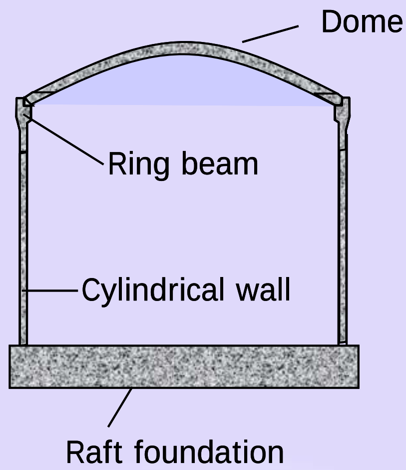



**Figure 9-6.8** Cross-section of a nuclear containment structure

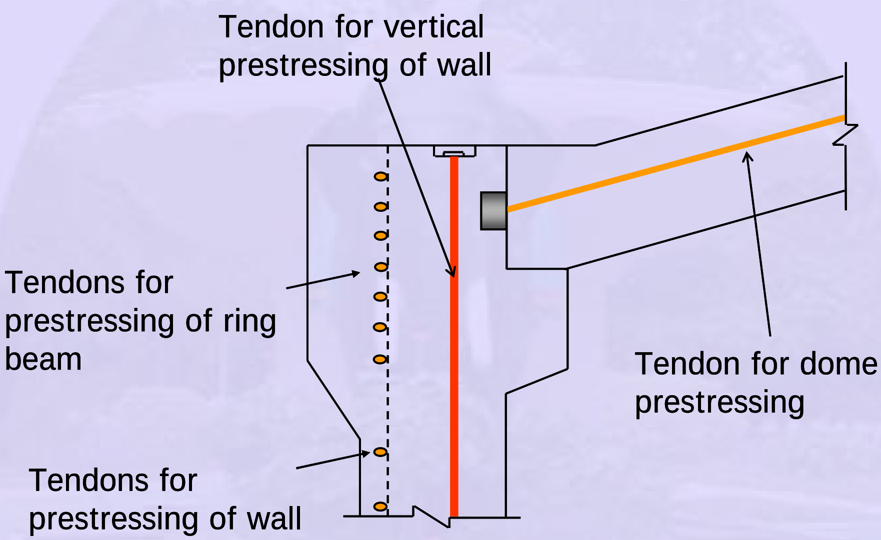



**Figure 9-6.9** Typical layout of prestressing tendons at dome and ring beam junction

The following photo shows a prestressed nuclear containment structure.

**Figure 9-6.10** Containment Structure, Kaiga Atomic Power Plant, Karnataka (Reference: Larsen & Toubro)

**Analysis**

The analysis of a ring beam is based on a load symmetric about the vertical axis. Since the dome is not supposed to carry any moment at the edge, the resultant reaction at the ring beam is tangential. The following figure shows the forces at the base of dome.

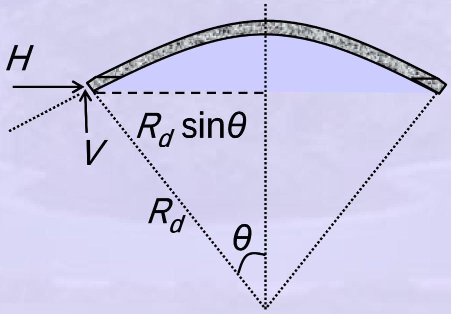



**Figure 9-6.11** Forces at the base of dome

Let the total vertical load from the dome be $W$. The vertical reaction per unit length ($V$) is given as follows.

$$V = \frac{W}{2\pi R_d \sin\theta} \quad \textbf{(9-6.19)}$$

Here,

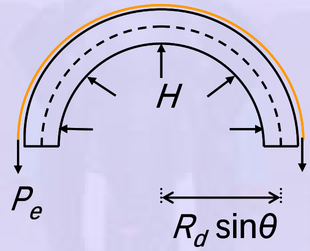
$R_d$ = radius of the dome

$\theta$ = half of the angle subtended by the dome.

The horizontal thrust ($H$) is calculated from the condition of the reaction to be tangential. The value per unit length is given as follows.

$$H = V\cot\theta = \frac{W\cot\theta}{2\pi R_d \sin\theta} \quad \textbf{(9-6.20)}$$

The thrust is resisted by the effective prestressing force ($P_e$) in the ring beam. $P_e$ can be estimated from the equilibrium of half of the ring beam as shown in the following sketch.




Plan of ring beam

**Figure 9-6.12** Forces in the ring beam

$$P_e = H R_d \sin\theta = \frac{W\cot\theta}{2\pi} \quad \textbf{(9-6.21)}$$

## 9.6.6 Conclusion

Prestressing of concrete is observed in other types of structural elements, such as bridge decks, shells and folded plates, offshore concrete gravity structures, pavements and raft foundations. The analysis of special structures is based on advanced theory of structural analysis or the finite element method. After the analysis, the design of such structures follows the basic principles of prestressed concrete design. It is expected that in future, further innovations in structural form, prestressing systems and construction technology will promote the application of prestressed concrete.

A few photos of recent applications follows.

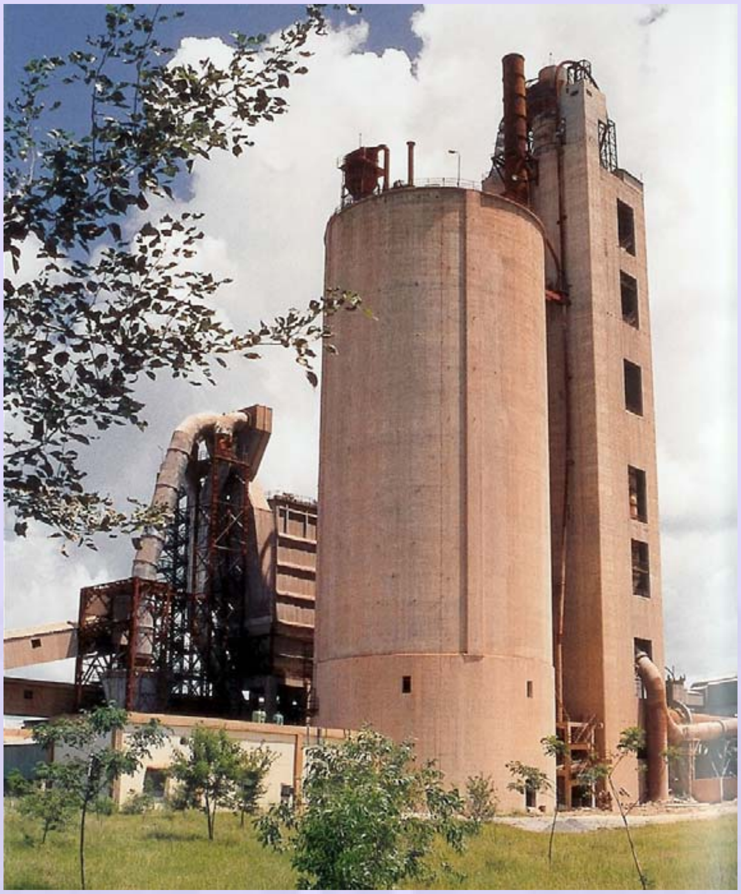


**Figure 9-6.13** Cement silo, Jayanthipuram, Tamilnadu
(Reference: Larsen & Toubro)

**Figure 9-6.14** Curved box-girder bridge, Jaipur-Kishangarh Highway, Rajasthan
(Reference: L & T Ramboll)

(a) Exterior view

(b) Interior view

**Figure 9-6.15** Folded plate

(Department of Ocean Engineering, Indian Institute of Technology Madras)