Recursion

- **Recursion is a fundamental programming technique that can provide an elegant solution certain kinds of problems**

Recursive Thinking

- **A *recursive definition* is one which uses the word or concept being defined in the definition itself**
- **When defining an English word, a recursive definition is often not helpful**
- **But in other situations, a recursive definition can be an appropriate way to express a concept**
- **Before applying recursion to programming, it is best to practice thinking recursively**

Recursive Definitions

- **Consider the following list of numbers:**

```
24, 88, 40, 37
```

- **Such a list can be defined as follows:**

```
A LIST is a:  number
        or a:  number  comma  LIST
```

- **That is, a LIST is defined to be a single number, or a number followed by a comma followed by a LIST**

- **The concept of a LIST is used to define itself**

Recursive Definitions

- **The recursive part of the LIST definition is used several times, terminating with the non-recursive part:**

```
number comma LIST
  24     ,   88, 40, 37

             number comma LIST
               88     ,   40, 37

                          number comma LIST
                            40     ,   37

                                       number
                                         37
```

Infinite Recursion

- All recursive definitions have to have a non-recursive part
- If they didn't, there would be no way to terminate the recursive path
- Such a definition would cause *infinite recursion*
- This problem is similar to an infinite loop, but the non-terminating "loop" is part of the definition itself
- The non-recursive part is often called the *base case*

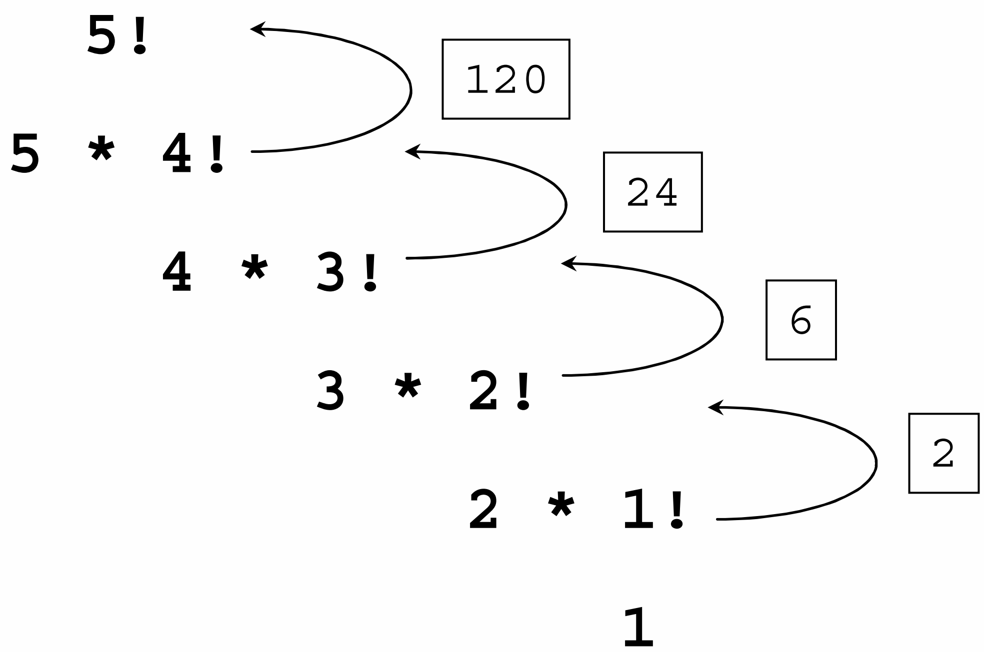
Recursive Definitions

- N!, for any positive integer N, is defined to be the product of all integers between 1 and N inclusive
- This definition can be expressed recursively as:

```
1!   =  1
N!   =  N * (N-1)!
```

- A factorial is defined in terms of another factorial
- Eventually, the base case of 1! is reached

Recursive Definitions



Recursive Programming

- **A Function can invoke itself; if set up that way, it is called a *recursive function***
- **The code of a recursive function must be structured to handle both the base case and the recursive case**
- **As with any function call, when the function completes, control returns to the function that invoked it (which may be an earlier invocation of itself)**

Recursive Programming

- **Consider the problem of computing the sum of all the numbers between 1 and any positive integer N**
- **This problem can be recursively defined as:**

$$\sum_{i=1}^{N} i = N + \sum_{i=1}^{N-1} i = N + N-1 + \sum_{i=1}^{N-2} i$$

$$= N + N-1 + N-2 + \sum_{i=1}^{N-3} i$$

$$\vdots$$

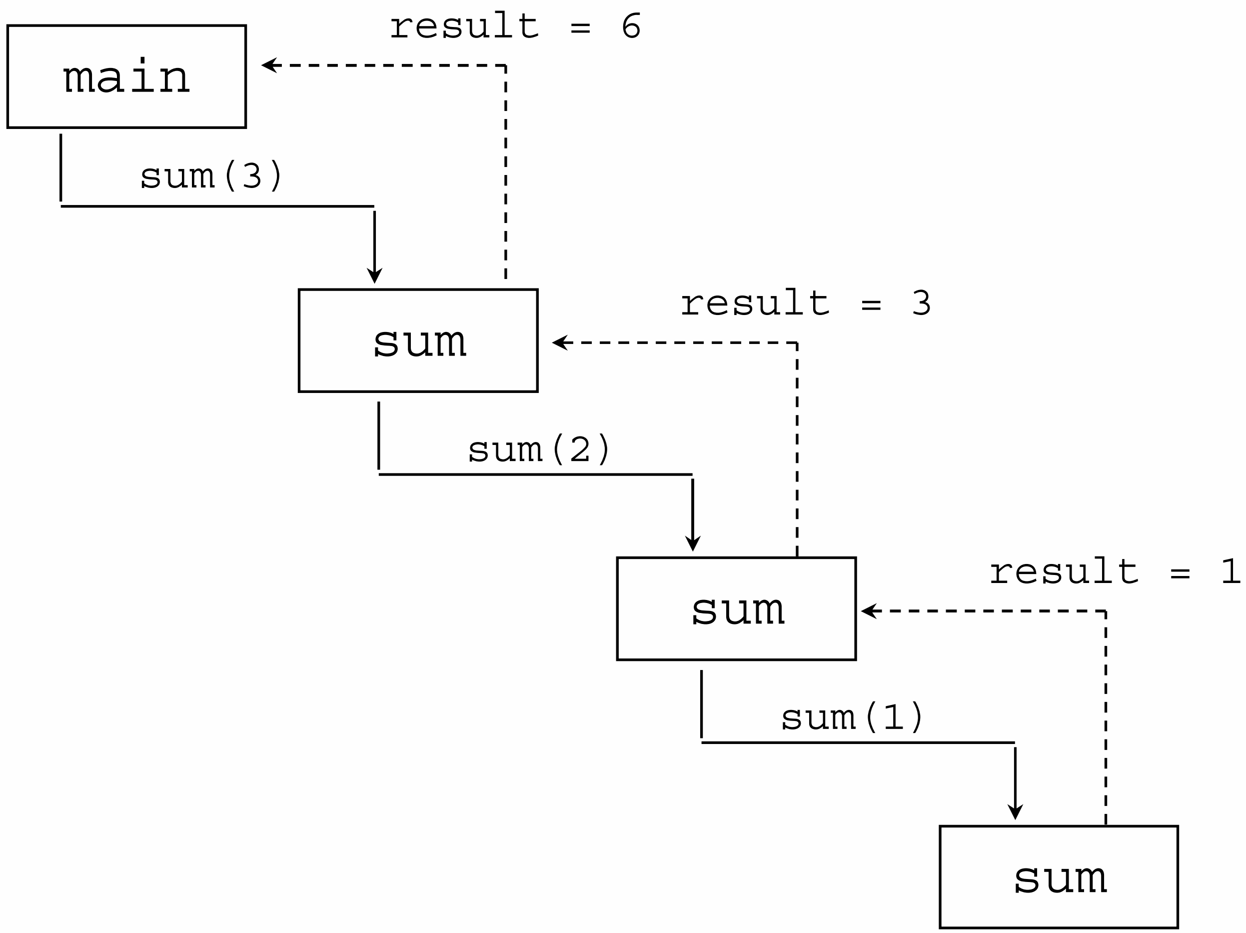
Recursive Programming

```
// This function returns the sum of 1 to num
int sum (int num)
{
    int result;

    if (num == 1)
        result = 1;
    else
        result = num + sum (n-1);

    return result;
}
```

Recursive Programming



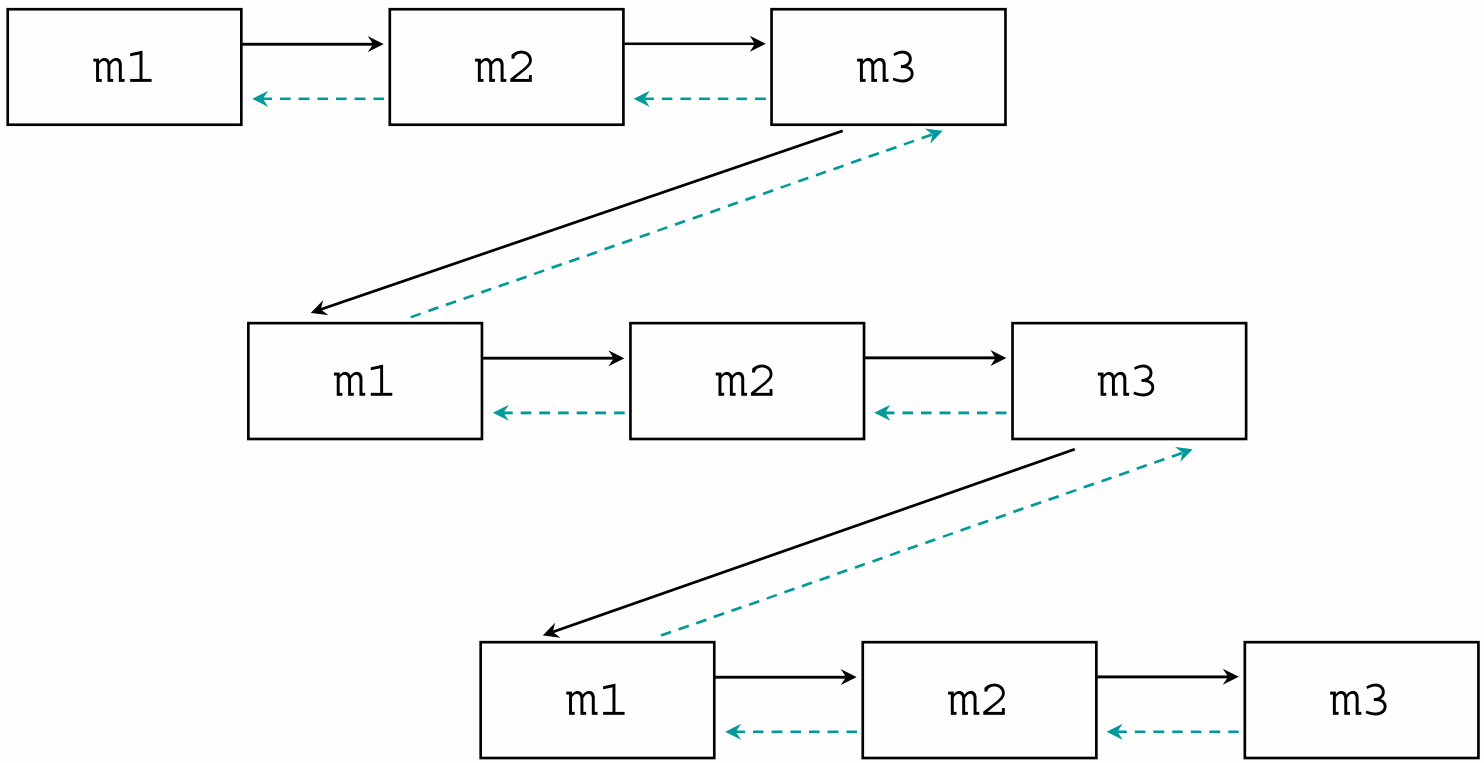
Recursive Programming

- Note that just because we can use recursion to solve a problem, doesn't mean we should
- For instance, we usually would not use recursion to solve the sum of 1 to N problem, because the iterative version is easier to understand
- However, for some problems, recursion provides an elegant solution, often cleaner than an iterative version
- You must carefully decide whether recursion is the correct technique for any problem

Indirect Recursion

- **A function invoking itself is considered to be *direct recursion***
- **A function could invoke another function, which invokes another, etc., until eventually the original function is invoked again**
- **For example, function `m1` could invoke `m2`, which invokes `m3`, which in turn invokes `m1` again**
- **This is called *indirect recursion*, and requires all the same care as direct recursion**
- **It is often more difficult to trace and debug**

Indirect Recursion

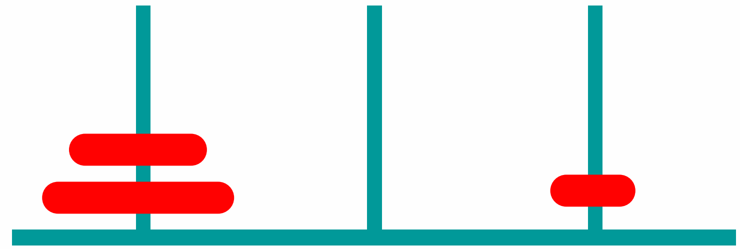


Towers of Hanoi

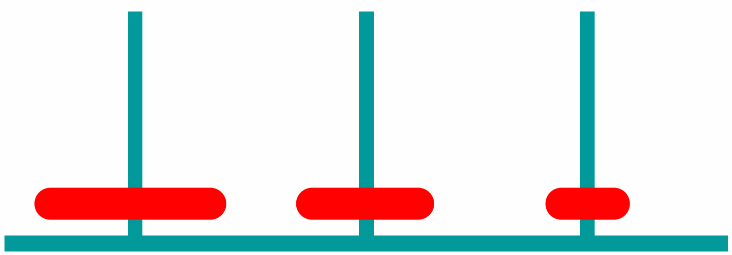
- The *Towers of Hanoi* is a puzzle made up of three vertical pegs and several disks that slide on the pegs
- The disks are of varying size, initially placed on one peg with the largest disk on the bottom with increasingly smaller ones on top
- The goal is to move all of the disks from one peg to another under the following rules:
  - We can move only one disk at a time
  - We cannot move a larger disk on top of a smaller one

Towers of Hanoi

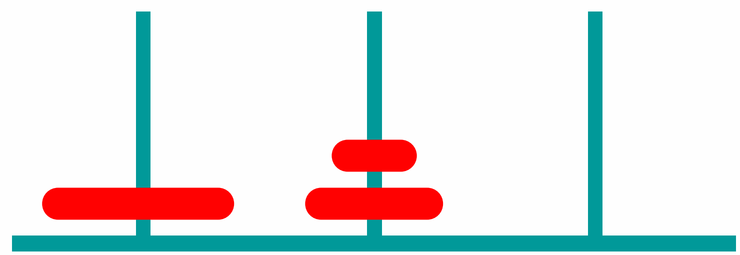
Original Configuration



Move 1

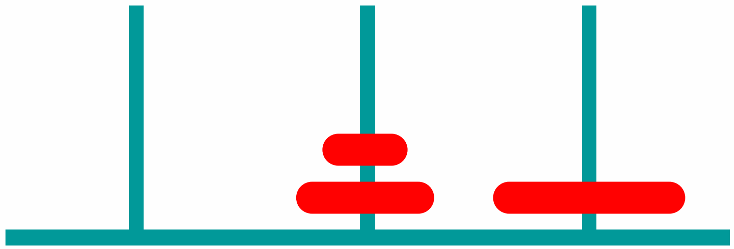


Move 2

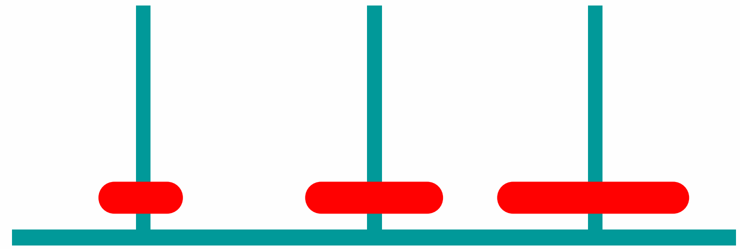


Move 3

Towers of Hanoi

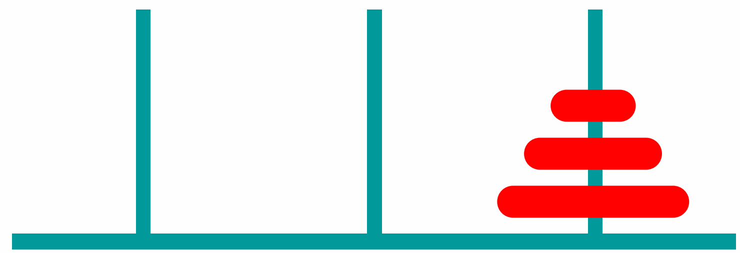


Move 4



Move 5

Move 6



Move 7 (done)

Towers of Hanoi

- **An iterative solution to the Towers of Hanoi is quite complex**
- **A recursive solution is much shorter and more elegant**

Towers of Hanoi

```
#include <stdio.h>
#include <conio.h>

void transfer(int,char,char,char);

int main()
{
        int n;
        printf("Recursive Solution to Towe of Hanoi Problem\n");
        printf("enter the number of Disks");
        scanf("%d",&n);
        transfer(n,'L','R','C');
        getch();
        return 0;
}
void transfer(int n,char from,char to,char temp)
{
        if (n>0)
        {
                transfer(n-1,from,temp,to);     /* Move n-1 disk from origin to temporary */
                printf("Move Disk %d from %c to %c\n",n,from,to);
                transfer(n-1,temp,to,from);     /* Move n-1 disk from temporary to origin */
        }
        return;
}
```

Drawbacks of Recursion

Regardless of the algorithm used, recursion has two important drawbacks:

- Function-Call Overhead
- Memory-Management Issues

Eliminating Recursion — Tail Recursion

A special kind of recursion is tail recursion.

- *Tail recursion* is when a recursive call is the last thing a function does.

Tail recursion is important because it makes the recursion → iteration conversion very easy.

- That is, we like tail recursion because it is easy to eliminate.
- In fact, tail recursion is such an obvious thing to optimize that some compilers automatically convert it to iteration.

Eliminating Recursion — Tail Recursion

For a void function, tail recursion looks like this:

```
void foo(TTT a, UUU b)
{
   …
   foo(x, y);
}
```

For a function returning a value, tail recursion looks like this:

```
SSS bar(TTT a, UUU b)
{
   …
   return bar(x, y);
}
```

A tail-recursive Factorial Function

We will use an auxiliary function to rewrite factorial as tail-recursive:

```
int factAux (int x, int result)
{
  if (x==0) return result;
  return factAux(x-1, result * x);
}
int tailRecursiveFact( int x)
{
    return factAux (n, 1);
}
```