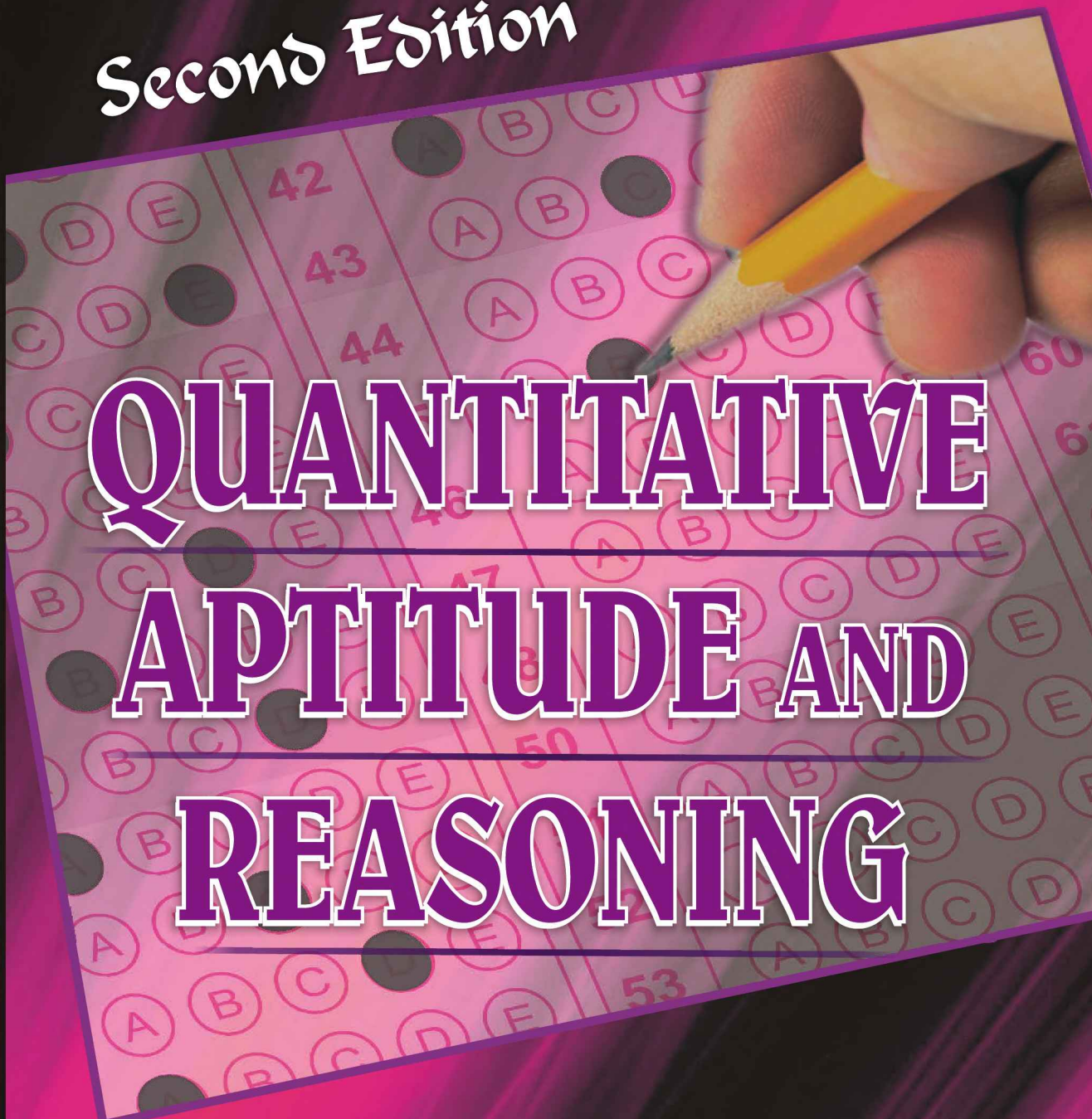


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QUANTITATIVE APTITUDE AND REASONING



R.V. Praveen

Quantitative Aptitude *and* Reasoning

Second Edition

R.V. Praveen

PHI Learning Private Limited

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QUANTITATIVE APTITUDE AND REASONING, Second Edition

R.V. Praveen

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Preface

This well-received book, now in its second edition, is revised as per the feedback received from students and readers.

This new edition contains five new chapters:

- Simplification
- Races and Games
- Stocks and Shares
- Discount
- Logarithm

Each chapter includes several new solved examples and exercises. Besides, it also includes previous year's questions that are asked in various competitive examinations such as Civil Services, Bank PO, CDS, MAT and so on. Many short-cut methods are used to solve objective type questions within a fraction of a minute.

The book is recommended to all engineering colleges in Kerala by the Director of Technical Education, Kerala. This volume is useful for aspirants of all graduate level competitive examinations in which speed and accuracy are the main factors to get success in it.

I have done this revision to the maximum extent possible within the limited time. I hope that this book will be a valuable resource for the students preparing for graduate level competitive examinations and campus recruitment test as well.

Constructive feedback from the readers for the improvement of this book is highly appreciated.

R.V. Praveen

Preface to the First Edition

Almost all competitive and job recruitment examinations have sections on quantitative aptitude and reasoning. Keeping its importance and the current pattern of examination in mind, this book caters to the needs of the students preparing for competitive examinations.

The book is organized in two parts: Part I (Quantitative Aptitude) and Part II (Reasoning). The reasoning section treats both verbal and nonverbal reasoning. The main aim of this text is to guide the students to solve the problems within the stipulated time and that too with a higher degree of accuracy. For ease of understanding, numerous notes and short-cut methods are given. Each chapter contains theory, illustrative examples, worked-out examples and exercises followed by necessary explanations. It contains numerous solved previous years' questions from various competitive examinations. Model test papers are also included to sharpen the analytical and logical thinking of the students.

This book is mainly intended exclusively for the aspirants of graduate level competitive examinations such as Civil Services, LIC—Assistant Administrative Officer (AAO), GIC—Assistant Manager, Common Admission Test (CAT), Management Aptitude Test (MAT), State Eligibility Test (SET), National Eligibility Test (NET), Bank Probationary Officer (PO), Staff Selection Commission (SSC), National Defence Academy (NDA), Combined Defence Services (CDS), Income Tax and Central Excise, etc. Besides, it would be equally useful for campus recruitment screening tests, career competency development courses as well as undergraduate students of Mathematics for their course in Aptitude Examination.

I extend my thanks to all those who took pains and interest in the publication of this book. Any constructive suggestion or feedback from the reader about the contents of the book or further improvement of the book is always welcome.

Finally, I thank PHI Learning for publishing this book.

R.V. Praveen

■ Part I

QUANTITATIVE APTITUDE

HCF and LCM

HIGHEST COMMON FACTOR (HCF): The highest common factor (HCF) of two or more numbers is the greatest number that divides each of them completely. There are two methods for finding the HCF of two or more numbers.

Split Method: In the split method, we split all the given numbers into prime factors and then take the product of all prime factors common to all the given numbers. The product is the HCF.

EXAMPLE 1 Find the HCF of 84 and 90.

Solution $84 = 2 \times 2 \times 3 \times 7$

$$90 = 2 \times 3 \times 3 \times 5$$

$$\text{HCF} = 2^1 \times 3^1 = 6$$

EXAMPLE 2 Find the HCF of 1365, 1584 and 1872.

Solution $1365 = 3 \times 5 \times 7 \times 13$

$$1584 = 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 11$$

$$1872 = 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 13$$

$$\text{HCF} = 3^1 = 3$$

EXAMPLE 3 Find the HCF of 792, 1155 and 1716.

Solution $792 = 2 \times 2 \times 2 \times 3 \times 3 \times 11$

$$1155 = 3 \times 5 \times 7 \times 11$$

$$1716 = 2 \times 2 \times 3 \times 11 \times 13$$

$$\text{HCF} = 3^1 \times 11^1 = 33$$



Note: Find out the prime factors of one of the given numbers and check which of those factors, by which the other numbers, are completely divisible. The product of these numbers is the HCF.

EXAMPLE 4 Find the HCF of 72 and 1452.

Solution The prime factors of 72 are 2, 2, 2, 3 and 3

i.e. $72 = 2 \times 2 \times 2 \times 3 \times 3$

of these, 1452 is divisible by $2 \times 2 \times 3$

$\therefore$ HCF is **12**

EXAMPLE 5 Find the HCF of 84 and 1584.

Solution $84 = 2 \times 2 \times 3 \times 7$

$$1584 = 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 11$$

$\therefore$ $\text{HCF} = 2^2 \times 3^1 = \mathbf{12}$

EXAMPLE 6 Find the HCF of 396 and 612.

Solution $396 = 2 \times 2 \times 3 \times 3 \times 11$

$$612 = 2 \times 2 \times 3 \times 3 \times 17$$

$\therefore$ $\text{HCF} = 2^2 \times 3^2 = \mathbf{36}$

Division Method: In the division method, we divide the greater number by the smaller number; then divide the divisor by the remainder; then divide the remainder with the next remainder and continue the process until no remainder is left. The last divisor in this process is the required HCF.

EXAMPLE 7 Find the HCF of 84 and 96.

Solution

$$\begin{array}{r} 84) 96 (1 \\ \underline{84} \\ 12) 84 (7 \\ \underline{84} \\ 0 \end{array}$$

$\therefore$ HCF = **12**

This method is based on the following two principles.

1. A number that divides another number also divides any multiple of that number. For example, 6 divides 24. Therefore, 6 divides any multiple of 24.
2. A number that divides each of the given two numbers, also divides their sum, difference and the sum and difference of any multiple of that numbers.

HCF of 84 and 210

$$= \text{HCF of } 84 \text{ and } 126 (126 = 210 - 84)$$

$$= \text{HCF of } 42 \text{ and } 84 (42 = 126 - 84)$$

$$= \text{HCF of } 42 \text{ and } 42 (42 = 84 - 42)$$

$\therefore$ The HCF of 84 and 210 = **42**

EXAMPLE 8 Find the HCF of 1672 and 374.

Solution 374) 1672 (4

$$\begin{array}{r}
 1496 \\
 \hline
 176 \overline{) 372} \text{ (2)} \\
 352 \\
 \hline
 20 \overline{) 176} \text{ (8)} \\
 160 \\
 \hline
 16 \overline{) 20} \text{ (1)} \\
 16 \\
 \hline
 4 \overline{) 16} \text{ (4)} \\
 16 \\
 \hline
 0
 \end{array}$$

$\therefore \text{HCF} = 4$

EXAMPLE 9 Find the HCF of 3672 and 2496.

Solution 2496) 3672 (1

$$\begin{array}{r}
 2496 \\
 \hline
 1176 \overline{) 2496} \text{ (2)} \\
 2352 \\
 \hline
 144 \overline{) 1176} \text{ (8)} \\
 1152 \\
 \hline
 24 \overline{) 144} \text{ (6)} \\
 144 \\
 \hline
 0
 \end{array}$$

$\therefore \text{HCF} = 24$

EXAMPLE 10 Find the HCF of 2648 and 3972.

Solution 2648) 3972 (1

$$\begin{array}{r}
 2648 \\
 \hline
 1324 \overline{) 2648} \text{ (2)} \\
 2648 \\
 \hline
 0
 \end{array}$$

$\therefore \text{HCF} = 1324$

HCF OF MORE THAN TWO NUMBERS: Find the HCF of any two of the given numbers and then find the HCF of this HCF and the third number. Continue this process and the last HCF will be the required HCF.

EXAMPLE 11 Find the HCF of 1266, 1461 and 1863.

Solution 1266) 1461 (1

$$\begin{array}{r}
 1266 \\
 195 \overline{) 1266} \text{ (6} \\
 \underline{1170} \\
 96 \overline{) 195} \text{ (2} \\
 \underline{192} \\
 3 \overline{) 96} \text{ (32} \\
 \underline{96} \\
 0
 \end{array}$$

∴ 3 is the HCF of 1266 and 1461.

$$\begin{array}{r}
 3 \overline{) 1863} \text{ (621} \\
 \underline{1863} \\
 0
 \end{array}$$

∴ The required HCF is 3.

 RULE

Method of finding the HCF may be simplified by the following rules:

1. Prime factors that are not common to the given numbers are rejected.
2. Common factors may be removed before the rule is applied. This factor should be multiplied with the HCF of the quotients.
3. Any factor of the divisor not contained in the dividend may be rejected.

EXAMPLE 12 Find the HCF of 84474 and 75582.

Solution $84474 = 2 \times 9 \times 4693$

$$75582 = 2 \times 9 \times 4199$$

Here, 2 and 9 are common factors.

∴ Find the HCF of 4693 and 4199 and then multiply the common factors with the HCF obtained.

$$\begin{array}{r}
 4199 \overline{) 4693} \text{ (1} \\
 \underline{4199} \\
 494
 \end{array}$$

494 is divisible by 2, but 4199 is not.

∴ Divide 494 by 2 and then proceed with 247 and 4199.

$$\begin{array}{r}
 247 \overline{) 4199} \text{ (17} \\
 \underline{247} \\
 1729 \\
 \underline{1729} \\
 0
 \end{array}$$

∴ The HCF of 4199 and 4693 is **247**.

∴ The HCF of the given numbers is $247 \times 9 \times 2 = \mathbf{4446}$.

$$\begin{array}{r} \text{or} \quad 494) 4199 (8 \\ \underline{3952} \\ 247) 494 (2 \\ \underline{494} \\ 0 \end{array}$$

EXAMPLE 13 Find the HCF of 372, 954 and 1728.

Solution

$$\begin{array}{r} 954) 1728 (1 \\ \underline{954} \\ 774) 774 (1 \\ \underline{774} \\ 180) 774 (4 \\ \underline{720} \\ 54) 54 (1 \\ \underline{54} \\ 0 \end{array}$$

$\therefore$ HCF of 1728 and 954 = 18.

Now, HCF of 372, 954 and 1728 = HCF of 372 and 18.

$$\begin{array}{r} 18) 372 (2 \\ \underline{36} \\ 12) 12 (1 \\ \underline{12} \\ 0 \end{array}$$

$\therefore$ HCF of 372, 954 and 1728 = 6

EXAMPLE 14 Find the HCF of 896, 1084 and 2372.

Solution

$$\begin{array}{r} 1084) 2372 (2 \\ \underline{2168} \\ 204) 204 (1 \\ \underline{204} \\ 0 \end{array}$$

$\therefore$ HCF of 2372 and 1084 = 4.

$\therefore$ HCF of 896, 1084 and 2372.

= HCF of 896 and 3.

$$\begin{array}{r} 3) 896 (298 \\ \underline{6} \\ 29 \\ \underline{27} \\ 26 \\ \underline{24} \\ 2) 3 (1 \\ \underline{2} \\ 1) 2 (2 \\ \underline{2} \\ 0 \end{array}$$

Hence, the HCF of 896, 1084 and 2372 = 1.

HCF OF DECIMALS: Make the same number of decimal places in all the given numbers; then find their HCF as if they are integers and mark off in the result as many decimal places as there are in each of the given numbers.

EXAMPLE 15 Find the HCF of 25.5, 42.5 and 170.

Solution The given numbers are equivalent to 255, 425 and 1700.

- (i) First, find the HCF of 255, 425 and 1700 which comes to 85 and then divide by 10.
- (ii) The required HCF is **8.5**.

EXAMPLE 16 Find the HCF of 36.54, 48.42 and 58.41.

Solution

- (i) The given numbers are equivalent to 3654, 4842 and 5841.
- (ii) Find the HCF of 3654, 4842 and 5841 which comes to about 9 and then divide the result so got by 100.
- (iii) Therefore, the required HCF is

$$\begin{array}{r} 4842) 5841 (1 \\ \underline{4842} \\ 999) 4842 (4 \\ \underline{3996} \\ 846) 999 (1 \\ \underline{846} \\ 153) 846 (5 \\ \underline{765} \\ 81) 153 (1 \\ \underline{81} \\ 72) 81 (1 \\ \underline{72} \\ 9) 72 (8 \\ \underline{72} \\ 0 \end{array}$$

$$\therefore \text{HCF} = 9$$

$$\therefore \text{Required HCF} = \text{HCF of } 3654 \text{ and } 9.$$

$$\begin{array}{r} 9 \overline{) 3654} \quad (46 \\ \underline{36} \\ 054 \\ \underline{54} \\ 0 \end{array}$$

$$\therefore \text{HCF} = 9$$

$$\therefore \text{Required HCF} = \frac{9}{100} = 0.09$$

EXAMPLE 17 Find the HCF of 78.9, 165.6 and 376.2.

Solution

- (i) The given numbers are equivalent to 789, 1656 and 3762.
- (ii) First find HCF of above three numbers and then divide the result by 10.
- (iii) Therefore, required HCF is

$$\begin{array}{r} 1656 \overline{) 3762} \quad (2 \\ \underline{3312} \\ 450 \overline{) 1656} \quad (3 \\ \underline{1350} \\ 306 \overline{) 450} \quad (1 \\ \underline{306} \\ 144 \overline{) 306} \quad (2 \\ \underline{288} \\ 18 \overline{) 144} \quad (8 \\ \underline{144} \\ 0 \end{array}$$

$$\therefore \text{HCF} = 18$$

$$\text{Now, Required HCF} = \text{HCF of } 789 \text{ and } 18.$$

$$\begin{array}{r} 18 \overline{) 789} \quad (43 \\ \underline{72} \\ 69 \\ \underline{54} \\ 15 \overline{) 18} \quad (1 \\ \underline{15} \\ 3 \overline{) 15} \quad (5 \\ \underline{15} \\ 0 \end{array}$$

$$\therefore \text{Required HCF} = \frac{3}{10} = 0.3$$

EXAMPLE 18 Find the HCF of 24.3, 68.4 and 127.8

Solution

- (i) The given numbers are equivalent to 243, 684 and 1278.
 (i) Find the HCF of the above three numbers and divide the result by 10.
 (i) Then the required HCF is

$$\begin{array}{r}
 684 \overline{)1278} \text{ (1)} \\
 \underline{684} \\
 594 \overline{)684} \text{ (1)} \\
 \underline{594} \\
 90 \overline{)594} \text{ (6)} \\
 \underline{540} \\
 54 \overline{)90} \text{ (1)} \\
 \underline{54} \\
 36 \overline{)54} \text{ (1)} \\
 \underline{36} \\
 18 \overline{)36} \text{ (2)} \\
 \underline{36} \\
 0
 \end{array}$$

Hence HCF = 18

The HCF of 243 and 18 is given by

$$\begin{array}{r}
 18 \overline{)243} \text{ (13)} \\
 \underline{18} \\
 63 \\
 \underline{54} \\
 9 \overline{)18} \text{ (2)} \\
 \underline{18} \\
 0
 \end{array}$$

$$\therefore \text{ Required HCF} = \frac{9}{10} = 0.9$$

HCF OF FRACTIONS: The HCF of two or more fractions is the highest fraction by which each of the fractions is completely divisible.

Express the given fractions in their lowest terms and then find the HCF given by the following formula

$$\text{HCF} = \frac{\text{HCF of Numerator}}{\text{LCM of Denominator}}$$

EXAMPLE 19 Find the HCF of $\frac{63}{9}$, $5\frac{6}{17}$ and $\frac{42}{51}$.

Solution $\frac{63}{9} = \frac{7}{1}$; $5\frac{6}{17} = \frac{91}{17}$ and $\frac{42}{51} = \frac{14}{17}$.

The fractions are $\frac{7}{1}$, $\frac{91}{17}$ and $\frac{14}{17}$.

$$\text{HCF} = \frac{\text{HCF of 7, 91 and 14}}{\text{LCM of 1, 17, 17}} = \frac{7}{17}$$

Each of the given numbers is completely divisible by $\frac{7}{17}$.

$$\therefore \text{Required HCF} = \frac{7}{17}.$$

EXAMPLE 20 Find the HCF of $\frac{31}{9}$, $2\frac{5}{7}$ and $3\frac{5}{11}$.

Solution $2\frac{5}{7} = \frac{19}{7}$; $3\frac{5}{11} = \frac{38}{11}$

Given fraction are $\frac{31}{9}$, $\frac{19}{7}$ and $\frac{38}{11}$.

$$\therefore \text{HCF} = \frac{\text{HCF of numerators}}{\text{LCM of denominators}} = \frac{\text{HCF of 31, 19 and 38}}{\text{LCM of 9, 7 and 11}} = \frac{1}{9 \times 7 \times 11} = \frac{1}{693}$$

Each of given fractions is completely divisible by $\frac{1}{693}$.

$$\therefore \text{Required HCF} = \frac{1}{693}$$

EXAMPLE 21 What is the greatest number that will divide 3600 and 2715 and leave remainders 9 and 6, respectively?

Solution On dividing 3600, 9 is left as remainder.

The required number must divide $(3600 - 9)$ completely.

Similarly, it must divide $(2715 - 6)$ or 2709 completely.

Hence, the required greatest number is the HCF of 3591 and 2709.

$$\begin{array}{r} 2709 \text{) } 3591 \text{ (1} \\ \underline{2709} \\ 882 \text{) } 2709 \text{ (3} \\ \underline{2646} \\ 63 \text{) } 882 \text{ (14} \\ \underline{63} \\ 252 \\ \underline{252} \\ 0 \end{array}$$

HCF of 3591 and 2709, is 63 which is the last divisor in the above process of division.

$$\therefore \text{HCF} = 63.$$

EXAMPLE 22 Find the greatest number which will divide 415, 748 and 1007 so as to leave a remainder 8 in each case?

Solution Required greatest number = HCF of $(415 - 8)$, $(748 - 8)$ and $(1007 - 8)$
i.e., HCF of 407, 740 and 999

$$\begin{array}{r} 740 \overline{) 999} (1 \\ 740 \\ \hline 259 \end{array}$$

$$\begin{array}{r} 259 \overline{) 740} (2 \\ 518 \\ \hline 222 \end{array}$$

$$\begin{array}{r} 222 \overline{) 259} (1 \\ 222 \\ \hline 37 \end{array}$$

$$\begin{array}{r} 37 \overline{) 222} (6 \\ 222 \\ \hline 0 \end{array}$$

$$\therefore \text{HCF of 37 and 407 is}$$

$$\begin{array}{r} 37 \overline{) 407} (11 \\ 37 \\ \hline 37 \\ 37 \\ \hline 0 \end{array}$$

$$\therefore \text{HCF is } 37.$$

HCF of 999 and 740 is 37.

Hence, the required greatest number is 37.

EXAMPLE 23 The numbers 12284 and 8655 when divided by a certain number of three digits, leave the same remainder. Find that number of three digits.

Solution The required number must be a factor of $(12284 - 8655)$ or 3629.

$$3629 = 19 \times 191$$

$\therefore$ 191 is the required number.

EXAMPLE 24 The product of two numbers is 5376 and their HCF is 16. Find the number.

Solution The numbers are multiples of 16. Let it be $16a$ and $16b$.

$$\therefore 16a \times 16b = 5376 \text{ or } ab = 21$$

$$\therefore a = 3; b = 7.$$

The numbers are 16×3 , 16×7 , i.e., **48** and **112**.

COMMON MULTIPLE: A common multiple of two or more numbers is a number which is exactly divisible by all of the given numbers.

LEAST COMMON MULTIPLE (LCM): The LCM of two or more given numbers is the least number which is exactly divisible by all the given numbers.

For example,

35 is a common multiple of 5 and 7.

70 is a common multiple of 5 and 7.

105 is a common multiple of 5 and 7.

But, 35 is the least common multiple (LCM) of 5 and 7.

The following are the methods to find the LCM of two or more given numbers.

Method of Prime Factors: Resolve the given numbers into prime factors and then take the product of the highest powers of all the factors and this product is the required LCM.

EXAMPLE 25 Find the LCM of 12, 16, 21 and 36.

$$\text{Solution } 12 = 2 \times 2 \times 3 = 2^2 \times 3$$

$$16 = 2 \times 2 \times 2 \times 2 = 2^4$$

$$21 = 3 \times 7$$

$$36 = 2 \times 2 \times 3 \times 3 = 2^2 \times 3^2$$

Here, the prime factors are 2, 3 and 7 and their highest powers are 2^4 , 3^2 and 7^1 .

$$\begin{aligned}
 \text{Therefore, the required LCM} &= 2^4 \times 3^2 \times 7 \\
 &= 16 \times 9 \times 7 \\
 &= \mathbf{1008}
 \end{aligned}$$

EXAMPLE 26 Find the LCM of 27, 36, 80 and 175.

$$\begin{aligned}
 \text{Solution} \quad 27 &= 3 \times 3 \times 3 &= 3^3 \\
 36 &= 2 \times 2 \times 3 \times 3 &= 2^2 \times 3^2 \\
 80 &= 2 \times 2 \times 2 \times 2 \times 5 &= 2^4 \times 5^1 \\
 175 &= 5 \times 5 \times 7 &= 5^2 \times 7^1
 \end{aligned}$$

Here, the prime factors in the highest powers occurring in the given numbers are 2^4 , 3^3 , 5^2 and 7^1 .

$$\begin{aligned}
 \text{Therefore, the required LCM} &= 2^4 \times 3^3 \times 5^2 \times 7^1 \\
 &= 16 \times 27 \times 25 \times 7 \\
 &= \mathbf{75600}
 \end{aligned}$$

EXAMPLE 27 Find the LCM of 42, 56, 91 and 126.

$$\begin{aligned}
 \text{Solution} \quad 42 &= 2 \times 3 \times 7 &= 2^1 \times 3^1 \times 7^1 \\
 56 &= 2 \times 2 \times 2 \times 7 &= 2^3 \times 7^1 \\
 91 &= 7 \times 13 &= 7^1 \times 13^1 \\
 126 &= 2 \times 3 \times 3 \times 7 &= 2^1 \times 3^2 \times 7^1
 \end{aligned}$$

Taking highest powers of all prime factors in the above expression, we get

$$\text{HCF} = 2^0 \times 7^1 \times 13^0 = 7$$

Division Method: Write down the numbers in a single line separated by comas, and then divide these numbers by a prime number which will divide at least two of the given numbers completely. Continue the division, till no two numbers are having common factors further. Those numbers which cannot be divided should be written as such during each division. Then the required LCM will be the product of all divisors and the numbers written in the last line.

EXAMPLE 28 Find the LCM of 15, 18, 54, 135 and 180.

Solution	2	15,	18,	54,	135,	180
	3	15,	9,	27,	135,	90
	3	5,	3,	9,	45,	30
	5	5,	1,	3,	15,	10
	3	1,	1,	3,	3,	2
	2	1,	1,	1,	1,	2
		1,	1,	1,	1,	1

$$\begin{aligned}
 \text{Hence, the required LCM} &= 2 \times 3 \times 3 \times 5 \times 3 \times 2 \\
 &= 2^2 \times 3^3 \times 5 \\
 &= \mathbf{540}
 \end{aligned}$$

EXAMPLE 29 Find the LCM of 18, 32, 72 and 96.

Solution

2	18,	32,	72,	96
2	9,	16,	36,	48
3	9,	8,	18,	24
3	3,	8,	6,	8
2	1,	8,	2,	8
2	1,	4,	1,	4
2	1,	2,	1,	2
	1,	1,	1,	1

$$\therefore \text{LCM} = 2 \times 2 \times 3 \times 3 \times 2 \times 2 \times 2 \times 1 \times 1 \times 1 \times 1 = 288$$



Note: The product of any two numbers is equal to the product of their HCF and LCM.

LCM of Decimals: First, make the same number of decimal places in all the given numbers and find their LCM as they are integers and mark in the result as many decimal places as there are in each of the given numbers.

EXAMPLE 30 Find the LCM of 0.8, 0.9 and 0.36.

Solution The given numbers are equivalent to 0.80, 0.90 and 0.36.

The LCM of 80, 90 and 36 = 720

$\therefore$ The required LCM = 7.2

EXAMPLE 31 Find the LCM of 0.72, 0.64 and 0.968.

Solution The given numbers are equivalent to 0.720, 0.640 and 0.968

The LCM of 720, 640 and 968 = 696960

$\therefore$ The LCM of 0.720, 0.640 and 0.968 = **696.96**

EXAMPLE 32 Find the LCM of 0.96, 1.28 and 0.84.

Solution The given numbers are equivalent to 96, 128 and 84. LCM of 96, 128 and 84 is given by

2	96,	128,	84
2	48,	64,	42
7	24,	32,	21
3	24,	32,	3
2	8,	32,	1
2	4,	16,	1
2	2,	8,	1
2	1,	4,	1
	1,	2,	1

$$\begin{aligned}
 \therefore \text{LCM} &= 2 \times 2 \times 7 \times 3 \times 2 \times 2 \times 2 \times 2 \times 1 \times 2 \times 1 \\
 &= 2^7 \times 3^1 \times 7^1 \\
 &= 128 \times 21
 \end{aligned}$$

LCM of 96, 128 and 84 = 2688

$$\therefore \text{LCM of } 0.96, 1.28 \text{ and } 0.84 = \mathbf{26.88}$$

LCM of Fractions: The LCM of two or more fractions is the least fraction or integer which is exactly divisible by each of them.

First, express the fractions in their lowest terms. Then the LCM of fraction is given by ratio of LCM of numerators to HCF of denominators of the given fraction.

$$\text{i.e.} \quad \text{LCM} = \frac{\text{LCM of Numerators}}{\text{HCF of Denominators}}$$

EXAMPLE 33 Find the LCM of

$$(i) \frac{2}{3}, \frac{6}{7} \quad (ii) \frac{7}{12}, \frac{16}{27} \quad (iii) \frac{8}{17}, \frac{18}{51}$$

Solution

$$(i) \text{ The required LCM} = \frac{\text{LCM of 2 and 6}}{\text{HCF of 3 and 7}} = \frac{6}{1} = \mathbf{6}$$

$$(ii) \text{ The required LCM} = \frac{\text{LCM of 7 and 16}}{\text{HCF of 12 and 27}} = \frac{\mathbf{112}}{\mathbf{3}}$$

$$(iii) \text{ The required LCM} = \frac{\text{LCM of 8 and 18}}{\text{HCF of 17 and 51}} = \frac{\mathbf{72}}{\mathbf{17}}$$

EXAMPLE 34 The LCM of two numbers is 5301 and their HCF is 9. If one of the numbers is 171, find the other number.

$$\text{Solution} \quad \text{The required number} = \frac{\text{LCM} \times \text{HCF}}{\text{First number}} = \frac{5301 \times 9}{171} = \mathbf{279}$$

EXAMPLE 35 What is the least number which when divided by 48, leaves 33 as the remainder and when divided by 78 leaves 63 as remainder, and when divided by 126 leaves 111 as the respective remainders?

Solution Here, $48 - 33 = 15$; $78 - 63 = 15$; $126 - 111 = 15$.

The remainder in each case is less than the divisor by 15. Therefore, if 15 is added to the required number, the resulting number will become exactly divisible by 48, 78 and 126, i.e., the required number is 15 less than the LCM of 48, 78 and 126.

The LCM of 48, 78 and 126 = 13, 104.

$$\therefore \text{The required number} = 13, 104 - 15 = \mathbf{13,089}$$

EXAMPLE 36 Find the greatest number of six digits which on being divided by 5, 6, 7 and 8 leaves remainders 3, 4, 5 and 6, respectively.

Solution The LCM of 5, 6, 7 and 8 = 840

The greatest number of 6 digits = 999999

On dividing 999999 by 840, the remainder will be 399.

Therefore, the greatest six digit number divisible by 840 is

$$999999 - 399 = \mathbf{999600}$$

Since $5 - 3 = 6 - 4 = 7 - 5 = 8 - 6 = 2$, the remainder in each case is less than the divisor by 2.

∴ The required number = $999600 - 2 = \mathbf{999598}$

EXAMPLE 37 What is the least multiple of 11, which when divided by 3, 4, 5, 6 and 7, leaves the remainder 2, 3, 4, 5 and 6 respectively?

Solution The LCM of 3, 4, 5, 6 and 7 = 420.

Since $3 - 2 = 4 - 3 = 5 - 4 = 6 - 5 = 7 - 6 = 1$, if you add one to the required number, the resulting number will be divisible by the given divisors.

But we require the number as a multiple of 11.

Assume the number to be $420 \times n - 1$

i.e. $(38 \times 11 + 2)n - 1$

∴ $38 \times 11n$ is divisible by 11, the number to be checked for divisibility by 11 is $2n - 1$.

The least value of $2n - 1$ to be divisible by 11 is 11

i.e. $2n - 1 = 11 \Rightarrow n = \frac{1 + 11}{2} = \mathbf{6}$

∴ The required number = $420 \times 6 - 1$
 $= \mathbf{2519}$

EXAMPLE 38 What is the least number which when divided by 12, 16 and 18 leaves 9 as remainder in each case, but when divided by 27 leaves no remainder.

Solution The number which is divisible by all the three given numbers is of the form $144k$, when k is an integer.

Here, the number leaves 9 as remainder on division

∴ Required number = $144k + 9$.

Given $(144k + 9) \div 27 = 0$

If $k = 5$, value of number will be equal to $144 \times 5 + 9 = 729$

Hence, required number = $\mathbf{729}$



EXERCISES

- The sum of two numbers is 468 and their HCF is 39. How many pairs of such numbers can be formed?
 (a) 11 (b) 22 (c) 33 (d) 36 (e) None of these
- Find the sum of the numbers between 500 and 700 such that when they are divided by 6, 8 and 12
 (i) it leaves no remainder
 (a) 3200 (b) 4600 (c) 5300 (d) 5400 (e) None of these
 (ii) it leaves 5 as remainder in each case
 (a) 4600 (b) 4447 (c) 4744 (d) 4848 (e) None of these

3. Is it possible to divide 1,000 into two parts such that their HCF is 15?
(a) Yes, it is possible (b) No, it is sometimes possible
(c) Yes, it may be possible (d) Never possible
4. In a school, 442 boys and 374 girls have been divided into the largest possible equal classes, so that each class of boys numbers the same as each class of girls. What is the number of classes?
(a) 16 (b) 18 (c) 20 (d) 24 (e) None of these
5. Find the greatest number of 4 digits and the least number of 5 digits that have 196 as their HCF.
(a) 9998, 10184 (b) 9996, 10192 (c) 9994, 10194 (d) 9998, 10196 (e) None of these
6. The product of two numbers is 4212 and their HCF is 9. Find the possible pair of numbers.
(a) 32, 121 (b) 36, 117 (c) 42, 114 (d) 45, 110 (e) None of these
7. The HCF and LCM of two numbers are 36 and 756 respectively. Find the possible pair of numbers.
(a) 72, 378 (b) 108, 252 (c) 144, 658 (d) 180, 622 (e) None of these
8. Find the maximum possible length which can be used to measure exactly the lengths 6 m, 4 m 75 cm, and 10 m 25 cm.
(a) 20 cm (b) 25 cm (c) 30 cm (d) 35 cm (e) None of these
9. The LCM of two numbers is 1008 and their HCF is 36. If one of the two numbers is 144, find the other number.
(a) 288 (b) 252 (c) 576 (d) 504 (e) None of these
10. Three bells ring together and they will ring at intervals of 6 seconds, 7 seconds and 8 seconds respectively. After what interval will they again ring together?
(a) 178 seconds (b) 174 seconds (c) 168 seconds (d) 164 seconds (e) None of these
11. Find the least number of square tiles required to pave the ceiling of a hall 17 m 55 cm long and 8 m 10 cm broad.
(a) 78 (b) 64 (c) 84 (d) 92 (e) None of these
12. Find the smallest whole number which is exactly divisible by 6, 8, 15, 18 and 24.
(a) 120 (b) 240 (c) 740 (d) 360 (e) None of these
13. Find the highest common factor of 36 and 84. [RRB]
(a) 4 (b) 6 (c) 12 (d) 18
14. Find the least common multiple of 24, 36 and 40. [RRB]
(a) 120 (b) 240 (c) 360 (d) 480
15. The ratio of two numbers is 3 : 4 and their HCF is 4. Their LCM is [SSC]
(a) 12 (b) 16 (c) 24 (d) 48
16. The product of two numbers is 4107. If the HCF of these numbers is 37, then the greater number is [SSC]
(a) 101 (b) 107 (c) 111 (d) 185
17. The HCF and LCM of two numbers are 21 and 84 respectively. If the ratio of the two numbers is 1 : 4, then the larger of the two numbers is [MAT]
(a) 12 (b) 48 (c) 84 (d) 108
18. The HCF of two numbers is 8. Which one of the following can never be their LCM ? [SSC]
(a) 24 (b) 48 (c) 56 (d) 60
19. Two numbers, both greater than 29, have HCF 29 and LCM 4147. The sum of the numbers is [SSC]
(a) 666 (b) 669 (c) 696 (d) 966
20. The least number which when divided by 5, 6, 7 and 8 leaves 3 as remainder, but when divided by 9 leaves no remainder, is [LIC AAO]
(a) 1677 (b) 1683 (c) 2523 (d) 3363

21. The least multiple of 7, which leaves a remainder of 4, when divided by 6, 9, 15 and 18 is [LIC AAO]
(a) 74 (b) 94 (c) 184 (d) 364
22. The least number which when divided by 12, 15, 20 and 54 leaves in each case a remainder of 8, is [RRB]
(a) 504 (b) 536 (c) 544 (d) 548
23. The smallest number which when diminished by 7, is divisible by 12, 16, 18, 21 and 28 is [LIC]
(a) 1008 (b) 1015 (c) 1022 (d) 1032
24. What will be the least number which when doubled will be exactly divisible by 12, 18, 21 and 30?
(a) 196 (b) 630 (c) 1260 (d) 2520
25. The greatest number which on dividing 1657 and 2037 leaves remainders 6 and 5 respectively is [RRB]
(a) 123 (b) 127 (c) 235 (d) 305
26. Let N be the greatest number that will divide 1305, 4665 and 6905, leaving the same remainder in each case. Then sum of the digits in N is [SSC]
(a) 4 (b) 5 (c) 6 (d) 8
27. Find the greatest number that will divide 43, 91 and 183 so as to leave the same remainder in each case. [LIC]
(a) 4 (b) 7 (c) 9 (d) 13
28. The maximum number of students among them 1001 pens and 910 pencils can be distributed in such a way that each student gets the same number of pens and same number of pencils is [SSC]
(a) 91 (b) 910 (c) 1001 (d) 1911
29. The greatest possible length which can be used to measure exactly the lengths 7 m, 3 m 85 cm, 12 m 95 cm is [RRB]
(a) 15 cm (b) 25 cm (c) 35 cm (d) 42 cm
30. The HCF and LCM of two numbers are 11 and 385 respectively. If one number lies between 75 and 125, then that number is [CBI]
(a) 77 (b) 88 (c) 99 (d) 110
31. The HCF of two numbers is 23 and the other two factors of their LCM are 13 and 14. Then larger of the two number is [SSC]
(a) 276 (b) 299 (c) 322 (d) 345
32. Product of two co-prime numbers is 117. Their LCM should be [CBI]
(a) 1 (b) 117
(c) equal to their HCF (d) cannot be calculated
33. If the sum of two numbers is 55 and the HCF and LCM of these numbers are 5 and 120 respectively, then the sum of the reciprocals of the numbers is equal to [CDS]
(a) $\frac{55}{601}$ (b) $\frac{601}{55}$ (c) $\frac{11}{120}$ (d) $\frac{120}{11}$
34. The LCM of two numbers is 495 and their HCF is 5. If the sum of the numbers is 100, then their difference is [SSC]
(a) 10 (b) 46 (c) 70 (d) 90
35. The HCF of two numbers is 11 and their LCM is 7700. If one of the numbers is 275, then the other is [Section Officer]
(a) 279 (b) 283 (c) 308 (d) 318

36. The product of two numbers is 2028 and their HCF is 13. The number of such pairs is [CBI]
 (a) 1 (b) 2 (c) 3 (d) 4
37. The sum of two numbers is 528 and their HCF is 33. The number of pairs of numbers satisfying the above conditions is [CBI]
 (a) 4 (b) 6 (c) 8 (d) 12
38. Three numbers are in the ratio 1 : 2 : 3 and their HCF is 12. The numbers are [Section Officer]
 (a) 4, 8, 12 (b) 5, 10, 15 (c) 10, 20, 30 (d) 12, 24, 36
39. The LCM of 22, 54, 108, 135 and 198 is [MBA]
 (a) 330 (b) 1980 (c) 5940 (d) 11880
40. HCF of $4 \times 27 \times 3125$, $8 \times 9 \times 25 \times 7$ and $16 \times 81 \times 5 \times 11 \times 49$ is [CBI]
 (a) 180 (b) 360 (c) 540 (d) 1260
41. $\frac{1095}{1168}$ when expressed in simplest form is [MBA]
 (a) $\frac{13}{16}$ (b) $\frac{15}{16}$ (c) $\frac{17}{26}$ (d) $\frac{25}{26}$
42. Three numbers which are co-prime to each other are such that the product of the first two is 551 and that of the last two is 1073. The sum of the three number is [SSC]
 (a) 75 (b) 81 (c) 85 (d) 89
43. The LCM of two numbers is 48. The numbers are in the ratio 2 : 3. The sum of the numbers is [SSC]
 (a) 28 (b) 32 (c) 40 (d) 64
44. Three numbers are in the ratio 3 : 4 : 5 and their LCM is 2400. Their HCF is [MBA]
 (a) 40 (b) 80 (c) 120 (d) 200
45. The LCM of two prime numbers x and y ($x > y$) is 161. The value of $3y - x$ is [SSC]
 (a) -2 (b) -1 (c) 1 (d) 2
46. The least number, which when divided by 48, 60, 72, 108 and 140 leaves 38, 50, 62, 98 and 130 as remainders respectively is [CBI]
 (a) 11115 (b) 15110 (c) 15120 (d) 15210
47. Let the least number of six digits, which when divided by 4, 6, 10 and 15, leave in each case the same remainder of 2, be N . The sum of the digits in N is [SSC]
 (a) 3 (b) 4 (c) 5 (d) 6
48. Find the smallest number of five digits exactly divisible by 12, 16, 18 and 27.
 (a) 10432 (b) 10342 (c) 10368 (d) 10638
49. Find the largest number of four digits exactly divisible by 14, 24, 27 and 32.
 (a) 9968 (b) 9048 (c) 6048 (d) 6408
50. The LCM of $2^2 \times 3^2 \times 5 \times 11$, $2^4 \times 3^4 \times 5^2 \times 7$ and $2^5 \times 3^3 \times 5^2 \times 7^2 \times 11$ is
 (a) $2^3 \times 3^2 \times 5$ (b) $2^5 \times 3^4 \times 5^2$
 (c) $2^3 \times 3^2 \times 5 \times 7 \times 11$ (d) $2^5 \times 3^4 \times 5^3 \times 7^2 \times 11$



1. (e) 2. (i) (d), (ii) (c) 3. (d) 4. (d) 5. (b) 6. (b) 7. (b) 8. (b) 9. (b)
 10. (c) 11. (a) 12. (d) 13. (c) 14. (c) 15. (d) 16. (c) 17. (c) 18. (d) 19. (c)

20. (b) 21. (d) 22. (d) 23. (b) 24. (b) 25. (b) 26. (a) 27. (a) 28. (a) 29. (c)
 30. (a) 31. (c) 32. (b) 33. (c) 34. (a) 35. (c) 36. (c) 37. (c) 38. (d) 39. (c)
 40. (a) 41. (b) 42. (c) 43. (c) 44. (a) 45. (a) 46. (b) 47. (c) 48. (c) 49. (c)
 50. (a)

Solutions with Necessary Explanation

1. $468 \div 39 = 12$

Therefore, 11 numbers are available and the number of pair of numbers having HCF = 39 is

given by $\frac{11+1}{2} = 6$

2. (i) The least number which is divisible by 6, 8 and 12 is the LCM of 6, 8 and 12 = 24.

The numbers greater than 500 and less than 700 which are divisible by 24 are

$$504, 528, 552, \dots, 696.$$

$$\begin{aligned}\text{The sum of the numbers} &= \frac{9}{2} (504 + 696) \\ &= 9 \times 600 = 5400\end{aligned}$$

- (ii) The number leaves remainder as 5 in each case, the required numbers are greater by 5 than the above numbers

$$\begin{aligned}\text{i.e., } &504 + 5, 528 + 5, \dots, 696 + 5 - 24 \\ &= 509, 533, \dots, 677.\end{aligned}$$

$$\begin{aligned}\text{The sum of the numbers} &= \frac{8}{2} (509 + 677) \\ &= 4 \times 1186 \\ &= 4744\end{aligned}$$

3. Since 1000 is not divisible by 15, it is not possible to divide 1000 into two parts such that their HCF may be 15. Hence the correct answer is (d).

4. Number of boys = 442; Number of girls = 374

As the number of boys in the class is same as that of girls.

$$\text{Strength of one class} = \text{HCF of } 442 \text{ and } 374 = 34$$

$$\therefore \text{Number of classes of boys} = 442 \div 34 = 13$$

$$\text{Number of classes of girls} = 374 \div 34 = 11$$

$$\therefore \text{Required number of classes} = 13 + 11 = 24$$

5. The greatest number of 4 digits = 9999

On dividing 9999 by 196, 3 will be left as remainder.

$$\therefore \text{The required greatest number} = 9996.$$

$$\text{Least number of 5 digits} = 10000$$

On dividing 10000 by 196, 4 will be left as remainder.

$$\therefore \text{The required 5 digit number} = 10000 - 4 + 196 = 10192$$

6. Let x and y be the two numbers and their HCF = 9; product = 4212.

$$\therefore x \times y = 9^2 \quad (\text{product in lower terms})$$

$$\text{i.e. } 4212 = 81 \quad (\text{product in lower terms})$$

$$\text{Product in lower terms} = 52$$

The possible values are (1, 52) or (4, 13).

∴ The required numbers are 4×9 and 13×9 , i.e. **36 and 117**.

7. The given HCF is 36 and LCM is 756.

$$\begin{aligned}\therefore \text{Product of two numbers} &= \text{HCF} \times \text{LCM} \\ &= 36 \times 756\end{aligned}$$

$$(\text{Product in lower terms}) \times (\text{HCF})^2 = \text{HCF} \times \text{LCM}$$

$$\text{Product in lower terms} = \frac{\text{LCM}}{\text{HCF}} = \frac{756}{36} = 21$$

The numbers are multiples of 3 and 7.

∴ The required numbers are 3×36 and 7×36 , i.e. **108, 252**.

8. Required maximum length = HCF of 6 m, 4 m 75 cm and 10 m 25 cm
i.e. the HCF of 600 cm, 475 cm and 1025 cm = **25 cm**

9. Let the second number be x .

$$x \times \text{the first number} = \text{HCF} \times \text{LCM}$$

$$\therefore x \times 144 = 36 \times 1008$$

$$\therefore x = \frac{36 \times 1008}{144} = 252$$

∴ Required number = **252**

10. Since the bells ring at the intervals of 6 seconds, 7 seconds and 8 seconds, respectively, next time they will ring together after an interval equal to the LCM of 6, 7 and 8 seconds.

$$\begin{aligned}\text{Therefore, the required time} &= \text{LCM of 6, 7 and 8 seconds} \\ &= \mathbf{168 \text{ seconds}}\end{aligned}$$

11. The maximum size of square tile
= HCF of length and breadth of hall
= HCF of 17 m 55 cm and 8 m 10 cm
= HCF of 1755 cm and 810 cm
= 135 cm

$$\text{The number of square tiles required} = \frac{\text{Area of hall}}{\text{Area of one tile}} = \frac{1755 \times 810}{(135)^2} = \mathbf{78}$$

12. Required number = LCM of 6, 8, 15, 18 and 24
= **360**

13. We have

$$36 = 2 \times 2 \times 3 \times 3$$

$$84 = 2 \times 2 \times 3 \times 7$$

$$\therefore \text{HCF} = 2 \times 2 \times 3 = \mathbf{12}$$

- 14.
2	24,	36,	40
2	12,	18,	20
2	6,	9,	10
3	3,	9,	5
	1,	3,	5

$$\therefore \text{LCM} = 2 \times 2 \times 2 \times 3 \times 3 \times 5 = \mathbf{360}$$

15. Let the numbers be $3x$ and $4x$.

HCF of above two numbers = x (given as 4)

$\therefore$ Number are 12 and 16.

LCM of 12 and 16 = **48**.

16. Let the numbers be $37x$ and $37y$.

$$\therefore 37x \times 37y = 4107$$

$$\Rightarrow x \times y = \frac{4107}{37 \times 37} = 3$$

$$\text{i.e. } x \times y = 1 \times 3 = 3$$

$$\therefore \text{Greater number} = 37y = 37 \times 3 = \mathbf{111}$$

17. Given HCF = 21; LCM = 84;

Let the numbers be $21x$ and $21y$.

$$\therefore \frac{21x}{21y} = \frac{1}{4}$$

$$\text{or } \frac{x}{y} = \frac{1}{4}$$

$$\therefore \text{Larger number} = 21 \times 4 = \mathbf{84}$$

18. Given HCF of two numbers = 8

LCM is completely divisible by HCF

$\therefore$ 24, 48 and 56 are completely divisible by 8, the value which can never be LCM = **60**.

19. Let the numbers be $29x$ and $29y$

$$\therefore \text{LCM} = 29 \times x \times y = 4147$$

$$\Rightarrow xy = \frac{4147}{29} = 143$$

Values of x and y can be (1, 143) and (11, 13).

Given that both numbers are greater than 29.

$\therefore$ Required numbers are 29×11 , 29×13 . i.e. 319, 377

$\therefore$ Sum of numbers = $319 + 377 = \mathbf{696}$

20. LCM of 5, 6, 7 and 8

$$\begin{array}{c|cccc} 2 & 5, & 6, & 7, & 8 \\ \hline & 5, & 3, & 7, & 4 \end{array}$$

$$= 2 \times 5 \times 3 \times 7 \times 4 = \mathbf{840}$$

$\therefore$ The required number = $840k + 3$.

For the number to be divisible by 9, $k = 2$

i.e., $840 \times 2 + 3 = \mathbf{1683}$

21. LCM of 6, 9, 15 and 18

$$\begin{array}{c|cccc} 2 & 6, & 9, & 15, & 18 \\ \hline 3 & 3, & 9, & 15, & 9 \\ \hline 3 & 1, & 3, & 5, & 3 \\ \hline & 1, & 1, & 5, & 1 \end{array}$$

$$= 2 \times 3 \times 3 \times 1 \times 1 \times 5 \times 1 = \mathbf{90}$$

Let the number be $90k + 4$.

$\therefore$ Number is divisible by 7, least value of $k = 4$.

$\therefore$ Required number = $90 \times 4 + 4 = 364$

22. LCM of 12, 15, 20 and 54

2	12,	15,	20,	54
2	6,	15,	10,	27
3	3,	15,	5,	27
5	1,	5,	5,	9
3	1,	1,	1,	9
	1,	1,	1,	3

$$= 2 \times 2 \times 3 \times 3 \times 3 \times 5 = 540$$

$\therefore$ Least number which leaves remainder 8 = $540 + 8 = 548$

23. LCM of 12, 16, 18, 21 and 28

2	12,	16,	18,	21,	28
2	6,	8,	9,	21,	14
3	3,	4,	9,	21,	7
7	1,	4,	3,	7,	7
2	1,	4,	3,	1,	1
	1,	2,	3,	1,	1

$$= 2 \times 2 \times 3 \times 7 \times 2 \times 2 \times 3 = 2^4 \times 3^2 \times 7 = 1008$$

$\therefore$ Smallest number diminished by 7 which is divisible by 12, 16, 18, 21 and 28

$$= 1008 + 7 = 1015$$

24. LCM of 12, 18, 21 and 30

2	12,	18,	21,	30
2	6,	9,	21,	15
3	3,	9,	21,	15
	1,	3,	7,	5

$$= 2 \times 2 \times 3 \times 3 \times 5 \times 7 = 1260$$

$$\therefore \text{Required number} = \frac{1260}{2} = 630$$

25. HCF of $(1657 - 6)$ and $(2057 - 5) = \text{HCF of } 1651 \text{ and } 2032$

$$\begin{array}{r}
 1651 \overline{) 2032} \quad (1 \\
 \underline{1651} \\
 381 \\
 \underline{381} \\
 0 \\
 \underline{0} \\
 0 \\
 \underline{0} \\
 0
 \end{array}$$

$\therefore \text{HCF} = 127$

26. $N = \text{HCF of } (6905 - 4665), (4665 - 1305) \text{ and } (6905 - 1305)$

i.e. $N = \text{HCF of } 2240, 3360 \text{ and } 5600 = 1120$

$\therefore$ Sum of digits of $N = 1 + 1 + 2 + 0 = 4$

27. Required number = HCF of $(91 - 43), (183 - 91) \text{ and } (183 - 43)$

= HCF of 48, 92 and 140 = 4

28. HCF of 1001 and 910 = 91

$$\begin{array}{r} 910 \overline{) 1001} \quad (1 \\ \underline{910} \\ 910 \quad (10 \\ \underline{910} \\ 0 \end{array}$$

$\therefore$ Required number of students = 91

29. Required length = HCF of 7 m, 3 m 85 cm and 12 m 95 cm

i.e. HCF of 700 cm, 385 cm, and 1295 cm

$$\begin{array}{r} 385 \overline{) 700} \quad (1 \\ \underline{385} \\ 315 \quad (1 \\ \underline{315} \\ 0 \end{array}$$

HCF = 35

HCF of 1295 and 35 = 35

$$\begin{array}{r} 35 \overline{) 1295} \quad (37 \\ \underline{705} \\ 245 \\ \underline{245} \\ 0 \end{array}$$

30. Given HCF = 11; LCM = 385;

$$385 = 5 \times 7 \times 11$$

$\therefore$ Number lies between 75 and 125 = $7 \times 11 = 77$

31. Given HCF = 23; LCM = $13 \times 14 \times 23$;

$\therefore$ Larger of the two numbers = $14 \times 23 = 322$

32. HCF of co-prime numbers = 1

Product of co-prime numbers = 117

$$\therefore \text{LCM of co-prime numbers} = \frac{117}{1} = 117$$

33. Let the numbers be x and y .

Given, $x + y = 55$ (1)

HCF = 5; LCM = 120;

We have $\text{HCF} \times \text{LCM} = x \times y$

i.e. $5 \times 120 = xy$ (2)

$$\begin{aligned} \text{i.e.} \quad x + y &= 55 & (1) \\ xy &= 600 & (2) \end{aligned}$$

$$\therefore \frac{1}{x} + \frac{1}{y} = \frac{y+x}{xy} = \frac{55}{600} = \frac{11}{120}$$

34. Let the numbers be x and y .

$$\text{Given} \quad x + y = 100 \quad (1)$$

LCM = 495; HCF = 5;

$$\therefore \text{LCM} \times \text{HCF} = xy$$

$$\text{i.e.,} \quad 495 \times 5 = xy$$

$$\text{or} \quad xy = 495 \times 5 \quad (2)$$

$$x - y = ?$$

$$\begin{aligned} (x-y)^2 &= (x+y)^2 - 4xy \\ &= (100)^2 - 4 \times 495 \times 5 \\ &= (100)^2 - 4 \times 5^2 \times 99 \\ &= 10^2 (10^2 - 99) = 100 \end{aligned}$$

$$\therefore x - y = 10 \quad (3)$$

$\therefore$ Difference between numbers = 10

35. Given HCF = 11; LCM = 7700; $x = 275$; $y = ?$

$$\text{We have,} \quad x \times y = \text{HCF} \times \text{LCM}$$

$$\text{i.e.,} \quad 275 \times y = 11 \times 7700$$

$$\therefore y = \frac{11 \times 7700}{275} = \frac{7700}{25} = 77 \times 4 = 308$$

36. Given $x \times y = 2028$; HCF = 13;

$$\therefore \text{LCM} = \frac{x \times y}{\text{HCF}} = \frac{2028}{13} = 156$$

Let $x = 13a$ and $y = 13b$

$$\therefore x \times y = 13 \times 13 \ ab = 13 \times 156 = 13 \times 13 \times 12$$

$$\therefore ab = 12$$

Factors of 12 are (1, 12), (2, 6) and (3, 4).

There are 3 such pairs of numbers.

37. Let the numbers be $33a$ and $33b$.

$$\therefore 33a + 33b = 528$$

$$\text{i.e.,} \quad a + b = \frac{528}{33} = 16$$

$\therefore$ Values of (a, b) can be (1, 15), (2, 14), (3, 13), (4, 12), (5, 11), (6, 10), (7, 9) and (8, 8)

$\therefore$ Number of pairs of numbers satisfying the given condition = 8

38. Given ratio of numbers = 1 : 2 : 3;

Their HCF = 12

$\therefore$ Numbers are 12, 24 and 36. Hence the correct answer is (d).

39. $\text{LCM} = 2^2 \times 3^3 \times 5 \times 11 = 5940$

2	22,	54,	108,	135,	198
2	11,	27,	54,	135,	99
3	11,	27,	27,	135,	99
3	11,	9,	9,	45,	33
3	11,	3,	3,	15,	11
5	11,	1,	1,	5,	11
11	11,	1,	1,	1,	11
	1,	1,	1,	1,	1

40. $4 \times 27 \times 3125 = 2^2 \times 3^3 \times 5^5$
 $8 \times 9 \times 25 \times 7 = 2^3 \times 3^2 \times 5^2 \times 7$
 $16 \times 81 \times 5 \times 11 \times 49 = 2^4 \times 3^4 \times 5 \times 11 \times 7^2$
 $\therefore \text{HCF of above three numbers} = 2^2 \times 3^2 \times 5 = 180$

41. $\frac{1095}{1168} = \frac{15}{16}$

42. Let co-prime numbers be x, y and z .
 Given $xy = 551$ and $yz = 1073$.

$$\frac{yz}{xy} = \frac{1073}{551};$$

$$\begin{array}{r} 551 \overline{) 1073} \quad (1 \\ \underline{551} \\ 522 \overline{) 551} \quad (1 \\ \underline{522} \\ 29 \overline{) 522} \quad (18 \\ \underline{29} \\ \underline{232} \\ \underline{232} \\ \underline{0} \end{array}$$

HCF = 29

$$\frac{z}{x} = \frac{1073}{551} = \frac{37}{19}$$

and $y = \text{HCF} = 29$.

$$\therefore x + y + z = 19 + 29 + 37 = 85$$

43. $\text{LCM} = 48$; Ratio of numbers = $2 : 3$.

Let numbers be $2x$ and $3x$.

$$\therefore \text{LCM of numbers} = 2 \times 3 \times x = 48$$

$$\therefore x = 8$$

$$\therefore \text{Sum of numbers} = 2x + 3x = 5x = 5 \times 8 = 40$$

44. Given $\text{LCM of three numbers} = 2400$;

Ratio of numbers = $3 : 4 : 5$.

$$\begin{array}{r} 29 \overline{) 1073} \quad (37 \\ \underline{87} \\ \underline{203} \\ \underline{203} \\ \underline{0} \end{array}$$

$$\begin{array}{r} 29 \overline{) 551} \quad (19 \\ \underline{29} \\ \underline{261} \\ \underline{261} \\ \underline{0} \end{array}$$

Let numbers be $3x$, $4x$ and $5x$.

$$\therefore \text{LCM of numbers} = 3 \times 4 \times 5 \times x = 2400$$

$$\therefore x = \frac{2400}{60} = 40$$

i.e. HCF of numbers = 40

45. Given LCM of two prime numbers x and $y = 161$

$$\therefore \text{HCF of } x \text{ and } y = 1,$$

$$\text{LCM} = x \times y = 161;$$

Factorising, we get

$$x \times y = 161 = 7 \times 23$$

$$\therefore x = 23; y = 7$$

($\because x > y$)

$$\therefore 3y - x = 3 \times 7 - 23 = -2$$

46. $\therefore$ Difference between divisor and remainder = 10,

$$\text{Required number} + 10 = \text{LCM of } 48, 60, 72, 108 \text{ and } 140$$

$$\text{LCM} = 2^4 \times 3^3 \times 5 \times 7 = 15120$$

$$\therefore \text{Required number} = \text{LCM} - 10 = 15120 - 10 = 15110$$

2	48,	60,	72,	108,	140
2	24,	30,	36,	54,	70
2	12,	15,	18,	27,	35
2	6,	15,	9,	27,	35
3	3,	15,	9,	27,	35
3	1,	5,	3,	9,	35
5	1,	5,	1,	3,	35
	1,	1,	1,	3,	7

47. LCM of 4, 6, 10 and 15 = $2^2 \times 3^2 \times 5 = 60$

2	4,	6,	10,	15
5	2,	3,	5,	15
3	2,	3,	1,	3
	2,	1,	1,	1

Least number of six digits = 100000

Least number of six digits divisible by above 4 numbers

$$= 100000 - 40 + 60 = 100020$$

$$60) 100000 \text{ (1666}$$

$$\begin{array}{r} 60 \\ \underline{60} \\ 400 \\ \underline{360} \\ 400 \\ \underline{360} \\ 400 \\ \underline{360} \\ 40 \end{array}$$

$\therefore$ Number leaves a remainder 2.

$\therefore$ Required number = $100020 + 2 = \mathbf{100022}$

$\therefore$ Sum of digits of number = $1 + 2 + 2 = 5$

48. Smallest number of five digits = 10000

LCM of 12, 16, 18 and 27

2	12,	16,	18,	27
2	6,	8,	9,	27
2	3,	4,	9,	27
3	3,	2,	9,	27
3	1,	2,	3,	9
	1,	2,	1,	3

$$= 2^4 \times 3^3 = \mathbf{432}$$

432) 10000 (23

$$\begin{array}{r} 864 \\ 1360 \\ 1296 \\ \hline 64 \end{array}$$

Smallest number of five digits exactly divisible by above numbers = $10000 - 64 + 432 = \mathbf{10368}$

49. Largest number of four digits = 9999

LCM of 14, 24, 27 and 32

2	14,	24,	27,	32
2	7,	12,	27,	16
2	7,	6,	27,	8
2	7,	3,	27,	4
3	7,	3,	27,	2
3	7,	1,	9,	2
	7,	1,	3,	2

$$= 2^5 \times 3^3 \times 7 = \mathbf{6048}$$

$\therefore$ Largest number of four digits exactly divisible by 14, 24, 27 and 32 = $\mathbf{6048}$

50. LCM of $2^2 \times 3^2 \times 5 \times 11$, $2^4 \times 3^4 \times 5^2 \times 7$ and $2^5 \times 3^2 \times 5^2 \times 7^2 \times 11$

$$= 2^2 \times 3^2 \times 5 = \mathbf{180}$$

Permutation and Combination

PERMUTATION: Different arrangements which can be made by taking some or all of the given things or objects at a time, is called *permutation*.

${}^nP_r$ denotes the number of permutations of n different things, taken r at a time. P denotes permutation.

$$\begin{aligned} {}^nP_r &= \frac{n!}{(n-r)!} = \frac{1 \times 2 \times 3 \times \cdots \times n}{1 \times 2 \times 3 \times \cdots \times (n-r)} \\ &= \frac{1 \times 2 \times 3 \times \cdots \times (n-r)(n-r+1)(n-r+2) \times \cdots \times n}{1 \times 2 \times 3 \times \cdots \times (n-r)} \\ &= (n-r+1)(n-r+2) \cdots \times n \end{aligned}$$

EXAMPLE 1 Find ${}^6P_2$.

Solution ${}^6P_2 = \frac{6!}{(6-2)!} = 6 \times 5 = 30$

EXAMPLE 2 Find ${}^8P_3$.

Solution ${}^8P_3 = \frac{8!}{(8-3)!} = 8 \times 7 \times 6 = 336$

FACTORIAL NOTATION: The product of n consecutive positive integers beginning with 1 is denoted by $n!$ or $\lfloor n$ and is read as factorial n .

$\therefore$ $n! = 1 \times 2 \times 3 \times \cdots \times (n-2) \times (n-1) \times n$
For example,

$$5! = 5 \times 4 \times 3 \times 2 \times 1 = 120$$

$$4! = 4 \times 3 \times 2 \times 1 = 24$$

If r and n are two positive integers such that $r < n$, then

$$\begin{aligned}\frac{n!}{r!} &= \frac{n \times (n-1)(n-2) \cdots \times 2 \times 1}{r \times (r-1)(r-2) \cdots \times 2 \times 1} \\ &= n \times (n-1) \times (n-2) \times \cdots \times (r+1)\end{aligned}$$

EXAMPLE 3 Find $6!$

Solution $6! = 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 720$

EXAMPLE 4 Find $7!$

Solution $7! = 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 5040$

COMBINATION: Different selections or groups which can be made by taking some or all of a number of given things or objects at a time, is called the *combination*.

${}^nC_r$ denotes the number of combinations of n different things taken r at a time.

$${}^nC_r = \frac{n!}{r!(n-r)!} = \frac{n(n-1)(n-2) \cdots (n-r+1)}{1 \times 2 \times 3 \times \cdots \times r}$$



Note: Total number of arrangements = Total number of groups or selection $\times r!$ where r is the number of objects in each group or selection.

So,

$${}^nP_r = {}^nC_r \times r!$$

For example,

$${}^5P_3 = {}^5C_3 \times 3!$$

ARRANGEMENTS: We have 5 different objects P, Q, R, S , and T . We have to form a group of 3 objects out of these 5 objects. We can have 10 such groups:

(i) P, Q and R ,	(ii) P, Q and S ,	(iii) P, Q and T ,
(iv) P, R and S ,	(v) P, R and T ,	(vi) P, S and T ,
(vii) Q, R and S ,	(viii) Q, R and T ,	(ix) R, S and T ,
(x) Q, S and T .		

$$\therefore \text{Total number of such groups} = {}^5C_3 = \frac{5!}{3!(5-3)!} = \frac{5!}{3! \times 2!} = \frac{5 \times 4}{1 \times 2} = 10$$

These three objects in each of these groups can be arranged in 6 different ways.

$$\therefore \text{Total number of arrangements} = \text{Total number of groups} \times r!$$

where r is the number of objects in the group.

In the above example, total number of arrangements = $10 \times 3! = 60$ ways.

${}^5P_3$ denotes the number of permutations or arrangements of 5 different things taken 3 at a time and

$${}^5P_3 = \frac{5!}{(5-3)!} = \frac{5!}{2!} = 5 \times 4 \times 3 = 60$$

EXAMPLE 5 You are given with four different colour pencils. You have to select two of them at a time. In how many ways, you can make such selection?

Solution Required number of selections = ${}^4C_2 = \frac{4 \times 3}{1 \times 2} = 6$

EXAMPLE 6 You are supplied with six different coloured balls. You have to select three of them at a time. In how many ways, you can make such selection?

Solution Required number of selections = ${}^6C_3 = \frac{6 \times 5 \times 4}{1 \times 2 \times 3} = 20$

EXAMPLE 7 In how many ways can the letters of the word 'PERFECT' be arranged?

Solution Since the word contains 7 characters, its letters can be arranged in $7!$ ways. It contains two number of E. Therefore, you have to divide the above value by $2!$.

$$\therefore \text{Required number of arrangements} = \frac{7!}{2!}$$

$$\frac{7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1}{2 \times 1} = 7 \times 6 \times 5 \times 4 \times 3 = 2520$$

EXAMPLE 8 In how many ways can the letters of the word 'ENGINEER' be arranged?

Solution Since the word contains 8 characters, its letters can be arranged in $8!$ ways.

It contains three E's and two N's. Therefore, you have to divide the above value by $(2! \times 2!)$.

$$\begin{aligned} \therefore \text{Required number of arrangements} &= \frac{8!}{2! \times 2!} = \frac{8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1}{2 \times 1 \times 2 \times 1} \\ &= 2 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 \\ &= 2 \times 5040 = 10080 \end{aligned}$$

FUNDAMENTAL PRINCIPLES OF COUNTING

Multiplication Rule: A person wants to go from place X to place Z through place Y .

From X , the person can go to Y either by bus or by train only. From Y , the person can go to Z either by train, bus, or aeroplane. There is no direct transport facility from place X to place Z . We have to find the maximum possible number of ways by which a person can travel from X to Z .

Here the person can go from X to Y in two ways (i.e. either by bus or by train) and he can go to Z from Y in three ways (i.e. either by bus or by train, or by aeroplane). Therefore, a person can reach Z from X in $2 \times 3 = 6$ ways in all.

$\therefore$ Maximum number of possible ways is 6.

Addition Rule: If there are two events and each event can be completed in m and n ways (maximum possible), then the maximum number of possible ways by which both events are completed will be $m + n$ ways.

For example, there are 20 men and 12 women in a conference. Each man and woman shakes their hands only among them. Find the maximum number of handshakes that has taken place at the conference.

Case 1: Total number of handshakes among 20 men.

$${}^{20}C_2 = \frac{20!}{2!(20-2)!} = \frac{20 \times 19}{1 \times 2} = 190$$

Case 2: Total number of handshakes among 16 women

$${}^{16}C_2 = \frac{16!}{2!(16-2)!} = \frac{16 \times 15}{1 \times 2} = 120$$

$\therefore$ Maximum number of handshakes = $190 + 120 = 310$



Notes:

1. The number of arrangements of n different things taken all at a time = $n!$
2. The number of permutations or arrangements of n different things taken r at a time when each thing can be repeated any number of times is

(a) $\frac{n!}{(n-r)!}$ (when repetition is not allowed.)

(b) n^r (when repetition is allowed.)

EXAMPLE 9 Suppose 4 digits 2, 4, 6, and 9 are given. You have to make a four digit number. In how many ways it can be done, if

- (i) repetition of digits is allowed.
- (ii) repetition of digits is not allowed.

Solution

(i) Required number of 4 digit numbers $n^r = 4^4 = 256$

(ii) Required number of 4 digit numbers = $\frac{n!}{(n-r)!} = \frac{4!}{(4-4)!} = 24$



Important Results

1. If ${}^nC_x = {}^nC_y$, then either $x = y$ or $x + y = n$.
2. The number of permutations of n things out of which p are alike and are of one type, q are alike and are of the other type and the remaining are different.

$$\frac{n!}{p!q!}$$

3. The number of selections of r things ($r \leq n$) out of n identical things is 1.
4. The total number of selections of zero or more things from n identical things = $n + 1$.
5. The total number of selections of zero or more things from n different things

$$= {}^nC_0 + {}^nC_1 + {}^nC_2 + \dots + {}^nC_n = 2^n.$$

6. The number of ways to distribute (or divide) n identical things among r persons, where any person may get any number of things

$${}^{n+r-1}C_{r-1}$$

EXAMPLE 10 How many numbers of five digits can be formed with the digits 2, 4, 6, 8 and 9, provided that the repetition of digits is not allowed?

Solution Given number of digits = 5

$\therefore$ Required number = ${}^5P_5 = 5! = 120$



Note: If repetition of digits was allowed, then required number = $5^5 = 3125$.

EXAMPLE 11 How many numbers of 4 digits can be formed with the digits 0, 1, 3, 4 and 6?

Solution Repetition of digit is not allowed as it is not mentioned in the question.

5	4	3	2	1

Box 5 cannot be filled with zero as the number becomes invalid.

∴ Box 5 can be filled in ${}^4P_1 = 4$ ways.

Box 4 to 1 can be filled with remaining 4 numbers in $4!$ ways $= 4 \times 3 \times 2 \times 1 = 24$ ways

∴ Required number of numbers $= 4 \times 24 = \mathbf{96}$

EXAMPLE 12 How many even numbers of three digits can be formed with the digits 0, 1, 2, 3, 4 and 6?

Solution Case 1: When 0 occurs at unit place.

4	5	1
3	2	1

Here, Box 1 is filled with zero.

i.e. in 1 way.

Box 2 can be filled in ${}^5P_1$ ways $= 5$ ways

Box 3 can be filled in ${}^4P_1$ ways $= 4$ ways

∴ Required number of numbers $= 1 \times 5 \times 4 = \mathbf{20}$

Case 2: When 0 does not occur at the unit place.

4	4	3
3	2	1

Here Box 1 can be filled in ${}^3P_1 = 3$ ways

Box 3 can be filled in ${}^4P_1 = 4$ ways

(∵ 0 cannot be filled in that box.)

Box 2 can be filled in ${}^4P_1 = 4$ ways

(∵ this box can be filled with remaining numbers.)

∴ Required number $= 4 \times 4 \times 3 = \mathbf{48}$

The total number of such numbers $= 20 + 48 = \mathbf{68}$

EXAMPLE 13 How many numbers greater than 2000 and less than 7,000 can be made with the digits 0, 1, 2, 4, 5, 6, 7 and 9, provided that the repetition of numbers is not allowed?

Solution Here box 1 can be filled

1	2	3	4

in ${}^4P_1 = 4$ ways [as 0, 1, 7 and 9 are to be excluded from filling in that box as per the conditions given].

Box 2 can be filled in ${}^7P_1 = 7$ ways

Box 3 can be filled in ${}^6P_1 = 6$ ways

Box 4 can be filled in ${}^5P_1 = 5$ ways

∴ Required number of numbers $= 4 \times 7 \times 6 \times 5 = \mathbf{840}$

EXAMPLE 14 How many words beginning with vowels can be formed with letters of the word 'EQUATOR'?

Solution There are 4 vowels in the word 'EQUATOR'

Box 1 can be filled in ${}^4P_1 = 4$ ways

Remaining boxes (2–7) can be filled in $6! = 720$ ways.

∴ Required number = $4 \times 720 = 2880$

1	2	3	4	5	6	7

EXAMPLE 15 In a party every person shakes hands with every other person. If there was a total of 171 handshakes in the party, find the number of persons who were present in the party.

Solution One handshake takes place between two persons. Therefore, the number of handshakes

$$= {}^nC_2 \text{ where } n \text{ is the number of persons}$$

$$\therefore {}^nC_2 = 171 \text{ (given)}$$

$$\text{i.e. } \frac{n(n-1)}{1 \times 2} = 171$$

$$n(n-1) = 171 \times 2 = 342 = 19 \times 18$$

$$\therefore n = 19$$

i.e. number of persons who was present in the party = **19**.

EXAMPLE 16 How many words can be formed with the letters of the word 'ENTERPRISE'?

Solution There are 10 letters in the word 'ENTERPRISE', out of which E occurs thrice, R occurs twice and the remaining are different.

$$\therefore \text{Required number} = \frac{10!}{3!2!} = 10 \times 9 \times 8 \times 7 \times 6 \times 5 \times 2 = \mathbf{302400}$$

EXAMPLE 17 In how many ways, 7 girls and 6 boys can be seated in a row such that they are in the alternate position?

Solution

G	B	G	B	G	B	G	B	G	B	G	B	G

1 2 3 4 5 6 7 8 9 10 11 12 13

In the above 13 positions, girls and boys can be seated in alternate positions as shown.

∴ 7 girls can be arranged in 7 places in ${}^7P_7 = 7!$ ways.

and 6 boys can be arranged in 6 places in ${}^6P_6 = 6!$ ways.

$$\begin{aligned} \therefore \text{Required number of ways} &= 7! \times 6! \\ &= 5040 \times 720 = \mathbf{3628800} \end{aligned}$$

EXAMPLE 18 Find the number of diagonals of a polygon of 14 sides.

Solution A polygon having n sides will be having n vertices. A diagonal can be formed by joining any two vertices of the polygon.

$$\therefore \text{Number of diagonals of the polygon} = {}^nC_2 - n$$

where n is the number of sides of polygon.

$$\therefore \text{Required number of diagonals} = \frac{n!}{2!(n-2)!} - n$$

$$= \frac{n(n-1)}{1 \times 2} - n = \frac{n(n-1) - 2n}{2}$$

When $n = 14$, the required number of diagonals

$$= \frac{14(14-1) - 2 \times 14}{2} = \frac{14(13-2)}{2} = 7 \times 11 = 77$$

EXAMPLE 19 Five men and four women are seated in a row, such that 4 women are always together. In how many ways they can be seated?

Solution Assume that 4 women are together.

$\therefore$ Total number of persons seated = $5 + 1 = 6$

$\therefore$ Number of ways in which six persons can be seated = $6! = 720$ ways.

4 women can be arranged among themselves in $4!$ ways = **24 ways**.

$\therefore$ Total ways of arrangement = $720 \times 24 = 17280$ ways.

EXAMPLE 20

- (i) How many different arrangements can be made by using all the letters in the word 'ARITHMETIC'?
- (ii) How many of them begin with 'M' and with 'T'?
- (iii) How many words can be formed by taking three letters at a time out of the letters of the word 'ARITHMETIC'?

Solution

- (i) Out of 10 letters in 'ARITHMETIC', there are two I's, two T's and other six different letters A, R, H, M, E and C.

The number of words that can be formed by taking all at a time

$$= \frac{10!}{2! \times 2!} = \frac{10 \times 9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1}{1 \times 2 \times 1 \times 2} = 907200$$

- (ii) To begin with 'M', First place can be filled in only one way.

$$\therefore \text{The required number of words} = \frac{1 \times 9!}{2! \times 2!} = 90720$$

To begin with 'T', the first place can be filled in one way and T occurs twice.

$$\therefore \text{The required number of words} = 2! \times \frac{8!}{2!} = 8! = 40320$$

- (iii) We have 8 different letters and out of them choose 3 at a time

$$\therefore \text{Number of such words} = {}^8P_3 = \frac{8!}{(8-3)!} = 8 \times 7 \times 6 = 336$$

If two of the three are alike and remaining different, one pair of such letters from the available two pairs can be selected in ${}^2C_1 = 2$ ways.

The remaining one has to be selected from the remaining 7 letters in

$${}^7C_1 = \frac{7!}{6! \times 1!} = 7 \text{ ways}$$

∴ Required number of words = $2 \times 7 = 14$

But, in the three-letter group, letter can be grouped among themselves in

$$\frac{3!}{2!} = 3 \text{ ways}$$

∴ Total number of words = $14 \times 3 = 42$

∴ Total number of three letter words = $336 + 42 = 378$.



EXERCISES

- How many words can be formed out of the letters of the word 'EDUCATION' such that vowels occupy the odd positions?
(a) 1440 (b) 2880 (c) 2840 (d) 2480 (e) None of these
- A candidate is required to answer 6 out of 10 questions which are divided into two groups each containing 5 questions and he is not permitted to attempt more than 4 from each group. In how many ways can he make his choice?
(a) 160 (b) 180 (c) 192 (d) 200 (e) None of these
- How many numbers of five digits can be formed with the digits 0, 1, 2, 3, 5 and 6?
(a) 480 (b) 540 (c) 600 (d) 660 (e) None of these
- How many odd numbers of four digits can be formed with the digits 0, 1, 2, 3, 4, 7 and 8?
(a) 150 (b) 180 (c) 120 (d) 210 (e) None of these
- How many numbers of 4 digits, divisible by 5, can be formed with the digits 0, 3, 5, 7 and 9?
(a) 120 (b) 90 (c) 60 (d) 30 (e) None of these
- From 8 men and 6 women, a group of seven persons is to be formed. In how many ways the group can be formed such that the group should include at least two ladies?
(a) 3256 (b) 1628 (c) 6512 (d) 4884 (e) None of these
- How many words of 4 letters beginning with 'A' or 'E' can be formed with the letters of the word 'EQUATOR'?
(a) 280 (b) 160 (c) 240 (d) 180 (e) None of these
- In how many ways can the letters of the word 'EXCELLENT' be arranged?
(a) 30024 (b) 34200 (c) 30420 (d) 30240 (e) None of these
- How many groups can be selected for playing football out of 5 ladies and 4 gentlemen such that there should be one lady and one gentleman on each side?
(a) 420 (b) 240 (c) 360 (d) 120 (e) None of these
- In how many ways can 15 different books be divided equally among
 - 3 sets or groups
(a) 261126 (b) 121626 (c) 122661 (d) 126126 (e) None of these
 - 5 sets or groups
(a) 4104100 (b) 1401400 (c) 1041040 (d) 1201200 (e) None of these
- There are six members in a group which are to be sent abroad. The total number of members is 12. In how many ways can the selection be made such that a particular member is always
 - included
(a) 426 (b) 462 (c) 246 (d) 264 (e) None of these
 - excluded
(a) 264 (b) 246 (c) 462 (d) 426 (e) None of these

12. Find the number of triangles formed by 12 points (out of which 4 are collinear) in a plane:
(a) 216 (b) 126 (c) 612 (d) 621 (e) None of these
13. A box contains 2 white balls, 3 black balls and 4 red balls. In how many ways can 3 balls be drawn from the box, if at least one black ball is to be included in the draw? **[Bank PO]**
(a) 32 (b) 48 (c) 64 (d) 96 (e) None of these
14. In how many ways, can a group of 5 men and 2 women be made out of a total of 7 men and 3 women? **[Bank PO]**
(a) 63 (b) 90 (c) 126 (d) 45 (e) 135
15. From a group of 7 men and 6 women, five persons are to be selected to form a committee so that at least 3 men are there on the committee. In how many ways can it be done? **[MBA]**
(a) 564 (b) 645 (c) 735 (d) 756 (e) None of these
16. How many three digit numbers can be formed from the digits 2, 3, 5, 6, 7 and 9, which are divisible by 5 and none of the digits is repeated? **[SSC]**
(a) 5 (b) 10 (c) 15 (d) 20
17. How many words can be formed by using all the letters of the word 'TRIVANDRUM' using each letter exactly once?
(a) $10!$ (b) $8!$ (c) $9!$ (d) $6!$
18. In how many ways can the letters of the word 'LEADER' be arranged? **[Bank PO]**
(a) 72 (b) 144 (c) 360 (d) 720 (e) None of these
19. In how many ways a committee, consisting of 5 men and 6 women, can be formed from 8 men and 10 women? **[Bank PO]**
(a) 266 (b) 5040 (c) 11760 (d) 86400 (e) None of these
20. In how many different ways can the letters of the word 'AUCTION' be arranged in such a way that the vowels always come together? **[SBI PO]**
(a) 30 (b) 48 (c) 144 (d) 576 (e) None of these
21. Out of 7 consonants and 4 vowels, how many words of 3 consonants and 2 vowels can be formed?
(a) 210 (b) 1050 (c) 25200 (d) 21400 (e) None of these
22. In a group of 6 boys and 4 girls, four children are to be selected. In how many different ways can they be selected such that at least one boy should be there? **[SBI PO]**
(a) 159 (b) 194 (c) 205 (d) 209 (e) None of these
23. In how many different ways can the letters of the word 'BANKING' be arranged so that the vowels always come together? **[Bank PO]**
(a) 120 (b) 240 (c) 360 (d) 540 (e) 720
24. In how many different ways can the letters of the word 'JUDGE' be arranged in such a way that the vowels always come together? **[SBI PO]**
(a) 48 (b) 120 (c) 124 (d) 160 (e) None of these
25. In how many different ways can the letters of the word 'OPTICAL' be arranged so that the vowels always come together? **[MBA]**
(a) 120 (b) 720 (c) 4320 (d) 2160 (e) None of these
26. In how many ways, a committee of 3 members can be selected from 5 men and 4 women, consisting of 2 men and 1 woman?
(a) 60 (b) 72 (c) 40 (d) 32
27. In how many ways a team of 11 members can be formed from a group of 15 persons?
(a) 3270 (b) 2730 (c) 1365 (d) 1635 (e) None of these

28. How many 3-letter words can be formed from the letters of the word 'EXPERTISE', if repetition of letters is not allowed?
 (a) 420 (b) 240 (c) 210 (d) 120 (e) None of these
29. In how many ways can 21 books on Sanskrit and 19 books on French be placed in a row on a shelf such that two books on French may not be together?
 (a) 3990 (b) 1540 (c) 1995 (d) 3672 (e) None of these
30. In how many different ways can the letters of the word 'DETAIL' be arranged in such a way that the vowels occupy only the odd positions?
 (a) 32 (b) 48 (c) 36 (d) 60 (e) 120



1. (b) 2. (d) 3. (c) 4. (a) 5. (c) 6. (a) 7. (c) 8. (d) 9. (b)
 10. (i) (d), (ii) (b) 11. (i) (b), (ii) (c) 12. (a) 13. (c) 14. (a) 15. (d) 16. (d) 17. (c)
 18. (c) 19. (c) 20. (d) 21. (a) 22. (d) 23. (e) 24. (a) 25. (b) 26. (c) 27. (c)
 28. (c) 29. (b) 30. (c)



Solutions with Necessary Explanation

1. Out of the nine letters, five are vowels and other four are consonants. Therefore, 5 vowels can be arranged in 5 odd positions in ${}^5P_5 = 5!$ ways. Similarly, 4 consonants can be arranged in 4 alternate positions (even positions) in $4!$ ways.
 $\therefore$ Total number of words = $5! \times 4! = 2880$.
2. There are 10 questions in total from two groups A and B each consisting of 5 questions. A candidate can attempt a maximum of 4 questions from one group and 6 questions altogether

Group A	Group B
4	2
3	3
2	4

$\therefore$ Number of possible combinations

$$\begin{aligned}
 &= {}^5C_4 \times {}^5C_2 + {}^5C_3 \times {}^5C_3 + {}^5C_2 \times {}^5C_4 \\
 &= 5 \times 10 + 10 \times 10 + 10 \times 5 \\
 &= 50 + 100 + 50 = 200
 \end{aligned}$$

3. Required number of numbers = ${}^5P_1 \times {}^5P_4$
 $= 5 \times 5! = 5 \times 120 = 600$
4. The unit place of the number can be filled in ${}^3P_1 = 3$ ways.
 The thousandth place of the number can be filled in ${}^5P_1 = 5$ ways.
 (Because 0 is excluded from selection to the thousandth position.)

The remaining two places can be filled in ${}^5P_2 = \frac{5 \times 4}{1 \times 2} = 10$ ways.

$\therefore$ The total number of 4 digit odd numbers $3 \times 5 \times 10 = 150$.

5. For a 4-digit number to be divisible by 5, it should end with either '5' or '0'. Therefore, the unit place of the number can be filled in ${}^2P_1 = 2$ ways.

Case (a): The remaining 3 positions can be filled in ${}^4P_3$ ways = $4!$ ways (if 0 is filled in the units place)

$\therefore$ Required number of numbers = $2 \times 4! = 48$.

Case (b): The remaining 3 positions can be filled in ${}^3P_3$ ways = $3!$ ways (if 0 is not filled in the units place)

$\therefore$ Required number of numbers = $2 \times 3! = 12$.

$\therefore$ Total number of such numbers = $48 + 12 = 60$.

6. A group of seven persons is to be formed from 8 men and 6 women. It is given that there should be at least two women in the group.

The possible combinations are

(2W, 5M), (3W, 4M), (4W, 3M), (5W, 2M) and (6W, 1M)

The number of combinations

$$\begin{aligned}
 &= {}^6C_2 \times {}^8C_5 + {}^6C_3 \times {}^8C_4 + {}^6C_4 \times {}^8C_3 + {}^6C_5 \times {}^8C_2 + {}^6C_6 \times {}^8C_1 \\
 &= \frac{6 \times 5}{1 \times 2} \times \frac{8 \times 7 \times 6 \times 5 \times 4}{1 \times 2 \times 3 \times 4 \times 5} + \frac{6 \times 5 \times 4}{1 \times 2 \times 3} \times \frac{8 \times 7 \times 6 \times 5}{1 \times 2 \times 3 \times 4} + \frac{6 \times 5 \times 4 \times 3}{1 \times 2 \times 3 \times 4} \times \frac{8 \times 7 \times 6}{1 \times 2 \times 3} \\
 &\quad + \frac{6 \times 5 \times 4 \times 3 \times 2}{1 \times 2 \times 3 \times 4 \times 5} \times \frac{8 \times 7}{1 \times 2} + 1 \times 8 \\
 &= 840 + 1400 + 840 + 168 + 8 = \mathbf{3256}
 \end{aligned}$$

$\therefore$ Required number of groups = **3256**

7. Since the 4-letter word has to begin with 'A' or 'E', the first letter can be arranged in ${}^2P_1 = 2$ ways.

The remaining 3 letters can be arranged from the six letters of the word 'EQUATOR' in ${}^6P_3 = 6 \times 5 \times 4 = 120$ ways.

$\therefore$ Required number of words = $2 \times 120 = \mathbf{240}$.

8. The given word contains six different letters with the letter 'E' repeating thrice and 'L' repeating twice.

$$\therefore \text{Required number of arrangements} = \frac{9!}{3! \times 2!} = \mathbf{30240}.$$

9. The groups that are possible from 5 ladies and 4 gentlemen are

1 lady, 1 gentleman groups from 4 ladies and 3 gentlemen

Side A: One lady and one gentleman can be selected in ${}^5C_1 \times {}^4C_1$ ways = $5 \times 4 = 20$ ways.

Side B: For selection of one lady and one gentleman in side (B), there are only 4 ladies and 3 gentlemen.

$\therefore$ Required number of selection = ${}^4C_1 \times {}^3C_1 = 4 \times 3 = 12$

$\therefore$ Total number of ways of selection for forming the team = $20 \times 12 = \mathbf{240}$

$$10. \quad (i) \text{ Required number of ways} = \frac{{}^{15}C_5 \times {}^{10}C_5 \times {}^5C_5}{3!} = \mathbf{126126}$$

$$(ii) \text{ Required number of ways} = \frac{{}^{15}C_3 \times {}^{12}C_3 \times {}^9C_3 \times {}^6C_3 \times {}^3C_3}{5!} = \mathbf{1401400}$$

11. (i) Here six members are to be selected from 12 members and a particular member is to be included.

The selection of one particular member can be done in ${}^1C_1$ or 1 way.

The selection of remaining 5 members from the 11 members can be done in ${}^{11}C_5$ ways.

$\therefore$ The required number of ways of selection = ${}^1C_1 \times {}^{11}C_5 = 462$

- (ii) When a particular member is to be excluded the required number of ways of selection = ${}^{11}C_6 = 462$.

12. The number of triangles which can be formed from 12 points when all of them are not collinear = ${}^{12}C_3$

$\because$ 4 points are collinear, the number of triangles that have to be formed = ${}^4C_3$.

$\therefore$ The number of triangles formed = ${}^{12}C_3 - {}^4C_3 = \frac{12 \times 11 \times 10}{1 \times 2 \times 3} - 4 = 220 - 4 = 216$

13. Required selection can be (1 black and 2 non-black) or (2 black and 1 non-black) or 3 black.

$\therefore$ The required number of ways in which balls can be drawn = ${}^3C_1 \times {}^6C_2 + {}^3C_2 \times {}^6C_1 + {}^3C_3$

$$= 3 \times \frac{6 \times 5}{1 \times 2} + \frac{3 \times 2}{1 \times 2} \times 6 + 1 = 45 + 18 + 1 = 64$$

14. 5 men can be selected from 7 men in ${}^7C_5$ ways.

2 women can be selected from 3 women in ${}^3C_2$ ways.

$\therefore$ Total number of ways in which group can be formed = ${}^7C_5 \times {}^3C_2$

$$= {}^7C_2 \times {}^3C_1$$

$$= \frac{7 \times 6}{1 \times 2} \times 3 = 63 \text{ ways}$$

15. Required grouping can be (3 men, 2 women) or (4 men, 1 woman) or 5 men.

$\therefore$ The number of ways in which above grouping can be made

$$= ({}^7C_3 \times {}^6C_2) + ({}^7C_4 \times {}^6C_1) + ({}^7C_5 \times {}^6C_0)$$

$$= \left(\frac{7 \times 6 \times 5}{1 \times 2 \times 3} \times \frac{6 \times 5}{1 \times 2} \right) + \left(\frac{7 \times 6 \times 5}{1 \times 2 \times 3} \times 6 \right) + \frac{7 \times 6}{1 \times 2} \times 1$$

$$= 35 \times 15 + 35 \times 6 + 21 = 35 \times 21 + 21 = 36 \times 21 = 756$$

16. Here 6 digits are given and you have to select 3 at a time.

1	2	3
4	5	1

Since the number is divisible by 5,

Box 3 can be filled with '5' in only one way.

Box 2 can be filled in ${}^5C_1 = 5$ ways

Box 1 can be filled in ${}^4C_1 = 4$ ways

$\therefore$ Total number of 3 digit numbers divisible by 5 = $1 \times 5 \times 4 = 20$

17. 'TRIVANDRUM' consists of 10 characters in which R repeats twice.

Therefore, 9 different letters are available. By taking all characters at a time without repetition, the number of words that can be formed = $9!$

18. 'LEADER' consists of 6 letters namely L, E, A, D and R in which E repeats twice.

$\therefore$ Required number of ways = $\frac{6!}{2!} = 5! \times 3 = 360$

19. 5 men can be selected from 8 men in ${}^8C_5$ way. 6 women can be selected from 10 women in ${}^{10}C_6$ ways.

$$\therefore \text{Total number of groups that can be formed} = {}^8C_5 \times {}^{10}C_6 = {}^8C_3 \times {}^{10}C_4$$

$$= \frac{8 \times 7 \times 6}{1 \times 2 \times 3} \times \frac{10 \times 9 \times 8 \times 7}{1 \times 2 \times 3 \times 4}$$

$$= (8 \times 7) \times (10 \times 3 \times 7) = 56 \times 210 = \mathbf{11760}$$

20. In the word 'AUCTION', there are 7 characters in which 4 are vowels.

i.e., 4 vowels and 3 consonants.

Taking 4 vowels together, there are four characters and that can be arranged in $4!$ ways.

Among 4 vowels, it can be arranged among them in $4!$ ways.

$$\therefore \text{Total number of ways} = 4! \times 4! = 24 \times 24 = \mathbf{576 \text{ ways.}}$$

21. 3 consonants can be selected from the given 7 consonants in ${}^7C_3$ ways.

2 vowels can be selected from the given 4 vowels in ${}^4C_2$ ways.

$$\therefore \text{Total number of ways} = {}^7C_3 \times {}^4C_2$$

$$= \frac{7 \times 6 \times 5}{1 \times 2 \times 3} \times \frac{4 \times 3}{1 \times 2} = (7 \times 5) \times (2 \times 3) = \mathbf{210}$$

22. The required selection can be (1 boy, 3 girls) or (2 boys, 2 girls) or (3 boys, 1 girl) or 4 boys.

$$\therefore \text{Number of ways} = ({}^6C_1 \times {}^4C_3) + ({}^6C_2 \times {}^4C_2) + ({}^6C_3 \times {}^4C_1) + ({}^6C_4 \times {}^4C_0)$$

$$= (6 \times 4) + \left(\frac{6 \times 5}{1 \times 2} \times \frac{4 \times 3}{1 \times 2} \right) + \left(\frac{6 \times 5 \times 4}{1 \times 2 \times 3} \times 4 \right) + \left(\frac{6 \times 5}{1 \times 2} \times 1 \right)$$

$$= 24 + (15 \times 6) + (5 \times 4 \times 4) + (3 \times 5)$$

$$= 24 + 90 + 80 + 15 = \mathbf{209}$$

23. In the word 'BANKING', there are 7 characters in total in which 'N' repeats twice and there are two vowels.

Letters can be grouped such that 6 characters can be arranged in $6!$ ways and as 'N' repeats twice.

$$\text{Number of grouping} = \frac{6!}{2!} \text{ ways}$$

$\therefore$ Two vowels, come together, it can be arranged among themselves in $2! = 2$ ways.

$$\therefore \text{Total number of ways} = \frac{6!}{2!} \times 2! = 6! \text{ ways} = \mathbf{720 \text{ ways}}$$

24. Required number of arrangements of 3 consonants + (2 vowels group) = $4! \times 2 = \mathbf{48 \text{ ways}}$

25. Required number of arrangements = $5! \times 3! = 120 \times 6 = \mathbf{720 \text{ ways}}$

26. 2 men can be selected from 5 men in ${}^5C_2$ ways

1 woman can be selected from 4 women in ${}^4C_1$ ways

$$\therefore \text{Total number of ways} = {}^5C_2 \times {}^4C_1 = \frac{5 \times 4}{1 \times 2} \times 4 = 5 \times 2 \times 4 = \mathbf{40 \text{ ways}}$$

27. Required number of ways = ${}^{15}C_{11} = {}^{15}C_4 = \frac{15 \times 14 \times 13 \times 12}{1 \times 2 \times 3 \times 4} = 15 \times 7 \times 13 = \mathbf{1365}$

28. Here, word 'EXPERTISE' contains 9 characters in which 'E' repeats thrice.

1	2	3
7	6	5

∴ Box 1 can be filled in ${}^7C_1 = 7$ ways

Box 2 can be filled in ${}^6C_1 = 6$ ways

Box 3 can be filled in ${}^5C_1 = 5$ ways

∴ Required number of 3-letter words so formed = $7 \times 6 \times 5 = 210$

29. Here Sanskrit and French books are to be placed in alternate positions.

∴ For 21 books in Sanskrit, alternately placed, there will be 22 places for French books.

Taking 19 places out of 22, number of arrangements = ${}^{22}C_{19} = {}^{22}C_3$

$$= \frac{22 \times 21 \times 20}{1 \times 2 \times 3} = 1540 \text{ ways}$$

30. Here 6 characters are there, in which 3 are vowels.

1	2	3	4	5	6

∴ Vowels can be placed in positions marked 1, 3 and 5.

∴ No. of arrangements = ${}^3P_3 = 3! = 6$

3 consonants can be arranged at positions marked 2, 4 and 6.

∴ Number of arrangements = ${}^3P_3 = 3! = 6$

∴ Total number of ways = $6 \times 6 = 36$

Probability

Probability is the chance of happening of an event in a random experiment.

RANDOM EXPERIMENT: An experiment when conducted repeatedly under the same conditions, if it does not give the same result, it is called the *random experiment*.

Let us take an example. When a coin is thrown, it will not fall with a particular face always up. Any of the two faces marked head (H) or tail (T) come up each time.

EVENT: The outcome of a random experiment is called the *event*.

Simple or Compound Event: An event is called the *simple event*, if it corresponds to a single outcome of a random experiment.

For example, while tossing a die the chance of getting face marked 5 is a *simple event*.

An event is called a *compound event* if it corresponds to more than one possible outcome. For example, in the above example, the chance of getting any face marked even is a compound event (since even numbers are more than one, i.e. 2, 4 and 6).

Exhaustive Events: All possible outcomes of an event are known as *exhaustive events*.

For example, when a coin is thrown, there are only two exhaustive events H and T (Head and Tail).

When two coins are thrown, the number of exhaustive events is 4 (2×2), i.e. HH, HT, TH, TT.

When a die is thrown, there are six exhaustive events (i.e. the occurrence of faces marked 1, 2, 3, 4, 5 and 6).

Mutually Exclusive Events: Two or more events are said to be mutually exclusive events; if the happening of any one of them excludes the happening of all the others in a single experiment.

For example,

- (a) In throwing a coin, events H and T are mutually exclusive. Here if Head occurs, it prevents the occurrence of Tail.
- (b) When a die is thrown, the occurrence of faces marked 1, 2, 3, 4, 5 and 6 is mutually exclusive. Here the occurrence of one face prevents the occurrence of the other.

Equally Likely Events: Two or more events are said to be *equally likely events*, if the chances of their happening are equal. There is no preference to any one event to the other.

If a coin is thrown, coming up of head or tail is equally likely.

If a die is thrown, coming up of faces marked 1, 2, 3, 4, 5 and 6 are equally likely.

Independent and Dependent Events: If the happening of an event is not affected by the happening of other events, it is said to be an *independent event*.

For example, a coin is thrown twice.

The occurrence of the head on the first throw is independent of the occurrence of the head in the second throw.

If the happening of an event is affected by the happening of the other events, it is said to be *dependent event*.

For example, five red balls and four black balls are kept in a box. You have to draw 2 balls out of them successively.

Here the chance of drawing a red/black ball first is $\frac{5}{9}$ or $\frac{4}{9}$. If the ball is not replaced and another ball is drawn, then the chance of drawing a red or black ball is $\frac{4}{8} = \frac{1}{2}$.

SAMPLE SPACE: A set containing all possible outcomes of a random experiment is known as *sample space*.

For example, in a coin throw the number of events in the sample space is 2. i.e. {H, T}.

Probability of an event A can be denoted as $P(A)$ and can be defined as

$$P(A) = \frac{n(A)}{n(S)} = \frac{\text{Number of favourable events}}{\text{Number of events in sample space}}$$

EXAMPLE 1 Two coins are tossed. What is the probability that (i) at least one head occurs (ii) at least one tail occurs?

Solution $n(S) = 4$. i.e. {(T, T), (H, H), (H, T), (T, H)}

$$(i) \ P(\text{occurrence of at least one head}) = \frac{n(A)}{n(S)} = \frac{3}{4}$$

$$(ii) \ P(\text{occurrence of at least one tail}) = \frac{n(A)}{n(S)} = \frac{3}{4}$$

EXAMPLE 2 A positive number is selected at random and is divided by 6. What is the probability that the remainder is (i) 1; (ii) not 1?

Solution If a number is divided by 6, the remainder will be 0, 1, 2, 3, 4 or 5.

$\therefore n(S) = 6$

$$(i) P(A) = \frac{n(A)}{n(S)}; n(A) = 1;$$

$$\therefore P(A) = \frac{1}{6}$$

$$(ii) P(B) = \frac{n(B)}{n(S)}; n(B) = 5;$$

$$\therefore P(\text{not } 1) = \frac{n(B)}{n(S)} = \frac{5}{6}$$

$$\text{i.e. } P(A') = 1 - P(A) = 1 - \frac{1}{6} = \frac{5}{6}$$



Note: If an event is represented by 'A', then the event, 'not A' is represented by A' or $\bar{A}$ or A^C and is known as *compliment of A*.

$$\therefore P(A) + P(A') = 1 \quad \text{or} \quad P(A) = 1 - P(A')$$

$0 \leq \text{Probability of an event} \leq 1$.

EXAMPLE 3 A card is drawn from a set of playing cards. What is the probability that card drawn is

- (i) a king or a queen?
- (ii) a card having the number a multiple of 2?
- (iii) a diamond card?
- (iv) a card of red suit?
- (v) a digit card of spade?
- (vi) a card having a number less than 9?
- (vii) a king of red suit?

Solution For all the above cases $n(S) = {}^{52}C_1 = 52$

$$(i) P(\text{king}) = \frac{4}{52} = \frac{1}{13}; P(\text{queen}) = \frac{4}{52} = \frac{1}{13}.$$

$$\therefore P(\text{a king or a queen}) = \frac{1}{13} + \frac{1}{13} = \frac{2}{13}$$

$$(ii) \text{ Required probability} = \frac{5 \times 4}{52} = \frac{5}{13}$$

$$(iii) \text{ Required probability} = \frac{13}{52} = \frac{1}{4}$$

$$(iv) \text{ Required probability} = \frac{26}{52} = \frac{1}{2}$$

$$(v) \text{ Required probability} = \frac{9}{52}$$

$$(vi) \text{ Required probability} = \frac{7 \times 4}{52} = \frac{7}{13}$$

$$(vii) \text{ Required probability} = \frac{2}{52} = \frac{1}{26}$$

EXAMPLE 4 From a pack of 52 cards, 4 cards are drawn. What is the probability that it has

- (i) all four kings?
- (ii) no ace?
- (iii) one king, one queen, one ace and one jack?
- (iv) one jack and three aces?
- (v) two digit cards and two honours card of red suit?

Solution For all the above cases,

$$n(S) = {}^{52}C_4 = \frac{52 \times 51 \times 50 \times 49}{1 \times 2 \times 3 \times 4} = 13 \times 17 \times 25 \times 49 = 54145$$

$$(i) P(\text{four kings}) = \frac{{}^4C_4}{{}^{52}C_4} = \frac{1}{13 \times 17 \times 25 \times 49} = \frac{1}{54145}$$

$$(ii) P(\text{no ace}) = \frac{{}^{48}C_4}{{}^{52}C_4} = \frac{48 \times 47 \times 46 \times 45}{1 \times 2 \times 3 \times 4} \times \frac{1}{13 \times 17 \times 25 \times 49} = \frac{38912}{54145}$$

$$(iii) P(\text{one king, one queen, one ace and one jack}) = \frac{{}^4C_1 \times {}^4C_1 \times {}^4C_1 \times {}^4C_1}{{}^{52}C_4}$$

$$= \frac{4 \times 4 \times 4 \times 4}{13 \times 17 \times 25 \times 49} = \frac{256}{54145}$$

$$(iv) P(\text{one jack and three aces}) = \frac{{}^4C_1 \times {}^4C_3}{{}^{52}C_4} = \frac{4 \times 4}{13 \times 17 \times 25 \times 49} = \frac{16}{54145}$$

$$(v) P(\text{two digit cards and two honours cards of red suit}) = \frac{{}^{36}C_2 \times {}^8C_2}{{}^{52}C_4}$$

$$= \frac{\frac{36 \times 35}{1 \times 2} \times \frac{8 \times 7}{1 \times 2}}{\frac{52 \times 51 \times 50 \times 49}{1 \times 2 \times 3 \times 4}} = \frac{18 \times 35 \times 4 \times 7}{13 \times 17 \times 25 \times 49} = \frac{17640}{54145}$$

EXAMPLE 5 A four-digit number is formed with the given six digits 1, 2, 3, 4, 5 and 7 at random. What is the probability that the number formed is

- (i) divisible by 2?
- (ii) not divisible by 2?
- (iii) divisible by 5?

Solution A four-digit number can be formed from the given six digits in ${}^6P_4$ ways, i.e. ${}^6P_4 = 6 \times 5 \times 4 \times 3 = 360$ ways.

- (i) For the number to be divisible by 2, either 2 or 4 should come in the unit place. Therefore, the unit place can be filled in ${}^2C_1 = 2$ ways. The remaining places can be filled in ${}^5P_3$ ways = $5 \times 4 \times 3 = 60$ ways.

$$\therefore P(\text{number divisible by 2}) = \frac{2 \times 60}{360} = \frac{1}{3}$$

$$(ii) P(\text{number not divisible by 2}) = 1 - \frac{1}{3} = \frac{2}{3}$$

- (iii) For the number to be divisible by 5, the unit place should be either 5 or 0. In the given set of six digits, only '5' can be used to fill in the unit place of the number to be formed. Therefore, the unit place can be filled in only one way. The remaining 3 places can be filled in ${}^5P_3$ ways.

$$\text{i.e. } {}^5P_3 = 5 \times 4 \times 3 = \mathbf{60 \text{ ways}}$$

$$\therefore P(\text{number divisible by 5}) = \frac{1 \times {}^5P_3}{{}^6P_4} = \frac{1 \times 60}{360} = \frac{1}{6}$$

EXAMPLE 6 In a box carrying one dozen of mangoes, one-third has become bad. If three mangoes are taken out from the box at random, what is the probability that at least one mango out of the three mangoes picked up is good?

Solution $P(\text{at least one good}) = 1 - P(\text{all bad})$

$$P(\text{all bad}) = \frac{{}^4C_3}{{}^{12}C_3} = \frac{4}{\left(\frac{12 \times 11 \times 10}{1 \times 2 \times 3}\right)} = \frac{1}{55}$$

$$\therefore P(\text{at least one good}) = 1 - \frac{1}{55} = \frac{54}{55}$$

EXAMPLE 7 A bag contains 5 red and 7 green balls. Two balls are drawn from it provided that the first ball is replaced before the second ball is drawn.

What is the probability, if

- (i) both balls drawn were red;
- (ii) both balls drawn were green;
- (iii) the first ball was red and second green;
- (iv) the first ball was green and second red.

Solution

$$(i) \text{ Probability that both balls drawn were red} = \frac{5}{12} \times \frac{5}{12} = \frac{25}{144}$$

$$(ii) \text{ Probability that both balls drawn were green} = \frac{7}{12} \times \frac{7}{12} = \frac{49}{144}$$

$$(iii) \text{ Probability that the first ball drawn was red and second green} = \frac{5}{12} \times \frac{7}{12} = \frac{35}{144}$$

$$(iv) \text{ Probability that the first ball drawn was green and second red} = \frac{7}{12} \times \frac{5}{12} = \frac{35}{144}$$

EXAMPLE 8 A box contains 5 red, 6 yellow and 7 green balls. Three balls are drawn at random. What is the probability that they are of different colour?

Solution Total number of balls = $5 + 6 + 7 = 18$

$$3 \text{ balls can be drawn from the 18 balls in } {}^{18}C_3 \text{ ways} = \frac{18 \times 17 \times 16}{1 \times 2 \times 3} = 3 \times 17 \times 16 = \mathbf{816}$$

If all the three balls are of same colour it can be done in ${}^5C_3 + {}^6C_3 + {}^7C_3$ ways

$$= \frac{5 \times 4}{1 \times 2} + \frac{6 \times 5 \times 4}{1 \times 2 \times 3} + \frac{7 \times 6 \times 5}{1 \times 2 \times 3}$$

$$= 10 + 20 + 35 = \mathbf{65 \text{ ways}}$$

$\therefore P(\text{all balls are of different colour})$

$$= 1 - P(\text{all balls are of same colour})$$

$$= 1 - \frac{65}{816} = \frac{\mathbf{751}}{\mathbf{816}}$$

EXAMPLE 9 A box contains 9 green and 7 yellow balls. Two balls are drawn in succession at random. What is the probability that one of them is green and the other yellow?

Solution Two balls can be drawn from $9 + 7 = 16$ balls in ${}^{16}C_2$ ways.

i.e.
$$\frac{16 \times 15}{1 \times 2} = \mathbf{120 \text{ ways}}$$

One green ball can be drawn from the given 9 green balls in ${}^9C_1 = 9$ ways.

One yellow ball can be drawn from the given 7 yellow balls in ${}^7C_1 = 7$ ways.

$$\therefore \text{Required probability} = \frac{{}^9C_1 \times {}^7C_1}{{}^{16}C_2} = \frac{9 \times 7}{120} = \frac{\mathbf{21}}{\mathbf{40}}$$

EXAMPLE 10 The letter of the word 'EQUATION' is arranged in different ways randomly. What is the probability that the vowels are (i) together, (ii) not together.

Solution The eight letters of the word 'EQUATION' can be arranged in $8!$ ways. There are 5 vowels in the word 'EQUATION'. If they are together the word 'EQUATION' can be arranged in ${}^4P_4 \times {}^5P_5 = 4! \times 5! = 24 \times 120 = \mathbf{2880 \text{ ways}}$

$$(i) \therefore P(\text{vowels occur together}) = \frac{4! \times 5!}{8!} = \frac{2880}{1 \times 2 \times 3 \times 4 \times 5 \times 6 \times 7 \times 8}$$

$$= \frac{4!}{6 \times 7 \times 8} = \frac{\mathbf{1}}{\mathbf{14}}$$

$$(ii) \therefore P(\text{vowels not together}) = 1 - \frac{1}{14} = \frac{\mathbf{13}}{\mathbf{14}}$$

EXAMPLE 11 What is the probability that a card drawn from a pack of 52 cards will be a diamond or a queen?

Solution Out of 52 cards, 13 cards are of diamond series and 3 cards are queen

$$\therefore n(E) = 13 + 3 = 16; n(S) = 52$$

$$\therefore P(E) = \frac{16}{52} = \frac{\mathbf{4}}{\mathbf{13}}$$

EXAMPLE 12 A box contains 10 red and 10 green balls. What is the probability that two balls drawn from the box is of the same colour?

Solution Number of possible ways in which two red balls are drawn from given 10 balls

$$= {}^{10}C_2 = \frac{10 \times 9}{1 \times 2} = 45$$

Similarly number of possible ways in which two green balls are drawn from given 10 balls

$$= {}^{10}C_2 = \frac{10 \times 9}{1 \times 2} = 45$$

$\therefore$ Total possible ways = $45 + 45 = 90$

Total number of possible ways in which two balls can be drawn from given 20 balls

$${}^{20}C_2 = \frac{20 \times 19}{1 \times 2} = 190$$

E = event of drawing 2 balls of same colour

$$\therefore P(E) = \frac{{}^{10}C_2 + {}^{10}C_2}{{}^{20}C_2} = \frac{90}{190} = \frac{9}{19}$$

EXAMPLE 13 Two dice are thrown at a time. What is the probability that sum of results obtained is 8 or 9?

Solution When two dice are thrown, total number of results, $n(S) = 6 \times 6 = 36$

Event of getting total of 8 or 9

$$E = \{(2, 6), (3, 5), (4, 4), (3, 6), (4, 5)\}$$

$$\therefore P(E) = \frac{n(E)}{n(S)} = \frac{5}{36}$$

EXAMPLE 14 If two coins are thrown simultaneously, find the probability of getting at least one head.

Solution Total number of possible events $n(S) = \{(H, H), (H, T), (T, H), (T, T)\} = 4$

Event of getting at least one head $E = \{(H, H), (H, T), (T, H)\} = 3$

$$\therefore P(E) = \frac{3}{4}$$

EXAMPLE 15 Two cards are drawn from a pack of 52 cards. What is the probability that either both are black or both are queen?

Solution Out of 52 cards, 26 cards are of black suit. There are 4 queens in all.

Let E be the event of getting both black cards or both queens.

E_1 = Event of getting both black suits

E_2 = Event of getting both queens

$$n(E_1) = {}^{26}C_2 = \frac{26 \times 25}{1 \times 2} = 325;$$

$$n(E_2) = {}^4C_2 = \frac{4 \times 3}{1 \times 2} = 6;$$

$$n(E_1 \cap E_2) = {}^2C_2 = 1$$

$$\therefore P(E) = \frac{n(E_1) + n(E_2) - n(E_1 \cap E_2)}{n(S)} = \frac{325 + 6 - 1}{1326};$$

$$P(E_2) = \frac{n(E_2)}{n(S)} = \frac{6}{1326};$$

$$P(E_1 \cap E_2) = \frac{1}{1326};$$

∴

$$\begin{aligned} P(E_1 \cup E_2) &= P(E_1) + P(E_2) - P(E_1 \cap E_2) \\ &= \frac{325}{1326} + \frac{6}{1326} - \frac{1}{1326} = \frac{330}{1326} = \frac{55}{221} \end{aligned}$$



EXERCISES

- A bag contains 7 black and 5 white balls. A ball is drawn out and replaced in the bag. Then a ball is drawn again. What is the probability that
 - both the balls drawn were white?
(a) 49/144 (b) 25/144 (c) 36/144 (d) 64/144 (e) None of these
 - both balls were black?
(a) 25/144 (b) 36/144 (c) 49/144 (d) 64/144 (e) None of these
 - the first ball was black and the second white?
(a) 35/144 (b) 45/144 (c) 65/144 (d) 55/144 (e) None of these
 - the first ball was white and the second black?
(a) 65/144 (b) 55/144 (c) 45/144 (d) 35/144 (e) None of these
- A box contains 5 red, 4 green and 3 black balls. 3 balls were drawn at random. What is the probability that they are not of the same colour?
(a) 13/55 (b) 41/44 (c) 13/44 (d) 52/55 (e) None of these
- A positive integer is selected at random and is divided by 9. What is the probability that the remainder is
 - equal to 1?
(a) 4/9 (b) 3/9 (c) 2/9 (d) 1/9 (e) None of these
 - not 1?
(a) 4/9 (b) 2/9 (c) 8/9 (d) 1/9 (e) None of these
- Two coins are tossed. What is the probability of appearing of
 - atmost one head?
(a) 1/4 (b) 2/4 (c) 3/4 (d) 1 (e) None of these
 - atmost two heads?
(a) 1/4 (b) 2/4 (c) 3/4 (d) 1 (e) None of these
- What is the chance that a leap year selected randomly will have 53 Wednesdays?
(a) 1/7 (b) 2/7 (c) 3/7 (d) 4/7 (e) None of these
 - What is the chance, if the year selected is not a leap year?
(a) 4/7 (b) 3/7 (c) 1/7 (d) 2/7 (e) None of these
- When two dice are thrown simultaneously, what is the probability that
 - the sum of numbers appeared is an odd number?
(a) 1/4 (b) 1/2 (c) 3/4 (d) 1 (e) None of these

- (ii) the sum of numbers appeared is an even number?
 (a) $1/4$ (b) $3/4$ (c) $1/2$ (d) 1 (e) None of these
- (iii) the sum of numbers appeared is a multiple of 3?
 (a) $2/3$ (b) $3/4$ (c) 1 (d) $1/3$ (e) None of these
- (iv) the sum of numbers appeared ≤ 10 ?
 (a) $5/12$ (b) $11/12$ (c) $6/12$ (d) $7/12$ (e) None of these
- (v) the sum of numbers appeared is 6 and 7?
 (a) $5/6$ (b) $3/6$ (c) $4/6$ (d) $1/6$ (e) None of these
- (vi) numbers appeared are equal?
 (a) $2/6$ (b) $1/6$ (c) $3/6$ (d) $4/6$ (e) None of these
7. A box contains 6 white and 8 red balls. Two balls are drawn at random in succession. What is the probability that one of them is white and the other red?
 (a) $72/91$ (b) $24/91$ (c) $48/91$ (d) $36/91$ (e) None of these
8. A group of 4 students is to be formed from among 4 girls and 6 boys. What is the probability that the group has less number of boys than the number of girls.
 (a) $9/42$ (b) $11/42$ (c) $5/42$ (d) $13/42$ (e) None of these
9. You are given digits 2, 3, 4, 6 and 7. In how many ways you can form a 3-digit number without repetition of digits which is odd and greater than 300?
 (a) 27 (b) 18 (c) 12 (d) 15 (e) None of these
10. From a pack of 52 cards, 4 cards are drawn. What is the probability that it has
 (i) no queen?
 (a) $\frac{53}{54,145}$ (b) $\frac{52}{54,145}$ (c) $\frac{104}{54,145}$ (d) $\frac{208}{54,145}$ (e) None of these
- (ii) one ace, one king, one queen and one jack?
 (a) $\frac{64}{54,145}$ (b) $\frac{128}{54,145}$ (c) $\frac{256}{54,145}$ (d) $\frac{512}{54,145}$ (e) None of these
- (iii) two digit-cards and one honours card of black and red suit?
 (a) $\frac{1125}{7735}$ (b) $\frac{1512}{7735}$ (c) $\frac{1152}{7735}$ (d) $\frac{2511}{7735}$ (e) None of these
11. In a box, there are 8 red, 7 blue and 6 green balls. One ball is picked up at random. What is the probability that it is neither red nor green? **[Bank PO]**
 (a) $\frac{1}{3}$ (b) $\frac{3}{4}$ (c) $\frac{7}{19}$ (d) $\frac{8}{21}$ (e) $\frac{9}{21}$
12. A bag contains 2 red, 3 green and 2 blue balls. Two balls are drawn at random. What is the probability that none of the balls drawn is blue? **[Bank PO]**
 (a) $\frac{10}{21}$ (b) $\frac{11}{21}$ (c) $\frac{2}{7}$ (d) $\frac{5}{7}$
13. A box contains 5 green, 4 yellow and 3 white marbles. Three marbles are drawn at random. What is the probability that they are not of the same colour? **[Bank PO]**
 (a) $\frac{3}{44}$ (b) $\frac{3}{55}$ (c) $\frac{52}{55}$ (d) $\frac{41}{44}$
14. What is the probability of getting a sum 9 from two throws of a dice? **[MBA]**
 (a) $\frac{1}{6}$ (b) $\frac{1}{8}$ (c) $\frac{1}{9}$ (d) $\frac{1}{12}$

15. From a pack of 52 cards, two cards are drawn together at random. What is the probability of both cards being kings? **[MBA]**
 (a) $\frac{1}{15}$ (b) $\frac{25}{57}$ (c) $\frac{35}{256}$ (d) $\frac{1}{221}$
16. Two cards are drawn together from a pack of 52 cards. The probability that one is a spade and one is a heart, is **[MBA]**
 (a) $\frac{3}{20}$ (b) $\frac{29}{34}$ (c) $\frac{47}{100}$ (d) $\frac{13}{102}$
17. A bag contains 4 white, 5 red and 6 blue balls. Three balls are drawn at random. What is the probability that all of them are red?
 (a) $\frac{1}{22}$ (b) $\frac{3}{22}$ (c) $\frac{2}{91}$ (d) $\frac{2}{77}$
18. In a class, 30% of students passed English, 20% passed French and 10% passed both. If a student is selected at random, what is the probability that he has passed English or French?
 (a) $\frac{2}{5}$ (b) $\frac{3}{4}$ (c) $\frac{3}{5}$ (d) $\frac{3}{10}$
19. Four persons are selected at random from a group of 3 men, 2 women and 4 children. The chance that exactly two of them are children is
 (a) $\frac{1}{9}$ (b) $\frac{1}{5}$ (c) $\frac{1}{12}$ (d) $\frac{10}{21}$
20. A box contains 4 red balls, 5 green balls and 6 white balls. A ball is drawn at random from the box. What is the probability that the ball drawn is either red or green?
 (a) $\frac{2}{5}$ (b) $\frac{3}{5}$ (c) $\frac{1}{5}$ (d) $\frac{7}{15}$
21. A box contains 6 black and 4 red balls. Three balls are drawn at random. What is the probability that one ball is red and the other two are black?
 (a) $\frac{1}{2}$ (b) $\frac{1}{12}$ (c) $\frac{3}{10}$ (d) $\frac{7}{12}$
22. Two dice are thrown simultaneously. What is the probability of getting two numbers whose product is even? **[Asst. PF Commissioner's Exam]**
 (a) $\frac{1}{2}$ (b) $\frac{3}{4}$ (c) $\frac{3}{8}$ (d) $\frac{5}{16}$



1. (i) (b), (ii) (c), (iii) (a), (iv) (d) 2. (b) 3. (i) (d), (ii) (c) 4. (i) (c), (ii) (d)
 5. (i) (b), (ii) (c) 6. (i) (b), (ii) (c), (iii) (d), (iv) (b), (v) (d), (vi) (b)
 7. (c) 8. (c) 9. (b) 10. (i) (e), (ii) (e), (iii) (c) 11. (a) 12. (a) 13. (d)
 14. (c) 15. (d) 16. (d) 17. (c) 18. (a) 19. (d) 20. (b) 21. (a) 22. (b)



Solutions with Necessary Explanation

1. (i) Both balls were white = $\frac{5}{12} \times \frac{5}{12} = \frac{25}{144}$
 (ii) Both balls were black = $\frac{7}{12} \times \frac{7}{12} = \frac{49}{144}$

$$(iii) \text{ The first ball was black and the second white} = \frac{7}{12} \times \frac{5}{12} = \frac{35}{144}$$

$$(iv) \text{ The first ball was white and the second black} = \frac{5}{12} \times \frac{7}{12} = \frac{35}{144}$$

2. Total number of balls = $5 + 4 + 3 = 12$

$P(\text{balls are not of the same colour}) = ?$

$$n(S) = {}^{12}C_3 = \frac{12 \times 11 \times 10}{1 \times 2 \times 3} = 220$$

i.e. 3 balls can be drawn from the given 12 balls in 220 ways.

If all are of same colour, it can be done in ${}^5C_3 + {}^4C_3 + {}^3C_3 = 10 + 4 + 1 = 15$ ways.

$P(\text{all balls are of same colour}) + P(\text{all balls are not of same colour}) = 1.$

$\therefore P(\text{all balls are not of same colour})$

$$= 1 - P(\text{all balls are of same colour})$$

$$= 1 - \frac{15}{220} = \frac{205}{220} = \frac{41}{44}.$$

3. When a positive integer is divided by 9, the remainder may be any integer from 0 to 8.

$\therefore n(S) = 9$

(i) $E(1) = \{1\}; n(E) = 1;$

$$\therefore P(E) = \frac{1}{9}$$

(ii) $E(\text{not } 1) = \{0, 2, 3, 4, 5, 6, 7, 8\};$

i.e. $n(E) = 8;$

$$\therefore P(E) = \frac{8}{9}$$

4. $n(S) = 4 = \{(H, H), (H, T), (T, H), (T, T)\}$

(i) $E(\text{appearing at most one head}) = \{HT, TH, TT\}$

$\therefore n(E) = 3;$

$$\therefore P(E) = \frac{3}{4}$$

(ii) $E(\text{appearing atmost two heads}) = \{HH, HT, TH, TT\}$

$\therefore n(E) = 4;$

$$\therefore P(E) = \frac{4}{4} = 1$$

5. (i) A leap year has 366 days and, therefore, it has 52 weeks and 2 more days. Two days can be $\{(Sunday, Monday), (Monday, Tuesday), (Tuesday, Wednesday), (Wednesday, Thursday), (Thursday, Friday), (Friday, Saturday), (Saturday, Sunday)\}.$

$\therefore n(S) = 7.$

Out of the above 7 cases, cases favourable for more Wednesdays are $\{(Tuesday, Wednesday), (Wednesday, Thursday)\}$

i.e. $n(E) = 2$

$$\therefore P(E) = \frac{2}{7}$$

- (ii) When the year is not a leap year, it has 52 complete weeks and 1 more day.

It can be {Sunday, Monday, Tuesday, Wednesday, Thursday, Friday, Saturday}

$$\therefore n(S) = 7$$

$\therefore$ From the above 7 cases, cases favourable for one more Wednesday is {Wednesday}.

$$\text{i.e. } n(E) = 1.$$

$$\therefore P(E) = \frac{1}{7}$$

6. (i) The sum of numbers appeared = odd numbers from 3 to 11 (both inclusive)

So the number of events $n(E) = 2 + 4 + 6 + 4 + 2 = 18$

$$\therefore \text{Sum} = 3 \rightarrow (1, 2) \text{ or } (2, 1)$$

$$= 5 \rightarrow (1, 4) \text{ or } (4, 1), (2, 3) \text{ or } (3, 2)$$

$$= 7 \rightarrow (1, 6) \text{ or } (6, 1), (2, 5) \text{ or } (5, 2), (3, 4) \text{ or } (4, 3)$$

$$= 9 \rightarrow (3, 6) \text{ or } (6, 3), (4, 5) \text{ or } (5, 4)$$

$$= 11 \rightarrow (5, 6) \text{ or } (6, 5)$$

$$n(s) = 6 \times 6 = 36$$

$$\therefore \text{Required probability} = \frac{n(E)}{n(S)} = \frac{18}{36} = \frac{1}{2}$$

- (ii) The sum of numbers appeared is even.

The required sums are 2, 4, 6, 8, 10, 12.

Events are (1, 1), (1, 3), (2, 2), (3, 1), (1, 5), (2, 4), (3, 3), (4, 2), (5, 1),
(2, 6), (3, 5), (4, 4), (5, 3), (6, 2), (4, 6), (5, 5), (6, 4), (6, 6).

So $n(E) = 18$

$$n(S) = 36.$$

$$\therefore \text{Required probability} = \frac{n(E)}{n(S)} = \frac{18}{36} = \frac{1}{2}$$

- (iii) The sum of numbers is a multiple of 3.

$\therefore$ The required sums are 3, 6, 9 and 12.

$\therefore$ Events are (1, 2), (2, 1), (1, 5), (5, 1), (2, 4), (4, 2), (3, 3), (3, 6), (6, 3), (4, 5), (5, 4), (6, 6)

$$\therefore n(E) = 12; n(S) = 36$$

$$\text{So, the required probability} = \frac{n(E)}{n(S)} = \frac{12}{36} = \frac{1}{3}$$

- (iv) The sum of numbers appeared ≤ 10 .

$\therefore$ The required sums are 2, 3, 4, 5, 6, 7, 8, 9 and 10

$\therefore$ Events are (1, 1), (1, 2), or (2, 1), (1, 3), (3, 1), (2, 2), (1, 4), (4, 1), (2, 3), (3, 2), (1, 5),
(5, 1), (2, 4), (4, 2), (3, 3), (1, 6), (6, 1), (2, 5), (5, 2), (4, 3), (3, 4), (2, 6),
(6, 2), (3, 5), (5, 3), (4, 4), (3, 6), (6, 3), (4, 5), (5, 4), (4, 6), (6, 4), (5, 5)

$$\therefore n(E) = 1 + 2 + 3 + 4 + 5 + 6 + 5 + 4 + 3$$

$$= 33$$

$$n(S) = 36$$

$$\therefore \text{The required probability} = \frac{33}{36} = \frac{11}{12}$$

- (v) The sum of numbers appeared is 6 and 7.

$\therefore$ The required sums are 6 and 7.

i.e. the required events are (1, 5), (5, 1), (2, 4), (4, 2), (3, 3), (1, 6),
(6, 1), (2, 5), (5, 2), (3, 4), (4, 3)

i.e. for 6, $n(E) = 5$

for 7, $n(E) = 6$

$$\therefore \text{for 6, the required probability} = \frac{n(E)}{n(S)} = \frac{5}{36}$$

$$\text{for 7, the required probability} = \frac{n(E)}{n(S)} = \frac{6}{36} = \frac{1}{6}$$

(vi) The numbers appeared are equal.

$\therefore$ The required events are (1, 1), (2, 2), (3, 3), (4, 4), (5, 5) and (6, 6).

$\therefore n(E) = 6$

$$\therefore \text{The required probability} = \frac{n(E)}{n(S)} = \frac{6}{36} = \frac{1}{6}$$

7. The number of ways that two balls can be drawn from (6 + 8) or 14 balls is ${}^{14}C_2 = \frac{14 \times 13}{1 \times 2} = 91$

The number of ways in which one white ball can be drawn from the six white balls is ${}^6C_1 = 6$

The number of ways in which one red ball can be drawn from the eight red balls is ${}^8C_1 = 8$

$\therefore$ The number of ways in which one white and one red ball are drawn is $\frac{{}^6C_1 \times {}^8C_1}{{}^{14}C_2} = \frac{6 \times 8}{91} = \frac{48}{91}$

8. The selection of 1 boy and 3 girls can be done in ${}^6C_1 \times {}^4C_3 = 6 \times 4 = 24$ ways

Selection of 4 girls and no boy can be done in ${}^6C_0 \times {}^4C_4 = 1 \times 1 = 1$ way.

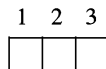
$\therefore n(E) = 24 + 1 = 25$ ways.

Without any restriction the group can be formed in ${}^{10}C_4 = \frac{10 \times 9 \times 8 \times 7}{1 \times 2 \times 3 \times 4} = 210$ ways

$$\therefore P(E) = \frac{n(E)}{n(S)} = \frac{25}{210} = \frac{5}{42}$$

9. The given digits are 2, 3, 4, 6 and 7.

Box 1 can be filled in 4 ways, i.e. 3, 4, 6 and 7.



When box 1 is having 3,

box '3' can be filled in using 7 only and box '2' can be filled in 3 ways.

So the number of such numbers formed = $1 \times 3 \times 1 = 3$

When box 1 is having 4,

box '3' can be filled in 2 ways (i.e. 3 or 7)

box '2' can be filled in 3 ways.

So the number of such numbers formed = $1 \times 3 \times 2 = 6$

When box 1 is having 6,

box '3' can be filled in 2 ways (i.e. 3 or 7)

box '2' can be filled in 3 ways.

So the number of such numbers formed = $1 \times 3 \times 2 = 6$

When box 1 is having 7,

box '3' can be filled in 1 way (i.e. 3 only)

box '2' can be filled in 3 ways.

∴ The number of such numbers formed = $1 \times 3 \times 1 = 3$

∴ The total number of such numbers = $3 + 6 + 6 + 3 = 18$

$$10. \text{ For all cases, } n(S) = {}^{52}C_4 = \frac{52 \times 51 \times 50 \times 49}{1 \times 2 \times 3 \times 4} = 13 \times 17 \times 25 \times 49$$

$$(i) \quad n(E) = {}^{48}C_4 = \frac{48 \times 47 \times 46 \times 45}{1 \times 2 \times 3 \times 4} = 2 \times 47 \times 46 \times 45$$

$$P(E) = \frac{2 \times 47 \times 46 \times 45}{13 \times 17 \times 25 \times 49} = \frac{39916}{54145}$$

$$(ii) \quad n(E) = {}^4C_1 \times {}^4C_1 \times {}^4C_1 \times {}^4C_1 \\ = 4 \times 4 \times 4 \times 4 = 256$$

$$P(E) = \frac{4 \times 4 \times 4 \times 4}{13 \times 17 \times 25 \times 49} = \frac{256}{270725}$$

$$(iii) \quad n(E) = {}^{36}C_2 \times {}^8C_1 \times {}^8C_1 \\ = \frac{36 \times 35}{1 \times 2} \times 8 \times 8 = 18 \times 35 \times 8 \times 8$$

$$\therefore P(E) = \frac{18 \times 35 \times 8 \times 8}{13 \times 17 \times 25 \times 49} = \frac{18 \times 8 \times 8}{13 \times 17 \times 35} = \frac{1152}{7735}$$

$$11. \text{ Total number of favourable events } n(S) = 8 + 7 + 6 = 21$$

Total number of possible events = 7

E = event of getting neither red nor green – event of getting blue.

$$\therefore P(E) = \frac{7}{21} = \frac{1}{3}$$

$$12. \text{ Total number of possible events } = 2 + 3 + 2 = 7$$

The number of ways in which 2 balls can be drawn out of given 7 = ${}^7C_2 = \frac{7 \times 6}{1 \times 2} = 21$

i.e. $n(S) = 21$

E = event of getting balls not to be blue

$n(E)$ = ways of drawing 2 balls from given 5

$$= {}^5C_2 = \frac{5 \times 4}{1 \times 2} = 10$$

$$\therefore P(E) = \frac{n(E)}{n(s)} = \frac{10}{21}$$

$$13. \text{ The number of ways in which 3 green marble can be drawn from the given 5 } = {}^5C_3$$

$$= {}^5C_2 = \frac{5 \times 4}{1 \times 2} = 10$$

Similarly, from 4 yellow marbles, marble can be drawn in ${}^4C_3 = {}^4C_1 = 4$ ways

Similarly, from 3 white marbles, 3 white marble can be drawn in ${}^3C_3 = 1$ way

The number of ways in which 3 marbles can be drawn from $5 + 4 + 3 = 12$ marbles given = ${}^{12}C_3$

$$= \frac{12 \times 11 \times 10}{1 \times 2 \times 3} = 220$$

∴ Required probability = 1 – Probability (of getting all the three with same colour)

$$= 1 - \frac{{}^5C_3 + {}^4C_3 + {}^3C_3}{220}$$

$$= 1 - \frac{10 + 4 + 1}{220} = 1 - \frac{3}{44} = \frac{41}{44}$$

14. The number of possible events when a dice is thrown twice = $6 \times 6 = 36$;

Number of possible events to get a sum 9 when a dice is thrown twice

$$= \{(3, 6), (4, 5), (5, 4), (6, 3)\}$$

E = Event of getting sum 9

∴
$$P(E) = \frac{4}{36} = \frac{1}{9}$$

15. The number of possible events when two cards are drawn together from a pack of 52 cards

$$= {}^{52}C_2 = \frac{52 \times 51}{1 \times 2} = 26 \times 51 = 1326$$

The number of possible events when two cards are drawn together to get both cards of king suit

$$= {}^4C_2 = \frac{4 \times 3}{1 \times 2} = 6$$

E = Event of getting both cards of king suit.

∴
$$P(E) = \frac{6}{1326} = \frac{1}{221}$$

16. The number of possible events when two cards are drawn together from a pack of 52 cards

$$= {}^{52}C_2 = \frac{52 \times 51}{1 \times 2} = 26 \times 51 = 1326$$

Number of possible events to draw a spade = ${}^{13}C_1 = 13$ ways.

Number of possible events to draw a heart = ${}^{13}C_1 = 13$ ways.

E = Event of getting 1 spade and 1 heart

$$n(E) = 13 \times 13 = 169$$

∴
$$P(E) = \frac{n(E)}{n(S)} = \frac{169}{1326} = \frac{13}{102}$$

17. Number of possible events to draw three balls from 4 white, 5 red and 6 blue balls

$$= {}^{15}C_3 = \frac{15 \times 14 \times 13}{1 \times 2 \times 3} = 5 \times 7 \times 13 = 455$$

E = Event of getting 3 balls red

The number of favourable event to get 3 red balls = ${}^5C_3 = \frac{5 \times 4}{1 \times 2} = 10$

∴
$$P(E) = \frac{n(E)}{n(S)} = \frac{10}{455} = \frac{2}{91}$$

18. Probability that student passed in English $P(E) = \frac{30}{100} = \frac{3}{10}$;

Probability that student passed in French $P(F) = \frac{20}{100} = \frac{2}{10} = \frac{1}{5}$;

Probability that student passed both English and French $P(E \cap F) = \frac{10}{100} = \frac{1}{10}$;

$$\therefore P(E \text{ or } F) = P(E) + P(F) - P(E \cap F) = \frac{3}{10} + \frac{1}{5} - \frac{1}{10} = \frac{3+2-1}{10} = \frac{4}{10} = \frac{2}{5}$$

19. The number of possible ways to select 4 persons from 3 men, 2 women and 4 children

$$= {}^9C_4 = \frac{9 \times 8 \times 7 \times 6}{1 \times 2 \times 3 \times 4} = 9 \times 7 \times 2 = 126$$

The number of ways in which 2 children can be drawn from given 4 children $= {}^4C_2 = \frac{4 \times 3}{1 \times 2} = 6$

Number of ways in which 2 person can be drawn from 3 men and 2 women $= {}^5C_2 = \frac{5 \times 4}{1 \times 2} = 10$

E = Event of selecting 4 persons with exactly two children

$$P(E) = \frac{{}^4C_2 \times {}^5C_2}{{}^9C_4} = \frac{6 \times 10}{126} = \frac{60}{126} = \frac{10}{21}$$

20. The number of possible events to draw a ball from 4 red, 5 green and 6 white balls $= {}^{15}C_1 = 15$

The number of ways in which a red ball is drawn from given 4 red balls $= {}^4C_1 = 4$

The number of ways in which a green ball is drawn from 5 green balls $= {}^5C_1 = 5$

$\therefore$ Total number of favourable events $= 4 + 5 = 9$

E = Event of getting either a red or a green ball

$$P(E) = \frac{n(E)}{n(S)} = \frac{4+5}{15} = \frac{9}{15} = \frac{3}{5}$$

21. Number of possible ways to draw three balls from 6 black and 4 red balls

$$= {}^{10}C_3 = \frac{10 \times 9 \times 8}{1 \times 2 \times 3} = 10 \times 3 \times 4 = 120$$

Number of favourable ways to get one red ball from given 4 red balls $= {}^4C_1 = 4$

Number of favourable ways to get two black balls from given 6 black balls $= {}^6C_2 = \frac{6 \times 5}{1 \times 2} = 15$

E = event of getting one red and two black balls

$$P(E) = \frac{n(E)}{n(S)} = \frac{{}^4C_1 \times {}^6C_2}{{}^{10}C_3} = \frac{4 \times 15}{120} = \frac{1}{2}$$

22. Number of possible events $= 36$

E = Event of getting product of 2 results even

Number of favourable events $= \{(1, 6), (1, 4), (1, 2), (2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6), (3, 2), (3, 4), (3, 6), (4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6), (5, 2), (5, 4), (5, 6), (6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6)\} = 27$

$$\therefore P(E) = \frac{n(E)}{n(S)} = \frac{27}{36} = \frac{3}{4}$$

Ratio and Proportion

RATIO: Ratio of two quantities represents how many times a quantity contains another quantity of the same kind.

In a ratio p/q , p and q are called terms of the ratio. The number in the first term p or numerator is called the *antecedent* and the second term q or denominator is called the *consequent*.

The ratio $7 : 11$ represents $\frac{7}{11}$ with antecedent = 7, consequent = 11.

RULE

The multiplication or division of each term of a ratio by the same non-zero number does not affect the ratio.

Compound Ratio: Ratios are compounded by multiplying together the antecedents for a new antecedent, and the consequents for a new consequent.

The compounded ratio of the ratios $(a : b)$, $(c : d)$, $(e : f)$ is $(ace : bdf)$.

For example, compounded ratio of $(2 : 3)$, $(4 : 5)$ and $(6 : 7)$ is $2 \times 4 \times 6 : 3 \times 5 \times 7 = 48 : 105$.

EXAMPLE 1 Find the compounded ratio of $3 : 4$, $13 : 9$, $5 : 26$ and $15 : 2$

Solution The required ratio = $\frac{3}{4} \times \frac{13}{9} \times \frac{5}{26} \times \frac{15}{2} = \frac{25}{16}$

EXAMPLE 2 Find the compounded ratio of $4 : 5$, $8 : 9$ and $12 : 13$

Solution The required ratio = $\frac{4}{5} \times \frac{8}{9} \times \frac{12}{13} = \frac{128}{195}$



Important Formulae

Duplicate ratio of $a : b$ is $a^2 : b^2$

Sub-duplicate ratio of $a : b$ is $\sqrt{a} : \sqrt{b}$

Triplicate ratio of $a : b$ is $a^3 : b^3$

Sub-triplicate ratio of $a : b$ is $a^{1/3} : b^{1/3}$

Inverse Ratio: If $a : b$ is the given ratio, then $\frac{1}{a} : \frac{1}{b}$ or $b : a$ is called its *inverse* or *reciprocal ratio*.

EXAMPLE 3 Find three numbers in the ratio $1 : 2 : 3$, so that the sum of their squares is equal to 224.

Solution Let the numbers be x , $2x$ and $3x$.

$$\therefore (x)^2 + (2x)^2 + (3x)^2 = 224$$

$$\therefore x^2 (1 + 4 + 9) = 224$$

$$x^2 = \frac{224}{14} = 16$$

$$\therefore x = 4$$

$\therefore$ The required numbers are **4, 8 and 12**.

 RULE

If the ratio between the first and the second quantities is $a : b$ and the ratio between second and third quantities is $c : d$, then the ratio among the first, the second and the third quantities is given by $ac : bc : bd$.

Proof: First : Second = $a : b$

Second : Third = $c : d$.

To combine the two ratios, consequent (b) of the first ratio should be made equal to the antecedent (c) of the second ratio. Therefore, multiply first ratio by c and second ratio by b .

$$\therefore \text{First : Second} = ac : bc$$

$$\text{Second : Third} = bc : bd$$

$$\therefore \text{First : Second : Third} = ac : bc : bd.$$

EXAMPLE 4 The sum of three numbers is 138. If the ratio between the first and the second is $2 : 3$ and that between the second and the third be $5 : 7$, find the second number.

Solution Let the numbers be a , b and c .

$$a : b = 2 : 3; b : c = 5 : 7$$

Combining these ratios, we get

$$a : b : c = 10 : 15 : 21$$

$$\text{The second number } b = \frac{15}{10 + 15 + 21} \times 138 = \frac{15}{46} \times 138 = \mathbf{45}$$

EXAMPLE 5 Ratio of money with Arun and Sunil is $3 : 4$ and that with Sunil and Suresh is $3 : 7$. If money with Sunil is ₹ 120, how much money does Suresh have?

Solution

Arun	Sunil	Suresh
3	4	
	3	7
9	12	28

The ratio of money with Arun, Sunil and Suresh is $9 : 12 : 28$.

Money with Sunil (12 parts) = ₹ 120

$\therefore$ Money with Suresh (28 parts) = ₹ 280

 **Theorem:** If the first and second quantities are in the ratio $a : b$; the second and third are in the ratio $c : d$ and the third and fourth quantities are in the ratio $e : f$, then the ratio among the first, second, third and fourth quantities is given by $ace : bce : bde : bdf$.

EXAMPLE 6 If $A : B = 3 : 4$; $B : C = 8 : 10$ and $C : D = 15 : 17$, then find $A : B : C : D$.

Solution $A : B = 3 : 4$

$$B : C = 8 : 10$$

$$C : D = 15 : 17$$

$$\begin{aligned} A : B : C : D &= 3 \times 8 \times 15 : 4 \times 8 \times 15 : 4 \times 10 \times 15 : 4 \times 10 \times 17 \\ &= 9 : 12 : 15 : 17 \end{aligned}$$

EXAMPLE 7 ₹ 960 is divided among 4 men, 5 women and 8 boys such that the share of a man, a woman and a boy may be in the ratio of $5 : 4 : 3$. What is the share of a woman?

Solution The ratio of shares among 4 men, 5 women and 8 boys is

$$4 \times 5 : 5 \times 4 : 8 \times 3 = 20 : 20 : 24$$

$$\therefore \text{Shares of 5 women} = \frac{20}{20 + 20 + 24} \times 960 = \mathbf{300}$$

$$\therefore \text{Share of a woman} = \frac{300}{5} = \mathbf{60}$$

PROPORTION: Equality of ratios is called the *proportion*. The numbers a, b, c and d are said to be in proportion if $a : b = c : d$. Here, d is called the *fourth proportional*.

$$\therefore ad = bc$$



Notes:

1. If any four quantities are in proportion, the product of the extremes is equal to the product of the mean, i.e. if $a : b = c : d$ then $ad = bc$.
2. Three quantities are said to be in *continued proportion* when the ratio of the first to the second is equal to the ratio of the second to the third.
The second quantity is called the *mean proportional* between the first and the third; and the third quantity is called the *third proportional* to the first and the second.
For example: 16, 12 and 9 are in continued proportion for $16 : 12 :: 12 : 9$.
Hence, 12 is the mean proportional between 16 and 9, and 9 is third proportional to 16 and 12.

EXAMPLE 8 Find the mean proportional between 6 and 96.

Solution Let x be the required mean proportional.

Then $6 : x :: x : 96$

or $\frac{6}{x} = \frac{x}{96}$

i.e. $x^2 = 6 \times 96$

$\therefore x = \sqrt{6 \times 96} = \mathbf{24}$

i.e. the mean proportional between two numbers is equal to the square root of their product.

EXAMPLE 9 Find the third proportional to 18 and 24.

Solution Let the third proportional be x .

Then 18, 24 and x are in proportion

$$\text{i.e.} \quad 18 : 24 :: 24 : x$$

$$\therefore \quad x = \frac{24 \times 24}{18} = 32$$

EXAMPLE 10 Find the fourth proportional to 16, 28 and 36.

Solution Let x be the fourth proportional

$$\therefore \quad 16 : 28 = 36 : x$$

$$\therefore \quad x = \frac{28 \times 36}{16} = 63$$

Direct proportion: The proportion by which the value of one variable varies directly as the value of another variable.

EXAMPLE 11 If 3 books cost ₹ 18, then what will be the cost of 21 books?

Solution If the number of books is increased n times, price of books will also increase n times where $n = 1, 2, 3, 4 \dots$

$$\therefore \quad 3 \text{ books} : 21 \text{ books} :: ₹ 18 : ₹ x$$

where x is the required cost.

$$\therefore \quad \text{Required cost } x = \frac{21 \times 18}{3} = ₹ 126$$

Above example illustrates the *direct proportion*.

Inverse proportion: The proportion by which the value of one variable varies inversely as another variable.

EXAMPLE 12 If 12 men can do a certain work in 25 days, in how many days, the same work will be completed by 10 men?

Solution Let the number of days required be x , so that work will be over by 10 men.

Here the inverse ratio of number of men is equal to the ratio of corresponding number of days

$$\therefore \quad \frac{1}{12} : \frac{1}{10} :: 25 : x$$

$$\text{or} \quad 10 : 12 :: 25 : x$$

$\therefore$ the required number of days

$$x = \frac{12 \times 25}{10} = 30$$

The above example illustrates the *inverse proportion*.

RULE

To find the 4th proportion when the other three are given is called the *simple proportion*. If a , b and c are the given quantities and you have to find the fourth quantity x ,

$$x = \frac{\text{Multiplication of means}}{\text{1st term}}$$

If a and b are of same kind and $a > b$, then $\frac{a}{b} = \frac{c}{x}$ from which x can be calculated.

EXAMPLE 13 If 96 mangoes can be bought for ₹ 288, how many can be bought for ₹ 48?

Solution Let the required number of mangoes be x . As the required quantity is less,

$$288 : 48 = 96 : x$$

$$\therefore x = \frac{48 \times 96}{288} = \mathbf{16}$$

EXAMPLE 14 If 8 men can do a piece of work in 25 days working 8 hours per day, how many men will do the same work for 10 days working for 5 hours?

Solution Let the required number of men be x .

As the number of days is less, the number of men will be more. Less working hours, more men.

$$\left. \begin{array}{l} \text{Days : } 10 : 25 \\ \text{Hours : } 5 : 8 \\ \text{Work : } 1 : 1 \end{array} \right\} :: 8 : x$$

$$\therefore x = \frac{\text{Third term} \times \text{Multiplication of means}}{\text{Multiplication of first terms}}$$

$$\text{or } x = \frac{8 \times 25 \times 8 \times 1}{10 \times 5 \times 1} = \mathbf{16 \text{ men}}$$

EXAMPLE 15 If 12 men can reap 80 hectares in 16 days, how many hectares 20 men can reap in 27 days?

Solution Let the required area be x hectares. As the number of men and the number of days increase, they will reap more hectares.

$$\therefore \text{Area required } x = \frac{80 \times 20 \times 27}{12 \times 16} = \mathbf{225 \text{ hectares}}$$

Proportional division: Proportion may be applied to divide a given quantity into parts which are proportional to the given numbers. This is termed the proportional division.

EXAMPLE 16 Divide ₹ 1848 into three shares proportional to 3, 4 and 5.

$$\begin{aligned} \text{Solution } \text{1st share} &= \frac{3}{3+4+5} \times 1848 \\ &= \frac{3}{12} \times 1848 = \frac{1848}{4} = \mathbf{₹ 462} \end{aligned}$$

$$\begin{aligned} \text{2nd share} &= \frac{4}{3+4+5} \times 1848 \\ &= \frac{4}{12} \times 1848 = \frac{1848}{3} = \mathbf{₹ 616} \end{aligned}$$

$$\begin{aligned} \text{3rd share} &= \frac{5}{3+4+5} \times 1848 \\ &= \frac{5}{12} \times 1848 = 5 \times 154 = \mathbf{₹ 770} \end{aligned}$$

EXAMPLE 17 A certain sum of money is divided among P , Q and S such that for each rupee P has, Q has 35 paise and S has 60 paise. If Q 's share is ₹ 7, find the sum of money.

Solution $P : Q : S = 100 : 35 : 60 = 20 : 7 : 12$

$$\therefore 20 + 7 + 12 = 39$$

Given $\frac{7}{39}$ of sum of money = ₹ 7

$$\therefore \text{The sum of money} = \frac{7 \times 39}{7} = ₹ 39$$

EXAMPLE 18 Divide ₹ 1391 into three parts such that 5 times the first may be equal to 6 times the second and 7 times the third.

Solution 5 times 1st part = 6 times 2nd part = 7 times 3rd part

$$\therefore \text{first part} = \frac{1}{5}; \text{second part} = \frac{1}{6}; \text{third part} = \frac{1}{7};$$

$$\text{first part} : \text{second part} : \text{third part} = \frac{1}{5} : \frac{1}{6} : \frac{1}{7} = 42 : 35 : 30$$

Now, divide 1391 in the proportion of these numbers. The numbers will be

$$\frac{42(1391)}{42+35+30}, \frac{35(1391)}{42+35+30} \text{ and } \frac{30(1391)}{42+35+30}$$

i.e. 546, 455 and 390.

EXAMPLE 19 Divide ₹ 1540 among 8 men, 9 women and 5 boys such that each woman gets 2 times as much as a boy, and a man as much as a woman and 2 boys together. Find how much each person receives?

Solution 1 man = 1 woman + 2 boys

$$1 \text{ woman} = 2 \text{ boys}$$

$$\therefore 1 \text{ man} = 4 \text{ boys}$$

$$\therefore 8 \text{ men} : 9 \text{ women} : 5 \text{ boys} = 32 \text{ boys} : 18 \text{ boys} : 5 \text{ boys} = 32 : 18 : 5$$

Dividing ₹ 1540 in the ratio of 32 : 18 : 5, we get,

$$5 \text{ boy's share} = \frac{5 \times 1540}{32 + 18 + 5} = ₹ 140$$

$$\therefore 1 \text{ boy's share} = \frac{140}{5} = ₹ 28$$

$$1 \text{ woman's share} = 2 \times 28 = ₹ 56$$

$$1 \text{ man's share} = 4 \times 28 = ₹ 112$$

EXAMPLE 20 One rupee coins, 50 paise coins and 25 paise coins, whose numbers are proportional to 3, $3\frac{1}{2}$ and 4 are together worth ₹ 161. Find the number of each coin.

Solution Coins are in the ratio $3 : 3\frac{1}{2} : 4$, i.e. 6 : 7 : 8

$$\therefore \text{Their proportional value} = 6 \times 1 : 7 \times \frac{1}{2} : 8 \times \frac{1}{4}$$

$$= 6 : 3\frac{1}{2} : 2$$

$$= 12 : 7 : 4$$

$$\therefore \text{The value of one rupee coins} = \frac{12}{12+7+4} \times 161 = \text{₹ } 84$$

$$\text{The value of 50 paise coins} = \frac{7}{12+7+4} \times 161 = \text{₹ } 49$$

$$\text{The value of 25 paise coins} = \frac{4}{12+7+4} \times 161 = \text{₹ } 28$$

$\therefore$ There are 84 one-rupee coins, 98 fifty-paise coins and 112 twenty-five-paise coins.



Note: If milk and water are in the proportion $a : b$ in x litres mixture of milk and water, the quantity of water to be added in order to make this ratio $c : d$ is $\frac{x(ad - bc)}{c(a + b)}$.

EXAMPLE 21 In the 45 litres mixture of milk and water, the ratio of milk and water is 5 : 4. Find the quantity of water required to be added so that the resultant mixture will be in the ratio 4 : 5.

Solution Quantity of water required to be added

$$\frac{x(ad - bc)}{c(a + b)} = \frac{45(5 \times 5 - 4 \times 4)}{4(5 + 4)} = \frac{45 \times 9}{4 \times 9} = \mathbf{11.25 \text{ litres}}$$



Note: A mixture contains milk and water in the ratio $a : b$. If x litres of water is added to the mixture, milk and water mixture will be in the ratio $a : c$. Then the quantity of milk in the mixture is given by $\frac{ax}{c - b}$ and that of water is given by $\frac{bx}{c - b}$.

EXAMPLE 22 A mixture contains milk and water in the ratio 3 : 2. If 4 litres of water is added to the mixture, the proportion becomes 3 : 4. Find the quantities of milk and water in the mixture.

$$\text{Quantity of milk} = \frac{ax}{c - b} = \frac{3 \times 4}{4 - 2} = \mathbf{6 \text{ litres}}$$

$$\text{Quantity of water} = \frac{bx}{c - b} = \frac{2 \times 4}{4 - 2} = \mathbf{4 \text{ litres}}$$

EXAMPLE 23 A mixture contains milk and water in the ratio 7 : 4. On adding 9 litres of water, quantity of milk and water in the mixture becomes equal. Find the quantity of milk and water in the mixture.

Solution Now, the new proportion of milk and water is 1 : 1 or 7 : 7. Therefore, proportional ratios are 7 : 4 and 7 : 7.

$$\therefore \text{Quantity of milk} = \frac{ax}{c - b} = \frac{7 \times 9}{7 - 4} = \frac{63}{3} = \mathbf{21 \text{ litres}}$$

$$\text{Quantity of water} = \frac{bx}{c-b} = \frac{4 \times 9}{7-4} = \frac{36}{3} = \mathbf{12 \text{ litres}}$$

The ratio between the two numbers is $a : b$. If each number is increased by x , then the ratio becomes $c : d$. Then the two numbers are given by

$$\frac{xa(c-d)}{ad-bc} \text{ and } \frac{xb(c-d)}{ad-bc}$$

EXAMPLE 24 The ratio between two numbers is $3 : 4$. If each number is increased by 3, the ratio becomes $4 : 5$. Find the numbers.

Solution The numbers are

$$(i) \quad \frac{xa(c-d)}{ad-bc} = \frac{3 \times 3(4-5)}{3 \times 5 - 4 \times 4} = \frac{9(-1)}{-1} = \mathbf{9}$$

$$(ii) \quad \frac{xb(c-d)}{ad-bc} = \frac{3 \times 4(4-5)}{3 \times 5 - 4 \times 4} = \frac{12(-1)}{-1} = \mathbf{12}$$

$\therefore$ The numbers are **9 and 12**.



Note: The incomes of two persons are in the ratio $a : b$ and their expenditures are in the ratio $c : d$. If each of them saves ₹ X , then their incomes are given by $\frac{Xa(d-c)}{(ad-bc)}$ and $\frac{Xb(d-c)}{(ad-bc)}$.

Their expenditures are given by $\frac{Xc(b-a)}{ad-bc}$ and $\frac{Xd(b-a)}{ad-bc}$

EXAMPLE 25 The incomes of Ashok and Suresh are in the ratio $4 : 3$ and their expenditures are in the ratio $3 : 2$. If each saves ₹ 800, what will be their income?

Solution The incomes are in the ratio $a : b = 4 : 3$

Expenditures are in the ratio $c : d = 3 : 2$

Savings $X = 800$

$$\therefore \text{Ashok's income} = \frac{Xa(d-c)}{(ad-bc)} = \frac{800 \times 4(2-3)}{4 \times 2 - 3 \times 3} = \frac{3200(-1)}{(-1)} = \mathbf{₹ 3200}$$

$$\text{Suresh's income} = \frac{Xb(d-c)}{(ad-bc)} = \frac{800 \times 3(2-3)}{4 \times 2 - 3 \times 3} = \frac{2400(-1)}{(-1)} = \mathbf{₹ 2400}$$

EXAMPLE 26 The incomes of Satheesh and Subhash are in the ratio $7 : 9$ and their expenditures are in the ratio $5 : 7$. If each of them saves ₹ 1200, what are their incomes and expenditure?

Solution The incomes are in the ratio $a : b = 7 : 9$

The expenditures are in the ratio $c : d = 5 : 7$

$$\text{Satheesh's income} = \frac{Xa(d-c)}{(ad-bc)} = \frac{1200 \times 7(7-5)}{7 \times 7 - 9 \times 5} = \frac{8400 \times 2}{4} = \mathbf{₹ 4200}$$

$$\text{Subhash's income} = \frac{Xb(d-c)}{(ad-bc)} = \frac{1200 \times 9(7-5)}{7 \times 7 - 9 \times 5} = \frac{10800 \times 2}{4} = \mathbf{₹ 5400}$$

$$\text{Satheesh's expenditure} = \frac{Xc(b-a)}{(ad-bc)} = \frac{1200 \times 5(9-7)}{7 \times 7 - 9 \times 5} = \frac{6000 \times 2}{4} = \text{₹ } 3000$$

$$\text{Subhash's expenditure} = \frac{Xd(b-a)}{(ad-bc)} = \frac{1200 \times 7(9-7)}{7 \times 7 - 9 \times 5} = \frac{8400 \times 2}{4} = \text{₹ } 4200$$



Note: A number which when added to the terms of the ratio $a : b$ makes it equal to the ratio $c : d$ is $\frac{ad-bc}{c-d}$.

EXAMPLE 27 Find the number which when added to the ratio $7 : 9$, makes it equal to $4 : 5$.

Solution $a : b = 7 : 9$; $c : d = 4 : 5$

$$\begin{aligned} \therefore \text{The required number} &= \frac{ad-bc}{c-d} = \frac{7 \times 5 - 9 \times 4}{4-5} \\ &= \frac{-1}{-1} = +1 \end{aligned}$$

A number which when subtracted from the terms of the ratio $a : b$, makes it equal to the ratio $c : d$ is $\frac{bc-ad}{c-d}$.

EXAMPLE 28 The ratio of incomes of Ajith and Sunil last year was $4 : 5$. The ratio of their own incomes of last year and this year are $6 : 7$ and $5 : 6$ respectively. If the total sum of their present incomes is ₹ 6,400, find the present income of Ajith.

$$\begin{aligned} \text{Solution} \quad \text{The ratio of present income} &= 4 \times \frac{7}{6} : 5 \times \frac{6}{5} \\ &= \frac{28}{6} : 6 = \frac{14}{3} : 6 \\ &= 14 : 18 \text{ or } 7 : 9 \end{aligned}$$

$$\therefore \text{Ajith's present income} = \frac{7}{7+9} \times 6400 = \text{₹ } 2800$$

Otherwise, Ratio of Ajith's incomes (last year and this year) = $6 : 7$

$$= 6 \times \frac{4}{6} : 7 \times \frac{4}{6} = 4 : \frac{14}{3}$$

Ratio of Sunil's incomes (last year and this year) = $5 : 6$

$$\therefore \text{Ratio of Ajith's income to Sunil's this year} = \frac{14}{3} : 6 = 7 : 9$$

$$\therefore \text{Ajith's present income} = \frac{7 \times 6400}{7+9} = \text{₹ } 2800$$

EXAMPLE 29 A bucket contains a mixture of milk and water in the proportion $2 : 1$. If 6 litres of mixture is replaced by 6 litres of water, then the ratio of the two liquids becomes $4 : 5$, what was the quantity of water contained in the bucket?

Solution Given 1st ratio = 2 : 1

2nd ratio = 4 : 5

D = Difference of cross products of ratios = $2 \times 5 - 1 \times 4 = 10 - 4 = 6$

$$\begin{aligned}\therefore \text{Common factor of 1st ratio} &= \left[\frac{\text{Quantity replaced}}{\text{Sum of terms in 1st ratio}} \right] + \left[\frac{\text{Quantity replaced} \times 4}{D} \right] \\ &= \left[\frac{6}{3} \right] + \left[\frac{6 \times 4}{6} \right] = 2 + 4 = 6\end{aligned}$$

$\therefore$ The quantity of milk = $2 \times 6 = 12$ litres

Similarly, the quantity of water = $1 \times 6 = 6$ litres

EXAMPLE 30 A vessel contains milk and water in the ratio 5 : 4. If 27 litres of mixture is removed and the same quantity of water is added, the ratio becomes 4 : 5. What quantity does the vessel contain?

Solution Let the vessel contain $5x$ litres of milk and $4x$ litres of water.

The removed quantity of mixture contains $\frac{5}{5+4} \times 27 = 15$ litres of milk and $27 - 15 = 12$ litres of water.

Now, $5x - 15 : (4x - 12 + 27) = 4 : 5$

i.e. $(5x - 15) : (4x + 15) = 4 : 5$

$$\frac{5x - 15}{4x + 15} = \frac{4}{5}$$

$$\Rightarrow 25x - 75 = 16x + 60$$

$$\Rightarrow 25x - 16x = 75 + 60 = 135$$

$$9x = 135$$

$$\therefore x = \frac{135}{9} = 15 \text{ litres}$$

$\therefore$ Vessel contains $(5 + 4)x = 9 \times 15 = 135$ litres.



Short cut: When the ratio is reversed (i.e. 5 : 4 becomes 4 : 5)

$$\begin{aligned}\text{Total quantity} &= \frac{(5+4)^2}{5^2 - 4^2} \times \text{Quantity of first liquid in the removed mixture} \\ &= \frac{9^2}{9} \times 15 = 135 \text{ litres}\end{aligned}$$



EXERCISES

- If $A : B = 7 : 9$ and $B : C = 8 : 11$, find $A : B : C$.
(a) $72 : 56 : 99$ (b) $56 : 72 : 99$ (c) $99 : 56 : 72$ (d) $56 : 99 : 72$ (e) None of these
- The sum of two numbers is 25 and their difference is $7\frac{1}{2}$. Find the ratio of the numbers.
[Hint : If $\frac{a}{b} = \frac{c}{d}$; $\frac{a+b}{a-b} = \frac{c+d}{c-d}$]
(a) $6 : 13$ (b) $7 : 13$ (c) $13 : 7$ (d) $13 : 6$ (e) None of these
- ₹ 120 is divided among 6 men, 8 women and 6 boys so that a man gets as much as 2 boys and 2 women get as much as 3 boys. Find the share of a boy.
(a) ₹ 3 (b) ₹ 6 (c) ₹ 12 (d) ₹ 4 (e) None of these
- What should be subtracted from the terms of the ratio $19 : 23$ makes it equal to the ratio of $3 : 4$?
(a) 7 (b) 6 (c) 5 (d) 4 (e) None of these
- An employer reduces the number of his employees in the ratio $9 : 7$ and increases their wages in the ratio $14 : 15$. Find the ratio in which the bill of wages increases or decreases.
(a) $1/3$ (b) $1/4$ (c) $1/5$ (d) $1/8$ (e) None of these
- When wheat is ₹ 13 per kg, 75 men can be fed for 15 days at a certain cost. How many men can be fed for 45 days at the same cost, when wheat is ₹ 1 per kg?
(a) 225 (b) 250 (c) 325 (d) 175 (e) None of these
- If 1,000 copies of a book of 25 sheets requires 50 reams of paper, how much paper is required for 5,000 copies of a book of 32 sheets?
(a) 160 (b) 320 (c) 480 (d) 640 (e) None of these
- Divide ₹ 1,000 among P , Q and R such that P gets $2/3$ of Q 's share and R gets $5/3$ of Q 's share. Then what is the share of Q ?
(a) 100 (b) 200 (c) 300 (d) 400 (e) None of these
- If 0.6 time one number is equal to 0.025 time the other, what is the ratio of two numbers?
(a) $1/24$ (b) $5/24$ (c) $7/24$ (d) $11/24$ (e) None of these
- Calculate the fourth proportional to the numbers 286, 78, 1342
(a) 732 (b) 671 (c) 183 (d) 366 (e) None of these
- How many one-rupee coins, 50-paise coins and 25-paise coins in total of which the numbers are proportional to 5, 7 and 12 are together worth ₹ 115?
(a) 50, 70, 120 (b) 60, 70, 110 (c) 70, 70, 100 (d) 70, 80, 90 (e) None of these
- Two numbers are in the ratio $3 : 5$. If 9 is subtracted from each, the new numbers are in the ratio $12 : 23$. The smaller number is [SSC]
(a) 27 (b) 33 (c) 49 (d) 55
- Two numbers are in the ratio $1 : 2$. If 7 is added to both, their ratio changes to $3 : 5$. The greatest number is [Hotel Management]
(a) 24 (b) 26 (c) 28 (d) 32
- If $0.75 : x :: 5 : 8$, then x is equal to [LIC]
(a) 1.12 (b) 1.20 (c) 1.25 (d) 1.30
- If $x : y = 5 : 2$, then $(8x + 9y) : (8x + 2y)$ is [SSC]
(a) $22 : 29$ (b) $26 : 61$ (c) $29 : 22$ (d) $61 : 26$

16. Salaries of Ravi and Sumit are in the ratio 2 : 3. If the salary of each is increased by ₹ 4000, the new ratio becomes 40 : 57. What is Sumit's present salary? [Bank PO]
(a) ₹ 17000 (b) ₹ 20000 (c) ₹ 25500 (d) None of these
17. If 10% of $x = 20\%$ of y , then $x : y$ is equal to [CDS]
(a) 1 : 2 (b) 2 : 1 (c) 5 : 1 (d) 10 : 1
18. Ratio of earnings of A and B is 4 : 7. If the earnings of A increase by 50% and those of B decrease by 25%, the new ratio of their earnings becomes 8 : 7. What are A's earnings? [Bank PO]
(a) ₹ 21000 (b) ₹ 26000 (c) ₹ 28000 (d) Data inadequate
19. If $A : B = 8 : 15$; $B : C = 5 : 8$ and $C : D = 4 : 5$, then $A : D$ is equal to
(a) 2 : 7 (b) 4 : 15 (c) 8 : 15 (d) 15 : 4
20. If $A : B = 2 : 3$, $B : C = 4 : 5$ and $C : D = 6 : 7$, then $A : B : C : D$ is [SSC]
(a) 16 : 22 : 30 : 35 (b) 16 : 24 : 15 : 35
(c) 16 : 24 : 30 : 35 (d) 18 : 24 : 30 : 35
21. The salaries of A, B, C are in the ratio 2 : 3 : 5. If the increments of 15%, 10% and 20% are allowed respectively in their salaries, then what will be the new ratio of their salaries? [Bank PO]
(a) 3 : 3 : 10 (b) 10 : 11 : 20 (c) 23 : 33 : 60 (d) cannot be determined
22. If ₹ 782 be divided into three parts, proportional to $\frac{1}{2} : \frac{2}{3} : \frac{3}{4}$, then the first part is: [CBI]
(a) ₹ 182 (b) ₹ 190 (c) ₹ 196 (d) ₹ 204
23. If $A : B = \frac{1}{2} : \frac{3}{8}$, $B : C = \frac{1}{3} : \frac{5}{9}$ and $C : D = \frac{5}{6} : \frac{3}{4}$, then the ratio $A : B : C : D$ is [SSC]
(a) 4 : 6 : 8 : 10 (b) 6 : 4 : 8 : 10 (c) 6 : 8 : 9 : 10 (d) 8 : 6 : 10 : 9
24. The sum of three numbers is 98. If the ratio of the first to the second is 2 : 3 and that of the second to the third is 5 : 8, then the second number is: [SSC]
(a) 20 (b) 30 (c) 48 (d) 58
25. A sum of ₹ 1,300 is divided amongst P, Q, R and S such that
$$\frac{P's\ share}{Q's\ share} = \frac{Q's\ share}{R's\ share} = \frac{R's\ share}{S's\ share} = \frac{2}{3}$$

Then P's share is [LIC]
(a) ₹ 140 (b) ₹ 160 (c) ₹ 240 (d) ₹ 320
26. A and B together have ₹ 1210. If $\frac{4}{15}$ of A's amount is equal to $\frac{2}{5}$ of B's amount, how much amount does B have? [AAO]
(a) ₹ 460 (b) ₹ 484 (c) ₹ 550 (d) ₹ 664
27. Two numbers are respectively 20% and 50% more than a third number. The ratio of the two numbers is [SSC]
(a) 2 : 5 (b) 3 : 5 (c) 4 : 5 (d) 6 : 7
28. The ratio of the number of boys and girls in a college is 7 : 8. If the percentage increase in the number of boys and girls is 20% and 10% respectively, what will be the new ratio? [RBI]
(a) 8 : 9 (b) 17 : 18 (c) 21 : 22 (d) cannot be determined
29. A sum of money is to be distributed among A, B, C and D in the proportion of 5 : 2 : 4 : 3. If C gets ₹ 1000 more than D, what is B's share? [RBI]
(a) ₹ 500 (b) ₹ 1500 (c) ₹ 2000 (d) None of these

30. The fourth proportional to 5, 8, 15 is [RBI]
 (a) 18 (b) 24 (c) 19 (d) 20 (e) 21
31. If 40% of a number is equal to two-thirds of another number, what is the ratio of first number to the second number? [Bank PO]
 (a) 2 : 5 (b) 3 : 7 (c) 5 : 3 (d) 7 : 3
32. x varies inversely as square of y . Given that $y = 2$ for $x = 1$. The value of x for $y = 6$ will be equal to [CDS]
 (a) 3 (b) 9 (c) $\frac{1}{3}$ (d) $\frac{1}{9}$
33. The compounded ratio of (2 : 5), (6 : 11) and (11 : 3) is
 (a) 2 : 11 (b) 11 : 5 (c) 4 : 5 (d) 5 : 11
34. The least whole number when subtracted from both the terms of the ratio 4 : 5 gives a ratio equal to 3 : 4 is
 (a) 3 (b) 6 (c) 9 (d) 12
35. A certain sum was divided among P and Q in the ratio 5 : 6. If Q's share was ₹ 2400, the total sum was
 (a) ₹ 4800 (b) ₹ 4400 (c) ₹ 3200 (d) ₹ 2800
36. The sides of a triangle are in the ratio $\frac{1}{2} : \frac{1}{3} : \frac{1}{4}$ and its perimeter is 156 cm. The length of the largest side is
 (a) 54 cm (b) 68 cm (c) 72 cm (d) 84 cm
37. The ratio of incomes of P and Q is 3 : 4 and the ratio of their expenditures is 2 : 3. If at the end of the year, each saves ₹ 2000, then the income of P is
 (a) ₹ 9000 (b) ₹ 8000 (c) ₹ 7000 (d) ₹ 6000
38. The ratio of the number of boys to girls in an institution is 6 : 5. If there are 192 boys in total, find the total number of students in the institution.
 (a) 250 (b) 325 (c) 352 (d) 425
39. A fraction which bears the same ratio to $\frac{1}{27}$ that $\frac{3}{11}$ does to $\frac{5}{9}$ is equal to [SSC]
 (a) $\frac{1}{55}$ (b) $\frac{1}{11}$ (c) $\frac{3}{11}$ (d) 55
40. If $A : B : C = 2 : 3 : 4$, then $\frac{A}{B} : \frac{B}{C} : \frac{C}{A}$ is equal to
 (a) 4 : 9 : 16 (b) 8 : 9 : 12 (c) 8 : 9 : 16 (d) 8 : 9 : 24



1. (b) 2. (c) 3. (d) 4. (a) 5. (e) 6. (c) 7. (b) 8. (c) 9. (a) 10. (d)
 11. (a) 12. (b) 13. (c) 14. (b) 15. (c) 16. (d) 17. (b) 18. (d) 19. (b) 20. (c)
 21. (c) 22. (d) 23. (d) 24. (b) 25. (b) 26. (b) 27. (c) 28. (c) 29. (c) 30. (b)
 31. (c) 32. (d) 33. (c) 34. (a) 35. (b) 36. (c) 37. (d) 38. (c) 39. (a) 40. (d)



Solutions with Necessary Explanation

1. Given $A : B = 7 : 9$; $B : C = 8 : 11$

$$A : B = 7 : 9$$

$$B : C = 8 : 11$$

Making the second proportion equal in both ratios, we get

$$A : B = 7 : 9 = 7 \times 8 : 9 \times 8 = 56 : 72$$

$$B : C = 8 : 11 = 8 \times 9 : 11 \times 9 = 72 : 99$$

$$\therefore A : B : C = 56 : 72 : 99$$

2. Given $a + b = 25$, $a - b = 7\frac{1}{2}$; $a/b = ?$

$$\frac{a+b}{a-b} = \frac{25}{7\frac{1}{2}} = \frac{50}{15} = \frac{10}{3}$$

$$\frac{(a+b)+(a-b)}{(a+b)-(a-b)} = \frac{10+3}{10-3} = \frac{13}{7}$$

$$\text{i.e.,} \quad \frac{2a}{2b} = \frac{13}{7} \quad \text{or} \quad \frac{a}{b} = \frac{13}{7}$$

3. Given share to men, women and boys as follows.

share for 2 women = share for 3 boys

share for 1 man = share for 2 boys.

There are 6 men, 8 women and 6 boys.

$$\therefore \text{Total share} = 6(2 \text{ boys}) + \frac{8}{2}(3 \text{ boys}) + 6(\text{boys}) \text{ share} = ₹ 120$$

i.e. Share of $(12 + 12 + 6)$ boys = ₹ 120

$$\therefore \text{Share of a boy} = \frac{120}{130} = ₹ 4$$

4. If a number is subtracted from $a : b$ and as a result it become $c : d$, then the number

$$\text{subtracted} = \frac{bc - ad}{c - d}$$

Here $a : b = 19 : 23$; $c : d = 3 : 4$

$$\begin{aligned} \therefore \text{Number subtracted} &= \frac{bc - ad}{c - d} = \frac{23 \times 3 - 19 \times 4}{3 - 4} \\ &= \frac{69 - 76}{-1} = 7 \end{aligned}$$

5. New number of employees = $\frac{7}{9}$ of old number of employees.

$$\text{New wages} = \frac{15}{14} \times \text{old wages}$$

$$\begin{aligned} \therefore \text{Resultant wages} &= \frac{7}{9} \times \frac{15}{14} \times \text{old total wages} \\ &= \frac{5}{6} \times \text{old total wages} \end{aligned}$$

$\therefore$ Bill of wages decrease by $1/6$ or **16.17%**

$$6. \frac{75 \text{ men} \times 15 \text{ days}}{Q_w} = \frac{x \text{ men} \times 45 \text{ days}}{13 Q_w}$$

where Q_w , quantity of wheat got for ₹ 13/kg.

Let x be number of men in the second case.

$$\therefore x = \frac{75 \times 15 \times 13}{45} = 25 \times 13 = \mathbf{325 \text{ men.}}$$

$$7. \frac{1000 \times 25}{50} = \frac{5000 \times 32}{x}$$

$$\therefore x = \frac{5000 \times 32 \times 50}{1000 \times 25} = \mathbf{320}$$

$\therefore$ 320 ream of paper is required.

8. Let the share of Q be x .

$$\text{Then } \frac{2}{3}x + x + \frac{5}{3}x = 1000$$

$$\Rightarrow \frac{10}{3}x = 1000 \Rightarrow x = \frac{3 \times 1000}{10} = \mathbf{300}$$

$$\therefore P\text{'s share} = \frac{2}{3}x = \frac{2}{3} \times 300 = \mathbf{₹ 200}$$

$$Q\text{'s share} = x = \mathbf{₹ 300}$$

$$R\text{'s share} = \frac{5}{3}x = \frac{5}{3} \times 300 = \mathbf{₹ 500}$$

9. Let the numbers be x and y .

$$0.6x = 0.025y$$

$$\therefore \frac{x}{y} = \frac{0.025}{0.6} = \frac{25}{600} = \frac{1}{24}$$

$\therefore$ Ratio of 2 numbers = 1 : 24.

10. Let the fourth proportional be x .

$$\text{Now } 286 : 78 = 1342 : x$$

$$\text{i.e. } \frac{286}{78} = \frac{1342}{x}$$

Cross multiplying, we get $286 \times x = 78 \times 1342$

$$\text{or } x = \frac{78 \times 1342}{286} = \mathbf{366}$$

$$11. (5 \times 1 + 7 \times 0.5 + 12 \times 0.25)x = 115$$

$$(5 + 3.5 + 3) = 115$$

$$11.5x = 115$$

$$\therefore x = \mathbf{10}$$

$$\therefore \text{Number of one-rupee coin} = 5x = \mathbf{50}$$

$$\text{Number of 50-paise coin } 7x = \mathbf{70}$$

$$\text{Number of 25-paise coin } 12x = \mathbf{120}$$

12. Let the numbers be $3k$ and $5k$.

$$\text{Now, } \frac{3k - 9}{5k - 9} = \frac{12}{23}$$

$$\Rightarrow 23(3k - 9) = 12(5k - 9)$$

$$\Rightarrow (69 - 60)k = (23 - 12)9$$

$$\text{or } k = 11$$

$$\therefore \text{ Smaller number} = 3 \times 11 = \mathbf{33}$$

13. Let the numbers be k and $2k$

$$\text{Now, } \frac{k + 7}{2k + 7} = \frac{3}{5}$$

$$\Rightarrow 5(k + 7) = 3(2k + 7)$$

$$\Rightarrow (6 - 5)k = 7(5 - 3)$$

$$\therefore k = \frac{7 \times 2}{1} = 14$$

$$\therefore \text{ Greatest number} = 2k = 14 \times 2 = \mathbf{28}$$

14. Given $0.75 : x :: 5 : 8$

$$\text{Here, } 0.75 \times 8 = x \times 5$$

$$\text{or } x = \frac{0.75 \times 8}{5} = \frac{6}{5} = \mathbf{1.2}$$

15. Given $x : y = 5 : 2 \Rightarrow x = 5k$ and $y = 2k$

$$\therefore \frac{8x + 9y}{8x + 2y} = \frac{8 \times 5k + 9 \times 2k}{8 \times 5k + 2 \times 2k} = \frac{58k}{44k} = \frac{\mathbf{29}}{\mathbf{22}}$$

16. Let the salary of Sumit be x .

$$\text{Given salary of Ravi to Sumit} = \frac{2}{3}$$

$$\therefore \text{ Salary of Ravi} = \frac{2}{3} \times x = \frac{2x}{3}$$

On increasing salary of both of the above by ₹ 4000, new ratio will be

$$\left(\frac{2}{3}x + 4000\right) : (x + 4000) = 40 : 57$$

$$\text{Solving, } 57\left(\frac{2}{3}x + 4000\right) = 40(x + 4000)$$

$$\Rightarrow x\left(\frac{57 \times 2}{3} - 40\right) = 4000(40 - 57)$$

$$x(38 - 40) = 4000(-17)$$

$$x = \frac{4000(-17)}{-2} = 34000$$

$$\therefore \text{ Sumit's present salary} = 34000 + 4000 = \mathbf{₹ 38000}$$

17. Given $\frac{10}{100} \times x = \frac{20}{100} \times y$

$$\therefore \frac{x}{y} = \frac{20/100}{10/100} = \mathbf{2 : 1}$$

18. Given Earnings of A : Earnings of $B = 4 : 7$

Let earnings of $A = 4k$ and that of $B = 7k$

$$\text{Now, } \frac{4k(1.5)}{7k(0.75)} = \frac{8}{7}$$

$$\text{or } \frac{6k}{5.25k} = \frac{8}{7}$$

Value of k cannot be determined.

Therefore data inadequate.

19. Given $A : B = 8 : 15$; $B : C = 5 : 8$; $C : D = 4 : 5$

$$A : B = 8 : 15$$

$$B : C = 15 : 24$$

$$C : D = 24 : 30$$

$$\therefore A : D = 8 : 30 \text{ or } 4 : 15$$

20. Given $A : B = 2 : 3$; $B : C = 4 : 5$ and $C : D = 6 : 7$;

$$A : B : C : D = ?$$

$$A : B = 2 : 3 \rightarrow 2 \times 4 : 3 \times 4 \rightarrow 2 \times 2 \times 4 : 2 \times 3 \times 4$$

$$B : C = 4 : 5 \rightarrow 4 \times 3 : 5 \times 3 \rightarrow 2 \times 4 \times 3 : 2 \times 5 \times 3$$

$$C : D = 6 : 7 \rightarrow 6 \times 5 : 7 \times 5 \rightarrow 6 \times 5 : 7 \times 5$$

$$\therefore A : B : C : D = 16 : 24 : 30 : 35$$

21. Salary of A : Salary of B : Salary of $C = 2 : 3 : 5$

Let their salaries be $2k$, $3k$ and $5k$. New Salaries are respectively $2k(1.15)$, $3k(1.10)$ and $5k(1.20)$

$$\therefore \text{Ratio of new salaries} = 2.3k : 3.3k : 6k = 23 : 33 : 60$$

22. Given ratio is $\frac{1}{2} : \frac{2}{3} : \frac{3}{4}$

$$= 12 \times \frac{1}{2} : 12 \times \frac{2}{3} : 12 \times \frac{3}{4}$$

$$= 6 : 8 : 9$$

$$\therefore \text{First part} = \frac{6}{6+8+9} \times ₹ 782 = 6 \times 34 = ₹ 204$$

23. Given, $A : B = \frac{1}{2} : \frac{3}{8} = 4 : 3$

$$B : C = \frac{1}{3} : \frac{5}{9} = 3 : 5$$

$$C : D = \frac{5}{6} : \frac{3}{4} = 10 : 9$$

$$\therefore A : B : C : D = 2 \times 4 : 2 \times 3 : 2 \times 5 : 9 = 8 : 6 : 10 : 9$$

24. Let the number be x , y and z

$$\text{Then } x + y + z = 98 \quad (1)$$

$$\frac{x}{y} = \frac{2}{3} \quad (2)$$

$$\frac{y}{z} = \frac{5}{8} \quad (3)$$

$$y = ?$$

$$\therefore x : y = 2 : 3$$

$$y : z = 5 : 8$$

$$\Rightarrow x : y : z = 2 \times 5 : 3 \times 5 : 3 \times 8 = 10 : 15 : 24$$

$$\therefore y = \frac{15}{10 + 15 + 24} \times 98 = \mathbf{30}$$

25. We have $P : Q = 2 : 3 \rightarrow 2 \times 2 : 3 \times 2 \rightarrow 2 \times 2 \times 2 : 2 \times 3 \times 2$

$$Q : R = 2 : 3 \rightarrow 2 \times 3 : 3 \times 3 \rightarrow 2 \times 2 \times 3 : 2 \times 3 \times 3$$

$$R : S = 2 : 3 \rightarrow 2 \times 3 \times 3 : 3 \times 3 \times 3$$

$$\therefore P : Q : R : S = 8 : 12 : 18 : 27$$

$$\therefore P\text{'s share} = \frac{8}{8 + 12 + 18 + 27} \times 1300 = \mathbf{\text{₹ } 160}$$

26. Let the amount with B be x and amount with A be y .

$$\therefore x + y = 1210 \quad (1)$$

$$\frac{4}{15}y = \frac{2}{5}x \quad (2)$$

$$\Rightarrow 4y = 6x \quad (3)$$

Substituting in (1), we get

$$x + \frac{6x}{4} = 1210$$

$$\Rightarrow \frac{10x}{4} = 1210$$

$$\Rightarrow x = 1210 \times \frac{4}{10} = \mathbf{\text{₹ } 484}$$

27. Let the third number be x .

Then first number = $1.2x$ and second be = $1.5x$

$$\therefore \text{First number} : \text{Second number} = 1.2x : 1.5x = \mathbf{4 : 5}$$

28. Ratio of boys : girls = $7 : 8$

Let number of boys be $7k$ and girls be $8k$.

$$\therefore \text{New ratio} = \frac{7k(1.2)}{8k(7.1)} = \frac{8.4k}{8.8k} = \mathbf{\frac{21}{22}}$$

29. Amount of money distributed between A, B, C and D is in the ratio $5 : 2 : 4 : 3$.

$$\therefore \text{Difference in money between C and D} = \left(\frac{4}{5 + 2 + 4 + 3} - \frac{3}{5 + 2 + 4 + 3} \right) \text{ of total amount}$$

$$= \frac{4 - 3}{14} \times \text{total amount} = \mathbf{\text{₹ } 1000}$$

$$\therefore \text{Total amount} = 14 \times 1000 = \mathbf{\text{₹ } 14000}$$

$$\therefore \text{B's share} = \frac{2}{5 + 2 + 4 + 3} \times 14000 = \mathbf{\text{₹ } 2000}$$

30. Fourth proportional to 5, 8, 15 is = $\frac{8 \times 15}{5} = \mathbf{24}$

31. Let the number be x and y

Now, $x(0.4) = \frac{2}{3} \times y$

$$\therefore \frac{x}{y} = \frac{2}{3 \times 0.4} = \frac{10}{6} = \frac{5}{3}$$

$$\therefore \text{Ratio of first number to second number} = \frac{5}{3}$$

32. Given $x \propto \frac{1}{y^2}$ or $x = k \times \frac{1}{y^2}$ (1)

Given, $y = 2$ for $x = 1$, for $y = 6$, $x = ?$

From (1), $1 = k \times \frac{1}{2^2}$

or $k = 1 \times 2^2 = 4$

$$\therefore x = 4 \times \frac{1}{6^2} = \frac{1}{9}$$

33. Compounded ratio of $(2 : 5)$, $(6 : 11)$ and $(11 : 3) = \frac{2}{5} \times \frac{6}{11} \times \frac{11}{3} = \frac{4}{5}$ or **4 : 5**

34. Given ratio = 4 : 5

Let terms of ratio be $4k$ and $5k$.

If a number x is subtracted from both terms, we get

$$\frac{6 - x}{7 - x} = \frac{3}{4}$$

$$\Rightarrow (6 - x)4 = 3(7 - x)$$

$$\Rightarrow 6 \times 4 - 7 \times 3 = x(4 - 3)$$

$$24 - 21 = x$$

or $x = 3$

$\therefore$ Least whole number = 3

35. $P : Q = 5 : 6$

Given $\frac{6}{5+6}$ of total sum = 2400

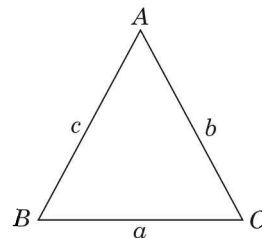
$$\therefore \text{Total sum} = 2400 \times \frac{11}{6} = \text{₹ } 4400$$

36. $a : b : c = \frac{1}{2} : \frac{1}{3} : \frac{1}{4}$ or $6 : 4 : 3$

$$\text{Given perimeter} = (a + b + c) = 156$$

$$\therefore \text{Length of largest side} = \frac{6}{6+4+3} \times 156$$

$$= \frac{6}{13} \times 156 = 72 \text{ cm}$$



37. Given ratio of income of P and Q as $3 : 4$

i.e. $p_i : q_i = 3 : 4$ or $3x : 4x$

Ratio of expenditures = $2 : 3$.

i.e., $p_e : q_e = 2 : 3$ or $2x : 3x$

Amount of savings of $P = p_i - p_e = 3x - 2x = x$

Amount of savings of $Q = q_i - q_e = 4x - 3x = x$

Given that $x = ₹ 2000$

$$p_i = ?$$

$\therefore p_i = 3x = \mathbf{6000}$

38. Let number of boys and girls be $6k$ and $5k$ respectively.

$\therefore$ Total number of students = $6k + 5k = 11k$

Given number of boys = $6k = 192$

or $k = 32$

$\therefore$ Total number of students = $11k = 11 \times 32 = \mathbf{352}$

39. Let the fraction be $\frac{p}{q}$.

Then $\frac{p}{q} \div \frac{1}{27} = \frac{3}{11} \div \frac{5}{9}$

or $\frac{p}{q} \times 27 = \frac{3}{11} \times \frac{9}{5}$

or $\frac{p}{q} = \frac{1}{27} \times \frac{3}{11} \times \frac{9}{5} = \frac{1}{55}$

40. Given $A : B : C = 2 : 3 : 4$

Then
$$\begin{aligned} \frac{A}{B} : \frac{B}{C} : \frac{C}{A} &= \frac{2}{3} : \frac{3}{4} : \frac{4}{2} \\ &= \frac{2}{3} : \frac{3}{4} : 2 = \mathbf{8 : 9 : 24} \end{aligned}$$

Percentage

A fraction whose denominator is expressed as 100 is called the *percentage* and the numerator of the fraction is called the *rate percentage*.

For example, a man spends 10% of his monthly income on dress materials means every ten rupee out of hundred rupees he gets, goes to the purchase of dress materials, i.e. if the monthly income of the man is ₹ 5000, then the spending towards dress materials comes

about $\frac{10}{100} \times 5000 = \text{₹ } 500$.

Important Prime Fractions and their Equivalent Percentage

$\frac{1}{32} = 3\frac{1}{8}\%$	$\frac{1}{20} = 5\%$	$\frac{1}{16} = 6\frac{1}{4}\%$	$\frac{2}{25} = 8\%$	$\frac{1}{12} = 8\frac{1}{3}\%$	$\frac{1}{10} = 10\%$
$\frac{3}{25} = 12\%$	$\frac{1}{8} = 12\frac{1}{2}\%$	$\frac{2}{15} = 13\frac{1}{3}\%$	$\frac{1}{7} = 14\frac{2}{7}\%$	$\frac{3}{20} = 15\%$	$\frac{4}{25} = 16\%$
$\frac{1}{4} = 25\%$	$\frac{1}{3} = 33\frac{1}{3}\%$	$\frac{3}{8} = 37\frac{1}{2}\%$	$\frac{2}{5} = 40\%$	$\frac{3}{5} = 60\%$	$\frac{5}{8} = 62\%$
$\frac{2}{3} = 66\frac{2}{3}\%$	$\frac{3}{4} = 75\%$	$\frac{7}{8} = 87\frac{1}{2}\%$			

EXAMPLE 1 In an election, there are two candidates. The candidate who gets 46% is rejected by a majority of 2688 votes. Find the total number of votes polled.

Solution Out of the total votes polled (100%)

The winning candidate got $(100 - 46) = 54\%$ votes and the other candidate got 46% votes.

∴ Majority = $54 - 46 = 8\%$ of total votes
= 2688

∴ Votes polled = $\frac{2688}{8/100} = \frac{2688 \times 100}{8} = \mathbf{33600}$

EXAMPLE 2 What percentage is equivalent to $\frac{5}{8}$?

Solution $\frac{5}{8} \times 100 = \frac{125}{2} = 62\frac{1}{2}\%$

EXAMPLE 3 The salary of Kannan is increased from ₹ 3500 to ₹ 4025. Find the increase per cent.

Solution Increase in salary = ₹ 4025 – ₹ 3500 = ₹ 525

Percentage increase in salary = $\frac{525}{3500} \times 100 = 15\%$

EXAMPLE 4 The price of sugar in the market rises by 25%. A person whose income is fixed, has to reduce the consumption of sugar so that the expenditure remains the same. By what percentage, he should reduce the consumption of sugar?

Solution The percentage reduction to be made in his consumption = $25 \times \frac{100}{100 + 25} = 20\%$.

➤ Theorem 1: If the price of a certain item is increased by $x\%$ and then increased by $y\%$, then the resultant increase in its price is given by

$$\left(x + y + \frac{xy}{100}\right)\%$$

Choose sign of x and y appropriately (i.e. +ve for increase and –ve for decrease).

➤ Theorem 2: If two values are respectively $x\%$ and $y\%$ more than the third value, then the first is $\frac{100 + x}{100 + y} \times 100\%$ of the second.

EXAMPLE 5 Two numbers are 25% and 50% more than a third number. What percentage is the first of the second?

Solution By Theorem 2,

The required value = $\frac{100 + 25}{100 + 50} \times 100 = \frac{5}{6} \times 100 = 83\frac{1}{3}\%$

➤ Theorem 3: If P is $x\%$ of R and Q is $y\%$ of R , then P is $\frac{x}{y} \times 100\%$ of Q .

EXAMPLE 6 Two numbers are 24% and 30% of a third number. What percentage is the first of the second?

Solution By Theorem 3,

The required value = $\frac{24}{30} \times 100 = \frac{4}{5} \times 100 = 80\%$

EXAMPLE 7 Two numbers are 20% and 40% less than a third number. What per cent is the second of the first?

Solution The per cent of the second number with respect to the first number is given by

$$\frac{100 - 40}{100 - 20} \times 100 = \frac{60}{80} \times 100 = 75\%$$

PERCENTAGE EXPENDITURE AND SAVINGS: If a person spends $x\%$ of his money on lodging and $y\%$ of the remaining on food and cloth expenses and ₹ m is the amount remaining with him, then the amount he had at the beginning is given by

$$\frac{m \times 100 \times 100}{(100 - x)(100 - y)}$$

EXAMPLE 8 A person spends 5% of his income on income tax and 12.5% of the remaining is saved. The amount spent was ₹ 29925. What was his income?

Solution Income = $\frac{29,925 \times 100 \times 100}{(100 - 5)(100 - 12.5)} = \text{₹ } 36000$

➤ Theorem 3: If $x\%$ of a quantity is taken by the first, $y\%$ of the remaining is taken by the second and $z\%$ of the remaining is taken by a third person and if the amount left is ₹ P , then the amount at the beginning is given by

$$\frac{P \times 100 \times 100 \times 100}{(100 - x)(100 - y)(100 - z)}$$

➤ Theorem 3: If $x\%$ of a quantity is added, $y\%$ of the increased quantity is added and $z\%$ of the first increased quantity is added and the amount becomes P , then the initial amount is given by

$$\frac{P \times 100 \times 100 \times 100}{(100 + x)(100 + y)(100 + z)}$$

EXAMPLE 9 A certain sum was first increased by 10% and then by 20%, and it become finally ₹ 5280. Find the initial sum.

Solution The required initial amount = $\frac{5280 \times 100 \times 100}{110 \times 120} = \text{₹ } 4000$

POPULATION FORMULA: If the original population of a town is P and the annual increase in population is $r\%$, population after n years will be $P \left(1 + \frac{r}{100}\right)^n$.

EXAMPLE 10 Annual increase in population of a town is 4%, and the present population is 40000. What will be the population of town after 2 years?

Solution Population after 2 years = $40000 \left(1 + \frac{4}{100}\right)^2$

$$= \frac{40000 \times 104 \times 104}{100 \times 100} = 43264$$

Theorem 6: The population of a town is P . It increases by $x\%$ during the first year, $y\%$ during the second year and $z\%$ during the third year. Population after 3 years will be

$$\frac{P(100 + x)(100 + y)(100 + z)}{100 \times 100 \times 100}$$

EXAMPLE 11 The population of a town is 15000. It increases by 10% for the first year, 20% for the second year and 40% for the third year. What will be the population after 3 years?

Solution The required population = $\frac{15000 \times 110 \times 120 \times 140}{100 \times 100 \times 100} = 27720$

EXAMPLE 12 The population of a town increases at the rate of 10% during the first year and it decreases at the rate of 5% during the second year. If it has 20900 inhabitants now, what was the population of town 2 years ago?

Solution The required population = $\frac{20900 \times 100 \times 100}{110 \times 95} = 20000$

Theorem 7: Let the present value of a machine be P . Let the depreciation of the machine be at the rate of $r\%$ per annum.

Then

1. Value of machine after n years = $P \left(1 - \frac{r}{100}\right)^n$

2. Value of machine n years ago = $\frac{P}{\left(1 - \frac{r}{100}\right)^n}$

EXAMPLE 13 A machine costs ₹ 320000 at present. If the value of the machine depreciates at the rate of 5% per annum. What will be its value 6 years hence?

Solution Present value of machine = ₹ 320000

$$\begin{aligned} \text{6 years hence, value of machine} &= 320000 \left(1 - \frac{5}{100}\right)^6 \\ &= 320000 \times \left(\frac{19}{20}\right)^2 \times \left(\frac{19}{20}\right)^2 \times \left(\frac{19}{20}\right)^2 \\ &= 32 \times \frac{19^2}{4} \times \frac{19^2}{4} \times \frac{19^2}{400} = \frac{19^2 \times 19^2 \times 19^2}{200} \\ &= \frac{19^6}{200} = \frac{47045881}{200} = ₹ 235229.41 \end{aligned}$$

Percentage Relationship: If A is $x\%$ more than B , then B is $\left(\frac{x}{100+x} \times 100\right)\%$ less than A .

If A is $x\%$ less than B , then B is $\left(\frac{x}{100-x} \times 100\right)\%$ more than A .



Note: If P is $r\%$ more than Q , then Q is less than P by $\left[\frac{r}{100+r} \times 100\right]\%$.

If P is $r\%$ less than Q , then Q is more than P by $\left[\frac{r}{100-r} \times 100\right]\%$.

➤ Theorem 8: If the price of a commodity increases by $r\%$, then decrease in consumption, so as not to increase the expenditure on this item is $\left(\frac{r}{100+r} \times 100\right)\%$.

If the price of a commodity decreases by $r\%$, then increase in consumption, so as not to decrease the expenditure on this item is $\left(\frac{r}{100-r} \times 100\right)\%$.

EXAMPLE 14 The salary of A is 20% more than that of B . Find how much per cent is B 's salary less than that of A .

Solution Let B 's salary be ₹ 100 per month.

Then A 's salary = $100 + 20\%$ of $100 = 120$

∴ B 's salary is ₹ 20 less than that of A when A 's salary is ₹ 120.

∴ When A 's salary is ₹ 100, B 's salary is $\frac{20}{120} \times 100 = \frac{100}{6} = ₹ \frac{50}{3}$ less than that of A , i.e.

B 's salary is $\frac{50}{3}\%$ less than that of A .

➤ Theorem 9: If a number is first increased by $x\%$ and then decreased by $x\%$, the net change is always a decrease which is equal to $x\%$ of x or $\frac{x^2}{100}$ percentage.

EXAMPLE 15 The salary of a worker is ₹ 2000. It is first increased by 10% and then decreased by 10%. What is the change in his salary?

Solution The salary of the worker = ₹ 2000

After 10% increase, salary = $\frac{110}{100} \times 2000 = ₹ 2200$

After 10% decrease, salary = $\frac{90}{100} \times 2200 = ₹ 1980$

∴ The net reduction in salary = ₹ 20

i.e. the percentage reduction in salary = $\frac{20}{2000} \times 100 = 1\%$

i.e. $\% \text{ reduction} = \frac{x^2}{100} \% = \frac{10^2}{100} \% = 1\%$

➤ **Theorem 10:** The pass mark in an examination is $x\%$. If a candidate who secures y marks fails by z marks, then the maximum mark is given by $M = \frac{100(y+z)}{x}$.

EXAMPLE 16 In an examination, a candidate requires 35% marks to pass. If he gets 90 marks and fails by 50 marks, find the maximum marks.

Solution Maximum marks, $M = \frac{100(y+z)}{x} = \frac{100(90+50)}{35} = 400$

EXAMPLE 17 In an examination, a candidate got 110 marks and failed by 70 marks. If the required pass percentage is 40, find the maximum marks.

Solution Maximum marks, $M = \frac{100(110+70)}{40} = 450$

➤ **Theorem 11:** A candidate scoring $x\%$ marks in an examination fails by a marks and another candidate scoring $y\%$ marks gets b marks more than the minimum required pass marks. Then the maximum mark for that examination is $M = \frac{100(a+b)}{(y-x)} = 100 \times \frac{\text{Difference of their scores}}{\text{Difference of their \% marks}}$.

EXAMPLE 18 A candidate scores 20% and fails by 45 marks and another candidate scores 45% marks gets 30 marks more than the minimum required to pass the examination. Find the maximum mark for the examination.

Solution Maximum marks = $\frac{100(45+30)}{(45-20)} = \frac{100 \times 75}{25} = 300$

➤ **Theorem 12:** In measuring the sides of a rectangle, one side is taken $x\%$ in excess and the other $y\%$ in deficit. The error per cent in the area calculated from the measurement is $x - y - \frac{xy}{100}$ in excess or deficit.

EXAMPLE 19 If the sides of a square are increased by 20%, find the percentage increase in its area.

Solution The percentage increase in area = $20 + 20 + \frac{20 \times 20}{100} = 40 + 4 = 44\%$

➤ **Theorem 13:** If the sides of a triangle, rectangle, square, circle, rhombus are increased by $x\%$, its area is increased by $\frac{x(x+200)}{100} \%$ or $\left(2x + \frac{x^2}{100}\right) \%$.

EXAMPLE 20 If the radius of a circle is increased by 10%, find the percentage increase in its area.

Solution Let the area of circle be πr^2 .

$$\text{After 10\% increase in radius, new area} = \pi \left(\frac{110r}{100} \right)^2 = 1.21 \pi r^2$$

$$\therefore \text{Percentage increase in area} = \frac{(1.21 - 1)\pi r^2}{\pi r^2} \times 100 = \mathbf{21\%}$$

 **Theorem 14:** In an examination, $x\%$ candidates failed in English and $y\%$ candidates failed in Maths. If $z\%$ of candidates failed in both the subjects, the percentage of students who passed in both the subjects is $100 - (x + y - z)$.

EXAMPLE 21 In an examination, 40% of the candidates failed in Maths, 30% failed in English and 20% failed in both. Find the percentage of candidates who passed in both the subjects.

Solution Required percentage = $100 - (40 + 30 - 20) = \mathbf{50\%}$

Percentage Increase in Savings: If income of a person increases by $y\%$ and his expenditure increases by $z\%$ and his present expenditure is $x\%$ of his income, find the percentage increase in his savings.

$$\text{Percentage increase in savings} = \frac{y \times 100 - z \times x}{100 - x}$$

EXAMPLE 22 A person spends 60% of his income. His income increases by 15% and his expenditure increases by 5%, find the percentage increase in his savings.

$$\text{Solution} \quad \text{Percentage increase in savings} = \frac{15 \times 100 - 5 \times 60}{100 - 60} = \frac{1500 - 300}{40} = \mathbf{30\%}$$

If the income is first increased by $x\%$ and then decreased by $y\%$, then the effective increase or decrease is given by $\left(x - y - \frac{xy}{100} \right)\%$.

EXAMPLE 23 The salary of a worker is first increased by 10% and then decreased by 20%. What is the percentage effect in his salary?

$$\begin{aligned} \text{Solution} \quad \text{Percentage effect} &= \% \text{ Increase} - \% \text{ Decrease} - \frac{\% \text{ Increase} \times \% \text{ Decrease}}{100} \\ &= 10 - 20 - \frac{10 \times 20}{100} = 10 - 20 - 2 = \mathbf{-12\%} \end{aligned}$$

i.e. **12% decrease** is the resultant effect.

 **Theorem 15:** If a value is first increased by $x\%$ and then by $y\%$ the final increase is given by $\left(x + y + \frac{xy}{100} \right)\%$.

EXAMPLE 24 A trader sells his articles at 10% higher than their original price. Due to higher demand, he again increases the price by 15%. What will be the percentage profit the trader make?

Solution

$$\begin{aligned}\% \text{ Profit} &= \% \text{ First increase} + \% \text{ Second increase} + \frac{\% \text{ First increase} \times \% \text{ Second increase}}{100} \\ &= 10 + 15 + \frac{10 \times 15}{100} = 10 + 15 + 1.5 = \mathbf{26.5\%}\end{aligned}$$

Effect on Revenue: If the price of a commodity is increased by $x\%$ and its consumption is decreased by $y\%$, then the effect on revenue

$$\begin{aligned}&= \text{Increase \% value} - \text{Decrease \% value} - \frac{\text{Increase \% value} \times \text{Decrease \% value}}{100} \\ \text{i.e. } &\left(x - y - \frac{xy}{100}\right)\%\end{aligned}$$

EXAMPLE 25 The price of an item is increased by 15% and its sale gets decreased by 10%. What will be the percentage effect on the income?

Solution $\% \text{ Effect} = 15 - 10 - \frac{15 \times 10}{100} = 5 - 1.5 = 3.5\%$

i.e. a profit of 3.5%

$\therefore$ His income is increased by 3.5%.

EXAMPLE 26 The price of an item is decreased by 10% and the sale is increased by 25%. What will be the percentage effect on income?

Solution $\% \text{ Effect} = 25 - 10 - \frac{25 \times 10}{100} = 15 - 2.5 = \mathbf{12.5\%}$

$\therefore$ His income is increased by 12.5%

➤ Theorem 16: When a certain quantity of goods B is added to change the percentage of goods A in a mixture of A and B , then the quantity of B to be added is

$$\left(\frac{\text{Previous \% value of } A}{\text{Changed \% value of } A} \times \text{Mixture quantity} \right) - \text{Mixture quantity}$$

EXAMPLE 27 In 1 kg mixture of sand and cement, 20% is cement. How much sand should be added so that the proportion of cement becomes 10%?

Solution Quantity of cement in 1 kg mixture $= \frac{20}{100} \times 1000 = 200 \text{ g}$

$\therefore$

$$\text{Sand} = 1000 - 200 = \mathbf{800 \text{ g}}$$



Short cut:

$$\begin{aligned}\text{Required quantity} &= \left(\frac{\text{Previous \% value of A}}{\text{Changed \% value of A}} \times \text{Mixture quantity} \right) - \text{Mixture quantity} \\ &= \left(\frac{20}{10} \times 1 \right) - 1 = 2 - 1 = 1 \text{ kg}\end{aligned}$$

EXAMPLE 28 Express the following as a decimal.

- (i) 12% (ii) 16% (iii) 0.3% (iv) 0.06%

Solution

$$(i) \ 12\% = \frac{12}{100} = \mathbf{0.12}$$

$$(ii) \ 16\% = \frac{16}{100} = \mathbf{0.16}$$

$$(iii) \ 0.3\% = \frac{0.3}{100} = \mathbf{0.003}$$

$$(iv) \ 0.06\% = \frac{0.06}{100} = \mathbf{0.0006}$$

EXAMPLE 29 Express the following as rate percent: (i) $\frac{13}{25}$ (ii) $5\frac{3}{7}$ (iii) 0.007

Solution

$$(i) \ \frac{13}{25} = \left(\frac{13}{25} \times 100 \right) \% = 52\%$$

$$(ii) \ 5\frac{3}{7} = \frac{38}{7} = \left(\frac{38}{7} \times 100 \right) \% = 542.85\%$$

$$(iii) \ 0.007 = \frac{7}{1000} = \left(\frac{7}{1000} \times 100 \right) \% = 0.7\%$$

EXAMPLE 30 Express the following as a fraction

- (i) 84% (ii) 6% (iii) 0.9% (iv) 0.08%

Solution

$$(i) \ 84\% = \frac{84}{100} = \frac{\mathbf{21}}{\mathbf{25}}$$

$$(ii) \ 6\% = \frac{6}{100} = \mathbf{0.06}$$

$$(iii) \ 0.9\% = \frac{0.9}{100} = \frac{\mathbf{9}}{\mathbf{1000}}$$

$$(iv) \ 0.08\% = \frac{0.08}{100} = \frac{8}{10000} = \frac{\mathbf{1}}{\mathbf{1250}}$$



EXERCISES

1. An empty fuel tank of a car was filled with ordinary type of petrol. When the tank was one-third empty, it was filled with high speed petrol. Again when the tank was one-third empty, it was filled with an ordinary type of petrol. Again, when the tank was one-third empty it was filled with high speed petrol. At this time what was the percentage of high speed petrol in the tank?

- (a) $51\frac{23}{27}\%$ (b) $48\frac{4}{27}\%$ (c) $49\frac{4}{27}\%$ (d) $50\frac{23}{27}\%$ (e) None of these

2. The length of a rectangle increases by 20% and the breadth by 30%. Then the perimeter of the rectangle will increase by
(a) 56% (b) 25% (c) $22\frac{2}{3}\%$ (d) Data inadequate (e) None of these
3. In a company there are 75% skilled workers and remaining are unskilled. The 80% of skilled workers and 20% of unskilled workers are permanent. If the number of temporary workers is 126, what is the number of total workers?
(a) 480 (b) 510 (c) 360 (d) 377 (e) None of these
4. If the height of a triangle is decreased by 40% and its base is increased by 40%, what will be the effect on its area?
(a) No change (b) 8% decrease (c) 16% decrease (d) 16% increase (e) 8% increase
5. The population of a town is 123,456,789. It increases by 10% during the first year, 20% during the second year and 30% during the third year. What is the average per cent increase during the three years?
(a) 20% (b) $23\frac{11}{15}\%$ (c) $23\frac{13}{15}\%$ (d) $26\frac{2}{3}\%$ (e) None of these
6. The monthly income and monthly expenditure of a person were ₹ 13500 and ₹ 9000 respectively. In the next year his income is increased by 14% while his expenditure is increased by 7%. The percentage increase in his savings is
(a) 7% (b) 21% (c) 28% (d) 35% (e) None of these
7. $60 = ?\%$ of 400
(a) 6 (b) 12 (c) 20 (d) 15 (e) None of these
8. 40% of $? = 240$
(a) 60 (b) 6000 (c) 960 (d) 600 (e) None of these
9. 23% of 8040 + 42% of 545 = $?\%$ of 3000
(a) 56.17 (b) 63.54 (c) 71.04 (d) 69.27 (e) None of these
10. Sunil decided to donate 5% of his salary. On the day of donation he changed his mind and donated ₹ 1687.50, which was 75% of what he had decided earlier. How much is Sunil's salary?
(a) ₹ 37500 (b) ₹ 45000 (c) ₹ 33750
(d) Cannot be determined (e) None of these
11. Suresh gave 40% of his income to his wife. He gave 25% of the remaining amount to his son. Of the remaining amount he spent 60% on education. Now the amount with him is ₹ 2700. What is the amount he gave to his wife?
(a) ₹ 4000 (b) ₹ 5000 (c) ₹ 8000 (d) ₹ 6000
12. In an examination 70% of the candidates passed in English, 80% passed in Mathematics, 10% failed in both the subjects. If 144 candidates passed in both, the total number of candidates was
(a) 125 (b) 200 (c) 240 (d) 375
13. About 8% of the voters in an election did not cast their votes. In this election, there were only two candidates. The winner by obtaining 48% of the total votes defeated his contestant by 1100 votes. The total number of voters in the election was
(a) 21000 (b) 23500 (c) 22000 (d) 27500
14. If all sides of a hexagon are increased by 7%, find the % increase in its area.
(a) 42.49% (b) 32.14% (c) 25.7% (d) 14.49% (e) None of these

15. Anil's monthly income is 30% more than Ashok's. Ashok's monthly income is 10% less than Manoj's. If the difference between the monthly income of Anil and Manoj is ₹ 1360, what is the monthly income of Ashok?
 (a) ₹ 8100 (b) ₹ 10430 (c) ₹ 9000 (d) Data inadequate (e) None of these
16. The 75% of a number is 380 more than 35% of the same number. What is 20% of that number?
 (a) 190 (b) 195.5 (c) 189.5 (d) 180 (e) None of these
17. The length of a rectangle is increased by 30% and the breadth by 20%. Then the perimeter of the rectangle will increase by what per cent.
 (a) 56% (b) 25% (c) 26% (d) Data inadequate (e) None of these
18. Two litres of water evaporate from 10 litres of a solution containing 10% salt. The percentage of salt in the remaining solution is
 (a) 12% (b) 12.5% (c) 8% (d) 8.5% (e) None of these
19. A speaks truth in 80% of the cases and B in 90% of the cases. Find the percentage when they contradict each other in describing the same facts.
 (a) 25% (b) 26% (c) 30% (d) 35% (e) None of these
20. 125% of $320 + ?\%$ of $125 = 440$
 (a) 46 (b) 42 (c) 50 (d) 32 (e) None of these
21. In a class 70% of the students read Hindi 40% of the students are boys and 20% of the girls do not read Hindi. Then what percentage of boys will read Hindi?
 (a) 16% (b) 22% (c) 55%
 (d) 48% (e) Can't be determined
22. If the height of a triangle is decreased by 40% and its base is increased by 30%, what will be the effect on the area?
 (a) Decrease by 22% (b) Increase by 22% (c) Decrease by 2%
 (d) Increase by 2% (e) None of these
23. Evaluate 28% of $450 + 45\%$ of 280 . [Bank PO]
 (a) 225 (b) 252 (c) 525 (d) 522
24. When the price of a product was decreased by 10%, the number sold increased by 30%. What was the effect on the total revenue? [RBI]
 (a) 20% (b) 18% (c) 17% (d) -10%
25. The salary of a person was reduced by 10%. By what per cent should his reduced salary be raised so as to bring it at par with his original salary? [SSC]
 (a) 10% (b) 9% (c) 11.11% (d) 9.99%
26. During one year, the population of a town was increased by 5% and during the next year, the population decreased by 5%. If the total population is 9975 at the end of the second year, then what was the population size in the beginning of the first year? [Hotel Management]
 (a) 8500 (b) 9000 (c) 9500 (d) 10000
27. 3.5 can be expressed in terms of percentage as: [SSC]
 (a) 0.35% (b) 3.5% (c) 35% (d) 350%
28. 88% of $370 + 24\%$ of $210 - ? = 118$. [Bank PO]
 (a) 256 (b) 258 (c) 268 (d) 358
29. 860% of $50 + 50\%$ of $860 = ?$ [RBI]
 (a) 430 (b) 516 (c) 860 (d) 960
30. 45% of $750 - 25\%$ of $480 = ?$ [Bank PO]
 (a) 216 (b) 217.50 (c) 236.50 (d) 245

31. 40% of 1640 + ? = 35% of 980 + 150% of 850. [SBI PO]
(a) 372 (b) 842 (c) 962 (d) 1052
32. 60% of 264 is the same as: [Hotel Management]
(a) 10% of 44 (b) 15% of 1056 (c) 30% of 132 (d) None of these
33. 0.01 is what per cent of 0.1? [SSC]
(a) $\frac{1}{100}$ (b) $\frac{1}{10}$ (c) 10 (d) 100
34. What per cent of ₹ 2650 is ₹ 1987.50? [Hotel Management]
(a) 60% (b) 75% (c) 80% (d) 90%
35. Rajeev buys goods worth ₹ 6650. He gets a rebate of 6% on it. After getting the rebate, he pays sales tax @ 10%. Find the amount he will have to pay for the goods. [MAT]
(a) ₹ 6876.10 (b) ₹ 6999.20 (c) ₹ 6654 (d) ₹ 7000
36. 4598 is 95% of? [Bank PO]
(a) 4800 (b) 4840 (c) 4850 (d) 4880
37. ?% of 360 = 129.6 [Bank PO]
(a) 36 (b) 64 (c) 72 (d) 77
38. 65% of ? = 20% of 422.50 [Bank PO]
(a) 84.5 (b) 130 (c) 139.425 (d) 200
39. ?% of 932 + 30 = 309.60 [Bank PO]
(a) 25 (b) 30 (c) 35 (d) 40
40. 45% of 1500 + 35% of 1700 = ?% of 3175 [Bank PO]
(a) 30 (b) 35 (c) 45 (d) None of these
41. If ₹ 2800 is $\frac{2}{7}$ per cent of the value of a house, the worth of the house (in ₹) is [LIC]
(a) 8,00,000 (b) 9,80,000 (c) 10,00,000 (d) 12,00,000
42. If 120 is 20% of a number, then 120% of that number will be [Section Officer]
(a) 20 (b) 120 (c) 360 (d) 720
43. Two-fifths of one-third of three-seventh of a number is 15. What is 40 per cent of that number? [Bank PO]
(a) 72 (b) 84 (c) 136 (d) 140 (e) None of these
44. If 35% of a number is 12 less than 50% of that number, then the number is [CBI]
(a) 40 (b) 50 (c) 60 (d) 80
45. The number which exceeds 16% of it by 42 is: [CBI]
(a) 50 (b) 52 (c) 58 (d) 60
46. If a number x is 10% less than another number y and y is 10% more than 125, then x is equal to [SSC]
(a) 123.75 (b) 140.55 (c) 143 (d) 150
47. A number, when 35 is subtracted from it, reduces to its 80 percent. What is four-fifths of that number? [BSRB]
(a) 70 (b) 90 (c) 120 (d) 140
48. The sum of two numbers is 2490. If 6.5% of one number is equal to 8.5% of the other, then the numbers are [IGNOU]
(a) 989, 1501 (b) 1011, 1479 (c) 1401, 1089 (d) 1411, 1079

49. A student multiplied a number by $\frac{3}{5}$ instead of $\frac{5}{3}$. What is the percentage error in the calculation? [SSC]
 (a) 34% (b) 44% (c) 54% (d) 64%
50. The population of a town increased from 1,75,000 to 2,62,500 in a decade. The average per cent increase of population per year is [CBI]
 (a) 4.37% (b) 5% (c) 6% (d) 8.75%
51. Two numbers A and B are such that the sum of 5% of A and 4% of B is two-third of the sum of 6% of A and 8% of B . Find the ratio of $A : B$. [MBA]
 (a) 2 : 3 (b) 1 : 1 (c) 3 : 4 (d) 4 : 3
52. The difference of two numbers is 20% of the larger number. If the smaller number is 20, then the larger number is [SSC]
 (a) 25 (b) 45 (c) 50 (d) 80
53. 8% of the people eligible to vote are between 18 and 21 years of age. In an election, 85% of those eligible to vote, who were between 18 and 21, actually voted. In that election, the number of persons between 18 and 21, who actually voted, was that per cent of those eligible to vote? [RRB]
 (a) 4.2 (b) 6.4 (c) 6.8 (d) 8
54. In a certain school, 20% of students are below 8 years of age. The number of students above 8 years of age is $\frac{2}{3}$ of the number of students of 8 years age which is 48. What is the total number of students in the school? [Bank PO]
 (a) 72 (b) 80 (c) 120 (d) 150 (e) None of these
55. Two tailors X and Y are paid a total of ₹ 550 per week by their employer. If X is paid 120 per cent of the sum paid to Y, how much is Y paid per week? [NIFT]
 (a) ₹ 200 (b) ₹ 250 (c) ₹ 300 (d) None of these
56. If $x\%$ of y is 100 and $y\%$ of z is 200, then find a relation between x and z . [SSC]
 (a) $z = \frac{x}{2}$ (b) $z = 2x$ (c) $z = \frac{x}{4}$ (d) $z = 4x$
57. If $A = x\%$ of y and $B = y\%$ of x , then which of the following is true? [Bank PO]
 (a) A is smaller than B
 (b) A is greater than B
 (c) Relationship between A and B cannot be determined
 (d) If x is smaller than y , then A is greater than B
 (e) None of these
58. If x is 80% of y , then what per cent of $2x$ is y ? [CBI]
 (a) 40% (b) $62\frac{1}{2}\%$ (c) $66\frac{2}{3}\%$ (d) 80%
59. If x is 90% of y , then what per cent of x is y ? [SSC]
 (a) 90% (b) $101\frac{1}{9}\%$ (c) $111\frac{1}{9}\%$ (d) 190%
60. A student secures 90%, 60% and 54% marks in test papers with 100, 150 and 200 respectively as maximum marks. The percentage of his aggregate is [Hotel Management]
 (a) 64 (b) 68 (c) 70 (d) None of these

61. In an examination, 5% of the applicants were found ineligible and 85% of the eligible candidates belonged to the general category. If 4275 eligible candidates belonged to other categories, then how many candidates applied for the examination? **[Hotel Management]**
(a) 30000 (b) 35000 (c) 37000 (d) None of these
62. 405 sweets were distributed equally among children in such a way that the number of sweets received by each child is 20% of the total number of children. How many sweets did each child receive? **[Bank PO]**
(a) 9 (b) 15 (c) 18 (d) 45
63. From the salary of an officer, 10% is deducted as house rent, 20% of the rest, he spends on conveyance, 20% of the rest he pays as income tax and 10% of the balance, he spends on clothes. Then, he is left with ₹ 15552. Find his total salary. **[LIC AAO]**
(a) ₹ 25000 (b) ₹ 30000 (c) ₹ 35000 (d) ₹ 40000
64. Sameer spends 24% of his monthly income on food and 15% on the education of his children. Of the remaining salary, he spends 25% on entertainment and 20% on conveyance. He is now left with ₹ 10736. What is the monthly salary of Sameer? **[Bank PO]**
(a) ₹ 27600 (b) ₹ 28000 (c) ₹ 31200 (d) ₹ 32000
65. Rohit spends 40% of his salary on food, 20% on house rent, 10% on entertainment and 10% on conveyance. If his savings at the end of a month are ₹ 1500, then his monthly salary is **[SSC]**
(a) ₹ 6000 (b) ₹ 7500 (c) ₹ 8000 (d) ₹ 10000
66. Peter could save 10% of his income. But two years later when his income was increased by 20%, he could save the same amount only as before. By how much per cent has his expenditure increased? **[RRB]**
(a) 22% (b) $22\frac{2}{9}\%$ (c) $23\frac{1}{3}\%$ (d) 24%
67. The price of an article was increased by $r\%$. Later the new price was decreased by $r\%$. If the latest price was ₹ 1, then the original price was **[SSC]**
(a) ₹ 1 (b) $\left(\frac{1-r^2}{100}\right)$ (c) $\frac{\sqrt{1-r^2}}{100}$ (d) $\left(\frac{10000}{10000-r^2}\right)$
68. A number is decreased by 10% and then increased by 10%. The number so obtained is 10 less than the original number what was the original number? **[SSC]**
(a) 1000 (b) 1050 (c) 1500 (d) 2000
69. If the price of a book is first decreased by 25% and then increased by 20%, then the net change in the price will be **[SSC]**
(a) No change (b) 5% increase (c) 5% decrease (d) 10% decrease
70. $37\frac{1}{2}\%$ of the candidates in an examination were girls, 75% of the boys and $62\frac{1}{2}\%$ of the girls passed and 342 girls failed. The number of boys failed was **[SSC]**
(a) 350 (b) 360 (c) 370 (d) 380
71. The price of a shirt is increased by 15% and then reduced by 15%. The final price of the shirt **[Hotel Management]**
(a) does not change (b) increases by 2.25%
(c) decreases by 2.25% (d) None of these

72. The weight of a container alone is 25% of the container filled with a certain fluid. When some fluid is removed, the weight of the container and remaining fluid is 50% of the original total weight. What fractional part of the liquid has been removed? [DMRC]
 (a) $\frac{1}{3}$ (b) $\frac{1}{2}$ (c) $\frac{2}{3}$ (d) $\frac{3}{4}$
73. One type of liquid contains 20% water and the second type of liquid contains 35% of water. A glass is filled with 10 parts of first liquid and 4 parts of second liquid. The percentage of water in the new mixture in the glass is [CBI]
 (a) 20% (b) $24\frac{2}{7}\%$ (c) 37% (d) 40%
74. Milk contains 5% water. What quantity of pure milk should be added to 10 litres of milk to reduce this to 2%? [Bank PO]
 (a) 5 litres (b) 7 litres (c) 15 litres
 (d) cannot be determined
75. How much pure alcohol has to be added to 400 ml of a solution containing 15% alcohol to change the concentration of alcohol in the mixture to 32%? [SSC]
 (a) 60 ml (b) 68 ml (c) 100 ml (d) 128 ml
76. Fresh fruits contain 68% water and dry fruits contain 20% water. How much dry fruit can be obtained from 100 kg of fresh fruits? [SSC]
 (a) 32 kg (b) 40 kg (c) 52 kg (d) 80 kg
77. In an examination, the percentage of students qualified to the number of students appeared from school A is 70%. In school B, the number of students appeared is 20% more than the students appeared from school A and the number of students qualified from school B is 50% more than the students qualified from school A. What is the percentage of students qualified to the number of students appeared from school B? [Bank PO]
 (a) 30% (b) 70% (c) 78.5% (d) 87.5%
78. In an examination, 65% students passed in Civics and 60% in History, 40% passed in both of these subjects. If 90 students failed in History and Civics both, then what is the total number of students? [RRB]
 (a) 600 (b) 650 (c) 700 (d) 750
79. In a hotel, 60% had vegetarian lunch while 30% had non-vegetarian lunch and 15% had both types of lunch. If 96 people were present, how many did not eat either type of lunch? [SSC]
 (a) 20 (b) 24 (c) 26 (d) 28
80. In an examination, 34% of students failed in Mathematics and 42% failed in English. If 20% of the students failed in both the subjects, then the percentage of students who passed in both the subject was [SSC]
 (a) 44 (b) 50 (c) 54 (d) 56
81. In an examination in which full marks were 800, A gets 20% more than B, B gets 20% more than C and C gets 15% less than D. If A got 576, what percentage of full marks did D get (approximately)? [RRB]
 (a) 45.7 (b) 51.2 (c) 58.8 (d) 61.7
82. Amit's monthly income is 30% more than that of Raunaq. Raunaq's monthly income is 20% less than that of Deepak. If the difference between the monthly incomes of Amit and Deepak is ₹ 800, what is the monthly income of Raunaq? [Bank PO]
 (a) ₹ 12000 (b) ₹ 16000 (c) ₹ 20000
 (d) Data inadequate

83. Out of 450 students of a school, 325 play football, 175 play cricket and 50 neither play football nor cricket. How many students play both football and cricket? [SSC]
(a) 50 (b) 75 (c) 100 (d) 225
84. The price of rice is reduced by 2%. How many kilograms of rice can now be bought for the money which was sufficient to buy 49 kg of rice earlier? [SSC]
(a) 48 kg (b) 49 kg (c) 50 kg (d) 51 kg
85. An empty fuel tank of a car was filled with A type petrol. When the tank was half-empty, it was filled with B type petrol. Again when the tank was half-empty, it was filled with A type petrol. When the tank was half-empty again, it was filled with B-type petrol. What is the percentage of A type petrol at present in the tank? [Bank PO]
(a) 33.5% (b) 37.5% (c) 40% (d) 50%
86. From a container having pure milk, 20% is replaced by water and the process is repeated thrice. At the end of the third operation, the milk is [SSC]
(a) 40% pure (b) 50% pure (c) 51.2% pure (d) 58.8% pure
87. Due to an increase of 30% in the price of eggs, 3 eggs less are available for ₹ 7.80. The present rate of egg per dozen is [NIFT]
(a) ₹ 8.64 (b) ₹ 8.88 (c) ₹ 9.36 (d) ₹ 10.40
88. The ratio 5 : 4 expressed in percentage as [SSC]
(a) 12.5% (b) 40% (c) 80% (d) 125%
89. Half of 1 per cent written as a decimal is [SSC]
(a) 0.005 (b) 0.05 (c) 0.02 (d) 0.2
90. What is 15 per cent of ₹ 34? [IMT]
(a) ₹ 3.40 (b) ₹ 3.75 (c) ₹ 4.50 (d) ₹ 5.10
91. What per cent of 7.2 kg is 18 gm? [SSC]
(a) 0.025% (b) 0.25% (c) 2.5% (d) 25%
92. What per cent of a day is 3 hours? [RRB]
(a) $12\frac{1}{2}\%$ (b) $16\frac{2}{3}\%$ (c) $18\frac{2}{3}\%$ (d) $22\frac{1}{2}\%$
93. A housewife saved ₹ 2.50 in buying an item on sale. If she spent ₹ 25 for the item, approximately how much per cent she saved in the transaction? [Section Officer's]
(a) 8% (b) 9% (c) 10% (d) 11%
94. 5% of (25% of ₹ 1600) is
(a) ₹ 5 (b) ₹ 17.50 (c) ₹ 20 (d) ₹ 25
95. 0.15% of $33\frac{1}{3}\%$ of ₹ 10000 is [SSC]
(a) ₹ 0.05 (b) ₹ 5 (c) ₹ 105 (d) ₹ 150
96. The difference between number and its two-fifth is 510. What is 10% of that number? [Bank PO]
(a) 12.75 (b) 85 (c) 204 (d) None of these
97. What percentage of numbers from 1 to 70 has squares that end in the digit 1? [MBA]
(a) 1 (b) 14 (c) 20 (d) 21
98. The sum of two numbers is $\frac{28}{25}$ of the first number. The second number is what per cent of the first? [Hotel Management]
(a) 12% (b) 14% (c) 16% (d) 18%

99. In a competitive examination in state A, 6% candidates got selected from the total appeared candidates. State B had an equal number of candidates appeared and 7% candidates got selected with 80 more candidates got selected than A. What was the number of candidates appeared from each state? [SBI PO]
 (a) 7600 (b) 8000 (c) 8400 (d) Data inadequate
100. The price of a car is ₹ 325000. It was insured to 85% of its price. The car was damaged completely in an accident and the insurance company paid 90% of the insurance. What was the difference between the price of the car and the amount received? [Bank PO]
 (a) ₹ 32500 (b) ₹ 48750 (c) ₹ 76375 (d) ₹ 81250
101. Three candidates contested an election and received 1136 7636 and 11628 votes respectively. What percentage of the total votes did the winning candidate get? [IMT]
 (a) 57% (b) 60% (c) 65% (d) 90%
102. If one number is 80% of the other and 4 times the sum of their squares is 656, then the numbers are [Hotel Management]
 (a) 4, 5 (b) 8, 10 (c) 16, 20 (d) None of these
103. When any number is divided by 12, then the dividend becomes $\frac{1}{4}$ th of the other number. By how much per cent first number is greater than the second number? [Bank PO]
 (a) 150 (b) 200 (c) 300 (d) Data inadequate
104. If 25% of a number is subtracted from a second number, the second number reduces to its five-sixths. What is the ratio of the first number to the second number? [SBI PO]
 (a) 1 : 3 (b) 2 : 3 (c) 3 : 2 (d) Data inadequate
105. Gauri went to the stationers and bought things worth ₹ 25, out of which 30 paise went on sales tax on taxable purchases. If the tax rate was 6%, then what was the cost of the tax free items? [MAT]
 (a) ₹ 15 (b) ₹ 15.70 (c) ₹ 19.70 (d) ₹ 20
106. A batsman scored 110 runs which included 3 boundaries and 8 sixes. What per cent of his total score did he make by running between the wickets? [SSC]
 (a) 45% (b) $45\frac{5}{11}\%$ (c) $54\frac{6}{11}\%$ (d) 55%
107. A fruit seller had some apples. He sells 40% apples and still has 420 apples. Originally, he had [SSC]
 (a) 588 apples (b) 600 apples (c) 672 apples (d) 700 apples
108. A student has to obtain 33% of the total marks to pass. He got 125 marks and failed by 40 marks. The maximum marks are [CBI]
 (a) 300 (b) 500 (c) 800 (d) 1000
109. 10% of the voters did not cast their vote in an election between two candidates. 10% of the votes polled were found invalid. The successful candidate got 54% of the valid votes and won by a majority of 1620 votes. The number of voters enrolled on the voter's list was [SSC]
 (a) 25000 (b) 33000 (c) 35000 (d) 40000
110. In an election between two candidates, one got 55% of the total valid votes, 20% of the votes were invalid. If the total number of votes was 7500, the number of valid votes that the other candidate got was [RRB]
 (a) 2700 (b) 2900 (c) 3000 (d) 3100
111. In an examination, 35% of the students passed and 455 failed. How many students appeared for the examination? [SSC]
 (a) 490 (b) 700 (c) 845 (d) 1300

112. A person who spends $66\frac{2}{3}\%$ of his income is able to save ₹ 1200 per month. His monthly expenses (in ₹) are [SSC]
(a) ₹ 1200 (b) ₹ 2400 (c) ₹ 3000 (d) ₹ 3200
113. In an election a candidate who gets 84% of the votes is elected by a majority of 476 votes. What is the total number of votes polled? [Hotel Management]
(a) 672 (b) 700 (c) 749 (d) 848
114. If $P\%$ of P is 36, then P is equal to [SSC]
(a) 15 (b) 60 (c) 600 (d) 3600
115. If $x\%$ of y is equal to z , what per cent of z is x ? [SSC]
(a) $\frac{y^2}{100}$ (b) $\frac{y}{100^2}$ (c) $\frac{100}{y}$ (d) $\frac{100^2}{y}$
116. Aman gave 40% of the amount he had to Rohan. Rohan in turn gave one-fourth of what he received from Aman to Sahil. After paying ₹ 200 to the taxi driver out of the amount he got from Rohan, Sahil now has ₹ 600 left with him. How much amount did Aman have? [Bank PO]
(a) ₹ 4000 (b) ₹ 8000 (c) ₹ 12000 (d) Data inadequate
117. A spider climbed $62\frac{1}{2}\%$ of the height of the pole in one hour and in the next hour it covered $12\frac{1}{2}\%$ of the remaining height. If the height of the pole is 192 m, then distance climbed in second hour is [Section Officer's]
(a) 3 m (b) 5 m (c) 7 m (d) 9 m
118. Of the 1000 inhabitants of a town, 60% are males of whom 20% are literate. If, of all the inhabitants, 25% are literate, then what per cent of the females of the town are literate? [MAT]
(a) 22.5 (b) 27.5 (c) 32.5 (d) 37.5
119. In a recent survey, 40% houses contained two or more people. Of those houses containing only one person, 25% were having only a male. What is the percentage of all houses, which contain exactly one female and no males? [SBI PO]
(a) 15 (b) 40 (c) 75
(d) Cannot be determined (e) None of these
120. A scored 30% marks and failed by 15 marks. B scored 40% marks and obtained 35 marks more than that required to pass. The pass percentage is [SSC]
(a) 33% (b) 38% (c) 43% (d) 46%
121. In a History examination, the average for the entire class was 80 marks. If 10% of the students scored 95 marks and 20% scored 90 marks, what was the average marks of the remaining students of the class? [DMRC]
(a) 65.5 (b) 72.5 (c) 75 (d) 85
122. In an examination, there are three papers and a candidate has to get 35% of the total to pass. In one paper, he gets 62 out of 150 and in the second 35 out of 150. How much must he get, out of 180, in the third paper to just qualify for a pass? [RRB]
(a) 60.5 (b) 68 (c) 70 (d) 71
123. The sum of the number of boys and girls in a school is 150. If the number of boys is x , then the number of girls becomes $x\%$ of the total number of students. The number of boys is [SSC]
(a) 40 (b) 50 (c) 60 (d) 90

124. A salesman is allowed $5\frac{1}{2}\%$ discount on the total sales made by him plus a bonus of $\frac{1}{2}\%$ on the sales over ₹ 10000. If his total earnings were ₹ 1990, then his total sales (in ₹) were [CBI]
 (a) 30000 (b) 32000 (c) 34000 (d) 35000
125. Peter earned 40% more money than Albert. Albert earned 20% less than Michael. Peter earned more than Michael by [SSC]
 (a) 10% (b) 12% (c) 20% (d) 25%
126. A's salary is 40% of B's salary which is 25% of C's salary. What percentage of C's salary is A's salary? [MBA]
 (a) 5% (b) 10% (c) 15% (d) 20%
127. In some quantity of ghee, 60% is pure ghee and 40% is vanaspati. If 10 kg of pure ghee is added, then the strength of vanaspati ghee becomes 20%. The original quantity was [Hotel Management]
 (a) 10 kg (b) 15 kg (c) 20 kg (d) 25 kg
128. Two numbers are respectively $12\frac{1}{2}\%$ and 25% more than a third number. The first number as a percentage of the second number is [CBI]
 (a) 50 (b) 60 (c) 75 (d) 90
129. Two numbers are less than a third number by 30% and 37% respectively. How much per cent is the second number less than the first? [SSC]
 (a) 3% (b) 4% (c) 7% (d) 10%
130. If A's height is 40% less than that of B, how much per cent B's height is more than that of A? [SSC]
 (a) $33\frac{1}{3}\%$ (b) 40% (c) 60% (d) $66\frac{2}{3}\%$
131. A's salary is 50% more than B's. How much per cent is B's salary less than A's? [SSC]
 (a) 33% (b) $33\frac{1}{4}\%$ (c) $33\frac{1}{3}\%$ (d) $33\frac{1}{2}\%$
132. The population of a town increases 4% annually but is decreased by emigration annually to the extent of $(\frac{1}{2})\%$. What will be the increase per cent in 3 years?
 (a) 9.8 (b) 10 (c) 10.5 (d) 10.8
133. The total population of a village is 5000. The number of males and females increases by 10% and 15% respectively and consequently the population of the village becomes 5600. What was the number of males in the village? [Bank PO]
 (a) 2000 (b) 2500 (c) 3000 (d) 4000
134. Depreciation applicable to an equipment is 20%. The value of the equipment 3 years from now will be less by: [MBA]
 (a) 45% (b) 48.8% (c) 51.2% (d) 60%
135. The population of a town 2 years ago was 62500. Due to migration to big cities, it decreases every year at the rate of 4%. The present population of the town is: [SSC]
 (a) 56700 (b) 57600 (c) 58800 (d) 60000
136. A district has 64000 inhabitants. If the population increases at the rate of $2\frac{1}{2}\%$ per annum, then the number of inhabitants at the end of 3 years will be [SSC]
 (a) 68911 (b) 68921 (c) 69200 (d) 70000

137. The value of a machine depreciates at the rate of 10% every year. It was purchased 3 years ago. If its present value is ₹ 8748, its purchase price was [AAO Exam]
 (a) ₹ 10000 (b) ₹ 11372 (c) ₹ 12000 (d) ₹ 12500
138. If inflation increases at a rate of 8% p.a., what will a ₹ 20 article cost at the end of two years? [Bank PO]
 (a) Between ₹ 20 and ₹ 21 (b) Between ₹ 21 and ₹ 22
 (c) Between ₹ 22 and ₹ 23 (d) Between ₹ 23 and ₹ 24
139. The population of a town was 160000 three years ago. If it was increased by 3%, 2.5% and 5% respectively in the last three years, then the present population is [IGNOU]
 (a) 177000 (b) 177366 (c) 177461 (d) 177596
140. The population of a town increases by 5% annually. If its population in 2001 was 138915, what it was in 1998? [RRB]
 (a) 100000 (b) 108000 (c) 110000 (d) 120000
141. The production of a company has ups and downs every year. The production increases for two consecutive years consistently by 15% and in the third year it decreases by 10%. Again in the next two years it increases by 15% each year and decreases by 10% in the third year. If we start counting from the year 1998, approximately what will be the effect on production of the company in 2002? [Bank PO]
 (a) 27% increase (b) 32% increase (c) 37% increase (d) 42% increase (e) 52% increase
142. In the month of January, the Railway Police caught 4,000 ticketless travellers. In February, the number rose by 5%. However, due to constant vigil by the Police and the Railway staff, the number reduced by 5% and in April it further reduced by 10%. The total number of ticketless travellers caught in the month of April was [MBA]
 (a) 3125 (b) 3255 (c) 3575 (d) 3591
143. The population of a variety of tiny bushes in an experimental field increased by 10% in the first year, increased by 8% in the second year but decreased by 10% in the third year. If the present number of bushes in the experimental field is 26730, then the number of bushes in the beginning was [MAT]
 (a) 25000 (b) 27000 (c) 28000 (d) 24600
144. The quantity of water needed to reduce 9 ml shaving lotion containing 50% alcohol to a lotion containing 30% alcohol is
 (a) 4 ml (b) 5 ml (c) 6 ml (d) 7 ml
145. A bag contains 600 coins of 25 paise, 1200 coins of 50 paise. If 12% of 25p coins and 24% of 50p coins are removed, the percentage of money removed from the bag is nearly
 (a) 15.6% (b) 17.8% (c) 21.6% (d) 30%



1. (b)	2. (d)	3. (c)	4. (c)	5. (c)	6. (c)	7. (d)	8. (d)	9. (d)	10. (b)
11. (d)	12. (c)	13. (d)	14. (d)	15. (e)	16. (a)	17. (d)	18. (b)	19. (b)	20. (d)
21. (c)	22. (a)	23. (b)	24. (c)	25. (c)	26. (d)	27. (d)	28. (b)	29. (a)	30. (b)
31. (c)	32. (b)	33. (c)	34. (b)	35. (a)	36. (b)	37. (a)	38. (b)	39. (b)	40. (d)
41. (b)	42. (d)	43. (e)	44. (d)	45. (a)	46. (a)	47. (d)	48. (d)	49. (d)	50. (b)
51. (d)	52. (a)	53. (c)	54. (e)	55. (b)	56. (b)	57. (e)	58. (b)	59. (c)	60. (a)
61. (a)	62. (d)	63. (b)	64. (d)	65. (b)	66. (b)	67. (d)	68. (a)	69. (d)	70. (d)
71. (c)	72. (c)	73. (b)	74. (c)	75. (c)	76. (c)	77. (d)	78. (a)	79. (b)	80. (a)

81. (c) 82. (b) 83. (c) 84. (c) 85. (b) 86. (c) 87. (c) 88. (d) 89. (a) 90. (d)
 91. (d) 92. (a) 93. (c) 94. (c) 95. (b) 96. (d) 97. (c) 98. (a) 99. (b) 100. (c)
 101. (a) 102. (b) 103. (b) 104. (b) 105. (c) 106. (b) 107. (d) 108. (b) 109. (a) 110. (a)
 111. (b) 112. (b) 113. (b) 114. (b) 115. (d) 116. (b) 117. (d) 118. (c) 119. (e) 120. (a)
 121. (c) 122. (d) 123. (c) 124. (c) 125. (b) 126. (b) 127. (a) 128. (a) 129. (d) 130. (d)
 131. (c) 132. (d) 133. (c) 134. (b) 135. (b) 136. (b) 137. (c) 138. (d) 139. (b) 140. (d)
 141. (c) 142. (d) 143. (a) 144. (c) 145. (c)



Solutions with Necessary Explanation

1. Let the capacity of tank be 27 litres.

	Ordinary	High speed
I	27	Nil
II	18	9
III	$12 + 9 = 21$	6
IV	14	$4 + 9 = 13$

Hence, the required per cent of high speed petrol in the tank

$$\frac{13}{14 + 13} \times 100 = \frac{13}{27} \times 100 = 48 \frac{4}{27} \%$$

2. Let length and breadth of rectangle be ' l ' and ' b ' units and perimeter be ' p ' units.

Then $p = 2(l + b)$

(1)

Here l is increased to $1.2l$ and b is increased to $1.3b$

$$\begin{aligned} \therefore \text{New value of perimeter} &= 2(1.2l + 1.3b) \\ &= 2.4l + 2.6b \end{aligned}$$

Hence answer is **data inadequate**.

3. Suppose the total number of workers in the company is 1000. Then the number of

(i) Skilled workers = 75% of 1000 = 750

(ii) Unskilled workers = 25% of 1000 = 250

(iii) Permanent workers = $\frac{80}{100} \times 750 + \frac{20}{100} \times 250 = 600 + 50 = 650$

(iv) Temporary workers = $1000 - 650 = 350$

Again, since the number of temporary workers is 126, the total number of workers in

$$\text{the company} = \frac{1000}{350} \times 126 = 360$$

4. Required percentage in the area = $-40 + 40 + \frac{(-40) \times 40}{100} = -16\%$ (16% decrease)

5. The population after the three years will be

$$123456789 \times \frac{110}{100} \times \frac{120}{100} \times \frac{130}{100} = 123456789 \times \frac{171.6}{100}$$

$\therefore$ Overall percentage increase in the previous population during the three years = 71.6%

$$\text{Average per cent increase during the three years} = \frac{71.6}{3} = \frac{716}{30} = 23 \frac{13}{15} \%$$

6. Monthly income of the person = ₹ 13500

Monthly expenditure of the person = ₹ 9000

Savings of the person = ₹ 4500

Next year his monthly income = $\frac{114}{100} \times 13500 = 15,390$

His monthly expenditure = $\frac{107}{100} \times 9000 = ₹ 9630$

His savings = $114 \times 135 - 107 \times 90$
 $= 45(342 - 214)$
 $= 45 \times 128$

∴ Percentage increase in savings = $\frac{45 \times 128 - 45 \times 100}{45 \times 100} = \frac{45 \times 28}{45 \times 100} = 28\%$

7. $60 = ?\% \text{ of } 400$

$$\Rightarrow 60 = ? \times \frac{400}{100}$$

$$\Rightarrow 60 = ? \times 4$$

$$\Rightarrow ? = \frac{60}{4} = 15$$

8. $40\% \text{ of } ? = 240$

$$\Rightarrow \frac{40 \times ?}{100} = 240$$

$$\Rightarrow ? = \frac{240 \times 100}{40} = 600$$

9. $23\% \text{ of } 8040 + 42\% \text{ of } 545 = ?\% \text{ of } 3000$

$$\Rightarrow 23 \times \frac{8040}{100} + 42 \times \frac{545}{100} = \frac{3000}{100} \times ?$$

$$\Rightarrow 3000 \times ? = 23 \times 8040 + 42 \times 545$$

$$\Rightarrow 3000 \times ? = 184920 + 22890$$

$$\Rightarrow 3000 \times ? = 207810$$

$$\Rightarrow ? = \frac{207810}{3000} = 69.27$$

10. Let Sunil's salary be ₹ x

$$5\% \text{ of } x = \frac{₹ 5x}{100} = \frac{₹ x}{20}$$

$$\text{i.e. } 1687.50 = \frac{75}{100} \times \frac{x}{20} = \frac{3x}{80}$$

$$\Rightarrow 3x = 1687.50 \times 80$$

$$\Rightarrow x = \frac{1,687.50 \times 80}{3} = ₹ 45000$$

11. The amount given to wife $\rightarrow 40\%$ (remaining 60%)

The amount given to son $\rightarrow 25\%$ of $60\% = 15\%$ (remaining 45%)

The amount spent on education $\rightarrow 60\%$ of $45\% = 27\%$ (remaining 18%)

$\therefore 18\% \rightarrow ₹ 2700$

$$\therefore 40\% \rightarrow \frac{2700}{18} \times 40 = ₹ 6000$$

12. Let the total number of candidates be 100

Number of candidates passed in English = 70

$\therefore$ Failed candidates in English = 30

$\therefore$ Number of failed candidates in English alone = $30 - 10 = 20$

Number of candidates passed in Maths = 80

Number of candidates failed in Maths = 20

Number of candidates failed in Maths alone = $20 - 10 = 10$

Number of candidates failed in both = 10

$\therefore$ Total number of failures = $20 + 10 + 10 = 40$

$\therefore$ Total number of candidates passed = $100 - 40 = 60$

$\therefore$ Strength of candidates attended the examination for a pass of 144 candidates

$$= \frac{100}{60} \times 144 = 240$$

13. Let the total number of votes be 100.

$\therefore$ Number of uncast votes = 8

Votes polled = $100 - 8 = 92$

Number of votes obtained by the winner = 48

Number of votes obtained by the loser = $92 - 48 = 44$

$\therefore$ The difference in votes obtained by the winner and loser = $48 - 44 = 4$

$\therefore$ Given difference in votes = 1100

$$= \frac{100}{4} \times 1100 = 27500$$

14. The required percentage increase = $7 + 7 + \frac{7 \times 7}{100} = 14.49\%$

15. Let the monthly income of Manoj be 100.

$$\therefore \text{Ashok's income} = \frac{90}{100} \times 100 = ₹ 90$$

$$\text{Monthly income of Anil} = \frac{130}{100} \times 90 = ₹ 117$$

Difference between income of Anil and Manoj = $117 - 100 = ₹ 17$

$\therefore$ Monthly income of Ashok for a difference in salary of ₹ 1360

$$= \frac{90}{17} \times 1360 = ₹ 7200$$

16. Let the number be x .

$$\therefore (75 - 35)\% \text{ of } x = 380$$

$$\text{i.e. } \frac{40}{100} \times x = 380$$

$$\therefore \frac{20}{100} \times x = \frac{380}{2} = 190$$

17. Require more information like ratio of the length of the rectangle to its breadth.

18. Required per cent = $\frac{10}{(10-2)} \times 10 = 12.5\%$

19. Required percentage = $\frac{8}{10} \times \frac{1}{10} + \frac{9}{10} \times \frac{2}{10}$
 $= \frac{8}{100} + \frac{18}{100} = \frac{26}{100} = 26\%$

20. $\frac{125}{100} \times 320 + \frac{?}{100} \times 125 = 440$
 $\Rightarrow \frac{?}{100} \times 125 = 440 - \frac{125}{100} \times 320$
 $= 440 - 400 = 40$
 $\therefore ? = \frac{40 \times 100}{125} = 32$

21. The students who do not read Hindi = $100 - 70 = 30$
 Number of boys = 40; Number of girls = 60
 Number of girls who does not read Hindi = 20% of 60 = 12
 Number of boys who does not read Hindi = $30 - 12 = 18$
 $\therefore$ No. of boys who reads Hindi = $40 - 18 = 22$

$\therefore$ Percentage of boys who read Hindi = $\frac{22}{40} \times 100 = 55\%$

22. Net effect on area = $\left(-40 + 30 - \frac{40 \times 30}{100}\right)\% = -22\%$

i.e. decrease by 22%

23. 28% of 450 + 45% of 280 = $\frac{28}{100} \times 450 + \frac{45}{100} \times 280 = 126 + 126 = 252$

24. Let the original price of commodity be ₹ x .
 Due to decrease in price by 10%, new price = $0.9x$.
 Selling quantity increased by 30%
 $\therefore$ Effect on revenue = $0.9x \times 1.3 = 1.17x$
 $\therefore$ Selling of commodity increased by 17%.
 Effect on revenue = $(-10 + 30 - 3)\% = 17\%$

25. Let x be the per cent by which reduced salary be raised so as to bring it to the original

Then $\left(-10 + x - \frac{10x}{100}\right)\% = 0$

i.e. $x - \frac{x}{10} - 10 = 0$

or $\frac{9x}{10} = 10$

or $x = \frac{10 \times 10}{9} = 11.11\%$

26. Let the population at the beginning of a year be x

$$\text{Then } x \left(1 + \frac{5}{100}\right) \left(1 - \frac{5}{100}\right) = 9975$$

$$\therefore x = \frac{9975 \times 100 \times 100}{105 \times 95} = 10000$$

27. $3.5 = \frac{350}{100} = 350\%$

28. $88\% \text{ of } 370 + 24\% \text{ of } 210 - ? = 118$

$$\begin{aligned} \therefore ? &= \frac{88}{100} \times 370 + \frac{24}{100} \times 210 - 118 \\ &= 325.6 + 50.4 - 118 = 258 \end{aligned}$$

29. $860\% \text{ of } 50 + 50\% \text{ of } 860 = \frac{860}{100} \times 50 + \frac{50}{100} \times 860 = 430 + 430 = 860$

30. $45\% \text{ of } 750 - 25\% \text{ of } 480 = \frac{45}{100} \times 750 - \frac{25}{100} \times 480 = 337.50 - 120 = 217.50$

31. $40\% \text{ of } 1640 + ? = 35\% \text{ of } 980 + 150\% \text{ of } 850$

$$\begin{aligned} \therefore ? &= 35\% \text{ of } 980 + 150\% \text{ of } 850 - 40\% \text{ of } 1640 \\ &= \frac{35}{100} \times 980 + \frac{150}{100} \times 850 - \frac{40}{100} \times 1640 \\ &= 343 + 1275 - 656 = 962 \end{aligned}$$

32. $60\% \text{ of } 264 = 15\% \text{ of } 1056$

33. $0.01 = x\% \text{ of } 0.1 = \frac{x}{100} \times 0.1$

$$\therefore x = \frac{0.01 \times 100}{0.1} = 10$$

34. $x\% \text{ of } 2650 = 1987.50$

i.e. $\frac{x}{100} \times 2650 = 1987.50$

$$\therefore x = \frac{1987.5}{2650} \times 100 = 75$$

35. Rajeev got 6% rebate as ₹ 6650

$$\therefore \text{Net price} = 6650 \times 0.94 = ₹ 6251$$

$$\text{Tax on net price @ } 10\% = \frac{10}{100} \times 6251 = ₹ 625.10$$

$$\therefore \text{Total price} = 6251 + 625.10 = ₹ 6876.10$$



Short cut: Total effect on the price due to rebate and sales tax in percentage

$$\left(-6 + 10 - \frac{6 \times 10}{100}\right)\% = 3.4\%$$

i.e. $6650 \times \frac{3.4}{100} = 226.10$

$$\therefore \text{Total price} = 6650 + 226.10 = ₹ 6876.10$$

36. Let $4598 = \frac{95}{100}$ of $x = \frac{95x}{100}$

or $x = \frac{4598 \times 100}{95} = 242 \times 20 = \mathbf{4840}$

37. $x\%$ of 360 = 129.6

i.e. $\frac{x}{100} \times 360 = 129.6$

or $x = \frac{129.6 \times 100}{360} = \mathbf{36}$

38. 65% of $x = 20\%$ of 422.50

i.e. $\frac{65}{100} \times x = \frac{20}{100} \times 422.50$

or $x = \frac{20}{100} \times 422.50 \times \frac{100}{65}$
 $= \frac{4 \times 422.5}{13} = \mathbf{130}$

39. Let $x\%$ of $932 + 30 = 309.6$

i.e. $\frac{x}{100} \times 932 + 30 = 309.6$

or $x = (309.6 - 30) \times \frac{100}{932}$
 $= \frac{27960}{932} = \mathbf{30}$

40. Let 45% of 1500 + 35% of 1700 = $x\%$ of 3175

i.e. $\frac{45}{100} \times 1500 + \frac{35}{100} \times 1700 = \frac{x}{100} \times 3175$

i.e. $x = (45 \times 15 + 35 \times 17) \frac{100}{3175}$
 $= (675 + 595) \frac{100}{3175} = 1270 \times \frac{100}{3175} = \mathbf{40}$

41. Let the value of house be ₹ x .

Given $2800 = \frac{2}{700} \times x$

or $x = \frac{2800 \times 700}{2} = \mathbf{₹ 980000}$

42. Let the number be x .

$$120 = \frac{20}{100} \text{ of } x$$

$$\therefore \frac{120}{100} \text{ of } x = 6 \times 120 = \mathbf{720}$$

43. Let the number be x .

$$\frac{2}{5} \text{ of } \frac{1}{3} \text{ of } \frac{3}{7} \text{ of } x = 15$$

$$\therefore x = 15 \times \frac{7}{3} \times 3 \times \frac{5}{2}$$

$$\begin{aligned} \therefore \frac{40}{100} \text{ of } x &= \frac{40}{100} \times 15 \times \frac{7}{3} \times 3 \times \frac{5}{2} \\ &= 5 \times 7 \times 3 = \mathbf{105} \end{aligned}$$

44. Let the number be x .

$$\therefore \frac{35}{100} \times x = \frac{50}{100} \times x - 12$$

$$\text{i.e. } \left(\frac{50 - 35}{100} \right) x = 12$$

$$\text{or } x = 12 \times \frac{100}{15} = \mathbf{80}$$

45. Let the number be x .

$$\therefore x - \frac{16}{100} x = 42$$

$$\text{or } x = 42 \times \frac{100}{84} = \mathbf{50}$$

46. Given $x = 0.9y$

$$y = 125 \times 1.1$$

$$\begin{aligned} \therefore x &= 0.9 \times 125 \times 1.1 \\ &= 125 \times 0.99 = \mathbf{123.75} \end{aligned}$$

47. Let the number be x .

$$x - 35 = 0.8x$$

$$\text{or } x(1 - 0.8) = 35$$

$$x = \frac{35}{0.2}$$

$$\therefore \frac{4}{5} x = \frac{4}{5} \times \frac{35}{0.2} = \mathbf{140}$$

48. Let x and y be the numbers.

$$x + y = 2490 \tag{1}$$

$$\frac{6.5}{100} x = \frac{8.5}{100} \times y \tag{2}$$

$$(2) \Rightarrow x = \frac{8.5}{6.5} y = \frac{17}{13} y \tag{3}$$

Substituting value of x from (3) in (1),

$$\frac{17}{13} y + y = 2490$$

$$\Rightarrow \frac{30}{13} y = 2490$$

$$\Rightarrow y = \frac{2490 \times 13}{30}$$
$$= 83 \times 13 = \mathbf{1079}$$

$$\therefore (1) \Rightarrow x = 2490 - 1079 = \mathbf{1411}$$

$$\mathbf{49. \% \text{ error}} = \frac{3/5 - 5/3}{5/3} \times 100 = \frac{(9 - 25)/15}{5/3} \times 100$$
$$= \frac{-16}{15} \times \frac{3}{5} \times 100 = \frac{-16}{25} \times 100 = \mathbf{-64\%}$$

50. Average percentage increase of population/year

$$= \frac{262500 - 175000}{175000 \times 10} \times 100$$
$$= \frac{87500}{175000} \times \frac{100}{10} = \mathbf{5\%}$$

51. 5% of A + 4% of B = $\frac{2}{3}$ of (6% of A + 8% of B)

$$\text{i.e.} \quad \frac{5}{100} A + \frac{4}{100} B = \frac{2}{3} \left(\frac{6}{100} A + \frac{8}{100} B \right)$$

$$\text{i.e.} \quad A \left(\frac{5}{100} - \frac{2}{3} \times \frac{6}{100} \right) = B \left(\frac{8}{100} \times \frac{2}{3} - \frac{4}{100} \right)$$

$$A \left(\frac{5 - 4}{100} \right) = B \left(\frac{16 - 12}{300} \right)$$

$$\therefore \frac{A}{B} = \frac{4}{300} \times \frac{100}{1} = \frac{\mathbf{4}}{\mathbf{3}}$$

52. Let the numbers be x and y .

$$x - y = \frac{20}{100} \times x \tag{1}$$

Given $y = 20$;

Then,

$$(1) \Rightarrow x \left(1 - \frac{20}{100} \right) = y$$

$$\Rightarrow x \times \frac{80}{100} = 20$$

or $x = \frac{100 \times 20}{80} = \mathbf{25}$

$$\mathbf{53.} \quad \frac{85}{100} \times \frac{8}{100} \times x = \frac{680}{10000} x$$

$$= \frac{6.8}{100} \times x$$

i.e. $\mathbf{6.8\%}$

54. Let the total number of students be x .

$$\text{Number of students below 8 years} = \frac{20}{100} \times x$$

$$\text{Number of students of 8 years age} = 48$$

$$\therefore \text{Number of students above 8 years} = x - \frac{20}{100}x - 48 = 0.8x - 48$$

$$\text{Given} \quad 0.8x - 48 = \frac{2}{3}(48) = 32$$

$$\text{or} \quad 0.8x = 32 + 48 = 80$$

$$\text{or} \quad x = \frac{80}{0.8} = \mathbf{100}$$

55. Let amount paid be x and y .

$$x + y = 550 \quad (1)$$

$$\text{Given} \quad x = \frac{120}{100}y \quad (2)$$

$$\therefore (1) \Rightarrow 1.2y + y = 550$$

$$\Rightarrow y = \frac{550}{2.2} = \mathbf{250}$$

$$56. \text{ Given} \quad \frac{x}{100} \times y = 100 \quad (1)$$

$$\frac{y}{100} \times z = 200 \quad (2)$$

$$\frac{(1)}{(2)} \Rightarrow \frac{x}{z} = \frac{1}{2}$$

$$\text{or} \quad z = \mathbf{2x}$$

$$57. \text{ Given } A = \frac{x}{100} \times y; \quad B = \frac{y}{100} \times x;$$

$$\therefore A = B$$

$$58. \text{ Given } x = \frac{80}{100} \times y \quad (1)$$

$$\text{Let } P\% \text{ of } 2x = y$$

$$\frac{P}{100} \times 2x = y$$

$$\Rightarrow P = \frac{100 \times y}{2x} \quad (2)$$

Solving (1) and (2),

$$\begin{aligned} P &= \frac{100}{2} \times \frac{100}{80} \\ &= \frac{100 \times 10}{16} = \frac{25 \times 5}{2} = \mathbf{62.5} \end{aligned}$$

59. Given $x = \frac{90}{100} \times y$ (1)

Let $p\%$ of $x = y$

i.e. $\frac{p}{100} \times x = y$ (2)

Solving (1) and (2)

$$(2) \Rightarrow p = 100 \times \frac{y}{x}$$
$$= 100 \times \frac{100}{90} = 111\frac{1}{9}\%$$

60. Percentage of aggregate = $\frac{90 + 90 + 108}{450} \times 100\%$

$$= \frac{288}{450} \times 100\%$$
$$= \frac{32}{50} \times 100\% = 64\%$$

61. Number of general category candidates = 85% of $95\% = \frac{85}{100} \times \frac{95}{100}$

Number of other category candidates = 15% of $95\% = \frac{15}{100} \times \frac{95}{100}$

Given $\frac{15}{100} \times \frac{95}{100} \times x = 4275$

$$\therefore x = \frac{4275 \times 100 \times 100}{95 \times 15} = \frac{225 \times 20 \times 100}{15} = 75 \times 4 \times 100 = 30000$$

$\therefore$ Total number of candidates applied for the examination = **30000**

62. Let the number of children be x .

$$\text{Number of sweets/children} = \frac{20}{100} \times x = \frac{x}{5}$$

$$\therefore x \times \frac{x}{5} = 405$$

or $x^2 = 2025$

$$\therefore x = 45$$

63. Let the total salary of officer be x .

$$\therefore \text{House rent} = \frac{10}{100}x$$

$$\text{Conveyance} = \frac{20}{100} \left(x - \frac{10}{100}x \right)$$
$$= \frac{20}{100} \times \frac{90}{100}x = \frac{18}{100}x$$

$$\begin{aligned}\text{Income tax} &= \frac{20}{100} \left(x - \frac{10x}{100} - \frac{18x}{100} \right) \\ &= \frac{20}{100} \times \frac{72x}{100} = \frac{14.4x}{100}\end{aligned}$$

$$\begin{aligned}\text{Clothes} &= \frac{10}{100} \left(x - \frac{18x}{100} - \frac{10x}{100} - \frac{14.4x}{100} \right) \\ &= \frac{10x}{100} \times \frac{57.6}{100} = \frac{576x}{10000}\end{aligned}$$

$$\begin{aligned}\text{Amount left} &= \frac{57.6x}{100} - \frac{5.76x}{100} \\ &= \frac{51.84x}{100} = 15552\end{aligned}$$

$$\therefore x = \frac{15552 \times 100}{51.84} = \frac{15552 \times 100}{5184} \times 100 = \text{₹ } 30000$$

64. Let the monthly salary of Sameer be x .

Food = 24%; education = 15%

$\therefore$ For entertainment = 25% of $(100 - 24 - 15)\% = 25\% \times 61\%$

Conveyance = 20% of $(100 - 24 - 15)\% = 20\% \times 61\%$

$$\begin{aligned}\text{Amount left} &= \left(100 - 24 - 15 - \frac{61}{4} - \frac{61}{5} \right) \% \text{ of } x \\ &= 10736\end{aligned}$$

$$\text{i.e. } 61 \left(1 - \frac{1}{4} - \frac{1}{5} \right) \times \frac{x}{100} = 10736$$

$$\text{or } x \left(\frac{20 - 5 - 4}{20} \right) = \frac{10736 \times 100}{61}$$

$$\begin{aligned}\text{or } x &= \frac{10736 \times 100 \times 20}{61 \times 11} \\ &= \frac{976 \times 100 \times 20}{61} = 16 \times 100 \times 20 = \text{₹ } 32000\end{aligned}$$

65. Rohit spends

for food = 40%

rent = 20%

entertainment = 10%

conveyance = 10%

Total = 80%

$\therefore$ Savings = $100 - 80 = 20\%$ of his salary = 1500

$$\begin{aligned}\therefore \text{His monthly salary} &= \frac{1500}{\frac{20}{100}} \\ &= 1500 \times \frac{100}{20} = \text{₹ } 7500\end{aligned}$$

66. Let the income of Peter be x .

$$\text{Savings} = \frac{10}{100} \times x = 0.1x$$

$$\text{After increase, new income, } x + \frac{20}{100}x = 1.2x$$

$$\text{Savings} = 0.1x$$

$$\begin{aligned} \text{\% change in expenditure} &= \frac{(1.2 - 0.1)x - (1 - 0.1)x}{(1 - 0.1)} \times 100 \\ &= \frac{(1.1 - 0.9)x}{0.9x} \times 100 = \frac{0.2}{0.9} \times 100 = 22\frac{2}{9}\% \end{aligned}$$

67. Let the original price be P

$$\therefore P \left(1 + \frac{r}{100}\right) \left(1 - \frac{r}{100}\right) = 1$$

$$\text{i.e.} \quad P = \frac{1 \times 100 \times 100}{(100 + r)(100 - r)} = \frac{10000}{100^2 - r^2}$$

68. Let the number be n

$$n + n \left(-10 + 10 - \frac{100}{100}\right)\% = n - 10$$

$$\text{i.e.} \quad n - n\% = n - 10$$

$$\text{or} \quad n\% = 10$$

$$\frac{n}{100} = 10$$

$$\text{or} \quad n = 10 \times 100 = 1000$$

69. Net change in price = $\left(-25 + 20 - \frac{500}{100}\right)\% = -10\%$ or **10% decrease**

$$70. \text{ Girls} = 37\frac{1}{2}\% \text{ of total;} \quad \text{Boys} = 62\frac{1}{2}\% \text{ of total}$$

$$62\frac{1}{2}\% \text{ passed} \quad 75\% \text{ passed}$$

$$37\frac{1}{2}\% \text{ failed} \quad 25\% \text{ failed}$$

$$\text{i.e. Number of girls failed} = \frac{37\frac{1}{2}}{100} \times \frac{37\frac{1}{2}}{100} \times \text{total} = 342;$$

$$\text{Similarly } \frac{25}{100} \times \frac{62\frac{1}{2}}{100} \times \text{total} = \text{Number of boys failed}$$

$$\text{i.e.} \quad \frac{25}{100} \times \frac{62\frac{1}{2}}{100} \times \frac{342 \times 100 \times 100}{37\frac{1}{2} \times 37\frac{1}{2}} = \text{number of boys failed}$$

$$\text{i.e. Number of boys failed} = \frac{25 \times 62\frac{1}{2} \times 342}{37\frac{1}{2} \times 37\frac{1}{2}}$$

$$\begin{aligned}
 &= \frac{25 \times 125 \times 342}{75 \times 37 \frac{1}{2}} = \frac{125 \times 114}{37 \frac{1}{2}} \\
 &= \frac{125 \times 228}{75} = \frac{5 \times 228}{3} = \mathbf{380}
 \end{aligned}$$

71. Net effect on price of shirt = $\left(+15 - 15 - \frac{225}{100}\right)\% = -2.25\%$ or **2.25% decrease**

72. Weight of container = 25% of total weight of container with fluid

After remaining part of fluid,

$$= \text{Weight of container} + \text{Fluid} = 50\% \text{ of total weight of container} + \text{Fluid}$$

$$\therefore \text{Quantity of fluid removed} = \frac{50\%}{75\%} = \frac{2}{3}$$

$$\begin{aligned}
 \frac{\text{Quantity displaced}}{\text{Original capacity of tank}} &= \frac{50\% \text{ of total weight}}{75\% \text{ of total weight}} \\
 &= \frac{2}{3}
 \end{aligned}$$

73. $\frac{1}{14} \left(10 \times \frac{20}{100} + 4 \times \frac{35}{100} \right) = (2 + 1.4) \frac{1}{14} = \frac{3.4}{14} = \frac{1.7}{7}$

$$\text{Expressed as percentage} = \frac{1.7}{7} \times 100 = \mathbf{24 \frac{2}{7} \%}$$

74. Quantity of pure milk in 10 litres of milk = 9.5 litres

Quantity of milk to be added to:

10 litres of milk of 95% strength to convert it of 98% strength = ?

Quantity of water in the milk = 0.5 litre

$$\text{i.e. } 2\% = 0.5 \text{ litre}$$

$$\therefore 100\% = 25 \text{ litre}$$

$\therefore$ Required quantity of milk to be added = **15 litres**

75. Quantity of pure alcohol in 400 ml sold = $\frac{15}{100} \times 400 = 60 \text{ ml}$

$$\therefore \text{Quantity of water in the alcohol} = 340 \text{ ml}$$

$$\text{In the new solution, } 68\% = 340 \text{ ml}$$

$$\therefore 100\% = 500 \text{ ml}$$

$$\therefore \text{Quantity of alcohol to be added to make concentration of } 32\% = 500 - 400 = \mathbf{100 \text{ ml}}$$

76. Fresh fruit contains = 68% water

Dry fruit contains = 20% water

$$\begin{aligned}
 \text{From 100 kg of fresh fruit} &= 100 \left(1 - \frac{48}{100} \right) \\
 &= 52 \text{ kg dry fruits can be obtained}
 \end{aligned}$$

77. Let x be the number of students appeared for the examination in school A.

$$\therefore \text{Number of students qualified in school A} = 0.7x$$

$$\text{Number of students appeared in school B} = 1.2x$$

$$\text{Number of students qualified in school B} = 1.5 \times 0.7x = 1.05x$$

$$\begin{aligned}\therefore \% \text{ of students qualified to students appeared from school B} &= \frac{1.05x}{1.2x} \times 100 \\ &= \frac{3.5}{4} \times 100 = \mathbf{87.5\%}\end{aligned}$$

78. Students failed in Civics = $100 - 65 = 35\%$
 Students failed in History = $100 - 60 = 40\%$
 Students failed in Civics or History = $100 - 40 = 60\%$
 $\therefore$ Students failed in both = $35 + 40 - 60 = 15\%$

i.e. $15\% = 90$

$$\therefore 100\% = \frac{90}{15} \times 100 = \mathbf{600}$$

79. The percentage of people did not eat either type of lunch = ?

$$\text{Number of persons took vegetarian food} = \frac{60}{100} \times 96$$

$$V_n = \frac{288}{5}$$

$$\text{Number of persons took non-vegetarian} = \frac{30}{100} \times 96$$

$$N_v = \frac{144}{5}$$

$$V_n \cap N_v = \frac{15}{100} \times 96 = \frac{72}{5}$$

$$\begin{aligned}V_n \cup N_v &= V_n + N_v - (V_n \cap N_v) \\ &= \frac{288}{5} + \frac{144}{5} - \frac{72}{5} = \frac{360}{5} = 72\end{aligned}$$

$$\therefore \text{People did not took any type of food} = 96 - 72 = \mathbf{24}$$

80. % of students failed in Mathematics $F_M = 34\%$

$$\% \text{ of students passed in Mathematics } P_M = 66\%$$

$$\% \text{ of students failed in English } F_E = 42\%$$

$$\% \text{ of students passed in English } P_E = 58\%$$

$$\% \text{ of students failed in both subjects } F_B = 20\%$$

$$\% \text{ of students passed in both subjects } P_B = ?$$

$$\begin{aligned}\text{i.e. } P_B &= P_M + P_E - (100 - F_B) = 66 + 58 - (100 - 20) \\ &= 66 + 58 - 80 = \mathbf{44\%}\end{aligned}$$

81. Let the marks got for D be x .

$$\text{Marks for C} = 0.85x$$

$$\text{Marks for B} = 1.2 \times 0.85x$$

$$\text{Marks for A} = 1.2 \times 1.2 \times 0.85x$$

Given Marks for A = 576

$$\therefore 576 = 1.2 \times 1.2 \times 0.85x$$

$$\begin{aligned}\therefore x &= \frac{576}{1.2 \times 1.2 \times 0.85} = \frac{400}{0.85} \\ &= 400 \times \frac{100}{85} = 400 \times \frac{20}{17} = \frac{8000}{17}\end{aligned}$$

$$\therefore \% \text{ of marks} = \frac{8000}{17} \times \frac{1}{800} \times 100 = \frac{1000}{17} \% = \mathbf{58.82\%}$$

82. Let the monthly income of Deepak be x .

$\therefore$ Monthly income of Raunaq = $0.8x$

Monthly income of Amit = $1.3 \times (0.8x) = 1.04x$

Given $(1.04x - x) = 800$

or $0.04x = 800$

or $x = \frac{800}{0.04} = 20000$

$\therefore$ Monthly income of Raunaq = $0.8 \times 20000 = \mathbf{16000}$

83. Number of students in the school = 450

Number of students play neither football nor cricket = 50

$\therefore$ Number of students who play either football or cricket = $n(F \cup C) = 450 - 50 = 400$

Number of students who play football $n(F) = 325$

Number of students who play cricket $n(C) = 175$

Number of students who play both $n(F \cap C) = n(F) + n(C) - n(F \cup C)$
 $= 325 + 175 - 400 = \mathbf{100}$

84. Let the price of rice be ₹ x and quantity purchased be y kg.

$\therefore x \times y = 0.98x \times y_1$

Given $y = 49$

$\therefore x \times 49 = 0.98x \times y_1$

or $y_1 = \frac{49}{0.98} = \mathbf{50 \text{ kg}}$

85. At the time of first filling,

quantity of A-type petrol in the tank = 100%

At the time of second filling,

quantity of A-type petrol in the tank = 50%

quantity of B-type petrol in the tank = 50%

At the time of third filling,

quantity of A-type petrol in the tank = $25 + 50 = 75\%$

quantity of B-type petrol in the tank = 25%

At the time of fourth filling,

quantity of A-type petrol in the tank = $\frac{75}{2} \% = 37.5\%$

quantity of B-type petrol in the tank = $\frac{25}{2} + 50 = 62.5\%$

86. Quantity of milk in the container after third operation = $0.8 \times 0.8 \times 0.8 = 0.512 = \mathbf{51.2\% \text{ pure}}$

87. Let the price of egg = x

$\therefore$ Increased price = $1.3x$

$\therefore$ For ₹ 7.8, number of eggs available before increase in price = $\frac{7.8}{x}$

$\therefore$ Number of eggs available after increase in price = $\frac{7.8}{1.3x}$

$\therefore$ $\frac{7.8}{x} - \frac{7.8}{1.3x} = 3$

i.e. $\frac{7.8}{x} - \frac{6}{x} = 3$

$\Rightarrow (7.8 - 6) = 3 \times x$

or $x = \frac{7.8 - 6}{3} = 0.6$

$\therefore$ New rate/dozen = $12 \times 0.6 \times 1.3 = \text{₹ } 9.36$

88. $5 : 4 = \frac{5}{4} = \frac{5}{4} \times 100\% = 125\%$

89. $\frac{1}{2} \times 1\% = \frac{1}{2}\% = \frac{1}{200} = 0.005$

90. 15% of ₹ 34 = $\frac{15}{100} \times 34 = \text{₹ } 5.1$

91. Let x be the required percentage.

$x\%$ of 7.2 kg = 18 g

i.e. $\frac{x}{100} \times 7.2 \text{ kg} = 18 \text{ g}$

or $\frac{x}{100} \times 7200 \text{ g} = 18 \text{ g}$

or $x = \frac{18 \times 100}{7200} = \frac{1}{4}$

$= \frac{1}{4} \times 100\% = 25\%$

92. Required percentage = $\frac{3}{24} \times 100\% = 12\frac{1}{2}\%$

93. Actual price of the item = $25 + 2.50 = 27.50$

Savings on selling the item = $\frac{27.50 - 25}{25} \times 100\% = 10\%$

94. 5% of (25% of ₹ 1600) = $\frac{5}{100} \times \frac{25}{100} \times ₹ 1600 = \text{₹ } 20$

95. 0.15% of $33\frac{1}{3}\%$ of ₹ 10,000

$= \frac{0.15}{100} \times \frac{33\frac{1}{3}}{100} \times ₹ 10000$

$$= \frac{15}{10000} \times \frac{100}{300} \times ₹ 10000$$

$$= \frac{15}{3} = ₹ 5$$

96. Let the number be x

$$x - \frac{2}{5}x = 510$$

or $x \left(1 - \frac{2}{5}\right) = 510$

or $x = \frac{510 \times 5}{(5 - 2)}$

$\therefore$ 10% of number = $0.1x$

i.e. $0.1x = 0.1 \times \frac{510 \times 5}{3} = \frac{51 \times 5}{3} = 85$

97. Number that ends with '1' or '9' when squared we get the result ending with digit '1'.

Therefore, every 10 consecutive numbers consists of two such numbers whose square ends with digit '1'.

Here 1, 9, 11, 19, 21, 29, 31, 39, 41, 49, 51, 59, 61, 69 are these numbers.

$\therefore$ Percentage of such numbers = $\frac{14}{70} \times 100 = 20\%$

98. Let the first number be x and second number be y .

Then $x + y = \frac{28}{25} \times x$ (1)

or $y = \left(\frac{28}{25} - 1\right)x$

$$= \left(\frac{28 - 25}{25}\right)x = \frac{3x}{25}$$

$\therefore y = x \times \frac{3}{25} = x \times \frac{12}{100}$

i.e. second number is 12% of the first.

99. Let x be the total number of candidates appeared from each state A and B.

Number of candidates selected from state A = $\frac{6}{100}x$ (1)

Number of candidates selected from state B = $\frac{7}{100}x$ (2)

Given that $\frac{7x}{100} - \frac{6x}{100} = 80$

$\therefore (7 - 6)x = 80 \times 100$

or $x = 8000$

100. Given price of car = ₹ 325000

Insured amount = $0.85 \times ₹ 325000 = ₹ 276250$

Insurance claim amount = $0.90 \times 276250 = ₹ 248625$

∴ Difference between price and claim amount = $325000 - 248625 = ₹ 76375$

101. Percentage of total votes winning candidate got = $\frac{11628}{1136 + 7636 + 11628} \times 100$
 $= \frac{11628}{20400} \times 100 = \frac{11628}{204} = 57\%$

102. Let one number be x

∴ Second number = $0.8x$

Then $4[x^2 + (0.8x)^2] = 656$

or $x^2 + (0.8x)^2 = \frac{656}{4}$

or $x^2(1 + 0.64) = 164$

or $x^2 = 100$

or $x = 10$

∴ The numbers are 8 and 10.

103. Let x and y be the numbers.

$$\frac{x}{12} = \frac{1}{4}y$$

or $x = 3y$

∴ Required percentage = $\left(\frac{x - y}{y}\right) \times 100\% = \frac{(3y - y)}{y} \times 100\% = 200\%$

104. Let the numbers be x and y .

Then $y - \frac{25}{100}x = \frac{5}{6}y$

i.e. $y - \frac{5}{6}y = \frac{25}{100}x$

$$\frac{1}{6}y = \frac{1}{4}x$$

or $\frac{x}{y} = \frac{6}{4} = \frac{3}{2}$

∴ First number : Second number = $x : y = 2 : 3$

105. Let the cost of taxable item be ₹ x .

∴ Tax paid = $\frac{6}{100} \times x = ₹ \frac{6x}{100} = 30 \text{ paise}$

or $\frac{6x}{100} = \frac{30}{100}$

or $x = \frac{30}{6} = ₹ 5$

∴ Cost of tax free item = $(25 - 5 - 0.30) = ₹ 19.70$

106. Total score of batsman = 110 runs

$$\begin{aligned}\text{Score by running between wickets} &= 110 - 3 \text{ boundaries} - 8 \text{ sixes} \\ &= 110 - 3 \times 4 - 8 \times 6 = 50\end{aligned}$$

$$\therefore \text{Required percentage} = \frac{50}{110} \times 100 = 45 \frac{5}{11} \%$$

107. Let the number of apples originally with fruit seller be x .

$$x - \frac{40}{100} \times x = 420$$

$$\text{i.e.} \quad x \times \frac{60}{100} = 420$$

$$\text{or} \quad x = \frac{420 \times 100}{60} = \mathbf{700}$$

108. Let maximum marks be x .

$$\text{Pass mark required} = \frac{33x}{100} = 125 + 40 = 165$$

$$\text{i.e.} \quad \frac{33x}{100} = 165$$

$$\text{or} \quad x = \frac{165 \times 100}{33} = \mathbf{500}$$

109. Out of 100% voters, 10% voters did not cast votes. 10% votes were invalid

$$\text{Net votes polled} = 90\%$$

$$\therefore \text{Net valid votes polled} = 90\% \text{ of } 90\% = 81\%$$

$$\text{Vote polled for successful candidate} = \frac{54}{100} \times \frac{81}{100} = 43.74\%$$

$$\text{Majority by which candidate won} = \frac{81}{100} \times (54 - 46)\% = \frac{81}{100} \times \frac{8}{100} = 6.48\%$$

$$\text{Given majority} = 1620 = 6.48\% \text{ of total voters}$$

$$\therefore \text{Total number of voters} = 1620 \times \frac{100}{6.48} = \mathbf{25000}$$

110. Let total polled votes be x .

$$\text{Invalid votes} = 20\%x$$

$$\text{Votes successful candidate got} = \frac{55}{100} \times (x - 0.2x)$$

$$\text{Given } x = 7500$$

$$\therefore \text{Valid votes scored by failed candidate} = \frac{45}{100} (x - 0.2x) = \frac{45}{100} \times 0.8x$$

$$= \frac{45}{100} \times 0.8 \times 7500 = 45 \times 60 = \mathbf{2700}$$

111. In the examination, let total appeared candidates be x .

$$\text{Number of failed candidate} = 455$$

$$\therefore \quad \frac{65}{100}x = 455$$

$$\text{or} \quad x = \frac{455 \times 100}{65} = \mathbf{700}$$

112. $\left(100 - 66\frac{2}{3}\right)\%$ of person's income = 1200

i.e. $33\frac{1}{3}\%$ of person's income = 1200

$\therefore$ Expense = $66\frac{2}{3}\%$ of person's income = $2 \times 1200 = ₹ 2400$

113. Let total number of votes polled be x .

Winning candidate got $\frac{84}{100} \times x$ votes

Failed candidate got $\frac{16}{100} \times x$ votes

Given majority = $\frac{84 - 16}{100} \times x = 476$

or $\frac{68}{100}x = 476$

or $x = \frac{476 \times 100}{68} = 700$

114. $\frac{P}{100}$ of $P = 36$

or $\frac{P}{100} \times P = 36$

or $P^2 = 36 \times 100$

or $P = \sqrt{36 \times 100} = 60$

115. Given $\frac{x}{100} \times y = z$

or $x = \frac{z}{y} \times 100 = \frac{100}{y} \times z = \frac{100^2}{y} \% \text{ of } z$

116. Let the amount with Aman be x .

Amount paid to Rohan = $0.4x$

Amount paid to Sahil by Rohan = $\frac{1}{4}(0.4x)$

Amount left with Sahil after paying taxi fare of ₹ 200 = 600

i.e. $\frac{1}{4}(0.4x) - 200 = 600$

i.e. $0.1x = 600 + 200 = 800$

or $x = ₹ 8000$

117. Given height of pole = 192 m

Distance climbed in second hours = $12\frac{1}{2}\%$ of $\left(100 - 62\frac{1}{2}\right)\%$ of 192

$= \frac{12.5}{100} \times \frac{37.5}{100} \times 192$

$$= \frac{25}{200} \times \frac{75}{200} \times 192$$

$$= \frac{1}{8} \times \frac{3}{8} \times 192 = 9 \text{ m}$$

118. Number of literate inhabitant male = $1000 \times \frac{60}{100} \times \frac{20}{100} = 120$

Number of literate inhabitants in total = $1000 \times \frac{25}{100} = 250$

$\therefore$ Number of literate female inhabitants = $250 - 120 = 130$

$\therefore$ Percentage of females who are literate = $\frac{130}{\frac{40}{100} \times 1000} \times 100$

i.e. Required percentage = $\frac{130}{400} \times 100 = 32\frac{1}{2}\%$

119. A house with two or more people

B houses with only a male

C house with only one female and no male

Let number of houses in total be x .

$$n(A) = 40\%$$

$$n(B) = 25\% \times 60\% = 15\%$$

$$n(C) = ?$$

$$n(A) + n(B) + n(C) = 100$$

$$= 100 - 40 - 15 = 45\%$$



Short cut: Number of houses with only one female

$$= (100 - 25)\% \text{ of } (100 - 40)\% \text{ of } x$$

$$= 75\% \times 60\% \times x$$

$$= \frac{75}{100} \times \frac{60}{100} \times x = \frac{3}{4} \times \frac{3}{5} x = \frac{9}{20} x$$

$$\text{Required maximum percentage} = \left(\frac{9}{20} x \times \frac{1}{x} \times 100 \right) \% = 45\%$$

120. Let x be maximum marks.

$\therefore$ Pass mark required = $\frac{30}{100}x + 15 = \frac{40}{100}x - 35$

Solving, we get,

$$\left(\frac{40 - 30}{100} \right) x = 15 + 35 = 50$$

or

$$x = \frac{50 \times 100}{10} = 500$$

$$\begin{aligned}\therefore \text{Required pass percentage} &= \frac{\frac{30}{100}(500) + 15}{500} \times 100 \\ &= \frac{150 + 15}{100} \times 100 = \mathbf{33\%}\end{aligned}$$

121. Average score in the class = 80 marks

Let x be the required average

$$\text{Average} = \frac{10}{100} \times 95 + \frac{20}{100} \times 90 + \frac{70}{100} \times x = 80$$

$$\text{i.e.} \quad \frac{70x}{100} = 80 - 18 - 9.5 = 52.5$$

$$\text{or} \quad x = \frac{52.5 \times 100}{70} = \frac{3 \times 100}{4} = \mathbf{75 \text{ marks}}$$

122. For pass required percentage = 35%

Let x be the required marks in third paper to pass.

$$\text{Given, score for 1st paper} = \frac{62}{150}$$

$$\text{Score for 2nd paper} = \frac{35}{150}$$

$$\text{Score for 3rd paper} = \frac{x}{180}$$

$$\therefore \frac{62 + 35 + x}{150 + 150 + 180} \times 100 = 35\%$$

$$\frac{97 + x}{480} \times 100 = 35\%$$

$$\text{or} \quad 97 + x = \frac{480 \times 35}{100}$$

$$\begin{aligned}\text{or} \quad x &= \frac{480 \times 35}{100} - 97 \\ &= 24 \times 7 - 97 = 168 - 97 = \mathbf{71}\end{aligned}$$

123. Let the number of girls be y .

$$x + y = 150$$

(1)

$$\text{Then} \quad y = \frac{x}{100} \times 150 = 1.5x$$

$$\therefore (1) \Rightarrow x + 1.5x = 150$$

$$\text{or} \quad 2.5x = 150$$

$$\text{or} \quad x = \frac{150}{2.5} = \mathbf{60}$$

i.e. Number of boys = **60**

124. Let total sales be for ₹ x .

$$\therefore \text{Total earnings of salesmen} = \frac{5 \frac{1}{2}}{100} \times x + \frac{\frac{1}{2}}{100} \times (x - 10000) = 1990$$

$$\begin{aligned}
 \text{i.e.} \quad & \frac{5.5x}{100} + \frac{x - 10000}{200} = 1990 \\
 \Rightarrow & \frac{11x + x - 10000}{200} = 1990 \\
 \Rightarrow & 12x - 10000 = 1990 \times 200 \\
 \Rightarrow & 12x = 398000 + 10000 = 408000 \\
 \Rightarrow & x = \frac{408000}{12} = \mathbf{34000}
 \end{aligned}$$

125. Let earning by Michael be x .

$\therefore$ Earning of Albert = $0.8x$

Earning of Peter = $1.4 \times 0.8x = 1.12x$

$\therefore$ Peter earned more than Michael by **12%**

126. Let C's salary be x .

$\therefore$ B's salary = $\frac{25}{100} \times x$

$\therefore$ A's salary = $\frac{40}{100} \times \frac{25}{100} \times x$

$= \frac{x}{10} = \mathbf{10\%}$ of C's salary

127. Let the original quantity of ghee be x kg.

$\therefore$ Pure ghee = $0.6x$

Vanspati = $0.4x$

After addition of 10 kg of pure ghee

New quantity of pure ghee = $0.6x + 10$

New quantity of Vanaspati = $0.4x$

Now, $\frac{0.4x}{x + 10} \times 100 = 20$

or $\frac{0.4x \times 5}{x + 10} = 1$

or $2x = x + 10$

or $x = \mathbf{10 \text{ kg}}$

128. Let the third number be x .

$\therefore$ First number = $\frac{12.5}{100}x$

Second number = $\frac{25}{100}x$

$\therefore$ Required percentage = $\frac{12.5x/100}{25x/100} \times 100 = \mathbf{50\%}$

129. Let x be the third number.

First number = $(1 - 0.3)x = 0.7x$

Second number = $(1 - 0.37)x = 0.63x$

$$\begin{aligned}\therefore \text{ Required percentage} &= \frac{(0.7 - 0.63)x}{0.7x} \times 100 \\ &= \frac{0.07}{0.7} \times 100 = \mathbf{10\%}\end{aligned}$$

130. Given A's height = $\left(1 - \frac{40}{100}\right)$ B's height

$$\therefore \text{ B's height} = \frac{100}{60} \times \text{A's height}$$

$$\therefore \text{ Required percentage} = \frac{100 - 60}{60} \times 100 = \mathbf{66\frac{2}{3}\%}$$

131. A's salary is 50% more than B's

$$\therefore \text{ B's salary is less than A's by } \left[\frac{50}{100 + 50} \times 100 \right] \% = \frac{100}{3} = \mathbf{33\frac{1}{3}\%}$$

132. Let the population of town be P .

$$\text{Net increase of population per annum} = 4 - \frac{1}{2} = 3\frac{1}{2}\%$$

$$\therefore \text{ Population after 3 years} = P \left(1 + \frac{3\frac{1}{2}}{100} \right)^3$$

$$\therefore \text{ Increase percentage} = \frac{P \left(1 + \frac{3\frac{1}{2}}{100} \right)^3 - P}{P} \times 100$$

$$\begin{aligned}&= \left[\left(1 + \frac{3\frac{1}{2}}{100} \right)^3 - 1^3 \right] \times 100 \\ &= \left[\frac{207}{200} \times \frac{207}{200} \times \frac{207}{200} - 1 \right] \times 100 \\ &= \left[\frac{207^3}{200^3} - 1 \right] \times 100 \\ &= [1.1087 - 1] \times 100 = 10.87\% = \mathbf{10.8\%}\end{aligned}$$

133. Total population $P = 5000$

Let number of males be M .

$$\therefore \text{ Number of females} = 5000 - M$$

$$\text{New number of males} = M \times \left(1 + \frac{10}{100} \right) = 1.1 M$$

$$\text{New number of females} = (5000 - M) \left(1 + \frac{15}{100} \right) = (5000 - M) 1.15$$

$$\text{Total population} = 1.1M + (5000 - M) 1.15 = 5600$$

$$\text{i.e. } 5750 + (1.1 - 1.15) M = 5600$$

$$\text{i.e. } (1.15 - 1.1) M = 5750 - 5600$$

$$0.05 M = 150$$

$$\therefore M = \frac{150}{0.05} = \frac{30}{0.01} = \mathbf{3000}$$

134. Let the value of machine be x .

$$\begin{aligned} \text{Value after 3 years of depreciation is 20\% per annum} &= x \left(1 - \frac{R}{100} \right)^N \\ &= x \left(1 - \frac{20}{100} \right)^3 \\ &= x \times \frac{80}{100} \times \frac{80}{100} \times \frac{80}{100} \\ &= x \times \frac{512}{1000} = 0.512x \end{aligned}$$

i.e. value is less by **48.8%**

135. Population 2 years ago = 62500

Decrease in population @ 4% per annum

$$\begin{aligned} \therefore \text{Present population} &= 62500 \left(1 - \frac{4}{100} \right)^2 \\ &= 62500 \left(\frac{96}{100} \right)^2 = 62500 \left(\frac{24}{25} \right)^2 \\ &= 100 \times (24)^2 = \mathbf{57600} \end{aligned}$$

136. Present population of district = 64000

$$\begin{aligned} \text{Population after 3 years with its increase @ } 2\frac{1}{2}\% \text{ per annum} &= 64000 \left(1 + \frac{2\frac{1}{2}}{100} \right)^3 \\ &= 64000 \times \left(1 + \frac{10}{400} \right)^3 = 64000 \times \left(\frac{410}{400} \right)^3 \\ &= \frac{64,000 \times (410)^3}{64,00,0000} = (41)^3 = \mathbf{68921} \end{aligned}$$

137. Value of a machine at present after depreciation @ 10% per annum for 3 years = ₹ 8784

Let x be the purchase price

$$\therefore x \left(1 - \frac{10}{100} \right)^3 = 8748$$

$$\begin{aligned}
 &= 8748 \times \frac{100^3}{90^3} = 8748 \times \frac{10^3}{9^3} \\
 &= 972 \times \frac{10^3}{9^2} = 12 \times 1000 = \text{₹ } 12000
 \end{aligned}$$

138. If inflation increases @ 8% per annum, the cost of article worth ₹ 20, at present will be having a value after 2 years equal to $20 \times \left(1 + \frac{8}{100}\right)^2$.

$$20 \times \left(\frac{108}{100}\right)^2 = 20 \times \left(\frac{27}{25}\right)^2 = 27^2 \times \frac{4}{125} = \frac{729 \times 4}{125} = 23.328$$

Its value will be between ₹ 23 and ₹ 24.

139. Population 3 years ago = 160000

$$\begin{aligned}
 \text{Present population} &= 160000 \times \left(1 + \frac{3}{100}\right) \left(1 + \frac{2.5}{100}\right) \left(1 + \frac{5}{100}\right) \\
 &= 160000 \times \left(\frac{103}{100}\right) \left(\frac{102.5}{100}\right) \left(\frac{105}{100}\right) \\
 &= 16 \times 103 \times 102.5 \times \frac{21}{20} = 16 \times 103 \times \frac{205}{2} \times \frac{21}{20} \\
 &= 4 \times 103 \times 205 \times \frac{21}{10} = 177366
 \end{aligned}$$

140. Let the population in 1998 be P .

$$\begin{aligned}
 \therefore P \left(1 + \frac{5}{100}\right)^3 &= 138915 \\
 \therefore P &= 138915 \times \frac{100^3}{105^3} \\
 &= 138915 \times \frac{20^3}{21^3} = 6615 \times \frac{20^3}{21^2} \\
 &= 315 \times \frac{20^3}{21} = 15 \times 20^3 = 120000
 \end{aligned}$$

141. Let production of a company last year be P .

∴ For 4 years from 1998 to 2002, resultant effect in production

$$\begin{aligned}
 &= P \left(1 + \frac{15}{100}\right) \left(1 + \frac{15}{100}\right) \left(1 - \frac{10}{100}\right) \left(1 + \frac{15}{100}\right) \\
 &= P \times \frac{115}{100} \times \frac{115}{100} \times \frac{90}{100} \times \frac{115}{100} \\
 &= P \times \frac{23}{20} \times \frac{23}{20} \times \frac{9}{10} \times \frac{23}{20} \\
 &= P \times \frac{12149 \times 9}{80000} = P \times \frac{109341}{80000} = P \times 1.3667625
 \end{aligned}$$

∴ Resultant effect = 36.68% Increase = 37% Increase

142. In January, number of ticketless travellers = 4000

$$\text{In February, number of ticketless travellers} = 4000 \times \left(1 + \frac{5}{100}\right) = 4200$$

$$\text{In March, number of ticketless travellers} = 4200 \times \left(1 - \frac{5}{100}\right) = 3990$$

$$\text{In April, number of ticketless travellers} = 3990 \times \left(1 - \frac{10}{100}\right) = \mathbf{3591}$$

143. Let the number of bushes at beginning be x .

$$\therefore x \left(1 + \frac{10}{100}\right) \left(1 + \frac{8}{100}\right) \left(1 - \frac{10}{100}\right) = 26730$$

$$\begin{aligned} \therefore x &= \frac{26730}{110 \times 108 \times 90} \times 100^3 \\ &= \frac{26730}{11 \times 108 \times 9} \times 100^2 = \frac{2430}{108 \times 9} \times 100^2 \\ &= \frac{270}{108} \times 100^2 = \frac{5}{2} \times 100^2 = \mathbf{25000} \end{aligned}$$

144. Alcohol content in 9 ml shaving lotion of 50% concentration = $9(0.5) = 4.5$ ml

To reduce it to 30% concentration, water required = ?

In 30% lotion, alcohol content = 4.5 ml

i.e. 30% quantity = 4.5 ml

$$\therefore 100\% \text{ quantity} = \frac{4.5}{30} \times 100 = 15 \text{ ml}$$

$\therefore$ Quantity of water in the mixture = $15 - 4.5 = 10.5$ ml

Quantity of water now added = $15 - 9 = \mathbf{6}$ ml

145. $600 + 0.25 + 1200 \times 0.5 = 150 + 600 = \text{₹ } 750$

$$\text{Amount removed} = \frac{12}{100} (150) + \frac{24}{100} (600) = 18 + 144 = \text{₹ } 162$$

$$\begin{aligned} \therefore \text{Percentage of money removed} &= \frac{162}{750} \times 100 \\ &= \frac{5.4}{250} \times 100 = \frac{216}{1000} \times 100 = 21.6\% \end{aligned}$$

Average

Average can be defined as the ratio of sum of given data to the number of data. For example, if $x_1, x_2, x_3, \dots, x_n$ are n different values of a variable, then the average value of that variable is given by $\frac{x_1 + x_2 + x_3 + \dots + x_n}{n}$.



Note: If a group of n people is having an average height of h and if a new person joined and the average height of people in the group is increased by x , then the height of the new person is given by

New average + Number of old members $\times$ Increase in average

i.e. $h = x + n \times x$

i.e. $h = (n + 1)x$

Age of new entrant = New average + Number of old members $\times$ Change in average

If average is increased, change is +ve, else it is -ve.

When a person of known height is replaced (from a group) by another person and the average height of the group of person is increased by x , then the height of new person is given by

Height of new person = Height of replaced person + Number of persons
 $\times$ Increase in average

EXAMPLE 1 The average age of 25 boys in a class is equal to 16. When a new boy was admitted to the class, the average age got decreased by one month. Find the age of new boy admitted.

Solution Total age of 25 boys = $25 \times 16 = 400$ years

When a new boy was admitted,

Total age of 26 boys = 26×15 years 11 months
= **413 years 10 months**

Age of new entrant = 413 years 10 months – 400 years
= **13 years 10 months**



Short cut: Age of new entrant = New average + Number of old members $\times$ Change in average
 $= 15 \text{ years } 11 \text{ months} + 25 (-1 \text{ month})$
 $= \mathbf{13 \text{ years } 10 \text{ months}}$

EXAMPLE 2 Average weight of 8 men is increased by 2 kg when one of them who weighs 130 kg is replaced by another man. What is the weight of the new man?

Solution Average weight is increased by 2 kg.

$\therefore$ Total weight is increased by $8 \times 2 = 16 \text{ kg}$

This is due to the extra weight of person included at present.

$\therefore$ Weight of new man = $130 + 16 = \mathbf{146 \text{ kg}}$



Short cut: The weight of new person = Weight of replaced person + Number of persons $\times$ Change in average
 $= 130 + 8 \times 2 = \mathbf{146 \text{ kg}}$

EXAMPLE 3 The average of 15 results is 60. If the average of first eight results is 58 and that of last eight is 61, find the eighth result.

Solution Sum of 15 results = $15 \times 60 = 900$

Sum of first 8 results = $8 \times 58 = 464$

Sum of last 8 results = $8 \times 61 = 488$

Eighth result = $464 + 488 - 900 = 952 - 900 = \mathbf{52}$



Short cut: Eighth result = $60 + (61 - 60)8 + (58 - 60)8$
 $= 60 + 1 \times 8 + (-2) \times 8$
 $= 60 + 8 - 16 = \mathbf{52}$

EXAMPLE 4 The average age of three members of a family was 24 five years ago. At present the average age of the family including a new member is 22. Find the age of the new member.

Solution Total age of family members 5 years before = $24 \times 3 = 72 \text{ years}$

Total age of family members at present = $(24 + 5) \times 3 = 87 \text{ years}$

Total age of family members including a new member at present = $22 \times 4 = 88 \text{ years}$

$\therefore$ Age of new member = $88 - 87 = \mathbf{1 \text{ year}}$



Short cut: Age of new member = New average + Old number of members $\times$ Change in average
 $= 22 + 3 \times (-7) = \mathbf{1 \text{ year}}$

EXAMPLE 5 The average of 15 results is 50, that of the first seven is 45 and that of last seven is 48. Find the value of the 8th number.

Solution 8th number = Sum of 15 results – (Sum of first 7 results + Sum of last 7 results)
 $= 15 \times 50 - (7 \times 45 + 7 \times 48)$
 $= 750 - (7 \times 93) = 750 - 651 = 99$



Short cut: 8th number = $50 + (50 - 45)7 + (50 - 48)7 = 50 + 35 + 14 = 99$

EXAMPLE 6 The average age of 16 boys in a class is decreased by 3 months, when a boy aged 17 years is replaced by a new boy. Find the age of new boy.

Solution Age of new boy = Age of replaced boy + Number of boys $\times$ Change in average

$$= 17 + 16 \times \left(\frac{-3}{12} \right) = 17 - 4 = 13 \text{ years}$$

RULES

Average Speed

Rule 1: If a man travels a certain distance at a speed of x km/h and the same distance at a speed of y km/h, then the average speed during the whole journey is given by $\frac{2xy}{x+y}$ km/h.

Proof: Let the distance travelled by the man be S km.

$\therefore$ Time taken for the first journey at a speed of x km/h = $\frac{S}{x}$ h

Time taken for the second journey at a speed of y km/h = $\frac{S}{y}$ h

$\therefore$ Total time take for the whole journey = $\frac{S}{x} + \frac{S}{y}$ h

Total distance travelled = $S + S = 2S$

$$\begin{aligned} \text{Average speed} &= \frac{\text{Total distance travelled}}{\text{Total time taken}} \\ &= \frac{2S}{S/x + S/y} = \frac{2S \times xy}{S \times y + S \times x} \\ &= \frac{S \times 2xy}{S \times (x + y)} = \frac{2xy}{x + y} \text{ km/h} \end{aligned}$$

Rule 2: If a man travels three equal distances at a speed of x km/h, y km/h and z km/h respectively, then the average speed of the man for the whole journey is given by $\frac{3xyz}{xy + yz + zx}$ km/h.

Proof: Let the three equal distances be S km.

Time taken for travelling the first distance = $\frac{S}{x}$ h

Time taken for travelling the second distance = $\frac{S}{y}$ h

Time taken for travelling the second distance = $\frac{S}{y}$ h

$\therefore$ Total distance travelled = $S + S + S = 3S$ km

Total time taken = $\frac{S}{x} + \frac{S}{y} + \frac{S}{z}$ h

$\therefore$ Average speed = $\frac{\text{Total distance travelled}}{\text{Total time taken}} = \frac{3S}{\left(\frac{S}{x} + \frac{S}{y} + \frac{S}{z}\right)}$
 $= \frac{3xyz}{yz + xz + xy}$ km/h

EXAMPLE 7 A person travels from a place P to Q at the speed of 40 km/h and returns at a speed of 10 km/h. What is the average speed of the person for the whole journey?

Solution Average speed = $\frac{2 \times 40 \times 10}{40 + 10} = 16$ km/h

EXAMPLE 8 A train travels three equal distances at a speed of 80 km/h and 90 km/h and 120 km/h respectively. Find the average speed of the train for the whole journey.

Solution Average speed = $\frac{3 \times 80 \times 90 \times 120}{80 \times 90 + 90 \times 120 + 120 \times 80}$
 $= \frac{3 \times 8 \times 9 \times 12 \times 1000}{100(72 + 108 + 96)} = 93\frac{21}{23}$ km/h

EXAMPLE 9 In an office, there are officers and workers. The average salary of officers and workers is ₹ 2000. The average salary of officers alone is ₹ 5500 and that of workers is ₹ 1900. If the number of officers is 16, find the number of workers in the office.

Solution Let the required number of workers be x .

Then

$$1900x + 5500 \times 16 = 2000(16 + x)$$

$$1900x - 2000x = 2000 \times 16 - 5500 \times 16$$

$$-100x = -3500 \times 16$$

$$\therefore x = \frac{3500 \times 16}{100} = 560$$



Short cut:

$$\begin{aligned} \text{Number of workers} &= \text{Number of officers} \times \frac{(\text{Average salary of officers} - \text{Mean average})}{(\text{Mean average} - \text{Average salary of workers})} \\ &= 16 \times \left(\frac{5500 - 2000}{2000 - 1900} \right) \\ &= 16 \times \frac{3500}{100} = 16 \times 35 = 560 \end{aligned}$$

EXAMPLE 10 The average of ten two-digit numbers is calculated. While calculating the average one number was mistakenly reversed and the average got reduced by 2.7. If the sum of the digits of the reversed number is 11, find the number.

Solution Let the number be xy .

$$\therefore \text{Value of number} = 10 \times x + 1 \times y$$

$$\therefore \text{Reversed number} = yx$$

$$\therefore \text{Value of reversed number} = 10 \times y + 1 \times x$$

$$\therefore \text{Difference of number and its reverse}$$

$$= (10x + y) - (10y + x)$$

$$= 10(x - y) + (y - x)$$

$$= (10 - 1)(x - y) = 9(x - y)$$

i.e. difference of number and its reverse is a multiple of 9.

In this problem, the given difference of number and its reverse $= 2.7 \times 10 = 27$

$$\text{i.e.} \quad 9(x - y) = 27$$

$$\therefore x - y = 3 \quad (1)$$

Sum of the digits is 11.

$$\text{i.e.} \quad x + y = 11 \quad (2)$$

Solving Eqs. (1) and (2), we get

$$(1) + (2) \Rightarrow 2x = 14 \quad \text{or} \quad x = 7$$

$$(1) - (2) \Rightarrow 2y = 8 \quad \text{or} \quad y = 4$$

$\therefore$ The required number is 74.



Short cut: Difference of digits = $\frac{\text{Difference of number and its reverse}}{9}$

The sum of digits will be given.

If $x_1, x_2, x_3, \dots, x_n$ are in Harmonic Progression (H.P.), their reciprocals will be in Arithmetic Progression (A.P.).

i.e. $\frac{1}{x_1}, \frac{1}{x_2}, \frac{1}{x_3}, \dots, \frac{1}{x_n}$ are in A.P.

$$\therefore \text{Arithmetic mean (A.M.)} = \frac{1}{n} \left[\frac{1}{x_1} + \frac{1}{x_2} + \dots + \frac{1}{x_n} \right]$$

$$\text{Harmonic Mean (H.M.)} = \frac{1}{\frac{1}{n} \left(\frac{1}{x_1} + \frac{1}{x_2} + \dots + \frac{1}{x_n} \right)}$$

EXAMPLE 11 A man goes to a place at the rate of 4 km/h. He comes back on a bicycle at 16 km/h. What is his average speed for the complete to and fro journey?

$$\text{Solution} \quad \text{Average speed} = \frac{2xy}{x + y} = \frac{2 \times 4 \times 16}{4 + 16} = \frac{32}{5} = 6.4 \text{ km/h}$$



Important Formulae

Average of consecutive natural numbers upto $n = \frac{(n+1)}{2}$

Average of squares of all natural numbers till $n = \frac{(n+1)(2n+1)}{6}$

Average of cubes of first n natural numbers = $\frac{n(n+1)^2}{4}$

Average of first n consecutive odd numbers = n

Average of squares of all consecutive odd numbers till $n = \frac{n(n+1)}{3}$

Average of squares of all consecutive even numbers till $n = \frac{(n+1)(n+2)}{3}$



Note: Average of any n consecutive odd numbers

or

Average of any n consecutive even numbers = First term + $(n-1)$



Short cut: When a new member joins a group without replacing any one,

Age of new person = Initial average of group + New number of members in the group
× Change in average

When a person leaves a group without any replacement being made, the age of the separated person
= Initial average – New number of persons × Change in average

EXAMPLE 12 If a person covers X km at x km/h and Y km at y km/h and Z km at z km/h, find his average speed in covering the complete distance.

Solution Average speed = $\frac{\text{Total distance covered}}{\text{Total time}} = \frac{X + Y + Z}{\left(\frac{X}{x} + \frac{Y}{y} + \frac{Z}{z}\right)} \text{ km/h}$

EXAMPLE 13 If a person covers A th part of the distance at x km/h, B th part of the distance at y km/h and the remaining C th part at z km/h, find his average speed.

Solution Let the total distance covered be D km.

Time taken to cover A th part (i.e. $A \times D$ km) = $\frac{AD}{x}$ h

Time taken to cover B th part (i.e. $B \times D$ km) = $\frac{BD}{y}$ h

Time taken to cover C th part (i.e. $C \times D$ km) = $\frac{CD}{z}$ h

Total time taken = $\left(\frac{AD}{x} + \frac{BD}{y} + \frac{CD}{z}\right)$ h

$$\begin{aligned}
 &= D \left(\frac{A}{x} + \frac{B}{y} + \frac{C}{z} \right) h \\
 \text{Average speed} &= \frac{D}{D \left(\frac{A}{x} + \frac{B}{y} + \frac{C}{z} \right)} \\
 &= \frac{1}{\frac{A}{x} + \frac{B}{y} + \frac{C}{z}} \text{ km/h}
 \end{aligned}$$



Short cut: If A, B, C are given as percentage, instead of part, expression change to

$$\text{Average speed} = \frac{100}{\left(\frac{A}{x} + \frac{B}{y} + \frac{C}{z} \right)} \text{ km/h}$$

EXAMPLE 14 A batsman makes a score of 82 runs in the 17th inning and thus increases his average by 2. Find his average after 17th inning.

Solution Let his average after 17th inning be x

$$\therefore \text{Average after 16th inning} = (x - 2)$$

$$\therefore 16(x - 2) + 82 = 17x$$

$$\text{or} \quad x = 82 - 32 = 50$$

EXAMPLE 15 Among the given three numbers, the second is twice the first and is also thrice the third. If the average of three numbers is 55, find the largest number.

Solution Let the first number be x

$$\begin{aligned}
 \therefore \text{Second number} &= 2x \\
 &= 3 \times \text{third number}
 \end{aligned}$$

$$\text{or} \quad \text{third number} = \frac{2x}{3}$$

$$\therefore x + 2x + \frac{2x}{3} = 55 \times 3 = 165$$

$$\text{i.e.} \quad \frac{(3x + 6x + 2x)}{3} = 165$$

$$\text{or} \quad 11x = 165 \times 3$$

$$\text{or} \quad x = \frac{165 \times 3}{11} = 45$$

$$\therefore \text{Largest number} = \text{second number} = 2x = 2 \times 45 = 90$$

EXAMPLE 16 Average of 11 results is 65. If the average of first six results is 63 and the last six results is 67, find the sixth result.

$$\begin{aligned}
 \text{Solution} \quad \text{Sixth result} &= (6 \times 63 + 6 \times 67 - 11 \times 65) \\
 &= 378 + 402 - 715 = 65
 \end{aligned}$$

EXAMPLE 17 The average weight of P , Q and R is 50 kg. If average weight of P and Q is 55 kg and that of Q and R is 43 kg, find the weight of Q .

Solution Let p , q and r be individual weight

$$\therefore p + q = 2 \times 55 = 110 \text{ kg} \quad (1)$$

$$q + r = 2 \times 43 = 86 \text{ kg} \quad (2)$$

$$p + q + r = 3 \times 50 = 150 \text{ kg} \quad (3)$$

$$(2) + (1) - (3) \Rightarrow (p + q) + (q + r) - (p + q + r) = 110 + 86 - 150 \text{ kg}$$

$$\text{i.e. } q = 46 \text{ kg}$$

EXAMPLE 18 A class consists of 36 girls and 44 boys. If the average weight of girls is 40 kg and that of boys is 35 kg, find the average weight of all students in the class?

Solution Total weight of students = $36 \times 40 + 44 \times 35 = 2980 \text{ kg}$

$$\text{Average weight of whole class} = \frac{2980}{36 + 44} = \frac{2980}{80} = 37.25 \text{ kg}$$

EXAMPLE 19 While entering marks scored by students of a class, mark of one student was mistakenly entered as 83 instead of 63. So the average marks for the class got increased by half. The number of students in the class is

Solution Let n be the number of students in the class

$$\text{Change in average} = \frac{1}{2} = \frac{83 - 63}{N} = \frac{20}{N}$$

$$\therefore N = 40$$



EXERCISES

- In an examination the average marks was found to be 60. The marks of 80 candidates had to be changed from 95 to 70 each and the average came down to 55 marks. The total number of candidates attended the examination were
(a) 250 (b) 300 (c) 400 (d) 450 (e) None of these
- The average age of a group of 15 students is 10 years. When 5 more students joined the group, the average got increased by 1 year. The age of the new student is
(a) 18 years (b) 16 years (c) 14 years (d) 12 years (e) None of these
- A person travels $\frac{1}{3}$ of his journey by bus at 60 km/h, $\frac{1}{3}$ by scooter at 30 km/h and the rest by walking at 10 km/h. Find his average speed for the whole journey.
(a) 30 km/h (b) $33\frac{1}{3}$ km/h (c) 20 km/h (d) 50 km/h (e) None of these
- The average speed of a train in the onward journey is 25% less than that of the return journey. The train halts for one hour on reaching the destination. The total time taken for the complete to and fro journey is 17 hours, covering a distance of 800 km. The speed of the train in the onward journey is [IAS]
(a) 45 km/h (b) 47.06 km/h (c) 43.75 km/h (d) 56.25 km/h
- The average of X_1 , X_2 and X_3 is 14. Twice the sum of X_2 and X_3 is 30. What is the value of X_1 ? [IAS]
(a) 20 (b) 27 (c) 16 (d) 12

6. A person travelled from one place to another at an average speed of 40 km/h and back to the original place at an average speed of 50 km/h. What is his average speed in km/h during the entire trip? [IAS]
(a) 45 (b) $20\sqrt{5}$ (c) $400/9$
(d) Impossible to find out unless the distance between the two place is known.
7. The average age of a cricket team reduces by 2 when two senior players aged 32 and 30 are replaced by 2 youngsters of equal age. What is the age of these players? [IAS]
(a) 20 (b) 22 (c) 24 (d) 26
8. The average height of the students of a class of 20 is 105 cm. If ten more students of average height of 120 cm join this class, the average height of the class will become
(a) 117.5 cm (b) 115 cm (c) 110 cm (d) 112.5 cm (e) None of these
9. The average of first 46 natural number is
(a) 23 (b) 23.5 (c) 24 (d) 25 (e) None of these
10. The average of 13 results is 50. If the average of first 7 results is 52 and that of last 7 results is 49, find the 7th result.
(a) 50 (b) 52 (c) 56 (d) 57 (e) None of these
11. The average age of a group of 18 persons is 32 years and 6 months. If two persons of age 65 years each and one person of age 50 years leave the group, the average age of the remaining persons in the group will be
(a) 26 years (b) 27 years (c) 28 years (d) 29 years (e) None of these
12. The average marks of a student obtained in Physics, Chemistry and Biology is 70. If the student obtained 90 marks in Mathematics, then the new average is
(a) 80 (b) 78 (c) 76 (d) 75 (e) None of these
13. The average weight of 3 person is 65 kg. If a fourth person having a weight of 45 kg joins the group, what is the average weight of the group?
(a) 65 kg (b) 63 kg (c) 61 kg (d) 60 kg (e) None of these
14. A man covers $\frac{1}{3}$ of his journey by train at 80 km/h, next $\frac{1}{3}$ by 60 km/h and the rest by bicycle at 30 km/h. Find the average speed of the man during the whole journey.
(a) 65 km/h (b) 45 km/h (c) 48 km/h (d) $53\frac{1}{3}$ km/h (e) None of these
15. The average weight of 6 persons is increased by 5 kg when one of them who weighs 40 kg is replaced by a new man. The weight of new man is
(a) 76 kg (b) 74 kg (c) 72 kg (d) 71 kg (e) None of these
16. The average age of a group of 9 students is 20 years. When 6 more students joined the group, the average age falls by 2 years. Then the average age of new students is
(a) 20 years (b) 18 years (c) 19 years (d) 16 years (e) None of these
17. A batch of 60 students made an average score of 50 marks and another batch of 40 students made it only 45. What is the overall average score?
(a) 49 (b) 48 (c) 47 (d) 46 (e) None of these
18. A man travels by a car to his office at 60 km/h and returns home along the same route at 20 km/h. Find the average speed of his whole journey.
(a) 40 km/h (b) 50 km/h (c) 30 km/h (d) 25 km/h (e) None of these
19. The arithmetic mean of the scores of a group of students in a test was 52. The brightest 20% of them secured a mean score of 80 and the duller 25% a mean score of 31. The mean score of remaining 55% is [SSC]
(a) 45 (b) 50 (c) 51.4 approx (d) 54.6 approx.

20. 10 years ago, the average age of a family of 4 members was 24 years. Two children having been born (with age difference of 2 years), the present average age of the family is the same. The present age of the youngest child is [SSC]
 (a) 1 year (b) 2 years (c) 3 years (d) 5 years
21. The average weight of 3 men A, B and C is 84 kg. Another man D joins the group and the average now becomes 80 kg. If another man E, whose weight is 3 kg more than that of D, replaces A, then the average weight of B, C, D and E becomes 79 kg. The weight of A is [Bank PO]
 (a) 70 kg (b) 72 kg (c) 75 kg (d) 80 kg
22. A cricketer has a certain average for 10 innings. In the eleventh inning, he scored 108 runs, thereby increasing his average by 6 runs. His new average is [AAO Exam]
 (a) 48 runs (b) 52 runs (c) 55 runs (d) 60 runs
23. The average age of 8 men is increased by 2 years when two of them whose ages are 21 years and 23 years are replaced by two new men. The average age of the two new men is [SSC]
 (a) 22 years (b) 24 years (c) 28 years (d) 30 years
24. The average weight of 45 students in a class is 52 kg. Five of them whose average weight is 48 kg leave the class and other 5 students whose average weight is 54 kg join the class. What is the new average weight (in kg) of the class [RRB]
 (a) $52\frac{1}{3}$ (b) $52\frac{1}{2}$ (c) $52\frac{2}{3}$ (d) None of these
25. The average age of 36 students in a group is 14 years. When teacher's age is included to it, the average increases by one what is the teacher's age in years? [RBI]
 (a) 31 (b) 36 (c) 51
 (d) Cannot be determined (e) None of these
26. The average weight of three boys A, B and C is $54\frac{1}{3}$ kg, while the average weight of three boys B, D and E is 53 kg. What is the average weight of A, B, C, D and E? [SBI PO]
 (a) 52.4 kg (b) 53.2 kg (c) 53.8 kg
 (d) Data inadequate (e) None of these
27. The mean of 50 observations was 36. It was found later that an observation 48 was wrongly taken as 23. The corrected new mean is [SSC]
 (a) 35.2 (b) 36.1 (c) 36.5 (d) 39.1
28. Of the three numbers, the first is twice the second and the second is twice the third. The average of the reciprocal of the numbers is $\frac{7}{72}$. The numbers are [CBI]
 (a) 16, 8, 4 (b) 20, 10, 5 (c) 24, 12, 6 (d) 36, 18, 9
29. In the first 10 overs of a cricket game, the run rate was only 3.2. What should be the run rate in the remaining 40 overs to reach the target of 282 runs? [MAT]
 (a) 6.25 (b) 6.5 (c) 6.75 (d) 7
30. The average of 6 numbers is 3.95. The average of two of them is 3.4, while the average of the other two is 3.85. What is the average of the remaining two numbers? [Bank PO]
 (a) 4.5 (b) 4.6 (c) 4.7 (d) 4.8
31. 16 children are to be divided into two groups A and B of 10 and 6 children. The average per cent marks obtained by the children of group A is 75 and the average per cent marks of all the 16 children is 76. What is the average per cent marks of children of group B? [BSRB]
 (a) $77\frac{1}{3}$ (b) $77\frac{2}{3}$ (c) $78\frac{1}{3}$ (d) $78\frac{2}{3}$

32. The average of five numbers is 27. If one number is excluded, the average becomes 25. The excluded number is [Section Officer's]
(a) 25 (b) 27 (c) 30 (d) 35
33. Out of 9 persons, 8 persons spent ₹ 30 each for their meals. The ninth one spent ₹ 20 more than the average expenditure of all the nine. The total money spent by all of them was [CBI]
(a) ₹ 260 (b) ₹ 290 (c) ₹ 292.50 (d) ₹ 400.50
34. A family consists of grand parents, parents and three grand children. The average age of the grand parents is 67 years, that of the parents is 35 years and that of the grand children is 6 years. What is the average age of the family? [RRB]
(a) $28\frac{4}{7}$ years (b) $31\frac{5}{7}$ years (c) $32\frac{1}{7}$ years (d) None of these
35. The sum of three consecutive odd numbers is 38 more than the average of these numbers. What is the first of these numbers [Bank PO]
(a) 13 (b) 17 (c) 19
(d) Data inadequate (e) None of these
36. The average of 7 consecutive numbers is 20. The largest of these numbers is [SSC]
(a) 20 (b) 22 (c) 23 (d) 24
37. The average of the two-digit numbers, which remain the same when the digits interchange their position is [CDS]
(a) 33 (b) 44 (c) 55 (d) 66
38. If the mean of a, b, c is M and $ab + bc + ca = 0$, then the mean of a^2, b^2, c^2 is [ISTTM]
(a) M^2 (b) $3M^2$ (c) $6M^2$ (d) $9M^2$
39. The average of first nine prime numbers is [CBI]
(a) 9 (b) 11 (c) $11\frac{1}{9}$ (d) $11\frac{2}{9}$
40. Find the average of all the numbers between 6 and 34 which are divisible by 5 [CBI]
(a) 18 (b) 20 (c) 24 (d) 30
41. A student was asked to find the arithmetic mean of the numbers 3, 11, 7, 9, 15, 13, 8, 19, 17, 21, 14 and x . He found the mean to be 12. What should be the number in place of x ? [Section Officer's]
(a) 3 (b) 7 (c) 17 (d) 31
42. If the mean of 5 observation $x, x + 2, x + 4, x + 6$ and $x + 8$ is 11, then the mean of the last three observations is [CDS]
(a) 11 (b) 13 (c) 15 (d) 17
43. The average of a non-zero number and its square is 5 times the number. The number is [SSC]
(a) 9 (b) 17 (c) 29 (d) 295
44. The average of the boys in a class is 16 years and that of girls is 15 years. The average age for the whole class is [SSC]
(a) 15 years
(b) 15.5 years
(c) 16 years
(d) Cannot be computed with the given information
45. A library has an average of 510 visitors on Sundays and 240 on other days. The average number of visitors per day in a month of 30 days beginning with a Sunday is [MAT]
(a) 250 (b) 275 (c) 280 (d) 285

46. If the average marks of three batches of 55, 60 and 45 students respectively is 50, 55 and 60, then the average marks of all the students is [CBI]
 (a) 53.33 (b) 54.68 (c) 55 (d) None of these
47. A car owner buys petrol at ₹ 7.50, ₹ 8 and ₹ 8.50 per litre for three successive years. What approximately is the average cost per litre of petrol if he spends ₹ 4000 each year? [MAT]
 (a) ₹ 7.98 (b) ₹ 8 (c) ₹ 8.50 (d) ₹ 9
48. The average of 50 numbers is 30. If two numbers, 35 and 40 are discarded, then the average of the remaining numbers is nearly. [RRB]
 (a) 28.32 (b) 28.78 (c) 29.27 (d) 29.68
49. The average age of 35 students in a class is 16 years. The average age of 21 students is 14. What is the average age of remaining 14 students? [SBI PO]
 (a) 15 years (b) 17 years (c) 18 years (d) 19 years
50. The average monthly salary of 20 employees in an organisation is ₹ 1500. If the manager's salary is added, then the average salary increases by ₹ 100. What is the manager's monthly salary? [RRB]
 (a) ₹ 2000 (b) ₹ 2400 (c) ₹ 3600 (d) ₹ 4800
51. The average monthly income of P and Q is ₹ 5050. The average monthly income of Q and R is ₹ 6250 and the average monthly income of P and R is ₹ 5200. The monthly income of P is [RRB]
 (a) ₹ 3500 (b) ₹ 4000 (c) ₹ 4050 (d) ₹ 5000
52. Average age of 15 students of a class is 15 years. Out of these the average age of 5 students is 14 years and that of the other 9 students is 16 years. The age of the 15th student is [SSC]
 (a) 11 years (b) 14 years (c) 15 years (d) $15\frac{2}{7}$ years
53. The average weight of A, B and C is 45 kg. If the average weight of A and B is 40 kg and that of B and C is 43 kg, then the weight of B is [SSC]
 (a) 17 kg (b) 20 kg (c) 26 kg (d) 31 kg
54. The average of 8 numbers is 20. The average of first two numbers is $15\frac{1}{2}$ and that of next three is $21\frac{1}{3}$. If the sixth number is less than the seventh and eighth numbers by 4 and 7 respectively, then the eighth number is [SSC]
 (a) 18 (b) 22 (c) 25 (d) 27
55. Of the three numbers, the average of the first and the second is greater than the average of the second and the third by 15. What is the difference between the first and the third of the three numbers? [SBI PO]
 (a) 15 (b) 45 (c) 60
 (d) Data inadequate (e) None of these
56. The average of runs of a cricket player of 10 innings was 32. How many runs must he make in his next innings so as to increase his average of runs by 4? [SSC]
 (a) 2 (b) 4 (c) 70 (d) 76
57. The average salary of all the workers in a workshop is ₹ 8000. The average salary of 7 technicians is ₹ 12000 and the average salary of the rest is ₹ 6000. The total number of workers in the workshop is [SSC]
 (a) 20 (b) 21 (c) 22 (d) 23

58. Three years ago, the average age of a family of 5 members was 17 years. A baby having been born, the average age of the family is the same today. The present age of the baby is [SSC]
 (a) 1 years (b) $1\frac{1}{2}$ years (c) 2 years (d) 3 years
59. A cricketer whose bowling average is 12.4 runs per wicket takes 5 wickets for 26 runs and thereby decreases his average by 0.4. The number of wickets taken by him till the last match was [SSC]
 (a) 64 (b) 72 (c) 80 (d) 85
60. The average weight of 8 persons increases by 2.5 kg when a new person comes in place of one of them weighing 65 kg. What might be the weight of the new person? [Bank PO]
 (a) 76 kg (b) 76.5 kg (c) 85 kg
 (d) Data inadequate (e) None of these
61. The average weight of a class of 24 students is 35 kg. If the weight of the teacher is included, the average rises by 400 g. The weight of the teacher is [SSC]
 (a) 45 kg (b) 50 kg (c) 53 kg (d) 55 kg
62. David obtained 76, 65, 82, 67 and 85 marks (out of 100) in English, Mathematics, Physics, Chemistry and Biology. What is his average marks? [Bank PO]
 (a) 65 (b) 69 (c) 72 (d) 76 (e) None of these
63. The average annual income (in ₹) of certain agricultural workers is S and that of other workers is T . The number of agricultural workers is 11 times that of other workers. Then the average annual income (in ₹) of all the workers is [SSC]
 (a) $\frac{S+T}{2}$ (b) $\frac{S+11T}{2}$ (c) $\frac{1+T}{11S}$ (d) $\frac{11S+T}{12}$
64. The average age of students of a class is 15.8 years. The average age of boys in the class is 16.4 years and that of the girls is 15.4 years. The ratio of the number of boys to the number of girls in the class is [SBI PO]
 (a) 1 : 2 (b) 2 : 3 (c) 3 : 4 (d) 3 : 5
65. Average of ten numbers is calculated. If each number was increased by 12%, then the average
 (a) will remain unchanged (b) may decrease
 (c) may increase (d) increased by 12%



1. (c) 2. (c) 3. (c) 4. (c) 5. (b) 6. (c) 7. (a) 8. (c) 9. (b) 10. (d)
 11. (b) 12. (d) 13. (d) 14. (c) 15. (e) 16. (e) 17. (b) 18. (c) 19. (c) 20. (c)
 21. (c) 22. (a) 23. (d) 24. (c) 25. (c) 26. (d) 27. (c) 28. (c) 29. (a) 30. (b)
 31. (b) 32. (d) 33. (c) 34. (b) 35. (b) 36. (c) 37. (c) 38. (d) 39. (c) 40. (b)
 41. (b) 42. (b) 43. (a) 44. (d) 45. (d) 46. (d) 47. (b) 48. (d) 49. (d) 50. (c)
 51. (b) 52. (a) 53. (d) 54. (c) 55. (e) 56. (d) 57. (b) 58. (c) 59. (d) 60. (c)
 61. (a) 62. (e) 63. (d) 64. (b) 65. (d)



Solutions with Necessary Explanation

1. Let the number of candidates attended the examination be ' n '.

$\therefore$ Total marks in the examination = $60n$.

80 candidates marks are changed from 95 to 70 and average reduced to 55 marks.

$\therefore$ Old average = 60 $\Rightarrow$ Total = $60n$ (1)

As per new average, total marks in the examination = $55n$

$$= 60n - 80 \times (95 - 70)$$

$$= 60n - 2000$$

$$\Rightarrow 60n - 55n = 2000$$

$$\Rightarrow 5n = 2000$$

$$\Rightarrow n = 400$$

2. Old average of 15 students = 10 years.

5 more students joined the group.

New average = $10 + 1 = 11$ years.

Let Age of new student = x years

$$\therefore \frac{15 \times 10 + 5 \times x}{20} = 11$$

$$\Rightarrow 5x = 20 \times 11 - 15 \times 10 = 70$$

$$\therefore x = \frac{70}{5} = 14 \text{ years}$$

3. Let ' S ' be the total distance travelled.

$$\text{Time taken for 1st } 1/3\text{rd of journey} = \frac{S/3}{60} = \frac{S}{180} \text{ hour.}$$

$$\text{Time taken for 2nd } 1/3\text{rd of journey} = \frac{S/3}{30} = \frac{S}{90} \text{ hour.}$$

$$\text{Time taken for 3rd } 1/3\text{rd of journey} = \frac{S/3}{10} = \frac{S}{30} \text{ hour.}$$

$$\therefore \text{Average speed} = \frac{\text{Total distance travelled}}{\text{Total time taken}}$$

$$= \frac{S}{\frac{S}{180} + \frac{S}{90} + \frac{S}{30}}$$

$$= \frac{S}{S \left(\frac{1}{180} + \frac{1}{90} + \frac{1}{30} \right)}$$

$$= \frac{180 \times 90 \times 30}{90 \times 30 + 180 \times 30 + 180 \times 90}$$

$$\begin{aligned}
&= \frac{180 \times 9 \times 3}{27 + 54 + 162} \\
&= \frac{180}{1 + 2 + 6} \\
&= \mathbf{20 \text{ km/hr.}}
\end{aligned}$$

4. Let the average speed of train for return journey be 'x' km/hr

$$\therefore \text{Average speed in the onward journey} = \frac{3}{4}x \text{ km/hr}$$

$$\text{i.e. Average speed for the whole journey} = \frac{800}{16} = 50 \text{ km/hr}$$

$$\text{i.e. } 50 = \frac{2(x) \left(\frac{3}{4}x \right)}{x + \frac{3}{4}x} = \frac{\frac{3}{2}x^2}{\frac{7}{4}x} = \frac{3}{2} \times \frac{4}{7}x = \frac{6}{7}x$$

$$\therefore x = \frac{7 \times 50}{6} \text{ km/hr.}$$

$$\therefore \text{Speed of train in onward journey} = \frac{3}{4} \times \frac{7 \times 50}{6} = \frac{175}{4} = \mathbf{43.75 \text{ km/hr}}$$

5. Given average of X_1 , X_2 and $X_3 = 14$.

$$\text{i.e. } X_1 + X_2 + X_3 = 14 \times 3 = 42 \quad (1)$$

$$2(X_2 + X_3) = 30$$

$$\Rightarrow X_2 + X_3 = 15 \quad (2)$$

$$\therefore (1) - (2) \Rightarrow X_1 = 42 - 15 = \mathbf{27}$$

6. Average speed of the person for his whole journey = $\frac{2 \times 40 \times 50}{(40 + 50)} = \frac{4000}{90} = \frac{\mathbf{400}}{\mathbf{9}} \text{ km/hr}$

7. A cricket team consists of 11 players.

Let its average age be x, and age of replaced player be y.

$$\therefore 11x - 32 - 30 + 2y = 11(x - 2)$$

$$\text{i.e. } 32 + 30 - 22 = 2y$$

$$\text{or } y = \frac{32 + 30 - 22}{2} = \frac{40}{2} = \mathbf{20 \text{ years}}$$

8. Average height of 20 students in a class = 105 cm.

After joining 10 students of average height 120 cm, average height of students of the class

$$= \frac{20 \times 105 + 10 \times 120}{30} = 2 \times 35 + 10 \times 4 = 70 + 40 = \mathbf{110 \text{ cm}}$$

9. Average of first 46 natural numbers = $\frac{1}{46} \left[\frac{46}{2} \times (1 + 46) \right] = \frac{47}{2} = \mathbf{23.5}$

10. Average of 13 results = 50

Average of first 7 results = 52

Average of last 7 results = 49

$$\therefore \text{The 7th result} = 7 \times 52 + 7 \times 49 - 13 \times 50 = 57$$

11. Average age of 18 persons = 32 years 6 months

Age of persons left = 65 years $\times$ 2 + 50 years $\times$ 1

$\therefore$ The sum of ages of persons remaining

$$= 18 \times 32 \text{ years 6 months} - 2 \times 65 - 1 \times 50$$

$$= 585 - 130 - 50 = 405$$

$$\therefore \text{Average age of remaining persons} = \frac{405}{15} = 27 \text{ years}$$

12. Average marks in Physics, Chemistry and Biology = 70

Marks obtained in Maths = 90

$$\therefore \text{New average for the 4 subjects} = \frac{70 \times 3 + 90 \times 1}{4} = \frac{300}{4} = 75 \text{ marks.}$$

13. Average weight of 3 persons = 65 kg

Weight of 4th person = 45 kg

$$\text{Average weight of new group} = \frac{65 \times 3 + 45 \times 1}{4} = \frac{195 + 45}{4} = \frac{240}{4} = 60 \text{ kg}$$

14. Average speed of the man for the whole journey = $\frac{3 \times 80 \times 60 \times 30}{80 \times 60 + 60 \times 30 + 30 \times 80}$

$$= \frac{3 \times 80 \times 6 \times 3}{8 \times 6 + 6 \times 3 + 3 \times 8} = \frac{3 \times 80 \times 3}{8 + 3 + 4} = \frac{9 \times 80}{15} = 3 \times 16 = 48 \text{ km/h}$$



15. **Short cut:** Weight of new man = Weight of replaced man + Increase in average $\times$ Number of persons

$$= 40 + 5 \times 6$$

$$= 40 + 30 = 70 \text{ kg}$$

16. Average age of new student =

$$\frac{\text{Old number of student} \times \text{Old average of students} - \text{New average} \times \text{New number of students}}{\text{Number of new students}}$$

$$= \frac{9 \times 20 - 18 \times 15}{6} = \frac{180 - 270}{6} = \frac{90}{6} = 15 \text{ years}$$

17. Overall average score = $\frac{60 \times 50 + 40 \times 45}{60 + 40}$

$$= \frac{60 \times 50 + 40 \times 45}{100}$$

$$= 30 + 18 = 48$$

18. Average speed = $\frac{20 \times 60 \times 20}{60 + 20} = \frac{2 \times 60 \times 20}{80} = 30 \text{ km/h}$

19. Let the number of students be 100.

$$\text{Then total score} = 52 \times 100 = 5200$$

$$\text{first 20\% scored} = 20 \times 80 = 1600$$

$$\text{last 25\% scored} = 25 \times 31 = 775$$

$$\therefore \text{Remaining 55\% student's score} = 5200 - 1600 - 775 = 2825$$

$$\therefore \text{Average score among 55\%} = \frac{2825}{55} = \mathbf{51.4 \text{ approx}}$$

20. Average age of 4 members (10 years ago) = 24 years

$$\therefore \text{Average age of 4 members at present} = 34 \text{ years}$$

Let the age of children be x and $x + 2$ years

$$\text{Present average age of family} = \frac{34 \times 4 + x + x + 2}{6} = 24$$

$$\text{i.e.} \quad \frac{136 + 2 + 2x}{6} = 24$$

$$\text{or} \quad 138 + 2x = 144$$

$$\text{or} \quad 2x = 6$$

$$\text{or} \quad x = 3 \text{ years}$$

$$\therefore x + 2 = \mathbf{5 \text{ years}}$$

$$\therefore \text{Age of youngest child} = \mathbf{3 \text{ years}}$$

21. Average weight of A, B and C = 84 kg

On joining D, average reduces to 80 kg.

Let weight of A, B, C, D and E be a, b, c, d and e respectively

$$\therefore \frac{a + b + c}{3} = 84 \quad (1)$$

$$\frac{a + b + c + d}{4} = 80 \quad (2)$$

$$a + b + c = 84 \times 3 \quad (a)$$

$$a + b + c + d = 80 \times 4 \quad (b)$$

$$(b) - (a) \Rightarrow d = 80 \times 4 - 84 \times 3$$

$$= 320 - 252 = 68 \text{ years}$$

$$e = d + 3 = 68 + 3 = 71 \text{ years}$$

$$\therefore \frac{b + c + d + 71}{4} = 79 \quad (3)$$

$$\text{or} \quad b + c + d + 71 = 79 \times 4$$

$$\text{or} \quad b + c + d = 79 \times 4 - 71 = 245 \quad (c)$$

$$(b) - (c) \Rightarrow a = 320 - 245 = \mathbf{75 \text{ years}}$$

22. Let his average in the tenth innings be x .

His score in 11th innings = 108 runs

$$\therefore \text{Average in eleventh innings} = \frac{x \times 10 + 108}{11} = x + 6$$

$$\text{i.e.} \quad 10x + 108 = (x + 6)11$$

$$11x - 10x = 108 - 66$$

$$x = 42$$

$$\therefore \text{New average} = x + 6 = 42 + 6 = \mathbf{48 \text{ runs}}$$

23. Let average age of 8 men be x .

Let the average age of two men joining be y .

On joining 2 persons by replacing two persons of ages 21 and 23 years

$$\therefore \text{New average} = \frac{8 \times x - 21 - 23 + 2y}{8} = x + 2$$

$$\text{i.e.} \quad 8x - 44 + 2y = 8x + 16$$

$$\text{or} \quad 2y = 44 + 16 = 60$$

$$\therefore y = \mathbf{30 \text{ years}}$$

24. Average weight of 45 students = 52 kg

$$\therefore \text{Total weight} = 45 \times 52 \text{ kg}$$

$$\text{Weight of students left} = 5 \times 48 \text{ kg}$$

$$\text{Weight of students joined} = 5 \times 54 \text{ kg}$$

$$\begin{aligned} \therefore \text{New average weight} &= \frac{45 \times 52 - 5 \times 48 + 5 \times 54}{45} \\ &= 52 + \frac{5(54 - 48)}{45} \\ &= 52 + \frac{30}{45} = \mathbf{52 \frac{2}{3} \text{ kg}} \end{aligned}$$

25. Average age of 36 students = 14 years

Let teacher's age be x years.

$$\therefore \frac{14 \times 36 + x}{37} = 14 + 1 = 15$$

$$\text{or} \quad 14 \times 36 + x = 15 \times 37$$

$$\therefore x = 15 \times 37 - 14 \times 36 = 51$$

26. Average weight of A, B and C = $\frac{a + b + c}{3} = 54 \frac{1}{3} \text{ kg}$

$$\text{Average weight of B, D and E} = \frac{b + d + e}{3} = 53 \text{ kg}$$

Data inadequate

27. Corrected new mean = $\frac{50 \times 36 - 23 + 48}{50} = 36 + \frac{25}{50} = \mathbf{36.5}$

28. Let the third number be x

$$\therefore \text{Second number} = 2 \times x = 2x$$

$$\text{First number} = 2 \times 2x = 4x$$

$$\begin{aligned} \therefore \text{Average of reciprocal of numbers} &= \frac{1}{3} \left(\frac{1}{x} + \frac{1}{2x} + \frac{1}{4x} \right) \\ &= \frac{1}{3x} \left(\frac{4 + 2 + 1}{4} \right) = \frac{7}{12x} = \frac{7}{72} \end{aligned}$$

$$\therefore x = 6$$

$$\therefore \text{Numbers are } \mathbf{6, 12, 24}$$

29. Score in the first 10 overs = $3.2 \times 10 = 32$

To achieve a target of 282 runs,

$$\text{Balance runs required} = 282 - 32 = 250$$

$$\text{Balance overs available} = 40$$

$$\therefore \text{Required run rate} = \frac{250}{40} = \mathbf{6.25}$$

$$\begin{aligned} 30. \text{ Required average} &= \frac{3.95 \times 6 - 3.4 \times 2 - 3.85 \times 2}{2} \\ &= \frac{23.70 - 6.8 - 7.7}{2} \\ &= \frac{23.70 - 14.50}{2} = \frac{9.20}{2} = \mathbf{4.60} \end{aligned}$$

31. Group A average = 75

$$\text{Group (A + B) average} = 76$$

$$\begin{aligned} \therefore \text{Required average} &= \frac{16 \times 76 - 10 \times 75}{6} \\ &= \frac{760 + 456 - 750}{6} = \frac{466}{6} \\ &= 77 \frac{4}{6} = \mathbf{77 \frac{2}{3} \%} \end{aligned}$$

32. Total of 5 numbers = 27×5

$$\text{Excluded number} = 27 \times 5 - 25 \times 4 = 135 - 100 = \mathbf{35}$$

33. Let the total money spent be x

$$\therefore \text{Average expenditure} = \text{₹} \frac{x}{9}$$

$$\therefore 8 \times 30 + \left(\frac{x}{9} + 20 \right) = x$$

$$\text{i.e.} \quad 260 = x - \frac{x}{9}$$

$$260 = \frac{8x}{9}$$

$$\text{or} \quad x = \frac{260 \times 9}{8} = 32.5 \times 9 = \text{₹} \mathbf{292.5}$$

34. Total age of a family = $2 \times 67 + 2 \times 35 + 3 \times 6$
 $= 134 + 70 + 18 = 222 \text{ years}$

$$\therefore \text{Average age of family} = \frac{222}{7} = \mathbf{31 \frac{5}{7} \text{ years}}$$

35. Let $(x - 2)$, x , $(x + 2)$ be the numbers.

$$\therefore \text{Sum of numbers} = x - 2 + x + x + 2 = 3x$$

$$\text{Average of numbers} = \frac{3x}{3} = x$$

$$\therefore 3x = x + 38$$

$$\therefore 2x = 38$$

$$\text{or } x = 19$$

$$\therefore \text{First number} = 19 - 2 = 17$$

36. Let number be x , $(x + 1)$, $(x + 2)$, ..., $(x + 6)$.

$$\therefore \text{Average} = \frac{1}{7} \times \frac{7}{2} (x + x + 6) = x + 3$$

$$\text{i.e. } x + 3 = 20$$

$$\therefore x = 20 - 3 = 17$$

$$\therefore \text{Largest number} = x + 6 = 17 + 6 = 23$$

37. Such two digit numbers are 11, 22, 33, 44, 55, 66, 77, 88 and 99.

$$\therefore \text{Sum of these number} = 11(1 + 2 + 3 + \dots + 9)$$

$$= 11 \times \frac{9}{2} (1 + 9) = 11 \times \frac{9}{2} \times 10 = 495$$

$$\therefore \text{Average} = \frac{495}{9} = 55$$

38. Given $\frac{a + b + c}{3} = M$

$$\text{or } (a + b + c) = 3M$$

$$(a + b + c)^2 = 9M^2$$

$$\text{i.e. } a^2 + b^2 + c^2 + 2(ab + bc + ca) = 9M^2$$

$$\therefore ab + bc + ca = 0$$

$$a^2 + b^2 + c^2 = 9M^2$$

39. First nine prime numbers are 2, 3, 5, 7, 11, 13, 17, 19 and 23.

$$\text{Average of above numbers} = \frac{2 + 3 + 5 + 7 + 11 + 13 + 17 + 19 + 23}{9} = \frac{100}{9} = 11 \frac{1}{9}$$

40. Numbers between 6 and 34, which are divisible by 5 are 10, 15, 20, 25 and 30.

$$\therefore \text{Average of above numbers} = \frac{10 + 15 + 20 + 25 + 30}{5} = \frac{100}{5} = 20$$

41. Given 12 numbers and mean to be 12.

$$\therefore \text{Sum of 12 numbers} = 12 \times 12 = 144$$

$$\therefore x = 144 - (3 + 11 + 7 + 9 + 15 + 13 + 8 + 19 + 17 + 21 + 14) = 144 - 137 = 7$$

42. Mean of 5 observations given,

$$\frac{x + x + 2 + x + 4 + x + 6 + x + 8}{5} = 11$$

$$\text{i.e. } x + 4 = 11$$

$$\text{or } x = 11 - 4 = 7$$

$$\therefore \text{Mean of last 3 observations} = \frac{x + 4 + x + 6 + x + 8}{3} = x + 6 = 7 + 6 = 13$$

43. Let the non-zero number be x .

$$\frac{x + x^2}{2} = 5x$$

$$\begin{aligned}\text{i.e.} \quad & x + x^2 = 10x \\ \text{or} \quad & x^2 - 9x = 0 \\ \text{or} \quad & x - 9 = 0 \\ \therefore \quad & x = 9\end{aligned}$$

44. Cannot be computed as the number of boys and girls are not given.

45. In a month starting with Sunday, there will be 5 Sundays and 25 week days

$$\therefore \text{Average visitor/day} = \frac{5 \times 510 + 25 \times 240}{30} = 5 \times 17 + 25 \times 8 = 85 + 200 = 285$$

$$\begin{aligned}46. \text{Average marks of all students} &= \frac{55 \times 50 + 60 \times 15 + 45 \times 60}{160} \\ &= \frac{55 \times 5 + 6 \times 15 + 45 \times 6}{16} = \frac{275 + 90 + 270}{16} = \frac{635}{16} = 39.68\end{aligned}$$

$$47. \text{In the first year petrol consumed} = \frac{4000}{7.5} \text{ litres}$$

$$\text{In the second year petrol consumed} = \frac{4000}{8} \text{ litres}$$

$$\text{In the third year petrol consumed} = \frac{4000}{8.5} \text{ litres}$$

$$\begin{aligned}\text{Average price/litre of petrol} &= \frac{4000 \times 3}{\frac{4000}{7.5} + \frac{4000}{8} + \frac{4000}{8.5}} \\ &= \frac{3 \times 7.5 \times 8 \times 8.5}{8 \times 8.5 + 7.5 \times 8.5 + 7.5 \times 8} \\ &= \frac{1530}{68 + 63.75 + 60} = \frac{1530}{191.75} = ₹ 8/\text{litre}\end{aligned}$$



Short cut: Price/litre of petrol for 3 successive years are ₹ 7.5, ₹ 8 and ₹ 8.5/ litre

$$\therefore \text{Average price} = \frac{7.5 + 8 + 8.5}{3} = ₹ 8/\text{litre}$$

48. The sum of 50 numbers = $50 \times 30 = 1500$

Discarding 35 and 40, we get

$$\begin{aligned}\text{Average of remaining 48 numbers} &= \frac{1500 - 35 - 40}{48} \\ &= \frac{1500 - 75}{48} = \frac{1425}{48} = 29.68\end{aligned}$$

49. Average age of 35 students = 16

Average age of 21 students = 14

$$\begin{aligned}\therefore \text{Average age of remaining 14 students} &= \frac{35 \times 16 - 21 \times 14}{14} \\ &= 5 \times 8 - 21 = 40 - 21 = 19 \text{ years}\end{aligned}$$

50. Average monthly salary of 20 employees = ₹ 1,500

On adding manager's salary (let it be x)

$$\text{New average} = \frac{1500 \times 20 + x}{21} = 1500 + 100 = 1600$$

$$\therefore 1500 \times 20 + x = 21 \times 1600$$

$$\begin{aligned} \text{or } x &= 21 \times 1600 - 20 \times 1500 \\ &= 33600 - 30000 = \text{₹ } 3600 \end{aligned}$$

51. Monthly income of P and $Q = 2 \times 5050 = 10100$ (1)

$$\text{Monthly income of } Q \text{ and } R = 2 \times 6250 = 12500 \quad (2)$$

$$\text{Monthly income of } P \text{ and } R = 2 \times 5200 = 10400 \quad (3)$$

$$\frac{(1) + (2) + (3)}{2} - (2) \Rightarrow$$

$$\begin{aligned} \text{Monthly income of } P &= \frac{10100 + 12500 + 10400}{2} - 12500 \\ &= 5050 + 6250 + 5200 - 12500 \\ &= 16500 - 12500 = \text{₹ } 4000 \end{aligned}$$

52. Total age of 15 students = $15 \times 15 = 225$ years (1)

$$\text{Sum of ages of 5 students} = 5 \times 14 = 70 \text{ years} \quad (2)$$

$$\text{Sum of ages of 9 students} = 9 \times 16 = 144 \text{ years} \quad (3)$$

$$\therefore \text{Age of 15th student} = (1) - (2) - (3) = 225 - (70 + 144) = 11 \text{ years}$$

53. Weight of $A + B + C = 45 \times 3 = 135$ kg (1)

$$\text{Weight of } A + B = 40 \times 2 = 80 \text{ kg} \quad (2)$$

$$\text{Weight of } B + C = 43 \times 2 = 86 \text{ kg} \quad (3)$$

Subtracting (2) from (1),

$$\therefore \text{Weight of } C = 135 - 80 = 55 \text{ kg} \quad (4)$$

Subtracting (3) from (1),

$$\text{Weight of } A = 135 - 86 = 49 \text{ kg} \quad (5)$$

Subtracting (5) from (2),

$$\therefore \text{Weight of } B = 80 - 49 = 31 \text{ kg} \quad (6)$$

54. Sum of 8 numbers = $8 \times 20 = 160$ (1)

$$\text{Sum of first 2 numbers} = 2 \times 15 \frac{1}{2} = 31 \quad (2)$$

$$\text{Sum of next 3 numbers} = 3 \times 21 \frac{1}{3} = 64 \quad (3)$$

$$\text{Sixth number} = \text{Seventh number} - 4 \quad (4)$$

$$\text{Sixth number} = \text{Eighth number} - 7 \quad (5)$$

$$\begin{aligned} (1) - (2) - (3) &\Rightarrow \text{6th number} + \text{7th number} + \text{8th number} \\ &= 160 - (31 + 64) = 160 - 95 = 65 \end{aligned} \quad (6)$$

$$\therefore (6) \Rightarrow (\text{8th number} - 7) + (\text{8th number} - 3) + (\text{8th number}) = 65$$

$$\text{or } 3 \times \text{8th number} = 65 + 7 + 3 = 75$$

$$\therefore \text{8th number} = 25$$

55. Let x , y and z be the numbers.

$$\frac{x+y}{2} = \frac{y+z}{2} + 15$$

$$\therefore \frac{x-z}{2} = 15$$

$$\text{or } x - z = 2 \times 15 = 30$$

56. Total score in 10 innings = $32 \times 10 = 320$

To raise the average to 36,

Total score after 11 innings = $36 \times 11 = 396$

$$\therefore \text{Runs he has to take in the 11th innings} = 396 - 320 = 76$$

57. Let the total number of workers be x .

$$\therefore \text{Number of workers except technician} = x - 7$$

$$\therefore 8000x = 7 \times 12000 + (x - 7) 6000$$

$$\text{i.e. } (8000 - 6000)x = 7 \times 12000 - 7 \times 6000$$

$$2000x = 7 \times 6000$$

$$\text{or } x = \frac{7 \times 6000}{2000} = 21$$

58. Average of age of family of 5 members (3 years ago) = 17 years

$$\therefore \text{Present age of family of 6 members with a new born baby} = 17 \text{ years}$$

Let the age of baby be x

$$\therefore \frac{(17 + 3) \times 5 + x}{6} = 17$$

$$\text{i.e. } \frac{100 + x}{6} = 17$$

$$\text{or } 100 + x = 102$$

$$\text{or } x = 2 \text{ years}$$

59. Let the number of wickets taken be x .

$$\therefore \frac{12.4 \times x + 26}{x + 5} = 12$$

$$\text{i.e. } 12.4x + 26 = 12x + 60$$

$$\text{or } 0.4x = 60 - 26 = 34$$

$$\text{or } x = \frac{34}{0.4} = 85$$

60. Let the weight of new person be x kg.

Weight of person replaced = 65 kg

i.e. Weight of new person = Weight of replaced person

+ old number of persons $\times$ change in average

$$x = 65 + 8 \times 2.5 = 85 \text{ kg}$$

61. Total weight of 24 students = $24 \times 35 = 840$

Let the weight of teacher be x kg.

$$\therefore \frac{840 + x}{25} = 35.4 \text{ kg}$$

$$\text{or } 840 + x = 25 \times 35.4 = 885$$

$$\therefore x = 885 - 840 = 45 \text{ kg}$$

$$62. \text{ Average marks} = \frac{76 + 65 + 82 + 67 + 85}{5} = \frac{375}{5} = 75$$

$$63. \text{ Average annual income of agricultural worker} = S$$

$$\text{Average annual income of other workers} = T$$

Let other workers be n in number and agricultural works be $11n$ in number.

$$\therefore \frac{11n \times S + n \times T}{12n} = \frac{11S + T}{12}$$

$$64. \text{ Average age of boys} = 16.4 \text{ years}$$

$$\text{Average age of girls} = 15.4 \text{ years}$$

Let m be number of boys and n be number of girls.

$$\therefore \frac{m \times 16.4 + n (15.4)}{m + n} = 15.8$$

$$16.4m + 15.4n = 15.8m + 15.8n$$

$$\text{i.e. } 0.6m = 0.4n$$

$$\text{or } \frac{m}{n} = \frac{0.4}{0.6} = \frac{2}{3}$$

65. Since all numbers were increased by 12%, the average will also **increase by 12%**.

Problems Based on Ages

To solve the problems based on ages, the student should know how to solve linear equations within a short span of time. This chapter provides some typical questions based on ages.

EXAMPLE 1 The age of a person A was 7 times that of B 5 years ago. At present the age of A is 4 times that of B. What are the present ages of A and B?

Solution Let the present age of B be x years.

$\therefore$ The present age of A = $4 \times x = 4x$ years

5 years ago

$$\text{Age of A} = 4x - 5$$

$$\text{Age of B} = x - 5$$

$$\therefore 4x - 5 = 7(x - 5)$$

$$\text{i.e. } 7x - 4x = 35 - 5 = 30$$

$$\text{i.e. } 3x = 30$$

$$\therefore x = 10 \text{ years}$$

$$\text{i.e. Present age of B} = 10 \text{ years}$$

$$\text{Present age of A} = 4 \times 10 = 40 \text{ years}$$

EXAMPLE 2 At present the age of a father is five times that of his son. Three years hence, the father's age would be four times that of his son. Find the present ages of the father and the son.

Solution Let the present age of son be x years.

Then the present age of father = $5x$ years

Three years hence

$$4(x + 3) = 5x + 3$$

$$\text{or } 4x + 12 = 5x + 3$$

$$\therefore x = 9 \text{ years}$$

$$\therefore \text{Son's age} = 9 \text{ years}$$

$$\text{Father's age} = 5 \times 9 = 45 \text{ years}$$

EXAMPLE 3 Three years ago, the age of a father was 7 times that of his son. Three years hence the father's age would be four times that of his son. What are the present ages of the father and his son?

Solution Let the present age of son be x years.

Three years ago

$$\text{Age of son} = (x - 3) \text{ years}$$

$$\text{Age of father} = 7(x - 3) \text{ years}$$

Three years hence

$$\text{Age of son} = (x + 3) \text{ years}$$

$$\text{Age of father} = 7(x - 3) + 6 = 7x - 15$$

$$= 4(x + 3)$$

$$\text{i.e.} \quad 7x - 15 = 4(x + 3)$$

$$\text{i.e.} \quad 7x - 4x = 15 + 12 = 27$$

$$3x = 27$$

$$\text{i.e.} \quad x = 9 \text{ years}$$

$$\therefore \quad \text{Age of son} = \mathbf{9 \text{ years}}$$

$$\text{Age of father} = 7(x - 3) + 3$$

$$= 7(9 - 3) + 3$$

$$= \mathbf{45 \text{ years}}$$



Short cut

1. t_1 years earlier the father's age was x times that of his son. At present the father's age is y times that of his son. What is the present age of his son?

$$\text{The son's age is given by } t_1 \frac{(x - 1)}{(x - y)}$$

2. The present age of the father is y times the age of his son. t_2 years hence, the father's age becomes z times the age of his son. What are present ages of the father and his son?

$$\text{Son's age is given by } \frac{t_2(z - 1)}{(y - z)}$$

3. t_1 years earlier the age of the father was x times the age of his son. t_2 years hence, the age of the father becomes z times the age of his son. What is the present age of the son?

$$\text{Son's age is given by } \frac{t_2(z - 1) + t_1(x - 1)}{(x - z)}$$

EXAMPLE 4 The age of a man is 4 times that of his son. Five years ago, the man was 7 times as old as his son at that time. What is the present age of the man?

Solution Son's age = $\frac{5(7 - 1)}{(7 - 4)} = \mathbf{10 \text{ years}}$

$\therefore$ Father's age = $4 \times 10 = \mathbf{40 \text{ years}}$

EXAMPLE 5 After 10 years, the age of a father will be twice that of his son. Five years ago, he was 5 times as old as his son. What are their present ages?

Solution Son's age = $\frac{5(5-1) + 10(2-1)}{(5-2)} = \frac{30}{3} = \mathbf{10 \text{ years}}$

$\therefore$ Present age of father = $5(10-5) + 5 = 25 + 5 = \mathbf{30 \text{ years}}$

EXAMPLE 6 Ten year's ago, Anil's father was 4 times as old as his son. After 10 years, the father will be twice as old as his son. What is the present age of Anil?

Solution Age of Anil = $\frac{10(4-1) + 10(2-1)}{(4-2)} = \frac{30+10}{2} = \mathbf{20 \text{ years}}$

EXAMPLE 7 Two years ago the ratio of Ashok's age to Sanal's age was 4 : 3. Two years hence the ratio of their ages will be 6 : 5. What is the sum of their present ages in years?

Solution Two years ago, Ashok's age was $\frac{4}{3}$ of Sanal's age.

Two years hence, Ashok's age will be $\frac{6}{5}$ of Sanal's age.

$$\therefore \text{Sanal's age} = \frac{2\left(\frac{4}{3}-1\right) + 2\left(\frac{6}{5}-1\right)}{\frac{4}{3}-\frac{6}{5}} = \frac{2/3 + 2/5}{2/15} = \frac{16/15}{2/15} = 8 \text{ years}$$

$$\therefore \frac{A-2}{S-2} = \frac{4}{3}$$

$$\text{i.e.} \quad \frac{A-2}{8-2} = \frac{4}{3}$$

$$\therefore A-2 = \frac{4}{3} \times 6 = 8$$

$$\therefore A = 8 + 2 = 10 \text{ years}$$

i.e. Ashok's age = 10 years.

$$\therefore \text{The sum of their present ages} = 10 + 8 = \mathbf{18 \text{ years}}$$

EXAMPLE 8 The sum of the ages of a father and his son is 60 years. Five years ago, father's age was 4 times that of his son. What are the present ages of the father and son?

Solution Let the age of son be x years.

$$\therefore \text{The age of father} = (60 - x) \text{ years}$$

Five years ago

$$[(60 - x) - 5] = 4(x - 5)$$

$$\text{i.e.} \quad 55 - x = 4x - 20$$

$$\text{i.e.} \quad 5x = 75$$

$$\text{or} \quad x = \mathbf{15 \text{ years}}$$

$$\therefore \text{The present age of son} = \mathbf{15 \text{ years}}$$

$$\text{The present age of father} = 60 - 15 = \mathbf{45 \text{ years}}$$

EXAMPLE 9 The sum of ages of a mother and her daughter is 52 years. After 4 years, the age of the mother will be three times that of the daughter. What is the age of the daughter?

Solution Let the age of daughter be x years.

$\therefore$ The age of mother = $(52 - x)$ years

After 4 years

$$(52 - x) + 4 = 3(x + 4)$$

$$\text{or} \quad 56 - x = 3x + 12$$

$$\text{or} \quad 4x = 56 - 12 = 44$$

$$\therefore \quad x = 11 \text{ years}$$

i.e. The age of daughter = 11 years



Short cut: Daughter's age = $\frac{\text{Total ages} - \text{Number of years after} \times (\text{Times} - 1)}{\text{Times} + 1}$

$$= \frac{52 - 4 \times (3 - 1)}{3 + 1}$$

$$= \frac{52 - 8}{4} = 11 \text{ years}$$

EXAMPLE 10 The ratio of father's age to son's age is 5 : 1. The product of their ages is 245. What will be the ratio of their ages after 5 years?

Solution Let the ages of son and father be x and $5x$ respectively.

$$\text{Then} \quad 5x \times x = 245$$

$$\text{or} \quad 5x^2 = 245$$

$$\text{i.e.} \quad x^2 = \frac{245}{5} = 49$$

$$\therefore \quad x = 7$$

$\therefore$ Son's age = 7 years

Father's age = 35 years

After 5 years

$$\text{Father's age} = 40 \text{ years}$$

$$\text{Son's age} = 12 \text{ years}$$

$\therefore$ The ratio of their ages = 40 : 12 = 10 : 3

EXAMPLE 11 The ratio of Ram's age to his mother's age is 4 : 13. The difference of their ages is 27. What will be the ratio of their ages after 6 years?

Solution The present age ratio = 4 : 13

The difference of their ages = $4k - 13k = 9k$

$$\therefore \quad 9k = 27$$

$$\therefore \quad k = 3$$

After 6 years, age ratio = $4k + 6 : 13k + 6$

$$\text{i.e.} \quad 4 \times 3 + 6 : 13 \times 3 + 6$$

$$\text{i.e.} \quad 18 : 45 = 2 : 5$$

EXAMPLE 12 The ratio of the ages of a father and his son is 7 : 2 at present. After 5 years the ratio will become 8 : 3. What is the present age of the son?

Solution The present age ratio between father and son = 7 : 2
After 5 years, age ratio = 8 : 3

$$\therefore \text{Son's age} = \frac{2 \times 5(8 - 3)}{7 \times 3 - 8 \times 2} = \frac{2 \times 5 \times 5}{21 - 16} = \frac{2 \times 5 \times 5}{5} = \mathbf{10 \text{ years}}$$

$$\text{Father's age} = \frac{7 \times 5(8 - 3)}{7 \times 3 - 8 \times 2} = \frac{7 \times 5 \times 5}{21 - 16} = \frac{7 \times 5 \times 5}{5} = \mathbf{35 \text{ years}}$$



Short cut: If the present age ratio between father and son is $x : y$ and after t years, age ratio becomes $a : b$, then

$$\text{Son's age} = \frac{y \times t(a - b)}{\text{Difference of cross product}}$$

$$\text{Father's age} = \frac{x \times t(a - b)}{\text{Difference of cross product}}$$

EXAMPLE 13 The ratio of the ages of a father and his son at present is 3 : 1, 6 years earlier the ratio was 4 : 1. What are the present ages of the son and the father?

Solution The present age ratio = 3 : 1
6 years before age ratio = 4 : 1

$$\therefore \text{Son's age} = \frac{1 \times 6(4 - 1)}{4 \times 1 - 3 \times 1} = \mathbf{18 \text{ years}}$$

$$\text{Father's age} = \frac{3 \times 6(4 - 1)}{4 \times 1 - 3 \times 1} = \mathbf{54 \text{ years}}$$

EXAMPLE 14 A father said to his son, "I was as old as you are at present at the time of your birth". If the father's age is 38 years now, the son's age five years back was.

[Assistant Grade Exam]

Solution Let the son's age 5 years back be x .

$$\therefore \text{Present age of son} = x + 5$$

$$\therefore 38 - (x + 5) = (x + 5)$$

$$\text{or } 38 - 5 - 5 = 2x$$

$$\text{or } x = \frac{28}{2} = \mathbf{14 \text{ years}}$$

EXAMPLE 15 The sum of the ages of 5 children born at the intervals of 3 years each is 50 years what is the age of the youngest child? [SSC]

Solution Let the age of youngest child be x .

$$\therefore x + (x + 3) + (x + 6) + (x + 9) + (x + 12) = 50$$

$$\text{i.e. } 5x + 30 = 50$$

$$\text{or} \quad 5x = 50 - 30$$

$$\text{or} \quad 5x = 20$$

$$\text{or} \quad x = \frac{20}{5} = 4 \text{ years}$$

Hence, age of youngest child = 4 years.

EXAMPLE 16 The sum of the present ages of a father and his son is 60 years. Six years ago, father's age was five times the age of his son. After six years, son's age will be.

[RRB]

Solution Let father's age be x and son's age be y

$$\therefore \quad x + y = 60 \quad (1)$$

Six years before,

$$x - 6 = 5(y - 6) \quad (2)$$

Solving (1) and (2),

$$x + y = 60 \quad (1)$$

$$(2) \Rightarrow x - 5y = 6 - 30 = -24 \quad (2)$$

$$(1) - (2) \Rightarrow y + 5y = 60 - (-24) = 84$$

$$\text{i.e.} \quad 6y = 84$$

$$\text{or} \quad y = \frac{84}{6} = 14$$

$$\therefore \quad (1) \Rightarrow x = 60 - y \\ = 60 - 14 = 46$$

After six years,

Age of father will be $x + 6 = 46 + 6 = 52$ years

Age of son will be $y + 6 = 14 + 6 = 20$ years

EXAMPLE 17 A person's present age is two-fifth of the age of his mother. After 8 years, he will be one-half of the age of his mother. How old is the mother at present?

[IGNOU]

Solution Let the present age of mother be x .

$$\therefore \quad \text{Person's age at present} = \frac{2}{5}x$$

After 8 years,

$$\text{Person's age will be } \frac{2}{5}x + 8 = \frac{1}{2}(x + 8)$$

$$\text{i.e.} \quad \frac{2}{5}x + 8 = \frac{x}{2} + 4$$

$$\text{or} \quad \frac{x}{2} - \frac{2}{5}x = 8 - 4$$

$$\text{or} \quad \frac{5x - 4x}{10} = 4$$

$$\begin{aligned} \text{or} \quad & \frac{x}{10} = 4 \\ \text{or} \quad & x = 40 \text{ years} \end{aligned}$$

EXAMPLE 18 The ratio between the present ages of A and B is 5 : 3 respectively. The ratio between A's age 4 years ago and B's age 4 years hence is 1 : 1. What is the ratio between A's age 4 years hence and B's age 4 years ago? [SIDBI]

Solution Let the present ages of A and B be x and y .

$$\therefore \quad \frac{x}{y} = \frac{5}{3} \quad (1)$$

$$\frac{x-4}{y+4} = \frac{1}{1} \quad (2)$$

$$\frac{x+4}{y-4} = ?$$

$$\text{From (1),} \quad x = \frac{5}{3}y$$

$$\text{From (2),} \quad \frac{5}{3}y - 4 = y + 4$$

$$\left(\frac{5}{3} - 1\right)y = 4 + 4 = 8$$

$$\frac{2}{3}y = 8$$

$$y = \frac{8 \times 3}{2} = 12$$

$$\begin{aligned} \text{From (1),} \quad x &= \frac{5}{3}y \\ &= \frac{5}{3} \times 12 = 20 \end{aligned}$$

$$\therefore \quad \frac{x+4}{y-4} = \frac{20+4}{12-4} = \frac{24}{8} = 3 \text{ or } 3:1$$



EXERCISES

- The ratio of ages of Sunil and Suresh is 2 : 3. Four years earlier their age ratio was 5 : 8. Find the present ages of Sunil and Suresh.
 - 12 years, 18 years
 - 24 years, 36 years
 - 20 years, 32 years
 - 30 years, 48 years
 - None of these
- The ratio of ages of Mohan and Madhu is 3 : 4. After 8 years their age ratio becomes 4 : 5. What are the present ages of Mohan and Madhu?
 - 21 years, 28 years
 - 27 years, 36 years
 - 24 years, 32 years
 - 30 years, 40 years
 - None of these

3. In a family, a couple has a son and a daughter. The age of the father is three times that of his daughter and the age of son is half of his mother. The wife is 9 years younger to her husband and the brother is seven years older than his sister. What is the age of the mother? [IAS]
 (a) 40 years (b) 45 years (c) 50 years (d) 60 years
4. A father is now three times as old as his son. Five years back, he was four times as old as his son. The age of the son is [IAS]
 (a) 12 (b) 15 (c) 18 (d) 20
5. The age of a father is twice that of his elder son. Ten years hence, the age of the father will be 3 times that of the younger son. If the difference of ages of two sons is 15 years, the age of the father is [IAS]
 (a) 60 years (b) 90 years (c) 80 years (d) 50 years
6. If the present age of X is half that of Y and after 20 years if the age of Y is $1\frac{1}{2}$ times more than X, what is the age of X now? [IAS]
 (a) 20 (b) 15 (c) 10 (d) 25
7. Father is aged three times more than his son Ronit. After 8 years, he would be two and a half times of Ronit's age. After further 8 years, how many times would he be of Ronit's age? [CBI]
 (a) 2 times (b) $2\frac{1}{2}$ times (c) $2\frac{3}{4}$ times (d) 3 times
8. Q is as much younger than R as he is older than T. If the sum of the ages of R and T is 60 years, what is definitely the difference between R and Q's age? [Bank PO]
 (a) 1 year (b) 2 years (c) 25 years
 (d) Data inadequate (e) None of these
9. The age of the father 10 years ago was thrice the age of his son. Ten years hence, father's age will be twice that of his son. The ratio of their present age is [LIC AAO]
 (a) 5 : 2 (b) 7 : 3 (c) 9 : 2 (d) 13 : 4
10. Eighteen years ago, a father was three times as old as his son. Now the father is only twice as old as his son. Then the sum of the present ages of the son and the father is [SSC]
 (a) 54 (b) 72 (c) 105 (d) 108
11. The ratio between the school ages of Neelam and Shaan is 5 : 6. If the ratio between the one-third age of Neelam and half of Shann's age is 5 : 9, then what is the school age of Shaan? [Bank PO]
 (a) 25 years (b) 30 years (c) 36 years
 (d) Cannot be determined (e) None of these
12. The ratio of the present ages of two brothers is 1 : 2 and 5 years back, the ratio was 1 : 3. What will be the ratio of their ages after 5 years? [SSC]
 (a) 1 : 4 (b) 2 : 3 (c) 3 : 5 (d) 5 : 6
13. A man is 24 years older than his son. In two years, his age will be twice the age of his son. The present age of the son is [RRB]
 (a) 14 years (b) 18 years (c) 20 years (d) 22 years
14. Present ages of X and Y are in the ratio 5 : 6. Seven years hence the ratio will become 6 : 7. What is present age of X in years? [Bank PO]
 (a) 35 (b) 42 (c) 49
 (d) Cannot be determined (e) None of these

15. The ratio between the present ages of P and Q is 5 : 7. If the difference between Q's present age and P's age after 6 years is 2, what is the sum of ages of P and Q at present? **[RRB]**
 (a) 48 years (b) 52 years (c) 56 years
 (d) Cannot be determined (e) None of these
16. The ratio between the ages of Arun and Deepak at present is 4 : 3. After 6 years, Arun's age will be 26 years. What is the age of Deepak at present? **[RRB]**
 (a) 12 years (b) 15 years (c) $19\frac{1}{2}$ years (d) 21 years
17. Present ages of Sameer and Anand are in the ratio 5 : 4. Three years hence, the ratio of their ages will become 11 : 9. What is Anand's present age in years? **[RBI]**
 (a) 24 (b) 27 (c) 40
 (d) Cannot be determined (e) None of these
18. The ratio between the present ages of P and Q is 6 : 7. If Q is 4 years older than P, what will be the ratio of ages of P and Q after 4 years? **[SBI PO]**
 (a) 3 : 4 (b) 3 : 5 (c) 4 : 3
 (d) Data inadequate (e) None of these
19. Six years ago, the ratio of the ages of Kunal and Sagar was 6 : 5. Four years hence, the ratio of their ages will be 11 : 10. What is Sagar's age at present? **[Bank PO]**
 (a) 16 years (b) 18 years (c) 20 years
 (d) Cannot be determined (e) None of these
20. A person was asked to state his age in years. His reply was "Take my age three years hence, multiply it by 3 and then subtract three times my age three years ago and you will know how old I am" What was the age of the person? **[SSC]**
 (a) 18 years (b) 20 years (c) 24 years (d) 32 years



1. (b) 2. (c) 3. (d) 4. (b) 5. (d) 6. (a) 7. (a) 8. (e) 9. (b) 10. (d)
 11. (d) 12. (c) 13. (d) 14. (a) 15. (a) 16. (b) 17. (a) 18. (e) 19. (a) 20. (a)



Solutions with Necessary Explanation

1. Let the ages of Sunil and Suresh at present be $2x$ and $5x$ respectively

Four years before, age ratio will be given by

$$\frac{2x - 4}{3x - 4} = \frac{5}{8}$$

or

$$8(2x - 4) = 5(3x - 4)$$

$\Rightarrow$

$$16x - 32 = 15x - 20$$

or

$$x = 32 - 20 = 12$$

$\therefore$ Present age of Sunil = $2x = 2 \times 12 = 24$ years

Present age of Suresh = $3x = 3 \times 12 = 36$ years

2. Let the ages of Mohan and Madhu be $3x$ and $4x$ respectively

After 8 years, their age ratio will be given by

$$\frac{3x + 8}{4x + 8} = \frac{4}{5}$$

$$\begin{aligned}\text{or} \quad & 5(3x + 8) = 4(4x + 8) \\ \text{or} \quad & 5(3x) - 4(4x) = 4 \times 8 - 5 \times 8 \\ & -x = (4 - 5)8 = -8 \\ \text{or} \quad & x = 8\end{aligned}$$

Age of Mohan = $3x = 3 \times 8 = 24$ years

Age of Madhu = $4x = 4 \times 8 = 32$ years

3. Let the age of mother be x years.

$$\begin{aligned}\text{Age of son} &= \frac{x}{2} \\ \text{Age of daughter} &= \frac{x}{2} - 7 \\ \therefore \text{Age of father} &= 3\left(\frac{x}{2} - 7\right) \\ 3\left(\frac{x}{2} - 7\right) - x &= 9 \\ \Rightarrow \frac{3x}{2} - 21 - x &= 9 \\ \text{or} \quad \frac{x}{2} &= 9 + 21 = 30 \\ \text{or} \quad x &= 60 \text{ years}\end{aligned}$$

4. Let the age of son be x .

$\therefore$ Age of father = $3x$

Five years ago,

$$\begin{aligned}\text{Age of father} &= 3x - 5 \\ &= 4(x - 5) \\ \text{i.e.} \quad 3x - 5 &= 4x - 20 \\ \text{or} \quad x &= 20 - 5 = 15 \text{ years}\end{aligned}$$

5. Let the ages of elder and younger sons be x_1 and x_2 .

$\therefore$ Age of father = $2x_1$

Ten years hence,

$$\text{Age of father} = 2x_1 + 10 = 3(x_2 + 10) \quad (1)$$

$$\text{Given} \quad x_1 - x_2 = 15$$

$$\text{From (1), } 2x_1 + 10 - (3x_2 + 30) = 0$$

$$2x_1 - 3x_2 - 20 = 0$$

$$(2x_1 - 2x_2) - x_2 - 20 = 0$$

$$2(15) - x_2 - 20 = 0$$

$$10 - x_2 = 0$$

$$\text{or} \quad x_2 = 10$$

$$\therefore \text{Age of elder son } x_1 = x_2 + 15 = 10 + 15 = 25 \text{ years}$$

$$\therefore \text{Age of father} = 2x_1 = 2 \times 25 = 50 \text{ years}$$

6. Let their present ages be x and y respectively

$$\therefore x = \frac{1}{2}y \quad (1)$$

After 20 years,

$$y + 20 = 1\frac{1}{2}(x + 20) \quad (2)$$

Multiplying (1) and (2), $y = 2x$ (3)

Subtracting (3) from (2), $20 = \frac{3}{2}(x + 20) - 2x$

or $20 = \frac{-x}{2} + 30$

or $\frac{x}{2} = 30 - 20 = 10$

or $x = 10 \times 2 = 20$

7. Let the age of Ronit be x .

$\therefore$ Age of father $y = x + 3x = 4x$

After 8 years,

$$\text{Age of father} = y + 8 = 2\frac{1}{2}(x + 8)$$

i.e. $4x + 8 = 2\frac{1}{2}(x + 8)$

i.e. $4x + 8 = \frac{5}{2}x + 20$

i.e. $4x - \frac{5}{2}x = 20 - 8$

$$\frac{3}{2}x = 12$$

i.e. $x = \frac{12 \times 2}{3} = \mathbf{8 \text{ years}}$

After next 8 years,

$$\begin{aligned} \text{Age of Ronit} &= x + 16 \\ &= 8 + 16 = 24 \text{ years} \end{aligned}$$

$$\begin{aligned} \text{Age of father} &= y + 16 = 4x + 16 \\ &= 4(8) + 16 = \mathbf{48 \text{ years}} \end{aligned}$$

$\therefore$ Age of father = $2 \times$ age of Ronit

$\therefore$ Age of father = **Two times** Ronit's age.

8. Given $T < Q < R$.

Let their ages be t , q and r .

$\therefore$ $r + t = 60$ (1)

$$r - q = q - t$$

or $2q = r + t = 60$

$\therefore$ $q = 30$

$\therefore$ $r = 45$ and $t = 15$ years

i.e. $r - q = 45 - 30 = \mathbf{15 \text{ years}}$

9. Let the age of son and father at present be x and y respectively

$\therefore$ 10 years ago,

$$y - 10 = 3(x - 10) \quad (1)$$

10 years hence,

$$y + 10 = 2(x + 10) \quad (2)$$

Subtracting (1) from (2), $10 - 10 = (2x + 20) - (3x - 30)$

$$20 = -x + 20 + 30$$

or

$$x = 30 \text{ years}$$

From (2),

$$y =$$

$$2(x + 10) - 10$$

$$= 2(x + 10 - 5) = 2(x + 5)$$

$$= 2(30 + 5) = 70 \text{ years}$$

$\therefore$

$$y : x = 7 : 3$$

10. Let the ages of son and father at present be x and y respectively

Eighteen years ago,

$$y - 18 = 3(x - 18) \quad (1)$$

$$y = 2x \quad (2)$$

$$x + y = ?$$

Subtracting (1) from (2), $18 = 2x - 3x + 54$

or

$$x = 54 - 18 = 36$$

From (2),

$$y = 2x = 2 \times 36 = 72$$

$\therefore$

$$x + y = 36 + 72 = 108$$

11. Let school ages of Neelam and Shaan be x and y respectively

$\therefore$

$$x : y = 5 : 6 = 5k : 6k$$

Given,

$$\frac{5k}{3} : \frac{6k}{2} = 5 : 9$$

i.e.

$$\frac{\frac{5k}{3}}{\frac{6k}{2}} = \frac{5}{9}$$

or

$$\frac{5 \times 9}{3} k = \frac{6 \times 5}{2} k$$

$$15k = 15k$$

Shaan's school age **cannot be determined**.

12. Let the present ages of brothers be x and y respectively

$\therefore$

$$x : y = 1 : 2$$

5 years ago,

$$(x - 5) : (y - 5) = 1 : 3$$

Then

$$(x + 5) : (y + 5) = ?$$

$$\frac{x}{y} = \frac{1}{2} \quad (1)$$

$$\frac{x - 5}{y - 5} = \frac{1}{3} \quad (2)$$

From (1),

$$2x = y \quad (A)$$

From (2),

$$3(x - 5) = y - 5 \quad (B)$$

Subtracting (B) from (A)

$$2x - 3(x - 5) = y - (y - 5)$$

or

$$2x - 3x + 15 = y - y + 5$$

$$\begin{aligned} \text{or} & \quad -x + 15 = 5 \\ \text{or} & \quad x = 15 - 5 = \mathbf{10 \text{ years}} \\ \text{From (A),} & \quad y = 2x = \mathbf{20 \text{ years}} \end{aligned}$$

$$\therefore \frac{(x+5)}{(y+5)} = \frac{10+5}{20+5} = \frac{15}{25} = \frac{3}{5}$$

13. Let the age of son at present be x .

$$\therefore \text{Age of man } y = x + 24 \quad (1)$$

After 2 years,

$$y + 2 = 2(x + 2) \quad (2)$$

$$\text{Subtracting (1) from (2), } (y + 2) - y = 2(x + 2) - (x + 24)$$

$$\text{i.e. } 2 = 2x + 4 - x - 24$$

$$\text{i.e. } 2 = x - 20$$

$$\text{or } x = 2 + 20 = \mathbf{22 \text{ years}}$$

14. Given $x : y = 5 : 6$

$$\text{or } \frac{x}{y} = \frac{5}{6} \quad (1)$$

After 7 years,

$$\frac{x+7}{y+7} = \frac{6}{7} \quad (2)$$

$$\text{From (1), } 6x = 5y \quad (a)$$

$$\text{From (2), } 7(x+7) = 6(y+7) \quad (b)$$

$$\text{Multiplying (a) by 6 } 36x = 30y \quad (i)$$

$$\text{Multiplying (b) by 5 } 35(x+7) = 30(y+7) \quad (ii)$$

$$\text{Subtracting (i) from (ii), } 35(x+7) - 36x = 30(y+7) - 30y$$

$$\text{i.e. } 35x - 36x + 35 \times 7 = 30y - 30y + 30 \times 7$$

$$\text{or } -x + 35 \times 7 = 30 \times 7$$

$$\text{or } x = (35 - 30) \times 7 = \mathbf{35 \text{ years}}$$

15. Let the ages of P and Q be $5k$ and $7k$ respectively.

$$\therefore 7k - (5k + 6) = 2$$

$$\Rightarrow 2k - 6 = 2$$

$$\text{or } k = \frac{2+6}{2} = 4$$

$$\therefore \text{Required sum} = 5k + 7k = 12k = 12 \times 4 = \mathbf{48 \text{ years}}$$

16. Let ages of Arun and Deepak be $4k$ and $3k$ respectively

After 6 years,

$$\text{Arun's age} = 4k + 6 = 26$$

$$\therefore k = \frac{26-6}{4} = 5$$

$$\therefore \text{Present age of Deepak} = 3k = 3 \times 5 = \mathbf{15 \text{ years}}$$

17. Let present ages of Sameer and Anand be $5k$ and $4k$.

$$\text{After 3 years, their age ratio will be given by } \frac{5k+3}{4k+3} = \frac{11}{9}$$

i.e. $9(5k + 3) = 11(4k + 3)$

$$45k + 27 = 44k + 33$$

$\therefore k = 33 - 27 = 6$

$\therefore$ Present age of Anand $= 4k = 4 \times 6 = 24$ years

18. Let the present ages of P and Q be $6k$ and $7k$ respectively

Given $7k - 6k = 4$

or $k = 4$

After 4 years, their age ratio will be given by,

$$\frac{6k + 4}{7k + 4} = \frac{6(4) + 4}{7(4) + 4} = \frac{28}{32} = \frac{7}{8}$$

19. Let ages of Kunal and Sagar at present be x and y .

Six years ago,

$$\frac{x - 6}{y - 6} = \frac{6}{5} \quad (1)$$

Four years after,

$$\frac{x + 4}{y + 4} = \frac{11}{10} \quad (2)$$

From (1), $5(x - 6) = 6(y - 6)$ (a)

From (2), $10(x + 4) = 11(y + 4)$ (b)

Multiplying (a) by 2 $10(x - 6) = 12(y - 6)$ (c)

Subtracting (b) from (c) $10\{(x - 6) - (x + 4)\} = 12(y - 6) - 11(y + 4)$

or $10\{-6 - 4\} = 12y - 72 - 11y - 44$

$$-100 = y - 116$$

or $y = 116 - 100 = 16$

$\therefore$ Present age of Sagar = 16 years

20. Let the present age of person be x .

$$x = (x + 3) \times 3 - (x - 3)3$$

$$= 3x + 9 - 3x + 9$$

$$= 18$$

$\therefore$ Present age of person = 18 years

Profit and Loss

Profit and loss are calculated on the cost price. The price at which a trader buys an article is called the *cost price* (C.P.). The price at which a trader sells the article is called the *selling price* (S.P.)



Important Formulae

$$\% \text{ profit} = \frac{\text{S.P.} - \text{C.P.}}{\text{C.P.}} \times 100$$

$$\% \text{ loss} = \frac{\text{C.P.} - \text{S.P.}}{\text{C.P.}} \times 100$$

$$\text{S.P.} = \left(\frac{100 + \% \text{ profit}}{100} \right) \times \text{C.P. in case of profit}$$

$$\text{S.P.} = \left(\frac{100 - \% \text{ loss}}{100} \right) \times \text{C.P. in case of loss}$$

or

$$\text{C.P.} = \frac{100}{(100 + \% \text{ profit})} \times \text{S.P. in case of profit}$$

$$\text{C.P.} = \frac{100}{(100 - \% \text{ loss})} \times \text{S.P. in case of loss}$$

If a person sells two similar items one at a gain of $x\%$ and the other at a loss of $x\%$, then the seller will be always at a loss of $\left(\frac{x}{10} \right)^2$

$$\text{i.e. Loss \%} = \left(\frac{\text{Common loss and profit \%}}{10} \right)^2 = \left(\frac{x}{10} \right)^2$$

EXAMPLE 1 A man buys x tables for ₹ y and sells w tables for ₹ z . Find the net and % profit or loss that he makes.

Solution S.P. of w table = ₹ z .

$$\text{S.P. of } x \text{ table} = ₹ \frac{z}{w} \cdot x$$

$$\text{Net profit} = \left(\frac{z}{w} \right) \cdot x - y$$

$$\% \text{ Profit} = \frac{\left(\frac{z}{w} \right) x - y}{y} \times 100\%$$

i.e.
$$\% \text{ Profit} = \left(\frac{zx}{wy} - 1 \right) \times 100\%$$



Note: Successive profits and losses

S.P. = C.P. $\times$ Product of change factors for each stage

$$\frac{1}{\text{Change factor}} = \frac{1}{\left(\frac{100 + \% \text{ profit}}{100} \right)} \text{ or } \frac{1}{\left(\frac{100 - \% \text{ loss}}{100} \right)}$$

$$\% \text{ gain} = \frac{\text{True weight} - \text{False weight}}{\text{False weight}} \times 100$$

$$= \frac{\text{Error}}{\text{True value} - \text{Error}} \times 100$$

EXAMPLE 2 A dishonest dealer professes to sell his goods at the cost price, but he uses a weight of 950 g for 1 kg. Find his gain per cent.

Solution Let the goods cost be ₹ 10 per kg.

He sells for ₹ 10 goods worth ₹ 9.50

$$\therefore \text{Gain} = ₹ 10 - ₹ 9.50 = ₹ 0.50$$

$$\therefore \% \text{ Gain} = \left(\frac{0.5}{10 - 0.5} \right) \times 100 = 5 \frac{5}{19} \%$$

$$\text{or } \left(\frac{1000 - 950}{950} \right) \times 100 = \left(\frac{50}{950} \right) \times 100 = \frac{100}{19} = 5 \frac{5}{19} \%$$

EXAMPLE 3 A dishonest dealer sells goods at 4% loss on the cost price but uses 16 g instead of 18 g. What is his percentage profit or loss?

Solution Let the cost price of goods be ₹ x/kg .

$$\text{Selling price} = \frac{₹ x(100 - 4)}{100} = ₹ \frac{24}{25} x/\text{kg}$$

Let the quantity of goods bought be y kg.

$$\therefore \text{Total investment} = ₹ xy$$

$$\text{Total return} = ₹ \frac{24}{25} x \times y \times \frac{18}{16} = ₹ \frac{27}{25} xy$$

$$\therefore \% \text{ profit} = \frac{\frac{27}{25} xy - xy}{xy} \times 100 = \frac{2xy/25}{xy} \times 100 = \frac{200}{25} = 8\%$$



Short cut: $\% \text{ profit or loss} = (100 - 4) \left[\frac{18}{16} \right] - 100 = 108 - 100 = 8\%$

To find the selling price:

EXAMPLE 4 A person bought an article for ₹ 240. For how much should he sell it so as to gain 20%?

Solution If the C.P. is ₹ 100, then S.P. will be ₹ 120.

Here C.P. is ₹ 240.

$$\therefore \text{S.P.} = 240 \times \frac{120}{100} = ₹ 288$$

Rule of fraction

If $x\%$ is the loss in the trade, then the calculating figures are 100 and $(100 - x)$.

$$\therefore \text{S.P.} = \frac{100 - x}{100} \times \text{C.P.}$$

If $x\%$ is the gain in the trade, then the calculating figures are 100 and $(100 + x)$.

$$\therefore \text{S.P.} = \frac{100 + x}{100} \times \text{C.P.}$$

EXAMPLE 5 By selling goods for ₹ 356.89, a trader lost 11%. Find the cost price.

Solution Here the C.P. is more than S.P.

$$\text{C.P.} = \frac{100}{100 - x} \times \text{S.P.} = \frac{100}{100 - 11} \times 356.89 = \frac{100}{89} \times 356.89 = ₹ 401$$

RULES

Rule 1: When an article is passed through three persons, and each time trade was at a profit of $x\%$ and $y\%$ respectively. If the cost price for the last person is C rupees, the cost price for the first person is given by

$$\text{the cost price for the first person} = \text{cost price for the third person} \times \frac{100}{100 + x} \times \frac{100}{100 + y}.$$

Rule 2: When there are two successive profits of $x\%$ and $y\%$, then the resultant profit per cent is given by $\left(x + y + \frac{xy}{100} \right)$.

Rule 3: When there is a profit of $x\%$ and loss of $y\%$ in a transaction, then the resultant profit or loss per cent is given by $\left(x - y - \frac{xy}{100} \right)$.

Thus in Q. 6., the resultant profit or loss per cent = $\left(20 - 10 - \frac{20 \times 10}{100} \right) = 8\% \text{ profit}$

EXAMPLE 6 By selling an article for ₹ 570, a tradesman would lose 5%. At what price must he sell it to gain 10%?

Solution $(100 - 5)\% \text{ C.P.} = 570$

$$\therefore \text{C.P.} = 570 \times \frac{100}{95}$$

$$\therefore (100 + 10)\% \text{ of C.P.} = \frac{570}{95} \times 110 = \text{₹ } 660$$

EXAMPLE 7 A person sold an article at a profit of 15%. If he had bought it at 10% less and sold it for ₹ 4.5 more, he would have gained 25%. Find the cost price.

Solution Let the cost of article be ₹ 100

$$\therefore \text{Selling price} = (100 + 15)\% \text{ of } 100 = \text{₹ } 115$$

$$\text{If C.P. is less by } 10\%, \text{ then new C.P.} = 100 - 10 = \text{₹ } 90$$

$$\therefore \text{Selling price at } 25\% \text{ gain} = 90 \times \frac{125}{100} = \text{₹ } 112.5$$

$$\therefore \text{Difference in S.P.} = 112.5 - 110 = \text{₹ } 2.5$$

$$\therefore \text{For difference of ₹ } 4.5, \text{ C.P.} = \frac{100}{2.5} \times 4.5 = \text{₹ } 180$$



Short cut: If a person sells an article at $x\%$ profit and if he bought it at $y\%$ less and sold it for ₹ z less he would have gained $k\%$. Then the cost price is given by

$$\begin{aligned} \text{C.P.} &= \frac{z \times 100 \times 100}{(100 + x)100 - (100 - y)(100 + k)} \\ &= \frac{4.5 \times 100 \times 100}{(100 + 15)100 - (100 - 10)(100 + 25)} \\ &= \frac{4.5 \times 100 \times 100}{115 \times 100 - 90 \times 125} = \frac{45000}{11500 - 11250} \\ &= \frac{45000}{250} = \text{₹ } 180 \end{aligned}$$

EXAMPLE 8 A buys a watch for ₹ 600 and sells it to B at 10% profit. B sells it to C for 5% profit. C uses it for two years and then sells it to D at a loss of 20%. For how much does C sell the watch to D?

Solution C.P. = ₹ 600

$$\text{For the first stage, i.e. A to B; change factor} = \frac{100 + 10}{100} = \frac{110}{100}$$

$$\text{For the second stage, i.e. B to C, change factor} = \frac{100 + 5}{100} = \frac{105}{100}$$

$$\text{For the third stage, i.e. C to D; change factor} = \frac{100 - 20}{100} = \frac{80}{100}$$

$$\begin{aligned}\text{Cost price to C} &= 600 \times \frac{110}{100} \times \frac{105}{100} \\ &= \frac{6 \times 11 \times 105}{10} = 3 \times 11 \times 21 = \text{₹ } 693\end{aligned}$$

$$\text{S.P. to D by C} = 693 \times \frac{80}{100} = \text{₹ } 554.4$$

Dishonest seller/buyer using faulty measures of length/weight

EXAMPLE 9 A dishonest shopkeeper professes to sell cloth at cost price, but he uses faulty metre rod. His metre rod measures 95 cm only. Find his gain per cent.

Solution Profit = Price of 5 cm cloth.

C.P. of the cloth sold as one metre = price of 95 cm cloth.

$$\therefore \quad \% \text{ profit} = \frac{5}{95} \times 100\% = 5 \frac{5}{19} \% \text{ or } 5.26 \%$$



Short cut: The general relationship can be written as

$$\% \text{ gain} = \frac{\text{Difference between true measure and False measure}}{\text{Amount for which payment made}}$$

EXAMPLE 10 A person gains $x\%$ when he sells an article for ₹ z . What would be the profit or loss per cent, if he sold it for ₹ w .

$$\text{Solution} \quad \text{C.P.} = z \times \left(\frac{100}{100 + x} \right)$$

$$\therefore \quad \text{profit} = \frac{w - z \left(\frac{100}{100 + x} \right)}{z \left(\frac{100}{100 + x} \right)} \times 100\% = \left[\frac{w}{z} \left(\frac{100 + x}{100} \right) - 1 \right] \times 100\%$$

$$\text{or} \quad \% \text{ profit} = \left[\frac{w}{z} (100 + x) - 100 \right] \%$$



Short cut: A man sells an article at a profit of $x\%$. If he sells it for ₹ z more, he would make a profit of $y\%$. Find the cost price of the article.

Solution $(y\% - x\%) \text{ of C.P.} = \text{₹ } z$

$$\text{or} \quad \text{C.P.} = \frac{z}{y\% - x\%}$$

$$\therefore \quad \text{Cost price of article} = \frac{z}{y - x} \times 100$$

EXAMPLE 11 A man sold an article at a loss of $x\%$. If he sells it for ₹ z more, he would gain $y\%$. Find the cost price of the article.

Solution Initial S.P. = $\frac{100 - x}{100} \times \text{C.P.}$

New S.P. = $\frac{100 + y}{100} \times \text{C.P.}$

So, $\left(\frac{100 + y}{100}\right) \text{C.P.} - \left(\frac{100 - x}{100}\right) \text{C.P.} = z$

or $\left[\frac{(100 + y) - (100 - x)}{100}\right] \times \text{C.P.} = z$

or $\frac{y + x}{100} \times \text{C.P.} = z$

or $\text{C.P.} = \boxed{\frac{z}{y + x} \times 100}$

EXAMPLE 12 A man sold an article at a profit of $x\%$. If he had bought it for $w\%$ less and sold it for ₹ z more, he would have made a profit of $y\%$. Find the cost price of the article.

Solution Let the original cost price be ₹ P .

Initial S.P. = ₹ $\left(\frac{100 + x}{100}\right)P$

New C.P. = ₹ $\left(\frac{100 - w}{100}\right)P$

New S.P. = ₹ $\left(\frac{100 + y}{100}\right)\left(\frac{100 - w}{100}\right)P$

Difference between the two selling prices = ₹ z

i.e. $\left(\frac{100 + y}{100}\right)\left(\frac{100 - w}{100}\right)P - \left(\frac{100 + x}{100}\right)P = z$

or $P = \frac{z}{\left(\frac{100 + y}{100}\right)\left(\frac{100 - w}{100}\right) - \left(\frac{100 + x}{100}\right)}$

or $P = \boxed{\frac{z \times 100 \times 100}{(100 + y)(100 - w) - 100(100 + x)}}$

EXAMPLE 13 A man buys certain quantity of an article for ₹ z . He sells m th part of the stock at a loss of $x\%$. At what per cent gain should he sell the remaining stock so as to make an overall profit of $y\%$ on the total transaction?

Solution The C.P. of m th part of stock = ₹ $m \cdot z$.

The S.P. of m th part of stock = $\left(\frac{100 - x}{100}\right)m \cdot z$.

For $y\%$ profit on total transaction, its S.P. = $\left(\frac{100 + y}{100}\right)z$

The S.P. of remaining $(1 - m)$ th part of stock

$$= \left(\frac{100 + y}{100}\right)z - \left(\frac{100 - x}{100}\right)m \cdot z = \left[\frac{(100 + y) - (100 - x)m}{100}\right]z$$

C.P. of the remaining $(1 - m)$ th part of stock = $(1 - m)z$.

$$\therefore \% \text{ profit} = \frac{\text{S.P.} - \text{C.P.}}{\text{C.P.}} \times 100\%$$

$$\begin{aligned} \% \text{ profit} &= \frac{\left[\frac{(100 + y) - (100 - x)m}{100}\right]z - (1 - m)z}{(1 - m)z} \times 100\% \\ &= \left[\frac{(100 + y) - (100 - x)m - 100(1 - m)}{(1 - m)z \times 100}\right]z \times 100\% \end{aligned}$$

$$\text{The required \% profit} = \boxed{\frac{(y + xm)}{(1 - m)}\%}$$

EXAMPLE 14 A man invests ₹ z in shares. He sells m th part at a profit of $x\%$, and the remaining at a loss of $y\%$. Find his overall per cent profit or loss.

Solution C.P. of m th part = mz

$$\text{S.P. of } m\text{th part} = \left(\frac{100 + x}{100}\right)mz$$

$$\text{C.P. of } (1 - m)\text{th part} = (1 - m)z$$

$$\text{S.P. of } (1 - m)\text{th part} = \frac{100 - y}{100}(1 - m)z$$

$$\begin{aligned} \text{Total S.P.} &= \left(\frac{100 + x}{100}\right)mz + \left(\frac{100 - y}{100}\right)(1 - m)z \\ &= \frac{[(100 + x)m + (100 - y)(1 - m)]z}{100} \end{aligned}$$

$$\text{Total C.P.} = ₹ z$$

$$\% \text{ profit} = \frac{\text{S.P.} - \text{C.P.}}{\text{C.P.}} \times 100\%$$

$$\begin{aligned} &= \frac{\left[\left(\frac{100 + x}{100}\right)m + \left(\frac{100 - y}{100}\right)(1 - m)\right]z - z}{z} \times 100\% \\ &= \left[\frac{(100 + x)m + (100 - y)(1 - m)}{100} - 1\right] \times 100\% \end{aligned}$$

$$= [(100 + x)m + (100 - y)(1 - m) - 100]\%$$

$$= [xm - y + ym]\%$$

i.e. % profit = $[mx - y(1 - m)]\%$

For no profit, no loss,

$$mx - y(1 - m) = 0$$

or
$$\frac{x}{y} = \frac{1 - m}{m}$$

EXAMPLE 15 A man buys certain quantity of an article for ₹ z . He sells $1/4$ th part of the stock at a loss of 20%. At what per cent gain should he sell the remaining stock so as to make an overall profit of 10% on the total transaction.

Solution The required % profit = $\frac{y + xm}{1 - m}\%$

Here $x = 20$; $y = 10$; $m = 1/4$;

The required % profit = $\frac{10 + 20/4}{1 - 1/4} = \frac{10 + 5}{3/4} = 20\%$

EXAMPLE 16 A reduction of $x\%$ in the price of an article enables a person to buy y kg more for ₹ z . Find the reduced and the original price per kg of the article.

Solution Saving due to price reduction = $\frac{xz}{100}$

Cost of y kg at the reduced price = ₹ $\frac{xz}{100}$

∴ Cost of 1 kg at the reduced price = ₹ $\frac{xz}{100y}$

∴ Reduced price = ₹ $\frac{xz}{100y}$ per kg

∴ Original price = $\frac{100}{100 - x} \times$ Reduced price

$$= \frac{100}{100 - x} \times \frac{xz}{100y}$$

Original price = $\frac{xz}{(100 - x)y}$ per kg

If $x\%$ is hike, then

Original price = $\frac{xz}{(100 + x)y}$ per kg

EXAMPLE 17 A man purchased m oranges at x a rupee and n oranges at y a rupee. He mixed them together and sold them at z a rupee. What is his per cent loss or gain. What happens when $m = n$?

Solution C.P. of m oranges at x a rupee = ₹ $\frac{m}{x}$

C.P. of n oranges at y a rupee = ₹ $\frac{n}{y}$

Total C.P. of $(m + n)$ oranges = ₹ $\left(\frac{m}{x} + \frac{n}{y}\right)$

Total S.P. of $(m + n)$ oranges at z a rupee = ₹ $\frac{m + n}{z}$

$$\% \text{ profit} = \frac{\text{S.P.} - \text{C.P.}}{\text{C.P.}} \times 100\%$$

$$= \left(\frac{\text{S.P.}}{\text{C.P.}} - 1 \right) \times 100\%$$

i.e.

$$\% \text{ profit} = \left(\frac{\frac{m + n}{z}}{\frac{m}{x} + \frac{n}{y}} - 1 \right) \times 100\%$$

$$= \left(\frac{\frac{m + n}{z}}{\frac{m y + n x}{x y}} - 1 \right) \times 100\%$$

$$= \left[\frac{(m + n)xy}{(m y + n x)z} - 1 \right] \times 100\%$$

When $m = n$,

$$\% \text{ profit} = \left[\frac{2mxy}{m(x + y)z} - 1 \right] \times 100\%$$

$$= \left[\frac{2xy}{(x + y)z} - 1 \right] \times 100\%$$

EXAMPLE 18 A man buys a certain number of mangoes at 6 a rupee and the same number at 9 a rupee too. He mixes and sells them at 8 a rupee. Find his per cent profit or loss.

Solution $\% \text{ profit} = \left[\frac{2xy}{(x + y)z} - 1 \right] \times 100\%$

Let the man purchase N number at each rate.

$$\therefore \text{Total C.P.} = \frac{N}{6} + \frac{N}{9} = \frac{5}{18}N;$$

$$x = 6; y = 9; z = 18$$

$$\therefore \text{Total S.P.} = \frac{2N}{8} = \text{₹} \frac{N}{4}$$

$$\begin{aligned} \therefore \% \text{ profit} &= \left(\frac{N/4 - 5N/18}{5N/18} \right) \times 100\% \\ &= \left[\frac{N/4}{5N/18} - 1 \right] \times 100\% \\ &= \left[\frac{18}{4 \times 5} - 1 \right] \times 100\% \\ &= -10\% \end{aligned}$$

$$\therefore \text{loss} = 10\%$$

EXAMPLE 19 m kg of rice costs n kg of wheat; p kg of wheat costs q kg of sugar; r kg of sugar costs s kg of coffee; t kg of coffee costs u kg of milk; v kg of milk costs w kg of tea. If x kg of tea cost ₹ y , find the cost of z kg of rice.

Solution Cost of m kg of rice = Cost of n kg of wheat;

Cost of p kg of wheat = Cost of q kg of sugar;

Cost of r kg of sugar = Cost of s kg of coffee;

Cost of t kg of coffee = Cost of u kg of milk;

Cost of v kg of milk = Cost of w kg of tea;

Cost of x kg of tea = ₹ y

? = Cost of z kg of rice

$$\therefore m \times p \times r \times t \times v \times x ? = n \times q \times s \times u \times w \times y \times z$$

$$\therefore ? = \frac{n \times q \times s \times u \times w \times y \times z}{m \times p \times r \times t \times v \times x}$$

EXAMPLE 20 Alfred buys an old scooter for ₹ 4700 and spends ₹ 800 on its repairs. If he sells the scooter for ₹ 5800, what is his gain per cent?

Solution C.P. = ₹ 4700 + ₹ 800 = ₹ 5500

S.P. = ₹ 5800

$$\begin{aligned} \therefore \text{gain \%} &= \frac{\text{S.P.} - \text{C.P.}}{\text{C.P.}} \times 100 = \frac{5800 - 5500}{5500} \times 100 \\ &= \frac{300}{5500} \times 100 = 5 \frac{25}{55} = 5 \frac{5}{11} \% \end{aligned}$$

EXAMPLE 21 If the selling price of an article is $\frac{5}{3}$ of its cost price, find the profit in the transaction.

$$\text{Solution Profit} = \frac{\text{S.P.} - \text{C.P.}}{\text{C.P.}} \times 100 = \frac{\frac{5}{3} \text{C.P.} - \text{C.P.}}{\text{C.P.}} \times 100 = \frac{2}{3} \times 100 = 66 \frac{2}{3} \%$$

**EXERCISES**

1. A trader sells rice at a profit of 5% and uses a weight which is 25% less. Find the total percentage gain.
(a) 20% (b) 30% (c) 40% (d) 50% (e) None of these
2. A person purchases 8 pens for ₹ 10 and sells 10 pens for ₹ 8. How much profit or loss does he make?
(a) 30% gain (b) 30% loss (c) 36% gain (d) 36% loss (e) None of these
3. A shopkeeper buys an article at 8 for ₹ 16 and sells them at 9 for ₹ 20. What is his gain or loss per cent?
(a) 10% loss (b) 10% gain (c) $11\frac{1}{9}\%$ gain (d) $11\frac{1}{9}\%$ loss (e) None of these
4. A person bought an article for ₹ 240. For how much should he sell it so as to lose 20%?
(a) 120 (b) 180 (c) 240 (d) 192 (e) None of these
5. By selling an article for ₹ 384, a trader gains 20%. Find the cost price of article.
(a) 160 (b) 240 (c) 320 (d) 480 (e) None of these
6. A sells an article to B at a profit of 20% and B sells it to C at a loss of 10%. If C pays ₹ 540 for it, at what price did A buy?
(a) 400 (b) 500 (c) 600 (d) 700 (e) None of these
7. A man sold a cow at a loss of 12%. If he had been able to sell it at a gain of 6%, it would have fetched ₹ 72 more than it did. What was the cost price?
(a) 300 (b) 500 (c) 400 (d) 600 (e) None of these
8. An article is sold for ₹ 4950 at a gain of 10%. What would have been the gain or loss per cent if it had been sold for ₹ 4275?
(a) 10% gain (b) 10% loss (c) 5% gain (d) 5% loss (e) None of these
9. A machine is sold at a profit of 15%. Had it been sold for ₹ 540 more 24% would have been gained. Find the cost price.
(a) 4800 (b) 5400 (c) 6000 (d) 6400 (e) None of these
10. A person sold a radio at a loss of 5%. Had he sold it for ₹ 210 more, he would have gained 25%. For what value should he sell it in order to gain 35%?
(a) 645 (b) 954 (c) 654 (d) 945 (e) None of these
11. An article is sold at a profit of 20%. If both the cost price and selling price are ₹ 150 less, profit would be 5% more. Find the cost price.
(a) 750 (b) 650 (c) 950 (d) 850 (e) None of these
12. A trader sells his table at a profit of 15% and the chair at a loss of $7\frac{1}{2}\%$, but on the whole he gains ₹ 50. If he sells the table at a loss of $7\frac{1}{2}\%$, and chair at a profit of 15%, then he neither gains nor loses. Find the cost price of the table
(a) $\frac{5000}{9}$ (b) $\frac{4000}{9}$ (c) $\frac{4400}{9}$ (d) $\frac{5300}{9}$ (e) None of these
13. A vendor started selling vegetables at ₹ 10 per kg, but couldn't find buyers at this rate. So, he reduced the price to 8.10/kg, but uses a faulty weight of 900 g for 1 kg. Find the per cent change in the actual price or loss.
(a) 5% (b) 10% (c) 15% (d) 20% (e) None of these

14. The cost price of 6 pens is equal to the selling price of 4 pens. Find the profit per cent.
(a) 25% (b) 60% (c) 50% (d) 80% (e) None of these
15. If cost price of 20 tables is equal to selling price of 25 tables, find the loss%
(a) 10% (b) 20% (c) 30% (d) 40% (e) None of these
16. A man sells an article at a loss of 5%. If he had bought it at 20% more and sold it for ₹ 115 less, he would have made a loss of 40%. Find the cost price of the article.
(a) 400 (b) 450 (c) 500 (d) 600 (e) None of these
17. When Ram sold his book for ₹ 255 he made a loss of 15%. For how much he should have sold it if he wanted to make a profit of 5%?
(a) 275 (b) 315 (c) 295 (d) 335 (e) None of these
18. A man invests ₹ 21,000 in shares of a company. He sells one-third of total shares at a profit of 10%. But the remaining shares he has to sell at 5% loss. Find his overall profit or loss%.
(a) 10% profit (b) 10% loss (c) No profit no loss
(d) 5% loss (e) None of these
19. A man buys 100 kg of sugar for ₹ 2400. He sold $\frac{1}{4}$ th of the stock at a loss of 5%. At what per cent profit should he sell the remaining stock so as to make an overall profit of 20% on the whole transaction?
(a) 25% (b) $27\frac{1}{3}\%$ (c) 28% (d) $28\frac{1}{3}\%$ (e) None of these
20. A man buys 300 dozen of bananas for ₹ 3,600. He sold $\frac{1}{3}$ rd of the stock at a loss of 10% and the remaining $\frac{2}{3}$ rd stock at a gain of 10%. Find his overall per cent profit or loss.
(a) $\frac{10}{3}\%$ gain (b) $\frac{10}{3}\%$ loss (c) $\frac{14}{3}\%$ loss (d) $\frac{14}{3}\%$ gain (e) None of these
21. A shopkeeper mixes 20 kg of rice which cost him ₹ 5 per kg with 35 kg of rice which cost him ₹ 7 per kg. Then he sells the mixture at ₹ 6.5 per kg. Find his total profit and per cent profit.
(a) $17,5\frac{41}{69}\%$ (b) $15,4\frac{47}{69}\%$ (c) $12,5,3\frac{43}{69}\%$ (d) $10,2\frac{37}{69}\%$ (e) None of these
22. A shopkeeper sold an article offering a discount of 5% and earned a profit of 23.5%. What would have been the percentage of profit earned if no discount was offered? [Bank PO]
(a) 24.5 (b) 28.5 (c) 30
(d) Data inadequate (e) None of these
23. The labelled price of a cupboard is ₹ 6500. The shopkeeper sold it by giving 5% discount on the labelled price and earned a profit of 15%. What approximately is the cost price of the cupboard? [Bank PO]
(a) ₹ 5000 (b) ₹ 5350 (c) ₹ 5600 (d) ₹ 5800 (e) ₹ 6000
24. A trader marked the selling price of an article at 10% above the cost price. At the time of selling, he allows certain discount and suffers a loss of 1%. He allowed a discount of [SSC]
(a) 9% (b) 10% (c) 10.5% (d) 11%
25. At what price should a shopkeeper mark a radio that costs him ₹ 1200 in order that he may offer a discount of 20% on the marked price and still make a profit of 25%? [Bank PO]
(a) ₹ 1675 (b) ₹ 1875 (c) ₹ 1900 (d) ₹ 2025 (e) None of these
26. By selling an umbrella for ₹ 300, a shopkeeper gains 20%. During a clearance sale, the shopkeeper allows a discount of 10% on the marked price. His gain per cent during the sale is [MBA]
(a) 7 (b) 7.5 (c) 8 (d) 9

27. The price of an article is raised by 30% and then two successive discount of 10% each are allowed. Ultimately, the price of the article is [SSC]
(a) decreased by 5.3% (b) increased by 3%
(c) increased by 5.3% (d) increased by 10%
28. The difference between a discount of 35% and two successive discount of 20% on a certain bill was ₹ 22. Find the amount of the bill [Bank PO]
(a) ₹ 200 (b) ₹ 1100 (c) ₹ 2200
(d) Data inadequate (e) None of these
29. Successive discounts of 10%, 12% and 15% amount to a single discount of [RRB]
(a) 32.68% (b) 35.28% (c) 36.68% (d) None of these
30. A fan is listed at ₹ 1500 and a discount of 20% is offered on the list price. What additional discount must be offered to the customer to bring the net price to ₹ 1104? [SSC]
(a) 8% (b) 10% (c) 12% (d) 15%
31. A pair of article, was bought for ₹ 37.40 at a discount of 15%. What must be the marked price of each of the articles? [AAO Exam]
(a) ₹ 11 (b) ₹ 22 (c) ₹ 33 (d) ₹ 44
32. Albert buys 4 horses and 9 cows for ₹ 13400. If he sells the horse at 10% profit and the cows at 20% profit, then he earns a total profit of ₹ 1880. The cost of a horse is [CDS]
(a) ₹ 1000 (b) ₹ 2000 (c) ₹ 2500 (d) ₹ 3000
33. The cash difference between the selling prices of an article at a profit of 4% and 6% is ₹ 3. The ratio of the two selling price is [CBI]
(a) 51 : 52 (b) 52 : 53 (c) 51 : 53 (d) 52 : 55
34. If a man reduces the selling price of a fan from ₹ 400 to ₹ 380, his loss increases by 2%. The cost price of the fan is [RRB]
(a) ₹ 480 (b) ₹ 500 (c) ₹ 600 (d) None of these
35. A sells a bicycle to B at a profit of 20%. B sells it to C at a profit of 25%. If C pays ₹ 225 for it, the cost price of the bicycle for A is [BSF]
(a) ₹ 110 (b) ₹ 120 (c) ₹ 125 (d) ₹ 150
36. A shopkeeper cheats to the extent of 10% while buying as well as selling, by using false weights. His total gain is [Bank PO]
(a) 10% (b) 11% (c) 20% (d) 21% (e) $22\frac{2}{9}\%$
37. A dishonest dealer use a scale of 90 cm instead of a metre scale and claims to sell at cost price. His profit is [NIFT]
(a) 9% (b) 10% (c) 12% (d) None of these
38. A dairyman pays ₹ 6.40 per litre of milk. He adds water and sells the mixture at ₹ 8 per litre, thereby making 37.5% profit. The proportion of water to milk received by the customers is [MAT]
(a) 1 : 10 (b) 1 : 12 (c) 1 : 15 (d) 1 : 20
39. Arun purchased 30 kg of wheat at the rate of ₹ 11.50 per kg and 20 kg of wheat at the rate of ₹ 14.25 per kg. He mixed the two and sold the mixture. Approximately at what price per kg should he sell the mixture to make 30% profit? [Bank PO]
(a) ₹ 14.80 (b) ₹ 15.40 (c) ₹ 15.60 (d) ₹ 16.30 (e) ₹ 18.20
40. A man bought apples at the rate of 8 for ₹ 34 and sold them at the rate of 12 for ₹ 57. How many apples should be sold to earn a net profit of ₹ 45? [SSC]
(a) 90 (b) 100 (c) 135 (d) 150

41. If on selling 12 note books, a seller makes a profit equal to selling price of 4 notebooks. What is his per cent profit? **[Bank PO]**
 (a) $16\frac{2}{3}$ (b) 25 (c) 50
 (d) Data inadequate (e) None of these
42. The cost price of 20 articles is the same as selling price of x articles. If the profit is 25%, then the value of x is **[MAT]**
 (a) 15 (b) 16 (c) 18 (d) 25
43. The percentage profit earned by selling an article for ₹ 1920 is equal to the percentage loss incurred by selling the same article for ₹ 1280. At what price should the article be sold to make 25% profit? **[SIDBI]**
 (a) ₹ 2000 (b) ₹ 2200 (c) ₹ 2400
 (d) Data inadequate (e) None of these
44. In a certain store, the profit is 320% of the cost. If the cost increases by 25% but the selling price remains constant, approximately what percentage of the selling price is the profit? **[MAT]**
 (a) 30% (b) 70% (c) 100% (d) 250%
45. If selling price is doubled, profit triples. Find the profit per cent. **[MAT]**
 (a) $66\frac{2}{3}$ (b) 100 (c) $105\frac{1}{3}$ (d) 120
46. The ratio between selling price and cost price of an article is 7 : 5. What is the ratio between the profit and the cost price of that article? **[Bank PO]**
 (a) 2 : 7 (b) 5 : 2 (c) 7 : 2
 (d) Data inadequate (e) None of these
47. When a plot was sold for ₹ 18700, the owner lost 15%. At what price must the plot be sold in order to gain 15%? **[AAO Exam]**
 (a) ₹ 21000 (b) ₹ 22500 (c) ₹ 25300 (d) ₹ 25800
48. A manufacturer undertakes to supply 2000 pieces of a particular component at ₹ 25 per piece. According to his estimates, even if 5% fail to pass the quality tests, then he will make a profit of 25%. However, as it turned out, 50% of the components were rejected. What is the loss to the manufacturer? **[MAT]**
 (a) ₹ 12000 (b) ₹ 13000 (c) ₹ 14000 (d) ₹ 15000
49. A fruitseller sells mangoes at the rate of ₹ 9 per kg and thereby loses 20%. At what price per kg he should have sold them to make a profit of 5%? **[RRB]**
 (a) ₹ 11.81 (b) ₹ 12 (c) ₹ 12.25 (d) ₹ 12.31
50. Peter purchased a machine for ₹ 80000 and spent ₹ 5000 on repair and ₹ 1000 on transport and sold it with 25% profit. At what price did he sell the machine? **[Bank PO]**
 (a) ₹ 105100 (b) ₹ 106250 (c) ₹ 107500 (d) ₹ 117500 (e) None of these
51. Hundred oranges are bought at the rate of ₹ 350 and sold at the rate of ₹ 48 per dozen. The percentage of profit or loss is **[SSC]**
 (a) $14\frac{2}{7}\%$ gain (b) 15% gain (c) $14\frac{2}{7}\%$ loss (d) 15% loss
52. A shopkeeper purchased 70 kg of potatoes for ₹ 420 and sold the whole lot at the rate of ₹ 6.5 per kg. What will be his gain per cent? **[SSC]**
 (a) $4\frac{1}{6}\%$ (b) $6\frac{1}{4}\%$ (c) $8\frac{1}{3}\%$ (d) 20%

53. The ratio of the cost price and the selling price is 4 : 5. The profit per cent is [Hotel Management]
(a) 10% (b) 20% (c) 25% (d) 30%
54. The profit earned by selling an article for ₹ 832 is equal to the loss incurred when the same article is sold for ₹ 448. What should be the selling price for making 50% profit? [Bank PO]
(a) ₹ 920 (b) ₹ 960 (c) ₹ 1060 (d) ₹ 1200 (e) None of these
55. If the selling price of 50 articles is equal to the cost price of 40 articles, then the loss or gain per cent is [Hotel Management]
(a) 20% loss (b) 20% gain (c) 25% loss (d) 25% gain
56. On an order of 5 dozen boxes of a consumer product, a retailer receives an extra dozen free. This is equivalent to allowing him a discount of [CBI]
(a) 15% (b) $16\frac{1}{6}\%$ (c) $16\frac{2}{3}\%$ (d) 20%
57. On selling 17 balls at ₹ 720, there is a loss equal to the cost price of 5 balls. The cost price of a ball is [SSC]
(a) ₹ 45 (b) ₹ 50 (c) ₹ 55 (d) ₹ 60
58. Some articles were bought at 6 for ₹ 5 and sold at 5 for ₹ 6. Gain per cent is [SSC]
(a) 30% (b) $33\frac{1}{3}\%$ (c) 35% (d) 44%
59. A man bought some fruits at the rate of 16 for ₹ 24 and sold them at the rate of 8 for ₹ 18. What is the profit per cent? [Bank PO]
(a) 25% (b) 40% (c) 50% (d) 60% (e) None of these
60. A trader mixes three varieties of groundnuts costing ₹ 50, ₹ 20 and ₹ 30 per kg in the ratio 2 : 4 : 3 in terms of weight and sells the mixture at ₹ 33 per kg. What percentage of profit does he make? [Hotel Management]
(a) 8% (b) 9% (c) 10% (d) None of these
61. A trader mixes 26 kg of rice at ₹ 20 per kg with 30 kg of rice of another variety at ₹ 36 per kg and sells the mixture at ₹ 30 per kg. His profit per cent is [Bank PO]
(a) No profit, no loss (b) 5% (c) 8%
(d) 10% (e) None of these
62. A vendor bought toffees at 6 for a rupee. How many for a rupee must he sell to gain 20%? [CBI]
(a) 3 (b) 4 (c) 5 (d) 6
63. A shopkeeper professes to sell his goods at cost price but uses a weight of 800 g instead of kilogram weight. Thus, he makes a profit of [CBI]
(a) 20% (b) $16\frac{2}{3}\%$ (c) 25% (d) None of these
64. A fair price shopkeeper makes 10% profit on his goods. He lost 20% goods during theft. His loss per cent is [SSC]
(a) 8 (b) 10 (c) 11 (d) 12
65. A man buys an article for 10% less than its value and sells it for 10% more than its value. His gain or loss per cent is [SSC]
(a) No profit, no loss (b) 20% profit
(c) Less than 20% profit (d) More than 20% profit
66. If 5% more is gained by selling an article for ₹ 350 than by selling it for ₹ 340, the cost of the article is [CBI]
(a) ₹ 50 (b) ₹ 160 (c) ₹ 200 (d) ₹ 225

67. A shopkeeper sells two watches for ₹ 308 each. On one he gets 12% profit and on the other 12% loss. His profit or loss in the entire transaction was [BSF]
- (a) Neither profit, nor loss (b) $1\frac{11}{25}\%$ loss
(c) $1\frac{11}{25}\%$ profit (d) $3\frac{2}{25}\%$ loss
68. A cloth merchant sold half of his cloth at 20% profit, half of the remaining at 20% loss and the rest was sold at the cost price. In the total transaction, his gain or loss will be [SSC]
- (a) Neither loss nor gain (b) 5% loss
(c) 5% gain (d) 10% gain
69. If a company sells a car with a marked price of ₹ 272000 and gives a discount of 4% on ₹ 200000 and 2.5% on the remaining amount of ₹ 72000, then the actual price charged by the company for the car is [SSC]
- (a) ₹ 250000 (b) ₹ 255000 (c) ₹ 260100 (d) ₹ 262200
70. A shopkeeper gives 12% additional discount on the discounted price, after giving an initial discount of 20% on the labelled price of a radio. If the final selling price of the radio is ₹ 704, then what is its labelled price? [RRB]
- (a) ₹ 844.80 (b) ₹ 929.28 (c) ₹ 1000 (d) ₹ 1044.80
71. A discount of 15% on one article is the same as a discount of 20% on another article. The costs of the two articles can be [SSC]
- (a) ₹ 40, ₹ 20 (b) ₹ 60, ₹ 40 (c) ₹ 80, ₹ 60 (d) ₹ 50, ₹ 30
72. The list price of an article at a showroom is ₹ 2000 and it is being sold at successive discounts of 20% and 10%. Its net selling price will be [SSC]
- (a) ₹ 1400 (b) ₹ 1440 (c) ₹ 1520 (d) ₹ 1700
73. The marked price of a watch was ₹ 720. A man bought the same for ₹ 550.80 after getting two successive discounts, the first being 10%. What was the second discount rate? [SSC]
- (a) 12% (b) 14% (c) 15% (d) 18%
74. By selling an article at $\frac{2}{5}$ of the marked price, there is a loss of 25%. The ratio of the marked price and the cost price of the article is [SSC]
- (a) 2 : 5 (b) 5 : 2 (c) 8 : 15 (d) 15 : 8
75. The cost price of an article is 64% of the marked price. Calculated the gain per cent after allowing a discount of 12% [CBI]
- (a) 37.5% (b) 48% (c) 50.5% (d) 52%
76. A trader marked the price of his commodity so as to include a profit of 25%. He allowed discount of 16% on the marked price. His actual profit was [SSC]
- (a) 5% (b) 9% (c) 16% (d) 25%
77. A shopkeeper sold Sarees at ₹ 266 each after giving 5% discount on labelled price. Had he not given the discount, he would have earned a profit of 12% on the cost price. What was the cost price of each Saree? [SBI PO]
- (a) ₹ 240 (b) ₹ 260 (c) ₹ 280
(d) Data inadequate (e) None of these
78. A shopkeeper sells 25 articles at ₹ 45 per article after giving 10% discount and earns 50% profit. If the discount is not given, the profit gained is
- (a) 60% (b) $60\frac{2}{3}\%$ (c) 66% (d) $66\frac{2}{3}\%$ (e) None of these

79. A shopkeeper sold a TV set for ₹ 17940 with a discount of 8% and earned a profit of 19.6%. What would have been the percentage of profit earned if no discount was offered?

[Bank PO]

- (a) 24.8% (b) 25% (c) 26.4%
(d) Cannot be determined (e) None of these

80. A product when sold with 10% rebate on the listed price gave a profit of ₹ 70. What was its cost price?

[Bank PO]

- (a) ₹ 200 (b) ₹ 350 (c) ₹ 700
(d) Cannot be determined (e) None of these

81. A shopkeeper fixes the marked price of an item 35% above its cost price. The percentage of discount allowed to gain 8% is

[Assistant Grade]

- (a) 20% (b) 27% (c) 31% (d) 43%

82. Kunal bought a suitcase with 15% discount on the labelled price. He sold the suitcase for ₹ 2880 with 20% profit on the labelled price. At what price did he buy the suitcase?

[SBI PO]

- (a) ₹ 2040 (b) ₹ 2400 (c) ₹ 2604 (d) ₹ 2640 (e) None of these

83. A trader marked his goods at 20% above the cost price. He sold half the stock at the marked price, one quarter at a discount of 20% on the marked price and the rest at a discount of 40% on the marked price. His total gain is

[SSC]

- (a) 2% (b) 4.5% (c) 13.5% (d) 15%

84. A shopkeeper sells a badminton racket, whose marked price is ₹ 30, at a discount of 15% and gives a shuttle cock costing ₹ 1.50 free with each racket. Even then he makes a profit of 20%. His cost price per racket is

[SSC]

- (a) ₹ 19.75 (b) ₹ 20 (c) ₹ 21 (d) ₹ 21.25

85. A shopkeeper expects a gain of $22\frac{1}{2}\%$ on his cost price. If in a week, his sale was of ₹ 392, what was his profit?

[Bank PO]

- (a) ₹ 18.20 (b) ₹ 70 (c) ₹ 72 (d) ₹ 88.25

86. A man buys a cycle for ₹ 1400 and sells it at a loss of 15%. What is the selling price of the cycle?

[SSC]

- (a) ₹ 1090 (b) ₹ 1160 (c) ₹ 1190 (d) ₹ 1202

87. A man gains 20% by selling an article for a certain price. If he sells it at double the price, what will be the percentage of profit?

[SSC]

- (a) 10% (b) 30% (c) 15% (d) 140%

88. If the cost price of 12 pens is equal to selling price of 8 pens, find the gain per cent.

[SSC]

- (a) 25% (b) 33% (c) 50% (d) $66\frac{2}{3}\%$



1. (c)	2. (d)	3. (c)	4. (d)	5. (c)	6. (b)	7. (c)	8. (d)	9. (c)	10. (d)
11. (a)	12. (b)	13. (b)	14. (c)	15. (b)	16. (c)	17. (b)	18. (c)	19. (d)	20. (b)
21. (c)	22. (c)	23. (b)	24. (b)	25. (b)	26. (c)	27. (c)	28. (c)	29. (a)	30. (a)
31. (b)	32. (b)	33. (b)	34. (d)	35. (d)	36. (d)	37. (d)	38. (a)	39. (d)	40. (a)
41. (c)	42. (b)	43. (a)	44. (b)	45. (b)	46. (e)	47. (c)	48. (b)	49. (a)	50. (c)
51. (a)	52. (c)	53. (c)	54. (b)	55. (a)	56. (c)	57. (d)	58. (d)	59. (c)	60. (c)

61. (b) 62. (c) 63. (c) 64. (d) 65. (d) 66. (c) 67. (b) 68. (c) 69. (d) 70. (c)
 71. (c) 72. (b) 73. (c) 74. (d) 75. (a) 76. (a) 77. (c) 78. (d) 79. (e) 80. (d)
 81. (a) 82. (a) 83. (a) 84. (b) 85. (c) 86. (c) 87. (d) 88. (c)



Solutions with Necessary Explanation

1. Let the cost price of rice be ₹ x/kg .

$$\therefore \text{Selling price of rice} = ₹ \left(\frac{105x}{100} \right) \text{ per } \frac{75}{100} \text{ kg}$$

$$\text{i.e. S.P.} = ₹ \left(\frac{105x}{100} \times \frac{100}{75} \right) / \text{kg}$$

$$= ₹ \frac{7x}{5} / \text{kg}$$

$$\therefore \% \text{ profit} = \frac{\frac{7x}{5} - x}{x} \times 100 = \frac{200}{5} = 40\%$$



Short cut: Total percentage profit = $100 \times \frac{\% \text{ profit} + \% \text{ less in weight}}{100 - \% \text{ less in weight}}$

$$= 100 \times \left(\frac{5 + 25}{100 - 25} \right) = \left(\frac{100 \times 30}{75} \right) = 40\%$$

If a shopkeeper sells his goods at $x\%$ loss on the cost price but uses y g instead of z g then his % profit or loss is

$$(100 - x) \frac{z}{y} - 100 \text{ as the sign is +ve or -ve.}$$

2. Price at which he bought the pencil = ₹ $\frac{10}{8} = ₹ 1.25$

$$\text{Price at which he sold the pencil} = ₹ \frac{8}{10} = ₹ 0.80$$

$$\% \text{ Profit or loss} = \frac{0.8 - 1.25}{1.25} = \frac{-0.45}{1.25} = -36\%$$

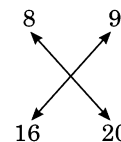
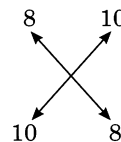
$$\% \text{ Profit or loss} = \frac{8 \times 8 - 10 \times 10}{10 \times 10} \times 100 = \frac{-3600}{100} = -36\%$$

$$\therefore \% \text{ loss} = 36\%$$

3. % profit or loss = $\frac{8 \times 20 - 9 \times 16}{9 \times 16} \times 100$

$$= \frac{1600}{9 \times 16} = \frac{100}{9} = 11\frac{1}{9}\%$$

$$\therefore \text{Profit} = 11\frac{1}{9}\%$$



4. If C.P. is ₹ 100, S.P. will be ₹ 80.

$$\therefore \text{S.P.} = \frac{80}{100} \times 240 = \text{₹ } 192$$

5. If the C.P. is ₹ 100, the S.P. will be ₹ 120.

$$\therefore \text{For S.P.} = \text{₹ } 384, \text{ C.P. will be } \frac{100}{120} \times 384 = 5 \times 64 = \text{₹ } 320$$



Short cut: By rule of fraction

$$\text{C.P.} = \text{S.P.} \times \frac{100}{100 + x} = 384 \times \frac{100}{100 + 20} = 384 \times \frac{100}{120} = \text{₹ } 320$$

6. In the transaction there is a profit of 20% and a loss of 10%.

$$\text{B's cost price} = 540 \times \frac{100}{90} = \text{₹ } 600$$

$$\text{A's cost price} = 600 \times \frac{100}{120} = \text{₹ } 500$$

7. (106)% of cost price – (100 – 12)% of cost price = ₹ 72

$$\therefore \text{Cost price} = \frac{72}{106 - 88} \times \frac{72 \times 100}{18} = \text{₹ } 400$$

8. C.P. = S.P. $\times \frac{100}{100 + 10} = 4950 \times \frac{100}{110} = \text{₹ } 4500$

$$\text{If S.P.} = 4275, \text{ loss \%} = \frac{4500 - 4275}{4500} \times 100 = \frac{225}{4500} \times 100 = 5\%$$

9. 124% of cost – 115% of cost = ₹ 540

$$\therefore 9\% \text{ of cost} = \text{₹ } 540$$

$$\therefore \text{Cost} = \frac{540 \times 100}{9} = \text{₹ } 6000$$



Short cut: Cost price = $\frac{\text{More gain} \times 100}{\text{Difference in percentage profit}}$

$$\therefore \text{Cost price} = \frac{540 \times 100}{24 - 15} = \frac{540 \times 100}{9} = \text{₹ } 6000$$

10. Let the cost price of radio be ₹ x .

$$\text{S.P. at 5\% loss} = \text{₹ } \frac{95}{100}x$$

$$\text{S.P. at 25\% gain} = \text{₹ } \frac{125}{100}x$$

$$\therefore \frac{125x}{100} - \frac{95x}{100} = \text{₹ } 210$$

$$\therefore 30x = ₹ 210 \times 100$$

$$= ₹ 21,000$$

$$\therefore x = ₹ 700$$

$$\therefore \text{To gain 35\% he would sell at } ₹ 700 \times \frac{135}{100} = 7 \times 135 = ₹ 945$$



Short cut: Selling price = More rupees $\frac{(100 + \% \text{ final gain})}{(\% \text{ gain} + \% \text{ loss})}$

$$= 210 \times \frac{(100 + 35)}{(25 + 5)} = \frac{210 \times 135}{30} = ₹ 945$$

11. The cost price of the article is ₹ x .

$$\therefore \text{The selling price} = ₹ \frac{120}{100} \times x$$

$$\text{New cost price} = ₹ (x - 150)$$

$$\text{New selling price} = ₹ \left(\frac{120x}{100} - 150 \right)$$

$$\text{New profit \%} = \frac{\left(\frac{120x}{100} - 150 \right) - (x - 150)}{(x - 150)} \times 100$$

$$= \frac{\left(\frac{120x}{100} - x \right) \times 100}{(x - 150)} = \frac{\frac{20x}{100} \times 100}{x - 150} = \frac{20x}{x - 150} \%$$

$$\text{New \% profit} = 20 + 5 = 25\%$$

$$\therefore \frac{20x}{x - 150} = 25 \text{ or } (x - 150)25 = 20x$$

$$25x - 20x = 150 \times 25$$

$$\therefore 5x = 150 \times 25$$

$$\text{or } x = 150 \times 5 = ₹ 750$$



Short cut: When cost price and selling price are reduced by the same amounts (say D),

$$\text{then the cost price} = \frac{[\text{Initial profit \%} + \text{Increase in profit \%}] \times D}{\text{Increase in profit \%}}$$

$$\therefore \text{Cost price} = \frac{[20 + 5] \times 150}{5} = ₹ 750$$

12. Let the cost price of table be ₹ t and that of chair ₹ c .

$$\text{Then } 15\% \text{ of } t - 7\frac{1}{2}\% \text{ of } c = 50$$

$$\text{i.e.} \quad 15t - 7\frac{1}{2}c = 5000 \quad (1)$$

$$-7\frac{1}{2}t + 15c = 0 \quad (2)$$

Solving Eqs. (1) and (2), we get

$$c = \frac{\text{₹ } 2000}{9} \text{ and } t = \frac{\text{₹ } 4000}{9}$$

13. After reduction, 900 g costs ₹ 8.10.

$$\therefore 1000 \text{ g will cost } \frac{1000}{900} \times 8.10 = \text{₹ } 9$$

$$\% \text{ Change in actual price or loss} = \frac{10 - 9}{10} \times 100\% = 10\%$$

14. S.P. of 4 pens = C.P. of 6 pens.

i.e. S.P. of 2 pens = C.P. of 3 pens

Profit = C.P. of 3 pens

Investment = C.P. of 6 pens

$$\% \text{ profit} = \frac{6 - 4}{4} \times 100 = 50\%$$

15. The C.P. of 20 tables = S.P. of 25 tables

$$\% \text{ loss} = \frac{25 - 20}{25} \times 100\% = 20\%$$



Note: In general, if the C.P. of x articles = S.P. of y articles, then profit % = $\left(\frac{x}{y} - 1\right) \times 100\%$

If result is negative, (then it represents loss)

16. Let the cost price of the article be 'P'.

$$\text{Initial S.P.} = \frac{95}{100}P$$

$$\text{Final S.P.} = \left(\frac{120}{100} \times \frac{60}{100}\right)P$$

$$\therefore \frac{95}{100}P - \left(\frac{120}{100} \times \frac{60}{100}\right)P = 115$$

$$\therefore P = \frac{115 \times 100 \times 100}{95 \times 100 - 120 \times 60} = \frac{115 \times 100 \times 100}{2300} = \text{₹ } 500$$

$$17. \text{ S.P.} = \left(\frac{255}{85} \times 100\right) \times \frac{105}{100} = \frac{255}{85} \times 105 = \text{₹ } 315$$

$$18. \text{ C.P. of one-third shares} = \frac{\text{₹ } 2100}{3} = \text{₹ } 7000$$

$$\text{S.P. of one-third shares} = \frac{110}{100} \times 7000 = \text{₹ } 7700$$

$$\text{C.P. of remaining } \frac{2}{3} \text{rd shares} = ₹ (21000 - 7000) = ₹ 14000$$

$$\text{S.P. of remaining } \frac{2}{3} \text{rd shares} = \frac{95}{100} \times 14000 = ₹ 13300$$

$$\text{Total S.P.} = ₹ 7700 + 13300 = ₹ 21000$$

$$\text{Total C.P.} = ₹ 21000$$

$$\text{S.P.} = \text{C.P.}$$

or there is **no profit no loss**.

19. Given $x = -5$, $y = +20$, $m = 1/4$

The required profit percentage on the remaining stock

$$\frac{y - xm}{1 - m} \% = \frac{20 + 5/4}{1 - 1/4} = \frac{85/4}{3/4} = \frac{85}{3} = 28\frac{1}{3} \%$$

Alternative method:

Let the required % profit be z

$$\frac{1}{4}(100 - 5) + \frac{3}{4}(100 + z) = 100 + 20$$

$$\text{or} \quad \frac{95}{4} + 75 + \frac{3}{4}z = 120$$

$$\begin{aligned} \text{or} \quad \frac{3}{4}z &= 120 - 75 - \frac{95}{4} \\ &= \frac{180 - 95}{4} = \frac{85}{4} \end{aligned}$$

$$\therefore z = \frac{85}{4} \times \frac{4}{3} = \frac{85}{3} = 28\frac{1}{3} \%$$

20. % profit = $[mx - (1 - m)y]\%$

$$x = 10; y = 10; m = \frac{1}{3}; 1 - m = \frac{2}{3}$$

$$\therefore \% \text{ profit} = \frac{1}{3} \times 10 - \frac{2}{3} \times 10 = \frac{-10}{3} \%$$

$$\therefore \text{loss} = \frac{10}{3} \%$$

21. Total C.P. = $20 \times 5 + 35 \times 7 = ₹ 345$

$$\text{Total S.P.} = (20 + 35) \times 6.5 = ₹ 357.5$$

$$\text{Total profit} = ₹ 357.5 - ₹ 345 = ₹ 12.5$$

$$\text{Per cent profit} = \frac{12.5}{345} \times 100\% = 3\frac{43}{69} \%$$

22. Given discount = 5% and profit = 23.5%

Let the cost price of article be ₹ 100.

$$\therefore \text{Selling price of article} = ₹ 123.50$$

Let the marked price of article be x

$$\therefore \frac{95}{100}x = 123.50$$

$$\begin{aligned} \text{or } x &= \frac{123.50 \times 100}{95} \\ &= \frac{12350}{95} = \frac{2470}{19} = ₹ 130 \end{aligned}$$

$\therefore$ Without discount, S.P. = ₹ 130, C.P. = ₹ 100

$$\begin{aligned} \therefore \text{Profit \%} &= \frac{\text{S.P.} - \text{C.P.}}{\text{C.P.}} \times 100 \\ &= \frac{130 - 100}{100} \times 100 = \mathbf{30\%} \end{aligned}$$

23. Labelled price of cupboard = ₹ 6500

$$\text{Selling price (S.P.)} = \frac{95}{100} \times 6500 = ₹ 6175$$

Profit % = 15;

Let x be the cost price

$$\therefore \frac{\text{S.P.} - \text{C.P.}}{\text{C.P.}} \times 100 = \% \text{ profit}$$

$$\text{i.e. } \frac{6175 - x}{x} \times 100 = 15$$

$$\text{i.e. } \frac{6175 - x}{x} = \frac{15}{100}$$

$$\text{i.e. } \frac{6175}{x} = 1 + \frac{15}{100} = \frac{115}{100}$$

$$\therefore x = \frac{6175 \times 100}{115} = 5369.56 \approx ₹ \mathbf{5350}$$

24. Let the cost price of article be ₹ 100.

$\therefore$ Marked price = ₹ 110

Let $x\%$ be the discount given

$$\therefore \text{S.P.} = 110 \left(1 - \frac{x}{100} \right)$$

$$\% \text{ loss} = \frac{\text{C.P.} - \text{S.P.}}{\text{C.P.}} \times 100$$

$$\text{or } 100 \left(1 - \frac{\text{S.P.}}{\text{C.P.}} \right) = 1$$

$$\text{or } 1 - \frac{\text{S.P.}}{\text{C.P.}} = \frac{1}{100}$$

$$\text{or } \frac{\text{S.P.}}{\text{C.P.}} = 1 - \frac{1}{100} = \frac{\mathbf{99}}{\mathbf{100}}$$

$$\text{i.e.} \quad \text{S.P.} = 110 \left(1 - \frac{x}{100} \right) = 99$$

$$\text{or} \quad 1 - \frac{x}{100} = \frac{99}{110}$$

$$\text{or} \quad \frac{x}{100} = 1 - \frac{99}{110} = \frac{11}{110} = \frac{1}{10}$$

$$\text{or} \quad x = 10\%$$

25. Let the marked price of radio be ₹ x .

$$\text{C.P.} = ₹ 1200; \quad \text{S.P.} = 0.8 \times x; \quad \% \text{ profit} = 25\%$$

$$\therefore \% \text{ profit} = \frac{\text{S.P.} - \text{C.P.}}{\text{C.P.}} \times 100$$

$$\text{i.e.} \quad \frac{0.8x - 1200}{1200} \times 100 = 25$$

$$\text{i.e.} \quad \frac{0.8x}{1200} = \frac{25}{100} + 1$$

$$\text{i.e.} \quad \frac{0.8x}{1200} = \frac{125}{100}$$

$$\text{or} \quad x = \frac{125 \times 1200}{100 \times 0.8} = 125 \times 15 = ₹ 1875$$

$$26. \quad \text{S.P.} = \text{C.P.} \left(1 + \frac{x}{100} \right) \text{ where } x \text{ is profit } \%$$

$$\therefore 300 = \text{C.P.} \left(1 + \frac{20}{100} \right)$$

$$\text{or} \quad \text{C.P.} = \frac{300}{1.2} = ₹ 250$$

$$\text{After discount, S.P.} = 300 \times \left(1 - \frac{10}{100} \right) = ₹ 270$$

$$\begin{aligned} \therefore \text{Gain } \% &= \frac{\text{S.P.} - \text{C.P.}}{\text{C.P.}} \times 100 \\ &= \frac{270 - 250}{250} \times 100 = 8\% \end{aligned}$$

27. Let the initial price of article be ₹ 100.

$$\text{After 1st rise of 30\%, price} = 1.3 \times 100 = ₹ 130$$

$$\text{After 1st discount of 10\%, price} = 130 \left(1 - \frac{10}{100} \right) = ₹ 117$$

$$\text{After 2nd discount of 10\%, price} = 117 \left(1 - \frac{10}{100} \right) = 117 \times 0.9 = ₹ 105.3$$

$\therefore$ Price is increased by 5.3%

or

Let the initial price of article be ₹ 100

$$\therefore \text{Marked price} = 1.3 \times 100 = ₹ 130$$

$$\begin{aligned} \therefore \text{Price after 2 discounts of 10\% each} &= 130 \left\{ 1 + \left(-10 - 10 + \frac{10^2}{100} \right) \% \right\} \\ &= 130 \left(\frac{100 - 19}{100} \right) = 130 \times \frac{81}{100} = ₹ 105.3 \end{aligned}$$

$$\therefore \% \text{ profit} = \frac{105.3 - 100}{100} \times 100 = 5.3\%$$

i.e. price is increased by 5.3%

28. Let the amount of bill be ₹ x .

1st discount = 35%

2nd discount = 20%, each two times in succession.

$$\therefore \text{Equivalent discount for 2nd one} = \left(-20 - 20 + \frac{400}{100} \right) \% = -36\% \quad 36\% \text{ decrease}$$

$$\therefore \text{Difference of discounts} = x(36 - 35)\% = 22$$

$$x \times \frac{1}{100} = 22$$

$$\therefore x = ₹ 2200$$

29. Let marked price be ₹ 100.

$$\therefore \text{S.P.} = \frac{85}{100} \times \frac{88}{100} \times \frac{90}{100} \times 100 = ₹ 67.32$$

$$\therefore \text{Single discount} = 100 - 67.32 = 32.68\%$$

30. Listed price = ₹ 1500

First discount = 20%

Let additional discount be $x\%$

$$\therefore 1500 \times \frac{80}{100} \times \left(\frac{100 - x}{100} \right) = 1104$$

$$\therefore 1 - \frac{x}{100} = \frac{1104 \times 100}{1500 \times 80}$$

$$\therefore \frac{x}{100} = 1 - \frac{1104 \times 100}{1500 \times 80}$$

$$\text{or} \quad x = \left(1 - \frac{1104 \times 100}{1500 \times 80} \right) \times 100 = \left(1 - \frac{368}{400} \right) \times 100 = 8\%$$

31. Let marked price be ₹ x .

$$\therefore \left(x \times \frac{85}{100} \right) \times 2 = ₹ 37.40$$

$$\therefore x = \frac{37.40 \times 100}{170} = ₹ 22$$

32. Let the C.P. of each horse be ₹ x and that of each cow be ₹ y .

$$\therefore x + y = 13400 \quad (1)$$

$$\therefore \text{Profit earned} = \frac{10}{100} \times 4x + \frac{20}{100} \times 9y = 1880$$

$$\text{i.e.} \quad 4x + 18y = 18800 \quad (2)$$

Solving (1) and (2), we get $x = ₹ 2000$

$\therefore$ Cost of horse = ₹ 2000

33. Let ₹ x be the cost price.

$$\therefore x \times \left\{ \left(1 + \frac{6}{100} \right) - \left(1 + \frac{4}{100} \right) \right\} = ₹ 3$$

$$\text{i.e.} \quad x \left\{ \frac{6-4}{100} \right\} = ₹ 3$$

$$\therefore x = \frac{3 \times 100}{2} = ₹ 150$$

$$\therefore \text{Ratio of selling price} = \frac{150 \times 1.04}{150 \times 1.06} = \frac{52}{53}$$

$$\therefore \text{Required ratio} = 52 : 53$$

34. Let ₹ x be the cost price of the fan.

$$\therefore 400 - 380 = 20 = 2\% \text{ of } x = \frac{2}{100} x$$

$$\therefore x = \frac{20 \times 100}{2} = ₹ 1000$$

35. Let the cost price of bicycle to A be ₹ x .

$$\therefore x \times \frac{120}{100} \times \frac{125}{100} = ₹ 225$$

$$\begin{aligned} \therefore x &= \frac{225 \times 100 \times 100}{120 \times 125} \\ &= \frac{9 \times 100 \times 100}{120 \times 5} = \frac{300}{2} = ₹ 150 \end{aligned}$$

36. The shopkeeper pays for 900 g instead of 1000 g while buying and sells the item weighing 900 gm for 1000 g or 1 kg.

$$\text{Gain \%} = \frac{(100 + \text{common gain \%})^2}{100} - 100$$

$$= \left[\frac{(100 + 10)^2}{100} - 100 \right] \%$$

$$= \left[\frac{12100}{100} - 100 \right] \% = 21 \%$$

37. Gain % = $\left(\frac{10}{90} \times 100 \right) \% = 11 \frac{1}{9} \%$

38. Profit = 37.5%

C.P. = 6.4; S.P. = 8

$$\therefore \text{Mean cost price} = \frac{100}{137.5} \times 8 = ₹ \frac{64}{11}$$

$$\therefore \text{Required ratio} = \frac{64}{110} : \frac{64}{11} = 1 : 10$$

39. C.P. =
- $30 \times 11.50 + 20 \times 14.25 = 345 + 285 = ₹ 630/50 \text{ kg} = ₹ 12.6 \text{ kg}$

S.P. = $1.3 \times ₹ 12.6/\text{kg} = ₹ 16.38/\text{kg} \approx ₹ 16.30/\text{kg}$

40. C.P. = ₹
- $\frac{34}{8}$
- ; S.P. = ₹
- $\frac{57}{12}$

Let x be the number of apples sold

$$\therefore 45 = (\text{S.P.} - \text{C.P.}) x$$

$$\text{or } x = \frac{45}{\text{S.P.} - \text{C.P.}} = \frac{45}{\frac{57}{12} - \frac{34}{8}} = \frac{45}{\frac{114 - 102}{24}} = \frac{45 \times 24}{12} = ₹ 90$$

41. The cost price of 12 note books = Selling price of 8 note books

$$\therefore \text{Profit} = \text{Selling price of 4 note books} = \text{Cost price of 6 note books}$$

$$\therefore \% \text{ profit} = \frac{\text{Cost price of 6 note books}}{\text{Cost price of 12 note books}} \times 100 = 50\%$$

42. C.P. of 20 articles = S.P. of
- x
- articles

$$\therefore \text{Profit (25\%)} = \text{S.P. of } (20 - x) \text{ articles}$$

$$25\% \text{ profit} = \text{C.P. of 5 articles} = \text{S.P. of } \frac{x}{4} \text{ articles}$$

$$\therefore \frac{x}{4} = 20 - x$$

$$\text{or } \frac{5}{4}x = 20$$

$$\text{or } x = \frac{20 \times 4}{5} = 16$$

43. Let ₹
- x
- be the C.P. of article.

$$\text{Then } \frac{1920 - x}{x} = \frac{x - 1280}{x}$$

$$\frac{1920}{x} + \frac{1280}{x} = 1 + 1 = 2$$

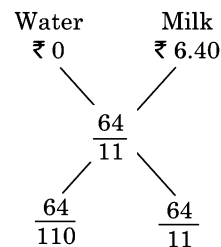
$$\text{or } x = \frac{1920 + 1280}{2} = 1600$$

$$\text{For 25\% profit S.P.} = 1600 \times 1.25 = ₹ 2000$$

44. Let
- x
- be the initial cost price.

$$\therefore \text{S.P.} = 420\% \text{ of } x = 4.2x, \quad \% \text{ profit} = 3.2x$$

If C.P. is increased by 25%



$$\text{New C.P.} = 1.25x, \quad \text{New S.P.} = 4.2x$$

$$\therefore \text{Profit} = \text{S.P.} - \text{C.P.} = (4.2 - 1.25)x = 2.95x$$

$$\therefore \text{Required percentage} = \left(\frac{2.95x}{4.2x} \times 100 \right) \% = \frac{1475}{21} = 70\%$$

$$45. \text{ Profit} = \text{S.P.} - \text{C.P.}$$

$$\text{If S.P. is doubled, new profit} = 2\text{S.P.} - \text{C.P.} = 3(\text{S.P.} - \text{C.P.})$$

$$\text{i.e.} \quad 3\text{C.P.} - \text{C.P.} = 3\text{S.P.} - 2\text{S.P.}$$

$$\text{or} \quad 2\text{C.P.} = \text{S.P.}$$

$$\therefore \text{Profit \%} = \frac{\text{S.P.} - \text{C.P.}}{\text{C.P.}} \times 100 = \frac{2\text{C.P.} - \text{C.P.}}{\text{C.P.}} \times 100 = 100\%$$

$$46. \quad \frac{\text{S.P.}}{\text{C.P.}} = \frac{7}{5}$$

$$\therefore \frac{\text{Profit}}{\text{C.P.}} = \frac{\text{S.P.} - \text{C.P.}}{\text{C.P.}} = \frac{7 - 5}{5} = \frac{2}{5}$$

$$\therefore \text{Profit} : \text{C.P.} = 2 : 5$$

$$47. \text{ S.P.} = ₹ 18700; \quad \text{loss \%} = 15$$

$$\therefore \frac{85}{100} \text{C.P.} = ₹ 18700$$

$$\text{To gain 15\% it should be sold @ } ₹ \frac{115}{100} \text{ C.P.}$$

$$\therefore \text{Required S.P.} = \frac{18700}{85} \times 115 = ₹ 25300$$

$$48. \text{ S.P. of one piece} = ₹ 25$$

$$\therefore \text{Total S.P.} = 2000 \times 25; \quad \text{Profit \%} = 25$$

5% failed in quality test

$$\therefore \text{Net S.P.} = \left(2000 \times \frac{95}{100} \right) \times 25 = 1900 \times 25$$

$$\therefore \text{Cost price} = \frac{100}{125} \times 1900 \times 25 = ₹ 38000$$

If 50% of components rejected,

$$\text{So, loss incurred to manufacturer} = (38000 - 25 \times 1000) = ₹ 13000$$

$$49. \text{ Required selling price} = \frac{9}{80} \times 105 = ₹ 11.81/\text{kg}$$

$$50. \text{ Total cost price} = 80000 + 5000 + 1000 = ₹ 86000$$

$$\text{Required selling price @ 25\% profit} = 86000 \times \frac{125}{100} = ₹ 107500$$

$$51. \text{ C.P.} = ₹ 350/100 = ₹ 3.5/\text{each}$$

$$\text{S.P.} = ₹ 48/12 = ₹ 4/\text{each}$$

$$\therefore \frac{\text{S.P.} - \text{C.P.}}{\text{C.P.}} \times 100 = \frac{4 - 3.5}{3.5} \times 100 = \frac{100}{7} = 14\frac{2}{7} \% \text{ profit}$$

52. Cost price of potatoes = ₹ 420/70 kg = ₹ 6/kg

Selling price of potatoes = ₹ 6.5/kg

$$\therefore \text{Gain per cent} = \frac{\text{S.P.} - \text{C.P.}}{\text{C.P.}} \times 100 = \frac{6.5 - 6}{6} \times 100 = \frac{1}{12} \times 100 = 8\frac{1}{3}\%$$

53.
$$\frac{\text{Cost price}}{\text{Selling price}} = \frac{4}{5}$$

$$\therefore \text{Profit per cent} = \frac{\text{S.P.} - \text{C.P.}}{\text{C.P.}} \times 100 = \frac{5 - 4}{4} \times 100 = 25\%$$

54. Let x be the C.P. of article.

$$\therefore 832 - x = x - 448$$

or
$$x = \frac{832 + 448}{2} = ₹ 640$$

S.P. with 50% profit = C.P. $\times$ 1.5 = $640 \times 1.5 = ₹ 960$

55. S.P. of 50 articles = C.P. of 40 articles

$\therefore$ C.P. of 1 article = S.P. of 1.25 article

$$\begin{aligned} \% \text{ loss} &= \frac{\text{C.P.} - \text{S.P.}}{\text{C.P.}} \times 100 \\ &= \frac{\text{S.P. of 1.25 article} - \text{S.P. of 1 article}}{\text{S.P. of 1.25 article}} \times 100 \\ &= \frac{0.25}{1.25} \times 100 = 20\% \end{aligned}$$

56. Discount = $\frac{1}{5+1} \times 100 = \frac{1}{6} \times 100 = 16\frac{2}{3}\%$

57. S.P. of 17 balls = ₹ 720

(C.P. - S.P.) of 17 balls = C.P. of 5 balls

$$\begin{aligned} \therefore \text{S.P. of 17 balls} &= \text{C.P. of } (17 - 5) \text{ balls} \\ &= \text{C.P. of 12 balls} = 720 \end{aligned}$$

$$\therefore \text{C.P. of ball} = \frac{720}{12} = ₹ 60$$

58. C.P. = ₹ 5/6; S.P. = ₹ 6/5

$$\begin{aligned} \text{Gain \%} &= \frac{\text{S.P.} - \text{C.P.}}{\text{C.P.}} \times 100 \\ &= \frac{\frac{6}{5} - \frac{5}{6}}{\frac{5}{6}} \times 100 = \frac{\frac{36 - 25}{30}}{\frac{5}{6}} \times 100 \\ &= \frac{11}{30} \times \frac{6}{5} \times 100 = \frac{11}{25} \times 100 = 44\% \end{aligned}$$

59. C.P. of fruits = ₹ 24/16 = ₹ 3/2;

S.P. of fruits = ₹ 18/8 = ₹ 9/4;

$$\text{Profit \%} = \frac{\text{S.P.} - \text{C.P.}}{\text{C.P.}} \times 100$$

$$= \frac{\frac{9}{4} - \frac{3}{2}}{\frac{3}{2}} \times 100 = \frac{(9-6)}{\frac{3}{2}} \times 100$$

$$= \frac{\frac{3}{2}}{\frac{3}{2}} \times 100 = \frac{3}{4} \times \frac{2}{3} \times 100 = 50\%$$

60. Cost price = $50 \times 2 + 20 \times 4 + 30 \times 3 = ₹ 270$

Selling price = $33 \times 9 = ₹ 297$

$$\therefore \text{Profit \%} = \frac{\text{S.P.} - \text{C.P.}}{\text{C.P.}} \times 100 = \frac{297 - 270}{270} \times 100 = 10\%$$

61. C.P. of (26 + 30 = 56 kg) of rice = $26 \times 20 + 30 \times 36$

$$= 520 + 1080 = ₹ 1600$$

S.P. of 56 kg of rice @ ₹ 30/kg = $56 \times 30 = ₹ 1680$

$$\therefore \text{Profit \%} = \frac{1680 - 1600}{1600} \times 100 = \frac{80}{1600} \times 100 = 5\%$$

62. C.P. of toffees = ₹ 1/6

S.P. of toffees with 20% gain = $1.2 \times \frac{1}{6} = ₹ 0.2/\text{each} = ₹ 1/5$

$$63. \text{Profit \%} = \frac{\text{True weight} - \text{False weight}}{\text{True weight}} \times 100\% = \frac{1000 \text{ g} - 800 \text{ g}}{800 \text{ g}} \times 100\% = 25\%$$

64. Let C.P. of goods be ₹ x/each .

S.P. of goods = $1.1 \times x$

Let number of items be y

$\therefore$ Total C.P. = ₹ $x \times y$

S.P for $\frac{4y}{5}$ items = $1.1x \times \frac{4y}{5}$

$$\text{S.P.} = \frac{4.4}{5} xy$$

$$\therefore \text{loss \%} = \frac{\text{C.P.} - \text{S.P.}}{\text{C.P.}} \times 100$$

$$= \frac{xy - \frac{4.4}{5} xy}{xy} \times 100$$

$$= \left(1 - \frac{4.4}{5}\right) \times 100 = \frac{0.6}{5} \times 100 = 12\%$$

$$65. \text{ Gain or loss \%} = \frac{(1.1 - 0.9)}{0.9} \times 100 = \frac{1100 - 900}{9} = \frac{200}{9} = 22\frac{2}{9}\%$$

more than 20% profit

$$66. \text{ C.P.} \times 0.05 = 350 - 340 = 10$$

$$\therefore \text{ C.P.} = \frac{10}{0.05} = \text{₹ } 200$$

$$67. \text{ Profit or loss \% in the transaction} = \left(\frac{\text{Common loss and gain \%}}{10} \right)^2 = \left(\frac{12}{10} \right)^2$$

$$= 1.44\% = 1\frac{11}{25}\% \text{ loss}$$

$$68. \text{ S.P.} = 50\% \times 120\% \times \text{C.P.} + 25\% (80\%) \text{ C.P.} + 25\% \times 100\% \text{ C.P.}$$

$$\text{S.P.} = \left(\frac{6}{10} + \frac{2}{10} + \frac{1}{4} \right) \text{C.P.} = \left(\frac{4}{5} + \frac{1}{4} \right) \text{C.P.} = \left(\frac{16+5}{20} \right) \text{C.P.} = \left(\frac{21}{20} \right) \text{C.P.}$$

$$\therefore \text{ profit \%} = \left(\frac{21}{20} - 1 \right) \times 100 = 5\%$$

$$69. \text{ Discount} = \frac{4}{100} \times 200000 + \frac{2.5}{100} \times 72000 = 8000 + 1800 = \text{₹ } 9800$$

$$\therefore \text{ Price changed by company for the car} = 272000 - 9800 = \text{₹ } 262200$$

$$70. \text{ Net discount} = \left(-12 - 20 + \frac{240}{100} \right) \% = -29.6\%$$

$$\therefore (100 - 29.6)\% \text{ of labelled price} = 704$$

$$\therefore \text{ Labelled price} = \frac{704}{70.4} \times 100 = \text{₹ } 1000$$

$$71. \frac{15}{100} \times x = \frac{20}{100} \times y$$

$$\text{or } \frac{x}{y} = \frac{20}{100} \times \frac{100}{15}$$

$$\text{or } \frac{x}{y} = \frac{4}{3}$$

$$\text{i.e. } x = 80, y = 60$$

$$72. \text{ Net discount} = \left(-20 - 10 + \frac{200}{100} \right) \% = -28\%$$

$$\therefore \text{ Selling price} = (100 - 28)\% \text{ of list price}$$

$$= 72\% \text{ of } 2000$$

$$= 72 \times 20 = \text{₹ } 1440$$

$$73. \text{ Total discount} = \frac{720 - 550.80}{720} \times 100$$

$$= \frac{169.20}{720} \times 100 = \frac{16920}{720} = 23.5\%$$

Given first discount = 10%

Let second discount be $x\%$

$$\therefore \left(-10 - x + \frac{10x}{100} \right) \% = 23.5\%$$

$$\text{i.e.} \quad x - \frac{x}{10} = 23.5 - 10 = 13.5$$

$$\text{or} \quad \frac{9}{10}x = 13.5$$

$$\text{or} \quad x = 13.5 \times \frac{10}{9} = 15\%$$

74. $\frac{2}{5}$ of marked price = 75% of C.P.

$$\therefore \frac{\text{Marked price}}{\text{Cost price (or C.P.)}} = \frac{75\%}{2/5} = \frac{3/4}{2/5} = \frac{3}{4} \times \frac{5}{2} = \frac{15}{8}$$

75. C.P. of article = $\frac{64}{100}$ of marked price

S.P. of article = (100 – 12)% of marked price

$$\therefore \text{gain \%} = \frac{\text{S.P.} - \text{C.P.}}{\text{C.P.}} \times 100$$

$$= \frac{\frac{88}{100} - \frac{64}{100}}{\frac{64}{100}} \times 100 = \frac{24}{64} \times 100 = \frac{3}{8} \times 100 = 37.5\%$$

76. Marked price = $1.25 \times$ cost price

S.P. = (100 – 16)% of marked price

= 84% of marked price

$$= \frac{84}{100} \times 1.25 \times \text{cost price}$$

$$= \frac{21}{25} \times 1.25 \times \text{cost price} = 21 \times 0.05 \times \text{cost price}$$

$$= 1.05 \times \text{cost price}$$

$$\therefore \text{profit \%} = \frac{1.05 - 1}{1} \times 100 = 5\%$$

77. Let the cost price of saree be ₹ x .

$$\text{Labelled price} = 266 \times \frac{100}{95} = ₹ 280$$

$$\text{profit \%} = 12 = \frac{280 - x}{x} \times 100$$

$$\text{or} \quad x \left(\frac{12}{100} + 1 \right) = 280$$

$$\text{or} \quad x = \frac{100}{112} \times 280 = ₹ 250$$

78. S.P. = ₹ 45/each; profit = 50%

$$\text{S.P. without discount} = 45 \times \frac{100}{90} = ₹ 50$$

$$\therefore \text{C.P.} = \frac{\text{S.P.}}{1.5} = \frac{45}{1.5} = ₹ 30$$

$$\begin{aligned} \therefore \text{Without discount profit \%} &= \frac{\text{S.P.} - \text{C.P.}}{\text{C.P.}} \times 100 \\ &= \frac{50 - 30}{30} \times 100 = \frac{2}{3} \times 100 = 66\frac{2}{3}\% \end{aligned}$$

79. S.P. = ₹ 17940; profit % = 19.6

$$\therefore \text{C.P.} = \frac{17940}{1.196} = ₹ 15000$$

$$\text{Marked price} = 17940 \times \frac{100}{92} = 195 \times 100 = ₹ 19500$$

$$\therefore \text{Profit \% without giving discount} = \frac{19500 - 15000}{15000} \times 100 = \frac{4500}{15000} \times 100 = 30\%$$

80. Data given is inadequate and therefore cannot be determined.

81. Marked price = $1.35 \times$ cost price

For a gain of 8%,

$$\text{S.P.} = 1.08 \times \text{cost price}$$

$$\therefore \% \text{ discount allowed on marked price} = \frac{1.35 - 1.08}{1.35} \times 100 = \frac{0.27}{1.35} \times 100 = 20\%$$

82. C.P. = $\frac{85}{100} \times$ labelled price

$$\text{S.P.} = 2880 = \frac{120}{100} \text{ labelled price}$$

$$\therefore \text{C.P.} = \frac{2880 \times 100}{120} \times \frac{85}{100} = 24 \times 85 = ₹ 2040$$

83. Marked price = $1.2 \times$ C.P.

$$\begin{aligned} \text{Total S.P.} &= \frac{1}{2} \times (1.2 \times \text{C.P.}) + \frac{1}{4} \times 0.8 \times 1.2 \times \text{C.P.} + \frac{1}{4} \times 0.6 \times 1.2 \times \text{C.P.} \\ &= (0.6 + 0.24 + 0.18) \text{C.P.} = 1.02 \text{ C.P.} \end{aligned}$$

$$\therefore \text{Total gain} = 2\%$$

84. Marked price = ₹ 30

$$\text{S.P. after discount of 15\%} = 0.85 \times 30 = ₹ 25.5$$

$$\text{Less cost of shuttle cock (free) worth ₹ 1.50} = \underline{-1.5}$$

$$\text{Net price} = \underline{₹ 24.00}$$

$$\therefore 24 = \text{C.P.} \times 1.2$$

$$\text{or } \text{C.P.} = \frac{24}{1.2} = ₹ 20$$

85. Let his profit be ₹ x .

$$\text{Cost price} \times \left(1 + \frac{22.5}{100}\right) = \text{Selling price} = ₹ 392$$

$$\therefore \text{Cost price} = \frac{392 \times 100}{122.5} = \frac{392 \times 1000}{1225} = \mathbf{₹ 320}$$

$$\therefore \text{Profit} = \text{S.P.} - \text{C.P.} = 392 - 320 = \mathbf{₹ 72}$$

86. C.P. of cycle = ₹ 1400

$$\text{S.P. cycle} = \frac{85}{100} \times 1400 = \mathbf{₹ 1190}$$

$$87. \quad \text{S.P.}_1 = \frac{120}{100} \times \text{C.P.}$$

$$\text{S.P.}_2 = 2 \text{ S.P.}_1 = \frac{240}{100} \times \text{C.P.}$$

$$\therefore \% \text{ profit} = \frac{\text{S.P.}_2 - \text{C.P.}}{\text{C.P.}} \times 100 = \mathbf{140\%}$$

88. Let C.P. of 1 pen be ₹ x .

$$\text{C.P. of 12 pens} = \text{S.P. of 8 pens} = ₹ 12x$$

$$\text{Profit} = \text{S.P. of 4 pens}$$

$$= \text{C.P. of 6 pens} = ₹ 6x$$

$$\therefore \text{gain \%} = \frac{\text{C.P. of 6 pens}}{\text{C.P. of 12 pens}} \times 100$$

$$= \frac{6x}{12x} \times 100 = \mathbf{50\%}$$

Squares and Square Roots

DUPLEX: For a single central digit, duplex is its square. For example, duplex of 7 is $7^2 = 49$.

The duplex of a number with even number of digits is double the cross-product of digits equidistant from the two ends.

For example,

$$\text{Duplex of } 79 = 2(7 \times 9) = 126$$

$$\text{Duplex of } 34 = 2(3 \times 4) = 24$$

$$\text{Duplex of } 109 = 2(1 \times 9) + 0^2 = 18$$

$$\text{Duplex of } 1357 = 2(1 \times 7) + 2(3 \times 5) = 44$$

EXAMPLE 1 Find the value of $(209)^2$.

$$\begin{aligned} \text{Solution } (209)^2 &= \text{D for } 2 \mid \text{D for } 20 \mid \text{D for } 209 \mid \text{D for } 09 \mid \text{D for } 9 \\ &= 2^2 \mid 2(2 \times 0) \mid 2(2 \times 9) + 0^2 \mid 2(0 \times 9) \mid 9^2 \\ &= 4 \mid 0 \mid 36 \mid 0 \mid 81 \\ &= \mathbf{43681} \end{aligned}$$

$$\begin{array}{r} 81 \\ 0 \\ 36 \\ 0 \\ 4 \\ \hline 43681 \end{array}$$

EXAMPLE 2 Find the value of $(873)^2$.

$$\begin{aligned} \text{Solution } (873)^2 &= \text{D for } 8 \mid \text{D for } 87 \mid \text{D for } 873 \mid \text{D for } 73 \mid \text{D for } 3 \\ &= 64 \mid 2(8 \times 7) \mid 2(8 \times 3) + 7^2 \mid 2(7 \times 3) \mid 3^2 \\ &= 64 \mid 112 \mid 97 \mid 42 \mid 9 \\ &= \mathbf{762129} \end{aligned}$$

$$\begin{array}{r} 9 \\ 42 \\ 97 \\ 112 \\ 64 \\ \hline 762129 \end{array}$$

Square of a two-digit number beginning with 5

Take the number as $5A$.

$$\begin{aligned} (5A)^2 &= 5^2 \mid 2 \times 5 \times A \mid A^2 \\ &= 25 \mid 10A \mid A^2 \end{aligned}$$

$$\begin{aligned}
 &= 25 | A0 | A^2 \\
 &= (25 + A) | A^2, A^2 \text{ is written as a two-digit number.}
 \end{aligned}$$

EXAMPLE 3 Find the value of the following:

(a) $(54)^2$ (b) $(59)^2$ (c) $(52)^2$

Solution

(a) $(54)^2 = 25 + A | A^2 = 25 + 4 | 4^2 = 29 | 16 = \mathbf{2916}$

(b) $(59)^2 = 25 + A | A^2 = 25 + 9 | 9^2 = 34 | 81 = \mathbf{3481}$

(c) $(52)^2 = 25 + A | A^2 = 25 + 2 | 2^2 = 27 | 04 = \mathbf{2704}$

Square of a number with unit digit as 5

Let the number be represented as $A5$.

$$\begin{aligned}
 (A5)^2 &= A^2 | 2 \times A \times 5 | 5^2 \\
 &= A^2 | 10A | 25 \\
 &= A^2 | A0 | 25 \\
 &= A^2 + A | 25 \\
 &= \mathbf{A(A + 1) | 25}
 \end{aligned}$$

EXAMPLE 4 (a) $(15)^2 = ?$ (b) $(25)^2 = ?$ (c) $(115)^2 = ?$ (d) $(225)^2 = ?$

Solution

(a) $(15)^2 = 1(1 + 1) | 25 = 2 | 25 = \mathbf{225}$

(b) $(25)^2 = 2(2 + 1) | 25 = 6 | 25 = \mathbf{625}$

(c) $(115)^2 = 11(11 + 1) | 25 = 132 | 25 = \mathbf{13225}$

(d) $(225)^2 = 22(22 + 1) | 25 = 506 | 25 = \mathbf{50625}$

Square of a number which is nearer to 10^x

Use the algebraic formula

$$x^2 = (x^2 - y^2) + y^2 = (x + y)(x - y) + y^2$$

EXAMPLE 5 Find the value of the following

(a) $(98)^2$ (b) $(103)^2$ (c) $(993)^2$ (d) $(1007)^2$

Solution

(a) $(98)^2 = (98)^2 - 2^2 + 2^2 = (98 + 2)(98 - 2) + 4 = 9600 + 4 = \mathbf{9604}$

(b) $(103)^2 = (103)^2 - 3^2 + 3^2 = 100 \times 106 + 9 = \mathbf{10609}$

(c) $(993)^2 = (993)^2 = (993^2 - 7^2) + 7^2 = 1000 \times 986 + 49 = \mathbf{986049}$

(d) $(1007)^2 = (1007^2 - 7^2) + 7^2 = 1000 \times 1014 + 49 = \mathbf{1014049}$

SQUARE ROOTS: To find the square root easily, the following points are to be kept in mind.

1. Make thorough squares of all natural numbers up to 32.
2. Perfect squares will never end with digits 2, 3, 7, 8.

The square root of a perfect square can be identified at a glance using the end digits of those numbers as shown in the following table.

If square ends with the digit	Square root will end with digit
1	1 or 9
4	2 or 8
5	5
6	4 or 6
9	3 or 7

EXAMPLE 6 Find the value of $\sqrt{4096}$.

Solution $\sqrt{4096} = 10 \times \sqrt{40.96}$

$\sqrt{40.96}$ lies between 6 and 7. The root can end with either 4 or 6 as square ends with 6.

But 40.96 is nearer to $6^2 = 36$.

$\therefore$ The root is $6.4 \times 10 = 64$

EXAMPLE 7 Find the value of $\sqrt{35344}$.

Solution $\sqrt{35344} = 10 \times \sqrt{353.44}$

$\sqrt{353.44}$ lies between 18 and 19.

Now the root can end with 2 or 8 as square ends with 4

$\therefore$ 353.44 is nearer to $19^2 = 361$

The root will be ending with 8

$\therefore$ root = $18.8 \times 10 = 188$



EXERCISES

Q (1–5) Find the squares of the following numbers:

1. 847

2. 8347

3. 54613

4. 480028

5. 63040586

Q (6–15) Find values of the following:

6. 108^2

7. 96^2

8. 997^2

9. 1004^2

10. 1009^2

11. 999^2

12. 991^2

13. 94^2

14. 106^2

15. 109^2

Q (16–30) Find the square roots of the following:

16. $\sqrt{4624}$

17. $\sqrt{20044}$

18. $\sqrt{12321}$

19. $\sqrt{3481}$

20. $\sqrt{7569}$

21. $\sqrt{61009}$

22. $\sqrt{143641}$

23. $\sqrt{481636}$

24. $\sqrt{906304}$

25. $\sqrt{619369}$

26. $\sqrt{767376}$

27. $\sqrt{357604}$

28. $\sqrt{474721}$

29. $\sqrt{802816}$

30. $\sqrt{636804}$

Q (31–82) Find the most correct answer from the given choices:

31. If $x = \frac{\sqrt{3} + 1}{\sqrt{3} - 1}$ and $y = \frac{\sqrt{3} - 1}{\sqrt{3} + 1}$, then the value of $(x^2 + y^2)$ is

[SSC]

(a) 10

(b) 13

(c) 14

(d) 15

32. If $x = (7 - 4\sqrt{3})$, then the value of $\left(x + \frac{1}{x}\right)$ is [SSC]
 (a) $3\sqrt{3}$ (b) $8\sqrt{3}$ (c) 14 (d) $14 + 8\sqrt{3}$
33. If $\sqrt{2} = 1.414$, the square root of $\frac{\sqrt{2}-1}{\sqrt{2}+1}$ is nearest to [CBI]
 (a) 0.172 (b) 0.414 (c) 0.586 (d) 1.414
34. If $\frac{5+2\sqrt{3}}{7+4\sqrt{3}} = a + b\sqrt{3}$, then [RRB]
 (a) $a = -11, b = -6$ (b) $a = -11, b = 6$
 (c) $a = 11, b = -6$ (d) $a = 6, b = 11$
35. $\left[\frac{3\sqrt{2}}{\sqrt{6}-\sqrt{3}} - \frac{4\sqrt{3}}{\sqrt{6}-\sqrt{2}} - \frac{6}{\sqrt{8}-\sqrt{12}}\right] = ?$ [RRB]
 (a) $\sqrt{3} - \sqrt{2}$ (b) $\sqrt{3} + \sqrt{2}$ (c) $5\sqrt{3}$ (d) 1
36. $\left(2 + \sqrt{2} + \frac{1}{2 + \sqrt{2}} + \frac{1}{\sqrt{2} - 2}\right) =$ [SSC]
 (a) $2 - \sqrt{2}$ (b) 2 (c) $2 + \sqrt{2}$ (d) $2\sqrt{2}$
37. Given $\sqrt{5} = 2.2361, \sqrt{3} = 1.7321$, then $\frac{1}{\sqrt{5}-\sqrt{3}} =$ [SSC]
 (a) 1.98 (b) 1.984 (c) 1.9841 (d) 2
38. What is the least number which should be subtracted from 0.000326 to make it a perfect square? [SSC]
 (a) 0.000002 (b) 0.000004 (c) 0.02 (d) 0.04
39. Find the smallest number by which 5808 should be multiplied so that the product becomes a perfect square. [SSC]
 (a) 2 (b) 3 (c) 7 (d) 11
40. If $\sqrt{5} = 2.236$, then the value of $\frac{\sqrt{5}}{2} - \frac{10}{\sqrt{5}} + \sqrt{125} =$ [MBA]
 (a) 5.59 (b) 7.826 (c) 8.944 (d) 10.062
41. The value of $\frac{1 + \sqrt{0.01}}{1 - \sqrt{0.1}}$ is nearly equal to [CBI]
 (a) 0.6 (b) 1.1 (c) 1.6 (d) 1.7
42. Square root of $0.\bar{4}$ is [SSC]
 (a) $0.\bar{6}$ (b) $0.\bar{7}$ (c) $0.\bar{8}$ (d) $0.\bar{9}$
43. If $a = 0.1039$, then the value of $\sqrt{4a^2 - 4a + 1 + 3a}$ is [CBI]
 (a) 0.1039 (b) 0.2078 (c) 1.1039 (d) 2.1039

44. $\left(\sqrt{3} - \frac{1}{\sqrt{3}}\right)^2$ simplifies to [RRB]
 (a) $\frac{3}{4}$ (b) $\frac{4}{\sqrt{3}}$ (c) $\frac{4}{3}$ (d) None of these
45. Square root of $(7 + 3\sqrt{5})(7 - 3\sqrt{5})$ is [SSC]
 (a) $\sqrt{5}$ (b) 2 (c) 4 (d) $3\sqrt{5}$
46. The value of $\sqrt{\frac{(0.03)^2 + (0.21)^2 + (0.065)^2}{(0.003)^2 + (0.021)^2 + (0.0065)^2}}$ is [SSC]
 (a) 0.1 (b) 10 (c) 10^2 (d) 10^3
47. If $3\sqrt{5} + \sqrt{125} = 17.88$, then what will be the value of $\sqrt{80} + 6\sqrt{5}$? [Bank PO]
 (a) 13.41 (b) 20.46 (c) 21.66 (d) 22.35
48. If $\sqrt{1 + \frac{55}{729}} = 1 + \frac{x}{27}$, then the value of x is [CDS]
 (a) 1 (b) 3 (c) 5 (d) 7
49. $\sqrt{\frac{48.4}{0.289}}$ is equal to [SSC]
 (a) $1\frac{5}{17}$ (b) $12\frac{1}{17}$ (c) $12\frac{16}{17}$ (d) $129\frac{7}{17}$
50. If $\sqrt{x} \div \sqrt{441} = 0.02$, then the value of x is [SSC]
 (a) 0.1764 (b) 1.764 (c) 1.64 (d) 2.64
51. $28\sqrt{?} + 1426 = \frac{3}{4}$ of 2872. The value in place of (?) is [BSRB]
 (a) 576 (b) 676 (c) 1296 (d) 1444
52. What number should be divided by $\sqrt{0.25}$ to give the result as 25? [CBI]
 (a) 12.5 (b) 25 (c) 50 (d) 125
53. If $\frac{52}{x} = \sqrt{\frac{169}{289}}$, then the value of x is [CBI]
 (a) 52 (b) 58 (c) 62 (d) 68
54. $\sqrt{1.5625} = ?$
 (a) 1.05 (b) 1.25 (c) 1.45 (d) 1.55
55. $1.5^2 \times \sqrt{0.0225} = ?$ [Bank PO]
 (a) 0.0375 (b) 0.3375 (c) 3.275 (d) 32.75
56. The value of $\sqrt{0.000441}$ is [SSC]
 (a) 0.00021 (b) 0.0021 (c) 0.021 (d) 0.21
57. The digit in the unit's place in the square root of 15876 is [SSC]
 (a) 2 (b) 4 (c) 6 (d) 8

58. If $x * y = x + y + \sqrt{xy}$, the value of $6 * 24$ is [CBI]
 (a) 41 (b) 42 (c) 43 (d) 44
59. $\left(\frac{\sqrt{625}}{11} \times \frac{14}{\sqrt{25}} \times \frac{11}{\sqrt{196}}\right)$ is equal to [SSC]
 (a) 5 (b) 6 (c) 8 (d) 11
60. Evaluate $\sqrt{41 - \sqrt{21 + \sqrt{19 - \sqrt{9}}}}$ [CBI]
 (a) 3 (b) 5 (c) 6 (d) 6.4
61. The value of $\sqrt{10 + \sqrt{25 + \sqrt{108 + \sqrt{154 + \sqrt{225}}}}}$ is [SSC]
 (a) 4 (b) 6 (c) 8 (d) 10
62. Square root of $(272^2 - 128^2)$ is [SSC]
 (a) 144 (b) 200 (c) 240 (d) 256
63. $\sqrt{0.00004761} =$ [CBI]
 (a) 0.00069 (b) 0.0069 (c) 0.0609 (d) 0.069
64. The value of $\sqrt{0.01} + \sqrt{0.81} + \sqrt{1.21} + \sqrt{0.0009}$ is [SSC]
 (a) 2.03 (b) 2.1 (c) 2.11 (d) 2.13
65. $\sqrt{1.5625} = ?$ [SBI PO]
 (a) 1.05 (b) 1.25 (c) 1.45 (d) 1.55
66. Three-fifth of square of a certain number is 126.15. What is the number? [SSC]
 (a) 14.5 (b) 75.69 (c) 145 (d) 210.25
67. $\sqrt{\frac{96.8}{0.578}} =$ [SSC]
 (a) $1\frac{5}{17}$ (b) $12\frac{1}{17}$ (c) $12\frac{16}{17}$ (d) $129\frac{7}{17}$
68. The value of $\frac{\sqrt{80} - \sqrt{112}}{\sqrt{45} - \sqrt{63}}$ is [SSC]
 (a) $\frac{3}{4}$ (b) $1\frac{1}{3}$ (c) $1\frac{7}{9}$ (d) $1\frac{3}{4}$
69. The smallest number added to 680621 to make the sum a perfect square is [SSC]
 (a) 4 (b) 5 (c) 6 (d) 8
70. $\left(\frac{2 + \sqrt{3}}{2 - \sqrt{3}} + \frac{2 - \sqrt{3}}{2 + \sqrt{3}} + \frac{\sqrt{3} - 1}{\sqrt{3} + 1}\right)$ simplifies to [SSC]
 (a) $16 - \sqrt{3}$ (b) $4 - \sqrt{3}$ (c) $2 - \sqrt{3}$ (d) $2 + \sqrt{3}$
71. What is the least number which should be subtracted from 0.000365 to make it a perfect square? [SSC]
 (a) 0.000002 (b) 0.000004 (c) 0.02 (d) 0.04

72. If $\sqrt{5} = 2.236$, then value of $\frac{1}{\sqrt{5}}$ is
 (a) 0.447 (b) 0.367 (c) 0.745 (d) None of these
73. The value of $\frac{1 + \sqrt{0.04}}{1 - \sqrt{0.1}}$ is nearly equal to [CBI]
 (a) 0.6 (b) 1.1 (c) 1.6 (d) 1.7
74. Square root of $\frac{(0.75)^3}{1 - 0.75} + [0.75 + 0.75^2 + 1]$ is [SSC]
 (a) 1 (b) 2 (c) 3 (d) 4
75. Given $\sqrt{2} = 1.414$. The value of $\sqrt{8} + 2\sqrt{32} - 3\sqrt{128} + 4\sqrt{50}$ is [SSC]
 (a) 8.426 (b) 8.484 (c) 8.526 (d) 8.876
76. The least perfect square, which is divisible by 21, 36 and 66 completely is [CBI]
 (a) 213444 (b) 214344 (c) 214434 (d) 231444
77. The value of $\sqrt{\frac{0.16}{0.4}}$ is [IGNOU]
 (a) 0.02 (b) 0.2 (c) 0.63 (d) None of these
78. $\sqrt{\frac{0.081 \times 0.484}{0.0064 \times 6.25}}$ is equal to [NIFT]
 (a) 0.9 (b) 0.99 (c) 9 (d) 99
79. If $0.13 + p^2 = 13$, then p equals [SSC]
 (a) 0.01 (b) 0.1 (c) 10 (d) 100
80. What should come in place of both the question marks in the equation $\frac{?}{\sqrt{128}} = \frac{\sqrt{162}}{?}$ [Bank PO]
 (a) 12 (b) 14 (c) 144 (d) 196
81. For what value of ? the statement $\left(\frac{?}{15}\right)\left(\frac{?}{135}\right) = 1$ is true? [SSC]
 (a) 15 (b) 25 (c) 35 (d) 45
82. Given that $\sqrt{13} = 3.605$ and $\sqrt{130} = 11.40$, find the value of $\sqrt{1.3} + \sqrt{1300} + \sqrt{0.013}$ [SSC]
 (a) 36.164 (b) 36.304 (c) 37.164 (d) 37.304



- ANSWERS**
31. (c) 32. (c) 33. (a) 34. (c) 35. (c) 36. (b) 37. (c) 38. (a) 39. (b) 40. (b)
 41. (c) 42. (a) 43. (c) 44. (c) 45. (b) 46. (b) 47. (d) 48. (a) 49. (c) 50. (a)
 51. (b) 52. (a) 53. (d) 54. (b) 55. (c) 56. (c) 57. (c) 58. (b) 59. (a) 60. (c)
 61. (a) 62. (c) 63. (b) 64. (d) 65. (b) 66. (a) 67. (c) 68. (b) 69. (a) 70. (a)
 71. (b) 72. (a) 73. (d) 74. (d) 75. (b) 76. (a) 77. (c) 78. (b) 79. (b) 80. (a)
 81. (d) 82. (d)



Solutions with Necessary Explanation

1. $(847)^2 = ?$

$$\begin{aligned}(847)^2 &= \text{D for } 8 \mid \text{D for } 84 \mid \text{D for } 847 \mid \text{D for } 47 \mid \text{D for } 7 \\ &= 8^2 \mid 2(8 \times 4) \mid 2(8 \times 7) + 4^2 \mid 2(4 \times 7) \mid 7^2 \\ &= 64 \mid 64 \mid 128 \mid 56 \mid 49 \\ &= \mathbf{717409}\end{aligned}$$

$$\begin{array}{r} 49 \\ 56 \\ 128 \\ 64 \\ 64 \\ \hline 717409 \end{array}$$

2. $(8347)^2 = ?$

$$\begin{aligned}(8347)^2 &= \text{D for } 8 \mid \text{D for } 83 \mid \text{D for } 834 \mid \text{D for } 8347 \mid \text{D for } 347 \mid \text{D for } 47 \mid \text{D for } 7 \\ &= 8^2 \mid 2(8 \times 3) \mid 2(8 \times 4) + 3^2 \mid 2(8 \times 7) + 2(3 \times 4) \mid 2(3 \times 7) + 4^2 \\ &\quad \mid 2(4 \times 7) \mid 7^2 \\ &= 64 \mid 48 \mid 73 \mid 136 \mid 58 \mid 56 \mid 49 \mid \\ &= \mathbf{69672409}\end{aligned}$$

$$\begin{array}{r} 49 \\ 56 \\ 58 \\ 136 \\ 73 \\ 48 \\ 64 \\ \hline 69672409 \end{array}$$

3. $(54613)^2 = ?$

$$\begin{aligned}(54613)^2 &= \text{D for } 5 \mid \text{D for } 54 \mid \text{D for } 546 \mid \text{D for } 5461 \mid \text{D for } 54613 \mid \text{D for } 4613 \\ &\quad \mid \text{D for } 613 \mid \text{D for } 13 \mid \text{D for } 3. \\ &= 5^2 \mid 2(5 \times 4) \mid 2(5 \times 6) + 4^2 \mid 2(5 \times 1) + 2(4 \times 6) \mid 2(5 \times 3) + 2(4 \times 1) \\ &\quad + 6^2 \mid 2(4 \times 3) + 2(6 \times 1) \mid 2(6 \times 3) + 1^2 \mid 2(1 \times 3) \mid 3^2 \\ &= 25 \mid 40 \mid 76 \mid 58 \mid 74 \mid 36 \mid 37 \mid 6 \mid 9 \mid \\ &= \mathbf{2982579769}\end{aligned}$$

$$\begin{array}{r} 09 \\ 06 \\ 37 \\ 36 \\ 74 \\ 58 \\ 76 \\ 40 \\ 25 \\ \hline 2982579769 \end{array}$$

4. $(480028)^2 = ?$

$$\begin{aligned}(480028)^2 &= \text{D for } 4 \mid \text{D for } 48 \mid \text{D for } 480 \mid \text{D for } 4800 \mid \text{D for } 48002 \mid \text{D for } 480028 \\ &\quad \mid \text{D for } 80028 \mid \text{D for } 0028 \mid \text{D for } 028 \mid \text{D for } 28 \mid \text{D for } 8 \\ &= 4^2 \mid 2(4 \times 8) \mid 2(4 \times 0) + 8^2 \mid 2(4 \times 0) + 2(8 \times 0) \mid 2(4 \times 2) \\ &\quad + 2(8 \times 0) + 0^2 \mid 2(4 \times 8) + 2(8 \times 2) + 2(0 \times 0) \mid 2(8 \times 8) \\ &\quad + 2(0 \times 2) + 0^2 \mid 2(0 \times 8) + 2(0 \times 2) \mid 2(0 \times 8) \\ &\quad + 2^2 \mid 2(2 \times 8) \mid 8^2 \\ &= 16 \mid 64 \mid 64 \mid 0 \mid 16 \mid 96 \mid 128 \mid 0 \mid 4 \mid 32 \mid 64 \\ &= \mathbf{230426880784}\end{aligned}$$

$$\begin{array}{r} 64 \\ 32 \\ 04 \\ 00 \\ 128 \\ 96 \\ 16 \\ 00 \\ 64 \\ 64 \\ 16 \\ \hline 230426880784 \end{array}$$

5. $(63040586)^2 = ?$

$$\begin{aligned}(63040586)^2 &= \text{D for } 6 \mid \text{D for } 63 \mid \text{D for } 630 \mid \text{D for } 6304 \mid \text{D for } 63040 \mid \text{D for } 630405 \\ &\quad \mid \text{D for } 6304058 \mid \text{D for } 63040586 \mid \text{D for } 3040586 \mid \text{D for } 040586 \mid \\ &\quad \text{D for } 40586 \mid \\ &\quad \text{D for } 0586 \mid \text{D for } 586 \mid \text{D for } 86 \mid \text{D for } 6 \\ &= 6^2 \mid 2(6 \times 3) \mid 2(6 \times 0) + 3^2 \mid 2(6 \times 4) + 2(3 \times 0) \mid 2(6 \times 0) + 2(3 \times 4) + 0^2 \\ &\quad \mid 2(6 \times 5) + 2(3 \times 0) + 2(0 \times 4) \mid 2(6 \times 8) + 2(3 \times 5) + 2(0 \times 0) + 4^2 \\ &\quad \mid 2(6 \times 6) + 2(3 \times 8) + 2(0 \times 5) + 2(4 \times 0) \mid 2(3 \times 6) + 2(0 \times 8) + 2(4 \times 5) + 0^2 \\ &\quad \mid 2(0 \times 6) + 2(4 \times 8) + 2(0 \times 5) \mid 2(4 \times 6) + 2(0 \times 8) + 5^2 \mid 2(0 \times 6) + 2(5 \times 8) \\ &\quad \mid 2(5 \times 6) + 8^2 \mid 2(8 \times 6) \mid 6^2 \\ &= 36 \mid 36 \mid 9 \mid 48 \mid 24 \mid 60 \mid 142 \mid 120 \mid 76 \mid 64 \mid 73 \mid 80 \mid 124 \mid 96 \mid 36 \\ &= \mathbf{3974115483223396}\end{aligned}$$

$$\begin{array}{r}
 36 \\
 96 \\
 124 \\
 80 \\
 73 \\
 64 \\
 76 \\
 120 \\
 142 \\
 60 \\
 24 \\
 48 \\
 09 \\
 36 \\
 \hline
 3974115483223396
 \end{array}$$

6. $108^2 = (100 + 8)^2 = 10000 + 1600 + 64 = 11664$
7. $96^2 = (100 - 4)^2 = 10000 - 800 + 16 = 9216$
8. $997^2 = (1000 - 3)^2 = 1000000 - 6000 + 9 = 944009$
9. $1004^2 = (1000 + 4)^2 = 1000000 + 8000 + 16 = 1008016$
10. $1009^2 = (1000 + 9)^2 = 1000000 + 18000 + 81 = 1018081$
11. $999^2 = (1000 - 1)^2 = 1000000 - 2000 + 1 = 998001$
12. $991^2 = (1000 - 9)^2 = 1000000 - 18000 + 81 = 982081$
13. $94^2 = (100 - 6)^2 = 10000 - 1200 + 36 = 8836$
14. $106^2 = (100 + 6)^2 = 10000 + 1200 + 36 = 11236$
15. $109^2 = (100 + 9)^2 = 10000 + 1800 + 81 = 11881$
16. $\sqrt{4624} = 10 \times \sqrt{46.24}$
 $\sqrt{46.24}$ lies between 6 and 7. The root will end with either 2 or 8 as square ends with 4. Here 46.24 is nearer to $7^2 = 49$.
 $\therefore$ Root is $6.8 \times 10 = 68$
17. $\sqrt{20044} = 10 \times \sqrt{200.44}$
 $\sqrt{200.44}$ lies between 14 and 15. The root will end with either 2 or 8 as square ends with 4. Here 200.44 is nearer to $14^2 = 196$.
Hence root is $10 \times 14.2 = 142$
18. $\sqrt{12321} = 10 \times \sqrt{123.21}$
 $\sqrt{123.21}$ lies between 11 and 12. The root will end with 1 or 9 as square ends with 1. Here 123.21 is nearer to $11^2 = 121$.
Hence root is $10 \times 11.1 = 111$
21. $\sqrt{61009} = 10 \times \sqrt{610.09}$
 $\sqrt{610.09}$ lies between 24 and 25. Root will end with 3 or 7 as square ends with 9. Here 610.09 is nearer to $25^2 = 625$.
Hence root is $10 \times 24.7 = 247$
23. $\sqrt{481636} = 10 \times \sqrt{4816.36}$
 $\sqrt{4816.36}$ lies between 69 and 70. Root will end with either 4 or 6 as square ends with 6.

Here 4816.36 is nearer to $69^2 = 4761$

Hence root is $10 \times 69.4 = \mathbf{694}$

24. $\sqrt{906304} = 10 \times \sqrt{9063.04}$

$\sqrt{9063.04}$ lies between 95 and 96. Root will end with 2 or 8 as square ends with 4.

Here 9063.04 is nearer to $95^2 = 9025$

Hence root = $95.2 \times 10 = \mathbf{952}$

25. $\sqrt{619369} = 10 \times \sqrt{6193.69}$

$\sqrt{6193.69}$ lies between 78 and 79. Root will end with 3 or 7 as square ends with 9.

Here 6193.69 is nearer to $79^2 = 6241$

Hence root = $78.7 \times 10 = \mathbf{787}$

29. $\sqrt{802816} = 10 \times \sqrt{8028.16}$

$\sqrt{8028.16}$ lies between 89 and 90. Root will end with 4 or 6 as square ends with 6.

Here 8028.16 is nearer to $90^2 = 8100$.

Hence root = $89.6 \times 10 = \mathbf{896}$

30. $\sqrt{636804} = 10 \times \sqrt{6368.04}$

$\sqrt{6368.04}$ lies between 79 and 80. Root will end with 2 or 8 as square ends with 4.

Here 6368.04 is nearer to $80^2 = 6400$.

Hence root = $10 \times 79.8 = \mathbf{798}$

31. $x = \frac{(\sqrt{3} + 1)}{(\sqrt{3} - 1)} \times \frac{(\sqrt{3} + 1)}{(\sqrt{3} + 1)} = \frac{(\sqrt{3} + 1)^2}{(\sqrt{3})^2 - 1} = \frac{4 + 2\sqrt{3}}{3 - 1} = 2 + \sqrt{3}$

$$y = \frac{(\sqrt{3} - 1)}{(\sqrt{3} + 1)} \times \frac{(\sqrt{3} - 1)}{(\sqrt{3} - 1)} = \frac{(\sqrt{3} - 1)^2}{(\sqrt{3})^2 - 1^2} = \frac{4 - 2\sqrt{3}}{2} = 2 - \sqrt{3}$$

$$\therefore x^2 + y^2 = (2 + \sqrt{3})^2 + (2 - \sqrt{3})^2 = 2 \times [2^2 + (\sqrt{3})^2] = \mathbf{14}$$

32. $x = 7 - 4\sqrt{3}$

$$\therefore \frac{1}{x} = \frac{1}{7 - 4\sqrt{3}} = \frac{(7 + 4\sqrt{3}) \times 1}{(7 + 4\sqrt{3}) \times (7 - 4\sqrt{3})} = \frac{7 + 4\sqrt{3}}{7^2 - (4\sqrt{3})^2} = \frac{7 + 4\sqrt{3}}{49 - 48} = 7 + 4\sqrt{3}$$

$$\therefore x + \frac{1}{x} = 7 - 4\sqrt{3} + 7 + 4\sqrt{3} = \mathbf{14}$$

33. $\frac{\sqrt{2} - 1}{\sqrt{2} + 1} = \frac{(\sqrt{2} - 1)}{(\sqrt{2} + 1)} \times \frac{(\sqrt{2} - 1)}{(\sqrt{2} - 1)} = \frac{(\sqrt{2} - 1)^2}{(\sqrt{2})^2 - 1^2} = \frac{2 + 1 - \sqrt{2}}{(2 - 1)} = 3 - 2\sqrt{2}$

Given $\sqrt{2} = 1.414$

$$\therefore 3 - 2\sqrt{2} = 3 - 2 \times 1.414 = 3 - 2.828 = \mathbf{0.172}$$

34.
$$\frac{5 + 2\sqrt{3}}{7 + 4\sqrt{3}} = \frac{(5 + 2\sqrt{3})}{(7 + 4\sqrt{3})} \times \frac{(7 - 4\sqrt{3})}{(7 - 4\sqrt{3})}$$

$$= \frac{5 \times 7 + 7 \times 2\sqrt{3} - 5 \times 4\sqrt{3} - 2 \times 4 \times 3}{7^2 - (4\sqrt{3})^2}$$

$$= \frac{35 - 24 + \sqrt{3}(14 - 20)}{49 - 48} = 11 - 6\sqrt{3}$$

$$\begin{aligned} \therefore a + b\sqrt{3} &= 11 - 6\sqrt{3} \\ \Rightarrow a &= 11; \quad b = -6 \end{aligned}$$

$$\begin{aligned} 35. \quad & \frac{3\sqrt{2}}{\sqrt{6} - \sqrt{3}} - \frac{4\sqrt{3}}{\sqrt{6} - \sqrt{2}} - \frac{6}{\sqrt{8} - \sqrt{12}} \\ &= \frac{3\sqrt{2} \times (\sqrt{6} + \sqrt{3})}{(\sqrt{6} - \sqrt{3})(\sqrt{6} + \sqrt{3})} - \frac{4\sqrt{3}}{(\sqrt{6} - \sqrt{2})} \times \frac{(\sqrt{6} + \sqrt{2})}{(\sqrt{6} + \sqrt{2})} - \frac{6}{(\sqrt{8} - \sqrt{12})} \frac{(\sqrt{8} + \sqrt{12})}{(\sqrt{8} + \sqrt{12})} \\ &= \frac{3\sqrt{2}(\sqrt{6} + \sqrt{3})}{6 - 3} - \frac{4\sqrt{3}(\sqrt{6} + \sqrt{2})}{6 - 2} - \frac{6(\sqrt{8} + \sqrt{12})}{8 - 12} \\ &= \frac{3\sqrt{12} + 3\sqrt{6}}{3} - \frac{(4\sqrt{18} + 4\sqrt{6})}{4} - \frac{(6\sqrt{8} + 6\sqrt{12})}{-4} \\ &= \sqrt{12} + \sqrt{6} - \sqrt{18} - \sqrt{6} + \frac{3\sqrt{8}}{2} + \frac{3}{2}\sqrt{12} \\ &= 2\sqrt{3} + \sqrt{6} - 3\sqrt{2} - \sqrt{6} + 3\sqrt{2} + 3\sqrt{3} \\ &= 2\sqrt{3} + 3\sqrt{3} = 5\sqrt{3} \end{aligned}$$

$$\begin{aligned} 36. \quad & 2 + \sqrt{2} + \frac{1}{2 + \sqrt{2}} + \frac{1}{\sqrt{2} - 2} = 2 + \sqrt{2} + \frac{1}{(2 + \sqrt{2})} \times \frac{(2 - \sqrt{2})}{(2 - \sqrt{2})} + \frac{1}{(\sqrt{2} - 2)} \times \frac{(2 + \sqrt{2})}{(2 + \sqrt{2})} \\ &= (2 + \sqrt{2}) + \frac{2 - \sqrt{2}}{2^2 - (\sqrt{2})^2} + \frac{-1(2 + \sqrt{2})}{2^2 - (\sqrt{2})^2} \\ &= 2 + \sqrt{2} + \frac{2 - \sqrt{2}}{4 - 2} - \frac{1(2 + \sqrt{2})}{4 - 2} \\ &= 2 + \sqrt{2} + 1 - \frac{\sqrt{2}}{2} - 1 - \frac{\sqrt{2}}{2} = 2 \end{aligned}$$

$$\begin{aligned} 37. \quad & \frac{1}{\sqrt{5} - \sqrt{3}} = \frac{1}{(\sqrt{5} - \sqrt{3})} \times \frac{(\sqrt{5} + \sqrt{3})}{(\sqrt{5} + \sqrt{3})} \\ &= \frac{\sqrt{5} + \sqrt{3}}{(\sqrt{5})^2 - (\sqrt{3})^2} \\ &= \frac{\sqrt{5} + \sqrt{3}}{5 - 3} = \frac{\sqrt{5} + \sqrt{3}}{2} \end{aligned}$$

$$\text{Given } \sqrt{5} = 2.2361; \quad \sqrt{3} = 1.7321$$

$$\therefore \frac{\sqrt{5} + \sqrt{3}}{2} = \frac{2.2361 + 1.7321}{2} = \frac{3.9682}{2} = 1.9841$$

$$38. \quad 0.000326 = 326 \times 10^{-6}$$

324×10^{-6} is a perfect square

$$\therefore \text{Required number to be subtracted} = (326 - 324) \times 10^{-6} = 2 \times 10^{-6}$$

39. $5808 = 2 \times 2 \times 2 \times 2 \times 3 \times 11 \times 11 = 2^4 \times 3 \times 11^2$

To make the above number a perfect square, it should be multiplied by 3.

40. $\frac{\sqrt{5}}{2} - \frac{10}{\sqrt{5}} + \sqrt{125} = \frac{\sqrt{5}}{2} - 2\sqrt{5} + 5\sqrt{5} = \left(5 - 2 + \frac{1}{2}\right)\sqrt{5} = \frac{7}{2}\sqrt{5}$

Given $\sqrt{5} = 2.236$

$\therefore \frac{7\sqrt{5}}{2} = 7 \times \frac{2.236}{2} = 7 \times 1.118 = \mathbf{7.826}$

41. $\frac{1 + \sqrt{0.01}}{1 - \sqrt{0.1}} = \frac{1 + 0.1}{1 - \sqrt{0.1}}$

$$\frac{(1 + 0.1)(1 + \sqrt{0.1})}{(1 - \sqrt{0.1})(1 + \sqrt{0.1})} = \frac{1 + \sqrt{0.1} + 0.1 + 0.1\sqrt{0.1}}{1^2 - 0.1}$$

$$= \frac{1.1 + 1.1\sqrt{0.1}}{0.9} = \frac{1.1 \times 1.361}{0.9}$$

($\because \sqrt{0.1} = 0.361$ approx.)

$$= \frac{1.4971}{0.9} = \frac{1.4971}{9} = 1.663 \approx \mathbf{1.6}$$

42. $0.\overline{4} = 0.444 \dots = \frac{4}{9}$

$\therefore \frac{\sqrt{4}}{9} = \frac{2}{3} = 0.666 \dots = \mathbf{0.\overline{6}}$

43. $\sqrt{4a^2 - 4a + 1} + 3a = \sqrt{(2a - 1)^2} + 3a = \pm(2a - 1) + 3a$
 $= 5a - 1 \quad \text{or} \quad 3a - (2a - 1) = (a + 1)$

Given $a = 0.1039$

$\therefore 5a - 1 = 5 \times 0.1039 - 1 = 0.5195 - 1 = \mathbf{0.4705}$

$a + 1 = 0.1039 + 1 = \mathbf{1.1039}$

44. $\left(\sqrt{3} - \frac{1}{\sqrt{3}}\right)^2 = 3 + \frac{1}{3} - 2 \times \sqrt{3} \times \frac{1}{\sqrt{3}} = 3 + \frac{1}{3} - 2 = 1 + \frac{1}{3} = \frac{4}{3}$

45. $\sqrt{(7 + 3\sqrt{5})(7 - 3\sqrt{5})} = \sqrt{(7^2 - (3\sqrt{5})^2)} = \sqrt{49 - 45} = \sqrt{4} = \mathbf{2}$

46. $\sqrt{\frac{(0.03)^2 + (0.21)^2 + (0.065)^2}{(0.003)^2 + (0.021)^2 + (0.0065)^2}} = \sqrt{\frac{10^2 [(0.003)^2 + (0.021)^2 + (0.0065)^2]}{(0.003)^2 + (0.021)^2 + (0.0065)^2}} = \mathbf{10}$

47. $3\sqrt{5} + \sqrt{125} = 3\sqrt{5} + 5\sqrt{5} = 8\sqrt{5}$

Given that $8\sqrt{5} = 17.88$

$\therefore \sqrt{80} + 6\sqrt{5} = 4\sqrt{5} + 6\sqrt{5} = 10\sqrt{5} = \frac{17.88}{8\sqrt{5}} \times 10\sqrt{5} = \mathbf{22.35}$

48. $\sqrt{1 + \frac{55}{729}} = 1 + \frac{x}{27}$

$\Rightarrow 1 + \frac{55}{729} = \left(1 + \frac{x}{27}\right)^2 = 1 + \frac{x^2}{27^2} + \frac{2x}{27}$

$$\Rightarrow \frac{x^2}{27^2} + \frac{2x}{27} = \frac{55}{729}$$

$$\Rightarrow \frac{x^2 + 27 \times 2x}{27^2} = \frac{55}{729}$$

$$\Rightarrow x^2 + 54x - 55 = 0$$

$$\therefore (x + 55)(x - 1) = 0 \Rightarrow x = 1 \text{ or } x = -55$$

$$\therefore x = 1$$

$$49. \sqrt{\frac{48.4}{0.289}} = \sqrt{\frac{48400}{289}} = \frac{220}{17} = 12 \frac{16}{17}$$

$$50. \sqrt{x} + \sqrt{441} = 0.02$$

$$\therefore x = (0.02 \times \sqrt{441})^2 = (0.02 \times \sqrt{441})^2 = (0.42)^2 = 0.1764$$

$$51. 28\sqrt{?} = \frac{3}{4} \times 2872 - 1426 = 3 \times 718 - 1426 = 728$$

$$\therefore \text{Value in place of ?} = \left(\frac{728}{28}\right)^2 = (26)^2 = 676$$

$$52. \text{The required number} = 25 \times \sqrt{0.25} = 25 \times 0.5 = 12.5$$

$$53. \frac{52}{x} = \sqrt{\frac{169}{289}} = \frac{13}{17}$$

$$\therefore x = \frac{52 \times 17}{13} = 68$$

$$54. \sqrt{1.5625} = 1.25$$

$$55. 1.5^2 \times \sqrt{0.0225} = 1.5^2 \times 10^{-2} \times 15 = 3.275$$

$$56. \sqrt{0.000441} = \sqrt{10^{-6} \times 441} = 10^{-3} \times 21 = 0.021$$

$$57. 15876 = 158.76 \times 10^2$$

158.76 in between squares 144 and 169 and is nearer to 169. Therefore square root should end with 6.

$$58. x * y = x + y + \sqrt{xy}$$

$$\therefore 6 * 24 = 6 + 24 + \sqrt{6 \times 24}$$

$$= 30 + \sqrt{144} = 30 + 12 = 42$$

$$59. \left(\frac{\sqrt{625}}{11} \times \frac{14}{\sqrt{25}} \times \frac{11}{\sqrt{196}} \right) = \frac{25}{11} \times \frac{14}{5} \times \frac{11}{14} = \frac{25}{5} = 5$$

$$60. \sqrt{41 - \sqrt{21 + \sqrt{19 - \sqrt{9}}}} = \sqrt{41 - \sqrt{21 + \sqrt{(19 - 3)}}}$$

$$= \sqrt{41 - \sqrt{21 + 4}} = \sqrt{41 - 5} = \sqrt{36} = 6$$

$$\begin{aligned}
 61. \quad \sqrt{10 + \sqrt{25 + \sqrt{108 + \sqrt{154 + \sqrt{225}}}}} &= \sqrt{10 + \sqrt{25 + \sqrt{108 + \sqrt{154 + 15}}}} \\
 &= \sqrt{10 + \sqrt{25 + \sqrt{108 + 13}}} \\
 &= \sqrt{10 + \sqrt{25 + 11}} = \sqrt{10 + 6} = \sqrt{16} = 4
 \end{aligned}$$

$$62. \quad \sqrt{272^2 - 128^2} = \sqrt{(272 + 128)(272 - 128)} = \sqrt{400 \times 144} = 20 \times 12 = \mathbf{240}$$

$$63. \quad \sqrt{0.00004761} = \sqrt{10^{-6} \times 47.61} = 10^{-3} \times 6.9 = \mathbf{0.0069}$$

$$64. \quad \sqrt{0.01} + \sqrt{0.81} + \sqrt{1.21} + \sqrt{0.0009} = 0.1 + 0.9 + 1.1 + 0.03 = \mathbf{2.13}$$

$$65. \quad \sqrt{1.5625} = \mathbf{1.25}$$

$$66. \quad \frac{3}{5} \times x^2 = 126.15$$

$$\therefore x^2 = \frac{126.15 \times 5}{3} = 42.05 \times 5$$

$$x^2 = 210.25$$

$$\text{or } x = \sqrt{210.25} = \mathbf{14.5}$$

$$67. \quad \sqrt{\frac{96.8}{0.578}} = \sqrt{\frac{96800}{578}} = \sqrt{\frac{48400}{289}} = \mathbf{12 \frac{16}{17}}$$

$$68. \quad \frac{\sqrt{80} - \sqrt{112}}{\sqrt{45} - \sqrt{63}} = \frac{4\sqrt{5} - 4\sqrt{7}}{3\sqrt{5} - 3\sqrt{7}} = \frac{4(\sqrt{5} - \sqrt{7})}{3(\sqrt{5} - \sqrt{7})} = \frac{4}{3} = \mathbf{1 \frac{1}{3}}$$

69. 4 is to be added to 680621 to become a perfect square

$$\begin{array}{r}
 825 \\
 8 \overline{) 680621} \\
 \underline{64} \\
 162 \\
 \underline{160} \\
 22 \\
 1645 \\
 \underline{1645} \\
 0
 \end{array}$$

$$70. \quad \frac{2 + \sqrt{3}}{2 - \sqrt{3}} + \frac{2 - \sqrt{3}}{2 + \sqrt{3}} + \frac{\sqrt{3} - 1}{\sqrt{3} + 1}$$

$$= \frac{(2 + \sqrt{3})(2 + \sqrt{3})}{(2 - \sqrt{3})(2 + \sqrt{3})} + \frac{(2 - \sqrt{3})}{2 + \sqrt{3}} \times \frac{(2 - \sqrt{3})}{(2 - \sqrt{3})} + \frac{(\sqrt{3} - 1)(\sqrt{3} - 1)}{(\sqrt{3} + 1)(\sqrt{3} - 1)}$$

$$= \frac{(2 + \sqrt{3})^2}{(2)^2 - (\sqrt{3})^2} + \frac{(2 - \sqrt{3})^2}{(2)^2 - (\sqrt{3})^2} + \frac{(\sqrt{3} - 1)^2}{(\sqrt{3})^2 - (1)^2}$$

$$= \frac{4 + 3 + 4\sqrt{3}}{4 - 3} + \frac{4 + 3 - 4\sqrt{3}}{4 - 3} + \frac{3 + 1 - 2\sqrt{3}}{3 - 1}$$

$$\begin{aligned}
&= 7 + 4\sqrt{3} + 7 - 4\sqrt{3} + \frac{4 - 2\sqrt{3}}{2} \\
&= 7 + 7 + 2 + 4\sqrt{3} - 4\sqrt{3} - \sqrt{3} \\
&= 16 - \sqrt{3}
\end{aligned}$$

71. $0.000365 = 365 \times 10^{-6}$
 4×10^{-6} should be subtracted to get a perfect square.

72. $\frac{1}{\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}} = \frac{\sqrt{5}}{5} = \frac{2.236}{5} = 0.4472 \approx \mathbf{0.447}$

73. $\frac{1 + \sqrt{0.04}}{1 - \sqrt{0.1}} = \frac{1 + 0.2}{1 - 0.316} = \frac{1.2}{0.684} = \mathbf{1.7 \text{ approx.}}$

74. $\frac{(0.75)^3}{1 - 0.75} + [0.75 + 0.75^2 + 1] = (0.75)^2 \times 3 + 0.75 + 0.75^2 + 1$
 $= 4 \times (0.75)^2 + 1.75$
 $= 4 \times \frac{9}{16} + 1.75 = 2.25 + 1.75 = 4$

75. $\sqrt{8} + 2\sqrt{32} - 3\sqrt{128} + 4\sqrt{50}$
 $= 2\sqrt{2} + 8\sqrt{2} - 24\sqrt{2} + 20\sqrt{2}$
 $= 6\sqrt{2} = 6 \times 1.414 = \mathbf{8.484}$

76. LCM of 21, 36 and 66 = $2^2 \times 3^2 \times 7 \times 11$

3	21, 36, 66
2	7, 12, 22
2	7, 6, 11
	7, 3, 11

$\therefore$ Least perfect square divisible by 21, 36 and 66 = $2^2 \times 3^2 \times 7^2 \times 11^2 = \mathbf{213444}$

77. $\sqrt{\frac{0.16}{0.4}} = \sqrt{0.4} = \mathbf{0.63}$

78. $\sqrt{\frac{0.081 \times 0.484}{0.0064 \times 6.25}} = \sqrt{\frac{0.81 \times 0.0484}{0.0064 \times 6.25}} = \frac{0.9 \times 0.22}{0.08 \times 2.5}$
 $= \frac{0.9 \times 0.22}{0.2} = 0.9 \times 1.1 = \mathbf{0.99}$

79. $0.13 \div p^2 = 13$

or $\frac{0.13}{p^2} = 13$

or $p^2 = \frac{0.13}{13} = 0.01$

or $p = \mathbf{0.1}$

$$80. \frac{?}{\sqrt{128}} = \frac{\sqrt{162}}{?}$$

$$\therefore (?)^2 = \sqrt{128 \times 162} = \sqrt{64 \times 324} = 8 \times 18$$

$$\begin{aligned}\therefore \quad ? &= \sqrt{18 \times 8} \\ &= \sqrt{36 \times 4} = \mathbf{12}\end{aligned}$$

$$81. \left(\frac{?}{15}\right)\left(\frac{?}{135}\right) = 1$$

$$\begin{aligned}\text{or} \quad (?)^2 &= 15 \times 135 \\ &= 15^2 \times 9\end{aligned}$$

$$\begin{aligned}\text{or} \quad ? &= \sqrt{15^2 \times 9} \\ &= 15 \times 3 = \mathbf{45}\end{aligned}$$

$$\begin{aligned}82. \quad \sqrt{1.3} + \sqrt{1300} + \sqrt{0.013} &= \sqrt{10^{-2} \times 130} + 10\sqrt{13} + \sqrt{10^{-4} \times 130} \\ &= 0.1\sqrt{130} + 10\sqrt{13} + 0.01\sqrt{130} \\ &= 10\sqrt{13} + 0.11\sqrt{130}\end{aligned}$$

$$\text{Given } \sqrt{13} = 3.605; \quad \sqrt{130} = 11.4$$

$$\begin{aligned}\therefore \quad 10\sqrt{13} + 0.11\sqrt{130} &= 10 \times 3.605 + 0.11 \times 11.4 \\ &= 36.05 + 1.254 = \mathbf{37.304}\end{aligned}$$

Cubes and Cube Roots

To find out the cube root of a number, the reader is advised to recall the cubes of first ten natural numbers.

The cube root of a number can be identified using the end digit of those perfect cubes. The following table illustrates how to find easily the cube roots of the given numbers (perfect cubes).

If the number ends with digit	Its cube root will end with digit
1	1
2	8
3	7
4	4
5	5
6	6
7	3
8	2
9	9

TO FIND CUBES OF TWO-DIGIT NUMBERS

Step 1 Write down the cubes of digit in tenth place in a row of 4 figures. The other three numbers should be written in a geometrical ratio which exists between the unit's place and tenth place of given number.

Step II Write down under the second and third numbers, just two times of second and third number. Then add up the two rows.

EXAMPLE 1 Find the value of $(12)^3$.

Solution Step I Tens' digit is 1. So write the cube of 1. The ratio between the unit's digit and ten's digit is 2 : 1, so, the next digit will be double the previous one.

So the first row is

1 2 4 8

Step II In the above row, the 2nd and 3rd elements are 2 and 4. Double these numbers and write below them. Then add up the two rows.

1	2	4	8
	4	8	
1	6	12	8
1	7	2	8

∴ The number is **1728**

EXAMPLE 2 Find the value of $(11)^3$.

Solution

Step I	1	1	1	1
Step II		2	2	
	1	3	3	1

∴ $(11)^3 = 1331$

EXAMPLE 3 Find the value of $(16)^3$.

Solution

Step I	1^3	$1^2 \cdot 6^1$	$1^1 \cdot 6^2$	$1^0 \cdot 6^3$
i.e.	1	6	36	216
Step II		12	72	
	1	18	108	216

Place carry over to }
left in each place. }

4 30 129 216

Read all end digits i.e. **4096**

∴ $(16)^3 = 4096$

EXAMPLE 4 Find the value of $(18)^3$.

Solution

Step I	1^3	$1^2 \cdot 8^1$	$1^1 \cdot 8^2$	$1^0 \cdot 8^3$
i.e.	1	8	64	512
Step II		16	128	

Add up the rows

1 24 192 512

Place carry over to }
left in each place. }

5 48 243 512

Read all end digits i.e. **5832**

∴ $(18)^3 = 5832$

EXAMPLE 5 Find the value of $(17)^3$.

Solution

Step I	1^3	$1^2 \cdot 7^1$	$1^1 \cdot 7^2$	$1^0 \cdot 7^3$
i.e.	1	7	49	343
Step II		7×2	49×2	
i.e.		14	98	

Add up to two rows

1 7 49 343

14 98

1 21 147 343

4 39 181 343

- (i) In the unit's place, write down 3 and carry 34.
- (ii) In the tens' place $49 + 98 + 34 = 181$. Write down 1 and carry 18.
- (iii) In the hundredth place, $7 + 14 + 18 = 39$. Write down 9 and carry 3.
- (iv) In the thousandth place, $1 + 3 = 4$.

$$\therefore (17)^3 = 4913$$

EXAMPLE 6 Find the value of $(19)^3$.

Solution

Step I	1^3	$1^2 \cdot 9^1$	$1^1 \cdot 9^2$	$1^0 \cdot 9^3$
i.e.	1	9	81	729
Step II		9×2	81×2	
i.e.		18	162	

Add upto two rows,

	1	9	81	729
		18	162	
	1+	27+	243+	729
	5	31	72	
	6	58	315	729

- (i) In the unit's place write 9 and carry 72.
- (ii) In the tens' place, $81 + 162 + 72 = 315$. Write down 5 and carry 31.
- (iii) In the hundredth place, $9 + 18 + 31 = 58$. Write down 8 and carry 5.
- (iv) $1 + 5 = 6$. Write it down in the thousandth place.

$$\therefore (19)^3 = 6859$$

EXAMPLE 7 Find $\sqrt[3]{1331}$.

Solution $\sqrt[3]{1331} = 10 \times \sqrt[3]{1.331}$

$\sqrt[3]{1.331}$ lies between 1 and 2. Since the end digit of the cube is 1, the cube root will also end with 1.

$$\therefore \text{root is } 10 \times 1.1 = 11$$

$$\therefore \sqrt[3]{1331} = 11$$

EXAMPLE 8 Find $\sqrt[3]{103823}$.

Solution $\sqrt[3]{103823} = 10 \times \sqrt[3]{103.823}$

$\sqrt[3]{103.823}$ lies between 4 and 5 since the end digit of perfect cube is 3, the cube root will end with 7.

$$\therefore \text{The root is } 10 \times 4.7 = 47$$

EXAMPLE 9 Find $\sqrt[3]{1092727}$.

Solution $\sqrt[3]{1092727} = 10 \times \sqrt[3]{1092.727}$

$\sqrt[3]{1092.727}$ lies between 10 and 11. Since the end digit of perfect cube is 7, the cube root will end with 3.

$$\therefore \text{The root is } 10 \times 10.3 = 103$$



EXERCISES

- Find the cubes of the following numbers:
 (a) 26 (b) 27 (c) 32 (d) 49 (e) 43
 (f) 51 (g) 53 (h) 57 (i) 64 (j) 69
 (k) 73 (l) 92 (m) 95
- Find the cube roots of the following:
 (a) 941192 (b) 681472 (c) 884736 (d) 405224 (e) 250047
 (f) 551368 (g) 753571 (h) 614125 (i) 300763 (j) 389017
 (k) 704969
- Find the cube root of 0.000216.
 (a) 0.6 (b) 0.06 (c) 0.006 (d) None of these
- Find the cube root of 0.000729.
 (a) 0.9 (b) 0.09 (c) 0.009 (d) None of these
- Smallest five-digit number which is a perfect cube is
 (a) 10961 (b) 10648 (c) 10968 (d) 10689
- What is the smallest number by which 5600 must be divided to make it a perfect cube?
 (a) 7 (b) 70 (c) 14 (d) 700
- The least perfect cube which is completely divisible by 21, 24 and 27 is
 (a) 70488 (b) 78048 (c) 74088 (d) 78084
- Find the least number by which 4800 should be multiplied to make it a perfect cube.
 (a) 75 (b) 50 (c) 45 (d) 30



3. (b) 4. (b) 5. (b) 6. (d) 7. (c) 8. (c)



Solutions with Necessary Explanation

[**Hint:** Students are advised to workout yourself remaining exercises for which only answers are given in the following manner.]

$$\begin{array}{rcccc}
 1. & (a) & (26)^3 = & 2^3 & 2^2 \times 6 & 2 \times 6^2 & 6^3 \\
 & & \text{Row I} & 8 & 24 & 72 & 216 \\
 & & \text{Row II} & & 48 & 144 & \\
 & & \text{Add I and II} & 8 & 72 & 216 & 216
 \end{array}$$

Write '6' in the unit place and carry 21 to 10th place. Then $216 + 21 = 237$.

Write '7' in the tenth place and carry 23 to 100th place.

i.e. $72 + 23 = 95$

Write '5' in the hundredth place and carry 9 to thousandth place i.e. $8 + 9 = 17$.

Hence $(26)^3 = 17576$

$$\begin{array}{rcll}
 \text{(c) } (32)^3 = & 3^3 & 3^2 \times 2 & 3 \times 2^2 & 2^3 \\
 \text{Row I i.e.} & 27 & 18 & 12 & 8 \\
 \text{Row II} & & 36 & 24 & \\
 \text{I + II} & 27+ & 54+ & 36 & 8 \\
 & 5 & 3 & & \\
 & 32 & 57 & & \\
 & & 7 & 6 & 8
 \end{array}$$

Hence $(32)^3 = 32768$

$$\begin{array}{rcll}
 \text{(d) } (49)^3 = & 4^3 & 4^2 \times 9 & 4 \times 9^2 & 9^3 \\
 \text{Row I} & 64 & 144 & 324 & 729 \\
 \text{II} & & 288 & 648 & \\
 \text{I + II} & 64 & 432+ & 972+ & 729 \\
 \text{Carry} \} & 53 & 104 & 72 & \\
 & 117 & 536 & 1044 & \\
 & & 6 & 4 & 9
 \end{array}$$

Hence $(49)^3 = 117649$

$$\begin{array}{rcll}
 \text{(h) } (57)^3 = & 5^3 & 5^2 \times 7 & 5 \times 7^2 & 7^3 \\
 \text{Row I} & 125 & 175 & 245 & 343 \\
 \text{II} & & 350 & 490 & \\
 \text{I + II} & 125 & 525+ & 735+ & 343 \\
 \text{Carry} \} & 60 & 76 & 34 & \\
 & 185 & 601 & 769 & \\
 & & 1 & 9 & 3
 \end{array}$$

Hence $(57)^3 = 185193$

$$\begin{array}{rcll}
 \text{(j) } (69)^3 = & 6^3 & 6^2 \times 9 & 6 \times 9^2 & 9^3 \\
 \text{Row I} & 216 & 324 & 486 & 729 \\
 \text{II} & & 648 & 972 & \\
 \text{I + II} & 216+ & 972+ & 1458+ & 729 \\
 \text{Carry} \} & 112 & 153 & 72 & \\
 & 328 & 1125 & 1530 & \\
 & & 5 & 0 & 9
 \end{array}$$

Hence $(69)^3 = 328509$

$$\begin{array}{rcll}
 \text{(m) } (95)^3 = & 9^3 & 9^2 \times 5 & 9 \times 5^2 & 5^3 \\
 \text{Row I} & 729 & 405 & 225 & 125 \\
 \text{Row II} & & 810 & 450 & \\
 \text{I + II} & 729 & 1215+ & 675+ & 125 \\
 \text{Carry} \} & 128 & 68 & 12 & \\
 & 857 & 1283 & 687 & \\
 & & 3 & 7 & 5
 \end{array}$$

Hence $(95)^3 = 857375$

2. (a) $\sqrt[3]{941192} = 10 \times \sqrt[3]{941.192}$

$\sqrt[3]{941.192}$ lies between 9 and 10. Cube ends with 2 and so cube root will end with 8.

$$\therefore \sqrt[3]{941.192} = 9.8$$

Hence $\sqrt[3]{941192} = 10 \times 9.8 = 98$

$$(c) \sqrt[3]{884736} = 10 \times \sqrt[3]{884.736}$$

$\sqrt[3]{884.736}$ lies between 9 and 10. Cube ends with 6 and so cube root will end with '6'.

$$\therefore \sqrt[3]{884.736} = 9.6$$

$$\text{Hence } \sqrt[3]{884736} = 10 \times 9.6 = \mathbf{96}$$

$$(e) \sqrt[3]{250047} = 10 \times \sqrt[3]{250.047}$$

$\sqrt[3]{250.047}$ lies between 6 and 7. Cube root will end with '3' as cube end with '7'.

$$\therefore \sqrt[3]{250.047} = 6.3$$

$$\therefore \sqrt[3]{250047} = 10 \times 6.3 = \mathbf{63}$$

$$(g) \sqrt[3]{753571} = 10 \times \sqrt[3]{753.571}$$

$\sqrt[3]{753.571}$ lies between 9 and 10. Cube root will end with '1' as cube end with '1'.

$$\therefore \sqrt[3]{753.571} = 9.1$$

$$\therefore \sqrt[3]{753571} = 10 \times 9.1 = \mathbf{91}$$

$$(h) \sqrt[3]{614125} = 10 \times \sqrt[3]{614.125}$$

$\sqrt[3]{614.125}$ lies between 8 and 9. Cube root will end with '5' as cube end with '5'.

$$\therefore \sqrt[3]{614.125} = 8.5$$

$$\therefore \sqrt[3]{614125} = 10 \times 8.5 = \mathbf{85}$$

$$(j) \sqrt[3]{389017} = 10 \times \sqrt[3]{389.017}$$

$\sqrt[3]{389.017}$ lies between 7 and 8. Cube root will end with '3' as cube end with '7'.

$$\therefore \sqrt[3]{389.017} = 7.3$$

$$\therefore \sqrt[3]{389017} = 10 \times 7.3 = \mathbf{73}$$

$$(k) \sqrt[3]{704969} = 10 \times \sqrt[3]{704.969}$$

$\sqrt[3]{704.969}$ lies between 8 and 9. Cube root will end with '9' as cube end with '9'.

$$\therefore \sqrt[3]{704.969} = 8.9$$

$$\text{Hence } \sqrt[3]{704969} = 10 \times 8.9 = \mathbf{89}$$

$$3. \sqrt[3]{0.000216} = \sqrt[3]{216 \times 10^{-6}} = 6 \times 10^{-2} = \mathbf{0.06}$$

$$4. \sqrt[3]{0.000729} = \sqrt[3]{729 \times 10^{-6}} = 9 \times 10^{-2} = \mathbf{0.09}$$

$$5. (22)^3 = 484 \times 22 = \mathbf{10648}$$

Hence smallest five-digit number which is a perfect cube is 10648.

$$6. 5600 = 2^3 \times 7 \times 10^2 = 2^5 \times 7 \times 5^2$$

$\therefore$ The least number by which 5600 is to be divided to get a perfect cube

$$= 2^2 \times 7^1 \times 5^2$$

$$= 7 \times 10^2 = \mathbf{700}$$

7. The least number completely divisible by 21, 24 and 27 is

3	21,	24,	27
3	7,	8,	9
2	7,	8,	3
2	7	4	3
	7	2	3

LCM of 21, 24 and 27 = $2^3 \times 3^3 \times 7$

∴ Least perfect cube completely divisible by 21, 24 and 27 is

$$2^3 \times 3^3 \times 7^3 = \text{LCM} \times 7^2$$

$$= \text{LCM} \times 49 = 74088$$

8. $4800 = 2^3 \times 2 \times 3 \times 2^2 \times 5^2 = 2^6 \times 3^1 \times 5^2$

∴ Least number which should be multiplied with given number make it a perfect cube is $3^2 \times 5 = 45$

Series

A *series* is a sequence or ordered collection of numbers.

PRIME NUMBERS: A prime number is a counting number greater than 1, which is divisible only by 1 and by itself. For example,

2, 3, 5, 7, 11, etc.

PERFECT SQUARE: A whole number whose square root is also a whole number is called a *perfect square*. For example,

4, 9, 16, etc.

PERFECT CUBE: A whole number whose cube root is also a whole number is called a *perfect cube*. For example,

1, 8, 27, 64, etc.

ARITHMETIC PROGRESSION (AP)/ARITHMETIC SERIES: An arithmetic progression is the sequence of numbers in which a number in the series is obtained by adding a constant number called *common difference* to the previous number in the series.

The sum of first n terms of the series $S = \frac{n}{2} \{2a + (n - 1)d\}$

where a is the first term and d is the common difference.

The sum of the series (of all n terms) $= \frac{n}{2} \{a + l\} = \frac{n}{2} \{2a + (n - 1)d\}$

where ' a ' is the first term and ' l ' is the last term

For example,

- (i) 2, 5, 8, 11 ... common difference = 3
- (ii) 9, 15, 21, 27 ... common difference = 6 are in A.P.

GEOMETRIC PROGRESSION (GP)/GEOMETRIC SERIES: A sequence of numbers is said to be in *geometric progression* (GP), if the ratio of two consecutive terms in the sequence is always the same and is called the *common ratio*.

The n th term of a series is given by

$$a_n = a \cdot r^{n-1};$$

$$l = a \cdot r^{N-1};$$

$$S_n = a(r^n - 1)/(r - 1);$$

$$S_N = a(r^N - 1)/(r - 1), r \neq 1$$

where a = first term

r = common ratio

l = last term

S_n = Sum of first n terms

S_N = Sum of all N terms of the series

For example,

- (i) 3, 15, 75, 375, ... common ratio = 5 (ii) 2, 6, 18, 54 ... common ratio = 3

COMBINATION SERIES: This can be a series combining different rules in a logical way.

- (a) **Series of perfect squares:** For example, 9, 25, 49, 81, 121 ...

The series can be represented by the mathematical expression $f(x) = (1 + 2x)^2$ where $x = 1, 2, 3 \dots$

- (b) **Series of perfect cubes:** For example,

- (i) 1, 27, 729, 19683 ...

can be represented by $f(x) = (27)^{x-1}$

where $x = 1, 2, 3, 4$

- (ii) 1, 64, 373, 1000, ...

can be represented by $f(x) = [1 + 3(x - 1)]^3$

where $x = 1, 2, 3, 4 \dots$

Second Degree Arithmetic Series: A series in which the differences between the successive terms themselves are in AP and forms an arithmetic series is called the *second degree arithmetic series*. For example, 2, 3, 6, 11, 18 ...

The difference of successive terms of the above series 1, 3, 5, 7 ... which forms an arithmetic series with common difference 2.

The given series can be represented by a quadratic mathematical function:

$$f(x) = x^2 + 2$$

where $x = 0, 1, 2, 3, 4 \dots$

Third Degree Arithmetic Series: A series in which the difference of the successive terms forms a second degree arithmetic series is termed *third degree arithmetic series*. Mathematically, such a series can be represented by a general cubic function and hence the name third degree. For example,

- (a) 20, 30, 42, 59, 84, 120, 170, ...

The difference of successive terms of the above series is 10, 12, 17, 25, 36, 50... and 2, 5, 8, 11, 14 ... and this forms an arithmetic series with common difference 3.

- (b) 1, 15, 53, 127, 249, 431, 685, ...

The difference of successive terms of the above series is 14, 38, 74, 122, 182, 254. The difference of successive terms of the above series is 24, 36, 48, 60, 72 and this forms an arithmetic series with common difference 12.

Mathematically, the given series can be represented by a cubic function

$$f(x) = 2x^3 - 1 \quad \text{where } x = 1, 2, 3, 4 \dots$$

ARITHMETICO–GEOMETRIC (A–G) SERIES: The series is formed by combination of arithmetic and geometric progressions. Each succeeding term is obtained by first adding (or subtracting) the common difference of its AP to the previous term and then multiplying (or dividing) the sum by common ratio of its GP. An important property of such an A–G series is that the difference of successive terms are in GP.

For example,

$$1, 8, 22, 50, 106, 218$$

Series is obtained by adding 3 to previous term and then multiplying the sum by 2.
i.e. $a_2 = (a_1 + 3) \times 2$

GEOMETRICO–ARITHMETIC (G–A) SERIES: Each succeeding term in G–A series is found by first multiplying (or dividing) the previous term by the common ratio of its GP and then adding (or subtracting) to it the common difference of its AP. The important property of G–A series, like A–G series, is that the difference of its successive terms are in geometric progression. For example, 1, 2, 6, 22, 86, 342, ...

The terms in the above series are obtained by first multiplying the previous term by 4 and then subtracting 2 from the resultant product.

TWIN SERIES: As the name suggests, it consists of two series combined into a single series. The alternating terms of this series form an independent series. For example,

$$2, 3, 7, 9, 12, 27, 17, 81, 22, 243 \dots$$

Odd terms of the series 2, 7, 12, 17, 22, ... is an arithmetic series with common difference 5.

Even terms of the series 3, 9, 27, 81, 243 ... is a geometric series with common ratio 3.

ALTERNATING SERIES: In an alternating series, the successive terms increase and decrease alternately. It differs from irregular increase (or decrease) series wherein the successive terms increase (or decrease) continuously but the ratio of increase does not follow a particular trend.

While solving an alternate series we should focus on the following possibility of its formation:

1. Whether it is a twin series which has been formed by combining two different series.
or
2. It has been formed by performing two different kinds of operations alternately on successive terms. For example,

$$6, 13, 12, 40, 24, 104, 48, 229, 96 \dots$$

In the above series, there are two series 6, 12, 24, 48, 96 ... and 13, 40, 104, 229 ...
i.e. the first one is a geometric series with first term 6 and common ratio 2.

In the second series, terms are having differences 27, 64, 125, ...
i.e., $3^3, 4^3, 5^3, \dots$

ODD MAN OUT (WRONG NUMBER) IN A SERIES

For example,

- (i) 3, 5, 7, 9, 11 ...

9 is the odd man out because all others are prime numbers.

- (ii) 2, 3, 7, 10, 11 ...
Except 10, all are prime numbers.
- (iii) 3, 10, 29, 66, 127, 220, 345 ...
Except 220, all other terms can be expressed as $x^3 + 2$, where $x = 1, 2, 3, \dots$

SPECIAL TYPE OF SERIES

- (a) Numbers followed by their LCM or HCF.

For example,

1, 2, 3, 6, 4, 5, 12, 60, 5, 6, 7 ...

In the above series, every four numbers forms a block in which the fourth number is the LCM of first three numbers. Therefore, the number next to 7 is 210.

i.e. LCM of 5, 6, 7 = 210

- (b) Numbers followed by their product.

For example,

1, 2, 2, 4, 8, 32, ...

$1 \times 2 = 2$; $2 \times 2 = 4$; $2 \times 4 = 8$; $4 \times 8 = 32$; $8 \times 32 = 256$,

which is the next number in the series.

- (c) By use of digit sum.

For example,

12, 15, 21, 24, 30, ...

Next number = Previous number + Digit-sum of previous number.

i.e., $15 = 12 + (1 + 2)$

$21 = 15 + (1 + 5)$

$24 = 21 + (2 + 1)$

$30 = 24 + (2 + 4)$

$\therefore$ Next number = $30 + (3 + 0) = 33$



EXERCISES

In the following number series, one of the number does not fit into the series. Mark the wrong number in the series.

- 1788 892 444 220 112 52 24
(a) 52 (b) 112 (c) 220 (d) 444 (e) 892
- 225 289 338 374 397 415 424
(a) 415 (b) 289 (c) 338 (d) 374 (e) 397
- 5 7.5 11.25 17.5 29.75 50 91.25
(a) 7.5 (b) 17.5 (c) 29.75 (d) 91.25 (e) None of these
- 35 118 280 600 1238 2504 5036
(a) 118 (b) 280 (c) 600 (d) 1238 (e) 2504
- 10 12 28 90 368 1840 1112
(a) 1840 (b) 368 (c) 90 (d) 28 (e) 12

In each of the following questions, a number series is given. After the series, a number is given followed by (A), (B), (C), (D) and (E). You have to complete the series starting

with the number given following the sequence of the given series. Then answer the given questions.

6. 80 50 130 100 180 150 230
900 (A) (B) (C) (D) (E)

Which of the following numbers will come in place of (E)?

- (a) 1050 (b) 970 (c) 1000 (d) 950 (e) None of these

7. 60 121 131 264 284 571 601
120 (A) (B) (C) (D) (E)

Which of the following numbers will come in place of (D)?

- (a) 524 (b) 1011 (c) 1010 (d) 514 (e) None of these

8. 2 4 9 20 43 90
3 (A) (B) (C) (D) (E)

Which of the following numbers will come in place of (D)?

- (a) 58 (b) 99 (c) 48 (d) 59 (e) None of these

9. 200 184 193 157 182 118 167
150 (A) (B) (C) (D) (E)

Which of the following numbers will come in place of (E)?

- (a) 132 (b) 115 (c) 68 (d) 117 (e) None of these

10. 4 14 42 147 588 2058 10290
8 (A) (B) (C) (D) (E)

Which of the following numbers will come in place of (C)?

- (a) 28 (b) 1176 (c) 343 (d) 216 (e) None of these

A number series is given in each of the following question. A number is given after the series and after that (A), (B), (C), (D) and (E) are given. Find the number in place of the specified letter.

11. 13 14 5 18 0.5
19 (A) (B) (C) (D) (E)

What will come in place of (E)?

- (a) 12.75 (b) 25.5 (c) 18.75 (d) 15.25 (e) None of these

12. 17 21.5 30.5 44 62
21 (A) (B) (C) (D) (E)

What will come in place of (E)?

- (a) 84.5 (b) 88.5 (c) 90 (d) 95 (e) None of these

13. 12 26 11 36 9
7 (A) (B) (C) (D) (E)

What will come in place of (C)?

- (a) 25 (b) 21 (c) 27 (d) 30 (e) None of these

14. What should come in the place of the question mark(?) in the following number series?

11 28 55 92 139 ?

- (a) 196 (b) 100 (c) 210 (d) 216 (e) None of these

15. In the following number series only one number is wrong. Find out that wrong number?

2 6 30 210 2520 20790

- (a) 30 (b) 6 (c) 210 (d) 2520 (e) None of these

16. The next number of the sequence 0, 3, 8, 15, 24, 35, ... is

- (a) 46 (b) 47 (c) 48 (d) 50 (e) None of these

Fill in the blanks with appropriate choice

17. 1 6 27 108 _____
(a) 405 (b) 402 (c) 256 (d) 162
18. 4 10 33 136 _____
(a) 385 (b) 685 (c) 786 (d) 812
19. 2 12 36 80 _____
(a) 120 (b) 140 (c) 150 (d) 170
20. 2 18 66 210 642 _____
(a) 1842 (b) 1648 (c) 1798 (d) 1938
21. 8 64 216 512 _____
(a) 1000 (b) 1110 (c) 1090 (d) 1024
22. 3 13 27 50 87 143 _____
(a) 148 (b) 223 (c) 316 (d) 412
23. 4 30 160 810 4060 _____
(a) 10160 (b) 15180 (c) 20310 (d) 22168

Fill in the blanks with appropriate choice

24. 243, _____, 108, 72, 48
(a) 81 (b) 162 (c) 324 (d) 120
25. _____, 24, 96, 384, 1536
(a) 12 (b) 18 (c) 6 (d) 8
26. 336, 210, 120, _____, 24
(a) 40 (b) 60 (c) 48 (d) 30
27. 5, 14, 32, _____, 140
(a) 36 (b) 40 (c) 68 (d) 84
28. 2, 20, _____, 380, 1532
(a) 92 (b) 60 (c) 112 (d) 124

Find the odd man out:

29. 5, 10, 13, 20, 29
(a) 13 (b) 20 (c) 10 (d) 29
30. 4096, 3072, 2048, 1728, 1296
(a) 3072 (b) 2048 (c) 1728 (d) 1296
31. 2, 10, 30, 70, 130
(a) 10 (b) 30 (c) 70 (d) 130
32. 10, 25, 49, 81, 121
(a) 81 (b) 49 (c) 25 (d) 10
33. 190, 166, 145, 128, 112, 100, 91
(a) 166 (b) 145 (c) 128 (d) 112
34. 1, 3, 10, 21, 64, 129, 356, 777
(a) 64 (b) 129 (c) 356 (d) 777
35. 445, 221, 109, 46, 25, 11, 4
(a) 221 (b) 109 (c) 46 (d) 25

In each of the following questions, a number series is given. After the series, a number is given below which is followed by letters A, B, C, D and E. You have to complete the

series starting with the number given and following the same property as in the given number series.

36. 4 6 15 49 201 1011
15 A B C D E
What should come in place of D?
(a) 115 (b) 465 (c) 455 (d) 475 (e) None of these
37. 0 1 0 9 32 825
4 A B C D E
What should come in the place of C?
(a) 279 (b) 81 (c) 269 (d) 272 (e) None of these
38. 2 -1 28 -9 86 -25
122 A B C D E
What should come in place of B?
(a) 124 (b) 328 (c) 9 (d) 228 (e) None of these
39. 101 323 545 767 989 111011
34 A B C D E
What should come in place of A?
(a) 56 (b) 78 (c) 43 (d) 100 (e) None of these
40. -1 0 10 65 345 1750
-2 A B C D E
What should come in place of E?
(a) -1750 (b) 1375 (c) -1325 (d) -1475 (e) None of these
41. 4 29 74 231 928 4641
3 A B C D E
What will come in place of D?
(a) 696 (b) 904 (c) 902 (d) 4520 (e) None of these
42. 600 360 360 450 600 800
1200 A B C D E
What will come in place of B?
(a) 600 (b) 660 (c) 560 (d) 720 (e) None of these
43. 43 49 2197 130321 100000 1
25 A B C D E
What will come in place of E?
(a) 1 (b) 0.8 (c) 0.75 (d) 0.63 (e) None of these
44. 703 803 924 1068 1237 1433
307 A B C D E
What will come in place of E?
(a) 641 (b) 1037 (c) 841 (d) 672 (e) None of these
45. 14 71 428.5 3004 24039 216361
4 A B C D E
What will come in place of B?
(a) 125 (b) 128 (c) 125.5 (d) 128.5 (e) None of these
- What will come in place of the question mark (?) in the following number series?
46. 283 505 ? 1282 1837 2503
(a) 828 (b) 848 (c) 882 (d) 884 (e) None of these

47. 7 36 125 256 ? 64 1
(a) 81 (b) 162 (c) 128 (d) 243 (e) None of these
48. 5 6 ? 87 412 2185
(a) 16 (b) 13 (c) 20 (d) 18 (e) 21
49. 60 20 ? 15 60 12
(a) 60 (b) 40 (c) 75 (d) 50 (e) 48
50. 2 5 17.5 43.75 ?
(a) 151.125 (b) 151.135 (c) 151.175 (d) 153.125 (e) 153.175
51. 3 20 87 392 ?
(a) 1612 (b) 1963 (c) 847 (d) 2025 (e) None of these
52. 12 28 75 316 1555 ?
(a) 9366 (b) 9294 (c) 7811 (d) 9336 (e) None of these
53. 112 111 119 92 156 31 ?
(a) 1375 (b) 287 (c) 387 (d) 247 (e) None of these
54. 1 15 16 31 47 78 125 ?
(a) 172 (b) 203 (c) 139 (d) 167 (e) None of these
55. 55 60 67 78 91 108 ?
(a) 125 (b) 121 (c) 127 (d) 89 (e) None of these



1. (b) 2. (e) 3. (c) 4. (d) 5. (a) 6. (b) 7. (a) 8. (d) 9. (c) 10. (e)
11. (c) 12. (b) 13. (b) 14. (a) 15. (d) 16. (c) 17. (a) 18. (b) 19. (c) 20. (d)
21. (a) 22. (b) 23. (c) 24. (b) 25. (c) 26. (b) 27. (c) 28. (a) 29. (c) 30. (b)
31. (c) 32. (d) 33. (c) 34. (c) 35. (c) 36. (b) 37. (b) 38. (d) 39. (a) 40. (e)
41. (b) 42. (c) 43. (a) 44. (b) 45. (d) 46. (e) 47. (d) 48. (c) 49. (a) 50. (d)
51. (e) 52. (a) 53. (d) 54. (b) 55. (c)



Solutions with Necessary Explanation

- The series is $(a_1 + 2) - 2 = a_2$
Hence the number 112 is wrong.
- The series is formed by addition of $8^2, 7^2, 6^2 \dots$ to the previous term.
Hence the wrong number is 397.
- The series is formed by first adding 2.5,
Then $(2.5)2 - 1.25 = 3.75$;
 $(3.75)2 - 1.25 = 6.25$;
 $(6.25)2 - 1.25 = 11.25$ and so on.
Hence the number 29.75 is wrong.
- The series is formed by first adding 83.
Then $(83)2 - 4 = 162$;
 $162 \times 2 - 4 = 320$;
 $320 \times 2 - 4 = 636$;

$$636 \times 2 - 4 = 1268;$$

$$1268 \times 2 - 4 = 2532$$

Hence the wrong number is **1238**.

5. The series is formed by applying $\times 1 + 2, \times 2 + 4, \times 3 + 6, \times 4 + 8, \times 5 + 10, \times 6 + 12$.

Hence the wrong number is **1840**.

6. The series is an alternate series and is formed by successive addition of -30 and $+80$ alternatively to the previous term.

Therefore 900, 870, 950 ... is the series in which (E) will be **970**.

7. Series is obtained as follows:

$$\text{1st term} = 60;$$

$$\text{2nd term} = 60 \times 2 + 1 = 121;$$

$$\text{3rd term} = 121 + 10 = 131;$$

$$\text{4th term} = 131 \times 2 + 2 = 264;$$

$$\text{5th term} = 264 + 20 = 284;$$

$$\text{6th term} = 284 \times 2 + 3 = 571;$$

$$\text{7th term} = 571 + 30 = 601.$$

$\therefore$ Here, 1st term = 120;

$$\text{2nd term} = 120 \times 2 + 1 = 241;$$

$$\text{3rd term} = 241 + 10 = 251;$$

$$\text{4th term} = 251 \times 2 + 2 = 504;$$

$$\text{5th term} = 504 + 20 = \mathbf{524}.$$

8. The series is obtained as follows:

$$\text{1st term} = 2;$$

$$\text{2nd term} = 2 \times 2 = 4;$$

$$\text{3rd term} = 4 \times 2 + 1 = 9;$$

$$\text{4th term} = 9 \times 2 + 2 = 20;$$

$$\text{5th term} = 20 \times 2 + 3 = 43;$$

$$\text{6th term} = 43 \times 2 + 4 = 90.$$

Similarly, from the given numbers, series is obtained as follows:

$$\text{1st term} = 3;$$

$$\text{2nd term} = 3 \times 2 = 6;$$

$$\text{3rd term} = 6 \times 2 + 1 = 13;$$

$$\text{4th term} = 13 \times 2 + 2 = 28;$$

$$\text{5th term} = 28 \times 2 + 3 = \mathbf{59};$$

$$\text{6th term} = 59 \times 2 + 4 = 122.$$

9. The given series is obtained as follows:

$$\text{1st term} = 200;$$

$$\text{2nd term} = 200 - 16 = 184;$$

$$\text{3rd term} = 184 + 9 = 193;$$

$$\text{4th term} = 193 - 36 = 157;$$

$$\text{5th term} = 157 + 25 = 182;$$

$$\text{6th term} = 182 - 64 = 118;$$

$$\text{7th term} = 118 + 49 = 167.$$

Similarly when series is started with a number 150, the number to be obtained as 6th term is given by $150 - 16 + 9 - 36 + 25 - 64 = \mathbf{68}$

10. Series is obtained as follows:

$$\text{1st term} = 4;$$

$$\text{2nd term} = 4 \times 3.5 = 14;$$

$$\text{3rd term} = 14 \times 3 = 42;$$

$$\text{4th term} = 42 \times 3.5 = 147;$$

$$\text{5th term} = 147 \times 4 = 588;$$

$$\text{6th term} = 588 \times 3.5 = 2058;$$

$$\text{7th term} = 2058 \times 5 = 10290.$$

Similarly given series starts with 8.

i.e. $\text{1st term} = 8;$

$$\text{2nd term} = 8 \times 3.5 = 28;$$

$$\text{3rd term} = 28 \times 3 = 84;$$

$$\text{4th term} = 84 \times 3.5 = 293;$$

i.e. number in place of (C) = 293.

Here correct answer is not given among choice.

$\therefore$ Answer is (e).

11. Series is $\times 1 + 1, \div 2 - 2, \times 3 + 3, \div 4 - 4, \dots$

$$\therefore A = 19 \times 1 + 1 = 20;$$

$$B = 20 \div 2 - 2 = 8;$$

$$C = 8 \times 3 + 3 = 27;$$

$$D = 27 \div 4 - 4 = 2.75;$$

$$E = 2.75 \times 5 + 5 = 18.75$$

$\therefore$ Number in place of (E) = 18.75.

12. Series is $+4.5, +9.0, +13.5, +18.0, \dots$

$$\therefore A = 21 + 4.5 = 25.5;$$

$$B = 25.5 + 9 = 34.5;$$

$$C = 34.5 + 13.5 = 48.0;$$

$$D = 48.0 + 18 = 66.0;$$

$$E = 66.0 + 22.5 = 88.5.$$

$\therefore$ Number in place of (E) = 88.5

13. The series is $\times 2 + 2, \div 2 - 2, \times 3 + 3, \div 3 - 3 \dots$

$$\therefore A = 7 \times 2 + 2 = 16;$$

$$B = 16 \div 2 - 2 = 6;$$

$$C = 6 \times 3 + 3 = 21;$$

$$D = 21 \div 3 - 3 = 4;$$

$$E = 4 \times 4 + 4 = 20.$$

$\therefore$ Number in place of (C) = 21

14. The series is 11, 28, 55, 92, 139 ?

Difference between consecutive term of the series are in AP is

i.e. 17, 27, 37, 47, 57, ...

$\therefore$ Number in place of (?) = $139 + 57 = 196$

15. The series is 2, 6, 30, 210, 2520, 20790

$$\text{1st term} = 2;$$

$$\text{2nd term} = 2 \times 3 = 6;$$

$$\text{3rd term} = 6 \times 5 = 30;$$

$$\text{4th term} = 30 \times 7 = 210;$$

$$\text{5th term} = 210 \times 9 = 1890;$$

$$\text{6th term} = 1890 \times 11 = 20790.$$

$\therefore$ The number **2520** is wrong and it should be replaced by 1890.

16. The series is 0, 3, 8, 15, 24, 35 ...

$$\text{1st term} = 0;$$

$$\text{2nd term} = 0 + 3 = 3;$$

$$\text{3rd term} = 3 + 5 = 8;$$

$$\text{4th term} = 8 + 7 = 15;$$

$$\text{5th term} = 15 + 9 = 24;$$

$$\text{6th term} = 24 + 11 = 35;$$

$$\text{7th term} = 35 + 13 = 48.$$

$\therefore$ The required number (next number) in the sequence is **48**.

17. The series is 1, 2×3 , 3×3^2 , 4×3^3 , 5×3^4

$$\therefore \text{Next term in the series} = 5 \times 3^4 = \mathbf{405}$$

18. The series is $4 \times 2 + 2$, $10 \times 3 + 3$, $33 \times 4 + 4$, $136 \times 5 + 5$.

$$\therefore \text{Next term in the series} = 136 \times 5 + 5 = \mathbf{685}$$

19. The series is $1^3 + 1^2$, $2^3 + 2^2$, $3^3 + 3^2$, $4^3 + 4^2$, $5^3 + 5^2$

$$\therefore \text{Next term in the series} = 5^3 + 5^2 = \mathbf{150}$$

20. This is an A–G series (arithmetico–geometric series). Each term is first added by 4 and then sum is multiplied by 3.

$$\therefore \text{Next term in the series} = (642 + 4) \times 3 = \mathbf{1938}$$

21. The series is cubes of even numbers

$$\therefore \text{Next term in the series is } 10^3 = \mathbf{1000}$$

22. The given series is a third degree arithmetic series

i.e. given series is 3, 13, 27, 50, 87, 143

Their differences are 10, 14, 23, 37, 56

Difference of the above series is 4, 9, 14, 19 (i.e. this series is in AP)

$$\therefore \text{Next term in the given series} = 24 + 56 + 143 = \mathbf{223}$$

23. The given series is an A–G series (arithmetico–geometric series). Each term is first added by 2 and then sum is multiplied by 5.

$$\therefore \text{Next term in the series} = (4060 + 2) \times 5 = \mathbf{20310}$$

24. 162

$$\mathbf{25. 6}$$

$$\mathbf{26. 60 \text{ (Third degree AP)}}$$

27. 68 (G–A series $\times 2 + 4$)

$$\mathbf{28. 92 \text{ (A–G series} + 3 \times 4)}$$

29. The series is $1^2 + 4$, $2^2 + 4$, $3^2 + 4$, $4^2 + 4$, $5^2 + 4$.

$$\therefore \text{Odd man out} = \mathbf{10}$$

30. The series is a GP with common ratio = $3/4$

$$\therefore \text{Odd man out} = \mathbf{2048}$$

31. The series is $1^3 + 1$, $2^3 + 2$, $3^3 + 3$, $4^3 + 4$, $5^3 + 5$.

$$\therefore \text{Odd man out} = \mathbf{70}$$

32. The sequence forms square of consecutive odd numbers

$$\therefore \text{Odd man out} = \mathbf{10}$$

33. This is a second degree arithmetic series with their differences in AP.

In this 128 is wrong.

∴ Odd man out = **128**

34. This is a Geometrico-Arithmetic series ($\times 2 + 1, \times 3 + 1, \times 2 + 1, \dots$)

Here **356** is odd.

35. This is a A-G series ($-3 \div 2$)

Here the term 46 is odd.

36. The series is $\times 1 + 2, \times 2 + 3, \times 3 + 4, \times 4 + 5 \dots$

$$\therefore A = 15 \times 1 + 2 = 17$$

$$B = 17 \times 2 + 3 = 37$$

$$C = 37 \times 3 + 4 = 115$$

$$D = 115 \times 4 + 5 = \mathbf{465}$$

Hence value of D = **465**

37. The series is $\times 1^2 + 1^2, \times 2 - 2, \times 3^2 + 3^2, \times 4 - 4 \dots$

$$\therefore A = 4 \times 1^2 + 1^2 = 5$$

$$B = 5 \times 2 - 2 = 8$$

$$C = 8 \times 3^2 + 3^2 = 81$$

Hence value of C = **81**

38. The series is $1^2 + 1, 3 - 2^2, 5^2 + 3, 7 - 4^2, 9^2 + 5, \dots$

$$\text{Here } 2 = 1^2 + 1; \quad 122 = 1171$$

$$-1 = 3 - 2^2; \quad A = 13 - 2^2 = 9$$

$$28 = 5^2 + 3; \quad B = 15^2 + 3 = 228$$

$$-9 = 7 - 4^2; \quad C = 17 - 4^2 = 1.$$

∴ Answer is **228**

39. In this series, every digit of the number increases by 2 in each step. i.e. 3 4 5 6

Hence A = **56**

40. The series is $(+1) \times 5, (+2) \times 5, (+3) \times 5, (+4) \times 5, (+5) \times 5$

∴ Ans. = **-1375**

Here correct answer is not given. Hence answer is (e).

41. Here $4 \times 1 + 5^2 = 29; \quad 29 \times 2 + 4^2 = 74;$

$$74 \times 3 + 3^2 = 231; \quad 231 \times 4 + 2^2 = 928;$$

$$928 \times 5 + 1^2 = 4641.$$

$$\text{Similarly } 3 \times 1 + 5^2 = 28; \quad 28 \times 2 + 4^2 = 72;$$

$$72 \times 3 + 3^2 = 225; \quad 225 \times 4 + 2^2 = 904.$$

∴ Answer is **904**

42. Here $600 \times \frac{1}{2} + 60 = 360; \quad 360 \times \frac{2}{3} + 120 = 360;$

$$360 \times \frac{3}{4} + 180 = 450; \quad 450 \times \frac{4}{5} + 240 = 600;$$

$$600 \times \frac{5}{6} + 300 = 800.$$

Similarly, $1200 \times \frac{1}{2} + 60 = 660$; $660 \times \frac{2}{3} + 120 = 560$;

$\therefore$ Answer is **560**.

43. Here $(4 + 3)^2 = 49$; $(4 + 9)^3 = 2197$; $(2 + 1 + 9 + 7)^4 = 130321$; $(1 + 3 + 0 + 3 + 2 + 1)^5 = 100000$; $(1 + 0 + 0 + 0 + 0 + 0)^6 = 1$.

Since digit sum of 43 and 25 are the same, remaining terms of the series will be in the same pattern.

$\therefore$ Answer is **1**

44. Here $703 + 10^2 = 803$; $803 + 11^2 = 924$;
 $924 + 12^2 = 1068$; $1068 + 13^2 = 1237$;
 $1237 + 14^2 = 1433$.

$\therefore$ Required answer $= 307 + (1433 - 703) = 1037$

45. Here, $14 \times 5 + 100 \times 1 = 71$; $71 \times 6 + 1.25 \times 2 = 428.5$;
 $428.5 \times 7 + 1.50 \times 3 = 3004$; $3004 \times 8 + 1.75 \times 4 = 24039$;
 $24039 \times 9 + 2.00 \times 5 = 216361$

Similarly, $4 \times 5 + 1.00 \times 1 = 21$; $21 \times 6 + 1.25 \times 2 = 128.5$

46. The series is obtained by adding + 222, + 333, + 444 ... in succession to the previous term. Hence question mark (?) will be replaced by **838**.

47. The series is $7^1, 6^2, 5^3, 4^4, 3^5, 2^6$ and 1^7 ,

$\therefore$ Answer is $3^5 = 243$

48. The series is $\times 1 + 1^3, \times 2 + 2^3, \times 3 + 3^3, \dots$

$\therefore$ Answer is **20**

49. The series is $\times \frac{1}{3}, \times 3, \times \frac{1}{4}, \times 4, \frac{1}{5}, \times 5 \dots$

$\therefore$ Answer is $20 \times 3 = 60$

50. This is an alternate series $\times 2.5, \times 3.5, \times 2.5, \times 3.5, \dots$

$\therefore$ Answer is **153.125**

51. $3 + 1^2 + 16 = 20$; $20 + 9^2 - 14 = 87$;
 $87 + 17^2 + 16 = 392$; $392 + 25^2 - 14 = 1003$

The answer is **1003**.

Here correct answer is not given. Hence answer is (e).

52. $12 \times 2 + 2^2 = 28$; $28 \times 3 - 3^2 = 75$;
 $75 \times 4 + 4^2 = 316$; $316 \times 5 - 5^2 = 1555$;
 $1555 \times 6 + 6^2 = 9366$

53. $112 - 1^3 = 111$; $111 + 2^3 = 119$; $119 - 3^3 = 92$; $92 + 4^3 = 156$; $156 - 5^3 = 31$; $31 + 6^3 = 247$.

54. Here $1 + 15 = 16$; $15 + 16 = 31$; $16 + 31 = 47$; $31 + 47 = 78$; $47 + 78 = 125$; $78 + 125 = 203$.
The series is similar to Fibonacci series i.e. each term of the series is obtained by taking sum of previous two terms.

55. $55 + 5 = 60$; $60 + 7 = 67$; $67 + 11 = 78$; $78 + 13 = 91$; $91 + 17 = 108$; $108 + 19 = 127$.

Here the numbers added are consecutive prime numbers.

Progression and Sequence

ARITHMETIC PROGRESSION (AP): A sequence of numbers is said to be in Arithmetic Progression if the difference between any two consecutive numbers is always the same. The difference between two consecutive terms of the series is termed the *common difference* (d), for example, 2, 6, 10, 14 ... is an AP with first term $a = 2$ and common difference $d = 6 - 2 = 10 - 6 = 4$.

If in an AP, there are n terms, and if the first term is a and common difference is d , last term $l = a + (n - 1)d$.

r th term from beginning $= a + (r - 1)d$

$$\begin{aligned}\text{Sum of the series} &= \frac{1}{2}n(a + l) \\ &= \frac{1}{2}n\{2a + (n - 1)d\}\end{aligned}$$

GEOMETRIC PROGRESSION (GP): A sequence of numbers (or terms) is said to be in geometric progression if the ratio of two consecutive numbers (or terms) is constant. This constant ratio is called the *common ratio*.

If in a GP, there are n terms, and if the first term is a and common ratio is r , then n th term is given by $a \cdot r^{n-1}$.

$$\begin{aligned}\text{Sum of } n \text{ terms } S_n &= \frac{a(r^n - 1)}{(r - 1)} \text{ if } r > 1 \text{ or } r < -1 \\ S_n &= \frac{a(1 - r^n)}{(1 - r)} \text{ if } -1 < r < 1\end{aligned}$$

EXAMPLE 1 The sum of 3 numbers in a GP is 26 and their product is 216. Find the numbers.

Solution Let the numbers be $\frac{a}{r}$, a , ar .

The sum of 3 numbers is given by $\frac{a}{r} + a + ar = 26$ (1)

The product of 3 numbers is given by $\frac{a}{r} \times a \times ar = 216$ (2)

$$(2) \Rightarrow a^3 = 216 \Rightarrow a = \sqrt[3]{216} = 6$$

$$\therefore (1) \Rightarrow 6 \left(\frac{1}{r} + r \right) = 26 - 6 = 20$$

$$\Rightarrow \frac{1}{r} + r = \frac{20}{6} = \frac{10}{3}$$

$$\therefore r = 3$$

$\therefore$ The numbers are **2, 6, 18**.

EXAMPLE 2 The product of three numbers in a GP is 1728 and the sum of the products of them taken in pairs is 756. Find the numbers.

Solution Let the numbers be $\frac{a}{r}, a, ar$.

$$\therefore \frac{a}{r} \cdot a \cdot ar = 1728 \quad (1)$$

$$\Rightarrow a^3 = 1728$$

$$\Rightarrow a = 12$$

$$\frac{a}{r} \cdot a + a \cdot ar + ar \cdot \frac{a}{r} = 756 \quad (2)$$

$$\Rightarrow \frac{a^2}{r} + a^2 \cdot r + a^2 = 756$$

$$\begin{aligned} \text{i.e.} \quad a^2 \left(r + \frac{1}{r} \right) &= 756 - a^2 \\ &= 756 - 12^2 = 756 - 144 = 612 \end{aligned}$$

$$\therefore r + \frac{1}{r} = \frac{612}{144} = 4 \frac{1}{4}$$

$$\therefore r = 4$$

$\therefore$ The numbers are $\frac{12}{4}, 12$ and 12×4 , i.e. **3, 12 and 48**

HARMONIC PROGRESSION (HP): A sequence of numbers is said to be in HP, if the reciprocals of its terms form an AP. Conversely, if the terms of a sequence are in AP, their reciprocals form an HP.

If $x_1, x_2, x_3, \dots$ are in HP, then $\frac{1}{x_1}, \frac{1}{x_2}, \frac{1}{x_3} \dots$ are in AP.

For example, $\frac{1}{3}, \frac{1}{7}, \frac{1}{11}, \frac{1}{15}$ are in HP.



Note: Three quantities a , b and c are in HP, then $a : c = (a - b) : (b - c)$.

The n th term of an HP = $\frac{1}{a + (n-1)d}$.

HARMONIC MEAN (HM): The Harmonic Mean (HM) of two given quantities a and b is given by $HM = \frac{2ab}{a+b}$.

ARITHMETIC MEAN (AM): If x_1 , x_2 , and x_3 are in AP, then the Arithmetic Mean between x_1 and $x_2 = \bar{x} = \frac{x_1 + x_2}{2}$, such that x_1 , $\bar{x}$, and x_2 are in AP.

For example, Arithmetic Mean of 4 and 8 = $\frac{4+8}{2} = 6$

$\therefore$ 4, 6, 8 are in AP with common difference 2.

GEOMETRIC MEAN (GM): The geometric mean between the two numbers a and $ar^2 = \sqrt{a \times ar^2} = ar$. Then a , ar , ar^2 are in GP.

For example, the GM between 3 and 27 = $\sqrt{3 \times 27} = 9$

$\therefore$ 3, 9, and 27 are in GP.

If x and y are two unequal positive distinct quantities, A, G, and H are their arithmetic mean, geometric mean and harmonic mean respectively, then

(i) $AH = G^2$

(ii) $A > G > H$ or $A \geq G \geq H$.



Important Results

Finite Series

1. The sum of first n natural numbers, $S_n = \frac{n(n+1)}{2}$.
2. The sum of squares of first n natural numbers, $S_n = \frac{n(n+1)(2n+1)}{6}$.
3. The sum of cubes of first n natural numbers, $S_n = \frac{n^2(n+1)^2}{4} = \left[\frac{n(n+1)}{2} \right]^2$.

EXAMPLE 3 Find the sum of first n terms of the following series $\frac{1}{3.4}, \frac{1}{4.5}, \frac{1}{5.6}, \frac{1}{6.7}, \dots, \frac{1}{7.8}, \dots$

Solution $x_m = \frac{1}{(m+2) \cdot (m+3)} = \frac{1}{m+2} - \frac{1}{m+3}$

where $m = 1, 2, 3, \dots$

$$x_1 = \frac{1}{3} - \frac{1}{4}; \quad x_2 = \frac{1}{4} - \frac{1}{5}; \quad x_3 = \frac{1}{5} - \frac{1}{6}, \text{ etc.}$$

$$\therefore x_n = \frac{1}{n+2} - \frac{1}{n+3}$$

$$\begin{aligned} S_n &= \frac{1}{3} + \left(\frac{-1}{4} + \frac{1}{4} \right) + \left(\frac{-1}{5} + \frac{1}{5} \right) + \frac{-1}{n+3} \\ &= \frac{1}{3} - \frac{1}{n+3} = \frac{n+3-3}{3(n+3)} = \frac{n}{3(n+3)} \end{aligned}$$

EXAMPLE 4 Find the sum of squares of first n natural numbers.

Solution
$$\begin{aligned} S_n &= 1^2 + 2^2 + 3^2 + 4^2 + \dots + n^2 \\ &= \frac{n(n+1)(2n+1)}{6} \end{aligned}$$

EXAMPLE 5 Find the sum of the series $5 + 55 + 555 + 5555 + \dots$ to n terms.

Solution
$$\begin{aligned} &= 5(1 + 11 + 111 + \dots \text{ to } n \text{ terms}) \\ &= \frac{5}{9}(9 + 99 + 999 + \dots \text{ to } n \text{ terms}) \\ &= \frac{5}{9} \{ (10 - 1) + (10^2 - 1) + (10^3 - 1) + \dots \text{ to } n \text{ terms} \} \\ &= \frac{5}{9} \{ 10 + 10^2 + 10^3 + \dots + 10^n - n \times 1 \} \\ &= \frac{5}{9} \left\{ 10 \times \frac{10^n - 1}{10 - 1} - n \right\} \\ &= \frac{5}{9} \left\{ \frac{10}{9} (10^n - 1) - n \right\} \end{aligned}$$

EXAMPLE 6 Find the sum of squares of first n odd natural numbers.

Solution The series of odd natural numbers is $1^2, 3^2, 5^2, 7^2, \dots$ to n terms.

The terms of the series is given by $(2m - 1)^2$, where $m = 1, 2, 3, \dots$,

$$(2m - 1)^2 = 4m^2 - 4m + 1$$

$$\begin{aligned} \therefore S_n &= \sum_{m=1}^n (2m - 1)^2 = 4 \sum_{m=1}^n m^2 - 4 \sum_{m=1}^n m + \sum_{m=1}^n 1 \\ &= 4 \times \frac{n(n+1)(2n+1)}{6} - 4 \frac{n(n+1)}{2} + n \\ &= \frac{2n}{3} (n+1)(2n+1) - 2n(n+1) + n \end{aligned}$$

$$\begin{aligned}
&= 2n(n+1) \left\{ \frac{2n+1}{3} - 1 \right\} + n \\
&= 2n(n+1) \frac{(2n+1-3)}{3} + n \\
&= n \left\{ 2(n+1) \frac{(2n-2)}{3} + 1 \right\} \\
&= \frac{n}{3} \{ 2(n+1)(2n-2) + 3 \} \\
&= \frac{n}{3} \{ 4n^2 - 4n + 4n - 4 + 3 \} \\
&= \frac{n}{3} \{ 4n^2 - 1 \}
\end{aligned}$$

EXAMPLE 7 Find the sum of squares of first n even natural numbers.

Solution The series is $2^2, 4^2, 6^2, 8^2, \dots$ The term of the series are given by $(2m)^2 = 4m^2$ where $m = 1, 2, 3, 4 \dots$

$$S_n = 4 \sum_{m=1}^n m^2 = \frac{4n(n+1)(2n+1)}{6} = \frac{2n(n+1)(2n+1)}{3}$$

EXAMPLE 8 Find the sum of first n terms of the series $1, 2x, 3x^2, 4x^3, \dots$

Solution $S_n = 1 + 2x + 3x^2 + \dots + nx^{n-1}$

$$\therefore xS_n = x + 2x^2 + 3x^3 + \dots + nx^n$$

$$S_n - xS_n = 1 + x + x^2 + \dots + x^{n-1} - nx^n$$

$$\begin{aligned}
\therefore S_n &= \frac{1 + x + x^2 + \dots + x^{n-1} - nx^n}{(1-x)} \\
&= \frac{1 + x + x^2 + \dots + x^{n-1}}{1-x} - \frac{nx^n}{1-x} \\
&= \frac{1(1-x^n)}{(1-x)^2} - \frac{nx^n}{(1-x)} \\
&= \frac{1-x^n}{(1-x)^2} - \frac{nx^n}{(1-x)}
\end{aligned}$$

EXAMPLE 9 Find the value of $\left(1 + \frac{1}{x}\right) \left(1 + \frac{1}{x+1}\right) \left(1 + \frac{1}{x+2}\right) \left(1 + \frac{1}{x+3}\right)$.

Solution The given expression = $\left(1 + \frac{1}{x}\right) \left(1 + \frac{1}{x+1}\right) \left(1 + \frac{1}{x+2}\right) \times \left(1 + \frac{1}{x+3}\right)$

$$= \frac{x+1}{x} \times \frac{x+2}{x+1} \times \frac{x+3}{x+2} \times \frac{x+4}{x+3} = \frac{x+4}{x}$$

EXAMPLE 10 Find the sum of the series $5^3 + 6^3 + \dots + 10^3$.

Solution The sum of cubes of first n natural numbers

$$S_n = \left\{ \frac{n(n+1)}{2} \right\}^2$$

$\therefore$

$$S_{10} = 1^3 + 2^3 + \dots + 10^3$$

$$S_4 = 1^3 + 2^3 + \dots + 4^3$$

$$S_{10} - S_4 = 5^3 + 6^3 + \dots + 10^3$$

$$\begin{aligned} &= \left[\frac{10 \times 11}{2} \right]^2 - \left[\frac{4 \times 5}{2} \right]^2 = 5^2 \times 11^2 - 2^2 \times 5^2 \\ &= 5^2 [121 - 4] = 5^2 \times 117 = \mathbf{2925} \end{aligned}$$

EXAMPLE 11 The sum $9 + 16 + 25 + 36 + \dots + 100$ is equal to _____.

Solution $S = 9 + 16 + 25 + 36 + 49 + 64 + 81 + 100$

i.e. sum of squares of first n natural numbers $= \frac{n(n+1)(2n+1)}{6}$

$\therefore$ Required sum $= S_{10} - S_2$

$$\begin{aligned} &= \frac{10(10+1)(2 \times 10+1)}{6} - \frac{2(2+1)(2 \times 2+1)}{6} \\ &= \frac{10 \times 11 \times 21}{6} - \frac{2 \times 3 \times 5}{6} \\ &= 5 \times 11 \times 7 - 5 \\ &= 5(77 - 1) = \mathbf{380} \end{aligned}$$

EXAMPLE 12 In the expansion of $\left(x^3 - \frac{3}{x^2}\right)^{15}$, the term free from x is _____.

Solution $\left(x^3 - \frac{3}{x^2}\right)^{15}$

Let its r th term be independent of x .

$$\therefore (x^3)^{15-r+1} \cdot \left(\frac{1}{x^2}\right)^{r-1} = x^0$$

$$\text{i.e. } 3(15 - r + 1) + (r - 1)(-2) = 0$$

$$\text{i.e. } 48 - 3r + 2 - 2r = 0$$

$$\text{i.e. } 50 - 5r = 0$$

$$\therefore r = 50/5 = \mathbf{10}$$

$\therefore$ The answer is 10th term.

**Notes:**

1. If the p th term of an AP is q and q th term of the AP is p , then $(p + q)$ th term of the AP is zero.
2. If the p th term of an AP is x and the q th term of the AP is y , then the sum of its first $(p + q)$ terms is

$$\frac{1}{2}(p + q) \left\{ x + y + \frac{x + y}{p - q} \right\}$$

3. If the sum of n terms of an AP is m and the sum of m terms is n , then the sum of $(m + n)$ terms is $-(m + n)$.
4. The ratio of the sum of p arithmetic means to the sum of q arithmetic means inserted between any two numbers is $p : q$.
5. If a , b and l are the first, second and last terms of a GP respectively, then the sum of the GP is

$$\frac{bl - a^2}{(b - a)}.$$

6. A GP of n terms has the first term a and common ratio r and S_n is the sum of the GP. If S'_n is the sum of the reciprocals of the terms, then

$$\frac{S_n}{S'_n} = a^2 r^{(n-1)}$$

7. If S_n , S_{2n} and S_{3n} are the sum of first n , $2n$ and $3n$ terms respectively of a GP, then

$$S_n(S_{3n} - S_{2n}) = (S_{2n} - S_n)^2$$

8. A GP contains n terms and n is an odd number. The product of n terms of the GP is equal to the n th power of its middle term.
9. If a , b , c and d are in GP and x is any even number, then

$$\frac{a - bx}{a + bx} = \frac{b - cx}{b + cx} = \frac{c - dx}{c + dx}$$

10. If S_n denotes the sum of n terms of a GP, whose first term is a and common ratio is r , then

$$\begin{aligned} S_T &= S_1 + S_2 + S_3 + \dots + S_n \\ &= \frac{n \cdot a}{1 - r} - \frac{ar(1 - r^n)}{(1 - r)^2} \end{aligned}$$

S_T is the sum total of sum of G.P., with 1, 2, 3, ..., n terms.

i.e. $S_T = \sum_{i=1}^n S_i$, S_i is the sum of G.P. with number of terms = i .

11. If the first term of a GP is a , then the n th term is b and the product of first n terms is p , then $p^2 = (ab)^n$.
12. If S_n , S_{2n} , S_{3n} are the sums of first n , $2n$, and $3n$ terms respectively of an AP, then $S_{3n} = 3(S_{2n} - S_n)$.
13. The sum of first n odd integers/natural numbers = $\frac{n^2}{2}$.
14. The sum of first n even natural numbers = $n(n + 1)$.
15. The sum of squares of first n natural numbers

$$\sum_{m=1}^n m^2 = \frac{n(n + 1)(2n + 1)}{6}.$$

16. The sum of first n natural numbers

$$\sum_{m=1}^n m = \frac{n(n + 1)}{2}$$

17. The sum of cubes of first n natural numbers

$$\sum_{m=1}^n m^3 = \left[\frac{n(n+1)}{2} \right]^2 = \left[\sum_{m=1}^n m \right]^2$$

18. The sum of squares of first n odd natural numbers

$$\sum_{m=1}^n (2m-1)^2 = \frac{n(4n^2-1)}{3} = \frac{n(2n+1)(2n-1)}{3}$$

19. The sum of squares of first n even natural numbers

$$\sum_{m=1}^n (2m)^2 = 4 \sum_{m=1}^n m^2 = \frac{2n(n+1)(2n+1)}{3}$$

20. The sum of first n terms of the series

$$\{1^2 - 2^2 + 3^2 - 4^2 + \dots\} = \frac{1}{2} (-1)^{n+1} n(n+1)$$

21. The sum of first n terms of Arithmetico-Geometric Series

$$a, (a+d)r, (a+2d)r^2, \dots, [a+(n-1)d]r^{n-1}$$

$$\text{is } S_n = \frac{a - \{a + (n-1)d\}r^n}{1-r} + \frac{dr(1-r^{n-1})}{(1-r)^2}$$

22. The sum of first n terms of the Arithmetico-Geometric Series

$$(1 + 2x + 3x^2 + \dots + nx^{n-1}) = \frac{1-x^n}{(1-x)^2} - \frac{nx^n}{1-x}$$

Sum of typical infinite series

$$23. 1 + \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + \dots + \frac{1}{n!} = e$$

$$24. 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots + \frac{x^n}{n!} = e^x$$

$$25. x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \dots = \log_e(1+x) \quad \text{where } -1 < x \leq 1$$

$$26. x + \frac{x^2}{2} + \frac{x^3}{3} + \frac{x^4}{4} + \dots = -\log_e(1-x)$$

$$27. 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} - \frac{1}{6} + \dots = \log 2$$

$$28. x + \frac{x^3}{3} + \frac{x^5}{5} + \frac{x^7}{7} + \dots = \frac{1}{2} \log \left(\frac{1+x}{1-x} \right) \quad \text{where } -1 < x < 1$$

$$29. 1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \frac{x^6}{6!} + \dots = \frac{1}{2} (e^x + e^{-x})$$

$$30. x + \frac{x^3}{3!} + \frac{x^5}{5!} + \frac{x^7}{7!} + \dots = \frac{1}{2} (e^x - e^{-x})$$

$$31. 1 - x + x^2 - x^3 + \dots = \frac{1}{1+x}$$

$$32. 1 + x + x^2 + x^3 + \dots = \frac{1}{1-x}$$

$$33. 1 - 2x + 3x^2 - 4x^3 + \dots = \frac{1}{(1+x)^2}$$

$$34. 1 + 2x + 3x^2 + 4x^3 + \dots = \frac{1}{(1-x)^2}$$



EXERCISES

- Find the n th term of the following series:
 - 1, 7, 17, 31, 49, ...
 (a) $2n^2 + 1$ (b) $2n^2 - 1$ (c) $n^2 + 1$ (d) $n^2 - 1$ (e) None of these
 - 12, 72, 232, 2592, ...
 (a) $2 \times 6^{n-1}$ (b) $2 \times 6^{n-2}$ (c) 2×6^n (d) $2 \times 6^{n+1}$ (e) None of these
 - 5, 14, 29, 50, 77, ...
 (a) $3n^3 + 1$ (b) $3n^3 - 1$ (c) $3n^3 + 2$ (d) $3n^3 - 2$ (e) None of these
- Insert 6 arithmetic means between 20 and 76.
 (a) 26, 38, 44, 52, 60, 68 (b) 24, 36, 38, 50, 62, 68 (c) 28, 36, 44, 52, 60, 68
 (d) 26, 38, 44, 50, 62, 68 (e) 28, 34, 46, 50, 62, 68
- The first and last terms of an AP are 75 and 375 respectively. If its sum is 3600, find the number of terms in the AP and its common difference.
 (a) 14, 22 (b) 16, 20 (c) 18, 18 (d) 20, 16 (e) None of these
- Find the sum of an AP of 40 terms whose first and last terms are 80 and 275.
 (a) 8400 (b) 7200 (c) 7100 (d) 4800 (e) None of these
- Write the seventh term of the following GP.
 - 6, 12, 24, 48, ...
 (a) 192 (b) 96 (c) 384 (d) 48 (e) None of these
 - 39366, 13122, 4374, 1458, ...
 (a) 27 (b) 54 (c) 108 (d) 216 (e) None of these
 - $21a, 84a^3, 336a^5, 1344a^7$
 (a) a^{11} (b) a^{12} (c) a^{13} (d) None of these
- Find the sum of first 7 terms of a GP, whose first term is 1024 and the common ratio is $\frac{1}{2}$.
 (a) 1016 (b) 508 (c) 2032 (d) 4064 (e) None of these
- The sum of three numbers in a GP is 105 and their product is 8000. Find the numbers.
 (a) 5, 20, 40 (b) 5, 20, 80 (c) 4, 16, 64 (d) None of these
- The AM and GM of two numbers are 17 and 8 respectively. Find their Harmonic Mean and write the AP, GP and HP. Harmonic mean is _____.
 (a) $\frac{16}{17}$ (b) $\frac{32}{17}$ (c) $\frac{64}{17}$ (d) $\frac{128}{17}$ (e) None of these
- Find the sum of first n terms of the AP $\frac{n-1}{n}, \frac{n+1}{n}, \frac{n+2}{n}, \frac{n+3}{n}, \dots$
 (a) $\frac{n-1}{2}$ (b) $\frac{n(n+1)}{2}$ (c) $\frac{3(n-1)}{2}$ (d) $\frac{3(n+1)}{2}$ (e) None of these
- If the sum of first n terms of two arithmetic progressions 3, 7, 11, 15, ... and 30, 33, 36, 39 ... are equal, then find the value of n .
 (a) 45 (b) 50 (c) 55 (d) 60 (e) None of these
- The sum of first 20 terms of a GP is 1025 times the sum of its first 10 terms. Find the common ratio of the GP.
 (a) 6 (b) 2 (c) 24 (d) 18 (e) None of these

12. The sum of first n terms of the series $1, 1 + m, 1 + m + m^2, 1 + m + m^2 + m^3$, will be: _____.
[Hint: Try yourself]

13. Find the sum of $1 + \frac{1}{2} + \frac{1}{2^2} + \frac{1}{2^3} + \dots$ to infinite terms.

$$\left[\text{Hint: } \frac{1}{1-x} = 1 + x + x^2 + \dots \text{ Put } x = \frac{1}{2} \right]$$

- (a) 1 (b) $\frac{3}{2}$ (c) 2 (d) $\frac{5}{2}$ (e) None of these

14. Find the sum of first n terms of the series $(3^3 - 2^3) + (5^3 - 4^3) + (7^3 - 6^3) + (9^3 - 8^3) + \dots$

[Hint: Try yourself]

15. Find the sum to n terms of the series $0.9 + 0.99 + 0.999 + \dots$

[Hint: Try yourself]

16. Find the sum of n terms of the series $1 \cdot 2 \cdot 3 + 4 \cdot 5 \cdot 6 + 7 \cdot 8 \cdot 9 + \dots$

[Hint: Try yourself]

17. Find the sum of n terms of the series $\frac{1}{1 \cdot 2 \cdot 3} + \frac{1}{2 \cdot 3 \cdot 4} + \frac{1}{3 \cdot 4 \cdot 5} + \frac{1}{4 \cdot 5 \cdot 6}$

[Hint: Try yourself]

18. Find the sum of infinite terms of a GP $1 + \frac{2}{3} + \frac{3}{3^2} + \frac{4}{3^3} + \frac{5}{3^4} + \dots$

- (a) $\frac{3}{4}$ (b) $\frac{5}{4}$ (c) $\frac{7}{4}$ (d) $\frac{9}{4}$ (e) None of these

19. Find the geometric mean between $\frac{3}{2}$ and $\frac{27}{2}$.

- (a) $\frac{7}{2}$ (b) $\frac{9}{2}$ (c) $\frac{11}{2}$ (d) $\frac{13}{2}$ (e) None of these



1. (i) (b), (ii) (c), (iii) (c) 2. (c) 3. (b) 4. (c) 5. (i) (c), (ii) (b), (iii) (d) 6. (c)
 7. (b) 8. (c) 9. (c) 10. (c) 11. (b) 12. (d) 13. (c) 14. (d) 15. (b)



Solutions with Necessary Explanation

1. (i) n th term of the series is $2n^2 - 1$ where $n = 1, 2, 3, \dots$

(ii) n th term of the series is 2×6^n where $n = 1, 2, 3, \dots$

(iii) n th term of the series is $3n^3 + 2$, where $n = 1, 2, 3, \dots$

2. The given two numbers are 20 and 76.

Let six arithmetic means between 20 and 76 differ by a common difference of d between consecutive terms.

$$\therefore 76 = 20 + 7d$$

$$\Rightarrow d = \frac{76 - 20}{7} = 8$$

$\therefore$ Arithmetic means are 28, 36, 44, 52, 60 and 68

3. Let the number of terms and common difference of AP be n and d respectively.

$$\therefore a_1 = 75 \text{ and } a_n = 375$$

$$\Rightarrow a_n = a_1 + (n - 1)d$$

$$375 = 75 + (n - 1)d$$

$$\Rightarrow (n - 1)d = 300 \quad (1)$$

$$\text{Given } S_n = \frac{n}{2} \{2a_1 + (n - 1)d\} = 3600$$

$$\frac{n}{2} \{150 + (n - 1)d\} = 3600 \quad (2)$$

Equations (1) and (2)

$$\Rightarrow \frac{n}{2} \{150 + 300\} = 3600$$

$$\Rightarrow n = \frac{2 \times 3600}{450} = 16$$

$$\therefore (1) \Rightarrow d = \frac{300}{n - 1} = \frac{300}{16 - 1} = 20$$

4. Given $a_1 = 80$ and $a_{40} = 275$

$$\therefore a_1 + 39d = 940$$

$$\Rightarrow 80 + 39d = 275$$

$$\therefore d = \frac{275 - 80}{39} = 5$$

$$\therefore S_{40} = \frac{n}{2} \{a_1 + a_{40}\} = \frac{40}{2} \{80 + 275\} = 7100$$

5. (i) 7th term of GP = $a \times r^6$
 $= 6 \times 2^6 = 6 \times 64 = 384$

$$(ii) \text{ 7th term of GP} = 39366 \times \left(\frac{1}{3}\right)^6 = 54$$

$$(iii) \text{ 7th term of GP} = 21 \times a \times 4^6 \times a^{2 \times 6} = 21 \times 4^6 \times a^{1+12} = 21 \times 4^6 a^{13} = 86016a^{13}$$

6. Given $a = 1024$; $r = \frac{1}{2}$

$$\begin{aligned} S_7 &= a \frac{(1 - r^7)}{(1 - r)} = \frac{1024 \left(1 - \left(\frac{1}{2}\right)^7\right)}{1 - \frac{1}{2}} \\ &= 1024 \times \frac{127}{128} \times 2 \\ &= 16 \times 127 = 2032 \end{aligned}$$

7. In the GP, $S_3 = 105$ (1)

$$\frac{a}{r} \times a \times ar = 8000 \quad (2)$$

$$(2) \Rightarrow a^3 = 8000 \Rightarrow a = 20$$

$$\therefore (1) \Rightarrow \frac{20}{r} + 20 + 20r = 105$$

$$\Rightarrow 20\left(\frac{1}{r} + r\right) = 105 - 20 = 85$$

$$\Rightarrow r + \frac{1}{r} = \frac{85}{20} = \frac{17}{4} = 4\frac{1}{4}$$

$$\Rightarrow r = 4$$

Here $a = 20$; $r = 4$

$\therefore$ numbers are $\frac{a}{r}$, a and ar .

i.e. $\frac{20}{4}$, 20 and 20×4

i.e. 5, 20 and 80.

8. Given AM = 17; GM = 8

Let a and b be the two numbers

$$\therefore \text{AM} = \frac{a+b}{2} = 17 \quad \text{or} \quad a+b = 34 \quad (1)$$

$$\text{GM} = \sqrt{ab} = 8 \quad \text{or} \quad ab = 64 \quad (2)$$

$$\begin{aligned} \therefore (a-b) &= \sqrt{(a+b)^2 - 4ab} = \sqrt{(34)^2 - 4 \times 64} \\ &= \sqrt{34^2 - 16^2} = \sqrt{(34+16)(34-16)} \\ &= \sqrt{50 \times 18} = \sqrt{900} = 30 \end{aligned}$$

$$\therefore a - b = 30 \quad (3)$$

$$\text{Solving (1) and (3), } a = \frac{34+30}{2} = 32;$$

$$b = \frac{34-30}{2} = 2$$

$$\begin{aligned} \therefore \text{Harmonic Mean} &= \frac{2ab}{a+b} = \frac{2 \times 32 \times 2}{32+2} \\ &= \frac{2 \times 64}{34} = \frac{64}{17} \end{aligned}$$

$\therefore$ AP is 2, 17, 32

GP is 2, 8, 32

HP is $2, \frac{64}{17}, 32$

$$\text{Harmonic Mean} = \frac{64}{17}$$

9. $S_n = \frac{n-1}{n} + 1 + \frac{n+1}{n} + \frac{n+2}{n} + \dots$ up to n terms

$$\text{Here } a = \frac{n-1}{n}; d = 1 - \frac{n-1}{n} = \frac{1}{n} = \frac{n+1}{n} - 1$$

$$\begin{aligned}\therefore S_n &= \frac{n}{2} \{2a + (n-1)d\} = \frac{n}{2} \left\{ \frac{2(n-1)}{n} + (n-1) \frac{1}{n} \right\} \\ &= \frac{n}{2} \times \frac{3(n-1)}{n} = \frac{3(n-1)}{2}\end{aligned}$$

10. Given two Arithmetic Progressions.

For the first AP = 3, 7, 11, 15 ... sum of n terms is given by

$$\begin{aligned}S_{n_1} &= \frac{n}{2} \{2a + (n-1)d\} \\ &= \frac{n}{2} \{2(3) + (n-1)4\} \\ &= \frac{n}{2} \{6 + 4n - 4\} \\ &= \frac{n}{2} \{4n + 2\} = n(2n + 1)\end{aligned}$$

For the second AP = 30, 33, 36, 39 ... sum of n terms is given by

$$\begin{aligned}S_{n_2} &= \frac{n}{2} \{2a + (n-1)d\} \\ &= \frac{n}{2} \{2 \times 30 + (n-1)3\} \\ &= \frac{n}{2} \{60 + 3n - 3\} \\ &= \frac{n}{2} \{57 + 3n\} \\ &= \frac{3n(19 + n)}{2}\end{aligned}$$

Given that

$$S_{n_1} = S_{n_2}$$

$$\therefore n(2n + 1) = \frac{3n}{2} (19 + n)$$

$$\text{i.e. } (2n + 1)2 = 3(19 + n)$$

$$4n + 2 = 57 + 3n$$

$$\therefore 4n - 3n = 57 - 2 = 55$$

$$\text{i.e. } n = 55$$

$$\therefore \text{Value of } n = 55$$

$$\mathbf{11.} \quad S_{10} = \frac{a(r^{10} - 1)}{(r - 1)}; S_{20} = \frac{a(r^{20} - 1)}{(r - 1)} = 1025 S_{10}$$

$$\text{i.e. } \frac{a(r^{20} - 1)}{(r - 1)} = 1025 \times \frac{a(r^{10} - 1)}{(r - 1)}$$

$$\text{i.e. } (r^{20} - 1) = 1025 (r^{10} - 1)$$

$$\Rightarrow r^{20} - 1025r^{10} = 1 - 1025 = -1024$$

$$\text{i.e. } r^{10}(r^{10} - 1025) = -1024$$

$$(1025 - r^{10})r^{10} = 1024 = 32^2 = 2^{10}$$

$$(2^{10} + 1 - r^{10})r^{10} = 2^{10}$$

Solving by trial and error,

$$\Rightarrow r = \pm 2$$

$$12. \quad S_n = 1 + (1 + m) + (1 + m + m^2) + (1 + m + m^2 + m^3) + \dots \quad (1)$$

$$mS_n = m + (m + m^2) + (m + m^2 + m^3) + \dots \quad (2)$$

$$(1) - (2) \Rightarrow (1 - m)S_n = 1(1 - m) + (1 + m)(1 - m) + (1 + m + m^2)(1 - m) + \dots$$

$$\left\{ \begin{aligned} (1 - m)S_n &= (1 - m) + (1 - m^2) + (1 - m^3) + \dots + (1 - m^n) \end{aligned} \right.$$

$$(1 - m)S_n = 1 \times n - (m + m^2 + m^3 + \dots + m^n)$$

$$= n - m \frac{(1 - m^n)}{(1 - m)}$$

$$\text{i.e. } S_n = \frac{n}{1 - m} - \frac{m(1 - m^n)}{(1 - m)^2} \quad \left[\because S_n = \frac{a(1 - r^n)}{(1 - r)}, \text{ if } r < 1 \right]$$

$$\therefore S_n = \frac{n}{1 - m} + \frac{m(1 - m^n)}{(1 - m)^2}$$

$$13. \quad S_n = 1 + \frac{1}{2} + \frac{1}{2^2} + \frac{1}{2^3} + \dots$$

$$= 1 + x + x^2 + x^3 + \dots \quad \text{where } x = \frac{1}{2}$$

$$= \frac{1}{1 - x} = \frac{1}{1 - \frac{1}{2}} = 2$$

$$14. \quad (3^3 - 2^3) + (5^3 - 4^3) + (7^3 - 6^3) + \dots$$

$$= \sum_{m=1}^n (2m+1)^3 - (2m)^3 = \sum_{m=1}^n [(2m+1) - 2m][(2m+1)^2 + (2m+1)2m + (2m)^2]$$

$$= \sum_{m=1}^n 1 \times [4m^2 + 1 + 4m + 2m(2m+1) + 4m^2]$$

$$= \sum_{m=1}^n 12m^2 + 6m + 1$$

$$= 12 \sum_{m=1}^n m^2 + 6 \sum_{m=1}^n m + \sum_{m=1}^n 1$$

$$= 12 \times \frac{n(n+1)(2n+1)}{6} + 6 \frac{n(n+1)}{2} + n$$

$$= 2n(n+1)(2n+1) + 3n(n+1) + n$$

$$= n(n+1)\{2(2n+1) + 3\} + n$$

$$= n(n+1)\{4n+2+3\} + n$$

$$\begin{aligned}
 &= n\{(n+1)(4n+5)+1\} \\
 &= n\{4n^2+9n+5+1\} \\
 &= 4n^3+9n^2+6n
 \end{aligned}$$

$$15. S_n = 0.9 + 0.99 + 0.999 + \dots$$

$$\begin{aligned}
 &= 9\{0.1 + 0.11 + 0.111 + \dots\} \\
 &= \frac{9}{10^{n+1}} \{10^n + 11 \times 10^{n-1} + 111 \times 10^{n-2} + \dots\} \\
 &= \frac{1}{10^{n+1}} \{9 \times 10^n + 99 \times 10^{n-1} + 999 \times 10^{n-2} + \dots\} \\
 &= \frac{1}{10^{n+1}} \{(10-1)10^n + (100-1)10^{n-1} + (1000-1)10^{n-2} + \dots\} \\
 &= \frac{1}{10^{n+1}} \{n \times 10^{n+1} - (10^n + 10^{n-1} + 10^{n-2} + \dots + 10^1)\} \\
 &= \frac{1}{10^{n+1}} \left\{ n \times 10^{n+1} - \frac{10^n [1 - (1/10)^n]}{1 - \frac{1}{10}} \right\} \\
 &= \frac{1}{10^{n+1}} \left\{ n \times 10^{n+1} - \frac{10^n - 1}{\frac{10-1}{10}} \right\} \\
 &= \frac{1}{10^{n+1}} \left\{ n \times 10^{n+1} - \frac{10^{n+1} - 10}{9} \right\} \\
 &= n - \frac{1 - 10^{-n}}{9}
 \end{aligned}$$

$$16. S_n = 1 \cdot 2 \cdot 3 + 4 \cdot 5 \cdot 6 + 7 \cdot 8 \cdot 9 + \dots$$

$$\begin{aligned}
 &= \sum_{m=1}^{3n-2} m(m+1)(m+2) = \sum_{m=1}^{3n-2} m(m^2 + 3m + 2) \\
 &= \sum_{m=1}^{3n-2} m^3 + 3m^2 + 2m \\
 &= \left[\frac{(3n-2)(3n-1)}{2} \right]^2 + \frac{3(3n-2)(3n-1)(6n-3)}{6} + 2(3n-2) \\
 &= \frac{9n^2 - 9n + 2}{4} + \frac{3(9n^2 - 9n + 2)(2n-1)}{2} + 2(3n-2) \\
 &= \frac{9n^2 - 9n + 2}{4} \{6(2n-1) + 1\} + 6n - 4 \\
 &= \frac{(9n^2 - 9n + 2)(12n-5) + 24n - 16}{4} \\
 &= \frac{108n^3 - 108n^2 + 24n - 45n^2 + 45n - 10 + 24n - 16}{4}
 \end{aligned}$$

$$\begin{aligned}
&= \frac{108n^3 - 153n^2 + 93n - 26}{4} \\
&= \frac{3(3n-2)(n+1)(9n^2-9n+4)}{4}
\end{aligned}$$

$$\begin{aligned}
17. S_n &= \frac{1}{1 \cdot 2 \cdot 3} + \frac{1}{2 \cdot 3 \cdot 4} + \frac{1}{3 \cdot 4 \cdot 5} + \cdots \\
&= \sum_{m=1}^n \frac{1}{m(m+1)(m+2)} = \frac{1}{m} \left(\frac{1}{m+1} - \frac{1}{m+2} \right) \\
&= \sum_{m=1}^n \left(\frac{1}{m} - \frac{1}{m+1} \right) - \frac{1}{2} \left(\frac{1}{m} - \frac{1}{m+2} \right) \\
&= \sum_{m=1}^n \frac{1}{2m} - \frac{1}{m+1} + \frac{1}{m+2} \\
&= \left(\frac{1}{2} + \frac{1}{4} + \frac{1}{6} + \cdots + \frac{1}{2n} \right) + \left(\frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \cdots + \frac{1}{n+2} \right) - \left(\frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \cdots + \frac{1}{n+1} \right) \\
&= \left(\frac{1}{2} + \frac{1}{4} + \frac{1}{6} + \cdots + \frac{1}{2n} \right) + \frac{1}{n+2} - \frac{1}{2} \\
&= \sum \frac{1}{2n} - \frac{n}{2(n+2)} = \frac{1}{2} \sum \frac{1}{n} - \frac{n}{2(n+2)}
\end{aligned}$$

$$\begin{aligned}
18. S_n &= 1 + \frac{2}{3} + \frac{3}{3^2} + \frac{4}{3^3} + \frac{5}{3^4} + \cdots \\
&= 1 + 2x + 3x^2 + 4x^3 + 5x^4 + \cdots \quad \text{where } x = \frac{1}{3} \\
&= \frac{1}{(1-x)^2} = \frac{1}{\left(1 - \frac{1}{3}\right)^2} = \frac{9}{4}
\end{aligned}$$

$$19. \text{ GM between } \frac{3}{2} \text{ and } \frac{27}{2} \text{ is given by } \text{GM} = \sqrt{\frac{3}{2} \times \frac{27}{2}} = \sqrt{\frac{81}{4}} = \frac{9}{2}$$

Fractions

If any unit is divided into a number of equal parts, one or more of these parts is called a *fraction* of the unit. For example, $\frac{1}{3}, \frac{2}{3}, \frac{3}{4}, \frac{4}{7}$, etc. are fractions.

The numerator and denominator of a fraction are called its terms.

PROPER FRACTION: A proper fraction is one whose numerator is less than the denominator. For example, $\frac{1}{2}, \frac{1}{3}, \frac{2}{5}, \frac{3}{8}$, etc. are proper fractions.

The value a proper fraction is always less than 1.

IMPROPER FRACTION: A fraction, whose numerator is equal to or greater than the denominator, is called an improper fraction. For example, $\frac{11}{9}, \frac{7}{3}, \frac{13}{6}, \frac{19}{3}$, etc. are improper fractions.

The value of improper fraction is always more than 1.

MIXED FRACTIONS: An improper fraction can be expressed as a whole number and a proper fraction. This expression is called mixed fraction. For example, $2\frac{3}{4}, 3\frac{5}{7}, 7\frac{2}{9}$, etc. are mixed fractions.

LIKE FRACTIONS: Fractions in which denominators are the same are called like fractions. For example, $\frac{11}{17}, \frac{14}{17}, \frac{9}{17}$, etc. are like fractions.

UNLIKE FRACTIONS: Fractions in which denominators are different, are called unlike fractions. For example, $\frac{3}{8}, \frac{4}{11}, \frac{5}{17}, \frac{7}{19}$, etc. are unlike fractions.

COMPOUND FRACTIONS: The fraction of a fraction is called a compound fraction. For example,

$$\frac{1}{3} \text{ of } \frac{4}{7} = \frac{1}{3} \times \frac{4}{7} = \frac{4}{21}$$

$$\frac{3}{7} \text{ of } \frac{5}{11} = \frac{3}{7} \times \frac{5}{11} = \frac{15}{77}$$

COMBINED OPERATIONS: In simplifying the fractions involving the various signs and brackets the following points should be remembered.

1. The operations of multiplication and division should be performed before those of addition and subtraction.
2. Each of the signs '×' or '÷' should be applied only to the number which immediately follows it. For example,

$$(i) \quad \frac{4}{5} \times \frac{7}{12} \div \frac{5}{24} = \frac{4}{5} \times \frac{7}{12} \times \frac{24}{5} = \frac{56}{25}$$

$$(ii) \quad \frac{3}{7} \div \frac{5}{12} \times \frac{5}{24} = \frac{3}{7} \times \frac{12}{5} \times \frac{5}{24} = \frac{3}{14}$$

3. The operations within brackets are to be carried out first.
4. The rule of 'BODMAS' is applied for combined operations.

Complex Fractions: A complex fraction is one in which the numerator or denominator or both are fractions. For example, $\frac{3/5}{7}, \frac{7}{3/5}, \frac{5/9}{7/9}, \frac{1/2 + 3/5}{3/4 - 2/5}$ are complex fractions.

EXAMPLE 1 Simplify the following:

$$(a) \quad \frac{3/15}{2/5}$$

$$(b) \quad \frac{1/2 + 2/3}{3/4 - 2/9}$$

$$(c) \quad \frac{7}{5 - 8/3} \div \frac{3 - \frac{2}{3 - 1/2}}{4 - 1/2}$$

Solution

$$(a) \quad \frac{3/15}{2/5} = \frac{3}{15} \times \frac{5}{2} = \frac{1}{2}$$

$$(b) \quad \frac{1/2 + 2/3}{3/4 - 2/9} = \frac{\frac{1 \times 3 + 2 \times 2}{6}}{\frac{3 \times 9 - 4 \times 2}{36}} = \frac{7/6}{19/36} = \frac{7}{6} \times \frac{36}{19} = \frac{42}{19}$$

$$\begin{aligned}
 \text{(c)} \quad & \frac{7}{5 - 8/3} \div \frac{3 - \frac{2}{3 - 1/2}}{4 - 1/2} \\
 &= \frac{7}{\frac{15 - 8}{3}} \div \frac{3 - \frac{2}{(6 - 1)/2}}{(8 - 1)/2} = \frac{7 \times 3}{7} \div \frac{3 - \frac{4}{5}}{7/2} \\
 &= 3 \div \frac{(15 - 4) \frac{5}{7/2}}{2} = 3 \div \frac{11}{5} \times \frac{2}{7} \\
 &= \left(3 + \frac{11}{5}\right) \times \frac{2}{7} = \left(3 \times \frac{5}{11}\right) \times \frac{2}{7} = \frac{30}{77}
 \end{aligned}$$

Continued Fraction: A fraction that contains an additional fraction in the numerator (or the denominator) is called the continued fraction.

EXAMPLE 2 Solve $2 + \frac{1}{1 + \frac{1}{2 + 2/3}}$

Solution $2 + \frac{1}{1 + \frac{1}{2 + \frac{2}{3}}} = 2 + \frac{1}{1 + \frac{1}{8/3}} = 2 + \frac{1}{1 + \frac{3}{8}} = 2 + \frac{1}{11/8} = 2 + \frac{8}{11} = \frac{30}{11}$

EXAMPLE 3 Solve $3 + \frac{2}{3 + \frac{1}{5 - \frac{4}{3}}}$

Solution $3 + \frac{2}{3 + \frac{1}{5 - \frac{4}{3}}} = 3 + \frac{2}{3 + \frac{1}{11/3}} = 3 + \frac{2}{3 + \frac{3}{11}}$
 $= 3 + \frac{2}{36/11} = 3 + \frac{2 \times 11}{36} = 3 + \frac{11}{18} = \frac{65}{18}$



Note: To simplify a continued fraction begin at the bottom and work upwards.

EXAMPLE 4 Simplify $\frac{1}{2 + \frac{1}{2 + \frac{1}{1 + 1/3}}}$

$$\begin{aligned}
 \text{Solution } \frac{1}{2 + \frac{1}{2 + \frac{1}{1 + 1/3}}} &= \frac{1}{2 + \frac{1}{2 + \frac{1}{4/3}}} = \frac{1}{2 + \frac{1}{2 + 3/4}} \\
 &= \frac{1}{2 + \frac{1}{11/4}} = \frac{1}{2 + \frac{4}{11}} = \frac{1}{\frac{22 + 4}{11}} = \frac{11}{26}
 \end{aligned}$$

EXAMPLE 5 Simplify $3 + \frac{1}{5 + \frac{1}{4 + 2/7}}$

$$\begin{aligned}
 \text{Solution } 3 + \frac{1}{5 + \frac{1}{4 + \frac{2}{7}}} &= 3 + \frac{1}{5 + \frac{1}{\frac{30}{7}}} = 3 + \frac{1}{5 + \frac{7}{30}} \\
 &= 3 + \frac{1}{\frac{157}{30}} = 3 + \frac{30}{157} = \frac{501}{157}
 \end{aligned}$$

EXAMPLE 6 Simplify $4 + \frac{1}{5 + \frac{1}{3 + \frac{1}{4 + 1/3}}}$

$$\begin{aligned}
 \text{Solution } 4 + \frac{1}{5 + \frac{1}{3 + \frac{1}{4 + 1/3}}} &= 4 + \frac{1}{5 + \frac{1}{3 + \frac{1}{13/3}}} = 4 + \frac{1}{5 + \frac{1}{3 + 3/13}} = 4 + \frac{1}{5 + \frac{1}{\frac{42}{13}}} \\
 &= 4 + \frac{1}{5 + \frac{13}{42}} = 4 + \frac{1}{\frac{223}{42}} = 4 + \frac{42}{223} = 4 \frac{42}{223}
 \end{aligned}$$

EXAMPLE 7 A sum of money gets increased by its seventh part and amounts to ₹ 40. Find the sum.

Solution Let the sum be S .

Then $S + \frac{S}{7} = 40$

$$\Rightarrow \frac{8S}{7} = 40$$

$$\Rightarrow S = \frac{40 \times 7}{8} = 35$$

$\therefore$ Sum = ₹ 35

EXAMPLE 8 A train starts with full of passengers. At the first station it drops one-third of passengers and takes in 96 more. At the next, it drops one-half of the new total and takes in 12 more. On reaching the next station, there are found to be 248 passengers left. With how many passengers did the train start?

Solution Let the number of passengers be x when the train started off.

$$\therefore \text{At the first station, the number of passengers} = x - \frac{x}{3} + 96 = \frac{2x}{3} + 96$$

$$\text{At the second station, the number of passengers} = \left(\frac{2x}{3} + 96 \right) \frac{1}{2} + 12$$

$$\text{At the third station, the number of passengers left} = \frac{x}{3} + 48 + 12 = 248$$

$$\therefore \quad \frac{x}{3} + 12 = 200$$

$$x = (200 - 12)3 = 564$$

DECIMAL FRACTION: Fraction in which the denominators are powers of 10 are called decimal fractions.

For example,

$$\frac{1}{10} = 0.1; \quad \frac{1}{100} = 0.01; \quad \frac{5}{1000} = 0.005; \quad \frac{11}{1000} = 0.011 \text{ etc.}$$



RULE

Converting a decimal into a vulgar fraction.

Write down the given number without decimal point in the numerator, and in the denominator write 1 followed by as many zeros as the number of figures after the decimal point in the given decimal fraction.

EXAMPLE 9 Reduce the following decimal fraction into vulgar fraction.

(i) 0.89

(ii) 2.078

(iii) 0.00056

Solution

(i) $0.89 = \frac{89}{100}$

(ii) $2.078 = \frac{2078}{1000} = 2 \frac{78}{1000}$

(iii) $0.00056 = \frac{56}{100000}$

Addition and Subtraction of Decimals



RULE

Write down the numbers under one another, placing the decimal points in one column. The number can now be added or subtracted as usual.

EXAMPLE 10 Add 4.076, 2.374, 0.56 and 15.

Solution

$$\begin{array}{r} 4.076 \\ 2.374 \\ 0.56 \\ 15.0 \\ \hline 22.010 \end{array}$$

EXAMPLE 11 Add 2.057, 14.146, 0.078 and 12.

Solution

$$\begin{array}{r}
 2.057 + \\
 14.146 \\
 0.078 \\
 12.0 \\
 \hline
 28.281
 \end{array}$$

EXAMPLE 12 Subtract 17.078 from 24.35

Solution

$$\begin{array}{r}
 24.350 - \\
 17.078 \\
 \hline
 7.272
 \end{array}$$

Multiplication of Decimal Fraction by a Power of 10: Shift the decimal point to the right as many places as in the power of 10.

For example,

$$7.6342 \times 100 = 763.42;$$

$$0.0876 \times 1000 = 87.6;$$

$$0.0963 \times 10000 = 963;$$

Multiplication of Decimal Fractions: Multiply the given numbers without considering the decimal point. In the product, decimal point is marked off by shifting as many places of decimal to the left of the number as the sum of number of decimals in the given numbers.

For example,

$$0.3 \times 0.03 \times 0.003 = ?$$

$$3 \times 3 \times 3 = 27$$

$$\text{Sum of decimal places} = 1 + 2 + 3 = 6$$

$$\therefore 0.3 \times 0.03 \times 0.003 = \mathbf{0.000027}$$

Dividing a Decimal Fraction by a Counting Number: Divide the given number without considering the decimal point, by the given counting number. In the quotient so obtained mark the decimal point to give as many places of decimal as there are in the given decimal fraction.

For example,

$$\text{Find } 0.0234 \div 18$$

$$234 \div 18 = 13$$

The given number contains 4 decimal places

$$\therefore 0.0234 \div 18 = \mathbf{0.0013}$$

Dividing a Decimal Fraction by a Decimal Fraction: Multiply both dividend and divisor by a suitable power of 10 to make divisor a whole number.

For example,

$$0.00088 \div 0.11 = ?$$

Solution $\frac{0.00088}{0.11} = \frac{0.00088}{0.11} \times \frac{100}{100} = \frac{0.088}{11} = \mathbf{0.008}$

COMPARISON OF FRACTIONS: If the given fractions are to be arranged in ascending or descending order of magnitude, then convert each one of them in the decimal format and then arrange accordingly.

For example,

Arrange $\frac{3}{7}$, $\frac{7}{11}$ and $\frac{5}{9}$ in ascending order.

We have, $\frac{3}{7} = 0.428$; $\frac{5}{9} = 0.555\dots$; $\frac{7}{11} = 0.6363\dots$;

$$0.428 < 0.555 < 0.6363$$

or $\frac{3}{7} < \frac{5}{9} < \frac{7}{11}$

RECURRING DECIMAL: In a decimal fraction, if a digit or set of digits is repeated continuously, then it is called a *recurring decimal*.

For example,

$$\frac{1}{3} = 0.333\dots; \quad \frac{1}{6} = 0.1666\dots; \quad \frac{2}{3} = 0.666\dots$$

Pure Recurring Decimal: A decimal fraction in which all the figures after the decimal point are repeated, is called a *pure recurring decimal*.

For example,

$$\frac{1}{3} = 0.333\dots; \quad \frac{1}{9} = 0.111\dots; \text{ are pure recurring decimals.}$$

Mixed Recurring Decimal: A decimal fraction in which some figures do not repeat and some of them are repeated, is called a *mixed recurring decimal*.

For example,

$$0.01633\dots = 0.016\overline{3}$$

Converting a Pure Recurring Decimal into Vulgar Fraction: Write the repeated figures only once in the numerator and write as many mines in the denominator as the number of repeating figures.

For example,

$$0.\overline{5} = \frac{5}{9}; \quad 0.0\overline{67} = \frac{67}{999}; \quad 0.\overline{059} = \frac{59}{999}; \quad 0.\overline{74} = \frac{74}{99} \text{ etc.}$$

Converting a Mixed Recurring Decimal into Vulgar Fraction: In the numerator, write the difference between the number formed by all the digits after decimal point and that formed by digits which are not repeated. In the denominator, write as many nines as the number of repeating digits followed by as many zeros as the number of non-repeating digits.

For example,

$$0.\overline{19} = \frac{19 - 1}{90} = \frac{18}{90} = \frac{1}{5};$$

$$0.4\overline{465} = \frac{4465 - 44}{9900} = \frac{4421}{9900}$$

EXAMPLE 13 Arrange the fractions $\frac{3}{5}$, $\frac{4}{7}$, $\frac{8}{9}$ and $\frac{9}{11}$ in their descending order. [RBI]

Solution $\frac{3}{5} = 0.6$; $\frac{4}{7} = 0.5711$; $\frac{8}{9} = 0.88$; $\frac{9}{11} = 0.8181$

$$0.88 > 0.8181 > 0.6 > 0.5711$$

or $\frac{8}{9} > \frac{9}{11} > \frac{3}{5} > \frac{4}{7}$

EXAMPLE 14 What value will replace the question mark in the following equation?

$$5172.49 + 378.352 + ? = 9318.678 \quad \text{[BSRB]}$$

Solution $? = 9318.678 - [5172.49 + 378.352]$
 $= 9318.678 - 5550.842 = \mathbf{3767.836}$

EXAMPLE 15 Evaluate $2.5 \div 0.0005$ [MBA]

Solution $2.5 \div 0.0005 = \frac{2.5 \times 10000}{0.0005 \times 10000} = \frac{25000}{5} = \mathbf{5000}$

EXAMPLE 16 Evaluate $136.09 \div 43.9$ [Hotel Management]

Solution $\frac{136.09}{43.9} = \frac{136.09 \times 10}{43.9 \times 10} = \frac{1360.9}{439} = \mathbf{3.1}$

EXAMPLE 17 Simplify: $\frac{0.05 \times 0.05 \times 0.05 + 0.04 \times 0.04 \times 0.04}{0.05 \times 0.05 - 0.05 \times 0.04 + 0.04 \times 0.04}$ [IGNOU]

Solution Given expression is of the form $\frac{a^3 + b^3}{a^2 - ab + b^2}$

where $a = 0.05$ and $b = 0.04$

$$\frac{a^3 + b^3}{a^2 - ab + b^2} = a + b = 0.05 + 0.04 = \mathbf{0.09}$$

EXAMPLE 18 Evaluate: $6202.5 + 620.25 + 62.025 + 6.2025 + 0.62025$ **Solution** Adding, we get

$$\begin{array}{r}
 6202.5 \\
 620.25 \\
 62.025 \\
 6.2025 \\
 0.62025 \\
 \hline
 6891.59775
 \end{array}$$

Sum = **6891.59775****EXERCISES****What will come in the place of question mark (?) in the following?**

1. $6\frac{1}{7} + 15\frac{2}{3} + 11\frac{1}{6} = 33\frac{1}{21} - ?$

(a) $\frac{13}{14}$

(b) $1\frac{1}{42}$

(c) $\frac{1}{14}$

(d) $\frac{1}{7}$

(e) None of these

2. $57\frac{4}{7} + 29\frac{1}{21} + ? = 90\frac{3}{35}$

(a) $4\frac{7}{15}$

(b) $4\frac{9}{15}$

(c) $3\frac{1}{15}$

(d) $3\frac{7}{15}$

(e) None of these

3. $12\frac{5}{7} + 23\frac{3}{5} - 14\frac{1}{3} = 28\frac{4}{7} - ?$

(a) $3\frac{4}{105}$

(b) $4\frac{62}{105}$

(c) $6\frac{62}{105}$

(d) $4\frac{61}{105}$

(e) None of these

4. $7^5 \div 7^3 \div 7^2 \times 7^4 \times \frac{1}{7^3} = 7^?$

(a) 1

(b) 2

(c) 0

(d) 3

(e) None of these

5. $13\frac{4}{?} + 5\frac{3}{4} \text{ of } 3\frac{1}{5} \div 4\frac{3}{5} = 17\frac{4}{5}$

(a) 1

(b) 0

(c) 5

(d) 7

(e) None of these

6. $47\frac{1}{17} \div 1\frac{49}{51} + 23\frac{5}{7} + ? = 67\frac{4}{9}$

(a) 0

(b) $19\frac{46}{63}$

(c) $20\frac{46}{63}$

(d) 19

(e) None of these

7. $22\frac{2}{9} + 33\frac{4}{7} - ? = 28\frac{4}{45}$

(a) $27\frac{37}{105}$

(b) $29\frac{74}{105}$

(c) $27\frac{74}{105}$

(d) $28\frac{74}{105}$

(e) None of these

8. $1\frac{4}{7} + 1\frac{3}{5} + 1\frac{1}{3} = ?$

- (a) $5\frac{47}{105}$ (b) $4\frac{58}{105}$ (c) $4\frac{53}{105}$ (d) $5\frac{43}{105}$ (e) None of these

9. $12\frac{1}{3} + 10\frac{5}{6} - 7\frac{2}{3} - 1\frac{4}{7} = ?$

- (a) $13\frac{13}{14}$ (b) $13\frac{11}{14}$ (c) $11\frac{13}{14}$ (d) $14\frac{11}{13}$ (e) None of these

10. $4\frac{3}{4} + 2\frac{1}{8} + 7\frac{1}{4} + 3\frac{7}{8} + 11\frac{12}{13} = ?$

- (a) $27\frac{12}{13}$ (b) $28\frac{12}{13}$ (c) $29\frac{12}{13}$ (d) $29\frac{11}{13}$ (e) None of these

11. $7\frac{11}{18} + 13\frac{1}{9} + 5\frac{7}{9} - 16\frac{2}{3} = ?$

- (a) $9\frac{5}{6}$ (b) $9\frac{5}{18}$ (c) $9\frac{1}{6}$ (d) $8\frac{5}{6}$ (e) $10\frac{5}{6}$

12. $1\frac{7}{9} + 3\frac{5}{8} - 2\frac{1}{18} + 4\frac{7}{16} - 9\frac{5}{18} + ? = 0$

- (a) $1\frac{71}{144}$ (b) $1\frac{73}{144}$ (c) $1\frac{37}{144}$ (d) $2\frac{71}{144}$ (e) None of these

13. $425 \div 16.95 \times ? = 225$

- (a) 11 (b) 0.8 (c) 9 (d) 19 (e) 0.9

14. $198.995 \times 12.005 + 16.25 \times 6.95 = ?$

- (a) 2580 (b) 2550 (c) 2400 (d) 1450 (e) 2500

15. $1.2 \times 1.02 \times 1.002 = ?$

- (a) 1.226448 (b) 1.246448 (c) 1.2468 (d) 1.2648 (e) None of these

16. $6.4 - 1.6 \div 0.2 = ?$

- (a) 16 (b) 4 (c) 1.6 (d) 3.2 (e) None of these

17. $8.88 \times 88.8 \times 88 = ?$

- (a) 68301.142 (b) 79391.642 (c) 65365.824 (d) 76218.414 (e) None of these

18. $989.001 + 1.00982 \times 76.792 = ?$

- (a) 1000 (b) 1100 (c) 1065 (d) 110 (e) 100

19. $375 + 2.832 - 1.001 + 1.803 = ?$

- (a) 8.385 (b) 4.009 (c) 7.384 (d) 5.01 (e) None of these

20. Which one of the following is the smallest fraction?

$\frac{6}{11}, \frac{13}{17}, \frac{19}{27}, \frac{21}{23}, \frac{5}{7}$

- (a) 13/17 (b) 6/11 (c) 19/27 (d) 21/23 (e) None of these

21. Find the smallest fraction which when added to $\frac{2}{5} \times \frac{15}{21} \times \frac{7}{10} \times \frac{3}{8}$ gives a whole number.

- (a) 23/40 (b) 35/40 (c) 37/40 (d) 31/40 (e) None of these

22. The value of $\frac{(0.96)^3 - (0.1)^3}{(0.96)^2 + 0.096 + (0.1)^2}$ is: [SSC]
(a) 0.86 (b) 0.95 (c) 0.97 (d) 1.06
23. The value of $\frac{(2.3)^3 - 0.027}{(2.3)^2 + 0.69 + 0.09}$ is: [SSC]
(a) 0 (b) 1.6 (c) 2 (d) 3.4
24. Simplify $\frac{0.2 \times 0.2 + 0.2 \times 0.02}{0.044}$ [SSC]
(a) 0.004 (b) 0.4 (c) 1 (d) 2
25. Evaluate $\frac{(2.39)^2 - (1.61)^2}{2.39 - 1.61}$ [SSC]
(a) 2 (b) 4 (c) 6 (d) 8
26. On simplification of $\frac{(2.644)^2 - (2.356)^2}{0.288}$, we get: [SSC]
(a) 1 (b) 4 (c) 5 (d) 6
27. $\frac{1.49 \times 14.9 - 0.51 \times 5.1}{14.9 - 5.1}$ [SSC]
(a) 0.20 (b) 2.00 (c) 20 (d) 22
28. $\frac{4.2 \times 4.2 - 1.9 \times 1.9}{2.3 \times 6.1} = ?$ [RRB]
(a) 0.5 (b) 1 (c) 1.9 (d) 4.2
29. $(7.5 \times 7.5 + 37.5 + 2.5 \times 2.5)$ is equal to [SSC]
(a) 30 (b) 60 (c) 80 (d) 100
30. The value of $\left(\frac{0.051 \times 0.051 \times 0.051 + 0.041 + 0.041 \times 0.041}{0.051 \times 0.051 - 0.051 \times 0.041 + 0.041 \times 0.041} \right)$ is: [SSC]
(a) 0.00092 (b) 0.0092 (c) 0.092 (d) 0.92
31. The value of $\frac{0.125 + 0.027}{0.5 \times 0.5 + 0.09 - 0.15}$ is [SSC]
(a) 0.08 (b) 0.2 (c) 0.8 (d) 1
32. $\left[\frac{8(3.75)^3 + 1}{(7.5)^2 - 6.5} \right]$ is equal to: [SSC]
(a) $\frac{9}{5}$ (b) 2.75 (c) 4.75 (d) 8.5
33. $\left(\frac{10.3 \times 10.3 \times 10.3 + 1}{10.3 \times 10.3 - 10.3 + 1} \right)$ is equal to: [SSC]
(a) 9.3 (b) 10.3 (c) 11.3 (d) 12.3
34. $\frac{5 \times 1.6 - 2 \times 1.4}{1.3} = ?$ [Bank PO]
(a) 0.4 (b) 1.2 (c) 1.4 (d) 4

35. The value of $\frac{3.157 \times 4126 \times 3.198}{63.972 \times 2835.121}$ is nearer to: [CBI]
 (a) 0.002 (b) 0.02 (c) 0.2 (d) 2
36. $\frac{0.0203 \times 2.92}{0.0073 \times 14.5 \times 0.7} = ?$ [RRB]
 (a) 0.8 (b) 1.45 (c) 2.40 (d) 3.25
37. The value of $\frac{3.6 \times 0.48 \times 2.50}{0.12 \times 0.09 \times 0.5}$ is: [SSC]
 (a) 80 (b) 800 (c) 8000 (d) 80000
38. $\frac{(0.1667)(0.8333)(0.3333)}{(0.2222)(0.6667)(0.1250)}$ is approximately equal to: [MBA]
 (a) 2 (b) 2.40 (c) 2.43 (d) 2.50
39. If $1^3 + 2^3 + \dots + 9^3 = 2025$, then the value of $(0.11)^3 + (0.22)^3 + \dots + (0.99)^3$ is close to: [SSC]
 (a) 0.2695 (b) 0.3695 (c) 2.695 (d) 3.695
40. If $\frac{1}{6.198} = 0.16134$, then the value of $\frac{1}{0.0006198}$ is: [SSC]
 (a) 0.016134 (b) 0.16134 (c) 1613.4 (d) 16134
41. If $2994 \div 14.5 = 172$, then $29.94 \div 1.45 = ?$ [LIC]
 (a) 0.172 (b) 1.72 (c) 17.2 (d) 172
42. $(0.\overline{09} \times 7.\overline{3})$ is equal to: [SSC]
 (a) $0.\overline{6}$ (b) $0.\overline{657}$ (c) $0.\overline{67}$ (d) $0.\overline{657}$
43. $3.\overline{87} - 2.\overline{59} = ?$ [LIC AAO]
 (a) 1.20 (b) $1.\overline{2}$ (c) $1.\overline{27}$ (d) $1.\overline{28}$
44. The value of $2.1\overline{36}$ is:
 (a) $\frac{47}{220}$ (b) $\frac{68}{495}$ (c) $2\frac{3}{22}$ (d) None of these
45. The correct expression of $6.\overline{46}$ in the fractional form is [CBI]
 (a) $\frac{646}{99}$ (b) $\frac{64640}{100}$ (c) $\frac{640}{100}$ (d) $\frac{640}{99}$
46. The rational number for the recurring decimal $0.125125\dots$ is: [MBA]
 (a) $\frac{63}{487}$ (b) $\frac{119}{993}$ (c) $\frac{125}{999}$ (d) None of these
47. When $0.232323\dots$ is converted into a fraction, then the result is: [CBI]
 (a) $\frac{1}{5}$ (b) $\frac{2}{9}$ (c) $\frac{23}{99}$ (d) $\frac{23}{100}$
48. If $\frac{144}{0.144} = \frac{14.4}{x}$, then the value of x is: [CBI]
 (a) 0.0144 (b) 1.44 (c) 14.4 (d) 144

49. $\frac{0.009}{?} = 0.01$ [MBA]
 (a) 0.0009 (b) 0.09 (c) 0.9 (d) 9
50. The correct expression of $5.\overline{45}$ in the fractional form is [CBI]
 (a) $\frac{545}{99}$ (b) $\frac{54540}{1000}$ (c) $\frac{540}{100}$ (d) $\frac{540}{99}$
51. Write down the following quotients in the descending order [CDS]
 1. $\frac{368.39}{17}$ 2. $\frac{170.50}{62}$ 3. $\frac{875.65}{83}$
 (a) 1, 3, 2 (b) 2, 1, 3 (c) 2, 3, 1 (d) 3, 1, 2
52. $0.04 \times 0.0162 = ?$ [MBA]
 (a) 6.48×10^{-3} (b) 6.48×10^{-4} (c) 6.48×10^{-5} (d) 6.48×10^{-6}
53. $40.83 \times 1.02 \times 1.2 = ?$ [SBI PO]
 (a) 41.64660 (b) 42.479532 (c) 49.97592 (d) 58.7952
54. $16.02 \times 0.001 = ?$ [Bank PO]
 (a) 0.001602 (b) 0.01602 (c) 0.1602 (d) 1.6021
55. $0.002 \times 0.5 = ?$ [Bank PO]
 (a) 0.0001 (b) 0.001 (c) 0.01 (d) 0.1
56. In which of the following fractions are written in ascending order? [Bank PO]
 (a) $\frac{1}{3}, \frac{2}{5}, \frac{4}{7}, \frac{3}{5}, \frac{5}{6}, \frac{6}{7}$ (b) $\frac{1}{3}, \frac{2}{5}, \frac{3}{5}, \frac{4}{7}, \frac{5}{6}, \frac{6}{7}$
 (c) $\frac{1}{3}, \frac{2}{5}, \frac{3}{5}, \frac{5}{6}, \frac{4}{7}, \frac{6}{7}$ (d) $\frac{2}{7}, \frac{3}{5}, \frac{1}{3}, \frac{4}{7}, \frac{5}{6}, \frac{6}{7}$
57. What is the difference between the biggest and the smallest fraction among $\frac{2}{3}, \frac{3}{4}, \frac{4}{5}$ and $\frac{5}{6}$? [CBI]
 (a) $\frac{1}{6}$ (b) $\frac{1}{12}$ (c) $\frac{1}{20}$ (d) $\frac{1}{30}$
58. Which of the following are in descending order of their value? [RRB]
 (a) $\frac{5}{9}, \frac{7}{11}, \frac{8}{15}, \frac{11}{17}$ (b) $\frac{5}{9}, \frac{8}{15}, \frac{11}{17}, \frac{7}{11}$
 (c) $\frac{11}{17}, \frac{7}{11}, \frac{8}{15}, \frac{5}{9}$ (d) $\frac{11}{17}, \frac{7}{11}, \frac{5}{9}, \frac{8}{15}$
59. $832.58 - 242.31 = 779.84 - ?$ [BSRB]
 (a) 179.57 (b) 199.57 (c) 295.05 (d) None of these
60. $138.009 + 341.981 - 146.305 = 123.6 + ?$ [Bank PO]
 (a) 120.085 (b) 120.85 (c) 220.085 (d) None of these
61. $12.1212 + 17.0005 - 9.1102 = ?$ [BSRB]
 (a) 20.0015 (b) 20.0105 (c) 20.0115 (d) 20.1015
62. $48.95 - 32.006 = ?$ [IBPS]
 (a) 16.089 (b) 16.35 (c) 16.89 (d) 16.944

63. $617 + 6.017 + 0.617 + 6.0017 = ?$ [MBA]
 (a) 6.2963 (b) 62.965 (c) 629.6357 (d) None of these
64. $34.95 + 240.016 + 23.98 = ?$ [Bank PO]
 (a) 298.0946 (b) 298.111 (c) 298.946 (d) 299.09
65. $3889 + 12.952 - ? = 3854.002$ [Bank PO]
 (a) 47.095 (b) 47.752 (c) 47.932 (d) 47.95
66. $337.62 + 8.591 + 34.4 = ?$ [SSC]
 (a) 370.611 (b) 380.511 (c) 380.611 (d) 426.97
67. The sum of first 20 terms of the series $\frac{1}{5 \times 6} + \frac{1}{6 \times 7} + \frac{1}{7 \times 8} + \dots$ is [Hotel Management]
 (a) 0.16 (b) 1.6 (c) 16 (d) None of these
68. When 52416 is divided by 312, the quotient is 168. What will be the quotient when 52.416 is divided by 0.0168? [Hotel Management]
 (a) 3.12 (b) 312 (c) 3120 (d) None of these
69. In which of the following fractions are written in ascending order? [NABARD]
 (a) $\frac{2}{3}, \frac{3}{5}, \frac{7}{9}, \frac{9}{11}, \frac{8}{9}$ (b) $\frac{3}{5}, \frac{2}{3}, \frac{9}{11}, \frac{7}{9}, \frac{8}{9}$ (c) $\frac{3}{5}, \frac{2}{3}, \frac{7}{9}, \frac{9}{11}, \frac{8}{9}$
 (d) $\frac{8}{9}, \frac{9}{11}, \frac{7}{9}, \frac{2}{3}, \frac{3}{5}$ (e) $\frac{8}{9}, \frac{9}{11}, \frac{7}{9}, \frac{3}{5}, \frac{2}{3}$
70. Find the value of the following expression upto four places of decimals

$$\left[1 + \frac{1}{1 \times 2} + \frac{1}{1 \times 2 \times 4} + \frac{1}{1 \times 2 \times 4 \times 8} + \frac{1}{1 \times 2 \times 4 \times 8 \times 16} \right]$$
 [Hotel Management]
 (a) 1.6414 (b) 1.6415 (c) 1.6416 (d) 1.6428
71. The denominator of a fraction is 3 more than the numerator. If the numerator as well as the denominator is increased by 4, the fraction becomes $\frac{4}{5}$. What was the original fraction? [SBI PO]
 (a) $\frac{8}{11}$ (b) $\frac{5}{8}$ (c) $\frac{10}{13}$ (d) $\frac{7}{10}$
72. The product of two fractions is $\frac{14}{15}$ and their quotient is $\frac{35}{24}$. The greater fraction is: [SSC]
 (a) $\frac{4}{5}$ (b) $\frac{7}{6}$ (c) $\frac{7}{4}$ (d) $\frac{7}{3}$
73. If the numerator of a fraction is increased by 2 and the denominator is increased by 3, the fraction becomes $\frac{7}{9}$ and if both the numerator and the denominator are decreased by 1, the fraction becomes $\frac{4}{5}$. What is the original fraction? [SBI PO]
 (a) $\frac{5}{6}$ (b) $\frac{9}{11}$ (c) $\frac{13}{16}$ (d) $\frac{17}{21}$

74. In a pair of fractions, fraction A is twice the fraction B and the product of two fractions is $\frac{2}{25}$. What is the value of fraction A ? [Bank PO]
- (a) $\frac{1}{5}$ (b) $\frac{1}{25}$ (c) $\frac{2}{5}$ (d) Data inadequate
75. The difference between the numerator and the denominator of a fraction is 5. If 5 is added to its denominator, the fraction is decreased by $1\frac{1}{4}$. Find the value of fraction [MBA]
- (a) $\frac{1}{6}$ (b) $2\frac{1}{4}$ (c) $3\frac{1}{4}$ (d) 6
76. If 1 is added to the denominator of a fraction, the fraction becomes $\frac{1}{2}$. If 1 is added to the numerator of the fraction, the fraction becomes 1. The fraction is: [CBI]
- (a) $\frac{1}{3}$ (b) $\frac{2}{3}$ (c) $\frac{3}{4}$ (d) $\frac{3}{2}$



1. (c) 2. (d) 3. (c) 4. (a) 5. (c) 6. (b) 7. (c) 8. (c) 9. (a) 10. (c)
 11. (a) 12. (a) 13. (c) 14. (e) 15. (a) 16. (e) 17. (e) 18. (c) 19. (c) 20. (b)
 21. (c) 22. (a) 23. (c) 24. (c) 25. (b) 26. (c) 27. (b) 28. (b) 29. (d) 30. (c)
 31. (c) 32. (d) 33. (c) 34. (d) 35. (c) 36. (a) 37. (b) 38. (d) 39. (c) 40. (c)
 41. (c) 42. (a) 43. (d) 44. (c) 45. (d) 46. (c) 47. (c) 48. (a) 49. (c) 50. (d)
 51. (a) 52. (b) 53. (c) 54. (b) 55. (b) 56. (a) 57. (a) 58. (d) 59. (d) 60. (d)
 61. (c) 62. (d) 63. (c) 64. (c) 65. (d) 66. (c) 67. (a) 68. (c) 69. (c) 70. (c)
 71. (a) 72. (b) 73. (a) 74. (c) 75. (b) 76. (b)



Solutions with Necessary Explanation

1. $\frac{1}{14}$

2. $57\frac{4}{7} + 29\frac{1}{21} + ? = 90\frac{3}{35}$

or
$$\begin{aligned} ? &= (90 - 57 - 29) + \frac{3}{35} - \frac{4}{7} - \frac{1}{21} \\ &= 4 + \frac{3 \times 3}{35 \times 3} - \frac{4 \times 15}{105} - \frac{5}{105} = 4 + \frac{9 - 65}{105} = 4 - \frac{56}{105} \\ &= 3\frac{49}{105} = 3\frac{7}{15} \end{aligned}$$

3. $? = 28\frac{4}{7} + 14\frac{1}{3} - 23\frac{3}{5} - 12\frac{5}{7}$

$$= (28 + 14 - 23 - 12) + \left(\frac{4}{7} + \frac{1}{3} - \frac{3}{5} - \frac{5}{7} \right)$$

$$= 7 - \frac{1}{7} + \frac{5-9}{15} = 7 - \frac{1}{7} - \frac{4}{15}$$

$$= 7 - \left(\frac{1 \times 15 + 4 \times 7}{105} \right) = 7 - \frac{43}{105} = 6 \frac{62}{105}$$

4. $7^5 \div 7^3 \div 7^2 \times 7^4 \times \frac{1}{7^3} = 7^?$

$$7^{5-3-2+4-3} = 7^{9-8} = 7^1$$

$$\therefore \text{Answer} = 1$$

5. $13 \frac{4}{?} + 5 \frac{3}{4} \text{ of } 3 \frac{1}{5} \div 4 \frac{3}{5} = 17 \frac{4}{5}$

$$\Rightarrow 13 \frac{4}{?} = 17 \frac{4}{5} - \left(5 \frac{3}{4} \text{ of } 3 \frac{1}{5} \div 4 \frac{3}{5} \right)$$

$$= 17 \frac{4}{5} - \left(\frac{23}{4} \times \frac{16}{5} \div \frac{23}{5} \right) = 17 \frac{4}{5} - \left(\frac{23}{4} \times \frac{16}{5} \times \frac{5}{23} \right) = 17 \frac{4}{5} - 4 = 13 \frac{4}{5}$$

$$\therefore ? = 5$$

6. $47 \frac{1}{17} \div 1 \frac{49}{51} + 23 \frac{5}{7} + ? = 67 \frac{4}{9} \quad \Rightarrow \quad 47 \frac{1}{17} \times \frac{1}{1 \frac{49}{51}} + 23 \frac{5}{7} + ? = 67 \frac{4}{9}$

$$\Rightarrow \frac{800}{17} \times \frac{1}{\frac{100}{51}} + 23 \frac{5}{7} + ? = 67 \frac{4}{9} \quad \Rightarrow \quad \frac{800}{17} \times \frac{51}{100} + \frac{166}{7} + ? = \frac{607}{9}$$

$$\Rightarrow ? = \frac{607}{9} - \frac{166}{7} - \frac{800}{17} \times \frac{51}{100} = \frac{607}{9} - \frac{166}{7} - 24 = \frac{607 \times 7 - 166 \times 9 - 24 \times 63}{63} = \frac{1243}{63} = 19 \frac{46}{63}$$

7. $22 \frac{2}{9} + 33 \frac{4}{7} - ? = 28 \frac{4}{45}$

$$\Rightarrow ? = 22 \frac{2}{9} + 33 \frac{4}{7} - 28 \frac{4}{45} = 22 + 33 - 28 + \frac{2}{9} + \frac{4}{7} - \frac{4}{45}$$

$$= 27 + \frac{2 \times 35 + 4 \times 45 - 4 \times 7}{315} = 27 + \frac{70 + 180 - 4 \times 7}{315}$$

$$= 27 + \frac{70 + 180 - 28}{315} = 27 + \frac{222}{315} = 27 + \frac{74}{105} = 27 \frac{74}{105}$$

8. $? = 1 + 1 + 1 + \frac{4}{7} + \frac{3}{5} + \frac{1}{3} = 3 + \left(\frac{60 + 63 + 35}{105} \right) = 3 + \frac{158}{105} = 3 + 1 + \frac{53}{105} = 4 \frac{53}{105}$

9. $? = 12 \frac{1}{3} + 10 \frac{5}{6} - 7 \frac{2}{3} - 1 \frac{4}{7}$

$$= (12 + 10 - 7 - 1) + \left(\frac{1}{3} + \frac{5}{6} - \frac{2}{3} - \frac{4}{7} \right) = 14 + \left(\frac{14 + 35 - 28 - 24}{42} \right) = 14 - \frac{3}{42}$$

$$= 14 - \frac{1}{14} = 13 \frac{13}{14}$$

$$10. 4\frac{3}{4} + 2\frac{1}{8} + 7\frac{1}{4} + 3\frac{7}{8} + 11\frac{12}{13}$$

$$= 4\frac{3}{4} + 7\frac{1}{4} + 2\frac{1}{8} + 3\frac{7}{8} + 11\frac{12}{13}$$

$$= (4 + 7 + 2 + 3 + 11) + \left(\frac{3}{4} + \frac{1}{4}\right) + \left(\frac{1}{8} + \frac{7}{8}\right) + \frac{12}{13}$$

$$= 27 + 1 + 1 + \frac{12}{13} = 29\frac{12}{13}$$

$$11. (7 + 13 + 5 - 16) + \left(\frac{11}{18} + \frac{1}{9} + \frac{7}{9} - \frac{2}{3}\right) = 9 + \left(\frac{11 + 16 - 12}{18}\right) = 9 + \frac{15}{18} = 9\frac{5}{6}$$

$$12. 1\frac{7}{9} + 3\frac{5}{8} - 2\frac{1}{18} + 4\frac{7}{16} - 9\frac{5}{18} + x = 0$$

$$\text{or } (1 + 3 - 2 + 4 - 9) + \frac{7}{9} + \frac{5}{8} + \frac{7}{16} - \frac{1}{18} - \frac{5}{18} + x = 0$$

$$-3 + \left(\frac{7}{9} - \frac{6}{18}\right) + \frac{5}{8} + \frac{7}{16} + x = 0$$

$$\Rightarrow -3 + \frac{14 - 6}{18} + \frac{10 + 7}{16} + x = 0 \quad \Rightarrow -3 + \frac{4}{9} + \frac{17}{16} + x = 0$$

$$\Rightarrow -3 + \frac{64 + 153}{144} + x = 0 \quad \Rightarrow -3 + \frac{217}{144} + x = 0$$

$$\Rightarrow -3 + 1 + \frac{73}{144} + x = 0 \quad \Rightarrow x = 2 - \frac{73}{144} = 1\frac{71}{144}$$

$$13. 425 \div 16.95 \times ? = 225$$

$$\text{or } \frac{425 \times ?}{17} = 225 \quad \text{or } ? = \frac{225 \times 17}{425} = 9$$

$$14. 198.995 \times 12.005 + 16.25 \times 6.95 = ?$$

$$\therefore ? \approx 199 \times 12 + 16 \times 7 \approx 2388 + 112$$

$$\text{or } ? = 2500$$

$$15. 1.2 \times 1.02 \times 1.002 = ?$$

$$1.2 \times 1.02 \times 1.002 = 1.226448$$

$$16. 6.4 - 1.6 \div 0.2 = 6.4 - 1.6 \times \frac{1}{0.2} = 6.4 - 8 = -1.6$$

$$\therefore \text{None of these}$$

$$17. ? = 8.88 \times 88.8 \times 88 = 69391.872$$

$$\therefore \text{None of these}$$

$$18. 989.001 + 1.00982 \times 76.792 = ?$$

$$\Rightarrow ? \approx 989 + 77 \approx 1066 \approx 1065$$

$$19. 3.75 + 2.832 - 1.001 + 1.803 = 7.384$$

$$20. \quad \frac{6}{11}, \frac{13}{17}$$

$$6 \times 17 < 11 \times 13$$

$$\therefore \frac{6}{11} < \frac{13}{17}$$

$$\text{Then } \frac{6}{11}, \frac{19}{27}$$

$$6 \times 27 < 11 \times 19$$

$$\therefore \frac{6}{11} < \frac{19}{27}$$

$$\frac{6}{11}, \frac{21}{23}$$

$$6 \times 23 < 11 \times 21$$

$$\therefore \frac{6}{11} < \frac{21}{23}$$

$$\frac{6}{11}, \frac{5}{7}$$

$$6 \times 7 < 11 \times 5$$

$$\therefore \frac{6}{11} < \frac{5}{7}$$

$$\therefore \text{Smallest fraction is } \frac{6}{11}$$

$$21. \quad \frac{2}{5} \times \frac{15}{21} \times \frac{7}{10} \times \frac{3}{8} = \frac{3}{40} < 1$$

To get whole a number, the number to be added

$$= 1 - \frac{3}{40} = \frac{37}{40}$$

$$22. \quad \text{The given expression is of the form } \frac{a^3 - b^3}{a^2 + ab + b^2} \text{ and is equal to } (a - b)$$

$$\text{Here } a = 0.96; \quad b = 0.1$$

$$\therefore a - b = 0.96 - 0.1 = 0.86$$

$$23. \quad \frac{(2.3)^3 - 0.027}{(2.3)^2 + 0.69 + 0.09} = \frac{(2.3)^3 - (0.3)^3}{(2.3)^2 + (2.3)(0.3) + (0.3)^2}$$

$$\therefore \text{Given expression} = \frac{a^3 - b^3}{a^2 + ab + b^2} = a - b$$

$$\text{Here } a = 2.3; \quad b = 0.3$$

$$\therefore a - b = 2.3 - 0.3 = 2$$

$$24. \quad \frac{0.2 \times 0.2 + 0.2 \times 0.02}{0.044} = \frac{0.04 + 0.004}{0.044} = \frac{0.044}{0.044} = 1$$

$$25. \frac{(2.39)^2 - (1.61)^2}{2.39 - 1.61} = \frac{a^2 - b^2}{a - b} = a + b$$

Here $a = 2.39$; $b = 1.61$

$$\therefore a + b = 2.39 + 1.61 = 4$$

$$26. \frac{(2.644)^2 - (2.356)^2}{0.288} = \frac{(2.644)^2 - (2.356)^2}{(2.644 - 2.356)} = 2.644 + 2.356 = 5$$

$$27. \frac{1.49 \times 14.9 - 0.51 \times 5.1}{14.9 - 5.1} = \frac{1.49 \times 1.49 \times 10 - 0.51 \times 0.51 \times 10}{10(1.49 - 0.51)} \\ = \frac{(1.49)^2 - (0.51)^2}{(1.49 - 0.51)} = 1.49 + 0.51 = 2$$

$$28. \frac{4.2 \times 4.2 - 1.9 \times 1.9}{2.3 \times 6.1} = \frac{(4.2)^2 - (1.9)^2}{(4.2 - 1.9)(4.2 + 1.9)} = 1$$

$$29. 7.5 \times 7.5 + 37.5 + 2.5 \times 2.5 = (7.5)^2 + 2(7.5)(2.5) + (2.5)^2 \\ = (7.5 + 2.5)^2 = 10^2 = 100$$

$$30. \text{ Given expression} = \frac{(0.051)^3 + (0.041)^3}{(0.051)^2 - (0.051)(0.041) + (0.041)^2}$$

is of the form $\frac{a^3 + b^3}{a^2 - ab + b^2} = a + b$

where $a = 0.051$ and $b = 0.041$

$$\therefore \text{ Given expression} = a + b = 0.051 + 0.041 = 0.092$$

$$31. \text{ Given expression} = \frac{(0.5)^3 + (0.3)^3}{(0.5)^2 + (0.3)^2 - (0.5)(0.3)} = 0.5 + 0.3 = 0.8$$

$$32. \text{ Given expression} = \frac{(2 \times 3.75)^3 + 1^3}{(7.5)^2 + 1^2 - (7.5)(1)} = \frac{(7.5)^3 + 1^3}{(7.5)^2 - (7.5)(1) + 1^2} = 7.5 + 1 = 8.5$$

$$33. \text{ Given expression} = \frac{10.3^3 + 1^3}{10.3^2 - (10.3)(1) + (1)^2} = 10.3 + 1 = 11.3$$

$$34. \text{ Given expression} = \frac{8 - 2.8}{1.3} = \frac{5.2}{1.3} = 4$$

$$35. \text{ Given expression} \approx \frac{3.2 \times 4126 \times 3.2}{64 \times 2835} \approx \frac{32 \times 4126 \times 32}{64 \times 2835 \times 100} \\ \approx \frac{16 \times 4126}{2835 \times 100} \approx \frac{66016}{2835 \times 100} = \frac{23.28}{100} \approx 0.23 \approx 0.2$$

$$36. \frac{0.0203 \times 2.92}{0.0073 \times 14.5 \times 0.7} = \frac{203 \times 292}{73 \times 145 \times 7} = \frac{4}{5} = 0.8$$

$$37. \frac{3.6 \times 0.48 \times 2.50}{0.12 \times 0.09 \times 0.5} = \frac{36 \times 48 \times 250}{12 \times 9 \times 5} = 4 \times 4 \times 50 = 800$$

$$\begin{aligned}
 38. \text{ Given expression} &= \frac{(0.1667)(0.8333)(0.3333)}{(0.2222)(0.6667)(0.1250)} \\
 &= \left(\frac{1}{6} \times \frac{5}{6} \times \frac{1}{3} \right) \div \left(\frac{2}{9} \times \frac{2}{3} \times \frac{1}{8} \right) = \frac{1}{6} \times \frac{5}{6} \times \frac{1}{3} \times \frac{9}{2} \times \frac{3}{2} \times 8 = \frac{5}{2} = \mathbf{2.5}
 \end{aligned}$$

$$\begin{aligned}
 39. \text{ Given } 1^3 + 2^3 + \dots + 9^3 &= 2025 \\
 \therefore (0.11)^3 + (0.22)^3 + \dots + (0.99)^3 \\
 &= (0.11)^3 \{1^3 + 2^3 + \dots + 9^3\} \\
 &= (0.11)^3 \times 2025 = 0.001331 \times 2025 = \mathbf{2.695}
 \end{aligned}$$

$$\begin{aligned}
 40. \text{ Given } \frac{1}{6.198} &= 0.16134 \\
 \therefore \frac{1}{0.0006198} &= \frac{10000}{6.198} = 10000 \times 0.16134 = \mathbf{1613.4}
 \end{aligned}$$

$$\begin{aligned}
 41. \text{ Given } \frac{2994}{14.5} &= 172 \\
 \therefore \frac{29.94}{1.45} &= \frac{2994}{100} \times \frac{10}{14.5} = \frac{172 \times 10}{100} = \mathbf{17.2}
 \end{aligned}$$

$$\begin{aligned}
 42. \quad 0.\overline{09} \times 7.\overline{3} &= ? \\
 0.\overline{09} &= \frac{9}{99} = \frac{1}{11}; \quad 7.\overline{3} = 7\frac{1}{3} = \frac{22}{3} \\
 \therefore 0.\overline{09} \times 7.\overline{3} &= \frac{1}{11} \times \frac{22}{3} = \frac{2}{3} = \mathbf{0.\overline{6}}
 \end{aligned}$$

$$\begin{aligned}
 43. \quad 3.\overline{87} - 2.\overline{59} &= ? \\
 3.\overline{87} - 2.\overline{59} &= \mathbf{1.\overline{28}}
 \end{aligned}$$

$$\begin{aligned}
 44. \quad 2.\overline{136} &= 2 + 0.\overline{136} \\
 &= 2 + \frac{136 - 1}{990} = 2 + \frac{3}{22} = 2\frac{\mathbf{3}}{\mathbf{22}}
 \end{aligned}$$

$$45. \quad 6.\overline{46} = 6 + \frac{46}{99} = \frac{\mathbf{640}}{\mathbf{99}}$$

$$\begin{aligned}
 46. \quad 0.125125\dots &= 0.\overline{125} \\
 0.\overline{125} &= \frac{\mathbf{125}}{\mathbf{999}}
 \end{aligned}$$

$$\begin{aligned}
 47. \quad 0.232323\dots &= 0.\overline{23} \\
 0.\overline{23} &= \frac{\mathbf{23}}{\mathbf{99}}
 \end{aligned}$$

$$\begin{aligned}
 48. \quad \frac{144}{0.144} &= \frac{14.4}{x} \\
 \text{or } x &= \frac{14.4 \times 0.144}{144} = \mathbf{0.0144}
 \end{aligned}$$

49. $?\ = \frac{0.009}{0.01} = \mathbf{0.9}$

50. $5.\overline{45} = 5 + 0.\overline{45}$

$$0.\overline{45} = \frac{45}{99}$$

$$\therefore \text{Given expression} = 5 + \frac{45}{99} = \frac{\mathbf{540}}{99}$$

51. 1. $\frac{368.39}{17} = \mathbf{21.67}$

2. $\frac{170.50}{62} = \mathbf{2.75}$

3. $\frac{875.65}{83} = \mathbf{10.55}$

$$21.67 > 10.55 > 2.75$$

Hence answer is (a) **1, 3, 2**

52. $0.04 \times 0.0162 = 4 \times 10^{-2} \times 162 \times 10^{-4}$
 $= 648 \times 10^{-6}$
 $= 6.48 \times 10^{-4}$

53. $40.83 \times 1.02 \times 1.2 = ?$

$$4083 \times 102 \times 12 = 4997592$$

$$\therefore 40.83 \times 1.02 \times 1.2 = \mathbf{49.97592}$$

54. $16.02 \times 0.001 = \mathbf{0.01602}$

55. $0.002 \times 0.5 = \mathbf{0.001}$

56. $\frac{1}{3} = 0.333...;$ $\frac{2}{5} = 0.4$

$$\frac{4}{7} = 0.57142...; \quad \frac{3}{5} = 0.6$$

$$\frac{5}{6} = 0.8333...; \quad \frac{6}{7} = 0.8571...$$

$$0.8571... > 0.8333... > 0.6 > 0.57142 > 0.4 > 0.333$$

$$\text{i.e. } \frac{6}{7} > \frac{5}{6} > \frac{3}{5} > \frac{4}{7} > \frac{2}{5} > \frac{1}{3}$$

57. $\frac{2}{3} = 0.666...;$ $\frac{3}{4} = 0.75$

$$\frac{4}{5} = 0.8; \quad \frac{5}{6} = 0.8333...$$

$$\text{Required difference} = 0.833... - 0.666...$$

$$= \frac{5}{6} - \frac{2}{3} = \frac{5-4}{6} = \frac{\mathbf{1}}{6}$$

58. $\frac{5}{9} = 0.555\dots$; $\frac{7}{11} = 0.6363\dots$

$\frac{8}{15} = 0.533\dots$; $\frac{11}{17} = 0.64705\dots$

$\therefore 0.647 > 0.63 > 0.55 > 0.533\dots$

i.e. $\frac{11}{17} > \frac{7}{11} > \frac{5}{9} > \frac{8}{15}$

59. $832.58 - 242.31 = 779.84 - ?$

$\therefore ? = 779.84 + 242.31 - 832.58$
 $= 779.84 - (832.58 - 242.31)$
 $= 779.84 - 590.27$
 $= \mathbf{189.57}$

60. $138.009 + 341.981 - 146.305 = 123.6 + ?$

$\therefore ? = 138.009 + 341.981 - 146.305 - 123.6$
 $= 138.009 + 341.981 - (146.305 + 123.6)$
 $= 138.009 + (341.981 - 269.905)$
 $= 138.009 + 72.076 = \mathbf{210.085}$

61. $12.1212 + 17.0005 - 9.1102$

$= 17.0005 + (12.1212 - 9.1102)$
 $= 17.0005 + 3.0110 = \mathbf{20.0115}$

62. $48.95 - 32.006 = \mathbf{16.944}$

63. $617 + 6.017 + 0.617 + 6.0017 = \mathbf{629.6357}$

$$\begin{array}{r} 617.0 \\ 6.017 \\ 0.617 \\ 6.0017 \\ \hline \mathbf{629.6357} \end{array}$$

64. $34.95 + 240.016 + 23.98 = \mathbf{298.946}$

$$\begin{array}{r} 34.95 \\ 240.016 \\ 23.98 \\ \hline \mathbf{298.946} \end{array}$$

65. $3889 + 12.952 - ? = 3854.002$

$\therefore ? = 3889 + 12.952 - 3854.002 = \mathbf{47.95}$

$$\begin{array}{r} 3889.0 \\ 12.952 \\ \hline 3901.952 \\ 3854.002 \\ \hline \mathbf{47.950} \end{array}$$

66. $337.62 + 8.591 + 34.4 = 380.611$

$$\begin{array}{r} 337.62 \\ 8.591 \\ 34.4 \\ \hline 380.611 \end{array}$$

67. $\frac{1}{5 \times 6} + \frac{1}{6 \times 7} + \frac{1}{7 \times 8} + \dots + \frac{1}{24 \times 25}$

$$= \frac{1}{5} - \frac{1}{6} + \frac{1}{6} - \frac{1}{7} + \frac{1}{7} - \frac{1}{8} + \dots + \frac{1}{24} - \frac{1}{25}$$

$$= \frac{1}{5} - \frac{1}{25} = \frac{5-1}{25} = \frac{4}{25} = \frac{16}{100} = 0.16$$

68. Given $\frac{52416}{312} = 168$

Let $\frac{52.416}{0.0168} = x$

or $x = \frac{52.416}{0.0168} = \frac{52416}{1000} \times \frac{10000}{168} = \frac{52416}{168} \times 10 = 312 \times 10 = 3120$

69. $\frac{2}{3} = 0.666\dots; \quad \frac{3}{5} = 0.6$

$\frac{7}{9} = 0.777\dots; \quad \frac{9}{11} = 0.8181\dots$

$\frac{8}{9} = 0.888\dots$

$\therefore 0.6 < 0.666\dots < 0.777\dots < 0.8181\dots < 0.888\dots$

i.e. $\frac{3}{5} < \frac{2}{3} < \frac{7}{9} < \frac{9}{11} < \frac{8}{9}$

70. $1 + \frac{1}{1 \times 2} + \frac{1}{1 \times 2 \times 4} + \frac{1}{1 \times 2 \times 4 \times 8} + \frac{1}{1 \times 2 \times 4 \times 8 \times 16}$

$$= 1 + 0.5 + 0.125 + \frac{16+1}{1 \times 2 \times 4 \times 8 \times 16}$$

$$= 1.625 + \frac{17}{1024} = 1.625 + 0.0166 = 1.6416$$

71. Let the numerator be a and denominator $= a + 3$

$\therefore \frac{a+4}{[(a+3)+4]} = \frac{4}{5}$

$\Rightarrow 5(a+4) = 4(a+7)$

$\Rightarrow a = 28 - 20 = 8$

$\therefore$ Required fraction $= \frac{8}{8+3} = \frac{8}{11}$

72. Let the two fractions be x and y .

$$\therefore xy = \frac{14}{15} \quad (1)$$

$$\frac{x}{y} = \frac{35}{24} \quad (2)$$

$$\frac{(1)}{(2)} \Rightarrow \frac{xy}{x/y} = \frac{14}{15} \div \frac{35}{24}$$

$$\begin{aligned} \Rightarrow y^2 &= \frac{14}{15} \times \frac{24}{35} \\ &= \frac{2}{5} \times \frac{8}{5} = \frac{16}{25} \end{aligned}$$

$$\therefore y = \frac{4}{5}$$

$$\therefore (1) \Rightarrow x = \frac{14}{15} \div y = \frac{14}{15} \times \frac{5}{4} = \frac{7}{6}$$

$\therefore$ Greater fraction is $\frac{7}{6}$

73. Let $\frac{x}{y}$ be the fraction.

$$\text{Now} \quad \frac{x+2}{y+3} = \frac{7}{9} \quad (1)$$

$$\frac{x-1}{y-1} = \frac{4}{5} \quad (2)$$

$$\begin{aligned} (1) \quad &\Rightarrow 9(x+2) = 7(y+3) \\ &\Rightarrow 9x - 7y = 21 - 18 = 3 \\ &9x - 7y = 3 \end{aligned} \quad (A)$$

$$\begin{aligned} (2) \quad &\Rightarrow 5(x-1) = 4(y-1) \\ &\Rightarrow 5x - 4y = 5 - 4 = 1 \\ &\Rightarrow 5x - 4y = 1 \end{aligned} \quad (B)$$

Solving (A) and (B), we get

$$4 \times (A) \Rightarrow 36x - 28y = 12 \quad (C)$$

$$7 \times (B) \Rightarrow 35x - 28y = 7 \quad (D)$$

$$(C) - (D) \Rightarrow x = 5$$

$$\therefore (B) \Rightarrow y = \frac{5x-1}{4} = \frac{25-1}{4} = 6$$

$\therefore$ Required fraction is $\frac{5}{6}$

74. Let the fractions B be x and A be $2x$.

$$\therefore x \times 2x = \frac{2}{25}$$

$$\Rightarrow 2x^2 = \frac{2}{25}$$

$$\Rightarrow x^2 = \frac{1}{25}$$

$$\Rightarrow x = \frac{1}{5}$$

$$\therefore \text{Required value of fraction } A = 2 \times \frac{1}{5} = \frac{2}{5}$$

75. Let $\frac{x}{y}$ be the fraction.

$$\text{Then} \quad x - y = 5 \quad (1)$$

$$\frac{x}{y+5} = \frac{x}{y} - \frac{5}{4} \quad (2)$$

$$\Rightarrow \frac{-x}{y+5} + \frac{x}{y} = \frac{5}{4}$$

$$\Rightarrow 4[x(y+5) - xy] = 5y(y+5) \quad (A)$$

Substitute value of y in (A) from (1), we get

$$4[x\{x\} - x(x-5)] = 5(x-5)x$$

$$\Rightarrow 4(x^2 - x^2 + 5x) = 5x^2 - 25x$$

$$\Rightarrow 20x + 25x = 5x^2$$

$$\Rightarrow 9x = x^2$$

$$\Rightarrow x = 9$$

$$\therefore (1) \Rightarrow y = x - 5 = 9 - 5 = 4$$

$$\therefore \text{Required fraction is } \frac{9}{4} = 2\frac{1}{4}$$

76. Let the fraction be $\frac{x}{y}$.

$$\text{Then} \quad \frac{x}{y+1} = \frac{1}{2} \quad (1)$$

$$\frac{x+1}{y} = 1 \quad (2)$$

$$(1) \Rightarrow 2x = y + 1$$

$$\Rightarrow 2x - y = 1 \quad (A)$$

$$(2) \Rightarrow x + 1 = y$$

$$\Rightarrow y - x = 1 \quad (B)$$

Solving (A) and (B), we get

$$(A) + (B) \Rightarrow x = 2$$

$$\therefore (B) \Rightarrow y = x + 1 = 2 + 1 = 3$$

$$\therefore \text{Required fraction} = \frac{2}{3}$$

Elementary Algebra I

ALGEBRAIC EXPRESSIONS: A number along with literal numbers and signs of fundamental operations is called an *algebraic expression*.

It may be **monomial** if it has only one term and **binominal** if it has two terms. It may be even polynomial if it has more than two terms. For example,

Monomial: $+2x, -3y$, etc.

Binomial: $x + 2y, 2x + 3y$, etc.

Polynomial: $2x + 3y + 4z, 3x - 2y + 5z$, etc.

ADDITION AND SUBTRACTION OF POLYNOMIALS: Here the sum or difference of coefficients of like terms is performed. For example, if $3x^2 + 4y + 5z$ and $2x^2 - 3y + 4z$ are two polynomials, then

$$\text{Sum of these two polynomials} = (3x^2 + 4y + 5z) + (2x^2 - 3y + 4z) = 5x^2 + y + 9z.$$

$$\text{Difference of these two polynomials} = (3x^2 + 4y + 5z) - (2x^2 - 3y + 4z) = x^2 + 7y + z.$$

EXAMPLE 1 Find the sum of $(4x^2 + 3y - 2z)$, $(3x^2 - 4y + 5z)$ and $(5x^2 + 6y - 7z)$.

Solution Sum of polynomials $= (4x^2 + 3y - 2z) + (3x^2 - 4y + 5z) + (5x^2 + 6y - 7z)$

$$= (4 + 3 + 5)x^2 + (3 - 4 + 6)y + (-2 + 5 - 7)z$$

$$= 12x^2 + 5y - 4z$$

EXAMPLE 2 If $f(x) = 2x^2 + 3y - 4z$, $g(x) = 5x^2 - 6y + 7z$ and $h(x) = 3x^2 - 8y + 5z$. Find $f(x) + g(x) + h(x)$.

Solution $f(x) + g(x) + h(x) = 2x^2 + 3y - 4z + 5x^2 - 6y + 7z + 3x^2 - 8y + 5z$

$$= 10x^2 - 11y + 8z$$

FACTORIZATION OF POLYNOMIALS

For factorization of polynomials, factor theorem and remainder theorem are useful.

➤ Factor Theorem: If $(x - a)$ is a factor of the polynomial $P(x)$, then the remainder obtained when $P(x)$ is divided by $(x - a)$ is given by $P(a) = 0$. For example, if $(x + 2)$ is a factor of $x^2 + 5x + 6$, then $P(-2) = 0$.

∴ $P(-2) = (-2)^2 + 5(-2) + 6 = 0$. Hence $(x + 2)$ is a factor of $x^2 + 5x + 6$.

EXAMPLE 3 Let $f(x) = x^3 - 12x^2 + 44x - 48$. Find out whether $(x - 2)$ and $(x - 3)$ are factors of $f(x)$.

Solution

(a) If $f(2) = 0$, then $(x - 2)$ is a factor of $f(x)$

$$\begin{aligned} f(2) &= (2)^3 - 12(2)^2 + 44(2) - 48 \\ &= 8 - 48 + 88 - 48 \\ &= 0 \end{aligned}$$

∴ **$(x - 2)$ is a factor of $f(x)$**

(b) If $f(3) = 0$, then $(x - 3)$ is a factor of $f(x)$

$$\begin{aligned} f(3) &= (3)^3 - 12(3)^2 + 44(3) - 48 \\ &= 27 - 108 + 132 - 48 \\ &= 3 \neq 0 \end{aligned}$$

∴ **$f(3) \neq 0$, $(x - 3)$ is not a factor of $f(x)$.**

EXAMPLE 4 Find out whether $(3x - 1)$ is a factor of $27x^3 - 9x^2 - 6x + 2$ by factor theorem.

Solution If $(3x - 1)$ is a factor of $f(x) = 27x^3 - 9x^2 - 6x + 2$, then $f\left(\frac{1}{3}\right) = 0$

$$\begin{aligned} f\left(\frac{1}{3}\right) &= 27\left(\frac{1}{3}\right)^3 - 9\left(\frac{1}{3}\right)^2 - 6\left(\frac{1}{3}\right) + 2 \\ &= 1 - 1 - 2 + 2 = 0 \end{aligned}$$

∴ **$(3x - 1)$ is a factor of $f(x)$.**

➤ Remainder Theorem: If $P(x)$ is a polynomial in x and if it is divided by $(x - a)$, then the remainder will be $P(a)$. For example, when $P(x) = 3x^2 - 2x + 2$ is divided by $(x - 2)$, then the remainder is given by $P(2)$.

$$\begin{aligned} \text{Here } P(2) &= 3(2)^2 - 2(2) + 2 \\ &= 3(4) - 4 + 2 \\ &= 12 - 4 + 2 = 10 \end{aligned}$$

EXAMPLE 5 Without using the division process, find the remainder when $x^3 + 4x^2 - 5x + 3$ is divided by $x + 3$.

Solution For $x + 3 = 0$, $x = -3$

$$\begin{aligned}\therefore \text{Remainder will be } P(-3) &= (-3)^3 + 4(-3)^2 - 5(-3) + 3 \\ &= -27 + 36 + 15 + 3 = 27\end{aligned}$$

EXAMPLE 6 Find the remainder when $3x^3 - 4x^2 + 5x - 2$ is divided by $(x + 3)$.

Solution Remainder will be $P(-3)$

$$\begin{aligned}\therefore P(-3) &= 3(-3)^3 - 4(-3)^2 + 5(-3) - 2 \\ &= -81 - 36 - 15 - 2 = -134\end{aligned}$$

EXAMPLE 7 If the expression $Px^3 + 4x^2 - 26$ and $2x^3 - 6x + P$ when divided by $(x - 3)$ leave the same remainder, find the value of P .

Solution

$$\begin{aligned}P_1(x) &= P(3)^3 + 4(3)^2 - 26 \\ &= 27P + 36 - 26 \\ &= 27P + 36 - 26 \\ &= 27P + 10\end{aligned}$$

$$\begin{aligned}P_2(x) &= 2(3)^2 - 6(3) + P \\ &= 54 - 18 + P \\ &= 36 + P\end{aligned}$$

$$\begin{aligned}\therefore P_1(x) &= P_2(x) \\ 27P + 10 &= 36 + P \\ 27P - P &= 36 - 10\end{aligned}$$

$$\Rightarrow 26P = 26$$

$$\therefore P = \frac{26}{26} = 1$$

EXAMPLE 8 Find the values of p and q when $px^3 + x^2 - 2x - q$ is exactly divisible by $(x - 1)$ and $(x + 1)$.

Solution When the given expression is exactly divisible by the given divisor, the remainder will be zero.

When it is divisible by $(x - 1)$,

$$\begin{aligned}\Rightarrow P(1) &= p(1)^3 + (1)^2 - 2(1) - q = 0 \\ \Rightarrow p + 1 - 2 - q &= 0 \\ \Rightarrow p - q &= 1\end{aligned}\tag{1}$$

When it is divisible by $(x + 1)$,

$$\begin{aligned}\text{i.e. } P(-1) &= p(-1)^3 + (-1)^2 - 2(-1) - q = 0 \\ \Rightarrow -p + 1 + 2 - q &= 0 \\ \Rightarrow p + q &= 3\end{aligned}\tag{2}$$

Solving (1) and (2), we have

$$p = 2, q = 1$$

**Conditions of Divisibility**

1. $x^n + a^n$ is exactly divisible by $(x + a)$ only when n is odd.
For example, $a^3 + b^3$ is exactly divisible by $(a + b)$.
2. $x^n + y^n = (x + y)(x^{n-1} - x^{n-2}y + x^{n-3} \cdot y^2 - \dots + y^{n-1})$
3. $x^n + a^n$ is not exactly divisible by $(x + a)$ when n is even.
For example, $a^4 + b^4$ is not exactly divisible by $(a + b)$.
4. $x^n + a^n$ is never divisible by $(x - a)$.
For example, $a^3 + b^3$ or $a^6 + b^6$ is not divisible by $(a - b)$.
5. $x^n - a^n$ is exactly divisible by $x + a$ when n is even.
For example, $x^4 - a^4$ is exactly divisible by $x + a$.
6. $x^n - y^n = (x + y)(x^{n-1} - x^{n-2} \cdot y + x^{n-3} \cdot y^2 - \dots - y^{n-1})$ when n is even.
7. $x^n - a^n$ is exactly divisible by $(x - a)$ whether n is odd or even.
For example, $a^7 - b^7$ and $a^8 - b^8$ are exactly divisible by $a - b$.
8. $x^n - y^n = (x - y)(x^{n-1} + x^{n-2} \cdot y + \dots + y^{n-1})$ when n is either odd or even.

The 5th power of any single digit number has the same right-hand digit as the number itself.

$x^3 - x$ is divisible by 3.

$x^5 - x$ is divisible by 5.

$x^7 - x$ is divisible by 7.

$x^{11} - x$ is divisible by 11

$x^{13} - x$ is divisible by 13.

In general, if n is prime, then for any whole number, $x^n - x$ is divisible by n .

If n is odd, $n(n^2 - 1)$ is divisible by 24.

If n is an odd prime greater than 3, $n^2 - 1$ is divisible by 24. For example, $n = 11$, $11^2 - 1 = 120 = 24 \times 5$, divisible by 24.

If n is odd, $2^n + 1$ is divisible by 3 and if n is even, $2^n - 1$ is divisible by 3.

If n is prime, then $n(n^4 - 1)$ is divisible by 30.

If n is odd, $2^{2n} + 1$ is divisible by 5 and if n is even, $2^{2n} - 1$ is divisible by 5.

If n is odd, $5^{2n} + 1$ is divisible by 13 and if n is even, $5^{2n} - 1$ is divisible by 13.

EXAMPLE 9 The expression $5^{2n} - 2^{3n}$ has a factor

- (a) 3 (b) 7 (c) 10 (d) 17 (e) None of these

Solution $5^{2n} - 2^{3n} = (5^2)^n - (2^3)^n = (25)^n - (8)^n$

The above expression has a factor $(25 - 8) = 17$

17 is a factor of $5^{2n} - 2^{3n}$. **Therefore, the answer is (d).**

EXAMPLE 10 The last digit in the expansion of $(41)^n - 1$ when n is any positive integer is

- (a) 2 (b) 1 (c) 0 (d) -1 (e) None of these

Solution The last digit in $(41)^n$ for any value of n is 1.

$\therefore$ The last digit in $(41)^n - 1$ is 0. **Hence the answer is (c).**

EXAMPLE 11 Find the last two digits of the expression of $2^{12n} - 6^{4n}$ when n is any positive integer.

- (a) 36 (b) 64 (c) 10 (d) 00 (e) None of these

Solution $(2^6)^{2n} - (6^2)^{2n} = (64)^{2n} - (36)^{2n}$

$\therefore (64 + 36)$ is a factor of $2^{12n} - 6^{4n}$. (by rule 5)

i.e., last two digits of the expression must be **00** as the number is a multiple of 100.

Hence answer is (d).

EXAMPLE 12 For all integral values of n , the expression $7^{2n} - 3^{3n}$ is a multiple of

- (a) 22 (b) 12 (c) 10 (d) 11 (e) 24

Solution $7^{2n} - 3^{3n} = (7^2)^n - (3^3)^n$

$$= (49)^n - (27)^n$$

$\therefore (49 - 27) = 22$ is a factor of the expression $7^{2n} - 3^{3n}$. (by rule 7)

Hence answer is (a).

EXAMPLE 13 What should be subtracted from $27x^3 - 9x^2 - 6x - 5$ to make it exactly divisible by $(3x - 1)$?

Solution $3x - 1 = 0 \Rightarrow x = 1/3$

$$\begin{aligned} \therefore \text{Remainder is } f\left(\frac{1}{3}\right) &= 27\left(\frac{1}{3}\right)^3 - 9\left(\frac{1}{3}\right)^2 - 6\left(\frac{1}{3}\right) - 5 \\ &= 1 - 1 - 2 - 5 \\ &= -7. \end{aligned}$$

If we reduce remainder = -7 from the given expression, the expression will be exactly divisible by the given divisor.

EXAMPLE 14 What should be subtracted from $x^2 + 7x + 16$ to make it exactly divisible by $(x + 2)$?

Solution If $P(x) = x^2 + 7x + 16$ is divisible by $(x + 2)$, $P(-2) = 0$

Here $P(-2) = (-2)^2 + 7(-2) + 16 = 6$

$\therefore 6$ is to be subtracted from the given polynomial.

➤ Theorem: For Zero of a Polynomial

Let $f(x) = a_0x^n + a_1x^{n-1} + \dots + a_n$ be a polynomial with integral coefficients. If an integer k is a zero polynomial then k is a factor of a_n . For this value of k , $f(k) = 0$. Consider the following example:

$$f(y) = y^3 - 2y^2 + y + 4$$

Suppose k is an integral zero of polynomial $f(y)$. Then by the theorem, k is a factor of a_n .

Hence possible value of k are 1, -1 , 2, -2 , 4 and -4 . Here we get $f(-1) = 0$ and $f(k)$ for all other values are non zero.

$\therefore -1$ is the zero of $f(y)$.

EXAMPLE 15 Find integral zeros of the polynomial $f(x) = x^3 - 2x^2 + x + 4$.

Solution Let m be an integral zero of the polynomial $f(x)$. Then by the theorem m is a factor of a_{n+1} , i.e., 4.

Hence possible values of m are 1, -1, 2, -2, 4 and -4.

Now,

$$\begin{aligned} f(1) &= (1)^3 - 2(1)^2 + (1) + 4 \\ &= 1 - 2 + 1 + 4 = 4 \neq 0 \end{aligned}$$

$$\begin{aligned} f(-1) &= (-1)^3 - 2(-1)^2 + (-1) + 4 \\ &= -1 - 2 - 1 + 4 = 0 \end{aligned}$$

$\therefore$ (-1) is the zero of $f(x)$.

Similarly, 2, -2, 4 and -4 are not the zeros of $f(x)$. Thus, the only integral zero of $f(x)$ is -1.

THEORY OF INDICES: The expression a^m means the product of m factors each equal to a . When m is a positive integer, it is called the *exponent* or *index* or the *power* of a .

- Any quantity raised to the power zero is always equal to 1.

For example, $a^0 = 1$ ($a \neq 0$), $1^0 = 1$, $(-1)^0 = 1$, $\left(\frac{1}{2}\right)^0 = 1$

- Whenever any index or power is taken from the numerator to denominator or from the denominator to numerator, the sign of the power changes. For example,

$$a^m = \frac{1}{a^{-m}}; \frac{1}{a^m} = a^{-m};$$

$$\frac{a^m}{a^n} = a^m \cdot a^{-n} = a^{m-n}$$

LAW OF INDICES

In algebra, root problems are solved by laws of indices. For example,

$$\sqrt[2]{a} = a^{1/2}, \sqrt[4]{a} = a^{1/4}, \sqrt[4]{a^3} = a^{3/4}$$

Laws of Indices:



1. $x^m \times x^n = x^{m+n}$

2. $(x^m)^n = x^{mn}$

3. $x^{m^n} = x^{(m^n)}$

4. $(xy)^m = x^m \cdot y^m$

5. $\frac{x^m}{x^n} = x^{m-n}$, if $m > n$

6. $\frac{x^m}{x^n} = \frac{1}{x^{n-m}}$, if $n > m$

HCF: The highest common factor of two or more algebraic expressions can be determined by the following methods:

- Factor method
- Division method

In case of factor method, the factors of the given expressions are found separately. Then the maximum number of common factors are taken out and multiplied, the product becomes the HCF.

For example, find HCF of $8(x^2 - 5x + 6)$ and $12(x^2 - 9)$ is obtained as:

$$8(x^2 - 5x + 6) = 4 \times 2 (x - 3) (x - 2)$$

$$12(x^2 - 9) = 4 \times 3 (x - 3) (x + 3)$$

$$\therefore \text{HCF} = 4(x - 3)$$

LCM: The minimum multiplied quantities out of two or more algebraic expressions are called the LCM.

If only two expressions are given, first find their HCF by factor method or by division method, and apply the following formula to find their LCM:

$$\text{LCM of two expressions} = \frac{\text{Product of two expressions}}{\text{Their HCF}}$$

For example, LCM of $8(x^2 - 5x + 6)$ and $12(x^2 - 9)$ is obtained as:

$$\begin{aligned} \text{LCM} &= \frac{8(x^2 - 5x + 6) \times 12(x^2 - 9)}{4(x - 3)} \\ &= 24(x - 2)(x + 3)(x - 3) \\ &= \mathbf{24(x^3 - 2x^2 - 9x + 18)} \end{aligned}$$



EXERCISES

- $$\frac{27^{3n+1} \cdot 81^{-n}}{9^{n+5} \cdot 3^{3n-7}}$$

(a) 0 (b) 1 (c) 2 (d) 3 (e) None of these
- If $x = 12$, $y = 4$; then find the value of $(x + y)^{x/y}$.

(a) 144 (b) 288 (c) 256 (d) 4096 (e) None of these
- If y varies as x and $y = 7$ when $x = 2$, find y when $x = 5$.

(a) 7 (b) 9 (c) 14 (d) 49 (e) None of these
- If $x + 2$ exactly divides $x^3 + 6x^2 + 11x + 6k$, find the value of k .

(a) 0 (b) 1 (c) -1 (d) 2 (e) None of these
- What must be added to $\frac{x}{y}$ to make it $\frac{y}{x}$?

(a) $\frac{y^2 + x^2}{xy}$ (b) $\frac{y^2 - x^2}{xy}$ (c) $\frac{xy}{x^2 + y^2}$ (d) $\frac{xy}{y^2 - x^2}$ (e) None of these
- What is the square root of $(x^2 + 4x + 4)(x^2 + 6x + 9)$?

(a) $x^2 + 4x + 4$ (b) $x^2 + 6x + 9$ (c) $x^2 + 5x + 6$ (d) $x^2 + 5x + 4$ (e) None of these
- If $3^{x-y} = 27$ and $3^{x+y} = 243$, find the value of x .

(a) 3 (b) 9 (c) 4 (d) 2 (e) None of these
- If $x = 9$, $y = \sqrt{17}$, the value of $(x^2 - y^2)^{-1/3}$ is equal to _____?

(a) 3 (b) 4 (c) 5 (d) 9 (e) None of these
- Find the value of $[x^{b+c}]^{b-c} \times [x^{c+a}]^{c-a} \times [x^{a+b}]^{a-b}$.

(a) 0 (b) -1 (c) 1 (d) 2 (e) None of these

10. Find the value of $\frac{3^{n+4} - 6 \times 3^{n+1}}{3^{n+2}}$.
 (a) 3 (b) 2 (c) 5 (d) 7 (e) None of these
11. Find the least number which when divided by 3, 5, 6 and 11 in succession the remainders got were 1, 2, 4 and 9 respectively.
 (a) 1688 (b) 6188 (c) 1866 (d) 6818 (e) None of these
12. Find the least number, which when divided by 11, 13, 17 and 19 in succession the remainders obtained were 7, 9, 14 and 15 respectively.
 (a) 87455 (b) 84575 (c) 85745 (d) 84755 (e) None of these
13. Find the least number, which when divided by 2, 5, 7 and 11 in succession, remainders obtained were 1, 4, 6 and 10 respectively.
 (a) 1358 (b) 5138 (c) 3158 (d) 1538 (e) None of these
14. Find the greatest 4 digit number, which when divided by 3, 7, 11 and 13 in succession, remainders obtained were 1, 5, 9 and 12 respectively.
 (a) 7959 (b) 9579 (c) 5979 (d) 9759 (e) None of these
15. Find the greatest 4 digit number, which when divided by 2, 7, 11 and 13 in succession remainders got were 1, 6, 10 and 12 respectively.
 (a) 6008 (b) 8060 (c) 8600 (d) 8006 (e) None of these
16. The values of a and b for which $3x^3 - ax^2 - 74x + b$ is a multiple of $x^2 + 2x - 24$ are:
 (a) $a = -5, b = 24$ (b) $a = 5, b = 24$ (c) $a = 13, b = 16$
 (d) $a = -13, b = 16$ (e) None of these
17. Find the values of a and b when $f(x) = 2x^3 + ax^2 - 11x + b$ is exactly divisible by $(x - 2)(x + 3)$.
 (a) $a = 3, b = 6$ (b) $a = 3, b = -6$ (c) $a = -3, b = 6$
 (d) $a = -3, b = -6$ (e) None of these
18. If $f(x) = 4x^3 - 2x^2 + 5x - 8$ is divided by $x - 2$, what will be the remainder?
 (a) 25 (b) 42 (c) 16 (d) 26 (e) None of these



1. (b) 2. (d) 3. (e) 4. (b) 5. (b) 6. (c) 7. (c) 8. (e) 9. (c) 10. (d)
 11. (c) 12. (d) 13. (d) 14. (e) 15. (d) 16. (a) 17. (b) 18. (d)



Solutions with Necessary Explanation

1. $\frac{27^{3n+1} \cdot 81^{-n}}{9^{n+5} \cdot 3^{3n-7}} = \frac{(3^3)^{3n+1} \cdot (3^4)^{-n}}{(3^2)^{n+5} \cdot 3^{3n-7}} = \frac{3^{9n+3} \cdot 3^{-4n}}{3^{2n+10} \cdot 3^{3n-7}} = 3^{9n+3-4n-(2n+10+3n-7)} = 3^0 = 1$
2. $x = 12, y = 4$
 $\therefore (x + y)^{x/y} = (12 + 4)^{12/4} = 16^3 = 4096$.
3. $y \propto x; \quad y = \frac{7}{2} \times 5 = \frac{35}{2}$
4. $P(x) = x^3 + 6x^2 + 11x + 6k$
 $\therefore \quad x + 2$ is a factor of $P(x), P(-2) = 0$.

$$\begin{aligned}\text{i.e. } P(-2) &= (-2)^3 + 6(-2)^2 + 11(-2) + 6k = 0 \\ \Rightarrow & -8 + 24 - 22 + 6k = 0 \\ \Rightarrow & 6k = 8 + 22 - 24 = 6\end{aligned}$$

$$\therefore k = \frac{6}{6} = 1$$

5. Let it be $f(x, y)$

$$\text{Then } f(x, y) + \frac{x}{y} = \frac{y}{x}$$

$$\therefore f(x, y) = \frac{y}{x} - \frac{x}{y} = \frac{y^2 - x^2}{xy}$$

$$6. \sqrt{(x^2 + 4x + 4)(x^2 + 6x + 9)} = \sqrt{(x+2)^2(x+3)^2} = (x+2)(x+3) = x^2 + 5x + 6$$

$$7. 3^{x-y} = 27 = 3^3 \Rightarrow x - y = 3 \quad (1)$$

$$3^{x+y} = 243 = 3^5 \Rightarrow x + y = 5 \quad (2)$$

Solving (1) and (2)

$$x = 4$$

$$8. x = 9, y = \sqrt{17}$$

$$\therefore (x^2 - y^2)^{-1/3} = (9^2 - 17)^{-1/3} = (81 - 17)^{-1/3} = (64)^{-1/3} = 4^{-1} = \frac{1}{4}.$$

$$\begin{aligned}9. [x^{b+c}]^{b-c} \cdot [x^{c+a}]^{c-a} \cdot [x^{a+b}]^{a-b} &= x^{b^2-c^2} \cdot x^{c^2-a^2} \cdot x^{a^2-b^2} \\ &= x^{(b^2-c^2)+(c^2-a^2)+(a^2-b^2)} = x^0 = 1\end{aligned}$$

$$\begin{aligned}10. \frac{3^{n+4} - 6 \times 3^{n+1}}{3^{n+2}} &= \frac{3^{n+4} - 2 \times 3^{n+2}}{3^{n+2}} \\ &= \frac{3^{n+2}(3^2 - 2)}{3^{n+2}} \\ &= 9 - 2 = 7\end{aligned}$$

11. Each time, on division remainders got were less than the divisor by 2.

Let the number be m .

$$m \div 3 = n \quad \text{remainder } 1$$

$$n \div 5 = p \quad \text{remainder } 2$$

$$p \div 6 = q \quad \text{remainder } 4$$

$$q \div 11 = r \quad \text{remainder } 9.$$

$$\Rightarrow q = 11r + 9 \Rightarrow p = 6q + 4$$

$$\text{i.e. } p = 6(11r + 9) + 4$$

$$\Rightarrow n = 5p + 2 \Rightarrow n = 5[6(11r + 9) + 4] + 2$$

$$m = 3n + 1 \Rightarrow m = 3 \times \{5[6(11r + 9) + 4] + 2\}$$

$$= 3 \times 5 \times 6 \times 11r + 3 \times 5 \times 6 \times 9 + 3 \times 5 \times 4 + 3 \times 2$$

$$= \text{Last quotient} \times (\text{Product of all divisors}) + \text{Last remainder}$$

$$\times (\text{Product of divisors except last one}) + \text{Last but one remainder}$$

$$\times (\text{Product of divisors except last two}) + (\text{Remainder before last but one})$$

$$\times (\text{Product of divisors except last three}) + \dots + \text{Second remainder} \times \text{First divisor}.$$

Required number can be obtained as shown below.

$\therefore$ Number is least, $\therefore$ last quotient = 1.

$$\begin{aligned}\therefore \text{ Required number} &= 1 \times 3 \times 5 \times 6 \times 11 + 9 \times 3 \times 5 \times 6 + 4 \times 3 \times 5 + 2 \times 3 \\ &= 990 + 810 + 60 + 6 = 1866\end{aligned}$$

12. Follow the above rule,

$$\begin{aligned}\text{Required number} &= 1 \times 11 \times 13 \times 17 \times 19 + 15 \times 11 \times 13 \times 17 + 14 \times 11 \times 13 + 9 \times 11 \\ &= 46189 + 36465 + 2002 + 99 = 84755\end{aligned}$$

$$\begin{aligned}13. \text{ Required number} &= 1 \times 2 \times 5 \times 7 \times 11 + 10 \times 2 \times 5 \times 7 + 6 \times 2 \times 5 + 4 \times 2 \\ &= 770 + 700 + 60 + 8 = 1538\end{aligned}$$

$$\begin{aligned}14. \text{ Least number} &= 1 \times 3 \times 7 \times 11 \times 13 + 12 \times 3 \times 7 \times 11 + 9 \times 3 \times 7 + 5 \times 3 + 1 \\ &= 3003 + 2772 + 189 + 15 = 5979 + 1 = 5980\end{aligned}$$

$$\text{Greatest number} = 5980 + (3 \times 7 \times 11 \times 13) = 8983$$

$$\begin{aligned}15. \text{ Least number} &= 1 \times 2 \times 7 \times 11 \times 13 + 12 \times 2 \times 7 \times 11 + 10 \times 2 \times 7 + 6 \times 2 \\ &= 2002 + 1848 + 140 + 12 = 4002\end{aligned}$$

$$\therefore \text{Greatest number of 4 digit} = 4002 + 2(2002) = 8006$$

$$16. \text{ Given } f(x) = 3x^3 - ax^2 - 74x + b$$

$$x^2 + 2x - 24 = (x + 6)(x - 4)$$

For $(x + 6)$ to be factor of $f(x)$, $f(-6) = 0$

$$f(-6) = 3(-6)^3 - a(-6)^2 - 74(-6) + b = -648 - 36a + 444 + b = 0$$

$$\Rightarrow -36a + b - 204 = 0 \quad (1)$$

For $(x - 4)$ to be factor of $f(x)$, $f(4) = 0$

$$f(4) = 3(4)^3 - a(4)^2 - 74(4) + b = 0$$

$$\Rightarrow 192 - 16a - 296 + b = 0$$

$$\Rightarrow -16a + b - 104 = 0 \quad (2)$$

$$(1) \Rightarrow b = 204 + 36a \quad (A)$$

$$(2) \Rightarrow b = 104 + 16a \quad (B)$$

Equating (A) and (B),

$$36a + 204 = 16a + 104$$

$$\Rightarrow 20a = -100$$

$$\Rightarrow a = -5$$

$$\therefore (B) \Rightarrow b = 104 + 16(-5)$$

$$= 104 - 80 = 24$$

$$\therefore a = -5, b = 24$$

$$17. \text{ Given } f(x) = 2x^3 + ax^2 - 11x + b$$

For $f(x)$ to be divisible by $(x - 2)(x + 3)$,

$$f(2) = 0 \text{ and } f(-3) = 0$$

$$f(2) = 2(2)^3 + a(2)^2 - 11(2) + b = 0$$

$$\Rightarrow 16 + 4a - 22 + b = 0$$

$$\text{i.e. } 4a + b - 6 = 0$$

$$f(-3) = 0 \Rightarrow 2(-3)^3 + a(-3)^2 - 11(-3) + b = 0 \quad (1)$$

$$\Rightarrow -54 + 9a + 33 + b = 0$$

$$\Rightarrow 9a + b - 21 = 0 \quad (2)$$

$$(1) \Rightarrow b = 6 - 4a \quad (A)$$

$$(2) \Rightarrow b = 21 - 9a \quad (B)$$

Equating (A) and (B),

$$6 - 4a = 21 - 9a$$

$$\therefore 9a - 4a = 21 - 6 = 15$$

$$\therefore 5a = 15$$

$$\therefore a = \frac{15}{5} = 3$$

$$\therefore (A) \Rightarrow b = 6 - 4 \times 3 = -6$$

$$\therefore a = 3, b = -6$$

18. Given $f(x) = 4x^3 - 2x^2 + 5x - 8$. If $f(x)$ is divided by $(x - 2)$, remainder will be $f(2)$.

$$\begin{aligned} \therefore \text{remainder} &= f(2) = 4(2)^3 - 2(2)^2 + 5(2) - 8 \\ &= 32 - 8 + 10 - 8 = 26 \end{aligned}$$

Elementary Algebra II

LINEAR EQUATIONS

Equations with One Variable: A statement of equality which contains only one unknown quantity or variable is called an equation. The graph of such equation is a straight line, whose abscissa x or ordinate y satisfies the given equation.

Root or solution: Any value of the variable that makes the statement of equation true is called the *root of the equation*.

EXAMPLE 1 Ashok's father is four times as old as him. Four years ago, his father was six times as old as he was then. Find their present ages.

Solution Let the present age of Ashok be x years, and the present age of his father be $4x$ years.

Four years ago, the age of father was $4x - 4$ and age of Ashok was $(x - 4)$.

$$\text{i.e.} \quad 4x - 4 = 6(x - 4)$$

$$\Rightarrow 4x - 4 = 6x - 24$$

$$\text{i.e.} \quad 2x = 20$$

$$\therefore x = 10 \text{ years}$$

$$\text{i.e.} \quad \text{Age of Ashok} = 10 \text{ years}$$

$$\text{Age of father} = 40 \text{ years}$$

QUADRATIC EQUATION: An expression of form $ax^2 + bx + c$, ($a \neq 0$) where a , b and c are real numbers is called a quadratic expression in x . Corresponding equation of expression is $ax^2 + bx + c = 0$. In other words, an equation in which the highest power of the variable is 2, is called a quadratic equation.

For example, the equation of the type $3x^2 + 6x + 9 = 0$ denotes a quadratic equation.

SOLVING QUADRATIC EQUATIONS: A quadratic equation can be solved either by factor method or by formula. When solved, it gives two values for the variable, which are called roots of the equation.

By Factor Method: In this method, find the factors of the given equation making the right-hand side equal to zero. Then by equating the factors to zero, we get values of the variable.

For example, the factors of equation $(x^2 - 5x + 6) = 0$ is obtained by the factor method. On factorizing, $(x - 2)(x - 3) = 0$

$$\text{i.e.} \quad x - 2 = 0 \quad \Rightarrow \quad x = 2$$

$$\text{and} \quad x - 3 = 0 \quad \Rightarrow \quad x = 3$$

$\therefore$ Solution is $x = 2$ and $x = 3$.

By Formula: $ax^2 + bx + c = 0$ is the standard form of a quadratic equation. The roots of the equation are given by the formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

The quantity $b^2 - 4ac$ is called the *discriminant (D)*. The two values of x obtained from a quadratic equation are called the *roots of the equation*.

If α and β are the roots of the equation, the sum of roots of the equation $\alpha + \beta = \frac{-b}{a}$

and product of roots $= \alpha\beta = \frac{c}{a}$.

For a quadratic equation $ax^2 + bx + c = 0$,

- (i) The roots will be equal if $b^2 - 4ac = 0$.
- (ii) The roots will be unequal and real if $b^2 - 4ac > 0$.
- (iii) The roots will be unequal and imaginary if $b^2 - 4ac < 0$.

FACTORIZATION OF QUADRATIC EQUATIONS: The quadratic polynomial $ax^2 + bx + c$ can be factorized only if there exists two numbers α and β such that

$$(i) \text{ Sum of roots } = \alpha + \beta = \frac{-b}{a}$$

$$(ii) \text{ Product of roots } = \alpha\beta = \frac{c}{a}$$

EXAMPLE 2 Factorize $2x^2 + 11x + 5$.

Solution Here $a = 2$; $b = 11$; $c = 5$

$$\therefore \quad \text{Sum of roots } = \alpha + \beta = \frac{-b}{a} = \frac{-11}{2}$$

$$\text{Product of roots } = \alpha\beta = \frac{c}{a} = \frac{5}{2}$$

$$2(\alpha + \beta) = -11 \quad (1)$$

$$2\alpha \times 2\beta = 10 \quad (2)$$

$$\therefore 2\alpha = -1 \text{ and } 2\beta = -10$$

$$\text{or } \alpha = \frac{-1}{2} \text{ and } \beta = -5$$

$$\text{i.e. } 2x^2 + 11x + 5 = \left(x + \frac{1}{2}\right)(x + 5) \times 2 = (2x + 1)(x + 5)$$

CONDITIONS FOR FACTORIZATION OF A QUADRATIC EQUATION: All quadratic equations cannot be factorized. For that, following conditions are to be tested:

- (i) If $b^2 - 4ac > 0$, then the quadratic equation can be factorized.
- (ii) If $b^2 - 4ac < 0$, then the quadratic equation cannot be factorized.

Some important formulae that are used in basic operations and in finding the factors of an expression are given below:



Important Formulae

1. $a^2 - b^2 = (a + b)(a - b)$
2. $(a + b)^2 = a^2 + 2ab + b^2$
3. $(a - b)^2 = a^2 - 2ab + b^2$
4. $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$
5. $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$
6. $a^3 + b^3 = (a + b)^3 - 3ab(a + b)$
7. $a^3 - b^3 = (a - b)^3 + 3ab(a + b)$
8. $(a + b)^2 = (a - b)^2 + 4ab$
9. $(a - b)^2 = (a + b)^2 - 4ab$
10. $a^3 + b^3 + c^3 - 3abc = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$

$$= \frac{1}{2}(a + b + c)[(a - b)^2 + (b - c)^2 + (c - a)^2]$$
11. $(a + b + c)^3 = a^3 + b^3 + c^3 + 3(b + c)(c + a)(a + b)$
12. $a^2 + b^2 = (a + b)^2 - 2ab$
13. $a^2 + b^2 = (a - b)^2 + 2ab$
14. If $a + b + c = 0$ then the value of $a^3 + b^3 + c^3$ is $3abc$
15. $x^2 + (a + b)x + ab = (x + a)(x + b)$
16. $a^2(b + c) + b^2(c + a) + c^2(a + b) + 3abc = (a + b + c)(ab + bc + ca)$
17. $ab(a + b) + bc(b + c) + ca(c + a) + 2abc = (a + b)(b + c)(c + a)$
18. $a^2(b - c) + b^2(c - a) + c^2(a - b) = -(a - b)(b - c)(c - a)$
19. $(a + b + c)^2 = a^2 + b^2 + c^2 + 2(ab + bc + ca)$

Conditions for Maximum and Minimum Values of Quadratic Expression

$ax^2 + bx + c$ (= y suppose)

For minimum value of y , $a > 0$.

$$\text{Minimum value of } y = \frac{-D}{4a} \quad \left(\text{which is possible when } x = \frac{-b}{2a} \right)$$

For maximum value of y , $a < 0$

$$\text{Maximum value of } y = \frac{-D}{4a} \quad \left(\text{which is possible when } x = \frac{b}{2a} \right)$$

EXAMPLE 3 Find the minimum value of $2x^2 - 5x - 3$ and also find x .

Solution Minimum value of the expression $= \frac{-D}{4a} = \frac{-49}{4 \times 2} = -6.125$

$$\therefore x = \frac{-b}{2a} = \frac{-(-5)}{2 \times 2} = \frac{5}{4} = 1.25$$

EXAMPLE 4 A certain number of tennis balls were purchased for ₹ 450. Five more balls could have been purchased for the same amount if each ball was cheaper by ₹ 15. Find the number of balls purchased.

Solution Let the number of balls purchased be x .

$$\therefore \text{Cost price of a ball} = \frac{450}{x} = \frac{450}{x+5} + 15$$

$$\Rightarrow x = 10 \quad \text{or} \quad -15$$

$$\text{Cost price of a ball} = \frac{450}{10} = 45$$

$$\therefore \text{Number of balls purchased} = 10$$

FORMULATING A QUADRATIC EQUATION FROM GIVEN ROOTS: Whenever the roots of the quadratic equations are given, the quadratic equation is given by

$$x^2 - (\text{Sum of roots})x + \text{Products of roots}$$

For example, if roots are given as 2 and 3, then the quadratic equation will be

$$x^2 - (3 + 2)x + 3 \times 2 = 0$$

$$\text{i.e.} \quad x^2 - 5x + 6 = 0$$



Notes:

1. A quadratic equation $ax^2 + bx + c = 0$ will have reciprocal roots, if $a = c$.
2. When a quadratic equation $ax^2 + bx + c = 0$ has one root equal to zero, then $c = 0$.
3. When the roots of the quadratic equation $ax^2 + bx + c = 0$ are negative reciprocals of each other, then $c = a$.
4. When both the roots are equal to zero, $b = 0$ and $c = 0$.
5. When one root is infinite, then $a = 0$ and when both the roots are infinite, then $a = 0$ and $b = 0$.
6. When the roots are equal in magnitude but opposite in sign, then $b = 0$.
7. If two quadratic equations $ax^2 + bx + c = 0$ and $a_1x^2 + b_1x + c_1 = 0$ have a common root (i.e. one root common), then $(bc_1 - b_1c)(ab_1 - a_1b) = (ca_1 - c_1a)^2$.
8. If they have both the roots common, then $\frac{a}{a_1} = \frac{b}{b_1} = \frac{c}{c_1}$.
9. The square root of any negative number is an imaginary number.

For example, $\left(\sqrt{-3}, \sqrt{-5}, \sqrt{-25}, \sqrt{-0.04}, \sqrt{-3^4}, \sqrt{-\frac{17}{15}} \text{ etc.} \right)$

For problem solving purposes, take $\sqrt{-1} = i$

So $\sqrt{-25} = \sqrt{(-1) \times 25} = \sqrt{-1} \times \sqrt{25} = 5i$, which is an imaginary number.

EXAMPLE 5 Find the discriminant of the following quadratic equations. Also find the nature of roots of the equation. Verify them.

- (i) $x^2 - 5x + 4 = 0$
- (ii) $3x^2 + 4x - 3 = 0$
- (iii) $x^2 + 5x + 6 = 0$
- (iv) $-5x^2 + 7x = 0$
- (v) $2x^2 - 6x + 7 = 0$.

Solution

- (i) In $x^2 - 5x + 4 = 0$, $a = 1$, $b = -5$, $c = 4$.
 $b^2 - 4ac = (-5)^2 - 4 \times 1 \times 4 = 25 - 16 = 9 > 0$ and a perfect square.
 $\therefore$ Roots will be distinct and rational numbers.
 Here roots are:

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{5 \pm \sqrt{9}}{2 \times 1} = \frac{5 \pm 3}{2} = \frac{5+3}{2} \text{ or } \frac{5-3}{2}$$

$$= 4 \text{ or } 1$$

- (ii) In $3x^2 + 4x - 3 = 0$, $a = 3$, $b = 4$, $c = -3$
 $\therefore b^2 - 4ac = (4)^2 - 4 \times 3 \times (-3) = 16 + 36 = 52 > 0$ and not a perfect square.
 $\therefore$ Roots will be distinct and irrational numbers.

$$\begin{aligned} \text{Roots are } \frac{-4 \pm \sqrt{52}}{2 \times 3} &= \frac{-4 \pm 2\sqrt{13}}{6} = \frac{-4 + 2\sqrt{13}}{6} \text{ or } \frac{-4 - 2\sqrt{13}}{6} \\ &= \frac{-2 + \sqrt{13}}{3} \text{ or } \frac{-2 - \sqrt{13}}{3} \end{aligned}$$

- (iii) In $x^2 + 5x + 6 = 0$, $a = 1$, $b = 5$, $c = 6$
 $\therefore b^2 - 4ac = (5)^2 - 4 \times 1 \times 6 = 25 - 24 = 1 > 0$, a perfect square
 $\therefore$ Roots will be distinct and rational numbers.

$$\begin{aligned} \therefore \text{Roots are } \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} &= \frac{-5 \pm \sqrt{1}}{2 \times 1} = \frac{-5 \pm 1}{2} \\ &= \frac{-5+1}{2} \text{ or } \frac{-5-1}{2} \\ &= -2 \text{ or } -3 \end{aligned}$$

- (iv) In $-5x^2 + 7x = 0$, $a = -5$, $b = 7$, $c = 0$
 $\therefore b^2 - 4ac = 7^2 - 4(-5)(0) = 7^2 = 49 > 0$, a perfect square.
 $\therefore$ Roots will be distinct and rational numbers.

$$\begin{aligned} \text{Here roots are } \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} &= \frac{-7 \pm \sqrt{49}}{2 \times (-5)} = \frac{-7 \pm 7}{-10} \\ &= \frac{-7+7}{-10} \text{ or } \frac{-7-7}{-10} \\ &= 0 \text{ or } \frac{7}{5} \end{aligned}$$

- (v) In $2x^2 - 6x + 7 = 0$, $a = 2$, $b = -6$, $c = 7$
 $\therefore b^2 - 4ac = (-6)^2 - 4(2)(7) = 36 - 56 = -20 < 0$
 $\therefore$ Roots will be imaginary.

EXAMPLE 6 Form a quadratic equation whose roots are (i) 3 and -5 (ii) 2 and 7; also verify them.

Solution

- (i) The quadratic equation will be

$$x^2 - (3 + (-5))x + 3 \times (-5) = 0$$

i.e., $x^2 + 2x - 15 = 0$

- (ii) The quadratic equation will be

$$x^2 - (2 + 7)x + 2 \times 7 = 0$$

i.e., $x^2 - 9x + 14 = 0$

EXAMPLE 7

- (i) If one of the roots of the equation $x^2 - 19x + 88 = 0$ is 8, find the other root.
(ii) If one of the roots of the equation $4x^2 - 27x + 18 = 0$ is 6, find the other root.

Solution

- (i) Here the product of roots $\frac{c}{a} = \frac{88}{1} = 88$

Given one of the roots = 8

$$\therefore \text{Second root} = \frac{88}{8} = 11$$

or

$$\text{Sum of the roots } \frac{-b}{a} = \frac{19}{1} = 19$$

$$\therefore \text{One root is 8, second root will be } 19 - 8 = 11$$

- (ii) Here the product of roots $\frac{c}{a} = \frac{18}{4} = \frac{9}{2}$

Given one of the roots = 6

$$\therefore \text{Second root} = \frac{9/2}{6} = \frac{3}{4}$$

or

$$\text{Sum of the roots} = \frac{-b}{a} = \frac{27}{4}$$

$$\therefore \text{One of the roots} = 6,$$

$$\text{Second root will be } \frac{27}{4} - 6 = \frac{3}{4}$$

EXAMPLE 8 Find the roots of the equations (i) $2x^2 + 3x - 5 = 0$ (ii) $x^2 - 8x + 7 = 0$.



Note: Whenever we get $a + b + c = 0$, one of the roots will always be 1.

Solution

- (i) Here $a + b + c = 2 + 3 + (-5) = 0$
i.e. one of the roots is 1

$$\text{Here the sum of the roots} = \frac{-b}{a} = \frac{-3}{2}$$

$$\therefore \text{Other root} = \frac{-3}{2} - 1 = \frac{-5}{2}$$

$$\text{So the required roots are 1 and } \frac{-5}{2}$$

- (ii) Here $a + b + c = 1 + (-8) + 7 = 0$
i.e. one of the roots is 1.

$$\text{Here, the sum of the roots} = \frac{-b}{a} = \frac{8}{1} = 8$$

$$\therefore \text{The other root} = 8 - 1 = 7$$

$$\text{So the required roots are 1 and 7.}$$

EXAMPLE 9 A motorcycle travels 20 km an hour faster than a cycle over a journey of 600 km. The cycle takes 15 hours more than the motorcycle. Find their speed.

Solution Let the speed of cycle be x km/h

$$\therefore \text{Speed of motorcycle} = (20 + x) \text{ km/h}$$

$$\text{Time taken by the cycle} = \frac{\text{Distance}}{\text{Speed}} = \frac{600}{x}$$

$$\text{Time taken by motorcycle} = \frac{600}{x + 20} = \frac{600}{x} - 15$$

$$\text{i.e.} \quad \frac{600}{x + 20} = \frac{600 - 15x}{x}$$

$$\text{i.e.} \quad \frac{40}{x + 20} = \frac{40 - x}{x}$$

$$\Rightarrow 40x = (40 - x)(x + 20)$$

$$\Rightarrow 40x = 40x + 800 - x^2 - 20x$$

$$\Rightarrow x^2 + 20x - 800 = 0$$

$$x = \frac{-20 \pm \sqrt{(20)^2 - 4 \times 1 \times -800}}{2 \times 1}$$

$$= \frac{-20 \pm \sqrt{400 + 4 \times 800}}{2}$$

$$\begin{aligned}
 &= \frac{-20 \pm 60}{2} = \frac{-20 + 60}{2} \text{ or } \frac{-20 - 60}{2} \\
 &= \frac{40}{2} \text{ or } \frac{-80}{2} \\
 &= \mathbf{20 \text{ or } -40}
 \end{aligned}$$

$\therefore x = 20 \text{ km/h}$ and $x + 20 = 40 \text{ km/h}$

$\therefore$ Speed of cycle = **20 km/h**

Speed of motorcycle = $x + 20 = \mathbf{40 \text{ km/h}}$.

EXAMPLE 10 Solve the equation $3^{2x+1} - 3^x = 3^{x+3} - 3^2$.

Solution

$$3^{2x+1} - 3^x = 3^{x+3} - 3^2$$

$$\Rightarrow 3^{2x} \times 3^1 - 3^x = 3^x \times 3^3 - 3^2$$

$$\text{i.e. } 3 \times (3^x)^2 - 3^x = 3^3 \times 3^x - 3^2$$

$$\text{Put } 3^x = k$$

$$\text{Now } 3k^2 - k = 27k - 9$$

$$\text{i.e. } 3k^2 - 28k + 9 = 0$$

$$\begin{aligned}
 \therefore k &= \frac{28 \pm \sqrt{(-28)^2 - 4(3)(9)}}{2 \times 3} \\
 &= \frac{28 \pm \sqrt{(28)^2 - 108}}{6} = \frac{28 \pm \sqrt{676}}{6} \\
 &= \frac{28 \pm 26}{6} = \frac{28 + 26}{6} \text{ or } \frac{28 - 26}{6} \\
 &= \mathbf{9 \text{ or } \frac{1}{3}}
 \end{aligned}$$

When $3^x = 9$, $x = 2$

$$3^x = \frac{1}{3}, 3^x = 3^{-1} \text{ or } x = -1$$

$\therefore$ Roots are **2 and -1**.

EXAMPLE 11 For the equation $-9x^2 + 12x - m = 0$ be a perfect square of a linear expression, what will be the value of m ?

Solution A linear expression is of the form $ax + b = 0$, $a \neq 0$, a and b are constant. A quadratic equation whose roots are α and β is given by $(x - \alpha)(x - \beta) = 0$.

If $\alpha = \beta$, then the equation becomes $(x - \alpha)^2 = 0$. For both the roots to be equal, we have $D = 0$.

$\therefore -9x^2 + 12x - m = 0$ can be a perfect square of a linear expression if discriminant = 0

$$\text{or } b^2 - 4ac = (12)^2 - 4(-9)(-m) = 0$$

$$\Rightarrow 144 - 36m = 0$$

$$\Rightarrow m = \frac{144}{36} = \mathbf{4}$$

(Examples 12–16) In each question one or more equation(s) is /are provided. On the basis of these you have to find out the relation between p and q .

Give answer (1) if $p = q$

Give answer (2) if $p > q$

Give answer (3) if $q > p$

Give answer (4) if $p \geq q$ and

Give answer (5) if $q \geq p$

12. I. $2p + \frac{5}{2} = p + 3$

II. $q - \frac{5}{2} = 1$

13. I. $\frac{p}{2} - \frac{p}{3} = 1$

II. $q^2 + 36 = 12q$

14. I. $\frac{p}{5} - \frac{2}{7} = 0$

II. $q^2 = 2q - 1$

15. I. $p^2 + 3 = 12$

II. $3q - 5 = 1 + q$

16. I. $7p^2 - 8p + 1 = 0$

II. $\frac{5q}{2} - \frac{q}{4} = \frac{1}{8}$

Solution

12. I. $2p - p = 3 - \frac{5}{2} \Rightarrow p = \frac{1}{2}$

II. $q = 1 + \frac{5}{2} = 3.5$ Therefore $q > p$.

$\therefore$ Answer (3)

13. I. $\frac{p}{2} - \frac{p}{3} = 1 \Rightarrow \frac{p}{6} = 1 \therefore p = 6$

II. $q^2 - 12q + 36 = 0 \Rightarrow (q - 6)^2 = 0$
 $\therefore q = 6$ Therefore $p = q$.

$\therefore$ Answer (4)

14. I. $\frac{p}{5} - \frac{2}{7} = 0 \Rightarrow p = \frac{5 \times 2}{7} = \frac{10}{7}$

II. $q^2 - 2q + 1 = 0 \Rightarrow (q - 1)^2 = 0 \Rightarrow q = 1$
 Therefore $p > q$.

$\therefore$ Answer (2)

15. I. $p^2 = 12 - 3 = 9 \therefore p = \pm 3$

II. $3q - 5 = 1 + q$
 $2q = 6 \therefore q = 3$

Therefore $q \geq p$.

$\therefore$ Answer (5)

$$\begin{aligned}
 16. \quad \text{I. } p &= \frac{8 \pm \sqrt{8^2 - 4(7)(1)}}{2 \times 7} \\
 &= \frac{8 \pm \sqrt{64 - 28}}{2 \times 7} = \frac{8 \pm 6}{2 \times 7} \\
 &= \frac{8+6}{14} \quad \text{or} \quad \frac{8-6}{14} \\
 &= 1 \quad \text{or} \quad \frac{1}{7}
 \end{aligned}$$

$$\begin{aligned}
 \text{II. } \frac{5q}{2} - \frac{q}{4} &= \frac{1}{8} \\
 \Rightarrow \frac{9q}{4} &= \frac{1}{8} \Rightarrow q = \frac{1}{18} \quad \therefore p > q
 \end{aligned}$$

$\therefore$ Answer (2)

(Examples 17–21) In each question one or more equation(s) is/are provided. On the basis of these, you have to find out relation between p and q .

Give answer (1) if $p = q$

(2) if $p > q$

(3) if $q > p$

(4) if $p \geq q$ and

(5) if $q \geq p$

$$17. \quad \text{(i)} \quad \frac{5}{28} \times \frac{9}{8} p = \frac{15}{14} \times \frac{13}{16} q$$

$$18. \quad \text{(i)} \quad p - 7 = 0 \quad \text{(ii)} \quad 3q^2 - 10q + 7 = 0$$

$$19. \quad \text{(i)} \quad 4p^2 = 16 \quad \text{(ii)} \quad q^2 - 10q + 25 = 0$$

$$20. \quad \text{(i)} \quad 4p^2 - 5p + 1 = 0 \quad \text{(ii)} \quad q^2 - 2q + 1 = 0$$

$$21. \quad \text{(i)} \quad q^2 - 11q + 30 = 0 \quad \text{(ii)} \quad 2p^2 - 7p + 6 = 0$$

Solution

$$17. \quad \text{(i)} \quad \frac{5}{28} \times \frac{9}{8} p = \frac{15}{14} \times \frac{13}{16} q$$

$$\Rightarrow 3p = 13q \quad \text{or} \quad \frac{p}{q} = \frac{13}{3}$$

$\therefore p > q$. The answer is (2).

$$18. \quad \text{(i)} \quad p - 7 = 0 \Rightarrow p = 7; \quad \text{(ii)} \quad 3q^2 - 10q + 7 = 0$$

$$\Rightarrow 3q(q - 1) - 7q + 7 = 0 \Rightarrow (3q - 7)(q - 1) = 0$$

$$\Rightarrow q = \frac{7}{3} \quad \text{or} \quad 1$$

$\therefore p > q$.

The answer is (2).

$$19. (i) 4p^2 = 16 \Rightarrow p = \pm 2 ; (ii) q^2 - 10q + 25 = 0 \Rightarrow (q - 5)^2 = 0 \Rightarrow q = 5.$$

$$\therefore q > p.$$

The answer is (3).

$$20. (i) 4p^2 - 5p + 1 = 0$$

$$\begin{aligned} \Rightarrow p &= \frac{5 \pm \sqrt{5^2 - 4 \times 4 \times 1}}{2 \times 4} \\ &= \frac{5 \pm \sqrt{25 - 16}}{8} \\ &= \frac{5 \pm 3}{8} = \frac{5 + 3}{8} \quad \text{or} \quad \frac{5 - 3}{8} \\ &= 1 \quad \text{or} \quad \frac{1}{4} \end{aligned}$$

$$(ii) q^2 - 2q + 1 = 0 \Rightarrow (q - 1)^2 = 0 \Rightarrow q = 1$$

$$\therefore q \geq p.$$

The answer is (5).

$$21. (i) q^2 - 11q + 30 = 0 \Rightarrow (q - 6)(q - 5) = 0 \Rightarrow q = 6 \quad \text{or} \quad 5$$

$$(ii) 2p^2 - 7p + 6 = 0$$

$$\begin{aligned} \Rightarrow p &= \frac{7 \pm \sqrt{7^2 - 4 \times 2 \times 6}}{2 \times 2} = \frac{7 \pm \sqrt{49 - 48}}{4} \\ &= \frac{7 \pm 1}{4} = \frac{7 + 1}{4} \quad \text{or} \quad \frac{7 - 1}{4} = 2 \quad \text{or} \quad \frac{3}{2} \end{aligned}$$

$$\therefore q > p.$$

The answer is (3).

Conditions for the maximum and the minimum values of the quadratic expression $ax^2 + bx + c$ ($= y$, suppose).

For the minimum value of y , the condition is $a > 0$ (if $a < 0$, y has no minimum value).

The minimum value of $y = \frac{-D}{4a}$ which is possible when $x = \frac{b}{2a}$.

EXAMPLE 22 If x be real, find the maximum value of $-2x^2 + x + 3$ and also find the corresponding value of x .

Solution The corresponding equation is $-2x^2 + x + 3 = 0$.

$$D = (1)^2 - 4(-2)(3) = 25.$$

$$\therefore \text{The required maximum value of the given expression} = \frac{-D}{4a} = \frac{-25}{4 \times (-2)} = \frac{25}{8} \quad \text{and}$$

$$\text{corresponding value of } x = \frac{-b}{2a} = \frac{-1}{2(-2)} = \frac{1}{4}$$

Explanation:

$$-2x^2 + x + 3 = y \quad (1)$$

$$\text{or} \quad -2x^2 + x + (3 - y) = 0$$

$$\begin{aligned}\therefore D &= (+1)^2 - 4(-2)(3 - y) \\ &= 1 + 8(3 - y) = 1 + 24 - 8y = 25 - 8y\end{aligned}$$

Given that x is real, so $D \geq 0$ or $25 - 8y \geq 0$ or $25 \geq 8y$.

$$\therefore y \leq \frac{25}{8} \text{ i.e. maximum value of } y \text{ is } \frac{25}{8}.$$

$$\text{Substituting } y = \frac{25}{8} \text{ in (1), we get } -2x^2 + x + 3 = \frac{25}{8}$$

$$\Rightarrow 16x^2 - 8x + 1 = 0$$

$$\Rightarrow (4x - 1)^2 = 0 \quad \text{or} \quad (4x - 1) = 0$$

$$\therefore x = \frac{1}{4}.$$

EXAMPLE 23 A certain number of balls were purchased for ₹ 450. Five more balls could have been purchased for the same amount if each ball was cheaper by ₹ 15. Find the number of balls purchased.

- (a) 15 (b) 20 (c) 10 (d) 25 (e) None of these

Solution Let the number of balls purchased be x . Then comparing price of balls, in both conditions

$$\begin{aligned}\frac{450}{x} &= \frac{450}{x+5} + 15 \\ \Rightarrow \frac{30}{x} &= \frac{30}{x+5} + 1 = \frac{30+x+5}{x+5} = \frac{35+x}{x+5}\end{aligned}$$

$$\text{i.e. } 30(x+5) = x(35+x)$$

$$\Rightarrow 30x + 150 = 35x + x^2$$

$$\Rightarrow x^2 + 5x - 150 = 0$$

$$\text{i.e. } (x+15)(x-10) = 0$$

$$\therefore x = 10 \text{ or } -15$$

$\therefore$ No. of balls is a +ve value, $x = 10$.

$$\therefore \text{Cost price of a ball} = \frac{450}{x} = \frac{450}{10} = \text{₹ } 45$$

When we get 5 balls more for the same amount, the cost price of a ball = $\frac{450}{x+5}$ which is less by 15 compared to the actual cost price.



Short cut:

$$\begin{aligned}\frac{450}{x} &= \frac{450}{x+5} + 15 \\ \Rightarrow \frac{30}{x} &= \frac{30}{x+5} + 1\end{aligned}$$

Put the value of x such that the ratio on the left side is greater than the right by 1.

$\therefore x = 10$ will satisfy.

EXAMPLE 24 A businessman knows that the price of a commodity will increase by ₹ 5 per packet. He bought some packets of this commodity for ₹ 4500. If he bought this packet on the new price then he gets 10 packets less. What is the number of packets bought by him?

- (a) 90 (b) 100 (c) 50 (d) 125 (e) None of these

Solution Let the businessman bought x packets.

Then the cost price of a packet = $\frac{4500}{x}$.

When he get 10 packets less then cost price of a packet = $\frac{4500}{x - 10}$

Here
$$\frac{4500}{x} + 5 = \frac{4500}{x - 10}$$

From the choices given above, number x and $x - 10$ both should divide 4500 completely.

∴ 100 is the correct answer.



EXERCISES

- Roots of $2kx^2 + 5kx + 2 = 0$ are equal if k is equal to
(a) 9/25 (b) 16/25 (c) 7/25 (d) 18/25 (e) None of these
- The area of a rectangle is same as that of a circle of radius $\frac{\sqrt{35}}{11}$ cm. If the length of the rectangle exceeds its breadth by 3 cm, find the dimensions of rectangle.
(a) 6×3 cm (b) 5×2 cm (c) 7×4 cm (d) 4×1 cm (e) None of these
- The surface area of a pipe, open at both ends is equal to 628 m^2 . Difference between its radius and its length is 15 m. If the pipe was closed at one end, what amount of water it can hold?
(a) 1750 m^3 (b) 7150 m^3 (c) 1570 m^3 (d) 7510 m^3 (e) None of these
- The roots of $2kx^2 + 5kx + 2 = 0$ are equal if k is equal to _____.
(a) 4/5 (b) -4/5 (c) 16/25 (d) -16/25 (e) None of these
- What is the condition that one of the roots of the equation is double the other in $ax^2 + bx + c = 0$?
(a) $ab^2 = 2ac$ (b) $b^2 = 18ac$ (c) $2b^2 = 9ac$ (d) $b^2 = 4ac$ (e) None of these
- The sum of the roots of the equation $\frac{1}{x+a} + \frac{1}{x+b} = \frac{1}{c}$ is zero.
(a) $a^2 + b^2$ (b) $-(a^2 + b^2)$ (c) $\frac{1}{2}(a^2 + b^2)$ (d) $-\frac{1}{2}(a^2 + b^2)$ (e) None of these
- Find the roots of the equation, $\sqrt{3y+1} = \sqrt{y-1}$.
(a) $\sqrt{2}$ (b) $\sqrt{3}$ (c) 1 (d) 0 (e) None of these
- If α and β are the roots of the equation $x^2 + 3ax + c = 0$ and if $\alpha^2 + \beta^2 = 5$, find the value of a .
(a) $\sqrt{\frac{2+5c}{9}}$ (b) $\sqrt{\frac{5+2c}{9}}$ (c) $\sqrt{\frac{2-5c}{9}}$ (d) $\sqrt{\frac{5-2c}{9}}$ (e) None of these

9. If α and β are the roots of the equation $x^2 + 3ax + 2a^2 = 0$ and if $\alpha^2 + \beta^2 = 5$, find the value of a .
 (a) 0 (b) -1 (c) +1 (d) ± 1 (e) None of these
10. If $x + \frac{1}{x} = 2$, then the value of $x^2 + \frac{1}{x^2}$ will be _____.
 (a) 1 (b) 2 (c) 3 (d) 4 (e) None of these
11. If $x + \frac{1}{x} = 3$, then the value of $x^8 + \frac{1}{x^8}$ will be _____.
 (a) 3 (b) 9 (c) 27 (d) 81 (e) None of these
12. If $x + y = 3$, $xy = 2$; then find the value of $x^3 - y^3$.
 (a) 2 (b) 3 (c) 4 (d) 7 (e) None of these
13. If $x - \frac{1}{x} = \sqrt{21}$, then the value of $\left(x^2 + \frac{1}{x^2}\right)\left(x + \frac{1}{x}\right)$ will be _____.
 (a) 151 (b) 511 (c) 115 (d) 165 (e) None of these
14. If $a + b + c = 0$, then find the value of $a^3 + b^3 + c^3$.
 (a) $2abc$ (b) $3abc$ (c) $a^2b^2c^2$ (d) $2a^2b^2c^2$ (e) None of these
15. If $x = \frac{\sqrt{3} + 1}{\sqrt{3} - 1}$, $y = \frac{\sqrt{3} - 1}{\sqrt{3} + 1}$ find the value of $x^2 + y^2$.
 (a) 7 (b) 9 (c) 14 (d) 49 (e) None of these
16. Find the combined product of $(a^4 + b^4)(a^2 + b^2)(a + b)(a - b)$
 (a) $a^4 - b^4$ (b) $a^4 + b^4$ (c) $a^8 + b^8$ (d) $a^8 - b^8$ (e) None of these
17. Find the value of $x^3 - \frac{1}{x^3}$ when $x - \frac{1}{x} = a$.
 (a) $a^3 - 3a$ (b) $3a^3 - a$ (c) $a^3 + 3a$ (d) $3a^3 + a$ (e) None of these
18. If $x = 2^{1/3} - 2^{-1/3}$, find value of $2x^3 + 6x$.
 (a) 1 (b) 2 (c) 3 (d) 4 (e) None of these
19. Find the value of $\frac{(a-b)^2}{(b-c)(c-a)} + \frac{(b-c)^2}{(a-b)(c-a)} + \frac{(a-c)^2}{(a-b)(b-c)}$.
 (a) 0 (b) 1 (c) 2 (d) 3 (e) None of these
20. If $a + b + c = 0$, then find the value of $\frac{1}{b^2 + c^2 - a^2} + \frac{1}{c^2 + a^2 - b^2} + \frac{1}{a^2 + b^2 - c^2}$.
 (a) 0 (b) 1 (c) 2 (d) 3 (e) None of these
21. If $x + y + z = 0$, then find the value of $\frac{(x+y)(y+z)(z+x)}{xyz}$.
 (a) 0 (b) 1 (c) -1 (d) 2 (e) None of these
22. If $a + b + c = 0$, then find the value of $\frac{a^4 + b^4 + c^4}{b^2c^2 + c^2a^2 + a^2b^2}$.
 (a) 0 (b) 1 (c) 2 (d) 3 (e) None of these

23. If $x + y = 2z$, find the value of $\frac{x}{x-z} + \frac{z}{y-z}$.
 (a) 0 (b) 1 (c) 2 (d) 3 (e) None of these
24. If $x + \frac{1}{y} = 1$ and $y + \frac{1}{z} = 1$ then find value of $z + \frac{1}{x}$.
 (a) 0 (b) 1 (c) 2 (d) 3 (e) None of these
25. If $a = b = c$, then find the value of $\frac{(a+b+c)^2}{a^2+b^2+c^2}$.
 (a) 1 (b) 2 (c) 3 (d) 4 (e) None of these
26. Find the value of $a^2 + b^2 + c^2 - 2ab + 2ac - 2bc$, if $a = x + y$, $b = x - y$ and $c = 2x - 1$.
 (a) $(2x + 2y + 1)^2$ (b) $(2x - 2y + 1)^2$ (c) $(2x - 2y - 1)^2$ (d) $(1 - 2x - 2y)^2$ (e) None of these
27. Find the value of $x^3 + y^3 + z^3 - 3xyz$, when $x + y + z = 16$ and $xy + yz + zx = 78$.
 (a) 176 (b) 352 (c) 716 (d) 532 (e) None of these
28. $\frac{1}{2}(a+b+c)\{(a-b)^2 + (b-c)^2 + (c-a)^2\} = ?$
 (a) $a^3 + b^3 + c^3 + 3abc$ (b) $a^3 + b^3 + c^3 - 3abc$ (c) $a^3 + b^3 + c^3 + 3abc(a+b+c)$
 (d) $3abc$ (e) 3
29. Find the value of $x^3 + y^3 + z^3 - 3xyz$ when $x = 89$, $y = 87$, $z = 84$.
 (a) 260 (b) 19 (c) 4940 (d) 4490 (e) None of these
30. If $x = a(b-c)$, $y = b(c-a)$ and $z = c(a-b)$, then $\left(\frac{x}{a}\right)^3 + \left(\frac{y}{b}\right)^3 + \left(\frac{z}{c}\right)^3 = ?$
 (a) $\frac{3xyz}{abc}$ (b) $\frac{xyz}{abc}$ (c) $3xyzabc$ (d) 3 (e) None of these
31. When $x + \frac{1}{x} = 3$, find $x^2 + \frac{1}{x^2}$.
 (a) 3 (b) 6 (c) 9 (d) 27 (e) None of these
32. Find the value of $a^2 + b^2 + c^2 - 2ab - 2bc - 2ca$ when $a = 17$, $b = 15$ and $c = 13$.
 (a) 111 (b) 121 (c) 225 (d) 361 (e) None of these
33. When $a = -5$, $b = -6$ and $c = 10$ find the value of $\frac{a^3 + b^3 + c^3 - 3abc}{(ab + bc + ca - a^2 - b^2 - c^2)}$.
 (a) -1 (b) 1 (c) 2 (d) -2 (e) 3
34. Find the value of $(a+b+c)^3 - 3(b+c)(c+a)(a+b)$ if $a = 5$, $b = 3$ and $c = 2$.
 (a) 120 (b) 180 (c) 160 (d) 240 (e) None of these



1. (b) 2. (b) 3. (c) 4. (c) 5. (c) 6. (d) 7. (e) 8. (b) 9. (d) 10. (b)
 11. (e) 12. (d) 13. (c) 14. (b) 15. (c) 16. (d) 17. (c) 18. (c) 19. (d) 20. (a)
 21. (c) 22. (c) 23. (b) 24. (b) 25. (c) 26. (d) 27. (b) 28. (b) 29. (c) 30. (a)
 31. (e) 32. (e) 33. (b) 34. (c)



Solutions with Necessary Explanation

1. Roots of $2kx^2 + 5kx + 2 = 0$ are equal

$$\therefore \text{discriminant} = 0$$

$$\text{i.e. } (5k)^2 - 4(2k)(2) = 0$$

$$\text{i.e. } 25k^2 - 16k = 0$$

$$\therefore (25k - 16)k = 0$$

$$\Rightarrow k = \frac{16}{25}$$

2. Let the breadth of rectangle be b

$$\therefore \text{length} = b + 3$$

$$\therefore \text{Area of rectangle} = b(b + 3) = \pi \left(\sqrt{\frac{35}{11}} \right)^2$$

$$b^2 + 3b = \frac{22}{7} \times \frac{35}{11} = \frac{70}{7} = 10$$

$$\text{i.e. } b^2 + 3b = 10$$

$$\text{i.e. } b^2 + 3b - 10 = 0$$

$$(b + 5)(b - 2) = 0$$

$$\therefore b = 2 \text{ cm}$$

$$\therefore \text{Length} = b + 3 = 2 + 3 = 5 \text{ cm}$$

Hence dimension of rectangle is $5 \times 2 \text{ cm}$.

3. Let r be the radius of the pipe

$$\therefore 2\pi rl = 628 \text{ m}^2$$

Substituting value of l in terms of r , we get

$$l - r = 15 \text{ m or } l = (15 + r)$$

$$\therefore 2\pi r(15 + r) = 628 = 200\pi \text{ m}^2$$

$$(15 + r)r = 100$$

$$\Rightarrow r = 5 \text{ cm}$$

$$\therefore l = 15 + r = 20 \text{ cm}$$

$$\begin{aligned} \therefore \text{Volume of the pipe} &= \pi r^2 l = \pi (5)^2 \times 20 \\ &= 500\pi = 1570 \text{ m}^3 \end{aligned}$$

4. Roots of $2kx^2 + 5kx + 2 = 0$ are equal if discriminant = 0.

$$\text{i.e. } (5k)^2 - 4(2k)(2) = 0$$

$$25k^2 - 16k = 0$$

$$\Rightarrow 25k - 16 = 0$$

$$\therefore k = \frac{16}{25}$$

5. Let one root of the equation $ax^2 + bx + c = 0$ be α .

Then other root be 2α .

$$\therefore \text{Sum of roots} = \frac{-b}{a} = \alpha + 2\alpha = 3\alpha \quad (1)$$

$$\text{and product of roots} = \frac{c}{a} = \alpha(2\alpha) = 2\alpha^2 \quad (2)$$

$$(1) \Rightarrow \alpha = \frac{-b}{3a}$$

$$(2) \Rightarrow \alpha^2 = \frac{c}{2a} = \left(\frac{-b}{3a}\right)^2 = \frac{b^2}{9a^2}$$

$$\text{i.e.} \quad 2b^2 = 9ac$$

$$6. \frac{1}{x+a} + \frac{1}{x+b} = \frac{1}{c} \quad (1)$$

Let the roots of eqn. be α and $-\alpha$

$$\therefore \text{Sum of roots} = \alpha - \alpha = 0$$

$$\text{Product of roots} = \alpha(-\alpha) = -\alpha^2$$

$$(1) \Rightarrow c(x+b) + c(x+a) = (x+a)(x+b)$$

$$\text{i.e.} \quad x^2 + x(a+b-2c) + (ab-bc-ca) = 0$$

$$\therefore \text{Sum of roots} = -(a+b-2c) = 0 \Rightarrow a+b=2c$$

$$\text{Product of roots} = (ab-bc-ca) = ab-c(a+b)$$

$$= ab - \frac{(a+b)^2}{2}$$

$$= \frac{-1}{2}(a^2 + b^2)$$

$$7. \sqrt{3y+1} = \sqrt{y-1}$$

Squaring both sides,

$$3y+1 = y-1$$

$$\Rightarrow 3y-y = -1-1 = -2$$

$$2y = -2$$

$$\therefore y = \frac{-2}{2} = -1$$

$$\therefore y = -1 \Rightarrow \sqrt{3y+1} = \sqrt{3(-1)+1} = \sqrt{-2} \text{ which is not a real number.}$$

Hence no real root exists.

$$8. \text{ Given } \alpha \text{ and } \beta \text{ are roots of equation } x^2 + 3ax + c = 0.$$

$$\alpha^2 + \beta^2 = 5; a = ?$$

$$\text{Here} \quad \alpha + \beta = -3a \quad (1)$$

$$\alpha\beta = c \quad (2)$$

$$\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$$

$$= (-3a)^2 - 2c$$

$$\Rightarrow = 9a^2 - 2c = 5$$

$$\therefore a = \sqrt{\frac{5+2c}{9}}$$

9. In the above equation, put $c = 2a^2$

$$\therefore 9a^2 = 5 + 2(2a^2)$$

$$\Rightarrow 9a^2 = 5 + 4a^2$$

$$\Rightarrow 5a^2 = 5$$

$$\Rightarrow a^2 = \frac{5}{5} = 1$$

$$\Rightarrow a = \pm 1$$

$$10. \quad x + \frac{1}{x} = 2; \Rightarrow x^2 + \frac{1}{x^2} = \left(x + \frac{1}{x}\right)^2 - 2 = 2^2 - 2 = 2$$

$$11. \quad x + \frac{1}{x} = 3; \Rightarrow x^2 + \frac{1}{x^2} = \left(x + \frac{1}{x}\right)^2 - 2 = 3^2 - 2 = 7$$

$$x^4 + \frac{1}{x^4} = \left(x^2 + \frac{1}{x^2}\right)^2 - 2 = 7^2 - 2 = 49 - 2 = 47$$

$$x^8 + \frac{1}{x^8} = \left(x^4 + \frac{1}{x^4}\right)^2 - 2 = 47^2 - 2 = 2207.$$

$$12. \quad x + y = 3; xy = 2; \Rightarrow (x - y)^2 = (x + y)^2 - 4xy = 3^2 - 4(2) = 1 \Rightarrow x - y = 1$$

$$\begin{aligned} \Rightarrow x^3 - y^3 &= (x - y)(x^2 + y^2 + xy) \\ &= (x - y)[(x + y)^2 - xy] \\ &= 1 \times [3^2 - 2] \\ &= 7 \end{aligned}$$

$$13. \quad x - \frac{1}{x} = \sqrt{21}, \quad \left(x^2 + \frac{1}{x^2}\right)\left(x + \frac{1}{x}\right) = ?$$

$$\left(x + \frac{1}{x}\right)^2 = \left(x - \frac{1}{x}\right)^2 + 4 = 21 + 4 = 25$$

$$\Rightarrow x + \frac{1}{x} = \sqrt{25} = 5$$

$$x^2 + \frac{1}{x^2} = \left(x + \frac{1}{x}\right)^2 - 2 = 5^2 - 2 = 23$$

$$\therefore \left(x^2 + \frac{1}{x^2}\right) \cdot \left(x + \frac{1}{x}\right) = 23 \times 5 = 115$$

$$14. \quad a + b + c = 0;$$

$$\begin{aligned} \therefore a^3 + b^3 + c^3 &= (a + b + c)^3 - 3(b + c)(c + a)(a + b) \\ &= 0^3 - 3(-a)(-b)(-c) \\ &= 3abc \end{aligned}$$

$$15. \quad x = \frac{\sqrt{3} + 1}{\sqrt{3} - 1}; y = \frac{\sqrt{3} - 1}{\sqrt{3} + 1}$$

$$x = \frac{\sqrt{3} + 1}{\sqrt{3} - 1} = \frac{(\sqrt{3} + 1)^2}{(\sqrt{3})^2 - 1^2} = \frac{4 + 2\sqrt{3}}{3 - 1} = \frac{4 + 2\sqrt{3}}{2} = 2 + \sqrt{3}$$

$$y = \frac{\sqrt{3} - 1}{\sqrt{3} + 1} = \frac{(\sqrt{3} - 1)^2}{(\sqrt{3})^2 - 1^2} = \frac{4 - 2\sqrt{3}}{3 - 1} = \frac{4 - 2\sqrt{3}}{2} = 2 - \sqrt{3}$$

$$\therefore \quad x^2 + y^2 = (2 + \sqrt{3})^2 + (2 - \sqrt{3})^2 = 2(2)^2 + 2(\sqrt{3})^2 \\ = 8 + 6 = 14$$

$$\begin{aligned} 16. \quad & (a^4 + b^4)(a^2 + b^2)(a + b)(a - b) \\ &= (a^4 + b^4)(a^2 + b^2)(a^2 - b^2) \\ &= (a^4 + b^4)(a^4 - b^4) \\ &= (a^4)^2 - (b^4)^2 = a^8 - b^8 \end{aligned}$$

$$17. \text{ Given } x - \frac{1}{x} = a; \quad x^3 - \frac{1}{x^3} = ?$$

$$(x - y)^3 = x^3 - y^3 - 3xy(x - y)$$

$$\text{i.e.} \quad \left(x - \frac{1}{x}\right)^3 = x^3 - \frac{1}{x^3} - 3x \cdot \frac{1}{x} \left(x - \frac{1}{x}\right)$$

$$\Rightarrow \quad a^3 = x^3 - \frac{1}{x^3} - 3(a)$$

$$\text{i.e.} \quad x^3 - \frac{1}{x^3} = a^3 + 3a$$

$$18. \text{ Given } x = 2^{1/3} - 2^{-1/3}; \quad 2x^3 + 6x = ?$$

$$2x^3 + 6x = 2x(x^2 + 3)$$

$$\text{Let } x = a - \frac{1}{a}$$

$$\begin{aligned} \therefore \quad 2x^3 + 6x &= 2 \left\{ \left(a - \frac{1}{a}\right)^3 + 3 \left(a - \frac{1}{a}\right) \right\} \\ &= 2 \left\{ a^3 - \frac{1}{a^3} - 3a \times \frac{1}{a} \left(a - \frac{1}{a}\right) + 3 \left(a - \frac{1}{a}\right) \right\} \\ &= 2 \left\{ a^3 - \frac{1}{a^3} \right\} = 2 \left\{ (2^{1/3})^3 - \frac{1}{(2^{1/3})^3} \right\} \\ &= 2 \left\{ 2 - \frac{1}{2} \right\} = 3 \end{aligned}$$

$$\begin{aligned} 19. \quad & \frac{(a-b)^2}{(b-c)(c-a)} + \frac{(b-c)^2}{(a-b)(c-a)} + \frac{(a-c)^2}{(a-b)(b-c)} = \frac{(a-b)^3 + (b-c)^3 + (a-c)^3 \cdot (c-a)}{(a-b)(b-c)(c-a)} \\ &= \frac{(a-b)^3 + (b-c)^3 + (c-a)^3}{(a-b)(b-c)(c-a)} = \frac{3(a-b)(b-c)(c-a)}{(a-b)(b-c)(c-a)} = 3 \end{aligned}$$

$$20. \quad a + b + c = 0$$

$$\begin{aligned} & \frac{1}{b^2 + c^2 - a^2} + \frac{1}{c^2 + a^2 - b^2} + \frac{1}{a^2 + b^2 - c^2} = ? \\ & a + b + c = 0 \Rightarrow a + b = (-c) \end{aligned}$$

$$\begin{aligned}\text{or} \quad & (a+b)^2 = (-c)^2 \\ \text{or} \quad & a^2 + 2ab + b^2 = c^2 \\ \text{or} \quad & a^2 + b^2 - c^2 = -2ab\end{aligned}$$

Similarly,

$$\begin{aligned} & a^2 + c^2 - b^2 = -2ac \\ \text{and} \quad & b^2 + c^2 - a^2 = -2bc\end{aligned}$$

$\therefore$ Given expression becomes

$$\frac{1}{-2bc} + \frac{1}{-2ac} + \frac{1}{-2ab} = \frac{-(a+b+c)}{2abc} = 0$$

$$21. \quad x + y + z = 0, \quad \frac{(x+y)(y+z)(z+x)}{xyz} = ?$$

$$\text{i.e.} \quad \frac{(-z)(-x)(-y)}{xyz} = \frac{-xyz}{xyz} = -1$$

$$22. \quad a + b + c = 0, \quad \frac{a^4 + b^4 + c^4}{b^2c^2 + c^2a^2 + a^2b^2} = ?$$

$$(a + b + c) = 0$$

$$(a + b + c)^2 = 0$$

$$\Rightarrow \quad a^2 + b^2 + c^2 = -2(ab + bc + ca)$$

Squaring both sides,

$$(a^2 + b^2 + c^2)^2 = [-2(ab + bc + ca)]^2$$

$$a^4 + b^4 + c^4 + 2a^2b^2 + 2b^2c^2 + 2c^2a^2 = 4[a^2b^2 + b^2c^2 + a^2c^2 + 2abc(a + b + c)]$$

$$\begin{aligned}\therefore \quad \frac{a^4 + b^4 + c^4}{b^2c^2 + c^2a^2 + a^2b^2} &= \frac{4[a^2b^2 + b^2c^2 + a^2c^2 + 2abc(a + b + c)] - 2(a^2b^2 + b^2c^2 + c^2a^2)}{(b^2c^2 + c^2a^2 + a^2b^2)} \\ &= (4 - 2) + 8abc(0) \\ &= 2\end{aligned}$$

$$23. \quad x + y = 2z; \quad \frac{x}{x-z} + \frac{z}{y-z} = ?$$

$$\begin{aligned}\frac{x}{x-z} + \frac{z}{y-z} &= \frac{x}{z-y} + \frac{z}{y-z} \\ &= \frac{x}{z-y} - \frac{z}{z-y} \\ &= \frac{x-z}{z-y} = 1\end{aligned}$$

$$24. \quad x + \frac{1}{y} = 1; \quad y + \frac{1}{z} = 1; \quad z + \frac{1}{x} = ?$$

$$x + \frac{1}{y} = 1 \quad \Rightarrow \quad x = 1 - \frac{1}{y}$$

or

$$\frac{1}{x} = \frac{1}{1 - \frac{1}{y}} = \frac{y}{y-1}$$

$$y + \frac{1}{z} = 1 \quad \Rightarrow \quad \frac{1}{z} = 1 - y$$

or
$$z = \frac{1}{1 - y}$$

$$\therefore z + \frac{1}{x} = \frac{1}{1 - y} + \frac{y}{y - 1} = \frac{1}{1 - y} + \frac{-y}{1 - y} = \frac{1 - y}{1 - y} = 1$$

25. $a = b = c$; $\frac{(a + b + c)^2}{a^2 + b^2 + c^2} = ?$

$$\frac{(a + b + c)^2}{a^2 + b^2 + c^2} = \frac{(3a^2)}{3a^2} = \frac{9a^2}{3a^2} = 3$$

26. $a = x + y$; $b = x - y$; $c = 2x - 1$;

$$\begin{aligned} a^2 + b^2 + c^2 - 2ab + 2ac - 2bc &= (b - a - c)^2 \\ &= [x - y - (x + y) - (2x - 1)]^2 \\ &= [x - y - x - y - 2x + 1]^2 \\ &= [-2x - 2y + 1]^2 \end{aligned}$$

27. $x + y + z = 16$; $xy + yz + zx = 78$;

$$\begin{aligned} x^3 + y^3 + z^3 - 3xyz &= (x + y + z) [(x^2 + y^2 + z^2 - (xy + yz + zx))] \\ &= 16[100 - 78] \quad [\because x^2 + y^2 + z^2 = (x + y + z)^2 - 2(xy + yz + zx)] \\ &= 352 = 16^2 - 2(78) = 256 - 156 = 100 \end{aligned}$$

28. By standard identity,

$$a^3 + b^3 + c^3 - 3abc = \frac{1}{2}(a + b + c) \{(a - b)^2 + (b - c)^2 + (c - a)^2\}$$

$\therefore$ Answer is $a^3 + b^3 + c^3 - 3abc$

29. $x^3 + y^3 + z^3 - 3xyz = \frac{1}{2}(x + y + z) \{(x - y)^2 + (y - z)^2 + (z - x)^2\}$

Given $x = 89$, $y = 87$, $z = 84$

$$\begin{aligned} \therefore \text{RHS} &= \frac{1}{2}(89 + 87 + 84) \{(89 - 87)^2 + (87 - 84)^2 + (84 - 89)^2\} \\ &= \frac{1}{2} \times 260 \times \{2^2 + 3^2 + (-5)^2\} \\ &= 130 \times 38 = 3800 + 1140 = 4940 \end{aligned}$$

30. Given $x = a(b - c)$, $y = b(c - a)$, $z = c(a - b)$

$$\therefore \left(\frac{x}{a}\right)^3 + \left(\frac{y}{b}\right)^3 + \left(\frac{z}{c}\right)^3 = (b - c)^3 + (c - a)^3 + (a - b)^3$$

If $p + q + r = 0$, $p^3 + q^3 + r^3 = 3pqr$

$\therefore$ RHS = $3(b - c)(c - a)(a - b)$

$$= 3 \times \frac{x}{a} \times \frac{y}{b} \times \frac{z}{c} = \frac{3xyz}{abc}$$

31. Given $x + \frac{1}{x} = 3$

$$\therefore x^2 + \frac{1}{x^2} = \left(x + \frac{1}{x}\right)^2 - 2 = 3^2 - 2 = 7$$

32. Given $a = 17$, $b = 15$ and $c = 13$

$$\begin{aligned} a^2 + b^2 + c^2 - 2ab - 2bc - 2ca &= ? \\ &= (a + b + c)^2 - 4ab - 4bc - 4ca \\ &= (a + b + c)^2 - 4b(a + c) - 4ac \\ &= (17 + 15 + 13)^2 - 4 \times 15(17 + 13) - 4 \times 17 \times 13 \\ &= 45^2 - 60 \times 30 - 4 \times 17 \times 13 \\ &= 45(45 - 40) - 4 \times 17 \times 13 = 45 \times 5 - 68 \times 13 = 225 - 884 = -\mathbf{659} \end{aligned}$$

33. Given $a = -5$, $b = -6$, $c = 10$

$$\therefore \frac{(a^3 + b^2 + c^3 - 3abc)}{(ab + bc + ca - a^2 - b^2 - c^2)} = -(a + b + c) = -[-5 + (-6) + 10] = 1$$

34. Given $a = 5$, $b = 3$, $c = 2$

$$\therefore (a + b + c)^3 - 3(b + c)(c + a)(a + b) = a^3 + b^3 + c^3$$

i.e. $\text{RHS} = 5^3 + 3^3 + 2^3 = 125 + 27 + 8 = \mathbf{160}$

Partnership

Partnership is an association of two or more persons who put their money together in order to carry on a certain business. Person entered into such an association are called **Partners**. It is of two kinds: (i) Simple partnership, and (ii) Compound partnership.

SIMPLE PARTNERSHIP: If the capital of the partners is invested for the same period, the partnership is called the simple partnership. In such a partnership, profit and loss is distributed among the partners in the ratio of their investments.

For example, if A invests ₹ x and B invest ₹ y , then their profit or loss at the end of a year will be in the ratio $x : y$.

COMPOUND PARTNERSHIP: If the capital of the partners is invested for different lengths of time, the partnership is called the compound partnership.

If a group of n persons invested different amounts for different periods, then their profit ratio is

$$At_1 : Bt_2 : Ct_3 : Dt_4 \dots : Xt_n$$

[Here the first person invested an amount A for t_1 years, second person invested an amount B for t_2 years, and so on]. When investments are of different periods, then equivalent capitals are calculated for a unit of time by taking (capital $\times$ number of units of time).

If A invests ₹ x for t_1 months and B invest ₹ y for t_2 months in a year, then

A's profit or loss or B's profit or loss = $xt_1 : yt_2$



Note: A partner who manages the business is known as a working partner and are who simply invests the money is a sleeping partner.

EXAMPLE 1 Three partners A, B, and C invest ₹ 1600, ₹ 1800, and ₹ 2300 respectively in a business. How should they divide the profit of ₹ 1938?

Solution The profit should be divided in the ratios of the capital, i.e. 16 : 18 : 23.

$$16 + 18 + 23 = 57.$$

$$\therefore \text{A's share} = \frac{16}{57} \text{ of ₹ 1938} = \text{₹ 544}$$

$$\text{B's share} = \frac{18}{57} \text{ of ₹ 1938} = \text{₹ 612}$$

$$\text{C's share} = \frac{23}{57} \text{ of ₹ 1938} = \text{₹ 782}$$

EXAMPLE 2 X, Y, and Z enter into partnership. X advances ₹ 12,000 for 4 months, Y ₹ 14,000 for 8 months and Z ₹ 10,000 for 10 months. They gain ₹ 5850 together. Find the share of each.

Solution ₹ 12000 in 4 months earns a profit equal to profit for ₹ 48000 in 1 month.

Similarly, profit earned by ₹ 14000 in 8 months = profit earned by ₹ 14000 × 8 in 1 month.

Similarly, profit earned by ₹ 10000 in 10 months = profit earned by ₹ 10000 × 10 in 1 month.

∴ Profit is divided in the ratio 48000, 112000, and 100000, i.e. in the ratio of 12, 28 and 25.

Now $12 + 28 + 25 = 65$

$$\therefore \text{A's share} = \frac{12}{65} \times 5850 = \text{₹ 1080}$$

$$\text{B's share} = \frac{28}{65} \times 5850 = \text{₹ 2520}$$

$$\text{C's share} = \frac{25}{65} \times 5850 = \text{₹ 2250}$$



Note: In compound partnership, the ratio of profits is directly proportional to both money and time, so they are multiplied together to get the corresponding shares in the ratio of profits.

EXAMPLE 3 A started a business with ₹ 2000. B joined him after 3 months with ₹ 4000. C put a sum of ₹ 10,000 in the business for 2 months only. At the end of the year, the business gave a profit of ₹ 5600. How should the profit be divided among them?

Solution Ratio of their profits (A's : B's : C's)

$$= 2 \times 12 : 4 \times 9 : 10 \times 2$$

$$= 6 : 9 : 5$$

Now, $6 + 9 + 5 = 20$

$$\therefore \text{A's share} = \frac{6}{20} \times 5600 = \text{₹ 1680}$$

$$\text{B's share} = \frac{9}{20} \times 5600 = \text{₹ 2520}$$

$$\text{C's share} = \frac{5}{20} \times 5600 = \text{₹ 1400}$$

EXAMPLE 4 A and B entered into a partnership for a year. A contributed ₹ 1500 and B ₹ 2000. After 4 months they admitted C, who contributed ₹ 2250. If B withdraw his contribution after 9 months, how would they share a profit of ₹ 900 at the end of the year?

Solution A's share : B's share : C's share

$$\begin{aligned} &= 1500 \times 12 : 2000 \times 9 : 2250 \times 8 \\ &= 15 \times 12 : 20 \times 9 : 22.5 \times 8 \\ &= 180 : 180 : 180 \\ &= 1 : 1 : 1 \end{aligned}$$

∴ each of them got ₹ $\frac{900}{3} = ₹ 300$

EXAMPLE 5 A and B entered into a speculation. A put in ₹ 500 and B put in ₹ 450. At the end of 4 months A withdrew half of his capital and at the end of 6 months B withdrew half of his capital, then C entered with a capital of ₹ 700. At the end of 12 months, in what ratio would the profit be divided?

Solution A's share : B's share : C's share

$$\begin{aligned} &= \left(500 \times 4 + \frac{500}{2} \times 8 \right) : \left(450 \times 6 + \frac{450}{2} \times 6 \right) : 700 \times 6 \\ &= 500 \times 8 : 450 \times 9 : 700 \times 6 \\ &= 80 : 81 : 84 \end{aligned}$$

∴ Profit will be divided in the ratio 80 : 81 : 84



Important Formula

Formula for compound partnership

$$\frac{\text{A's capital} \times \text{A's time in partnership}}{\text{B's capital} \times \text{B's time in partnership}} = \frac{\text{A's profit}}{\text{B's profit}}$$

EXAMPLE 6 A began a business with ₹ 4500 and was joined afterwards by B with ₹ 3,000. When did B join if the profits at the end of the year were divided in the ratio 2 : 1?

Solution If B joined the business for x months

$$\therefore \frac{4500 \times 12}{3000 \times x} = \frac{2}{1}$$

$$\text{or} \quad 3000 \times x \times 2 = 4500 \times 12 \times 1$$

$$\text{or} \quad x = \frac{45 \times 6}{30} = 9 \text{ months.}$$

∴ B joined after $(12 - 9) = 3$ months.

EXAMPLE 7 A and B entered into a partnership with their capitals in the ratio 7 : 9. At the end of 8 months, A withdrew his capital. If they receive the profits in the ratio 8 : 9, find how long B's capital was used.

Solution If B's capital was used for x months

$$\therefore \frac{7 \times 8}{9 \times x} = \frac{8}{9} \quad \text{or} \quad x = \frac{7 \times 8 \times 9}{9 \times 8} = 7 \text{ months}$$

$\therefore$ B's capital was used for **7 months**.

EXAMPLE 8 A, B and C invested capitals in the ratio 2 : 3 : 5, the timing of their investments being in the ratio 4 : 5 : 6. In what ratio would their profit be disbursed?

Solution If three investments are in the ratio $a : b : c$ and duration of their investments be in the ratio $x : y : z$, then the profit would be disbursed in the ratio $ax : by : cz$.

$$\therefore \quad \text{Required ratio} = 2 \times 4 : 3 \times 5 : 5 \times 6 = \mathbf{8 : 15 : 30}$$

EXAMPLE 9 A, B, and C are partners. A receives $\frac{2}{5}$ of the profit and B and C share the remaining profit equally. A's income is increased by ₹ 220 when the profit rises from 8% to 10%. Find the capitals invested by A, B, and C.

$$\text{Solution} \quad \text{A's share} = (10 - 8)\% = ₹ 220$$

$$\therefore \quad 100\% = ₹ 11000$$

$$\therefore \quad \text{A's capital} = ₹ 11000$$

$$\text{i.e.,} \quad \frac{2}{5} \text{ of share} = ₹ 11000$$

$$\frac{3}{5} \text{ of share} = 11000 \times \frac{3}{2} = ₹ 16500$$

$\therefore$ B's and C's capitals are ₹ 8250 each.

EXAMPLE 10 Two partners invest ₹ 125000 and ₹ 85000 respectively in a business and agree that 60% of the profit should be divided equally between them and the remaining profit is to be treated as interest on capital. If one partner gets ₹ 300 more than the other, find the total profit made in the business.

Solution The difference counts only due to the 40% of the profit distributed among them according to their investments.

Let the total profit be ₹ x

Then 40% of profit = $0.4x$

Ratio of profit = 125000 : 85000 = 25 : 17

$$\therefore \quad \text{Share of first partner} = \frac{25}{25 + 17} \times (40\% \text{ of } x)$$

$$= \frac{25}{42} \times \frac{4}{10} x = \frac{5}{21} x.$$

$$\text{Share of second partner} = \frac{17}{25 + 17} (40\% \text{ of } x) = \frac{17}{42} \times \frac{4}{10} x = \frac{17x}{105}$$

$$\text{Difference in share} = \frac{5x}{21} - \frac{17x}{105} = 300$$

$$\text{or} \quad \frac{(25x - 17x)}{105} = 300$$

or

$$8x = 300 \times 105$$

 $\therefore$

$$x = \frac{300 \times 105}{8} = \text{₹ } 3937.50$$



Short cut: Ratio of profit = 125000 : 85000 = 25 : 17

 $\therefore$

$$\text{Total profit} = 300 \left(\frac{100}{40} \right) \left(\frac{25 + 17}{25 - 17} \right) = \text{₹ } 3937.50$$

EXAMPLE 11 A, B, and C are partners in a business. A, whose money has been used for 4 months, claims $\frac{1}{8}$ of the profit. B, whose money has been used for 6 months, claims $\frac{1}{3}$ of the profit. C had invested ₹ 1560 for 8 months. How much did A and B contribute?

Solution Ratio of their shares in profit = $\frac{1}{8} : \frac{1}{3} : \left(1 - \frac{1}{8} - \frac{1}{3} \right)$

$$= \frac{1}{8} : \frac{1}{3} : \frac{13}{24} = 3 : 8 : 13$$

Let contribution of A and B be a and b .

 $\therefore$

$$a \times 4 : b \times 6 : 1560 \times 8 = 3 : 8 : 13.$$

 $\therefore$

$$\frac{a \times 4}{3} = \frac{1560 \times 8}{13} \quad \text{or} \quad a = \frac{3}{4} \times \frac{1560 \times 8}{13}$$

i.e.

$$a = 3 \times 120 \times 2 = \text{₹ } 720$$

$\therefore$ Contribution of A = ₹ 720

$$\text{Contribution of B} = \left(\frac{720 \times 8}{3} \right) \times \frac{4}{6} = 80 \times 8 \times 2 = \text{₹ } 1280$$

EXAMPLE 12 Two partners invested ₹ 50000 and ₹ 70000 respectively in a business and agreed that 70% of the profit should be divided equally between them and the remaining profit in the ratio of investment. If one partner gets ₹ 90 more than the other, find the total profit made in the business.

Solution 30% of the profit will be distributed among the partners proportional to their investment.

$\therefore$ Ratio of profit = 50000 : 70000 = 5 : 7.

Let x be the total profit.

$$\text{Then difference of their profit} = \frac{5-7}{5+7}(0.3x) = 90$$

 $\therefore$

$$x = \frac{90 \times 12}{2 \times 0.3} = 900 \times 2 = \text{₹ } 1800$$

 $\therefore$

$$\text{Total profit} = \text{₹ } 1800$$



Short cut: Ratio of profit = 50000 : 70000 = 5 : 7

$$\therefore \text{Total profit} = 90 \times \left(\frac{100}{30} \right) \left(\frac{5+7}{7-5} \right) = \text{₹ } 1800$$

EXAMPLE 13 A, B, and C invested capitals in the ratio 2 : 3 : 4. At the end of the business term, they received the profits in the ratio 3 : 6 : 10. Find the ratio of periods for which they contributed their capitals.

Solution If investments are in the ratio $x : y : z$ and profits in the ratio $a : b : c$, then the

$$\text{ratio of periods} = \frac{a}{x} : \frac{b}{y} : \frac{c}{z}$$

$$\begin{aligned} \therefore \text{Required ratio} &= \frac{3}{2} : \frac{6}{3} : \frac{10}{4} \\ &= \frac{3}{2} \times 12 : \frac{6}{3} \times 12 : \frac{10}{4} \times 12 = 18 : 24 : 30 = \mathbf{3 : 4 : 5} \end{aligned}$$

EXAMPLE 14 In a partnership, A invested $\frac{1}{6}$ of the capital for $\frac{1}{6}$ of the time, B invested $\frac{1}{3}$ of the capital for $\frac{1}{3}$ of the time, and C invested the rest of the capital for the whole period. At the end of the period, they earned a profit of ₹ 4600. Find the share of B.

Solution C invested $1 - \left(\frac{1}{6} + \frac{1}{3} \right) = 1 - \frac{1}{2} = \frac{1}{2}$ part of the capital

$$\begin{aligned} \therefore \text{Ratio of profit} &= \frac{1}{6} \times \frac{1}{6} : \frac{1}{3} \times \frac{1}{3} : \frac{1}{2} \times 1 \\ &= \frac{1}{36} : \frac{1}{9} : \frac{1}{2} = 1 : 4 : 18 \end{aligned}$$

$$\therefore \text{B's share} = \left(\frac{4}{1+4+18} \right) 4600 = \frac{4}{23} \times 4600 = \text{₹ } \mathbf{800}$$

EXAMPLE 15 P, Q and R enter into a partnership. P invests 3 times as much as Q invests and Q invests two-thirds of what R invests. At the end of the year, the profit earned is ₹ 8800. What is the share of Q?

Solution Let the investment made by R be ₹ x .

$$\therefore \text{Investment by Q} = \text{₹ } \frac{2}{3}x$$

$$\therefore \text{Investment by P} = 3 \times \frac{2}{3}x = \text{₹ } 2x$$

$\therefore$ Ratio of investments of P, Q and R is

$$2x : \frac{2x}{3} : x = 6 : 2 : 3$$

$$\begin{aligned}
 \therefore \quad \text{Share of Q} &= \frac{2}{6+2+3} \times 8800 \\
 &= \frac{2}{11} \times 8800 = \text{₹ } 1600
 \end{aligned}$$

EXAMPLE 16 X, Y and Z start a business each investing ₹ 30000. After 6 months X withdrew ₹ 5000, Y withdrew ₹ 6000 and Z invests ₹ 6000 more. At the end of the year, a total profit of ₹ 17500 was recorded. Find the share of each.

Solution Ratio of capitals of X, Y and Z

$$\begin{aligned}
 &= 30000 \times 6 + 25000 \times 6 : 30000 \times 6 + 24000 \times 6 : 30000 \times 6 + 36000 \times 6 \\
 &= 55000 \times 6 : 54000 \times 6 : 66000 \times 6 = 55 : 54 : 66
 \end{aligned}$$

$$\therefore \quad \text{Share of X} = \text{₹} \left(17500 \times \frac{55}{175} \right) = \text{₹ } 5500$$

$$\text{Share of Y} = \text{₹} \left(17500 \times \frac{54}{175} \right) = \text{₹ } 5400$$

$$\text{Share of Z} = \text{₹} \left(17500 \times \frac{66}{175} \right) = \text{₹ } 6600$$

EXAMPLE 17 P invested ₹ 64000 in a business. Q joined him with ₹ 48000 after a few months. If the total profit was divided among them in the ratio 2 : 1 towards the end of year. After how many months did Q join.

Solution Let Q joined the business after x months.

$\therefore$ Q invested his money for $(12 - x)$ months.

$$\therefore \quad \frac{64000 \times 12}{48000 \times (12 - x)} = \frac{2}{1}$$

$$\Rightarrow 48000 (12 - x) \times 2 = 64000 \times 12$$

$$\Rightarrow 6(12 - x) = 4 \times 12$$

$$\Rightarrow 6x = 6 \times 12 - 4 \times 12$$

$$\Rightarrow = 24$$

$$\text{or} \quad x = \frac{24}{6} = 4 \text{ months}$$

EXAMPLE 18 A, B, and C enter into a partnership. They invest ₹ 40000, ₹ 80000 and ₹ 120000 respectively. At the end of the first year, B withdraws ₹ 40000, while at the end of the second year, C withdraws ₹ 80000. In what ratio will the profit be shared at the end of 3 years? **[Hotel Management]**

Solution Ratio of shares of profit between A, B and C is

$$\begin{aligned}
 &(40000 \times 36) : (80000 \times 12) + (40000 \times 24) : (1,20,000 \times 24) + (40000 \times 12) \\
 &= 4 \times 36 : (8 \times 12 + 4 \times 24) : (12 \times 24 + 4 \times 12) \\
 &= 4 \times 36 : 4 \times 48 : 4 \times 84 = 3 : 4 : 7
 \end{aligned}$$



EXERCISES

- How should a profit of ₹ 450 be divided between two partners, one of whom has contributed ₹ 1200 for 5 months and the other ₹ 750 for 4 months?
(a) 1 : 1 (b) 1 : 2 (c) 2 : 1 (d) 2 : 3 (e) None of these
- A and B enter into a partnership for a year. A contributes ₹ 1500 and B ₹ 2000. After 4 months, they admit C, who contributes ₹ 2250. If B withdraws his contribution after 9 months, how would they share a profit of ₹ 900 at the end of the year?
(a) 6 : 8 : 9 (b) 8 : 6 : 9 (c) 1 : 1 : 1 (d) 1 : 2 : 3 (e) None of these
- A and B enter into a partnership; A puts in ₹ 50 and B puts in ₹ 45. At the end of 4 months, A withdraws half his capital and at the end of 5 months B withdraws $\frac{1}{2}$ of his; C then enters with a capital of ₹ 70; at the end of 12 months, the profit of concern is ₹ 254; how can the profit be divided among A, B and C?
(a) 76, 80 and 98 (b) 80, 76 and 98 (c) 76, 98 and 80 (d) 80, 98 and 76 (e) None of these
- Three partners altogether invested ₹ 114000 in a business. At the end of the year, one got ₹ 337.50, the second ₹ 1125.00 and the third ₹ 675.00 as profit. How much amount did each invest? What is the percentage of profit?
(a) 75% (b) 3.75% (c) 1.875% (d) 15% (e) None of these
- A and B enter into a partnership with ratio of capitals as 5 : 6; and at the end of 8 months, A withdraws. If they receive profits in the ratio of 5 : 9, find how long B's capital was used?
(a) 9 months (b) 8 months (c) 10 months (d) 12 months (e) None of these
- Two friends P and Q started a business investing in the ratio of 5:6. R joined them after six months investing an amount equal to that of Q. At the end of the year, 20% profit was earned which was equal to ₹ 98000. What was the amount invested by R? [SBI PO]
(a) ₹ 105000 (b) ₹ 175000 (c) ₹ 210000 (d) Data inadequate
(e) None of these
- Shekhar started a business investing ₹ 25000 in 1999. In 2000, he invested an additional amount of ₹ 10000 and Rajeev joined him with an amount of ₹ 35000. In 2001, Shekhar invested another additional amount of ₹ 10000 and Jatin joined them with an amount of ₹ 35000. What will be Rajeev's share in the profit of ₹ 150000 earned at the end of 3 years from the start of the business in 1999? [SBI PO]
(a) ₹ 45000 (b) ₹ 50000 (c) ₹ 70000 (d) ₹ 75000 (e) None of these
- A started business with ₹ 3500 and after 5 months, B joins with A as his partner. After a year, profit is divided in the ratio 2:3. What is B's contribution in the capital? [SSC]
(a) ₹ 7500 (b) ₹ 8000 (c) ₹ 8500 (d) ₹ 9000
- A, B and C jointly thought of engaging themselves in a business venture. It was agreed that A would invest ₹ 6500 for 6 months, B ₹ 8400 for 5 months and C, ₹ 10000 for 3 months. A wants to be the working member for which he was to receive 5% of the profits. The profit earned was ₹ 7400. Calculate the share of B in the profit. [MBA]
(a) ₹ 1900 (b) ₹ 2660 (c) ₹ 2800 (d) ₹ 2840
- A, B and C entered into a partnership in the ratio $\frac{1}{2} : \frac{1}{3} : \frac{1}{4}$. After 2 months, A withdraws half of his capital and after 10 months, a profit of ₹ 378 is divided among them. What is B's share? [SSC]
(a) ₹ 129 (b) ₹ 144 (c) ₹ 156 (d) ₹ 168

11. A, B and C entered into a partnership in the ratio $\frac{7}{2} : \frac{4}{3} : \frac{6}{5}$. After 4 months, A increases his share by 50%. If the total profit at the end of one year be ₹ 21600, then B's share in the profit is: [LIC AAO]
 (a) ₹ 2100 (b) ₹ 2400 (c) ₹ 3600 (d) ₹ 4000
12. A and B entered into partnership with capitals in the ratio 4:5. After 3 months, A withdrew $\frac{1}{4}$ of his capital and B withdrew $\frac{1}{5}$ of his capital. The gain at the end of 10 months was ₹ 760. A's share in the profit is: [AAO]
 (a) ₹ 330 (b) ₹ 360 (c) ₹ 380 (d) ₹ 430
13. In a business, A and C invested amounts in the ratio 2:1, whereas the ratio between amounts invested by A and B was 3:2. If ₹ 1,57,300 was their profit, how much amount did B receive? [Bank PO]
 (a) ₹ 24200 (b) ₹ 36300 (c) ₹ 48400 (d) ₹ 72600
14. Arun, Kamal and Vinay invested ₹ 8000, ₹ 4000 and ₹ 8000 respectively in a business. Arun left after six months. If after eight months, there was a gain of ₹ 4005, then what will be the share of Kamal? [RRB]
 (a) ₹ 890 (b) ₹ 1335 (c) ₹ 1662 (d) ₹ 1780
15. Kamal started a business investing ₹ 9000. After five months, Sameer joined with a capital of ₹ 8000. If at the end of the year, they earn a profit of ₹ 6970, then what will be the share of Sameer in the profit? [RRB]
 (a) ₹ 1883.78 (b) ₹ 2380 (c) ₹ 3690 (d) ₹ 3864
16. A, B and C enter into a partnership investing ₹ 35000, ₹ 45000 and ₹ 55000 respectively. The respective shares of A, B and C in an annual profit of ₹ 40500 are: [CBI]
 (a) ₹ 10500, ₹ 13500, ₹ 16500 (b) ₹ 11500, ₹ 13000, ₹ 16000
 (c) ₹ 11000, ₹ 14000, ₹ 15500 (d) ₹ 11500, ₹ 12500, ₹ 16500
17. Simran started a software business by investing ₹ 50000. After six months, Nanda joined her with a capital of ₹ 80000. After 3 years, they earned a profit of ₹ 24500. What was Simran's share in the profit? [Bank PO]
 (a) ₹ 9423 (b) ₹ 10250 (c) ₹ 12500 (d) ₹ 14000 (e) None of these
18. A and B start a business with investments of ₹ 5000 and ₹ 4500 respectively. After 4 months, A takes out half of his capital. After two more months, B takes out one-third of his capital while C joins them with a capital of ₹ 7000. At the end of a year, they earn a profit of ₹ 5080. Find the share of each member in the profit. [Bank PO]
 (a) A – ₹ 1400, B – ₹ 1900, C – ₹ 1780
 (b) A – ₹ 1600, B – ₹ 1800, C – ₹ 1680
 (c) A – ₹ 1800, B – ₹ 1500, C – ₹ 1780
 (d) A – ₹ 1680, B – ₹ 1600, C – ₹ 1800
19. Three partners shared the profit in a business in the ratio 5:7:8. They had partnered for 14 months, 8 months and 7 months respectively. What was the ratio of their investments? [Hotel Management]
 (a) 5:7:8 (b) 28:49:64 (c) 38:28:21 (d) None of these
20. A and B are partners in a business. A contributes $\frac{1}{4}$ of the capital for 15 months and B received $\frac{2}{3}$ of the profit. For how long B's money was used? [SSC]
 (a) 6 months (b) 9 months (c) 10 months (d) one year

21. A began a business with ₹ 85000. He was joined afterwards by B with ₹ 42500. For how much period does B join, if the profits at the end of the year are divided in the ratio of 3:1? [NIFT]
 (a) 4 months (b) 5 months (c) 6 months (d) 8 months
22. A and B started a business in partnership investing ₹ 20000 and ₹ 15000 respectively. After six months, C joined them with ₹ 20000. What will be B's share in the total profit of ₹ 25000 earned at the end of 2 years from the starting of the business? [SBI PO]
 (a) ₹ 7500 (b) ₹ 9000 (c) ₹ 9500 (d) ₹ 10000 (e) None of these
23. Aman started a business investing ₹ 70000. Rakhi joined him after six months with an amount of ₹ 1,05,000 and Sagar joined them with ₹ 1.4 lakhs after another six months. The amount of profit earned should be distributed in what ratio among Aman, Rakhi and Sagar respectively, 3 years after Aman started the business? [Bank PO]
 (a) 7:6:10 (b) 12:15:16 (c) 42:45:56
 (d) Cannot be determined (e) None of these
24. Satheesh started a business investing ₹ 60000. After 3 months, Sunil joined him with a capital of ₹ 75000. After another 6 months, Subhash joined them with a capital of ₹ 105000. At the end of the year, they made a profit of ₹ 19000. Find the share of each.
 (a) ₹ 8500, ₹ 7000, ₹ 3500 (b) ₹ 8000, ₹ 7500, ₹ 3500
 (c) ₹ 8000, ₹ 6500, ₹ 4500 (d) ₹ 8500, ₹ 6500, ₹ 4000
25. A and B started a partnership business investing some amount in the ratio of 3:5. C joined them after six months with an amount equal to that of B. In what proportion should the profit at the end of one year be distributed among A, B and C? [Bank PO]
 (a) 3:5:2 (b) 3:5:5 (c) 6:10:5
 (d) Data inadequate (e) None of these



1. (c) 2. (c) 3. (b) 4. (c) 5. (d) 6. (c) 7. (b) 8. (d) 9. (b) 10. (b)
 11. (d) 12. (a) 13. (c) 14. (a) 15. (b) 16. (a) 17. (e) 18. (b) 19. (d) 20. (c)
 21. (d) 22. (a) 23. (b) 24. (b) 25. (c)



Solutions with Necessary Explanation

1. Ratio of profit = $1200 \times 5 : 750 \times 4 = 2 : 1$

$$\therefore \text{1st partner gets } 450 \times \frac{2}{2+1} = \text{₹ } 300$$

$$\therefore \text{2nd partner gets } 450 \times \frac{1}{2+1} = \text{₹ } 150$$

2. A's share : B's share : C's share

$$= 1500 \times 12 : 2000 \times 9 : 2250 \times 8$$

$$= 18000 : 18000 : 18000 = 1 : 1 : 1$$

$$\therefore \text{Share of A} = \text{Share of B} = \text{Share of C}$$

i.e. profit of ₹ 900 is divided into 3 equal parts equal to ₹ 300

3. A's share : B's share : C's share

$$= (50 \times 4 + 25 \times 8) : (45 \times 5 + 22.5 \times 7) : 70 \times 7$$

$$= 400 : 382.5 : 490 = 800 : 765 : 980$$

$$= 160 : 153 : 196$$

Total profit = ₹ 254

$$\therefore \text{Profit of A} = \frac{160}{160 + 153 + 196} \times 254 = \frac{160}{509} \times 254 \approx \mathbf{80}$$

$$\text{Profit of B} = \frac{153}{509} \times 254 \approx \mathbf{76}$$

$$\text{Profit of C} = \frac{196}{509} \times 254 \approx \mathbf{98}$$

4. Total investment = ₹ 114000

Ratio of investment = 337.50 : 1125 : 675

$$= 1 : \frac{10}{3} : 2$$

$$= \mathbf{3 : 10 : 6}$$

$$\therefore \text{Percentage profit} = \frac{337.5 + 1125 + 675}{114000} \times 100\%$$

$$= \frac{2137.5}{1140} \% = \mathbf{1.875\%}$$

5. Ratio of capitals = 5 : 6

Let the ratio of time = 8 : x

ratio of profits = 5 : 9

$$\therefore 5 \times 8 : 6 \times x = 5 : 9$$

$$\text{or } 40 : 6x = 5 : 9 \quad \text{or } \frac{40}{6x} = \frac{5}{9}$$

$$\therefore x = \frac{40 \times 9}{6 \times 5} = 4 \times 3 = \mathbf{12 \text{ months}}$$

6. Let the total profit be ₹ x.

$$\therefore 20\% \text{ of } x = 98000$$

$$\Rightarrow x = 98000 \times \frac{100}{20} = 490000$$

Ratio of investments between P and Q = 5 : 6. R joined 6 months after starting the business.

Let the capital of P, Q and R be ₹ 5y, 6y and 6y respectively.

$$\therefore (5y \times 12) + (6y \times 12) + (6y \times 6) = 490000 \times 12$$

$$\Rightarrow 168y = 490000 \times 12$$

$$\Rightarrow y = \frac{490000 \times 12}{168} = \mathbf{₹ 35000}$$

$$\therefore \text{R's investment} = 6y = \mathbf{₹ 6 \times 35000}$$

$$= \mathbf{₹ 210000}$$

7. Shekhar : Rajeev : Jatin

$$= (25000 \times 12 + 35000 \times 12 + 45000 \times 12) : 35000 \times 24 : 35000 \times 12$$

$$= 105000 \times 12 : 35000 \times 24 : 35000 \times 12$$

$$= \mathbf{3 : 2 : 1}$$

$$\therefore \text{Rajeev's share} = \frac{2}{3+2+1} \times 150000 = \mathbf{₹ 50000}$$

8. Let B's contribution be ₹ x .

$$\therefore A : B = 3500 \times 12 : x \times 7 = 2 : 3$$

$$\therefore \frac{3500 \times 12}{7x} = \frac{2}{3}$$

$$\begin{aligned} \text{or } x &= \frac{3500 \times 12 \times 3}{7 \times 2} \\ &= 250 \times 12 \times 3 = \text{₹ } 9000 \end{aligned}$$

9. Ratio of investments between A, B and C

$$= 6500 \times 6 : 8400 \times 5 : 10000 \times 3$$

$$= 65 \times 6 : 84 \times 5 : 100 \times 3$$

$$= 390 : 420 : 300 = 13 : 14 : 10$$

Profit earned = ₹ 7400

$$\therefore \text{A receives 5\% of profit} = \frac{5}{100} \times 7400 = \text{₹ } 370$$

$$\therefore \text{Net profit to be shared} = 7400 - 370 = \text{₹ } 7030$$

$$\therefore \text{Share of profit to B} = \frac{14}{37} \times 7030 = 14 \times 190 = \text{₹ } 2660$$

10. Ratio of investment between A, B and C at the start of business

$$= \frac{1}{2} : \frac{1}{3} : \frac{1}{4} = 6 : 4 : 3$$

Let their investments be ₹ 6k, ₹ 4k and ₹ 3k..

$$\text{Intestment by A} = 6k \times 2 + 3k \times 10 = 42k$$

$$\text{Intestment by B} = 4k \times 12 = 48k$$

$$\text{Intestment by C} = 3k \times 12 = 36k$$

$$\text{Ratio between investments} = 42k : 48k : 36k = 7 : 8 : 6$$

Share of profit to B from ₹ 378

$$= \frac{8}{7+8+6} \times 378 = \frac{8}{21} \times 378$$

$$= 8 \times 18 = \text{₹ } 144$$

11. Ratio of investments between A, B and C

$$= \frac{7}{2} : \frac{4}{3} : \frac{6}{5} = 7 \times 15 : 4 \times 10 : 6 \times 6 = 105 : 40 : 36$$

Let their investments be ₹ 105k, ₹ 40k and ₹ 36k respectively at the beginning

After 4 months, investment by A = $1.5 \times 105k = 157.5k$

Total profit = ₹ 21600.

$\therefore$ Ratio of investments

$$= (105k \times 4 + 157.5k \times 8) : 40k \times 12 : 36k \times 12$$

$$= (420k + 1260k) : 490k : 36 \times 12k$$

$$= 1680k : 480k + 36 \times 12k = 140 : 40 : 36 = 35 : 10 : 9$$

$$\begin{aligned}\therefore \text{Share of B} &= \frac{10}{35+10+9} \times 21600 \\ &= \frac{10 \times 21600}{54} \\ &= \text{₹ } 4000\end{aligned}$$

12. Initial ratio of investment = 4 : 5. Let the initial investment of A and B be 4k and 5k respectively.

$\therefore$ Ratio of investments between A and B

$$(4k \times 3 + 3k \times 7) : (5k \times 3 + 4k \times 7) = \mathbf{33k : 43k} = 33 : 43$$

$$\begin{aligned}\therefore \text{Share of profit to A} &= \frac{33}{33+43} \times 760 \\ &= \frac{33}{76} \times 760 = \text{₹ } 330\end{aligned}$$

13. Ratio of investments between A and C = 2 : 1 = 6 : 3

Ratio of investments between A and B = 3 : 2 = 6 : 4

Ratio of investments between A, B and C = 6 : 4 : 3

Total profit = ₹ 157300

$$\therefore \text{Share of B} = \frac{4}{6+4+3} \times 157300 = 4 \times 12100 = \text{₹ } 48400$$

14. Arun : Kamal : Vinay

$$= (8000 \times 6) : (4000 \times 8) : (8000 \times 8)$$

$$8 \times 6 : 4 \times 8 : 8 \times 8 = 3 : 2 : 4$$

Total gain = ₹ 4005

$$\begin{aligned}\therefore \text{Share of Kamal} &= \frac{2}{3+2+4} \times 4005 = \frac{2}{9} \times 4005 \\ &= \text{₹ } 890\end{aligned}$$

15. Kamal : Sameer = 9000 × 12 : 8000 × 7

$$= 9 \times 12 : 8 \times 7 = 27 : 14$$

$$\therefore \text{Share of Sameer} = \frac{14}{41} \times 6970 = 14 \times 170 = \text{₹ } 2380$$

16. A : B : C = 35000 : 45000 : 55000

$$= 35 : 45 : 55 = 7 : 9 : 11$$

$$\therefore \text{Share of A} = \frac{7}{27} \times 40500 = 7 \times 1500 = \text{₹ } 10500$$

$$\text{Share of B} = \frac{9}{27} \times 40500 = \text{₹ } 13500$$

$$\text{Share of C} = \frac{11}{27} \times 40500 = 11 \times 1500 = \text{₹ } 16500$$

17. Simran : Nanda = $50000 \times 36 : 80000 \times 30 = 5 \times 36 : 8 \times 30 = 3 : 4$
 Total profit = ₹ 24500

$$\therefore \text{Simran's share of profit} = \frac{3}{7} \times 24500 = ₹ 3 \times 3500 = ₹ 10500$$

18. A : B : C = $(5000 \times 4 \times 2500 \times 8) : (4500 \times 6 + 3000 \times 6) : 7000 \times 6$
 $= 40000 : 45000 : 42000 = 40 : 45 : 42$

Total profit = ₹ 5080

$$\therefore \text{Share of profit to A} = \frac{40}{127} \times 5080 = ₹ 1600$$

$$\text{Share of profit to B} = \frac{45}{127} \times 5080 = ₹ 1800$$

$$\text{Share of profit to C} = \frac{42}{127} \times 5080 = ₹ 1680$$

19. Let x , y and z be their investments.

$$\therefore x \times 14 : y \times 8 : z \times 7 = 5 : 7 : 8$$

$$\therefore \frac{14x}{7z} = \frac{5}{8}$$

$$\text{or } 16x = 5z$$

$$\text{or } z = \frac{16}{5}x$$

$$\frac{14x}{8y} = \frac{5}{7}$$

$$\Rightarrow 14 \times 7x = 40y$$

$$\text{or } y = \frac{98}{40}x$$

$$\begin{aligned} \therefore x : y : z &= x : \frac{98}{40}x : \frac{16}{5}x \\ &= x : \frac{49}{20}x : \frac{16}{5}x = 20 : 49 : 64 \end{aligned}$$

20. Let total capital be ₹ x .

Let B's money was used for y months.

$$\therefore A : B = \left(\frac{x}{4} \times 15 \right) : \frac{3x}{4} \times y = 1 : 2$$

$$\text{i.e. } 15x : 3xy = 1 : 2$$

$$\text{or } 5 : y = 1 : 2$$

$$\text{or } y = 5 \times 2 = 10 \text{ months}$$

21. Let B was in the business for x months.

$$\therefore A : B = 85000 \times 12 : 42500 \times x = 3 : 1$$

$$\text{i.e. } 24 : x = 3 : 1$$

$$\text{or} \quad \frac{24}{x} = \frac{3}{1}$$

$$\text{or} \quad x = \frac{24 \times 1}{3} = \mathbf{8 \text{ months}}$$

$$\begin{aligned} 22. \text{ A : B : C} &= (20000 \times 24) : (15000 \times 24) : (20000 \times 18) \\ &= 48 : 36 : 36 = 4 : 3 : 3 \end{aligned}$$

Total profit = ₹ 25000

$$\therefore \text{ Share of profit to B} = \frac{3}{4+3+3} \times 25000 = \mathbf{₹ 7,500}$$

$$\begin{aligned} 23. \text{ Aman : Rakhi : Sagar} \\ &= (70000 \times 36) : (105000 \times 30) : (140000 \times 24) \\ &= 36 \times 7 : 315 : 14 \times 24 = 36 : 45 : 48 \\ &= \mathbf{12 : 15 : 16} \end{aligned}$$

$$\begin{aligned} 24. \text{ Satheesh : Sunil : Subhash} \\ &= 60000 \times 12 : 75000 \times 9 : 105000 \times 3 \\ &= 720 : 9 \times 75 : 315 = 144 : 135 : 63 = 16 : 15 : 7 \end{aligned}$$

Total profit = ₹ 19000

$$\therefore \text{ Share of Satheesh} = \frac{16}{38} \times 19000 = \mathbf{₹ 8000}$$

$$\text{Share of Sunil} = \frac{15}{38} \times 19000 = \mathbf{₹ 7500}$$

$$\text{Share of Subhash} = \frac{7}{38} \times 19000 = \mathbf{₹ 3500}$$

$$25. \text{ Let the investment by A and B be ₹ } 3x \text{ and ₹ } 5x.$$

$$\therefore \text{ Investment by C} = ₹ 5x$$

$$\begin{aligned} \therefore \text{ A : B : C} &= (3x \times 12) : (5x \times 12) : (5x \times 6) \\ &= 36 : 60 : 30 = 6 : 10 : 5 \end{aligned}$$

Ratio by which profit is to be shared = **6 : 10 : 5**

Simple Interest

When an amount is deposited in a bank or lent to a person for an interest of $r\%$ per annum, the amount of interest will be calculated for the total number of days lent or deposited at the specified rate using the formula.

$$I = \frac{PRN}{100} \text{ and } A = P \left(1 + \frac{RN}{100} \right)$$

where P = Principal amount

R = Rate of interest per annum

N = Number of years (period of deposit)

A = Total amount accrued at the end of the period

(The interest thus accrued is called the *simple interest*.)

When time is given in days (D), convert it into years by dividing it by 365 $\left[\text{i.e. } \frac{D}{365} \right]$

$$I = \frac{PR}{100} \times \frac{D}{365}$$

If I_1 and I_2 are interests for 2 different periods, find the rate/annum.

$$\begin{aligned} I &= \frac{P_1 R N_1}{100} + \frac{P_2 R N_2}{100} \\ &= \frac{R}{100} (P_1 N_1 + P_2 N_2) \end{aligned}$$

$\therefore$

$$R = \frac{100I}{(P_1 N_1 + P_2 N_2)}$$

If R_1 and R_2 are rate of interest for same duration, principal being different, duration is given by

$$N = \frac{100I}{P_1 R_1 + P_2 R_2}$$

Find the annual instalment that will discharge a debt of ₹ A due in t years at $R\%$ per annum simple interest.

$$\text{Annual instalment required } x = \frac{100A}{100N + R \frac{(N-1)N}{2}}$$

where A = Total amount

N = Number of years/instalments

R = Rate of interest per annum

$$x = \frac{A}{N + \frac{R}{100} [1 + 2 + 3 + \dots (N-1)]}$$

A sum of money becomes n times in t years at simple interest. Find the rate of interest per annum in terms of n and t .

$$r = \frac{100(n-1)}{t} \%$$

In what time will an investment at $r\%$ per annum simple interest become n times?

$$\text{Period of investment } t = \frac{100(n-1)}{r} \text{ years}$$

$$\text{Investment} = \frac{\text{Additional income} \times 100}{\text{Time} \times \text{Difference in rate of interest}}$$

The difference in interests on a certain sum at $r_1\%$ for t_1 years and $r_2\%$ for t_2 years is ₹ I . Find the sum.

$$\therefore P = \frac{100I}{|r_1t_1 - r_2t_2|}$$

If a certain sum of money amounts to ₹ A_1 in t_1 years at $r\%$ p.a., it would amount to ₹ A_2 in $t_2 = \frac{1}{r} \left\{ \frac{A_2}{A_1} (100 + rt_1) - 100 \right\}$ years.

EXAMPLE 1 The simple interest on a sum of money is $1/9$ of the principal, and the number of years is equal to the rate per cent per annum. Find the rate per cent.

Solution Let Principal = P , Time = t years, rate = $t\%$

$$\therefore \frac{P \times t \times t}{100} = \frac{P}{9}$$

$$\therefore \frac{t^2}{100} = \frac{1}{9} \quad \text{or} \quad t^2 = \frac{100}{9}$$

$$\therefore t = \frac{10}{3} = 3\frac{1}{3}$$

$$\therefore \text{Rate per cent} = 3\frac{1}{3} \% \quad (\because \text{rate} = t\%)$$



Short cut: Rate = time = $\sqrt{100 \times \frac{1}{9}} = \frac{10}{3} = 3\frac{1}{3}$

Rate = $3\frac{1}{3}\%$; time = $3\frac{1}{3}$ years.

EXAMPLE 2 What annual payment will discharge a debt of ₹ 770 due in 5 years, the rate of interest being 5% p.a.?

Solution Let annual payment be P rupees.

Interest accrued in 4 years at 5% p.a. along with the amount credited

$$A = P \left(1 + \frac{NR}{100} \right) = P \left(\frac{100 + 4 \times 5}{100} \right) = \frac{120P}{100}$$

$$\text{Similarly in 3 years, amount} = P \left(\frac{100 + 3 \times 5}{100} \right) = \frac{115P}{100}$$

$$\text{Similarly in 2 years, amount} = P \left(\frac{100 + 2 \times 5}{100} \right) = \frac{110P}{100}$$

$$\text{Similarly in 1 year, amount} = P \left(\frac{100 + 1 \times 5}{100} \right) = \frac{105P}{100}$$

Total amount accrued in 4 years along with the last annual payment of ₹ P .

$$= \frac{120P}{100} + \frac{115P}{100} + \frac{110P}{100} + \frac{105P}{100} + P = 770 \quad \therefore \quad \frac{550P}{100} = 770$$

$$\therefore \quad P = \frac{770 \times 100}{550} = 140$$

Hence, the annual payment = ₹ 140.



Theorem: The annual payment that will discharge a debt of ₹ A due in t years at the rate of interest $r\%$ per annum is $\frac{100A}{100t + \frac{rt(t-1)}{2}}$.

EXAMPLE 3 What annual payment will discharge a debt of ₹ 848 in 4 years at 4% p.a.?

Solution By the theorem: Annual payment = $\frac{100A}{100N + \frac{R(N-1)N}{2}}$

Here $A = ₹ 848$; $N = 4$ years; $R = 4\%$.

$$\therefore \quad \text{Annual payment} = \frac{848 \times 100}{4 \times 100 + \frac{4(3)(4)}{2}} = ₹ 200$$

EXAMPLE 4 The rate of interest for the first two years is 3% per annum, for the next three years is 8% per annum and for the period beyond 5 years 10% per annum. If a man gets ₹ 1,520 at simple interest for six years, how much money will he deposit?

Solution Let the deposit be ₹ 100.

Then interest for first 2 years @ 3% p.a = ₹ 6.

Interest for next 3 years @ 8% p.a. = ₹ 24.

Interest for the last year = ₹ 10.

Total interest = ₹ 40

∴ Deposited amount when interest is ₹ 1520 = $\frac{100}{40} \times 1520 = ₹ 3800$



Short cut: Deposited amount or Principal =
$$\frac{\text{Interest} \times 100}{(r_1 t_1 + r_2 t_2 + r_3 t_3 + \dots)}$$

$$= \frac{1520 \times 100}{3 \times 2 + 8 \times 3 + 10 \times 1}$$

$$= \frac{1520 \times 100}{40} = ₹ 3800$$

EXAMPLE 5 A sum of money doubles itself in 10 years at simple interest. What is the rate of interest?

Solution Let the sum be ₹ 100.

After 10 years amount = ₹ 200

∴ Interest accrued $I = 200 - 100 = ₹ 100$

∴ Rate $r = \frac{100I}{Pt} = \frac{100 \times 100}{100 \times 10} = 10\%$



Short cut: Rate =
$$\frac{100(\text{Multiple number of Principal} - 1)}{\text{Time}}$$

∴ Rate = $\frac{100(2 - 1)}{10} = 10\%$

EXAMPLE 6 A sum of money becomes four times at simple interest rate at 5% per annum in what time?

Solution Time =
$$\frac{100(\text{Multiple number of principal} - 1)}{\text{Rate}}$$

$$= \frac{100(4 - 1)}{5} = 60 \text{ years}$$

EXAMPLE 7 A certain sum of money amounts to ₹ 756 in 2 years and to ₹ 873 in 3.5 years. Find the sum and the rate of interest.

Solution $P + SI$ for 2 years = 756 (1)

$P + SI$ for 3.5 years = 873 (2)

$$I = \frac{Prt}{100} 9$$

∴ SI for 1.5 years = $873 - 756 = 117$

$$\begin{aligned} \text{i.e.,} \quad r &= \frac{I \times 100}{P \times t} = \frac{117 \times 100}{P \times 1.5} \\ Pr &= \frac{11700}{1.5} = \text{₹ } 7800 \end{aligned} \quad (3)$$

$$\therefore \quad \text{For 2 years, interest} = \frac{P \times 2 \times r}{100} = 756 - P$$

$$\text{i.e.,} \quad \frac{2 \times 7800}{100} = 756 - P$$

$$\Rightarrow \quad P = 756 - 156 = \text{₹ } 600$$

$$\therefore \quad (3) \Rightarrow r = \frac{7800}{P} = \frac{7800}{600} = 13\% \text{ p.a.}$$

EXAMPLE 8 A sum was put at simple interest at a certain rate for 2 years. Had it been put at 3% higher rate, it would have fetched ₹ 300 more. Find the sum.

Solution Let the sum be x deposited at $y\%$ per annum.

$$\text{For simple interest deposit for 2 years, } I_1 = \frac{x \times 2 \times y}{100}$$

When the interest rate is 3% more (i.e. $y + 3$)

$$I_2 = \frac{x \times 2 \times (y + 3)}{100} = 300 + I_1$$

$$\text{i.e.,} \quad I_2 - I_1 = \frac{2x}{100}[(y + 3) - y] = 300$$

$$\Rightarrow \quad 3x = \frac{100 \times 300}{2} = \text{₹ } 15000$$

$$\text{or} \quad x = \text{₹ } 5000$$



$$\text{Short cut: Sum} = \frac{\text{More interest} \times 100}{\text{Time} \times \text{More rate}} = \frac{300 \times 100}{2 \times 3} = \text{₹ } 5000$$

➤ Theorem: A sum X is lent out in n parts in such a way that the interest on the first part at $r_1\%$ for t_1 years, the interest on the second part at $r_2\%$ for t_2 years, the interest on the third part at $r_3\%$ for t_3 years and so on are equal. The ratio in which the sum was divided in n parts is given by $\frac{1}{r_1 t_1} : \frac{1}{r_2 t_2} : \frac{1}{r_3 t_3} : \dots : \frac{1}{r_n t_n}$.

EXAMPLE 9 A sum of ₹ 26000 is lent out in two parts in such a way that the interest on one part at 10% for 5 years is equal to that on another part at 9% for 6 years. Find the two sums.

$$\text{Solution} \quad \text{Interest} = \frac{P_1 \times 5 \times 10}{100} = \frac{P_2 \times 6 \times 9}{100}$$

$$\text{or} \quad \frac{P_1}{P_2} = \frac{6 \times 9}{5 \times 10} = \frac{27}{25} = 27 : 25$$

$$\therefore \text{1st part} = 26000 \times \frac{27}{(27 + 25)} = \text{₹ } 13500$$

$$\therefore \text{2nd part} = 26000 - 13,500 = \text{₹ } 12500$$

Using the above shortcut,

$$\begin{aligned} \text{1st part} : \text{2nd part} &= \frac{1}{r_1 t_1} : \frac{1}{r_2 t_2} \\ &= \frac{1}{10 \times 5} : \frac{1}{9 \times 6} = \frac{1}{50} : \frac{1}{54} = 54 : 50 = 27 : 25 \end{aligned}$$

EXAMPLE 10 A certain sum amounted to ₹ 575 at 5% in a time in which ₹ 750 amounted to ₹ 840 at 4%. If the rate of interest is simple, find the sum.

Solution Interest = 840 – 750 = 90

$$\therefore \text{Time} = \frac{90 \times 100}{750 \times 4} = 3 \text{ years}$$

$$\therefore \text{Sum} = \frac{100 \times \text{Amount}}{100 + rt} = \frac{100 \times 575}{100 + 5 \times 3} = \text{₹ } 500$$



Short cut: $\text{Sum} = \frac{100 \times \text{Amount}}{100 + rt}$.

EXAMPLE 11 A person lent a certain sum at 4% per annum simple interest, and in 8 years the interest amounted to ₹ 340 less than the sum lent. Find the sum lent.

Solution Let the sum be ₹ x .

$$\begin{aligned} \therefore \text{Interest} &= \frac{Prt}{100} = \frac{x \times 4 \times 8}{100} = \frac{32x}{100} \\ &= x - 340 \end{aligned}$$

$$\therefore x - \frac{32x}{100} = 340$$

$$\Rightarrow \frac{68x}{100} = 340$$

$$\Rightarrow x = \frac{340 \times 100}{68} = \text{₹ } 500$$



Short cut: $\text{Sum} = \frac{100}{100 - 8 \times 4} \times 340 = \text{₹ } 500$

i.e. $\text{Sum} = \frac{100}{100 - rt} \times (\text{Difference in Principal and Interest})$

➤ Theorem: When different amounts mature at the same amount at simple rate of interest, the ratio of amounts invested is in inverse ratio of $(100 + \text{rate} \times \text{time})$. That is, the ratio in which

amounts are invested is $\frac{1}{100 + r_1 t_1} : \frac{1}{100 + r_2 t_2} : \frac{1}{100 + r_3 t_3} : \dots : \frac{1}{100 + r_n t_n}$

EXAMPLE 12 Two equal amounts of money are deposited in two banks each at 15% per annum for 3.5 years and 5 years respectively. If the difference between their interests is ₹ 144, find each sum.

Solution Let the sum be ₹ x .

$$\text{Then} \quad \frac{x \times 15 \times 5}{100} - \frac{x \times 15 \times 7}{200} = 144$$

$$\therefore \quad \frac{x}{200} (150 - 105) = 144$$

$$\text{i.e.} \quad x = \frac{200 \times 144}{45} = \text{₹ } 640$$



Short cut: Two equal amounts are deposited at $r_1\%$ and $r_2\%$ for t_1 years and t_2 years respectively. If difference in interest is I_d , then

$$\text{Sum} = \frac{I_d \times 100}{r_1 t_1 - r_2 t_2}$$

$$\text{Here,} \quad \text{Sum} = \frac{144 \times 100}{15 \times 5 - 15 \times 3.5} = \frac{144 \times 100}{22.5} = \text{₹ } 640$$

EXAMPLE 13 Some amount out of ₹ 7000 was lent at 6% per annum and the remaining at 4% per annum. If the total simple interest from both the fractions in 5 years was ₹ 1600, find the sum lent at 6% per annum.

Solution Let the amount be ₹ x that was lent at 6% per annum simple interest.

$$\therefore \quad \frac{x \times 6 \times 5}{100} + \frac{(7000 - x) \times 4 \times 5}{100} = 1600$$

$$\text{i.e.,} \quad \frac{3x}{10} + \frac{7000 - x}{5} = 1600$$

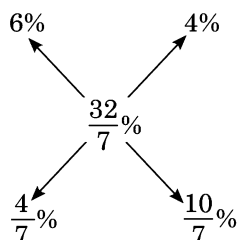
$$\text{or} \quad 3x + 2(7000 - x) = 16000$$

$$\text{or} \quad x + 14000 = 16000$$

$$\text{or} \quad x = 16000 - 14000 = \text{₹ } 2000$$

By method of Alligation:

$$\text{Overall rate of interest} = \frac{1600 \times 100}{5 \times 7000} = \frac{32}{7}\%$$



∴ Ratio of two amounts = 2 : 5.

∴ Amount lent at 6% = $7000 \times \frac{2}{7} = ₹ 2000$

EXAMPLE 14 A person borrows ₹ 5,000 for 2 years at 4% p.a. simple interest. He immediately lends it to another person at $6\frac{1}{4}$ % p.a. for 2 years. Find his gain in the transaction per year. [SSC]

Solution

$$\begin{aligned}\text{Gain in 2 years} &= \left(5000 \times \frac{25}{4} \times \frac{1}{100} \times 2\right) - \left(5000 \times 4 \times \frac{2}{100}\right) \\ &= 5000 \times \frac{2}{100} \times \left(\frac{25}{4} - 4\right) = 5000 \times \frac{2}{100} \times \left(\frac{25 - 16}{4}\right) \\ &= 100 \times \frac{9}{4} = ₹ 225\end{aligned}$$

∴ Gain in 1 year = $\frac{225}{2} = ₹ 112.50$

EXAMPLE 15 A sum of ₹ 12,500 amounts to ₹ 15,500 in 4 years at the rate of simple interest. What is the rate of interest? [Bank PO]

Solution Let $R\%$ be the rate of interest p.a.

$$\therefore A = P + \frac{PNR}{100}$$

$$\text{or } I = \frac{PNR}{100}$$

$$\therefore 15500 - 12500 = \frac{12500 \times 4 \times R}{100}$$

$$\text{or } R = \frac{3000 \times 100}{50000} = 6\%$$

EXAMPLE 16 A sum fetched a total simple interest of ₹ 4016.25 at the rate of 9% p.a. in 5 years. What is the sum? [NABARD]

Solution Let P be the sum.

$$\therefore P \times 5 \times \frac{9}{100} = 4016.25$$

$$\begin{aligned}\therefore P &= \frac{4016.25 \times 100}{9 \times 5} = \left(\frac{401625}{9 \times 5}\right) \\ &= ₹ \left(\frac{401625}{45}\right) = ₹ 8925\end{aligned}$$

EXAMPLE 17 Simple interest on a certain amount is $\frac{9}{16}$ of the principal. If the numbers representing the rate of interest in per cent and time in years be equal, then time, for which the principal is lent out is: **[RRB]**

Solution Let P be the principal and N be the period in years

$$\therefore \text{Simple interest} = \frac{P \times N \times R}{100} = \frac{9}{16}P$$

$$\text{or} \quad \frac{NR}{100} = \frac{9}{16}$$

Given that $N = R$.

$$\therefore \quad \frac{N^2}{100} = \frac{9}{16}$$

Taking square root on both sides,

$$\frac{N}{10} = \frac{3}{4}$$

$$\text{or} \quad N = \frac{10 \times 3}{4} = 7\frac{1}{2} \text{ years}$$

EXAMPLE 18 The difference between the simple interest received from two different sources on ₹ 1500 for 3 years is ₹ 13.50. The difference between their rates of interest is: **[RRB]**

Solution Let R_1 and R_2 be the rates of interest.

$$\therefore \quad \frac{1500 \times 3 \times R_1}{100} - \frac{1500 \times 3 \times R_2}{100} = 13.50$$

$$\therefore \quad R_1 - R_2 = \frac{13.50 \times 100}{1500 \times 3} = \frac{1350}{4500} = 0.3\%$$



EXERCISES

- If ₹ 640 amounts to ₹ 832 in 2 years, what will ₹ 860 amount to in 4 years at the same rate of interest per annum?
(a) ₹ 1736 (b) ₹ 1376 (c) ₹ 3176 (d) ₹ 3716 (e) None of these
- A sum of ₹ 25400 was lent out in two parts, one at 12% and the other at $12\frac{1}{2}\%$. If the total annual income is 3124.2, the money lent at 12% is _____.
(a) 15240 (b) 25400 (c) 10160 (d) 31242 (e) None of these
- A sum of ₹ 26000 was lent out in two parts in such a way that the interest on one part at 10% for 5 years is equal to that on the other part at 9% for 6 years. The sum lent out at 10% is _____.
(a) 12500 (b) 13500 (c) 26000 (d) 39000 (e) None of these

4. A sum of money will double itself in 16 years at simple interest at an yearly rate of _____.
(a) 25% (b) $12\frac{1}{2}\%$ (c) $6\frac{1}{4}\%$ (d) $8\frac{1}{3}\%$ (e) None of these
5. A sum of money put at simple interest trebles itself in 15 years. The rate per cent per annum is _____.
(a) $6\frac{1}{3}\%$ (b) $8\frac{1}{3}\%$ (c) $13\frac{1}{3}\%$ (d) $12\frac{1}{3}\%$ (e) None of these
6. ₹ 8000 amounts to ₹ 9200 in 3 years at simple interest. If the interest rate is increased by 3%, to how much would it amount?
(a) 9400 (b) 9600 (c) 9720 (d) 9920 (e) None of these
7. A man invested $\frac{1}{3}$ of his capital at 7%, $\frac{1}{4}$ at 8% and the remainder at 10%. If the annual income is ₹ 561, the capital is _____.
(a) 4400 (b) 5500 (c) 6600 (d) 5800 (e) None of these
8. If the interest on ₹ 12000 be more than the interest on ₹ 10000 by ₹ 500 in 3 years, the rate per cent is _____.
(a) $6\frac{1}{4}\%$ (b) $12\frac{1}{2}\%$ (c) $8\frac{1}{3}\%$ (d) $16\frac{1}{3}\%$ (e) None of these
9. A sum was put at simple interest at a certain rate for 2 years. Had it been put at 1% higher rate, it would have fetched ₹ 24 more. The sum is _____.
(a) ₹ 600 (b) ₹ 800 (c) ₹ 1200 (d) ₹ 1600 (e) None of these
10. A sum of money becomes $\frac{8}{5}$ of itself in 5 years at a certain rate of interest. The rate per cent per annum is _____.
(a) 8% (b) 10% (c) 12% (d) $12\frac{1}{2}\%$ (e) None of these
11. The difference between the interest received from two different banks on ₹ 5,000 for 2 years is ₹ 25. The difference between their rate is _____.
(a) 3% (b) 2% (c) $1\frac{1}{2}\%$ (d) 0.25% (e) None of these
12. Two equal amounts are deposited in two banks, each at 15% per annum, for $3\frac{1}{2}$ years and 5 years respectively. If the difference between their interests is ₹ 1440, each sum is _____.
(a) ₹ 4600 (b) ₹ 5000 (c) ₹ 6400 (d) ₹ 7200
13. The rate of interest on a sum is 4% per annum for the first 2 years, 6% per annum for the next 4 years and 8% per annum for the period beyond 6 years. If the simple interest accrued by the sum for a total period of 9 years is ₹ 1120, find the sum.
(a) ₹ 1000 (b) ₹ 2000 (c) ₹ 3000 (d) ₹ 4000 (e) None of these
14. The simple interest on a sum is $\frac{1}{16}$ of the principal and the number of years is equal to the rate per cent per annum. The rate per cent per annum is _____.
(a) $1\frac{1}{2}\%$ (b) $2\frac{1}{2}\%$ (c) $3\frac{1}{2}\%$ (d) $4\frac{1}{2}\%$ (e) None of these
15. A lent ₹ 6000 to B for 2 years and ₹ 1500 to C for 4 years and received altogether from both ₹ 900 as simplex interest. The rate of interest is _____.
(a) 2% (b) 3% (c) 4% (d) 5% (e) None of these
16. Divide ₹ 2379 into 3 parts so that their amounts after 2, 3 and 4 years respectively may be equal, the rate of interest being 5% per annum at simple interest. The first part is: [CBI]
(a) ₹ 759 (b) ₹ 792 (c) ₹ 818 (d) 828

17. David invested certain amount in three different schemes A, B and C with the rate of interest 10% p.a., 12% p.a. and 15% p.a. respectively. If the total interest accrued in one year was ₹ 3200 and the amount invested in scheme C was 150% of the amount invested in scheme A and 240% of the amount invested in scheme B, what was the amount invested in scheme B?

[Bank PO]

- (a) ₹ 5000 (b) ₹ 6500 (c) ₹ 8000
(d) Cannot be determined (e) None of these

18. A money lender finds that due to a fall in the annual rate of interest from 8% to $7\frac{3}{4}\%$, his yearly income diminishes by ₹ 61.50. His capital is: **[SSC]**

- (a) ₹ 22400 (b) ₹ 23800 (c) ₹ 24600 (d) ₹ 26000

19. If the annual rate of simple interest increases from 10% to $12\frac{1}{2}\%$, a man's yearly income increases by ₹ 1250. His principal (in ₹) is: **[SSC]**

- (a) ₹ 45000 (b) ₹ 50000 (c) ₹ 60000 (d) ₹ 65000

20. Peter invested an amount of ₹ 12,000 at the rate of 10% p.a. simple interest and another amount at the rate of 20% p.a. simple interest. The total interest earned at the end of one year on the total amount invested became 14% p.a. Find the total amount invested.

[SBI PO]

- (a) ₹ 20000 (b) ₹ 22000 (c) ₹ 24000 (d) ₹ 25000 (e) None of these

21. A lent ₹ 5,000 to B for 2 years and ₹ 3,000 to C for 4 years on simple interest at the same rate of interest and received ₹ 2200 in all from both of them as interest. The rate of interest per annum is: **[CBI]**

- (a) 5% (b) 7% (c) $7\frac{1}{8}\%$ (d) 10%

22. A lends ₹ 2500 to B and a certain sum to C at the same time at 7% p.a. simple interest. If after 4 years, A altogether receives ₹ 1120 as interest from B and C, then the sum lent to C is: **[SSC]**

- (a) ₹ 700 (b) ₹ 1500 (c) ₹ 4000 (d) ₹ 6500

23. At what rate per cent per annum will the simple interest on a sum of money be $\frac{2}{5}$ of the amount in 10 years? **[SSC]**

- (a) 4% (b) $5\frac{2}{3}\%$ (c) 6% (d) $6\frac{2}{3}\%$

24. At what rate per cent of simple interest will a sum of money double itself in 12 years?

[SSC]

- (a) $8\frac{1}{4}\%$ (b) $8\frac{1}{3}\%$ (c) $8\frac{1}{2}\%$ (d) $9\frac{1}{2}\%$

25. A sum of money lent out at simple interest amounts to ₹ 720 after 2 years and to ₹ 1,020 after a further period of 5 years. The sum is: **[SSC]**

- (a) ₹ 500 (b) ₹ 600 (c) ₹ 710 (d) ₹ 700

26. A sum of money amounts to ₹ 9,800 after 5 years and ₹ 12,005 after 8 years at the same rate of simple interest. The rate of interest per annum is: **[SSC]**

- (a) 5% (b) 8% (c) 12% (d) 15%

27. Nitin borrowed some money at the rate of 6% p.a. for the first three years, 9% p.a. for the next five years and 13% p.a. for the period beyond 8 years. If the total interest paid by him at the end of eleven years is ₹ 8,160, how much money did he borrow? [Bank PO]
(a) ₹ 8000 (b) ₹ 10000 (c) ₹ 12000 (d) Data inadequate
(e) None of these
28. A sum invested at 5% simple interest per annum grows to ₹ 504 in 4 years. The same amount at 10% simple interest per annum in $2\frac{1}{2}$ years will grow to: [CDS]
(a) ₹ 420 (b) ₹ 450 (c) ₹ 525 (d) ₹ 550
29. In how many years, ₹ 150 will produce the same interest @8% as ₹ 800 produce in 3 years @ $4\frac{1}{2}$ %? [RRB]
(a) 6 (b) 8 (c) 9 (d) 12
30. What will be the ratio of simple interest earned by certain amount at the same rate of interest for 6 years and that of 9 years? [Bank PO]
(a) 1:3 (b) 1:4 (c) 2:3 (d) Data inadequate
(e) None of these
31. ₹ 800 becomes ₹ 956 in 3 years at a certain rate of simple interest. If the rate of interest is increased by 4%, what amount will ₹ 800 become in 3 years? [Bank PO]
(a) ₹ 1020.80 (b) ₹ 1025 (c) ₹ 1052 (d) Data inadequate
(e) None of these
32. A certain amount earns simple interest of ₹ 1,750 after 7 years. Had the interest been 2% more, how much more interest would it have earned? [Bank PO]
(a) ₹ 35 (b) ₹ 245 (c) ₹ 350 (d) Cannot be determined
(e) None of these
33. What is the present worth of ₹ 132 due in 2 years at 5% simple interest per annum? [CBI]
(a) ₹ 112 (b) ₹ 118.80 (c) ₹ 120 (d) ₹ 122
34. A man took a loan from a bank at the rate of 12% p.a. simple interest. After 3 years he had to pay ₹ 5400 interest only for the period. The principal amount borrowed by him was: [SSC]
(a) ₹ 2000 (b) ₹ 10000 (c) ₹ 15000 (d) ₹ 20000
35. Reena took a loan of ₹ 1,200 with simple interest for as many years as the rate of interest. If she paid ₹ 432 as interest at the end of the loan period, what was the rate of interest? [RBI]
(a) 3.6 (b) 6 (c) 18 (d) Cannot be determined
(e) None of these
36. Adam borrowed some money at the rate of 6% p.a. for the first two years, at the rate of 9% p.a. for the next three years, and at the rate of 14% p.a. for the period beyond five years. If he pays a total interest of ₹ 11,400 at the end of nine years, how much money did he borrow? [Bank PO]
(a) ₹ 10000 (b) ₹ 11000 (c) ₹ 12000 (d) ₹ 13000 (e) None of these
37. At what rate per cent per annum will a sum of money double in 16 years? [RRB]
(a) $5\frac{1}{4}$ % (b) $6\frac{1}{4}$ % (c) $7\frac{1}{4}$ % (d) $8\frac{1}{4}$ % (e) None of these
38. A sum of ₹ 1,550 is lent out into two parts, one at 8% and another one at 6%. If the total annual income is ₹ 106, find the money lent at each rate. [LIC AAO]
(a) ₹ 600, 950 (b) ₹ 650, 900 (c) ₹ 700, 850 (d) ₹ 550, 1,000 (e) None of these

39. The simple interest on a sum of money is $\frac{4}{9}$ of the principal. Find the rate per cent and time, if both are numerically equal. [SSC]

- (a) $6\frac{1}{3}\%$, 6 years 4 months (b) $6\frac{2}{3}\%$, 6 years 8 months
(c) $5\frac{2}{3}\%$, 5 years 8 months (d) $5\frac{1}{3}\%$, 5 years 4 months (e) None of these

40. An automobile financier claims to be lending money at simple interest, but he includes the interest every six months for calculating the principal. If he is charging an interest of 10%, the effective rate of interest becomes: [NIFT]

- (a) 10% (b) 10.25% (c) 10.5% (d) None of these



1. (b) 2. (c) 3. (b) 4. (c) 5. (c) 6. (d) 7. (c) 8. (c) 9. (c) 10. (c)
11. (d) 12. (c) 13. (b) 14. (b) 15. (d) 16. (d) 17. (a) 18. (c) 19. (b) 20. (a)
21. (d) 22. (b) 23. (a) 24. (b) 25. (b) 26. (c) 27. (a) 28. (c) 29. (c) 30. (c)
31. (c) 32. (d) 33. (c) 34. (c) 35. (b) 36. (c) 37. (b) 38. (b) 39. (b) 40. (b)



Solutions with Necessary Explanation

1. Rate of interest = $\frac{192 \times 100}{640 \times 2} = 15\%$

$\therefore$ Interest on ₹ 860 = $\frac{860 \times 15 \times 4}{100} = ₹ 516$

$\therefore$ Amount = $860 + 516 = ₹ 1376$



Short cut: Interest on ₹ 640 for 2 years is ₹ 192.

$\therefore$ Interest on ₹ 860 for 4 years

$$= 192 \left[\frac{860}{640} \right] \times \left[\frac{4}{2} \right] = ₹ 516$$

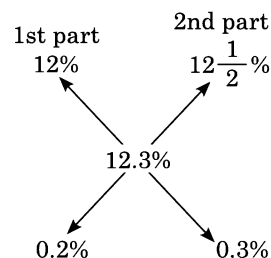
$\therefore$ Amount = $860 + 516 = ₹ 1376$

2. Overall rate of interest = $\frac{3124.2}{25400} \times 100 = 12.3\%$

$\therefore$ The sum will be divided in the ratio 0.2 : 0.3 or 2 : 3.

$\therefore$ The sum lent at 12% = $25400 \times \frac{2}{5} = ₹ 10160$

The sum lent at $12\frac{1}{2}\%$ = $25400 \times \frac{3}{5} = ₹ 15240$





3. **Short cut:** Ratio of two parts = $r_2 t_2 : r_1 t_1$
 $= 9 \times 6 : 10 \times 5 = 54 : 50 = 27 : 25$

$$\therefore \text{Sum lent out at 10\%} = 26000 \times \frac{27}{(27 + 25)} = \text{₹ } 13500$$

4. The rate of interest = $\frac{100(2 - 1)}{16} = 6\frac{1}{4}\%$

5. The rate of interest = $\frac{100(3 - 1)}{15} = \frac{200}{15} = 13\frac{1}{3}\%$

6. The rate of interest = $\frac{1200 \times 100}{8000 \times 3} = 5\%$

New rate becomes $5 + 3 = 8\%$

$$\therefore \text{Interest} = \frac{8000 \times 8 \times 3}{100} = \text{₹ } 1920$$

$$\therefore \text{Amount} = 8000 + 1920 = \text{₹ } 9920$$



Short cut: Increase in interest = $\frac{8000 \times 3 \times 3}{100} = \text{₹ } 720$

$$\therefore \text{Increased amount} = 9200 + 720 = \text{₹ } 9920$$

7. Let the capital be ₹ 120

$$\text{Then total interest} = 7\% \text{ of } \frac{120}{3} + 8\% \text{ of } \frac{120}{4} + 10\% \text{ of remainder.}$$

$$= 7\% \text{ of } 40 + 8\% \text{ of } 30 + 10\% \text{ of } 50$$

$$= 2.8 + 2.4 + 5 = \text{₹ } 10.2$$

$$\therefore \text{Actual capital} = \frac{120}{10.2} \times 561 = \text{₹ } 6600$$



Short cut: Capital = $\frac{561}{\left(7\% \text{ of } \frac{1}{3} + 8\% \text{ of } \frac{1}{4} + 10\% \text{ of } \frac{5}{12}\right)}$

$$= \frac{561 \times 100}{\frac{7}{3} + \frac{8}{4} + \frac{25}{6}} = \frac{561 \times 100 \times 12}{28 + 24 + 50} = \text{₹ } 6600$$

8. Let the rate of interest be $x\%$

$$\text{Then} \quad \frac{12000 \times 3x}{100} = \frac{10000 \times 3x}{100} + 500$$

$$\therefore 360x = 300x + 500$$

i.e. $60x = 500$

$\therefore x = \frac{500}{60} = 8\frac{1}{3}\%$



Short cut: $\text{Rate} = \frac{\text{Difference in interest} \times 100}{\text{Time (Difference in principal)}}$

$$= \frac{500 \times 100}{3(12000 - 10000)} = \frac{25}{3} = 8\frac{1}{3}\%$$

9. Let the sum deposited be P at $r\%$ per annum

$\therefore \frac{P \times r \times 2}{100} = \frac{P(r+1) \times 2}{100} - 24$

i.e. $\frac{2P}{100} - 24 = 0$

or $P = \frac{24 \times 100}{2} = \text{₹ } 1200$

$\therefore$ The sum deposited = ₹ 1200



Short cut: $\text{Sum} = \frac{(\text{Difference in interests}) \times 100}{\text{Time} \times \text{Difference in rate}}$

$$= \frac{24 \times 100}{2 \times 1} = \text{₹ } 1200$$

10. $\text{Rate} = \frac{100 \left(\frac{8}{5} - 1 \right)}{\text{Time}} = \frac{100 \times \frac{3}{5}}{5} = \frac{300}{25} = 12\%$

11. $\text{Sum} = \frac{\text{Difference in interest} \times 100}{\text{Time} \times \text{Difference in rate}}$

i.e. $5000 = \frac{25 \times 100}{2 \times x}$

i.e. $x = \frac{2500}{5000 \times 2} = \frac{1}{4} = 0.25\%$

12. $\text{Sum} = \frac{\text{Difference in interests} \times 100}{\text{Rate} \times \text{Difference in times}} = \frac{1440 \times 100}{15 \times 1.5} = \text{₹ } 6400$

13. $\text{Sum} = \frac{\text{Interest} \times 100}{r_1 t_1 + r_2 t_2 + r_3 t_3 + \dots} = \frac{1120 \times 100}{4 \times 2 + 6 \times 4 + 8 \times 3} = \frac{1120 \times 100}{56} = \text{₹ } 2000$

14. Let the rate of interest be $r\%$.

$\therefore$ time = r years

$\therefore \frac{S}{16} = \frac{S \times r \times r}{100}$

$$\text{or} \quad r^2 = \frac{100}{16} = \frac{25}{4}$$

$$\text{or} \quad r = \sqrt{\frac{25}{4}} = \frac{5}{2} = 2\frac{1}{2}\%$$

15. Let the rate of interest be $r\%$ per annum.

$$\text{Then} \quad \frac{6000 \times 2 \times r}{100} + \frac{1500 \times 4 \times r}{100} = 900$$

$$\text{i.e.} \quad 120r + 60r = 900$$

$$\therefore \quad r = \frac{900}{180} = 5\%$$

16. x, y and z be the three parts such that $x + y + z = 2379$

$\therefore$ As the three amounts with interest are separately equal,

$$x + \frac{x \times 2 \times 5}{100} = y + \frac{y \times 3 \times 5}{100} = z + \frac{z \times 4 \times 5}{100}$$

$$\text{i.e.} \quad \frac{11x}{10} = \frac{23y}{20} = \frac{6z}{5} = k$$

$$\text{or} \quad x = \frac{10k}{11}; y = \frac{20k}{23}; z = \frac{5k}{6};$$

$$\therefore \quad x + y + z = \frac{10k}{11} + \frac{20k}{23} + \frac{5k}{6} = 2379$$

$$\Rightarrow \quad 1380k + 1320k + 1265k = 2379 \times 11 \times 23 \times 6$$

$$\Rightarrow \quad 3965k = 2379 \times 11 \times 23 \times 6$$

$$\Rightarrow \quad k = \frac{2379 \times 11 \times 23 \times 6}{3965} = \frac{3 \times 11 \times 23 \times 6}{5}$$

$$\therefore \quad x = \frac{10k}{11} = \frac{10 \times 3 \times 11 \times 23 \times 6}{11 \times 5} = ₹ 828$$

17. Let the amount invested in scheme B be x .

$$\therefore \quad \text{Investment in scheme C} = \frac{240}{100} \times x = 2.4x$$

$$\therefore \quad \text{Investment in scheme A} = \frac{240}{100} \times x \times \frac{100}{150} = 1.6x$$

$$\text{Total interest accrued} = 1.6x \times \frac{10}{100} + x \times \frac{12}{100} + 2.4x \times \frac{15}{100} = 3200$$

$$\text{i.e.,} \quad 0.16x + 0.12x + 0.36x = 3200$$

$$\text{or} \quad 0.64x = 3200$$

$$\text{or} \quad x = \frac{3200}{0.64} = 5000$$

i.e. Investment in scheme B = ₹ 5000

18. Let the capital be x .

$$x \times \left(8 - 7\frac{3}{4}\right)\% = 61.50$$

$$\text{or} \quad x \times \frac{1}{400} = 61.50$$

$$\text{or} \quad x = 400 \times 61.50 = \text{₹ } 24600$$

19. Let his principal (in ₹) be x .

$$\therefore x \times \left(12\frac{1}{2} - 10\right)\% = 1250$$

$$\text{or} \quad x = \frac{1250}{2\frac{1}{2}\%}$$

$$= \frac{1250}{2.5} \times 100 = \text{₹ } 50000$$

20. Let the total amount invested be x .

$$\therefore 12000 \times \frac{10}{100} + (x - 12000) \times \frac{20}{100} = x \times \frac{14}{100}$$

$$\Rightarrow 1200 + \frac{x - 12000}{5} = \frac{14x}{100}$$

$$\Rightarrow \frac{x}{5} - \frac{14x}{100} = \frac{12000}{5} - 1200$$

$$\Rightarrow \frac{x}{5} - \frac{7x}{50} = 2400 - 1200$$

$$\Rightarrow \frac{10x - 7x}{50} = 1200$$

$$\Rightarrow 3x = 1200 \times 50$$

$$\Rightarrow x = \frac{1200 \times 50}{3}$$

$$= \text{₹ } 20000$$

21. Let the rate of simple interest be $x\%$.

$$\therefore 5000 \times 2 \times \frac{x}{100} + 3000 \times 4 \times \frac{x}{100} = 2200$$

$$\text{i.e.} \quad 100x + 120x = 2200$$

$$x = \frac{2200}{100 + 120} = 10\%$$

22. Let the sum lent to C be x .

$$\therefore 2500 \times \frac{7}{100} \times 4 + x \times \frac{7}{100} \times 4 = 1120$$

$$\Rightarrow 700 + \frac{7x}{25} = 1120$$

$$\Rightarrow \frac{7x}{25} = 1120 - 700$$

$$\Rightarrow x = \frac{420 \times 25}{7}$$

$$= \text{₹ } 1500$$

23. Let x be the rate per cent per annum and P be the principal.

$$\therefore P \times 10 \times \frac{x}{100} = \frac{2}{5}P$$

$$\Rightarrow x = \frac{2}{5} \times \frac{100}{10} = 4\%$$

24. Let x be the rate per cent per annum and P be the principal.

$$\therefore P \times 12 \times \frac{x}{100} = P$$

$$\Rightarrow x = \frac{100}{12} \% = 8\frac{1}{3} \%$$

25. Let the sum be P and rate be $x\%$.

$$\therefore P \times 2 \times \frac{x}{100} + P = 720$$

$$\text{or } P \left(1 + \frac{x}{50} \right) = 720 \quad (1)$$

$$P \times 7 \times \frac{x}{100} + P = 1020$$

$$\text{or } P \left(1 + \frac{7x}{100} \right) = 1020 \quad (2)$$

$$\frac{(1)}{(2)} \Rightarrow \frac{\left(1 + \frac{x}{50} \right)}{\left(1 + \frac{7x}{100} \right)} = \frac{720}{1020}$$

$$\Rightarrow \frac{\left(\frac{100 + 2x}{100} \right)}{\left(\frac{100 + 7x}{100} \right)} = \frac{720}{1020}$$

$$\Rightarrow \frac{(100 + 2x)}{(100 + 7x)} = \frac{12}{17}$$

$$\Rightarrow x = 10\%$$

$$\therefore (1) \Rightarrow P = \frac{720}{\left(1 + \frac{x}{50} \right)} = \frac{720}{\left(1 + \frac{10}{50} \right)}$$

$$= \frac{720}{60/50} = \frac{720 \times 50}{60} = \text{₹ } 600$$

26. Let the rate of interest be $x\%$ and sum be ₹ P .

$$\therefore P + P \times 5 \times \frac{x}{100} = 9800$$

$$\text{or} \quad P \left(1 + \frac{x}{20} \right) = 9800 \quad (1)$$

$$\text{Similarly} \quad P + P \times \frac{8x}{100} = 12005$$

$$\text{or} \quad P \left(1 + \frac{8x}{100} \right) = 12005 \quad (2)$$

$$\frac{(2)}{(1)} \Rightarrow \frac{\left(1 + \frac{8x}{100} \right)}{\left(1 + \frac{x}{20} \right)} = \frac{12005}{9800}$$

$$\Rightarrow \frac{(100 + 8x)/100}{(20 + x)/20} = \frac{12005}{9800}$$

$$\Rightarrow \frac{100 + 8x}{100 + 5x} = \frac{12005}{9800} = \frac{2401}{1960} = \frac{343}{280}$$

$$\frac{4(25 + 2x)}{5(20 + x)} = \frac{49}{40}$$

$$\text{i.e.} \quad \left(\frac{25 + 2x}{20 + x} = \frac{49}{32} \right)$$

$$\Rightarrow x = 12\%$$

27. Let the amount borrowed by Nitin be ₹ x .

$$\therefore x \times 3 \times \frac{6}{100} + x \times \frac{5 \times 9}{100} + x \times \frac{3 \times 13}{100} = 8160$$

$$\text{i.e.} \quad x \left(\frac{18}{100} + \frac{45}{100} + \frac{39}{100} \right) = 8160$$

$$\text{or} \quad x \times \frac{102}{100} = 8160$$

$$\text{or} \quad x = \frac{8160 \times 100}{102} = 8000$$

$\therefore$ Amount borrowed by Nitin = ₹ 8000

28. Let P be the sum.

$$P + P \times 4 \times \frac{5}{100} = 504$$

$$\text{or} \quad \frac{6P}{5} = 504 \quad (1)$$

$$\begin{aligned}
 P + P \times \frac{5}{2} \times \frac{10}{100} &= \frac{5P}{4} \\
 &= \frac{5}{4} \times \left(\frac{5 \times 504}{6} \right) \quad [\text{value of } P \text{ from Eq. (1)}] \\
 &= 5 \times 5 \times \frac{504}{24} = 25 \times 21 = \text{₹ } 525
 \end{aligned}$$

29. Let the number of years be N .

$$\begin{aligned}
 150 \times N \times \frac{8}{100} &= 800 \times 3 \times \frac{9}{200} \\
 \Rightarrow 12N &= 108 \\
 \text{or } N &= 9 \text{ years}
 \end{aligned}$$

30. $N_1 = 6$ years; $N_2 = 9$ years; $R_1 = R_2$;

$$\begin{aligned}
 \frac{PN_1R_1}{100} + \frac{PN_2R_2}{100} &= \frac{N_1R_1}{N_2R_2} = \frac{N_1}{N_2} \quad (\text{as } R_1 = R_2) \\
 \frac{6}{9} &= \frac{2}{3} \\
 \therefore N_1 : N_2 &= 2 : 3
 \end{aligned}$$

31. $800 \times 3 \times \frac{x}{100} = 956 - 800 = 156$

or $24x = 156$

or $2x = 13$

or $x = \frac{13}{2}\% = 6\frac{1}{2}\%$

$\therefore$ Increased rate $= \left(6\frac{1}{2} + 4 \right) = 10\frac{1}{2}\% = \frac{21}{2}\%$

$\therefore$ Amount $= 800 \times 3 \times \frac{21}{200} + 800 = 252 + 800 = \text{₹ } 1052$



Short cut: $x = \frac{156}{800 \times 3} \times 100$
 $= \frac{156}{24} = 6\frac{1}{2}\%$

$\therefore$ New amount $= 800 + 800 \times 3 \times \frac{10\frac{1}{2}}{100}$
 $= 800 + 252 = \text{₹ } 1052$

32. Interest/year $= \frac{1750}{7} = \text{₹ } 250$

Hence interest cannot be determined as data is inadequate.

33. Let the present worth be ₹ x .

$$\therefore x \left(1 + \frac{2 \times 5}{100} \right) = 132$$

$$\Rightarrow x = \frac{132 \times 100}{100 + 10}$$

$$\Rightarrow x = \frac{1320}{11}$$

$$\Rightarrow x = 120$$

$\therefore$ Present worth = ₹ 120

34. Let the principal amount borrowed be ₹ x .

$$\therefore x \times 3 \times \frac{12}{100} = 5400$$

$$\text{or } x = ₹ 15000$$

35. Let the loan period and interest be x .

$$\therefore 1200 \times x \times \frac{x}{100} = 432$$

$$\text{or } x^2 = \frac{432}{12} = 36$$

$$\text{or } x = 6\%$$

$\therefore$ Rate of interest = 6%

36. Let the amount borrowed be ₹ x .

$$\therefore x \times \left(2 \times \frac{6}{100} + 3 \times \frac{9}{100} + 4 \times \frac{14}{100} \right) = 11,400.$$

$$x \times \frac{95}{100} = 11,400$$

$$\text{or } x = \frac{11400 \times 20}{19} = ₹ 12000$$

$\therefore$ Amount borrowed = ₹ 12000

37. Let P be the principal and x be rate of interest .

$$\therefore P \left(1 + \frac{16 \times x}{100} \right) = 2P$$

$$\text{or } \frac{16x}{100} = 1$$

$$\text{or } x = \frac{100}{16} \% = 6\frac{1}{4} \%$$

$\therefore$ Rate percent = $6\frac{1}{4} \%$

38. Let amounts be x and $(1550 - x)$.

$$\therefore x \times \frac{8}{100} + (1550 - x) \times \frac{6}{100} = 106$$

$$8x - 6x + 1550 \times 6 = 10600$$

$$\text{or} \quad x = \frac{10600 - 1550 \times 6}{2} = 5300 - 4650 = \text{₹ } 650$$

$\therefore$ Amounts are **650** and **900**

39. Let principal be P and rate per cent and time be x each.

$$\therefore \quad P \times x \times \frac{x}{100} = \frac{4}{9}P$$

$$\text{or} \quad x^2 = \frac{400}{9}$$

$$\text{or} \quad x = \frac{20}{3} = 6\frac{2}{3}$$

$\therefore$ Rate per cent = $6\frac{2}{3}\%$ and period = $6\frac{2}{3}$ years = 6 years and 8 months.

40. Let the amount lent be ₹ 100.

$$\therefore \quad \text{Total amount with interest at the end of 6 months} = 100 + 100 \times \frac{5}{100} = \text{₹ } 105$$

$\therefore$ Amount at the end of 12 months

$$= 105 + 105 \times \frac{5}{100}$$

$$= 105 + 5.25$$

$$= 110.25$$

$$\therefore \quad \text{Total effect} = 110.25 - 100 = \textbf{10.25\%}$$

Compound Interest

When an amount P is deposited in bank for a certain rate of interest ($r\%$ p.a.) on the condition that every year interest will be calculated and added to the principal. The way of calculating interest in this fashion is called compounding. The interest thus accrued is called compound interest.

$$A = P \left(1 + \frac{R}{100} \right)^N$$

where P = Principal amount (initial)

R = Rate of interest per annum

N = Duration of deposit in years

A = Amount accrued at the end of the period

N can also be denoted in terms of half year/quarter, etc. Then the corresponding rate will be $\frac{R}{2}$ or $\frac{R}{4}$.

An article worth (P) at present ₹ A , t years hence is given by

$$P = \frac{A}{\left(1 + \frac{r}{100} \right)^t}$$

A sum at a rate of interest compounded yearly becomes ₹ A_1 in n years and ₹ A_2 in $(n + 1)$ years, then

$$r = \frac{100(A_2 - A_1)}{A_1} \% \text{ p.a. and } P = A_1 \left(\frac{A_1}{A_2} \right)^n$$

A sum of loan amount A is to be paid back in n equal instalments of $R x$ each. If the rates of interest is $r\%$ per annum compounded annually, then

$$x = \frac{Ar}{100 \left[1 - \left(\frac{100}{100 + r} \right)^n \right]}$$

An amount A due n years hence is to be paid in n equal annual instalments of $R x$ each. If the rate of interest is $r\%$ per annum compounded annually, then

$$x = \frac{Ar}{100 \left[1 - \left(\frac{100 + r}{100} \right)^n \right]}$$

Money that was deposited at Compound Interest (CI) for which interest is calculated at the end of each year or a fixed period and the interest that was due to the depositor is not paid and the same is added to the sum deposited and the amount so obtained is the principal for the next period. The same process continues till the term of the deposit ends and the difference between the final amount and the initial principal is called Compound Interest (CI).



Important Formulae

Let the Principal = P , Time = t years and Rate = $r\%$ p.a.

Case I: When interest is compounded annually:

$$\text{Amount} = P \left[1 + \frac{r}{100} \right]^t$$

Case II: When interest is compounded half-yearly:

$$\text{Amount} = P \left[1 + \frac{r/2}{100} \right]^{2t} = P \left[1 + \frac{r}{200} \right]^{2t}$$

Case III: When interest is compounded quarterly:

$$\text{Amount} = P \left[1 + \frac{r/4}{100} \right]^{4t} = P \left[1 + \frac{r}{400} \right]^{4t}$$

Case IV: When the rate of interest is $r_1\%$, $r_2\%$ and $r_3\%$ for 1st year, 2nd year and 3rd year respectively, then

$$\text{Amount} = P \left(1 + \frac{r_1}{100} \right) \left(1 + \frac{r_2}{100} \right) \left(1 + \frac{r_3}{100} \right)$$

(a) To find the rate:

EXAMPLE 1 At what rate percentage compound interest does a sum become nine-fold in 2 years?

Solution Let the sum be ₹ x and rate be $r\%$ per annum, then

$$9x = x \left(1 + \frac{r}{100} \right)^2$$

or

$$9 = \left(1 + \frac{r}{100} \right)^2$$

or

$$3 = 1 + \frac{r}{100}$$

$$\begin{aligned}\text{or} \quad & \frac{r}{100} = 2 \\ \therefore & r = 200\%\end{aligned}$$



Short cut: If a certain sum becomes m times in t years, rate of compound interest r is equal to $100[(m)^{1/t} - 1]$

$$\text{Here, } r = 100[(9)^{1/2} - 1] = 100(3 - 1) = 200\%$$

EXAMPLE 2 At what rate percentage (compound interest) will a sum of money become eight times in three years?

$$\begin{aligned}\text{Solution} \quad \text{Rate\%} &= [(8)^{1/t} - 1] \times 100 \\ &= [(8)^{1/3} - 1] \times 100 \\ &= [2 - 1] \times 100 = 100\%\end{aligned}$$

EXAMPLE 3 At what rate per cent compounded yearly will ₹ 80000 amount to ₹ 88200 in 2 years?

Solution We have

$$80000 \left(1 + \frac{r}{100}\right)^2 = 88200$$

$$\text{or} \quad \left(1 + \frac{r}{100}\right)^2 = \frac{88200}{80000} = \frac{441}{400} = \left(\frac{21}{20}\right)^2$$

$$\text{or} \quad \left(1 + \frac{r}{100}\right) = \frac{21}{20}$$

$$\therefore r = \frac{100}{20} = 5\%$$

EXAMPLE 4 The difference between compound interest and simple interest on an amount of ₹ 15000 for 2 years is ₹ 96. What is the rate of interest per annum? **[RBI]**

Solution

$$\left[15000 \left(1 + \frac{R}{100}\right)^2 - 15000\right] - \frac{15000 \times R \times 2}{100} = 96$$

$$\Rightarrow 15000 \left[\left(1 + \frac{R}{100}\right)^2 - 1 - \frac{R}{50} \right] = 96$$

$$\Rightarrow 15000 \left[1 + \left(\frac{R}{100}\right)^2 + \frac{2R}{100} - 1 - \frac{2R}{100} \right] = 96$$

$$\Rightarrow 15000 \times \left(\frac{R}{100}\right)^2 = 96$$

$$\begin{aligned} \text{or} \quad R^2 &= \frac{96 \times 100^2}{15000} \\ \text{or} \quad R &= \sqrt{\frac{96 \times 10^4}{15 \times 10^3}} \\ &= \sqrt{\frac{960}{15}} = \sqrt{64} = \mathbf{8\%} \end{aligned}$$

EXAMPLE 5 At what rate of compound interest per annum will a sum of ₹ 1500 become ₹ 1749.60 in 2 years?

Solution Let $x\%$ be the rate of compound interest per annum.

$$\begin{aligned} \therefore 1500 \left(1 + \frac{x}{100}\right)^2 &= 1749.60 \\ \Rightarrow \left(1 + \frac{x}{100}\right)^2 &= \frac{1749.60}{1500} = 1.1664 \\ \Rightarrow x &= \mathbf{8\%} \end{aligned}$$

EXAMPLE 6 Determine the effective annual rate of interest corresponding to a nominal rate of 6% per annum payable half-yearly. [SSC]

Solution Let the principal be ₹ 100.

$\therefore$ Amount at the end of one year

$$\begin{aligned} &= 100 \times \left(1 + \frac{3}{100}\right)^2 \\ &= 100 \times (1.03)^2 = 100 \times 1.0609 = \mathbf{106.09} \end{aligned}$$

$\therefore$ Effective rate of interest = **6.09%**

(b) To find time:

EXAMPLE 7 In what time will ₹ 30000 amount to 34347 at 7% compound interest.

Solution Let the period be t years.

[LIC AAO]

$$\therefore \text{Compound interest} = 30000 \left(1 + \frac{7}{100}\right)^t = 34347$$

$$\text{i.e.} \quad \left(1 + \frac{7}{100}\right)^t = \frac{34347}{30000}$$

$$\left(\frac{107}{100}\right)^t = \frac{11449}{10000} = \left(\frac{107}{100}\right)^2$$

$$\therefore t = \mathbf{2 \text{ years}}$$

EXAMPLE 8 In how many years will a sum of ₹ 1600 at 10% per annum compound interest, compounded half yearly become ₹ 1944.81?

Solution Let t be the number of years.

∴ Number of half years = $2t$.

∴ Amount at the end of t years is:

$$1600 \left(1 + \frac{5}{100} \right)^{2t} = 1944.81$$

$$\Rightarrow 1600 \times \left(\frac{105}{100} \right)^{2t} = 1600 \times \left(\frac{21}{20} \right)^{2t} = 1944.81$$

$$\therefore \left(\frac{21}{20} \right)^{2t} = \frac{1944.81}{1600} = \frac{194481}{160000}$$

$$\left(\frac{21}{20} \right)^t = \sqrt{\frac{194481}{160000}} = \frac{441}{400}$$

$$\Rightarrow \left(\frac{21}{20} \right)^2 = \frac{441}{400}$$

$$\therefore t = 2 \text{ years}$$

EXAMPLE 9 In what time will ₹ 390625 amount to ₹ 456976 at 4% compound interest?

Solution Here $A = P \left(1 + \frac{r}{100} \right)^t$

$$\text{i.e. } 390625 \left(1 + \frac{4}{100} \right)^t = 456976$$

$$\therefore \left(1 + \frac{1}{25} \right)^t = \frac{456976}{390625}$$

$$\text{i.e. } \left(\frac{26}{25} \right)^t = \frac{456976}{390625}$$

$$\text{i.e. } \left(\frac{26}{25} \right)^t = \left(\frac{26}{25} \right)^4$$

$$\therefore t = 4$$

The required time is 4 years.

EXAMPLE 10 A sum placed at compound interest doubles itself in 4 years. In how many years will it amount to eight times itself?

Solution If P is the principal and r , rate of interest per annum,

$$P \left(1 + \frac{r}{100} \right)^4 = 2P$$

$$\therefore \left(1 + \frac{r}{100} \right)^4 = 2$$

Cubing both sides,

$$\left(1 + \frac{r}{100}\right)^{12} = 2^3 = 8$$

or
$$P \left(1 + \frac{r}{100}\right)^{12} = 8P$$

∴ The required time to become the sum eight times itself = **12 years**



Short cut: P becomes $2P$ in 4 years
 $2P$ becomes $4P$ in next 4 years
 $4P$ becomes $8P$ in next 4 years
 ∴ P becomes $8P$ in $4 + 4 + 4 = 12$ years

EXAMPLE 11 Find the least number of complete years in which a sum of money at 20% p.a. Compound interest will be more than double.

Solution We have,

$$P \left(1 + \frac{20}{100}\right)^t > 2P$$

∴
$$\left(\frac{6}{5}\right)^t > 2$$

By trial,
$$\left(\frac{6}{5}\right)^4 > 2$$

∴ The required time is 4 years.

EXAMPLE 12 A sum of money at compound interest amounts to four times itself in 4 years. In how many years will it be 16 times of itself?

Solution Let the sum be ₹ x .

Then,
$$4x = x \left(1 + \frac{r}{100}\right)^4$$

or
$$4 = \left(1 + \frac{r}{100}\right)^4$$

Squaring,
$$(4)^2 = \left[\left(1 + \frac{r}{100}\right)^4\right]^2$$

or
$$16 = \left[1 + \frac{r}{100}\right]^8$$

or
$$16x = x \left[1 + \frac{r}{100}\right]^8$$

∴ The sum will be 16 times in 8 years.



Short cut: If a sum becomes x times in y years at compound interest then it will be $(x)^n$ times in ny years.

∴ If sum becomes 4 times in 4 years, it will be $(4)^2$ times in $2 \times 4 = 8$ years.

To find the SI, if the CI is given, and vice versa

EXAMPLE 13 If the CI on a certain sum for 2 years at 3% be ₹ 1015, what would be the SI?

$$\begin{aligned}\text{Solution} \quad \text{CI on 1 rupee} &= \left(1 + \frac{3}{100}\right)^2 - 1 = \left(\frac{103}{100}\right)^2 - 1 \\ &= ₹ \frac{609}{10000}\end{aligned}$$

$$\text{SI on 1 rupee} = ₹ \frac{2 \times 3}{100} = ₹ \frac{6}{100}$$

$$\therefore \frac{\text{SI}}{\text{CI}} = \frac{6}{100} \times \frac{10000}{609} = \frac{200}{203}$$

$$\therefore \text{SI} = \frac{200}{203} \text{ of CI} = \frac{200}{203} \times 1015 = ₹ 1000$$



$$\text{Short cut: Simple interest} = \frac{rt}{100 \left[\left(1 + \frac{r}{100}\right)^t - 1 \right]} \times \text{Compound Interest.}$$

Find sum and rate if CI and SI are given

EXAMPLE 14 The compound interest on a certain sum for 2 years is ₹ 408 and simple interest is ₹ 400. Find the rate of interest per annum and the sum.

Solution Difference between the simple interest and compound interest for 2 years is the interest on the first year's interest.

$$\text{First year's SI} = \frac{400}{2} = ₹ 200$$

$$\text{CI} - \text{SI} = 408 - 400 = ₹ 8$$

$$\text{Interest on ₹ 200 for 1 year} = ₹ 8$$

$$\therefore \text{Interest on ₹ 100 for 1 year} = ₹ 4$$

$$\therefore \text{Rate} = 4\%$$

$$\therefore \text{Principal is given by } P = \frac{100 \times I}{rt} = \frac{100 \times 400}{4 \times 2} = ₹ 5000$$



Short cut: (for 2 years only)

$$\begin{aligned}\text{Rate} &= \frac{2 \times \text{Difference in CI and SI}}{\text{SI}} \times 100 \\ &= \frac{2 \times 8}{400} \times 100 = 4\% \\ \text{Sum} &= \frac{400 \times 100}{4 \times 2} = \text{₹ } 5000\end{aligned}$$

Division of Sum

EXAMPLE 15 Divide ₹ 5204 between X and Y, so that the share of X at the end of 7 years may be equal to the share of Y at the end of 9 years, compound interest being at 4%.

Solution We have

$$(\text{Share of X at present}) \left(1 + \frac{4}{100}\right)^7 = (\text{Share of Y at present}) \left(1 + \frac{4}{100}\right)^9$$

$$\therefore \frac{\text{Share of X at present}}{\text{Share of Y at present}} = \left(1 + \frac{4}{100}\right)^2 = \left(\frac{26}{25}\right)^2 = \frac{676}{625}$$

Dividing ₹ 5204 in the ratio 676 : 625.

$$\text{Present share of X} = \frac{676}{676 + 625} \times 5204 = \text{₹ } 2704$$

$$\text{Present share of Y} = \frac{625}{676 + 625} \times 5204 = \text{₹ } 2500$$

When difference between SI and CI is given

Theorem: When the difference between the compound interest and simple interest on a certain sum of money for 2 years at $r\%$ per annum is ₹ x , then the sum is given by:

$$\text{Sum} = \frac{(\text{CI} - \text{SI}) \times 100 \times 100}{\text{Rate} \times \text{Rate}} = x \left(\frac{100}{r}\right)^2$$

When the sum is given, difference in SI and CI is calculated by

$$\text{Difference} = \text{Sum} \times \left(\frac{r}{100}\right)^2$$

EXAMPLE 16 The difference between the compound interest and the simple interest on a certain sum of money at 5% per annum for 2 years is ₹ 15. Find the sum.

Solution $\text{Sum} = \text{Difference} \times \left(\frac{100}{r}\right)^2 = 15 \times \left(\frac{100}{5}\right)^2 = \text{₹ } 6000.$

EXAMPLE 17 Find the difference between the compound interest and the simple interest for the sum of ₹ 1500 at 10% per annum for 2 years.

Solution Difference = Sum $\times \left(\frac{r}{100}\right)^2 = 1500 \left(\frac{10}{100}\right)^2 = \text{₹ } 15$

➤ Theorem: If the difference between CI and SI on a certain sum for 3 years at $r\%$ is ₹ x , then the sum will be $\frac{\text{Difference} \times (100)^3}{r^2(300 + r)}$ and if sum is given, difference will be $\text{sum} \times \frac{r^2(300 + r)}{(100)^3}$.

EXAMPLE 18 If the difference between CI and SI on a certain sum of money for 3 years at 5% per annum is ₹ 1220, find the sum.

Solution Sum = $\frac{\text{Difference} \times (100)^3}{r^2(300 + r)} = \frac{1220 (100)^3}{5^2(300 + 5)} = \text{₹ } 160000.$

EXAMPLE 19 Find the difference between CI and SI on ₹ 8000 for 3 years at 2.5% per annum.

Solution Difference = Sum $\times \frac{r^2(300 + r)}{(100)^3} = \frac{8000 \times 2.5 \times 2.5 (300 + 2.5)}{100 \times 100 \times 100}$

$$= \frac{8 \times 25 \times 25 \times 3025}{100 \times 100 \times 100} = \frac{121}{8} = \text{₹ } 15.125$$

EXAMPLE 20 The simple interest on a sum at 4% per annum for 2 years is ₹ 80. Find the compound interest on the same sum for the same period.

Solution Rate = $\frac{2 \times \text{Difference in CI and SI}}{\text{SI}} \times 100$

or Difference in CI and SI = $\frac{\text{Rate} \times \text{SI}}{2 \times 100} = \frac{4 \times 80}{2 \times 100} = 1.6$

$\therefore$ CI = 80 + 1.6 = ₹ 81.6

EXAMPLE 21 The compound interest on a certain sum of money for 2 years at 10% per annum is ₹ 420. Find the simple interest at the same rate and for the same time.

Solution We have, $\text{SI} = \frac{rt}{100 \left[\left(1 + \frac{r}{100}\right)^t - 1 \right]} \times \text{CI}$

When $t = 2$,

$$\begin{aligned} \text{SI} &= \frac{2r}{100 \left[1 + \frac{r^2}{100^2} + \frac{2r}{100} - 1 \right]} \times \text{CI} \\ &= \frac{2r \times \text{CI} \times 100}{(r^2 + 200r)} \end{aligned}$$

$$= \frac{200r \times CI}{r(r + 200)}$$

Substituting, $SI = \frac{200 \times 10 \times 420}{10 \times 210} = ₹ 400$



Note: When CI is given and SI is asked, the above formulae may be used.

When relationship between CI and SI is asked for 3 years

$$SI = \frac{rt}{100 \left[\left(1 + \frac{r}{100} \right)^t - 1 \right]} \times CI$$

$$= \frac{3r}{100 \left[\left(1 + \frac{r}{100} \right)^3 - 1 \right]} \times CI$$

EXAMPLE 22 If the simple interest on a certain sum of money for 3 years at 5% is ₹ 150, find the corresponding compound interest.

Solution $SI = \frac{3r}{100 \left[\left(1 + \frac{r}{100} \right)^3 - 1 \right]} \times CI$

i.e. $150 = \frac{3 \times 5}{100 \left[\left(1 + \frac{5}{100} \right)^3 - 1 \right]} \times CI$

$\therefore CI = \frac{150}{15} \times 100 \left[\left(1 + \frac{5}{100} \right)^3 - 1 \right] = 1000 \left[\left(\frac{21}{20} \right)^3 - 1^3 \right]$

$$= 1000 \frac{(21 - 20)[21^2 + 21(20) + 20^2]}{8000}$$

$$= \frac{441 + 420 + 400}{8} = \frac{1261}{8} = ₹ 157.62$$

EXAMPLE 23 An amount of money grows up to ₹ 4840 in 2 years and up to ₹ 5324 in 3 years on compound interest. Find the rate per cent.

Solution We have,

$$CI \text{ for 3rd year} = 5324 - 4840 = ₹ 484$$

$$\therefore \text{Rate} = \frac{484 \times 100}{4840} = 10\%$$

(i.e., ₹ 484 is the interest accrued on the principal amount ₹ 4840 at the end of 2nd year.)



Short cut: $\text{Rate} = \frac{[\text{Difference of amount after } n \text{ years and } (n + 1) \text{ years}] \times 100}{\text{Amount after } n \text{ years}}$

In this case $n = 2$

$$\begin{aligned} \therefore \text{Rate} &= \frac{[\text{Difference of amount after 2 years and 3 years}] \times 100}{\text{Amount after 2 years}} \\ &= \frac{(5324 - 4840) \times 100}{4840} = 10\% \end{aligned}$$

EXAMPLE 24 Find the compound interest on ₹ 19375 in 2 years, the rate of interest being 4% for the first year and 8% for the second year.

Solution After first year the amount = $19375 \left(1 + \frac{4}{100}\right) = 19375 \left(\frac{104}{100}\right)$

$$\begin{aligned} \text{After second year the amount} &= 19375 \left(\frac{104}{100}\right) \left(\frac{108}{100}\right) \\ &= 19375 \left(\frac{26}{25}\right) \left(\frac{27}{25}\right) = ₹ 21762 \end{aligned}$$

$$\therefore \text{Compound interest} = 21762 - 19375 = ₹ 2387$$

EXAMPLE 25 A sum of ₹ 4800 becomes ₹ 6000 in 4 years at a certain rate of compound interest. What will be the sum after 12 years?

$$\text{Solution} \quad 4800 \left(1 + \frac{r}{100}\right)^4 = 6000$$

$$\text{or} \quad \left(1 + \frac{r}{100}\right)^4 = \frac{6000}{4800} = \frac{5}{4}$$

$$\therefore \left(1 + \frac{r}{100}\right)^{12} = \left(\frac{5}{4}\right)^3 = \frac{125}{64}$$

$$\therefore 4800 \left(1 + \frac{r}{100}\right)^{12} = 4800 \times \frac{125}{64} = ₹ 9375$$

$\therefore$ Amount after 12 years will be ₹ 9375



Short cut: Required amount = $\frac{(6000)^{12/4}}{(4800)^{12/4-1}} = \frac{(6000)^3}{(4800)^2} = ₹ 9375$



Note: If a sum x becomes y in t_1 years at compound rate of interest, then after t_2 years the sum becomes $\frac{y^{t_2/t_1}}{x^{t_2/t_1-1}}$ rupees.

EXAMPLE 26 Find the compound interest on ₹ 100000 for 3 years if the rate of interest is 4% for the first year, 5% for the second year and 6% for the third year.

Solution The compound interest on ₹ x in t years, if the rate of interest is $r_1\%$ for the first year, $r_2\%$ for the second year... and $r_t\%$ for the t th year, is given by

$$x \left(1 + \frac{r_1}{100}\right) \left(1 + \frac{r_2}{100}\right) \cdots \left(1 + \frac{r_t}{100}\right) - x$$

$$\begin{aligned} \text{Here, Compound interest} &= 100000 \left(1 + \frac{4}{100}\right) \left(1 + \frac{5}{100}\right) \left(1 + \frac{6}{100}\right) - 100000 \\ &= 100000 \left(\frac{26}{25}\right) \left(\frac{21}{20}\right) \left(\frac{53}{50}\right) - 100000 \\ &= 4 \times 26 \times 21 \times 53 - 100000 = 115752 - 100000 \\ &= \text{₹ } 15752 \end{aligned}$$

EXAMPLE 27 What sum of money at compound interest will amount to ₹ 224952 in 3 years, if the rate of interest is 3% for the first year, 4% for the second year and 5% for the third year?

Solution

$$A = P \left(1 + \frac{r_1}{100}\right) \left(1 + \frac{r_2}{100}\right) \left(1 + \frac{r_3}{100}\right)$$

Here,

$$\begin{aligned} 224952 &= P \left(1 + \frac{3}{100}\right) \left(1 + \frac{4}{100}\right) \left(1 + \frac{5}{100}\right) \\ &= P(1.03)(1.04)(1.05) \end{aligned}$$

$$\therefore P = \frac{224952}{(1.03)(1.04)(1.05)} = \text{₹ } 200000$$



Short cut: By rule of fraction;

$$\text{Principal} = 224952 \left(\frac{100}{103}\right) \left(\frac{100}{104}\right) \left(\frac{100}{105}\right) = \text{₹ } 200000$$

EXAMPLE 28 Anil deposits ₹ 20000 at the rate of 16% compounded annually and Ashok deposits an equal amount at the rate of 15% compounded half yearly. If both deposit their money for $1\frac{1}{2}$ years only, calculate which deposit earns better interest.

Solution Amount after $1\frac{1}{2}$ years in both the cases will be $20000\left(1 + \frac{16}{100}\right)^{3/2}$ and $20000\left(1 + \frac{7.5}{100}\right)^3$ respectively.

Comparing both quantities,

$$20000\left(1 + \frac{16}{100}\right)^{3/2} \Leftrightarrow 20000\left(1 + \frac{7.5}{100}\right)^3$$

or
$$\left(1 + \frac{16}{100}\right)^{3/2} \Leftrightarrow \left(1 + \frac{7.5}{100}\right)^3$$

Squaring both sides,

$$\left(1 + \frac{16}{100}\right)^3 \Leftrightarrow \left(1 + \frac{7.5}{100}\right)^6$$

Taking cube roots on both sides,

$$\left(1 + \frac{16}{100}\right)^1 \Leftrightarrow \left(1 + \frac{7.5}{100}\right)^2$$

$$\left(1 + \frac{16}{100}\right) \Leftrightarrow 1 + 2\left(\frac{7.5}{100}\right) + \left(\frac{7.5}{100}\right)^2$$

$$1 + \frac{16}{100} \Leftrightarrow 1 + \frac{15}{100} + \left(\frac{7.5}{100}\right)^2$$

$$\frac{1}{100} \Leftrightarrow \left(\frac{7.5}{100}\right)^2$$

i.e.
$$\frac{1}{100} > \left(\frac{7.5}{100}\right)^2$$

or
$$1 > \frac{7.5^2}{100}$$

∴ First quantity is higher than the second.

EXAMPLE 29 A sum of money is lent at compound interest rate of 20% per annum for 2 years. It would fetch ₹ 482 more if the interest is compounded half-yearly. Find the sum.

Solution Let the sum be P .

Compound interest when interest is compounded yearly = $P\left[1 + \frac{20}{100}\right]^2 - P$

Compounded interest when interest is compounded half-yearly = $P\left[1 + \frac{10}{100}\right]^4 - P$

Difference in interests is given by

$$P \left[1 + \frac{10}{100} \right]^4 - P \left[1 + \frac{20}{100} \right]^2 = 482$$

i.e. $P(1.1)^4 - P(1.2)^2 = 482$

i.e. $P(1.21^2 - 1.2^2) = 482$

$$\begin{aligned} \therefore P &= \frac{482}{(1.21 + 1.2)(1.21 - 1.2)} \\ &= \frac{482}{2.41 \times 0.01} = \text{₹ } 20000 \end{aligned}$$

EXAMPLE 30 In what time will ₹ 1000 become ₹ 1331 at 10% per annum compounded annually? [SSC]

Solution $P = \text{₹ } 1000$; $A = \text{₹ } 1331$; $R = 10\%$; $t = ?$

$$A = P \left(1 + \frac{r}{100} \right)^t$$

$$1331 = 1000 \left(1 + \frac{10}{100} \right)^t$$

or $\left(\frac{110}{100} \right)^t = \frac{1331}{1000}$

or $\left(\frac{11}{10} \right)^t = \frac{1331}{1000}$

or $\left(\frac{11}{10} \right)^3 = \frac{1331}{1000}$

$\therefore t = 3 \text{ years}$



EXERCISES

- If 64000 is deposited at 5% compound interest for a period of 18 months, interest being compounded half-yearly, find the amount.
(a) 69821 (b) 68921 (c) 62891 (d) 62981 (e) None of these
- An amount of ₹ 12500 is deposited for a period of 2 years at an interest of 4% per annum. Find the difference between simple interest and compound interest for the deposit.
(a) ₹ 10 (b) ₹ 20 (c) ₹ 50 (d) ₹ 40 (e) None of these
- An amount of ₹ 8000 was deposited at simple interest for 3 years at 7.5% per annum. How much more would have gained had it been deposited at the same rate per cent compound interest?
(a) ₹ 135.75 (b) ₹ 138.75 (c) ₹ 138.375 (d) ₹ 138.753 (e) None of these

4. A certain sum at compound interest amounts in two years to ₹ 2809 and in three years to ₹ 2977.54. Find the rate of interest and the original sum.
(a) ₹ 1500 (b) ₹ 2000 (c) ₹ 2500 (d) ₹ 3000 (e) None of these
5. Difference between simple interest and compound interest on a certain sum for 3 years at 5% per annum is ₹ 122. Find the sum.
(a) ₹ 14000 (b) ₹ 16000 (c) ₹ 18000 (d) ₹ 20000 (e) None of these
6. A sum of money invested at compound interest amounts to ₹ 800 in 3 years and to ₹ 840 in 4 years. The rate of interest per annum is: [SSC]
(a) $2\frac{1}{2}\%$ (b) 4% (c) 5% (d) $6\frac{2}{3}\%$
7. The difference between the simple interest on a certain sum at the rate of 10% per annum for 2 years and compound interest which is compounded every 6 months is ₹ 124.05. What is the principal? [SBI PO]
(a) ₹ 6000 (b) ₹ 8000 (c) ₹ 10000 (d) ₹ 12000 (e) None of these
8. The difference between simple interest and compound interest compounded annually on a certain sum of money for 2 years at 4% per annum is ₹ 1. The sum (in ₹) is: [SSC]
(a) 625 (b) 630 (c) 640 (d) 650
9. The simple interest on a certain sum of money for 3 years at 8% per annum is half the compound interest on ₹ 4000 for 2 years at 10% per annum. The sum placed on simple interest is: [SSC]
(a) ₹ 1550 (b) ₹ 1650 (c) ₹ 1750 (d) ₹ 2000
10. If the compound interest on a sum for 2 years at $12\frac{1}{2}\%$ per annum is ₹ 510, the simple interest on the same sum at the same rate for the same period of time is. [SSC]
(a) ₹ 400 (b) ₹ 450 (c) ₹ 460 (d) ₹ 480
11. The principal that amounts to ₹ 4913 in 3 years at $6\frac{1}{4}\%$ per annum compound interest compounded annually is. [SSC]
(a) ₹ 3096 (b) ₹ 4076 (c) ₹ 4085 (d) ₹ 4096
12. The compound interest on ₹ 30000 at 7% per annum is ₹ 4347. The period (in years) is: [LIC AAO]
(a) 2 (b) $2\frac{1}{2}$ (c) 3 (d) 4
13. What will be the difference between simple and compound interest @ 10% per annum on a sum of ₹ 1000 after 4 years? [Bank PO]
(a) ₹ 31 (b) ₹ 32.10 (c) ₹ 40.40 (d) ₹ 64.10 (e) None of these
14. Find the compound interest on ₹ 5625 for 9 months at 16% per annum compounded quarterly. [RRB]
(a) ₹ 1851 (b) ₹ 1941 (c) ₹ 1951 (d) ₹ 1961
15. A bank offers 5% compound interest calculated on half-yearly basis. A customer deposits ₹ 1600 each on 1st January and 1st July of a year. At the end of the year, the amount he would have gained by way of interest is: [NDA]
(a) ₹ 120 (b) ₹ 121 (c) ₹ 122 (d) ₹ 123
16. What will be the compound interest on a sum of ₹ 25000 after 3 years at the rate of 12% per annum? [SBI PO]
(a) ₹ 9000.30 (b) ₹ 9720 (c) ₹ 10123.20 (d) ₹ 10483.20 (e) None of these

17. Albert invested an amount of ₹ 8000 in a fixed deposit scheme for 2 years at compound interest rate of 5% per annum. How much amount will Albert get on maturity of the fixed deposit? **[Bank PO]**
 (a) ₹ 8600 (b) ₹ 8620 (c) ₹ 8800 (d) ₹ 8840 (e) None of these
18. Sam invested ₹ 15000 at 10% per annum for one year. If the interest is compounded half-yearly, then the amount received by Sam at the end of the year will be: **[SBI PO]**
 (a) ₹ 16500 (b) ₹ 16525.50 (c) ₹ 16537.50 (d) ₹ 18150 (e) None of these
19. What is the difference between the compound interests on ₹ 5000 for $1\frac{1}{2}$ years at 4% per annum compounded yearly end half-yearly? **[SSC]**
 (a) ₹ 2.04 (b) ₹ 3.06 (c) ₹ 4.80 (d) ₹ 8.30
20. What annual payment will discharge a debt of ₹ 1025 due in 2 years at the rate of 5% compound interest? **[SSC]**
 (a) ₹ 550 (b) ₹ 551.25 (c) ₹ 560 (d) ₹ 560.75
21. A sum of money placed at compound interest doubles itself in 5 years. It will amount to eight times itself at the same rate of interest in: **[Hotel Management]**
 (a) 7 years (b) 10 years (c) 15 years (d) 20 years
22. Mr. Dua invested money in two schemes A and B offering compound interest at 8% per annum and 9% per annum respectively. If the total amount of interest accrued through two schemes together in two years was ₹ 4818.30 and the total amount invested was ₹ 27000. What was the amount invested in scheme A. **[Bank PO]**
 (a) ₹ 12000 (b) ₹ 13500 (c) ₹ 15000
 (d) Cannot be determined (e) None of these
23. On a sum of money, the simple interest for 2 years is ₹ 660, while the compound interest is ₹ 696.30, the rate of interest being the same in both the cases. The rate of interest is: **[Hotel Management]**
 (a) 10% (b) 10.5% (c) 12% (d) None of these
24. There is 60% increase in an amount in 6 years at simple interest. What will be the compound interest of ₹ 12000 after 3 years at the same rate? **[SIDBI]**
 (a) ₹ 2160 (b) ₹ 3120 (c) ₹ 3972 (d) ₹ 6240 (e) None of these
25. The compound interest on a certain sum for 2 years at 10% per annum is ₹ 525. The simple interest on the same sum for double the time at half the rate per cent per annum is: **[CBI]**
 (a) ₹ 400 (b) ₹ 500 (c) ₹ 600 (d) ₹ 800
26. If the simple interest on a sum of money for 2 years at 5% per annum is ₹ 50, what is the compound interest on the same sum at the same rate and for the same time? **[CBI]**
 (a) ₹ 51.25 (b) ₹ 52 (c) ₹ 54.25 (d) ₹ 60
27. The least number of complete years in which a sum of money put out at 20% compound interest will be more than doubled is: **[NIFT]**
 (a) 3 (b) 4 (c) 5 (d) 6
28. The difference between the compound interest and the simple interest accrued on an amount of ₹ 18000 in 2 years was ₹ 405. What was the rate of interest per cent per annum? **[Bank PO]**
 (a) 12% (b) 13% (c) 14% (d) 15% (e) None of these

29. In how many years will a sum of ₹ 800 at 10% per annum compounded semi-annually become ₹ 926.10? **[Section Officer's]**
- (a) $1\frac{1}{3}$ (b) $1\frac{1}{2}$ (c) $2\frac{1}{3}$ (d) $2\frac{1}{2}$
30. A man borrows ₹ 3825 to be paid back with compound interest at the rate of 4% per annum by the end of 2 years into equal yearly instalments. How much will be each instalment?
- (a) ₹ 1275 (b) ₹ 1352 (c) ₹ 2028 (d) ₹ 2082
31. A sum of ₹ 15000 deposited at compound interest becomes double after 5 years. After 20 years, it will become: **[SSC]**
- (a) ₹ 120000 (b) ₹ 150000 (c) ₹ 155000 (d) ₹ 240000
32. The difference between compound interest and simple interest on a sum for 2 years at 10% per annum, when the interest is compounded annually, is ₹ 20. If the interest were compounded half yearly, the difference in two interests will be: **[SSC]**
- (a) ₹ 31.82 (b) ₹ 31.01 (c) ₹ 32.81 (d) ₹ 32.18
33. A man borrows ₹ 25000 at 20% compound interest. At the end of every year he pays ₹ 5000 as part repayment. How much does he still owe after three such instalments?
- (a) ₹ 25000 (b) ₹ 12864 (c) ₹ 15600 (d) None of these
34. A sum of money borrowed and paid back in two annual instalments of ₹ 882 each allowing 5% compound interest. The sum borrowed was: **[AIMATS]**
- (a) ₹ 1620 (b) ₹ 1640 (c) ₹ 1680 (d) ₹ 1700
35. What will be the compound interest on a sum of ₹ 50000 after 4 years at the rate of 10% per annum?
- (a) ₹ 20000 (b) ₹ 21205 (c) ₹ 22205 (d) ₹ 23205



1. (b) 2. (b) 3. (c) 4. (c) 5. (b) 6. (c) 7. (b) 8. (a) 9. (c) 10. (d)
 11. (d) 12. (a) 13. (d) 14. (c) 15. (b) 16. (c) 17. (e) 18. (c) 19. (a) 20. (b)
 21. (c) 22. (a) 23. (d) 24. (c) 25. (b) 26. (a) 27. (b) 28. (d) 29. (b) 30. (c)
 31. (d) 32. (b) 33. (a) 34. (b) 35. (d)



Solutions with Necessary Explanation

$$1. \text{ Amount} = 64000 \left(1 + \frac{2.5}{100}\right)^3 = 64000 \left(\frac{41}{40}\right)^3 = \text{₹ } 68921$$

$$2. \text{ Difference} = \text{Sum} \times \left(\frac{r}{100}\right)^2 = 12500 \left(\frac{4}{100}\right)^2 = \frac{12500}{625} = \text{₹ } 20$$

$$3. \text{ Difference} = \frac{\text{Sum} \times r^2 (300 + r)}{(100)^3} = \frac{8000 \times (7.5)^2 (300 + 7.5)}{(100)^3} = \text{₹ } 138.375$$

$$4. \text{ Difference} = 2977.54 - 2809 = \text{₹ } 168.54$$

$$\therefore \text{ Rate of interest} = \frac{168.54 \times 100}{2809} = 6\%$$

Let x be the amount deposited

$$\therefore 2809 = x \left(1 + \frac{6}{100} \right)^2$$

$$\text{or } x = \frac{2809 \times 100^2}{106^2} = 2809 \left(\frac{50}{53} \right)^2 = \text{₹ } 2500$$

$$5. \text{ Sum} = \frac{\text{Difference} \times 100^3}{r^2(300 + r)} = \frac{122 \times 100^3}{5^2 + (300 + 5)} = \frac{122}{61} \times \left(\frac{100}{5} \right)^3 = 2 \times 8000 = \text{₹ } 16000$$

6. Let R be the rate of interest. We have,

$$A1 = P \left(1 + \frac{R}{100} \right)^3 = 800$$

$$\text{or } P \left(1 + \frac{R}{100} \right)^3 = 800 \quad (1)$$

$$\text{Similarly } P \left(1 + \frac{R}{100} \right)^4 = 840 \quad (2)$$

$$\frac{(2)}{(1)} \Rightarrow \left(1 + \frac{R}{100} \right) = \frac{840}{800} = 1.05$$

$$\therefore R = 5\%$$

7. Let P be the principal.

$$\text{S.I. on first year} = P \times \frac{10 \times 1}{100} = \frac{P}{10}$$

$$\text{S.I. on second year} = P \times \frac{10 \times 1}{100} = \frac{P}{10}$$

$$\text{Total S.I.} = 2 \times \frac{P}{10} = \frac{P}{5} \quad (1)$$

$$\begin{aligned} \text{C.I. for 2 year} &= P \left(1 + \frac{R}{200} \right)^4 - P \\ &= P \left[\left(1 + \frac{10}{200} \right)^4 - 1 \right] \\ &= P \left[\left(1 + \frac{5}{100} \right)^4 - 1 \right] \end{aligned} \quad (2)$$

$$(2)-(1) \Rightarrow P \left[\left(1 + \frac{5}{100} \right)^4 - 1 - \frac{1}{5} \right] = 124.05$$

$$\therefore P \left[\left(\frac{21}{20} \right)^4 - \frac{6}{5} \right] = 124.05$$

$$\text{or} \quad P \left[\frac{1,94,481}{1,60,000} - \frac{6}{5} \right] = 124.05$$

$$\begin{aligned} \text{or} \quad P &= \frac{12405}{100} \times \frac{1,60,000}{1,94,481 - 1,92,000} \\ &= \text{₹ } 8000 \end{aligned}$$

8. Let the sum be P .

$$\therefore P \left[\left(1 + \frac{4}{100} \right)^2 - 1 \right] - P \times \frac{2 \times 4}{100} = 1$$

$$\text{or} \quad P \left[\left(\frac{26}{25} \right)^2 - 1 - \frac{2}{25} \right] = 1$$

$$\text{or} \quad P = \frac{1 \times (25)^2}{(26)^2 - (25)^2 - 50} = \frac{(25)^2}{(51)1 - 50} = (25)^2 = \text{₹ } 625$$

9. Let the sum be P .

$$\text{S.I. for 3 years} = \frac{P \times 3 \times 8}{100} \quad (1)$$

$$\begin{aligned} \text{C.I. (for ₹ 4000 @ 10% p.a for 2 year)} &= 4000 \left(1 + \frac{10}{100} \right)^2 - 4000 \\ &= 4000 \left[\left(\frac{11}{10} \right)^2 - 1 \right] = 4000 \times \left(\frac{11^2 - 10^2}{10^2} \right) \\ &= 40 \times 21 = 840 \end{aligned} \quad (2)$$

$\therefore$ From (1) and (2),

$$P \times \frac{3 \times 8}{100} = \frac{1}{2} \times \text{C.I.} = \frac{1}{2} \times 840 = 420$$

$$\therefore P = \frac{420 \times 100}{24} = 70 \times 25 = \text{₹ } 1750$$

10. Let the principal amount be P .

$$\therefore P \left[\left(1 + \frac{12(1/2)}{100} \right)^2 - 1 \right] = \text{₹ } 510$$

$$\text{i.e.} \quad P \left[\left(1 + \frac{1}{8} \right)^2 - 1 \right] = 510$$

$$\text{i.e.} \quad P \left[\left(\frac{9}{8} \right)^2 - 1 \right] = 510$$

$$\text{or} \quad P = \frac{510 \times (8)^2}{(9)^2 - (8)^2} = \frac{510 \times (8)^2}{(9+8) \times 1} = 30 \times 64 = \text{₹ } 1920$$

$$\therefore \text{S.I.} = 1920 \times 2 \times \frac{25}{200} = \text{₹ } 480$$

11. Let the principal be P .

$$\therefore P \left(1 + \frac{6\frac{1}{4}}{100} \right)^3 = 4913$$

$$\text{or } P \left(1 + \frac{25}{400} \right)^3 = 4913$$

$$\text{or } P = \frac{4913}{\left(1 + \frac{1}{16} \right)^3} = \frac{4913 \times 16^3}{17^3} = \text{₹ } 4096$$

12. Let the period of deposit be N years.

$$\therefore 30000 \left[\left(1 + \frac{7}{100} \right)^N - 1 \right] = 4347$$

$$\text{i.e. } 30000 \left(\frac{107}{100} \right)^N - 30000 = 4347$$

$$\text{or } 30000 \left(\frac{107}{100} \right)^N = 34347$$

$$\therefore \left(\frac{107}{100} \right)^N = \frac{34347}{30000} = \frac{11449}{10000} = \left(\frac{107}{100} \right)^2$$

$$\therefore N = 2 \text{ years}$$

13. S.I. on ₹ 1000 @ 10% p.a. after 4 years

$$= \frac{1000 \times 4 \times 10}{100} = \text{₹ } 400$$

C.I. on ₹ 1000 @ 10% p.a. after 4 years.

$$\begin{aligned} &= 1000 \left[\left(1 + \frac{10}{100} \right)^4 - 1 \right] = 1000 \left[\left(\frac{11}{10} \right)^4 - 1 \right] \\ &= \frac{1000}{10^4} (11^4 - 10^4) = \frac{1000}{10000} (14641 - 10000) \\ &= \frac{1}{10} \times 4641 \times 10 = \text{₹ } 464.10 \end{aligned}$$

Hence C.I. – S.I. = 464.10 – 400 = ₹ 64.10

14. Compound interest on ₹ 15625 for 9 months @ 16% per annum

$$\begin{aligned} &= 15625 \left[\left(1 + \frac{16}{100 \times 4} \right)^3 - 1 \right] = 15625 \left[\left(1 + \frac{1}{25} \right)^3 - 1 \right] \\ &= 15625 \times \left[\frac{26^3}{25^3} - 1 \right] = 25 \times 625 \times \left[\frac{26^3}{25^3} - 1 \right] \end{aligned}$$

$$= 26^3 - 25^3 = (26^2 + 25^2 + 26 \times 25)(26 - 25)$$

$$= 676 + 625 + 650 = \text{₹ } 1951$$

15. Total interest at the end of the year

$$= 1600 \left(1 + \frac{5}{200}\right)^2 + 1600 \left(1 + \frac{5}{200}\right) - 3200$$

$$= 1600 \left[\left(1 + \frac{1}{40}\right)^2 - 1 \right] + 1600 \times \frac{1}{40}$$

$$= 1600 \times \frac{(41^2 - 40^2)}{40^2} + 40 = (41 + 40)(41 - 40) + 40$$

$$= 81 + 40 = \text{₹ } 121$$

16. C.I. after 3 years $= 25000 \left[\left(1 + \frac{12}{100}\right)^3 - 1 \right] = 25000 \left[\left(\frac{28}{25}\right)^3 - 1 \right]$

$$= \frac{25000}{25^3} (28^3 - 25^3) = \frac{40}{25} (28 - 25)(28^2 + 25^2 + 28 \times 25)$$

$$= \frac{40 \times 3}{25} (784 + 625 + 700) = \frac{24}{5} \times 2109$$

$$= 4.8 \times 2109 = \text{₹ } 10123.20$$

17. $A = P \left(1 + \frac{R}{100}\right)^N$

$$= 8000 \left(1 + \frac{5}{100}\right)^2 = 8000 \times \left(\frac{21}{20}\right)^2$$

$$= 20 \times 21^2 = \text{₹ } 8820$$

18. $A = P \left(1 + \frac{R}{100}\right)^N$

$$= 15000 \left(1 + \frac{10}{200}\right)^2 = 15000 \left(1 + \frac{5}{100}\right)^2$$

$$= 15000 \times \left(\frac{21}{20}\right)^2 = \frac{15000 \times 441}{400}$$

$$= 37.5 \times 441 = \text{₹ } 16537.50$$

19. Difference in C.I. $= 5000 \left[\left(1 + \frac{2}{100}\right)^3 - 1 \right] - 5000 \left[\left(1 + \frac{4}{100}\right) \left(1 + \frac{2}{100}\right) - 1 \right]$

$$= 5000 \left[\left\{ \left(\frac{51}{50}\right)^3 - 1 \right\} - \left(\frac{26}{25} \times \frac{51}{50} - 1 \right) \right]$$

$$= 5000 \left[\left(\frac{51^3 - 50^3}{50^3} \right) - \frac{(13 \times 51 - 25^2)}{25^2} \right]$$

$$\begin{aligned}
 &= 5000 \left[\frac{51^2 + 50^2 + 51(50)}{50^3} - \frac{(663 - 625)}{25^2} \right] \\
 &= 5000 \left[\frac{2601 + 2500 + 2550}{50^3} - \frac{38}{25^2} \right] = 5000 \left[\frac{7651}{50^3} - \frac{38}{25^2} \right] \\
 &= \frac{5000}{50^3} [7651 - 38 \times 4 \times 50] = \frac{2}{50} [7651 - 7600] \\
 &= \frac{2}{50} \times 51 = \text{₹ } 2.04
 \end{aligned}$$

20. Let each annual instalment be ₹ x .

$$\therefore \frac{x}{\left(1 + \frac{5}{100}\right)^2} + \frac{x}{\left(1 + \frac{5}{100}\right)} = 1025$$

$$\text{or } x \times \left(\frac{20}{21}\right)^2 + x \left(\frac{20}{21}\right) = 1025$$

$$\text{or } \frac{400x}{441} + \frac{20x}{21} = 1025$$

$$\Rightarrow \frac{400x + 420x}{441} = 1025$$

$$\text{or } x = \frac{1025 \times 441}{820} = \text{₹ } 551.25$$

Hence each annual instalment = ₹ 551.25

$$21. 2P = P \left(1 + \frac{R}{100}\right)^5 \quad (1)$$

Let the number of years at which amount becomes 8 times be N .

$$\therefore 8P = P \left(1 + \frac{R}{100}\right)^N \quad (2)$$

$$(1) \Rightarrow \left(1 + \frac{R}{100}\right)^5 = 2 \quad (3)$$

$$\frac{(2)}{(1)} \Rightarrow \frac{8P}{2P} = \frac{P \left(1 + \frac{R}{100}\right)^N}{P \left(1 + \frac{R}{100}\right)^5}$$

on substitution from (3), we get,

$$4 = \frac{\left(1 + \frac{R}{100}\right)^N}{2}$$

$$\text{or } \left(1 + \frac{R}{100}\right)^N = 8 \quad (4)$$

$$(3) \Rightarrow \left(1 + \frac{R}{100}\right)^5 = 2$$

$$\therefore (4) \Rightarrow \left(1 + \frac{R}{100}\right)^N = 2^3 = \left[\left(1 + \frac{R}{100}\right)^5\right]^3 = \left(1 + \frac{R}{100}\right)^{15}$$

$$\therefore N = 15 \text{ years}$$

22. Let the amount invested in scheme A be ₹ x .

$\therefore$ Amount in scheme B = $(27000 - x)$

$\therefore$ C.I. for 2 years.

$$= x \left[\left(1 + \frac{8}{100}\right)^2 - 1 \right] + (27000 - x) \left[\left(1 + \frac{9}{100}\right)^2 - 1 \right] = 4818.30$$

$$\text{i.e. } x \left[\left(\frac{27}{25}\right)^2 - 1 \right] + (27000 - x) \left[\left(\frac{109}{100}\right)^2 - 1 \right] = 4818.30$$

$$\text{or } \frac{x}{25^2} [(27)^2 - (25)^2] + \frac{(27000 - x)}{100^2} [(109)^2 - (100)^2] = 4818.30$$

$$\text{or } \frac{x}{25^2} \times 52 \times 2 + \frac{27000 - x}{10000} \times 209 \times 9 = 4818.30$$

$$\text{or } \frac{x}{100^2} [104 \times (4)^2 - 1881] = 4818.30 - 1881 \times 2.7$$

$$-\frac{217x}{100^2} = -260.4$$

$$\text{or } x = \frac{260.4 \times (100)^2}{217} = \text{₹ } 12000$$

23. Let the rate of interest be $R\%$.

$$\text{S.I.} = \frac{P \times 2 \times R}{100} = 660 \quad (1)$$

$$\text{C.I.} = P \left[\left(1 + \frac{R}{100}\right)^2 - 1 \right] = 696.30 \quad (2)$$

$$\frac{(2)}{(1)} \Rightarrow \frac{\left[\left(1 + \frac{R}{100}\right)^2 - 1 \right]}{\frac{2R}{100}} = \frac{696.30}{660}$$

$$\Rightarrow \frac{100}{2R} \left(\frac{R^2}{100^2} + \frac{2R}{100} \right) = 1.055$$

$$\Rightarrow \frac{R}{200} + 1 = 1.055$$

$$\text{or } R = (1.055 - 1) \times 200 = 11\%$$

$$24. \text{ S.I. for 6 years} = \frac{P \times 6 \times R}{100} = 0.6P$$

$$\text{or} \quad R = \frac{0.6 \times 100}{6} = 10\%$$

C.I. for 3 years for ₹ 12000 @ 10% p.a.

$$\begin{aligned} &= 12000 \left[\left(1 + \frac{10}{100} \right)^3 - 1 \right] = 12000 \left[\left(\frac{11}{10} \right)^3 - 1 \right] \\ &= 12000 \times \frac{(11)^3 - (10)^3}{10^3} = 12 \times (11 - 10) [(11)^2 + (10)^2 + 110] \\ &= 12 \times 1 \times 331 = \text{₹ } 3972 \end{aligned}$$

$$25. \text{ C.I.} = P \left[\left(1 + \frac{10}{100} \right)^2 - 1 \right] = 525 \quad (1)$$

$$\text{S.I.} = \frac{P \times 4 \times 5}{100} = \frac{P}{5} \quad (2)$$

Substituting from (1), we get

$$\begin{aligned} \text{S.I.} = \frac{P}{5} &= \frac{525}{\left[\left(\frac{11}{10} \right)^2 - 1 \right] \times 5} \\ &= \frac{105 \times 10^2}{(11^2 - 10^2)} = \frac{10500}{21 \times 1} = \text{₹ } 500 \end{aligned}$$

$$26. \quad \text{S.I.} = P \times 2 \times \frac{5}{100} = 50$$

$$\Rightarrow \quad P = \text{₹ } 500$$

$$\begin{aligned} \text{C.I.} &= P \left[\left(1 + \frac{5}{100} \right)^2 - 1 \right] = 500 \times \left[\left(\frac{21}{20} \right)^2 - 1 \right] \\ &= 500 \times \frac{(21^2 - 20^2)}{20^2} \\ &= \frac{500 \times 41 \times 1}{400} = \frac{205}{4} = \text{₹ } 51.25 \end{aligned}$$

27. Let the number of year be N .

$$\therefore \quad P \left(1 + \frac{20}{100} \right)^N > 2P$$

$$\text{or} \quad \left(1 + \frac{20}{100} \right)^N > 2$$

$$\left(\frac{6}{5} \right)^N > 2$$

$$\therefore \quad N = 4 \text{ years}$$

28. Let rate of interest be $R\%$ p.a.

$$\text{S.I.} = \frac{18000 \times 2 \times R}{100} \quad (1)$$

$$\text{C.I.} = 18000 \left[\left(1 + \frac{R}{100} \right)^2 - 1 \right] \quad (2)$$

$$(2)-(1) \Rightarrow 18000 \left[\left(1 + \frac{R}{100} \right)^2 - 1 - \frac{2R}{100} \right] = 405$$

$$\text{or} \quad 18000 \left[\left(\frac{R}{100} \right)^2 \right] = 405$$

$$\begin{aligned} \text{or} \quad \left(\frac{R}{100} \right)^2 &= \frac{405}{18000} \\ &= \frac{45}{2000} \\ &= \frac{9}{400} \end{aligned}$$

$$\text{or} \quad \frac{R}{100} = \frac{3}{20}$$

$$\text{or} \quad R = 15\%$$

29. Let the required number of years be N .

$$\therefore 800 \left(1 + \frac{10}{200} \right)^{2N} = 926.10 \quad (1)$$

$$\text{i.e.} \quad 800 \left(\frac{105}{100} \right)^{2N} = 926.10$$

$$800 \left(\frac{21}{20} \right)^{2N} = 926.10$$

$$\begin{aligned} \therefore \left(\frac{21}{20} \right)^{2N} &= \frac{926.10}{800} \\ &= \frac{9261}{8000} \end{aligned}$$

$$= \left(\frac{21}{20} \right)^3$$

$$\therefore 2N = 3 \text{ years}$$

$$\text{or} \quad N = 1 \frac{1}{2} \text{ years}$$

30. Let each instalment be ₹ x .

$$\therefore \frac{x}{\left(1 + \frac{4}{100}\right)^2} + \frac{x}{\left(1 + \frac{4}{100}\right)} = 3825$$

$$\text{i.e.} \quad \frac{x}{\left(\frac{26}{25}\right)^2} + \frac{x}{\left(\frac{26}{25}\right)} = 3825$$

$$\text{or} \quad x + x \times \frac{26}{25} = 3825 \times \left(\frac{26}{25}\right)^2$$

$$\text{i.e.} \quad x \times \frac{51}{25} = 3825 \times \frac{(26)^2}{(25)^2}$$

$$\begin{aligned} x &= 3825 \times \frac{(26)^2}{(25)^2} \times \frac{25}{51} \\ &= \frac{153 \times (26)^2}{51} = 3 \times (26)^2 = \text{₹ } 2028 \end{aligned}$$

$$31. \quad 15000 \left(1 + \frac{R}{100}\right)^5 = 30000$$

$$\text{or} \quad \left(1 + \frac{R}{100}\right)^5 = 2 \quad (1)$$

$$\begin{aligned} \therefore 15000 \left(1 + \frac{R}{100}\right)^{20} &= 15000 \left[\left(1 + \frac{R}{100}\right)^5\right]^4 \\ &= 15000 \times 2^4 = \text{₹ } 2,40,000 \end{aligned}$$

$\therefore$ Amount after 20 years = ₹ 2,40,000

32. C.I. for 2 years @ 10% p.a. interest

$$= P \left[\left(1 + \frac{10}{100}\right)^2 - 1 \right] \quad (1)$$

$$\text{S.I.} = \frac{P \times 2 \times 10}{100} = \frac{P}{5} \quad (2)$$

$$(1)-(2) \Rightarrow P \left[\left(\frac{11}{10}\right)^2 - 1 - \frac{1}{5} \right] = 20$$

$$\text{or} \quad P \left(\frac{(11)^2 - (10)^2 - 20}{100} \right) = 20$$

$$\text{or} \quad P = \frac{20 \times 100}{1} = \text{₹ } 2000$$

$\therefore$ If interest is compounded half yearly,

$$\text{C.I.} - \text{S.I.} = P \left[\left(1 + \frac{5}{100}\right)^4 - 1 \right] - \frac{P}{5}$$

$$\begin{aligned}
&= P \left[\left(1 + \frac{5}{100} \right)^4 - 1 - \frac{1}{5} \right] \\
&= 2000 \left[\left(\frac{21}{20} \right)^4 - 1 - \frac{1}{5} \right] = 2000 \left[\frac{(21)^4 - (20)^4}{(20)^4} - \frac{1}{5} \right] \\
&= 2000 \left[\frac{(21^2 + 20^2)(21^2 - 20^2)}{20^4} - \frac{1}{5} \right] = 2000 \left[\frac{841 \times 41 \times 1}{20^4} - \frac{1}{5} \right] \\
&= 2000 \left[\frac{34481}{1,60,000} - \frac{1}{5} \right] = \frac{2000}{1,60,000} [34481 - 32000] \\
&= \frac{1}{80} \times 2481 = 31.0125
\end{aligned}$$

$\therefore$ Difference of interest = ₹ 31.01

$$\begin{aligned}
\text{33. Balance amount} &= 25000 \left(1 + \frac{20}{100} \right)^3 - 5000 \left(1 + \frac{20}{100} \right)^2 - 5000 \left(1 + \frac{20}{100} \right) - 5000 \\
&= 25000 \times \left(\frac{6}{5} \right)^3 - 5000 \left(\frac{6}{5} \right)^2 - 5000 \left(\frac{6}{5} \right) - 5000 \\
&= 200 \times 216 - 200 \times 36 - 6000 - 5000 \\
&= 43200 - 7200 - 6000 - 5000 = \text{₹ } 25000
\end{aligned}$$

34. Annual instalment = ₹ 882, R = 5%

$$\therefore \text{ Required amount} = \frac{882}{\left(1 + \frac{5}{100} \right)^2} + \frac{882}{\left(1 + \frac{5}{100} \right)}$$

$$\begin{aligned}
\text{i.e., Amount borrowed} &= \frac{882}{\left(\frac{21}{20} \right)^2} + \frac{882}{\left(\frac{21}{20} \right)} \\
&= 882 \left[\left(\frac{20}{21} \right)^2 + \left(\frac{20}{21} \right) \right] = 882 \left[\frac{20^2 + 20 \times 21}{21^2} \right] \\
&= 2[400 + 420] = \text{₹ } 1640
\end{aligned}$$

$$\begin{aligned}
\text{35. C.I.} &= 50000 \left[\left(1 + \frac{10}{100} \right)^4 - 1 \right] \\
&= 50000 \left[\left(\frac{11}{10} \right)^4 - 1 \right] = \frac{50000}{10^4} [11^4 - 10^4] \\
&= 5 \times [(11^2 + 10^2)(11^2 - 10^2)] = 5 \times [221 \times 21] = \text{₹ } 23205
\end{aligned}$$

$\therefore$ Compound interest = ₹ 23205.

Time and Work

If the number of men engaged to do a certain work is increased in the ratio $a : b$, then the time required to do the same work will get changed in the ratio $b : a$.



Important Formulae

1. If W represents an amount of work, P represents the number of persons doing the job and T represents the time period of doing work, then

$$\frac{PT}{W} = \text{constant}$$

2. A can do a work in D_1 days and B can do the same work in D_2 days. Then A and B if work together, then the number of days required to complete the work is

$$\frac{D_1 D_2}{D_1 + D_2}$$

3. A, B, and C can do a work in D_1 , D_2 and D_3 days respectively working alone. Time taken to complete the work if they work together is given by

$$\frac{D_1 D_2 D_3}{D_1 D_2 + D_2 D_3 + D_3 D_1} \text{ days}$$

4. A and B together can do a piece of work in D days. If A can do it alone in D_1 days, in how many days can B do it alone?

$$(A + B)\text{'s 1 day's work} = \frac{1}{D}$$

$$A\text{'s 1 day's work} = \frac{1}{D_1}$$

$$\therefore B\text{'s 1 day's work} = \frac{1}{D} - \frac{1}{D_1} = \frac{D_1 - D}{DD_1}$$

$$\therefore B \text{ alone can do this work in } \frac{DD_1}{D_1 - D} \text{ days}$$

5. A and B can do a piece of work in x days, B and C in y days, C and A in z days. How long will each take to do the same work separately?

$$(A + B)\text{'s 1 day's work} = \frac{1}{x}$$

$$(B + C)\text{'s 1 day's work} = \frac{1}{y}$$

$$(C + A)\text{'s 1 day's work} = \frac{1}{z}$$

$$\therefore \{(A + B) + (B + C) + (C + A)\}\text{'s 1 day's work} = \frac{1}{x} + \frac{1}{y} + \frac{1}{z}$$

$$\text{or } 2(A + B + C)\text{'s 1 day's work} = \frac{1}{x} + \frac{1}{y} + \frac{1}{z}$$

$$\therefore (A + B + C)\text{'s 1 day's work} = \frac{1}{2} \left(\frac{1}{x} + \frac{1}{y} + \frac{1}{z} \right)$$

$$\therefore \begin{aligned} A\text{'s 1 day's work} &= (A + B + C)\text{'s 1 day's work} \\ &\quad - (B + C)\text{'s 1 day's work} \end{aligned}$$

$$= \frac{1}{2} \left(\frac{1}{x} + \frac{1}{y} + \frac{1}{z} \right) - \frac{1}{y}$$

$$= \frac{1}{2} \left(\frac{1}{x} - \frac{1}{y} + \frac{1}{z} \right)$$

$$= \frac{yz - xz + xy}{2xyz}$$

$$\therefore A \text{ alone can do the work in } \frac{2xyz}{xy + yz - zx} \text{ days}$$

$$B \text{ alone can do the work in } \frac{2xyz}{yz + zx - xy} \text{ days}$$

$$C \text{ alone can do the work in } \frac{2xyz}{zx + xy - yz} \text{ days}$$

EXAMPLE 1 A and B can do a work in 15 days, B and C in 20 days, C and A in 12 days. In how many days can they complete it if:

- (i) They work together and (ii) they work separately.

Solution

- (i) If they work together, the number of days required to complete the work

$$= \frac{2xyz}{xy + yz + zx} = \frac{2(15)(20)(12)}{(15)(20) + (20)(12) + (12)(15)}$$

$$= \frac{10 \times 12}{5 + 4 + 3} = \frac{120}{12} = \mathbf{10 \text{ days}}$$

(ii) If they work separately

$$\begin{aligned}\text{A alone can complete the work in } & \frac{2xyz}{xy + yz - zx} \text{ days} \\ &= \frac{2(15)(20)(12)}{(15)(20) + (20)(12) - (12)(15)} = \frac{10 \times 12}{5 + 4 - 3} \\ &= \frac{120}{6} = \mathbf{20 \text{ days}}\end{aligned}$$

$$\begin{aligned}\text{B alone can complete the work in } & \frac{2xyz}{yz + zx - xy} \text{ days} \\ &= \frac{2(15)(20)(12)}{(20)(12) + (12)(15) - (15)(20)} = \frac{10 \times 12}{4 + 3 - 5} \\ &= \frac{120}{2} = \mathbf{60 \text{ days}}\end{aligned}$$

$$\begin{aligned}\text{C alone can complete the same work in } & \frac{2xyz}{zx + xy - yz} \text{ days} \\ &= \frac{2(15)(20)(12)}{(12)(15) + (15)(20) - (20)(12)} = \frac{10 \times 12}{3 + 5 - 4} \\ &= \frac{120}{4} = \mathbf{30 \text{ days}}\end{aligned}$$

EXAMPLE 2 A group of boys can do a certain piece of work in 40 days. If the group had 10 more boys, the work could be done in 5 days less. How many boys are there in the group?

Solution The original number of workers in the group

$$\begin{aligned}&= \frac{\text{Number of additional workers} \times \text{Number of days taken by second group}}{\text{Number of less days}} \\ &= \frac{10 \times 35}{5} = \mathbf{70 \text{ numbers}}\end{aligned}$$

EXAMPLE 3 A group of 20 boys took a contract to paint a shopping complex in 16 days. After 10 days they added 5 more boys and finished the work 2 days before the schedule. By how many days the schedule would have been delayed if these 5 boys were not added.

Solution Number of additional days needed

$$\begin{aligned}&= \frac{\text{Number of additional workers} \times \text{Number of days taken by the second group}}{\text{Original number of workers in the group}} \\ &= \frac{5 \times 4}{20} = \mathbf{1 \text{ day}}\end{aligned}$$

i.e. in all 15 days (i.e. 1 day in advance as per the schedule).



Important Formula

N men start to do a work in D days. After d days, n more men had to join in order to complete the work in scheduled time. If n men had not joined them, the number of additional days needed to complete the work is given by

$$\text{Over and above } D \text{ days } \frac{n \times (D - d)}{N} \text{ days}$$

EXAMPLE 4 50 men or 80 women can finish a job in 50 days. A contractor deploys 40 men and 48 women for this work, but after every duration of 10 days, 5 men and 8 women are removed till the work is completed. In how many days can the said work be completed?

Solution 50 men can finish the job in 50 days.

$\therefore$ 1 man can finish it in 50×50 days or part of work done by 1 man/day = $1/2500$.

Similarly 80 women can finish the job in 50 days.

$\therefore$ 1 woman can finish it in 80×50 days.

Part of work done by a woman/day = $1/4000$

At the start 40 men and 48 women were engaged.

$$50 \text{ men} = 80 \text{ women}$$

or

$$5 \text{ men} = 8 \text{ women}$$

$$\therefore \quad \text{At the start the number of men engaged} = 40 + \frac{48 \times 5}{8} = 70$$

$$\therefore \quad \text{After first 10 days work, part of work completed} = 10 \times 70 \times \frac{1}{2500} = \frac{7}{25}$$

$$\therefore \quad \text{The remaining part of work} = 1 - \frac{7}{25} = \frac{18}{25}$$

Then 5 men and 8 women are removed.

$$\therefore \quad \text{After second 10 days, part of work completed} = 10 \times 60 \times \frac{1}{2500} = \frac{6}{25}$$

$$\therefore \quad \text{The remaining part of work} = \frac{18}{25} - \frac{6}{25} = \frac{12}{25}$$

Then 5 men and 8 women are removed

$$\therefore \quad \text{After third 10 days, part of work completed} = 10 \times 50 \times \frac{1}{2500} = \frac{5}{25}$$

$$\therefore \quad \text{The remaining part of work} = \frac{12}{25} - \frac{5}{25} = \frac{7}{25}$$

Then 5 men and 8 women are removed.

$$\therefore \quad \text{After fourth 10 days, part of work completed} = 10 \times 40 \times \frac{1}{2500} = \frac{4}{25}$$

$$\therefore \quad \text{The remaining part of work} = \frac{7}{25} - \frac{4}{25} = \frac{3}{25}$$

Then 5 men and 8 women are removed.

$$\therefore \quad \text{After fifth 10 days, part of work completed} = 10 \times 30 \times \frac{1}{2500} = \frac{3}{25}$$

$\therefore$ The remaining part of work = Nil

i.e. work got completed in **50 days**.

If N_1 persons can do W_1 work in D_1 days and N_2 persons can do W_2 work in D_2 days then we have a very general formula in the relationship of $N_1 D_1 W_2 = N_2 D_2 W_1$.

or
$$\frac{N_1 D_1}{W_1} = \frac{N_2 D_2}{W_2}$$

From the above formula, we can arrive at the following results.



Important Results

1. The more men, the less days and conversely the more days the less men.
2. The more men, the more work and conversely the more work, the more men.
3. The more days, the more work and conversely the more work, the more days.

If we include working hours (T_1, T_2) also, then the above relationship becomes

$$\frac{N_1 D_1 T_1}{W_1} = \frac{N_2 D_2 T_2}{W_2}$$

If efficiency is also included as E_1 and E_2 for the persons in two groups, then

$$\frac{N_1 D_1 T_1 E_1}{W_1} = \frac{N_2 D_2 T_2 E_2}{W_2}$$

EXAMPLE 5 A can do a piece of work in 6 days. How many days will he take to complete four such works?

Solution From point 3 noted above, the more work, the more days.

Here $D_1 = 6$ days, $W_1 = 1$, $D_2 = ?$ $W_2 = 4$

$$\therefore \quad W_2 D_1 = W_1 D_2$$

$$\begin{aligned} \Rightarrow \quad D_2 &= \frac{W_2 D_1}{W_1} \\ &= \frac{4 \times 6}{1} = \mathbf{24 \text{ days}} \end{aligned}$$

EXAMPLE 6 Ten men can do a piece of work in 12 days. How many men are needed to complete the work in 20 days?

Solution To do a work in 12 days, 10 men are needed.

To do a work in 1 day, 10×12 men are needed.

$$\therefore \text{ To do the work in 20 days, } \frac{10 \times 12}{20} = 6 \text{ men are needed.}$$



Short cut: $N_1 D_1 = N_2 D_2$

Here $N_1 = 10$; $D_1 = 12$ days; $N_2 = ?$; $D_2 = 20$ days

$$\therefore N_2 = \frac{N_1 D_1}{D_2} = \frac{10 \times 12}{20} = 6$$

RULE OF FRACTIONS: To do the work in 20 days, we need less number of men than 10. So multiply 10 with a fraction which is less than 1.

$$\therefore \text{Required number} = 10 \times \frac{12}{20} = 6$$

EXAMPLE 7 30 men can plant 60 trees in 5 hours. If 5 men leave the job, how many trees will be planted in 10 hours?

Solution 30 men working 5 hours plant 60 trees

or 1 man working 1 hour, number of trees planted = $\frac{60}{30 \times 5} = \frac{2}{5}$ trees

$$\therefore 25 \text{ men working 10 hours, number of trees planted} = \frac{60 \times 25 \times 10}{30 \times 5} = \mathbf{100 \text{ trees}}$$



Short cut: $W N_1 D_1 W_2 = N_2 D_2 W_1$

$$30 \times 5 \times W_2 = 25 \times 10 \times 60$$

$$\therefore W_2 = \frac{25 \times 10 \times 60}{30 \times 5} = \mathbf{100 \text{ trees}}$$

There were 30 men, but when 5 men leaves the job, we are left with 25 men. As the number of men is reduced, less number of trees will be planted by them. So, 60 would be multiplied with less than one. And the fraction used is $25/30$. As the number of hours increases, the more number of trees will be planted. So the previous product is multiplied by a fraction

more than one, i.e. $\left(\frac{10}{5}\right)$.

$$\therefore \text{Required number of trees to be planted} = 60 \times \left(\frac{25}{30}\right) \times \left(\frac{10}{5}\right) = \mathbf{100 \text{ trees}}$$

EXAMPLE 8 5 men can prepare 20 items in 8 days working 6 hours a day. Then in how many days can 9 men prepare 30 items working 8 hours a day?

Solution We have, $\frac{N_1 D_1 T_1}{W_1} = \frac{N_2 D_2 T_2}{W_2}$

$$\text{i.e. } \frac{5 \times 8 \times 6}{20} = \frac{9 \times D_2 \times 8}{30}$$

$$\therefore D_2 = \frac{5 \times 8 \times 6 \times 30}{20 \times 9 \times 8} = \mathbf{5 \text{ days}}$$



Short cut: Required number of days,

$$D_2 = \text{First number of days } (D_1) \times \left(\frac{N_1}{N_2}\right) \times \left(\frac{W_2}{W_1}\right) \times \left(\frac{T_1}{T_2}\right)$$

$$= 8 \times \left(\frac{5}{9}\right) \times \left(\frac{30}{20}\right) \times \left(\frac{6}{8}\right) = 5 \text{ days}$$

Theorem: If A can do a piece of work in 'x' days and B can do it in 'y' days, then A and B together can do the same work in $\frac{xy}{x+y}$ days.

EXAMPLE 9 A can do a piece of work in 10 days and B can do the same work in 15 days. How long will they take if both work together?

Solution A can do $1/10$ part of the work in 1 day.

B can do $1/15$ part of the work in 1 day.

$\therefore$ A and B together can do $(1/10 + 1/15)$ part of the work in 1 day.

$\therefore$ Time taken to complete the work, when both A and B work together = $\frac{1}{1/10 + 1/15}$

$$= \frac{10 \times 15}{10 + 15} = \frac{150}{25} = 6 \text{ days}$$



Short cut: A and B can do the work in $\frac{10 \times 15}{10 + 15} = \frac{150}{25} = 6 \text{ days}$

Theorem: If A, B, and C can do a work in x, y, and z days respectively, then all of them working together can finish the work in $\frac{xyz}{xy + yz + zx}$ days.

EXAMPLE 10 If A can do a piece of work in 10 days, B can do the same work in 15 days and C can do the same work in 30 days, how long will they take to complete the work when they work together?

Solution By the above theorem, time taken to complete the work

$$= \frac{10 \times 15 \times 30}{10 \times 15 + 15 \times 30 + 30 \times 10}$$

$$= \frac{1 \times 15 \times 30}{15 + 45 + 30} = \frac{30}{1 + 3 + 2} = \frac{30}{6} = 5 \text{ days}$$

Theorem: If Sunil and Suresh together can do a piece of work in x days and Sunil alone can do it in y days, then Suresh alone can do the same work in $\frac{xy}{y-x}$ days.

EXAMPLE 11 Anil and Manoj together can do a piece of work in 6 days and Anil alone can do it in 8 days. In how many days can Manoj alone do it?

Solution Anil and Manoj can do $\frac{1}{6}$ part of the work in 1 day.

Anil alone can do $\frac{1}{8}$ part of the work in 1 day.

$\therefore$ Manoj alone can do $(\frac{1}{6} - \frac{1}{8})$ part of the work in 1 day.

$\therefore$ Manoj alone can do the same work in $\frac{1}{(\frac{1}{6} - \frac{1}{8})} = \frac{8 \times 6}{8 - 6} = \frac{48}{2} = \mathbf{24 \text{ days}}$



Short cut: Manoj alone can do the same work in $\frac{xy}{y - x}$ days.

$$= \frac{8 \times 6}{8 - 6} = \mathbf{24 \text{ days}}$$

EXAMPLE 12 P and Q can do a piece of work in 12 days, Q and R in 15 days, R and P in 20 days. How long would each take separately to do the same work?

Solution P and Q can complete the work in 12 days.

Q and R can complete the work in 15 days.

R + P can complete the work in 20 days.

$\therefore$ $2(P + Q + R)$ can complete the work in $\frac{12 \times 15 \times 20}{12 \times 15 + 15 \times 20 + 20 \times 12} = \mathbf{5 \text{ days}}$

$\therefore$ P, Q and R can do the work in $5 \times 2 = 10$ days

P and Q can do the work in 12 days

Q and R can do the work in 15 days

R and P can do the work in 20 days

$\therefore$ P alone can do the work in $\frac{15 \times 10}{15 - 10} = \frac{150}{5} = \mathbf{30 \text{ days}}$

$$[\because P = (P + Q + R) - (Q + R)]$$

Similarly Q alone can do the work in $\frac{20 \times 10}{20 - 10} = \frac{200}{10} = \mathbf{20 \text{ days}}$

Similarly R alone can do the work in $\frac{12 \times 10}{12 - 10} = \frac{120}{2} = \mathbf{60 \text{ days}}$

EXAMPLE 13 A and B together can do a piece of work in 16 days which B and C together can do in 12 days. After A has been working at it for 2 days and B for 7 days, C takes up and finishes it alone. In how many days could each do the work by himself?

Solution A and B can do $\frac{1}{16}$ part of the work in 1 day.

B and C can do $\frac{1}{12}$ part of the work in 1 day.

A's 2 day's + B's 7 day's + C's 16 day's work = 1

or A's 2 days + B's 2 day's + B's 5 day's + C's 5 day's + C's 11 day's work = 1

$\therefore$ (A + B)'s 2 day's + (B + C)'s 5 day's + C's 11 day's work = 1

$$\frac{2}{16} + \frac{5}{12} + \text{C's 11 day's work} = 1$$

$$\therefore \text{C's 11 day's work} = 1 - \left(\frac{2}{16} + \frac{5}{12} \right) = \frac{11}{24}$$

$$\therefore \text{C's 1 day's work} = \frac{11}{24 \times 11} = \frac{1}{24}$$

$$\text{B's 1 day's work} = \frac{1}{12} - \frac{1}{24} = \frac{1}{24}$$

$$\text{A's 1 day's work} = \frac{1}{16} - \frac{1}{24} = \frac{1}{48}$$

$\therefore$ A, B, and C can do the work in 48, 24 and 24 days respectively when they work alone.

EXAMPLE 14 If 3 men or 4 women can complete a certain job in 43 days, how long will 7 men and 5 women take to reap it?

Solution 3 men can do $1/43$ part of job in 1 day.

4 women can do $1/43$ part of job in 1 day.

$\therefore$ 1 man can do $1/(43 \times 3)$ part of job in 1 day.

1 woman can do $1/(43 \times 4)$ part of job in 1 day.

$\therefore$ (7 men + 5 women) can do $\left(\frac{7}{43 \times 3} + \frac{5}{43 \times 4} \right)$ part of the job in 1 day

$$\begin{aligned} \therefore \text{Time taken to complete the job by 7 men and 5 women} &= \frac{1}{\left(\frac{7}{43 \times 3} + \frac{5}{43 \times 4} \right)} \\ &= \frac{43 \times 3 \times 4}{7 \times 4 + 5 \times 3} = \frac{43 \times 3 \times 4}{28 + 15} \\ &= \frac{43 \times 3 \times 4}{43} = \mathbf{12 \text{ days}} \end{aligned}$$

Another method:

$$3 \text{ men} = 4 \text{ women}$$

$$\therefore 1 \text{ man} = 4/3 \text{ women}$$

$$\therefore 7 \text{ men} = 28/3 \text{ women}$$

$$\therefore 7 \text{ men} + 5 \text{ women} = 28/3 + 5 = 43/3 \text{ women}$$

Now we have,

4 women do the job in 43 days.

$$\therefore 43/3 \text{ women do the same job is } \frac{4 \times 43}{43/3} = 4 \times 3 = \mathbf{12 \text{ days}}$$



Short cut: Required number of days = $\frac{1}{\left[\frac{7}{43 \times 3} + \frac{5}{43 \times 4} \right]} = \frac{43 \times 3 \times 4}{7 \times 4 + 5 \times 3} = \mathbf{12 \text{ days}}$

EXAMPLE 15 A certain number of men can do a work in 60 days. If there were 8 men more it could have finished in 10 days less. How many men were there?

Solution Let the number of men be x

$(x + 8)$ men can finish the work in $(60 - 10) = 50$ days

i.e. $x \times 60 = (x + 8) \times 50$

On simplifying the equation,

$$\Rightarrow 10 \times x = 50 \times 8$$

$$\Rightarrow x = \frac{50 \times 8}{10} = \mathbf{40 \text{ men}}$$

Another method:

Let the original number of men be x .

x men can do the job in 60 days.

$(x + 8)$ men can do the same job in $(60 - 10) = 50$ days.

$\therefore$ Work done by 8 men in 50 days is same as what x men can do in 10 days.

$$\therefore 8 \times 50 = x \times 10$$

$$\therefore x = \frac{8 \times 50}{10} = \mathbf{40}$$



Short cut: Original number of workers

$$= \frac{\text{Number of more workers} \times \text{Number of days taken by second group}}{\text{Number of less days}}$$

$$= \frac{8 \times (60 - 10)}{10} = \frac{8 \times 50}{10} = \mathbf{40 \text{ men}}$$

EXAMPLE 16 X is twice as good a workman as Y. Together, they finish the work in 14 days. In how many days can it be done by each separately?

Solution Let Y finish the work in $2n$ days.

Therefore X can finish the same work in n days

$$\therefore (X + Y) \text{ can finish the work in } \frac{2n \times n}{2n + n} = 14$$

$$\text{i.e. } \frac{2n^2}{3n} = 14$$

$$\text{or } n = \mathbf{21}$$

$\therefore$ X finishes the work in 21 days and Y finishes the same work in 42 days when they work independently.



Short cut: $(2 + 1)$ men – one man work in 14 days

$$\therefore 1 \text{ man – one day work for } 14 \times 3 = 42 \text{ days}$$

$$\therefore 2 \text{ men – one day work for } 42/2 = \mathbf{21 \text{ days}}$$

EXAMPLE 17 A and B can do a work in 45 days and 40 days respectively. They began the work together, but A left after some time and B finished the remaining work in 23 days. After how many days did A leave?

Solution If A and B work together, time taken to complete the work

$$= \frac{40 \times 45}{40 + 45} = \frac{40 \times 45}{85} = \frac{40 \times 9}{17}$$

When B works alone for 23 days, the part of the work completed = $23 \times \frac{1}{40} = \frac{23}{40}$

$\therefore$ Remaining work = $1 - \frac{23}{40} = \frac{17}{40}$ **part**

$\therefore$ Time taken to complete the above part of the work when A and B work together

$$= \frac{40 \times 9}{17} \times \frac{17}{40} = \mathbf{9 \text{ days}}$$



Short cut: Required number of days = $\frac{40 \times 45}{40 + 45} \left(\frac{40 - 23}{40} \right) = \mathbf{9 \text{ days}}$

EXAMPLE 18 A group of men decided to do a work in 10 days. But five of them were absent. If the rest of the group did the work in 12 days, find the original number of men.

Solution Let the original number of men be n . Here number of man days is constant.

i.e. $10 \times n = 12(n - 5)$

$\therefore n = \frac{12 \times 5}{(12 - 10)} = \mathbf{30 \text{ men}}$

EXAMPLE 19 A person decided to build a farmhouse in 40 days. He employed 100 men in the beginning and 100 more after 35 days and completed the work in stipulated time. If he had not employed the additional men, how many days behind schedule would it have been finished?

Solution Let the time taken to complete the work by 100 men be t days.

$\therefore$ Work done by 100 men in 35 days + Work done by 200 men in $(40 - 35) = 5$ days = 1

or
$$\frac{100 \times 35 + 200 \times 5}{100t} = 1$$

$\Rightarrow \frac{45}{t} = 1$ or $t = \mathbf{45 \text{ days}}$

$\therefore$ If additional men were not employed, the work would have completed in $(45 - 40) = \mathbf{5 \text{ days}}$ behind the schedule.



Short cut: 200 men do the rest of the work in $40 - 35 = \mathbf{5 \text{ days}}$

$\therefore$ 100 men can do the rest of the work in $\frac{5 \times 200}{100} = \mathbf{10 \text{ days}}$

$\therefore$ Required number of days = $10 - 5 = \mathbf{5 \text{ days}}$

EXAMPLE 20 P can complete a work in 25 days and Q can complete the same work in 20 days. If they work together and P leaves after 5 days, in how many days Q will finish the rest of the work.

Solution Part of the work done by P and Q in 5 days = $5 \left[\frac{1}{25} + \frac{1}{20} \right] = 5 \times \frac{9}{100} = \frac{9}{20}$

$$\therefore \text{Rest of the work} = 1 - \frac{9}{20} = \frac{11}{20}$$

$$\text{Q will do the rest of work in } \frac{20}{1} \times \frac{11}{20} = \mathbf{11 \text{ days}}$$

EXAMPLE 21 30 men working 7 hours a day can complete a work in 20 days. In how many days will 21 men working 8 hours a day do the same work?

Solution We have $M_1 D_1 T_1 = M_2 D_2 T_2$

$$\therefore D_2 = \frac{M_1 D_1 T_1}{M_2 T_2} = \frac{30 \times 20 \times 7}{21 \times 8} = \mathbf{25 \text{ days}}$$

EXAMPLE 22 X takes as much time as Y and Z together take to complete a job. X and Y together can complete the job in 10 days. Z alone can complete the same job in 15 days. In how many days can Y alone do the same job?

Solution Given (X + Y) together can complete the job in 10 days.
Z alone can complete the same job in 15 days.

$$\therefore \text{Time taken to complete the job when all of them work together} = \frac{15 \times 10}{15 + 10} = \mathbf{6 \text{ days}}$$

But time taken by X is same as Y and Z together.

$$\therefore \text{Y and Z can complete the job in } 2 \times 6 = \mathbf{12 \text{ days}}$$

$$\therefore \text{Y alone can complete the work in } \frac{15 \times 12}{15 - 12} = \mathbf{60 \text{ days}} \quad [\because Y = (Y + Z) - Z]$$

EXAMPLE 23 Ashok can do a piece of work in 16 days. Raman can do the same work in 8 days while Satish can do it in 32 days. All of them worked together when they started, but Raman left after 2 days. Ashok left 3 days before the completion of the work. How long would work last?

Solution Let the work lasted for t days.

Then Raman's 2 days' work + Ashok's $(t - 3)$ days' work + Satish's t days' work = 1

$$\text{or} \quad \frac{2}{8} + \frac{t-3}{16} + \frac{t}{32} = 1$$

$$\text{i.e.} \quad \frac{1}{4} + \frac{t-3}{16} + \frac{t}{32} = 1$$

$$\text{i.e.} \quad \frac{8 + 2(t-3) + t}{32} = 1$$

$$\text{i.e.} \quad 3t + 2 = 32$$

or

$$3t = 32 - 2 = 30$$

 $\therefore$

$$t = \frac{30}{3} = 10 \text{ days}$$

EXAMPLE 24 P and Q undertake to do a job for ₹ 91. P can do it alone in 13 days and Q can do it alone in 7 days. If with the assistance of a boy they finish the work in 2 days then the boy gets ₹ _____.

Solution P's 2 days' work + Q's 2 days' work + Boy's 2 days' work = 1

$$\text{i.e. } \frac{2}{13} + \frac{2}{7} + \text{Boys 2 day's work} = 1$$

 $\therefore$

$$\text{Boy's 2 day's work} = 1 - \left(\frac{2}{13} + \frac{2}{7} \right)$$

$$= 1 - \left(\frac{14 + 26}{91} \right)$$

$$= \frac{91 - 40}{91} = \frac{51}{91}$$

 $\therefore$

$$\begin{aligned} \text{Ratio of shares} &= \frac{2}{13} : \frac{2}{7} : \frac{51}{91} \\ &= 2 \times 7 : 2 \times 13 : 51 \\ &= 14 : 26 : 51 \end{aligned}$$

 $\therefore$

$$\text{Boy's share} = \frac{51}{14 + 26 + 51} \times 91 = \frac{51}{91} \times 91 = \text{₹ } 51$$



EXERCISES

- P and Q working together can complete a certain work in 6 days. Q alone can do it in 8 days. In how many days P alone could finish the same work?
(a) 12 days (b) 16 days (c) 18 days (d) 24 days (e) None of these
- A certain sum of money is sufficient to pay wages for 21 days of X or 28 days of Y. The money is sufficient to pay the wages of both for _____ days.
(a) 8 days (b) 12 days (c) 16 days (d) 20 days (e) None of these
- If 3 men or 5 women can reap a field in 43 days, how long will 5 men and 6 women take to reap it?
(a) 10 days (b) 12 days (c) 15 days (d) 18 days (e) None of these
- 3 men and 4 boys can complete a certain work in 8 days, while 4 men and 4 boys can complete the same work in 6 days. Then 2 men and 4 boys will finish it in _____ days.
(a) 24 days (b) 8 days (c) 6 days (d) 4 days (e) None of these
- 2 men and 3 women can complete a work in 10 days, while 4 men can complete it in 10 days. In how many days will 3 men and 3 women complete the work?
(a) 6 days (b) 8 days (c) 10 days (d) 12 days (e) None of these

6. P and Q can do a piece of work in 30 and 40 days respectively. They began the work together, but P left after some days and Q finished the remaining work in 12 days. After how many days did P leave?
 (a) 8 days (b) 10 days (c) 12 days (d) 14 days (e) None of these
7. P is twice as good a workman as Q, and together they finish a work in 16 days. In how many days can it be done by each separately?
 (a) 16 days, 24 days (b) 24 days, 48 days (c) 20 days, 40 days
 (d) 24 days, 16 days (e) None of these
8. Ram, Rajan and Rangan can do a certain work in 6, 12 and 24 days respectively. In what time will they altogether do it?
 (a) $6/7$ days (b) $12/7$ days (c) $24/7$ days (d) $48/7$ days (e) None of these
9. P and Q complete a certain work in 30 days, Q and R in 40 days while R and P in 60 days. How long will they take to finish it together?
 (a) $40/3$ days (b) $80/3$ days (c) $160/3$ days (d) $20/3$ days (e) None of these
10. P, Q and R can finish a work in 10, 12 and 15 days respectively. If Q stops after 2 days, how long would it take P and R to finish the same work?
 (a) 6 days (b) 4 days (c) 3 days (d) 2 days (e) None of these
11. If 6 men and 8 boys can do a piece of work in 10 days while 26 men and 48 boys can do the same in 2 days, the time taken by 15 men and 20 boys in doing the same type of work will be [SSC]
 (a) 4 days (b) 5 days (c) 6 days (d) 7 days
12. Twenty four men can complete a work in sixteen days. Thirty-two women can complete the same work in twenty-four days. Sixteen men and sixteen women started working and worked for twelve days. How many more men are to be added to complete the remaining work in 2 days? [Bank PO]
 (a) 16 (b) 24 (c) 36 (d) 48 (e) None of these
13. 10 women can complete a work in 7 days and 10 children take 14 days to complete the work. How many days will 5 women and 10 children take to complete the work? [Bank PO]
 (a) 3 (b) 5 (c) 7
 (d) Cannot be determined (e) None of these
14. 4 men and 6 women can complete a work in 8 days, while 3 men and 7 women can complete it in 10 days. In how many days will 10 women complete it? [SSC]
 (a) 35 (b) 40 (c) 45 (d) 50
15. If 12 men and 16 boys can do a piece of work in 5 days; 13 men and 24 boys can do it in 4 days, then the ratio of the daily work done by a man to that of a boy is [SSC]
 (a) 2 : 1 (b) 3 : 1 (c) 3 : 2 (d) 5 : 4
16. Three men, four women and six children can complete a work in seven days. A woman does double the work a man does and a child does half the work a man does. How many women alone can complete this work in 7 days? [SBI PO]
 (a) 7 (b) 8 (c) 12
 (d) Cannot be determined (e) None of these
17. A, B and C can do a piece of work in 20, 30 and 60 days respectively. In how many days can A do the work if he is assisted by B and C on every third day? [RRB]
 (a) 12 days (b) 15 days (c) 16 days (d) 18 days
18. Twenty women can do a work in sixteen days. Sixteen men can complete the same work in fifteen days. What is the ratio between the capacity of a man and a woman? [BSRB]
 (a) 3 : 4 (b) 4 : 3 (c) 5 : 3 (d) Data inadequate

19. 12 men can complete a piece of work in 4 days, while 15 women can complete the same work in 4 days, 6 men start working on the job and after working for 2 days, all of them stopped working. How many women should be put on the job to complete the remaining work, if it is to be completed in 3 days? [SBI PO]
(a) 15 (b) 18 (c) 22 (d) Data inadequate
(e) None of these
20. 10 men and 15 women together can complete a work in 6 days. It takes 100 days for one man alone to complete the same work. How many days will be required for one woman alone to complete the same work? [Bank PO]
(a) 90 (b) 125 (c) 145 (d) 150 (e) None of these
21. A man, a woman and a boy can complete a job in 3, 4 and 12 days respectively. How many boys must assist 1 man and 1 woman to complete the job in $\frac{1}{4}$ of a day? [SSC]
(a) 1 (b) 4 (c) 19 (d) 41
22. 12 men complete a work in 9 days. After they have worked for 6 days, 6 more men join them. How many days will they take to complete the remaining work? [RRB]
(a) 2 days (b) 3 days (c) 4 days (d) 5 days (e) None of these
23. 10 men can complete a piece of work in 15 days and 15 women can complete the same work in 12 days. If all the 10 men and 15 women work together, in how many days will the work get completed? [SBI PO]
(a) 6 (b) $6\frac{1}{3}$ (c) $6\frac{2}{3}$ (d) $7\frac{2}{3}$
24. A alone can complete a work in 16 days and B alone in 12 days. Starting with A, they work on alternate days. The total work will be completed in [SSC]
(a) 12 days (b) 13 days (c) $13\frac{5}{7}$ days (d) $13\frac{3}{4}$ days
25. A, B and C are employed to do a piece of work for ₹ 529. A and B together are supposed to do $\frac{19}{23}$ of the work and B and C together $\frac{8}{23}$ of the work. What amount should A be paid? [CBI]
(a) ₹ 315 (b) ₹ 345 (c) ₹ 355 (d) ₹ 375
26. A and B together can complete a work in 12 days. A alone can complete it in 20 days. If B does the work only for half a day daily, then in how many days A and B together will complete the work? [RRB]
(a) 10 days (b) 11 days (c) 15 days (d) 20 days
27. A can do a piece of work in 14 days which B can do in 21 days. They begin together but 3 days before the completion of the work, A leaves off. The total number of days to complete the work is: [RRB]
(a) $6\frac{3}{5}$ (b) $8\frac{1}{2}$ (c) $10\frac{1}{5}$ (d) $13\frac{1}{2}$
28. A alone can do a piece of work in 6 days and B alone in 8 days. A and B under took to do it for ₹ 3200. With the help of C, they completed the work in 3 days. How much is to be paid to C? [SSC]
(a) ₹ 375 (b) ₹ 400 (c) ₹ 600 (d) ₹ 800
29. A and B can do a piece of work in 45 days and 40 days respectively. They began to do the work together but A leaves after some days and then B completed the remaining work in 23 days. The number of days after which A left the work was: [Bank PO]
(a) 6 (b) 8 (c) 9 (d) 12

30. Kim can do a work in 3 days while David can do the same work in 2 days. Both of them finish the work together and get ₹ 150. What is the share of Kim? [SSC]
 (a) ₹ 30 (b) ₹ 60 (c) ₹ 70 (d) ₹ 75
31. A and B together can do a piece of work in 30 days. A having worked for 16 days, B finishes the remaining work alone in 44 days. In how many days shall B finish the whole work alone? [CBI]
 (a) 30 days (b) 40 days (c) 60 days (d) 70 days
32. A can finish a work in 24 days, B in 9 days and C in 12 days. B and C start the work but are forced to leave after 3 days. The remaining work was done by A in: [SSC]
 (a) 5 days (b) 6 days (c) 10 days (d) $10\frac{1}{2}$ days
33. A does $\frac{4}{5}$ of a work in 20 days. He then calls in B and they together finish the remaining work in 3 days. How long B alone would take to do the whole work? [SSC]
 (a) 23 days (b) 37 days (c) $37\frac{1}{2}$ days (d) 40 days
34. A and B can together finish a work in 30 days. They worked together for 20 days and then B left. After another 20 days, A finished the remaining work. In how many days A alone can finish the job? [SSC]
 (a) 40 (b) 50 (c) 54 (d) 60
35. A can finish a work in 18 days and B can do the same work in 15 days. B worked for 10 days and left the job. In how many days, A alone can finish the remaining work? [Bank PO]
 (a) 5 (b) $5\frac{1}{2}$ (c) 6 (d) 8
36. Machine P can print one lakh books in 8 hours, machine Q can print the same number of books in 10 hours, while machine R can print them in 12 hours. All the machines are started at 9.00 a.m., while machine P is closed at 11.00 a.m. and the remaining two machines complete the work. Approximately at what time will the work be finished? [Bank PO]
 (a) 11.30 a.m. (b) 12.00 Noon (c) 12.30 p.m. (d) 01.00 p.m.
37. X and Y can do a piece of work in 20 days and 12 days respectively. X started the work alone and then after 4 days Y joined him till the completion of the work. How long did the work last? [Bank PO]
 (a) 6 days (b) 10 days (c) 15 days (d) 20 days
38. A and B can do a piece of work in 30 days, while B and C can do the same work in 24 days and C and A in 20 days. They all work together for 10 days when B and C leave. How many days more will A take to finish the work? [CBI]
 (a) 18 days (b) 24 days (c) 30 days (d) 36 days
39. A and B can complete a work in 15 days and 10 days respectively. They started doing the work together but after 2 days B had to leave and A alone completed the remaining work. The whole work was completed in: [SSC]
 (a) 8 days (b) 10 days (c) 12 days (d) 15 days
40. A and B can do a job together in 7 days. A is $1\frac{3}{4}$ times as efficient as B. The same job can be done by A alone in: [SSC]
 (a) $9\frac{1}{3}$ days (b) 11 days (c) $12\frac{1}{4}$ days (d) $16\frac{1}{3}$ days

41. A can do a certain work in the same time in which B and C together can do it. If A and B together could do it in 10 days and C alone in 50 days, then B alone could do it in: [SSC]
(a) 15 days (b) 20 days (c) 25 days (d) 30 days
42. A is thrice as good a workman as B and therefore is able to finish a job in 60 days less than B. Working together, they can do it in: [SSC]
(a) 20 days (b) $22\frac{1}{2}$ days (c) 25 days (d) 30 days
43. A can do a work in 15 days and B in 20 days. If they work on it together for 4 days, then the fraction of the work that is left is: [SSC]
(a) $\frac{1}{4}$ (b) $\frac{1}{10}$ (c) $\frac{7}{15}$ (d) $\frac{8}{15}$
44. A is 30% more efficient than B. How much time will they, working together, take to complete a job which A alone could have done in 23 days? [Hotel Management]
(a) 11 days (b) 13 days (c) $20\frac{3}{17}$ days (d) None of these
45. Sakshi can do a piece of work in 20 days. Tanya is 25% more efficient than Sakshi. The number of days taken by Tanya to do the same piece of work is [Hotel Management]
(a) 15 (b) 16 (c) 18 (d) 25
46. A and B can do a work in 8 days, B and C can do the same work in 12 days. A, B and C together can finish it in 6 days. A and C together will do it in: [RRB]
(a) 4 days (b) 6 days (c) 8 days (d) 12 days
47. P can complete a work in 12 days working 8 hours a day. Q can complete the same work in 8 days working 10 hours a day. If both P and Q work together, working 8 hours a day, in how many days can they complete the work? [Bank PO]
(a) $5\frac{5}{11}$ (b) $5\frac{6}{11}$ (c) $6\frac{5}{11}$ (d) $6\frac{6}{11}$
48. A takes twice as much time as B or thrice as much time as C to finish a piece of work. Working together, they can finish the work in 2 days. B can do the work alone in [SSC]
(a) 4 days (b) 6 days (c) 8 days (d) 12 days
49. A and B can do a work in 12 days. B and C in 15 days, C and A in 20 days. If A, B and C work together they will complete work in: [SSC]
(a) 5 days (b) $7\frac{5}{6}$ days (c) 10 days (d) $15\frac{2}{3}$ days
50. A can lay railway track between two given stations in 16 days and B can do the same job in 12 days. With the help of C, they did the job in 4 days only. Then C alone can do the job in: [SSC]
(a) $9\frac{1}{5}$ days (b) $9\frac{2}{5}$ days (c) $9\frac{3}{5}$ days (d) 10 days
51. A man can do a piece of work in 5 days, but with the help of his son, he can do it in 3 days. In what time can the son do it alone? [SSC]
(a) $6\frac{1}{2}$ days (b) 7 days (c) $7\frac{1}{2}$ days (d) 8 days
52. A can finish a work in 18 days and B can do the same work in half the time taken by A. Then, working together, what part of the same work they can finish in a day? [SSC]
(a) $\frac{1}{6}$ (b) $\frac{1}{9}$ (c) $\frac{2}{5}$ (d) $\frac{2}{7}$

53. A, B and C can complete a piece of work in 24, 6 and 12 days respectively. Working together, they will complete the same work in: **[CBI]**
- (a) $\frac{1}{24}$ day (b) $\frac{7}{24}$ day (c) $3\frac{3}{7}$ days (d) 4 days
54. A does a work in 10 days and B does the same work in 15 days. In how many days they together will do the same work? **[RRB]**
- (a) 5 days (b) 6 days (c) 8 days (d) 9 days
55. A tyre has two punctures. The first puncture alone would have made the tyre flat in 9 minutes and the second alone would have done it in 6 minutes. If air leaks out at a constant rate, how long does it take both the punctures together to make it flat? **[DMRC]**
- (a) $1\frac{1}{2}$ minutes (b) $3\frac{1}{2}$ minutes (c) $3\frac{3}{5}$ minutes (d) $4\frac{1}{4}$ minutes



1. (d) 2. (b) 3. (c) 4. (e) 5. (b) 6. (c) 7. (b) 8. (c) 9. (b) 10. (c)
 11. (a) 12. (b) 13. (c) 14. (b) 15. (a) 16. (a) 17. (b) 18. (b) 19. (a) 20. (e)
 21. (d) 22. (a) 23. (c) 24. (c) 25. (b) 26. (c) 27. (c) 28. (b) 29. (c) 30. (b)
 31. (c) 32. (c) 33. (c) 34. (d) 35. (c) 36. (d) 37. (a) 38. (a) 39. (b) 40. (b)
 41. (c) 42. (b) 43. (d) 44. (b) 45. (b) 46. (c) 47. (a) 48. (b) 49. (c) 50. (c)
 51. (c) 52. (a) 53. (c) 54. (b) 55. (c)



Solutions with Necessary Explanation

1. Required number of days = $\frac{6 \times 8}{8 - 6} = 24$ days.
2. As the given sum is for 21 days or 28 days of wages of X or Y, the sum will be completely divisible by 21 and 28. So let the sum be LCM of 21 and 28 = 84.

$$\therefore \quad X \text{ gets } \frac{84}{21} = ₹ 4/\text{day}$$

$$Y \text{ gets } \frac{84}{28} = ₹ 3/\text{day}$$

$$\therefore (X + Y) \text{ together gets } ₹ 7/\text{day}$$

$$\therefore ₹ 84 \text{ is sufficient for } \frac{84}{7} = 12 \text{ days to pay both of them.}$$



Short cut: Number of days = $\frac{\text{Multiplication of number of days}}{\text{Addition of number of days}}$

$$= \frac{21 \times 28}{21 + 28} = 12 \text{ days}$$

3. 3 men = 5 women

$$\therefore 5 \text{ men} = \frac{5}{3} \times 5 = \frac{25}{3} \text{ women}$$

$$\therefore 5 \text{ men} + 6 \text{ women} = \frac{25}{3} + 6 = \frac{43}{3} \text{ women}$$

5 women can reap in 43 days

$$W_1 D_1 = W_2 D_2$$

$$\therefore \frac{43}{3} \text{ women can reap in } \frac{43 \times 5}{43/3} = 5 \times 3 = \mathbf{15 \text{ days}}$$



Short cut: Required number of days = $\frac{1}{\left[\frac{5}{3 \times 43} + \frac{6}{5 \times 43} \right]} = \mathbf{15 \text{ days}}$

4. 3 men + 4 boys complete the work in 8 days (1)

4 men + 4 boys complete the work in 6 days (2)

Equation (2) and (1) $\Rightarrow$ 1 man complete the work in $\frac{8 \times 6}{8 - 6} = 24$ days (3)

$\therefore$ 3 men can complete the work in $\frac{24}{3} = 8$ days (4)

Equations (1) and (4) $\Rightarrow$ boys do not work.

$\therefore$ 2 men + 4 boys complete in $\frac{24}{2} = \mathbf{12 \text{ days}}$.

5. 4 men complete in 10 days

$\therefore$ 2 men complete in $2 \times 10 = 20$ days (1)

2 men and 3 women complete in 10 days (2)

Equations (2) and (1) $\Rightarrow$ 3 women complete in $\frac{10 \times 20}{20 - 10} = 20$ days (3)

Equation (1) $\Rightarrow$ 3 men complete in $\frac{20 \times 2}{3} = \frac{40}{3}$ days (4)

Equations (3) + (4) $\Rightarrow$ 3 men + 3 women complete in $\frac{20 \times 40/3}{20 + 40/3} = \frac{20 \times 40}{100} = \mathbf{8 \text{ days}}$

6. Work done by Q alone in 12 days =
- $\frac{12}{40} = \frac{3}{10}$
- work

Work done by P and Q together = $1 - \frac{3}{10} = \frac{7}{10}$ work

P and Q together can do the work in $\frac{40 \times 30}{40 + 30} = \frac{120}{7}$ days

$\therefore$ The number of days worked by P and Q together to complete $\frac{7}{10}$ work = $\frac{120}{7} \times \frac{7}{10} = \mathbf{12 \text{ days}}$



Short cut: Required number of days = $\frac{30 \times 40}{30 + 40} \times \frac{40 - 12}{40} = \mathbf{12 \text{ days}}$

7. Let Q do the work in $2n$ days

$\therefore$ P does the work in n days

$\therefore$ P and Q together can do it in $\frac{2n \times n}{2n + n} = \frac{2n^2}{3n} = \frac{2n}{3} = 16$ days

$\therefore n = \frac{3 \times 16}{2} = 24$ days

P completes in 24 days and Q completes in 48 days when worked independently.

8. Required number of days = $\frac{6 \times 12 \times 24}{6 \times 12 + 12 \times 24 + 24 \times 6} = \frac{24}{1 + 4 + 2} = \frac{24}{7}$ or $3\frac{3}{7}$ days

9. $2(P + Q + R)$ will do the work in $\frac{30 \times 40 \times 60}{30 \times 40 + 40 \times 60 + 60 \times 30} = \frac{120}{2 + 4 + 3} = \frac{120}{9} = \frac{40}{3}$ days.

$\therefore$ P + Q + R will do the work in $2 \times \frac{40}{3} = \frac{80}{3}$ or $26\frac{2}{3}$ days

10. Part of work completed by P, Q, and R in 2 days = $2 \left(\frac{1}{10} + \frac{1}{12} + \frac{1}{15} \right) = 2 \times \left(\frac{6+5+4}{60} \right) = \frac{1}{2}$ work

$\therefore$ If Q is withdrawn P and R have to complete the remaining half portion of the work.

Time taken to do 1 work by P + R = $\frac{10 \times 15}{10 + 15} = \frac{150}{25} = 6$ days

$\therefore$ 1/2 work will be completed in $6/2 = 3$ days.

11. Let the part of work done per day by a man = x and that by a boy = y

$$\therefore \quad 6x + 8y = \frac{1}{10} \quad (1)$$

$$26x + 48y = \frac{1}{2} \quad (2)$$

Solving (1) and (2), we get

$$x = \frac{1}{100} \text{ and } y = \frac{1}{200}$$

$\therefore$ 1 day's work of 15 men and 20 boys

$$= 15 \times \frac{1}{100} + 20 \times \frac{1}{200} = \frac{15+10}{100} = \frac{25}{100} = \frac{1}{4}$$

$\therefore$ 15 men and 20 boys will complete the work in 4 days.

12. 1 man's 1 day's work = $\frac{1}{16 \times 24}$ (1)

1 woman's 1 day's work = $\frac{1}{32 \times 24}$ (2)

$\therefore$ 16 men's + 16 women's 1 day's work

$$= 16 \times \frac{1}{16 \times 24} + 16 \times \frac{1}{32 \times 24} = \frac{1}{24} + \frac{1}{48}$$

$$= \frac{1}{16}$$

$$\therefore 12 \text{ day's work} = 12 \times \frac{1}{16} = \frac{3}{4}$$

$$\therefore \text{Remaining work} = 1 - \frac{3}{4} = \frac{1}{4}$$

Let x more men be required for 2 days work to complete

$$\therefore (16 + x) \times \frac{2}{16 \times 24} + 16 \times \frac{2}{32 \times 24} = \frac{1}{4}$$

$$\begin{aligned} \text{i.e.} \quad 2(16 + x) + 16 &= \frac{16 \times 24}{4} \\ &= 16 \times 6 \end{aligned}$$

$$\therefore 2x + 32 + 16 = 16 \times 6$$

$$\begin{aligned} \therefore 2x &= 16 \times 6 - 32 - 16 \\ &= 48 \end{aligned}$$

$$\therefore x = \frac{48}{2} = 24$$

$\therefore$ Required number of men for 2 day's work = 24

$$13. \therefore 1 \text{ woman's work/day} = \frac{1}{10 \times 7} = \frac{1}{70}$$

$$1 \text{ child's work/day} = \frac{1}{10 \times 14} = \frac{1}{140}$$

$$\begin{aligned} \therefore 5 \text{ women's} + 10 \text{ children's work/day} &= 5 \times \frac{1}{70} + 10 \times \frac{1}{140} \\ &= \frac{5}{70} + \frac{5}{70} \\ &= \frac{10}{70} = \frac{1}{7} \end{aligned}$$

$\therefore$ 7 days will be required to complete the work.

14. Let part of work/day by men be x and that by women be y .

$$\therefore 4x + 6y = \frac{1}{8} \quad (1)$$

$$3x + 7y = \frac{1}{10} \quad (2)$$

Solving (1) and (2), we get

$$x = \frac{11}{400}; \quad y = \frac{1}{400};$$

$\therefore$ Part of work done by 10 women/day is given by

$$10 \times \frac{1}{400} = \frac{1}{40}$$

$\therefore$ The work can be completed in 40 days.

15. Let x and y be part of work done/day by a man and a boy respectively.

$$\therefore 12x + 16y = \frac{1}{5} \quad (1)$$

$$13x + 24y = \frac{1}{4} \quad (2)$$

Solving (1) and (2), we get

$$x = \frac{1}{100}; \quad y = \frac{1}{200};$$

$$\therefore x : y = \frac{1}{100} : \frac{1}{200} \text{ or } 2 : 1$$

16. Let part of work done/day by a man be x .

$\therefore$ Part of work done/day by a woman = $2x$;

Part of work done/day by a child = $\frac{x}{2}$;

$$\therefore 3x + 4(2x) + 6\left(\frac{x}{2}\right) = \frac{1}{7}$$

$$\Rightarrow 14x = \frac{1}{7} \text{ or } x = \frac{1}{14 \times 7} = \frac{1}{98}$$

$$\therefore 2x = 2 \times \frac{1}{98} = \frac{1}{49}$$

$$\therefore \text{Work done by a woman for 7 days} = 7 \times \frac{1}{49} = \frac{1}{7}$$

$\therefore$ Number of women required to complete the work = 7

17. Part of work done by A/day = $\frac{1}{20}$

Part of work done by B/day = $\frac{1}{30}$

Part of work done by C/day = $\frac{1}{60}$

$\therefore$ Part of work done by A, B and C for first 3 days

$$= \left(\frac{3}{20} + \frac{1}{30} + \frac{1}{60} \right) = \frac{9}{60} + \frac{2}{60} + \frac{1}{60} = \frac{12}{60} = \frac{1}{5}$$

$\therefore$ Number of days required to complete the work = $5 \times 3 \text{ days} = 15 \text{ days}$

18. Part of work done/day by a woman = $\frac{1}{20 \times 16}$;

Part of work done/day by a man = $\frac{1}{16 \times 15}$;

Ratio of capacities from man to woman

$$= \frac{1}{16 \times 15} \div \frac{1}{20 \times 16} = \frac{1}{16 \times 15} \times 20 \times 16$$

$$= 4 : 3$$

19. 1 day's work by a man = $\frac{1}{12 \times 4}$

1 day's work by a woman = $\frac{1}{15 \times 4}$

Part of work done by 6 men for 2 days = $\left(6 \times \frac{1}{12 \times 4}\right) \times 2 = \frac{1}{4}$

$\therefore$ Remaining work = $1 - \frac{1}{4} = \frac{3}{4}$

Let number of women required for 3 days be x .

$\therefore x \times \frac{1}{15 \times 4} \times 3 = \frac{3}{4}$

or $x = 15$

20. $10x + 15y = \frac{1}{6}$ (1)

1 day's 1 man's work = $x = \frac{1}{100}$ (2)

$\therefore y = \left(\frac{1}{6} - 10 \times \frac{1}{100}\right) \frac{1}{15} = \left(\frac{1}{6} - \frac{1}{10}\right) \frac{1}{15}$
 $= \left(\frac{5-3}{30}\right) \times \frac{1}{15} = \frac{1}{225}$

$\therefore$ **225 days** are required for a woman to complete the work.

21. Let x , y and z be part of work done/day by a man, woman and a boy respectively.

$\therefore x = \frac{1}{3}; y = \frac{1}{4}; z = \frac{1}{12}$

Let N be the number of boys to assist 1 man and 1 woman for $\frac{1}{4}$ of a day.

$\therefore \left(1 \times \frac{1}{3} + 1 \times \frac{1}{4} + N \times \frac{1}{12}\right) \frac{1}{4} = 1$

i.e. $\frac{1}{12} + \frac{1}{16} + \frac{N}{48} = 1$

$\therefore N = \left(1 - \frac{1}{12} - \frac{1}{16}\right) \times 48 = \frac{48-4-3}{48} \times 48 = 41$

22. Part of work done by a man/day = $\frac{1}{12 \times 9}$

Let the number of days taken to complete the remaining work be x days.

Part of work done during first 6 days = $6 \times \left(12 \times \frac{1}{12 \times 9}\right)$
 $= \frac{6}{9} = \frac{2}{3}$

$\therefore$ Remaining work = $1 - \frac{2}{3} = \frac{1}{3}$

Number of men working after 6 days = $12 + 6 = 18$

$$\therefore 18 \times \frac{1}{12 \times 9} \times x = \frac{1}{3}$$

$$\text{or } x = \frac{12 \times 9}{18 \times 3} = 2$$

$\therefore$ Required number of days = 2

23. Part of work done by man/day = $\frac{1}{10 \times 15}$

$$\text{Part of work done by woman/day} = \frac{1}{15 \times 12}$$

Let number of days required be x

$$\therefore \left(10 \times \frac{1}{10 \times 15} + 15 \times \frac{1}{15 \times 12} \right) x = 1$$

$$\begin{aligned} \text{or } x &= \frac{1}{\frac{1}{15} + \frac{1}{12}} \\ &= \frac{15 \times 12}{15 + 12} = \frac{20}{3} = 6 \frac{2}{3} \text{ days} \end{aligned}$$

24. Part of work done by A per day = $\frac{1}{16}$

$$\text{Part of work done by B per day} = \frac{1}{12}$$

$$\begin{aligned} \therefore \text{For every two days, work done} &= \frac{1}{16} + \frac{1}{12} \\ &= \frac{3 + 4}{48} = \frac{7}{48} \end{aligned}$$

Let total number of days required to complete the work be x days.

$$\therefore \frac{7}{48} \times \frac{x}{2} = 1$$

$$\begin{aligned} \text{or } x &= \frac{48 \times 2}{7} \\ &= \frac{96}{7} = 13 \frac{5}{7} \text{ days} \end{aligned}$$

25. Part of work by A + B = $\frac{19}{23}$

$$\therefore \text{Part of work by C alone} = 1 - \frac{19}{23} = \frac{4}{23}$$

$$\text{Part of work by A alone} = 1 - \frac{8}{23} = \frac{15}{23}$$

$$\therefore \text{Part of work by B alone} = 1 - \left(\frac{4}{23} + \frac{15}{23} \right) = \frac{4}{23}$$

$$\begin{aligned} \therefore \text{Amount should be paid to A} &= 529 \times \frac{15}{23} \\ &= 23 \times 15 = \text{₹ } 345 \end{aligned}$$

26. Part of work done by A and B/day = $\frac{1}{12}$

$$\text{Part of work done by A alone/day} = \frac{1}{20}$$

$$\therefore \text{Part of work done by B alone/day} = \frac{1}{12} - \frac{1}{20} = \frac{5-3}{60} = \frac{1}{30}$$

$$\therefore \text{Part of work done by A for full day and B for half day} = \frac{1}{20} + \frac{1}{60} = \frac{3+1}{60} = \frac{1}{15}$$

Let the total number of days required to complete the work be x days.

$$\therefore \frac{1}{15} \times x = 1$$

$$\therefore x = \mathbf{15 \text{ days}}$$

27. Part of work done/day by A = $\frac{1}{14}$

$$\text{Part of work done/day by B} = \frac{1}{21}$$

Let the total number of days required to complete the work be x days.

$$\therefore \left(\frac{1}{14} + \frac{1}{21} \right) (x-3) + \frac{1}{21} \times 3 = 1$$

$$\text{i.e.} \quad \left(\frac{3+2}{42} \right) (x-3) + \frac{1}{7} = 1$$

$$\text{or} \quad (x-3) = \frac{6}{7} \times \frac{42}{5}$$

$$= 7.2$$

$$\therefore x = 7.2 + 3$$

$$= \mathbf{10.2}$$

$$= \mathbf{10 \frac{1}{5} \text{ days}}$$

28. Part of work/day by A = $\frac{1}{6}$

$$\text{Part of work/day by B} = \frac{1}{8}$$

$$\text{Time taken by A and B to complete the work} = \frac{8 \times 6}{8+6} = \frac{48}{14} = \frac{24}{7}$$

Time taken by A, B and C = 3 days

$$\begin{aligned}\therefore \text{Time taken by C alone} &= \frac{24/7 \times 3}{24/7 - 3} = \frac{72/7}{(24 - 21)/7} \\ &= \frac{72}{7} \times \frac{7}{3} \\ &= 24 \text{ days}\end{aligned}$$

$$\therefore \text{Part of work by C/day} = \frac{1}{24}$$

$$\frac{1}{6} : \frac{1}{8} : \frac{1}{24} = 4 : 3 : 1$$

$$\begin{aligned}\therefore \text{Amount to be paid to C} &= 3200 \times \frac{1}{4 + 3 + 1} \\ &= \frac{3200}{8} \\ &= \text{₹ } 400\end{aligned}$$

29. Part of work done/day by A = $\frac{1}{45}$

Part of work done/day by B = $\frac{1}{40}$

Let the number of days after which A left be x .

$$\therefore \left(\frac{1}{45} + \frac{1}{40} \right) x + \frac{1}{40} \times 23 = 1$$

$$\begin{aligned}\text{i.e.} \quad \left(\frac{8 + 9}{360} \right) x &= 1 - \frac{23}{40} \\ &= \frac{17}{40}\end{aligned}$$

$$\therefore x = \frac{17}{40} \times \frac{360}{17} = 9 \text{ days}$$

30. Share of Kim = $\frac{2}{2 + 3} \times 150 = \text{₹ } 60$

31. Let the number of days required for B to finish the whole work be x .

(A + B) → finishes in 30 days.

A for 16 days and B for 44 days → total 60 days.

∴ B finish the whole work in **60 days**.

32. Part of work by A alone/day = $\frac{1}{24}$

Part of work by B alone/day = $\frac{1}{9}$

Part of work by C alone/day = $\frac{1}{12}$

Let the number of days taken by A to complete the remaining work be x .

$$\therefore \left(\frac{1}{9} + \frac{1}{12}\right)3 + \frac{1}{24} \times x = 1$$

$$x = \left[1 - \left(\frac{3}{9} + \frac{3}{12}\right)\right] \times 24 = \left[1 - \left(\frac{1}{3} + \frac{1}{4}\right)\right] \times 24$$

$$\therefore = \left[1 - \left(\frac{4+3}{12}\right)\right] \times 24 = \frac{12-7}{12} \times 24 = \mathbf{10 \text{ days}}$$

33. Part of work completed in 20 days by A = $\frac{4}{5}$

$$\text{Remaining work} = 1 - \frac{4}{5} = \frac{1}{5}$$

Let x be the part of work/day by B

$$\therefore \left(\frac{1}{25} + x\right)3 = \frac{1}{5}$$

$$\text{or} \quad \frac{1}{25} + x = \frac{1}{15}$$

$$\text{or} \quad x = \frac{1}{15} - \frac{1}{25} = \frac{5-3}{75} = \frac{2}{75}$$

$\therefore$ Time taken by B alone to complete the work

$$= \frac{1}{2/75} = \frac{75}{2} = \mathbf{37 \frac{1}{2} \text{ days}}$$

34. Part of work done/day by A + B = $\frac{1}{30}$

Let part of work done/day by A alone be x .

$$\therefore \frac{1}{30} \times 20 + x \times 20 = 1$$

$$\therefore x = \left(1 - \frac{20}{30}\right) \frac{1}{20} = \frac{1}{3} \times \frac{1}{20} = \frac{1}{60}$$

$$\therefore \text{A alone can finish the work in } \frac{1}{1/60} = \mathbf{60 \text{ days}}$$

35. Part of work done/day by A = $\frac{1}{18}$

$$\text{Part of work done/day by B} = \frac{1}{15}$$

Let the number of days taken by A to finish remaining work be x .

$$\therefore \frac{1}{15} \times 10 + \frac{1}{18} \times x = 1$$

$$\therefore x = \left(1 - \frac{10}{15}\right) 18 = \frac{1}{3} \times 18 = \mathbf{6 \text{ days}}$$

36. Part of work done/hr by machine P = $\frac{1}{8}$

Part of work done/hr by machine Q = $\frac{1}{10}$

Part of work done/hr by machine R = $\frac{1}{12}$

$$\therefore \left(\frac{1}{8} + \frac{1}{10} + \frac{1}{12}\right)2 + \left(\frac{1}{10} + \frac{1}{12}\right)x = 1$$

$$\begin{aligned}\therefore x &= \left\{1 - \left(\frac{1}{4} + \frac{1}{5} + \frac{1}{6}\right)\right\} \div \left(\frac{1}{10} + \frac{1}{12}\right) = \left\{1 - \left(\frac{15+12+10}{60}\right)\right\} \div \left(\frac{6+5}{60}\right) \\ &= \frac{23}{60} \times \frac{60}{11} = \frac{23}{11} = 2\frac{1}{11} \text{ hours}\end{aligned}$$

$$\therefore 11 + 2\frac{1}{11} = \text{approximately 1 p.m.}$$

37. $\left(\frac{1}{20} + \frac{1}{12}\right)4 + \frac{1}{12} \times x = 1$

$$\begin{aligned}\therefore x &= \left\{1 - \left(\frac{1}{5} + \frac{1}{3}\right)\right\} \times 12 = \left\{1 - \left(\frac{3+5}{15}\right)\right\} \times 12 \\ &= \left(\frac{15-8}{15}\right) \times 12 = \frac{7 \times 4}{5} = 5.6 \approx \mathbf{6 \text{ days}}\end{aligned}$$

38. $(A + B) = \frac{1}{30}$; $(B + C) = \frac{1}{24}$; $(C + A) = \frac{1}{20}$;

$$\begin{aligned}\therefore (A + B + C) &= \frac{1}{2} \left(\frac{1}{30} + \frac{1}{24} + \frac{1}{20}\right) \\ &= \frac{1}{2} \left(\frac{4+5+6}{120}\right) = \frac{15}{240} = \frac{1}{16}\end{aligned}$$

Let x days be required by A to finish the remaining work.

$$\therefore \frac{1}{16} \times 10 + \left(\frac{1}{16} - \frac{1}{24}\right) \times x = 1$$

$$\begin{aligned}\therefore x &= \left(1 - \frac{10}{16}\right) \div \left(\frac{1}{16} - \frac{1}{24}\right) \\ &= \frac{6}{16} \times \frac{1}{\frac{3-2}{48}} = \frac{6}{16} \times \frac{48}{1} = \mathbf{18 \text{ days}}\end{aligned}$$

39. $\left(\frac{1}{15} + \frac{1}{10}\right) \times 2 + \frac{1}{15} \times x = 1$

$$\therefore x = \left\{1 - 2\left(\frac{2}{30} + \frac{3}{30}\right)\right\} \times 15 = \frac{2}{3} \times 15 = \mathbf{10 \text{ days}}$$

40. $A + B \rightarrow \frac{1}{7}$

Let part of work by B/day be x

$\therefore$ Part of work by A/day = $\frac{7}{4}x$

$\therefore x + \frac{7}{4}x = \frac{1}{7}$

or $x = \frac{1}{7} \div \left(1 + \frac{7}{4}\right)$
 $= \frac{1}{7} \times \frac{4}{11} = \frac{4}{77}$

$\therefore$ Part of work by A/day = $\frac{7}{4} \times \frac{4}{77} = \frac{1}{11}$

A alone can complete the work in $\frac{1}{1/11} = 11$ days.

41. $A + B \rightarrow \frac{1}{10}$

$C \rightarrow \frac{1}{50}$

$A = B + C = ?$

Let x be the number of days required for the work by B alone.

$\therefore$ Part of work by B alone = $\frac{1}{x}$

$A \rightarrow \frac{1}{10} - \frac{1}{x} = \frac{1}{x} + \frac{1}{50} \rightarrow B + C$

$\therefore \frac{1}{x} + \frac{1}{x} = \frac{1}{10} + \frac{1}{50}$

$\frac{2}{x} = \frac{5-1}{50}$

$\frac{2}{x} = \frac{4}{50}$

or $x = \frac{2 \times 50}{4} = 25$ days

42. Let number of days taken by B to complete a work be x .

$\therefore$ Number of days taken by A for completing the same work = $\frac{x}{3} = x - 60$

$\therefore \frac{2}{3}x = 60$

or $x = \frac{60 \times 3}{2} = 90$ days

Time taken by B = 90 days

Time taken by A = 30 days.

∴ Time taken by A and B together to complete the work

$$= \frac{30 \times 90}{30 + 90} = \frac{30 \times 90}{120} = 22\frac{1}{2} \text{ days}$$

43. Fraction of work left $= 1 - \left(\frac{1}{15} + \frac{1}{20} \right) 4$

$$= 1 - \frac{(4+3)}{60} \times 4 = 1 - \frac{7}{15} = \frac{8}{15}$$

44. B completes x part of a work in one day.

∴ B completes the same work in $\frac{1}{x}$ days

∴ A completes $1.3x$ part of work in a day

∴ A completes the work in $\frac{1}{1.3x}$ days

But $\frac{1}{1.3x} = 23$

or $x = \frac{1}{23 \times 1.3} \quad (1)$

$$\begin{aligned} \text{Time taken by A and B together} &= \frac{\frac{1}{x} \times \frac{1}{1.3x}}{\frac{1}{x} + \frac{1}{1.3x}} \\ &= \frac{\frac{1}{1.3x^2}}{\frac{2.3x}{1.3x^2}} = \frac{1}{2.3x} \end{aligned}$$

∴ From (1), required time = 13 days.

45. Part of work done/day by Sakshi = $\frac{1}{20}$

$$\text{Part of work done/day by Tanya} = \frac{5}{4} \times \frac{1}{20} = \frac{1}{16}$$

∴ Number of days taken by Tanya to complete the work = 16 days.

46. Part of work/day by A + B = $\frac{1}{8}$

$$\text{Part of work/day by B + C} = \frac{1}{12}$$

$$\text{Part of work/day by A + B + C} = \frac{1}{6}$$

Part of work/day by A + C = Part of work done by (A + B + C) — Part of work done by B alone

$$\begin{aligned}
&= \frac{1}{6} - \left\{ \frac{1}{6} - [2 \times (A + B + C) - (A + B) - (B + C)] \right\} \\
&= \frac{1}{6} - \left\{ \frac{1}{6} - \left(2 \times \frac{1}{6} - \frac{1}{8} - \frac{1}{12} \right) \right\} = \frac{1}{3} - \frac{1}{8} - \frac{1}{12} \\
&= \frac{8-3-2}{24} = \frac{3}{24} = \frac{1}{8}
\end{aligned}$$

A and C together will complete the job in $\frac{1}{1/8} = 8$ days

47. Quantity of work by P/hr = $\frac{1}{12 \times 8} = \frac{1}{96}$;

Quantity of work by Q/hr = $\frac{1}{10 \times 8} = \frac{1}{80}$;

Let the required number of days to complete the work be x . Then

$$\left(\frac{1}{96} + \frac{1}{80} \right) 8 \times x = 1$$

$$\begin{aligned}
\therefore x &= \frac{1}{8 \times \left(\frac{1}{96} + \frac{1}{80} \right)} \\
&= \frac{1}{8 \times \left(\frac{5+6}{480} \right)} = \frac{480}{8 \times 11} = \frac{60}{11} = 5 \frac{5}{11} \text{ days}
\end{aligned}$$

48. Let the time required to complete the work by B alone be $\frac{x}{2}$

Time required by A = x

Time required by C = $\frac{x}{3}$

$$\therefore \text{Time taken by A, B and C when they work together} = \frac{1}{\frac{1}{x} + \frac{2}{x} + \frac{3}{x}} = 2$$

or $\frac{x}{6} = 2$

or $x = 12$ days

$\therefore$ B takes $x/2 = 6$ days to complete the work.

49. (A + B) $\rightarrow$ part of work done/day $\rightarrow \frac{1}{12}$

(B + C) $\rightarrow$ part of work done/day $\rightarrow \frac{1}{15}$

(C + A) $\rightarrow$ part of work done/day $\rightarrow \frac{1}{20}$

(A + B + C) $\rightarrow$ part of work done/day $\rightarrow \frac{1}{2} \left(\frac{1}{12} + \frac{1}{15} + \frac{1}{20} \right)$

$$= \frac{1}{2} \left(\frac{5+4+3}{60} \right) = \frac{1}{10}$$

∴ A + B + C will complete the work in 10 days.

50. Part of work by A/day = $\frac{1}{16}$

Part of work by B/day = $\frac{1}{12}$

Part of work by A + B + C/day = $\frac{1}{4}$

$$\begin{aligned} \therefore \text{Part of work by C alone/day} &= \frac{1}{4} - \left(\frac{1}{16} + \frac{1}{12} \right) \\ &= \frac{1}{4} - \left(\frac{3+4}{48} \right) = \frac{12-7}{48} = \frac{5}{48} \end{aligned}$$

∴ C alone can do the job in $\frac{48}{5} = 9\frac{3}{5}$ days

51. Let the number of days required by son to do the work alone be x .

$$\therefore \frac{1}{5} + \frac{1}{x} = \frac{1}{3}$$

or $x = \frac{1}{\frac{1}{3} - \frac{1}{5}} = \frac{15}{5-3} = 7\frac{1}{2}$ days

52. Work done/day by A = $\frac{1}{18}$

Work done/day by B = $\frac{1}{9}$

When they work together, part of work done/day = $\frac{1}{18} + \frac{1}{9} = \frac{1}{6}$

53. Required time to complete the work = $\frac{abc}{ab+bc+ca}$

$$\begin{aligned} &= \frac{24 \times 6 \times 12}{24 \times 6 + 6 \times 12 + 12 \times 24} \\ &= \frac{24}{2+1+4} = \frac{24}{7} = 3\frac{3}{7} \text{ days} \end{aligned}$$

54. Required number of days = $\frac{10 \times 15}{10+15}$

$$= \frac{150}{25} = 6 \text{ days}$$

55. Required time = $\frac{9 \times 6}{9+6} = \frac{54}{15} = \frac{18}{5} = 3.6$ minutes

$$= 3\frac{3}{5} \text{ minutes}$$

Work and Wages

This chapter deals with the calculation of man-hours for a certain work and calculation of total wages paid for a work according to the principle “Total man-days for a certain work remains constant”.

For example, if X number of persons complete a certain job in x days and Y number of persons complete the same job in y days, then

$$X \cdot x = Y \cdot y$$

EXAMPLE 1 A can do certain piece of work in d_1 days and B in d_2 days. Both of them do the work together. If the total amount paid for the work is ₹ M , how much would each get?

Solution A's daily work = $\frac{1}{d_1}$

B's daily work = $\frac{1}{d_2}$

$$\text{A's share : B's share} = \frac{1}{d_1} : \frac{1}{d_2} = d_2 : d_1$$

$$\therefore \text{A's share} = ₹ \left(\frac{d_2}{d_2 + d_1} \right) M$$

$$\text{B's share} = ₹ \left(\frac{d_1}{d_2 + d_1} \right) M$$

EXAMPLE 2 The wages of 20 boys for 15 days is ₹ 9000. If the daily wage of a man is one and half times that of a boy, then how many men must work for 30 days to earn ₹ 13500?

Solution Total wage of a group = Number of persons in the group $\times$ One person's daily wage $\times$ Number of days.

$$\therefore \text{One person's daily wage} = \frac{\text{Total wage of group}}{\text{Number of persons in the group} \times \text{Number of days}}$$

$$\therefore \text{One boy's daily wage} = \frac{9000}{20 \times 15} = ₹ 30$$

$$\text{One man's daily wage} = 1.5 \times 30 = ₹ 45$$

$$\begin{aligned} \therefore \text{Number of men in the group} &= \frac{\text{Total wages of group of men}}{\text{One man's daily wage} \times \text{Number of days}} \\ &= \frac{13500}{45 \times 30} = 10 \end{aligned}$$



EXERCISES

- A person can do a work in 20 days. With the help of a boy he can do the same work in 12 days. If they get ₹ 500 for that work, what is the share of that boy?
(a) ₹ 100 (b) ₹ 200 (c) ₹ 250 (d) ₹ 300 (e) None of these
- Two person's agreed to do a work for ₹ 2000. One alone could do it in 6 days the other in 8 days. With the assistance of a boy they finish it in 3 days. How should the money be divided?
(a) 1 : 2 : 3 (b) 2 : 3 : 1 (c) 1 : 3 : 4 (d) 4 : 3 : 1 (e) None of these
- The wages for 12 women amount to ₹ 30240 in 24 days. How many men must work 16 days to receive ₹ 10080, the daily wages of a man being equal to that of a woman?
(a) 4 (b) 6 (c) 8 (d) 10 (e) None of these
- If 3 men and 4 boys can earn ₹ 4200 in 7 days, and 11 men and 13 boys can earn ₹ 16600 in 8 days, in what time will 7 men with 9 boys earn ₹ 19250?
(a) 10 days (b) 12 days (c) 14 days (d) 16 days (e) None of these
- Two persons P and Q together complete a piece of work which it would have taken them respectively 12 and 18 days to complete if they worked separately. They received a payment of ₹ 1200 . Find their shares.
(a) 500, 700 (b) 700, 500 (c) 480, 720 (d) 720, 480 (e) None of these



1. (b) 2. (d) 3. (b) 4. (c) 5. (d)



Solutions with Necessary Explanation

$$1. \text{ The boy can do the work in } \frac{12 \times 20}{20 - 12} = 30 \text{ days}$$

$$\therefore \text{Man's share : boy's share} = 30 : 20 = 3 : 2$$

$$\therefore \text{Boy's share} = \frac{2}{3+2} \times 500 = ₹ 200$$

$$2. \text{ 1st man's 3 day's work} = \frac{1}{6} \times 3 = \frac{3}{6} = \frac{1}{2}$$

$$\text{2nd man's 3 day's work} = \frac{1}{8} \times 3 = \frac{3}{8}$$

$$\therefore \text{ Boy's 3 day's work} = 1 - \left(\frac{3}{6} + \frac{3}{8} \right) = 1 - \frac{3 \times 4 + 3 \times 3}{24} = 1 - \frac{21}{24} = \frac{3}{24} = \frac{1}{8}$$

$$\therefore \text{ Their shares will be in the ratio } \frac{3}{6} : \frac{3}{8} : \frac{1}{8} = 3 \times 4 : 3 \times 3 : 3 \times 3 = 4 : 3 : 1$$

$$\therefore \text{ Share for 1st man} = \frac{4}{4+3+1} \times 2000 = \text{₹ } 1000$$

$$\text{Share for 2nd man} = \frac{3}{4+3+1} \times 2000 = \text{₹ } 750$$

$$\text{Boy's share} = \frac{1}{4+3+1} \times 2000 = \text{₹ } 250$$

$$3. \text{ Wages of a woman/day} = \frac{30240}{12 \times 24} = \text{₹ } 105$$

$$\text{Wages of a man/day} = \text{₹ } 105$$

$$\therefore \text{ Number of men} = \frac{\text{Total wage}}{\text{Number of days} \times \text{Wage of man/day}} = \frac{10080}{16 \times 105} = 6$$

$$4. \quad 3 \text{ men} + 4 \text{ boy in 1 day earn } \text{₹ } \frac{4200}{7} = \text{₹ } 600 \quad (1)$$

$$11 \text{ men} + 13 \text{ boy in 1 day earn } \text{₹ } \frac{16600}{8} = \text{₹ } 2075 \quad (2)$$

$$\text{Equations (1) and (2)} \Rightarrow \frac{3 \text{ men} + 4 \text{ boy}}{600} = \frac{11 \text{ men} + 13 \text{ boy}}{2075}$$

$$\Rightarrow \frac{3 \text{ men} + 4 \text{ boy}}{24} = \frac{11 \text{ men} + 13 \text{ boy}}{83}$$

$$\text{or} \quad \text{men} (3 \times 83 - 11 \times 24) = \text{boy} (13 \times 24 - 4 \times 83)$$

$$\text{or} \quad \frac{\text{men}}{\text{boy}} = \frac{-20}{-15} = \frac{4}{3}$$

$$\therefore (1) \quad \Rightarrow \quad 3 \text{ men} + 4 \text{ boy} = 3 \text{ men} + 4 \times \frac{3}{4} \text{ men} \\ = 3 \text{ men} + 3 \text{ men} = 6 \text{ men earns ₹ } 600/\text{day}$$

$$\therefore \quad \text{Wage/day of man} = \text{₹ } 100$$

$$\therefore \quad 7 \text{ men} + 9 \text{ boys earns in 1 day} = \left[7 + 9 \left(\frac{3}{4} \right) \right] \times 100 = \text{₹ } 1375$$

$$\therefore 7 \text{ men} + 9 \text{ boys earn ₹ } 19250 \text{ in } \frac{19250}{1375} = 14 \text{ days.}$$

5. Wages are distributed in inverse proportion of a number of days. Hence money will be divided between P and Q in the ratio 18 : 12.

$$\therefore \quad \text{P gets } \frac{18}{18+12} \times 1200 = \text{₹ } 720 \quad \text{and} \quad \text{Q gets } \frac{12}{18+12} \times 1200 = \text{₹ } 480$$

Pipes and Cistern

In this chapter, all problems are related to the pipe flow, calculation of filling and emptying time for a water tank or flush tank, etc.



Important Formulae

1. If a pipe can fill a tank in x hours, and another pipe can fill the same tank in y hours, when both pipes are open, time taken to completely fill the tank

$$= \frac{xy}{x + y}$$

2. If a pipe can fill a tank in x hours and another pipe can empty the full tank in y hours, the net part filled in 1 hour, when both pipes are open simultaneously is given by

$$\left(\frac{1}{x} - \frac{1}{y} \right) = \frac{y - x}{xy}$$

3. When both the pipes are open simultaneously, time taken to fill the tank completely

$$\frac{xy}{y - x} \text{ hours}$$

4. A pipe can fill a tank in x hours. But due to leakage in its bottom, the tank is now filled in y hours. Then the time taken by the leakage to empty the full tank

$$\frac{xy}{y - x} \text{ hours}$$

5. A pipe can fill a tank in x hours and another one fills it in y hours. A third pipe can empty the full tank in z hours. When all the three pipes are open simultaneously the net part of the tank filled in 1 hour

$$= \frac{1}{x} + \frac{1}{y} - \frac{1}{z} = \frac{yz + zx - xy}{xyz}$$

6. When all the three pipes are open, time taken to fill the tank

$$= \frac{xyz}{yz + zx - xy} \text{ hours}$$

EXAMPLE 1 A pipe can fill a tank in 20 hours. Due to a leak in the bottom of the tank, it is filled in 25 hours. If the tank is full, how much time will the leak take to empty it?

Solution Time taken = $\frac{xy}{y-x} = \frac{25 \times 20}{25-20} = \mathbf{100 \text{ hours}}$

EXAMPLE 2 8 taps are fitted to a water tank. Some of them are water taps to fill the tank and the remaining are outlet taps used to empty the tank. Each water tap can fill the tank in 12 hours and each outlet tap can empty it in 36 hours. On opening all the taps, tank is filled in 3 hours. Find the number of water taps.

Solution Let the number of inlet water taps be x .

$\therefore$ Number of outlet taps = $8 - x$

The tank can be filled by each tap in 12 hours.

$\therefore$ Part filled by x taps in 1 hour = $x/12$

The tank can be emptied by each outlet tap in 36 hours.

$\therefore$ Part emptied by $(8 - x)$ outlet taps in 1 hour = $\frac{8-x}{36}$

$\therefore$ Net part filled in 1 hour = $\frac{x}{12} - \frac{8-x}{36} = \frac{3x - (8-x)}{36} = \frac{4x-8}{36} = \frac{x-2}{9}$

$\therefore$ Tank will be filled in $\frac{9}{x-2}$ hours on opening all the taps together

$\therefore \frac{9}{x-2} = 3$

or $3(x-2) = 9 \Rightarrow x = 5$

$\therefore$ Number of water taps = **5**

EXAMPLE 3 Pipe A can fill a tank in 20 hours while pipe B alone can fill it in 30 hours and pipe C can empty the same tank in 40 hours. If all the pipes are opened together, what will be the time needed to make the tank fill?

Solution Net part filled in 1 hour = $\left(\frac{1}{20} + \frac{1}{30} - \frac{1}{40}\right) = \frac{7}{120}$

$\therefore$ The tank will be filled in $\frac{120}{7}$ hours i.e. **$17\frac{1}{7}$ hours**.



Short cut: Time required to completely fill the tank is $\frac{xyz}{yz + zx - xy}$ hours

$$= \frac{20 \times 30 \times 40}{30 \times 40 + 40 \times 20 - 20 \times 30} = \frac{120}{7} = \mathbf{17\frac{1}{7} \text{ hours}}$$

EXAMPLE 4 Two pipes A and B can fill a tank in 24 minutes and 32 minutes respectively. If both the pipes are opened simultaneously, after how much time should B be closed so that the tank is full in 18 minutes.

Solution A can fill $\frac{1}{24}$ part of tank in a minute.

B can fill $\frac{1}{32}$ part of tank in a minute.

Let after x minutes B be closed.

$$\therefore \left(\frac{1}{24} + \frac{1}{32} \right) x + \frac{1}{24} (18 - x) = 1$$

$$\Rightarrow x = 8 \text{ minutes}$$



Short cut: Pipe B should be closed after $\left(1 - \frac{18}{24} \right) \times 32 = 8 \text{ minutes}$

EXAMPLE 5 Three pipes A, B, and C can fill a cistern in 6 hours. After working together for 2 hours, C is closed and A and B fill the cistern in 8 hours. Find the time in which the cistern can be filled by pipe C.

Solution Part of the cistern that was filled by A, B, and C together in 1 hour = $\frac{1}{6}$.

$$\therefore \text{For the first 2 hours, filled portion} = 2 \times \frac{1}{6} = \frac{1}{3}$$

Part of cistern that was filled by A and B together in 1 hour = $\frac{1}{8}$

$$\therefore \left(1 - \frac{1}{3} \right) = \frac{2}{3} \text{ part should be filled in 8 hours}$$

$$\therefore \text{A + B can fill the cistern in } \frac{8 \times 3}{2} = 12 \text{ hours}$$

$$\therefore \text{Time in which cistern can be filled by pipe C alone} = \frac{12 \times 6}{12 - 6} = 12 \text{ hours.}$$



EXERCISES

- One tap can fill a cistern in 2 hours and another can empty the cistern in 3 hours. How long will they take to fill the cistern if both the taps are opened?
(a) 4 hours (b) 6 hours (c) 3 hours (d) 2 hours (e) None of these
- If two pipes function simultaneously, the reservoir will be filled in 6 hours. One pipe fills the reservoir 5 hours faster than the other. How many hours does the faster pipe take to fill the reservoir?
(a) 8 hours (b) 10 hours (c) 12 hours (d) 9 hours (e) None of these
- A cistern has a leak which would empty it in 8 hours. A tap is turned on which admits 6 litres a minute into the cistern, and it is now emptied in 12 hours. How many litres does the cistern hold?
(a) 5000 litres (b) 4680 litres (c) 6840 litres (d) 8640 litres (e) None of these
- A cistern has two taps which fill it in 12 minutes and 15 minutes respectively. There is also a waste pipe in the cistern. When all the pipes are opened, the empty cistern is filled in 20 minutes. How long will the waste pipe take to empty a full cistern?
(a) 5 minutes (b) 10 minutes (c) 15 minutes (d) 20 minutes (e) None of these

5. A tank can be filled by one tap in 20 minutes and by another in 25 minutes. Both the taps are kept open for 5 minutes and then the second is turned off. In how many minutes more will the tank be completely filled?
(a) 18 minutes (b) 16 minutes (c) 11 minutes (d) 8 minutes (e) None of these
6. A large tanker can be filled by two pipes A and B in 60 minutes and 40 minutes respectively. How many minutes will it take to fill the tanker from empty state if B is used for half the time and A and B fill it together for the other half? **[DMRC]**
(a) 15 minutes (b) 20 minutes (c) 27.5 minutes (d) 30 minutes
7. Two pipes can fill a tank in 20 minutes and 24 minutes respectively, and a waste pipe can empty 3 gallons per minute. All the three pipes working together can fill the tank in 15 minutes. The capacity of the tank is: **[Bank PO]**
(a) 60 gallons (b) 100 gallons (c) 120 gallons (d) 180 gallons
8. Two pipes A and B can fill a tank in 15 minutes and 20 minutes respectively. Both the pipes are opened together but after 4 minutes, pipe A is turned off. What is the total time required to fill the tank? **[UPSC]**
(a) 10 min 20 sec (b) 11 min 45 sec (c) 12 min 30 sec (d) 14 min 40 sec
9. Three pipes A, B and C can fill a tank in 6 hours. After working at it together for 2 hours, C is closed and A and B can fill the remaining part in 7 hours. The number of hours taken by C alone to fill the tank is: **[LIC AAO]**
(a) 10 (b) 12 (c) 14 (d) 16
10. A tank is filled by three pipes with uniform flow. The first two pipes operating simultaneously fill the tank in the same time during which the tank is filled by the third pipe alone. The second pipe fills the tank 5 hours faster than the first pipe and 4 hours slower than the third pipe. The time required by the first pipe is: **[MBA]**
(a) 6 hours (b) 10 hours (c) 15 hours (d) 30 hours
11. One pipe can fill a tank three times as fast as another pipe. If together the two pipes can fill the tank in 36 minutes, then the slower pipe alone will be able to fill the tank in: **[CBI]**
(a) 81 minutes (b) 108 minutes (c) 144 minutes (d) 192 minutes
12. Three pipes A, B and C can fill a tank from empty to full in 30 minutes, 20 minutes and 10 minutes respectively. When the tank is empty, all the three pipes are opened. A, B and C discharge chemical solutions P, Q and R respectively. What is the proportion of solution R in the liquid in the tank after 3 minutes? **[DMRC]**
(a) $\frac{5}{11}$ (b) $\frac{6}{11}$ (c) $\frac{7}{11}$ (d) $\frac{8}{11}$
13. Two pipes A and B can fill a cistern in $37\frac{1}{2}$ minutes and 45 minutes respectively. Both pipes are opened. The cistern will be filled in just half an hour, if pipe B is turned off after: **[SSC]**
(a) 5 minutes (b) 9 minutes (c) 10 minutes (d) 15 minutes
14. Three taps A, B and C can fill a tank in 12, 15 and 20 hours respectively. If A is opened all the time, and B and C are opened for one hour each alternately, the tank will be full in: **[SSC]**
(a) 6 hours (b) $6\frac{2}{3}$ hours (c) $5\frac{1}{2}$ hours (d) 7 hours
15. Two pipes A and B can separately fill a cistern in 60 minutes and 75 minutes respectively. There is a third pipe in the bottom of the cistern to empty it. If all the three pipes are simultaneously opened, then the cistern is full in 50 minutes. In how much time the third pipe alone can empty the cistern? **[SSC]**
(a) 90 minutes (b) 100 minutes (c) 110 minutes (d) 120 minutes

16. A tank is filled in 5 hours by three pipes A, B and C. Pipe C is twice as fast as B, and B is twice as fast as A. How much time will pipe A alone take to fill the tank? **[Bank PO]**
 (a) 20 hours (b) 25 hours (c) 35 hours
 (d) Cannot be determined (e) None of these
17. Two pipes A and B together can fill a cistern in 4 hours. Had they been opened separately, then B would have taken 6 hours more than A to fill the cistern. How much time will be taken by A to fill the cistern separately? **[NABARD]**
 (a) 1 hour (b) 2 hours (c) 6 hours (d) 8 hours
18. A pump can fill a tank with water in 2 hours. Because of a leak, it took $2\frac{1}{3}$ hours to fill the tank. The leak can drain all the water of the tank in: **[SSC]**
 (a) $4\frac{1}{3}$ hours (b) 7 hours (c) 8 hours (d) 14 hours
19. A water tank is two-fifth full. Pipe A can fill a tank in 10 minutes, and pipe B can empty it in 6 minutes. If both the pipes are opened, how long will it take to empty or fill the tank completely? **[Bank PO]**
 (a) 6 minutes to empty (b) 6 minutes to fill (c) 9 minutes to empty
 (d) 9 minutes to fill (e) None of these
20. Pipes A and B can fill a tank in 5 hours and 6 hours respectively. Pipe C can empty it in 12 hours. If all the three pipes are opened together, then the tank will be filled in: **[Bank PO]**
 (a) $1\frac{13}{17}$ hours (b) $2\frac{8}{11}$ hours (c) $3\frac{9}{17}$ hours (d) $4\frac{1}{2}$ hours
21. Two pipes A and B can fill a tank in 20 minutes and 30 minutes respectively. If both the pipes are used together, then how long will it take to fill the tank? **[MAT]**
 (a) 12 minutes (b) 15 minutes (c) 25 minutes (d) 50 minutes
22. Pipe A can fill a tank in 5 hours, pipe B in 10 hours and pipe C in 30 hours. If all the pipes are opened, in how many hours will the tank be filled? **[CBI]**
 (a) 2 hours (b) 2.5 hours (c) 3 hours (d) 3.5 hours
23. A cistern can be filled by a tap in 4 hours, while it can be emptied by another tap in 9 hours. If both the taps are opened simultaneously, then after how much time will the cistern get filled? **[Hotel Management]**
 (a) 4.5 hours (b) 5 hours (c) 6.5 hours (d) 7.2 hours
24. Two pipes A and B can fill a cistern in 18 minutes and 9 minutes respectively, while a third pipe C can empty the full tank in 5 minutes. A and B are kept open for 4 minutes in the beginning and then C is also opened. In what time is the cistern emptied?
 (a) 15 minutes (b) 20 minutes (c) 22 minutes (d) 24 minutes
25. 10 buckets of water fill a tank when the capacity of each bucket is 12 litres. How many buckets will be needed to fill the same tank, if the capacity of each bucket is 8 litres?
 (a) 8 (b) 12 (c) 15 (d) 18



1. (b) 2. (b) 3. (d) 4. (b) 5. (c) 6. (d) 7. (c) 8. (d) 9. (c) 10. (c)
 11. (c) 12. (b) 13. (b) 14. (d) 15. (b) 16. (c) 17. (c) 18. (d) 19. (a) 20. (c)
 21. (a) 22. (c) 23. (d) 24. (b) 25. (c)



Solutions with Necessary Explanation

- Time required to fill the cistern = $\frac{2 \times 3}{3 - 2} = 6$ hours.
- Let the faster pipe takes x hours to fill the tank.
 $\therefore$ The slower pipe takes $(x + 5)$ hours to fill the tank.
 $\therefore$ The two taps together fill the tank in $\frac{x(x+5)}{x + x + 5} = 6$ hours
 $\therefore x = 10$ or -3
 $\therefore$ The faster tap fills the tank in **10 hours**.
- The tap can fill the tank in $\frac{12 \times 8}{12 - 8} = 24$ hours
 $\therefore$ Capacity of tank = 6 litres/min $\times (60 \times 24) = 8640$ litres
- The two taps together can fill the cistern in $\frac{12 \times 15}{12 + 15} = \frac{20}{3}$ minutes
 Including a flow through waste pipe, cistern will be filled in 20 minutes.
 $\therefore$ Time taken to empty the cistern = $\frac{20 \times 20/3}{20 - 20/3} = \frac{20 \times 20/3}{40/3} = 10$ minutes
- For the first 5 minutes, part of the tank filled by both the pipes = $\frac{5}{20} + \frac{5}{25} = \frac{1}{4} + \frac{1}{5} = \frac{9}{20}$
 $\therefore$ Part of tank to be filled up = $1 - \frac{9}{20} = \frac{11}{20}$
 $\therefore$ First tap can fill $11/20$ of tank in $20 \times \frac{11}{20} = 11$ minutes
- Part of tanker filled by pipe A in 1 minute = $\frac{1}{60}$
 Part of tanker filled by pipe B in 1 minute = $\frac{1}{40}$
 Let the time taken to fill the tank be $2x$ minutes
 $\therefore$ Part of tanker filled = $\frac{1}{40} \times x + \left(\frac{1}{60} + \frac{1}{40}\right)x = 1$
 i.e. $\frac{x}{40} + \frac{(2+3)}{120}x = 1$
 $\Rightarrow \frac{x}{40} + \frac{x}{24} = 1$
 $\Rightarrow \frac{3x + 5x}{120} = 1$
 $\Rightarrow x = \frac{1 \times 120}{8} = 15$ minutes
 $\therefore$ Time taken to fill the tank = $2x = 30$ minutes

7. Part of tank filled in by first pipe/minute = $\frac{1}{20}$

Part of tank filled in by second pipe/minute = $\frac{1}{24}$

Part of tank filled in by all the two pipes and waste pipe = $\frac{1}{15}$

Let time taken to empty by waste pipe be x minute

$$\therefore \frac{1}{20} + \frac{1}{24} - \frac{1}{x} = \frac{1}{15}$$

$$\begin{aligned} \text{or} \quad \frac{1}{x} &= \frac{1}{20} + \frac{1}{24} - \frac{1}{15} \\ &= \frac{6 + 5 - 8}{120} = \frac{3}{120} = \frac{1}{40} \end{aligned}$$

$$\therefore x = 40 \text{ minutes}$$

$$\therefore \text{Capacity of tank} = 40 \times 3 = \mathbf{120 \text{ gallons}}$$

8. Part of the tank filled in by A and B in 1 minute = $\frac{1}{15} + \frac{1}{20} = \frac{4 + 3}{60} = \frac{7}{60}$

In the first 4 minutes, part of tank filled = $\frac{4 \times 7}{60} = \frac{28}{60} = \frac{7}{15}$

$$\text{Remaining part to be filled} = 1 - \frac{7}{15} = \frac{8}{15}$$

$$\therefore \frac{1}{20} \times x = \frac{8}{15}$$

$$\begin{aligned} \text{or} \quad x &= \frac{20 \times 8}{15} = \frac{4 \times 8}{3} \\ &= 10 \frac{2}{3} = 10 \text{ min } 40 \text{ sec.} \end{aligned}$$

$$\therefore \text{Total time taken to fill the tank} = x + 4 = \mathbf{14 \text{ minutes } 40 \text{ seconds}}$$

9. Part of tank filled during first 2 hours by A, B and C = $\frac{2}{6} = \frac{1}{3}$

$$\text{Remaining part to be filled} = 1 - \frac{1}{3} = \frac{2}{3}$$

$$\text{Part of tank filled by A and B in 7 hours} = \frac{2}{3}$$

$$\text{Part of tank filled by A and B in 1 hour} = \frac{2}{21}$$

Part of tank filled by C alone in 1 hour

$$= \frac{1}{6} - \frac{2}{21} = \frac{7 - 4}{42} = \frac{3}{42} = \frac{1}{14}$$

The tank can be filled by C alone in **14 hours**.

10. Let the second pipe fills the tank in x hours.

$\therefore$ The first pipe fills the tank in $x + 5$ hours

third pipe fills the tank in $x - 4$ hours

$$\therefore \quad \frac{1}{x} + \frac{1}{x+5} = \frac{1}{x-4}$$

$$\Rightarrow \quad \frac{1}{x} = \frac{1}{x-4} - \frac{1}{x+5}$$

$$\Rightarrow \quad \frac{1}{x} = \frac{(x+5) - (x-4)}{(x-4)(x+5)}$$

$$\frac{1}{x} = \frac{9}{(x-4)(x+5)}$$

$$\text{or} \quad (x-4)(x+5) = 9x$$

$$\Rightarrow \quad x^2 + x - 9x - 20 = 0$$

$$\Rightarrow \quad x^2 - 8x - 20 = 0$$

$$\Rightarrow \quad (x-10)(x+2) = 0$$

$$\therefore \quad x = 10 \quad \text{or} \quad -2$$

$\therefore$ Time required by first pipe = $x + 5 = 15$ hours

11. Let the first pipe fill a tank in x minutes
and the second pipe fill a tank in $3x$ minutes.

$$\therefore \quad \frac{1}{x} + \frac{1}{3x} = \frac{1}{36}$$

$$\text{or} \quad \frac{3+1}{3x} = \frac{1}{36}$$

$$\text{or} \quad x = \frac{36 \times 4}{3} = 48 \text{ minutes}$$

The slower pipe alone can fill the tank in $3x$ minutes = $3 \times 48 = 144$ minutes.

12. Part of tank filled in one minute by A, B and C

$$= \frac{1}{30} + \frac{1}{20} + \frac{1}{10} = \frac{2+3+6}{60} = \frac{11}{60}$$

$$\text{P, Q and R are in the ratio } \frac{1}{30} : \frac{1}{20} : \frac{1}{10} = 2 : 3 : 6$$

$$\therefore \text{Proportion of R in the liquid after 3 minutes} = \frac{6}{2+3+6} = \frac{6}{11}$$

13. Let pipe B be turned off after x minutes.

$$\therefore \quad \left(\frac{1}{37\frac{1}{2}} + \frac{1}{45} \right) x + \frac{1}{37\frac{1}{2}} \times (30 - x) = 1$$

$$\Rightarrow \quad \left(\frac{2}{75} + \frac{1}{45} \right) x + \frac{2}{75} (30 - x) = 1$$

$$\begin{aligned} \Rightarrow & \left(\frac{6+5}{225}\right)x + \frac{6}{225}(30-x) = 1 \\ \Rightarrow & \frac{11x+180-6x}{225} = 1 \\ \Rightarrow & 5x+180 = 225 \\ \Rightarrow & 5x = 45 \\ \Rightarrow & x = \mathbf{9 \text{ minutes}} \end{aligned}$$

14. When A and B are opened, part of tank filled in = $\left(\frac{1}{12} + \frac{1}{15}\right) = \frac{5+4}{60} = \frac{3}{20}$

When A and C are opened, part of tank filled in = $\left(\frac{1}{12} + \frac{1}{20}\right) = \frac{5+3}{60} = \frac{2}{15}$

$\therefore$ In first 2 hours, part of tank filled in = $\frac{3}{20} + \frac{2}{15} = \frac{9+8}{60} = \frac{17}{60}$

In 6 hours, part of tank filled = $\frac{17}{60} \times 3 = \frac{51}{60} = \frac{17}{20}$

Balance to be filled = $1 - \frac{17}{20} = \frac{3}{20}$

It will be filled in 1 hour when A and B are opened

$\therefore$ Total time taken = $6 + 1 = \mathbf{7 \text{ hours}}$

15. Let the time taken by C alone to empty the tank be x minute

Part of tank filled in 1 minute when A, B and C are opened = $\frac{1}{60} + \frac{1}{75} - \frac{1}{x} = \frac{1}{50}$

$$\begin{aligned} \therefore \quad \frac{1}{x} &= \frac{1}{60} + \frac{1}{75} - \frac{1}{50} \\ &= \frac{5+4-6}{300} = \frac{1}{100} \end{aligned}$$

$\therefore \quad x = \mathbf{100 \text{ minutes}}$

16. Let time taken by A alone to fill the tank be x hours

time taken by B alone to fill the tank be $\frac{x}{2}$ hours

time taken by C alone to fill the tank be $\frac{x}{4}$ hours

$$\therefore \quad \frac{1}{x} + \frac{1}{x/2} + \frac{1}{x/4} = \frac{1}{5}$$

$$\Rightarrow \quad \frac{1}{x} + \frac{2}{x} + \frac{4}{x} = \frac{1}{5}$$

$$\Rightarrow \quad \frac{7}{x} = \frac{1}{5}$$

$$\Rightarrow \quad x = \mathbf{35 \text{ hours}}$$

17. Let the time taken by A alone to fill the cistern be x hours
 $\therefore$ The time taken by B alone to fill the cistern = $(x + 6)$ hours

$$\therefore \quad \frac{1}{x} + \frac{1}{(x + 6)} = \frac{1}{4}$$

$$\Rightarrow \quad \frac{x + x + 6}{x(x + 6)} = \frac{1}{4}$$

$$\Rightarrow \quad (2x + 6)4 = x(x + 6)$$

$$\Rightarrow \quad x^2 + 6x = 8x + 24$$

$$\Rightarrow \quad x^2 - 2x - 24 = 0$$

$$(x - 6)(x + 4) = 0$$

$$\therefore \quad x = \mathbf{6 \text{ hours}}$$

18. Let the time taken to drain completely be x hours.

$$\therefore \quad \frac{1}{2} - \frac{1}{x} = \frac{1}{2\frac{1}{3}}$$

$$\Rightarrow \quad \frac{1}{2} - \frac{1}{x} = \frac{3}{7}$$

$$\Rightarrow \quad \frac{1}{x} = \frac{1}{2} - \frac{3}{7}$$

$$\Rightarrow \quad \frac{1}{x} = \frac{7 - 6}{14}$$

$$\Rightarrow \quad \frac{1}{x} = \frac{1}{14}$$

$$\therefore \quad x = \mathbf{14 \text{ hours}}$$

19. Part of tank filled by A in one minute = $\frac{1}{10}$

$$\text{Part of tank emptied by B in one minute} = \frac{1}{6}$$

$$\text{Part of tank emptied by A and B in one minute} = \frac{1}{6} - \frac{1}{10} = \frac{5 - 3}{30} = \frac{1}{15}$$

15 minutes will be taken to empty the tank, when it is full.

$$\therefore \frac{2}{5}^{\text{th}} \text{ of the tank can be emptied in } \frac{2}{5} \times 15 = \mathbf{6 \text{ minutes}}$$

20. Pipe A can fill $\frac{1}{5}$ part of tank in 1 hour

Similarly pipe B can fill $\frac{1}{6}$ part of tank in 1 hour

C can empty $\frac{1}{12}$ part of tank in 1 hour

When A, B and C are opened, part of tank filled in 1 hour

$$= \frac{1}{5} + \frac{1}{6} - \frac{1}{12}$$

$$= \frac{12 + 10 - 5}{60} = \frac{17}{60}$$

$$\text{Time taken to fill the tank} = \frac{1}{17/60} = \frac{60}{17} \text{ hours} = 3 \frac{9}{17} \text{ hours}$$

21. Time taken to fill the tank when A and B are opened

$$= \frac{1}{\frac{1}{20} + \frac{1}{30}} = \frac{1}{\frac{3+2}{60}} = \frac{60}{5} = 12 \text{ minutes}$$

22. Time taken to fill the tank, when A, B and C are opened

$$= \frac{1}{\frac{1}{5} + \frac{1}{10} + \frac{1}{30}} = \frac{5 \times 10 \times 30}{5 \times 10 + 10 \times 30 + 30 \times 5}$$

$$= \frac{30}{1 + 6 + 3} = 3 \text{ hours}$$

23. When both taps are opened, time taken to fill the cistern

$$= \frac{1}{\frac{1}{4} - \frac{1}{9}} = \frac{9 \times 4}{9 - 4} = \frac{36}{5} = 7.2 \text{ hours}$$

24. Part of tank filled in the first minute by A and B

$$= \left(\frac{1}{18} + \frac{1}{9} \right) = \frac{(1+2)}{18} = \frac{1}{6}$$

$$\therefore \text{Part of tank filled in first 4 minutes by A and B} = 4 \times \frac{1}{6} = \frac{2}{3}$$

Part of the tank emptied in 1 minute when A, B and C are opened

$$= \frac{1}{5} - \left(\frac{1}{18} + \frac{1}{9} \right) = \frac{1}{5} - \frac{1}{6} = \frac{1}{30}$$

$\therefore$ Time taken to empty the tank = 30 minutes

$\therefore \frac{2}{3}$ of the tank will be emptied in $\frac{2}{3} \times 30 = 20$ minutes

25. Capacity of tank = $10 \times 12 = 120$ litres

$$\therefore \text{Number of buckets needed to fill the tank with 8 litre capacity of bucket} = \frac{10 \times 12}{8} = 15$$

i.e. Number of buckets required = 15

Alligation

Alligation is an old and practical method of solving arithmetic problem related to mixtures of ingredients.



Important Formulae

$$\text{Rule of Alligation} = \frac{\text{Amount of Cheaper ingredient}}{\text{Amount of Dearer ingredient}} = \frac{\text{Cost price of Dearer} - \text{Mean price}}{\text{Mean price} - \text{Cost price of Cheaper}}$$

1. A vessel contains x litres of liquid A and y litres is withdrawn and replaced by liquid B and the operation is repeated n times in all. Find the ratio of liquid A left after n th operation to the whole quantity of liquid A initially present in the vessel.

Original quantity of liquid A = x litres

$$\text{Quantity withdrawn first time} = y \text{ litres} = \frac{y}{x} (x \text{ litres})$$

$$\therefore \text{Quantity of liquid A left after 1st operation} = \left(1 - \frac{y}{x}\right) \text{ part}$$

$$\text{Similarly quantity left after 2nd operation} = \left(1 - \frac{y}{x}\right)^2 \text{ part}$$

$$\text{Similarly quantity left after } n\text{th operation} = \left(1 - \frac{y}{x}\right)^n \text{ part}$$

$$\therefore \frac{\text{Quantity of A left after } n\text{th operation}}{\text{Quantity of A initially present}} = \frac{\left(1 - \frac{y}{x}\right)^n \times x}{x} = \left(1 - \frac{y}{x}\right)^n = \left(\frac{x - y}{x}\right)^n$$

2. If a group of 2 legged and 4 legged creatures gives a head count H and their total number of legs is C , then

$$\text{Number of 4 legged animals} = \frac{C - 2H}{2}$$

$$\text{Number of 2 legged animals} = \frac{4H - C}{2}$$

m gram of sugar solution has $x\%$ sugar in it. To increase the sugar content in the solution to $y\%$, quantity of sugar needed to be added = $\frac{m(y - x)}{100 - y}$

3. If contents of n vessels of sizes $c_1, c_2, \dots, c_n$ filled with mixture of liquids A and B in the ratio $a_1 : b_1, a_2 : b_2, \dots, a_n : b_n$ respectively are emptied into a single large vessel, then

$$\frac{\text{Quantity of liquid A}}{\text{Quantity of liquid B}} = \frac{\left[\frac{a_1 c_1}{a_1 + b_1} + \frac{a_2 c_2}{a_2 + b_2} + \dots + \frac{a_n c_n}{a_n + b_n} \right]}{\left[\frac{b_1 c_1}{a_1 + b_1} + \frac{b_2 c_2}{a_2 + b_2} + \dots + \frac{b_n c_n}{a_n + b_n} \right]}$$

4. If the contents of n vessel of equal size filled with mixtures of liquids A and B in the ratio $a_1 : b_1, a_2 : b_2, \dots, a_n : b_n$, respectively are emptied into a single large vessel, then

$$\frac{\text{Quantity of liquid A}}{\text{Quantity of liquid B}} = \frac{\left[\frac{a_1}{a_1 + b_1} + \frac{a_2}{a_2 + b_2} + \dots + \frac{a_n}{a_n + b_n} \right]}{\left[\frac{b_1}{a_1 + b_1} + \frac{b_2}{a_2 + b_2} + \dots + \frac{b_n}{a_n + b_n} \right]}$$

Alligation help us to

- find the mean or average value of mixture when two or more ingredients which may be mixed together and the proportion in which they are mixed are given (Alligation medial); and
- find the proportion in which the ingredients at given prices must be mixed to produce a mixture at a given price (Alligation alternate).

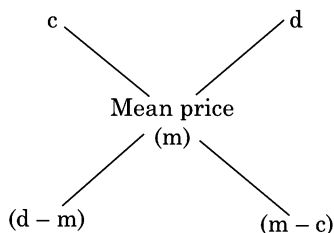
The word *alligation* literally means *linking*. The alligation method is applied to determine the percentage value, ratio, rate, price, speed, etc. and not for absolute values. i.e. whenever per cent, per hour, per kg, etc. are being compared, we can use alligation.

RULE OF ALLIGATION: If the items are mixed in a ratio, then

$$\frac{\text{Quantity of Cheaper}}{\text{Quantity of Dearer}} = \frac{\text{CP of Dearer} - \text{Mean price}}{\text{Mean price} - \text{CP of Cheaper}}$$

CP of cheaper item for unit quantity (c)

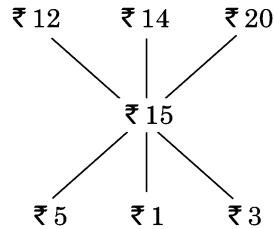
CP of unit quantity of dearer item (d).



∴ Cheaper quantity: dearer quantity = $(d - m) : (m - c)$

EXAMPLE 1 Find the proportion in which three types of sugar at ₹ 12, ₹ 14 and ₹ 20 may be mixed so as to obtain a mixture worth ₹ 15 per kg.

Solution Required proportion = 5 : 5 : (3 + 1) = 5 : 5 : 4

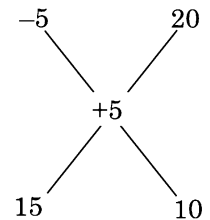


EXAMPLE 2 A person buys two watches for ₹ 1,000. He sells one watch at a loss of 5% and the other at a gain of 20% and on the whole he gains ₹ 50. Find the cost price of each watch.

Solution Overall percentage profit = $\frac{50}{1000} \times 100 = 5\%$

C.P. of watch sold at 5% less = $\frac{3}{3+2} \times 1000 = ₹ 600$

C.P. of watch sold at 20% gain = $\frac{2}{3+2} \times 1000 = ₹ 400$



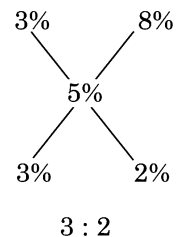
COMBINATION OF DIFFERENT RATES OF INTEREST

EXAMPLE 3 A person has ₹ 5000. He invests a part of it at 3% per annum and the remainder at 8% per annum simple interest. His total income as interest in 3 years is ₹ 750. Find the sum invested at different rates of interest.

Solution Average rate of interest = $\frac{750 \times 100}{5000 \times 3} = 5\%$

Investment at 3% per annum = $\frac{3}{3+2} \times 5000 = ₹ 3000$

Investment at 8% per annum = $\frac{2}{3+2} \times 5000 = ₹ 2000$



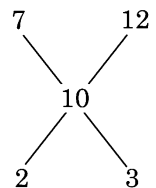
SPEED AND DISTANCE

EXAMPLE 4 A person covers a distance of 100 km in 10 hours, partly by walking at 7 km/h and the rest by running at 12 km/h. Find the distance covered in each part.

Solution Average speed = $\frac{100}{10} = 10$ km/h

Ratio of time taken at 7 km/h to 12 km/h = 2 : 3

∴ Time taken at 7 km/h = $\frac{2}{2+3} \times 10 = 4$ hours



Distance covered at 7 km/h = $7 \times 4 = 28$ km
 Distance covered at 12 km/h = $100 - 28 = 72$ km

WAGES AND SALARY, EXPENDITURE AND SAVINGS

EXAMPLE 5 The expenditure and savings of an employee are in the ratio 3 : 1. His income increases by 16%, but at the same time his expenditure also increases by 20%. Find the increase or decrease in his savings.

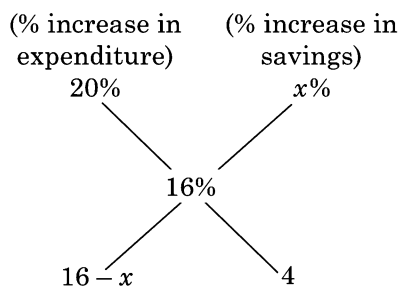
Solution Expenditure and savings are two constituents of income.

Let $x\%$ be the increase in his savings.

$$\frac{16 - x}{4} = \frac{3}{1}$$

$$\Rightarrow 16 - x = 12$$

$$\Rightarrow x = 4\%$$



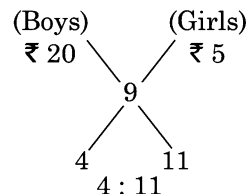
DIVISION OF SUM INTO PARTS

EXAMPLE 6 ₹ 675 was divided among 75 boys and girls. Each boy gets ₹ 20 whereas a girl gets ₹ 5. Find the number of boys and girls.

Solution Average money/head = $\frac{675}{75} = ₹ 9$

$$\therefore \text{Number of boys} = \frac{4}{4 + 11} \times 75 = 20$$

$$\text{Number of girls} = \frac{11}{4 + 11} \times 75 = 55$$



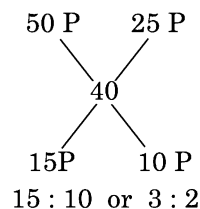
COINS

EXAMPLE 7 A sum of ₹ 12 is made up of 30 coins which consist of either 50 paise or 25 paise. How many are there of each kind?

Solution Average value/coin = $\frac{12 \times 100}{30} = 40$ paise

$$\therefore \text{Number of 50-paise coins} = \frac{3}{3 + 2} \times 30 = 18$$

$$\text{Number of 25-paise coins} = \frac{2}{3 + 2} \times 30 = 12$$



REPLACEMENT OF A PART OF MIXTURE BY ONE INGREDIENT

EXAMPLE 8 From a cask of wine containing 25 litres, 5 litres is withdrawn and the cask is refilled with water. The process is repeated a second time and then a third time.

Find the quantity of wine left in the cask and also the ratio of wine to water in the resulting mixture.

Solution Quantity of wine left after 1st withdrawal = $1 - \frac{1}{5} = \frac{4}{5}$ of the cask

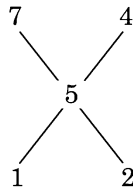
Quantity of wine left after 2nd withdrawal = $\left(\frac{4}{5}\right)^2$ of the cask

Quantity of wine left after 3rd withdrawal = $\left(\frac{4}{5}\right)^3$ of the cask
 $= \frac{64}{125} \times 25 = 12\frac{4}{5}$ litres

$$\begin{aligned}\text{Ratio of wine to water} &= \frac{64/125}{1 - 64/125} \\ &= \frac{64/125}{61/125} = 64/61\end{aligned}$$

EXAMPLE 9 How many kilograms of salt at 4/kg must a person mix with 25 kg of salt at ₹ 7/kg, so that he may on selling the mixture at ₹ 6/kg, gain 20% on the outlay?

Solution Cost price of mixture = $6 \times \frac{100}{120} = ₹ 5/\text{kg}$



∴ Ratio = 1 : 2

∴ For each kg of salt at ₹ 7 /kg, 2 kg of salt at ₹ 4/kg is used.

∴ Required weight of salt = $25 \times 2 = 50 \text{ kg}$

EXAMPLE 10 Milk and water are mixed in vessel A in the proportion 5 : 2, and in vessel B in the proportion 8 : 5. In what proportion should quantities be taken from two vessels so as to form a mixture in which milk and water will be in the proportion of 9 : 4?

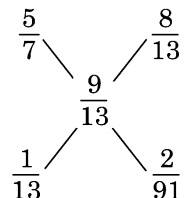
Solution In vessel A, quantity of milk = $\frac{5}{7}$ of the mixture.

In vessel B, quantity of milk = $\frac{8}{13}$ of the mixture.

In the new mixture, quantity of milk = $\frac{9}{13}$ of the mixture.

∴ Required proportion $\frac{1}{13} : \frac{2}{91} = 7 : 2$

∴ $\frac{7}{9}$ parts from vessel A and $\frac{2}{9}$ parts from vessel B.



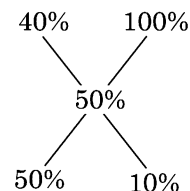
EXAMPLE 11 200 g of sugar solution has 40% sugar in it. How much sugar should be added to make it 50% in the solution?

Solution The existing solution has 40% sugar.

Sugar is to mixed, which is a 100% solution.

Required ratio = 5 : 1

$$\therefore \text{Required sugar} = \frac{200}{5} \times 1 = 40 \text{ g}$$

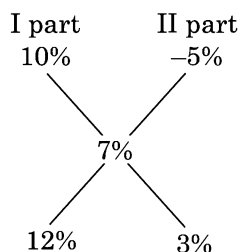


Short cut: Quantity of sugar added
 = Weight of sugar solution $\frac{(\text{Required \% value} - \text{Present \% value})}{(100 - \text{Required \% value})}$

i.e. $= 200 \frac{(50 - 40)}{(100 - 50)} = 40 \text{ g}$

EXAMPLE 12 A trader has 50 kg of rice, a part of which he sells at 10% profit and the rest at 5% loss. He gains 7% on the whole. What is the quantity sold at 10% gain and 5% loss?

Solution Ratio of quantities sold at 10% profit and 5% loss = 12 : 3 = 4 : 1



$$\therefore \text{Quantity sold at 10\% profit} = \frac{4}{5} (50 \text{ kg}) = 40 \text{ kg}$$

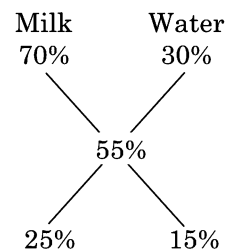
$$\text{Quantity sold at 5\% loss} = 50 - 40 = 10 \text{ kg}$$

EXAMPLE 13 A vessel of 80 litres is filled with milk and water. 70% of milk and 30% of water is taken out of the vessel. It is found that the vessel is emptied by 55%. Find the initial quantity of milk and water.

Solution Required ratio of milk water = 25 : 15 or 5 : 3

$$\text{Quantity of milk} = \frac{80}{5 + 3} \times 5 = 50 \text{ litres}$$

$$\text{Quantity of water} = \frac{80}{5 + 3} \times 3 = 30 \text{ litres}$$



EXAMPLE 14 A container contained 80 litres of milk. From this container 8 litres of milk was taken out and replaced by water. This process was further repeated two times. How much milk is now contained by the container?

Solution The amount of liquid left after n operations, when the container originally contain x units of liquid from which y units is taken out each time is $x \left(\frac{x-y}{x} \right)^n$ units.

$$\therefore \text{Amount of milk left} = 80 \left(\frac{80-8}{80} \right)^3 = 58.32 \text{ kg.}$$

EXAMPLE 15 ₹ 15000 is invested in two parts such that if one part is invested at 6%, and the other at 5%, the total amount in one year as interest from both investments is ₹ 850. How much is invested at 5%.

Solution Total interest at 5% for ₹ 15000 = ₹ 750

Balance interest at 1% excess is got from amount x is ₹ 100.

$$\therefore x \times 1 \times \frac{1}{100} = 100$$

$$\therefore x = ₹ 10000$$

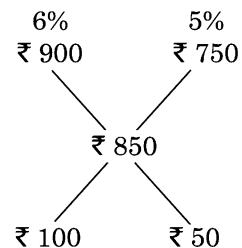
∴ ₹ 10000 is invested at 6% per annum and

₹ 5000 is invested at 5% per annum

Applying allegation rule:

Required ratio = 2 : 1

$$\therefore \text{Money invested at 5\%} = \frac{1}{2+1} \times ₹ 15000 = ₹ 5000$$



EXAMPLE 16 A jar contains a mixture of two liquids A and B in the ratio 4 : 1. When 10 litres of the mixture is taken out and 10 litres of liquid B is poured into the jar, the ratio becomes 2 : 3. How many litres of liquid A was contained in the jar?

Solution Method I: In original mixture, the % of liquid B = $\frac{1}{4+1} \times 100 = 20\%$

In the resultant mixture, % of liquid B = $\frac{3}{2+3} \times 100 = 60\%$

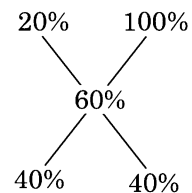
Replacement is made by liquid B, so that % of B in second mixture = 100%

By method of alligation:

∴ Ratio of first and second mixtures that should be added is 1 : 1.

∴ Total mixture = 10 + 10 = 20 litres

and liquid A = $\frac{4}{5} \times 20 = 16$ litres



Method II: This method is explained through fraction

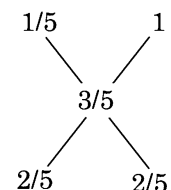
Fraction of B in original mixture = $1/5$

Fraction of B in second mixture = 1

Fraction of B in resulting mixture = $3/5$

Applying method of alligation:

Original mixture and liquid B are added in the same ratio.



10 litres of liquid B is added after taking out 10 litres of mixture from the jar. There should have been 10 litres of mixture left.

∴ Quantity of mixture in the jar = $10 + 10 = 20$ litres

$$\text{Quantity of A in the jar} = \frac{4}{4+1} \times 20 = \mathbf{16 \text{ litres}}$$

Method III: Let the quantity of mixture in the jar be $5x$ litres.

Then initial proportion of liquids A to B = $4x : x$

After taking 10 litres of mixture out, ratio becomes $4x - 10 \left(\frac{4}{4+1} \right) : x - \left(\frac{1}{4+1} \right) 10 + 10$

i.e. $(4x - 8) : x - 2 + 10 = 2 : 3$

i.e. $\frac{4x - 8}{x + 8} = \frac{2}{3} \Rightarrow x = 4$

∴ Quantity of A in the mixture = $4x = 4 \times 4 = 16$ litres



Note: Liquid A in original mixture = $4x$

Liquid A taken out with 10 litres of mixture = $10 \times \frac{4}{4+1} = 8$ litres

Remaining quantity of A = $4x - 10 \left(\frac{4}{5} \right) = 12$ litres

Liquid B in original mixture = x litres

Liquid B taken out with 10 litres of mixture = $10 \left(\frac{1}{5} \right) = 2$ litres

Liquid B added = 10 litres

Total quantity of liquid B = $x - 10 \left(\frac{1}{5} \right) + 10$

Ratio of liquid A : B = $2 : 3$



Theorem: If x glasses of equal size are filled with a mixture of spirit and water. The ratio of spirit and water in each glass is as follows $a_1 : b_1, a_2 : b_2, \dots, a_n : b_n$. If the contents of all the glasses are emptied into a single vessel, then proportion of spirit and water in it is given by

$$\left(\frac{a_1}{a_1 + b_1} + \frac{a_2}{a_2 + b_2} + \dots + \frac{a_n}{a_n + b_n} \right) : \left(\frac{b_1}{a_1 + b_1} + \frac{b_2}{a_2 + b_2} + \dots + \frac{b_n}{a_n + b_n} \right)$$

EXAMPLE 17 In three vessels each of 10 litres capacity, mixture of milk and water is filled. The ratios of milk and water are $2 : 1, 3 : 1$ and $3 : 2$ in the three respective vessels. If all the three vessels are emptied into a single vessel, find the proportion of milk and water in the mixture.

Solution Required ratio is $\left(\frac{2}{2+1} + \frac{3}{3+1} + \frac{3}{3+2} \right) : \left(\frac{1}{2+1} + \frac{1}{3+1} + \frac{2}{3+2} \right)$

$$= \left(\frac{2}{3} + \frac{3}{4} + \frac{3}{5} \right) : \left(\frac{1}{3} + \frac{1}{4} + \frac{2}{5} \right) = \frac{40 + 45 + 36}{60} : \frac{15 + 20 + 24}{60} = \frac{121}{60} : \frac{59}{60} = \mathbf{121 : 59}$$



Note: The capacity of vessel may be taken as LCM of (2 + 1), (3 + 1) and (3 + 2) i.e. 60 litres. So without using the theorem, the problem can be solved.

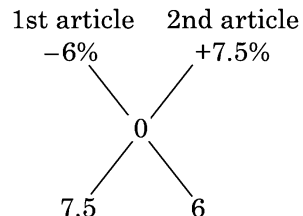
EXAMPLE 18 A person buys two articles for ₹ 1,350 and sells one at a loss of 6% and the other at a gain of 7.5% and on the whole he neither gains nor loses. What does each article cost?

Solution Cost of 1st article = $\frac{5}{5+4} \times 1350 = ₹ 750$

Cost of 2nd article = $\frac{4}{5+4} \times 1350 = ₹ 600$

∴ Required ratio is 7.5 : 6 or 5 : 4

$$= 1.5 \times 5 : 1.5 \times 4 = 5 : 4$$



EXERCISES

- How much water must be added to a vessel which contains 80 litres of milk at cost price ₹ 3.5/litre so that the cost of milk reduces to ₹ 2/litre?
(a) 20 litres (b) 40 litres (c) 60 litres (d) 70 litres (e) None of these
- A sum of ₹ 64 is made up of 80 coins which are either 100 paise or 50 paise coins. How many coins are of 50 paise?
(a) 22 (b) 32 (c) 42 (d) 52 (e) None of these
- Varun's expenditures and savings are in the ratio of 4 : 1. His income increases by 20%. If his savings increase by 12%, by how much % should his expenditure increase?
(a) 12% (b) 20% (c) 22% (d) 32% (e) None of these
- A dishonest trader professes to sell his article at cost price but he uses 900 g weight instead of 1 kg. What is the percentage gain of the trader in this trade?
(a) 80/9% (b) 100/9% (c) 110/9% (d) 70/9% (e) None of these
- A sum of ₹ 390 was divided among 450 boys and girls. Each girl gets 50 paise and each boy gets one rupee. How many girls are there among them?
(a) 100 (b) 110 (c) 120 (d) 130 (e) None of these
- An amount of ₹ 70,000 was lent at 6% per annum and the remaining at 4% per annum. If the total simple interest from both the fractions in 5 years was ₹ 16000, the sum lent at 6% per annum was _____.
(a) ₹ 10000 (b) ₹ 20000 (c) ₹ 30000 (d) ₹ 35000 (e) None of these
- Gold is 19 times as heavy as water and copper is 9 times. In what ratio should these metals be mixed so that the mixture may be 15 times as heavy as water?
(a) 5 : 4 (b) 4 : 3 (c) 3 : 2 (d) 2 : 1 (e) None of these
- A man has given ₹ 300000 as loan. He lent some of his capital to Ajay at an interest rate of 20% per annum and the rest to Ramesh at an interest rate of 12% per annum. At the end of one year, he got 17% of his capital as interest. How much did he lend to Ajay?
(a) ₹ 120000 (b) ₹ 140000 (c) ₹ 160000 (d) ₹ 180000 (e) None of these

9. Haresh earned ₹ 4000 per month. From the last month his income was increased by 8%. Due to rise in prices, his expenditure also got increased by 12% and his savings decreased by 4%. Find his increased expenditure and initial saving.
(a) 3000, 1000 (b) 3000, 1360 (c) 3360, 1000 (d) 3360, 1200 (e) None of these
10. A person has 60 pens. He sells some of these at a profit of 12% and the rest at 8% loss. On the whole, he gets a profit of 11%. How many pens were sold at 12% profit and how many at 8% loss?
(a) 60 (b) 58 (c) 57 (d) 55 (e) None of these
11. Tea worth ₹ 126/kg and ₹ 135/kg are mixed with a third variety in the ratio 1 : 1 : 2. If the mixture is worth ₹ 153/kg, the price of the third variety/kg will be [SSC]
(a) ₹ 169.50 (b) ₹ 170 (c) ₹ 175.50 (d) ₹ 180
12. In what ratio must a grocer mix two varieties of tea worth ₹ 60 a kg and ₹ 65 a kg so that by selling the mixture at ₹ 68.20 a kg he may gain 10%? [SSSC]
(a) 3 : 2 (b) 3 : 4 (c) 3 : 5 (d) 4 : 5
13. The cost of Type 1 rice is ₹ 15/kg and Type 2 rice is ₹ 20/kg. If both Type 1 and Type 2 are mixed in the ratio of 2 : 3, then the price per kg of the mixed variety of rice is [MBA]
(a) ₹ 18 (b) ₹ 18.50 (c) ₹ 19 (d) ₹ 19.50
14. In what ratio must water be mixed with milk to gain $16\frac{2}{3}\%$ on selling the mixture at cost price? [LIC AAO]
(a) 1 : 6 (b) 6 : 1 (c) 2 : 3 (d) 4 : 3
15. 8 litres are drawn from a cask full of wine and is then filled with water. This operation is performed three more times. The ratio of the quantity of wine now left in cask to that of the water is 16:65. How much wine did the cask hold originally? [NIFT]
(a) 18 litres (b) 24 litres (c) 32 litres (d) 42 litres
16. In what ratio must a grocer mix two varieties of pulses costing ₹ 15 and ₹ 20 per kg respectively so as to get a mixture worth ₹ 16.5/kg? [RRB]
(a) 3 : 7 (b) 5 : 7 (c) 7 : 3 (d) 7 : 5
17. Find the ratio in which rice at ₹ 7.20/kg be mixed with rice at ₹ 5.7/kg to produce a mixture worth ₹ 6.3/kg. [IGNOU]
(a) 1 : 3 (b) 2 : 3 (c) 3 : 4 (d) 4 : 5
18. A bottle full of whisky contains 50% alcohol. A part of this whisky is replaced by another containing 18% alcohol and the percentage of alcohol was found to be 26%. The quantity of whisky replaced is:
(a) $\frac{1}{4}$ (b) $\frac{3}{4}$ (c) $\frac{2}{5}$ (d) $\frac{3}{5}$
19. How many kg of sugar costing ₹ 11.70/kg must be mixed with 24 kg of sugar costing ₹ 8.1/kg so that there may be a gain of 15% by selling the mixture at ₹ 9/kg?
(a) 6 kg (b) 8 kg (c) 10 kg (d) 12 kg
20. A merchant has 1000 kg of rice, part of which he sells at 8% profit and the rest at 16% profit. He gains 14% on the whole. The quantity sold at 16% profit is:
(a) 500 kg (b) 650 kg (c) 750 kg (d) 800 kg

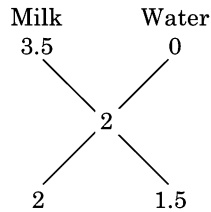


1. (c) 2. (b) 3. (c) 4. (b) 5. (c) 6. (b) 7. (c) 8. (e) 9. (c) 10. (c)
11. (c) 12. (a) 13. (a) 14. (a) 15. (b) 16. (c) 17. (b) 18. (b) 19. (b) 20. (c)



Solutions with Necessary Explanation

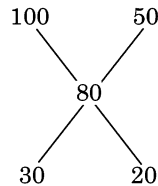
1.



Ratio of milk : water = 2 : 1.5 or **4 : 3**

Added water = $\frac{80}{4} \times 3 = \mathbf{60 \text{ litres}}$

2. Average value/coin = $\frac{6400}{80} = 80$ paise

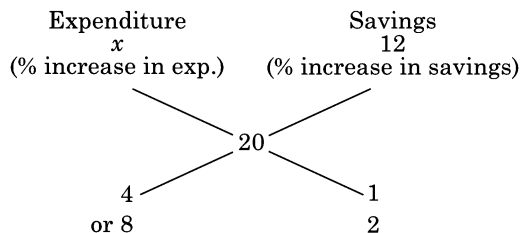


$\therefore 100 \text{ paise} : 50 \text{ paise} = 30 : 20$ or **3 : 2**

$\therefore \text{Number of 50 paise coins} = \frac{80 \times 2}{3 + 2} = \mathbf{32}$

3. $x - 20 = 2$

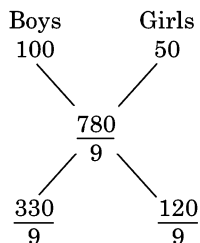
or $x = \mathbf{22\%}$



$\therefore \text{\% increase in expenditure} = \mathbf{22\%}$

4. % gain of the trader = $\frac{100}{900} \times 100 = \frac{100}{9} \% = 11\frac{1}{9} \%$

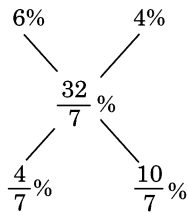
5. Paise per head = $\frac{39000}{450} = \frac{780}{9} \text{ paise}$



∴ i.e. required ratio = 330 : 120 or **11 : 4**

∴ Number of girls among them = $\frac{4}{11+4} \times 450 = \mathbf{120}$

6.

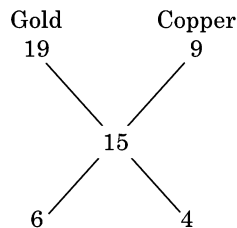


Total simple interest percentage = $\frac{16000 \times 100}{70000 \times 5} = \frac{32}{7} \%$

∴ Ratio of amounts = $\frac{4}{7} : \frac{10}{7} = \mathbf{2 : 5}$

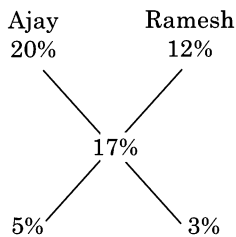
∴ Sum lent at 6% per annum = $\frac{2}{2+5} \times ₹ 70000 = \mathbf{₹ 20000}$

7.



Gold : Copper = 6 : 4 or **3 : 2**

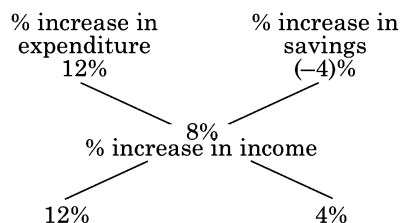
8.



Ratio of amounts that was given to Ajay and Ramesh are in the ratio 5 : 3

∴ Amount that was given to Ajay = $\frac{5}{5+3} \times 300000 = \mathbf{₹ 187500}$

9.



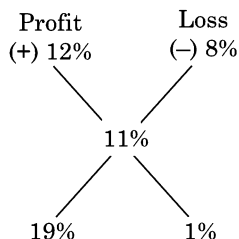
∴ Expenditure : Savings = 12 : 4 = **3 : 1**

$$\therefore \text{Expenditure} = 4000 \times \frac{3}{3+1} = \text{₹ } 3000$$

$$\text{Savings} = 4000 - 3000 = ₹ 1000$$

$$\therefore \text{Increased expenditure} = 3000 \times \left(\frac{112}{100} \right) = \text{₹ } 3360$$

10.



Ratio of pens sold at 12% profit and 8% loss = 19 : 1

$\therefore$ Number of pens sold at 12% profit

$$= \frac{19}{19+1} \times 60 = \mathbf{57}$$

11. Let the price of third variety be ₹ x

First two varieties are mixed in the proportion 1 : 1

$$\therefore \text{Average price} = \frac{\text{₹}126 + \text{₹}135}{2} = \text{₹}130.50$$

Mixture is in the proportion 1 : 1 : 2 i.e. 2 : 2 or 1 : 1

$$\therefore (x - 153) : 22.50 = 1 : 1$$

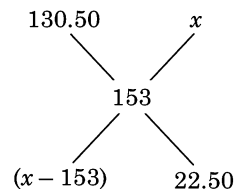
or

$$(x - 153) = 22.50$$

⇒

$$x = ₹ 175.50$$

Hence price of third variety = ₹ 175.50



12. Selling price of mixture = ₹ 68.20 at 10% gain

$$\therefore \text{Cost price of mixture} = \left(\frac{100}{110} \times 68.20 \right) = \text{₹ } 62$$

$\therefore$ Required ratio = **3 : 2**

13. Let the price of mixed variety be ₹ x/kg

$$\therefore \frac{20 - x}{x - 15} = \frac{2}{3}$$

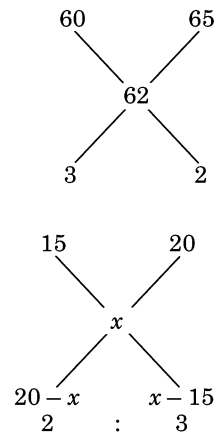
$$(20 - x)3 = 2(x - 15)$$

$$60 - 3x = 2x - 30$$

or

$$5x = 60 + 30 = 90$$

$$x = ₹ 18/\text{kg}$$



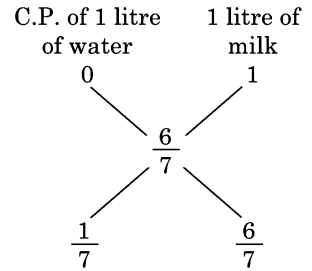
14. C.P. of 1 litre milk = ₹ 1

$$\text{Gain \%} = 16 \frac{2}{3} \% = \frac{50}{3} \%$$

S.P. of 1 litre mixture ₹ 1

$$\text{C.P. of 1 litre mixture} = \frac{300}{350} \times 1 = ₹ \frac{6}{7}$$

$$\therefore \text{Ratio of water and milk} = \frac{1}{7} : \frac{6}{7} = 1 : 6$$



15. Let the quantity of wine originally in the cask was x litres

$$\text{Quantity of wine left in the cask after 4 operations} = x \left(1 - \frac{8}{x}\right)^4 \text{ litres}$$

$$\therefore \frac{x \left(1 - \frac{8}{x}\right)^4}{x} = \frac{16}{81}$$

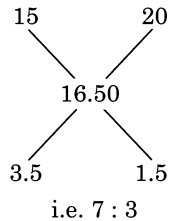
$$\text{or} \quad \left(1 - \frac{8}{x}\right)^4 = \frac{16}{81} = \left(\frac{2}{3}\right)^4$$

$$\therefore 1 - \frac{8}{x} = \frac{2}{3}$$

$$\text{or} \quad \frac{8}{x} = 1 - \frac{2}{3} = \frac{1}{3}$$

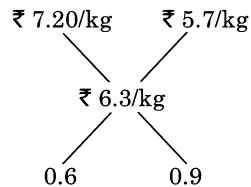
$$\therefore x = 8 \times 3 = 24 \text{ litres}$$

16. Required ratio = 7 : 3.



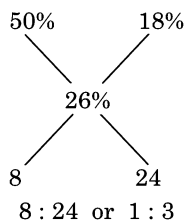
17. Required proportion

$$= (6.3 - 5.7) : (7.2 - 6.3) = 0.6 : 0.9 \quad \text{or} \quad 2 : 3$$



18. By rule of alligation, to get mean strength of 26%, given solution of strengths 50% and 18% are to be mixed in the proportion 1 : 3.

$$\therefore \text{Quantity of whisky replaced} = \frac{3}{4}$$



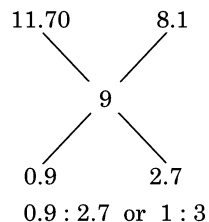
19. Required proportion by rule of alligation

= 1 part of sugar costing ₹ 11.70/kg and 3 parts of sugar costing ₹ 8.1/kg

i.e. Required proportion = 1 : 3

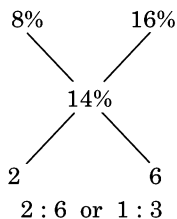
$\therefore$ Quantity of sugar to be mixed with 24 kg of sugar costing ₹ 8.1/kg

$$= \frac{1}{3} \times 24 = 8 \text{ kg}$$



20. Quantity of rice at 8% profit and 16% are in the ratio 1 : 3.

$$\text{Quantity of rice sold at 16\% profit} = \frac{3}{3+1} \times 1000 = 750 \text{ kg}$$



Problems on Trains



Important Formulae

1. Time taken (t) by a train x metres long to cross a stationary person or pole or signal post is equal to the time taken by the train to cover the distance x metres (equal to its length) with its own speed (v)

$$t = \frac{x}{v}$$

2. Time taken by a train x metres long to cross a stationary object (like platform, bridge, tunnel, standing train, etc.) y metres long is equal to time taken by the train to cover a total distance $(x + y)$ metres at its own speed (v).

$$t = \frac{x + y}{v}$$

3. (a) A train x metres long running at speed v passes an object of length y metres moving at speed u in the same direction.

$$\text{Time taken by the train to cross the moving object} = \frac{x + y}{v - u}$$

- (b) If the object is a man, signal post or things like this, $t = \frac{x}{v - u}$

4. If the train and object are moving in the opposite direction, i.e. they are approaching towards each other.

$$\text{Relative speed} = v + u$$

$$\text{Then time taken by the train to cross the moving object} = \frac{x + y}{v + u}$$

$$\text{Time taken by the train to cross a walking man/moving object} = \frac{x}{v + u}$$

5. Two trains A and B start from two points P and Q and move towards each other. After crossing they take a and b hours in reaching Q and P respectively. Then the ratio of their speed is given by

$$\text{A's speed} : \text{B's speed} = \sqrt{b} : \sqrt{a} = \sqrt{b/a}$$

6. Two trains A and B start simultaneously from stations P and Q and move towards each other at speed S_1 and S_2 km/h respectively. At the point where they meet, one train has covered x km more than the other. Then the distance between P and Q is $\left(\frac{S_1 + S_2}{S_1 - S_2}\right)x$ km.
7. Two trains A and B are moving in the same direction at speeds S_1 and S_2 km/h. The faster train passes a man sitting in the slower train in t seconds. Then the length of the faster train is $\frac{5}{18}(S_1 - S_2)t$ metres.
8. Two trains A and B start simultaneously from stations P and Q, x km apart and move towards each other. They meet after t hours. If the difference in speeds of the two trains is 5 km/h. Show that the speeds of the two trains are $1/2(x/t + 5)$ km/h and $1/2(x/t - 5)$ km/h

i.e. speed of faster train $S_1 = \frac{1}{2}\left(\frac{x}{t} + 5\right)$ km/h

Slower train $S_2 = \frac{1}{2}\left(\frac{x}{t} - 5\right)$ km/h

9. A train passes a man standing on the platform in t_1 seconds and the platform completely in t_2 seconds. If the length of platform is l , then the length of train is $l\left(\frac{t_1}{t_2 - t_1}\right)$ metres.
10. If a train covers a certain distance non-stop, its average speed is S_1 km/h, but when it covers the distance with stoppages, its average speed is reduced to S_2 km/h. Then the train stops for $\left(\frac{S_1 - S_2}{S_1}\right)$ hour for every hour travelled.
11. Two trains of lengths l_1 and l_2 metres run on parallel tracks. When running in the same direction, the faster train passes the slower one in t_1 seconds. But when they are running in the opposite directions at same speeds as earlier, they cross each other completely in t_2 seconds.

Then the speeds of the trains are $\left(\frac{l_1 + l_2}{2}\right)\left(\frac{t_1 + t_2}{t_1 t_2}\right)$ m/s and $\left(\frac{l_1 + l_2}{2}\right)\left(\frac{t_1 - t_2}{t_1 t_2}\right)$ m/s

12. (a) When two trains are moving in the same direction, time taken to cross each other

$$= \frac{2 \times \text{Product of two times}}{\text{Difference of two times}}$$

- (b) When the two trains are moving in the opposite direction, time taken to cross each other

$$= \frac{2 \times \text{Product of two times}}{\text{Sum of two times}}$$

13. A train overtakes two persons walking in the same direction at x km/h and y km/h respectively and completely passes them in t_1 seconds and t_2 seconds respectively. Then length of train

$$= \frac{\text{Difference of speeds of two man} \times \text{Product of two times}}{\text{Difference of times}}$$

$$= \frac{(x - y) \times t_1 t_2}{t_1 - t_2}$$

Let the speed of train be S km/h and its length be l metres.

$$\therefore (S - x)t_1 = (S - y)t_2 = l$$

$$\Rightarrow S(t_1 - t_2) = xt_1 - yt_2$$

$$S = \frac{(xt_1 - yt_2)}{(t_1 - t_2)}$$

But

$$l = (S - x)t_1$$

On simplifying the above equation using the expression for S , we get

$$l = \frac{(x - y)t_1t_2}{(t_1 - t_2)}$$

14. Two trains of equal length but travelling at different speeds cross a stationery pole in t_1 seconds and t_2 seconds respectively. They will cross each other in time

(a) $t = \frac{2t_1t_2}{t_2 - t_1}$ when they are travelling in the same direction.

(b) $t = \frac{2t_1t_2}{t_2 + t_1}$ when they are travelling in the opposite direction.

EXAMPLE 1 How long does a train 120 metres long running at the rate of 36 km/h take to cross a bridge 144 metres in length?

Solution In crossing the bridge, it must travel its own length plus the length of the bridge.

$$\therefore \text{Required time} = \frac{(120 + 144)\text{m}}{36 \text{ km/h}}$$

But $1 \text{ km/h} = \frac{5}{18} \text{ m/s}$

$$\therefore \text{Required time} = \frac{264}{36 \times \frac{5}{18}} = \mathbf{26.4 \text{ seconds}}$$

EXAMPLE 2 Two trains 141 metres and 119 metres in length respectively are running in opposite directions, one at the rate of 30 km and the other at the rate of 42 km an hour. In what time will they be completely clear of each other from the moment they meet?

Solution As the two trains are moving in opposite directions, their relative speed

$$= 42 + 30 = 72 \text{ km/h} = 72 \times \frac{5}{18} = 20 \text{ m/s}$$

$$\therefore \text{Required time} = \frac{\text{Total length}}{\text{Relative speed}} = \frac{141 + 119}{20} = \mathbf{13 \text{ seconds}}$$

EXAMPLE 3 A train 120 metres in length travels at 66 km/h. In what time will it pass a man who is walking at 6 km/h

- (i) against it, (ii) in the same direction?

Solution

(i) Relative speed = $66 + 6 = 72 \text{ km/h} = 72 \times \frac{5}{18} = 20 \text{ m/s}$

$$\text{Required time} = \frac{\text{Length of train}}{\text{Relative speed}} = \frac{120}{20} = \mathbf{6 \text{ seconds}}$$

$$(ii) \quad \text{Relative speed} = 66 - 6 = 60 \text{ km/h} = 60 \times \frac{5}{18} = \frac{50}{3} \text{ m/s}$$

$$\therefore \text{Required time} = \frac{120}{50/3} = \frac{120 \times 3}{50} = \mathbf{7.2 \text{ seconds}}$$

EXAMPLE 4 Two trains of the same length but with different speeds pass a static pole in 4 seconds and 5 seconds respectively. In what time will they cross each other when they are moving in

- (i) the same direction?
- (ii) the opposite direction?

Solution Let the length of train be l metres.

$\therefore$ The speeds of trains are $l/4$ m/s and $l/5$ m/s.

Total distance to be travelled = $2l$ metres.

$$(i) \quad \text{Relative speed} = \frac{l}{4} - \frac{l}{5} = \frac{l}{20} \text{ m/s}$$

$$\therefore \text{Required time} = 2l \div \frac{l}{20} = 2l \times \frac{20}{l} = \mathbf{40 \text{ seconds}}$$

$$(ii) \quad \text{Relative speed} = \frac{l}{4} + \frac{l}{5} = \frac{9l}{20} \text{ m/s}$$

$$\therefore \text{Required time} = 2l \div \frac{9l}{20} = 2l \times \frac{20}{9l} = \frac{40}{9} = \mathbf{4\frac{4}{9} \text{ seconds}}$$



Short cut:

$$1. \text{ Time} = \frac{2(\text{Product of time taken})}{\text{Difference of time taken}} = \frac{2(4 \times 5)}{(5 - 4)} = \mathbf{40 \text{ seconds}}$$

$$2. \text{ Time} = \frac{2(\text{Product of time taken})}{(\text{Sum of time taken})} = \frac{2(4 \times 5)}{(4 + 5)} = \frac{40}{9} \text{ seconds} = \mathbf{4\frac{4}{9} \text{ seconds}}$$



Note: If the trains are of different lengths and moving with the same speed:

1. When they are moving in the same direction, they can't pass each other.
2. When they are moving in the opposite direction, time taken to pass each other

$$= \frac{\text{Total length}}{\text{Speed}} = \frac{4 + 5}{2} = \mathbf{4.5 \text{ seconds}}$$



Short cut: Required time = Average of two times.

EXAMPLE 5 Two trains of length 100 m and 80 m respectively run on parallel tracks. When running in the same direction the faster train passes the slower one in 18 seconds, but when they are running in opposite directions with the same speeds as earlier, they pass each other in 9 seconds. Find the speed of each train.

Solution Let the speed of trains be S_1 and S_2 m/s.

When they are moving in the same direction, the relative speed

$$S_1 - S_2 = \frac{100 + 80}{18} = 10 \text{ m/s}$$

Similarly, when they are in the opposite direction, the relative speed

$$S_1 + S_2 = \frac{100 + 80}{9} = 20 \text{ m/s}$$

Solving, we get $S_1 = 15 \text{ m/s}$; $S_2 = 5 \text{ m/s}$.



Short cut: Speed of faster/slower train = Average length of trains

$$\times \left[\frac{1}{\text{Opposite direction's time}} \pm \frac{1}{\text{Same direction's time}} \right]$$

$$\text{Speed of faster train} = \frac{l_1 + l_2}{2} \left(\frac{t_1 + t_2}{t_1 \times t_2} \right) = \frac{100 + 80}{2} \left(\frac{18 + 9}{18 \times 9} \right) = \frac{90 \times 27}{18 \times 9} = \mathbf{15 \text{ m/s}}$$

$$\text{Speed of slower train} = \frac{100 + 80}{2} \left(\frac{18 - 9}{18 \times 9} \right) = \frac{90 \times 9}{18 \times 9} = \mathbf{5 \text{ m/s}}$$

EXAMPLE 6 A train overtakes two persons who are walking in the same direction as the train is moving, at the rate of 2 km/h and 4 km/h respectively and passes them completely in 9 and 10 seconds respectively. Find the speed and length of the train.

Solution Speeds of men are 2 km/h and 4 km/h

$$\text{i.e.} \quad 2 \times \frac{5}{18} = \frac{5}{9} \text{ m/s and } 4 \times \frac{5}{18} = \frac{10}{9} \text{ m/s}$$

Let the speed of train be x m/s

$\therefore$ Length of train = relative speed $\times$ time taken to pass a man

$$= \left(x - \frac{5}{9} \right) \times 9 = \left(x - \frac{10}{9} \right) 10 \quad (1)$$

$$\therefore \quad x = \frac{100}{9} - \frac{45}{9} = \frac{55}{9} \text{ m/s}$$

$$= \frac{55}{9} \times \frac{18}{5} \text{ km/h} = \mathbf{22 \text{ km/h}}$$

$$\text{Length of train} = \left(x - \frac{5}{9} \right) 9 = \left(\frac{55}{9} - \frac{5}{9} \right) 9 = \mathbf{50 \text{ m}}$$



Short cut: Length of train = $\frac{(\text{Difference in speeds of two men}) \times T_1 \times T_2}{T_2 - T_1}$

$$= \left(\frac{10}{9} - \frac{5}{9} \right) \times \frac{9 \times 10}{10 - 9} = \frac{5}{9} \times 9 \times 10 = \mathbf{50 \text{ m}}$$

EXAMPLE 7 A train passes a pole in 15 seconds and passes a platform 100 m long in 25 seconds. Find the length of train.

Solution Let the length of train be l metres.

$$\therefore \frac{l}{15} = \frac{l + 100}{25}$$

$$\text{or} \quad 25l = 15l + 1500$$

$$\therefore l = \frac{1500}{(25 - 15)} = \mathbf{150 \text{ m}}$$

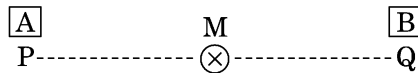


Short cut: Length of train = $\frac{(\text{Time to pass a pole} \times \text{Length of platform})}{(\text{Difference in time to cross a pole and platform})}$

$$= \frac{15 \times 100}{25 - 15} = \mathbf{150 \text{ m}}$$

EXAMPLE 8 Two trains A and B start from two places P and Q towards Q and P respectively. After passing each other they take 4 hours 48 minutes and 3 hours and 20 minutes to reach Q and P respectively. If the train from P is moving at 45 km/h then find the speed of other train.

Solution Let the speed of train B be x km/h and they meet at M.



$$\therefore 45 \text{ km/h} \times (4 \text{ hours} + 48 \text{ minutes})$$

$$\therefore MQ = 45 \times \left[4 \frac{4}{5} \right] = 45 \times \frac{24}{5} = \mathbf{216 \text{ km}}$$

$$\text{Distance MP} = x \times (3 \text{ hours} + 20 \text{ minutes})$$

$$= x \times 3 \frac{1}{3} = x \times \frac{10}{3} = \frac{10x}{3} \text{ km}$$

$\therefore$ Time to reach the train from Q to M

$$= \frac{QM}{x} = \frac{PM}{45}$$

$$\text{or} \quad \frac{216}{x} = \frac{10x/3}{45}$$

$$\begin{aligned}
 \text{or} \quad & 10x^2 = 216 \times 3 \times 45 \\
 \text{or} \quad & x^2 = 108 \times 3 \times 9 \\
 \therefore & x = 9 \times 2 \times 3 = \mathbf{54 \text{ km/h}}
 \end{aligned}$$



Short cut:

$$\text{Speed of second train} = \text{Speed of first train} \times \frac{\sqrt{\text{Time taken by first train after meeting}}}{\sqrt{\text{Time taken by second train after meeting}}}$$

$$\begin{aligned}
 &= 45 \times \frac{\sqrt{4\frac{4}{5}}}{\sqrt{3\frac{1}{3}}} = 45 \times \frac{\sqrt{24/5}}{\sqrt{10/3}} = 45 \times \sqrt{\frac{24}{5} \times \frac{3}{10}} = 45 \times \frac{6}{5} = \mathbf{54 \text{ km/h}}
 \end{aligned}$$

EXAMPLE 9 Two places P and Q are 162 km apart. A train leaves P for Q and at the same time another train leaves Q for P. Both the trains meet 6 hours after they start moving. If the train travelling from P to Q travels 8 km/h faster than the other train, find the speed of the two trains.

Solution Let the speeds of trains be S_1 km/h and S_2 km/h

$$\therefore S_1 + S_2 = \frac{162}{6} = 27 \text{ km/h} \quad (1)$$

$$S_1 - S_2 = 8 \text{ km/h} \quad (2)$$

Solving (1) and (2), we get,

$$S_1 = \frac{27 + 8}{2} = \mathbf{17.5 \text{ km/h}}$$

and

$$S_2 = \frac{27 - 8}{2} = \mathbf{9.5 \text{ km/h}}$$



Short cut: Speed of faster/slower train =

$$\frac{\text{Distance between the places} \pm \text{time taken to meet} \times \text{difference in speeds of trains}}{2 \times \text{time taken to meet}}$$

$$\text{Speeds of the trains} = \frac{162 + 6 \times 8}{2 \times 6} \text{ and } \frac{162 - 6 \times 8}{2 \times 6}$$

i.e. **17.5 km/h** and **9.5 km/h**

EXAMPLE 10 The speeds of two trains are in the ratio $x : y$. They are moving in the opposite direction on parallel tracks. The first train crosses a telegraph pole in a seconds whereas the second train crosses a telegraph pole in b seconds. Find the time taken by the trains to cross each other completely.

Solution Let the speeds of trains be kx and ky metres/second.

$\therefore$ Length of first train = $kx \times a = kax$ metres

Length of second train = $ky \times b = kby$ metres

$$\text{Time taken to cross each other} = \frac{\text{Sum of lengths}}{\text{Relative speed}} = \frac{kax + kby}{kx + ky} = \frac{ax + by}{x + y} \text{ seconds}$$

EXAMPLE 11 How many seconds will a train 100 metres long running at the rate of 36 km an hour take to pass a telegraph post?

Solution To pass the telegraph post, the train should cover its own length.

$$\therefore \quad \text{Speed of train} = 36 \text{ km/h} = 36 \times \frac{5}{18} = 10 \text{ m/s}$$

$$\text{Required time} = \frac{\text{Length of train}}{\text{Speed}} = \frac{100}{10} = \mathbf{10 \text{ seconds}}$$

EXAMPLE 12 How long does a train 180 metres long running at the rate of 36 km/h take to cross a bridge 100 metres in length?

Solution In crossing the bridge, the train must travel its own length plus the length of the bridge.

$$\text{Speed of train} = 36 \text{ km/h} = 36 \times \frac{5}{18} = 10 \text{ m/s}$$

$$\text{Required time} = \frac{180 + 100}{10} = \mathbf{28 \text{ seconds}}$$

EXAMPLE 13 Two trains are moving in the same direction at 50 km/h and 30 km/h. The faster train crosses a man in the slower train in 18 seconds. Find the length of the faster train.

Solution Relative speed = $(50 - 30)$ km/h

$$= 20 \times \frac{5}{18} = \frac{50}{9} \text{ m/s}$$

$$\text{Distance covered in 18 seconds at this speed} = \frac{50}{9} \times 18 = \mathbf{100 \text{ m}}$$

$\therefore$ Length of faster train = **100 m**

EXAMPLE 14 A train travelling at 36 km/h took 15 seconds in passing a standing man. Find the length of the train.

$$\text{Speed of train} = 36 \text{ km/h} = 36 \times \frac{5}{18} = \mathbf{10 \text{ m/s}}$$

Time taken to pass the man = 15 seconds.

$$\therefore \quad \text{Length of train} = 10 \times 15 = \mathbf{150 \text{ m}}$$

**EXERCISES**

1. How many seconds will a train 60 m in length, travelling at the rate of 42 km/h, take to pass another train 84 m long, proceeding in the same direction at the rate of 30 km/h?
(a) 20 seconds (b) 30 seconds (c) 40 seconds (d) 50 seconds (e) None of these
2. Two trains running at the rate of 45 km and 36 km an hour respectively, on parallel tracks in opposite directions; are observed to pass each other in 8 seconds, and when they are running in the same direction at the same rate as before, a person sitting in the faster train observes that he passes the other in 30 seconds. Find the length of trains.
(a) 70 m, 100 m (b) 65 m, 95 m (c) 75 m, 105 m (d) 80 m, 110 m (e) None of these
3. Two stations P and Q are 110 km apart on a straight line. One train starts from P at 7 am and travels towards Q at 20 km/h speed. Another train starts from Q at 8 am and travels towards P at a speed of 25 km/h. At what time will they meet?
(a) 9 am (b) 10 am (c) 10.30 am (d) 11 am (e) None of these
4. A train 75 metres long overtook a person who was walking at the rate of 6 km/h and passed him in $7\frac{1}{2}$ seconds. Subsequently, it overtook a second person, and passed him in $6\frac{3}{4}$ seconds. At what rate was the second person travelling?
(a) 1 km/h (b) 2 km/h (c) 3 km/h (d) 4 km/h (e) None of these
5. A train covers a distance between two stations A and B in 45 minutes. If the speed is reduced by 5 km/h, it will cover the same distance in 48 minutes. What is the distance between the two stations A and B (in km)? Also find the speed of the train.
(a) 80 km, 60 km/h (b) 90 km, 70 km/h (c) 60 km, 80 km/h
(d) 70 km, 70 km/h (e) None of these
6. Two trains are running at 40 km/hr and 20 km/hr respectively in the same direction. Faster train completely passes a man in the slower train in 5 seconds. What is the length of the faster train? **[RRB]**
(a) 23 m (b) $23\frac{2}{9}$ m (c) 27 m (d) $27\frac{7}{9}$ m
7. How long does a train 110 metres long running at the speed of 72 km/hr take to cross a bridge 132 metres in length? **[RRB]**
(a) 9.8 sec (b) 12.1 sec (c) 12.42 sec (d) 14.3 sec
8. A train running at the speed of 60 km/hr crosses a pole in 9 seconds. What is the length of the train? **[Bank PO]**
(a) 120 metres (b) 180 metres (c) 324 metres (d) Cannot be determined
(e) None of these
9. A train covers a distance of 12 km in 10 minutes. If it takes 6 seconds to pass a telegraph post, then the length of the train is **[Bank PO]**
(a) 90 m (b) 100 m (c) 120 m (d) 140 m
10. A 300 m long train crosses a platform in 39 seconds, while it crosses a signal pole in 18 seconds. What is the length of the platform? **[Bank PO]**
(a) 320 m (b) 350 m (c) 650 m
(d) Data inadequate (e) None of these

11. Two trains are moving in opposite directions at 60 km/hr and 90 km/hr respectively. Their lengths are 1.1 km and 0.9 km respectively. The time taken by the slower train to cross the faster train in seconds is: **[MBA]**
(a) 36 (b) 45 (c) 48 (d) 49
12. A train 110 metres long is running with a speed of 60 km/hr. In what time will it pass a man who is running at 6 km/hr in the direction opposite to that in which the train is going? **[MAT]**
(a) 5 sec (b) 6 sec (c) 7 sec (d) 10 sec
13. Two goods trains, each 500 m long, are running in opposite directions on parallel tracks. Their speeds are 45 km/hr and 30 km/hr respectively. Find the time taken by the slower train to pass the driver of the faster one. **[MAT]**
(a) 12 sec (b) 24 sec (c) 48 sec (d) 60 sec
14. Two trains of equal lengths take 10 seconds and 15 seconds respectively to cross a telegraph post. If the length of each train be 120 metres, in what time will they cross each other travelling in opposite direction? **[SSC]**
(a) 10 sec (b) 12 sec (c) 15 sec (d) 20 sec
15. How many seconds will a 500 metre long train take to cross a man walking with a speed of 3 km/hr in the direction of the moving train if the speed of the train is 63 km/hr? **[SSC]**
(a) 25 (b) 30 (c) 40 (d) 45
16. Two trains 140 and 160 metres long run at a speed of 60 km/hr and 40 km/hr respectively in opposite directions on parallel tracks. The time which they take to cross each other, is **[SSC]**
(a) 9 sec (b) 9.6 sec (c) 10 sec (d) 10.8 sec
17. Two trains are running in opposite directions with the same speed. If the length of each train is 120 metres, and they cross each other in 12 seconds, then the speed of each train is: **[SSC]**
(a) 10 km/hr (b) 18 km/hr (c) 36 km/hr (d) 72 km/hr
18. A train moves past a telegraph post and a bridge 264 m long in 8 seconds and 20 seconds respectively. What is the speed of the train? **[SSC]**
(a) 69.5 km/hr (b) 70 km/hr (c) 79 km/hr (d) 79.2 km/hr
19. A goods train runs at the speed of 72 kmph and crosses a 250 m long platform in 26 seconds. What is the length of the goods train? **[Bank PO]**
(a) 230 m (b) 240 m (c) 260 m (d) 270 m
20. A train 360 m long is running at a speed of 45 km/hr. In what time will it pass a bridge 140 m long? **[BSF]**
(a) 40 sec (b) 42 sec (c) 45 sec (d) 48 sec
21. Two trains of equal length are running on parallel lines in the same direction at 46 km/hr and 36 km/hr respectively. The faster train passes the slower train in 36 seconds. The length of each train is **[MAT]**
(a) 50 m (b) 72 m (c) 80 m (d) 82 m
22. A 270 m long train running at a speed of 120 km/hr crosses another train running in opposite direction at the speed of 80 km/hr in 9 seconds. What is the length of the other train? **[SBI PO]**
(a) 230 m (b) 240 m (c) 260 m (d) 320 m (e) None of these
23. Two trains, each 100 m long, moving in opposite directions, cross each other in 8 seconds. If one is moving twice as fast as the other, then the speed of the faster train is: **[CDS]**
(a) 30 km/hr (b) 45 km/hr (c) 60 km/hr (d) 75 km/hr

24. Two trains, one from Howrah to Patna and the other from Patna to Howrah, start simultaneously. After they meet, the trains reach their destinations after 9 hours and 16 hours respectively. The ratio of their speeds is [RRB]
(a) 2 : 3 (b) 4 : 3 (c) 6 : 7 (d) 9 : 16
25. A train 125 m long passes a man, running at 5 km/hr in the same direction in which the train is going, in 10 seconds. The speed of the train is [AAO]
(a) 45 km/hr (b) 50 km/hr (c) 54 km/hr (d) 55 km/hr
26. A train speeds past a pole in 15 seconds and a platform 100 m long in 25 seconds. Its length is [RRB]
(a) 50 m (b) 150 m (c) 200 m (d) Data Inadequate
27. A train 800 metres long is running at a speed of 78 km/hr. If it crosses a tunnel in 1 minute, then the length of tunnel is: [SSC]
(a) 130 m (b) 360 m (c) 500 m (d) 540 m
28. A train 240 m long passed a pole in 24 seconds. How long will it take to pass a platform 650 m long? [RRB]
(a) 65 sec (b) 89 sec (c) 100 sec (d) 150 sec
29. In what time will a train 100 m long cross an electric pole, if its speed be 144 km/hr? [SSC]
(a) 2.5 sec (b) 4.25 sec (c) 5 sec (d) 12.5 sec
30. A train 280 m long, running with a speed of 63 km/hr will pass a tree in [SSC]
(a) 15 sec (b) 16 sec (c) 18 sec (d) 20 sec
31. Two trains running in opposite directions cross a man standing on the platform in 27 seconds and 17 seconds respectively, and they cross each other in 23 seconds. The ratio of their speeds is [Hotel Management]
(a) 1 : 3 (b) 3 : 2 (c) 3 : 4 (d) None of these
32. A jogger running at 9 km/hr alongside a railway track is 240 m ahead of the engine of a 120 m long train running at 45 km/hr in the same direction. In how much time will the train pass the jogger? [IGNOU]
(a) 3.6 sec (b) 18 sec (c) 36 sec (d) 72 sec
33. A train passes a station platform in 36 seconds and a man standing on the platform in 20 seconds. If the speed of the train is 54 km/hr, what is the length of the platform?
(a) 120 m (b) 240 m (c) 300 m (d) None of these
34. The length of the bridge, which a train 130 metres long and travelling at 45 km/hr can cross in 30 seconds, is [Section Officer's]
(a) 200 m (b) 225 m (c) 245 m (d) 250 m
35. Two trains 100 metres and 120 metres long are running in the same direction with speeds of 72 km/hr and 54 km/hr. In how much time will the faster train cross the second? [CBI]
(a) 40 sec (b) 42 sec (c) 44 sec (d) 46 sec
36. A train 100 m long is running at the speed of 30 km/hr. Find the time taken by it to pass a man standing near the railway line.
(a) 10 sec (b) 12 sec (c) 14 sec (d) 16 sec
37. A train is moving at a speed of 120 km/hr. If the length of train is 120 m, how long will it take to cross a railway platform 130 metres long?
(a) 7 sec (b) $7\frac{1}{2}$ sec (c) 8 sec (d) $8\frac{1}{2}$ sec

38. Length of a train and that of a platform are equal. If with a speed of 135 km/hr, the train crosses the platform in 40 seconds, then the length of train is
 (a) 500 m (b) 600 m (c) 750 m (d) 900 m
39. Two trains 250 m and 200 m long are running on parallel track at the rate of 36 km/hr and 45 km/hr respectively. In how much time will they cross each other, if they are running in the same direction?
 (a) 72 sec (b) 144 sec (c) 180 sec (d) 252 sec
40. A train 120 m long moving at a speed of 60 km/hr crosses a train 130 m long coming from opposite direction in 6 seconds. The speed of the second train is
 (a) 72 km/hr (b) 78 km/hr (c) 84 km/hr (d) 90 km/hr



1. (e) 2. (c) 3. (b) 4. (b) 5. (c) 6. (d) 7. (b) 8. (e) 9. (c) 10. (b)
 11. (c) 12. (b) 13. (b) 14. (b) 15. (b) 16. (d) 17. (c) 18. (d) 19. (d) 20. (a)
 21. (a) 22. (a) 23. (c) 24. (b) 25. (b) 26. (b) 27. (c) 28. (b) 29. (a) 30. (b)
 31. (b) 32. (c) 33. (b) 34. (c) 35. (c) 36. (b) 37. (b) 38. (c) 39. (c) 40. (d)



Solutions with Necessary Explanation

1. Relative speed = $42 - 30 = 12$ km/h

$$= 12 \times \frac{5}{18} = \frac{10}{3} \text{ m/s}$$

$$\begin{aligned} \text{Time taken by the train to pass another train} &= \frac{\text{Distance travelled}}{\text{Speed}} \\ &= \frac{(60 + 84) \text{ m}}{10/3 \text{ m/s}} \\ &= \frac{432}{10} = 43.2 \text{ seconds} \end{aligned}$$

2. Relative speed of two trains = $45 + 36 = 81$ km/h

$$= 81 \times \frac{5}{18} = \frac{45}{2} = 22 \frac{1}{2} \text{ m/s}$$

$$\therefore \text{Length of both the trains} = \frac{45}{2} \times 8 = 180 \text{ m}$$

When two trains are moving in the same directions, relative speed = $45 - 36 = 9$ km/h

$$= 9 \times \frac{5}{18} = \frac{5}{2} \text{ m/s}$$

$$\text{Length of slower train} = \frac{5}{2} \times 30 = 75 \text{ m}$$

$$\therefore \text{Length of faster train} = 180 - 75 = 105 \text{ m}$$



Short cut: Length of slower train = (Relative speed of two train) $\times$ 30

$$= (45 - 36) \times \frac{5}{18} \times 30 = 75 \text{ m}$$

Length of faster train = Total length of both train – Length of slower train

$$= (45 + 36) \times \frac{5}{18} \times 8 - 75$$

$$= 180 - 75 = 105 \text{ m}$$

3. The train starts from P and covers a distance of 20 km till 8 am.

Remaining distance = $110 - 20 = 90$ km is covered by the trains with relative speed of $20 + 25 = 45$ km/h

$$\therefore \text{They meet after } \frac{90}{45} = 2 \text{ h}$$

i.e. at $8 + 2 = 10$ h

i.e. at **10 am**

4. Relative speed of train and first person = $\frac{75}{15/2} = 10 \text{ m/s} = 10 \times \frac{18}{5} = 36 \text{ km/h}$

$$\therefore \text{Speed of train} = 36 + 6 = 42 \text{ km/h}$$

$$\text{Relative speed of train and second person} = \frac{75}{27/4} = \frac{300}{27} \times \frac{18}{5} = 40 \text{ km/h}$$

$$\therefore \text{Speed of second person} = 42 - 40 = 2 \text{ km/h}$$

5. Let the distance between two stations A and B be x km

Let the speed of train be y km/h

$$\therefore \frac{x}{y} = \frac{45}{60} = \frac{3}{4} \Rightarrow x = \frac{3}{4}y \quad (1)$$

When speed is reduced by 5 km/h

$$\frac{x}{y-5} = \frac{48}{60} = \frac{4}{5} \Rightarrow x = \frac{4}{5}(y-5) \quad (2)$$

Solving (1) and (2),

$$\frac{3}{4}y = \frac{4}{5}(y-5)$$

$$\text{or } y \left(\frac{4}{5} - \frac{3}{4} \right) = \frac{4}{5} \times 5 = 4$$

$$\therefore y \times \frac{16-15}{20} = 4$$

$$\text{or } y = 4 \times 20 = 80 \text{ km/h}$$

$\therefore$ Speed of train = **80 km/h**

$$\text{Distance between two stations} = \frac{3}{4} \times 80 = 60 \text{ km}$$



Short cut: Speed of train = $\frac{48}{48 - 45} \times 5 = 80 \text{ km/h}$

$$\text{Distance} = \text{Speed} \times \text{time} = \left[\frac{48}{48 - 45} \right] \times 5 \times \frac{45}{60} = 60 \text{ km}$$

6. Relative speed of faster train w.r.t. slower train = $40 - 20 = 20 \text{ km/hr} = 20 \times \frac{5}{18} \text{ m/s} = \frac{50}{9} \text{ m/s}$

Let length of train be x metres.

$$\therefore \frac{x}{5} = \frac{50}{9}$$

$$\text{or } x = \frac{250}{9} = 27\frac{7}{9} \text{ m}$$

7. Speed of train = $72 \text{ km/hr} = 72 \times \frac{5}{18} \text{ m/s} = 20 \text{ m/s}$

Let the time taken by train to cross bridge be ' t ' seconds.

$$\therefore \frac{110 + 132}{20} = t$$

$$\text{i.e. } \frac{242}{20} = t$$

$$\text{or } t = 12.1 \text{ sec}$$

8. Let length of train be x metres.

$$\therefore x = \left(60 \times \frac{5}{18} \right) \times 9$$

$$= 150 \text{ metres}$$

9. Let length of train be x metres.

$$\text{Speed of train} = \frac{12 \text{ km}}{10 \text{ min}} = \frac{12}{1/6} \text{ km/hr} = 72 \text{ km/hr}$$

$$= 72 \times \frac{5}{18} \text{ m/s} = 20 \text{ m/s}$$

$$\therefore x = 20 \times 6 = 120 \text{ m}$$

10. Let length of platform be x metres.

$$\text{Speed of train} = \frac{300 + x}{39} = \frac{300}{18}$$

$$\text{i.e. } 300 + x = \frac{300 \times 39}{18}$$

$$\text{or } x = 300 \times \frac{39}{18} - 300$$

$$= 300 \times \frac{21}{18} = 50 \times 7 = 350 \text{ m}$$

11. Let time taken by slower train to cross faster train be t seconds

Relative speed = $60 + 90 = 150$ km/hr

Total length of train = $1.1 + 0.9 = 2$ km.

$$\begin{aligned}\therefore \text{ Time taken to cross each other } t &= \frac{\text{Length of train}}{\text{Relative speed}} \\ &= \frac{2 \text{ km}}{150 \text{ km/hr}} = \frac{1}{75} \text{ hour} \\ &= \frac{60 \times 60}{75} \text{ sec} = 12 \times 4 = \mathbf{48 \text{ sec}}\end{aligned}$$

12. Let the time taken by train to cross a running man be t sec.

Relative speed of train w.r.t. man

$$\begin{aligned}&= 60 + 6 = 66 \text{ km/hr} = 66 \times \frac{5}{18} \text{ m/s} \\ &= \frac{55}{3} \text{ m/s}\end{aligned}$$

$$\begin{aligned}\therefore \text{ Time taken to cross running man, } t &= \frac{\text{Length of train}}{\text{Speed}} \\ &= \frac{110}{55/3} = \mathbf{6 \text{ sec}}\end{aligned}$$

13. Relative speed of train = $45 + 30 = 75$ km/hr = $75 \times \frac{5}{18} = \frac{125}{6}$ m/s

Let the time taken by slower train to pass the driver of faster train be t seconds.

$$\begin{aligned}\therefore t &= \frac{\text{Length of slower train}}{\text{Relative speed of train}} \\ &= \frac{500 \text{ m}}{\frac{125}{6} \text{ m/s}} = \mathbf{24 \text{ sec}}\end{aligned}$$

14. Speed of first train = $\frac{120}{10} = 12$ m/s

$$\text{Speed of second train} = \frac{120}{15} = 8 \text{ m/s}$$

Let t sec be the time in which they cross each other.

$$\begin{aligned}\therefore t &= \frac{\text{Total length of trains}}{\text{Relative speed of trains}} \\ &= \frac{120 + 120}{(12 + 8)} = \frac{240}{20} = \mathbf{12 \text{ sec}}\end{aligned}$$

15. Let the time taken by the train to cross a walking man be t seconds.

$$\text{Relative speed of train w.r.t. man} = (63 - 3) = 60 \text{ km/hr} = 60 \times \frac{5}{18} \text{ m/s} = \frac{50}{3} \text{ m/s}$$

Length of train = 500 m

$$\therefore t = \frac{500}{50/3} = \frac{500 \times 3}{50} = \mathbf{30 \text{ sec}}$$

16. Relative speed of train, $v_r = 60 + 40 = 100$ km/hr

$$= 100 \times \frac{5}{18} \text{ m/s}$$

$$= \frac{250}{9} \text{ m/s}$$

Length of trains, $L = 140 + 160 = 300$ m.

Time taken by the trains to cross each other

$$\frac{L}{v_r} = \frac{300}{250/9} = \frac{300 \times 9}{250} = \frac{6 \times 9}{5} = \mathbf{10.8 \text{ sec}}$$

17. Let speed of trains be v m/s.

$\therefore$ Relative speed of trains $v_r = 2v$ m/s

Time taken to cross each other = 12 sec.

Length of trains = $120 + 120 = 240$ m.

$$\therefore \frac{L}{v_r} = t$$

$$\text{or } v_r = \frac{L}{t} = \frac{240}{12} = 20 \text{ m/s}$$

$$\therefore v = \frac{v_r}{2} = 10 \text{ m/s} = 10 \times \frac{18}{5} = \mathbf{36 \text{ km/hr}}$$

18. Let speed of train be v km/hr. Let length of train be L m.

$$\therefore \frac{L}{8} = \frac{L + 264}{20} = v \times \frac{5}{18}$$

$$\text{i.e. } \frac{L}{v \times \frac{5}{18}} = 8 \Rightarrow L = v \times \frac{5 \times 8}{18} \quad (1)$$

$$\text{similarly } \frac{L + 264}{20} = v \times \frac{5}{18} \Rightarrow L + 264 = v \times \frac{5}{18} \times 20 \quad (2)$$

$$(2) - (1) \Rightarrow 264 = v \times \frac{5}{18} (20 - 8) = v \times \frac{5}{18} \times 12$$

$$\text{or } v = 264 \times \frac{18}{60} = \mathbf{79.2 \text{ km/hr}}$$

19. Speed of train = $72 \text{ km/hr} = 72 \times \frac{5}{18} \text{ m/s} = 20 \text{ m/s}$

Let length of goods train be x metre.

$$\therefore \frac{x + 250}{26} = 20$$

$$\text{i.e. } x + 250 = 20 \times 26 = 520$$

$$\therefore x = 520 - 250 \\ = \mathbf{270 \text{ m}}$$

20. Speed of train = 45 km/hr = $45 \times \frac{5}{18} = 12.5$ m/s

Length of bridge = 140 m.

Let time taken by train to pass bridge be t seconds.

$$\therefore \frac{360 + 140}{t} = 12.5$$

or $t = \frac{500}{12.5} = \mathbf{40 \text{ seconds}}$

21. Let length of each train be l metres

$\therefore$ Total length of trains = $l + l = 2l$.

Relative speed of faster train w.r.t. slower train

$$= 46 - 36 = 10 \text{ km/hr} = 10 \times \frac{5}{18} \text{ m/s} = \frac{25}{9} \text{ m/s}$$

$$\therefore \text{Relative speed} = \frac{\text{Total length of trains}}{\text{Time taken to pass the train}}$$

i.e. $\frac{25}{9} = \frac{2l}{36}$

or $l = \frac{25 \times 36}{18} = \mathbf{50 \text{ m}}$

22. Let the length of second train be l metres.

Relative speed of train = $120 + 80 = 200$ km/hr

$$= 200 \times \frac{5}{18} \text{ m/s} = \frac{500}{9} \text{ m/s}$$

$$\therefore \frac{270 + l}{9} = \frac{500}{9}$$

or $270 + l = 500$

or $l = 500 - 270 = \mathbf{230 \text{ m}}$

23. Let speed of slower train be $x/2$ and that of faster train be x metres/second.

$$\therefore \frac{100 + 100}{8} = x + \frac{x}{2} = \frac{3x}{2}$$

or $x = \frac{200}{8} \times \frac{2}{3} = \frac{50}{3} \text{ m/s}$

$$= \frac{50}{3} \times \frac{18}{5} \text{ km/hr} = \mathbf{60 \text{ km/hr}}$$

24. Ratio of their speeds = inverse ratio of square root of time taken to reach destination

i.e. $\frac{v_1}{v_2} = \sqrt{\frac{t_2}{t_1}} = \sqrt{\frac{16}{9}} = \frac{4}{3}$

25. Let speed of train be x km/hr

$$\therefore (x - 5) \times \frac{5}{18} \text{ m/s} = \frac{125}{10} = 12.5$$

$$\therefore x - 5 = \frac{12.5 \times 18}{5} = 45$$

$$\therefore x = 45 + 5 = \mathbf{50 \text{ km/hr}}$$

26. Let length of train be l metres.

$$\therefore \frac{l}{15} = \frac{l+100}{25}$$

$$\text{i.e.} \quad \frac{l}{3} = \frac{l+100}{5}$$

$$\text{or} \quad 5l = 3l + 300$$

$$\text{or} \quad l = \frac{300}{(5-3)} = \mathbf{150 \text{ m}}$$

27. Let length of tunnel be x metres.

$$\text{Speed of train} = 78 \text{ km/hr} = 78 \times \frac{5}{18} \text{ m/s} = \frac{65}{3} \text{ m/s}$$

$$\therefore \frac{800+x}{60} = \frac{65}{3}$$

$$\text{or} \quad (800+x) = \frac{60}{3} \times 65$$

$$800+x = 1300$$

$$\text{or} \quad x = 1300 - 800 = \mathbf{500 \text{ m}}$$

28. Let the time taken by the train to pass a platform be t seconds.

$$\therefore \frac{240+650}{t} = \frac{240}{24} = 10$$

$$\text{or} \quad t = \frac{240+650}{10} = \mathbf{89 \text{ sec}}$$

29. Let time taken to cross a pole be t seconds.

$$\therefore \frac{100}{t} = 144 \times \frac{5}{18} \text{ m/s} = 40$$

$$\text{or} \quad t = \frac{100}{40} = \mathbf{2.5 \text{ sec}}$$

30. Time taken by the train to pass a tree

$$= \frac{\text{Length of train}}{\text{Speed of train}} = \frac{280 \text{ m}}{63 \times \frac{5}{18} \text{ m/s}} = \frac{280 \times 18}{63 \times 5} = 8 \times 2 = \mathbf{16 \text{ seconds}}$$

31. Let the speeds of first and second trains be x m/s and y m/s respectively

$$\therefore \text{Length of first train} = x \times 27 = 27x \text{ metres.}$$

$$\text{Length of second train} = y \times 17 = 17y \text{ metres.}$$

$$\text{Time taken to cross each other} = \frac{27x+17y}{x+y} = 23$$

Simplifying, we get

$$27x + 17y = 23x + 23y$$

$$\text{or} \quad 4x = 6y$$

$$\text{or} \quad \frac{x}{y} = \frac{6}{4} = \frac{3}{2}$$

32. Let the time taken by train to pass jogger be t seconds

Relative speed of train w.r.t. jogger = $45 - 9 = 36$ km/hr

$$= 36 \times \frac{5}{18} \text{ m/s} = 10 \text{ m/s}$$

$$t = \frac{\text{Distance travelled by train to pass jogger}}{\text{Relative speed}}$$

$$= \frac{240 + 120 \text{ m}}{10 \text{ m/s}} = \mathbf{36 \text{ sec}}$$

33. Speed of train = $54 \text{ km/hr} = 54 \times \frac{5}{18} \text{ m/s} = 15 \text{ m/s}$.

Let length of platform be x metres and y metres that of train.

$$\therefore \frac{y}{20} = 15$$

$$\text{or} \quad y = 300 \text{ m}$$

$$\text{Similarly, speed of train} = \frac{\text{Length of (train + platform)}}{\text{Time to cross platform}}$$

$$\text{i.e.} \quad 15 = \frac{x + 300}{36}$$

$$\text{or} \quad x = 15 \times 36 - 300 = 540 - 300 = \mathbf{240 \text{ metres.}}$$

34. Let length of bridge be x metres.

$$\therefore \frac{x + 130}{30} = 45 \times \frac{5}{18} = \frac{25}{2}$$

$$\text{or} \quad x + 130 = 25 \times 15 = 375$$

$$\therefore x = 375 - 130 = \mathbf{245 \text{ m}}$$

35. Relative speed of faster train w.r.t. slower train

$$= 72 - 54 = 18 \text{ km/hr} = 18 \times \frac{5}{18} \text{ m/s} = 5 \text{ m/s}$$

$$\text{Time taken by the faster train to cross the slower train} = \frac{(100 + 120) \text{ m}}{5 \text{ m/s}} = \mathbf{44 \text{ sec}}$$

36. Speed of train = $30 \text{ km/hr} = 30 \times \frac{5}{18} \text{ m/s} = \frac{25}{3} \text{ m/s}$

$$\text{Time taken by the train to pass standing man} = \frac{\text{Length of train}}{\text{Speed of train}} = \frac{100 \text{ m}}{\frac{25}{3} \text{ m/s}}$$

$$= \frac{100 \times 3}{25} = \mathbf{12 \text{ sec}}$$

37. Speed of train = 120 km/hr = $120 \times \frac{5}{18}$ m/s = $\frac{100}{3}$ m/s

$$\begin{aligned}\text{Time taken by train to cross a platform} &= \frac{\text{Length of train} + \text{Length of platform}}{\text{Speed of train}} \\ &= \frac{120 + 130}{\frac{100}{3}} = \frac{250 \times 3}{100} = 7\frac{1}{2} \text{ sec}\end{aligned}$$

38. Speed of train = 135 km/hr = $135 \times \frac{5}{18}$ m/s

Let length of train and length of platform be l metres each

$$\text{Speed of train} = \frac{\text{Length of train} + \text{Length of platform}}{\text{Time taken by train to cross platform}}$$

i.e. $\frac{75}{2} = \frac{l+l}{40}$

or $2(2l) = 75 \times 40$

or $l = \frac{75 \times 40}{2 \times 2} = 750 \text{ metres}$

39. Relative speed of faster train w.r.t. slower train = $45 - 36 = 9$ km/hr = $9 \times \frac{5}{18}$ m/s = 2.5 m/s

$$\text{Time taken to cross each other} = \frac{\text{Sum of lengths of train}}{\text{Relative speed of train}}$$

i.e. $t = \frac{250 + 200}{2.5} = \frac{450}{2.5} = 180 \text{ sec}$

40. Speed of first train = 60 km/hr = $60 \times \frac{5}{18}$ m/s = $\frac{50}{3}$ m/s

Let the speed of second train be x m/s.

$$\text{Relative speed of trains} = \left(\frac{50}{3} + x \right) \text{ m/s}$$

$$\text{Relative speed of train} = \frac{\text{Distance travelled}}{\text{Time taken to cross train}}$$

i.e. $\frac{50}{3} + x = \frac{120 + 130}{6} = \frac{250}{6}$

i.e. $\frac{50}{3} + x = \frac{125}{3}$

or $x = \frac{125 - 50}{3} = 25 \text{ m/s}$

i.e. Speed of second train = 25 m/s

$$= 25 \times \frac{18}{5} \text{ km/hr} = 90 \text{ km/hr}$$

Boats and Streams

Let the speed of a boat or swimmer in still water be Z km/h and the speed of stream or current be S km/h.

∴ Speed of the boat while going downstream = $(Z + S)$ km/h

Speed of the boat while going upstream = $(Z - S)$ km/h

∴ Speed of boat in still water = $\frac{1}{2}(\text{Downstream speed} + \text{Upstream speed})$

Speed of the stream = $\frac{1}{2}(\text{Downstream speed} - \text{Upstream speed})$



Important Formulae

1. A man swims downstream d_1 km in t_1 hours and upstream d_2 km in t_2 hours. Then his speed in still water is $\frac{1}{2} \left(\frac{d_1 t_2 + d_2 t_1}{t_1 t_2} \right)$ km/h and speed of current is $\frac{1}{2} \left(\frac{d_1 t_2 - d_2 t_1}{t_1 t_2} \right)$ km/h.
2. A man rows a certain distance downstream in t_1 hours and returns the same distance upstream in t_2 hours. If the speed of the stream is S km/h, speed of man in still water and speed of stream will be $S \left(\frac{t_2 + t_1}{t_2 - t_1} \right)$ km/h and $S \left(\frac{t_2 - t_1}{t_2 + t_1} \right)$ km/h.
3. A man can row a boat in still water at Z km/h. In a stream flowing at S km/h, if it takes him t hours to row to a point and come back, then the distance (D) between the two points is

$$\frac{t(Z^2 - S^2)}{2Z} \text{ km}$$

4. A person can row a boat u_1 km upstream and d_1 km downstream in t_1 hours. Also, he can row u_2 km upstream and d_2 km downstream in t_2 hours.

$$\text{Then his upstream speed} = \frac{u_1 d_2 - u_2 d_1}{t_1 d_2 - t_2 d_1}$$

$$\text{His downstream speed} = \frac{u_1 d_2 - u_2 d_1}{t_2 u_1 - t_1 u_2}$$

EXAMPLE 1 A man can row upstream at 10 km/h and downstream at 16 km/h. Find the man's rate in still water and the rate of current.

Solution Man's rate in still water = $\frac{1}{2}$ (Upstream speed + Downstream speed)

$$= \frac{1}{2} (10 + 16) = \mathbf{13 \text{ km/h}}$$

$$\text{Rate of current} = \frac{1}{2} (\text{Downstream speed} - \text{Upstream speed})$$

$$= \frac{1}{2} (16 - 10) = \mathbf{3 \text{ km/h}}$$

EXAMPLE 2 A man swims downstream 30 km and upstream 18 km, taking a time of 3 hours each. What is the velocity of the current?

Solution Downstream speed = $\frac{30}{3} = 10 \text{ km/h}$

$$\text{Upstream speed} = \frac{18}{3} = 6 \text{ km/h}$$

$$\text{Velocity of current} = \frac{1}{2} (\text{Downstream speed} - \text{Upstream speed})$$

$$= \frac{1}{2} (10 - 6) = \mathbf{2 \text{ km/h}}$$

EXAMPLE 3 A man can row at a speed of 6 km/h in still water. It takes him twice as long as to row up as to row down the river. Find the rate of stream.

Solution Speed of man in still water = 6 km/h

Let the rate of stream = x km/h

$\therefore$ Upstream speed = $(6 - x)$ km/h

Downstream speed = $(6 + x)$ km/h

But $(6 + x) = 2(6 - x)$

$\Rightarrow x + 2x = 2 \times 6 - 6$

$$3x = 6$$

$\therefore x = \mathbf{2 \text{ km/h}}$

EXAMPLE 4 A man can row 50 km upstream and 60 km downstream in 15 hours. Also he can row 30 km upstream and 48 km downstream in 12 hours. Find the speed of the man in still water and the speed of the current.

Solution Let upstream speed be u km/hr and downstream speed be v km/hr

$$\therefore \frac{50}{u} + \frac{60}{v} = 15 \quad (1)$$

$$\frac{30}{u} + \frac{48}{v} = 10 \quad (2)$$

Solving (1) and (2),

$$3 \times (1) - 5 \times (2) \Rightarrow$$

$$3 \times \frac{60}{v} - 5 \times \frac{48}{v} = 3 \times 15 - 5 \times 10$$

$$\frac{180 - 240}{v} = 45 - 50$$

$$\frac{-60}{v} = -5$$

or
$$v = \frac{-60}{-5} = 12 \text{ km/hr}$$

From (2),
$$\frac{30}{u} + \frac{48}{12} = 10$$

or
$$\frac{30}{u} = 10 - 4 = 6$$

or
$$u = \frac{30}{6} = 5 \text{ km/hr}$$

$$\text{Speed of man in still water} = \frac{u+v}{2} = \frac{12+5}{2} = \frac{17}{2} \text{ km/hr}$$

$$\begin{aligned} \text{Speed of current} &= \frac{1}{2}(v-u) = \frac{1}{2}(12-5) \\ &= \frac{7}{2} \text{ km/hr} \end{aligned}$$

EXAMPLE 5 A man can row upstream at 8 km/hr and downstream at 12 km/hr. Find man's rate in still water and the rate of current.

Solution Rate in still water $= \frac{1}{2}(12+8) = 10 \text{ km/hr}$

$$\text{Rate of current} = \frac{1}{2}(12-8) = 2 \text{ km/hr}$$

EXAMPLE 6 A man can row at 24 km/hr in still water. It takes him thrice as long to row up as to row down the river. Find the rate of stream.

Solution Let man's upstream speed be x km/hr and downstream speed be $3x$ km/hr.

$$\therefore \text{Rate in still water} = \frac{1}{2}(x + 3x) = 2x \text{ km/hr}$$

$$\text{i.e.} \quad 2x = 24 \quad \text{or} \quad x = 12 \text{ km/hr}$$

$$\therefore \text{Upstream speed} = 12 \text{ km/hr} \quad \text{and}$$

$$\text{Downstream speed} = 3x = 36 \text{ km/hr}$$

$$\therefore \text{Rate of stream} = \frac{1}{2}(36 - 12) = 12 \text{ km/hr}$$



EXERCISES

- The speed of a boat in still water is 6 km/h and the speed of the stream is 1.5 km/h. A man rows to a place at a distance of 22.5 km and comes back to the starting point. Find the total time taken by him.
(a) 4 hours (b) 6 hours (c) 8 hours (d) 10 hours (e) None of these
- A person can row a certain distance downstream in 6 hours and return the same distance in 9 hours. If the stream flows at the rate of $2\frac{1}{4}$ km/h, find how far he can row in an hour in still water.
(a) $10\frac{1}{4}$ km/h (b) $11\frac{1}{4}$ km/h (c) $12\frac{1}{4}$ km/h (d) $13\frac{1}{4}$ km/h (e) None of these
- A boat travels upstream from B to A and downstream from A to B in 3 hours. If the speed of the boat in still water is 9 km/h and the speed of the current is 3 km/h, find the distance between A and B in kilometres.
(a) 10 km (b) 12 km (c) 14 km (d) 16 km (e) None of these
- The current of a stream runs at 1 km/h. A motor boat goes 35 km upstream and back again to the starting point in 12 hours. The speed of the motor boat in still water in km/h is _____.
(a) 4 (b) 6 (c) 8 (d) 12 (e) None of these
- A boat covers 24 km upstream and 36 km downstream in 6 hours, while it covers 36 km upstream and 24 km downstream in $6\frac{1}{2}$ hours. Then the velocity of the current is _____.
(a) 1 km/h (b) 1.5 km/h (c) 2 km/h (d) 2.5 km/h
- If a man's rate with the current is 12 km/h and the rate of current is 1.5 km/h, then the man's rate against the current in km/h is _____.
(a) 10 (b) 9 (c) 8 (d) 6 (e) None of these
- A man can row at 5 km/h in still water and the velocity of the current is 1 km/h. It takes him 1 hour to row to a place and back. How far is the place ?
(a) 1.2 km (b) 1.8 km (c) 2.4 km (d) 3 km (e) None of these
- The speed of a boat in still water is 6 km/h. It takes the boat twice as long to row up as to row down the river. Find the rate of the stream.
(a) 1 km/hr, 6 km/hr (b) 2 km/hr, 9 km/hr (c) 3 km/hr, 8 km/hr
(d) 4 km/hr, 6 km/hr (e) None of these

9. A man can row 30 km upstream and 44 km downstream in 10 hours. Also, he can row 40 km upstream and 55 km downstream in 13 hours. Find the rate of the current and the speed of the man in still water.
(a) 1 km/h (b) 2 km/h (c) 3 km/h (d) 4 km/h (e) None of these
10. A person can row a certain distance downstream in 6 hours and return the same distance in 9 hours. If the stream flows at the rate of 3 km/h, find the speed of the person in still water.
(a) 12 km/h (b) 15 km/h (c) 18 km/h (d) 20 km/h (e) None of these
11. A boat takes 90 minutes less to travel 36 miles downstream than to travel the same distance upstream. If the speed of the boat in still water is 10 metre/hour, the speed of the stream is [MAT]
(a) 2 m/hr (b) 2.5 m/hr (c) 3 m/hr (d) 4 m/hr
12. A motor boat, whose speed is 15 km/hr in still water goes 30 km downstream and comes back in a total of 4 hours 30 minutes. The speed of the stream (in km/hr) is [RRB]
(a) 4 (b) 5 (c) 6 (d) 10
13. A man can row at 5 km/hr in still water. If the velocity of current is 1 km/hr and it takes him 1 hour to row to a place and come back, how far is the place? [SSC]
(a) 2.4 km (b) 2.5 km (c) 3 km (d) 3.6 km
14. A boat covers a certain distance downstream in 1 hour, while it comes back in $1\frac{1}{2}$ hours. If the speed of the stream be 3 km/hr, what is the speed of the boat in still water? [Bank PO]
(a) 12 km/hr (b) 13 km/hr (c) 14 km/hr (d) 15 km/hr (e) None of these
15. A boat running upstream takes 8 hours 48 minutes to cover a certain distance, while it takes 4 hours to cover the same distance running downstream. What is the ratio between the speed of the boat and speed of the current respectively? [Bank PO]
(a) 2 : 1 (b) 3 : 2 (c) 8 : 3
(d) Cannot be determined (e) None of these
16. A boat can travel with a speed of 13 km/hr in still water. If the speed of the stream is 4 km/hr, find the time taken by the boat to go 68 km downstream. [RRB]
(a) 2 hours (b) 3 hours (c) 4 hours (d) 5 hours
17. A man can row three-quarters of a kilometre against the stream in $11\frac{1}{4}$ minutes. The speed (in km/hr) of the man in still water is [LIC AAO]
(a) 2 (b) 3 (c) 4 (d) 5
18. A man's speed with the current is 15 km/hr, and the speed of the current is 2.5 km/hr. The man's speed against the current is [MAT]
(a) 8.5 km/hr (b) 9 km/hr (c) 10 km/hr (d) 12.5 km/hr
19. A boat running downstream covers a distance of 16 km in 2 hours, while for covering the same distance upstream, it takes 4 hours. What is the speed of the boat in still water? [SBI PO]
(a) 4 km/hr (b) 6 km/hr (c) 8 km/hr (d) Data inadequate
20. In one hour, a boat goes 11 km along the stream and 5 km against the stream. The speed of the boat in still water (in km/hr) is: [SSC]
(a) 3 (b) 5 (c) 8 (d) 9
21. A man takes twice as long to row a distance against the stream as to row the same distance in favour of the stream. The ratio of the speed of the boat (in still water) and the stream is [SSC]
(a) 2 : 1 (b) 3 : 1 (c) 3 : 2 (d) 4 : 3

22. A boatman goes 2 km against the stream in 1 hour and goes 1 km along the current in 10 minutes. How long will he take to go 5 km in stationary water? [RRB]
 (a) 40 minutes (b) 1 hour
 (c) 1 hour 15 minutes (d) 1 hour 30 minutes
23. Rahul can travel 12 miles downstream in a certain river in 6 hours less than it takes him to travel the same distance upstream. But if he could double his usual rowing rate for his 24-mile round trip, the downstream 12 miles would take only one hour less than the upstream 12 miles. What is the speed of the current in miles/hour? [MAT]
 (a) $1\frac{1}{3}$ (b) $1\frac{2}{3}$ (c) $2\frac{1}{3}$ (d) $2\frac{2}{3}$
24. A man can row $7\frac{1}{2}$ km/hr in still water. If in a river running at 1.5 km/hr, it takes him 50 minutes to row to a place and back, how far off is the place? [RRB]
 (a) 2 km (b) 3 km (c) 4 km (d) 5 km
25. There is a road beside a river. Two friends started from a place A, moved to a temple situated at another place B and then returned to A again. One of them moves on a cycle at a speed of 12 km/hr, while the other sails on a boat at a speed of 10 km/hr. If the river flows at the speed of 4 km/hr, which of the two friends will return to place A first? [RRB]
 (a) cyclist (b) sailor
 (c) both cyclist and sailor (d) data inadequate



1. (c) 2. (b) 3. (b) 4. (b) 5. (c) 6. (b) 7. (c) 8. (b) 9. (c) 10. (b)
 11. (a) 12. (b) 13. (a) 14. (d) 15. (c) 16. (c) 17. (d) 18. (c) 19. (b) 20. (c)
 21. (b) 22. (c) 23. (d) 24. (b) 25. (a)



Solutions with Necessary Explanation

1. Given speed of boat in still water = 6 km/h
 Speed of stream = 1.5 km/h
 $\therefore$ Upstream speed of boat = $6 - 1.5 = 4.5$ km/h
 $\therefore$ Time taken for upstream side = $22.5/4.5 = 5$ h
 Similarly, downstream speed of boat = $6 + 1.5 = 7.5$ km/h
 $\therefore$ Time taken for downstream side = $22.5/7.5 = 3$ h
 $\therefore$ Total time taken = $5 + 3 = 8$ h



Short cut: Total time =
$$\frac{2 \times \text{Distance} \times \text{Speed in still water}}{(\text{Speed in still water})^2 - (\text{Speed of current})^2}$$

$$= \frac{2 \times 22.5 \times 6}{(6)^2 - (1.5)^2} = 8 \text{ hours}$$

2. Let the speed of person in still water be x km/h.

$$\therefore \text{Downstream speed} = x + 2\frac{1}{4} = \left(x + \frac{9}{4}\right) \text{ km/h}$$

$$\text{Upstream speed} = \left(x - \frac{9}{4}\right) \text{ km/h}$$

$$\therefore \text{Distance} = \left(x + \frac{9}{4}\right)6 = \left(x - \frac{9}{4}\right)9$$

$$\Rightarrow 6x + \frac{27}{2} = 9x - \frac{81}{4}$$

$$\Rightarrow 9x - 6x = \frac{27}{2} + \frac{81}{4} = \frac{135}{4}$$

$$\Rightarrow 3x = \frac{135}{4}$$

$$\text{or } x = \frac{45}{4} = 11\frac{1}{4} \text{ km/h}$$



Short cut:

$$\text{Speed in still water} = \text{Rate of stream} \times \left(\frac{\text{Sum of upstream and downstream time}}{\text{Difference of upstream and downstream time}} \right)$$

$$= \frac{9}{4} \times \left(\frac{9+6}{9-6} \right) = \frac{9 \times 15}{4 \times 3} = \frac{45}{4} = 11\frac{1}{4} \text{ km/h}$$

3. Let the distance be x km.

$$\therefore \text{Upstream speed} = 9 - 3 = 6 \text{ km/h}$$

$$\text{Downstream speed} = 9 + 3 = 12 \text{ km/h}$$

$$\text{Total time taken for upstream and downstream journey} = \frac{x}{6} + \frac{x}{12} = 3$$

$$\text{or } 3x/12 = 3 \text{ or } x = 12 \text{ km}$$

4. Let the speed of boat in still water be x km/h.

$$\text{Upstream speed of boat} = (x - 1) \text{ km/h}$$

$$\text{Downstream speed of boat} = (x + 1) \text{ km/h}$$

$$\therefore \text{Total time taken} = \frac{35}{x-1} + \frac{35}{x+1} = 12 \text{ hours}$$

$$\therefore \frac{35(x+1) + 35(x-1)}{(x+1)(x-1)} = 12$$

$$\text{or } \frac{70x}{(x+1)(x-1)} = 12$$

$$\text{or } 70x = 12(x+1)(x-1)$$

$$35x = 6(x+1)(x-1)$$

$$\therefore x = 6 \text{ or } 1/6$$

i.e. **6 km/h** or **1/6 km/h**.

**Short cut:**

$$\text{Distance} = \text{Total time} \times \frac{(\text{Speed in still water})^2 - (\text{Speed of current})^2}{2 \times \text{speed in still water}}$$

$$35 = 12 \times \frac{(x^2 - 1^2)}{2 \times x}$$

On simplifying, $x = 6$ or $1/6$ i.e. Speed of boat in still water = **6 km/h** or **1/6 km/h** $\therefore x > 1$, speed of boat = **6 km/h**

5. Let the velocity of current be
- x
- km/h.

$$\frac{24}{(u - x)} + \frac{36}{(u + x)} = 6 \quad (1)$$

$$\frac{36}{(u - x)} + \frac{24}{(u + x)} = 6 \frac{1}{2} \quad (2)$$

$$1.5 \times (1) \Rightarrow \frac{36}{(u - x)} + \frac{36 \times 1.5}{(u + x)} = 9 \quad (3)$$

$$(3) - (2) \Rightarrow \frac{1.5 \times 36 - 24}{(u + x)} = 9 - 6 \frac{1}{2} = \frac{5}{2}$$

$$\text{or} \quad \frac{30}{u + x} = \frac{5}{2}$$

$$\text{or} \quad u + x = \frac{30 \times 2}{5} = 12 \text{ km/h}$$

$$\begin{aligned} \therefore (2) \Rightarrow \frac{36}{u - x} &= 6 \frac{1}{2} - \frac{24}{12} \\ &= 6 \frac{1}{2} - 2 = 4 \frac{1}{2} = \frac{9}{2} \end{aligned}$$

$$\therefore u - x = \frac{36 \times 2}{9} = 8 \text{ km/h}$$

$$\therefore x = \frac{(u + x) - (u - x)}{2} = \frac{12 - 8}{2} = 2 \text{ km/h}$$

6. Let the man's rate in still water be
- x
- km/h.

$$\therefore x + 1.5 = 12 \text{ km/h}$$

$$\therefore x = 12 - 1.5 = 10.5 \text{ km/h}$$

$$\therefore \text{The man's rate against the current} = 10.5 - 1.5 = 9 \text{ km/h}$$

**Short cut:** The man's rate against current = Man's rate with the current – $2 \times$ rate of current

$$= 12 - 2 \times 1.5 = 9 \text{ km/h}$$

$$7. \text{ Distance} = \text{Total time} \times \frac{(\text{Speed in still water})^2 - (\text{Speed of current})^2}{2 \times \text{Speed in still water}}$$

$$= 1 \times \frac{(5^2 - 1^2)}{2 \times 5} = 2.4 \text{ km}$$

8. Let the man's rate for upstream be x km/h.

$\therefore$ The man's rate for downstream = $2x$ km/h.

$$\therefore \text{Man's rate in still water} = \frac{x + 2x}{2} = \frac{3x}{2} \text{ km/h.}$$

Here, $3x/2 = 6$ or $x = 4$ km/h

$$\therefore \text{Rate of stream} = \frac{1}{2} (\text{Downstream rate} - \text{Upstream rate}) = \frac{1}{2} (2x - x) = \frac{x}{2} = \frac{4}{2} = 2 \text{ km/h}$$



Short cut: Let the rate of stream be x km/h

$$\therefore 6 + x = 2(6 - x)$$

$$\text{or } 3x = 2 \times 6 - 6 = 6$$

$$\therefore x = \frac{6}{3} = 2 \text{ km/h}$$

9. Let the speed of man in still water be u km/h and rate of current be v km/h.

$$\therefore \frac{30}{u - v} + \frac{44}{u + v} = 10 \quad (1)$$

$$\frac{40}{u - v} + \frac{55}{u + v} = 13 \quad (2)$$

$$4 \times (1) \Rightarrow \frac{120}{u - v} + \frac{176}{u + v} = 40 \quad (3)$$

$$3 \times (2) \Rightarrow \frac{120}{u - v} + \frac{165}{u + v} = 39 \quad (4)$$

Solving (3) and (4), we get,

$$\frac{176 - 165}{u + v} = 40 - 39 = 1$$

$$\therefore u + v = \frac{176 - 165}{1} = 11 \text{ km/h}$$

$$\begin{aligned} \therefore (1) \Rightarrow \frac{30}{u - v} &= 10 - \frac{44}{u + v} \\ &= 10 - \frac{44}{11} \\ &= 10 - 4 = 6 \end{aligned}$$

$$\therefore u - v = \frac{30}{6} = 5 \text{ km/h}$$

$$\therefore u = \frac{11+5}{2} = 8 \text{ km/h}$$

$$v = \frac{11-5}{2} = 3 \text{ km/h}$$



Short cut: (By use of multiple cross – multiplication)

Upstream distance	Downstream distance	Time
30	44	10
40	55	13

$$\text{Upstream speed of man} = \frac{30 \times 55 - 40 \times 44}{55 \times 10 - 44 \times 13} = 5 \text{ km/h}$$

$$\text{Downstream speed of man} = \frac{30 \times 55 - 40 \times 44}{30 \times 13 - 40 \times 10} = 11 \text{ km/h}$$

$$\therefore \text{Speed of man} = \frac{5+11}{2} = 8 \text{ km/h}$$

$$\text{Speed of stream} = \frac{11-5}{2} = 3 \text{ km/h}$$

10. The speed of the person in still water = Rate of current $\times \frac{(\text{Sum of times})}{(\text{Difference of times})}$

$$= 3 \times \frac{(9+6)}{(9-6)} = 15 \text{ km/h}$$

11. Let speed of stream be x km/hr

$$\therefore \text{Downstream speed} = (10 + x) \text{ miles/hr.}$$

$$\text{Upstream speed} = (10 - x) \text{ miles/hr.}$$

$$\therefore \frac{36}{(10-x)} - \frac{36}{(10+x)} = \frac{90}{60}$$

$$\text{i.e.} \quad \frac{\{(10+x)-(10-x)\}36}{(10+x)(10-x)} = \frac{3}{2}$$

$$\text{or} \quad \frac{72x}{(100-x^2)} = \frac{3}{2}$$

$$\text{or} \quad 144x = 300 - 3x^2$$

$$\text{or} \quad x^2 + 48x - 100 = 0$$

$$\therefore (x+50)(x-2) = 0$$

$$\Rightarrow x = 2 \text{ miles/hour}$$

12. Let the speed of stream be x km/hr.

$$\therefore \text{Downstream speed} = (15 + x) \text{ km/hr.}$$

$$\text{Upstream speed} = (15 - x) \text{ km/hr.}$$

$$\text{i.e.} \quad \frac{30}{(15+x)} + \frac{30}{(15-x)} = 4.5$$

$$\text{or} \quad \frac{30[(15-x) + (15+x)]}{(15+x)(15-x)} = 4.5$$

$$\text{or} \quad \frac{30 \times 30}{15^2 - x^2} = 4.5$$

$$\text{or} \quad 900 = 4.5 (15^2 - x^2)$$

$$\text{or} \quad x^2 = \frac{4.5 \times 15^2 - 900}{4.5}$$

$$= 15^2 - 200 = 25$$

$$\text{or} \quad x = 5 \text{ km/hr}$$

13. Let the distance of the place be x km.

$$\therefore \quad \text{Speed downstream} = 5 + 1 = 6 \text{ km/hr}$$

$$\text{Speed upstream} = 5 - 1 = 4 \text{ km/hr}$$

$$\therefore \quad \frac{x}{6} + \frac{x}{4} = 1$$

$$\text{or} \quad x = \frac{1 \times 6 \times 4}{(6+4)} = \frac{24}{10} = 2.4 \text{ km}$$

14. Let speed of boat in still water be x km/hr.

$$\therefore \quad \text{Upstream speed} = (x - 3) \text{ km/hr}$$

$$\text{Downstream speed} = (x + 3) \text{ km/hr}$$

$$\therefore \quad (x + 3)1 = (x - 3) 1.5$$

$$\text{or} \quad x + 3 = 1.5x - 4.5$$

$$\text{or} \quad 0.5x = 3 + 4.5 = 7.5$$

$$\text{or} \quad x = \frac{7.5}{0.5} = 15 \text{ km/hr}$$

15. Let speed of boat in still water be x km/hr.

$$\text{and} \quad \text{speed of current} = kx \text{ km/hr}$$

$$\therefore \quad \text{Upstream speed} = (x - kx) = (1 - k)x \text{ km/hr}$$

$$\text{downstream speed} = (x + kx) = (1 + k)x \text{ km/hr}$$

$$\therefore \quad (1 - k)x \times \left(8 + \frac{48}{60}\right) = (1 + k)x \times 4$$

$$\text{or} \quad (1 - k) 8.8 = (1 + k) 4$$

$$\text{or} \quad (1 - k) 2.2 = (1 + k)$$

$$\text{or} \quad (2.2 + 1) k = 2.2 - 1 = 1.2$$

$$\therefore \quad k = \frac{1.2}{3.2} = \frac{3}{8}$$

$$\therefore \quad \text{Speed of boat: Speed of current} = 1 : k = 1 : \frac{3}{8} \text{ or } 8 : 3$$

16. Downstream speed = $13 + 4 = 17$ km/hr

$\therefore$ Time taken by boat to go 68 km downstream = $\frac{68}{17} = 4$ hours

17. Upstream speed = $\frac{3/4 \text{ km}}{45/4 \times 60 \text{ hr}}$

$$= \frac{3}{4} \times \frac{4 \times 60}{45} = 4 \text{ km/hr}$$

18. Downstream speed = 15 km/hr

Speed of current = .5 km/hr

$\therefore$ Speed of man in still water = $15 - 2.5 = 12.5$ km/hr

$\therefore$ Speed against current = $12.5 - 2.5 = 10$ km/hr

19. Let speed of boat in still water be x km/hr.

$\therefore \quad \frac{1}{2} \left(\frac{16}{2} + \frac{16}{4} \right) = \text{Speed of boat in still water}$

i.e.
$$x = \frac{1}{2}(8 + 4)$$

$$= 6 \text{ km/hr}$$

20. Let speed of boat in still water be x km/hr

$\therefore \quad \text{Speed in still water } x = \frac{11 + 5}{2} = 8 \text{ km/hr}$

21. Let speed of boat in still water be x km/hr and that of stream be kx km/hr.

$\therefore \quad \frac{S}{(x - kx)} = 2 \frac{S}{(x + kx)}$

or
$$\frac{1}{1 - k} = \frac{2}{1 + k}$$

or
$$1 + k = 2(1 - k)$$

or
$$k + 2k = 2 - 1$$

$$3k = 1$$

or
$$k = \frac{1}{3}$$

$\therefore$ Required ratio = $1 : k = 1 : \frac{1}{3} = 3 : 1$

22. Downstream speed = $\frac{1}{10/60} = 6$ km/hr

Upstream speed = $\frac{2}{1} = 2$ km/hr

$\therefore \quad \text{Speed of boatman in still water} = \frac{\text{Upstream speed} + \text{Downstream speed}}{2}$
$$= \frac{2 + 6}{2} = 4 \text{ km/hr}$$

∴ Time taken to go 5 km in stationary water

$$= \frac{5}{4} = 1.25 \text{ hours} = \mathbf{1 \text{ hour } 15 \text{ minutes}}$$

23. Let the speed of current be u miles/hour and speed in still water be x miles/hour.

$$\therefore \frac{12}{x-u} - \frac{12}{x+u} = 6 \text{ hours}$$

$$\frac{12\{(x+u)-(x-u)\}}{(x+u)(x-u)} = 6 \text{ hours}$$

$$\frac{12 \times 2u}{x^2 - u^2} = 6 \text{ hours} \quad (1)$$

$$\frac{12}{2x-u} - \frac{12}{2x+u} = 1$$

$$\frac{12\{(2x+u)-(2x-u)\}}{(2x+u)(2x-u)} = 1$$

$$\frac{12 \times 2u}{4x^2 - u^2} = 1 \quad (2)$$

$$(1) \Rightarrow \frac{4u}{x^2 - u^2} = 1$$

$$\text{or } x^2 - u^2 = 4u$$

$$\text{or } x^2 = u^2 + 4u \quad (A)$$

$$(2) \Rightarrow 4x^2 - u^2 = 24u$$

$$4x^2 = u^2 + 24u$$

$$x^2 = \frac{u^2 + 24u}{4} \quad (B)$$

Equating (A) and (B), we get,

$$u^2 + 4u = \frac{u^2 + 24u}{4}$$

$$\Rightarrow 4u^2 + 16u = u^2 + 24u$$

$$\Rightarrow 3u^2 = 8u$$

$$\text{or } u = \frac{8}{3} \text{ miles/hour} = \mathbf{2\frac{2}{3} \text{ miles/hour}}$$

24. Let the place be at a distance of x km.

$$\text{Upstream speed} = 7\frac{1}{2} - 1\frac{1}{2} = 6 \text{ km/hr}$$

$$\text{Downstream speed} = 7\frac{1}{2} + 1\frac{1}{2} = 9 \text{ km/hr}$$

$$\therefore \frac{x}{6} + \frac{x}{9} = \frac{50}{60}$$

$$\text{i.e.} \quad \frac{3x + 2x}{18} = \frac{5}{6}$$

$$\text{or} \quad (3x + 2x) 6 = 5 \times 18$$

$$\text{or} \quad 5x = \frac{5 \times 18}{6}$$

$$\text{or} \quad 5x = 15$$

$$\text{or} \quad x = \mathbf{3 \text{ km}}$$

$$25. \quad \frac{x}{10+4} + \frac{x}{10-4} = \frac{x}{14} + \frac{x}{6}$$

$$= \frac{3x + 7x}{42} = \frac{10x}{42}$$

$$= \frac{5x}{21} \text{ hour (Time taken by sailor)}$$

$$\frac{2x}{12} = \frac{x}{6} \text{ hours (Time taken by cyclist)}$$

$$\therefore \frac{5x}{21} > \frac{x}{6}, \text{ Cyclist will return first.}$$



$$\text{Short cut: Average speed of sailor} = \frac{2uv}{u+v}$$

$$= \frac{2(10+4)(10-4)}{(10+4) + (10-4)}$$

$$= \frac{2 \times 14 \times 6}{(14+6)}$$

$$= \frac{2 \times 14 \times 6}{20} = 8.4 \text{ km/hr}$$

($\therefore$ His speed on both sides are the same)

Average speed of cyclist = 12 km/hr

$\therefore$ Cyclist will return first.

Elementary Mensuration I

(Measurement of Area)



Important Formulae

AREA AND PERIMETER

Triangle

1. (a) Area = $\frac{1}{2}bh$; when b is the base and h is the height.

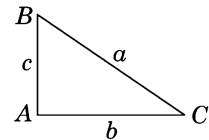
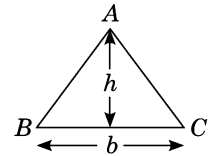
$$(b) \quad b = \frac{2A}{h} \quad \text{or} \quad b^2 = 2A \times \frac{b}{h} \quad \text{or} \quad b = \sqrt{2A \left(\frac{b}{h} \right)}$$

$$(c) \quad h = \frac{2A}{b} = \sqrt{\frac{2A}{(b/h)}}$$

2. $A = \sqrt{s(s-a)(s-b)(s-c)}$; where $s = \frac{1}{2}(a+b+c)$; $p = a+b+c$

3. $a^2 = b^2 + c^2$ (Pythagoras theorem)

4. $A = \frac{1}{2}bc$



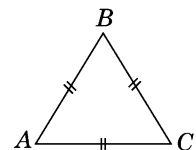
Isosceles triangle

5. $A = \frac{1}{4}b\sqrt{4a^2 - b^2}$

Equilateral triangle

$$\angle A = \angle B = \angle C = \frac{180^\circ}{3} = 60^\circ$$

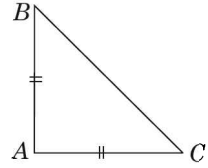
6. $A = \frac{\sqrt{3}}{4}a^2$



Right-angled isosceles triangle

$$\angle A = 90^\circ, b = c = \frac{BC}{\sqrt{2}}$$

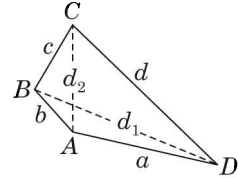
$$6. (a) A = \frac{1}{2} b^2 = \frac{1}{4} (\text{hypotenuse})^2$$



Quadrilateral

$$7. A = \sqrt{4(d_1 d_2)^2 - (b^2 + d^2 - a^2 - c^2)^2}$$

$$P = a + b + c + d$$



Special Case

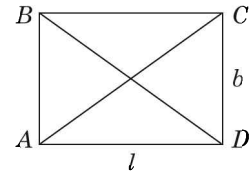
Rectangle

$$8. (a) \text{Area} = l \times b$$

$$\text{Length} = \frac{A}{b} \quad \text{or} \quad b = \frac{A}{l}$$

$$(b) \text{Diagonal } AC = BD = \sqrt{l^2 + b^2}$$

$$(c) P = 2(l + b)$$



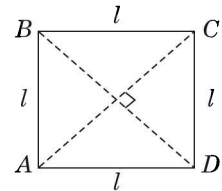
Square

$$9. (a) \text{Area} = l^2$$

$$(b) \text{Diagonal } (d) = AC = BD = \sqrt{2}l$$

$$\text{or } l = \frac{(\text{diagonal})}{\sqrt{2}} = \frac{d}{\sqrt{2}}$$

$$(c) A = l^2 = \frac{d^2}{2}; P = 4l$$

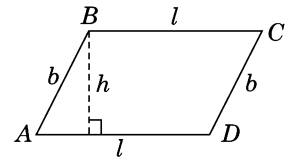


Parallelogram

$$10. \text{Area} = \text{Base} \times \text{Height}$$

$$\text{or } A = l \times h$$

$$P = 2(l + b)$$



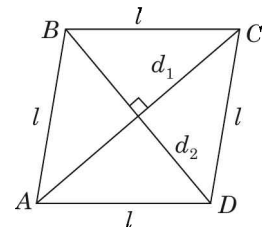
Rhombus

$$11. (a) \text{Area} = \frac{1}{2} (\text{Product of diagonals})$$

$$= \frac{1}{2} d_1 d_2$$

$$(b) \text{Side } l = \frac{1}{2} \sqrt{d_1^2 + d_2^2}$$

$$(c) P = 4l$$

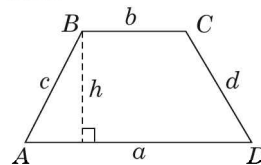


Trapezium

12. Area of trapezium = $\frac{1}{2}$ (Sum of parallel sides) $\times$ Distance between them

$$A = \frac{1}{2} (a + b) \times h$$

$$P = a + b + c + d$$

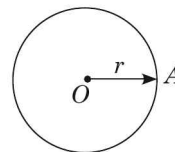
**Circle**

13. (a) Area = $\pi r^2 = \frac{\pi}{4} D^2$ where $\pi = \frac{22}{7}$ and $D = 2r$

$$D = \sqrt{\frac{4A}{\pi}}$$

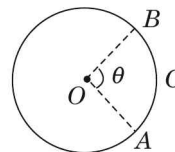
- (b) Perimeter $P = 2\pi r = \pi D$ and $D = \frac{P}{\pi}$

$$A = \frac{P^2}{4\pi} \Rightarrow P = \sqrt{4\pi \times A}$$

**Arc**

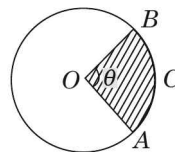
14. A part of the circumference of the circle is called arc of the circle.

$$\widehat{ACB} = \frac{\theta}{360^\circ} \times 2\pi r = r\theta$$

**Sector**

15. Area of sector $AOB = \pi r^2 \times \frac{\theta}{360^\circ}$

$$= \frac{1}{2} \times \text{arc } ACB \times r$$

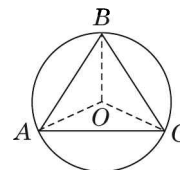
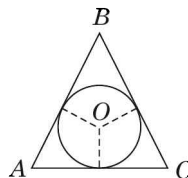
**Room**

16. (a) Area of 4 walls of room = $2h(l + b)$
 (b) Area of floor and 4 walls = $2h(l + b) + lb$
 (c) Area of floor, roof and 4 walls = $2\{h(l + b) + lb\}$
17. (a) **Polygon:** A plane figure bounded by multiple number of sides.
 (b) **Regular polygon:** Polygon whose all sides are equal.
18. **Incircle or Inscribed circle:** It is the circle inside the polygon whose all sides are tangent to the circle. For an equilateral triangle of side a , radius of inscribed circle = $\frac{a}{2\sqrt{3}}$
19. **Circumcircle:** It is the circle whose circumference touches all the vertices of the polygon.

$$\text{For an equilateral triangle, radius of circumcircle} = \frac{a}{\sqrt{3}}$$

$$(a) \text{ Area of a regular polygon} = \frac{1}{2} \times \text{Number of sides}$$

$$\times \text{Radius of the inscribed circle}$$



(b) Area of regular hexagon = $\frac{3\sqrt{3}}{2} (\text{side})^2 = 2.598 (\text{side})^2$

(c) Area of a regular octagon = $2(\sqrt{2} + 1) (\text{side})^2 = 4.828 (\text{side})^2$

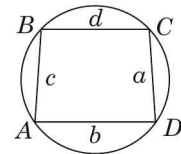
(d) For a regular polygon of n equal sides its vertex angle θ is given by $\theta = \left(\frac{n-2}{n}\right) 180^\circ$

20. **Cyclic quadrilateral:** It is a quadrilateral whose vertices lie on the circumference of the circle.

$$A = \sqrt{s(s-a)(s-b)(s-c)(s-d)}$$

where $s = \frac{a+b+c+d}{2}$

$$\angle A + \angle C = \angle B + \angle D$$



Isosceles triangle

21. (a) Area = $A = \frac{b}{4} \sqrt{4a^2 - b^2}$

(b) If the triangle is isosceles right triangle, $A = \frac{1}{2} (\text{side})^2 = \frac{1}{4} (\text{hypotenuse})^2$

Path, verandah, courtyard, etc. around a rectangular or square space

22. (a) If the garden is l metres long and b metre broad and a path having a width of w metre is constructed around the garden on its side, area of path = $2w(l + b + 2w) \text{ m}^2$
 (b) If the same path is inside, area of path = $2w[l + b - 2w] \text{ m}^2$
 (c) If the path outside and inside taken together, total area = $4w[l + b] \text{ m}^2$
 (d) A square field is surrounded by a path w metres wide on its outside. Area of path is $A_0 \text{ m}^2$. What is the area of the field?

$$\text{Area of field} = \left[\frac{A_0 - 4w^2}{4w} \right] \text{ m}^2$$

- (e) If in a parallelogram, two heights h_1 and h_2 and its area are given, then their sides are given by $l = A/h_1$; $b = A/h_2$
 (f) One of the diagonals of a rhombus of side ' s ' measures d_1 . The area of rhombus is given

$$\text{by } d_1 \sqrt{s^2 - \left(\frac{d_1}{2}\right)^2}.$$

- (g) If d_1 and d_2 are the diagonals of a rhombus, find area and perimeter.

$$s = \frac{1}{2} \sqrt{d_1^2 + d_2^2}$$

$$\text{Perimeter } P = 4s$$

$$A = \frac{1}{2} d_1 d_2$$

- (h) Area of a regular octagon

$$A = 2\sqrt{2} + 1 (\text{side})^2$$

All the three sides of a triangle is increased by $x\%$. Find the % change in its area.

Solution: $s = \frac{a+b+c}{2}$

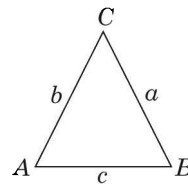
$$\% \text{ increase in area} = x \left(2 + \frac{x}{100} \right) \%$$

Similarly, when all sides of a square is increased by $x\%$, the % increase in

$$\text{area} = x \left(2 + \frac{x}{100} \right) \%$$

The % increase in area of any two dimensional figure when both dimensions

$$\text{are increased by } x\% \text{ is } x \left(2 + \frac{x}{100} \right) \%.$$



A square is inscribed in a circle of radius r . Then area of inscribed square is $2r^2$.

If the circumference and area of a circle are numerically equal, then the diameter is equal to 4 cm.

23. **Wheel:** The number of revolutions of the wheel needed to cover a distance

$$= \frac{\text{Distance to be covered}}{\pi \times \text{Diameter of wheel}}$$

Cuboid

24. Total surface area = $2(lb + bh + lh)$

25. Face diagonals = $\sqrt{l^2 + h^2}$, $\sqrt{b^2 + h^2}$ and $\sqrt{l^2 + b^2}$

26. Diagonal of the cuboid = $\sqrt{l^2 + b^2 + h^2}$

Cube

27. Volume = a^3 cubic units

28. Total surface area = $6a^2$ square units

$$\therefore \text{Volume} = \left(\sqrt{\frac{\text{Surface area}}{6}} \right)^3$$

29. Face diagonal = $\sqrt{2}a$

30. Body diagonal = $\sqrt{3}a$

Cylinder

31. Volume = $\pi r^2 h$ cubic units

32. Area of curved surface = $2\pi r h$ square units

33. Area of the base = Area of top = πr^2 square units

34. Total surface area = $2\pi r h + 2\pi r^2 = 2\pi r(h + r)$ square units

Sphere

35. Volume = $\frac{4}{3}\pi r^3$ cubic units

36. Surface area = $4\pi r^2$ square units

Hemisphere

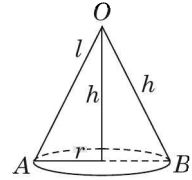
37. Volume = $\frac{2\pi r^3}{3}$ cubic units

38. Area of curved surface = $2\pi r^2$ square units

39. Total surface area = $3\pi r^2$ square units

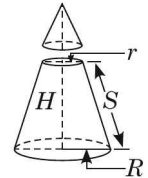
Cone

40. Volume = $\frac{1}{3}\pi r^2 h$ cubic units
41. Slant height $l = \sqrt{r^2 + h^2}$
42. Area of base = πr^2 square units
43. Area of curved surface = $\pi r l = \pi r \sqrt{r^2 + h^2}$ square units
44. Total surface area of cone = $\pi r l + \pi r^2 = \pi r(l + r)$ square units



Frustum of a right circular cone

45. $S = \sqrt{H^2 + (R - r)^2}$
46. Volume = $\frac{\pi H}{3} (R^2 + r^2 + Rr)$ cubic unit
47. Area of curved or slant surface = $\pi(R + r) S$ square units
48. Total surface area of the frustum = $\pi[(R^2 + r^2) + S(R + r)]$ square units
49. If the radius of a cylinder is increased by $x\%$ and its height increased by $y\%$ then volume



will be increased by $\left[2x + y + \frac{x^2 + 2xy}{100} + \frac{x^2 y}{(100)^2} \right] \%$

50. If the length, breadth and height of a cuboid are increased by $x\%$, $y\%$ and $z\%$ respectively, then percentage increase in its volume = $\left[x + y + z + \frac{xy + yz + zx}{100} + \frac{xyz}{(100)^2} \right] \%$
51. If the sides of a cube are increased by $x\%$, then the percentage increase in its surface area,
$$= \left[2x + \frac{x^2}{100} \right] \%$$

52. The radius (R) of the largest sphere that can be carved out of a cone of radius r , height h and slant height l is given by $R = \frac{rh}{r + l}$

53. Volume flow rate of a liquid through an opening = Cross sectional area of opening $\times$ Speed of liquid

54. (a) Area of a square = $(\text{side})^2 = \frac{1}{2} (\text{diagonal})^2$

(b) Perimeter of a square = $4 \times \text{side}$

55. Area of 4 walls of a room = $2(\text{length} + \text{breadth}) \times \text{height}$

56. Area of parallelogram = Base $\times$ Height

57. Area of a rhombus = $\frac{1}{2} \times (\text{Product of diagonals})$

When d_1 and d_2 are the two diagonals, then side of rhombus = $\frac{1}{2} \sqrt{d_1^2 + d_2^2}$

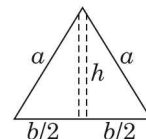
58. (a) Area of an equilateral triangle = $\frac{\sqrt{3}}{4} (\text{side})^2$

(b) Perimeter of an equilateral triangle = $3 \times \text{side}$

59. Area of an isosceles triangle

$$A = \frac{b}{4} \sqrt{4a^2 - b^2}$$

$$h = \sqrt{a^2 - (b/2)^2} = \frac{1}{2} \sqrt{4a^2 - b^2}$$



60. If
- a
- ,
- b
- , and
- c
- are lengths of the sides of a triangle and
- $s = \frac{1}{2}(a + b + c)$
- , then

$$\text{Area of triangle} = \sqrt{s(s-a)(s-b)(s-c)}$$

61. Area of triangle =
- $\frac{1}{2}$
- Base
- $\times$
- Height

62. Area of trapezium =
- $\frac{1}{2}$
- (Sum of parallel sides
- $\times$
- Perpendicular distance between them)
-
- $$= \frac{1}{2} (a + b)h$$

where a and b are parallel sides and h , perpendicular distance between the sides a and b .

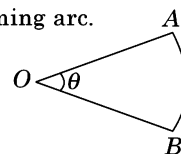
63. (a) Circumference of circle =
- $2\pi r$

- (b) Area of circle =
- πr^2

- (c) arc
- $AB = \frac{2\pi r \theta}{360^\circ}$
- where
- $\angle AOB = \theta$
- and
- r
- , radius of part of circle forming arc.

- (d) Area of sector
- $AOB = \frac{\pi r^2 \theta}{360^\circ}$

- (e) Area of sector
- $AOB = \frac{1}{2} \times \text{arc } AB \times r$



64. In a parallelogram

$$\text{Area} = 2\sqrt{s(s-a)(s-b)(s-d)}$$

where a and b are two adjacent sides and d is the diagonal connecting the ends of the two sides.

PROBLEMS ON RECTANGLES AND SQUARES

Type I Direct Application of Formula

EXAMPLE 1 Calculate the area of a rectangle 23 metres 7 decimetres long and 14 metres 4 decimetres 8 centimetres wide.

Solution Length = 23.70 m

Breadth = 14.48 m

Area = 23.70×14.48 square metres

= **343.18 square metres**

EXAMPLE 2 Find the diagonal of a rectangle whose sides are 12 metres and 5 metres.

Solution Length of diagonal = $\sqrt{12^2 + 5^2} = \sqrt{169} = \mathbf{13 \text{ metres}}$

Type II Carpeting a Floor

EXAMPLE 3 How many metres of a carpet 75 cm wide will be required to cover the floor of a room which is 20 metres long and 12 metres broad?

Solution Length required = $\frac{(\text{Length} \times \text{Breadth}) \text{ of room}}{\text{Width of carpet}} = \frac{20 \times 12}{0.75} = \mathbf{320 \text{ m}}$

EXAMPLE 4 What amount will be required in carpeting the floor in the above example if the carpet is available at ₹ 30/m?

Solution



Short cut: Amount required = Rate/m × Required length of carpet

$$= \text{Rate/m} \times \frac{\text{Length of room} \times \text{Breadth of room}}{\text{Width of carpet}}$$

$$= 30 \times \frac{20 \times 12}{0.75} = \mathbf{₹ 9600}$$

Type III Paving a Courtyard with Tiles

EXAMPLE 5 How many paving stones of size 2.5 m × 2 m are required to pave a rectangular courtyard 30 m long and 16.5 m wide?

Solution Number of paving stones required = $\frac{\text{Length} \times \text{Breadth of courtyard}}{\text{Length} \times \text{Breadth of each paving stone}}$

$$= \frac{30 \times 16.5}{2.5 \times 2} = \mathbf{99}$$

EXAMPLE 6 What will be the amount required to pave stones in Example 5, if the paving stone of the given dimension is available at ₹ 10 /piece?

Solution



Short cut:

$$\text{Amount required} = \text{Price per stone} \times \frac{\text{Length} \times \text{Breadth of courtyard}}{\text{Length} \times \text{Breadth of each paving stone}}$$

$$= 10 \times \frac{30 \times 16.5}{2.5 \times 2} = \mathbf{₹ 990}$$

Type IV Paving with Largest Square Tiles

EXAMPLE 7 A hall 39 m 10 cm long and 35 m 70 cm broad is to be paved with square tiles. Find the largest tile so that the tiles exactly fit and also find the number of tiles required.

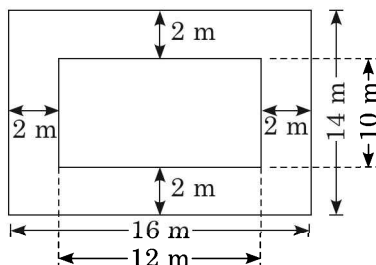
Solution

Short cut: Side of largest possible tile = HCF of length and breadth of room
 = HCF of 39.10 and 35.70 m
 = **1.70 m**

$$\begin{aligned}\text{Number of tiles required} &= \frac{\text{Length} \times \text{Breadth of room}}{(\text{HCF of length and breadth of room})^2} \\ &= \frac{39.10 \times 35.70}{1.70 \times 1.70} = \mathbf{483}\end{aligned}$$

Type V Path round a Garden, Verandah round a Room

EXAMPLE 8 A rectangular hall 12 m long and 10 m broad, is surrounded by a verandah 2 metres wide. Find the area of the verandah?

**Solution**

Short cut:

- (i) When verandah is outside the room, surrounding it,

$$\begin{aligned}\text{Area of verandah} &= 2 \times \text{Width of verandah} \\ &\quad \times [\text{Length} + \text{Breadth of room} + 2(\text{Width of verandah})]\end{aligned}$$

- (ii) When path is within the garden, surrounding by it,

$$\text{Area of path} = 2(\text{Width of path}) \times [\text{Length} + \text{Breadth of garden} - 2(\text{Width of path})]$$

$\therefore$ Verandah is outside the room, formula (i) may be applied.

$$\therefore \text{Area of verandah} = 2 \times 2 \times [10 + 12 + 2 \times 2] = 4 \times 26 = \mathbf{104 \text{ m}^2}$$

EXAMPLE 9 A rectangular grassy plot is 112 m $\times$ 78 m in dimension. It has a gravel path 2.5 m wide all round it on the inside. Find the area of the path and the cost of constructing it at ₹ 20/m²?

Solution Path is within the garden, above formula (ii) may be applied to find the area of path i.e.

$$\text{area of path} = 2 \times 2.5 \times [112 + 78 - 2 \times 2.5] = 5 \times 185 = \mathbf{925 \text{ m}^2}$$

$$\therefore \text{Cost of construction} = \text{Rate/m}^2 \times \text{area} = 20 \times 925 = \mathbf{₹ 18500}$$

I. When area of the path is given, to find the area of garden enclosed (garden is square in shape)

EXAMPLE 10 A path 2 m wide running all round a square garden has an area of 9680 m^2 . Find the area of the part of the garden enclosed by the path.

Solution



Short cut: Area of square garden = $\left[\frac{\text{Area of path} - 4 (\text{width of path})^2}{4 \times \text{width of path}} \right]^2$

$$\therefore \text{Area of garden} = \left[\frac{9680 - 4(2)^2}{4 \times 2} \right]^2 = \left[\frac{9664}{8} \right]^2 = (1208)^2 = \mathbf{1459264 \text{ m}^2}$$

II. When area of path is given to find the width of path

EXAMPLE 11 A path all around the inside of a rectangular park $37 \times 30 \text{ m}$ occupies 570 m^2 . Find the width of the path.

Solution By using formula (ii) in Example 8,

$$\text{Area of path} = 2 \times \text{Width of path} \times [\text{Length} + \text{Breadth of path} - 2 \times \text{Width of path}]$$

Let the width of path be x metres.

$$\therefore \text{Area of path} = 2 \times x \times [37 + 30 - 2 \times x]$$

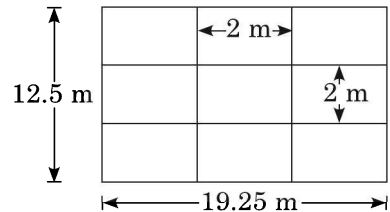
$$\text{i.e.} \quad 570 = 134x - 4x^2$$

$$\Rightarrow 4x^2 - 134x + 570 = 0$$

Solving we get, $x = 5 \text{ m}$

III. Paths crossing each other

EXAMPLE 12 An oblong piece of ground measures $19 \text{ m } 25 \text{ dm} \times 12 \text{ m } 5 \text{ dm}$. From the centre of each side a path 2 m wide goes across to the centre of the opposite side. What will be the area of the path? Find the cost of paving these paths at the rate of ₹ 13.2/m².



Solution Cost of paving = Area of path $\times$ Rate of paving
 $= 59.5 \times 13.2 = \mathbf{₹ 785}$



Short cut:

(a) Area of path = (Width of path) [Length + Breadth of park – Width of path]

(b) Area of park minus path = (Length of park – Width of path) $\times$ (Breadth of park – Width of path)

$$\text{Here, area of path} = 2(19.25 + 12.5 - 2) = 2 \times 29.75 = \mathbf{59.5 \text{ m}^2}$$

$$\text{Cost} = \text{Rate} \times \text{Area} = 13.2 \times 59.5 = \mathbf{₹ 785.4}$$

Type VI Area and Ratio

EXAMPLE 13 The sides of a rectangular field of 864 m^2 are in the ratio of 3 : 2. Find the sides.

Solution**Short cut:**

$$\text{First side} = \sqrt{\text{Area} \times \text{Ratio}}$$

$$\text{Second side} = \sqrt{\text{Area} \times \text{Inverse ratio}}$$

$$\therefore \quad \text{First side} = \sqrt{864 \times \frac{3}{2}} = \sqrt{1296} = 36 \text{ m}$$

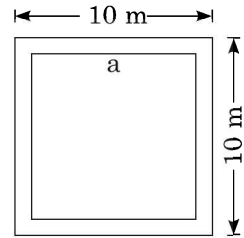
$$\text{Second side} = \sqrt{864 \times \frac{2}{3}} = \sqrt{576} = 24 \text{ m}$$

Type VII Miscellaneous Cases (Carpet and Oil Cloth)

EXAMPLE 14 In the centre of a room 10 metres square, there is a square of turkey carpet, and the rest of the floor is covered with oil cloth. The carpet and the oil cloth cost respectively ₹ 15 and ₹ 6.50 per square metre, and the total cost of the carpet and the oil cloth is ₹ 1338.50. Find the width of the oil cloth border.

Solution Area of square room = $10 \times 10 = 100 \text{ m}^2$

$$\text{Average cost of carpet and oil cloth/m}^2 = ₹ \frac{1338.50}{100} = ₹ 13.385$$



Carpet	Oil cloth
15	6.50
13.385	
6.885	1.615
81 : 19	

By alligation rule, area of square carpet is 81 m^2 . Therefore side of carpet is 9 m. Room is of size $10 \text{ m} \times 10 \text{ m}$. Area of oil cloth = 19 m^2

Therefore, $10 - 9 = 2 \times \text{Width of border}$

$$\therefore \quad \text{Width of border} = \frac{1}{2} \text{ metre} = 5 \text{ dm}$$

$$\therefore \quad \text{Width of oil cloth} = 5 \text{ dm}$$

EXAMPLE 15 A square field of 2 km^2 is to be divided into two equal parts by a fence which coincides with a diagonal. Find the length of the fence.

Solution Area of field = 2 km^2

$$\text{Diagonal} \left[\sqrt{2} \times (\text{side of field}) \right] = \sqrt{2} \times \sqrt{2} = 2 \text{ kilometres}$$

EXAMPLE 16 A square field of area 31684 m^2 is to be enclosed with wire placed at heights 1, 2, 3, 4 metres above the ground. What length of the wire will be required, if its length required for each circuit is 5% greater than the perimeter of the field?

Solution Area of the field = 31684 m^2

$$\therefore \text{Perimeter} = \sqrt{31684} \times 4 \text{ m} = 178 \times 4$$

$$\therefore \text{Length of each circuit} = 178 \times 4 \times \frac{105}{100} \text{ m}$$

Here wire is placed at 4 different heights

$$\therefore \text{Total length of wire required} = 178 \times 4 \times \frac{105}{100} \times 4 = \mathbf{2990.4 \text{ metres}}$$

PROBLEMS ON TRIANGLES

Type I Direct Application of Formulae

EXAMPLE 17 The base of a triangular field is 880 metres and its height 550 metres. Find the area of the field. Also calculate the charges for supplying water to the field at the rate of ₹ 24.25/ hectometre².

$$\begin{aligned} \text{Solution Area of the field} &= \frac{\text{Base} \times \text{Height}}{2} = \frac{880 \times 550}{2} \text{ m}^2 \\ &= \frac{440 \times 550}{100 \times 100} \text{ hectometre}^2 = \mathbf{24.20 \text{ hectometre}^2} \end{aligned}$$

$$\begin{aligned} \text{Cost of supplying water to the whole field} &= \text{Rate/sq. hectometre} \times \text{Area of field} \\ &= 24.25 \times 24.20 = \mathbf{₹ 586.85} \end{aligned}$$

EXAMPLE 18 The base of a triangular field is 3 times its height. If the cost of cultivating the field at ₹ 36.72/hectare is ₹ 495.72, find its base and height.

$$\begin{aligned} \text{Solution Area of the field} &= \frac{495.72}{36.72} \text{ hectares} \\ &= \frac{27}{2} \text{ hectares} \end{aligned}$$

$$\begin{aligned} \text{But, area of field} &= \frac{1}{2} \times \text{Base} \times \text{Height} \\ &= \frac{1}{2} \times (3 \times \text{Height}) \times \text{Height} \quad (\because \text{Base} = 3 \times \text{Height}) \\ &= \frac{3}{2} (\text{Height})^2 \end{aligned}$$

Equating R.H.S. of above two equations,

$$\frac{3}{2} (\text{Height})^2 = \frac{27}{2} \text{ hectares}$$

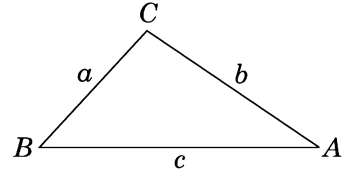
$$\therefore (\text{Height})^2 = \frac{27}{3} = 9 \text{ hectares}$$

$$\text{or} \quad \text{Height} = 3 \times 10^2 \text{ m} \\ = 300 \text{ m}$$

$$\therefore \text{Base} = 3 \times \text{Height} = 3 \times 300 = 900 \text{ m}$$

EXAMPLE 19 Find the area of a triangle whose sides are 50 metres, 78 metres, 112 metres respectively and also find the perpendicular from the opposite angle on the side 112 metres.

Solution Here $a = 50 \text{ m}$; $b = 78 \text{ m}$; $c = 112 \text{ m}$



$$\therefore s = \frac{1}{2}(a + b + c) \\ = \frac{1}{2}(50 + 78 + 112) = 120 \text{ metres}$$

$$\therefore s - a = 120 - 50 = 70 \text{ m};$$

$$s - b = 120 - 78 = 42 \text{ m};$$

$$s - c = 120 - 112 = 8 \text{ m};$$

$$\therefore \text{Area} = \sqrt{s(s-a)(s-b)(s-c)} \\ = \sqrt{120 \times 70 \times 42 \times 8} \\ = \sqrt{120 \times 70 \times 7 \times 6 \times 2 \times 4} \\ = \sqrt{7^2 \times 12^2 \times 2^2 \times 10^2} \\ = 7 \times 12 \times 2 \times 10 = 1680 \text{ m}^2$$

$$\therefore \text{Perpendicular} = \frac{\text{Area}}{1/2 \text{ base}} = \frac{1680}{1/2 \times 112} = 30 \text{ m}$$

Type II Faster Methods for Triangle Problems

Solve Examples (20 – 22) by faster methods.

Solution to Example 20: Since Example 17 involves direct application of the formula, a method faster than the one adopted can't be used.

Solution to Example 21: The ratio between base and height in Example 18 is 3 : 1.

$$\therefore \text{Base} = \sqrt{2 \times \text{Area} \times \text{Ratio}}$$

$$\text{Height} = \sqrt{2 \times \text{Area} \times \text{Inverse ratio}}$$

$$\therefore \text{Base} = \sqrt{2 \times \frac{27}{2} \times \frac{3}{1}} = 900 \text{ m}$$

$$\text{Height} = \sqrt{2 \times \text{Area} \times \text{Inverse ratio}}$$

$$= \sqrt{2 \times \frac{27}{2} \times \frac{1}{3}} = 300 \text{ m}$$

Solution to Example 22: Since Example 19 involves the direct application of formula, no short cut can be employed.

Problems on Parallelogram, Rhombus and Trapezium

Type I Direct Application of Formulae

EXAMPLE 23 Find the surface area of a metal sheet which is in the form of a parallelogram whose base is 12 cm and height is 7.8 cm.

Solution Surface area = Height $\times$ Base = $7.8 \times 12 = 93.6 \text{ cm}^2$

EXAMPLE 24 Find the area of a rhombus one of whose diagonals measures 8 cm and the other 10 cm.

Solution Area = $\frac{1}{2} \times \text{Product of diagonals} = \frac{1}{2} \times 8 \times 10 = 40 \text{ cm}^2$

EXAMPLE 25 Find the distance between the two parallel sides of a trapezium if the area of trapezium is 250 m^2 , and the two parallel sides are 30 m and 20 m respectively.

Solution Area = $\frac{1}{2} \times \text{Height} \times (\text{Sum of parallel sides})$

i.e. $250 = \frac{1}{2} \times h \times (30 + 20)$

$\therefore h = \frac{250 \times 2}{30 + 20} = \frac{500}{50} = 10 \text{ m}$



Short cut: To find the area of a rhombus with one side and one diagonal given.

EXAMPLE 26 Find the area of a rhombus one side of which measures 20 cm and one diagonal 24 cm.

Solution



Short cut: Area of rhombus = Diagonal $\times \sqrt{(\text{Side})^2 - \left(\frac{\text{Diagonal}}{2}\right)^2}$

i.e.
$$\begin{aligned} \text{Area} &= 24 \times \sqrt{(20)^2 - \left(\frac{24}{2}\right)^2} \\ &= 24 \times \sqrt{400 - 144} = 24 \times 16 = 384 \text{ cm}^2 \end{aligned}$$

EXAMPLE 27 The perimeter of a rhombus is 146 cm and one of its diagonal is 55 cm. Find the other diagonal and the area of the rhombus.

Solution



Short cut: Other diagonal = $2 \times \sqrt{(\text{Side})^2 - \left(\frac{\text{Diagonal}}{2}\right)^2}$

$$\text{Side of rhombus} = \frac{146}{4} = 36.5 \text{ cm}$$

$$\begin{aligned}\text{Other diagonal} &= 2 \times \sqrt{(36.5)^2 - \left(\frac{55}{2}\right)^2} \\ &= 2 \times \sqrt{64 \times 9} = 2 \times 8 \times 3 = \mathbf{48 \text{ cm}}\end{aligned}$$

$$\therefore \text{Area of rhombus} = \frac{1}{2} (\text{Product of diagonals}) = \frac{1}{2} (48 \times 55) = \mathbf{1320 \text{ cm}^2}$$

PROBLEMS ON REGULAR POLYGONS

A polygon with all sides equal is termed a regular polygon, for example, triangle, quadrilateral, pentagon, hexagon, etc. are polygons.

1. Area of a regular polygon = $\frac{1}{2} \times \text{Number of sides} \times \text{Radius of the inscribed circle}$
2. Area of a hexagon = $\frac{3\sqrt{3}}{2} \times (\text{side})^2$
3. Area of an octagon = $2(\sqrt{2} + 1)(\text{side})^2$

EXAMPLE 28 Find the area of a regular hexagon whose side measures 9 cm.

Solution Area of a regular hexagon = $\frac{3\sqrt{3}}{2} a^2$

Here $a = 9 \text{ cm}$

$$\therefore \text{Area} = \frac{3\sqrt{3}}{2} 9^2 \text{ cm}^2 = \mathbf{210.4 \text{ cm}^2 \text{ (approx.)}}$$

EXAMPLE 29 Find the side of a regular octagonal enclosure whose area is 1 hectare in nearest metre.

Solution Area of a regular octagon = $2(1 + \sqrt{2})a^2$

Given $2(1 + \sqrt{2})a^2 = 1 \text{ hectare}$

$$\therefore a^2 = \frac{10000}{2(1 + \sqrt{2})} \text{ m}^2 = 2071 \text{ m}^2 (\text{approx.})$$

or $a = \sqrt{\frac{10000}{2(1 + \sqrt{2})}} = \mathbf{46 \text{ metres (approx.)}}$

PROBLEMS ON ROOMS AND WALLS

EXAMPLE 30 A room 8 metres long, 6 metres wide and 3 metres high has two windows $1\frac{1}{2}\text{ m} \times 1\text{ m}$ and a door $2\text{ m} \times 1\frac{1}{2}\text{ m}$. Find the area of the wall and the cost of covering it with wall paper 50 cm wide at ₹ 2.5/metre.

Solution Area of wall = $2(8 + 6)3 = 84\text{ m}^2$

Area of two windows and a door = $2 \times 1\frac{1}{2} \times 1 + 1 \times 2 \times 1\frac{1}{2} = 6\text{ m}^2$

∴ Area to be covered = $84 - 6 = 78\text{ m}^2$

∴ Length of paper required = $\frac{78}{50/100} = \frac{7800}{50} = 156\text{ m}$

∴ Cost = ₹ $2.5/\text{m} \times 156 = ₹ 390$

LINING A BOX WITH METAL

EXAMPLE 31 A closed box measures externally 90 cm long, 60 cm wide, 45 cm high and is made of wood $2\frac{1}{2}\text{ cm}$ thick. Find the cost of lining it on the inside with metal at 6 paise/dm².

Solution Internal dimensions are 85 cm, 55 cm and 40 cm, i.e. $8\frac{1}{2}\text{ dm}$, $5\frac{1}{2}\text{ dm}$ and 4 dm.

∴ Area of 4 sides = $2\left(8\frac{1}{2} + 5\frac{1}{2}\right) \times 4\text{ dm}^2 = 112\text{ dm}^2$

Area of bottom and top = $2 \times 8\frac{1}{2} \times 5\frac{1}{2} = \frac{187}{2}\text{ dm}^2$

Total area = $112 + \frac{187}{2} = \frac{411}{2}\text{ dm}^2$

∴ Cost of lining = $6 \times \frac{411}{2}\text{ paise} = 1233\text{ paise} = ₹ 12.33$

PROBLEMS ON CIRCLES

Some formulae which are used to find quick solutions:

$$\begin{array}{lll} \text{(a) Area} = \pi(\text{radius})^2 & \text{(b) Radius} = \sqrt{\frac{\text{Area}}{\pi}} & \text{(c) Diameter} = 2\sqrt{\frac{\text{Area}}{\pi}} \\ \text{(d) Area} = \pi\left(\frac{\text{Diameter}}{2}\right)^2 & \text{(e) Perimeter} = 2\pi(\text{radius}) & \text{(f) Radius} = \left(\frac{\text{Perimeter}}{2\pi}\right) \end{array}$$

$$(g) \text{ Perimeter} = \pi(\text{diameter}) \quad (h) \text{ Diameter} = \left(\frac{\text{Perimeter}}{\pi} \right)$$

$$(i) \text{ Arc of a sector} = \left(\frac{\theta^\circ}{360^\circ} \right) \times \text{Circumference}$$

$$(j) \text{ Area of a sector} = \left(\frac{\theta^\circ}{360^\circ} \right) \times \pi \times (\text{radius})^2$$

I. Direct Application of Formulae:

EXAMPLE 32 Find the radius of a circular field whose circumference measures $5\frac{1}{2}$ km.

Solution Circumference (c) = $2\pi r$

$$\text{or} \quad r = \frac{c}{2\pi}$$

$$\begin{aligned} \therefore \text{ Required radius} &= \frac{\frac{11}{2} \times 1000 \text{ m}}{2\pi} \\ &= \frac{\frac{11}{2} \times 1000 \times 7}{2 \times 22} = \frac{7000}{8} = \mathbf{875 \text{ metres}} \end{aligned}$$

AREA OF A RING

EXAMPLE 33 The circumference of a circular garden is 1012 m. Find the area outside the garden, a road of 3.5 m width runs round it. Calculate the area of this road and find the cost of gravelling it at the rate of ₹ 3/m².

Solution



Short cut: $\text{Area} = \frac{(\text{Circumference})^2}{4\pi} = \frac{(1012)^2}{4 \times \frac{22}{7}} = \mathbf{81466 \text{ m}^2}$

$$\text{Area of ring} = \pi[(\text{width of ring}) (2 \times \text{Inner radius} + \text{Width of ring})]$$

$$\text{But, inner radius} = \sqrt{\frac{\text{Area}}{\pi}} = \sqrt{\frac{81466 \times 7}{22}} = \mathbf{161 \text{ m}}$$

$$\therefore \text{ Area of ring-shaped road} = \pi \times 3.5 \times [3.5 + 2 \times 161] = \frac{22}{7} \times 3.5 \times 325.5 = 3580.5 \text{ m}^2$$

$$\therefore \text{ Cost of gravelling} = 3580.5 \times 3 = \mathbf{₹ 10741.50}$$

IDENTICAL CIRCLES PLACED TOGETHER

EXAMPLE 34 An equilateral triangle of side 2 m is placed on the top of a group of 3 circles of radius 1 m each such that the three corners of triangle are at the centres of the

circle: (a) calculate the area common to all the circles and the triangle, (b) calculate the area of the remaining portion of the triangle.

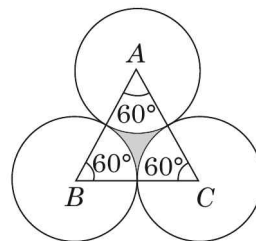
Solution



Short cut: Here, the side of equilateral triangle is double the radius of the circle and therefore all circles touch each other.

$$\therefore \text{Area of each sector} = \frac{1}{2} r^2 \theta = \frac{1}{2} r^2 \times \frac{\pi}{3} = \frac{\pi}{6} r^2$$

$$\begin{aligned} \text{Area of remaining portion} &= \frac{\sqrt{3}}{4} a^2 - 3 \times \frac{\pi}{6} r^2 = \frac{\sqrt{3}}{4} (2r)^2 - 3 \times \frac{\pi}{6} r^2 \\ &= \left[\sqrt{3} - \frac{\pi}{2} \right] r^2 = 0.162 r^2 \end{aligned}$$



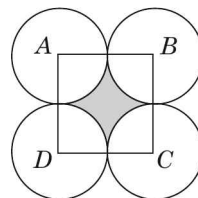
$$(a) \text{ Sum of areas of 3 sectors} = 3 \times \frac{\pi}{6} r^2 = \frac{\pi r^2}{2} = \frac{22}{7 \times 2} (1)^2 = 1.57 \text{ m}^2$$

$$(b) \text{ Area of remaining portion} = 0.162 r^2 = 0.162 \times (1)^2 = 0.162 \text{ m}^2$$



EXERCISES

- The radius of a coin is 1 cm. If four of these coins are placed on a table so that the rim of each touches that of the other two, find the area of the unoccupied space between them.
 - 0.25 cm^2
 - 0.5 cm^2
 - 1.57 cm^2
 - 0.86 cm^2
 - None of these
- The radius of a circular wheel is $1\frac{3}{4}$ m. How many revolutions will it make in travelling 11 km?
 - 500
 - 1000
 - 1500
 - 2000
 - None of these
- A room is 7 metres long and 5 metres broad, the doors and windows occupy 5 m^2 , and the cost of papering the remaining part of the surface of the walls with paper 75 cm wide, at ₹ 4.20 per piece of 13 m is ₹ 39.20. Find the height of room.
 - 3 m
 - 4 m
 - 4.5 m
 - 5 m
 - None of these
- A hall, whose length is 16 m and breadth twice its height takes 168 m of paper 2 m wide for its four walls. Find the area of the floor.
 - 188 m^2
 - 196 m^2
 - 192 m^2
 - 198 m^2
 - None of these
- Find the area of a rhombus one of whose diagonals measures 10 cm and the other 12 cm.
 - 120 cm^2
 - 60 cm^2
 - 50 cm^2
 - 40 cm^2
 - None of these
- The area of a square field is 69696 cm^2 . Its diagonal will be equal to [SSC]
 - 313.296 cm
 - 353.296 cm
 - 373.296 cm
 - 393.296 cm
- If the ratio of areas of two squares is 225 : 256, then the ratio of their perimeters is [SSC]
 - 225 : 256
 - 256 : 225
 - 15 : 16
 - 16 : 15



8. A man walked diagonally across a square plot. Approximately what was the per cent saved by not walking along the edges? [MBA]
(a) 20 (b) 24 (c) 30 (d) 33
9. A rectangular park 60 m long and 40 m wide has two concrete crossroads running in the middle of the park, and rest of the park has been used as a lawn. If the area of the lawn is 2109 m^2 , then what is the width of the road? [MAT]
(a) 2.91 m (b) 3 m (c) 5.82 m (d) None of these
10. The length of rectangle is increased by 60%. By what per cent would the width have to be decreased so as to maintain the same area? [MAT]
(a) $37\frac{1}{2}\%$ (b) 60% (c) 75% (d) 120%
11. Wheels of diameters 7 cm and 14 cm start rolling simultaneously from X and Y, which are 1980 cm apart, towards each other in opposite directions. Both of them make the same number of revolutions per second. If both of them meet after 10 seconds, the speed of the smaller wheel is [MAT]
(a) 22 cm/sec (b) 44 cm/sec (c) 66 cm/sec (d) 132 cm/sec
12. A man runs round a circular field of radius 50 m at the speed of 12 km/hr. What is the time taken by the man to take twenty rounds of the field? [MAT]
(a) 30 minutes (b) 32 minutes (c) 34 minutes (d) None of these
13. The length of a rectangular plot is 60 % more than its breadth. If the difference between the length and the breadth of that rectangle is 24 cm, what is the area of that rectangle? [Bank PO]
(a) 2400 cm^2 (b) 2480 cm^2 (c) 2560 cm^2
(d) Data inadequate (e) None of these
14. The length of a rectangular plot is 20 metres more than its breadth. If the cost of fencing the plot at ₹ 26.50/m is ₹ 5300, what is the length of plot in metres? [Bank PO]
(a) 40 (b) 50 (c) 120
(d) Data inadequate (e) None of these
15. A farmer wishes to start a 100 m^2 rectangular vegetable garden. Since he has only 30 m barbed wire, he fences three sides of the garden setting his house compound wall act as the fourth side fencing. The dimension of the garden is [RRB]
(a) $15 \text{ m} \times 6.67 \text{ m}$ (b) $20 \text{ m} \times 5 \text{ m}$
(c) $30 \text{ m} \times 3.33 \text{ m}$ (d) $40 \text{ m} \times 2.5 \text{ m}$
16. A rectangle has width a and length b . If the width is decreased by 20% and the length is increased by 10%, then what is the area of the new rectangle in percentage compared to ab ? [RRB]
(a) 80% (b) 88% (c) 110% (d) 120%
17. Each side of a rhombus is 26 cm and one of its diagonals is 48 cm long. The area of the rhombus is [RRB]
(a) 240 cm^2 (b) 300 cm^2 (c) 360 cm^2 (d) 480 cm^2
18. A circle and a rectangle have the same perimeter. The sides of the rectangle are 18 cm and 26 cm. What is the area of the circle? [Bank PO]
(a) 88 cm^2 (b) 154 cm^2 (c) 1250 cm^2
(d) Cannot be determined (e) None of these
19. A circular ground whose diameter is 35 metres, has a 1.4 m broad garden around it. What is the area of the garden in square metres? [SBI PO]
(a) 160.16 (b) 176.16 (c) 196.16
(d) Data inadequate (e) None of these

20. If the radius of a circle is increased by 75%, then its circumference will increase by [CDS]
(a) 25% (b) 50% (c) 75% (d) 100%
21. If an equilateral triangle of area X and a square of area Y have the same perimeter, then X is [CDS]
(a) equal to Y (b) greater than Y
(c) less than Y (d) less than or equal to Y
22. An error of 2% in excess is made while measuring the side of a square. The percentage of error in the calculated area of the square is [CDS]
(a) 2% (b) 2.02% (c) 4% (d) 4.04%
23. A can go round a circular path 8 times in 40 minutes. If the diameter of the circle is increased to 10 times the original diameter, then the time required by A to go round the new path once, travelling at the same speed as before, is [SSC]
(a) 20 min (b) 25 min (c) 50 min (d) 100 min
24. An equilateral triangle, a square and a circle have equal perimeters. If T denotes the area of triangle, S , the area of the square and C , the area of the circle, then [CDS]
(a) $S < T < C$ (b) $T < C < S$ (c) $T < S < C$ (d) $C < S < T$
25. The sides of a triangle are 6 cm, 11 cm and 15 cm. The radius of its incircle is [MAT]
(a) $3\sqrt{2}$ cm (b) $\frac{4\sqrt{2}}{5}$ cm (c) $\frac{5\sqrt{2}}{4}$ cm (d) $6\sqrt{2}$ cm
26. A square is inscribed in a circle whose radius is 4 cm. The areas of the portion between the circle and the square is
(a) $(8\pi - 16)$ (b) $(8\pi - 32)$ (c) $(16\pi - 16)$ (d) $(16\pi - 32)$
27. The area of a circle is 220 cm^2 . The area of a square inscribed in this circle will be [CBI]
(a) 49 cm^2 (b) 70 cm^2 (c) 140 cm^2 (d) 150 cm^2
28. A tank is 25 m long, 12 m wide and 6 m deep. The cost of plastering its walls and bottom at 75 paise/m² is [CBI]
(a) ₹ 456 (b) ₹ 458 (c) ₹ 558 (d) ₹ 568
29. If in a triangle, the area is numerically equal to the perimeter, then the radius of the inscribed circle of the triangle is [SSC]
(a) 1 (b) 1.5 (c) 2 (d) 3
30. The area of the incircle of an equilateral triangle of side 42 cm is [SSC]
(a) $22\sqrt{3} \text{ cm}^2$ (b) 231 cm^2 (c) 462 cm^2 (d) 924 cm^2
31. The areas of the largest circle, that can be drawn inside a rectangle with sides 18 cm and 14 cm, is [SSC]
(a) 49 cm^2 (b) 154 cm^2 (c) 378 cm^2 (d) 1078 cm^2
32. The area of a sector of a circle of radius, 5 cm, formed by an arc of length 3.5 cm, is [SSC]
(a) 7.5 cm^2 (b) 7.75 cm^2 (c) 8.5 cm^2 (d) 8.75 cm^2
33. What will be the area of a semi-circle whose perimeter is 36 cm? [BSRB]
(a) 154 cm^2 (b) 168 cm^2 (c) 308 cm^2
(d) Data inadequate (e) None of these
34. A circular park has a path of uniform width around it. The difference between outer and inner circumferences of the circular path is 132 m. Its width is [SSC]
(a) 20 m (b) 21 m (c) 22 m (d) 28 cm

35. Find the diameter of a wheel that makes 113 revolutions to go 2 km 26 decimetres. [SSC]
(a) $4\frac{4}{13}$ m (b) $6\frac{4}{11}$ m (c) $12\frac{4}{11}$ m (d) $12\frac{8}{11}$ m
36. Find the length of a rope by which a cow must be tethered in order that it may be able to graze an area of 9856 m^2 . [MAT]
(a) 64 m (b) 32 m (c) 56 m (d) 28 m
37. The radius of a wheel is 0.25 m. The number of revolutions it will make to travel a distance of 11 km will be [RRB]
(a) 2800 (b) 4000 (c) 5500 (d) 7000
38. If the ratio of areas of two circles is 4 : 9, then the ratio of their circumferences will be [RRB]
(a) 2 : 3 (b) 3 : 2 (c) 4 : 9 (d) 9 : 4
39. A wire bent in the form of a circle of radius 42 cm is bent in the form of a rectangle whose sides are in the ratio of 6 : 5. The smaller side of the rectangle is [SSC]
(a) 25 cm (b) 30 cm (c) 36 cm (d) 60 cm
40. Among a square of perimeter 44 cm and a circle of circumference 44 cm, which figure has larger area and by how much? [SSC]
(a) Both have equal area (b) Square, 33 cm^2
(c) Circle, 33 cm^2 (d) Square, 495 cm^2
41. In measuring the sides of a rectangle, one side is taken 5% in excess, and the other 4% in deficit. Find the error per cent in the area calculated from these measurements. [MBA]
(a) 2% (b) 1% (c) 0.8% (d) 0.6%
42. The area of a field in the shape of a trapezium measures 1440 m^2 . The perpendicular distance between its parallel sides is 24 m. If the ratio of the parallel sides is 5 : 3, the length of the larger parallel side is [SSC]
(a) 45 m (b) 60 m (c) 75 m (d) 120 m
43. One of the diagonals of a rhombus is double the other diagonal. Its area is 25 cm^2 . The sum of the diagonals is [SSC]
(a) 10 cm (b) 12 cm (c) 15 cm (d) 16 cm
44. The area of a rhombus is 150 cm^2 . The length of one of its diagonals is 10 cm. The length of the other diagonal is [SSC]
(a) 25 cm (b) 30 cm (c) 35 cm (d) 40 cm
45. A triangle and a parallelogram are constructed on the same base such that their areas are equal. If the altitude of the parallelogram is 100 m, then the altitude of the triangle is [MAT]
(a) $10\sqrt{2}$ m (b) 100 m (c) $100\sqrt{2}$ m (d) 200 m
46. If every side of a triangle is doubled, the area of the new triangle is K times the area of the old one. K is equal to [RRB]
(a) $\sqrt{2}$ (b) 2 (c) 3 (d) 4
47. The perimeter of a triangle is 30 cm and its area is 30 cm^2 . If the largest side measures 13 cm, then what is the length of the smallest side of the triangle? [SSC]
(a) 3 cm (b) 4 cm (c) 5 cm (d) 6 cm

48. The sides of a triangle are in the ratio of $\frac{1}{2} : \frac{1}{3} : \frac{1}{4}$. If the perimeter is 52 cm, then the length of the smallest side is [MAT]
 (a) 9 cm (b) 10 cm (c) 11 cm (d) 12 cm
49. The areas of a square and rectangle are equal. The length of the rectangle is greater than the length of any side of the square by 5 cm and the breadth is less by 3 cm. Find the perimeter of the rectangle. [SSC]
 (a) 17 cm (b) 26 cm (c) 30 cm (d) 34 cm
50. The length of one pair of opposite sides of a square is increased by 5 cm on each side, the ratio of the length and the breadth of the newly formed rectangle becomes 3 : 2. What is the area of the original square? [Bank PO]
 (a) 25 cm² (b) 81 cm² (c) 100 cm² (d) 225 cm² (e) None of these
51. If the area of a square increases by 69%, then the side of the square increases by [MAT]
 (a) 13% (b) 30% (c) 39% (d) 69%
52. A park square in shape has a 3 m wide road inside it running along its sides. The area occupied by the road is 1764 m². What is the perimeter along the outer edge of the road? [Bank PO]
 (a) 576 metres (b) 600 metres (c) 640 metres
 (d) Data inadequate (e) None of these
53. The perimeter of a square is 48 cm. The area of a rectangle is 4 cm² less than the area of the square. If the length of the rectangle is 14 cm, then its perimeter is [SSC]
 (a) 24 cm (b) 48 cm (c) 50 cm (d) 54 cm
54. A 2-metre broad pathway is to be constructed around a rectangular plot on the inside. The area of the plot is 96 m². The rate of construction is ₹ 50/m². Find the total cost of construction. [SBI PO]
 (a) ₹ 2400 (b) ₹ 4000 (c) ₹ 4800
 (d) Data inadequate (e) None of these
55. What will be the cost of gardening 1-metre broad boundary around a rectangular plot having perimeter of 340 metres at the rate of ₹ 10 per square metre? [Bank PO]
 (a) ₹ 1700 (b) ₹ 3400 (c) ₹ 3440
 (d) Cannot be determined (e) None of these
56. A typist uses a sheet measuring 20 cm × 30 cm lengthwise. If a margin of 2 cm is left on each side and a 3 cm margin on top and bottom, the per cent of the page used for typing is [MAT]
 (a) 40 (b) 60 (c) 64 (d) 72
57. The difference between two parallel sides of a trapezium is 4 cm. The perpendicular distance between them is 19 cm. If the area of the trapezium is 475 cm², find the length of the parallel sides. [RRB]
 (a) 29 cm, 25 cm (b) 28 cm, 24 cm (c) 27 cm, 23 cm (d) 26 cm, 22 cm
58. If the length and breadth of a rectangular plot be increased by 50% and 20% respectively, then how many times will its area be increased? [Bank PO]
 (a) $1\frac{1}{3}$ (b) 2 (c) $3\frac{2}{5}$ (d) $4\frac{1}{5}$ (e) None of these
59. The percentage increase in the area of a rectangle, if each of its sides is increased by 20%, is [MAT]
 (a) 40% (b) 42% (c) 44% (d) 46%

60. A rectangular paper, when folded into two congruent parts, had a perimeter of 34 cm for each part folded along one set of sides and the same is 38 cm when folded along the other set of sides. What is the area of the paper? [SSC]
(a) 140 cm^2 (b) 240 cm^2 (c) 560 cm^2 (d) None of these
61. A took 15 seconds to cross a rectangular field diagonally walking at the rate of 52 m/min and B took the same time to cross the same field along its sides walking at the rate of 68 m/min. The area of the field is [SSC]
(a) 30 m^2 (b) 40 m^2 (c) 50 m^2 (d) 60 m^2
62. If the length and breadth of a rectangular room are each increased by 1 m, then the area of floor is increased by 21 m^2 . If the length is increased by 1 m and breadth is decreased by 1 m, then the area is decreased by 5 m^2 . The perimeter of the floor is [MBA]
(a) 30 m (b) 32 m (c) 36 m (d) 40 m
63. The area of a triangle is 216 cm^2 and its sides are in the ratio 3 : 4 : 5. The perimeter of the triangle is [SSC]
(a) 6 cm (b) 12 cm (c) 36 cm (d) 72 cm
64. The ratio of bases of two triangles is $x : y$ and that of their areas is $a : b$. Then the ratio of their corresponding altitudes will be [SSC]
(a) $ax : by$ (b) $\frac{a}{x} : \frac{b}{y}$ (c) $ay : bx$ (d) $\frac{x}{a} : \frac{b}{y}$
65. A wire which when bent in the form of a square encloses an area of 484 cm^2 . What will be the enclosed area when the same wire is bent into the form of a circle? [SSC]
(a) 462 cm^2 (b) 539 cm^2 (c) 616 cm^2 (d) 693 cm^2
66. The number of revolutions of a wheel of diameter 40 cm makes in travelling a distance of 176 m, is [SSC]
(a) 140 (b) 150 (c) 160 (d) 166
67. If a wire is bent into the shape of a square, then the area of the square is 81 cm^2 . When the wire is bent into a semi-circular shape, the area of the semi-circle will be [SSC]
(a) 22 cm^2 (b) 44 cm^2 (c) 77 cm^2 (d) 154 cm^2
68. Four equal-sized circular plates of maximum diameter are cut off from a square paper sheet of area 784 cm^2 . The circumference of each plate is [SSC]
(a) 22 cm (b) 44 cm (c) 66 cm (d) 88 cm
69. If the perimeter of an isosceles right triangle is $(6 + 3\sqrt{2})\text{m}$, then the area of the triangle is [MAT]
(a) 4.5 m^2 (b) 5.4 m^2 (c) 9 m^2 (d) 81 m^2
70. The length of a rectangular hall is 5 m more than its breadth. The area of the hall is 750 m^2 . The length of the hall is [SSC]
(a) 15 m (b) 22.5 m (c) 25 m (d) 30 m
71. The area of a rectangle is 460 m^2 . If the length is 15% more than the breadth, what is the breadth of the rectangular field? [Bank PO]
(a) 15 metres (b) 26 metres (c) 34.5 metres
(d) Cannot be determined (e) None of these
72. The perimeters of five squares are 24 cm, 32 cm, 40 cm, 76 cm and 80 cm respectively. The perimeter of another square equal in area to the sum of the areas of these squares is [SSC]
(a) 31 cm (b) 62 cm (c) 124 cm (d) 961 cm

73. The number of marble slabs of size $20 \text{ cm} \times 30 \text{ cm}$ required to pave the floor of a square room of side 3 metres, is
 (a) 100 (b) 150 (c) 225 (d) 250
74. The difference of the areas of two squares drawn on two line segments of different lengths is 32 cm^2 . Find the length of the greater line segment if one is longer than the other by 2 cm. [SSC]
 (a) 7 cm (b) 9 cm (c) 11 cm (d) 16 cm
75. If the circumference and the area of a circle are numerically equal, then the diameter is equal to [SSC]
 (a) $\pi/2$ (b) 2π (c) 2 (d) 4



1. (d) 2. (b) 3. (b) 4. (c) 5. (b) 6. (c) 7. (c) 8. (c) 9. (b) 10. (a)
 11. (c) 12. (b) 13. (c) 14. (e) 15. (b) 16. (b) 17. (d) 18. (e) 19. (a) 20. (c)
 21. (c) 22. (d) 23. (c) 24. (c) 25. (c) 26. (d) 27. (c) 28. (c) 29. (c) 30. (c)
 31. (b) 32. (d) 33. (e) 34. (b) 35. (b) 36. (c) 37. (d) 38. (a) 39. (d) 40. (c)
 41. (c) 42. (c) 43. (c) 44. (b) 45. (d) 46. (d) 47. (c) 48. (d) 49. (d) 50. (c)
 51. (b) 52. (b) 53. (b) 54. (d) 55. (c) 56. (b) 57. (c) 58. (e) 59. (c) 60. (a)
 61. (d) 62. (d) 63. (d) 64. (c) 65. (c) 66. (a) 67. (c) 68. (b) 69. (a) 70. (d)
 71. (e) 72. (c) 73. (b) 74. (b) 75. (d)



Solutions with Necessary Explanation

1. Area of $\square ABCD = 2 \times 2 = 4 \text{ cm}^2$

$$\text{Area of 4 sectors} = 4 \times \frac{\pi}{4} r^2 = \pi r^2 = \pi(1)^2 = \pi \text{ cm}^2$$

$$\therefore \text{Unoccupied space} = 4 - 3.14 = 0.86 \text{ cm}^2$$

2. Distance to be travelled = 11 km = 11000 m

$$\text{Radius of wheel} = 1\frac{3}{4} \text{ m}$$

$$\therefore \text{Circumference of the wheel} = 2\pi r$$

$$= 2 \times \frac{22}{7} \times \frac{7}{4} = 11 \text{ m}$$

$$\therefore \text{In travelling 11000 m, the number of revolution required} = 11000/11 = 1000$$

3. Total cost of papering = ₹ 39.20

$$\text{Length of paper required} = \frac{13 \times 39.20}{4.20} = \frac{364}{3} \text{ m}$$

$$\text{Area of paper} = \frac{364}{3} \times \frac{75}{100} = 91 \text{ m}^2$$

$$\text{Area of walls} = 91 + 5 (\text{Opening}) = 96 \text{ m}^2$$

$$\therefore \text{Area of wall} = 2 \times (7 + 5) \times \text{Height} = 96 \text{ m}^2$$

$$\text{i.e. } 24 \times \text{height} = 96 \text{ m}^2$$

$$\therefore \text{height} = 96/24 = 4 \text{ m}$$

$$4. \quad \text{Let height} = h \text{ metres}$$

$$\therefore \text{Breadth} = 2h \text{ metres}$$

$$\text{Area of walls} = 2(l + b) \times h = 2(16 + 2h) \times h \text{ m}^2$$

$$\therefore \text{Area of paper} = 168 \times 2 = 336 \text{ m}^2$$

$$\therefore 2(16 + 2h)h = 336 \text{ m}^2$$

$$\therefore (8 + h)h = 84$$

$$\text{Solving } h = 6 \text{ m}$$

$$\therefore \text{Breadth } b = 2h = 12 \text{ m}$$

$$\text{Area of floor} = l \times b = 16 \times 12 = 192 \text{ m}^2$$

$$5. \text{ Area of rhombus} = \frac{1}{2} (\text{product of diagonals}) = \frac{1}{2} \times (10 \times 12) = 60 \text{ cm}^2$$

$$6. \alpha^2 = 69696 \text{ cm}^2; \quad \text{diagonal} = a\sqrt{2} = \sqrt{a^2} \times \sqrt{2} \\ = \sqrt{69696} \sqrt{2} = 264\sqrt{2} \text{ cm} = 373.296$$

$$7. \frac{A_1}{A_2} = \frac{a_1^2}{a_2^2} = \frac{225}{256} = \frac{15^2}{16^2}$$

$$\therefore \text{Ratio of perimeters} = \frac{4a_1}{4a_2} = \frac{a_1}{a_2} = \frac{15}{16}$$

$$8. \text{ Distance through sides} = a + a = 2a$$

$$\text{Distance along diagonal} = a\sqrt{2}$$

$$\therefore \text{Per cent saved} = \frac{2a - a\sqrt{2}}{2a} \times 100 \\ = \frac{2 - \sqrt{2}}{2} \times 100 \\ = \frac{0.586}{2} \times 100 = 29.3\% \approx 30\%$$

$$9. \text{ Let width of road be } x \text{ metres.}$$

$$\begin{aligned} \text{Area of road} &= 60 \times 40 - 2109 \\ &= 2400 - 2109 = 291 \text{ m}^2 \end{aligned}$$

$$\text{Length of road} = 60 + (40 - x) \text{ metres.}$$

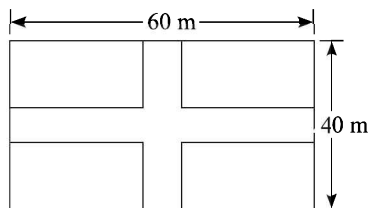
$$\therefore \text{Area of road} = (100 - x)x = 291$$

$$\text{i.e. } x^2 - 100x + 291 = 0$$

$$\therefore (x - 3)(x - 97) = 0$$

$$\therefore x = 3 \text{ or } 97 \text{ metres.}$$

$$\therefore \text{Width of road} = 3 \text{ metres}$$



10. Let the percentage by which width have to be decreased be x .

∴ Resultant effect due to increase and decrease

$$= 60 - x - \frac{60x}{100} = 0$$

i.e. $60 - x - \frac{3}{5}x = 0$

i.e. $60 - \frac{8}{5}x = 0$

$$\Rightarrow x = \frac{60 \times 5}{8} = 37.5\%$$

11. Let speed of smaller wheel be x cm/sec.

∴ Distance travelled in 10 sec

$$= x \times 10 = 10x \text{ cm}$$

∴ Speed of larger wheel = $2x$ cm/sec

∴ Distance travelled in 10 sec

$$= 2x \times 10 = 1980 - 10x$$

$$\therefore 30x = 1980$$

or $x = \frac{1980}{30} = 66 \text{ cm/sec}$

12. Circumference of field = $2\pi r = 2\pi \times 50 = 100\pi$ metres

$$\begin{aligned} \therefore \text{For 20 rounds, distance to be covered} &= 100\pi \times 20 \\ &= 2000\pi \text{ metres} \end{aligned}$$

$$\begin{aligned} \therefore \text{Time taken for 20 rounds} &= \frac{\text{Distance}}{\text{Speed}} \\ &= \frac{2000\pi \text{ m}}{12 \times \frac{5}{18} \text{ m/s}} = \frac{2000 \times 18}{60} \pi \text{ sec} \\ &= 600\pi \text{ sec} \\ &= \frac{600\pi}{60} \text{ min} = 10\pi = 31.4 \approx 32 \text{ minutes} \end{aligned}$$

13. Let breadth of rectangle be x centimetres

∴ Length of rectangle = $1.6x$ centimetres

Given $1.6x - x = 24 \text{ cm}$

or $0.6x = 24$

or $x = \frac{24}{0.6} = 40 \text{ cm}$

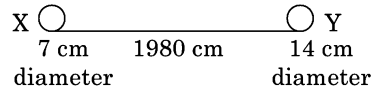
∴ Area of rectangle = $64 \times 40 = 2560 \text{ cm}^2$

14. Let length of plot be x metres

∴ Breadth of plot = $(x - 20)$ metres

∴ Perimeter of plot = $2[x + (x - 20)] = (4x - 40)$ metres

∴ Cost of fencing at ₹ 26.5/m = $(4x - 40) \times 26.5 = 5300$



$$\text{i.e.,} \quad 4x - 40 = \frac{5300}{26.5} = 200$$

$$\therefore \quad x = \frac{200 + 40}{4} = \mathbf{60 \text{ metres}}$$

15. $l + 2b = 30$.

From the options, $20 + 2(5) = 30$.

$\therefore$ Dimensions are $20 \text{ m} \times 5 \text{ m}$.

16. Area of rectangle = ab .

$$\begin{aligned} \text{Area of new rectangle} &= (1.1a)(0.8b) = 0.88ab \\ &= \mathbf{88\% \text{ of } ab} \end{aligned}$$

17. From $\triangle ABC$,

$$OA = OC = \frac{48}{2} = 24 \text{ cm.}$$

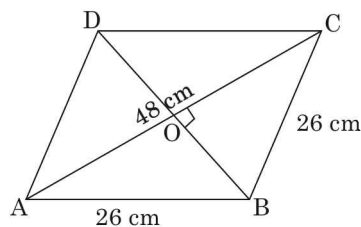
From $\triangle OAB$,

$$\begin{aligned} (OB)^2 &= (AB)^2 - (OA)^2 \\ &= (26)^2 - (24)^2 \\ &= (26 + 24)(26 - 24) \\ &= 50 \times 2 = 100 \end{aligned}$$

$$\therefore \quad OB = 10 \text{ cm}$$

$$\Rightarrow \quad DB = 2 \times OB = 2 \times 10 = 20 \text{ cm}$$

$$\therefore \quad \text{Area} = \frac{1}{2} \times AC \times DB = \frac{1}{2} \times 48 \times 20 = \mathbf{480 \text{ cm}^2}$$



18. Perimeter of circle = $2(18 + 26) = 88 \text{ cm} = 2\pi r$

or $r = \frac{44}{\pi} \text{ cm}$

$$\text{Area of circle} = \pi r^2 = 2\pi r \times \frac{r}{2} = 88 \times \frac{44}{2\pi} = \frac{88 \times 22}{\frac{22}{7}} = 88 \times 7 = \mathbf{616 \text{ cm}^2}$$

$$\begin{aligned} \text{19. Area of garden} &= \pi [(17.5 + 1.4)^2 - (17.5)^2] \\ &= \pi [18.9^2 - 17.5^2] \\ &= \pi [(18.9 + 17.5)(18.9 - 17.5)] = \pi \times 36.4 \times 1.4 \\ &= \frac{22}{7} \times 36.4 \times 1.4 = 22 \times 5.2 \times 1.4 = 114.4 \times 1.4 = \mathbf{160.16 \text{ m}^2} \end{aligned}$$

20. Circumference $\propto$ radius

$\therefore$ On increasing radius by 75%, its circumference will also increase by **75%**.

21. $X = \frac{\sqrt{3}}{4} a^2$ or $a^2 = \frac{4X}{\sqrt{3}}$

$$\text{Perimeter of triangle} = 3a = 3 \times (4x)^{1/2} \times 3^{-1/4}$$

$$\text{i.e.} \quad 3a = 6x^{1/2} 3^{-1/4} \quad (1)$$

$$Y = b^2 \quad \text{or} \quad b = Y^{1/2}$$

$$\text{Perimeter of square} = 4b = 4Y^{1/2} \quad (2)$$

Equating (1) and (2), since perimeters are the same.

$$6X^{1/2} 3^{-1/4} = 4Y^{1/2}$$

$$\begin{aligned} \text{i.e.} \quad 2 \times 3^{1-1/4} \cdot X^{1/2} &= 2^2 \cdot Y^{1/2} \\ 3^{3/4} \cdot X^{1/2} &= 2Y^{1/2} \end{aligned}$$

Squaring both sides,

$$\begin{aligned} (3^{3/4})^2 \cdot (X^{1/2})^2 &= (2Y^{1/2})^2 \\ \text{i.e.} \quad 3^{3/2} \cdot X &= 4Y \end{aligned}$$

$$\therefore X = \frac{4}{3^{3/2}} Y.$$

$$\therefore X < Y$$

$$22. \quad a^2 \left[\frac{(1.02)^2 - (1.0)^2}{a^2} \right] \times 100 = 4.04\%$$

Required percentage of error = **4.04%**

23. Circumference $\propto$ Diameter

$\therefore$ On increasing diameter to 10 times, circumference will increase to 10 times its original length.

Speed of A = 8 rounds/40 min = 1 round/5 min.

$\therefore$ Distance to be travelled increased to 10 times.

New speed of A = $\frac{1 \text{ round}}{5 \times 10 \text{ min}} = 1 \text{ round/50 min}$

24. Equating perimeters of equilateral triangle, square and circle, we have

$$3a = 4b = 2\pi r \tag{1}$$

$$\text{Area of equilateral triangle} = \frac{\sqrt{3}}{4} a^2$$

$$\text{i.e.} \quad T = \frac{\sqrt{3}}{4} a^2 \tag{2}$$

$$\text{Similarly} \quad S = b^2 \tag{3}$$

$$\text{Similarly} \quad C = \pi r^2 \tag{4}$$

$$\text{From (1),} \quad 3a = 4b$$

$$\text{or} \quad b = \frac{3}{4} a$$

$$\text{From (3),} \quad S = \left(\frac{3}{4} a \right)^2 = \frac{9}{16} a^2 \tag{5}$$

$$\text{From (1),} \quad 3a = 2\pi r \quad \text{or} \quad r = \frac{3a}{2\pi}$$

$$\text{From (4),} \quad C = \pi \times \left(\frac{3a}{2\pi} \right)^2 = \frac{9}{4} \frac{a^2}{\pi} \tag{6}$$

Comparing (2), (5) and (6),

$$T = \frac{\sqrt{3}}{4} a^2; \quad S = \frac{9}{16} a^2; \quad C = \frac{9}{4\pi} a^2$$

$$\therefore T < S < C$$

25. Let r be radius of incircle

i.e.

$$FO = OE = OD = Y$$

Given

$$a = 11 \text{ cm}; \quad b = 6 \text{ cm}; \quad c = 15 \text{ cm}$$

$\therefore$

$$S = \frac{a+b+c}{2} = \frac{11+6+15}{2} = 16 \text{ cm}$$

$$\begin{aligned} \text{Area} &= \sqrt{s(s-a)(s-b)(s-c)} \\ &= \sqrt{16 \times (16-11)(16-6)(16-15)} \\ &= \sqrt{16 \times 5 \times 10 \times 1} = 20\sqrt{2} \text{ cm}^2 \end{aligned}$$

$$\therefore \text{Radius of incircle} = \frac{\Delta}{S} = \frac{20\sqrt{2}}{16} = \frac{5\sqrt{2}}{4} \text{ cm}.$$

26. Area of circle $= \pi r^2 = \pi \times (4)^2 = 16\pi \text{ cm}^2$

Diagonal of inscribed square = Radius of circle = 4 cm

$$\therefore \text{Side of square} = \frac{4 \times 2}{\sqrt{2}} = 4\sqrt{2} \text{ cm}$$

$$\therefore \text{Area of square} = (4\sqrt{2})^2 = 32 \text{ cm}^2$$

$$\therefore \text{Required area} = (16\pi - 32) \text{ cm}^2/\text{m}$$

27. Area of circle, $\pi r^2 = \frac{22}{7} \times r^2 = 220 \text{ cm}^2$

$$\therefore r = \sqrt{\frac{220}{22} \times 7} = \sqrt{70} \text{ cm}.$$

Diagonal of square inscribed = Diameter of circle

$$= 2\sqrt{70} \text{ cm}$$

$$\therefore \text{Side of square inscribed} = \frac{2\sqrt{70}}{\sqrt{2}} = \sqrt{140} \text{ cm}$$

$$= 2\sqrt{35} \text{ cm}$$

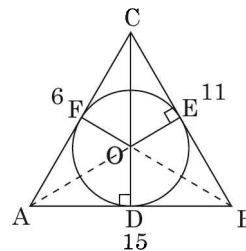
$$\begin{aligned} \therefore \text{Area of square} &= (2\sqrt{35})^2 \\ &= 4 \times 35 \\ &= 140 \text{ cm}^2 \end{aligned}$$

28. Total surface area of tank $= [2(25 + 12) \times 6] + 25 \times 12$

$$= 444 + 300 = 744 \text{ m}^2$$

$$\therefore \text{Cost of plastering at } 75 \text{ paise/m}^2 = 744 \times 0.75$$

$$= \frac{2232}{4} = \text{₹ } 558$$



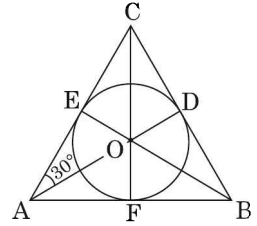
$$\begin{aligned} 29. \text{ Radius} &= \frac{\text{Area}}{\text{Semi-perimeter}} = \frac{\text{Area}}{\text{Area}/2} \\ &= \text{Area} \times \frac{2}{\text{Area}} = 2 \end{aligned}$$

30. $AB = BC = AC = 42 \text{ cm};$

$$OA = \frac{AE}{\cos 30^\circ} = \frac{AC}{2 \cos 30^\circ} = \frac{42}{2 \times \frac{\sqrt{3}}{2}} = 14\sqrt{3} \text{ cm}$$

$$\therefore OE = OA \sin 30^\circ = 14\sqrt{3} \times \frac{1}{2} = 7\sqrt{3} \text{ cm}$$

$$\begin{aligned} \text{Area of incircle} &= \pi r^2 = \pi (OE)^2 \\ &= \pi (7\sqrt{3})^2 \\ &= 49 \times 3 \times \frac{22}{7} \\ &= 462 \text{ cm}^2 \end{aligned}$$



31. Required area $= \pi \times \left(\frac{14}{2}\right)^2 = 7^2 \times \frac{22}{7} = 154 \text{ cm}^2$

32. Given $R = 5 \text{ cm}; S = 3.5 \text{ cm}$

$$\theta = \frac{S}{R} = \frac{3.5}{5} = 0.7 \text{ rad}$$

$$\text{Area} = \frac{1}{2} R^2 \theta = \frac{1}{2} \times 5^2 \times 0.7 = 8.75 \text{ cm}^2$$

33. Given $\pi r + 2r = 36 \text{ cm}$
i.e. $(\pi + 2)r = 36 \text{ cm}$

$$\therefore r = \frac{36}{\pi + 2} = \frac{36}{\frac{22}{7} + 2} = \frac{36 \times 7}{(22 + 14)} = 7 \text{ cm}$$

$$\text{Area of semi-circle} = \frac{1}{2} \pi r^2 = \frac{1}{2} \times \frac{22}{7} \times 7^2 = 77 \text{ cm}^2$$

34. Let width of path be x metres

$$2\pi R - 2\pi r = 2\pi (R - r) = 2\pi \times x = 132 \text{ m}$$

$$\therefore x = \frac{132}{2\pi} = \frac{132}{2 \times \frac{22}{7}} = \frac{132 \times 7}{2 \times 22} = 21 \text{ m}$$

35. Let diameter of wheel be x metres

$\therefore$ Circumference of wheel $= \pi \times x$ metres

$\therefore$ In 113 revolution, distance covered $= 113 \times \pi \times x$

$$\begin{aligned} \therefore 113 \times \pi \times x &= 2 \text{ km } 26 \text{ decametres} \\ &= 2000 \text{ m} + 260 \text{ m} = 2260 \text{ m} \end{aligned}$$

$$\therefore x = \frac{2260}{113 \times \frac{22}{7}} = \frac{2260 \times 7}{113 \times 22} = \frac{10 \times 7}{11} = 6 \frac{4}{11} \text{ m}$$

36. Let the length of rope be l metres

$$\therefore \pi l^2 = 9856$$

$$\Rightarrow l^2 = \frac{9856}{\pi} = \frac{9856}{\frac{22}{7}} = \frac{9856 \times 7}{22} = 448 \times 7 = 64 \times 7^2$$

$$\therefore l = 8 \times 7 = \mathbf{56 \text{ metres}}$$

37. $r = 0.25$ m;

Let number of revolutions made be N .

$$\therefore N \times 2\pi \times r = 11 \text{ km}$$

$$\text{i.e. } N \times 2 \times \frac{22}{7} \times 0.25 = 11 \times 1000 \text{ m.}$$

$$\therefore N = \frac{11000 \times 7}{2 \times 22 \times 0.25} = \mathbf{7000}$$

38. Area of circle $\propto (\text{Radius})^2$

$$\therefore \frac{a_1}{a_2} = \left(\frac{r_1}{r_2} \right)^2 = \frac{4}{9} \text{ or } \frac{r_1}{r_2} = \frac{2}{3}.$$

Perimeter $\propto$ Radius

$$\therefore \frac{p_1}{p_2} = \frac{r_1}{r_2} = \frac{2}{3} \text{ or } \mathbf{2 : 3}$$

39. Circumference of circle $= 2\pi \times r = 2\pi \times 42 = 2 \times \frac{22}{7} \times 42 = 264$ cm

Let smaller side of rectangle be x .

$$\therefore \text{Larger side} = \frac{6}{5}x$$

$$\therefore \text{Perimeter of rectangle} = 2 \left(x + \frac{6}{5}x \right) = 264$$

$$\text{i.e. } 2 \times \frac{11x}{5} = 264$$

$$\therefore x = \frac{264 \times 5}{22} = 12 \times 5 = \mathbf{60 \text{ cm}}$$

40. Perimeter of square $= 44$ cm

$$\therefore \text{Its area} = (11)^2 = 121 \text{ cm}^2$$

$$\text{Perimeter of circle} = 2\pi r = 44 \text{ cm}$$

$$\text{i.e. } 2 \times \frac{22}{7} \times r = 44$$

$$\text{or } r = 7 \text{ cm}$$

$$\therefore \text{Its area} = (\pi r)^2 = \pi \times (7)^2 = \frac{22}{7} \times (7)^2 = \mathbf{154 \text{ cm}^2}$$

Area of circle is larger by 33 cm^2 .

41. Let l and b be the length and breadth of rectangle.

Then $l(1.05) \times b(0.96) = 1.0080 lb$.

$\therefore$ Error per cent in area calculated

$$= \frac{1.0080 lb - lb}{lb} \times 100 = 0.8\%$$

42. Area of trapezium ABCD = $\frac{1}{2}(5x + 3x) \times 24 = 96x \text{ m}^2$

$$\therefore 96x = 1440$$

$$\therefore x = \frac{1440}{96} = 15 \text{ m}$$

$$\therefore \text{Larger side} = 5x = 5 \times 15 = 75 \text{ m}$$

43. Let the diagonals be d_1 and d_2 .

Given $d_1 = 2d_2$;

$$\text{Area} = \frac{1}{2}d_1d_2 = \frac{1}{2}(2d_2)(d_2) = 25 \text{ cm}^2$$

$$\therefore d_2^2 = 25 \text{ cm}^2$$

$$\Rightarrow d_2 = 5 \text{ cm}$$

$$\therefore d_1 = 2d_2 = 10 \text{ cm}$$

$$\therefore \text{Sum of diagonals} = 10 + 5 = 15 \text{ cm}$$

44. Area = $\frac{1}{2}d_1d_2 = \frac{1}{2} \times 10 \times d_2 = 150$

$$\therefore d_2 = \frac{150}{5} = 30 \text{ cm}$$

45. Area of triangle = $\frac{1}{2}bh_1$ (1)

$$\text{Area of parallelogram} = bh_2 \quad (2)$$

Equating (1) and (2),

$$\frac{1}{2}bh_1 = bh_2$$

$$\text{or } h_1 = 2h_2 \quad (3)$$

$$\text{Given } h_2 = 100 \text{ m.}$$

From (3), we have

$$h_1 = 2 \times 100 = 200 \text{ m}$$

46. Area of triangle = $\sqrt{s(s-a)(s-b)(s-c)} = \Delta$

If every side is doubled, new area will be

$$\sqrt{s(s-a)(s-b)(s-c) \times 2 \times 2 \times 2 \times 2} = 4\Delta$$

$$\therefore K = 4.$$

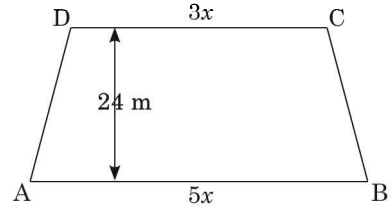
47. Perimeter $2s = a + b + c = 30 \text{ cm}$;

$$\text{Area} = \sqrt{s(s-a)(s-b)(s-c)} = 30$$

$$\text{i.e. } \sqrt{15(15-a)(15-b)(15-c)} = 30$$

$$(15-a)(15-b)(15-c) = 30$$

$$\therefore \text{Smallest side} = 5 \text{ cm.}$$



48. $a : b : c = \frac{1}{2} : \frac{1}{3} : \frac{1}{4} = 6 : 4 : 3$

Let sides be $6x$, $4x$ and $3x$ centimetres

$\therefore$ Perimeter $= 6x + 4x + 3x = 52$ cm

or $x = \frac{52}{13} = 4$

$\therefore$ Smallest side $= 3x = 3 \times 4 = 12$ cm

49. Let length of side of square be a centimetre.

$\therefore$ Length of rectangle $= (a + 5)$ cm

Breadth of rectangle $= (a - 3)$ cm

Given $a^2 = (a + 5)(a - 3)$

$\Rightarrow a^2 + 2a - 15 = a^2$

$\Rightarrow 2a - 15 = 0$

$\Rightarrow a = \frac{15}{2}$

$\therefore$ Perimeter of rectangle $= 2[(a + 5) + (a - 3)]$

$$= 2[2a + 2] = 4a + 4 = 4 \times \frac{15}{2} + 4 = 34 \text{ cm}$$

50. Let the side of square be x centimetres.

Length of rectangle $= (x + 5)$ centimetres

Now, $\frac{x+5}{x} = \frac{3}{2}$

or $3x = 2(x + 5)$

$\Rightarrow x = 10$ cm.

$\therefore$ Area of original square $= x^2 = 10^2 = 100$ cm²

51. Let the side of square be increased by $x\%$

$\therefore x + x + \frac{x^2}{100} = 69$

i.e. $2x + \frac{x^2}{100} = 69$

or $x^2 + 200x = 6900$

$x^2 + 200x - 6900 = 0$

or $(x + 230)(x - 30) = 0$

$\therefore x = 30\%$

52. Let the side along the outer edge of road be x .

$\therefore$ Side along the inner edge of road $= x - 6$

Area occupied by the road $= x^2 - (x - 6)^2 = 1764$

i.e. $[x + (x - 6)][x - (x - 6)] = 1764$

i.e. $(2x - 6)6 = 1764$

or $2x - 6 = 294$

$$\therefore x = \frac{294 + 6}{2} = 150$$

$$\therefore \text{Perimeter along the outer edge of road} \\ = 4 \times 150 = \mathbf{600 \text{ metres}}$$

53. Perimeter of square = 48 cm

$$\therefore \text{Side of square} = \frac{48}{4} = 12 \text{ cm}$$

$$\text{Area of square} = (12)^2 = 144 \text{ cm}^2$$

$$\text{Area of rectangle} = 144 - 4 = 140 \text{ cm}^2$$

$$\text{Given length of rectangle} = 14 \text{ cm}$$

$$\therefore \text{Breadth of rectangle} = \frac{\text{Area}}{\text{Length}} = \frac{140}{14} = 10 \text{ cm}$$

$$\therefore \text{Perimeter of rectangle} = 2(l + b) = 2(14 + 10) = \mathbf{48 \text{ cm}}$$

54. Let l and b be the length and breadth of plot.

$$\therefore lb = 96 \text{ m}^2$$

$$\text{Area of path} = (l - 4)(b - 4) - lb = 16 - 4(l + b)$$

$$\therefore l \text{ and } b \text{ are not known, value of area of path can't be determine.}$$

$$\therefore \mathbf{\text{Data is inadequate.}}$$

55. Perimeter = 340 cm; Width of garden = 1 m

$$\therefore \text{Perimeter of outer edge of garden} = 348 \text{ m}$$

$$\therefore \text{Average perimeter} = \frac{340 + 348}{2} = 344 \text{ m}$$

$$\therefore \text{Cost of gardening at ₹ } 10/\text{m}^2 = (344 \times 1) \times 10 \\ = \mathbf{₹ 3440}$$

56. Total area = $20 \times 30 = 600 \text{ cm}^2$

$$\therefore \text{Area left for margin} = (30 \text{ cm} \times 2 \text{ cm}) \times 2 + (20 \text{ cm} \times 3 \text{ cm}) \times 2 = 240 \text{ cm}^2$$

$$\therefore \text{Per cent of page used} = \frac{(600 - 240)}{600} \times 100 \\ = \frac{360}{600} \times 100 \\ = \mathbf{60\%}$$

57. Let l_1 and l_2 be the parallel sides of trapezium.

$$l_1 = l_2 + 4; \quad h = 19 \text{ cm};$$

$$\therefore \text{Area} = \frac{h}{2}(l_1 + l_2) = \frac{19}{2}(l_2 + 4 + l_2) = 475 \text{ cm}^2$$

$$\text{or } 2l_2 + 4 = \frac{475 \times 2}{19} = 25 \times 2 = 50$$

$$\therefore l_2 = \frac{50 - 4}{2} = \mathbf{23 \text{ cm}}$$

$$\therefore l_1 = l_2 + 4 = 23 + 4 = \mathbf{27 \text{ cm}}$$

$$\begin{aligned}
 \text{58. Increase in area by percentage} &= 50 + 20 + \frac{50 \times 20}{100} \\
 &= 70 + 10 = 80\%
 \end{aligned}$$

$$\therefore \text{Total increase} = \frac{80}{100} = \frac{4}{5} \text{ times}$$

59. Percentage increase in area of rectangle

$$= 20 + 20 + \frac{20 \times 20}{100} = 44\%$$

60. Let l and b be the length and breadth of paper.

When folded along breadth,

$$2\left(\frac{l}{2} + b\right) = 34 \quad \text{or} \quad l + 2b = 34 \quad (1)$$

When folded along length,

$$2\left(l + \frac{b}{2}\right) = 38 \quad \text{or} \quad 2l + b = 38 \quad (2)$$

Solving (1) and (2), we get,

$$2 \times (2) - (1) \Rightarrow 3l = 76 - 34 = 42$$

$$\text{or} \quad l = 14 \text{ cm.}$$

$$\therefore (2) \Rightarrow b = 38 - 2l$$

$$= 38 - 28 = 10 \text{ cm}$$

$$\therefore \text{Area of paper} = 14 \times 10 = 140 \text{ cm}^2$$

$$\begin{aligned}
 \text{61. Length of diagonal} &= \frac{52 \text{ m}}{60 \text{ sec}} \times 15 \text{ sec} \\
 &= 13 \text{ m}
 \end{aligned}$$

Let length of semi perimeter be x .

$$\begin{aligned}
 \therefore x &= \frac{68 \text{ m}}{60 \text{ sec}} \times 15 \text{ sec} \\
 &= 17 \text{ m}
 \end{aligned}$$

$$\therefore \text{Length} = 12 \text{ m and width} = 5 \text{ m}$$

$$\text{Area of field} = 12 \times 5 = 60 \text{ m}^2$$

62. Let length and breadth of rectangle be l and b .

$$\therefore (l + 1)(b + 1) - lb = 21 \text{ m}^2$$

$$\text{i.e.} \quad lb + l + b + 1 - lb = 21 \text{ m}^2$$

$$\text{or} \quad l + b + 1 = 21$$

$$\text{or} \quad l + b = 20 \quad (1)$$

$$(l + 1)(b - 1) - lb = -5$$

$$lb - l + b - 1 - lb = -5$$

$$\text{or} \quad -l + b - 1 = -5$$

$$l - b = 4 \quad (2)$$

Solving (1) and (2), we get

$$l = 12 \text{ m}; \quad b = 8 \text{ m};$$

$$\therefore \text{Perimeter of floor} = 2(l + b) = 2(12 + 8) = \mathbf{40 \text{ m}}$$

63. Area of triangle = Δ

$$S = \frac{3x + 4x + 5x}{2} = 6x$$

$$S - 3x = 3x; \quad S - 4x = 2x; \quad S - 5x = x;$$

$$\Delta = \sqrt{s(s-a)(s-b)(s-c)}$$

$$= \sqrt{6x \times 3x \times 2x \times x} = 6x^2$$

i.e.,

$$6x^2 = 216$$

or

$$x^2 = 36$$

or

$$x = 6 \text{ cm}$$

$$\therefore \text{Perimeter of triangle} = 3x + 4x + 5x = 12x$$

$$= 12 \times 6 = \mathbf{72 \text{ cm}}$$

$$\mathbf{64.} \quad \frac{b_1}{b_2} = \frac{x}{y}; \quad \frac{\Delta_1}{\Delta_2} = \frac{a}{b}$$

$$\therefore \quad \Delta_1 = \frac{1}{2} b_1 h_1 \quad (1)$$

$$\text{i.e.} \quad \Delta_2 = \frac{1}{2} b_2 h_2 \quad (2)$$

$$\frac{(1)}{(2)} \Rightarrow \frac{\Delta_1}{\Delta_2} = \frac{b_1 h_1}{b_2 h_2} = \frac{b_1}{b_2} \times \frac{h_1}{h_2}$$

$$\text{i.e.} \quad \frac{a}{b} = \frac{x}{y} \times \frac{h_1}{h_2}$$

$$\therefore \quad \frac{h_1}{h_2} = \frac{a y}{b x} = \mathbf{a y : b x}$$

65. Area of square = 484 cm^2

$$\therefore \text{Perimeter of square} = 4 \times \sqrt{484} = 4 \times 22 = 88 \text{ cm}$$

When it is bent into a circle,

$$\pi d = 88 \text{ cm}$$

$$\text{or} \quad \frac{22}{7} d = 88$$

$$\text{or} \quad d = \frac{88 \times 7}{22} = 28 \text{ cm}$$

$$\therefore \text{Area of circle} = \frac{\pi d^2}{4} = \frac{22}{7} \times \left(\frac{28}{2}\right)^2$$

$$= 22 \times 2 \times 14$$

$$= \mathbf{616 \text{ cm}^2}$$

66. Circumference of wheel having diameter 40 cm

$$= \pi d = \frac{22}{7} \times 40 = \frac{880}{7} \text{ cm}$$

Number of revolution required to travel 176 m

$$= \frac{176 \text{ m}}{\pi d} = \frac{176 \times 100}{\frac{880}{7}} = \frac{176 \times 100 \times 7}{880} = \mathbf{140}$$

67. Area of square = 81 cm^2

$$\therefore \text{Length of wire} = 4 \times \sqrt{81} = 36 \text{ cm}$$

$$\text{Periphery of semi-circle} = 2r + \pi r = (\pi + 2)r = 36$$

$$\therefore r = \frac{36}{(\pi + 2)} = \frac{36}{\left(\frac{22}{7} + 2\right)} = \frac{36 \times 7}{(22 + 14)} = 7 \text{ cm}$$

$$\begin{aligned} \therefore \text{Area of semi-circle} &= \frac{\pi r^2}{2} = \frac{\pi \times 7^2}{2} \\ &= \frac{22}{7} \times \frac{7^2}{2} = \mathbf{77 \text{ cm}^2} \end{aligned}$$

68. Side of square sheet = $\sqrt{784} = 28 \text{ cm}$

Maximum diameter of a circular plate = 14 cm

$$\therefore \text{Circumference of each plate} = \pi d = \frac{22}{7} \times 14 = \mathbf{44 \text{ cm}}$$

69. Side of isosceles triangle = $\frac{6 + 3\sqrt{2}}{(1 + 1 + \sqrt{2})} = 3 \text{ m}$

$$\therefore \text{Area of triangle} = \frac{1}{2}bh = \frac{1}{2} \times 3 \times 3 = \mathbf{4.5 \text{ m}^2}$$

70. Let length of hall be l metres

$$\therefore \text{breadth of hall} = (l - 5) \text{ metres.}$$

$$\text{Area of hall} = l(l - 5) = 750$$

$$\Rightarrow l^2 - 5l - 750 = 0$$

$$\text{i.e. } (l - 30)(l + 25) = 0$$

$$\therefore l = \mathbf{30 \text{ metres}}$$

71. Let breadth of rectangle be b metres

$$\therefore \text{Length of rectangle} = 1.15b \text{ metres.}$$

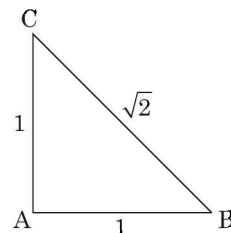
$$\text{Area of rectangle} = 1.15b \times b = 460$$

$$\text{i.e. } 1.15 b^2 = 460$$

$$\text{or } b^2 = \frac{460}{1.15} = 400$$

$$\therefore b = \mathbf{20 \text{ metres}}$$

$$\begin{aligned} 72. \text{ Sum of areas of squares} &= \left(\frac{24}{4}\right)^2 + \left(\frac{32}{4}\right)^2 + \left(\frac{40}{4}\right)^2 + \left(\frac{76}{4}\right)^2 + \left(\frac{80}{4}\right)^2 \\ &= \mathbf{6^2 + 8^2 + 10^2 + 19^2 + 20^2} \end{aligned}$$



$$= 36 + 64 + 100 + 361 + 400 \\ = 961$$

$$\therefore \text{Side of large square} = \sqrt{961} = 31 \text{ cm}$$

$$\therefore \text{Perimeter of large square} = 4 \times 31 = \mathbf{124 \text{ cm}}$$

73. Area of room = $3 \text{ m} \times 3 \text{ m} = 9 \text{ m}^2$

$$\text{Area of 1 marble slab} = 20 \text{ cm} \times 30 \text{ cm} = 600 \text{ cm}^2$$

$$\therefore \text{Number of slabs required to pave the floor} = \frac{9 \text{ m}^2}{600 \text{ cm}^2} = \frac{9 \times 100 \times 100}{600} = \mathbf{150}$$

74. Let length of greater line segment be x centimetres

$$\therefore \text{Length of smaller line segment} = (x - 2) \text{ cm}$$

$$\text{Area of square formed by first line segment} = (x)^2$$

$$\text{Area of square formed by second line segment} = (x - 2)^2$$

$$\text{Given} \quad (x)^2 - (x - 2)^2 = 32$$

$$\Rightarrow (x + x - 2)2 = 32$$

$$2x - 2 = 16$$

$$\text{or} \quad x = \frac{16 + 2}{2} = 9$$

$$\therefore \text{Length of greater line segment} = \mathbf{9 \text{ cm}}$$

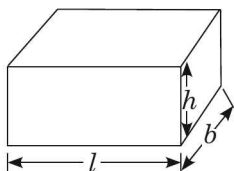
75. $2\pi r = \pi r^2$

$$\text{or} \quad r = 2$$

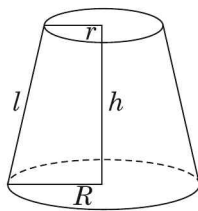
$$\therefore \text{Diameter} = 4$$

Elementary Mensuration II (Measurement of Volume and Surface Areas)

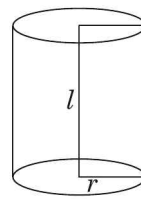
An object which occupies space has three dimensions: length, breadth and depth. Such an object is called a *solid*. Some commonly known solids are shown in the following figures:



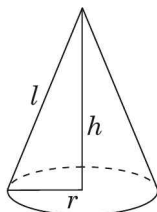
Cuboid



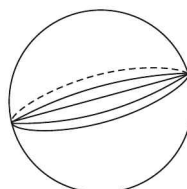
Frustum of a cone



Cylinder



Cone



Sphere



Important Formulae

CUBOID

Let length = l ; breadth = b and height = h units

1. Volume of cuboid = $l \times b \times h$ cubic units = $\sqrt{A_1 \times A_2 \times A_3}$
where A_1 = Area of base or top; A_2 = Area of side face; A_3 = Area of other side face.
2. Area of whole surface of cuboid = $2(lb + bh + lh)$ sq. units.

3. Diagonal of cuboid = $\sqrt{l^2 + b^2 + h^2}$ units.

CUBE

Let the edge of cube be a units. Then

4. Volume of cube = a^3 cubic units.
5. Whole surface area of cube = $6a^2$ sq. units
6. Diagonal of cube = $\sqrt{3} a$ units.

CYLINDER

Let the radius of the base of cylinder be r units and height be h units. Then

7. Volume of cylinder = $\pi r^2 h$ cubic units.
8. Curved surface area of cylinder = $2\pi r h$ sq. units.
9. Total surface area of cylinder = $(2\pi r h + 2\pi r^2) = 2\pi r(h + r)$ sq. units.

SPHERE

Let the radius of sphere be r units. Then

10. Volume of sphere = $4/3\pi r^3$ cubic units.
11. Surface area of sphere = $4\pi r^2$ sq. units.
12. Volume of a hemisphere = $2/3\pi r^3$ cubic units.
13. Curved surface area of hemisphere = $2\pi r^2$ sq. units.
14. Whole surface area of hemisphere = $3\pi r^2$ sq. units.

RIGHT CIRCULAR CONE

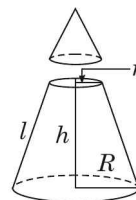
Let r be the radius of base, h the height and l the slant height of a cone. Then

15. Slant height $l = \sqrt{h^2 + r^2}$.
16. Volume of cone = $1/3 \pi r^2 h$ cubic units.
17. Curved surface area of the cone = $\pi r l$ sq. units = $(\pi r \sqrt{r^2 + h^2})$ sq. units.
18. Total surface area of cone = $\pi r l + \pi r^2 = \pi r(l + r)$ sq. units.

FRUSTUM OF A RIGHT CIRCULAR CONE

If a cone is cut by a plane parallel to the base so as to divide the cone into two parts as shown in the figure, the lower part is called the frustum of a cone.

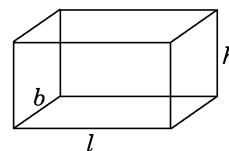
Let the radius of the base of frustum = R , radius of top = r , height = h and slant height = l units.



19. Slant height $l = \sqrt{h^2 + (R - r)^2}$ units.
20. Curved surface area = $\pi(R + r)l$ sq. units.
21. Total surface area = $\pi\{(R + r)l + r^2 + R^2\}$ sq. units.
22. Volume = $\frac{\pi h}{3} (r^2 + R^2 + rR)$ cubic units.

RIGHT PARALLELOPIPED

It is a type of cuboid in which the shape of a side face is rectangular whereas the shape of the base or the top face is a parallelogram (neither a rectangle nor a square).



23. Surface area (of side faces) = $2h(b + l)$ sq. units.

24. Surface area (of the base or the top face) = $2\sqrt{s(s-a)(s-b)(s-d)}$ sq. units

25. Total surface area = $2h(b+l) + 4\sqrt{s(s-a)(s-b)(s-d)}$ sq. units

26. Volume = Base area $\times$ height.

PROBLEMS ON CUBES AND CUBOIDS

Type I Direct Application of Formulae

EXAMPLE 1 Find the volume and the surface area of a slab of stone measuring 4 metres in length, 2 metres in width and $\frac{1}{4}$ metre in thickness.

Solution Volume = $4 \times 2 \times \frac{1}{4} = 2$ cubic metres

$$\begin{aligned}\text{Surface area} &= 2(lb + bh + lh) \\ &= 2(4 \times 2 + 2 \times \frac{1}{4} + 4 \times \frac{1}{4}) \\ &= 2(8 + \frac{1}{2} + 1) = \mathbf{19 \text{ sq. units}}\end{aligned}$$

EXAMPLE 2 A brick measures $20 \text{ cm} \times 10 \text{ cm} \times 7\frac{1}{2} \text{ cm}$. How many bricks will be required for a wall 25 m long, 2 m high and $\frac{3}{4} \text{ m}$ thick?

Solution Volume of wall = $25 \times 2 \times \frac{3}{4} = \frac{75}{2}$ cubic metres

$$\text{Volume of 1 brick} = \frac{20}{100} \times \frac{10}{100} \times \frac{15}{200} = \frac{3}{2000} \text{ cubic metres}$$

$$\therefore \text{Required number of bricks} = \frac{75}{2} \div \frac{3}{2000} = \frac{75}{2} \times \frac{2000}{3} = \mathbf{25000}$$

EXAMPLE 3 Find the volume of a cuboid whose areas of base and two adjacent faces are 180 cm^2 , 96 cm^2 and 120 cm^2 respectively.

Solution Volume of a cuboid = $\sqrt{\text{Area of base} \times \text{Area of one face} \times \text{Area of other face}}$

$$= \sqrt{180 \times 96 \times 120} \text{ cm}^3 = \mathbf{1440 \text{ cm}^3}$$

Type II Some Quicker Methods

EXAMPLE 4 A closed wooden box measures externally 9 cm long, 7 cm broad, 6 cm high. If the thickness of the wood is half a centimetre, find (i) the capacity of the box and (ii) the weight supposing that 1 m^3 of wood weighs 0.9 g.

Solution Capacity = (External length – 2 × thickness)
 × (External breadth – 2 × thickness)
 × (External height – 2 × thickness)

Volume of material = External volume – Capacity

Here, Capacity = $(9 - 2 \times 0.5) (7 - 2 \times 0.5) (6 - 2 \times 0.5)$
 $= 8 \times 6 \times 5 = 240 \text{ cm}^3$

Volume of wood = External volume – Capacity
 $= 9 \times 7 \times 6 - 240 = 378 - 240 = 138 \text{ cm}^3$

Weight of wood = Volume of wood × Density of wood
 $= 138 \times 0.9 = 124.2 \text{ g}$

EXAMPLE 5 The surface area of a cube is $30\frac{3}{8}$ sq. metres. Find its volume.

Solution Volume of cube = $\left(\sqrt{\frac{\text{Surface area}}{6}}\right)^3$

Here, Surface area = $\frac{243}{8} \text{ m}^2$

∴ Volume of cube = $\left(\sqrt{\frac{243/8}{6}}\right)^3$
 $= \left(\sqrt{\frac{243}{48}}\right)^3 = \left(\sqrt{\frac{81}{16}}\right)^3$
 $= \left(\frac{9}{4}\right)^3 = \frac{729}{64} = 11\frac{25}{64} \text{ cu.m}$

RAINFALL IN A GIVEN AREA AND SIMILAR PROBLEMS

EXAMPLE 6 The annual rainfall at a place is 43 cm. Find the weight in metric tonnes of the annual rainfall there on a hectare of land, taking the weight of water to be 1 metric tonne for 1 cubic metre.

Solution



Short cut: Volume of water = Height of water × Base area

Given water level = 43 cm; Area of land = 1 hectare = 10^4 m^2

∴ Volume of water = $\frac{43}{100} \times 10^4 \text{ m}^3 = 4300 \text{ m}^3$

∴ Weight of water = $4300 \times 1 = 4300 \text{ metric tonnes}$.

EXAMPLE 7 A rectangular tank is 50 metres long and 29 m deep. If 1000 m^3 of water be drawn off the tank, the level of the water in the tank goes down by 2 metres. How many cubic metres of water can the tank hold?

Solution



Short cut: Base area = $\frac{\text{Volume of water}}{\text{Level of water}} = \frac{1000}{2} = 500 \text{ m}^2$

$\therefore$ Total volume = Depth $\times$ Base area = $29 \times 500 = 14500 \text{ m}^3$

CONVERSION OF A SQUARE BAR INTO CUBES

EXAMPLE 8 A cubic metre of copper weighing 9000 kg is rolled into a square bar 9 m long. An exact cube is cut off from the bar. How much does it weigh?

Solution Here, the given volume of copper is rolled into a square bar of given length. Then an exact cube is cut off from this square bar.

Given volume = 1 m^3

i.e.,

$$1 \text{ m}^3 = \text{Area of square base} \times \text{length}$$

$$\Rightarrow \text{Area of square base} \times \text{length} = 1 \text{ m}^3$$

$$\Rightarrow \text{Area of square base} = \frac{1}{\text{length}} = \frac{1}{9} \text{ m}^2$$

$$\therefore \text{Side of square base} = \sqrt{\frac{1}{9}} = \frac{1}{3} \text{ m}$$

$$\therefore \text{Volume of cut off cube} = (\text{side})^3 = \left(\frac{1}{3}\right)^3 = \frac{1}{27} \text{ m}^3$$

$$\text{Weight of cube} = \frac{1}{27} \times 9000 = 333.3 \text{ kg}$$



Short cut: Volume of cube cut off = $\left(\sqrt{\frac{\text{Volume of original solid}}{\text{Length of solid}}} \right)^3$

$$\therefore \text{Volume} = \left(\sqrt{\frac{1}{9}} \right)^3 = \frac{1}{27} \text{ m}^3$$

$$\therefore \text{Weight of cube} = \frac{1}{27} \times 9000 = 333.3 \text{ kg}$$

OBJECT CHANGING ITS SHAPE

EXAMPLE 9 A cubic metre of gold is extended by hammering so as to cover an area of 6 hectares. Find the thickness of the gold.

Solution The underlying concept for these type of questions is *total volume of a solid remains the same even when its shape changes*.

$$\begin{aligned}\therefore & \text{Old Volume} = \text{New Volume} \\ \Rightarrow & 1\text{m}^3 = 60000 \times \text{Thickness} \\ \Rightarrow & \text{Thickness} = \frac{1}{60000} = 0.0017 \text{ cm}\end{aligned}$$

EXAMPLE 10 Three cubes of metals whose edges are 3, 4 and 5 cm respectively are melted and formed into a single cube. If there is no loss of metal in the process, find the side of the new cube.

Solution



Short cut: When many cubes integrate into one cube, the side of the new cube is given by side = $\sqrt{\text{Sum of cubes of sides of all the cubes}}$

$$\text{Here, side} = \sqrt{3^3 + 4^3 + 5^3} = \sqrt{27 + 64 + 125} = \sqrt{216} = 6 \text{ cm}$$

EXAMPLE 11 A cube of sides 3 cm is melted and similar cubes of sides 1 cm each are formed. How many such cubes are possible?

Solution Quicker method

$$\text{Number of cubes possible} = \left(\frac{\text{Original length of cube}}{\text{New length of cube}} \right)^3$$

$$\text{i.e.} \quad \text{Number of cubes} = \left(\frac{3}{1} \right)^3 = 27$$

PROBLEMS ON CYLINDERS

Type I Direct Application of Formula

EXAMPLE 12 Find the volume of a cylinder which has a height of 14 metres and a base of radius 3 metres. Find the curved surface of the cylinder.

$$\text{Solution} \quad \text{Volume} = \frac{22}{7} \times (3)^2 \times 14 = 396 \text{ m}^3$$

$$\text{Curved surface area} = \text{Circumference} \times \text{Height}$$

$$= 2 \times \frac{22}{7} \times 3 \times 14 = 264 \text{ m}^2$$

Type II Indirect Method

EXAMPLE 13 A hollow cylindrical tube opens at both ends is made of iron 2 cm thick. If the external diameter is 50 cm and the length of the tube be 140 cm, find the volume of iron in it.

Solution

Short cut: Volume of metal = $\pi[(\text{Outer radius})^2 - (\text{Inner radius})^2] \times \text{Height}$

Here outer radius = $50/2 = 25$ cm

Inner radius = $25 - 2 = 23$ cm

Height = 140 cm

$\therefore$

$$\text{Volume} = \pi[25^2 - 23^2] \times 140$$

$$= \frac{22}{7} [25^2 - 23^2] \times 140 \quad \left[\because \pi = \frac{22}{7} \right]$$

$$= 440 (25 + 23) (25 - 23)$$

$$= 440 \times 48 \times 2 = \mathbf{42240 \text{ m}^3}$$



Note: The above problem can also be solved by using the equation

$$\text{Volume} = \pi \times \text{Height} \times (2 \times \text{Outer radius} - \text{Thickness}) \times (\text{Thickness})$$

(when the outer radius is given)

$$\text{Volume} = \pi \times \text{Height} \times (2 \times \text{Inner radius} + \text{Thickness}) \times (\text{Thickness})$$

(when the inner radius is given)

Here using the first relation,

$$\text{Volume} = \frac{22}{7} \times 140 \times (2 \times 25 - 2) (2) = \frac{22}{7} \times 140 \times 48 \times 2 = \mathbf{42240 \text{ m}^3}$$

EXAMPLE 14 Find the weight of a lead pipe 3.5 cm long if the external diameter is 2.4 cm, the thickness of the lead is 2 mm and 1 cm³ of lead weighs 11.4 gm.

Solution Weight of lead pipe = Unit weight of pipe $\times$ Volume of pipe

$$= 11.4 \text{ g/cc} \times \frac{22}{7} \times 3.5 (2 \times 2.4 - 0.2) (0.2) \text{ cm}^3$$

$$= 11.4 \times \frac{22}{7} \times 3.5 \times 4.6 \times 0.2$$

$$= 11.4 \times 22 \times 0.5 \times 4.6 \times 0.2 = \mathbf{115.368 \text{ g}}$$

ROLLING A SQUARE INTO A CYLINDER

EXAMPLE 15 A rectangular sheet with dimensions 22 m $\times$ 10 m is rolled into a cylinder so that the smaller side becomes the height of the cylinder. What is the volume of the cylinder so formed?

Solution $\text{Volume} = \frac{\text{Height} \times (\text{Other side of the sheet})^2}{4\pi}$

Here, $\text{Volume} = \frac{10 \times (22)^2}{4 \times \frac{22}{7}} = \mathbf{385 \text{ m}^3}$

PROBLEM ON CONES

EXAMPLE 16 Find what length of canvas 2 metres in width is required to make a conical tent 8 metres in diameter and 5.6 metres in slant height. Also find the cost of the canvas at the rate of ₹ 3.20/m.

Solution Curved surface area = πrl

$$= \frac{22}{7} \times \left(\frac{1}{2} \times 8 \right) \times 5.6$$

$$= 22 \times 4 \times 0.8 = 70.4 \text{ m}^2$$

∴ Length of the canvas = $70.4 \div 2 = 35.2$ metres

Cost of the canvas = ₹ $35.2 \times 3.2 = ₹ 112.64$

EXAMPLE 17 The diameter of a right circular cone is 14 metres and its slant height is 12 metres. Find its

- (i) Curved surface area, (ii) Total surface area (iii) Volume, and
(iv) The cost of colouring its total surface at the rate of 14 paise/m².

Solution

(i) Curved surface area = $\pi rl = \frac{22}{7} \times \frac{14}{2} \times 12 = ₹ 264 \text{ m}^2$

(ii) Total surface area = $\pi r(r + l) = \frac{22}{7} \times \frac{14}{2} \left(\frac{14}{2} + 12 \right) = 22 \times 19 = 418 \text{ m}^2$

(iii) Volume = $\frac{1}{3} \pi r^2 h$

Here, $h = \sqrt{l^2 - r^2} = \sqrt{12^2 - 7^2} = 9.75 \text{ m}$

∴ Volume = $\frac{1}{3} \pi r^2 h = \frac{1}{3} \times \frac{22}{7} \times 7^2 \times 9.75 = 22 \times 7 \times 3.25 = 500.5 \text{ m}^3$

(iv) Required cost = ₹ $418 \times \frac{14}{100} = ₹ 58.52$

EXAMPLE 18 A frustum of a right circular cone has a diameter of base 10 cm, of top 6 cm and a height of 5 cm. Find the area of its whole surface and volume.

Solution $R = 5$ cm, $r = 3$ cm, and $h = 5$ cm

∴ $l = \sqrt{h^2 + (R - r)^2} = \sqrt{5^2 + (5 - 3)^2} = \sqrt{29} = 5.385 \text{ cm}$

∴ Whole surface area of the frustum = $\pi(R^2 + r^2 + Rl + rl)$

$$= \frac{22}{7} (5^2 + 3^2 + 5 \times 5.385 + 3 \times 5.385) = 242.25 \text{ cm}^2$$

$$\text{Volume} = \frac{\pi h}{3} (R^2 + r^2 + Rr)$$

$$\begin{aligned}
 &= \frac{22}{7} \times \frac{5}{3} (5^2 + 3^2 + 5 \times 3) \\
 &= \frac{22}{7} \times \frac{5}{3} \times 49 = \mathbf{256.67 \text{ cm}^3}
 \end{aligned}$$

PROBLEMS ON SPHERES

Type I Direct Application of Formula

EXAMPLE 19 Find the volume and the surface area of a sphere of diameter 42 cm.

Solution Radius = $42/2 = 21$ cm

$$\therefore \text{Volume} = \frac{4}{3} \pi r^3 = \frac{4}{3} \times \frac{22}{7} \times 21 \times 21 \times 21 = \mathbf{38808 \text{ cm}^3}$$

$$\text{Surface area} = 4\pi r^2 = 4 \times \frac{22}{7} \times 21 \times 21 = \mathbf{5544 \text{ cm}^2}$$

EXAMPLE 20 Find the volume, curved surface area and the total surface area of a hemisphere of radius 21 cm.

$$\begin{aligned}
 \text{Solution} \quad \text{Volume} &= \frac{2}{3} \pi r^3 \\
 &= \frac{2}{3} \times \frac{22}{7} \times 21 \times 21 \times 21 = \mathbf{19404 \text{ cm}^3}
 \end{aligned}$$

$$\text{Curved surface area} = 2\pi r^2 = 2 \times \frac{22}{7} \times 21 \times 21 = \mathbf{2772 \text{ cm}^2}$$

$$\text{Total surface area} = 3\pi r^2 = 3 \times \frac{22}{7} \times 21 \times 21 = \mathbf{4158 \text{ cm}^2}$$

Type II Indirect Method

Spheres Changing Shapes

The basic principle in solving questions of this type where solid in the shape of any object (sphere, cone etc.) changes into some other shape, the volume remains unchanged.

EXAMPLE 21 A metallic sphere of diameter 18 cm is drawn into a wire of diameter 4 mm. Find the length of the wire.

Solution When a sphere is converted into a cylinder, the length of wire is given by

$$\text{Length of cylinder} = \frac{4 \times (\text{Radius of sphere})^3}{3 \times (\text{Radius of cylinder})^2}$$

$$\begin{aligned}
 \text{i.e.,} \quad l &= \frac{4 \times (18/2)^3}{3 \times (4/20)^2} = \frac{4 \times 9^3 \times 20^2}{3 \times 4^2} \\
 &= \frac{4 \times 9^3 \times 10^2}{3 \times 2^2} = 9^2 \times 3 \times 100 = \mathbf{24300 \text{ cm}}
 \end{aligned}$$

**Note:**

$$1. \text{ Radius of sphere} = \sqrt{\frac{3}{4} [(\text{Length of cylinder}) \times (\text{Radius of cylinder})^2]}$$

$$2. \text{ Radius of cylinder} = \sqrt{\frac{4 \times (\text{Radius of sphere})^2}{3 \times (\text{Length of cylinder})}}$$

EXAMPLE 22 A copper sphere of 36 cm diameter is drawn into a cylindrical wire of length 2.43 m. What is the radius of wire?

$$\begin{aligned} \text{Solution} \quad \text{Radius of wire} &= \sqrt{\frac{4 \times (\text{Radius of sphere})^3}{3 \times \text{Length of wire}}} \\ &= \sqrt{\frac{4 \times 36^3}{3 \times 243}} = \sqrt{\frac{4 \times 4^3 \times 9^3}{9^3}} = 4^2 = \mathbf{16 \text{ cm}} \end{aligned}$$

EXAMPLE 23 A cylinder of radius 2 cm and height 15 cm is melted and the same mass is used to create a sphere. What will be the radius of the sphere?

$$\begin{aligned} \text{Solution} \quad \text{Radius of sphere} &= \sqrt{\frac{3}{4} (\text{Length of cylinder}) \times (\text{Radius of cylinder})^2} \\ &= \sqrt{\frac{3}{4} \times 15 \times 2^2} = \sqrt{45} = \mathbf{3\sqrt{5} \text{ cm}} \end{aligned}$$

EXAMPLE 24 How many bullets can be made out of a lead cylinder 28 cm high and 6 cm radius, each bullet being 1.5 cm in diameter?

$$\begin{aligned} \text{Solution} \quad \text{Number of bullets} &= \frac{\text{Volume of cylinder}}{\text{Volume of 1 bullet}} \\ &= \frac{\pi \times 6 \times 6 \times 28}{\frac{4}{3} \pi \times (0.75)^3} = \frac{6 \times 6 \times 28 \times 3}{4 \times \frac{3}{4} \times \frac{3}{4} \times \frac{3}{4}} \\ &= \mathbf{2 \times 2 \times 28 \times 4 \times 4 = 1792} \end{aligned}$$

EXAMPLE 25 Find the number of lead balls of diameter 1 cm each that can be made from a sphere of diameter 16 cm.

$$\begin{aligned} \text{Solution} \quad \text{Number of balls} &= \frac{\text{Volume of big sphere}}{\text{Volume of 1 small sphere}} \\ &= \frac{\frac{4}{3} \pi \times \frac{16}{2} \times \frac{16}{2} \times \frac{16}{2}}{\frac{4}{3} \pi \times 0.5 \times 0.5 \times 0.5} \\ &= \frac{8 \times 8 \times 8}{0.5 \times 0.5 \times 0.5} = \mathbf{4096} \end{aligned}$$



Short cut: Number of balls = $\left(\frac{\text{Bigger radius}}{\text{Smaller radius}} \right)^3 = \left(\frac{8}{0.5} \right)^3 = 16^3 = 4096$

TYPICAL CASES

Problems Involving Ratios

I. Two spheres

- (i) (Ratio of radii)² = Ratio of surface areas
- (ii) (Ratio of radii)³ = Ratio of volume
- (iii) (Ratio of surface areas)³ = (Ratio of volume)²

II. Two cylinders

A. When the volumes are equal

- (i) Ratio of radii = $\sqrt{\text{Inverse ratio of heights}}$
- (ii) Ratio of curved surface areas = Inverse ratio of radii
- (iii) Ratio of curved surface areas = $\sqrt{\text{Ratio of heights}}$

B. When the radii are equal

- (i) Ratio of volumes = Ratio of heights
- (ii) Ratio of curved surface areas = Ratio of heights
- (iii) Ratio of volumes = Ratio of curved surface areas

C. When the heights are equal

- (i) Ratio of volumes = (Ratio of radii)²
- (ii) Ratio of curved surface areas = Ratio of radii
- (iii) Ratio of volumes = (Ratio of curved surface areas)²

D. When the curved surface areas are equal

- (i) Ratio of volumes = Ratio of radii
- (ii) Ratio of volumes = Inverse ratio of heights
- (iii) Ratio of radii = Inverse ratio of heights

III. Two cubes

- (i) Ratio of volumes = (Ratio of sides)³
- (ii) Ratio of surface areas = (Ratio of sides)²
- (iii) (Ratio of surface areas)³ = (Ratio of volumes)²

IV. Two cones

A. When the volumes are equal

- (i) Ratio of radii = $\sqrt{\text{Inverse ratio of heights}}$

B. When the radii are equal

- (i) Ratio of volumes = Ratio of heights

C. When heights are equal

- (i) Ratio of volumes = (Ratio of radii)²

D. When curved surface areas are equal

- (i) Ratio of radii = Inverse ratio of slant heights

EXAMPLE 26 The curved surface areas of two spheres are in the ratio 1 : 4. Find the ratio of their volumes.

Solution We have (ratio of surface areas)³ = (Ratio of volumes)²

$$\therefore (1 : 4)^3 = (\text{Ratio of volumes})^2$$

$$\text{i.e. } 1 : 64 = (\text{ratio of volumes})^2$$

$$\therefore \text{Ratio of volumes} = \sqrt{1 : 64} = 1 : 8$$



Short cut: Ratio of radii = $\sqrt{\frac{1}{4}} = \frac{1}{2}$

$$\therefore \text{Ratio of volumes} = \left(\frac{1}{2}\right)^3 = \frac{1}{8} = 1 : 8$$

EXAMPLE 27 The radii of two spheres are in the ratio of 1 : 2. What is the ratio of their surface areas?

Solution We have ratio of surface areas = (Ratio of radii)² = $\left(\frac{1}{2}\right)^2 = 1 : 4$

EXAMPLE 28 Two cylinders of equal volume have their heights in the ratio of 1 : 2. What will be the ratio of their radii ?

Solution We have the ratio of radii = $\sqrt{\text{Inverse ratio of heights}} = \sqrt{2 : 1} = \sqrt{2} : 1$

EXAMPLE 29 If the heights of two cones are in the ratio 1 : 4 and their diameters are in the ratio 4 : 5, what is the ratio of their volumes?

Solution Volume of cone = $\frac{1}{3}\pi(\text{radius})^2 \times \text{height}$

$$\begin{aligned} \therefore \frac{\text{Volume of first cone}}{\text{Volume of second cone}} &= \frac{\frac{1}{3}\pi (\text{Radius of first cone})^2 \times (\text{Height})}{\frac{1}{3}\pi (\text{Radius of second cone})^2 \times (\text{Height})} \\ &= \left(\frac{\text{Radius of first cone}}{\text{Radius of second cone}} \right)^2 \left(\frac{\text{Height of 1st cone}}{\text{Height of 2nd cone}} \right) \end{aligned}$$

$$\text{Ratio of volumes} = (\text{ratio of radii})^2 (\text{ratio of heights})$$

$$= (4 : 5)^2 (1 : 4)$$

$$= \frac{16}{25} \times \frac{1}{4} = \frac{4}{25} = 4 : 25$$

V. For a cone and also for a cylinder

$$(i) \text{ Ratio of volumes} = (\text{Ratio of radii})^2 \times (\text{Ratio of heights})$$

$$(ii) \text{ Ratio of heights} = (\text{Inverse ratio of radii})^2 \times (\text{Ratio of volumes})$$

$$(iii) \text{ Ratio of radii} = \sqrt{(\text{Ratio of volumes}) \times (\text{Inverse ratio of heights})}$$

VI. For a cylinder

- (i) Ratio of curved surface areas = (Ratio of radii) $\times$ (Ratio of heights)
- (ii) Ratio of heights = (Ratio of curved surface areas) $\times$ (Inverse ratio of radii)
- (iii) Ratio of radii = (Ratio of curved surface areas) $\times$ (Inverse ratio of heights)

VII. For a cone

- (i) Ratio of curved surface areas = (ratio of radii) $\times$ (ratio of slant heights)
- (ii) Ratio of slant heights = (ratio of curved surface areas) $\times$ (inverse ratio of radii)
- (iii) Ratio of radii = (ratio of curved surface areas) $\times$ (inverse ratio of slant heights)

EXAMPLE 30 If the radius of a cylinder is doubled and the height is halved, what is the ratio between the new volume and the previous volume?

Solution Let the initial radius and height of the cylinder be r cm and h cm respectively

Then

$$V_1 = \pi r^2 h$$

$$V_2 = \pi(2r)^2 h/2 = 2\pi r^2 h$$

$$\frac{\text{New volume}}{\text{Previous volume}} = \frac{2\pi r^2 h}{\pi r^2 h} = \frac{2}{1} = 2 : 1$$

EXAMPLE 31 A well of 11.2 m diameter is dug 8 m deep. The earth taken out has been spread all round it to a width of 7 m to form a circular embankment. Find the height of this embankment.

Solution Volume of earth dug out = $\pi r^2 h = \frac{22}{7} \times \left(\frac{11.2}{2}\right)^2 \times 8$

$$= \frac{22}{7} \times 5.6 \times 5.6 \times 8 = 788.48 \text{ m}^3$$

$$\text{Area of embankment} = \pi[(5.6 + 7)^2 - (5.6)^2]$$

$$= \frac{22}{7} \times (5.6 + 7 + 5.6)(5.6 + 7 - 5.6)$$

$$= \frac{22}{7} \times 18.2 \times 7 = 400.4 \text{ m}^2$$

$$\text{Height of embankment} = \frac{\text{Volume of earth}}{\text{Area of embankment}} = \frac{788.48}{400.4} = 1.97 \text{ m}$$

EXAMPLE 32 A right-angled triangle having base 6.3 m and height equal to 10 cm is turned around the height. Find the volume of the cone thus formed. Also find the surface area.

Solution Height of cone thus formed = Height of triangle

Radius = Base of the triangle

Slant height = Hypotenuse of the triangle

$\therefore$

Surface area = $\pi(\text{radius}) \times (\text{slant height})$

$$\begin{aligned}
&= \frac{22}{7} (6.3) (\sqrt{6.3^2 + 10^2}) \\
&= 22 \times 0.9 \sqrt{139.69} \\
&= 22 \times 0.9 \times 11.82 = \mathbf{234.04 \text{ m}^2}
\end{aligned}$$

➤ Theorem: For Cuboid:

If the length, breadth and height of a cuboid are increased by $x\%$, $y\%$ and $z\%$ respectively, then its volume is increased by $\left[x + y + z + \frac{xy + yz + xz}{100} + \frac{xyz}{(100)^2} \right]\%$

For Cube:

If the side of a cube is increased by $x\%$, then its volume will be increased by

$$\left[3x + \frac{3x^2}{100} + \frac{x^3}{100^2} \right]\% \quad \text{or} \quad \left[\left(1 + \frac{x}{100} \right)^3 - 1 \right] \times 100\%$$

For Sphere:

If the radius of a sphere is changed by $x\%$ then its volume will be changed by $\left[3x + \frac{3x^2}{100} + \frac{x^3}{100} \right]\%$

For Cylinder:

- I. If the height is changed by $x\%$ and radius remains the same, then volume will be changed by $x\%$.
- II. If radius is changed by $x\%$ and the height remains the same, the volume will be changed by

$$\left[2x + \frac{x^2}{100} \right]\% \quad \text{or} \quad \left[\left(1 + \frac{x}{100} \right)^2 - 1 \right] \times 100\%$$

- III. If the radius is changed by $x\%$ and the height is changed by $y\%$ then volume will be changed by

$$\left[2x + y + \frac{x^2 + 2xy}{100} + \frac{x^2 y}{100} \right]\%$$

- IV. If height and radius both change by $x\%$, then volume will change by $\left[3x + \frac{3x}{100} + \frac{x^3}{100^2} \right]\%$

EXAMPLE 33 Each edge of a cube is increased by 50%. What is the percentage increase in its volume? Also find the percentage increase in its surface area?

Solution Percentage increase in volume

$$\begin{aligned}
&= \left[\left(1 + \frac{x}{100} \right)^3 - 1 \right] \times 100\% \\
&= \left[\left(1 + \frac{50}{100} \right)^3 - 1 \right] \times 100\% \\
&= [(1.5)^3 - 1] \times 100\%
\end{aligned}$$

$$= \left[\frac{6.75 - 2}{2} \right] \times 100\% = \frac{4.75}{2} \times 100\% = \mathbf{237.5\%}$$

$$\text{Percentage increase in area} = \left[2x + \frac{x^2}{100} \right]$$

$$= (2 \times 50) + \frac{(50 \times 50)}{100} = \mathbf{125\%}$$

EXAMPLE 34 Each of the radius and the height of a cone is increased by 20%. Find the percentage increase in volume.

Solution Percentage increase in volume = $\left[\left(1 + \frac{x}{100} \right)^3 - 1 \right] \times 100\%$

$$= \left[\left(1 + \frac{20}{100} \right)^3 - 1 \right] \times 100\%$$

$$= [(1.2)^3 - 1] \times 100\% = \mathbf{72.8\%}$$

EXAMPLE 35 The radius of a sphere is increased by 5%. Find the percentage increase in its surface area.

Solution Required percentage value = $2x + \frac{x^2}{100} = 2 \times 5 + \frac{5^2}{100} = \mathbf{10.25}$

i.e. surface area of sphere is increased by 10.25%.

EXAMPLE 36 Each edge of a cube is decreased by 50%. Find the percentage decrease in its surface area and volume.

Solution For a two dimensional figure, percentage increase or decrease in area is given

by $\left[2x + \frac{x^2}{100} \right] \%$

i.e. required percentage = $2(-50) + \frac{(-50)^2}{100} = -100 + 25 = \mathbf{-75\%}$

i.e. area decreases by 75%

Percentage decrease in volume

$$= \left[\left(1 + \frac{x}{100} \right)^3 - 1 \right] \times 100 = \left[\left(1 - \frac{50}{100} \right)^3 - 1 \right] \times 100$$

$$= [(0.5)^3 - 1] \times 100 = -0.875 \times 100 = \mathbf{-87.5}$$

i.e. volume decreases by 87.5%

EXAMPLE 37 Find the ratio of volumes of a cube to that of a sphere which will fit inside the cube.

Solution Let side of the cube be a units and diameter of the sphere be a units

$$\begin{aligned}\therefore \frac{\text{Volume of cube}}{\text{Volume of sphere}} &= \frac{a^3}{\frac{4}{3}\pi \left(\frac{a}{2}\right)^3} \\ &= \frac{1}{\frac{4}{3}\pi \times \frac{1}{8}} = \frac{6}{\pi} = 6 : \pi\end{aligned}$$

EXAMPLE 38 A cube of maximum volume (each corner touching the surface from inside) is cut from a sphere. Find the ratio of the volume of the cube and the sphere.

Solution Let the radius of the sphere be r cm and side of the cube be x cm.
Then diagonal of cube = diameter of sphere

or $\sqrt{3}x = 2r \Rightarrow x = \frac{2r}{\sqrt{3}}$

$$\begin{aligned}\text{Ratio of volume} &= \frac{\text{Volume of cube}}{\text{Volume of sphere}} \\ &= \frac{x^3}{\frac{4}{3}\pi r^3} = \frac{(2r/\sqrt{3})^3}{\frac{4}{3}\pi r^3} = \frac{8/(3\sqrt{3}) \times r^3}{\frac{4}{3}\pi \times r^3} \\ &= \frac{8}{\sqrt{3}} \div 4\pi = \frac{2}{\pi\sqrt{3}}\end{aligned}$$

$\therefore$ Ratio = $2 : \sqrt{3}\pi$

EXAMPLE 39 The volumes of two cubes are in the ratio of 8 : 125. Find the ratio of their edges and surface areas.

Solution We have,

$$\text{Ratio of sides} = (\text{Ratio of volumes})^{1/3}$$

$$\text{Ratio of surface areas} = (\text{Ratio of volumes})^{2/3}$$

$\therefore$ Ratio of sides = $(8 : 125)^{1/3} = 2 : 5$

$$\text{Ratio of surface areas} = (8 : 125)^{2/3} = 4 : 25$$

EXAMPLE 40 A circular wire of radius 42 cm is cut and bent in the form of a rectangle whose sides are in the ratio of 6 : 5. Find the smaller side of the rectangle.

Solution Length of wire = Circumference of the circle

$$= 2\pi \times 42 = 2 \times \frac{22}{7} \times 42 = 264 \text{ cm}$$

$$\text{Perimeter of rectangle} = 264 \text{ cm}$$

$$\text{Length of rectangle} = \frac{6}{2(6+5)} \times 264 = 72 \text{ cm}$$

$$\text{Breadth of rectangle} = \frac{5}{2(6+5)} \times 264 = 60 \text{ cm}$$

EXAMPLE 41 A right circular cone is exactly fitted inside a cube in such a way that the edges of the base of the cone are touching the edges of one of the faces of the cube and the vertex is on the opposite face of the cube. If the volume of the cube is 343 cc, what is the approximate volume of the cone?

Solution Edge of the cube = $\sqrt[3]{343} = 7$ cm

Radius of cone = $7/2 = 3.5$ cm

Height of cone = 7 cm

$\therefore$ Volume of cone = $\frac{1}{3} \pi r^2 h$

$$= \frac{1}{3} \times \frac{22}{7} \times \left(\frac{7}{2}\right)^2 \times 7 = \frac{11 \times 49}{6} \approx 90 \text{ cm}^3$$



Important Formulae

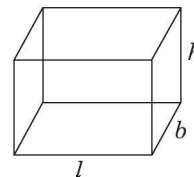
1. Right parallelepiped

(a) Surface area (of side faces) = $2h(l + b)$

(b) Surface area (of base or top face) = $2\sqrt{s(s-a)(s-b)(s-d)}$ sq. units .

(c) Total surface area = $2h(l + b) + 4\sqrt{s(s-a)(s-b)(s-d)}$ sq. units .

(d) Volume = Base area $\times$ Height.



2. Cylinder

Let the radius of base of cylinder be r units and its height be h units. Then

(a) Volume of the cylinder = $(\pi r^2 h)$ cubic units.

(b) Curved surface area of the cylinder = $(2\pi r h)$ sq. units.

(c) Total surface area of the cylinder = $(2\pi r h + 2\pi r^2)$ sq. units.

3. Hollow cylinder: Let the outer radius of cylinder be R units and inner radius of cylinder be r units. Let the height of cylinder be h units and thickness be $(R - r) = t$ units.

(a) Volume of cylinder = $\pi h(R^2 - r^2)$ cubic units.

$$= \pi h(R + r)t \text{ cubic units.}$$

(b) Outer curved surface area of cylinder = $\pi R h$ square units.

(c) Inner curved surface area = $\pi r h$ square units.

(d) Total curved surface area of cylinder = $\pi h(R + r)$ square units

(e) Total surface area of cylinder = $\pi h(R + r) + 2\pi(R^2 - r^2)$ sq. units

$$= \pi(R + r) \{h + 2(R - r)\} \text{ sq. units}$$

$$= \pi(R + r) \{h + 2t\} \text{ sq. units.}$$

PROBLEMS ON CYLINDERS

Type I Direct Application of Formulae

EXAMPLE 42 Find the volume of a cylinder which has a height of 14 metres and a base of radius 4 metres. Also find the curved surface area of the cylinder.

Solution Volume = $\pi r^2 h \text{ m}^3$

$$= \frac{22}{7} \times 4 \times 4 \times 14 = \mathbf{704 \text{ m}^3}$$

Curved surface area = Circumference $\times$ Height

$$= (2\pi r) \times h \text{ m}^2$$

$$= 2 \times \frac{22}{7} \times 4 \times 14 = \mathbf{352 \text{ m}^2}$$

EXAMPLE 43 A hollow cylindrical tube opens at both ends is made of iron 2 cm thick. If the external diameter is 50 cm and the length of the tube is 105 cm, find the volume of iron in it.

Solution Volume of metal = $\pi h(R^2 - r^2)$

External radius $R = 50 \div 2 = 25 \text{ cm}$

$\therefore$ Inner radius, $r = R - t$

$$= 25 - 2 = 23 \text{ cm}$$

$\therefore$ Volume = $\frac{22}{7} \times 105 \times (25^2 - 23^2)$

$$= 330 \times 96 = \mathbf{31680 \text{ m}^3}$$



Short cut: Volume = $\pi \times \text{Height} \times (2 \times \text{Outer radius} - \text{Thickness}) \times (\text{Thickness})$
(if outer radius is known)

Volume = $\pi \times \text{Height} \times (2 \times \text{Inner radius} + \text{Thickness}) \times (\text{Thickness})$
(if inner radius is given)

Here,

$$\text{Volume} = \frac{22}{7} \times 105 \times (2 \times 25 - 2) (2)$$

$$= \frac{22}{7} \times 105 \times 48 \times 2 = \mathbf{31680 \text{ cu. m}}$$

Rolling a Square into a Cylinder

EXAMPLE 44 A rectangular sheet with dimensions $22 \text{ m} \times 12 \text{ m}$ is rolled into a cylinder so that the smaller side becomes the height of the cylinder. What is the volume of the cylinder so formed?

Solution



Short cut: In this case, use the rule,

$$\text{Volume} = \frac{\text{Height} \times (\text{Other side of the sheet})^2}{4\pi}$$

$\therefore$ Here,

$$\text{Volume} = \frac{12 \times (22)^2}{4 \times \frac{22}{7}} = \mathbf{462 \text{ m}^3}.$$

Cones

EXAMPLE 45 Find the length of canvas 2 metres in width required to make a conical tent 8 metres in diameter and 5.6 metres in slant height. Find out the cost of the canvas at the rate of ₹ 3.20/metre.

Solution Curved surface area = πrl

$$\begin{aligned} &= \frac{22}{7} \times \frac{1}{2} \times 8 \times 5.6 \\ &= 22 \times 4 \times 0.8 = \mathbf{70.4 \text{ m}^2} \end{aligned}$$

$$\text{Length of canvas} = \frac{\text{Curved surface area}}{\text{Width of canvas}} = \frac{70.4}{2} = \mathbf{35.2 \text{ m}}$$

$$\text{Cost of canvas at ₹ 3.2/m} = 35.2 \times 3.2 = \mathbf{₹ 112.64}$$

EXAMPLE 46 The diameter of a right circular cone is 14 metres and its slant height is 12 metres. Find its

- (i) Curved surface area (ii) Total surface area (iii) Volume and,
(iv) Cost of colouring its total surface at the rate of ₹ 1.2/m²

Solution

$$\begin{aligned} \text{(i) Curved surface area} &= \pi rl = \frac{22}{7} \times \frac{14}{2} \times 12 \\ &= 22 \times 12 = \mathbf{264 \text{ m}^2} \end{aligned}$$

$$\begin{aligned} \text{(ii) Total surface area} &= \pi r(r + l) = \frac{22}{7} \times \frac{14}{2} \left(\frac{14}{2} + 12 \right) \\ &= \frac{22}{7} \times \frac{14}{2} \times 19 = 22 \times 19 = \mathbf{418 \text{ m}^2} \end{aligned}$$

$$\text{(iii) Volume} = \frac{1}{3} \pi r^2 h$$

$$\begin{aligned} \text{But } h &= \sqrt{l^2 - r^2} \\ &= \sqrt{(12)^2 - (7)^2} = 9.75 \text{ metres} \end{aligned}$$

$$\text{Volume} = \frac{1}{3} \times \frac{22}{7} \times 7^2 \times 9.75 = \mathbf{500.5 \text{ m}^3}$$

$$\text{(iv) Cost of colouring} = 418 \times 1.2 = \mathbf{₹ 501.6}$$

EXAMPLE 47 Frustum of a cone has a base diameter of 10 cm, and top of 6 cm diameter and a height of 5 cm. Find the area of its whole surface and volume.

Solution $R = 5 \text{ cm}$; $r = 3 \text{ cm}$; $h = 5 \text{ cm}$

∴

$$\begin{aligned} l &= \sqrt{h^2 + (R - r)^2} \\ &= \sqrt{5^2 + (5 - 3)^2} = \sqrt{29} = \mathbf{5.385 \text{ cm}} \end{aligned}$$

∴ Whole surface area of the frustum

$$= \pi[R^2 + r^2 + (R + r)l]$$

$$= \frac{22}{7}[5^2 + 3^2 + (5 + 3)5.385] = \mathbf{242.25 \text{ m}^2}$$

$$\text{Volume} = \frac{\pi h}{3}(R^2 + r^2 + Rr)$$

$$= \frac{22}{7} \times \frac{5}{3}(5^2 + 3^2 + 5 \times 3) = \mathbf{256.67 \text{ m}^3}$$



Important Formulae

I. Two Spheres

- (i) (Ratio of radii)² = Ratio of surface areas
- (ii) (Ratio of radii)³ = Ratio of volume
- (iii) (Ratio of surface areas)³ = (Ratio of volumes)²

II. Two Cubes

- (i) Ratio of volumes = (Ratio of sides)³
- (ii) Ratio of surface areas = (Ratio of sides)²
- (iii) (Ratio of surface areas)³ = (Ratio of volumes)²

III. Two cylinders

- (a) When the volumes are equal
 - (i) Ratio of radii = $\sqrt{\text{Inverse ratio of heights}}$
 - (ii) Ratio of curved surface areas = Inverse ratio of radii
 - (iii) Ratio of curved surface areas = $\sqrt{\text{Ratio of heights}}$
- (b) When the radii are equal
 - (i) Ratio of volumes = Ratio of heights
 - (ii) Ratio of curved surface areas = Ratio of heights
 - (iii) Ratio of volume = Ratio of curved surface areas
- (c) When the heights are equal
 - (i) Ratio of volumes = (Ratio of radii)²
 - (ii) Ratio of curved surface areas = Ratio of radii
 - (iii) Ratio of volumes = (Ratio of curved surface areas)²
- (d) When curved surface areas are equal
 - (i) Ratio of volumes = Ratio of radii
 - (ii) Ratio of radii = Inverse ratio of heights
 - (iii) Ratio of volumes = Inverse ratio of heights

IV. Two Cones

- (a) When the volumes are equal
Formula (i) of cylinders holds
- (b) When radii are equal
Formula (i) of cylinders holds
- (c) When the heights are equal
Formula (i) of cylinders holds
- (d) When the curved surface areas are equal
Formula (iii) of cylinders holds with a change
i.e. Ratio of radii = Inverse ratio of slant heights



EXERCISES

- A circular wire of radius 42 cm is cut and bent in the form of a rectangle whose sides are in the ratio 6 : 5. Find the smaller side of the rectangle.
(a) 40 cm (b) 60 cm (c) 80 cm (d) 100 cm (e) None of these
- The volumes of two cubes are in the ratio of 8 : 125. Then find the ratio of their edges and surface areas.
(a) 16 : 25 (b) 9 : 25 (c) 4 : 25 (d) 36 : 25 (e) None of these
- A cylinder, a hemisphere and a cone stand on the same base and have the same heights. Find the ratio of their volumes and also the ratio of the areas of their curved surface.
(a) 1 : 1 : 1, $1 : 1 : \sqrt{2}$ (b) 1 : 2 : 3, $1 : \sqrt{2} : \sqrt{2}$ (c) 3 : 2 : 1, $\sqrt{2} : \sqrt{2} : 1$
(d) 2 : 3 : 1, $\sqrt{2} : 1 : \sqrt{2}$ (e) None of these
- A right angled triangle having base 6.3 m and height equal to 10 cm is turned around the height. Find the volume of the cone thus formed. Also find the surface area.
(a) 253.83 m^2 (b) 233.83 m^2 (c) 213.83 m^2 (d) 223.83 m^2 (e) None of these
- If the heights of two cones are in the ratio 1 : 5 and their diameters are in the ratio 5 : 6, what is the ratio of their volumes?
(a) 1 : 6 (b) 1 : 36 (c) 5 : 6 (d) 5 : 36 (e) None of these
- The edges of a cuboid are in the ratio 1 : 2 : 3, and its surface area is 88 cm^2 . The volume of the cuboid is [SSC]
(a) 24 cm^3 (b) 48 cm^3 (c) 64 cm^3 (d) 120 cm^3
- The perimeter of one face of a cube is 20 cm. Its volume must be [SSC]
(a) 125 cm^3 (b) 400 cm^3 (c) 1000 cm^3 (d) 8000 cm^3
- If the volumes of two cubes are in the ratio 27 : 1, the ratio of their edges is [SSC]
(a) 1 : 3 (b) 1 : 27 (c) 3 : 1 (d) 27 : 1
- A cylindrical tank of diameter 35 cm is full of water. If 11 litres of water is drawn off the water level in the tank will drop by [SSC]
(a) $10\frac{1}{2} \text{ cm}$ (b) $11\frac{3}{7} \text{ cm}$ (c) $12\frac{6}{7} \text{ cm}$ (d) 14 cm
- The curved surface area of a right circular cone of height 15 cm and base diameter 16 cm is [SSC]
(a) $60\pi \text{ cm}^2$ (b) $68\pi \text{ cm}^2$ (c) $120\pi \text{ cm}^2$ (d) $136\pi \text{ cm}^2$
- A hemispherical bowl is filled to the brim with a beverage. The contents of the bowl are transferred into a cylindrical vessel whose radius is 50% more than its height. If the diameter is same for both the bowl and the cylinder, the volume of the beverage in the cylindrical vessel is [IAS]
(a) $66\frac{2}{3}\%$ (b) $78\frac{1}{2}\%$ (c) 100%
- A hollow sphere of internal and external diameters 4 cm and 8 cm respectively is melted into a cone of base diameter 8 cm. The height of the cone is [RRB]
(a) 12 cm (b) 14 cm (c) 15 cm (d) 18 cm
- The volumes of two spheres are in the ratio of 64 : 27. The ratio of their surface areas is [RRB]
(a) 1 : 2 (b) 2 : 3 (c) 9 : 16 (d) 16 : 9

14. The slant height of a right circular cone is 10 m, and its height is 8 m. Find the area of its curved surface [RRB]
(a) $30\pi \text{ m}^2$ (b) $40\pi \text{ m}^2$ (c) $60\pi \text{ m}^2$ (d) $80\pi \text{ m}^2$
15. How many cubes of 10 cm edge can be put in a cubical box of 1 m edge? [RRB]
(a) 10 (b) 100 (c) 1000 (d) 10000
16. A cylindrical vessel of radius 4 cm contains water. A solid sphere of radius 3 cm is lowered into the water until it is completely immersed. The water level in the vessel will rise by [MBA]
(a) $\frac{2}{9}$ cm (b) $\frac{4}{9}$ cm (c) $\frac{9}{4}$ cm (d) $\frac{9}{2}$ cm
17. The volume of the largest right circular cone that can be cut out of a cube of edge 7 cm is [MAT]
(a) 13.6 cm^3 (b) 89.8 cm^3 (c) 121 cm^3 (d) 147.68 cm^3
18. Three cubes with sides in the ratio 3 : 4 : 5 are melted to form a single cube whose diagonal is $12\sqrt{3}$ cm. The sides of the cubes are [MAT]
(a) 3 cm, 4 cm, 5 cm (b) 6 cm, 8 cm, 10 cm
(c) 9 cm, 12 cm, 15 cm (d) None of these
19. The length of an edge of a hollow cube opened at one face is $\sqrt{3}$ metres. What is the length of the largest pole that it can accommodate? [MAT]
(a) $\sqrt{3}$ m (b) 3 m (c) $3\sqrt{3}$ m (d) $\frac{3}{\sqrt{3}}$ m
20. A metallic sheet is of rectangular shape with dimensions 48 m $\times$ 36 m. From each of its corners, a square is cut off so as to make an open box. If the length of the square is 8 m, the volume of the box (in m^3) is [MAT]
(a) 4830 (b) 5120 (c) 6420 (d) 8960
21. The number of bricks, each measuring 25 cm $\times$ 12.5 cm $\times$ 7.5 cm, required to construct a wall 6 m long, 5 m high and 0.5 m thick, while the mortar occupies 5% of the volume of the wall, is [MBA]
(a) 3040 (b) 5740 (c) 6080 (d) 8120
22. A solid metallic spherical ball of diameter 6 cm is melted and recast into a cone with diameter of the base as 12 cm. The height of the cone is [CBI]
(a) 2 cm (b) 3 cm (c) 4 cm (d) 6 cm
23. If the measured value of the radius is 1.5% larger, the percentage error (correct to one decimal place) made in calculating the volume of a sphere is [CBI]
(a) 2.1 (b) 3.2 (c) 4.6 (d) 5.4
24. Consider the volumes of the following:
1. A parallelepiped of length 5 cm, breadth 3 cm and height 4 cm.
2. A cube of each side 4 cm.
3. A cylinder of radius 3 cm and length 3 cm.
4. A sphere of radius 3 cm.
The volumes of these in the decreasing order is [Civil Services]
(a) 1, 2, 3, 4 (b) 1, 3, 2, 4 (c) 4, 2, 3, 1 (d) 4, 3, 2, 1
25. The radii of two cones are in the ratio 2 : 1, their volumes are equal. Find the ratio of their heights. [CBI]
(a) 1 : 8 (b) 1 : 4 (c) 2 : 1 (d) 4 : 1
26. The radius and height of a right circular cone are in the ratio 3 : 4. If its volume is $96\pi \text{ cm}^3$, what is its slant height? [CBI]
(a) 8 cm (b) 9 cm (c) 10 cm (d) 12 cm

27. 66 cm^3 of silver is drawn into a wire 1 mm in diameter. The length of the wire in metres will be [CBI]
(a) 84 (b) 90 (c) 168 (d) 336
28. X and Y are two cylinders of the same height. The base of X has diameter which is equal to half the diameter of the base of Y. If the height of X is doubled, the volume of X becomes [CBI]
(a) equal to the volume of Y (b) double the volume of Y
(c) half the volume of Y (d) greater than the volume of Y
29. The height of a wall is six times its width, and the length of the wall is seven times its height. If volume of the wall be 16128 m^3 , its width is [CBI]
(a) 4 m (b) 4.5 m (c) 5 m (d) 6 m
30. The radius of cylinder is half its height and area of the inner part is 616 cm^2 . Approximately how many litres of milk can it contain? [SBI PO]
(a) 1.4 (b) 1.5 (c) 1.7 (d) 1.9 (e) 2.2
31. The capacity of a cylindrical tank is 246.4 litres. If the height is 4 metres, what is the diameter of the base? [Bank PO]
(a) 1.4 m (b) 2.8 m (c) 14 m (d) 28 m (e) None of these
32. The cost of paint in ₹ 36/kg. If 1 kg of paint covers 1.5 m^2 , how much will it cost to paint outside of a cube having 2.5 m side? [Bank PO]
(a) ₹ 700 (b) ₹ 784 (c) ₹ 884 (d) ₹ 900 (e) None of these
33. A hall is 15 m long and 12 m broad. If the sum of the areas of the floor and the ceiling is equal to the sum of areas of the four walls, the volume of the hall is [LIC AAO]
(a) 720 (b) 900 (c) 1200 (d) 1800
34. A cone of height 9 cm with diameter of its base 18 cm is carved out from a wooden solid sphere of radius 9 cm. The percentage of the wood wasted is [SSC]
(a) 25% (b) $25\pi\%$ (c) 50% (d) 75%
35. If three metallic spheres of radii 6 cm, 8 cm and 10 cm are melted to form a single sphere, the diameter of the new sphere will be: [DMRC]
(a) 12 cm (b) 24 cm (c) 30 cm (d) 36 cm
36. A large cube is formed from the material obtained by melting three smaller cubes of sides 3 cm, 4 cm and 5 cm respectively. What is the ratio of the total surface areas of the smaller cubes and the large cube? [MAT]
(a) 2 : 1 (b) 3 : 2 (c) 25 : 18 (d) 27 : 20
37. A rectangular water tank is $80 \text{ m} \times 40 \text{ m}$. Water flows into it through a pipe 40 cm^2 at the opening at a speed of 10 km/hr. By how much, the water level will rise in the tank in half an hour? [MBA]
(a) $\frac{3}{2} \text{ cm}$ (b) $\frac{4}{9} \text{ cm}$ (c) $\frac{5}{8} \text{ cm}$ (d) None of these
38. The volume of the greatest sphere that can be cut off from a cylindrical log of wood of base radius 1 cm and height 5 cm is [CBI]
(a) $\frac{4}{3}\pi$ (b) $\frac{10}{3}\pi$ (c) 5π (d) $\frac{20}{3}\pi$
39. A cylinder with base radius of 8 cm and height of 2 cm is melted to form a cone of height 6 cm. The radius of the cone will be: [RRB]
(a) 4 cm (b) 5 cm (c) 6 cm (d) 8 cm
40. How many bricks, each measuring $25 \text{ cm} \times 11.25 \text{ cm} \times 6 \text{ cm}$, will be needed to build a wall $8 \text{ m} \times 6 \text{ m} \times 22.5 \text{ cm}$? [BSF]
(a) 5600 (b) 6000 (c) 6400 (d) 7200



1. (b) 2. (c) 3. (c) 4. (b) 5. (d) 6. (b) 7. (a) 8. (c) 9. (b) 10. (d)
 11. (c) 12. (b) 13. (d) 14. (c) 15. (c) 16. (c) 17. (b) 18. (b) 19. (b) 20. (b)
 21. (c) 22. (b) 23. (c) 24. (d) 25. (b) 26. (c) 27. (a) 28. (c) 29. (a) 30. (e)
 31. (b) 32. (d) 33. (c) 34. (d) 35. (a) 36. (c) 37. (c) 38. (a) 39. (d) 40. (c)



Solutions with Necessary Explanation

1. Length of wire = Perimeter of rectangle
 = Circumference of circle

$$\text{i.e. } 2\pi \times 42 = 2 \times \frac{22}{7} \times 42 = 264 \text{ cm}$$

$$\therefore \text{Perimeter of rectangle} = 264$$

$$\therefore \text{Length} = \frac{6}{11 \times 2} \times 264 = 72 \text{ cm}$$

$$\text{Breadth} = \frac{5}{11 \times 2} \times 264 = 60 \text{ cm}$$

2. Ratio of volumes of cubes = (Ratio of sides)³

$$\text{i.e. } 8 : 125 = (2)^3 : (5)^3$$

$$\therefore \text{Edges are in the ratio } 2 : 5$$

$$\text{Ratio of surface areas} = (\text{Ratio of sides})^2 = (2)^2 : (5)^2 = 4 : 25$$

3. Let the heights be h units

Let the radii be r units

Let the volumes be respectively v_1 , v_2 and v_3 .

Let curved surface areas be respectively CSA_1 , CSA_2 and CSA_3 .

$$\therefore v_1 : v_2 : v_3 = \pi r^2 h : \frac{2}{3} \pi r^3 : \frac{1}{3} \pi r^2 h$$

$$= h : \frac{2}{3} r : \frac{h}{3}$$

or

$$3h : 2r : h \text{ or } 3 : 2 : 1$$

($\because h = r$)

$$\text{CSA}_1 : \text{CSA}_2 : \text{CSA}_3 = 2\pi r h = 2\pi r^2 : \pi r l$$

$$= 2h : 2r : l$$

$$= 2h : 2r : \sqrt{r^2 + h^2}$$

$$= 2h : 2r : \sqrt{2r^2}$$

($\because r = h$)

$$= 2h : 2r : \sqrt{2}r$$

or

$$2 : 2 : \sqrt{2}$$

or

$$\sqrt{2} : \sqrt{2} : 1$$

$$\begin{aligned}
 4. \text{ Volume of cone formed} &= \frac{1}{3} \pi \times (6.3)^2 \times 10 \\
 &= \frac{1}{3} (1247.4) \text{ m}^3 = \mathbf{415.8 \text{ m}^3}
 \end{aligned}$$

$$\text{Surface area of cone} = \pi r l$$

$$\text{But } l = \sqrt{r^2 + h^2} = \sqrt{(6.3)^2 + 10^2} = \mathbf{11.81 \text{ m}}$$

$$\therefore \text{Surface area} = \frac{22}{7} \times 6.3 \times 11.81 = \mathbf{233.83 \text{ m}^2}$$

$$\begin{aligned}
 5. \text{ Ratio of volumes of two cones} &= \frac{r_1^2 h_1}{r_2^2 h_2} \\
 &= \left(\frac{r_1}{r_2} \right)^2 \times \left(\frac{h_1}{h_2} \right) \\
 &= \left(\frac{5}{6} \right)^2 \times \left(\frac{1}{5} \right) = \mathbf{5 : 36}
 \end{aligned}$$

$$6. \text{ Surface area} = 2 \times l \times b + 2(l + b) \times h = 88 \text{ cm}^2$$

$$\text{Let } l = 3x; \quad b = 2x; \quad h = x;$$

$$\therefore \text{Surface area} = 2 \times 3x \times 2x + 2(3x + 2x)x = 88 \text{ cm}^2$$

$$\text{i.e. } 12x^2 + 10x^2 = 88$$

$$\therefore x^2 = \frac{88}{22} = 4$$

$$\therefore x = 2$$

$$\begin{aligned}
 \therefore \text{Volume of cuboid} &= 3x \times 2x \times x = 6x^3 \\
 &= 6(2)^3 = \mathbf{48 \text{ cm}^3}
 \end{aligned}$$

$$7. \text{ Given } 4a = 20 \text{ cm} \Rightarrow a = 5 \text{ cm};$$

$$\therefore \text{Volume} = a^3 = 5^3 = \mathbf{125 \text{ cm}^3}$$

$$8. \frac{a_1^3}{a_2^3} = \frac{27}{1} \Rightarrow \frac{a_1}{a_2} = \frac{3}{1}$$

$$9. \text{ Water drawn} = \pi \times \left(\frac{35}{2} \right)^2 \times h = 11000 \text{ cm}^3$$

$$\text{or } h = \frac{11000 \times 4}{\pi \times 35^2} = \frac{11000 \times 4}{\frac{22}{7} \times 35^2}$$

$$= \frac{11000 \times 4}{22 \times 35 \times 5} = \frac{80}{7} = \mathbf{11 \frac{3}{7} \text{ cm}}$$

$$10. \text{ Given } h = 15 \text{ cm}; \quad r = \frac{16}{2} = 8 \text{ cm}$$

$$\therefore l = \sqrt{h^2 + r^2} = \sqrt{15^2 + 8^2} = 17 \text{ cm}$$

$$\text{Curved surface area} = \pi r l = \pi \times 8 \times 17 = \mathbf{136 \pi \text{ cm}^2}$$

11. $\frac{2}{3}\pi r^3$ = Volume of hemispherical bowl.

Let h be the height of cylinder.

$$\therefore r = 1.5 h$$

$$\text{Volume of cylinder} = \pi r^2 h = \pi r^2 \times \frac{r}{1.5} = \frac{\pi r^3}{1.5} = \frac{2\pi r^3}{3}$$

$\therefore$ Both volumes are equal, required percentage = 100%

$$\begin{aligned} 12. \text{ Volume of sphere} &= \frac{4}{3}\pi \left[\left(\frac{8}{2}\right)^3 - \left(\frac{4}{2}\right)^3 \right] \\ &= \frac{4}{3}\pi(4^3 - 2^3) \\ &= \frac{4}{3}\pi \times 56 \text{ cm}^3 \end{aligned}$$

Diameter of cone = $2r = 8$ cm.

$$\therefore r = 4 \text{ cm.}$$

$$\therefore \text{Volume of cone} = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi \times (4)^2 \times h = \frac{4}{3}\pi \times 56$$

[$\because$ Volume of cone = Volume of sphere]

$$\text{i.e. } 16h = 4 \times 56$$

$$\begin{aligned} \therefore h &= \frac{4 \times 56}{16} \\ &= 14 \text{ cm} \end{aligned}$$

$$13. \frac{V_1}{V_2} = \frac{64}{27} = \frac{r_1^3}{r_2^3} \Rightarrow \frac{r_1}{r_2} = \frac{4}{3}$$

$$\text{Surface area} = 4\pi r^2$$

$$\therefore \text{area} \propto r^2$$

$$\therefore \frac{A_1}{A_2} = \frac{r_1^2}{r_2^2} = \frac{16}{9}$$

$$14. l = 10 \text{ m; } h = 8 \text{ m;}$$

Curved surface area of cone = $\pi r l$

$$r^2 = \sqrt{l^2 - h^2} = \sqrt{10^2 - 8^2} = \sqrt{36} = 6 \text{ cm}$$

$$\therefore \text{C.S.A} = \pi \times 6 \times 10 = 60\pi \text{ m}^2$$

$$15. \text{ Volume of cube of 1 m side } V_1 = 1^3 = 1 \text{ m}^3$$

$$\text{Volume of cube of 10 cm side } V_2 = 10^3 = 1000 \text{ cm}^3 = 0.001 \text{ m}^3$$

$$\therefore \text{Required number of cubes} = \frac{V_1}{V_2} = \frac{1}{0.001} = 1000$$

16. Volume of sphere of 3 cm radius $V_1 = \frac{4}{3}\pi(3)^3 = 36\pi \text{ cm}^3$

Sectional area of cylinder, $A_1 = \pi r^2 = \pi \times (4)^2 = 16\pi \text{ cm}^2$

$\therefore$ Height through which water will rise

$$= \frac{V_1}{A_1} = \frac{36\pi \text{ cm}^3}{16\pi \text{ cm}^2} = \frac{9}{4} \text{ cm}$$

17. $d = 7 \text{ cm}$; $h = 7 \text{ cm}$

$$\begin{aligned} \text{Volume of cone} &= \frac{1}{3}\pi r^2 h = \frac{1}{3} \times \frac{\pi d^2 h}{4} \\ &= \frac{1}{3} \times \frac{\pi(7^2) \times 7}{4} \\ &= \frac{1}{3} \times \frac{22}{7} \times \frac{7^3}{4} \\ &= \frac{11 \times 7^2}{6} \\ &= \frac{539}{6} = 89.83 \approx 89.8 \text{ cm}^3 \end{aligned}$$

18. Total volumes of cubes $= (3x)^3 + (4x)^3 + (5x)^3$
 $= (27 + 64 + 125)x^3$
 $= 216x^3$

$\therefore$ Side of single cube $= \sqrt[3]{216x^3} = 6x$

$\therefore$ Diagonal of cube $= 6x\sqrt{3} = 12\sqrt{3}$

$\therefore x = \frac{12\sqrt{3}}{6\sqrt{3}} = 2$

$\therefore$ Sides are **6 cm, 8 cm and 10 cm.**

19. Required length $= \sqrt{3} \times \sqrt{3} = 3 \text{ m}$

20. Size of sheet is $48 \text{ m} \times 36 \text{ m}$

After cutting squares of 8 m size from its corners,

$$\begin{aligned} \text{Size of sheet} &= (48 - 16) \times (36 - 16) \\ &= 32 \text{ m} \times 20 \text{ m} \end{aligned}$$

$\therefore$ Volume of box $= 32 \times 20 \times 8 = 5120 \text{ m}^3$

21. Volume of wall $= 6 \text{ m} \times 5 \text{ m} \times 0.5 \text{ m} = 15 \text{ m}^3$

Volume of 1 brick $= 25 \times 12.5 \times 7.5 = 2343.75 \text{ cm}^3$

$$\begin{aligned} \text{Required number of bricks} &= \frac{95\% \text{ of } 15 \text{ m}^3}{9375/4} \\ &= \frac{0.95 \times 15 \times 10^6 \text{ cm}^3}{2343.75 \text{ cm}^3} \end{aligned}$$

$$\begin{aligned}
&= \frac{95 \times 15 \times 10^4}{9375/4} \\
&= \frac{95 \times 15 \times 10^4 \times 4}{9375} \\
&= \frac{19 \times 3 \times 10^4 \times 4}{375} \\
&= \frac{19 \times 4 \times 10^4}{125} = 19 \times 4 \times 8 \times 10 = \mathbf{6080}
\end{aligned}$$

$$22. \text{ Volume of spherical ball} = \frac{4}{3} \pi \left(\frac{6}{2} \right)^3 = 36 \pi \text{ cm}^3 \quad (1)$$

$$\text{Volume of cone} = \frac{1}{3} \pi (6)^2 \times h = 12 \pi h \quad (2)$$

Equating (1) and (2), we get

$$12 \pi h = 36 \pi$$

$$\Rightarrow h = \frac{36 \pi}{12 \pi} = \mathbf{3 \text{ cm}}$$

$$\begin{aligned}
23. \text{ \% error} &= \frac{r_c^3 \sim r_e^3}{r_c^3} \times 100 \\
&= \frac{r_c^3 \sim (1.015)^3 r_c^3}{r_c^3} \times 100 \\
&= \frac{1.04567 \sim 1}{1} \times 100 = 4.567\% \approx \mathbf{4.6\%}
\end{aligned}$$

24. Required order is **4, 3, 2, 1**.

$$25. \frac{r_1}{r_2} = \frac{2}{1}; \quad \frac{V_1}{V_2} = 1$$

$$\text{i.e.} \quad V_1 = \frac{1}{3} \pi r_1^2 h_1; \quad V_2 = \frac{1}{3} \pi r_2^2 h_2$$

$$\text{i.e.} \quad \frac{1}{3} \pi r_1^2 h_1 = \frac{1}{3} \pi r_2^2 h_2$$

$$\therefore \frac{h_1}{h_2} = \frac{r_2^2}{r_1^2} = \frac{1^2}{2^2} = \frac{1}{4}$$

26. Let radius = $3x$ and height = $4x$;

$$\text{Volume of cone} = \frac{1}{3} \pi r^2 h = \frac{1}{3} \pi (3x)^2 \times 4x = 96 \pi$$

$$\text{i.e.} \quad \frac{1}{3} \pi \times 9x^2 \times 4x = 96 \pi$$

$$\text{i.e.} \quad 12x^3 = 96$$

$$\begin{aligned}\Rightarrow x^3 &= \frac{96}{12} = 8 \\ \therefore x &= 2 \\ \therefore h &= 4x = 8 \text{ cm}; \quad r = 3x = 6 \text{ cm.} \\ \therefore l &= \sqrt{h^2 + r^2} = \sqrt{8^2 + 6^2} = 10 \text{ cm}\end{aligned}$$

$$\begin{aligned}27. \text{ Required length} &= \frac{66 \text{ cm}^3}{\frac{\pi}{4}(0.1)^2} = \frac{66 \times 4}{\pi \times 0.01} \\ &= \frac{66 \times 4 \times 100}{\frac{22}{7}} = 3 \times 4 \times 7 \times 100 = 8400 \text{ cm} = \mathbf{84 \text{ m}}\end{aligned}$$

$$28. \quad h_1 = h_2; \quad d_1 = \frac{1}{2}d_2;$$

$$\text{Volume of cylinder X } V_1 = \pi d_1^2 h_1 \quad (1)$$

$$\text{Volume of cylinder Y } V_2 = \pi d_2^2 h_2 \quad (2)$$

$$V_1 = \pi d_1^2 h_1 = \pi d_1^2 h \quad (A)$$

$$\begin{aligned}V_2 &= \pi d_2^2 h_2 = \pi (2d_1)^2 \times h \\ &= 4\pi d_1^2 h \\ &= 4V_1 \quad (B)\end{aligned}$$

If height of X is doubled, volume of X will become double

Volume of X = Half the volume of Y

$$29. \quad h = 6b; \quad l = 7h$$

$$\text{Volume of wall} = lbh = (7h) \left(\frac{h}{6} \right) \times h = \frac{7}{6}h^3$$

$$\text{Given } \frac{7}{6}h^3 = 16128$$

$$\begin{aligned}\Rightarrow h^3 &= \frac{16128 \times 6}{7} \\ &= 2304 \times 6 = 13824\end{aligned}$$

$$\therefore h = 24$$

$$\therefore \text{Width } b = \frac{h}{6} = \frac{24}{6} = 4 \text{ m}$$

$$30. \quad r = \frac{h}{2};$$

$$A = 2\pi rh = 616 \text{ cm}^2$$

$$2\pi r(2r) = 616$$

$$r^2 = \frac{616}{4\pi} = \frac{154}{\frac{22}{7}} = \frac{154 \times 7}{22} = 49$$

$$\therefore r = 7 \text{ cm}$$

$$\text{Volume of cylinder} = \pi r^2 h = \pi(7)^2(2 \times 7)$$

$$= \frac{22}{7} \times 7^3 \times 2$$

$$= 44 \times 49$$

$$= 2156 \text{ cm}^3$$

$$\text{Approximate volume in litres} = \frac{2156}{1000} = 2.156 \approx \mathbf{2.2 \text{ litre}}$$

31. $V = 246.4 \text{ litre}; h = 4 \text{ m}; d = ?$

$$V = \frac{\pi d^2}{4} \times h$$

$$\therefore d = \sqrt{\frac{4V}{\pi h}} = \sqrt{\left(4 \times \frac{246.4 \text{ m}^3}{1000}\right) \div \frac{22}{7} \times 4 \text{ m}}$$

$$= \sqrt{\frac{7 \times 246.4}{22000}} \text{ m}$$

$$= \sqrt{\frac{7 \times 246.4 \times 1000}{22}} \text{ cm}$$

$$= \sqrt{7 \times 11.2 \times 1000} \text{ cm} = \sqrt{100 \times 784}$$

$$= 10 \times 28 = 280 \text{ cm} = \mathbf{2.8 \text{ m}}$$

32. Required cost = $\frac{6(2.5)^2 \times 1}{1.5} \times 36 = 25 \times 36 = \mathbf{₹900}$

33. $2 \times 15 \times 12 = 2(15 + 12) \times h$

$$\therefore h = \frac{2 \times 15 \times 12}{2 \times 27}$$

$$= \frac{20}{3} \text{ m}$$

$$\therefore \text{Volume of hall} = 15 \times 12 \times \frac{20}{3} = \mathbf{1200 \text{ m}^3}$$

34. Volume of wooden solid sphere = $\frac{4}{3} \pi (9)^3 = \frac{4}{3} \times \pi \times 9^3 = 972 \pi \text{ cm}^3$

$$\text{Volume of cone} = \frac{1}{3} \pi \left(\frac{18}{2}\right)^2 \times 9 = 243 \pi \text{ cm}^3$$

$$\therefore \text{Required percentage} = \frac{972 \pi - 243 \pi}{972 \pi} \times 100 = \mathbf{75\%}$$

35. Required diameter = $\sqrt[3]{6^3 + 8^3 + 10^3}$

$$= \sqrt[3]{216 + 512 + 1000}$$

$$= \sqrt[3]{1728} = \mathbf{12 \text{ cm}}$$

36. Side of large cube = $\sqrt[3]{3^3 + 4^3 + 5^3} = 6 \text{ cm}$

Total surface area = $6a^2$

Total surface area for smaller cubes = $6\{3^2 + 4^2 + 5^2\}$
 $= 6(9 + 16 + 25)$
 $= 300 \text{ cm}^2$

T.S.A of large cube = $6 \times 6^2 = 216 \text{ cm}^2$

Required ratio = $\frac{300}{216} = \frac{25}{18}$

37. Speed of water = $10 \text{ km/hr} = 10 \times \frac{5}{18} \text{ m/s} = \frac{25}{9} \text{ m/s}$

Let h be the height through which water will rise in half an hour.

$\therefore$ Quantity of inflow = AV

$$= \frac{40}{10000} \text{ m}^2 \times \frac{25}{9} \text{ m/s}$$

$$= \frac{1}{90} \text{ m}^3/\text{s}$$

$\therefore$ Quantity of inflow in 30 minutes

$$= \frac{1}{90} \text{ m}^3/\text{s} \times 30 \times 60 \text{ sec} = 20 \text{ m}^3$$

$\therefore$ Required height = $\frac{20}{80 \times 40} \text{ m} = \frac{20 \times 100}{80 \times 40} \text{ cm} = \frac{5}{8} \text{ cm}$

38. Volume of greatest sphere = $\frac{4}{3}\pi(1)^3 = \frac{4}{3}\pi \text{ cm}^3$

39. Volume of cylinder = $\pi(8)^2 \times 2 = 128\pi \text{ cm}^3$

Let radius of cone be $r \text{ cm}$.

$\therefore$ Volume of cone = $\frac{1}{3}\pi r^2 h$

$$= \frac{1}{3}\pi r^2 \times (6) \text{ cm}^3 = 2\pi r^2 \text{ cm}^3$$

Equating both volumes, we get

$$128\pi = 2\pi r^2$$

$\therefore r^2 = \frac{128\pi}{2\pi} = 64$

$\therefore r = \sqrt{64} = 8 \text{ cm}$

40. Volume of wall = $8 \text{ m} \times 6 \text{ m} \times 22.5 \text{ cm}$

$$= 48 \times 0.225 \text{ m}^3$$

Volume of brick = $25 \text{ cm} \times 11.25 \text{ cm} \times 6 \text{ cm}$

$\therefore$ Required number of bricks = $\frac{8 \times 6 \times 22.5 \times 10^4}{25 \times 11.25 \times 6} = \frac{8 \times 2 \times 10^4}{25}$
 $= 8 \times 2 \times 4 \times 10^2 = 6400$

Problems on Clock

A clock is a device which measures and shows time. In this chapter some facts related to a clock are included based on which you have to work out problems.



Important Facts

1. In every hour the two hands coincide once.
2. Each hour space equals $360^\circ/12 = 30^\circ$.
3. Each minute space equals $360^\circ/60 = 6^\circ$.
4. When the two hands are at right angles (90°), they are $90/6 = 15$ minutes apart. This occurs twice in every hour.
5. When two hands are in the opposite direction, they are $180/6 = 30$ minutes apart. This occurs only once in every hour.
6. When the hands are coincident, they are obviously zero degrees and zero minutes apart.
7. The two hands are in the same straight line when they are either coincident or opposite to each other.
8. The hands coincide 11 times in every 12 hours, because there is a common position 12 o'clock between 11 and 1 o'clock. Hence they coincide only 22 times a day.
9. The hands of a clock are at right angles twice in every hour but in 12 hours they are at right angles only 22 times. It is so because there are two positions common in every 12 hours. i.e. 3 o'clock and 9 o'clock.
10. Two hands of a clock will be together between h and $(h + 1)$ o'clock at $(60/11)h$ minutes past h o'clock.

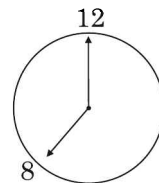
EXAMPLE 1 Find the time between 8 o'clock and 9 o'clock, when the two hands of a clock are in the same straight line.

Solution Two hands of a clock are in the same straight line in two cases.

- (i) When the two hands are in exactly opposite direction.

This equals $180^\circ/6^\circ = 30$ minutes apart.

At 8 o'clock, minute hand is at $8 \times 5 = 40$ minutes spaces behind the hour hand.



Therefore, the minute hand will have to gain $(40 - 30) = 10$ minutes space over the hour hand.

$\therefore$ Gain of 55 minutes space = 60 minutes

$\therefore$ Gain of 10 minutes space = $\frac{60}{55} \times 10 = \frac{120}{11}$ minutes

$\therefore$ The minute hand will be exactly opposite direction to the hour hand at $10 \frac{10}{11}$ minutes past 8 o'clock.

(ii) When the two hands coincide (i.e. 0 minutes apart)

In this case the minute hand will have to gain $8 \times 5 = 40$ minute space over the hour hand.

$\therefore$ Gain of 55 minutes space = 60 minutes

Gain of 40 minutes space = $\frac{60}{55} \times 40 = \frac{480}{11} = 43 \frac{7}{11}$ minutes

$\therefore$ Two hands will coincide at $43 \frac{7}{11}$ minutes past 8 o'clock.

Between h and $(h + 1)$ o'clock, the minute hand is $5h$ minute space behind the hour hand.

Case (a) The minute hand is m minutes behind the hour hand.

The minute hand must gain $(5h - m)$ minute space over the hour hand.

Case (b) The minute hand is m minutes ahead of the hour hand.

The minute hand must gain $(5h + m)$ minute space over the hour hand. Gain of 55 minutes space = 60 minutes.

Gain of $(5h \pm m)$ minute space = $60/55 (5h \pm m)$ minutes
 $= 12/11 (5h \pm m)$ minutes past h o'clock

When minute hand is behind the hour hand, angle between hour hand and minute hand is given by angle = $30 \left(H - \frac{M}{5} \right) + \frac{M}{2}$ degrees, where H represents hours and M represents minutes.

(iii) When the minute hand is ahead of the hour hand, the angle between the two hands

at M minutes past H hour is given by angle = $30 \left(\frac{M}{5} - H \right) - \frac{M}{2}$ degrees.

(iv) When the minute hand overtakes the hour hand of a clock at intervals of m , gain

or loss/day by the clock = $\left(\frac{720}{11} - m \right) \left(\frac{60 \times 24}{m} \right)$

EXAMPLE 2 At what time between 3 o'clock and 4 o'clock will the hands of a clock be together?

Solution At 3 o'clock, hour hand is at 3 and the minute hand is at 12. i.e., they are 15 minutes apart. Minute hand must gain 15 minutes over the hour hand so that both of them will be together.

But 55 minutes are gained by minute hand in 60 minute.

$$\therefore 15 \text{ minutes will be gained in } \left(\frac{60}{55} \times 15 \right) \text{ min} = 16 \frac{4}{11} \text{ min}$$

$$\therefore \text{The hands will coincide at } 16 \frac{4}{11} \text{ minutes past 3.}$$

EXAMPLE 3 The minute hand of a clock overtakes the hour hand at intervals of 65 minutes of the correct time. How much a day does the clock gain or lose?

Solution Usually minute hand gains 55 minutes over the hour hand in 60 minutes. To be together, minute hand must gain 60 minutes over the hour hand.

$$\therefore 60 \text{ minutes are gained in } \left(\frac{60}{55} \times 60 \right) \text{ min} = 65 \frac{5}{11} \text{ minutes}$$

$\therefore$ Both of them are together after 65 minutes,

$$\text{Gain in 65 min} = \left(65 \frac{5}{11} - 65 \right) = \frac{5}{11} \text{ minutes}$$

$$\therefore \text{Gain in 24 hours} = \frac{5}{11} \times \frac{60 \times 24}{65} = 10 \frac{10}{43} \text{ minutes}$$

$$\therefore \text{Clock gain } 10 \frac{10}{43} \text{ minutes in 24 hours.}$$

EXAMPLE 4 A clock is set at 9 a.m. The clock gains 10 minutes in 24 hours. What will be the true time when the clock indicates 2 p.m. on the following day?

Solution Time interval between today 9 a.m. and tomorrow 2 p.m. = 29 hours
24 hours 10 minute of the clock = 24 hours of correct clock

$$\therefore \frac{145}{6} \text{ hours of this clock} = 24 \text{ hours of the correct clock}$$

$$\therefore 29 \text{ hours of this clock} = \left(24 \times \frac{6}{145} \times 29 \right) \text{ hours of correct clock}$$

$$= \left(24 \times \frac{6}{145} \times 29 \right) \text{ hours of correct clock}$$

$$= 28 \text{ hours 48 min after 9 a.m.} = 32 \text{ hours 43 min.}$$

i.e. time is 48 minute past 1 p.m.

EXAMPLE 5 At what time between 5 o'clock and 6 o'clock will the hands of a clock be at right angle.

Solution At 5 o'clock minute hand will be 25 minutes space behind the hour hand.

When the two hands are at right angles, they are 15 minutes apart.

Therefore they can be at right angles in the following two cases:

Case I. When minute hand is 15 minutes behind the hour hand.

Here minute hand will have to gain $(25 - 15) = 10$ minutes space.

$$\therefore 10 \text{ minutes space will be gained by it in } \frac{60}{55} \times 10 = 10 \frac{10}{11} \text{ minutes.}$$

∴ They are at right angles at $10\frac{10}{11}$ minutes past 5.

Case II. When minute hand is 15 minutes space ahead of the hour hand:

Here minute hand will have to gain $(25 + 15) = 40$ minutes space.

∴ 40 minutes space will be gained in $\left(\frac{60}{55} \times 40\right) = 43\frac{7}{11}$ minutes.

∴ They are at right angles at $43\frac{7}{11}$ minutes past 5.

EXAMPLE 6 Find the angle between the hour hand and the minute hand of a clock when the time is 4.35.

Solution Angle traced by the hour hand in 12 hours = 360° .

∴ Angle traced by it in 4 hours 35 min $\left(\frac{55}{12} \text{ hours}\right)$

$$= \left(\frac{55}{12} \times \frac{360}{12}\right)^\circ = 137\frac{1}{2}^\circ$$

Angle traced by minute hand in 60 min = 360°

Angle traced by it in 35 minutes = $\left(\frac{360}{60} \times 35\right)^\circ = 210^\circ$

∴ Required angle = $\left(210 - 137\frac{1}{2}\right)^\circ = 72\frac{1}{2}^\circ$

EXAMPLE 7 At what time between 9 o'clock and 10 o'clock will the hand of a clock be in the same straight line but pointing in opposite direction.

Solution At 9 o'clock, the hour hand is at 9 and the minute hand is at 12, i.e., two hands are 15 minutes space apart.

To be in same straight line but pointing in the opposite direction, they will be at 30 minutes space apart.

∴ Minute hand will have to gain $(30 - 15) = 15$ minutes space over the hour hand.

55 minutes space is gained in 60 minutes.

∴ 15 minutes space will be gained in $\left(\frac{60}{55} \times 15\right) \text{ min} = \frac{180}{11} = 16\frac{4}{11} \text{ minutes}$

∴ The hands will be in the same straight line, but pointing in opposite direction at $16\frac{4}{11}$ minutes past 9.



EXERCISES

Study the following questions given below carefully and answer most appropriately from the given alternatives.

- The time shown by a clock was 2 o'clock. After a certain time the mirror image shown by the clock was 15 minutes past 7. What is the time lag between the two times?
(a) 5 h 15 min. (b) 4 h 45 min. (c) 3 h 15 min. (d) 2 h 45 min. (e) None of these
- A clock strikes for every half an hour once and for every hour for the number same as that of the hour. What was the number of strikes given by the clock from 8 hours to 12 hours?
(a) 45 (b) 50 (c) 54 (d) 60 (e) None of these
- The time taken by a clock between two single strike is 2 seconds. What is the time taken to complete its strike at 10 o'clock?
(a) 20 seconds (b) 18 seconds (c) 22 seconds (d) 16 seconds (e) None of these
- A thief runs at a speed of 5m/s. A policeman runs behind him 2 seconds after thief started at a speed of 10 m/s. After how much time, the policeman will catch the thief.
(a) 1 second (b) 2 seconds (c) 3 seconds (d) 4 seconds (e) None of these
- Between 1 hour and 12 hours how many times the hour needle and minute needle will be together?
(a) 11 times (b) 12 times (c) 13 times (d) 10 times (e) None of these
- A watch which gains uniformly is 2 minutes slow at noon on Monday and is 4 min 48 seconds fast at 2 p.m. on the following Monday. When was it correct? **[RRB]**
(a) 2 p.m. on Tuesday
(b) 2 p.m. on Wednesday
(c) 3 p.m. on Thursday
(d) 1 p.m. on Friday
- At what angle the hands of a clock are inclined at 15 minutes past 5? **[LIC AAO]**
(a) $58\frac{1}{2}^\circ$ (b) 64° (c) $67\frac{1}{2}^\circ$ (d) $72\frac{1}{2}^\circ$
- At what time (in minutes) between 3 o'clock and 4 o'clock, both the needles will coincide each other? **[RRB]**
(a) $5\frac{1}{11}$ " (b) $12\frac{4}{11}$ " (c) $13\frac{4}{11}$ " (d) $16\frac{4}{11}$ "
- At what time between 7 o'clock and 8 o'clock will the hands of a clock be in the same straight line but, not together? **[AAO]**
(a) 5 minute past 7
(b) $5\frac{2}{11}$ minute past 7
(c) $5\frac{3}{11}$ minute past 7
(d) $5\frac{5}{11}$ minute past 7

10. How many times in a day, are the hands of a clock in straight line but opposite in direction? **[RRB]**
 (a) 20 (b) 22 (c) 24 (d) 48
11. How many times in a day, are the hands of a clock at right angle? **[IAS]**
 (a) 22 (b) 24 (c) 44 (d) 48
12. The reflex angle between the hands of a clock at 10.25 is **[SCRA]**
 (a) 180° (b) $192\frac{1}{2}^\circ$ (c) 195° (d) $197\frac{1}{2}^\circ$
13. An accurate clock shows 8 o'clock in the morning. Through how many degrees will the hour hand rotate when the clock shows 2 o'clock in the afternoon? **[IAS]**
 (a) 144° (b) 150° (c) 168° (d) 180°
14. The angle between the minute hand and the hour hand of a clock when the time is 10.30 is
 (a) 120° (b) 135° (c) 55° (d) 60°
15. The angle between the hour hand and the minute hand of a clock when the time is 4.55 is:
 (a) $177\frac{1}{2}^\circ$ (b) $182\frac{1}{2}^\circ$ (c) 180° (d) 175°



1. (d) 2. (c) 3. (b) 4. (b) 5. (a) 6. (b) 7. (c) 8. (d) 9. (d) 10. (a)
 11. (c) 12. (d) 13. (d) 14. (b) 15. (b)



Solutions with Necessary Explanation

- Initial time = 2 o'clock.
 Mirror image = 7 h 15 min.
 $\therefore$ The time then will be 4 hours 45 minutes (i.e. 12 – 7 hours 15 minutes)
 $\therefore$ Time lag = 4 h 45 min – 2h = **2 h 45 min.**
- For all half an hour once
 $\therefore$ Total strikes for half hour = 4
 For hour from 8 to 12, number of hourly strikes

$$= 8 + 9 + 10 + 11 + 12 = 50$$
 $\therefore$ Total strikes = 4 + 50 = **54**
- Time taken for completing single strike = 2 seconds
 Time taken to complete its strike at 10 o'clock = 9×2 seconds = **18 seconds.**
- Relative speed between policeman and thief = $10 - 5 = 5$ m/s
 The policeman lags behind thief by 2 seconds at the start.
 $\therefore$ Time taken by policeman to catch the thief = $\frac{\text{Distance covered by thief in 2 seconds}}{\text{Relative speed of policeman}}$

$$= \frac{5 \text{ m/s} \times 2 \text{ sec}}{5 \text{ m/s}} = \mathbf{2 \text{ seconds}}$$

5. In every hour, the hour needle and minute needle will be together once. So, in 11 hours, 11 times and once for 12 o'clock.

∴ In total 12 times.

∴ between 1 hour and 12 hours, number of coincidence = **11 times**.

6. Time interval between Monday 12 noon to next Monday 2 p.m.

$$= 7 \text{ days } 2 \text{ hours}$$

$$= 170 \text{ hours.}$$

∴ Watch gains $\left(2 + 4\frac{4}{5}\right)$ minutes in 170 hours

i.e. $\frac{34}{5}$ minutes gained in 170 hours.

∴ Watch gained 2 minutes in $\left(\frac{170}{34/5} \times 2\right)$ hours

$$= \frac{170 \times 2 \times 5}{34}$$

$$= \mathbf{50 \text{ hours}}$$

∴ Watch is correct 2 days 2 hours after 12 noon of Monday

i.e. at 2 p.m. on Wednesday

7. Angle traced by hour hand in 5 hours 15 minutes $\left(\frac{21}{4} \text{ hours}\right)$

$$= \left(\frac{360}{12} \times \frac{21}{4}\right)^\circ = \mathbf{157\frac{1}{2}^\circ}$$

Angle traced by minute hand in 15 minutes

$$= \left(\frac{360}{60} \times 15\right)^\circ = 90^\circ$$

$$\therefore \text{Required angle} = 157\frac{1}{2} - 90 = \mathbf{67\frac{1}{2}^\circ}$$

8. At 3 o'clock, minute hand is at 15 minutes space apart from the hour hand.

In order to coincide, it should gain 15 minutes space

55 minutes space is gained in 60 minutes

∴ 15 minutes space is gained in $\left(\frac{60}{55} \times 15\right)$ minutes

$$= \frac{180}{11} = \mathbf{16\frac{4}{11} \text{ minutes}}$$

Both hands will coincide at $16\frac{4}{11}$ minutes past 3

9. At 7 o'clock minute hand is at 25 minute space apart.

In order that both hands to be in straight line, minute hand should gain 5 minute space.

55 minute space is gained in 60 minutes

∴ 5 minutes spaces is gained in $\left(\frac{60}{55} \times 5\right)$ minutes = $5\frac{5}{11}$ minutes

∴ Required time = $5\frac{5}{11}$ minutes past 7.

10. The hands of a clock point in opposite directions in the same straight line 11 times in every 12 hours. So in a day, hands point in the opposite direction **22 times**.
11. In 12 hours, they are at right angles 22 times.
In 24 hours, they are at right angles **44 times**.

12. Angle traced by hour hand in 10.25 hours or $\frac{125}{12}$ hours

$$= \left(\frac{360}{12} \times \frac{125}{12} \right)^\circ = 312\frac{1}{2}^\circ$$

Angle traced by minute hand in 25 minutes

$$= \left(\frac{360}{60} \times 25 \right)^\circ = 150^\circ$$

$$\begin{aligned} \therefore \text{Reflex angle} &= 360^\circ - \left(312\frac{1}{2} - 150 \right)^\circ \\ &= 360^\circ - 162\frac{1}{2}^\circ \\ &= 197\frac{1}{2}^\circ \end{aligned}$$

13. Time interval between 8 a.m. and 2 p.m. = 6 hours
Hours hand will move through 30° in every hours.
In 6 hours, movement of hour hand = $6 \times 30 = 180^\circ$

14. Angle traced by hour hand in $10\frac{1}{2}$ hours or $\frac{21}{2}$ hours

$$= \left(\frac{360}{12} \times \frac{21}{2} \right)^\circ = 315^\circ$$

Angle traced by minute hand in 30 minutes

$$= \left(\frac{360}{60} \times 30 \right)^\circ = 180^\circ$$

$$\therefore \text{Required angle} = (315 - 180)^\circ = 135^\circ$$

15. Angle traced by hour hand in 4 hours 55 min $\left(\frac{59}{12} \text{ hours} \right)$

$$= \left(\frac{360}{12} \times \frac{59}{12} \right)^\circ = \frac{295}{2} = 147\frac{1}{2}^\circ$$

Angle traced by minute hand in 55 minutes

$$= \left(\frac{360}{60} \times 55 \right)^\circ = 330^\circ$$

$$\therefore \text{Required angle} = \left(330 - 147\frac{1}{2} \right)^\circ = 182\frac{1}{2}^\circ$$

Problems on Calendar

ODD DAYS: The number of days more than the complete number of weeks in a given period are called odd days.

ORDINARY YEAR: The year with 365 days is called an ordinary year. It has one odd day.

LEAP YEAR: The year with 366 days is called a leap year. Every year divisible by 4, which is not a centenary year, is a leap year. Every fourth centenary year is a leap year.

Examples:

- (a) Years such as 1984, 1996, 2008, etc. are leap years.
- (b) Years such as 400, 800, 1600, etc. are leap years.
- (c) Years such as 600, 900, 1400, 1900, 2100, etc. are not leap years.

COUNTING OF ODD DAYS

1 ordinary year = 365 days = 52 weeks + 1 day

Ordinary year has 1 odd day.

1 leap year = 366 days = 52 weeks + 2 days

Leap year has 2 odd days.

100 years = 76 ordinary years + 24 leap years

= $(76 \times 1) + (24 \times 2)$ odd days

= $76 + 48 = 124$ odd days

= 17 weeks + 5 days

= **5 odd days.**

$\therefore$ Number of odd days in 100 years = 5 odd days.



Important Facts

1. An ordinary year contains 365 days, i.e. 52 weeks + 1 odd day.
2. A leap year contains 366 days, i.e. 52 weeks + 2 odd days.
A year divisible by 4 and a century year divisible by 400 will be a leap year.
3. 100 years contain 24 leap years and 76 ordinary years.
 $\therefore$ 100 years contain 5 odd days.
4. 200 years contain $(5 \times 2) = 10$ days, i.e. 3 odd days.
5. 300 years contain $(5 \times 3) = 15$ days, i.e. 1 odd day.
6. 400 years is a leap year and hence it will contain $(5 \times 4 + 1) = 21$ days, i.e. 0 odd day.

For finding the day of the week on a particular date when no reference day is given.

- (i) Count the net number of odd days for the period upto the given date, and
- (ii) Write Sunday for 0 odd day, Monday for 1 odd day, Tuesday for 2 odd days and so on.

If the reference day is given; then

- (a) Find the net number of odd days for the period between the reference day and the given date.
- (b) The day of the week on the particular date is equal to number of odd days ahead or behind the reference day depending on whether the reference day falls before or after the given date.

For the calendars of two different years to be the same, they must satisfy the following two conditions simultaneously.

- (i) Both the years must be of the same type, i.e. both must be ordinary years or both must be leap years. If one is ordinary year and the other a leap year, their calendars can never match.
- (ii) 1st January of both the years must fall on the same day of the week, i.e. the net number of odd days for the period between these two years should be zero.



Notes:

While counting the number of odd days, always check

- (i) Whether the interval has leap years and if yes,
- (ii) Whether February of the leap year falls in that interval or not.

Also remember that

- (i) The last day of a century year can only be either Sunday, Monday, Wednesday or Friday. It can't be Tuesday, Thursday or Saturday.
- (ii) Any date in March is the same day of the week as the corresponding date in November of that year. Another pair showing similar corresponding feature is April–July.

EXAMPLE 1 What was the day of the week on 13th September 1986?

Solution 13th September 1986 = 1985 years + Period from 1.1.86 to 13.9.86

Number of odd days in 1600 years = 0

Number of odd days in 300 years = 1

85 years = 21 leap years + 64 ordinary years

= $(21 \times 2 + 64 \times 1)$ odd days

= 106 odd days

$$= (15 \text{ weeks} + 1 \text{ day})$$

$$= 1 \text{ odd day.}$$

$$\therefore 1985 \text{ years} = (0 + 1 + 1) \text{ odd days} = 2 \text{ odd days.}$$

$$\text{Period from 1.1.86 to 13.9.86} = 31 + 28 + 31 + 30 + 31 + 30 + 31 + 31 + 13$$

$$= 256 \text{ days}$$

$$= (36 \text{ weeks} + 4 \text{ days})$$

$$= 4 \text{ odd days}$$

$$\therefore \text{Total number of odd days} = 2 + 4 = 6 \text{ odd days.}$$

$$\therefore \text{Required day is Saturday.}$$

EXAMPLE 2 What was the day of the week on 26th January 1950?

Solution 26th January 1950 = 1949 years + 26 days.

$$\text{Odd days in 1600 years} = 0$$

$$\text{Odd days in 300 years} = 1$$

$$50 \text{ years} = 12 \text{ leap years} + 38 \text{ ordinary years}$$

$$= (12 \times 2 + 38 \times 1) \text{ odd days}$$

$$= 62 \text{ odd days}$$

$$= (8 \text{ weeks} + 6 \text{ days})$$

$$= 6 \text{ odd days.}$$

$$1.1.1950 \text{ to } 26.1.1950 = 26 \text{ days} = 3 \text{ weeks} + 5 \text{ odd days.}$$

$$\therefore \text{Total number of odd days} = 0 + 1 + 6 + 5$$

$$= 12 \text{ odd days}$$

$$= (1 \text{ week} + 5 \text{ days})$$

$$= 5 \text{ odd days}$$

$$\therefore \text{Required day is Friday.}$$

EXAMPLE 3 January 1, 2013 was Tuesday. What day of the weeks lies on Jan. 1, 2014?

Solution Year 2013 is an ordinary year and therefore it is having 1 odd day. 1st day of the year 2013 is Tuesday.

$$\therefore \text{1st day of the year 2014 will be Wednesday}$$

$$(\because \text{Year 2013 is having 1 odd day})$$

Hence Jan 1, 2014 lies on Wednesday.



EXERCISES

1. The birthday of Arjun falls between 20th and 23rd of March. Anil remembers that birthday of Arjun falls between 21st and 24th of March. So the birthday of Arjun will be on
(a) 21st March (b) 22nd March (c) 23rd March (d) 24th March (e) None of these
2. The 11th October 1986 was a Saturday. What was the day on 30th January, 1948?
(a) Sunday (b) Monday (c) Tuesday (d) Wednesday (e) None of these
3. If 12th December 1985 was Thursday, what will be the day on 10th January 2100?
(a) Monday (b) Tuesday (c) Wednesday (d) Thursday (e) None of these

4. 11th October 1986 was Saturday. When will be the same day and same month fall on Saturday?
(a) in 1990 (b) in 1995 (c) in 2026 (d) in 2086 (e) None of these
5. 10th January 2105 will be a Wednesday. What was the day on 31st December 1704?
(a) Sunday (b) Monday (c) Tuesday (d) Thursday (e) None of these
6. How many days are there in m weeks m days?
(a) $7m^2$ (b) $8m$ (c) $14m$ (d) 7
7. Which of the following is not a leap year?
(a) 800 (b) 1600 (c) 600 (d) 2400
8. The last day of a century cannot be:
(a) Monday (b) Tuesday (c) Wednesday (d) Friday
9. The calendar of the year 2010 will be the same for the year:
(a) 2012 (b) 2014 (c) 2018 (d) 2021
10. 27th March 2005 was on Monday. What was the day of the week on 27th March 2002?
(a) Sunday (b) Tuesday (c) Thursday (d) Saturday
11. What was the day of the week on 30th January 1948?
(a) Wednesday (b) Thursday (c) Friday (d) Saturday
12. What was the day of the week on 13th September 2001?
(a) Wednesday (b) Thursday (c) Friday (d) Saturday
13. Today is Wednesday. After 96 days, it will be
(a) Friday (b) Saturday (c) Sunday (d) Monday
14. 26th January 2013 was Saturday. What was the day of the week on 26th January 2012?
(a) Monday (b) Thursday (c) Friday (d) Sunday
15. January 1, 2006 was Sunday. What was the day of the week on January 1, 2012?
(a) Sunday (b) Tuesday (c) Thursday (d) Saturday



1. (b) 2. (e) 3. (a) 4. (e) 5. (a) 6. (b) 7. (c) 8. (b) 9. (d) 10. (c)
11. (c) 12. (b) 13. (d) 14. (b) 15. (a)



Solutions with Necessary Explanation

1. Birthday of Arjun – either 21st or 22nd March.
As Anil remembers, birthday falls either on 22nd or 23rd of March.
As both the above statements are true, the birthday of Arjun is on **22nd March**.
2. 11th October 1986 – Saturday
11th October 1986 to 30th January 1948
30th January 1948 – 30th January 1986 + 30th January 1986 – 11th October 1986
= 38 years + 1 + 28 + 31 + 30 + 31 + 30 + 31 + 31 + 30 + 11 days
= 38 years + 254 days
= $2 + (9 \times 5) + 1 + 254$ odd days
= $48 + 255 = 302$ odd days
 $302 \div 7 = 2$ odd days.
 $\therefore$ 30th January 1948 will be 1 day back from Saturday i.e. **Friday**.

3. 12th December 1985 – Monday
10th January 2100 – ?
12th December 1985 – 10th January 2100
= 12th Dec 1985 – 12th Dec 2099 + 12th Dec 2099 – 10th Jan 2100
= 114 years + 29 days
= $2 + 1 + 6 + 3 \times 5 + 29$ odd days
= 53 odd days.
 $53 \% 7 = 4$ odd days
i.e. Jan 2100 will be on Monday. Here answer is **(a)**.
4. The same day same month and same week will be observed after 400 years. Here answer is **None of these**.
5. 10th Jan 2105 – Wednesday
31st Dec 1704 – ?
31st Dec 1704 – 31st Dec 2104 + 31st Dec 2104 – 10th Jan 2105
= 400 years + 10 days = 10 odd days
 $10 \% 7 = 3$ odd days. So required day = Wednesday – 3 = **Sunday**.
6. m weeks m day = $7 \times m + m = 8m$ days
7. A century year divisible by 400 is a leap year.
 $\therefore$ 600 is not a leap year
8. 100 years contain 5 odd days.
 $\therefore$ Last day of 1st century is Friday
200 years contain $5 \times 2 = 10$ days = 3 odd days.
 $\therefore$ Last day of 2nd century is Wednesday.
300 years contains $5 \times 3 = 15$ days = 1 odd day.
 $\therefore$ Last day of 3rd century is Monday.
400 years contain 0 odd days
 $\therefore$ Last day of 4th century year is Sunday
 $\therefore$ Last day of a century year cannot be **Tuesday**
Thursday or Saturday.
9. Number of odd days in 2010 = 1
Number of odd days in 2011 = 1
Number of odd days in 2012 = 2
Number of odd days in 2013 = 1
Number of odd days in 2014 = 1
Number of odd days in 2015 = 1
Number of odd days in 2016 = 2
Number of odd days in 2017 = 1
Number of odd days in 2018 = 1
Number of odd days in 2019 = 1
Number of odd days in 2020 = 2
Total number $\quad \quad \quad = 14$
Sum of odd days = 14 days = 0 odd day.
 $\therefore$ Calendar for 2021 will be the same as for the year **2010**.

10 27th March 2002 to 27th March 2005 = 3 years

= 2 ordinary year + 1 leap year

= 2 + 2 = 4 odd days.

∴ Day on which 27th March 2003 falls

= Monday – 4 = **Thursday**

11. Odd days in 1600 years = 0

Odd days in 300 years = 1

47 years = 11 leap years + 36 ordinary years

= $(11 \times 2 + 36 \times 1)$ odd days

= 58 days = (8 weeks + 2 days)

= 2 odd days

Period from 1.1.48 to 30.1.48 = 30 days 2 odd days.

∴ Total number of odd days = 0 + 1 + 2 + 2 = 5 odd days

Hence 30th January 1948 was on **Friday**.

12. 2000 years = 0 odd days.

Period from 1.1.2001 to 13.9.2001 = $31 + 28 + 31 + 30 + 31 + 30 + 31 + 31 + 13 = 256$ days.

= (36 weeks + 4 days)

= 4 odd days

∴ 13th September 2001 was **Thursday**.

13. 96 days = 13 weeks + 5 days

= 5 odd days

∴ Week after 96 days = Wednesday + 5 = **Monday**

14. 2012 was a leap year and so it is having 2 odd days.

26th January 2013 was Saturday

∴ 26th January 2012 = Saturday – 2 = **Thursday**

15. 1st January 2006 was Sunday

1.1.2006 to 1.1.2012 = 6 years = 1 leap year + 5 ordinary years

= $(1 \times 2 + 5 \times 1)$ odd days

= 7 days

= 0 odd day.

∴ 1st January 2012 was **Sunday**.

Time and Distances

The speed of a body is the rate at which it is moving.

$$\text{Speed} = \frac{\text{Distance}}{\text{Time}}$$

$$\text{Average speed} = \frac{\text{Total distance covered}}{\text{Total time taken}}$$

Speed is measured in km/hr or m/s.

$$1 \text{ km/hr} = \frac{5}{18} \text{ m/s}$$

$$\text{or} \quad 1 \text{ m/s} = \frac{18}{5} \text{ km/hr}$$

BODIES MOVING IN THE SAME DIRECTION

Relative speed = Difference in speed

Distance between moving objects = Relative speed $\times$ Time of travel

BODIES MOVING IN THE OPPOSITE DIRECTION

- (i) The relative speed of one w.r.t. the other is the sum of their speeds.
- (ii) Increase or decrease in distance between them is the product of their relative speed and time.
- (iii) The distance between two bodies moving towards each other will get reduced at the rate of their relative speed (i.e. sum of their speeds). The time of their meeting (or crossing) is given by

$$\text{Meeting time} = \frac{\text{Initial distance between the two bodies}}{\text{Sum of their speeds}}$$

AVERAGE SPEED: If a certain distance d_1 is covered at speed S_1 and distance d_2 at S_2 , their average speed for the whole journey is given by

$$\text{Average speed} = \frac{S_1 S_2 (d_1 + d_2)}{S_2 d_1 + S_1 d_2}$$

When $d_1 = d_2$,

$$\text{Average speed} = \frac{2S_1 S_2}{S_1 + S_2}$$

If a person covers $1/n$ th part of the total journey at speed S_1 and the remaining at speed S_2 , then his average speed for the whole journey is given by

$$\text{Average speed} = \frac{nS_1 S_2}{(n-1)S_1 + S_2}$$

If a person covers m/n th part of the total journey at speed S_1 and the remaining journey at speed S_2 , then his average speed for the total journey is given by

$$\text{Average speed} = \frac{nS_1 S_2}{(n-m)S_1 + mS_2}$$

When a person covers a certain distance D km at speed S_1 km/h he is late by l_1 hours. If he covers it at S_2 km/h, he will be early by l_2 hours. Then

$$D = \frac{S_1 S_2}{S_2 - S_1} (l_1 - l_2) \text{ km}$$

If a runner improves his timing by time t by running at n times his original speed, his usual time

$$= \frac{t}{1 - 1/n} = \frac{\text{Improvement in time}}{1 - \left(\frac{\text{Old speed}}{\text{New speed}} \right)}$$

Two persons A and B start their journey at the same time in opposite directions from two points and after passing each other they complete their remaining journey arriving at the two points in a and b hours respectively. Then the ratio of their speeds is

$$\frac{S_a}{S_b} = \frac{\sqrt{b}}{\sqrt{a}} = \sqrt{\frac{b}{a}}$$

Two persons cover a certain distance D at S_1 and S_2 km/h respectively. If the difference in time taken by the two is t hours, then the distance covered is given by

$$D = \frac{S_1 S_2}{S_1 - S_2} \times t \text{ km} = \frac{\text{Product of two speeds}}{\text{Difference of two speeds}} \times \text{Difference in time taken}$$

A train starts from P at time t_1 and reaches Q at time t_2 . Another train starts from Q at t_3 (later than t_1) and reaches P at t_4 . The two trains will meet on their way at

Meeting time = Second train's starting time

$$+ \frac{(\text{1st arrival} - \text{2nd's starting time})}{\text{Sum of time taken by both}} \times (\text{Time taken by the second})$$

or

$$\text{First train's starting time} + \frac{(\text{2nd's arrival} - \text{1st starting time})}{\text{Sum of time taken by both}} \times (\text{Time taken by first})$$

EXAMPLE 1 A and B walk around a circle of circumference 1800 m at the speed of 15 m/minutes and 60 m/minutes respectively. If both start simultaneously from the same point and walk in the same direction, when will they be together again at the starting point for the first time?

Solution Circumference of circle = 1800 m

Speed of A and B are 150m/min. and 60 m/min.

A gains $(150 - 60) = 90$ m in 1 min.

$\therefore$ A gains 1800 m in $1800/90 = 20$ min.

So A and B will be together again for the first time after 20 min.

Time taken by A to complete 1 full circle = $1800/150 = 12$ min.

Time taken by B to complete 1 full circle = $1800/60 = 30$ min.

Thus, A will be at the starting point after every 12 minutes and B after every 30 minutes. So they will be together at the starting point after every 60 minutes (i.e. LCM of 12 and 30 is 60).

Hence, A and B will be again at the starting point for the first time after 60 minutes.

➤ Theorem: If a person covers a certain distance at a speed of x km/h and he covers the same distance at a speed of y km/h, then the average speed of the person for the whole journey is $\left(\frac{2xy}{x+y}\right)$ km/h.

If S_1 km/h and S_2 km/h are two different speeds of a person and the difference between arrival times is given as t hours then the distance covered by the person is given by

$$\left(\frac{S_1 \times S_2}{S_1 - S_2} \times t\right) \text{ km.}$$

If S_1 km/h and S_2 km/h are speeds of a person for his onward and return journey respectively and t hours is the total time taken by him for his whole journey, then the distance travelled

by the person is given by $\left(t \times \frac{S_1 \times S_2}{S_1 + S_2}\right)$ km.

MEETING POINT: If two trains travel at speeds of S_1 km/h and S_2 km/h and time lag at start between the two trains is t hours, then the meeting point's distance is given by

$$\left(\frac{S_1 \times S_2 \times t}{S_1 - S_2}\right) \text{ km.}$$

➤ Theorem: If two persons start at the same time in opposite directions from two points and after passing each other they complete their journeys in a and b hours respectively, then speed of the first person : speed of the second person = $\sqrt{b} : \sqrt{a}$.

If two persons travel the same distance at speeds of S_1 km/h and S_2 km/h respectively and one of them takes t hours more than the other to complete his journey, then the distance

covered by them is given by $\frac{S_1 \times S_2}{S_1 - S_2} \times t$ km.

$$\text{Time of rest per hour} = \frac{\text{Difference of speed}}{\text{Speed without stoppage}}$$

A person travels a certain distance by two modes of conveyance. The whole journey took t hours, then the distance travelled by the person in one side

$$= \text{Total time} \times \frac{\text{Multiplication of speeds}}{\text{Sum of speeds}}$$

EXAMPLE 2 A person covers a certain distance at a speed of 60 km/h and returns to the starting point at a speed of 40 km/h. Find the average speed of the person for the whole journey.

Solution Average speed = $\frac{2 \times 60 \times 40}{60 + 40} = 48 \text{ km/h}$

EXAMPLE 3 P covers a certain distance at a speed of 25 km/h and he reached his destination 5 minutes late. With a speed of 30 km/h, he reaches his destination 10 minutes earlier. Find the distance between the starting point and destination.

Solution Let the required distance be x km.

$$\therefore \text{Time taken to cover } x \text{ km at 25 km/h speed} = \frac{x}{25} \text{ hours}$$

$$\text{Similarly time taken to cover } x \text{ km at 30 km/h speed} = \frac{x}{30} \text{ hours}$$

$$\text{Difference between the time taken} = 5 + 10 = 15 \text{ min} = 1/4 \text{ hour}$$

$$\therefore \frac{x}{25} - \frac{x}{30} = \frac{1}{4}$$

$$\text{or} \quad 6x - 5x = \frac{150}{4}$$

$$\text{i.e.} \quad x = \frac{150}{4} \text{ km}$$



Short cut: Required distance = $\frac{25 \times 30}{30 - 25} \text{ km/h} \times \left(\frac{1}{4} \text{ h}\right) = 150 \times \frac{1}{4} = \frac{150}{4} \text{ km}$

EXAMPLE 4 A bus travels for 7 hours, the first half at 30 km/h and the second half at 40 km/h. Find the distance

Solution Let the distance be x km

$$\text{Time taken during the first half of total distance} = \frac{(x/2)}{30} = \frac{x}{60} \text{ hour}$$

$$\text{Time taken during the second half of total distance} = \frac{(x/2)}{40} = \frac{x}{80} \text{ hour}$$

$$\therefore \text{Time taken for the whole journey} = \frac{x}{60} + \frac{x}{80} = 7 \text{ hours}$$

$$\text{i.e.} \quad \frac{4x + 3x}{240} = 7 \text{ hours}$$

$$\therefore \quad x = \mathbf{240 \text{ km}}$$



Short cut: Required distance = $\frac{2 \times 7 \times 30 \times 40}{30 + 40} = \mathbf{240 \text{ km}}$



Note: Here half of the journey is meant by half of the total distance covered and not the half of time taken for the whole journey.

EXAMPLE 5 Two persons X and Y walk from a place P at 5 km/h and 6 km/h respectively. How much distance will they be apart after 4 hours:

- (i) If they walk in the same direction?
- (ii) If they walk in opposite directions?

Solution

- (i) When they walk in the same direction, the relative speed of Y with respect to X is 1 km/h.

$\therefore$ After 4 hours they will be $4 \times 1 \text{ km/h} = \mathbf{4 \text{ km}}$ apart.

- (ii) When they walk in opposite directions, the relative speed of Y with respect to X is $6 + 5 = 11 \text{ km/h}$.

$\therefore$ After 4 hours they will be $4 \times 11 = \mathbf{44 \text{ km}}$ apart.

EXAMPLE 6 A thief runs at a speed of 10 m/s. A policeman runs behind him at a speed of 12.5 m/s after 10 seconds the thief has started running. After how many metres, will the policeman catch the thief?

Solution During the first 10 seconds, the thief has covered $10 \text{ m/s} \times 10 = 100 \text{ m}$.

The time required by the policeman to cover 100 m is given by $\frac{100}{(12.5 - 10)} = \mathbf{40 \text{ seconds}}$

$\therefore$ The policeman will catch the thief at a distance of $12.5 \text{ m/s} \times 40 = \mathbf{500 \text{ metres}}$



Short cut: Required distance = $\frac{S_1 \times S_2 \times \text{Difference in time } (t)}{S_1 - S_2}$

$$\text{Required distance} = \frac{10 \times 12.5 \times 10}{2.5} = \mathbf{500 \text{ metres}}$$

EXAMPLE 7 Two persons cover the same distance at speeds of 25 km/h and 30 km/h respectively. Find the distance travelled if one person takes 25 minutes more than the other.

Solution Let the distance be $x \text{ km}$.

$\therefore$ Time taken by the first person = $x/25 \text{ hour}$

Time taken by the second person = $x/30$ hour

$$\therefore \frac{x}{25} - \frac{x}{30} = \frac{25}{60}$$

$$\text{i.e.} \quad \frac{6x - 5x}{150} = \frac{25}{60}$$

$$\text{or} \quad x = \frac{25 \times 150}{60} = \mathbf{62.5 \text{ km}}$$



Short cut: Required distance = $\frac{S_1 \times S_2}{S_1 \sim S_2} \times (t_1 \sim t_2)$

$$= \frac{25 \times 30}{30 \sim 25} \times \frac{25}{60}$$

$$= \frac{25 \times 30 \times 5}{60} = \frac{125}{2} = \mathbf{62.5 \text{ km}}$$

EXAMPLE 8 A person travels a certain distance at a speed of 60 km/h and with stoppages he travels the same distance at a speed of 40 km/h. How many minutes/hour does he stop?

Solution Let the total distance be x km

Time taken at 60 km/h speed = $x/60$ hour

Time taken at 40 km/h speed = $x/40$ hour

$$\therefore \text{Time taken to rest} = \frac{x}{40} - \frac{x}{60} = \frac{3x - 2x}{120} = \frac{x}{120} \text{ hour}$$

$$\therefore \text{Rest/hour} = \frac{x}{120} \div \frac{x}{40} = \frac{x}{120} \times \frac{40}{x} = \frac{1}{3} \text{ h} = \mathbf{20 \text{ min}}$$



Short cut: Time of rest/hour = $\frac{S_1 \sim S_2}{\text{Speed without stoppage}}$

$$= \frac{60 - 40}{60} = \frac{1}{3} \text{ hour} = \mathbf{20 \text{ minutes}}$$

EXAMPLE 9 A bus running at a speed of 30 km/h leaves Trivandrum at 10 am and another bus running at a speed of 40 km/h leaves the same place at 3 pm in the same direction. How many kilometres from Trivandrum will they be together?

Solution The first bus has advanced 30×5 km and the second bus gains $(40 - 30)$ or 10 km/h.

$$\therefore \text{The second bus will gain } 30 \times 5 \text{ km in } \frac{30 \times 5}{10} \text{ or } 15 \text{ h}$$

$$\therefore \text{Required distance from Trivandrum} = 15 \text{ hours} \times 40 \text{ km/h} = \mathbf{600 \text{ km}}$$



Short cut: The meeting point's distance from starting point

$$= \frac{S_1 \times S_2 \times \text{Difference in time}}{\text{Difference in speed}}$$

where S_1 and S_2 are the speeds of the first and second buses respectively.

$$\therefore \text{ Required distance} = \frac{30 \times 40 \times (3 \text{ pm} \sim 10 \text{ am})}{(40 \sim 30)} = \frac{30 \times 40 \times 5}{10} = \mathbf{600 \text{ km}}$$



Theorem: When two persons A and B are running such that the ratio of their speeds is

$a : b$, then Distance travelled by A = $2 \times \text{Distance between two points} \left(\frac{a}{a+b} \right)$

Distance travelled by B = $2 \times \text{Distance between two points} \left(\frac{b}{a+b} \right)$

EXAMPLE 10 When two persons A and B walk from P to Q, which are at a distance of 21 km, at 3 km/h and 4 km/h respectively, B reaches Q, returns immediately and meets A at R. Find the distance from P to R.

Solution When A reaches R, B also reaches R after returning from Q.

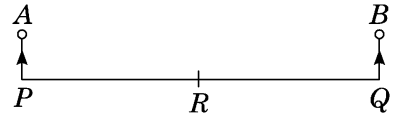
$\therefore$ Distance travelled by A = PR

Distance travelled by B = PQ + QR

$\therefore$ Distance travelled by A and B together = $2 \times PQ = 2 \times 21 = 42 \text{ km}$

i.e., as the ratio of speeds is 3 : 4

$\therefore$ PR, distance travelled by A = $\left(\frac{3}{3+4} \right) \times 42 = \mathbf{18 \text{ km}}$



Short cut: Distance travelled by A = $2 \times \text{Distance between two points} \times \left(\frac{a}{a+b} \right)$

$$= 2 \times 21 \times \frac{3}{(3+4)} = \mathbf{18 \text{ km}}$$



Theorem: If two persons A and B start at the same time in opposite directions from two points and after passing each other they complete the journey in a and b hours respectively, then the ratio of speeds from A to B is $\sqrt{b} : \sqrt{a}$.

EXAMPLE 11 A person starts by a car from Trivandrum to Kollam, and at the same time another person starts from Kollam by a car to Trivandrum. After passing each other they complete their journey in 2 hours and 1/2 hour respectively. At what rate does the second person drives the car if the first car runs at a speed of 40 km/h?

Solution Ratio of speeds = $\sqrt{b} : \sqrt{a}$

$$= \sqrt{1/2} : \sqrt{2} = \sqrt{1/4} = \frac{1}{2}$$

 $\therefore$ The second person's speed = First person's speed $\times 2$

$$= 40 \times 2 = 80 \text{ km/h}$$

➤ Theorem: If two persons cover the same distance at two different speeds a and b km/h and if t minutes is the difference in time taken by the two persons to cover the distance, then the distance covered is given by $\frac{a \times b}{a \sim b} \times \frac{t}{60}$ km.

EXAMPLE 12 Two buses travels to a place at speeds of 45 km/h and 60 km/h respectively. If the second bus takes 5 hours less than the first for completing the journey, find the total distance covered by the bus during the journey.

Solution As the second bus takes 5 hours less, distance covered = $\frac{45 \times 60}{45 \sim 60} \times 5 = 900 \text{ km}$.

➤ Theorem: Speed and time taken are inversely proportional.

$$\therefore S_1 T_1 = S_2 T_2 = S_3 T_3 = \dots$$

where $S_1, S_2, S_3, \dots$ are the speeds and $T_1, T_2, T_3, \dots$ are the time taken to travel the same distance.

EXAMPLE 13 A person covers a distance in 40 minutes if he runs at a speed of 45 km/h on an average. Find the speed at which he must run to reduce the time of journey to 30 minutes.

Solution Here $45 \times 40 = S_2 \times 30$

$$\therefore S_2 = \frac{45 \times 40}{30} = 60 \text{ km/h}$$



EXERCISES

- How many km/h does a man walk who passes through a street 600 m long in 5 minutes?
 (a) 9/5 km/h (b) 18/5 km/h (c) 27/5 km/h (d) 36/5 km/h (e) None of these
- A man takes 6 hours 30 minutes in walking to a certain place and riding back. He would have gained 2 hours 10 minutes by riding both ways. How long would he take to walk both ways?
 (a) 480 minutes (b) 520 minutes (c) 560 minutes (d) 600 minutes (e) None of these
- Two persons begin together to write out a booklet containing 8190 lines. The first person starts with the first line, writing at the rate of 200 lines an hour; and the second person starts with the last line, then writes 8189th line and so on, proceeding backward at the rate of 150 lines an hour. At what time will they meet?
 (a) 22.4 hours (b) 23.4 hours (c) 24.4 hours (d) 25.4 hours (e) None of these
- A motor car does a journey in 10 hours, the first half at 21 km/h, and the rest at 24 km/h. Find the distance.
 (a) 220 km (b) 224 km (c) 228 km (d) 232 km (e) None of these

5. Supposing that telegraph poles on a rail road are 50 metres apart, how many will be passed by a car in 4 hours if the speed of train is 45 km/h?
(a) 3599 (b) 3600 (c) 3601 (d) 3602 (e) None of these
6. A person has to reach a certain place and certain time and he finds that he will be 15 minutes late, if he walk at 4 km/h, and 10 minutes earlier, if he walk at 6 km/h. How far has he to walk?
(a) 3 km (b) 4 km (c) 5 km (d) 6 km (e) None of these
7. Two buses start at the same time from two bus stations and proceed towards each other at the rates of 20 km/h and 25 km/h respectively. When they meet, one bus may have travelled 80 km more than the other. Find the distance between the two bus stations.
(a) 680 km (b) 700 km (c) 710 km (d) 720 km (e) None of these
8. A person travelled a certain distance by train at a speed of 25 km/h and walked back at a speed of 4 km/h. The whole journey took 5 hours 48 minutes. What distance did he travel?
(a) 29 km (b) 25 km (c) 20 km (d) 14 km (e) None of these
9. A thief is spotted by a policeman from a distance of 200 metres. When the policeman started to catch the thief he also started running. Assume the speed of the thief to be 10 km/h and that of policeman 12 km/h, how far will the thief has run before he is overtaken by the policeman?
(a) 1 km (b) 2 km (c) 3 km (d) 4 km (e) None of these
10. A train travels at a speed of 10 m/s leaves Trivandrum for Nagercoil at 8 AM At what distance from Nagercoil will it meet a train which leaves Nagercoil for Trivandrum at 8.20 AM and travels one-third faster than the former does, the distance from Trivandrum to Nagercoil being 68 km?
(a) 26 km (b) 32 km (c) 46 km (d) 48 km (e) None of these
11. Two trains start from P and Q respectively and travel towards each other at a speed of 50 km/hr and 40 km/hr respectively. By the time they met the first train has travelled 100 km more than the second. The distance between P and Q is: [SSC]
(a) 500 km (b) 630 km (c) 660 km (d) 900 km
12. A thief steals a car at 2.30 p.m. and drives it at 60 km/hr. The theft is discovered at 3 p.m. and the owner sets off in another car at 75 km/hr. When will he overtake the thief? [RRB]
(a) 4.30 p.m. (b) 4.45 p.m. (c) 5 p.m. (d) 5.15 p.m.
13. With a uniform speed a car covers a certain distance in 8 hours. Had the speed been increased by 4 km/hr, the same distance could have been covered in $7\frac{1}{2}$ hours. What is the distance covered? [Bank PO]
(a) 420 km (b) 480 km (c) 640 km
(d) Cannot be determined (e) None of these
14. A man covered a certain distance at some speed. Had he moved 3 km/hr faster, he would have taken 40 minutes less. If he had moved 2 km/hr slower, he would have taken 40 minutes more. The distance (in km) is: [SSC]
(a) 35 (b) $36\frac{2}{3}$ (c) $37\frac{1}{2}$ (d) 40
15. A car travels the first one-third of a certain distance with a speed of 10 km/hr, the next one-third distance with a speed of 20 km/hr, and the last one-third distance with a speed of 60 km/hr. The average speed of the car for the whole journey is: [IAS]
(a) 18 km/hr (b) 24 km/hr (c) 30 km/hr (d) 36 km/hr
16. A boy goes to his school from his house at a speed of 3 km/hr and returns at a speed of 2 km/hr. If he takes 5 hours in going and coming, the distance between his house and school is: [SSC]
(a) 5 km (b) 5.5 km (c) 6 km (d) 6.5 km

17. A man on tour travels first 160 km at 64 km/hr and the next 160 km at 80 km/hr. The average speed for the first 320 km of the tour as: [RRB]
(a) 35.55 km/hr (b) 36 km/hr (c) 71.11 km/hr (d) 71 km/hr
18. A farmer travelled a distance of 61 km in 9 hours. He travelled partly on foot at 4 km/hr and partly on bicycle at 9 km/hr. The distance travelled on foot is: [UPSC]
(a) 14 km (b) 15 km (c) 16 km (d) 17 km
19. A person has to cover a distance of 6 km in 45 minutes. If he covers one-half of the distance in two-third of the total time, to cover the remaining distance in the remaining time, his speed (in km/hr) must be: [SSC]
(a) 6 (b) 8 (c) 12 (d) 15
20. A boy rides his bicycle 10 km at an average speed of 12 km/hr and again travels 12 km at an average speed of 10 km/hr. His average speed for the entire trip is approximately: [SSC]
(a) 10.4 km/hr (b) 10.8 km/hr (c) 11 km/hr (d) 12.2 km/hr
21. A person travels equal distances with speeds of 3 km/hr, 4 km/hr and 5 km/hr and takes a total time of 47 minutes. The total distance (in km) is: [RRB]
(a) 2 (b) 3 (c) 4 (d) 5
22. The distance between two cities A and B is 330 km. A train starts from A at 8 a.m. and travels towards B at 60 km/hr. Another train starts from B at 9 a.m. and travels towards A at 75 km/hr. At what time do they meet? [LIC AAO]
(a) 10 a.m (b) 10.30 a.m (c) 11 a.m. (d) 11.30 a.m.
23. In a flight of 600 km, an aircraft was slowed down due to bad weather. Its average speed for the trip as reduced by 200 km/hr and the time of flight increased by 30 minutes. The duration of the flight is: [MAT]
(a) 1 hour (b) 2 hours (c) 3 hours (d) 4 hours
24. In covering a distance of 30 km, Abhay takes 2 hours more than Sameer. If Abhay doubles his speed, then he would take 1 hour less than Sameer. Abhay's speed is [MAT]
(a) 5 kmph (b) 6 kmph (c) 6.25 kmph (d) 7.5 kmph
25. A train, when moves at an average speed of 40 km/hr, reaches its destination on time. When its average speed becomes 35 km/hr, then it reaches its destination 15 minutes late. Find the length of journey. [Bank PO]
(a) 30 km (b) 40 km (c) 70 km (d) 80 km (e) None of these
26. A walks around a circular field at the rate of one round/hour, while B runs around it at the rate of six rounds/hour. They start in the same direction from the same point at 7.30 a.m. They shall first cross each other at: [IAS]
(a) 7.42 a.m (b) 7.48 a.m (c) 8.10 a.m (d) 8.30 a.m
27. Two cars P and Q start at the same time from A and B which are 120 km apart. If the two cars travel in opposite directions, they meet after one hour and if they travel in same direction (from A towards B), then P meets Q after 6 hours. What is the speed of car P? [SBI PO]
(a) 60 km/hr (b) 70 km/hr (c) 120 km/hr
(d) Data inadequate (e) None of these
28. A thief is noticed by a policeman from a distance of 200 m. The thief starts running and the policeman chases him. The thief and the policeman run at the rate of 10 km and 11 km per hour respectively. What is the distance between them after six minutes? [SSC]
(a) 100 m (b) 150 m (c) 190 m (d) 200 m
29. If a person walks at 14 km/hr instead of 10 km/hr, he would have walked 20 km more. The actual distance travelled by him is: [RRB]
(a) 50 km (b) 56 km (c) 70 km (d) 80 km

30. A and B walk around a circular track. They start at 8 a.m. from the same point in the opposite directions. A and B walk at a speed of 2 rounds per hour and 3 rounds per hour respectively. How many times shall they cross each other before 9.30 a.m.? [UPSC]
 (a) 5 (b) 6 (c) 7 (d) 8
31. It takes 8 hours for a 600 km journey, if 120 km is done by train and the rest by car. It takes 20 minutes more, if 200 km is done by train and the rest by car. The ratio of the speed of the train to that of the car is: [MBA]
 (a) 2 : 3 (b) 3 : 2 (c) 3 : 4 (d) 4 : 3
32. A person travels from P to Q at a speed of 40 km/hr and returns by increasing his speed by 50%. What is his average speed for both the trips? [MBA]
 (a) 36 km/hr (b) 45 km/hr (c) 48 km/hr (d) 50 km/hr
33. Starting from his house, a student walks at a speed of 2.5 km/hr and reaches his school 6 minutes late. Next day he increases his speed by 1 km/hr and reaches the school 6 minutes early. How far is the school from his house? [SSC]
 (a) 1 km (b) 1.5 km (c) 1.75 km (d) 2 km
34. A car travelling with $\frac{5}{7}$ of its actual speed covers 42 km in 1 hour 40 minutes 48 seconds. Find the actual speed of the car. [SSC]
 (a) $17\frac{6}{7}$ km/hr (b) 25 km/hr (c) 30 km/hr (d) 35 km/hr
35. A man travels 600 km by train at 80 km/hr, 800 km by ship at 40 km/hr, 500 km by aeroplane at 400 km/hr and 100 km by car at 50 km/hr. What is the average speed for the entire distance? [SSC]
 (a) 60 km/hr (b) $60\frac{5}{123}$ km/hr (c) 62 km/hr (d) $65\frac{5}{123}$ km/hr
36. A is faster than B. A and B each walk 24 km. The sum of their speeds is 7 km/hr, and the sum of time taken by them is 14 hours. Then A's speed is equal to: [IAF]
 (a) 3 km/hr (b) 4 km/hr (c) 5 km/hr (d) 7 km/hr
37. Walking $\frac{6}{7}$ th of his usual speed, a man is 12 minutes late. The usual time taken by him to cover that distance is: [RRB]
 (a) 1 hour (b) 1 hour 12 min
 (c) 1 hour 15 min (d) 1 hour 20 min
38. Robert is travelling on his cycle and has calculated to reach point A at 2 p.m. if he travels at 10 km/hr; he will reach there at 12 noon if he travels at 15 km/hr. At what speed must he travel to reach A at 1 p.m.? [DMRC]
 (a) 8 km/hr (b) 11 km/hr (c) 12 km/hr (d) 14 km/hr
39. A train can travel 50% faster than a car. Both start from point A at the same time and reach point B 75 km away from A at the same time. On the way, however, the train lost about 12.5 minutes while stopping at the stations. The speed of the car is: [MAT]
 (a) 100 km/hr (b) 110 km/hr (c) 120 km/hr (d) 130 km/hr
40. The jogging track in a sports complex is 726 metres in circumference. Deepak and his wife start from the same point and walk in opposite direction at 4.5 km/hr and 9.75 km/hr respectively. They will meet for the first time in: [MAT]
 (a) 4.9 min (b) 5.28 min (c) 5.5 min (d) 6 min
41. A man can reach a certain place in 30 hours. If he reduces his speed by $\frac{1}{15}$ th, he goes 10 km less in that time. Find his speed. [SSC]
 (a) 4 km/hr (b) 5 km/hr (c) $5\frac{1}{2}$ km/hr (d) 6 km/hr

42. In covering a certain distance, the speeds of A and B are in the ratio of 3 : 4. A takes 30 minutes more than B to reach the destination. The time taken by A to reach the destination is: [SSC]
(a) 1 hour (b) $1\frac{1}{2}$ hours (c) 2 hours (d) $2\frac{1}{2}$ hours
43. Anna left for city A from city B at 5.20 a.m. She travelled at the speed of 80 km/hr for 2 hour 15 minutes. After that the speed was reduced to 60 km/hr. If the distance between two cities is 350 km, at what time did Anna reach city A? [Bank PO]
(a) 9.20 a.m. (b) 9.25 a.m. (c) 9.35 a.m.
(d) 10.05 a.m. (e) None of these
44. An express train travelled at an average speed of 100 km/hr, stopping for 3 minutes after every 75 km. How long did it take to reach its destination 600 km from the starting point? [MAT]
(a) 6 hour 21 min (b) 6 hours 24 min
(c) 6 hours 27 min (d) 6 hours 30 min
45. A car is running at a speed of 108 km/hr. What distance will it cover in 15 second? [RBI]
(a) 45 metres (b) 55 metres (c) 450 metres
(d) Cannot be determined (e) None of these
46. A person crosses a 600 m long street in 5 minutes. What is his speed in km/hr? [RRB]
(a) 3.6 (b) 7.2 (c) 8.4 (d) 10
47. How long will a boy take to run round a square field of side 35 metres, if he runs at the rate of 9 km/hr? [SSC]
(a) 50 sec (b) 52 sec (c) 54 sec (d) 56 sec
48. One of the two buses completes a journey of 300 km in $7\frac{1}{2}$ hours and the other a journey of 450 km in 9 hours. The ratio of their average speed is: [RRB]
(a) 2 : 3 (b) 3 : 4 (c) 4 : 5 (d) 8 : 9
49. A motor car starts with a speed of 70 km/hr with its speed increasing every 2 hours by 10 km/hr. In how many hours will it cover 345 km? [Bank PO]
(a) $2\frac{1}{4}$ hours (b) 4 hours 5 min (c) $4\frac{1}{2}$ hours
(d) Cannot be determined (e) None of these
50. A train covers a distance of 10 km in 12 minutes. If its speed is decreased by 5 km/hr, the time taken by it to cover the same distance will be: [SSC]
(a) 10 min (b) 11 min 20 sec
(c) 13 min (d) 13 min 20 sec
51. A man walking at the rate of 5 km/hr crosses a bridge in 15 minutes. The length of the bridge (in metres) is: [SSC]
(a) 600 (b) 750 (c) 1000 (d) 1250
52. A truck covers a distance of 550 metres in 1 minute, whereas a bus covers a distance of 33 km in 45 minutes. The ratio of their speeds is: [SSC]
(a) 3 : 4 (b) 4 : 3 (c) 3 : 5 (d) 50 : 3
53. The speed of a car increases by 2 km after every hour. If the distance travelled in the first one hour was 35 km, what the total distance travelled in 12 hours? [Bank PO]
(a) 456 km (b) 482 km (c) 552 km
(d) 556 km (e) None of these

54. An athlete runs 200 metres race in 24 seconds. His speed is: [SSC]
 (a) 20 km/hr (b) 24 km/hr (c) 28.5 km/hr (d) 30 km/hr
55. A man travelled from the village to the post office at the rate of 25 km/hr and walked back at the rate of 4 km/hr. If the whole journey took 5 hours 48 minutes, find the distance of the post office from the village. [SSC]
 (a) 15 km (b) 18 km (c) 20 km (d) 22 km



1. (d) 2. (b) 3. (b) 4. (b) 5. (c) 6. (c) 7. (d) 8. (c) 9. (a) 10. (b)
 11. (d) 12. (c) 13. (b) 14. (d) 15. (a) 16. (c) 17. (c) 18. (c) 19. (c) 20. (b)
 21. (b) 22. (c) 23. (a) 24. (a) 25. (c) 26. (a) 27. (b) 28. (a) 29. (a) 30. (c)
 31. (c) 32. (c) 33. (c) 34. (d) 35. (d) 36. (b) 37. (b) 38. (c) 39. (c) 40. (b)
 41. (b) 42. (c) 43. (e) 44. (a) 45. (c) 46. (b) 47. (d) 48. (c) 49. (c) 50. (d)
 51. (d) 52. (a) 53. (c) 54. (d) 55. (c)



Solutions with Necessary Explanation

1. Speed in m/s = $\frac{600}{5 \times 60} = 2$

$\therefore$ Speed in km/h = $2 \times \frac{18}{5} = 7\frac{1}{5}$ km/h

2. Walking + Riding = 6 hours 30 minutes (1)

2 Riding = 6 hours 30 min – 2 hours 10 min.

= 4 hours 20 min. (2)

Solving (1) and (2),

$2 \times (1) - (2) \Rightarrow 2 \text{ walking} = 13 \text{ hours} - 4 \text{ hours } 20 \text{ min.}$

= 8 hours 40 minutes.

= $(8 \times 60 + 40)$ minutes

= **520 minutes**



Short cut: One way riding saves 2 hours 10 minutes

$\Rightarrow$ One way walking takes 2 hours 10 minutes more than one way riding.

$\therefore$ Both way walking takes 6 hours 30 min + 2 hours 10 min = **8 hours 40 minutes**

= $(8 \times 60 + 40)$ minutes = **520 minutes**

3. Their relative speed = $200 + 150 = 350$ lines/h

Time at which they meet = $8190/350$ hour = $117/5$ hour = **23.4 hour**

4. Let the distance be x km.

Total time taken = $\frac{x}{2 \times 21} + \frac{x}{2 \times 24} = 10$ hours

or $\frac{x(24 + 21)}{2 \times 21 \times 24} = 10 \text{ hours}$

$\therefore x = \frac{10 \times 2 \times 21 \times 24}{45} = 2 \times 2 \times 7 \times 8 = \mathbf{224 \text{ km}}$

5. Total distance covered by the train in 4 hours = $45 \times 4 = 180 \text{ km}$

Telegraph poles are 50 m apart.

$\therefore$ Number of poles in 180 km will be $\frac{180000}{50} + 1 = \mathbf{3601}$

6. Required distance (Distance he has to walk) = $\frac{\text{Product of two speeds}}{\text{Difference of two speeds}} \times \text{Difference in time}$

$$= \frac{4 \times 6}{6 - 4} \times \left(\frac{15 + 10}{60} \right) = \frac{24}{2} \times \frac{25}{60} = \mathbf{5 \text{ km}}$$

7.



Short cut: Distance between two bus stations

$$= \text{Distance more travelled by one bus} \times \frac{\text{Sum of speeds}}{\text{Difference of speeds}}$$

$$= 80 \frac{(25 + 20)}{(25 - 20)} = 80 \times \frac{45}{5} = \mathbf{720 \text{ km}}$$

8. Total distance = Total time $\left(\frac{\text{Product of speeds}}{\text{Sum of speeds}} \right)$

$$= 5 \frac{48}{60} \left(\frac{25 \times 4}{25 + 4} \right) = 5 \frac{4}{5} \left(\frac{100}{29} \right) = \frac{29}{5} \times \frac{100}{29} = \mathbf{20 \text{ km}}$$

9. Relative speed of the policeman w.r.t. thief = $12 - 10 = 2 \text{ km/h}$

$\therefore$ Time taken by the policeman to catch the thief = $\frac{200/1000}{2} = \frac{1}{10} \text{ hour}$

$\therefore$ The distance thief has run before he was caught by the policeman = $10 \times \frac{1}{10} = \mathbf{1 \text{ km}}$

10. Speed of first train, $S_1 = 10 \text{ m/s} = 10 \times \frac{18}{5} = 36 \text{ km/h}$

Speed of second train, $S_2 = 36 \times 1 \frac{1}{3} = 48 \text{ km/h}$

Total distance between the two places = 68 km

Difference between first and second times of departure $T_1 - T_2 = -20 \text{ minutes} = -\frac{1}{3} \text{ hour}$

Distance of meeting point from Nagercoil = $68 - 36 \left(\frac{1}{3} \right) = \mathbf{56}$

$$84 \times t = 56$$

$$48t = \frac{56}{84} \times 48 = \frac{8}{12} \times 48 = \mathbf{32 \text{ km}}$$

$$\begin{aligned}\text{Distance of meeting point from Nagercoil} &= 48 \times \left[\frac{68 - 36 \left(\frac{1}{3} \right)}{36 + 48} \right] \\ &= 48 \left[\frac{68 - 12}{36 + 48} \right] = 32 \text{ km}\end{aligned}$$

$$11. \text{ Distance between P and Q} = \text{Distance more travelled by one train} \times \frac{\text{Sum of speeds}}{\text{Difference of speeds}}$$

$$\begin{aligned}&= 100 \text{ km} \times \frac{50 + 40}{50 - 40} \\ &= 100 \times \frac{90}{10} \\ &= 900 \text{ km}\end{aligned}$$

$$12. \text{ Relative speed of owner w.r.t. thief} = 75 - 60 = 15 \text{ km/hr.}$$

$$\text{The distance covered by thief in 30 minutes} = 60 \times \frac{1}{2} = 30 \text{ km}$$

$$\therefore \text{ Owner will overtake thief in } \frac{30}{15} = 2 \text{ hours}$$

$$\therefore \text{ He will overtake thief at 5 p.m.}$$



$$\text{Short cut: Required time} = \frac{S_2 \times \text{Difference in time } (t)}{S_1 - S_2}$$

where S_1 = Speed of man

S_2 = Speed of thief

t = Time lag between both

$$\text{Here } t = 3.00 - 2.30 = 30 \text{ minutes} = \frac{1}{2} \text{ hour; } S_1 = 75 \text{ km/hr; } S_2 = 60 \text{ km/hr}$$

$$\therefore \text{ Required time} = \frac{60 \times \frac{1}{2}}{15} = 2 \text{ hours.}$$

$$13. \text{ Let the distance covered be } S \text{ km.}$$

$$\therefore \left(\frac{S}{8} + 4 \right) \times \frac{15}{2} = S$$

$$\Rightarrow (S + 32) \times 15 = 16 S$$

$$\begin{aligned}\Rightarrow S &= 32 \times 15 \\ &= 480 \text{ km}\end{aligned}$$

$$14. \text{ Let distance be } S \text{ km and usual speed be } u \text{ km/hr}$$

$$\therefore \frac{S}{u} - \frac{S}{u + 3} = \frac{40}{60} = \frac{2}{3}$$

$$\text{or } \frac{3S}{u(u + 3)} = \frac{2}{3}$$

$$\text{or} \quad 9S = 2u(u + 3) \quad (1)$$

$$\text{Similarly by} \quad \frac{S}{u-2} - \frac{S}{u} = \frac{40}{60} = \frac{2}{3}$$

$$\text{or} \quad \frac{2S}{u(u-2)} = \frac{2}{3}$$

$$\text{or} \quad 3S = u(u-2) \quad (2)$$

Dividing (1) by (2), we get

$$3 = \frac{2(u+3)}{u-2}$$

$$\Rightarrow u = 12 \text{ km/hr}$$

and, therefore, from (2),

$$S = \frac{(12)(12-2)}{3} \\ = 4 \times 10 = 40 \text{ km/hr}$$

15. Let the total distance travelled be S km and average speed be x km/hr.

$$\therefore \frac{S/3}{10} + \frac{S/3}{20} + \frac{S/3}{60} = \frac{S}{x}$$

$$\text{or} \quad \frac{1}{30} + \frac{1}{60} + \frac{1}{180} = \frac{1}{x}$$

$$\text{or} \quad x = \frac{1}{\frac{1}{6} + \frac{1}{3} + \frac{1}{18}} = 18 \text{ km/hr}$$

16. Let the distance to his school be S km.

$$\text{Average speed} = \frac{2 \times 3 \times 2}{3 + 2} = 2.4 \text{ km/hr}$$

Time taken for whole journey = 5 hours.

$$\therefore 2S = \text{Average speed} \times \text{Time} \\ = 2.4 \times 5 = 12 \text{ km}$$

$$\text{or} \quad S = 6 \text{ km}$$

$$17. \text{ Average speed} = \frac{2 \times 64 \times 80}{64 + 80} = \frac{2 \times 64 \times 80}{144} \\ = \frac{1 \times 64 \times 10}{9} = \frac{640}{9} = 71.11 \text{ km/hr}$$

18. Let distance travelled by foot be x km, and therefore distance by bicycle = $(61 - x)$ km.

$$\therefore \frac{61-x}{9} + \frac{x}{4} = 9$$

$$\text{or} \quad (61-x)4 + 9x = 9 \times 36$$

$$\text{or} \quad 5x = 9 \times 36 - 4 \times 61 \\ = 4[81 - 61] \\ = 80$$

$$\text{or} \quad x = \frac{80}{5} = 16 \text{ km}$$

19. Required speed = $\frac{6 \text{ km}}{\frac{45}{60} \text{ hr}} = 8 \text{ km/hr}$

Half of journey (speed) = $\frac{3}{\frac{30}{60}} = 6 \text{ km/hr}$

Let speed for next half be $x \text{ km/hr}$

$\therefore$ Average speed = $\frac{2 \times 6 \times x}{6 + x} = 8$

i.e. $\frac{12x}{6 + x} = 8$

or $12x = 8 \times 6 + 8x$

or $4x = 8 \times 6$

or $x = 12 \text{ km/hr}$

20. Average speed = $\frac{\text{Total distance travelled}}{\text{Total time taken}}$

Total distance travelled = $10 + 12 = 22 \text{ km}$

Total time taken = $\left(\frac{10}{12} + \frac{12}{10}\right) = \frac{50 + 72}{60} = \frac{122}{60}$

= $\frac{61}{30}$ hours

$\therefore$ Average speed = $\frac{22}{\frac{61}{30}} = \frac{22 \times 30}{61} = 10.8 \text{ km/hr}$

21. Let total distance be $S \text{ km}$

If u_1, u_2, u_3 are their speeds for 3 equal distances and t minutes is the total time taken, then

$$\begin{aligned} \text{Total distance travelled} &= \frac{u_1 u_2 u_3 \times t}{20(u_1 u_2 + u_2 u_3 + u_3 u_1)} \\ &= \frac{3 \times 4 \times 5 \times 47}{20(3 \times 4 + 4 \times 5 + 5 \times 3)} = \frac{3 \times 47}{(12 + 20 + 15)} \\ &= \frac{3 \times 47}{47} = 3 \text{ km} \end{aligned}$$

22. Distance travelled by train from A from 8 a.m. to 9 a.m. = $60 \text{ km/hr} \times 1 \text{ hr} = 60 \text{ km}$

Distance between two trains at 9 a.m. = $330 - 60 = 270 \text{ km}$

Relative speed of second trains w.r.t first = $75 - (-60) = 135 \text{ km/hr}$

$\therefore$ Time taken by them to meet = $\frac{\text{Distance}}{\text{Speed}}$

= $\frac{270}{135}$

= 2 hours

At 11 a.m. both the trains will meet.

23. Let the duration of flight be t hours.

$$\therefore \frac{600}{t} - \frac{600}{t + \frac{1}{2}} = 200$$

$$\frac{600}{t} - \frac{2 \times 600}{2t + 1} = 200$$

$$(2t + 1) 600 - t \times 1200 = 200t (2t + 1)$$

$$\text{i.e.} \quad 3(2t + 1) - 6t = t(2t + 1)$$

$$\Rightarrow 6t + 3 - 6t = 2t^2 + t$$

$$\Rightarrow 2t^2 + t - 3 = 0$$

$$\therefore t = 1 \text{ hour}$$

Hence answer is (a).

24. Let Abhay's speed be x km/hr

$$\therefore \text{Time taken to travel 30 km by Abhay} = \frac{30}{x}$$

$$\therefore \text{Time taken to travel 30 km by Sameer} = \left(\frac{30}{x} - 2 \right)$$

$$\therefore \frac{30}{2x} = \left(\frac{30}{x} - 2 \right) - 1$$

$$\text{i.e.} \quad \frac{30}{2x} = \frac{30}{x} - 3$$

$$\text{or} \quad \frac{30}{x} - \frac{30}{2x} = 3$$

$$10 \left[\frac{1}{x} - \frac{1}{2x} \right] = 1$$

$$\text{or} \quad 10 \frac{(2-1)}{2x} = 1$$

$$\text{or} \quad x = \frac{10}{2} = 5 \text{ km/hr}$$

25. Let the length of journey be x km.

$$\frac{x}{35} = \frac{x}{40} + \frac{15}{60}$$

$$\frac{x}{35} - \frac{x}{40} = \frac{1}{4}$$

$$\frac{8x - 7x}{7 \times 8 \times 5} = \frac{1}{4}$$

$$\text{or} \quad x = \frac{7 \times 8 \times 5}{4} = 70 \text{ km}$$

26. Relative speed of B w.r.t A = $(6 - 1)$ rounds/hour
= 5 rounds/hour

∴ Time taken to complete one round = **12 minutes**

∴ They will meet for the first time after 12 minutes i.e. at **7.42 a.m.**

27. Relative speed of P w.r.t Q = 120 km/hr

i.e. $u + v = 120$

(1)

Relative speed of P w.r.t. Q, when they are in same direction

$$= u - v = \frac{120}{6} = 20 \quad (2)$$

Adding (1) and (2), we get

$$2u = 140$$

or

$$u = 70$$

∴ Speed of P, $u = 70$ km/hr

28. Relative speed of policeman w.r.t thief = $11 - 10 = 1$ km/hr

Distance already covered by thief = 200 m

After 6 minutes, distance covered by policeman

$$= 1 \text{ km/hr} \times \frac{6}{60} \text{ hr}$$

$$= \frac{1}{10} \text{ km} = 100 \text{ m}$$

∴ Distance between policeman and thief = $200 - 100 = 100 \text{ m}$

29. Let the actual distance travelled be x km.

$$\therefore \frac{x}{10} = \frac{x + 20}{14}$$

⇒

$$x = 50 \text{ km}$$

30. Relative speed of B w.r.t. A = $(3 + 2)$ rounds/hour = **5 rounds/hour**

They will cross each other 5 times in every hour and 2 times in half an hour. In total, they will cross each other 7 times upto 9.30 a.m.

31. Let speed of train be x km/hr and that of car be kx km/hr.

$$\therefore \frac{120}{x} + \frac{480}{kx} = 8 \quad (1)$$

$$\frac{200}{x} + \frac{400}{kx} = 8\frac{1}{3} \quad (2)$$

$$\text{Simplifying (1), } \Rightarrow 120k + 480 = 8kx \quad (A)$$

$$\text{Simplifying (2), } \Rightarrow 200k + 400 = \frac{25}{3}kx \quad (B)$$

Dividing (A) by (B),

$$\frac{120k + 480}{200k + 400} = \frac{8kx \times 3}{25kx}$$

$$\text{i.e. } \frac{120(k + 4)}{200(k + 2)} = \frac{24}{25}$$

$$\frac{5(k+4)}{8(k+2)} = 1$$

$$\Rightarrow \frac{k+4}{k+2} = \frac{8}{5}$$

Cross multiplying, we get,

$$8k + 16 = 5k + 20$$

$$8k - 5k = 20 - 16$$

$$3k = 4$$

$$k = \frac{4}{3}$$

$\therefore$ Speed of train : Speed of car = $x : kx = 1 : k$

$\therefore$ Required ratio = $1 : k = 1 : \frac{4}{3}$ or **3 : 4**

32. Average speed = $\frac{2(40)(60)}{(40+60)} = 2 \times 4 \times 6 = \mathbf{48 \text{ km/hr}}$

33. Let the distance to school be x km.

$$\frac{x}{2.5} - \frac{x}{3.5} = \frac{6+6}{60} = \frac{1}{5}$$

i.e. $\frac{x(7-5)}{7 \times 2.5} = \frac{1}{5}$

or $x = \frac{7 \times 2.5}{5 \times 2} = \mathbf{1.75 \text{ km}}$

34. $\frac{42 \text{ km}}{1 \text{ hr } 40 \text{ min } 48 \text{ sec}} = \frac{5}{7}$ of actual speed

$$\begin{aligned} \therefore \text{Actual speed} &= \frac{7}{5} \times \frac{42}{1 \text{ hr } 40 \text{ min } 48 \text{ sec}} \\ &= \frac{7 \times 42}{5 \text{ hr } 200 \text{ min } + 4 \text{ min}} \\ &= \frac{7 \times 42}{8 \text{ hr } 24 \text{ min}} = \frac{7 \times 42 \text{ km}}{8.4 \text{ hr}} \\ &= \mathbf{35 \text{ km/hr}} \end{aligned}$$

35. Total distance = $600 + 800 + 500 + 100 = 2000 \text{ km}$

$$\begin{aligned} \text{Total time taken} &= \frac{600}{80} + \frac{800}{40} + \frac{500}{400} + \frac{100}{50} \\ &= \frac{15}{2} + 20 + \frac{5}{4} + 2 \\ &= \mathbf{30.75 \text{ hours}} \end{aligned}$$

$$\begin{aligned} \therefore \text{Average speed} &= \frac{2000}{30.75} = \frac{8000}{123} \\ &= \mathbf{65 \frac{5}{123} \text{ km/hr}} \end{aligned}$$

36. Sum of speeds = 7 km/hr

∴ Speed of A is greater than $\frac{7}{2}$ km/hr.

If speed of A = 4 km/hr, that of B = 3 km/hr such that their sum = 7 km/hr

Sum of times = 14 hours is also satisfied with the above conditions.

∴ Speed of A = 4 km/hr.

37. New speed = $\frac{6}{7}$ of usual speed

New time = $\frac{7}{6}$ of usual time

$$\begin{aligned}\therefore \text{Difference in time} &= \frac{7}{6} \text{ of usual time} - \text{usual time} \\ &= \frac{1}{6} \text{ of usual time} \\ &= \frac{1}{5} \text{ of hour}\end{aligned}$$

$$\therefore \text{Usual time} = 6 \times \frac{1}{5} \text{ hour} = \mathbf{1 \text{ hour } 12 \text{ min}}$$

38. Relative speed = 15 – 10 = 5 km/hr

Difference in time = 2 hours.

∴ Relative distance = 5 × 2 = **10 km**

To reach at 1 p.m.

Let distance travelled be x km

$$\therefore \frac{x}{10} - \frac{x}{15} = 2$$

$$\Rightarrow x(3 - 2) = 2 \times 30$$

$$\text{or } x = \mathbf{60 \text{ km}}$$

$$\text{At 10 km/hr speed, time taken to travel} = \frac{60}{10} = 6 \text{ hours}$$

6 hours before 2 p.m. = 8 a.m.

$$\therefore \text{Required speed to reach at 1 a.m.} = \frac{60}{5} = \mathbf{12 \text{ km/hr}}$$

39. Let speed of car be x km/hr and that of train be $1.5x$ km/hr

Distance between A and B = 75 km

$$\therefore \frac{75}{x} = \frac{75}{1.5x} + \frac{12.5}{60} \text{ hours}$$

$$\text{i.e. } \frac{75}{x} - \frac{75}{1.5x} = \frac{5}{24}$$

$$\Rightarrow \frac{75(1.5 - 1)}{1.5x} = \frac{5}{24}$$

$$\Rightarrow x = \frac{75 \times 0.5 \times 24}{1.5 \times 5} = 5 \times 24 = \mathbf{120 \text{ km/hr}}$$

40. Relative speed of Deepak w.r.t. his wife

$$= 4.5 + 3.75$$

$$= \mathbf{8.25 \text{ km/hr}}$$

$$\therefore \text{Time taken to rotate periphery of sports complex} = \frac{726 \text{ m}}{8.25 \text{ km/hr}} = \frac{0.726 \text{ km}}{8.25 \text{ km/hr}}$$

$$= \frac{0.726}{8.25} \text{ hr}$$

$$= \frac{0.726}{8.25} \times 60 \text{ minutes}$$

$$= \mathbf{5.28 \text{ minutes}}$$

41. Let the speed of person be x km/hr

$$x \times 30 = \frac{14}{15}x \times 30 + 10$$

$$\therefore 30 \left(x - \frac{14}{15}x \right) = 10$$

$$\text{or} \quad \frac{30x}{15} = 10$$

$$\text{or} \quad x = \frac{10 \times 15}{30} = \mathbf{5 \text{ km/hr}}$$

42. Let time taken by A be x hours

$$\therefore \text{Time taken by B} = \left(x - \frac{1}{2} \right) \text{ hours}$$

$$\text{Distance covered by A} = 3k \times x \tag{1}$$

$$\text{Distance covered by B} = 4k \times \left(x - \frac{1}{2} \right) \tag{2}$$

Equating (1) and (2), we get

$$3k \times x = 4k \times \left(x - \frac{1}{2} \right)$$

$$3x = 4 \left(x - \frac{1}{2} \right)$$

$$\text{or} \quad 4x - 3x = 4 \times \frac{1}{2} = 2$$

$$\text{or} \quad x = \mathbf{2 \text{ hours}}$$

i.e. Time taken by A = **2 hours**

43. Let time taken by Anna at speed of 60 km/hr be x hours.

$$\therefore 80 \times 2 \frac{1}{4} + 60 \times x = 350$$

$$\therefore x = \frac{350 - 180}{60}$$

$$= \frac{170}{60} = \frac{17}{6} \text{ hours} = \mathbf{2 \text{ hours } 50 \text{ minutes}}$$

$$\therefore \text{Total time of travel} = 2 \text{ hours } 15 \text{ minutes} + 2 \text{ hours } 50 \text{ minutes} \\ = 5 \text{ hours } 5 \text{ minutes}$$

So Anna reached city A at 10.25 a.m.

Hence answer is (e).

44. Time of travel for a distance of 600 km at a speed of 100 km/hr = $\frac{600}{100} = 6 \text{ hours}$

$$\text{Time of halt at the rate of } 3 \text{ min/75 km} \\ = 3 \times 7 = 21 \text{ minutes}$$

$$\therefore \text{Total time of travel} = 6 \text{ hour } 21 \text{ min}$$

45. Speed of car = 108 km/hr = $108 \times \frac{5}{18} \text{ m/s} = 30 \text{ m/s}$

$$\therefore \text{Distance covered} = 30 \times 15 = 450 \text{ m}$$

46. Speed of person = $\frac{600 \text{ m}}{5 \text{ minutes}} = \frac{600 \text{ m}}{5 \times 60 \text{ sec}} = 2 \text{ m/s}$

$$\therefore \text{Speed of person (in km/hr)} = 2 \times \frac{18}{5} = 7.2 \text{ km/hr}$$

47. Time taken by boy to run round square field

$$= \frac{\text{Perimeter}}{\text{Speed}} = \frac{4 \times 35 \text{ m}}{9 \text{ km/hr}} = \frac{4 \times 35 \text{ m}}{9 \times \frac{5}{18} \text{ m/s}} \\ = \frac{4 \times 35 \times 2}{5} \text{ sec} = 56 \text{ seconds}$$

48. Ratio of their speeds = $\frac{300/7.5}{450/9} = \frac{40}{50} = \frac{4}{5}$ or **4 : 5**

49. In first 2 hours, distance covered = $70 \times 2 = 140 \text{ km}$

In the second 2 hours, distance covered = $80 \times 2 = 160 \text{ km}$

Remaining distance to be covered = $345 - (140 + 160) = 45 \text{ km}$

$$\therefore \text{Time taken by motor car to travel } 45 \text{ km at a speed of } 90 \text{ km/hr} = \frac{45}{90} = \frac{1}{2} \text{ hour}$$

$$\therefore \text{Required time} = 4 \frac{1}{2} \text{ hours}$$

50. Speed of train = $10 \text{ km/12 min} = \frac{10}{\frac{12}{60}} = 50 \text{ km/hr}$

If speed is decreased by 5 km/hr, new speed = 45 km/hr

$\therefore$ Time taken to cover same distance

$$= \frac{10}{45} \text{ hours} = \frac{10 \times 60}{45} \text{ minutes}$$

$$= \frac{120}{9} = \frac{40}{3} \text{ minutes}$$

$$= 13 \text{ minutes } 20 \text{ seconds}$$

51. Let length of bridge be L metres.

$$\therefore \left(\frac{L}{15 \times 60} \right) \times \frac{18}{5} = 5$$

$$\begin{aligned} \text{i.e.} \quad L &= \frac{5 \times 5 \times 60 \times 15}{18} \\ &= 5 \times 5 \times 10 \times 5 = \mathbf{1250 \text{ m}} \end{aligned}$$

52. Speed of truck = $\frac{550 \text{ m}}{60 \text{ sec}} = \frac{55}{6} \text{ m/s}$

$$\text{Speed of bus} = \frac{33 \text{ km}}{45 \text{ min}} = \frac{33}{45} \text{ km/hr} \times \frac{5}{18} \text{ m/s}$$

$$= \left(\frac{33 \times 60}{45} \right) \times \frac{5}{18} \text{ m/s}$$

$$= \left(11 \times 4 \times \frac{5}{18} \right) \text{ m/s} = \frac{110}{9} \text{ m/s}$$

$$\begin{aligned} \therefore \text{Ratio of their speed} &= \frac{55/6}{110/9} = \frac{55}{6} \times \frac{9}{110} \\ &= \frac{3}{4} \text{ or } \mathbf{3 : 4} \end{aligned}$$

53. Total distance travelled is 12 hours

$$= 35 + 37 + 39 + \dots \text{ up to 12 terms}$$

$$= \frac{n}{2} \{2a + (n-1)d\}$$

$$= \frac{12}{2} \{2 \times 35 + (12-1)2\}$$

$$= 6 \{70 + 22\} = \mathbf{552 \text{ km}}$$

54. Speed of athlete = $\frac{200 \text{ m}}{24 \text{ sec}} = \frac{25}{3} \text{ m/s} = \frac{25}{3} \times \frac{18}{5} \text{ km/hr} = \mathbf{30 \text{ km/hr}}$

55. Average speed = $\frac{2 \times 25 \times 4}{(25+4)} = \frac{200}{29} \text{ km/hr}$

$$\text{Distance travelled} = \text{Average speed} \times \text{Time}$$

$$= \frac{200}{29} \times 5 \frac{4}{5}$$

$$= \frac{200}{29} \times \frac{29}{5} = \mathbf{40 \text{ km}}$$

$$\therefore \text{Distance of post office to village} = \frac{40}{2} = \mathbf{20 \text{ km}}$$

Heights and Distances

ANGLES OF ELEVATION AND DEPRESSION: The line of sight or line of vision is a straight line from our eye to the object we are viewing. In the adjoining figure RP is the line of sight.

The angle of elevation is the angle through which our eyes move above from the horizontal. Here it is $\angle QRP = \theta$

The angle of depression is the angle through which our eyes move below the horizontal line.

Here it is $\angle RPQ = \alpha$

Generalised cases:

From $\triangle ABC$,

$$\tan \theta_2 = \frac{h_2}{d}$$

or

$$d = \frac{h_2}{\tan \theta_2} = h_2 \cot \theta_2 \quad (1)$$

Similarly from $\triangle BCD$,

$$\tan \theta_1 = \frac{h_1}{d}$$

or

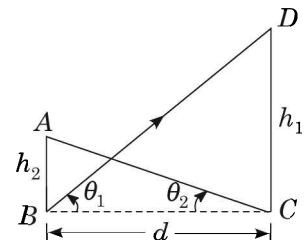
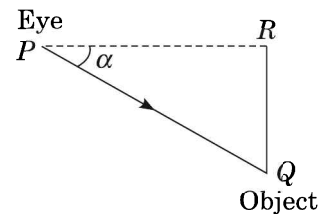
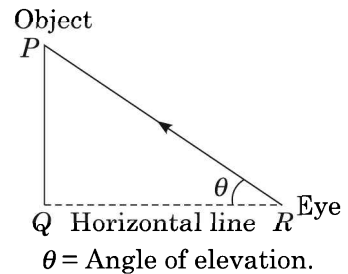
$$d = \frac{h_1}{\tan \theta_1} = h_1 \cot \theta_1 \quad (2)$$

From (1) and (2),

$$h_1 \cot \theta_1 = h_2 \cot \theta_2$$

$\therefore$ Height difference $h_1 \sim h_2 = d(\tan \theta_1 \sim \tan \theta_2)$

$$\therefore d = \frac{h_1 \sim h_2}{(\tan \theta_1 \sim \tan \theta_2)}$$

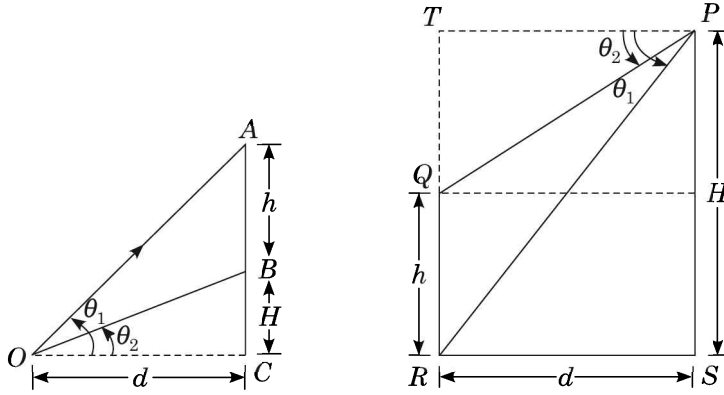


From $\triangle OCB$,

$$BC = d \tan \theta_2 \quad (1)$$

From $\triangle OAC$,

$$AC = d \tan \theta_1 \quad (2)$$



$$\begin{aligned} AB &= AC - BC \\ &= d \tan \theta_1 - d \tan \theta_2 \end{aligned}$$

$\therefore$

$$\begin{aligned} AB &= d \tan \theta_1 - H \\ &= H \left[\frac{d \tan \theta_1}{d \tan \theta_2} - 1 \right] \\ h &= H \left[\frac{\tan \theta_1}{\tan \theta_2} - 1 \right] \end{aligned}$$

From $\triangle PRT$ and $\triangle PQT$

$$h = d(\tan \theta_2 - \tan \theta_1)$$

$\therefore$

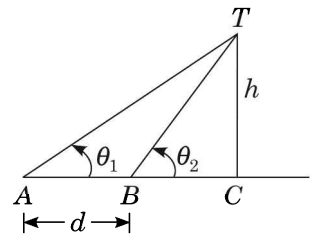
$$\begin{aligned} H &= d \tan \theta_2 \\ h &= H \left[1 - \frac{\tan \theta_1}{\tan \theta_2} \right] \end{aligned}$$

$\therefore$

$$d = \frac{h}{\tan \theta_2 - \tan \theta_1}$$

If A and B are two positions of a car or a ship or any other vehicle or any person and the distance $AB = d$ is covered in time t , the speed of the vehicle or ship is given by

$$S = \frac{d}{t} = \frac{h}{t} (\cot \theta_1 - \cot \theta_2)$$



A tower of height h stands vertical on the ground. Its angle of elevation from two points, distance d apart and lying along its mutually $\perp$ directions on the ground are θ_1 and θ_2 .

Then

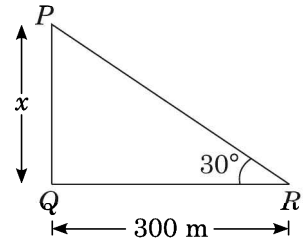
$$h = \frac{d}{\sqrt{\cot^2 \theta_1 + \cot^2 \theta_2}}$$

EXAMPLE 1 300 m from the foot of a cliff on the level ground, the angle of elevation of the top of a cliff is 30° . Find the height of this cliff.

Solution Let the height of cliff be x metres. From $\triangle PQR$,

$$\tan 30^\circ = \frac{PQ}{QR} = \frac{x}{300} = \frac{1}{\sqrt{3}}$$

$$\therefore x = \frac{300}{\sqrt{3}} = 100\sqrt{3} = 173.2 \text{ m}$$



EXAMPLE 2 The horizontal distance between two towers is $100\sqrt{3}$ metres. The angle of depression of the first tower when seen from the top of the second tower is 30° . If the height of the second tower is 160 m, find height of the first tower.

Solution Let RS be the second tower 160 m high.

Let PQ be the first tower of height x metres

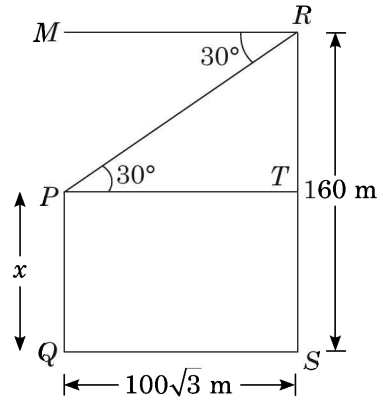
$MR \parallel PT$ and $\angle MRP = \angle RPT = 30^\circ$

$\therefore$ From $\triangle PRT$,

$$\tan 30^\circ = \frac{RT}{PT} = \frac{1}{\sqrt{3}} = \frac{RT}{100\sqrt{3}}$$

$$\therefore RT = 100 \text{ m}$$

$$\therefore PQ = RS - RT = 160 - 100 = 60 \text{ m}$$



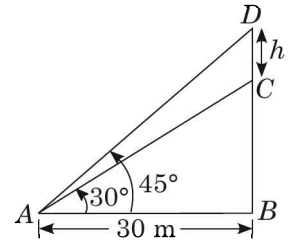
EXAMPLE 3 Angles of elevation of top and bottom of a flag kept on a flag post at 30 metres distance are 45° and 30° respectively. What is the height of the flag?

$$\text{Solution } \tan 45^\circ = \frac{DB}{AB} \quad \text{or} \quad DB = 30 \text{ m}$$

$$\tan 30^\circ = \frac{CB}{AB} \quad \text{or} \quad CB = \frac{30}{\sqrt{3}} \text{ m}$$

$$\text{Height of flag } CD = 30 - \frac{30}{\sqrt{3}} = 30 - 10\sqrt{3}$$

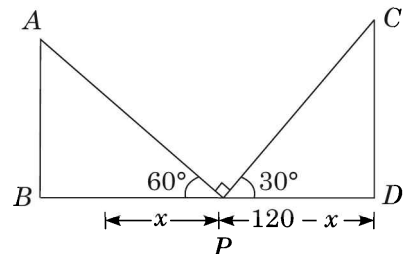
$$= 30 - 17.32 = 12.68 \text{ m}$$



EXAMPLE 4 Two poles of equal heights stand on either sides of a road way which is 120 m wide. At a point on the roadway between the poles, the elevations of the tops of the pole are 60° and 30° . Find the heights of the poles and the position of the point.

Solution Let AB and CD be the two poles of equal length l .

Let the point P be at a distance x metre from B such that $BP = x$ and $PD = 120 - x$.



From $\triangle ABP$,

$$\tan 60^\circ = \frac{l}{x} \quad \text{or} \quad x = \frac{l}{\tan 60^\circ} \quad (1)$$

From $\triangle PCD$,

$$\tan 30^\circ = \frac{l}{120 - x} \quad \text{or} \quad 120 - x = \frac{l}{\tan 30^\circ} \quad (2)$$

$$\begin{aligned} (1) + (2) &\Rightarrow 120 = \frac{l}{\tan 60^\circ} + \frac{l}{\tan 30^\circ} \\ &\Rightarrow l = \frac{120 \times \tan 60^\circ \tan 30^\circ}{\tan 30^\circ + \tan 60^\circ} \\ &= \frac{120 \times \sqrt{3} \times 1/\sqrt{3}}{(1/\sqrt{3} + \sqrt{3})} = \frac{120}{4/\sqrt{3}} = 30\sqrt{3} = \mathbf{52 \text{ m}} \\ x &= \frac{l}{\tan 60^\circ} = \frac{30\sqrt{3}}{\sqrt{3}} = \mathbf{30 \text{ m}} \end{aligned}$$

EXAMPLE 5 An aeroplane when 3000 m high passes vertically above another at an instant when the angles of elevation at the same observing point are 60° and 45° respectively. How many metres lower is one than the other?

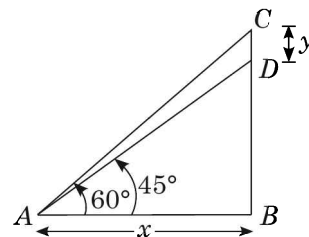
Solution Let C and D be the position of two aeroplanes, C being at a height of 3000 m from B .

Let D be y metre lower than C .

Let A be the point of observation.

Now $\angle DAB = 45^\circ$, $\angle CAB = 60^\circ$.

Let $AB = x$ metres.



From $\triangle ABC$,
$$\tan 60^\circ = \frac{BC}{AB} = \frac{3000}{x}$$

or
$$x = \frac{3000}{\tan 60^\circ} = \frac{3000}{\sqrt{3}} = 1000\sqrt{3} \quad (1)$$

From $\triangle ABD$,

$$\tan 45^\circ = \frac{BD}{AB} = \frac{3000 - y}{x} = 1$$

$\therefore x = 3000 - y \quad (2)$

Equating (1) and (2),

$$3000 - y = 1000\sqrt{3}$$

$$\begin{aligned} \therefore y &= 3000 - 1000\sqrt{3} \\ &= 3000 - 1732 = \mathbf{1268 \text{ m}} \end{aligned}$$

$\therefore$ Height difference between the two aeroplanes = **1268 m**

EXAMPLE 6 The shadow of a vertical pole is $\sqrt{3}$ of its height. Find the angle of elevation.

Solution Let BC be the height of pole = h .

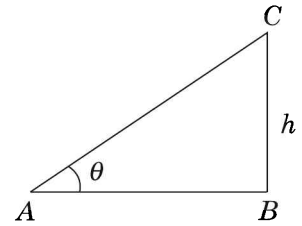
Length of shadow = $AB = h\sqrt{3}$

From $\triangle ABC$,

$$\tan \theta = \frac{BC}{AB} = \frac{h}{h\sqrt{3}} = \frac{1}{\sqrt{3}}$$

$$\therefore \theta = 30^\circ$$

$$\therefore \text{Angle of elevation} = 30^\circ$$



EXAMPLE 7 The angles of depression of two ships from the top of a lighthouse are 45° and 60° . If the ships are 240 m apart, find the height of lighthouse.

Solution Let $AQ = x$ m

$\therefore QB = (240 - x)$ m

From $\triangle APQ$

$$\tan 60^\circ = \frac{y}{x} = \sqrt{3}$$

$\Rightarrow$

$$y = x\sqrt{3}$$

From $\triangle BPQ$

$$\tan 45^\circ = \frac{y}{240 - x} = 1$$

$\Rightarrow$

$$y = 240 - x$$

From (1) and (2), we get

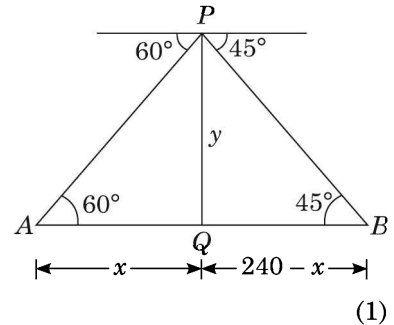
$$\frac{y}{\sqrt{3}} = 240 - y$$

or

$$y \left(1 + \frac{1}{\sqrt{3}} \right) = 240$$

$\Rightarrow$

$$y = \frac{240\sqrt{3}}{(1 + \sqrt{3})} \text{ m}$$



EXAMPLE 8 From the top of a cliff 200 m high the angles of depression of two boats which are due south of observer are 60° and 30° . Find the distance between the two boats.

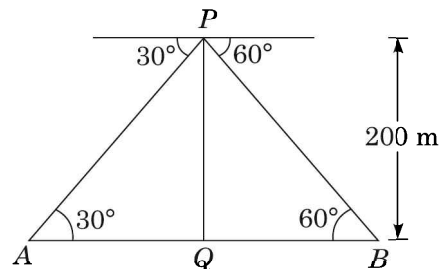
Solution Let $PQ = 200$ m, height of cliff.

From $\triangle APQ$,

$$\tan 30^\circ = \frac{PQ}{AQ} = \frac{1}{\sqrt{3}} = \frac{200}{AQ}$$

$\Rightarrow$

$$AQ = 200\sqrt{3} \text{ m}$$



From $\triangle BPQ$

$$\tan 60^\circ = \frac{PQ}{BQ} = \frac{200}{BQ} = \sqrt{3}$$

$$\Rightarrow BQ = \frac{200}{\sqrt{3}} \quad (2)$$

Distance between the two boats = $AQ + BQ = 200\sqrt{3} + \frac{200}{\sqrt{3}} = 460 \text{ m}$ (approx.)

EXAMPLE 9 A vertical tower stands on a horizontal plane and is surmounted by a vertical flagstaff of height h . At a point on the plane, the angles of elevation of the bottom of the flagstaff is α and that of the top of the flagstaff is β . Find the height of the tower.

Solution Let PQ be height of flagstaff = h and QR , height of tower = H .

From $\triangle QRS$,

$$\tan \alpha = \frac{QR}{RS} = \frac{H}{RS}$$

or

$$RS = \frac{H}{\tan \alpha} \quad (1)$$

From $\triangle PRS$,

$$\tan \beta = \frac{PQ + QR}{RS} = \frac{(h + H)}{H} \tan \alpha$$

or

$$H \tan \beta = (h + H) \tan \alpha \quad (2)$$

on rearranging, we get

$$H = \frac{h \tan \alpha}{\tan \beta - \tan \alpha}$$

EXAMPLE 10 From the top and bottom of a building of height h , the angles of elevation of the top of a tower are α and β respectively. Find the height of the tower.

Solution Let DC be the height of building.

Let AB be the height of tower = H .

From $\triangle ADE$,

$$\tan \alpha = \frac{AE}{DE} = \frac{H - h}{DE}$$

or

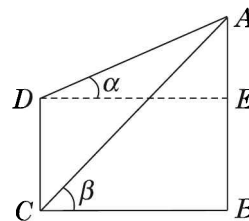
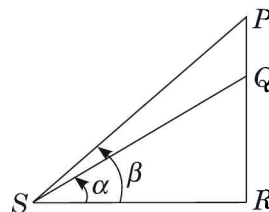
$$DE = \frac{H - h}{\tan \alpha} \quad (1)$$

From $\triangle ABC$,

$$\begin{aligned} \tan \beta &= \frac{AB}{BC} = \frac{H}{DE} = \frac{H}{(H - h)/\tan \alpha} \\ &= \frac{H \tan \alpha}{(H - h)} \end{aligned}$$

$\Rightarrow$

$$H = \frac{h \tan \beta}{\tan \beta - \tan \alpha}$$





EXERCISES

- A tower is 30 m high. An observer from the top of the tower makes an angle of depression 60° at the base of the building and angle of depression of 45° at the top of the building, what is the height of the building in metres?
(a) 18 (b) $12\sqrt{2}$ (c) $10\sqrt{3}$ (d) 15 (e) None of these
- Two towers of equal height stand on either side of a wide road which is 100 m wide. At a point on the road between the pillar, the elevations of the tops of the pillars are 60° and 30° . Find their heights in metres.
(a) $20\sqrt{3}$ (b) $26\sqrt{3}$ (c) $30\sqrt{3}$ (d) $22\sqrt{3}$ (e) None of these
- The angle of elevation of a ladder leaning against a house is 60° and the foot of the ladder is 6.5 m from the house. Find the length of the ladder in metres.
(a) 3.25 (b) $13/\sqrt{3}$ (c) 13 (d) 15 (e) None of these
- From an aeroplane vertically over a straight horizontal road, the angles of depression of two consecutive milestones on the opposite sides of the aeroplane are observed to be 30° and 60° . Find the height in miles of the aeroplane above the road.
(a) $\sqrt{3}/2$ (b) $\sqrt{3}/4$ (c) $\sqrt{3}/8$ (d) $2\sqrt{3}/12$ (e) None of these
- From the top of a cliff, 60 m high, the angle of depression of the top and bottom of a tower are observed to be 30° and 60° respectively. Find the height of the tower.
(a) 40 m (b) 50 m (c) 30 m (d) 35 m (e) None of these
- The angle of elevation of the top of an unfinished tower at a point 120 m from its base is 45° . How much higher must the tower be raised so that its angle of elevation is 60° at the same point?
(a) 90 m (b) 92 m (c) 97 m (d) 87.84 m (e) None of these
- An observer on the top of a cliff, 200 m above the sea-level, observes the angle of depression of two ships at anchor to be 45° and 30° respectively. Find the distance between the ships if the line joining them stretches to the base of cliff.
(a) 140 m (b) 150 m (c) 156 m (d) 146.4 m (e) None of these
- A man is watching from the top of a tower a boat speeding away from the tower. The boat makes an angle of depression of 45° with the man's eye when at a distance of 60 metres from the tower. After 5 seconds, the angle of depression becomes 30° . What is the approximate speed of the boat, assuming that it is running in still water? **[SBI PO]**
(a) 32 km/hr (b) 36 km/hr (c) 38 km/hr (d) 40 km/hr (e) 42 km/hr
- From a point P on a level ground, the angle of elevation of the top of a tower is 30° . If the tower is 100 m high, the distance of point P from the foot of the tower is **[RRB]**
(a) 149 m (b) 156 m (c) 173 m (d) 200 m
- A man on the top of a vertical observation tower observes a car moving at a uniform speed coming directly towards it. If it takes 12 minutes for the angle of depression to change from 30° to 45° , how soon after this will the car reach the observation tower? **[RRB]**
(a) 14 min 35 sec (b) 15 min 49 sec (c) 16 min 23 sec (d) 18 min 5 sec
- A man standing at a point P is watching the top of a tower, which makes an angle of elevation of 30° with the man's eye. The man walks some distance towards the tower to watch its top

and the angle of elevation becomes 60° . What is the distance between the base of the tower and the point P? **[Bank PO]**

- (a) $4\sqrt{3}$ units (b) 8 units (c) 12 units
(d) Data inadequate (e) None of these

12. The angle of elevation of the sun, when the length of the shadow of a tree is $\sqrt{3}$ times the height of the tree, is **[RRB]**

- (a) 30° (b) 45° (c) 60° (d) 90°

13. The angle of elevation of a ladder leaning against a wall is 45° , and the foot of the ladder is 4.242 m away from the wall. The length of ladder is

- (a) 8.4 m (b) 7.2 m (c) 6 m (d) 5.4 m

14. A man standing on the bank of a river observes that the angle subtended by a tree on the opposite bank is 60° . When he views the tower 36 m from the bank, he finds the angle to be 30° . Find the breadth of the river.

- (a) 12 m (b) 15 m (c) 18 m (d) 21 m

15. Two towers are situated at the opposite sides of bank of a river. One tower is 60 m high. From the top of this tower, angles of depression of top and bottom of the other tower are 30° and 60° respectively. Find the width of the river and the height of the other tower (in metres).

- (a) 20, $40\sqrt{3}$ (b) $40\sqrt{3}$, 20 (c) $20\sqrt{3}$, 40 (d) 40, $20\sqrt{3}$



1. (e) 2. (e) 3. (c) 4. (a) 5. (a) 6. (d) 7. (d) 8. (a) 9. (c) 10. (c)
11. (d) 12. (a) 13. (c) 14. (c) 15. (c)



Solutions with Necessary Explanation

1. Let the height of building AB be 'H' metres

$$\text{From } \triangle APQ, \quad \tan 60^\circ = \frac{QP}{AP} = \frac{30}{AP} = \sqrt{3}$$

$$\text{or} \quad AP = \frac{30}{\sqrt{3}} = 10\sqrt{3}$$

$$\text{From } \triangle BRQ, \quad \tan 45^\circ = \frac{QR}{BR} = \frac{QR}{AP} = 1 \quad (\because BR = AP)$$

$$\therefore QR = AP = 10\sqrt{3} \text{ m}$$

$$\therefore \text{Height of building } H = RP = QP - QR$$

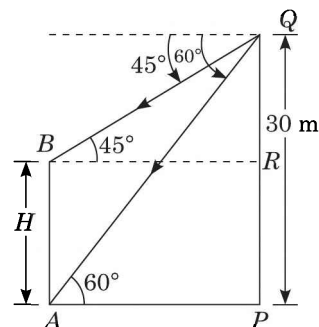
$$= 30 - 10\sqrt{3} = (10\sqrt{3} - 10)\sqrt{3} = 7.32 \times 1.732 = 12.68 \text{ m}$$

Hence answer is (e).

2. Let AB and PQ be the two towers and let its height be h metres each.

Let the point on the road be at a distance x metres from B.

$$\text{From } \triangle ABT, \quad \tan 60^\circ = \frac{AB}{BT} = \frac{h}{x}$$



or
$$x = \frac{h}{\tan 60^\circ} = \frac{h}{\sqrt{3}} \quad (1)$$

From $\triangle PQT$,
$$\tan 30^\circ = \frac{PQ}{QT} = \frac{h}{100 - x}$$

or
$$100 - x = \frac{h}{\tan 30^\circ} = \sqrt{3} h \quad (2)$$

(1) + (2) $\Rightarrow$
$$100 = \frac{h}{\sqrt{3}} + \sqrt{3} h = h \left(\sqrt{3} + \frac{1}{\sqrt{3}} \right)$$

or
$$h = \frac{100}{\left(\sqrt{3} + \frac{1}{\sqrt{3}} \right)} = \frac{100\sqrt{3}}{4} = 25\sqrt{3}$$

Hence answer is (e).

3. Let length of ladder AP be ' l ' metres

From $\triangle ABP$,
$$\cos 60^\circ = \frac{BP}{AP} = \frac{6.5}{l}$$

or
$$l = \frac{6.5}{1/2} = 13 \text{ m} \quad (\because \cos 60 = 1/2)$$

4. Let ' H ' be the elevation of aeroplane above the road in miles.

From $\triangle PQR$,
$$\tan 60^\circ = \frac{H}{QR}$$

or
$$QR = \frac{H}{\tan 60^\circ} = \frac{H}{\sqrt{3}} \quad (1)$$

From $\triangle PQS$,
$$\tan 30^\circ = \frac{H}{QS}$$

$$= \frac{H}{QR + RS} = \frac{H}{QR + 1}$$

or
$$QR + 1 = \frac{H}{\tan 30^\circ} = \sqrt{3} H \quad (2)$$

(2) - (1) $\Rightarrow$
$$1 = H \left(\sqrt{3} - \frac{1}{\sqrt{3}} \right) = \frac{2H}{\sqrt{3}}$$

or
$$H = \frac{\sqrt{3}}{2} \text{ m}$$

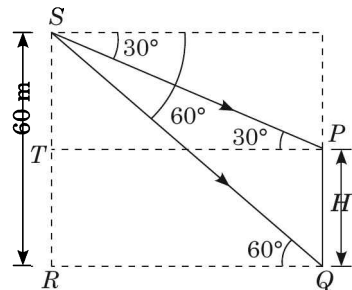
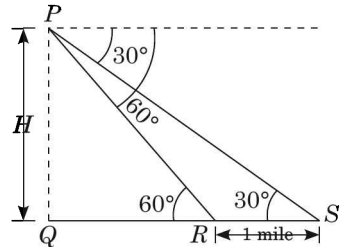
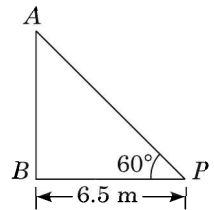
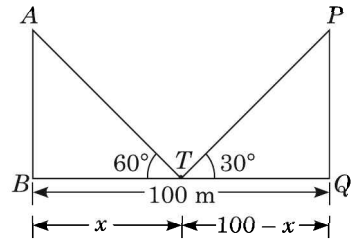
5. Let H be height of tower PQ .

From $\triangle QRS$,
$$\tan 60^\circ = \frac{60}{QR}$$

or
$$QR = \frac{60}{\tan 60} = \frac{60}{\sqrt{3}} = 20\sqrt{3} \quad (1)$$

From $\triangle PTS$,
$$\tan 30^\circ = \frac{ST}{PT} = \frac{60 - H}{QR} = \frac{60 - H}{20\sqrt{3}}$$

for
$$60 - H = 20\sqrt{3} \times \tan 30$$



$$\begin{aligned}\text{or} \quad & 60 - H = 20 \\ \text{or} \quad & H = 60 - 20 = 40 \text{ m.}\end{aligned}$$

$$6. \text{ From } \triangle PQR, \tan 45^\circ = \frac{PQ}{RQ} = 1$$

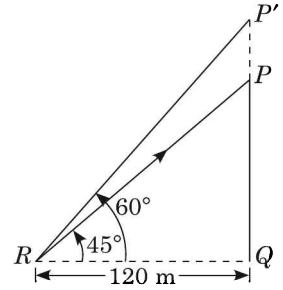
$$\text{or} \quad PQ = RQ = 120 \text{ m} \quad (1)$$

Let P' be the extended point of tower such that elevation to point P' is 60° .

$$\text{From } \triangle P'QR, \tan 60^\circ = \frac{P'Q}{RQ} = \frac{P'P + PQ}{RQ} = \sqrt{3}$$

$$\text{or} \quad P'P + PQ = RQ\sqrt{3}$$

$$P'P = RQ\sqrt{3} - PQ = 120\sqrt{3} - 120 = 87.84 \text{ m}$$



7. Let the distance between ships at QR be d metre.

$$\text{From } \triangle PQS, \tan 45^\circ = \frac{PS}{SQ} = 1$$

$$\text{or} \quad SQ = PS = 200 \text{ m} \quad (1)$$

$$\text{From } \triangle PRS, \tan 30^\circ = \frac{PS}{RS} = \frac{1}{\sqrt{3}}$$

$$\text{or} \quad \frac{PS}{SQ + QR} = \frac{1}{\sqrt{3}}$$

$$\text{or} \quad SQ + QR = PS\sqrt{3} = 200\sqrt{3}$$

$$\therefore QR = 200\sqrt{3} - 200$$

i.e. distance between two ships = $346.4 - 200 = 146.4 \text{ m}$

8. Let PQ be the tower and R and S be the two positions of the boat.

Given $RQ = 60 \text{ m}$; $\angle PSQ = 30^\circ$; $\angle PRQ = 45^\circ$

$$\text{From } \triangle PQR, \tan 45^\circ = \frac{PQ}{RQ} = 1$$

$$\text{i.e.} \quad PQ = RQ = 60 \text{ m} \quad (1)$$

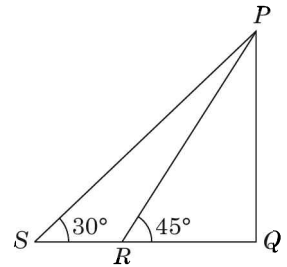
$$\text{From } \triangle PQS, \tan 30^\circ = \frac{PQ}{QS} = \frac{60}{QR + RS} = \frac{60}{60 + RS} = \frac{1}{\sqrt{3}} \quad (2)$$

$$(2) \Rightarrow 60 + RS = 60\sqrt{3}$$

$$\Rightarrow RS = 60\sqrt{3} - 60 = 60(\sqrt{3} - 1) \text{ m}$$

$$\therefore \text{ Required speed} = \frac{RS}{\text{Time}} = \frac{60(\sqrt{3} - 1)}{5} \text{ m/s} = 12 \times 0.73 \text{ m/s}$$

$$= 12 \times 0.73 \times \frac{18}{5} \text{ km/h} = 31.5 \text{ km/hr} \approx 32 \text{ km/hr}$$

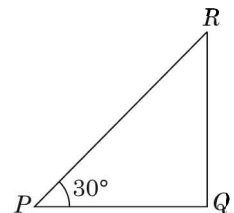


9. Given $QR = 100 \text{ m}$; Let $PQ = x$

$$\text{From } \triangle PQR, \tan 30^\circ = \frac{RQ}{PQ} = \frac{100}{x} = \frac{1}{\sqrt{3}}$$

$$\therefore x = 100\sqrt{3} \text{ m}$$

$$= 173.2 \text{ m} \approx 173 \text{ m}$$



10. Let the time taken be t seconds.

Let PQ be the tower and S and R be two positions of the car.

Now $\angle PSQ = 30^\circ$; $\angle PRQ = 45^\circ$;

Let $SR = x$ and $RQ = y$

Let height of tower $PQ = H$

$$\text{From } \triangle PQR, \quad \tan 45^\circ = \frac{PQ}{RQ} = 1$$

$$\text{i.e.} \quad PQ = RQ$$

$$\therefore \quad H = y \quad (1)$$

$$\text{From } \triangle PQS, \quad \tan 30^\circ = \frac{PQ}{SQ} = \frac{1}{\sqrt{3}}$$

$$\text{i.e.} \quad PQ\sqrt{3} = SQ$$

$$\text{i.e.} \quad H\sqrt{3} = x + y \quad (\because SQ = SR + RQ) \quad (2)$$

Solving (1) and (2)

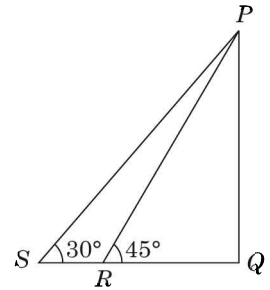
$$(2) - (1) \Rightarrow (x + y) - y = H\sqrt{3} - H$$

$$\Rightarrow x = H(\sqrt{3} - 1)$$

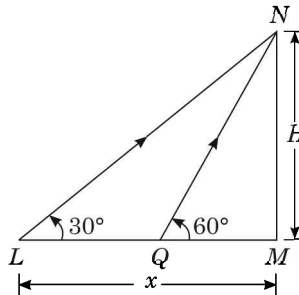
$$\therefore \text{Distance covered in 12 minutes} = H(\sqrt{3} - 1)$$

$\therefore$ Time taken to reach foot of tower after this point

$$= \frac{12}{H(\sqrt{3} - 1)} \times H = \frac{12}{\sqrt{3} - 1} \text{ min} = 16 \text{ min } 23 \text{ sec.}$$



11. Since no sides of triangle LMN or QMN are given, it is not possible to find the result.

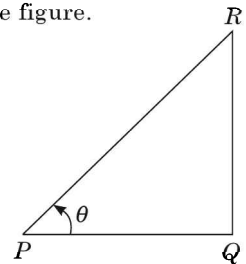


12. Let QR represents tree and PQ represents its shadow as shown in the figure.

It is given that $PQ = \sqrt{3} QR$

$$\therefore \quad \tan \theta = \frac{RQ}{PQ} = \frac{RQ}{\sqrt{3} QR} = \frac{1}{\sqrt{3}}$$

$$\therefore \quad \theta = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right) = 30^\circ$$



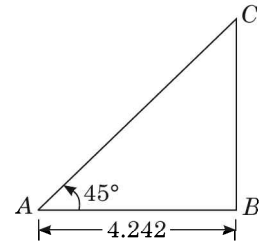
13. In figure, ladder is represented by AC .

$$\therefore \quad \cos 45^\circ = \frac{AB}{AC} = \frac{4.242}{AC}$$

or

$$AC = \frac{4.242}{\cos 45^\circ} = \frac{4.242}{1/\sqrt{2}}$$

$$= \frac{3\sqrt{2}}{1/\sqrt{2}} = 3 \times 2 = 6 \text{ m}$$



14. Let the tree be represented by BC .

AD is the distance between positions of man.

$DB = ?$

Let H be the height of tree and $DB = x$

$$\therefore \tan 30^\circ = \frac{H}{AB} = \frac{H}{36 + x}$$

or

$$H = (36 + x) \frac{1}{\sqrt{3}} \quad (1)$$

Similarly

$$\tan 60^\circ = \frac{H}{DB} = \frac{H}{x} = \sqrt{3}$$

or

$$H = x\sqrt{3} \quad (2)$$

Equating (1) and (2), we get

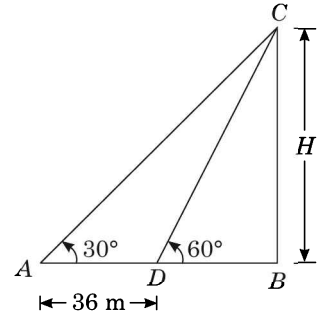
$$\frac{36 + x}{\sqrt{3}} = x\sqrt{3}$$

or

$$36 + x = 3x$$

or

$$x = \frac{36}{(3 - 1)} = 18 \text{ m}$$



15. Let AB and CD be the two towers.

$CD = 60 \text{ m}$; $\angle ECA = 30^\circ$; $\angle ECB = 60^\circ$

From $\triangle BDC$,

$$\cos 30^\circ = \frac{CD}{BC}$$

or

$$BC = \frac{CD}{\cos 30^\circ} = \frac{60}{\sqrt{3}/2}$$

$$= \frac{120}{\sqrt{3}} = 40\sqrt{3} \text{ m}$$

From $\triangle BCE$,

$$EC = BC \cos 60^\circ$$

$$= 40\sqrt{3} \times \frac{1}{2} = 20\sqrt{3} \text{ m}$$

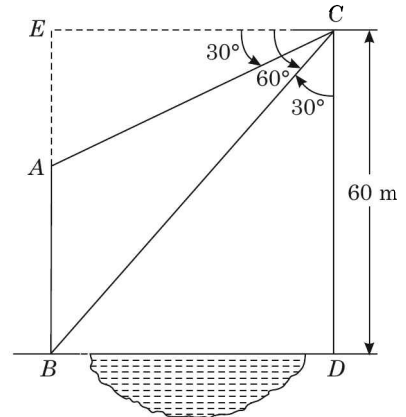
$\therefore$ Width of river = $20\sqrt{3} \text{ m}$

Height of tower = $AB = BE - AE$

$$= CD - AE$$

$$= 60 - EC \tan 30^\circ$$

$$= 60 - 20\sqrt{3} \times \frac{1}{\sqrt{3}} = 60 - 20 = 40 \text{ m}$$



Trigonometry

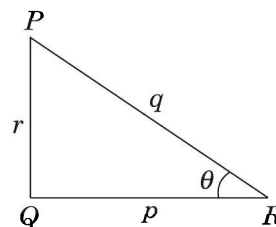
PYTHAGORUS THEOREM: In the right-angled triangle PQR ,

$$PR^2 = PQ^2 + QR^2$$

or

$$q^2 = r^2 + p^2$$

i.e. Square of hypotenuse = Sum of square of base and square of altitude.



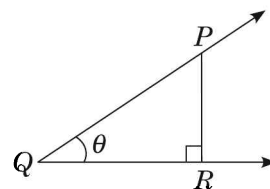
TRIGONOMETRIC RATIOS: Ratios of sides of right-angled triangles with respect to its angles are called *trigonometric ratios*.

Given $\angle PQR = \theta$

Then the ratio PR/QP is called sine of angle θ .

or
$$\sin \theta = \frac{PR}{QP}$$

i.e.
$$\sin \theta = \frac{\text{Opposite side to angle } \theta}{\text{Hypotenuse}}$$



Similarly,
$$\cos \theta = \frac{QR}{QP} = \frac{\text{Adjacent side to angle } \theta}{\text{Hypotenuse}}$$

$$\tan \theta = \frac{PR}{QR} = \frac{\text{Opposite side to angle } \theta}{\text{Adjacent side to angle } \theta}$$

Other three trigonometric ratios are cosecant, secant and cotangent of angle θ .

$$\operatorname{cosec} \theta = \frac{1}{\sin \theta}; \quad \sec \theta = \frac{1}{\cos \theta}; \quad \cot \theta = \frac{1}{\tan \theta}$$

If one of the six ratios is known, the other five ratios can be calculated.

TRIGONOMETRIC RATIO FOR $\theta = 30^\circ$

From $\triangle PSR$,

$$\sin 30^\circ = \frac{SR}{PR} = \frac{a}{2a} = \frac{1}{2}$$

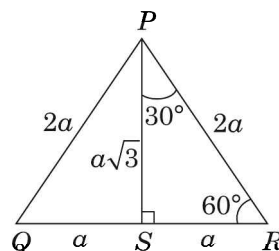
$$\cos 30^\circ = \frac{PS}{PR} = \frac{a\sqrt{3}}{2a} = \frac{\sqrt{3}}{2}$$

$$\tan 30^\circ = \frac{SR}{PS} = \frac{a}{a\sqrt{3}} = \frac{1}{\sqrt{3}}$$

$$\therefore \operatorname{cosec} 30^\circ = \frac{1}{\sin 30^\circ} = \frac{2a}{a} = 2$$

$$\sec 30^\circ = \frac{1}{\cos 30^\circ} = \frac{2a}{a\sqrt{3}} = \frac{2}{\sqrt{3}}$$

$$\cot 30^\circ = \frac{1}{\tan 30^\circ} = \frac{a\sqrt{3}}{a} = \sqrt{3}$$



Trigonometric ratio for $\theta = 60^\circ$.

From $\triangle PSR$,

$$\sin 60^\circ = \frac{PS}{PR} = \frac{a\sqrt{3}}{2a} = \frac{\sqrt{3}}{2}$$

$$\cos 60^\circ = \frac{SR}{PR} = \frac{a}{2a} = \frac{1}{2}$$

$$\tan 60^\circ = \frac{PS}{SR} = \frac{a\sqrt{3}}{a} = \sqrt{3}$$

$$\therefore \operatorname{cosec} 60^\circ = \frac{1}{\sin 60^\circ} = \frac{2a}{a\sqrt{3}} = \frac{2}{\sqrt{3}}$$

$$\sec 60^\circ = \frac{1}{\cos 60^\circ} = \frac{2a}{a} = 2$$

$$\cot 60^\circ = \frac{1}{\tan 60^\circ} = \frac{a}{a\sqrt{3}} = \frac{1}{\sqrt{3}}$$

Trigonometric ratio for $\theta = 45^\circ$.

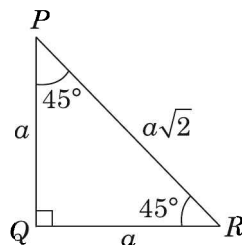
In the right-angled triangle PQR , right angled at Q and $\angle P = \angle R = 45^\circ$.

$$\therefore QP = QR$$

Let it be a units.

Then by Pythagorus theorem, $PR^2 = PQ^2 + QR^2 = a^2 + a^2 = 2a^2$

$$\therefore PR = a\sqrt{2}$$



$$\sin 45^\circ = \frac{PQ}{PR} = \frac{a}{a\sqrt{2}} = \frac{1}{\sqrt{2}}$$

$$\cos 45^\circ = \frac{QR}{PR} = \frac{a}{a\sqrt{2}} = \frac{1}{\sqrt{2}}$$

$$\tan 45^\circ = \frac{PQ}{QR} = \frac{a}{a} = 1$$

$$\therefore \operatorname{cosec} 45^\circ = \frac{1}{\sin 45^\circ} = \sqrt{2}$$

$$\sec 45^\circ = \frac{1}{\cos 45^\circ} = \sqrt{2}$$

$$\cot 45^\circ = \frac{1}{\tan 45^\circ} = 1$$

Trigonometric Ratio of Different Angles

$x =$		0/4	1/4	2/4	3/4	4/4	
θ	fn	0°	30°	45°	60°	90°	
sin		0	1/2	1/√2	√3/2	1	← √x
cos		1	√3/2	1/√2	1/2	0	← Write values of sine in reverse order
tan		0	1/√3	1	√3	∞	← sin/cos
cosec		∞	2	√2	2/√3	1	← 1/sin
sec		1	2/√3	√2	2	∞	← 1/cos or cosec written in reverse order
cot		∞	√3	1	1/√3	0	← 1/tan or tan written in reverse order

Trigonometric Ratio of Different Angles

		0	1	2	3	4
Angle	Function	0°	30°	45°	60°	90°
sin		√0/4	√1/4	√2/4	√3/4	√4/4
cos		1	√3/2	1/√2	1/2	0
tan		0	1/√3	1	√3	∞

ANGLES OF ELEVATION AND DEPRESSION

Suppose we are viewing an object. The line of sight or the line of vision is a straight line from our eye to the object we are viewing.

If the object is at a higher level from our eyes, the line of sight is above the horizontal line at the level of our eyes. Angle so subtended by the line of sight with the horizontal is called *Angle of Elevation* of the object.

If the line of sight is below the horizontal, angle so subtended by the line of sight with the horizontal is called *Angle of Depression* of the object.

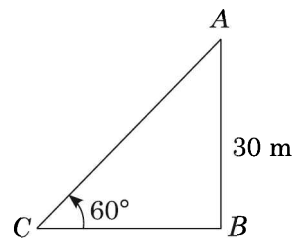
EXAMPLE 1 A person is standing at a certain distance from a tower. When the top of the tower is viewed, the angle of elevation is 60° . If the height of the tower is 30 m, what is the distance of the person from the tower?

Solution $\tan 60^\circ = \frac{AB}{BC}$

or $\frac{AB}{BC} = \sqrt{3}$

or $\frac{30}{BC} = \sqrt{3}$

or $BC = \frac{30}{\sqrt{3}} = 10\sqrt{3} \text{ m}$



EXAMPLE 2 The angles of elevation of top and bottom of a flag at a distance of 30 m are 45° and 30° respectively. What is the height of the flag AB ?

Solution $\tan 45^\circ = \frac{AC}{DC}$

or $AC = DC \tan 45^\circ$
 $= 30 \tan 45^\circ$
 $= 30 \text{ m}$

$\tan 30^\circ = \frac{BC}{DC}$

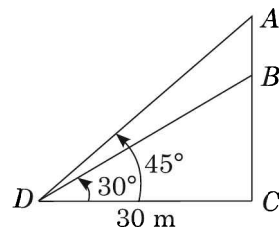
or $BC = DC \tan 30^\circ$

or $BC = 30 \tan 30^\circ$

$= 30 \times \frac{1}{\sqrt{3}} = 10\sqrt{3} \text{ m}$

$\therefore$ Height of flag $AB = 30 - 10\sqrt{3}$

$= 30 - 17.32 = 12.68 \text{ m}$



EXAMPLE 3 Two poles of equal heights stand on either side of a roadway which is 120 m wide. At a point on the roadway between the poles, the elevations of the top of the pole are 60° and 30° . Find the heights of the poles and the position of the point.

Solution Let PQ and RS be two poles and x m be the height of poles.

Let $QT = y$ m

$\therefore TS = (120 - y)$ m

In $\triangle PQT$,

$$\tan 60^\circ = \frac{PQ}{QT} = \frac{x}{y}$$

$$\Rightarrow x = y\sqrt{3} \quad (1)$$

In $\triangle RST$,

$$\tan 30^\circ = \frac{RS}{TS} = \frac{x}{120 - y}$$

$$\Rightarrow 120 - y = x\sqrt{3} \quad (2)$$

Solving (1) and (2), we get,

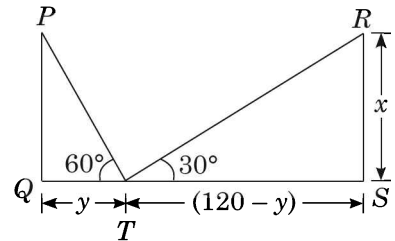
$$y\sqrt{3} \times \sqrt{3} = 120 - y$$

$$\Rightarrow 4y = 120$$

$$\text{or } y = 30 \text{ m}$$

$$\therefore (1) \Rightarrow x = y\sqrt{3}$$

$$\Rightarrow x = 30\sqrt{3} = 52 \text{ m}$$



EXAMPLE 4 The angles of depression of two ships from the top of a lighthouse are 45° and 30° . If the ships are 100 m apart, find the height of lighthouse.

Solution Let AB , the height of lighthouse be x m.

$$MN \parallel PQ$$

$$\therefore \angle MAP = \angle APB = 30^\circ.$$

$$\angle NAQ = \angle AQB = 45^\circ$$

$$\text{Let } PB = y \text{ m}$$

$$\therefore BQ = (100 - y) \text{ m}$$

In $\triangle ABP$,

$$\tan 30^\circ = \frac{AB}{BP} = \frac{1}{\sqrt{3}} = \frac{x}{y}$$

$$\text{or } y = x\sqrt{3}$$

In $\triangle ABQ$,

$$\tan 45^\circ = \frac{AB}{BQ} = 1$$

$$\text{or } x = (100 - y)$$

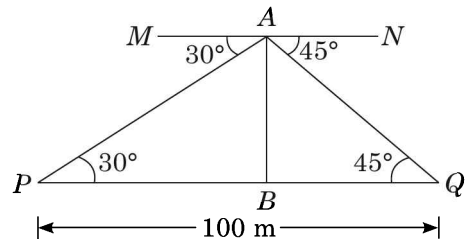
Solving (1) and (2), we get,

$$y = (100 - y)\sqrt{3} \quad \text{or} \quad y(1 + \sqrt{3}) = 100\sqrt{3}$$

$$\therefore y = \frac{100\sqrt{3}}{1 + \sqrt{3}} \text{ m}$$

$$\therefore x = \frac{y}{\sqrt{3}} = \frac{100}{1 + \sqrt{3}} \text{ m}$$

$$\text{i.e. Height of lighthouse} = \frac{100}{1 + \sqrt{3}} \text{ m.}$$

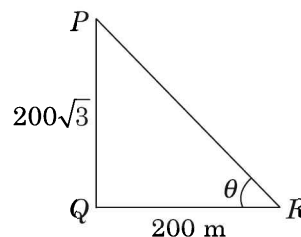


EXAMPLE 5 A tower is $200\sqrt{3}$ m high. Find the angle of elevation of its top from a point 200 m away from its roots.

Solution $\tan \theta = \frac{PQ}{QR}$

$$= \frac{200\sqrt{3}}{200} = \sqrt{3}$$

$\therefore Q = \tan^{-1}(\sqrt{3}) = 60^\circ$



Note: More applications on Trigonometry are included in the Chapter Heights and Distances.



EXERCISES

- Q is 14 km south and 16 km west of P . Find the distance of Q from P .
(a) 15.6 km (b) 18.8 km (c) 21.2 km (d) 24.4 km (e) 25.8 km
- The angle of elevation of the top of an unfinished tower at a point 120 m from its base is 45° . How much higher must the tower be raised so that its angle of elevation at the same point be 60° ?
(a) 90 m (b) 92 m (c) 97 m (d) 87.84 m (e) None of these
- The shadow of a tower standing on a level plane found to be 60 m longer when the angle of the sun is 30° than when it is 45° . Find the height of the tower when it is 45° .
(a) $60(\sqrt{3} + 1)$ (b) $30(\sqrt{3} + 1)$ (c) $\frac{60}{\sqrt{3} + 1}$ (d) $30(\sqrt{3} - 1)$ (e) None of these
- From an aeroplane vertically, over a straight horizontal road, the angles of depression of two consecutive milestones on the opposite sides of the aeroplane are observed to be 30° and 60° . Find the height in miles of the aeroplane above the road.
(a) $\sqrt{3}/2$ (b) $\sqrt{3}/4$ (c) $\sqrt{3}/8$ (d) $2\sqrt{3}/12$ (e) None of these
- An observer on the top of a cliff, 200 m above the sea level, observes the angle of depression of two ships at anchor to be 45° and 30° respectively. Find the distance between the ships, if the line joining them stretches to the base of the cliff.
(a) 40 m (b) 150 m (c) 156 m (d) 146.4 m (e) None of these
- A man observes angle of elevation of the top of a tower having a height of 100 m as 30° . After walking certain distance towards the tower, he observes the angle of elevation as 45° . What is the distance through which man walked towards the tower?
(a) 62.2 m (b) 73.2 m (c) 72.3 m (d) 82.4 m
- To an observer at the ground, angle of elevation of top of a tower is 30° . If the height of the tower is 100 m, find the distance between top of tower and point of observation (in metres)
(a) 100 (b) 150 (c) 200 (d) 250
- The angle of elevation of top of a tower is 30° . On walking 100 m towards the tower, the angle of elevation of top of a tower becomes 60° . Find the height of tower.
(a) 50 m (b) $50\sqrt{3}$ m (c) 100 m (d) $100\sqrt{3}$ m

9. A ladder having a length of 16 m is leaning against a wall at an angle of 60° with the ground. Find the distance between the wall and foot of ladder.
 (a) 12 m (b) 10 m (c) 8 m (d) 6 m
10. Two towers are situated at a certain distance apart. A person observed the angle of depression of top and bottom of second tower from the top of first tower as 30° and 60° respectively. If the height of first tower is 60 m, find the height of second tower and distance between the two towers.
 (a) $40\sqrt{3}$ m, 20 m (b) 40 m, $20\sqrt{3}$ m
 (c) 30 m, $20\sqrt{3}$ m (d) 20 m, $30\sqrt{3}$ m



1. (c) 2. (d) 3. (b) 4. (b) 5. (d) 6. (b) 7. (c) 8. (b) 9. (c) 10. (b)



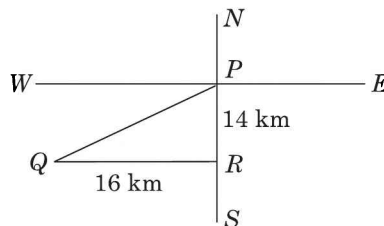
Solutions with Necessary Explanation

1. $PQ = \sqrt{PR^2 + QR^2}$

$$= \sqrt{14^2 + 16^2}$$

$$= \sqrt{196 + 256}$$

$$= \sqrt{452} = 21.2 \text{ km}$$



2. PQ is the unfinished tower.

$$QR = 120 \text{ m}$$

$\therefore$ From $\triangle PQR$,

$$\tan 45^\circ = \frac{PQ}{QR} = 1$$

$\therefore$

$$PQ = QR = 120 \text{ m}$$

From $\triangle SQR$,

$$\tan 60^\circ = \frac{SQ}{QR} = \frac{SP + PQ}{QR}$$

or

$$QR \tan 60^\circ = SP + PQ$$

$$120\sqrt{3} = SP + 120$$

$\therefore$

$$SP = 120(\sqrt{3} - 1)$$

$$= 120 \times 0.732 = 87.84 \text{ m}$$

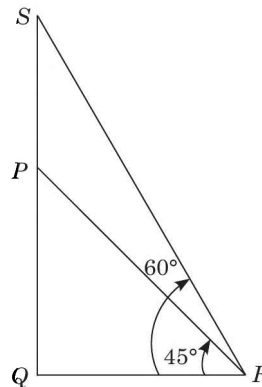
3. Height of tower = x m.

From $\triangle PQS$,

$$\tan 45^\circ = \frac{PQ}{SQ} = 1$$

$\therefore$

$$PQ = QS$$



i.e.

$$x = QS$$

From $\triangle PQR$,

$$\tan 30^\circ = \frac{PQ}{QR} = \frac{x}{QS + 60}$$

i.e.

$$\frac{1}{\sqrt{3}} = \frac{x}{x + 60}$$

Rearranging,

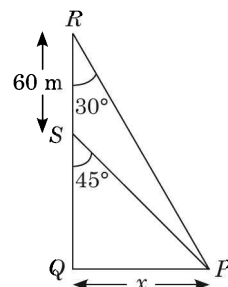
$$x\sqrt{3} = x + 60$$

or

$$x(\sqrt{3} - 1) = 60$$

 $\therefore$

$$\begin{aligned} x &= \frac{60}{(\sqrt{3} - 1)} = \frac{60}{(\sqrt{3} - 1)} \times \frac{(\sqrt{3} + 1)}{(\sqrt{3} + 1)} \\ &= \frac{60(\sqrt{3} + 1)}{(3 - 1)} = 30(\sqrt{3} + 1) \end{aligned}$$



4. Given
- $QS + SR = 1$
- mile.

(1)

$$PS = ?$$

From $\triangle PQS$,

$$\tan 30^\circ = \frac{PS}{QS}$$

or

$$QS = \frac{PS}{\tan 30^\circ} = \sqrt{3} PS$$

From $\triangle PSR$,

$$\tan 60^\circ = \frac{PS}{SR}$$

or

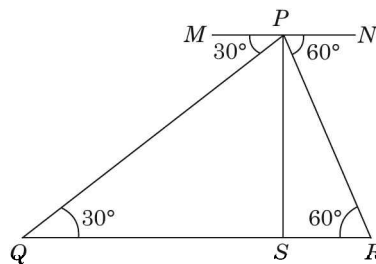
$$SR = \frac{PS}{\tan 60^\circ} = \frac{PS}{\sqrt{3}}$$

$$\therefore (1) \Rightarrow \sqrt{3} PS + \frac{PS}{\sqrt{3}} = 1$$

$$\Rightarrow (3 + 1)PS = \sqrt{3}$$

or

$$PS = \frac{\sqrt{3}}{4}$$



5. Cliff =
- $PQ = 200$
- m;
- $RS = ?$

From $\triangle PQS$,

$$\tan 45^\circ = \frac{PQ}{QS} = 1$$

 $\therefore$

$$QS = 200 \text{ m}$$

From $\triangle PQR$,

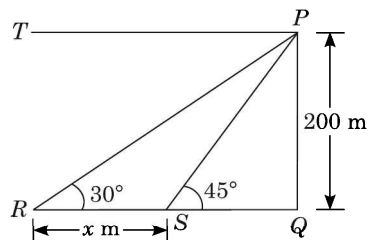
$$\tan 30^\circ = \frac{PQ}{QR} = \frac{1}{\sqrt{3}}$$

 $\therefore$

$$QR = PQ \sqrt{3} = 200\sqrt{3}$$

 $\therefore$

$$RS = QR - QS = 200\sqrt{3} - 200 = 146.4 \text{ m}$$



6. Let the distance through which man walked be x metres.

From $\triangle DBC$,

$$\tan 45^\circ = \frac{100}{DB} = 1$$

or

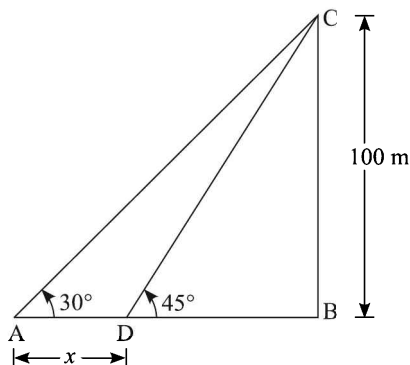
$$DB = 100 \text{ m}$$

From $\triangle ABC$,

$$\begin{aligned} \tan 30^\circ &= \frac{100}{AB} = \frac{100}{x + DB} \\ &= \frac{100}{x + 100} = \frac{1}{\sqrt{3}} \end{aligned}$$

$$\text{or} \quad \frac{x + 100}{100} = \sqrt{3}$$

$$\begin{aligned} \text{or} \quad x &= 100\sqrt{3} - 100 \\ &= 173.2 - 100 \\ &= 73.2 \text{ m} \end{aligned}$$

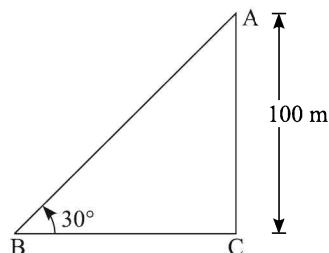


7. From the figure,

$$\sin 30^\circ = \frac{AC}{AB} = \frac{100}{AB}$$

$$\therefore AB = \frac{100}{\sin 30^\circ} = 200 \text{ m}$$

$\therefore$ Distance between top of tower and point of observation = $AB = 200 \text{ m}$.



8. Let H be the height of tower

From $\triangle ADB$,

$$\tan 60^\circ = \frac{AB}{DB} = \frac{H}{DB}$$

or

$$H = DB \tan 60^\circ$$

$$H = DB\sqrt{3}$$

or

$$DB = \frac{H}{\sqrt{3}} \quad (1)$$

From $\triangle ABC$,

$$\tan 30^\circ = \frac{AB}{BC} = \frac{H}{CD + DB} = \frac{H}{100 + DB}$$

or

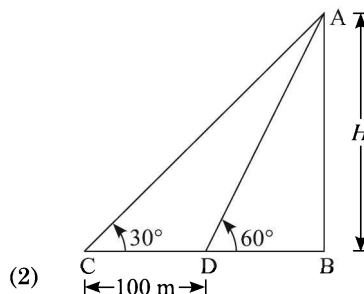
$$\frac{H}{(100 + DB)} = \frac{1}{\sqrt{3}}$$

or

$$H = \frac{100 + DB}{\sqrt{3}}$$

or

$$DB = H\sqrt{3} - 100$$



Equating (1) and (2), we get

$$\frac{H}{\sqrt{3}} = H\sqrt{3} - 100$$

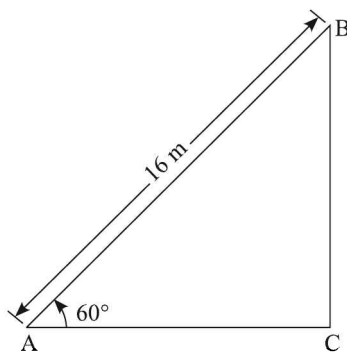
or $H\sqrt{3} - \frac{H}{\sqrt{3}} = 100$

or $\frac{2H}{\sqrt{3}} = 100$

or $H = \frac{100 \times \sqrt{3}}{2} = 50\sqrt{3} \text{ m}$

9. Distance between foot of ladder and wall is given by

$$AC = AB \cos 60^\circ = 16 \cos 60^\circ = 16 \frac{1}{2} = 8 \text{ m}$$



10. Let AB and CD be the two towers. Height of first tower = AB = 60 m.

Let H metres be the height of second tower and d metres be the distance between the towers AB and CD.

From $\triangle ABD$,

$$\tan 60^\circ = \frac{60}{d} = \sqrt{3}$$

or $d = \frac{60}{\sqrt{3}} = 20\sqrt{3} \text{ m}$

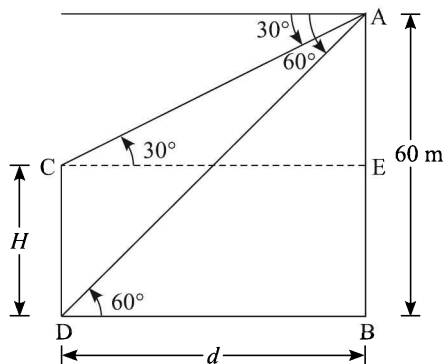
From $\triangle ACE$,

$$\begin{aligned} \tan 30^\circ &= \frac{AE}{CE} = \frac{AB - H}{d} \\ &= \frac{60 - H}{20\sqrt{3}} \end{aligned}$$

i.e. $\frac{1}{\sqrt{3}} = \frac{60 - H}{20\sqrt{3}}$

or $60 - H = \frac{20\sqrt{3}}{\sqrt{3}} = 20$

$\therefore H = 60 - 20 = 40 \text{ m}$



Odd Man Out and Series

A series is a sequence of numbers obtained by a pre-defined rule applied to the previous number.

In the given series one of the terms will not fit in the series, which is termed here *Odd Man Out*

In other words, 'Odd Man Out' can be defined as a number which does not belong to the set of the given numbers in which all others are alike in a particular manner.

EXAMPLE 1 Find the odd one out from the given series.

4, 27, 36, 81, 64

Solution All numbers except 27 are perfect squares.

EXAMPLE 2 Find the odd one out from the given series.

3, 7, 9, 11, 13, 17

Solution All numbers except 9 are prime numbers.

SERIES: A set of numbers or sequence of numbers formed by applying a particular pre-defined rule to a number in succession is termed series. For example,

(i) 2, 4, 6, 8, 10, ...,

i.e. the numbers are got added by 2 in each step.

(ii) 2, 6, 18, 54, ...,

i.e. the numbers in the sequence are obtained by multiplying each term with 3 in succession, i.e. $2 \times 3 = 6$, $6 \times 3 = 18$, $18 \times 3 = 54$.

Directions: In each of the following question one series is given. You have to fill in the space marked with a character choosing the correct alternative, in which it follows the same pattern as the first one.

EXAMPLE 3

4	14	36	114	460	
2	<i>a</i>	<i>b</i>	<i>c</i>	<i>d</i>	<i>e</i>

Find the value of *e*.

Solution The first series is $\times 1 + 10, \times 2 + 8, \times 3 + 6, \times 4 + 4 \dots$

$$\begin{aligned}\therefore a &= 2 \times 1 + 10 = 12 \\ b &= 12 \times 2 + 8 = 32 \\ c &= 32 \times 3 + 6 = 102 \\ d &= 102 \times 4 + 4 = 412 \\ \therefore e &= 412 \times 5 + 2 = \mathbf{2062}\end{aligned}$$

EXAMPLE 4 108 52 24 10 3
 64 *a* *b* *c* *d* *e*

What is the value of *c*?

Solution $a = (64 - 4) \div 2 = 30$
 $b = (30 - 4) \div 2 = 13$
 $c = (13 - 4) \div 2 = \mathbf{4.5}$

EXAMPLE 5 1296 864 576 384 256
 1080 *a* *b* *c* *d* *e*

What should replace *c*?

Solution First series is $\div 3 \times 2$
 $\therefore a = 1080 \div 3 \times 2 = 720$
 $b = 720 \div 3 \times 2 = 480$
 $c = 480 \div 3 \times 2 = \mathbf{320}$



EXERCISES

Directions (1–25): In each of the following questions, a number sequence is given in which except one of the terms in the sequence are alike. You have to find out the number which is odd in the sequence.

1. 4, 16, 25, 36, 64, 144
(a) 16 (b) 25 (c) 36 (d) 64
2. 8, 27, 64, 125, 216
(a) 27 (b) 64 (c) 125 (d) 216
3. 17, 35, 43, 53, 62, 80
(a) 35 (b) 43 (c) 53 (d) 62
4. 24, 18, 222, 82, 421
(a) 18 (b) 222 (c) 82 (d) 421
5. 6, 15, 21, 26, 33, 39
(a) 15 (b) 21 (c) 26 (d) 33
6. 14, 49, 63, 72, 77, 91
(a) 49 (b) 63 (c) 72 (d) 77
7. 8, 27, 125, 212, 343
(a) 8 (b) 27 (c) 125 (d) 212
8. 4, 9, 25, 49, 64, 121
(a) 9 (b) 25 (c) 49 (d) 64

9. 10, 25, 56, 70, 85, 95, 125
(a) 10 (b) 56 (c) 85 (d) 125
10. 2, 6, 12, 72, 824
(a) 6 (b) 12 (c) 72 (d) 824
11. 2, 3, 5, 8, 12, 17, 25, 30
(a) 8 (b) 12 (c) 17 (d) 25
12. 1, 5, 14, 30, 50, 91
(a) 14 (b) 30 (c) 50 (d) 91
13. 3, 5, 7, 15, 17, 23
(a) 7 (b) 15 (c) 17 (d) 23
14. 12, 16, 18, 22, 25, 28
(a) 18 (b) 22 (c) 25 (d) 28
15. 36, 64, 81, 125, 169
(a) 64 (b) 81 (c) 125 (d) 169
16. 4, 9, 16, 25, 32, 49
(a) 16 (b) 25 (c) 32 (d) 49
17. 263, 284, 393, 481, 482
(a) 284 (b) 393 (c) 481 (d) 482
18. 7, 12, 19, 26, 39, 52
(a) 19 (b) 26 (c) 39 (d) 52
19. 124, 133, 142, 152, 160
(a) 133 (b) 142 (c) 152 (d) 160
20. 12, 20, 30, 42, 54, 72
(a) 30 (b) 42 (c) 54 (d) 72
21. 69, 68, 65, 60, 54, 44
(a) 65 (b) 60 (c) 54 (d) 44
22. 27, 125, 343, 729, 1331
(a) 1331 (b) 343 (c) 729 (d) None of these
23. 216, 214, 211, 206, 200, 188
(a) 211 (b) 206 (c) 200 (d) 188
24. 2, 5, 11, 23, 45, 95
(a) 11 (b) 23 (c) 45 (d) 95
25. 12, 14, 31, 33, 69, 72, 145
(a) 33 (b) 69 (c) 72 (d) 145

Directions (26–50): In each of the following questions a number series is given. Just below the series, a number is given followed by (a), (b), (c), (d), and (e). You have to complete the series starting with the given number following the sequence for the given series. Then answer the questions given below it.

26. 18 22 38 74
121 (a) (b) (c) (d) (e)

Which of the following numbers will come in place of (c)?

[Bank PO]

- (a) 141 (b) 125 (c) 341 (d) 177 (e) 241

27. 4 7 24 93
2 (a) (b) (c) (d) (e)
Which of the following numbers will come in place of (d)?
(a) 12 (b) 230 (c) 3 (d) 51 (e) 1205
28. 4 2 2 3
12 (a) (b) (c) (d) (e)
Which of the following numbers will come in place of (e)?
(a) 45 (b) 6 (c) 9 (d) 18 (e) None of these
29. 264 136 72 40
488 (a) (b) (c) (d) (e)
Which of the following numbers will come in place of (a)?
(a) 128 (b) 248 (c) 38 (d) 23 (e) 68
30. 2 17 121 729
5 (a) (b) (c) (d) (e)
Which of the following numbers will come in place of (b)?
(a) 289 (b) 41 (c) 17393 (d) 1448 (e) 5796
31. 11 15 38 126
7 (a) (b) (c) (d) (e)
Which of the following will come in place of (c)? **[Bank PO]**
(a) 102 (b) 30 (c) 2140 (d) 80 (e) 424
32. 2 3 8 27
5 (a) (b) (c) (d) (e)
Which of the following will come in place of (e)?
(a) 184 (b) 6 (c) 925 (d) 45 (e) 14
33. 2 3 9 40.5
4 (a) (b) (c) (d) (e)
Which of the following will come in place of (b)?
(a) 486 (b) 81 (c) 3645 (d) 18 (e) 6
34. 12 28 64 140
37 (a) (b) (c) (d) (e)
Which of the following will come in place of (e)?
(a) 1412 (b) 164 (c) 696 (d) 78 (e) 340
35. 5 12 60 340
7 (a) (b) (c) (d) (e)
Which of the following will come in place of (d)?
(a) 172 (b) 5044 (c) 1012 (d) 20164 (e) 28
36. 1 9 65 393
2 (a) (b) (c) (d) (e)
Which of the following numbers would come in the place of (c)? **[BOB PO]**
(a) 490 (b) 853 (c) 731 (d) 729 (e) None of these
37. 8 8 12 24
36 (a) (b) (c) (d) (e)
Which of the following numbers would come in the place of (e)?
(a) 810 (b) 36 (c) 54 (d) 108 (e) None of these

38. 424 208 100 46
888 (a) (b) (c) (d) (e)
What number would come in the place of (b)?
(a) 20 (b) 440 (c) 216 (d) 56 (e) None of these
39. 4 5 9.75 23.5
7 (a) (b) (c) (d) (e)
What number would come in the place of (d)?
(a) 32.5 (b) 271.5 (c) 8 (d) 14.25 (e) None of these
40. 5 294 69 238
13 (a) (b) (c) (d) (e)
Which of the following numbers would come in the place of (e)?
(a) 246 (b) 206 (c) 125 (d) 302 (e) None of these
41. 15 16 25 50
189 (a) (b) (c) (d) (e)
Which of the following members will come in place of (e)? **[Bank PO]**
(a) 354 (b) 273 (c) 394 (d) 426 (e) None of these
42. 6 3.5 4.5 8.25
40 (a) (b) (c) (d) (e)
Which of the following numbers will come in place of (c)?
(a) 20.5 (b) 21.5 (c) 33.75 (d) 69.5 (e) None of these
43. 9 10 22 69
5 (a) (b) (c) (d) (e)
Which of the following numbers will come in place of (b)?
(a) 15 (b) 28 (c) 14 (d) 45 (e) None of these
44. 2 10 27 60
5 (a) (b) (c) (d) (e)
Which of the following numbers will come in place of (b)?
(a) 39 (b) 13 (c) 34 (d) 38 (e) None of these
45. 5 149 49 113
146 (a) (b) (c) (d) (e)
Which of the following numbers will come in place of (d)?
(a) 290 (b) 234 (c) 254 (d) 218 (e) None of these
46. 6 3.0 4.5 2.25
40 (a) (b) (c) (d) (e)
Which of the following numbers will come in place of (c)?
(a) 20.5 (b) 21.5 (c) 33.75 (d) 69.5 (e) 15
47. 5 9 26 90
13 (a) (b) (c) (d) (e)
Which of the following numbers will come in place of (e)?
(a) 2880 (b) 2292 (c) 1716 (d) 3432 (e) None of these
48. 4 9 25 103
3 (a) (b) (c) (d) (e)
Which of the following numbers will come in place of (c)?
(a) 391 (b) 81 (c) 91 (d) 79 (e) None of these

49. 6 10 32 126

2 (a) (b) (c) (d) (e)

Which of the following numbers will come in place of (a)?

(a) 4 (b) 6 (c) 2 (d) 3 (e) None of these

50. 1260 628 312 154

788 (a) (b) (c) (d) (e)

Which of the following numbers will come in place of (d)?

(a) 194 (b) 45.5 (c) 48 (d) 72.5 (e) None of these



1. (b)	2. (b)	3. (b)	4. (c)	5. (c)	6. (c)	7. (d)	8. (d)	9. (b)	10. (d)
11. (d)	12. (c)	13. (b)	14. (c)	15. (c)	16. (c)	17. (c)	18. (b)	19. (c)	20. (c)
21. (c)	22. (d)	23. (c)	24. (c)	25. (c)	26. (d)	27. (b)	28. (a)	29. (b)	30. (a)
31. (a)	32. (c)	33. (d)	34. (a)	35. (b)	36. (d)	37. (a)	38. (c)	39. (e)	40. (b)
41. (a)	42. (c)	43. (c)	44. (a)	45. (d)	46. (e)	47. (e)	48. (d)	49. (c)	50. (b)



Solutions with Necessary Explanation

- All numbers except 25 are perfect squares of even numbers.
- All numbers in the sequence are perfect cubes except 64 which is both perfect square and cube.
- All number except 43 are having digit-sum equal to 8.
i.e. $3 + 5 = 5 + 3 = 6 + 2 = 8$.
- All numbers except 82 are having product of digit equal to 8,
i.e. $2 \times 4 = 1 \times 8 = 2 \times 2 \times 2 = 4 \times 2 \times 1 = 8$.
- All numbers except 26 are multiplies of three.
- All numbers except 72 are multiplies of 7.
- Except 212, all numbers are cubes of prime numbers.
- Except 64, all numbers are squares of prime numbers.
- Except 56, all number are multiples of 5.
 $\therefore$ 56 is wrong.
- $2 \times 6 = 12$; $6 \times 12 = 72$; $12 \times 72 = 864$
 $\therefore$ 824 is wrong.
- The series is +1, +2, +3, +5, +6, +7.
 $\therefore 17 + 6 = 23 \neq 25$
 $\therefore$ 25 is wrong
- $+2^2, +3^2, +4^2, +5^2, +6^2$ is the series.
 $\therefore 30 + 5^2 = 55 \neq 50$
 $\therefore$ 50 is wrong
- Except 15 all number are prime.
- Except 25, all number are even.

15. The sequence is $6^2, 8^2, 9^2, 11^2, 13^2$.
All numbers except 125 are perfect squares.
 $\therefore$ 125 is wrong
16. Except 32, all numbers in the sequence are perfect squares.
 $\therefore$ 32 is wrong
17. Second digit = First digit $\times$ Third digit
Except 481, all other terms in the sequence follow the above condition.
 $\therefore$ 481 is wrong
18. The sequence is +5, +7, +9, +11, +13.
 $\therefore 19 + 9 = 28 \neq 26$
 $\therefore$ 26 is wrong
19. The sequence is an A.P. with common difference 9.
 $\therefore 142 + 9 = 151 \neq 152$
 $\therefore$ 152 is wrong
20. The sequence is +8, +10, +12, +14, +16.
 $\therefore 12 + 8 = 20; 20 + 10 = 30; 30 + 12 = 42;$
 $42 + 14 = 56 \neq 54; 56 + 16 = 72$
 $\therefore$ 54 is wrong
21. The sequence is -1, -3, -5, -7, -9.
 $\therefore 69 - 1 = 68; 68 - 3 = 65; 65 - 5 = 60; 60 - 7 = 53 \neq 54;$
 $53 - 9 = 44$
 $\therefore$ 54 is wrong
22. All terms in the given sequence are perfect cubes.
 $\therefore$ None of these numbers are wrong.
23. Subtract consecutive prime numbers successively.
i.e. -2, -3, -5, -7, -11
 $\therefore 216 - 2 = 214; 214 - 3 = 211; 211 - 5 = 206; 206 - 7 = 199 \neq 200;$
 $199 - 11 = 188$
 $\therefore$ 200 is wrong.
24. The sequence is $\times 2 + 1, \times 2 + 1, \dots$
 $\therefore 2 \times 2 + 1 = 5; 5 \times 2 + 1 = 11; 11 \times 2 + 1 = 23; 23 \times 2 + 1 = 47 \neq 45;$
 $47 \times 2 + 1 = 95$
 $\therefore$ 45 is wrong
25. The sequence is $\times 1 + 2, \times 2 + 3$, alternatively.
 $\therefore 12 \times 1 + 2 = 14; 14 \times 2 + 3 = 31;$
 $31 \times 1 + 2 = 33; 33 \times 2 + 3 = 69;$
 $69 \times 1 + 2 = 71 \neq 72; 71 \times 2 + 3 = 145$
 $\therefore$ 72 is wrong
26. The series is $+ 2^2, + 4^2, + 6^2, \dots$
 $\therefore$ required number $= 121 + 2^2 + 4^2 + 6^2 = 177$
27. The series is $\times 2 - 1, \times 3 + 3, \times 4 - 3, \times 5 + 5, \dots$
 $\therefore$ required number $= 230$
28. The series is $\times 0.5, \times 1, \times 1.5, \times 2 \dots$
 $\therefore$ required number $= 12 \times 0.5 \times 1 \times 1.5 \times 2.0 \times 2.5 = 45$

29. The series is $\div 2 + 4, \div 2 + 4 \dots$
 $\therefore$ required number $= 488 \div 2 + 4 = 244 + 4 = \mathbf{248}$
30. The series is $\times 8 + 1, \times 7 + 2, \times 6 + 3 \dots$
 $\therefore$ required number $= (5 \times 8 + 1) \times 7 + 2 = \mathbf{289}$
31. The series is $\times 1 + 4, \times 2 + 8, \times 3 + 12, \dots$
 $\therefore$ required number $= [(7 \times 1 + 4) \times 2 + 8] \times 3 + 12 = \mathbf{102}$
32. The series is $\times 1 + 1, \times 2 + 2, \times 3 + 3, \dots$
 $\therefore$ required number $= (((5 \times 1 + 1) \times 2 + 2) \times 3 + 3) \times 4 + 4) \times 5 + 5 = \mathbf{925}$
33. The series is $\times 1.5, \times 3, \times 4.5, \dots$
 $\therefore$ required number $= 4 \times 1.5 \times 3 = \mathbf{18}$
34. The series is $\times 2 + 4, \times 2 + 8, \times 2 + 12, \dots$
 $\therefore$ required number $= (((37 \times 2 + 4) \times 2 + 8) \times 2 + 12) \times 2 + 16) \times 2 + 20 = \mathbf{1412}$
35. The series is $\times 8 - 28, \times 7 - 24, \times 6 - 20, \dots$
 $\therefore$ required number $= (((7 \times 8 - 28) \times 7 - 24) \times 6 - 20) \times 5 - 16 = \mathbf{5044}$
36. The series is $\times 8 + 1, \times 7 + 2, \times 6 + 3, \dots$
 $\therefore$ required number $= ((2 \times 8 + 1) \times 7 + 2) \times 6 + 3 = \mathbf{729}$
37. The series is $\times 1, \times 1.5, \times 2, \dots$
 $\therefore$ required number $= 36 \times 1 \times 1.5 \times 2 \times 2.5 \times 3 = \mathbf{810}$
38. The series is $\div 2 - 4$.
 $\therefore$ required number $= (888 \div 2 - 4) \div 2 - 4 = \mathbf{216}$
39. The series is $\times 1 + 1, \times 1.5 + 2.25, \times 2 + 4, \times 2.5 + 6.25, \times 3 + 9, \dots$
 $\therefore$ required number $= (((7 \times 1 + 1) \times 1.5 + 2.25) \times 2 + 4) \times 2.5 + 6.25 = \mathbf{87.5}$
40. The series is $+(17)^2, -(15)^2, +(13)^2, -(11)^2, \dots$
 $\therefore$ required number $= 13 + 17^2 - 15^2 + 13^2 - 11^2 + 9^2 = \mathbf{206}$
41. The series is $+1^2, +3^2, +5^2, +7^2, \dots$
 $\therefore$ required number $= 189 + 1^2 + 3^2 + 5^2 + 7^2 + 9^2 = \mathbf{354}$
42. The series is $\times 0.5 + 0.5, \times 1 + 1, \times 1.5 + 1.5, \times 2 + 2, \dots$
 $\therefore$ required number $= ((40 \times 0.5 + 0.5) \times 1 + 1) \times 1.5 + 1.5 = \mathbf{33.75}$
43. The series is $\times 1 + 1, \times 2 + 2, \times 3 + 3, \dots$
 $\therefore$ required number $= (5 \times 1 + 1) \times 2 + 2 = \mathbf{14}$
44. The series is $\times 2 + 6, \times 2 + 7, \times 2 + 6, \dots$
 $\therefore$ required number $= (5 \times 2 + 6) \times 2 + 7 = \mathbf{39}$
45. The series is $+(12)^2, -(10)^2, +(8)^2, -(6)^2, \dots$
 $\therefore$ required number $= 146 + 12^2 - 10^2 + 8^2 - 6^2 = \mathbf{218}$
46. The series is $\div 2, \times 1.5, \dots$
 $\therefore$ required number $= ((40 \div 2) \times 1.5) \div 2 = \mathbf{15}$
47. Ans. 2860. The series is $\times 1 + 4, \times 2 + 8, \times 3 + 12, \dots$
 $\therefore$ required number $= (((13 \times 1 + 4) \times 2 + 8) \times 3 + 12) \times 4 + 16) \times 5 + 20 = \mathbf{2860}$
48. The series is $\times 2 + 1, \times 3 - 2, \times 4 + 3, \times 5 - 4, \dots$
 $\therefore$ required number $= ((3 \times 2 + 1) \times 3 - 2) \times 4 + 3 = \mathbf{79}$
49. The series is $\times 2 - 2, \times 3 + 2, \times 4 - 2, \times 6 + 2, \dots$
 $\therefore$ required number $= 2 \times 2 - 2 = \mathbf{2}$
50. The series is $\div 2 - 2$ in each step.
 $\therefore$ required number $= (((788 \div 2 - 2) \div 2 - 2) \div 2 - 2) \div 2 - 2 = \mathbf{45.5}$

Data Sufficiency

In this type of questions, you are required to judge whether the data given is sufficient to answer the question or not. Usually a question will be followed by two or three statements. These may contain data or some other related information by which the question can possibly be solved.

ANALYSIS: Questions in this chapter will cover topics such as percentage, time and work, time and distance, etc. We have already learnt how to solve this type of questions. These questions are asked in a different pattern.

Steps to Solve these Questions: A question of this pattern should be solved by following a systematic approach as follows.

Two statement data sufficiency

Directions: This type of questions consists of a question followed by two statements labelled (A) and (B). You have to decide if these statements are sufficient to answer the question. Give the answer as

- (a) If statement (A) alone is sufficient to answer the question but statement (B) alone is not sufficient to answer the question.
- (b) If statement (B) alone is sufficient to answer the question but statement (A) alone is not sufficient to answer the question.
- (c) If statement (A) and statement (B) taken together are sufficient to answer the question.
- (d) If statement (A) alone is sufficient and statement (B) alone is also sufficient to answer the question.
- (e) If statement (A) and statement (B) even taken together are not sufficient to answer the question.



Note: If a question involves two unknown, then two distinct equations are required for it. If this condition is fulfilled then (c) is the answer, otherwise (e) is the answer.

EXAMPLE 1 What is the volume of a rectangular box R?

- (A) The total surface area of R is 12 m^2
- (B) The height of R is 50 cm

Solution Volume of rectangular box = Length $\times$ Breadth $\times$ Height

The two pieces of information are not sufficient to answer the question.

$\therefore$ The answer is (e).

EXAMPLE 2 What is the value of $x + y$?

- (A) $x - 2y = 5$
- (B) $x^2 - 25 = 4xy - 4y^2$.

Solution This question involves two unknowns, but the two equations are not distinct. The second equation can be rearranged to $x^2 - 4xy + 4y^2 = 25$ which gets reduced to $(x - 2y)^2 = 5^2$ or $x - 2y = 5$, which is same as (A).

$\therefore$ The answer is (e).

EXAMPLE 3 What is the value of $(x - y)$?

- (A) $x - y = y - x$
- (B) $x - y = x^2 - y^2$

Solution The expression (A) involves two unknowns, but the first equation is sufficient.

i.e.

$$(x - y) = (y - x) = -(x - y)$$

$\Rightarrow$

$$2(x - y) = 0 \text{ or } (x - y) = 0$$

The second expression or expression (B) implies

$$(x - y) = (x^2 - y^2)$$

or

$$(x - y) = (x + y)(x - y)$$

$\Rightarrow$

$$(x - y)(x + y - 1) = 0$$

$\Rightarrow$

$$(x - y) = 0 \text{ or } (x + y - 1) = 0$$

As we don't know the accurate value of x and y , this expression can't be used or expression (B) is not sufficient.

Hence (a) is the answer.

Questions Based on Inequalities

EXAMPLE 4 If a and b are integers, i.e. $(a + b)$ an odd number?

- (A) $8 < a < 11$
- (B) $7 < b < 10$

Solution Statement (A) implies a may be 9 or 10. Statement (B) implies b may be 8 or 9. Hence $a + b$ may be either odd or even. Thus even with the help of both statements we can't answer the question. Hence the answer is (e).

EXAMPLE 5 Are two triangles congruent?

- (A) They are both equilateral triangles.
- (B) They both have equal bases and equal heights.

Solution Equilateral triangles with same bases are congruent by the side-side-side postulate. Hence the answer is (c).

EXAMPLE 6 What are the dimensions of a rectangle?

(A) The perimeter of the rectangle is 14.

(B) The diagonal of the rectangle is 5.

Solution Statement (A) implies $2(l + b) = 14$ or $l + b = 7$.

Statement (B) implies $l^2 + b^2 = 5^2$

Solving above two equations, we get x and y .

Hence answer is (c).

EXAMPLE 7 How long will it take two pipes P and Q to empty or fill a tank that is $\frac{3}{4}$ full?

(A) Pipe P can fill the tank in 12 minutes.

(B) Pipe Q can empty it in 8 minutes.

Solution The information contained in (A) and (B) together is needed to find the answer.

Hence the answer is (c).

EXAMPLE 8 Is x^2 an integer?

(A) x is a negative whole number.

(B) $4x^2$ is an integer.

Solution Statement (A) gives an affirmative answer.

Statement (B) may give an integer or a fraction. Hence (a) is the answer.

EXAMPLE 9 What percentage marks did a boy get in a test of 4 subjects?

(A) He got 90 in English and 84 in Maths.

(B) He got 75 in Hindi and 76 in Sanskrit.

Solution From statements (A) and (B), we get total marks of the student. But nothing is mentioned about the total marks for all the four individual subjects. So we can't answer the question. Hence the answer is (e).

EXAMPLE 10 How many letters can be typed by two typists in one day?

(A) A working day consists of 6 hours.

(B) Four typists can type 600 letters in 3 days.

Solution Only statement (B) is sufficient to answer the question. The length of working day is not relevant in this solution. Hence the answer is (b).



EXERCISES

Q. (1 – 10) Each of the questions below consists of a question followed by two statements numbered I and II. You have to decide whether the data provided in the statements are sufficient to answer the question. Read the statements and give answer.

- (a) If the data in statement I alone is sufficient to answer the question, while the data in statement II alone is not sufficient to answer the question.
- (b) If the data in statement II alone is sufficient to answer the questions, while the data in statement I alone is not sufficient to answer the question.
- (c) If the data either in statement I alone or in statement II alone is sufficient to answer the question.

- (d) If the data even in both statements I and II together are not sufficient to answer the question.
- (e) If the data in both statements I and II together are necessary to answer the question.
- The average of 8 results is 20. Find the last result, i.e. 8th result.
 - The average of the first two is $15\frac{1}{2}$ and that of the next three is $21\frac{1}{3}$.
 - The sixth result is less than the seventh by 4.
 - A tank has two taps. Tap A drains out at the rate of 5 litres per minute and tap B at the rate of 6 litres per minute. If the tank is full, in how many hours will it be emptied if both the taps are opened simultaneously?
 - The diameters of tap A and B are 2 cm and 2.5 cm respectively.
 - Capacity of the tank is given.
 - If x and y are positive integers, what is the unitary digit of the number x^y ?
 - x is divisible by 2, 3 and 8.
 - x^{2n} ends in 6 but x^{2n+1} ends in 4.
 - Suresh was 29 years old when his son Anil was born. What is the present age of Anil?
 - The sum of their ages now is 73 years.
 - The difference of their ages now is known.
 - A piece of cloth was washed. After washing, it was found to have lost 20% of its length and 10% of its breadth. Find the original length of the cloth.
 - The absolute decrease in its area is 1400 cm^2 .
 - The perimeter of the original cloth was 300 cm.
 - The simple interest of a certain sum for 10 years is ₹ 900. Find the sum.
 - The sum gets trebled in 5 years.
 - The sum at simple interest amounts to ₹ 405 in 2 years and ₹ 675 in 5 years.
 - Is a two-digit positive integer k divisible by 18?
 - When k is divided by 3, the remainder is 2.
 - When k is multiplied by 3, we get the result in three digits.
 - In a bag, there are coins of 50 paise, 25 paise and 20 paise. Find the number of 25 paise coins.
 - There are 8 rupees in the form of 20 paise coins.
 - There are ₹ 73 in the bag in all.
 - A goat is placed for grassing inside a triangular field and is tethered to one of the vertices of the field by a rope. What would be the area over which the goat can graze inside the field?
 - The rope is 7 metres long.
 - The field is equilateral triangle of side 10 metres.
 - Three men A, B and C go walking round a circle 1 km in circumference at different rates of speed. If they all start together and walk in the same direction, when will they first be together again?
 - Walking speeds of A, B and C are 80, 125 and 120 metres per minute respectively.
 - A, B and C complete the circle in $12\frac{1}{2}$ minutes, 8 minutes and $8\frac{1}{3}$ minutes respectively.
 - What is the two-digit number? [SBI PO]
 - The difference between the two digits is 9.
 - The sum of the digits is equal to the difference between the two digits.

12. What is the difference between the digits of a two-digit number? [Bank PO]
I. The sum of the digits of that number is 8.
II. One-fifth of that number is 15 less than half of 44.
13. What is the two-digit number whose first digit is a and the second digit is b ? The number is greater than 9. [MAT]
I. The number is a multiple of 51.
II. The sum of the digits a and b is 6.
14. Divya is twice as old as Shruti. What is the difference in their ages? [Bank PO]
I. Five years hence, the ratio of their ages would be 9:5.
II. Ten years back, the ratio of their ages was 3:1.
15. What is Reena's present age? [Bank PO]
I. Reena's present age is five times her son's present age.
II. Reena's age two years hence will be three times her daughter's age at that time.
16. Average age of employees working in a department is 30 years. In the next year, ten workers will retire. What will be the average age in the next year? [IMT]
I. Retirement age is 60 years.
II. There are 50 employees in the department.
17. What is Sonia's present age? [Bank PO]
I. Sonia's present age is five times Deepak's present age.
II. Five years ago her age was twenty five times Deepak's age at that time.
18. What is the speed of the boat in still water?
I. The speed downstream of the boat is thrice the speed upstream.
II. The sum of the speeds of the boat, upstream and downstream is 12 km/hr.
19. What is the speed of the boat in still water. [Bank PO]
I. It takes 2 hours to cover the distance between A and B downstream.
II. It takes 4 hours to cover the distance between A and B upstream.
20. A boat takes a total time of three hours to travel downstream from P to Q and upstream back from Q to P. What is the speed of the boat in still water? [SBI PO]
I. The speed of the current is 1 km/hr.
II. The distance between P and Q is 4 km.
21. What is the speed of the boat in still water? [Bank PO]
I. The boat covers a distance of 48 km in 6 hours while running upstream.
II. The boat covers the same distance in 4 hours while running downstream
22. A train crosses another train running in the opposite direction in x seconds. What is the speed of the train? [SBI PO]
I. Both the trains have the same length and are running in the same speed.
II. One train crosses a pole in 5 seconds.
23. What is the speed of a running train which takes 9 seconds to cross a signal post? [Bank PO]
I. The length of the train is 90 metres.
II. The train takes 27 seconds to cross a platform of 180 metres.
24. A train crosses a pole in 10 seconds. What is the length of train? [Bank PO]
I. The train crosses another train running in opposite direction with a speed of 80 km/hr in 22 seconds.
II. The speed of the train is 108 km/hr.

25. A train crosses a signal post in x seconds. What is the length of train? [NABARD]
I. The train crosses a platform of 100 metres in y seconds.
II. The train is running at the speed of 80 km/hr.
26. What is the speed of the train? [Bank PO]
I. 280 metres long train crosses a signal pole in 18 seconds.
II. 280 metres long train crosses a platform in 45 seconds.
27. What was the length of a running train crossing another 180 metre long train running in the opposite direction? [Bank PO]
I. The relative speed of the two trains was 150 km/hr.
II. The trains took 9 seconds to cross each other.
28. What is the speed of a train whose length is 210 metres? [Bank PO]
I. The train crosses another train of 300 metres length running in opposite direction in 10 seconds.
II. The train crosses another train running in the same direction at the speed of 60 km/hr in 30 seconds.
29. What is the length of a running train? [SBI PO]
I. The train crosses a man in 9 seconds.
II. The train crosses a 240 metre long platform in 24 seconds.
30. Towns A, B and C are on a straight line. Town C is between A and B. The distance from A to B is 100 km. How far is A from C? [MBA]
I. The distance from A to B is 25% more than the distance from C to B.
II. The distance from A to C is $\frac{1}{4}$ of the distance from C to B.
31. What is the usual speed of the train? [MBA]
I. The speed of the train is increased by 25 km/hr to reach the destination 150 km away in time.
II. The train is late by 30 minutes
32. Two cars pass each other in opposite direction. How long would they take to be 500 km apart? [MAT]
I. The sum of their speeds is 135 km/hr.
II. The difference of their speeds is 25 km/hr.
33. What is the percentage profit earned by selling the product? [Bank PO]
I. The profit earned was ₹ 50.
II. Had it been sold for ₹ 310, the profit would have been ₹ 70.
34. A man mixes two types of rices (X and Y) and sells the mixture at the rate of ₹ 17/kg. Find his profit percentage. [MBA]
I. The rate of X is ₹ 20/kg.
II. The rate of Y is ₹ 13/kg.
35. By selling a product for ₹ 100, how much profit was earned? [Bank PO]
I. 20% profit would have been earned if it were sold for ₹ 90.
II. The profit was one-third of the purchase price.
36. What is the cost price of the article?
I. The profit earned on the article is one-third of the cost price.
II. The article is sold for ₹ 400.

37. What was the cost price of the suitcase purchased by Richard? **[Bank PO]**
I. Richard got 20% concession on the labelled price.
II. Richard sold the suitcase for ₹ 2000 with 25% profit on the labelled price.
38. What would have been the selling price per kg of rice? **[Bank PO]**
I. 50 kg of rice was purchased for ₹ 3350 and ₹ 150 was spent on transport.
II. Profit earned was 5%.
39. By selling a product with 20% profit, how much profit was earned? **[SBI PO]**
I. The difference between cost and selling price is ₹ 40.
II. The selling price is 120% of the cost price.
40. The average age of teacher and students in a class is 3 years more than the average age of students. What is the age of the class teacher? **[Bank PO]**
I. There are 11 students in the class.
II. The average age of teacher and students is 14 years.
41. How many children are there in the group? **[Bank PO]**
I. Average age of the children in the group is 15 years. The total age of all the children in the group is 240 years.
II. The total age of all the children in the group and the teacher is 264 years. The age of the teacher is 9 years more than the average age of the children.
42. The average age of P, Q, R and S is 30 years. How old is R? **[RBI]**
I. The sum of ages of P and R is 60 years.
II. S is 10 years younger than R.
43. What is the average age of children in the class? **[Bank PO]**
I. Age of the teacher is as many years as the number of children.
II. Average age increased by 1 year if the teacher's age is also included.
44. How many candidates were interviewed everyday by panel A out of the three panels A, B and C? **[Bank PO]**
I. The three panels on an average interview 15 candidates everyday.
II. Out of a total of 45 candidates interviewed everyday by the three panels, the number of candidates interviewed by panel A is more by 2 than the candidates interviewed by panel C and is more by 1 than the candidates interviewed by panel B.
45. What will be the average weight of the remaining class? **[Bank PO]**
I. Average weight of 30 children out of total 46 in the class is 22.5 kg and that of the remaining children is 29.125 kg. A child having weight more than 40 kg is excluded.
II. Average weight of a class of 46 children is 23.5 kg. A child weighing 46 kg is dropped out.
46. Rahul, Anurag and Vivek started a business together. In what proportion would the annual profit be distributed among them? **[Bank PO]**
I. Rahul got one-fourth of the profit.
II. Rahul and Vivek contributed 75% of the total investment.
47. Ravi, Gagan and Nitin are running a business firm in partnership. What is Gagan's share in the profit earned by them? **[MBA]**
I. Ravi, Gagan and Nitin invested the amounts in the ratio of 2:4:7.
II. Nitin's share in the profit is ₹ 8750.

48. A and B together can complete a work in 7 days. B alone can do it in 20 days. What part of the work was carried out by A? [MBA]
 I. A completed the job alone after A and B worked together for 5 days.
 II. Part of the work done by A could have been done by B and C together in 6 days.
49. How long will machine Y, working alone, take to produce x candles? [MBA]
 I. Machine X produces x candles in 5 minutes.
 II. Machine X and machine Y working at the same time produce x candles in 2 minutes.
50. What percentage of simple interest per annum did Anand pay to Deepak? [IBPS]
 I. Anand borrowed ₹ 8000 from Deepak for four years.
 II. Anand returned ₹ 8800 to Deepak at the end of two years and settled the loan.
51. The simple interest on a sum of money is ₹ 50. What is the sum? [RBI]
 I. The interest rate is 10% p.a.
 II. The sum earned simple interest in 10 years.
52. What is the rate of simple interest? [Bank PO]
 I. The total interest earned was ₹ 4000.
 II. The sum was invested for 4 years.
53. What was the total compound interest on a sum after 3 years? [Bank PO]
 I. The interest after one year was ₹ 100 and the sum was ₹ 1000.
 II. The difference between simple and compound interest on a sum of ₹ 1000 at the end of 2 years was ₹ 10.
54. What was the rate of interest on a sum of money? [SBI PO]
 I. The sum fetched a total of ₹ 2522 as compound interest at the end of 3 years.
 II. The difference between the simple interest and the compound interest at the end of 2 years at the same rate was ₹ 40.
55. What will be the compounded amount? [Bank PO]
 I. ₹ 200 was borrowed for 192 months at 6% compounded annually.
 II. ₹ 200 was borrowed for 16 years at 6%.
56. What will be the cost of painting the inner walls of a room if the rate of painting is ₹ 20/ft²? [Bank PO]
 I. Circumference of the floor is 44 ft.
 II. Height of the wall of the room is 12 ft.
57. The area of a rectangle is equal to the area of a right angled triangle. What is the length of the rectangle? [Bank PO]
 I. The base of the triangle is 40 cm.
 II. The height of the triangle is 50 cm.
58. Area of a square is equal to the area of a circle. What is the circumference of the circle? [SBI PO]
 I. The diagonal of the square is x inches.
 II. The side of the square is y inches.
59. The area of a playground is 1600 m². What is its perimeter? [Bank PO]
 I. It is a perfect square playground.
 II. It costs ₹ 3200 to put a fence around the playground at the rate of ₹ 20/m.
60. What is the height of the triangle? [Bank PO]
 I. The area of the triangle is 20 times its base.
 II. The perimeter of the triangle is equal to the perimeter of a square of side 10 cm.

61. What is the height of a circular cone? [Bank PO]
I. The area of the cone is equal to the area of a rectangle whose length is 33 cm.
II. The area of the base of the cone is 154 cm^2 .
62. What is the capacity of a cylindrical tank? [IBPS]
I. Radius of the base is half of its height which is 28 metres.
II. Area of the base is 616 m^2 and its height is 28 metres.
63. Is a given rectangular block, a cube? [MAT]
I. At least 2 faces of the rectangular block are squares.
II. The volume of the block is 64.
64. What is the volume of 32 m high cylindrical tank? [Bank PO]
I. The area of its base is 154 m.
II. The diameter of the base is 14 m.
65. What is the volume of a cube? [Bank PO]
I. The area of each face of the cube is 64 m^2 .
II. The length of one side of the cube is 8 m.

Q. (66 – 70) In this type of questions, you have to determine which statement(s) is/are necessary/sufficient to answer the question. Questions will be accompanied by the statements A, B and C.

66. What is the minimum marks required to pass an examination?
A. A student gets 25% marks and fails by 40 marks
B. Another student who appeared in the same examination gets 70% of the pass marks and fails by 42 marks.
C. The total marks of the examination is 400.
(a) Any two of them (b) Either B alone or A and C together
(c) Only A and C together (d) Any of them
(e) None of these
67. Find the length of the side of a square.
A. Total cost of flooring the square is ₹ 1000 at the rate of ₹ 40/m².
B. A rectangle whose perimeter is 20 m is equal to the perimeter of the square.
C. A square of area equal to 50 m^2 can be made on the diagonal of the given square.
(a) Any of them (b) Any two of them
(c) Either B or C only (d) Either A or B only
(e) Either A or C only
68. In how many days can a work be completed by X, Y and, Z together?
A. The ratio of work efficiencies of X, Y and Z is 3 : 2 : 1.
B. Z works for 5 days and leaves the job and the remaining work is done by X and Y together in 5 days.
C. 60% of the work is done by X alone in 6 days
(a) Any two of them (b) Only A and C together
(c) Only B (d) Either B alone or A and C together
(e) Only A and either B or C
69. What will be the share of R in the profit earned by V, R and A together?
A. They together invested an amount of ₹ 54000 for a period of one year.

- B. R's investment was 25% less than V's and 50% more than A's.
C. The profit of V is ₹ 4000 more than that of A.
(a) Only A and B together (b) B and either A or C only
(c) Only B (d) Only B and C together
(e) Question can't be answered even after using all the information.

70. A shopkeeper sold an article and got ₹ 108 as profit. Find the profit percentage.
A. The selling price of the article is ₹ 828.
B. He gave 10% discount on the marked price, which is ₹ 920.
C. Cost price of the article is ₹ 720.
(a) Any two of them (b) Either A or C
(c) Any of them (d) Only B
(e) All statements are required.

Q. (71 – 80) The following questions are accompanied by three statements A, B and C. You have to determine which statement(s) is/are necessary/sufficient to answer the question.

71. X alone can do a piece of work in 20 days. In how many days can X and Y together finish the same work?
A. The ratio of the efficiencies of X and Y is 3 : 2.
B. Y can do four-fifths of the work in 24 days.
C. One-fourth of the work is done by X and Y together in 3 days.
(a) Only C (b) Any two of them
(c) Any of them (d) Only either A or B
(e) None of these
72. 40 students sit in rows and columns. How many students are seated in each column?
A. The number of rows is 62.5% of the number of columns.
B. The number of rows is $\frac{5}{8}$ of the number of columns.
C. The number of rows is less than the number of columns.
(a) Only A (b) C and either A or B
(c) Only B (d) Either A or B
(e) All the statements are required
73. What is height of triangle XYZ?
A. XY is the base and the perimeter of the triangle is 45 cm.
B. The ratio of the sides XY, YZ and ZX is 3 : 2 : 4.
C. The ratio of XY to the height of the triangle is $\frac{\sqrt{5}}{\sqrt{2}}$.
(a) B and either A or C (b) Only A and C together
(c) Any two of them (d) All the statements are required
(e) None of these
74. A person borrowed a sum of money on compound interest. What amount will the person pay after two years?
A. Simple interest fetched on the same sum in 2 years at the same rate of interest is ₹ 1250.
B. The simple interest in 4 years on the same sum and same rate of interest is one-fifth of the amount borrowed.

- C. The difference between compound interest and simple interest on the same sum after two years is ₹ 31.25.
- (a) Only A and B together (b) Only A and C together
(c) Only B and C together (d) Any two of them
(e) All the statements are required
75. At what time will a train reach Lucknow from Patna?
- A. The train crosses another train of equal length of 200 m and running in the opposite direction in 9 seconds.
B. The train leaves Patna at 11 : 15 A.M. for Lucknow, which is at a distance of 567 km.
C. The 97.50 m long train crosses a signal pole in 5 seconds.
- (a) Only A (b) B and C together
(c) A and C together (d) All statements are required
(e) Only B
76. Find the per cent profit on an article.
- A. The marked price of the article is ₹ 6000.
B. The difference between the selling price and cost price of the article is ₹ 800.
C. A discount of 20% is given on the marked price.
- (a) A and B together
(b) A and C together
(c) B and C together
(d) Question can't be answered even after using all the statements
(e) All statements are required.
77. What will be the sum of the present ages of P and Q?
- A. Ratio of present age of P to that of Q is 4 : 5.
B. Q's present age is 25% more than that of P.
C. Difference between the present ages of Q and P is 10 years.
- (a) Any two of them (b) C and either A or B
(c) Any of them (d) Only A and C together
(e) None of these
78. What is the length of a train?
- A. The train crosses a pole in 5 seconds.
B. The train crosses a platform of length 220 m in 15 seconds.
C. The speed of the train is 79.2 km/hr.
- (a) Any two of them (b) Only A and C together
(c) Only A and B together (d) All statements are required
(e) A and either B or C
79. How many ice cubes can be accommodated in a container?
- A. The length and the breadth of the container are 4 m and 5 m respectively.
B. The edge of the cube is 2 m.
C. The ratio of the volume of the container to that of the ice cube is 27 : 1.
- (a) A and B together (b) Only C is sufficient
(c) A and C together (d) B and C together
(e) None of these

80. The area of a rhombus is 1152 m^2 . What is the side of rhombus?
- A. One of its diagonal is 36 m.
 B. The other diagonal is 32 m.
 C. All the four sides are equal.
- (a) A and B together are sufficient (b) Only A alone is sufficient
 (c) Only B alone is sufficient (d) Either A alone or B alone is sufficient
 (e) All A, B and C are necessary



1. (d)	2. (b)	3. (b)	4. (a)	5. (e)	6. (c)	7. (a)	8. (d)	9. (e)	10. (c)
11. (e)	12. (e)	13. (e)	14. (c)	15. (d)	16. (e)	17. (e)	18. (b)	19. (d)	20. (e)
21. (e)	22. (d)	23. (a)	24. (b)	25. (c)	26. (a)	27. (e)	28. (e)	29. (e)	30. (c)
31. (e)	32. (a)	33. (b)	34. (d)	35. (c)	36. (e)	37. (e)	38. (e)	39. (a)	40. (e)
41. (a)	42. (d)	43. (d)	44. (b)	45. (b)	46. (e)	47. (e)	48. (a)	49. (e)	50. (e)
51. (e)	52. (d)	53. (c)	54. (e)	55. (c)	56. (c)	57. (d)	58. (c)	59. (c)	60. (a)
61. (d)	62. (c)	63. (d)	64. (c)	65. (c)	66. (b)	67. (d)	68. (b)	69. (d)	70. (c)
71. (c)	72. (d)	73. (e)	74. (d)	75. (b)	76. (e)	77. (b)	78. (a)	79. (b)	80. (d)



Solutions with Necessary Explanation

11. Let the unit digit of number be x and tenth digit of number be y .

$$\therefore \text{I} \rightarrow x \sim y = 9 \quad (1)$$

$$\text{II} \rightarrow x + y = x \sim y \quad (2)$$

Solving (1) and (2), we get

$$x \sim y = 9 \text{ and } x + y = 9$$

$$\Rightarrow x = 9 \text{ and } y = 0 \text{ or } y = 9 \text{ and } x = 0$$

$\therefore$ Required number is 90.

Thus both I and II are required to answer the question.

12. I $\rightarrow x + y = 8 \quad (1)$

$$\text{II} \rightarrow \frac{1}{5}(10x + y) = \left(\frac{1}{2} \times 44 - 15 \right)$$

$$\Rightarrow 10x + y = 35 \quad (2)$$

Solving (1) and (2), we get $x = 3$ and $y = 5$.

$\therefore$ Both I and II are required to answer the question.

13. Let the tenth place digit of number be a , and unit place digit of number be b .

$$\text{Number} = 10a + b$$

$$\text{I} \rightarrow 10a + b = 51 \times k \quad (1)$$

where $k = 1, 2, 3$ etc.

$$\text{II} \rightarrow a + b = 6 \quad (2)$$

When $k = 1$, solving (1) and (2), we get

$$a = 5 \text{ and } b = 1$$

$\therefore$ Number is 51

Therefore I and II together are necessary to answer the question.

14. Let the age of Shruti be x years and that of Divya be $2x$ years.

$$\text{I} \rightarrow 2x + 5 : x + 5 = 9 : 5$$

$$\text{II} \rightarrow 2x - 10 : x - 10 = 3 : 1$$

$$\text{i.e.} \quad \frac{2x + 5}{x + 5} = \frac{9}{5} \quad (1)$$

$$\frac{2x - 10}{x - 10} = \frac{3}{1} \quad (2)$$

From (1),

$$\frac{2x + 5}{x + 5} = \frac{9}{5}$$

$$\Rightarrow \frac{(2x + 5) - (x + 5)}{x + 5} = \frac{9 - 5}{5}$$

$$\Rightarrow \frac{x}{x + 5} = \frac{4}{5}$$

$$\therefore \Rightarrow \frac{x + 5}{x} = \frac{5}{4}$$

$$1 + \frac{5}{x} = \frac{1}{4} + 1$$

$$\text{or} \quad x = 5 \times 4 = \mathbf{20}$$

Difference in ages = $x = 20$ years

From (2),

$$\Rightarrow \frac{2x - 10}{x - 10} = \frac{3}{1}$$

$$\Rightarrow \frac{(2x - 10) - (x - 10)}{x - 10} = \frac{3 - 1}{1}$$

$$\Rightarrow \frac{x}{x - 10} = \frac{2}{1}$$

$$\Rightarrow \frac{x - 10}{x} = \frac{1}{2}$$

$$\Rightarrow 1 - \frac{10}{x} = \frac{1}{2}$$

$$\Rightarrow \frac{10}{x} = \frac{1}{2}$$

$$\Rightarrow x = \mathbf{20 \text{ years}}$$

$\therefore$ Either I or II alone is sufficient to answer the question.

15. Here data in both statements I and II together are not sufficient to answer the question.

16. I $\rightarrow$ Retirement age is 60 years

II $\rightarrow$ There are 50 employees in the department.

Average age of 50 employees = 30 years.

or total age of 50 employees = $50 \times 30 = 1500$ years

Number of employees next year = 40

Total age of 40 employees next year = $50 \times 31 - 10 \times 60 = 1550 - 600 = 950$

$$\therefore \text{Average age next year} = \frac{950}{40} = 23 \frac{3}{4} \text{ years}$$

$\therefore$ I and II together are necessary to answer the question.

17. I $\rightarrow$ Sonia's age = $5 \times$ Deepak's age

II $\rightarrow$ (Sonia's age - 5) = $25 \times$ (Deepak's age - 5)

$$\begin{aligned} \Rightarrow \quad \text{Sonia's age} - 5 &= 25 \times \left(\frac{\text{Sonia's age}}{5} - 5 \right) \\ &= 5 \times \text{Sonia's age} - 125 \end{aligned}$$

$$\text{or} \quad \text{Sonia's age} = \frac{125 - 5}{4} = 30 \text{ years}$$

$\therefore$ I and II together are necessary to answer the question.

18. Let speed of boat upstream be x km/hr

$\therefore$ I $\rightarrow$ Speed of boat downstream = $3x$ km/hr

II $\rightarrow$ $x + 3x = 12$ km/hr

$$\begin{aligned} \text{Speed of boat in still water} &= \frac{x + 3x}{2} \\ &= \frac{12}{2} = 6 \text{ km/hr} \end{aligned}$$

$\therefore$ II alone is sufficient to answer the question.

19. Let the distance covered be S km

$\therefore$ I $\rightarrow$ Downstream speed = $\frac{S}{2}$ km/hr

II $\rightarrow$ Upstream speed = $\frac{S}{4}$ km/hr

$$\begin{aligned} \therefore \text{Speed of boat in still water} &= \left(\frac{S}{2} + \frac{S}{4} \right) \frac{1}{2} \\ &= \frac{3}{8} S \text{ km/hr} \end{aligned}$$

$\therefore$ I and II even together are not sufficient to answer the question.

20. I $\rightarrow$ speed of current = 1 km/hr

II $\rightarrow PQ = 4$ km

Let speed of boat in still water be S km/hr

$$\therefore \quad \frac{PQ}{(S+1)} + \frac{PQ}{(S-1)} = 3$$

$$\text{i.e.} \quad \frac{4}{S+1} + \frac{4}{S-1} = 3$$

$$\Rightarrow \quad S = 3 \text{ km/hr}$$

$\therefore$ I and II together are sufficient to answer the question.

21. I $\rightarrow$ Upstream speed = $\frac{48}{6} = 8$ km/hr

II $\rightarrow$ Downstream speed = $\frac{48}{4} = 12$ km/hr

Speed of boat in still water = $\frac{1}{2}(8 + 12) = 10$ km/hr

$\therefore$ I and II together are necessary to answer the question.

22. I $\rightarrow L_1 = L_2; u_1 = u_2$

II $\rightarrow$ Speed of train = $\frac{L_1}{5}$ m/s

Speed of train = $\frac{L_1 + L_2}{x}$ m/s = $\frac{L_1}{5}$

From the given details, it is not possible to find speed of train, i.e. I and II taken even together is not sufficient to answer the question.

23. I $\rightarrow L = 90$ m; $u = \frac{90}{9} = 10$ m/s

II $\rightarrow u = \frac{90 + 180}{27} = 10$ m/s

$\therefore$ I alone gives the correct answer.

24. II $\rightarrow$ Speed of train = 108 km/hr = $108 \times \frac{5}{18} = 30$ m/s

$\therefore$ Length of train = Speed $\times$ Time = $30 \times 10 = 300$ m

$\therefore$ II alone is sufficient to answer the question.

25. II $\rightarrow$ Speed of train = 80 km/hr = $80 \times \frac{5}{18} = \frac{200}{9}$ m/s

I $\rightarrow$ Speed of train = $\frac{L + 100}{y}$ m/s = $\frac{200}{9}$ m/s

Also speed of train = $\frac{L}{x}$ m/s = $\frac{200}{9}$ m/s

It is not possible to find length of train from I or II alone.

26. I $\rightarrow$ Speed of train = $\frac{280 \text{ m}}{18 \text{ sec}} = \frac{140}{9}$ m/s

II $\rightarrow$ Speed of train = $\frac{280 + L}{45} = \frac{140}{9}$ m/s

$\Rightarrow L = 420$ m

I alone is sufficient to answer the question.

27. I $\rightarrow$ Relative speed of two trains = 150 km/hr = $150 \times \frac{5}{18}$ m/s = $\frac{125}{3}$ m/s

II $\rightarrow \frac{L + 180}{9} = \text{Speed of train} = \frac{125}{3}$ m/s

$\Rightarrow L = 195 \text{ m}$

Both I and II are necessary to answer the question.

28. I $\rightarrow \frac{l_1 + l_2}{u + v} = 10$ or $u + v = \frac{l_1 + l_2}{10} = 51 \text{ m/s}$

$\frac{210 + 300}{u + v} = 10$ or $\frac{210 + 300}{10} = u + v = 51 \text{ m/s}$

II $\rightarrow \frac{l_1 + l_2}{u - v} = 30$ or $\frac{210 + 300}{\left(4 - 60 \times \frac{5}{18}\right)} = 30$

or $u = \frac{210 + 300}{30} + 60 \times \frac{5}{18} = 17 + \frac{50}{3} = 33 \frac{2}{3} \text{ m/s}$

$\therefore$ I and II are necessary to answer the question.

29. Let length of train be L metres

I $\rightarrow \text{Speed} = \frac{L}{9} \text{ m/s}$ (1)

II $\rightarrow \text{Speed} = \frac{L + 240}{24} \text{ m/s}$ (2)

Solving (1) and (2), we get

$8L = 3L + 720$

$\Rightarrow 5L = 720$

$\Rightarrow L = 144 \text{ m}$

$\therefore$ I and II are necessary to answer the question.

30. $AB = 100 \text{ km}$; $AC = ?$

I $\rightarrow AB = 1.25 BC$

or $BC = \frac{1}{1.25} AB = 0.8 AB$

or $AC = 0.2 AB$
 $= 0.2 \times 100 = 20 \text{ km}$

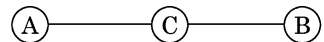
II $\rightarrow AC = \frac{1}{4} BC$ or $\frac{AC}{BC} = \frac{1}{4}$ or $AC = \frac{1}{1 + 4} \times 100 = 20 \text{ km}$

$\therefore$ I or II alone is sufficient to answer the question.

31. Let usual speed be u km/hr

$\therefore$ Time taken to cover 150 km = $\frac{150}{u}$ hour

I $\rightarrow$ Time taken to cover the distance at increased speed = $\frac{150}{(u + 25)}$ hour



$$\begin{aligned}
 \text{II} \rightarrow \quad & \frac{150}{u} - \frac{150}{u+25} = \frac{30}{60} \\
 \Rightarrow \quad & \frac{1}{u} - \frac{1}{u+25} = \frac{1}{300} \\
 \Rightarrow \quad & [(u+25) - u] \times 300 = u(u+25) \\
 \Rightarrow \quad & u^2 + 25u - 7500 = 0 \\
 \Rightarrow \quad & (u+100)(u-75) = 0 \\
 \therefore \quad & \text{Usual speed} = \mathbf{75 \text{ km/hr}}
 \end{aligned}$$

$\therefore$ I and II together are necessary to answer the question.

32. I $\rightarrow$ Relative speeds of faster car w.r.t. slower car = 135 km/hr

$$\therefore \text{Time taken by them to be 500 km apart} = \frac{\text{Distance}}{\text{Speed}} = \frac{500 \text{ km}}{135 \text{ km/hr}} = \frac{100}{27} \text{ hours} = 3 \frac{19}{27} \text{ hours}$$

Hence I alone is sufficient to answer the question, but II alone is not sufficient to answer the question.

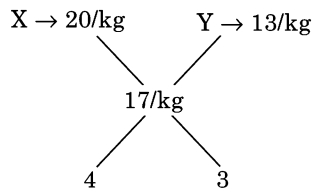
33. II $\rightarrow$ SP = ₹ 310; Profit = ₹ 70.

$$\therefore \text{CP} = \text{SP} - \text{Profit} = 310 - 70 = \text{₹ } 240$$

$$\therefore \text{Profit \%} = \frac{\text{SP} - \text{CP}}{\text{CP}} \times 100 = \left(\frac{310 - 240}{240} \right) \times 100\%$$

$\therefore$ II alone is sufficient to answer the question, but I alone is not sufficient.

34. Both I and II taken together is not sufficient to answer the question as the ratio in which two types of rices X and Y are mixed is not given.



35. SP = ₹ 100

$$\text{I} \rightarrow \text{If SP} = \text{₹ } 90, \text{ Gain \%} = 20\%$$

$$\therefore \text{CP} = \frac{100}{120} \times \text{SP} = \frac{100}{120} \times 90 = \text{₹ } 75$$

$$\therefore \text{If SP} = \text{₹ } 100, \text{ Profit \%} = \frac{100 - 75}{75} \times 100 = 33 \frac{1}{3} \%$$

$$\text{and Profit} = 100 - 75 = \text{₹ } 25$$

$$\text{II} \rightarrow \text{Profit is } \frac{1}{3} \text{rd of purchase price or } 33 \frac{1}{3} \%$$

$$\text{Let CP} = \text{₹ } x; \text{ Profit} = \frac{x}{3}$$

$$\therefore \text{SP} = \text{CP} + \text{Profit} = x + \frac{x}{3} = \text{₹ } \frac{4}{3} x = 100$$

$$\therefore x = \frac{3}{4} \times 100 = \text{₹ } 75$$

$$\text{Profit} = \frac{75}{3} = ₹ 25$$

∴ I alone or II alone is sufficient to answer the question.

36. Let CP of article be ₹ x

$$\therefore \text{I} \rightarrow \text{Profit earned} = ₹ \frac{x}{3}; \quad \text{SP} = x + \frac{x}{3} = \frac{4}{3}x;$$

$$\text{II} \rightarrow \text{SP} = ₹ 400$$

$$\therefore \frac{4}{3}x = 400 \quad \text{or} \quad x = \frac{3}{4} \times 400 = ₹ 300$$

$$\therefore \text{CP} = ₹ 300$$

∴ Both I and II together are sufficient to answer the question. Hence answer is (e).

37. Let labelled price of suitcase be ₹ x

$$\therefore \text{I} \rightarrow \text{CP} = \frac{80}{100} \text{ of } ₹ x = \frac{4}{5}x$$

$$\text{II} \rightarrow \text{SP} = ₹ 2000; \quad \text{SP} = \frac{125}{100} \times x = \frac{5}{4}x$$

$$\text{i.e.} \quad \frac{5}{4}x = 2000$$

$$\text{or} \quad x = \frac{4}{5} \times 2000 = ₹ 1600$$

$$\therefore \text{CP} = \frac{4}{5}x = \frac{4}{5} \times 1600 = ₹ 1280$$

∴ I and II together are sufficient to answer the question.

38. I → CP of 50 kg of rice = 3350 + 150 = ₹ 3500

$$\therefore \text{CP/kg of rice} = \frac{3500}{50} = ₹ 70$$

$$\text{II} \rightarrow \text{Profit} = 5\%$$

$$\therefore \text{SP} = \frac{105}{100} \times \text{CP} = \frac{105}{100} \times 70 = ₹ 73.50$$

∴ Both I and II together are necessary to answer the question.

39. Profit % = 20%

$$\text{I} \rightarrow \text{SP} - \text{CP} = ₹ 40$$

$$\text{i.e. Profit earned} = ₹ 40$$

$$\text{II} \rightarrow \text{SP} = \frac{120}{100} \times \text{CP}$$

II does not give the correct answer.

I alone is sufficient to answer the question.

40. Let the age of class teacher be x years.

$$\text{I} \rightarrow \text{Number of students} = 11$$

$$\text{II} \rightarrow \frac{\text{Sum of ages of 11 students} + x}{12} = 14 \text{ years}$$

Sum of ages of 11 students = $11 \times (14 - 3) = 121$

$$\therefore x = 12 \times 14 - 121 = 47 \text{ years}$$

$\therefore$ Both I and II are necessary to answer the question.

41. Let the number of children in the group be x .

$$\text{I} \rightarrow x = \frac{240}{15} = 16 \text{ numbers}$$

$$\text{II} \rightarrow \text{Age of teacher} = 15 + 9 = 24 \text{ years.}$$

$$\therefore \text{Age of } x \text{ children} = 264 - 24 = 240.$$

I alone gives the answer. II alone is not sufficient to answer the question.

$$42. \frac{P + Q + R + S}{4} = 30; \quad R = ?$$

$$\text{I} \rightarrow P + R = 60$$

$$\text{II} \rightarrow R - S = 10$$

It is not possible to find the age of R .

Hence both I and II taken together are not sufficient to answer the question.

43. Let the average age of children be x years, and number of children be y .

$$\text{I} \rightarrow \text{Age of teacher} = y \text{ years.}$$

$$\text{II} \rightarrow \text{New average} = x + 1 = \frac{xy + y}{y + 1} = \frac{(x + 1)y}{y + 1}$$

It is not possible to find age of teacher.

Both I and II taken together are not sufficient to answer the question.

44. II $\rightarrow$ Let the candidates interviewed by panel A per day be x , that by panel B be y and by C be $45 - (x + y)$

$$\text{Now,} \quad x = 45 - (x + y) + 2 \quad (1)$$

$$x = y + 1 \quad (2)$$

Solving (1) and (2), we get

$$45 + 2 - (x + y) = y + 1$$

$$\Rightarrow 47 - x - 1 = 2y$$

$$\text{or} \quad 46 - x = 2y$$

$$\text{But} \quad y = (x - 1), \text{ from (2)}$$

$$\therefore 46 - x = 2(x - 1)$$

$$46 - x = 2x - 2$$

$$\text{or} \quad 46 + 2 = 3x$$

$$\text{or} \quad x = \frac{48}{3} = 16$$

Hence II gives the answer. I does not give the correct answer.

45. I $\rightarrow$ Total weight of 46 children = $30 \times 22.5 + 16 \times 29.125 = 675 + 466 = 1141$

Weight excluded is not having correct value.

$\therefore$ Average of remaining class cannot be found.

$$\text{II} \rightarrow \text{Total weight of 46 children} = 46 \times 23.5 = 1081$$

$$\therefore \text{New average} = \frac{1081 - 46}{45} = \frac{1035}{45} = 23$$

$\therefore$ II gives correct answer, but I is not sufficient.

46. Let total investment be P .

$$\therefore \text{I} \rightarrow R = \frac{P}{4}$$

$$\text{II} \rightarrow R + V = \frac{75}{100} \times P = \frac{3}{4} P$$

$$\Rightarrow A = \frac{1}{4} P$$

$$\begin{aligned} \therefore R : A : V &= \frac{P}{4} : \frac{P}{4} : \frac{P}{2} \\ &= 1 : 1 : 2 \end{aligned}$$

$\therefore$ Both I and II together are necessary to answer the question.

47. $\text{I} \rightarrow R : G : N = 2 : 4 : 7$

$$\therefore \text{Gagan's share} = \frac{4}{2+4+7} \text{ of Total investment} = \frac{4}{13} \times \text{I}$$

$$\text{II} \rightarrow \frac{7}{13} \times \text{Investment amount} = 8750$$

$$\therefore \frac{4}{13} \times \text{Investment amount} = \frac{8750}{7} \times 4 = 1250 \times 4$$

i.e. Gagan's share in the profit = ₹ 5000

$\therefore$ Both I and II together are necessary to answer the question.

48. $A + B \rightarrow 7$ days

$B \rightarrow 20$ days

$$\text{I} \rightarrow \text{Total contribution by A and B for 5 days} = \frac{5}{7}$$

$$\text{Part of work by A alone} = 1 - \frac{5}{7} = \frac{2}{7}$$

$\text{II} \rightarrow B + C \rightarrow 6$ days

I alone gives answer. II is not sufficient to answer the question.

49. $\text{I} \rightarrow X \rightarrow x$ candles/5 minutes

$\text{II} \rightarrow X + Y \rightarrow x$ candles/2 minutes.

$X \rightarrow 5$ minutes

$X + Y \rightarrow 2$ minutes

$$\therefore Y \rightarrow \frac{5 \times 2}{5 - 2} = \frac{10}{3} \text{ minutes}$$

$\therefore$ Both I and II together are necessary to answer the question.

50. $\text{I} \rightarrow \text{Loan amount} = ₹ 8000$ for 4 years

$$\text{II} \rightarrow \text{Simple interest for 2 years} = \frac{8800 - 8000}{8000 \times 2} \times 100 = 5\%$$

$\therefore$ Both I and II taken together are sufficient to answer the question.

51. Simple interest on an amount = ₹ 50

Let amount be P

$\text{I} \rightarrow r = 10\% \text{ p.a.}$

II $\rightarrow$ Interest earned in 10 years.

$$\therefore I = \frac{PNR}{100}$$

$$\text{i.e. } 50 = \frac{P \times 10 \times 10}{100}$$

$$\therefore P = ₹ 50$$

$\therefore$ I and II together are necessary to answer the question.

52. I $\rightarrow I = ₹ 4000$

II $\rightarrow N = 4$ years

$$R = \frac{100 I}{P \times N}$$

But P is unknown. Therefore, R cannot be calculated.

I and II together are not sufficient to answer the question.

53. I $\rightarrow P = ₹ 1000$; $P \left(1 + \frac{R}{100}\right) = 1100$;

$$\therefore R = 10\%$$

$$\begin{aligned} \therefore \text{Sum after 3 years} &= P \left(1 + \frac{R}{100}\right)^N \\ &= 1000 \left(1 + \frac{10}{100}\right)^3 \\ &= 1000 \times \frac{11}{10} \times \frac{11}{10} \times \frac{11}{10} = \mathbf{1331} \end{aligned}$$

$\therefore$ I gives the answer

$$\text{II} \rightarrow 1000 \left[\left(1 + \frac{R}{100}\right)^2 - 1 \right] - \frac{1000 \times 2 \times R}{100} = 10$$

$$1000 \left[\left(1 + \frac{R}{100}\right)^2 - 1 \right] - 20R = 10$$

$$1000 \left(\frac{2R}{100} + \frac{R^2}{100^2} \right) - 20R = 10$$

$$20R + \frac{R^2}{10} - 20R = 10$$

$$\text{or } R^2 = 10 \times 10 = 100$$

$$\text{or } R = 10\%$$

$$\therefore \text{Sum after 3 years} = 1000 \times \left(1 + \frac{10}{100}\right)^3 = 1331$$

$\therefore$ II gives the answer

$\therefore$ I alone or II alone is sufficient to answer the question.

54. I $\rightarrow$ Let P be the sum and $R\%$ be the interest.

$$\therefore P \left[\left(1 + \frac{R}{100} \right)^3 - 1 \right] = 2522 \quad (1)$$

$$\text{II} \rightarrow P \left[\left(1 + \frac{R}{100} \right)^2 - 1 \right] - \frac{P \times 2R}{100} = 40 \quad (2)$$

Dividing (1) by (2), we get

$$\frac{P \left[\left(1 + \frac{R}{100} \right)^3 - 1 \right]}{P \left[\left(1 + \frac{R}{100} \right)^2 - 1 \right] - \frac{PR}{50}} = \frac{2522}{40} = \frac{1261}{20}$$

$$\text{i.e.} \quad \frac{\left(1 + \frac{R}{100} \right)^3 - 1}{\left(1 + \frac{R}{100} \right)^2 - 1 - \frac{R}{50}} = \frac{1261}{20}$$

$$\Rightarrow \frac{\frac{R^3}{100^3} + \frac{3R^2}{100^2} + \frac{3R}{100}}{\frac{R^2}{100^2} + \frac{2R}{100} - \frac{R}{50}} = \frac{\frac{R^3}{100^3} + \frac{3R^2}{100^2} + \frac{3R}{100}}{\frac{R^2}{100^2}} = \frac{1261}{20}$$

$$\text{i.e.} \quad \frac{R}{100} + 3 + \frac{300}{R} = \frac{1261}{20}$$

$$\text{or} \quad \frac{R}{100} + \frac{300}{R} = \frac{1261}{20} - 3 = \frac{1261 - 60}{20} = \frac{1201}{20}$$

$$\Rightarrow \frac{R^2 + 30000}{100R} = \frac{1201}{20}$$

$$\Rightarrow R^2 + 30000 = 1201 \times 5R$$

$$\text{i.e.} \quad R^2 - 6005R + 30000 = 0$$

$$\therefore (R - 6000)(R - 5) = 0$$

$$\therefore R = 5\%$$

I and II together are necessary to answer the question.

$$\begin{aligned} 55. \text{ I} \rightarrow A &= P \left(1 + \frac{R}{100} \right)^N = 200 \left(1 + \frac{6}{100} \right)^{192/12} \\ &= 200 \left(1 + \frac{6}{100} \right)^{16} \end{aligned}$$

$$\text{II} \rightarrow A = 200 \left(1 + \frac{6}{100} \right)^{16}$$

$\therefore$ I alone or II alone is sufficient to answer the question.

56. I $\rightarrow P = 44$ ft

II $\rightarrow h = 12$ ft

Area = $P \times h = 44 \times 12 = 528$ ft²

$\therefore$ Cost of painting = $528 \text{ ft}^2 \times 20/\text{ft}^2 = \text{₹ } 10560$

I and II together are necessary to answer the question.

57. $l \times b = \frac{1}{2} b_1 h$

I $\rightarrow b_1 = 40$ cm

II $\rightarrow h = 50$ cm

To find l , b is also needed.

b is not given, it is not possible to find l .

58. $a^2 = \pi r^2$

(1)

$p = 2\pi r = ?$

I $\rightarrow a\sqrt{2} = x$ inches;

II $\rightarrow a = y$ inches.

(1) $\Rightarrow \pi r^2 = a^2 = y^2 \Rightarrow r = \frac{y}{\sqrt{\pi}}$

Circumference of circle, $p = 2\pi r = 2\pi \times \frac{y}{\sqrt{\pi}} = 2y\sqrt{\pi}$

I $\rightarrow \pi r^2 = a^2 = \left(\frac{x}{\sqrt{2}}\right)^2 = \frac{x^2}{2}$

$\therefore r^2 = \frac{x^2}{2\pi} \quad \text{or} \quad r = \frac{x}{\sqrt{2\pi}}$

$\therefore p = 2\pi r = 2\pi \times \frac{x}{\sqrt{2\pi}} = x \times \sqrt{2\pi}$

$\therefore$ I alone or II alone is sufficient to answer the question.

59. $A = 1600$ m²; $P = ?$

I $\rightarrow a^2 = 1600 \Rightarrow a = 40$ m

$\therefore P = 4a = 4 \times 40 = 160$ m

II $\rightarrow p = \frac{3200}{20} = 160$ m

$\therefore$ I alone or II alone is sufficient to answer the question.

60. I $\rightarrow A = \frac{1}{2}bh = 20b$

(1)

$\Rightarrow h = 2 \times 20 = 40$ units

II $\rightarrow P = 4 \times 10 = 40$ cm

I alone gives the answer.

61. I $\rightarrow \pi r^2 = 154$ gives value of radius of cone.

II does not give height of cone.

I $\rightarrow A = \pi rl = \text{Area of rectangle with length} = 33$ cm

Area of cone cannot be found out as breadth of rectangle is not given.

Hence height of cone cannot be found out as statements I and II taken together are insufficient to answer the question.

62. I $\rightarrow r = \frac{h}{2} = \frac{28}{2} = 14$ cm

i.e. $r = 14$ m; $h = 28$ m

Capacity of cylindrical tank $= \pi r^2 h = \pi (14)^2 \times 28 = 17248 \text{ m}^2$

II $\rightarrow \pi r^2 = 616 \text{ m}^2$, $h = 28$ m

Capacity of tank $= \pi r^2 h = 616 \times 28 = 17248 \text{ m}^2$

$\therefore$ I alone or II alone is sufficient to answer the question.

63. I $\rightarrow$ Two quantities of l , b and h are equal.

II $\rightarrow lbh = 64$.

Thus the block may be a cuboid or cube.

64. Given $h = 32$ m

I $\rightarrow \pi r^2 = 154 \text{ m}^2$; Volume $= \pi r^2 h = 154 \times 32 = 4928 \text{ m}^2$

II $\rightarrow 2r = 14$ m; Volume $= \pi \times \left(\frac{14}{2}\right)^2 \times 32 = 4928 \text{ m}^2$

i.e. I alone or II alone is sufficient to answer the question.

65. I $\rightarrow a^2 = 64 \text{ m}^2 \rightarrow a = 8$ m

$\therefore$ Volume of cube $= a^3 = 8^3 = 512 \text{ m}^3$

II $\rightarrow a = 8$ m

Volume of cube $= a^3 = 8^3 = 512 \text{ m}^3$

I alone and II alone gives the answer.

80. Area of rhombus = Product of diagonals.

Data Analysis

Data analysis is an important topic in competitive examinations today. Usually, a table bar chart, pie-chart or a graph is given and candidates are asked questions to analyse the data given in order to test their ability.

In order to solve such questions you should have the following things in mind.

1. Try to understand quickly what the diagram represents.
2. If you are doing a table, make sure what the sum of all entries in a row and column individually represent. For example, if a table represents marks scored by 10 students in a class in three subjects (Physics, Chemistry and Biology), then the sum of entries in each row represents total marks scored by each student. The sum of entries in each column represents the total marks scored by all 10 students in three different subjects. Here, the sum of all entries in the last row is equal to the sum of all entries in the last column.

i.e. $\sum_{i=1}^n \text{total score by student } i = \sum_{j=1}^3 \text{total score for subject } j$ where i represents the number of students and j the number of subjects.

3. Do not make unnecessary steps. For example, if you are asked to find the subject in which all students have performed best, then look out for the total marks in each subject and locate the highest among them.
4. In the questions having diagrams, by just looking at the diagram, you can locate the most correct answer among the alternatives. Pen is not needed for answering such questions.

For example, you are given with a pie diagram showing the population of a town in 1991 and 2001 showing literacy of men, women and children in both the years. You have to find out by what percentage literacy of women has increased.

Now, you have to find out the percentage of literate women in both years and total women in both years and find the ratios separately.

i.e. percentage increase = $\frac{\text{Literate women (2001)}/\text{Total women (2001)}}{\text{Literate women (1991)}/\text{Total women (1991)}} - 1$

5. In some questions calculations are unavoidable. In this case short cuts can be applied if possible. Otherwise, some approximations can be made if it does not lead to wrong answers.

For example if you are asked to find the result on simplification like $\frac{1.2 \times 1.3 \times 1.5 \times 1.4}{2.1 \times 2.6}$. Now you have to go through the question and look for any possible cancellation among numerators and denominators. Then after such cancellation simplify the question and find the answer.

$$\frac{1.2 \times 1.3 \times 1.5 \times 1.4}{2.1 \times 2.6} = \frac{1.2}{2} = \mathbf{0.6}$$

6. In certain cases the result may not be finite or answer may be given as a value between 1.4 and 1.7 or 0.8 and 1.2. In that case approximation may be made and locate the answer from the given alternatives.

For example, $\frac{2}{13} + \frac{3}{14} = ?$

By approximating

$$2 \times 0.0775 + 3 \times 0.071 = 0.155 + 0.213 = \mathbf{0.368} \text{ (approx.)}$$

Given alternatives are (a) 0.3 (b) 0.35 (c) 0.36 (d) 0.37.

The answer will be (d) as it is the nearest value to approximated one.

TABLE: This section consists of questions that contain data (such as imports, exports, income of employees in a company, marks scored by students in a subject and so on) form of a table. You have to study the given information and then answer the given questions on the basis of comparative analysis of the data.

EXAMPLE 1 A class consists of 4 students and the marks obtained by them in Physics, Chemistry and Mathematics are given in the following table.

Students → Subjects ↓	S_1	S_2	S_3	S_4
Physics	60	50	70	75
Chemistry	70	45	60	80
Mathematics	80	55	55	75

Study the table carefully and answer the questions that follow.

- Find the average score in Physics by the students in the class.
- In which subject student S_2 got the highest score?
- In which subject average score in the class was highest?
- Who among the students is ranked first in the class?
- Who got the least marks in Chemistry in the class?

Solution

1. Average score in Physics in the class is given by $\frac{60 + 50 + 70 + 75}{4} = 63\frac{3}{4}$ marks

2. S_2 scored the highest in **Mathematics**

3. Average score for Physics = $\frac{60 + 50 + 70 + 75}{4} = 63\frac{3}{4}$

Average score for Chemistry = $\frac{70 + 45 + 60 + 80}{4} = 63\frac{3}{4}$

Average score for Mathematics = $\frac{80 + 55 + 55 + 75}{4} = 66\frac{1}{4}$

$\therefore$ Average score is highest for **Mathematics**.

4. Total score for $S_1 = 210$

Total score for $S_2 = 150$

Total score for $S_3 = 185$

Total score for $S_4 = 230$

$\therefore S_4$ is ranked **first** in the class.

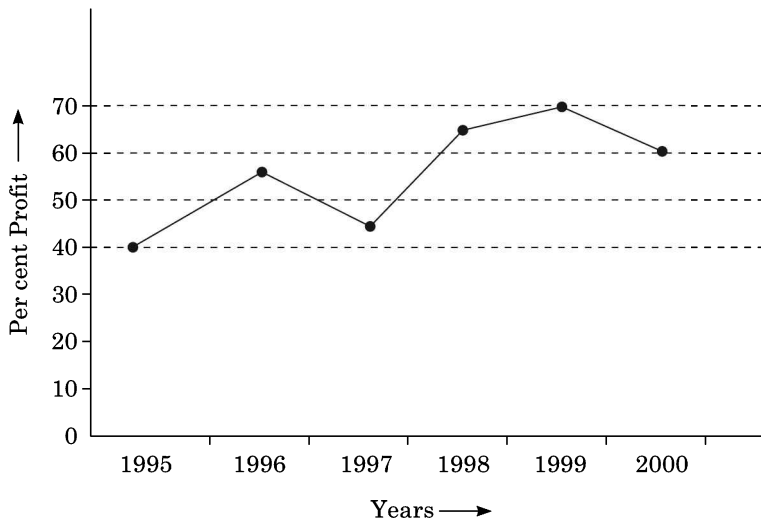
5. S_2 got the least marks in **Chemistry**.

LINE-GRAPH: A line-graph displays data that change over time. Line graphs compare two variables plotted along vertical axis and horizontal axis.

EXAMPLE 2 The following line-graph gives the annual profit earned by a company during the period 1995–2000. Study the line graph and answer the questions that are based on it. **[RBI]**

Per cent Profit Earned by a Company over the Years

$$\% \text{ Profit} = \frac{\text{Income} - \text{Expenditure}}{\text{Expenditure}} \times 100$$



- If the income in 1998 was ₹ 264 crores, what was the expenditure in 1998?
(a) ₹ 104 crores (b) ₹ 145 crores (c) ₹ 160 crores (d) ₹ 185 crores (e) None of these
- During which of the following years was the ratio of income to expenditure the minimum?
(a) 1996 (b) 1997 (c) 1998 (d) 1999 (e) 2000
- What is the average profit earned for the given years?
(a) $50\frac{2}{3}$ (b) $55\frac{5}{6}$ (c) $60\frac{1}{6}$ (d) $65\frac{1}{3}$ (e) None of these
- During which year the ratio of percentage profit earned to that in the previous year is the minimum?
(a) 1996 (b) 1997 (c) 1998 (d) 1999 (e) 2000
- If the expenditures in 1996 and 1999 are equal, then the approximate ratio of the income in 1996 and 1999 respectively is
(a) 1:1 (b) 2:3 (c) 9:10
(d) 13:14 (e) Cannot be determined
- If the expenditure in 2000 is 25% more than the expenditure in 1997, then the income in 1997 is what per cent less than the income in 2000?
(a) 22.5% (b) 25% (c) 27.5% (d) 31.25% (e) 32.5%
- If the profit in 1999 was ₹ 4 crores, what was the profit in 2000?
(a) ₹ 4.2 crores (b) ₹ 6.6 crores (c) ₹ 6.8 crores
(d) Cannot be determined (e) None of these
- In which year was the expenditure minimum?
(a) 2000 (b) 1997 (c) 1996
(d) Cannot be determined (e) None of these

Solution

- Let the expenditure in 1998 be ₹ x .

$$\therefore \frac{264 - x}{x} \times 100 = 65$$

$$\Rightarrow \frac{264}{x} - 1 = \frac{65}{100}$$

$$\Rightarrow \frac{264}{x} = \frac{65}{100} + 1 = \frac{165}{100}$$

$$\Rightarrow x = \frac{264 \times 100}{165} = \mathbf{160}$$

$\therefore$ Expenditure in 1998 = ₹ **160 crores**

- Given $\% \text{ Profit} = \frac{\text{Income} - \text{Expenditure}}{\text{Expenditure}} \times 100$

$$\Rightarrow \% \text{ Profit} = \left(\frac{\text{Income}}{\text{Expenditure}} - 1 \right) \times 100$$

$$\Rightarrow \frac{\text{Income}}{\text{Expenditure}} = \frac{\% \text{ Profit}}{100} + 1$$

$\therefore \frac{\text{Income}}{\text{Expenditure}}$ to be minimum, % profit should be minimum.

In the given years, % profit is minimum in 1997.

3. Average profit earned in the given years

$$\begin{aligned} &= \frac{1}{6} (40 + 55 + 45 + 65 + 70 + 60) \\ &= \frac{335}{6} = 55 \frac{5}{6} \end{aligned}$$

4. Ratio of percentage profit earned to that in the previous year for the given years:

$$1996 : \frac{55}{40} = 1.375; \quad 1997 : \frac{45}{55} = 0.818;$$

$$1998 : \frac{65}{45} = 1.414; \quad 1999 : \frac{70}{65} = 1.077;$$

$$2000 : \frac{60}{70} = 0.857;$$

$\therefore$ Required ratio is minimum in 1997.

5. Let the expenditure be ₹ x in 1996 and 1999.

Let the income be ₹ I_1 in 1996 and be ₹ I_2 in 1999.

$$\therefore \% \text{ Profit} = \frac{\text{Income} - \text{Expenditure}}{\text{Expenditure}} \times 100$$

$$\Rightarrow 55 = \frac{I_1 - x}{x} \times 100 \quad (\text{in the year 1996})$$

$$\Rightarrow 55 = \left(\frac{I_1}{x} - 1 \right) \times 100$$

$$\Rightarrow \frac{I_1}{x} = \frac{55}{100} + 1 = \frac{155}{100} \quad (1)$$

$$\Rightarrow 70 = \frac{I_2 - x}{x} \times 100 \quad (\text{in the year 1999})$$

$$\Rightarrow 70 = \left(\frac{I_2}{x} - 1 \right) \times 100$$

$$\Rightarrow \frac{I_2}{x} = \frac{70}{100} + 1 = \frac{170}{100} \quad (2)$$

Dividing (1) by (2), we have

$$\frac{I_1}{I_2} = \frac{155}{170} = \frac{31}{34} = 9 : 10$$

6. Let expenditure in 1997 be ₹ x

∴ Expenditure in 2000 = ₹ $1.25x$

Let the income in 1997 be ₹ I_1 and that in 2000 be ₹ I_2 .

In 1997,

$$45 = \left(\frac{I_1 - x}{x} \right) \times 100$$

$$\Rightarrow \frac{45}{100} = \frac{I_1}{x} - 1$$

$$\Rightarrow \frac{I_1}{x} = \frac{45}{100} + 1 = \frac{145}{100}$$

$$\Rightarrow I_1 = 1.45x$$

(1)

In 2000,

$$60 = \left(\frac{I_2 - 1.25x}{1.25x} \right) \times 100$$

$$\Rightarrow \frac{60}{100} = \frac{I_2}{1.25x} - 1$$

$$\Rightarrow \frac{I_2}{1.25x} = \frac{60}{100} + 1 = \frac{160}{100}$$

$$\Rightarrow I_2 = \frac{160}{100} \times 1.25x$$

$$= 2x$$

(2)

$$\therefore \frac{I_2 - I_1}{I_2} \times 100 = \frac{2x - 1.45x}{2x} \times 100\%$$

$$= \frac{0.55x}{2x} \times 100\% = \mathbf{27.5\%}$$

Required percentage = **27.5%**.

7. Let the profit in 2000 be ₹ x .

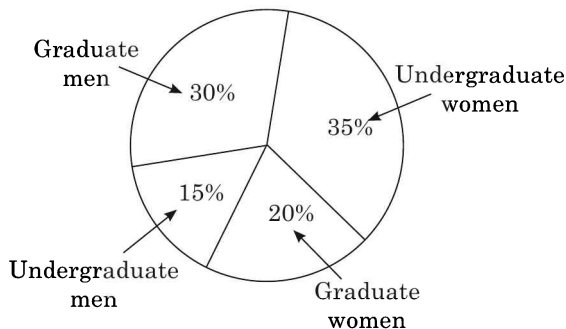
From the given line graph only % profit is available. Therefore, profit in 2000 cannot be determined.

8. From the line graph, expenditure cannot be determined. Hence answer is (d).

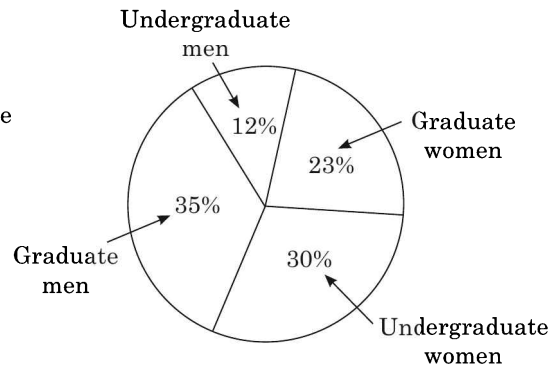
PIE-CHART: A pie-chart, also called a circle graph, is a circular chart that is divided into sectors. Each sector represents an item in a data to match the amount of item as a percentage of the total data. Pie-charts are useful to compare different parts of a whole amount.

EXAMPLE 3 Study the following pie-chart showing population of a town categorised according to their educational background.

Total population in 1991 = 40,000



Total population in 2001 = 1,20,000



- What is the percentage increase of graduate men to total population from 1991 to 2001?
(a) 33% (b) 45% (c) 16.67% (d) 25% (e) None of these
- What is the percentage increase in total population/year from 1991 to 2001?
(a) 1% (b) 2% (c) 3% (d) 4% (e) None of these
- What is the total number of graduate men in 2001?
(a) 12000 (b) 16000 (c) 32000 (d) 42000 (e) None of these
- What is the total number of undergraduate women in the year 2001?
(a) 16000 (b) 32000 (c) 36000 (d) 42000 (e) None of these
- What is the ratio between graduate men in 1991 to that in 2001?
(a) 5 : 7 (b) 6 : 7 (c) 2 : 7 (d) 3 : 7 (e) None of these

Solution

- % increase of graduate men to total population from 1991 to 2001

$$\begin{aligned}
 &= \left(\frac{\frac{35\% \text{ of } 1,20,000}{100\% \text{ of } 1,20,000} - \frac{30\% \text{ of } 40,000}{100\% \text{ of } 40,000}}{\frac{30\% \text{ of } 40,000}{100\% \text{ of } 40,000}} \right) \times 100 \\
 &= \left(\frac{\frac{35}{100} - \frac{30}{100}}{\frac{30}{100}} \right) \times 100 = \left(\frac{35 - 30}{30} \right) \times 100 = \frac{5}{30} \times 100 \\
 &= \frac{100}{6} = 16 \frac{4}{6}\% = 16 \frac{2}{3}\% = \mathbf{16.67\%}
 \end{aligned}$$

2. Required percentage = $\left(\frac{1,20,000 - 40,000}{40,000}\right) \times \frac{100}{10} = \frac{80,000}{40,000} \times 10 = \mathbf{20\%}$
3. Total number of graduate men in 2001
 $= 35\% \text{ of } 120000 = \frac{35}{100} \times 1,20,000 = \mathbf{42,000}$
4. Total number of undergraduate women in 2001
 $= 30\% \text{ of } 120000 = \frac{30}{100} \times 1,20,000 = \mathbf{36,000}$
5.
$$\frac{\text{Number of graduate men in 1991}}{\text{Number of graduate men in 2001}} = \frac{30\% \text{ of } 40,000}{35\% \text{ of } 1,20,000} = \frac{\frac{30}{100} \times 40,000}{\frac{35}{100} \times 1,20,000}$$

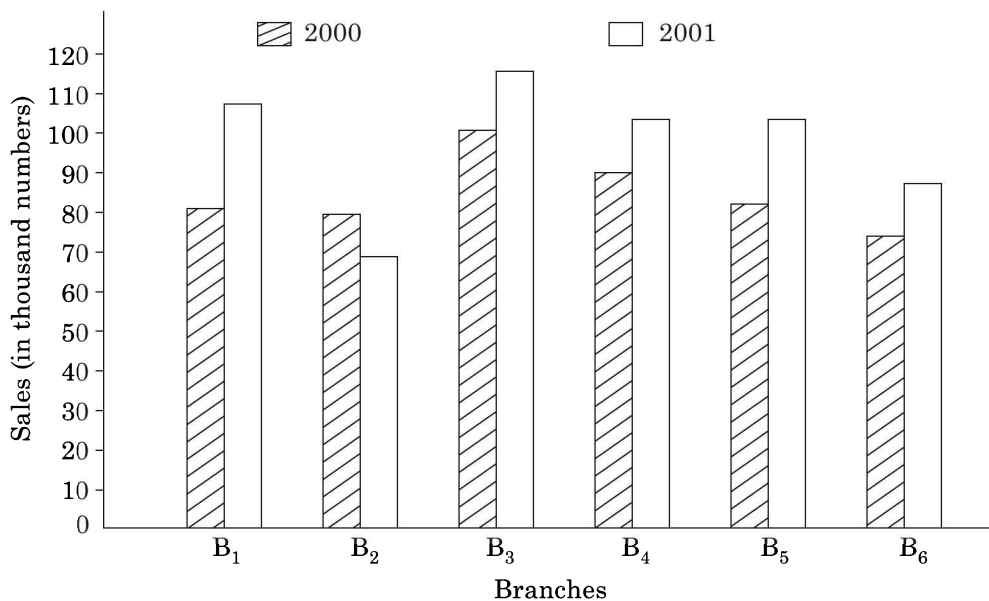
$$= \frac{30}{35 \times 3} = \frac{10}{35} = \frac{2}{7} = \mathbf{2 : 7}$$

BAR DIAGRAM This section consists of questions in which data collected is represented in the form of vertical or horizontal bars drawn by selecting a particular scale. One of the parameters is plotted against horizontal axis and the other parameter is plotted against vertical axis.

You have to study the given information carefully and then answer the given questions on the basis of data analysis.

EXAMPLE 4 The bar-graph provided below gives the sales of books (in thousand numbers) from six branches of a publishing company during two consecutive years 2000 and 2001. Answer the questions based on this bar-graph. **[Bank PO]**

Sales of Books (in thousand numbers) from six branches B_1, B_2, B_3, B_4, B_5 and B_6 of a publishing company in 2000 and 2001.



- Total sales of branches B₁, B₃ and B₅ together for both the years (in thousand numbers) is
(a) 250 (b) 310 (c) 435 (d) 560 (e) 585
- Total sales of branch B₆ for both the years is what per cent of the total sales of branch B₃ for both the years?
(a) 68.54% (b) 71.11% (c) 73.17% (d) 75.55% (e) 77.26%
- What is the average sale of all the branches (in thousand numbers) for the year 2000?
(a) 73 (b) 80 (c) 83 (d) 88 (e) 96
- What is the ratio of total sales of branch B₂ for both years to the total sales of branch B₄ for both years?
(a) 2 : 3 (b) 3 : 5 (c) 4 : 5 (d) 5 : 7 (e) 7 : 9
- What per cent of the average sales of branches B₁, B₂ and B₃ in 2001 is the average sales of branches B₁, B₃ and B₆ in 2000?
(a) 75% (b) 77.5% (c) 82.5% (d) 85% (e) 87.5%

Solution

- Total sales of branches B₁, B₃ and B₅ together for both the years (in thousand numbers)
= (80 + 105) + (95 + 110) + (75 + 95) = **560**

- Required percentage = $\frac{(70 + 80)}{(95 + 110)} \times 100\% = \frac{150}{205} \times 100\% = \mathbf{73.17\%}$

- Average sale of all the branches (in thousand numbers) for the year 2000
= $\frac{1}{6} (80 + 75 + 95 + 85 + 75 + 70) = \frac{1}{6} \times 480 = \mathbf{80}$

- Required ratio = $\frac{(75 + 65)}{(85 + 95)} = \frac{140}{180} = \frac{7}{9}$

- Average sales of branches B₁, B₂ and B₃ in 2001 (in thousand numbers)
= $\frac{1}{3} (105 + 65 + 110) = \frac{280}{3}$

Average sales of branches B₁, B₃ and B₆ in 2000 (in thousand numbers)

$$= \frac{1}{3} (80 + 95 + 70) = \frac{245}{3}$$

$$\begin{aligned} \therefore \text{Required percentage} &= \frac{(245/3)}{(280/3)} \times 100 = \frac{245}{280} \times 100 \\ &= \frac{49}{56} \times 100 = \frac{700}{8} = \mathbf{87.5\%} \end{aligned}$$



EXERCISES

Directions (Q. 1–5): The following table contains the percentage of literate population with respect to the total population of different states over the years.

States	1997	1998	1999	2000	2001	2002
A	67	72	69	78	79	82
B	82	81	85	85	87	88
C	78	81	84	87	89	91
D	56	65	69	71	75	77
E	89	93	94	95	95	97
F	85	88	91	93	95	96

- The populations of state C in 1997 and 1998 were in the ratio 2 : 3. What was the ratio of their literate population?
(a) 51 : 82 (b) 15 : 18 (c) 9 : 13 (d) 26 : 27 (e) None of these
- In 1998, all the six states had equal population of 12 lakhs. What was the average literate population of all the states in that year?
(a) 988000 (b) 1056000 (c) 960000 (d) 1028000 (e) None of these
- The populations of state D in 1998 and 2001 were equal. Approximately what was the percentage increase in literate population from 1998 to 2001?
(a) 17 (b) 15 (c) 16 (d) 18 (e) 19
- The population of state B increased every year by 10%. If its total population in 1999 was 10 lakhs, what was its literate population in 2001?
(a) 1052700 (b) 957000 (c) 1152400
(d) Cannot be determined (e) None of these
- If the average population of all the states in 1999 was 12.5 lakhs, then what was the average literate population in that year?
(a) 10.25 lakhs (b) 10.75 lakhs (c) 11 lakhs
(d) Cannot be determined (e) None of these

Directions (Q. 6–10): Study the following table carefully and answer the given questions. The details about the strength of six management institutes and the number of students getting placement over the years.

Institute →	A		B		C		D		E		F	
Year ↓	S	P	S	P	S	P	S	P	S	P	S	P
1998	480	350	525	470	350	300	560	500	380	320	400	300
1999	540	380	580	495	425	360	625	550	480	440	450	380
2000	580	400	700	600	410	370	580	540	450	420	480	420
2001	680	420	675	590	440	400	550	510	520	490	520	480
2002	650	450	750	680	520	480	600	530	500	470	550	500
2003	720	480	780	700	500	470	630	540	540	500	575	515

S = Strength, P = Number of students getting placement.

6. During which year the percentage of students getting placement from Institute B was the highest?
 (a) 1998 (b) 1999 (c) 2001 (d) 2002 (e) None of these
7. Approximately what is the overall percentage of students getting placement from all the institutes together in year 2000?
 (a) 80 (b) 78 (c) 75 (d) 89 (e) 86
8. The percentage of students getting placement was the highest for which of the institutes in the year 2002?
 (a) F (b) E (c) B (d) C (e) None of these
9. What is the percentage increase in the percentage of students getting placement from Institute F from 1998 to 1999 (rounded off to two digits after decimal)?
 (a) 12.59 (b) 13.12 (c) 12.89 (d) 13.29 (e) None of these
10. What is the ratio of the average strength of Institute E to that of Institute D?
 (a) 509 : 674 (b) 709 : 574 (c) 574 : 709 (d) 579 : 709 (e) None of these

Directions (Q. 11–15): Study the following table and answer the questions carefully. The following table shows the percentage of minor students and the proportion of boys and girls in six different schools.

Schools	% of minor students	Proportion of boys and girls	
		Minor B : G	Major B : G
A	92	12 : 11	3 : 1
B	96	5 : 3	1 : 1
C	68	3 : 1	5 : 3
D	84	2 : 5	3 : 1
E	88	3 : 1	7 : 5
F	80	2 : 3	3 : 2
Minor: Students whose age is less than 18.			

11. The average height of minor girls in school F is 5 ft and that of minor boys is 5ft 6 in. Find the average height of all minor students of school F.
 (a) $5\frac{1}{5}$ ft (b) $5\frac{1}{4}$ ft (c) $5\frac{2}{5}$ ft
 (d) Cannot be determined (e) None of these
12. The number of students in school B is 1000 and the average age of major boys is 18.2 year. When the age of the principal is included the average becomes 20 years. Find the age of the principal.
 (a) 36 years (b) 40 years (c) 56 years (d) 46 years (e) None of these
13. The number of minor girls in school C is 153. Find the total number of students in that school.
 (a) 800 (b) 850 (c) 900 (d) 1000 (e) None of these
14. There are 240 minor boys in school D. What is the number of girls who are not minors in that school?
 (a) 45 (b) 40 (c) 60 (d) 75 (e) None of these

15. If the total number of students in school F is 1250, then what is the number of girls in that school?

- (a) 600 (b) 700 (c) 650 (d) 750 (e) None of these

Directions (Q. 16–20): Study the following table and information to answer the questions.

Number of boys participating in different games

Games → Classes ↓	Cricket	Football	Hockey	Chess	Badminton
VII	22	18	24	12	6
VIII	20	16	16	12	8
IX	24	8	22	12	14
X	18	20	22	8	6
XI	12	14	18	8	14
XII	24	12	20	8	6

Information:

- (a) Every student of each class participates in a game.
 (b) Each student participates in one game only.
 (c) The total number of boys of the given classes is 444.
 (d) In each game, from every class, the number of girls participating is 50% of the number of boys participating in each game from the same class.
16. The girls of class VII and the boys of class XII together form what percentage of total number of students of the school?
- (a) 25% (b) 20% (c) $16\frac{2}{3}\%$ (d) $33\frac{1}{3}\%$ (e) None of these
17. Before starting a match, every girl hockey player of class X shakes hands with every other girl hockey players of that class. What would be the total number of hand shakes?
- (a) 66 (b) 55 (c) 231 (d) 253 (e) None of these
18. A committee of 5 students is to be formed out of the total badminton players of class XII. In how many ways can it be done so that a particular member is always included?
- (a) 70 (b) 56 (c) 126 (d) 66 (e) None of these
19. How many boys can't be paired for a game, if boys of classes IX and X are paired with girls of their classes?
- (a) 72 (b) 62 (c) 67 (d) 77 (e) None of these
20. How many students of the given classes play football?
- (a) 88 (b) 98 (c) 108 (d) 78 (e) None of these
21. In an office, the distribution of work hours is as shown in the following table. **[IAS]**

No. of staff members	No. of hours worked
5	0–19
12	20–24
25	25–29
40	30–34
15	35–39
8	40–45

Consider the following inferences drawn from the table.

- (i) The average number of hours worked by a staff member is about 30.
- (ii) The percentage of those who worked 35 or more hours is less than 25.
- (iii) At least 5 staff members worked more than 44 hours.

Which of these inference is valid?

- (a) (i) alone (b) (ii) alone (c) (i) and (ii) (d) (i), (ii), and (iii)

22. Consider the table given below providing details of traffic volume per hour for four locations.

[IAS]

Location	Total traffic volume	% of heavy vehicle	Average noise level in dB(A)	Noise pollution level in dB(A)
I	377	24.40	73.50	84.00
II	380	12.50	72.60	83.00
III	377	30.00	73.50	86.50
IV	225	12.50	72.98	80.90

When the total traffic volume is the same, the factor(s) which affect(s) the noise pollution level is/are:

- (a) % of heavy vehicles.
- (b) Noise pollution level and average noise level.
- (c) Average noise level and % of heavy vehicles.
- (d) Indeterminable on the basis of details given.

Directions (Q. 23–27): Study the following table, which contains classification of 100 students based on the marks obtained by them in physics and chemistry in an examination, carefully and answer the questions given below:

[Bank PO]

Marks out of 50 Subject	40 and above	30 and above	20 and above	10 and above	0 and above
Physics	9	32	80	92	100
Chemistry	4	21	66	81	100
(Aggregate) Average	7	27	73	87	100

23. The number of students scoring less than 40% marks in aggregate is
 (a) 13 (b) 19 (c) 20 (d) 27 (e) 34
24. If at least 60% marks in physics is required for pursuing higher studies in Physics, how many students will be eligible to pursue higher studies in Physics?
 (a) 27 (b) 32 (c) 34 (d) 41 (e) 68
25. What is the difference between the number of students passed with 30 as cut-off marks in Chemistry and those passed with 30 as cut-off marks in aggregate?
 (a) 3 (b) 4 (c) 5 (d) 6 (e) 7
26. The percentage of the number of students getting at least 60% marks in Chemistry over those getting at least 40% marks in aggregate, is approximately
 (a) 21% (b) 27% (c) 29% (d) 31% (e) 34%

27. If it is known that at least 23 students were eligible for a symposium on Chemistry, the minimum qualifying marks in Chemistry for eligibility to symposium would lie in the range
- (a) 40–50 (b) 30–40 (c) 20–30
(d) Below 20 (e) Cannot be determined

Directions (Q. 28–30): Refer to the following table and answer the questions that follow:

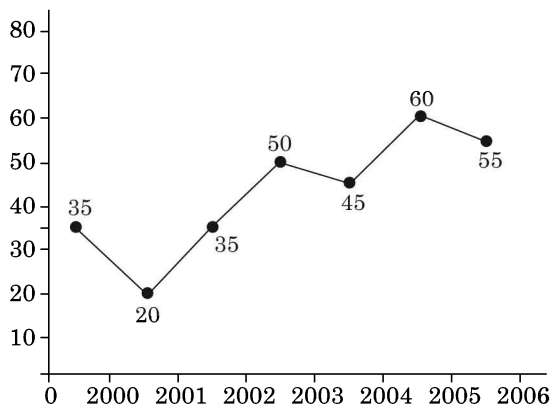
Distribution of marks obtained by 100 students in two papers (I and II) in Mathematics

Marks out of 50 → Paper ↓	40 and above	30 and above	20 and above	10 and above	0 and above
I	5	22	67	82	100
II	8	31	79	91	100
Aggregate (Average)	6	27	71	88	100

28. What should be the passing marks if minimum 80 students are required to be qualified with compulsory passing only in Paper I?
- (a) Below 20 (b) Above 20 (c) Below 40 (d) Above 40 (e) None of these
29. What will be the difference between the number of students passed with 30 as cut-off marks in Paper II and the number of students passed with same cut-off marks in aggregate?
- (a) 2 (b) 4 (c) 8 (d) 3 (e) None of these
30. What is the approximate percentage of students who have obtained 60% and more marks in paper II over the number of students who obtained 40% and more marks in aggregate?
- (a) 44 (b) 40 (c) 48
(d) Data inadequate (e) None of these

Directions (Q. 31–35): The following graph shows the percentage net profit of a certain company during the given period. Study it carefully and answer the questions given below

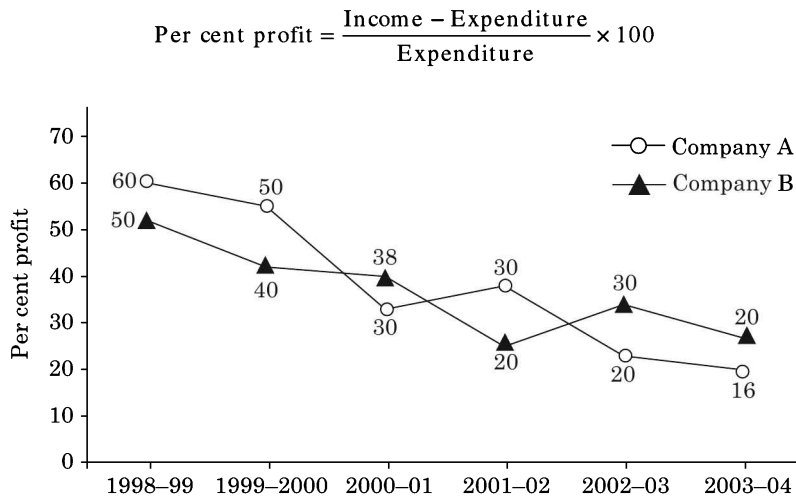
[Bank PO]



31. If the expenditure in 2002 was 20% more than the expenditure in 2000, by what per cent the income in 2002 was more/less than the income in 2000?
- (a) 25% less (b) 20% more (c) 27% more
(d) Data inadequate (e) None of these
32. During which of the following years was the ratio of income to expenditure the minimum?
- (a) 2000 (b) 2003 (c) 2004 (d) 2005 (e) None of these

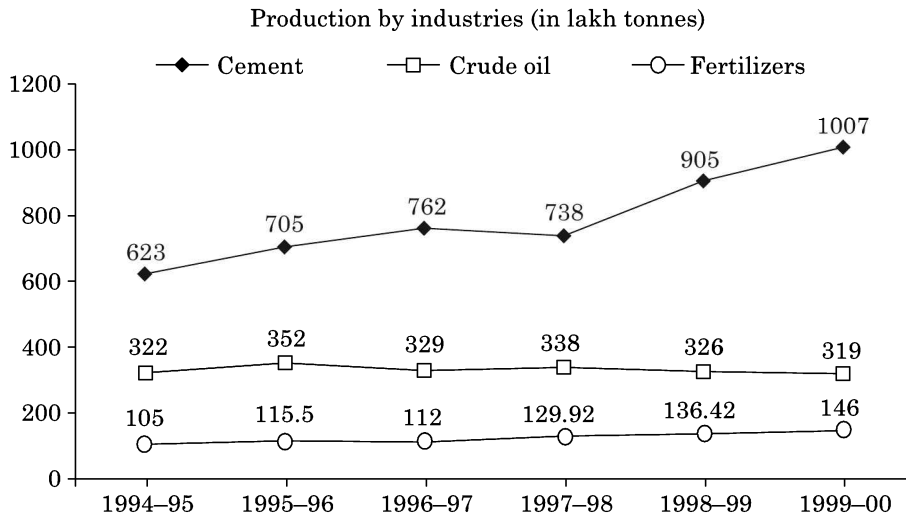
33. During which year the ratio of percentage net profit earned to that in the previous year was the minimum?
- (a) 2002 (b) 2003 (c) 2005
(d) 2002 and 2003 both (e) 2002, 2003 and 2005
34. If the expenditure in 2004 and 2006 was equal and income in 2006 was 15.5 lakhs, what was the income in 2004?
- (a) 12.5 lakhs (b) 13.5 lakhs (c) 14 lakhs
(d) Data inadequate (e) None of these
35. If the income in 2003 was ₹ 25 lakhs, what was the expenditure in that year?
- (a) 16 lakhs (b) 16.33 lakhs (c) 16.67 lakhs
(d) 15.67 lakhs (e) None of these

Directions (Q. 36–40): Study the following graph to answer the given questions
Per cent profit earned by two companies over the given years.



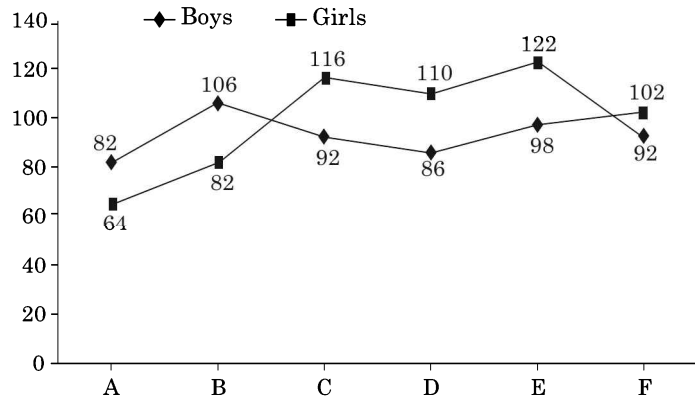
36. In 2000–01, the profit of company B was ₹ 10.5 crore. What was its income in that year?
- (a) ₹ 30 crores (b) ₹ 43.5 crores (c) ₹ 40.5 crores
(d) Can't be determined (e) None of these
37. If the income of company A in 1999–2000 was equal to the expenditure of company B in the same year, then the ratio of their respective profits was
- (a) 5 : 4 (b) 5 : 6 (c) 15 : 14
(d) Can't be determined (e) None of these
38. In a certain year, the expenditure and income of company A were ₹ 80 crores and ₹ 92 crores respectively. This can be true for which of the following years?
- (a) 2000–01 (b) 2001–02 (c) 2002–03 (d) 2003–04 (e) 1999–2000
39. In how many of the given years, the income of company A was more than that of company B?
- (a) Three (b) Two (c) Four
(d) Can't be determined (e) None of these
40. In a certain year, the income and profit of company B were ₹ 52 crores and ₹ 12 crores respectively. This can be true for which of the following years?
- (a) 1999–2000 (b) 2000–01 (c) 2001–02 (d) 2002–03 (e) None of these

Directions (Q. 41–45): Study the following graph and answer the questions carefully



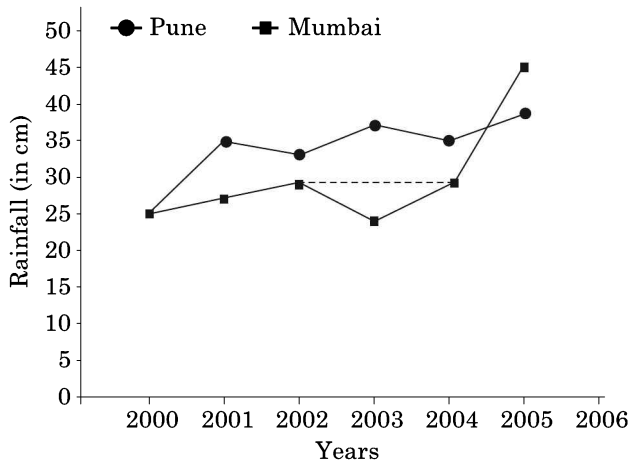
41. The difference between the production of crude oil in 1995–96 and that in 1999–2000 was exactly equal to the difference between the production of cement in which of the following pairs of years?
- (a) 1994–95 and 1997–98 (b) 1994–95 and 1999–00 (c) 1995–96 and 1997–98
 (d) 1998–99 and 1999–00 (e) None of these
42. In which of the following years was the percentage fluctuation (increase/decrease) in fertilizers the minimum from the previous year?
- (a) 1994–95 (b) 1996–97 (c) 1999–2000 (d) 1997–98 (e) 1998–99
43. What was the difference between the average cement production and the average crude oil production during the given years (in lakh tonne)?
- (a) 459 (b) 460 (c) 560 (d) 469 (e) 559
44. If there was 15% increase in the fertilizers production from 1999–2000 to 2000–01, what would be the difference between the cement and fertilizers production in 2000–01 (in lakh tonnes)?
- (a) 839.10 (b) 990.15 (c) 1012.05
 (d) Can't be determined (e) None of these
45. Find the approximate percentage increase in cement production from 1994–95 to 1999–2000.
- (a) 55% (b) 60% (c) 65% (d) 69% (e) 58%

Directions (Q. 46–50): Study the following graph carefully and answer the questions that follow.
The following diagram shows the passed candidates of class X in annual examination of six schools in a city.



46. Three-fourths of the number of boys of school C who passed is approximately what per cent more than four-sevenths of boys of school E who passed?
(a) 18% (b) 20% (c) 23% (d) 26% (e) 32%
47. If the total number of candidates appearing from schools A and F was 550, then find the approximate percentage of students failed in schools A and F together.
(a) 28% (b) 32% (c) 36% (d) 38% (e) 42%
48. In how many schools the number of boys passed in above average of the number of boys passed in all the six schools?
(a) 1 (b) 2 (c) 3 (d) 4 (e) 5
49. If total number of boys and girls appeared in school B are 150 and 120 respectively, then find the per cent difference of their passing.
(a) 2.33% (b) 5.3% (c) 8% (d) 12% (e) 7%
50. What is the difference between average number of boys and girls passed for all the schools?
(a) 2.6 (b) 3.3 (c) 4.9 (d) 6.8 (e) 7.2

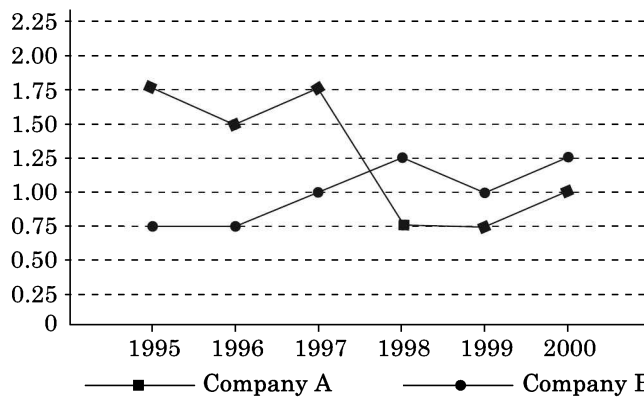
Directions (Q. 51–55): Study the following graph carefully and answer the questions given below.
Rainfall (in cm) in the cities of Mumbai and Pune over the given years.



51. What is the ratio of rainfall in the city of Mumbai to that in Pune in the year 2002?
 (a) 13 : 15 (b) 15 : 19 (c) 15 : 17 (d) 13 : 17 (e) 15 : 34
52. Which year showed the maximum rainfall in both the cities?
 (a) 2002 (b) 2004 (c) 2001 (d) 2003 (e) None of these
53. What is the average rainfall in the city of Mumbai over the years?
 (a) 30.5 cm (b) 28 cm (c) 35 cm (d) 26 cm (e) 38 cm
54. The rainfall in the city of Pune in the year 2001 is approximately, what per cent of the rainfall over the given years in the same city?
 (a) 13 (b) 17 (c) 20 (d) 25 (e) 11
55. What is the difference in the rainfall of the two cities in the year 2003?
 (a) 10 cm (b) 11 cm (c) 18 cm (d) 13 cm (e) 16 cm

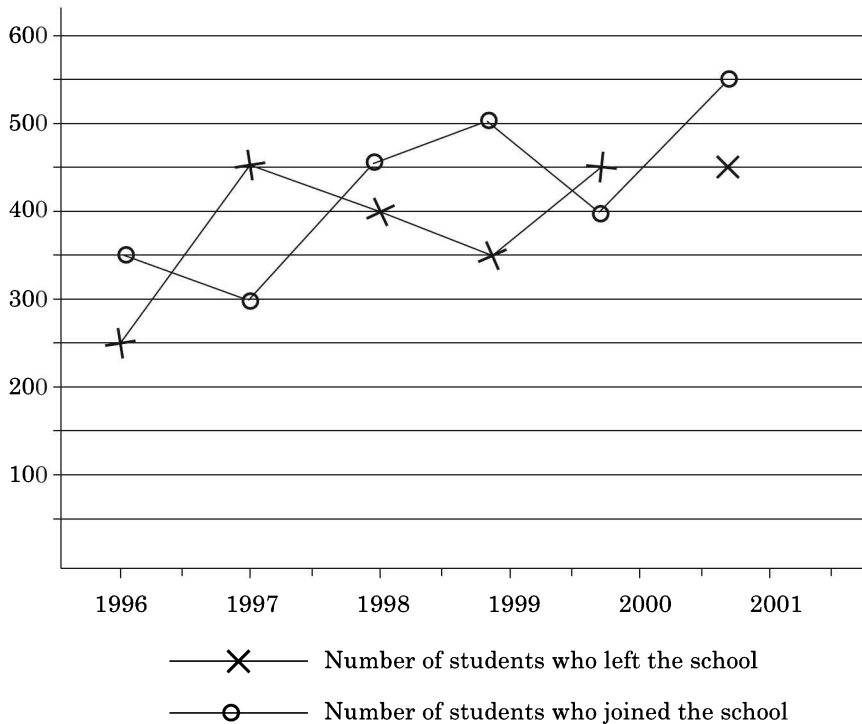
Directions (Q. 56–60): Answer the questions based on the line-graph below.

Ratio of Exports to Imports (in terms of money in ₹ crores) of two companies over the years.



56. In how many of the given years were the exports more than the imports for company A?
 (a) 2 (b) 3 (c) 4 (d) 5 (e) 6
57. In which year(s) was the difference between imports and exports of Company B the maximum?
 (a) 2000 (b) 1996 (c) 1998 and 2000
 (d) Cannot be determined (e) None of these
58. If the exports of Company A in 1998 were ₹ 237 crores, what was the amount of imports in that year?
 (a) ₹ 189.6 crores (b) ₹ 243 crores (c) ₹ 281 crores (d) ₹ 316 crores (e) None of these
59. If the imports of Company A in 1997 were increased by 40 per cent, what would be the ratio of exports to the increased imports?
 (a) 1.20 (b) 1.25 (c) 1.30
 (d) None of these (e) Cannot be determined
60. In 1995, the export of Company A was double that of Company B. If the imports of Company A during the year was ₹ 180 crores, what was the appropriate amount of imports of Company B during the year?
 (a) ₹ 190 crores (b) ₹ 210 crores (c) ₹ 225 crores
 (d) Cannot be determined (e) None of these

Directions (Q. 61–67): Study the following line-graph which gives the number of students who joined and left the school in the beginning of year for six years from 1996 to 2001. Initial strength of the school in 1995 = 3000. The questions given below the graph are based on the line-graph.



61. The strength of the school increased/decreased from 1997 to 1998 by approximately what per cent?
- (a) 1.2% (b) 1.7% (c) 2.1% (d) 2.4% (e) 2.6%
62. The number of students studying in the school during 1999 was:
- (a) 2950 (b) 3000 (c) 3100 (d) 3150 (e) 3200
63. During which of the following pairs of years, the strength of the school was same?
- (a) 1999 and 2001 (b) 1998 and 2000 (c) 1997 and 1998
(d) 1996 and 2000 (e) 1999 and 2000
64. The number of students studying in the school in 1998 was what per cent of the number of students studying in the school in 2001?
- (a) 92.13% (b) 93.75% (c) 96.88% (d) 97.25% (e) 99%
65. Among the given years, the largest number of students joined the school in the year.
- (a) 1996 (b) 1998 (c) 1999 (d) 2000 (e) 2001
66. For which year, the percentage rise/fall in the number of students, who left the school compared to the previous year, is maximum?
- (a) 1997 (b) 1998 (c) 1999 (d) 2000 (e) 2001
67. The ratio of the least number of students who joined the school to the maximum number of students who left the school in any of the years during the given period is:
- (a) 7 : 9 (b) 4 : 5 (c) 3 : 4 (d) 9 : 11 (e) 2 : 3

Directions (Q. 68–72): The following line-graph gives the percentage of the number of candidates who qualified an examination out of the total number of candidates who appeared for the examination over a period of seven years from 1994 to 2000. Study the graph and answer the questions based on it.

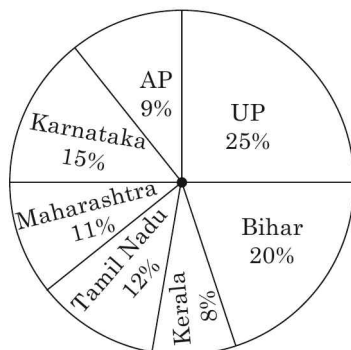
[Bank PO]

Percentage of Candidates Qualified to Appear in an Examination Over the Year.



68. The difference between the percentages of candidates qualified to appear was maximum in which of the following pairs of years?
 (a) 1994 and 1995 (b) 1997 and 1998 (c) 1998 and 1999
 (d) 1999 and 2000 (e) 1994 and 1997
69. In which pair of years was the number of candidates qualified the same?
 (a) 1995 and 1997 (b) 1995 and 2000 (c) 1998 and 1999
 (d) 1996 and 2000 (e) Data inadequate
70. If the number of candidates qualified in 1998 was 21200, what was the number of candidates appeared in 1998?
 (a) 32000 (b) 28500 (c) 26500 (d) 25000 (e) 24500
71. If the total number of candidates appeared in 1996 and 1997 together was 47400, then the total number of candidates qualified in these two years together was:
 (a) 34700 (b) 32100 (c) 31500
 (d) None of these (e) Data inadequate
72. The total number of candidates qualified in 1999 and 2000 together was 33500, and the number of candidates appeared in 1999 was 26500. What was the number of candidates appeared in 2000?
 (a) 24500 (b) 22000 (c) 20500 (d) 19000 (e) 18500

Directions (Q. 73–79): Study the following graph and table and answer the following questions given below.



The total population of the different states in 1993 was 25 lakhs.

Sex-and-literacy wise population ratio			
States	Sex	Literacy	
	M : F	Literate – Illiterate	
UP	5 : 3	2	: 7
Bihar	3 : 1	1	: 4
AP	2 : 3	2	: 1
Karnataka	3 : 5	3	: 2
Maharashtra	3 : 4	5	: 1
Tamil Nadu	3 : 2	7	: 2
Kerala	3 : 4	9	: 4

M = Male, F = Female

73. Approximately what is the total number of literate people in Maharashtra and Karnataka together?
 (a) 4.5 lakh (b) 8.5 lakh (c) 3 lakh (d) 3.5 lakh (e) 6 lakh
74. Approximately what will be the percentage of total male in UP, Maharashtra and Kerala of the total population of the given states?
 (a) 20% (b) 18% (c) 28% (d) 30% (e) 25%
75. If in the year 1993, there was an increase of 10% population of UP and 12% of Bihar compared to the previous year, then what was the ratio of the population of UP to Bihar in the year 1992?
 (a) 50 : 40 (b) 40 : 50 (c) 48 : 55 (d) 55 : 48 (e) None of these
76. What was the approximate percentage of women of Andhra Pradesh to the women of Tamil Nadu?
 (a) 90% (b) 110% (c) 120% (d) 85% (e) 95%
77. What is the ratio of the number of females in Tamil Nadu to the number of females in Kerala?
 (a) 1 : 2 (b) 1 : 1 (c) 2 : 1 (d) 2 : 3 (e) None of these
78. In Tamil Nadu, if 70% of the females are literate and 75% of the males are literate, what is the total number of illiterates in the state?
 (a) 75000 (b) 85000 (c) 71000 (d) 81000 (e) None of these
79. What is the ratio of literates in Andhra Pradesh to the literates in Bihar?
 (a) 2 : 5 (b) 3 : 5 (c) 3 : 2 (d) 2 : 3 (e) None of these

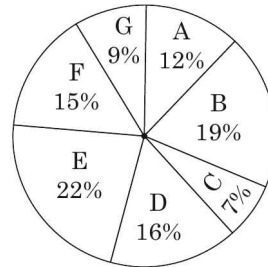
Directions (Q. 80–84): Study the following data carefully to answer these questions?

- Details of statewide distribution of candidates appeared in an examination.
- Statewise distribution of candidates appeared.

Total candidates appeared = 120000

Ratio of male and female candidates appeared

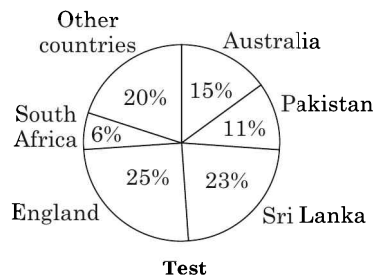
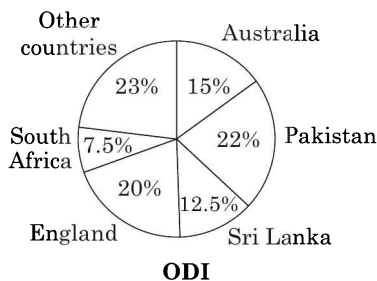
State	Ratio
	M : F
A	5 : 7
B	11 : 8
C	3 : 4
D	9 : 7
E	5 : 6
F	8 : 7
G	4 : 5



80. The number of female candidates appeared from state C is equal to the number of male candidates appeared in the examination from which state?
 (a) A (b) B (c) D (d) E (e) G
81. What is the ratio of number of male candidates appeared in the examination from state A to that from state E?
 (a) 1 : 2 (b) 1 : 1 (c) 2 : 1 (d) 6 : 7 (e) None of these
82. The total number of female candidates appeared from states E and G together is what per cent of the total number of candidates appeared in the examination from all the states?
 (a) 13 (b) 11 (c) 17 (d) 15 (e) None of these
83. What is the total number of male candidates appeared in the examination from states B and F together?
 (a) 16800 (b) 18400 (c) 13200 (d) 22800 (e) None of these
84. The number of female candidates appeared from state D is what per cent of the total number of candidates appeared in the examination from all the states?
 (a) 9 (b) 8 (c) 7.5 (d) 8.5 (e) None of these

Directions (Q. 85–89): Study the following pie charts and answer the questions that follow:

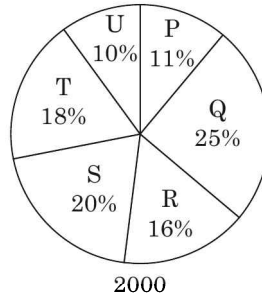
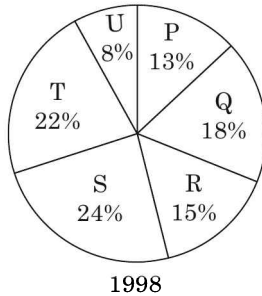
The following pie charts show the runs scored by a batsman against different countries in One-Day Internationals (ODI) and Test matches:



The runs scored by the batsman in ODI and Test matches are 8000 and 6000 respectively.

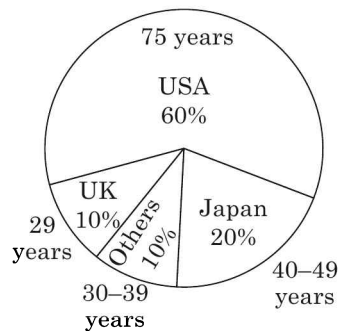
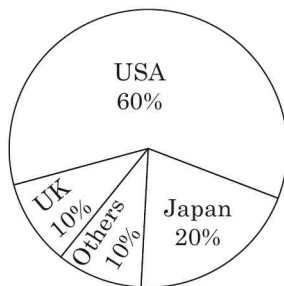
85. In the case of which of the following countries was there minimum difference in runs scored in ODI and that in Test matches?
 (a) Australia (b) Sri Lanka (c) England
 (d) South Africa (e) Pakistan
86. Find the ratio of runs scored against South Africa in ODI to that against England in Test Matches
 (a) 3 : 10 (b) 2 : 5 (c) 10 : 3 (d) 3 : 4 (e) None of these

87. If the batsman played 29 innings against Sri Lanka in ODI and remained not out in 4 innings, find his average runs scored against Sri Lanka in ODI.
 (a) 35 (b) 34.5 (c) 34.48 (d) 34.92 (e) None of these
88. If the batsman played 30 innings against Australia in ODI, how many runs must he make in his next innings (against Australia) so as to raise his average to 42 against that country?
 (a) 102 (b) 62 (c) 40 (d) 122 (e) 132
89. Find the total runs scored by the batsman in Test matches and ODI against Pakistan
 (a) 2240 (b) 3080 (c) 1540 (d) 2420 (e) None of these
- Directions (Q. 90–93):** Study the given pie diagrams carefully and answer the questions that follow.



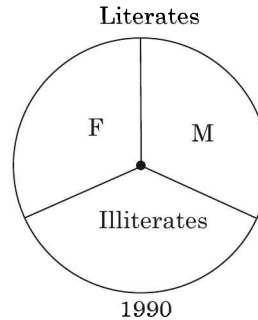
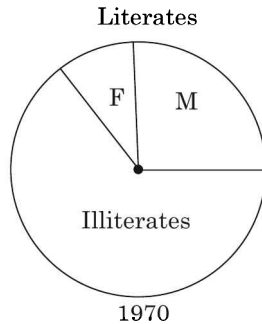
The given pie diagrams show the number of workers of different categories of a factory for the two different years. The total number of workers in 1998 and 2000 are 2000 and 2400 respectively.

90. Find the percentage increase in the number of workers in category U in 2000.
 (a) 25% (b) 50% (c) $33\frac{1}{3}\%$ (d) $66\frac{2}{3}\%$
91. What is the total number of increased workers for the categories in which the number of workers has increased?
 (a) 468 (b) 404 (c) 382 (d) None of these
92. Which category has shown decrease in the number of workers from 1998 to 2000?
 (a) P (b) R (c) S (d) None of these
93. Find the maximum difference between the number of workers of any two categories taken together for any one year and that of any two for the other year.
 (a) 660 (b) 636 (c) 416 (d) 502
94. The distribution of 100000 tourists who visited India during a particular year is shown in the given charts. Based on this, the number of Japanese tourists below the age of 39 who visited India in the year concerned is _____. [IAS]

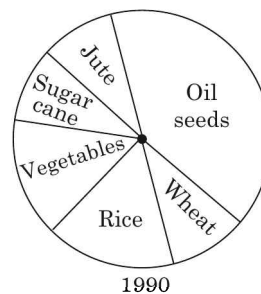
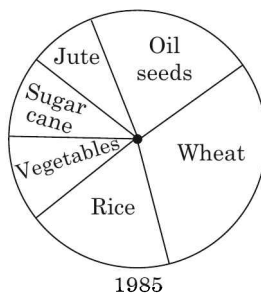


- (a) 10000 (b) 8000 (c) 6000 (d) 4000

95. The given pie charts show the proportion of literates and illiterates in a country, in the years 1970 and 1990, and also the proportion of males (M) and females (F) among the literates. Which one of the following statements can be said to be beyond any doubt? [IAS]

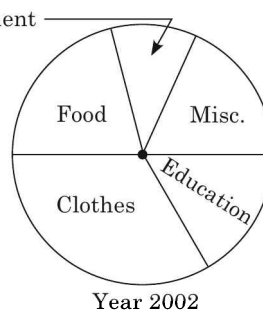
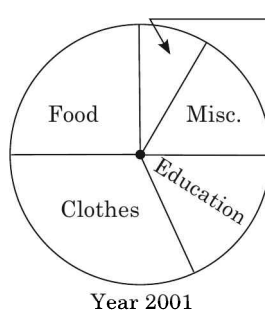


- (a) In 1970 half of the illiterates were women.
 (b) The proportion of literate males to the total population of males remained the same over the years.
 (c) The male literacy did not improve over this period.
 (d) The ratio of female literates to male literates improved significantly over this period.
96. Consider the following pie chart:



The pie charts given above depict the area under cultivation of different cash crops, for two years, 1985 and 1990. Which one of the following statement is FALSE? [IAS]

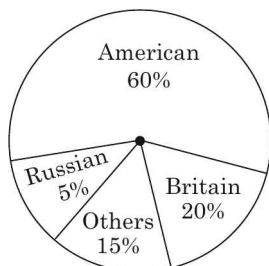
- (a) In 1985, the area under jute cultivation was less than that under rice cultivation.
 (b) More land was used for growing vegetables in 1990 than that of 1985.
 (c) In 1990, the area under rice and wheat cultivation taken together was less than $\frac{1}{2}$ the total cultivated area.
 (d) The percentage area under jute cultivation was less in 1988 than in 1990.
97. Which cannot be concluded from the following figures? [IAS]



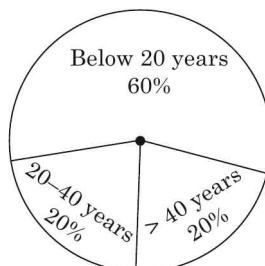
- (a) The expenditure towards entertainment is less.
 (b) The expenditure is same in both years.
 (c) The family spends more on clothes.
 (d) Each years expenditure on food is almost 25%.

98. The characteristics of foreign tourists in one year is given in the following pie chart [IAS]

Total number of tourists = 100000



Tourists countrywise



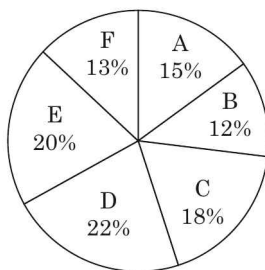
American tourists

From the chart, the number of American tourists in the age group 20–40 who visited during the year is _____.

- (a) 12000 (b) 20000 (c) 40000 (d) 60000

Directions (Q. 99–104): Study the following pie chart carefully and answer to the questions that follow.

There are six companies which are responsible for the production of two items. The pie chart shows the percentage of total production by six companies.



The cost of the total production (both items together) by six companies = ₹ 40 crores.

The following table shows the ratio of production between items I and II and the per cent profit earned for the two items.

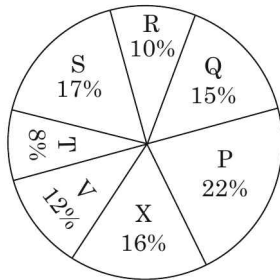
Company	Ratio of production		% profit earned	
	Item I	Item II	Item I	Item II
A	2	1	25	30
B	7	5	35	15
C	3	5	20	25
D	1	3	28	20
E	1	3	24	32
F	6	7	25	35

99. What is the total profit earned by company C for items I and II together (in ₹ crore)?
 (a) 1.166 (b) 1.665 (c) 1.125 (d) 1.765 (e) None of these
100. What is the total cost of the production of item I by companies B and F together (in ₹ crore)?
 (a) 10 (b) 4.8 (c) 5.2 (d) 12 (e) 9
101. The cost of production of item II of company D is what per cent more than the profit earned by company A on item I?
 (a) 550% (b) 580% (c) 560% (d) 490% (e) None of these
102. What is the total profit earned by company E on item II and the profit earned by company B on item I (in ₹ crore)?
 (a) 2.80 (b) 1.90 (c) 3.10 (d) 2.90 (e) None of these
103. What is the ratio of the cost of production of item I of company F to that of item II of company C?
 (a) 15 : 8 (b) 8 : 13 (c) 4 : 7 (d) 48 : 65 (e) None of these
104. Find the difference in the profit earned by company D on item II and by company A on item I.
 (a) 32 lakhs (b) 320 lakhs (c) 0.032 crore (d) 3.2 crore (e) None of these

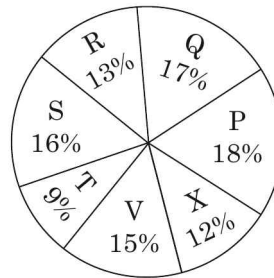
Directions (Q. 105–109): Study the following pie-chart carefully and answer the questions given below:

Distribution of candidates who were enrolled for MBA Entrance Exam and the candidates (Out of those enrolled) who passed the exam in different institutes. **[Bank PO]**

Candidates enrolled = 8550



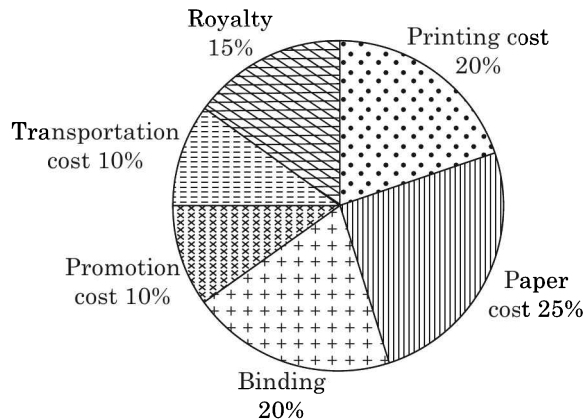
Candidates who passed the exam = 5700



105. What percentage of candidates passed the exam from institute T out of the total number of candidates enrolled from the same institute?
 (a) 50% (b) 62.5% (c) 75% (d) 80%
106. What is the ratio of candidates passed to the candidates enrolled from institute P?
 (a) 9 : 11 (b) 14 : 17 (c) 6 : 11 (d) 9 : 17
107. What is the percentage of candidates passed to the candidates enrolled for institutes Q and R together?
 (a) 68% (b) 80% (c) 74% (d) 65%
108. Which institute has the highest percentage of candidates passed to the candidates enrolled?
 (a) Q (b) R (c) V (d) T
109. The number of candidates passed from institutes S and P together exceeds the number of candidates enrolled from institutes T and R together by:
 (a) 228 (b) 279 (c) 399 (d) 407

Directions (Q. 110–118): The following pie-chart shows the percentage distribution of the expenditure incurred in publishing a book. Study the pie-chart and answer the questions based on it. **[Bank PO]**

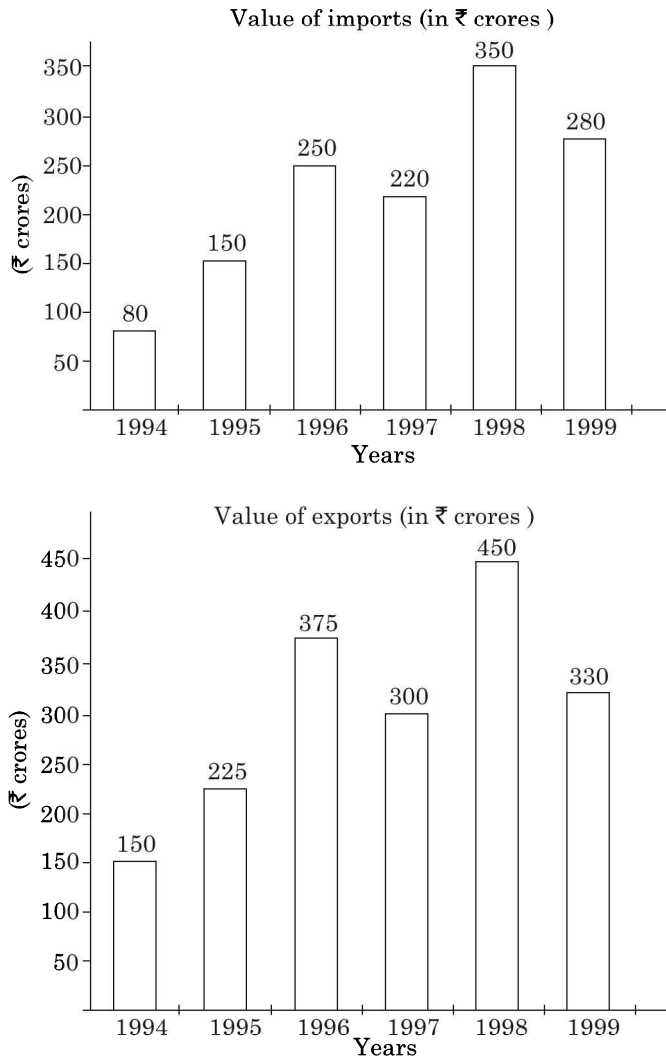
Various Expenditure (in percentage) incurred in publishing a book.



110. What is the central angle of the sector corresponding to the expenditure incurred on royalty?
 - (a) 15°
 - (b) 24°
 - (c) 54°
 - (d) 48°
111. Which two expenditures together have a central angle of 108° ?
 - (a) Binding cost and transportation cost
 - (b) Printing cost and paper cost
 - (c) Royalty and promotion cost
 - (d) Binding cost and paper cost
112. If for an edition of the book, the cost of paper is ₹ 56250, then find the promotion cost for this edition.
 - (a) ₹ 20000
 - (b) ₹ 22500
 - (c) ₹ 25500
 - (d) ₹ 28125
113. If the difference between the two expenditures are represented by 18° in the pie-chart, then these expenditures possibly are:
 - (a) Binding cost and promotion cost
 - (b) Paper cost and royalty
 - (c) Binding cost and printing cost
 - (d) Paper cost and printing cost
114. If for a certain quantity of books, the publisher has to pay ₹ 30600 as printing cost, then what will be the amount of royalty to be paid for these books?
 - (a) ₹ 19450
 - (b) ₹ 21200
 - (c) ₹ 22950
 - (d) ₹ 26150
115. The price of the book is marked 20% above the C.P. If the marked price of the book is ₹ 180, then what is the cost of paper used in a single copy of the book?
 - (a) ₹ 36
 - (b) ₹ 37.50
 - (c) ₹ 42
 - (d) ₹ 44.25
116. For an edition of 12500 copies, the amount of royalty paid by the publisher is ₹ 281250. What should be the selling price of the book if the publisher desires a profit of 5%?
 - (a) ₹ 152.50
 - (b) ₹ 157.50
 - (c) ₹ 162.50
 - (d) ₹ 167.50
117. If 5500 copies are published and the transportation cost on them amounts to ₹ 82500, then what should be the selling price of the book so that the publisher can earn a profit of 25%?
 - (a) ₹ 187.50
 - (b) ₹ 191.50
 - (c) ₹ 175
 - (d) ₹ 180
118. Royalty on the book is less than the printing cost by:
 - (a) 5%
 - (b) $33\frac{1}{3}\%$
 - (c) 20%
 - (d) 25%

Directions (Q. 119–123): Study the following graphs carefully and answer the questions that follow: **[Bank PO]**

The value of imports and exports by a company over the years.

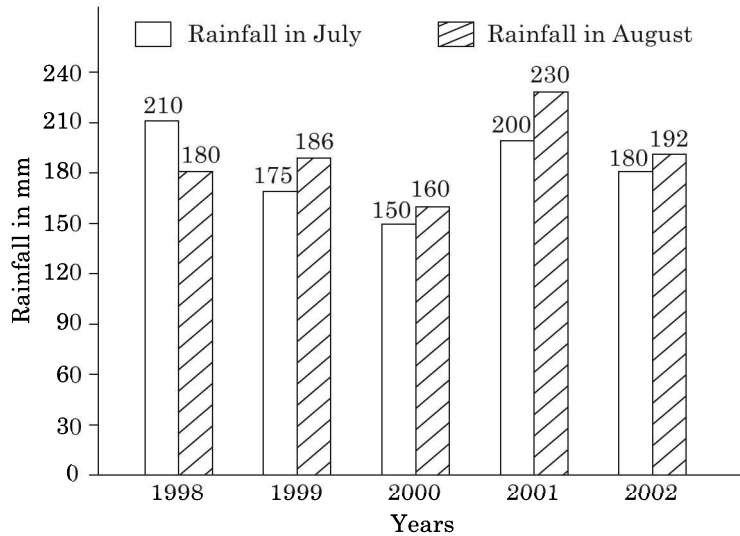


119. The value of exports in 1998 was, what percentage of the average value of imports in the years 1996, 1997 and 1999.
(a) 200 (b) 100 (c) 300 (d) 150 (e) None of these
120. The value of exports in 1996 was exactly, what percentage of the value of imports in the same year?
(a) 125 (b) 160 (c) 200 (d) 75 (e) None of these
121. What was the approximate difference between the value of average exports and the value of average imports of the given years?
(a) ₹ 85 cr. (b) ₹ 100 cr. (c) ₹ 75 cr. (d) ₹ 90 cr. (e) ₹ 80 cr.

122. In which of the following years was the difference between the value of exports and the value of imports exactly ₹ 100 crores?
 (a) 1995 (b) 1998 (c) 1997 (d) 1999 (e) None of these
123. What was the percentage increase in the value of exports from 1997 to 1998?
 (a) 150 (b) 100 (c) 75 (d) 50 (e) None of these

Directions (Q. 124–128): Study the following bar chart carefully and answer the questions that follow.

The bar chart shows the amount of rainfall (in mm) in a town in the months of July and August for different years.

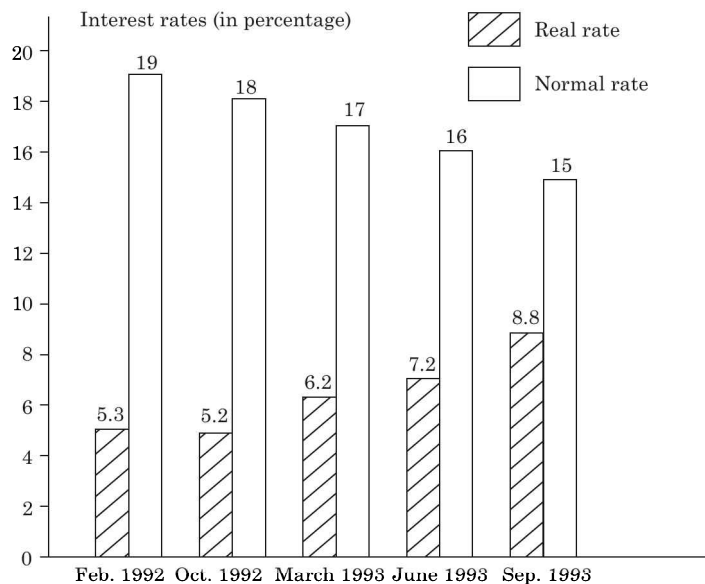


124. In August 1999, the average rainfall of first 16 days was 5 mm and that of last 16 days was 8 mm. Find the rainfall (in mm) of 16 August 1999.
 (a) 6 (b) 16 (c) 32 (d) 22 (e) None of these
125. For the given town, what is the percentage increase in the total rainfall of July and August from 2000 to 2001?
 (a) 36.7 (b) 38.7 (c) 32.6 (d) 39.6 (e) 37.6
126. The highest rainfall in a single month of August (among the given years) is how much more/less than the average rainfall of July for the given years?
 (a) 37 mm more (b) 47 mm more (c) 37 mm less
 (d) 47 mm less (e) None of these
127. The ratio of rainfall in July to that in August is the same for which of the following years?
 (a) 1998 and 2000 (b) 2000 and 1999 (c) 2000 and 2002
 (d) 2001 and 1999 (e) 1999 and 2002
128. On the basis of the given data, which of the following can be inferred?
 A. The town recorded the lowest rainfall in July 2000 among all the given months and years.
 B. The average rainfall in July (for all the given years only) is 183 mm.
 C. The average rainfall of the town during 2002 was 192 mm.
 (a) All the above (b) Only A (c) Only B and C
 (d) Only B (e) Only A and B

Years	Sales (in thousands)
1978	355
1979	340
1980	380
1981	400
1982	450
1983	400

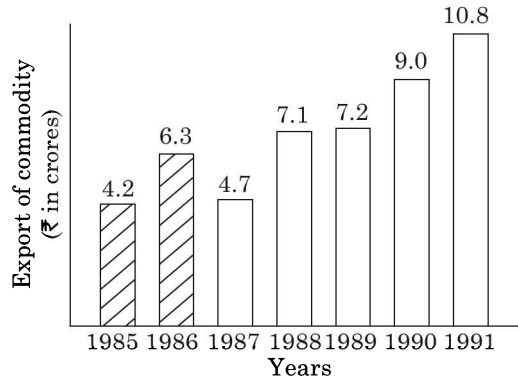
(d) ₹ Ten lakh

[IAS]

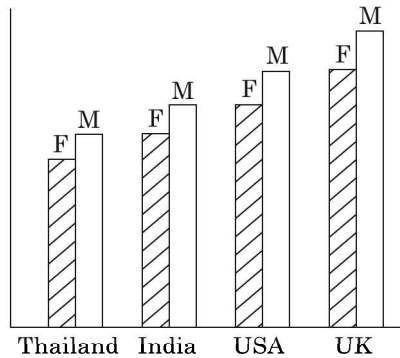


(d) The nominal rate of interest and real rate of interest are likely to become equal in 1994.

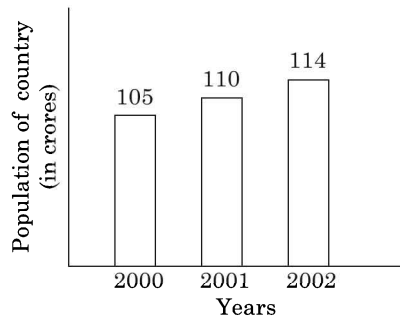
131. In which of the following pairs of years was the average export of the commodity around 6 crore? [IAS]



- (a) 1985 and 1986 (b) 1985 and 1991
(c) 1986 and 1987 (d) 1987 and 1988
132. The following figure corresponds to the heights of men and women of four different countries with standard deviation. Which of the following is true? [IAS]



- (a) All the women in the UK are taller than the men in the USA
(b) All the men in Thailand are shorter than the men in India
(c) All the men in UK are taller than the men in the USA
(d) Most men in the UK are taller than the men in India
133. Study the following bar chart and state which of the given statements are true.



- (a) The population remains the same in 2000 and 2001
 (b) The population is increased by 10% from year 2000 to 2001
 (c) The increase in population from 2000 to 2002 is nearly 9%
 (d) The increase in population from 2001 to 2002 is more than 5%



1. (e) 2. (c) 3. (b) 4. (a) 5. (d) 6. (d) 7. (e) 8. (b) 9. (a) 10. (c)
 11. (a) 12. (c) 13. (c) 14. (b) 15. (b) 16. (c) 17. (b) 18. (a) 19. (d) 20. (e)
 21. (c) 22. (d) 23. (d) 24. (b) 25. (d) 26. (c) 27. (c) 28. (e) 29. (b) 30. (a)
 31. (b) 32. (e) 33. (c) 34. (e) 35. (c) 36. (c) 37. (b) 38. (d) 39. (d) 40. (d)
 41. (c) 42. (b) 43. (a) 44. (d) 45. (b) 46. (c) 47. (d) 48. (c) 49. (a) 50. (b)
 51. (c) 52. (e) 53. (a) 54. (b) 55. (d) 56. (b) 57. (d) 58. (d) 59. (b) 60. (b)
 61. (b) 62. (d) 63. (d) 64. (b) 65. (e) 66. (a) 67. (e) 68. (b) 69. (e) 70. (c)
 71. (e) 72. (c) 73. (a) 74. (e) 75. (e) 76. (b) 77. (e) 78. (d) 79. (c) 80. (e)
 81. (a) 82. (c) 83. (d) 84. (e) 85. (c) 86. (b) 87. (e) 88. (a) 89. (d) 90. (b)
 91. (d) 92. (d) 93. (a) 94. (d) 95. (c) 96. (d) 97. (b) 98. (a) 99. (b) 100. (c)
 101. (c) 102. (d) 103. (e) 104. (a) 105. (c) 106. (c) 107. (b) 108. (b) 109. (c) 110. (c)
 111. (a) 112. (b) 113. (d) 114. (c) 115. (b) 116. (b) 117. (a) 118. (a) 119. (e) 120. (e)
 121. (a) 122. (b) 123. (d) 124. (d) 125. (b) 126. (b) 127. (c) 128. (e) 129. (c) 130. (b)
 131. (d) 132. (d) 133. (c)



Solutions with Necessary Explanation

1. Ratio of literate population of state C in the years 1997 and 1998 = $\frac{2 \times \frac{78}{100}}{3 \times \frac{81}{100}} = 52 : 81$. Hence

answer is (e).

2. The average literate population of all the states in the year of 1998

$$= 1200000 \times \frac{\left(\frac{72}{100} + \frac{81}{100} + \frac{81}{100} + \frac{65}{100} + \frac{93}{100} + \frac{88}{100} \right)}{6}$$

$$= \frac{1200000 \times 480}{600} = 960000$$

3. The required percentage increase in the literate population from 1998 to 2001 in the state 'D'

$$= \frac{(75 - 65)}{65} \times 100 = 15.38\% \approx 15\%$$

4. The literate population of state B in 2001 = $1000000 \times \frac{110}{100} \times \frac{110}{100} \times \frac{87}{100} = 1052700$

5. The ratio of populations of the given state is needed to calculate average literate population in the given year. Therefore, the answer is (d).

6. Percentage of students getting placement from institute B in the given years are as shown in the following table.

1998	1999	2000	2001	2002	2003
89.52%	85.34%	85.71%	87.40%	90.66%	89.74%

∴ Percentage of students getting placement from institute B was **highest in 2002**, i.e., 90.66%.

7. The overall percentage of students getting placement from all the institutes together the year 2000.

$$= \frac{400 + 600 + 370 + 540 + 420 + 420}{580 + 700 + 410 + 580 + 450 + 480} \times 100$$

$$= \frac{2750}{3200} \times 100 = 85.93 \approx \mathbf{86\%}$$

8. The percentage of students getting placement in the year 2002 can be seen in the following table.

A	B	C	D	E	F
69.23%	90.66%	92.30%	88.33%	94%	90.90%

∴ **Institute E** is the one with highest percentage of students getting placed.

9. Percentage of students getting placement in 1998 = $\frac{300}{400} \times 100 = \mathbf{75\%}$

Percentage of students getting placement in 1999 = $\frac{380}{450} \times 100 = \mathbf{84.44\%}$

Hence, the required percentage increase = $\left(\frac{84.44 - 75}{75} \right) \times 100 = \mathbf{12.59\%}$

10. The required ratio between the average strength of institutes

$$= \frac{380 + 480 + 450 + 520 + 500 + 540}{560 + 625 + 580 + 550 + 600 + 630} = \frac{2870}{3545} \approx \mathbf{574 : 709}$$

11. Ratio of minor boys to girls in school F = 2 : 3

Let the no. of minor boys = $2x$

and the no. of minor girls = $3x$

∴ Average height of all minor students = $\frac{2x \times \frac{11}{2} + 3x \times 5}{2x + 3x} = \frac{26x}{5x} = \mathbf{5\frac{1}{5} \text{ ft}}$

12. The number of major boys of the school = $1000 \times \frac{4}{100} \times \frac{1}{2} = \mathbf{20}$

Total age of 20 boys = $20 \times 18.2 = 364$ years

When principal's age is included, total age = $21 \times 20 = 420$ years

∴ Age of principal = $420 - 364 = \mathbf{56 \text{ years}}$

13. 900

14. 40

15. Minor girls in school F = $1250 \times \frac{80}{100} \times \frac{3}{5} = 600$

Major girls in school F = $1250 \times \frac{20}{100} \times \frac{2}{5} = 100$

∴ Total girls = $600 + 100 = \mathbf{700}$

16. Total number of girls of class 7th = 50% of $(22 + 18 + 24 + 12 + 6) = 41$

Total boys of class XII = $24 + 12 + 20 + 8 + 6 = 70$

$\therefore$ Total number of students = $444 + 50\%$ of $444 = 666$

$$\therefore \text{Required percentage} = \frac{70 + 41}{666} \times 100 = 16\frac{2}{3} \%$$

17. Girl hockey players of class X = 50% of $22 = 11$

$\therefore$ Number of handshakes = ${}^{11}C_2 = 55$

18. Badminton players of class XII = $6 + 3 = 9$

$$\therefore \text{Number of ways of selection} = {}^1C_1 \times {}^8C_4 = \frac{1 \times 8 \times 7 \times 6 \times 5}{1 \times 2 \times 3 \times 4} = 70$$

19. 77

20. Total football players = $88 + 44 = 132$. Hence answer is (e).

21. (i) and (ii). Hence answer is (c).

$$23. 40\% \text{ of } 50 = \frac{40}{100} \times 50 = 20$$

Required number = $100 - \text{Number of students scoring more than 20 marks}$
 $= 100 - 73 = 27$

24. 60% of $50 = 30$

Required number = Number of students scoring 30 and above marks in Physics
 $= 32$

25. Number of students passed with 30 as cut-off marks in Chemistry = 21

Number of students passed with 30 as cut-off marks in aggregate = 27

$\therefore$ Required difference = $27 - 21 = 6$

26. Number of students getting at least 60% marks in Chemistry

= Number of students scoring 30 marks and above in Chemistry = 21

Number of students getting at least 40% marks in aggregate

= Number of students scoring 20 marks and above in aggregate = 73

$$\text{Required percentage} = \left(\frac{21}{73} \times 100 \right) \% = 28.77\% \approx 29 \%$$

27. 23 students are to be selected for a symposium in Chemistry.

From the given ranges, only from 20–30 range, $66 - 21 = 45$ or less number of students can be selected. Hence answer is (c).

28. Here 80 students are required to be qualified. Therefore, correct answer is either **10 and above** or **0 and above** thereby 82 students or 100 students qualify paper I. Hence answer is (e).

29. Required difference = $31 - 27 = 4$

30. Students obtained 60% and more in paper II = 31

Students who obtained 40% and more in aggregate = 71

$$\therefore \text{Required percentage} = \frac{31}{71} \times 100 = 43.66\% = 44 \%$$

31. Let Income = I and Expenditure = E

$$\frac{I_{2002} - E_{2002}}{E_{2002}} = \frac{I_{2000} - E_{2000}}{E_{2000}} \quad (\because \text{net profit for both the years is the same})$$

$$\text{i.e.} \quad \frac{I_{2002}}{E_{2002}} = \frac{I_{2000}}{E_{2000}}$$

$$\frac{I_{2002}}{120\%E_{2000}} = \frac{I_{2000}}{E_{2000}}$$

$$\text{or} \quad \frac{I_{2002}}{I_{2000}} = 120\% \text{ or } 20\% \text{ more}$$

i.e. income in 2002 is **20%** more than income in 2000.



Short cut: Given net profit 2000 = Net profit 2002

$$\text{Expenditure 2002} = \frac{120}{100} \times \text{Expenditure 2000}$$

$$\therefore \text{Income 2002} = \frac{120}{100} \times \text{Income 2000}$$

32. When the percentage profit is minimum ratio of income to expenditure is also the minimum.

$$\text{i.e.} \quad \% \text{ profit} = \frac{I - E}{E} \times 100 = \left[\frac{I}{E} - 1 \right] \times 100$$

$$\text{i.e.} \quad \% \text{ profit} \propto \frac{I}{E}$$

Here (e) is the answer.

33. In 2002, ratio of net profit = $35/20 = 1.75$

In 2003, ratio of net profit = $50/35 = 1.43$

In 2005, ratio of net profit = $60/45 = 1.33$

$\therefore$ the answer is (c).

34. $E_{2004} = E_{2006} = k$; $I_{2006} = 15.5$ lakhs; $I_{2004} = ?$

$$\frac{I_{2006} - E_{2006}}{E_{2006}} = \frac{55}{100} = \frac{15.5 - k}{k} = \frac{15.5}{k} - 1 \quad (1)$$

$$\frac{I_{2004} - E_{2004}}{E_{2004}} = \frac{45}{100} = \frac{I_{2004} - k}{E_{2004}} = \frac{E_{2004}}{k} - 1 \quad (2)$$

$$(1) \Rightarrow \frac{15.5}{k} = 1 + \frac{55}{100} = \frac{155}{100} \quad (A)$$

$$(2) \Rightarrow \frac{I_{2004}}{k} = 1 + \frac{45}{100} = \frac{145}{100} \quad (B)$$

$$\frac{(A)}{(B)} \Rightarrow \frac{(15.5/k)}{(I_{2004}/k)}$$

$$\Rightarrow \frac{15.5}{I_{2004}} = \frac{155}{145} = \frac{31}{29}$$

$$\Rightarrow I_{2004} = \frac{29 \times 15.5}{31} = 14.5 \text{ lakhs}$$

Hence answer is (e).



Short cut: (By rule of fraction)

$$I_{2004} \left(\frac{100 + 45}{100 + 55} \right) I_{2006} = \frac{145}{155} \times 15.5 \text{ lakhs} = \mathbf{14.5 \text{ lakhs}}$$

Here expenditures are same in both years since the % profit is less in 2004 than in 2006, income in 2004 will also be less than that in 2006.

35. $I_{2003} = 25$ lakhs; $E_{2003} = ?$

$$\frac{I_{2003} - E_{2003}}{E_{2003}} = \frac{50}{100} = \frac{1}{2} \quad (1)$$

$$(1) \Rightarrow \frac{25 - E_{2003}}{E_{2003}} = \frac{25}{E_{2003}} - 1 = \frac{1}{2}$$

$$\Rightarrow 1 + \frac{1}{2} = \frac{25}{E_{2003}} = \frac{3}{2}$$

$$\Rightarrow E_{2003} = \frac{25 \times 2}{3} = \mathbf{16.67 \text{ lakhs}}$$



Short cut: Quicker Method

$$\text{Income} = \text{Expenditure} \times \left[\frac{100 + \% \text{ profit}}{100} \right]$$

$$\text{i.e.} \quad \text{Expenditure} = \text{Income} \times \frac{100}{[100 + \% \text{ profit}]}$$

$$\begin{aligned} \text{i.e.} \quad E_{2003} &= I_{2003} \times \frac{100}{100 + 50} \\ &= I_{2003} \times \frac{100}{150} = I_{2003} \times \frac{2}{3} \\ &= \frac{25 \times 2}{3} = \mathbf{16.67 \text{ lakhs}} \end{aligned}$$

Here expenditures are same in both years since the % profit is less in 2004 than in 2006, income in 2004 will also be less than that in 2006.

36. Required income = $\frac{135}{35} \times ₹ 10.5 \text{ crore} = ₹ 40.5 \text{ crore}$

37. Given income of A = Expenditure of B

or

Expenditure of A + Profit of A = Expenditure of B

or $\frac{100}{50} \times \text{Profit of A} + \text{Profit of A} = \frac{100}{40} \times \text{Profit of B}$

or $\frac{\text{Profit of A}}{\text{Profit of B}} = \frac{5}{6} = 5 : 6$

38. % profit of company A in that year = $\frac{92 - 80}{80} \times 100 = 15\%$

From the graph, profit of company A = 15% **during 2003–04.**

40. Expenditure of company B in that year = $52 - 12 = ₹ 40 \text{ crores.}$

$\therefore \text{Profit} = \frac{52 - 40}{40} \times 100 = 30\%$

and profit of company B is 30% **during 2002–03.**

41. Difference in the production of crude oil from 1995–96 to 1999–2000 = 33 = Difference in production of cement from 1995–96 to 1997–98. Hence answer is (c).

45. Required % increase = $\frac{1007 - 623}{623} \times 100 = 60\%$

46. $\frac{92 \times \frac{3}{4} - 98 \times \frac{4}{7}}{98 \times \frac{4}{7}} \times 100 = \frac{13}{56} \times 100 = 23\%$

47. Total number of passed students = $82 + 64 + 102 + 92 = 340$

$\therefore \text{Number of failed students} = 550 - 340 = 210$

$\therefore \text{Required percentage of failed students} = \frac{210}{550} \times 100 = 38.1\% \approx 38\%$

51. The required ratio = 30 : 34 or **15 : 17**

52. 2005. Hence answer is (e).

53. The required average = $\frac{25 + 29 + 30 + 24 + 30 + 45}{6} = \frac{183}{6} = 30.5$

54. The required per cent = $\frac{35}{(26 + 35 + 34 + 37 + 35 + 40)} \times 100 = \frac{35}{207} \times 100 = 16.9\%$

55. The required difference = $37 - 24 = 13 \text{ cm}$

56. Exports are more than imports for the years with exports to imports ratio greater than 1. Here for company A it is in 1995, 1996 and 1997, i.e. for **3 years.** Hence answer is (b).

57. From the given line graph only exports to imports ratio is available. The difference between imports and exports cannot be determined. Hence answer is (d).

58. In the year 1998, for company A, export to import ratio = 0.75;

Given that export in that year = ₹ 237 crores.

Let import for the year be ₹ x crores.

$$\therefore \frac{237}{x} = 0.75$$

$$\Rightarrow x = \frac{237}{0.75} = 79 \times 4 = \text{₹ } 316 \text{ crores}$$

59. In the year 1997, for company A, Export to Import ratio = 1.75;

$$\text{i.e. } \frac{E}{I} = 1.75$$

$$\text{New import } I_1 = I + \frac{40}{100} I = 1.4I.$$

$$\text{Required ratio} = \frac{E}{I_1} = \frac{1.75I}{1.4I} = 1.25$$

60. For company A in 1995,

$$\frac{E_A}{I_A} = 1.75 \quad (1)$$

For company B in 1995,

$$\frac{E_B}{I_B} = 0.75 \quad (2)$$

Given $I_A = \text{₹ } 180$ crores; $I_B = ?$

$$E_A = 2E_B \quad (3)$$

$$(1) \Rightarrow E_A = 1.75 I_A = 1.75 \times 180 = 315 \text{ crores}$$

$$\therefore (3) \Rightarrow E_B = \frac{E_A}{2} = \frac{315}{2} = \text{crores}$$

$$(2) \Rightarrow I_B = \frac{E_B}{0.75} = \frac{315}{2 \times 0.75} = \frac{630}{3} = 210 \text{ crores}$$

61. From the line graph,

In 1996: Number of students left school = 250

Number of students joined school = 350

In 1997: Number of students left school = 450

Number of students joined school = 300

In 1998: Number of students left school = 400

Number of students joined school = 450

In 1999: Number of students left school = 350

Number of students joined school = 500

In 2000: Number of students left school = 450

Number of students joined school = 400

In 2001: Number of students left school = 450

Number of students joined school = 550

$\therefore$ Strength of school in various years is:

$$1996: 3000 - 250 + 350 = 3100$$

$$1997: 3100 - 450 + 300 = 2950$$

$$1998: 2950 - 400 + 450 = 3000$$

$$\begin{aligned} 1999: & 3000 - 350 + 500 = 3150 \\ 2000: & 3150 - 450 + 400 = 3100 \\ 2001: & 3100 - 450 + 550 = 3200 \end{aligned}$$

$$\text{Required percentage} = \left(\frac{3000 - 2950}{2950} \right) \times 100 = 1.69\% = \mathbf{1.7\% \text{ decreased}}$$

62. In 1999, strength = **3150**.

63. In 1996 and 2000, strength was same. Hence answer is (d).

64. Required percentage = $\frac{3000}{3200} \times 100 = \mathbf{93.75\%}$.

65. Largest number of students joined the school in **2001**.

66. Percentage rise/fall in the number of students left the school (compared to previous year) in given years are:

$$1997: \left(\frac{450 - 250}{250} \right) \times 100 = 80\% \text{ (rise)}$$

$$1998: \left(\frac{450 - 400}{450} \right) \times 100\% = 11.11\% \text{ (fall)}$$

$$1999: \left(\frac{400 - 350}{400} \right) \times 100\% = 12.5\% \text{ (fall)}$$

$$2000: \left(\frac{450 - 350}{350} \right) \times 100\% = 28.57\% \text{ (rise)}$$

$$2001: \left(\frac{450 - 450}{450} \right) \times 100\% = 0\%$$

Per cent rise/fall is maximum in 1997 = 80%. Hence answer is (a).

67. From the above calculations,

$$\text{Required ratio} = \frac{300}{450} = \frac{2}{3} \text{ or } \mathbf{2 : 3}$$

68. Difference between percentages of candidates qualified to appear for the given pair of years are:

$$1994 \text{ and } 1995: 50 - 30 = 20\%$$

$$1997 \text{ and } 1998: 80 - 50 = 30\%$$

$$1998 \text{ and } 1999: 80 - 80 = 0\%$$

$$1999 \text{ and } 2000: 80 - 60 = 20\%$$

$$1994 \text{ and } 1997: 50 - 30 = 20\%$$

Here required percentage is maximum during **1997–1998**.

69. Number of candidates qualified cannot be determined as the line graph gives only percentage. Hence answer is (e).

70. Number of candidates qualified in 1998 = 21200

Let the number of candidates appeared be x .

$$\therefore \frac{21200}{x} = 0.8$$

$$\Rightarrow x = \frac{21200}{0.8} = \mathbf{26500}$$

71. From the total number of candidates appeared in 1996 and 1997, it is not possible to find out the number of candidates qualified in each year.

$\therefore$ Data is inadequate. Hence answer is (e).

72. Let number of candidates appeared in 2000 be x

Number of candidates qualified in 1999 and 2000 = 33500

Number of candidates appeared in 1999 = 26500

Number of candidates qualified in 1999 = $0.8 \times 26500 = 21200$

$\therefore$ Number of candidates qualified in 2000 = $33500 - 21200 = 12300$

$$\therefore x = \frac{12300}{0.6} = \frac{123000}{6} = \mathbf{20500}$$

73. Total number of literate people in Maharashtra and Karnataka.

$$\begin{aligned} &= \left[\frac{5}{6} \times 11\% + \frac{3}{5} \times 15\% \right] 25 \text{ lakhs} \\ &= \left[\frac{55}{6} + 9 \right] \% \times 25 \text{ lakhs} = \left[\frac{55}{6} + 9 \right] \times \frac{1}{4} \text{ lakhs} \\ &= \left[\frac{55 + 54}{6} \right] \times \frac{1}{4} = \frac{109}{24} \text{ lakhs} = \mathbf{4.54 \text{ lakh}} \end{aligned}$$

74. Total male in UP, Maharashtra and Kerala in percentage of total population

$$\begin{aligned} &= \frac{5}{8} \times 25\% + \frac{3}{7} \times 11\% + \frac{3}{7} \times 8\% \\ &= \left[\frac{125}{8} + \frac{33}{7} + \frac{24}{7} \right] \% = \left[\frac{128}{8} + \frac{57}{7} \right] \% \\ &= [15.63 + 8.14]\% = 23.77\% \approx 25\% \end{aligned}$$

$\therefore$ Approximate percentage = **25%**

$$75. \text{ Required ratio} = \frac{\frac{100}{110} \times \frac{25}{100} \times 25 \text{ lakhs}}{\frac{100}{112} \times \frac{20}{100} \times 25 \text{ lakhs}} = \frac{112}{110} \times \frac{25}{20} = \frac{112 \times 1}{11 \times 2 \times 4} = \frac{112}{88} = \mathbf{\frac{14}{11}}$$

or **14 : 11**. Hence answer is (e).

76. Approximate percentage of women (AP)/women (TN)

$$\begin{aligned} &= \frac{\frac{3}{5} \times 9\% \times 25 \text{ lakhs}}{\frac{2}{5} \times 12\% \times 25 \text{ lakhs}} \times 100 \\ &= \frac{3 \times 9}{2 \times 12} \times 100 = \frac{9}{2 \times 4} \times 100 = \frac{9}{8} \times 100\% = \mathbf{112.5\% \approx 110\%} \end{aligned}$$

$$\begin{aligned}
 77. \quad \frac{\text{Number of females (TN)}}{\text{Number of females (Kerala)}} &= \frac{\frac{2}{5} \times \frac{12}{100} \times 25 \text{ lakhs}}{\frac{4}{7} \times \frac{8}{100} \times 25 \text{ lakhs}} \\
 &= \frac{\frac{2}{5} \times 12}{\frac{4}{7} \times 8} = \frac{24}{5} \times \frac{7}{32} = \frac{3 \times 7}{5 \times 4} = \frac{21}{20}
 \end{aligned}$$

Hence answer is (e).

$$\begin{aligned}
 78. \quad \text{Total number of illiterates in Tamil Nadu} &= \left(1 - \frac{70}{100}\right) \text{ females} + \left(1 - \frac{75}{100}\right) \text{ males} \\
 &= 25 \text{ lakhs} \times \left[\frac{30}{100} \times \frac{2}{5} \times \frac{12}{100} + \frac{25}{100} \times \frac{3}{5} \times \frac{12}{100} \right] \\
 &= \frac{25 \text{ lakhs}}{100 \times 100} \left[\frac{30 \times 2 \times 12}{5} + \frac{25 \times 3 \times 12}{5} \right] \\
 &= \frac{25 \text{ lakhs}}{100 \times 100} [6 \times 24 + 15 \times 12] = \frac{25 \text{ lakhs}}{100 \times 100} [144 + 180] \\
 &= \frac{25 \times 324 \times 10^5}{10^4} = 25 \times 324 \times 10 = \mathbf{81000}
 \end{aligned}$$

$$79. \quad \text{Required ratio} = \frac{\frac{2}{3} \times 9\% \times 25 \text{ lakhs}}{\frac{1}{5} \times 20\% \times 25 \text{ lakhs}} = \frac{6\%}{4\%} = \frac{6}{4} = \frac{3}{2} \text{ or } \mathbf{3 : 2}$$

80. No. of female candidates appeared from state C

$$= 120000 \times \frac{7}{100} \times \frac{4}{7} = \mathbf{4800}$$

No. of male candidates appeared from state G

$$= 120000 \times \frac{9}{100} \times \frac{4}{9} = \mathbf{4800}$$

$$\text{i.e.} \quad \left(\frac{7}{100} \times \frac{4}{7} \right) = \left(\frac{9}{100} \times \frac{4}{9} \right)$$

Hence answer is (e).

$$81. \quad \text{Required ratio} = \frac{12 \times \left(\frac{5}{12} \right)}{22 \times \left(\frac{5}{11} \right)} = \mathbf{1 : 2}$$

82. Total number of female candidates appeared from states E and G together

$$= 120000 \times \frac{22}{100} \times \frac{6}{11} + 120000 \times \frac{9}{100} \times \frac{5}{9}$$

$$= 120000 \left[\frac{12}{100} + \frac{5}{100} \right] = \frac{120000 \times 17}{100}$$

∴ No. of female candidates = 17% of total candidates appeared from states E and G.



Short cut: Required percentage = $22 \times \frac{6}{5+6} + 9 \times \frac{5}{4+5} = 12 + 5 = 17\%$

83. The total no. of male candidates appeared in the examination from states B and F together

$$= 120000 \times \frac{19}{100} \times \frac{11}{19} + 120000 \times \frac{15}{1000} \times \frac{8}{15} = 13200 + 9600 = \mathbf{22800}$$

84. The percentage of female candidates appeared from state D with respect to the total no. of candidates appeared in the examination

$$= \frac{120000 \times \frac{16}{100} \times \frac{7}{16} \times 100}{120000} = 7\%$$

From the pie chart this can be directly obtained.

i.e. total candidates from state D = 16%. As ratio of male to female is given as 9 : 7.

∴ No. of female candidates = 7%. Hence answer is (e).

85. **England**

86. $\frac{\text{Runs against South Africa in ODI}}{\text{Runs against England in Test}} = \frac{7.5\% \text{ of } 8000}{25\% \text{ of } 6000} = \mathbf{2 : 5}$

87. His total runs against Sri Lanka in ODI = 12.5% of 8000 = **1000 runs**

$$\therefore \text{Average runs} = \frac{1000}{29 - 4} = \frac{1000}{25} = \mathbf{40 \text{ runs}}$$

Hence answer is (e).

88. His total runs against Australia in ODI = 15% of 8000 = 1200 runs

His runs after 31st innings = $31 \times 42 = \mathbf{1302 \text{ runs}}$

∴ Required answer = $1302 - 1200 = \mathbf{102 \text{ runs}}$

89. Total runs scored against Pakistan

$$= 22\% \text{ of } 8000 + 11\% \text{ of } 6000 = 1760 + 660 = \mathbf{2420 \text{ runs}}$$

90. Required increase = $\frac{(10\% \text{ of } 2400) - (8\% \text{ of } 2000)}{(8\% \text{ of } 2000)} \times 100 = \frac{240 - 160}{160} \times 100 = \mathbf{50\%}$

91. Workers have increased in categories of P, Q, R and U.

∴ Total no. of increased workers

$$= \left(\frac{11}{100} \times 2400 - \frac{13}{100} \times 2000 \right) + \left(\frac{25}{100} \times 2400 - \frac{18}{100} \times 2000 \right) \\ + \left(\frac{16}{100} \times 2400 - \frac{15}{100} \times 2000 \right) + \left(\frac{10}{100} \times 2400 - \frac{8}{100} \times 2000 \right)$$

$$\begin{aligned}
 &= (264 - 260) + (600 - 360) + (384 - 300) + (240 - 160) \\
 &= 4 + 240 + 84 + 80 = 408
 \end{aligned}$$

Hence answer is (d).

- 93. Case I:** Two categories having the highest number of workers in 1998 and simultaneously the two categories having the least number of workers in 2000.

$$\text{Difference} = (480 + 440) - (240 + 264) = 416$$

Case II: Two categories having the highest number of workers in 2000 and simultaneously the two categories having the least number of workers in 1998.

$$\therefore \text{Difference} = (600 + 480) - (160 + 260) = 660$$

- 94.** Number of Japanese tourists below the age of 39 who visited India.

$$= 100000 \times \frac{20}{100} \times \frac{20}{100} = 4000$$

i.e. 20% of total strength in Japanese.

20% of total strength agewise lies below 39 years.

$$\therefore \text{Required number} = 100000 \times \frac{20}{100} \times \frac{20}{100} = 4000$$

- 98.** Number of American tourists in the age group 20–40 years = $100000 \times \frac{60}{100} \times \frac{20}{100} = 12000$

$$\text{100. Total cost} = \left(\frac{7}{12} \times \frac{12}{100} + \frac{6}{13} \times \frac{13}{100} \right) 40 = \text{₹ } 5.2 \text{ crore}$$

$$\text{102. Total profit} = \left(\frac{3}{4} \times \frac{20}{100} \times \frac{32}{100} + \frac{7}{12} \times \frac{12}{100} \times \frac{35}{100} \right) 40 = \text{₹ } 2.90 \text{ crore}$$

$$\begin{aligned}
 \text{105. Required percentage} &= \left(\frac{9\% \text{ of } 5700}{8\% \text{ of } 8550} \right) \times 100 \\
 &= \left(\frac{9 \times 5700}{8 \times 8550} \right) \times 100 = 75\%
 \end{aligned}$$

$$\begin{aligned}
 \text{106. Required ratio} &= \left(\frac{18\% \text{ of } 5700}{22\% \text{ of } 8550} \right) \\
 &= \left(\frac{18 \times 5700}{22 \times 8550} \right) = 6 : 11
 \end{aligned}$$

$$\begin{aligned}
 \text{107. Required percentage} &= \left(\frac{30\% \text{ of } 5700}{25\% \text{ of } 8550} \right) \times 100 \\
 &= \left(\frac{30 \times 5700}{25 \times 8550} \right) \times 100 = 80\%
 \end{aligned}$$

- 108.** Percentage of candidates passed to candidates enrolled

$$\text{For Q: } \left(\frac{17\% \text{ of } 5700}{15\% \text{ of } 8550} \right) \times 100 = \left(\frac{17 \times 5700}{15 \times 8550} \right) \times 100 = \frac{17 \times 2}{15 \times 3} \times 100 = 75.56\%$$

$$\text{For R : } \left(\frac{13\% \text{ of } 5700}{10\% \text{ of } 8550} \right) \times 100 = \left(\frac{13 \times 5700}{10 \times 8550} \right) \times 100 = \frac{13 \times 2}{10 \times 3} \times 100 = \mathbf{86.67\%}$$

$$\text{For V : } \left(\frac{15\% \text{ of } 5700}{12\% \text{ of } 8550} \right) \times 100 = \left(\frac{15 \times 5700}{12 \times 8550} \right) \times 100 = \frac{15 \times 2}{12 \times 3} \times 100 = \mathbf{83.33\%}$$

$$\text{For T : } \left(\frac{9\% \text{ of } 5700}{8\% \text{ of } 8550} \right) \times 100 = \left(\frac{9 \times 5700}{8 \times 8550} \right) \times 100 = \left(\frac{9 \times 2}{8 \times 3} \right) \times 100 = \mathbf{75\%}$$

Among these the institute R has the highest percentage of candidates passed to candidates enrolled. Hence answer is (b).

- 109.** Number of candidates passed from institutes S and P

$$= (16 + 18)\% \text{ of } 5700 = 34\% \text{ of } 5700 \quad (1)$$

Number of candidates enrolled from institutes T and R

$$= (8 + 10)\% \text{ of } 8550 = 18\% \text{ of } 8550 \quad (2)$$

From (1) and (2),

$$\begin{aligned} \text{Required difference} &= \frac{34}{100} \times 5700 - \frac{18}{100} \times 8550 \\ &= 34 \times 57 - 18 \times 85.5 \\ &= 57 (34 - 18 \times 1.5) \\ &= 57 (34 - 27) = 57 \times 7 = \mathbf{399} \end{aligned}$$

- 110.** Central angle of sector corresponding to royalty = 15% of $360^\circ = 54^\circ$

- 111.** Central angle of 108° corresponds to $\left(108^\circ \times \frac{100}{360^\circ}\right)\% = \mathbf{30\%}$ of total expenditure.

From the pie-chart, Binding cost + Transportation cost

$$= 20\% + 10\% = 30\% \text{ of total expenditure.}$$

Hence answer is (a).

- 112.** Let the promotion cost be ₹ x .

Given Paper cost : Promotion cost = 25 : 10 = 5 : 2

We have Paper cost = ₹ 56250

$$\therefore \frac{\text{Promotion cost}}{\text{Paper cost}} = \frac{x}{56250} = \frac{2}{5}$$

$$\text{or } x = \frac{56250 \times 2}{5} = \mathbf{₹ 22500}$$

- 113.** Given difference in two expenditures corresponds to 18° in the pie-chart.

Percentage of expenditure corresponding to 18°

$$= \left(18^\circ \times \frac{100}{360^\circ}\right)\% = 5\%$$

From the given options difference between **paper cost and printing cost** is 5%.

- 114.** From the pie-chart, $\frac{\text{Printing cost}}{\text{Royalty}} = \frac{20\%}{15\%} = \frac{4}{3}$

Given printing cost = ₹ 30600. Let ₹ R be the royalty.

$$\therefore \frac{30600}{R} = \frac{4}{3}$$

$$\Rightarrow R = \frac{30600 \times 3}{4} = 7650 \times 3 = \text{₹ } 22950$$

115. Marked price of book = ₹ 180

$$\text{Cost price of book} = \frac{180 \times 100}{120} = \text{₹ } 150$$

Cost of paper used/copy of book = 25% of 150 = ₹ 37.50

116. Let the selling price of book be ₹ x

$$\therefore \text{Cost price of book} = x \times \frac{100}{105} = \frac{20x}{21}$$

$$\text{Royalty paid/Copy of book} = \frac{281250}{12500} = \frac{15}{100} \times \frac{20x}{21}$$

$$\text{i.e.} \quad \frac{281250}{12500} = \frac{x}{7}$$

$$\text{or} \quad x = \frac{7 \times 281250}{12500} = \text{₹ } 157.50$$

$$117. \text{Transportation cost/copy} = \frac{82500}{5500} = \text{₹ } 15$$

Let selling price of book be ₹ x .

$$\therefore \text{Cost price of book} = x \times \frac{100}{125} = \text{₹ } \frac{4x}{5}$$

$$\therefore \text{Transportation cost/book} = \frac{10}{100} \times \frac{4x}{5} = \text{₹ } 15$$

$$\text{i.e.} \quad \frac{2x}{25} = 15$$

$$\text{or} \quad x = \frac{25 \times 15}{2} = \text{₹ } 187.50$$

118. Printing cost – Royalty = 5%

$$119. \text{Average value of imports in the year 1996, 1997 and 1999} = \frac{250 + 220 + 280}{3} = \text{₹ } 250 \text{ cr}$$

Value of exports in 1998 = ₹ 450 cr

$$\therefore \text{Required percentage} = \frac{450}{250} \times 100 = 180\%$$

Hence answer is (e).

120. Value of exports in 1996 = ₹ 375 cr

Value of imports in 1996 = ₹ 250 cr

$$\therefore \text{Required percentage} = \frac{375}{250} \times 100 = 150\%$$

Hence answer is (e).

121. Value of average exports = $\frac{150 + 225 + 375 + 300 + 450 + 330}{6}$
= $\frac{150 + 300 + 450 + 930}{6} = 25 + 50 + 75 + 155 = ₹ 305 \text{ cr}$

Value of average imports = $\frac{80 + 150 + 250 + 220 + 350 + 280}{6}$
= $\frac{480 + 850}{6} = 80 + 141.67 = ₹ 221.67 \text{ cr} ; ₹ 222 \text{ crores}$
= Value of average exports – Value of average imports
= ₹ 305 cr – ₹ 222 cr
= 83 cr $\approx$ **85 cr**

122. 1998

123. Required percentage = $\frac{450 - 300}{300} \times 100\% = \frac{150}{300} \times 100\% = \mathbf{50\%}$

124. Total rainfall of 31 days in August 1999 = 186 cm

Total rainfall of first 16 days = $16 \times 5 = 80 \text{ mm}$

Total rainfall of last 16 days = $16 \times 8 = 128 \text{ mm}$

Rainfall of 16th August 1999 = $(128 + 80) - 186 = \mathbf{22 \text{ mm}}$

125. Percentage increase = $\frac{430 - 310}{310} \times 100 = \mathbf{38.7\%}$

126. Highest rainfall in August = 230 mm

Average rainfall in July = $\frac{210 + 175 + 150 + 200 + 180}{5} = 183 \text{ mm}$

$\therefore$ Difference = $230 - 183 = \mathbf{47 \text{ mm}}$ i.e., **47 mm more.**

127. i.e. in **2000 and 2002**

128. **Only A and B** can be inferred.

Mathematical Operations

ADDITION: In this type of problems, two factors taken into consideration are *speed* and *accuracy*.

Advantages of this method:

1. Save the labour of repeating all the work.
2. Locate the error, if any, in the column where it occurs.

EXAMPLE 1 Solve $3457 + 248 + 3746 + 5639 + 6854$

Solution Step I:

$$\begin{array}{r}
 3457 \\
 248 \\
 3746 \\
 5639 \\
 6854 \\
 \hline
 4
 \end{array}$$

[7 plus 8 is 15, mark a tick and add 5 to 6 which is 11, mark a tick and add 1 to 9 which is 10, mark a tick and add 0 to 4 which is 4 and write it down in the unit position].

Step II:

$$\begin{array}{r}
 3457 \\
 248 \\
 3746 \\
 5639 \\
 6854 \\
 \hline
 44
 \end{array}$$

[5 plus 3 (number of ticks in unit position) is 8; 8 plus 4 is 12; mark a tick and carry 2; 2 + 4 is 6; 6 + 3 is 9; 9 + 5 = 14; mark a tick and write down 4 in the tenth position].

Step III:

$$\begin{array}{r}
 3457 \\
 248 \\
 3746
 \end{array}$$

$$\begin{array}{r} 5639 \\ 6854 \\ \hline 944 \end{array}$$

[4 plus 2(number of ticks in tenth position) is 6; 6 + 2 is 8; 8 + 7 is 15; mark a tick and carry '5'. 5 + 6 is 11; mark a tick and carry '1'. 1 + 8 is 9; write down 9 in the hundredth position].

Step IV:

$$\begin{array}{r} 3457 \\ 248 \\ 3746 \\ 5639 \\ 6854 \\ \hline 19944 \end{array}$$

[3 + 2 (number of ticks in hundredth position) is 5; 5 + 3 is 8; 8 + 5 is 13; 13 + 6 is 19.]

ADDITION AND SUBTRACTION IN A SINGLE ROW

EXAMPLE 2 Find the value of $424 - 87 + 69 = \dots\dots\dots?$

Solution For units digit of answer, add and subtract the digits at unit places according to the signs attached with respective numbers

$$\begin{array}{l} 4 - 7 + 9 = 6 \\ 424 - 87 + 69 = \text{---} \text{---} (6) \end{array}$$

Similarly, the value of tenth place is

$$2 - 8 + 6 = 0$$

So $424 - 87 + 69 = \text{---} (0) (6)$

Similarly, the value at hundredth place is 4.

$$\therefore 424 - 87 + 69 = \mathbf{406}$$

EXAMPLE 3 Find the value of $4124 - 729 + 631 - 535 = \dots\dots\dots?$

Solution Step I:

$$\begin{array}{l} 4 - 9 + 1 - 5 = -9 \\ 2 - 2 + 3 - 3 = 0 \\ 1 - 7 + 6 - 5 = -5 \\ 4 - 0 + 0 - 0 = 4 \end{array}$$

$\therefore$ Temporary value is (4) (−5) (0) (−9).

Step II: Borrow 1 from 4 and add as 10 to −5 to get 5. Borrow 1 from 5 and add as 10 to 0 and borrow 1 from 10 and add as 10 to −9 to get 1.

$\therefore$ The number is **3491**.

EXAMPLE 4 Find the value of $89974 - 13568 - 46387 = \dots\dots\dots?$

Solution Step I:

$$\begin{array}{l} 4 - 8 - 7 = -11 \\ 7 - 6 - 8 = -7 \\ 9 - 5 - 3 = 1 \\ 9 - 3 - 6 = 0 \\ 8 - 1 - 4 = 3 \end{array}$$

$\therefore$ Temporary value is (3) (0) (1) (−7) (−11).

Step II: Borrow 1 from 1 and add it as 10 to -7 to get 3. Borrow 2 from 3 and add it as 20 to -11 so as to get 9.

i.e. At unit place add 20 which is borrowed from the tenth place as 2.

$$\therefore \qquad \qquad \qquad -11 + 20 = 9$$

From hundredth place borrow 1 and add it as 10 to -7 to get 3.

$$\text{i.e.} \qquad \qquad \qquad 10 - 7 - 2 = 1$$

$\therefore$ The hundredth place digit is 0.

$\therefore$ Number = **30019**.

DIGIT SUM METHOD

Rule: Add the digit sum of the numbers to be added till we get the result in single digit. Digit sum of the total should also be the same as the digit got after getting added the digit sums of individual numbers. For example,

$$\begin{aligned} 23 + 48 + 61 + 15 &= 147 \\ \text{Digit sum } 5 + 3 + 7 + 6 &= 12 \\ 3 &= 3 \end{aligned}$$

MULTIPLICATION

Rule I: The last digit of the multiplicand (number multiplied) is put down as the right-hand figure of the answer.

Rule II: Each successive digit of the multiplicand is added to its neighbour at the right.

EXAMPLE 5 Find the value of $2479 \times 11 = ?$

$$\begin{aligned} \text{Solution } 02479 \times 11 &= \text{---} 9 \quad (9 \times 1 = 9) \\ &= \text{---} 69 \quad (7 + 9 = 16. \text{ Write 6, carry '1'}) \\ &= \text{---} 269 \quad (4 + 7 = 11 + \text{'1'} = 12, \text{ write 2, carry '1'}) \\ &= \text{---} 7269 \quad (2 + 4 = 6 + \text{'1'} = 7) \\ &= 27269 \quad (0 + 2 = 2) \end{aligned}$$

MULTIPLICATION BY 12

Rule To multiply a number by 12, double each digit in turn and add its neighbour.

EXAMPLE 6 Find the value of $5328 \times 12 = \dots\dots\dots?$

$$\text{Solution } 05328 \times 12$$

$$\text{Step I:} \quad = \text{---} 6 \quad (8 \times 2 + 0 = 16. \text{ Write 6 and carry '1'})$$

$$\text{Step II:} \quad = \text{---} 36 \quad (2 \times 2 + 8 + \text{'1'} = 13. \text{ Write 3 and carry '1'})$$

$$\text{Step III:} \quad = \text{---} 936 \quad (3 \times 2 + 2 + \text{'1'} = 9)$$

$$\text{Step IV:} \quad = \text{---} 3936 \quad (5 \times 2 + 3 = 13. \text{ Write 3 and carry '1'})$$

$$\text{Step V:} \quad 63936 \quad (0 \times 2 + 5 + \text{'1'} = 6)$$

$\therefore$ Required number is **63936**.

MULTIPLICATION BY 13

Rule: To multiply by 13, treble each digit in turn and add its right neighbour.

EXAMPLE 7 Find the value of $9478 \times 13 = \dots\dots\dots?$

Solution $09478 \times 13 =$

Step I: = — — — — 4 ($8 \times 3 = 24$. Write 4, carry '2')

Step II = — — — 14 ($7 \times 3 + 8 = 29 + '2' = 31$. Write 1, carry '3')

Step III: = — — 214 ($4 \times 3 + 7 = 19 + '3' = 22$. Write 2, carry '2')

Step IV: = — 3214 ($9 \times 3 + 4 = 31 + '2' = 33$. Write 3, carry '3')

Finally **123214** ($0 \times 3 + 9 = 9 + '3' = 12$)

EXAMPLE 8 Multiply 9876 by 13.

Solution $09876 \times 13 = \dots\dots\dots$

Step I: = — — — — 8 ($6 \times 3 = 18$; Write 8, carry '1')

Step II: = — — — 88 ($7 \times 3 + 6 = 27 + '1' = 28$. Write 8, carry '2')

Step III: = — — 388 ($8 \times 3 + 7 = 31 + '2' = 33$. Write 3, carry '3')

Step IV: = — 8388 ($9 \times 3 + 8 = 35 + '3' = 38$. Write 8, carry '3')

Finally **128388** ($0 \times 3 + 9 = 9 + '3' = 12$)

MULTIPLICATION BY 9

Step I: Subtract the right-hand figure of the long number from 10. This gives the right-hand figure of the answer.

Step II: Taking the next digit from right, subtract it from 9 and add the neighbour on its right.

Step III: Continue the process till the last number.

Finally, when you are under the zero in front of the long number, subtract one from the neighbour and use that as the left-hand figure of the answer.

EXAMPLE 9 Find the value of $9768 \times 9 = \dots\dots\dots?$

Solution $09768 \times 9 = \dots\dots\dots$

Step I: Subtract 8 from 10, we get 2 as the right-end digit.

Step II: Subtract 6 from 9 and add neighbouring digit 8, then the result will be 11. Then write 1, carry '1'.

Step III: Subtract 7 from 9 and add neighbouring digit 6 and carry '1', then the result will be 9.

Step IV: Subtract 9 from 9 and add neighbouring digit 7, then the result will be 7.

Finally, subtract 1 from neighbouring digit 9 so that the result will be 8 and use that as the left-hand figure of the answer.

$\therefore$ The answer is **87912**

EXAMPLE 10 Find the value of $1347 \times 9 = \dots\dots\dots?$

Solution $01347 \times 9 = \dots\dots\dots?$

Step I: Subtract 7 from 10, we get 3 as the right-hand figure.

Step II: Subtract 4 from 9 and add neighbouring digit 7 so that the answer will be 12. Write 2 and carry '1'.

Step III: Subtract 3 from 9 and add neighbouring digit 4 and carry '1' in the last step so that the answer will be 11. Write 1, carry '1'.

Step IV: Subtract 1 from 9 and add neighbouring digit 3 and carry '1' in the last step so that the answer will be 12. Write 2, carry '1'.

Step V: Subtract 1 from neighbouring digit 1 and add carry '1' in the last step to get answer '1'.

Write the numbers got in all steps from right to left.

∴ The answer is **12123**.

Method II

Step I: Put a zero at the right end of the number.

Here the number is 1347. So write it as 13470.

Step II: Subtract the original number from this number

i.e. $13470 - 1347 = 12123$

MULTIPLICATION BY 25: Multiplication by 25 will be treated as multiplication by 100 followed by division by 4.

Step I: Put two zeros at the right end of the number.

Step II: Divide it by 4.

EXAMPLE 11 Multiply 1479 by 25.

Solution $\frac{147900}{4} = 36975$

GENERAL RULE FOR MULTIPLICATION: Multiplication by a two-digit number.

Step I: Multiply the right-hand digits of multiplicand and multiplier (unit-digit of multiplicand with unit-digit of the multiplier).

Step II: Do cross multiplication i.e. multiply unit-digit of one-number with tenth place digit of the other and sum up those terms with carry over from step I.

Step III: Multiply the left-hand figures of both multiplicand and multiplier and add carry over from last step with the above product.

The number obtained through the above steps when arranged from right to left, we will get the required number.

EXAMPLE 12 Find the value of $17 \times 18 = ?$

Solution Step I:

$$\begin{array}{r} 17 \\ \updownarrow \\ 18 \end{array}$$

Step II: $\begin{array}{r} 6 \\ \hline 17 \end{array}$ ($7 \times 8 = 56$ write 6, carry '5')

$$\begin{array}{r}
 \begin{array}{cc}
 \nearrow & \nwarrow \\
 1 & 8 \\
 \nwarrow & \nearrow
 \end{array} \\
 \hline
 0 \quad 6
 \end{array}$$

($1 \times 7 + 1 \times 8 + \text{carry '5'} = 20$. Write 0, carry '2')

Step III:

$$\begin{array}{r}
 1 \quad 7 \\
 \updownarrow \\
 1 \quad 8 \\
 \hline
 3 \quad 0 \quad 6
 \end{array}$$

($1 \times 1 + \text{carry '2'} = 3$)

$\therefore$ The answer is **306**. _____

EXAMPLE 13 Find the value of $78 \times 94 = \dots\dots\dots?$

Solution Step I:

$$\begin{array}{r}
 7 \quad 8 \\
 \updownarrow \\
 9 \quad 4 \\
 \hline
 2
 \end{array}$$

($8 \times 4 = 32$. Write 2, carry '3')

Step II:

$$\begin{array}{r}
 7 \quad 8 \\
 \begin{array}{cc}
 \nearrow & \nwarrow \\
 9 & 4 \\
 \nwarrow & \nearrow
 \end{array} \\
 \hline
 3 \quad 2
 \end{array}$$

($7 \times 4 + 9 \times 8 + \text{carry '3'} = 103$. Write 3, carry '10')

Step III:

$$\begin{array}{r}
 7 \quad 8 \\
 \updownarrow \\
 9 \quad 4 \\
 \hline
 73 \quad 3 \quad 2
 \end{array}$$

($7 \times 9 + 10 = 73$)

$\therefore$ The answer is **7332**.

MULTIPLICATION OF A 3-DIGIT NUMBER BY A 2-DIGIT NUMBER

EXAMPLE 14 Find the value of $347 \times 59 = \dots\dots\dots?$

Solution Step I:

$$\begin{array}{r}
 3 \quad 4 \quad 7 \\
 \updownarrow \\
 5 \quad 9 \\
 \hline
 3
 \end{array}$$

($7 \times 9 = 63$. Write 3, carry '6')

Step II:

$$\begin{array}{r}
 3 \quad 4 \quad 7 \\
 \begin{array}{cc}
 \nearrow & \nwarrow \\
 5 & 9 \\
 \nwarrow & \nearrow
 \end{array} \\
 \hline
 7 \quad 3
 \end{array}$$

($4 \times 9 + 5 \times 7 + \text{carry '6'} = 77$. Write 7, carry '7')

Step III:

$$\begin{array}{r}
 3 \quad 4 \quad 7 \\
 \quad \swarrow \downarrow \searrow \\
 \quad 5 \quad 9 \\
 \hline
 4 \quad 7 \quad 3
 \end{array}$$

 $(3 \times 9 + 4 \times 5 + \text{carry '7'} = 54. \text{ Write 4, carry '5'})$ **Step IV:**

$$\begin{array}{r}
 3 \quad 4 \quad 7 \\
 \quad \swarrow \downarrow \searrow \\
 \quad 5 \quad 9 \\
 \hline
 20 \quad 4 \quad 7 \quad 3
 \end{array}$$

 $(3 \times 5 = 15 + \text{carry '5'} = 20)$ $\therefore$ The answer is **20473**.**EXAMPLE 15** Find the value of 4967×68 **Solution Step I:**

$$\begin{array}{r}
 4 \quad 9 \quad 6 \quad 7 \\
 \quad \quad \quad \uparrow \downarrow \\
 \quad \quad 6 \quad 8 \\
 \hline
 \quad \quad 6
 \end{array}$$

 $(8 \times 7 = 56. \text{ Write 6, carry '5'})$ **Step II:**

$$\begin{array}{r}
 4 \quad 9 \quad 6 \quad 7 \\
 \quad \quad \quad \swarrow \searrow \\
 \quad \quad 6 \quad 8 \\
 \hline
 \quad \quad 5 \quad 6
 \end{array}$$

 $(6 \times 8 + 6 \times 7 + \text{carry '5'} = 95. \text{ Write 5, carry '9'})$ **Step III:**

$$\begin{array}{r}
 4 \quad 9 \quad 6 \quad 7 \\
 \quad \quad \quad \swarrow \downarrow \searrow \\
 \quad \quad 6 \quad 8 \\
 \hline
 7 \quad 5 \quad 6
 \end{array}$$

 $(9 \times 8 + 6 \times 6 + \text{carry '9'} = 117. \text{ Write 7, carry '11'})$ **Step IV:**

$$\begin{array}{r}
 4 \quad 9 \quad 6 \quad 7 \\
 \quad \quad \quad \swarrow \downarrow \searrow \\
 \quad \quad 6 \quad 8 \\
 \hline
 7 \quad 7 \quad 5 \quad 6
 \end{array}$$

 $(4 \times 8 + 9 \times 6 + \text{carry '11'} = 97. \text{ Write 7, carry '9'})$ **Step V:**

$$\begin{array}{r}
 4 \quad 9 \quad 6 \quad 7 \\
 \quad \quad \quad \swarrow \downarrow \searrow \\
 \quad \quad 6 \quad 8 \\
 \hline
 3 \quad 3 \quad 7 \quad 7 \quad 5 \quad 6
 \end{array}$$

 $(4 \times 6 + \text{carry '9'} = 33)$ $\therefore$ The answer is **337756**.

MULTIPLICATION BY A 3-DIGIT NUMBER**EXAMPLE 16** Find the value of 346×647 **Solution Step I:**

$$\begin{array}{r}
 3 \ 4 \ 6 \\
 \updownarrow \\
 6 \ 4 \ 7 \\
 \hline
 2
 \end{array}$$

 $(6 \times 7 = 42. \text{ Write 2, carry '4'})$ **Step II:**

$$\begin{array}{r}
 3 \ 4 \ 6 \\
 \nearrow \nwarrow \\
 6 \ 4 \ 7 \\
 \hline
 6 \ 2
 \end{array}$$

 $(4 \times 7 + 4 \times 6 + \text{carry '4'} = 56. \text{ Write 6, carry '5'})$ **Step III:**

$$\begin{array}{r}
 3 \ 4 \ 6 \\
 \nearrow \updownarrow \nwarrow \\
 6 \ 4 \ 7 \\
 \hline
 8 \ 6 \ 2
 \end{array}$$

 $(3 \times 7 + 4 \times 4 + 6 \times 6 + \text{carry '5'} = 78. \text{ Write 8, carry '7'})$ **Step IV:**

$$\begin{array}{r}
 3 \ 4 \ 6 \\
 \nearrow \nwarrow \\
 6 \ 4 \ 7 \\
 \hline
 3 \ 8 \ 6 \ 2
 \end{array}$$

 $(3 \times 4 + 6 \times 4 + \text{carry '7'} = 43. \text{ Write 3, carry '4'})$ **Step V:**

$$\begin{array}{r}
 3 \ 4 \ 6 \\
 \updownarrow \\
 6 \ 4 \ 7 \\
 \hline
 2 \ 2 \ 3 \ 8 \ 6 \ 2
 \end{array}$$

 $(3 \times 6 + \text{carry '4'} = 22)$ $\therefore$ The answer is **223862**.**EXAMPLE 17** Find the value of $4678 \times 376 = \dots\dots\dots ?$ **Solution Step I:**

$$\begin{array}{r}
 4 \ 6 \ 7 \ 8 \\
 \updownarrow \\
 3 \ 7 \ 6 \\
 \hline
 8
 \end{array}$$

 $(8 \times 6 = 48. \text{ Write 8, carry '4'})$ **Step II:**

$$\begin{array}{r}
 4 \ 6 \ 7 \ 8 \\
 \nearrow \nwarrow \\
 3 \ 7 \ 6 \\
 \hline
 2 \ 8
 \end{array}$$

 $(7 \times 6 + 7 \times 8 + \text{carry '4'} = 102. \text{ Write 2, carry '10'})$

Step III:

$$\begin{array}{rcccc}
 & 4 & 6 & 7 & 8 \\
 & & \nearrow & \downarrow & \nwarrow \\
 & & 3 & 7 & 6 \\
 \hline
 & 9 & 2 & 8 &
 \end{array}$$

$(6 \times 6 + 7 \times 7 + 3 \times 8 + \text{carry } 10 = 119$. Write 9, carry '11')

Step IV:

$$\begin{array}{rcccc}
 & 4 & 6 & 7 & 8 \\
 & \nwarrow & \nearrow & \downarrow & \nearrow \\
 & & 3 & 7 & 6 \\
 \hline
 & 8 & 9 & 2 & 8
 \end{array}$$

$(4 \times 6 + 6 \times 7 + 7 \times 3 + \text{carry '11'} = 98$. Write 8, carry '9')

Step V:

$$\begin{array}{rcccc}
 & 4 & 6 & 7 & 8 \\
 & \nwarrow & \nearrow & \downarrow & \nearrow \\
 & & 3 & 7 & 6 \\
 \hline
 5 & 8 & 9 & 2 & 8
 \end{array}$$

$(4 \times 7 + 6 \times 3 + \text{carry '9'} = 55$. Write 5, carry '5')

Step VI:

$$\begin{array}{rcccc}
 & 4 & 6 & 7 & 8 \\
 & \nwarrow & \nearrow & \downarrow & \nearrow \\
 & & 3 & 7 & 6 \\
 \hline
 1 & 7 & 5 & 8 & 9 & 2 & 8
 \end{array}$$

$(4 \times 3 + \text{carry '5'} = 17)$

$\therefore$ The answer is **1758928**.

MULTIPLICATION BY A 4-DIGIT NUMBER

EXAMPLE 18 Find the value of $5349 \times 4218 = \dots\dots\dots?$

Solution Step I:

$$\begin{array}{rcccc}
 & 5 & 3 & 4 & 9 \\
 & & & & \downarrow \\
 & & & & 8 \\
 \hline
 & & & & 2
 \end{array}$$

$(9 \times 8 = 72$. Write 2, carry '7')

Step II:

$$\begin{array}{rcccc}
 & 5 & 3 & 4 & 9 \\
 & & & \nearrow & \downarrow \\
 & & & 1 & 8 \\
 \hline
 & & & 8 & 2
 \end{array}$$

$(4 \times 8 + 9 \times 1 + \text{carry '7'} = 48$. Write 8, carry '4')

Step III:

$$\begin{array}{rcccc}
 5 & 3 & 4 & 9 \\
 & \nearrow & \downarrow & \nwarrow \\
 4 & 2 & 1 & 8 \\
 \hline
 & 0 & 8 & 2
 \end{array}$$

 $(3 \times 8 + 4 \times 1 + 9 \times 2 + \text{carry '4'} = 50. \text{ Write 0, carry '5'})$ **Step IV:**

$$\begin{array}{rcccc}
 5 & 3 & 4 & 9 \\
 & \nearrow & \downarrow & \nwarrow \\
 4 & 2 & 1 & 8 \\
 \hline
 2 & 0 & 8 & 2
 \end{array}$$

 $(5 \times 8 + 9 \times 4 + 3 \times 1 + 4 \times 2 + \text{carry '5'} = 92. \text{ Write 2, carry '9'})$ **Step V:**

$$\begin{array}{rcccc}
 5 & 3 & 4 & 9 \\
 & \nearrow & \downarrow & \nwarrow \\
 4 & 2 & 1 & 8 \\
 \hline
 6 & 2 & 0 & 8 & 2
 \end{array}$$

 $(5 \times 1 + 3 \times 2 + 4 \times 4 + \text{carry '9'} = 36. \text{ Write 6, carry '3'})$ **Step VI:**

$$\begin{array}{rcccc}
 5 & 3 & 4 & 9 \\
 & \nearrow & \downarrow & \nwarrow \\
 4 & 2 & 1 & 8 \\
 \hline
 5 & 6 & 2 & 0 & 8 & 2
 \end{array}$$

 $(5 \times 2 + 3 \times 4 + \text{carry '3'} = 25. \text{ Write 5, carry '2'})$ **Step VII:**

$$\begin{array}{rcccc}
 5 & 3 & 4 & 9 \\
 \updownarrow & & & \\
 4 & 2 & 1 & 8 \\
 \hline
 2 & 2 & 5 & 6 & 2 & 0 & 8 & 2
 \end{array}$$

 $(5 \times 4 + \text{carry '2'} = 22)$ $\therefore$ The answer is **22562082**.**EXAMPLE 19** Find the value of 568348×3147 .**Solution Step I:**

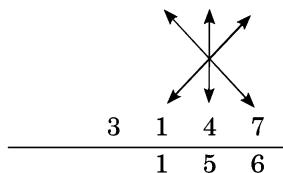
$$\begin{array}{rcccccc}
 & & & & 4 & 8 \\
 & & & & \updownarrow & \\
 & & & & 7 & \\
 \hline
 & & & & 6 &
 \end{array}$$

 $(8 \times 7 = 56. \text{ Write 6, carry '5'})$ **Step II:**

$$\begin{array}{rcccccc}
 & & & & 4 & 8 \\
 & & & & \nearrow & \downarrow \\
 & & & & 7 & \\
 \hline
 & & & & 5 & 6
 \end{array}$$

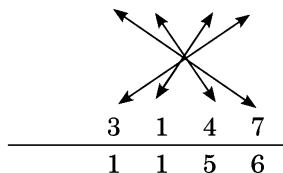
 $(4 \times 7 + 8 \times 4 + \text{carry '5'} = 65. \text{ Write 5, carry '6'})$

Step III: 5 6 8 3 4 8



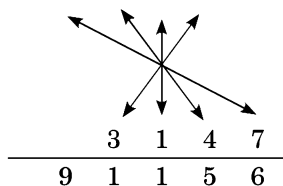
$(3 \times 7 + 4 \times 4 + 8 \times 1 + \text{carry '6'} = 51. \text{ Write 1, carry '5'})$

Step IV: 5 6 8 3 4 8



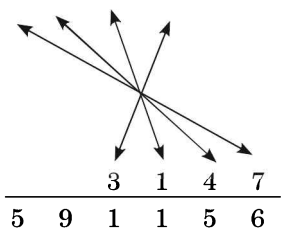
$(8 \times 7 + 3 \times 4 + 4 \times 1 + 8 \times 3 + \text{carry '5'} = 101. \text{ Write 1, carry '10'})$

Step V: 5 6 8 3 4 8



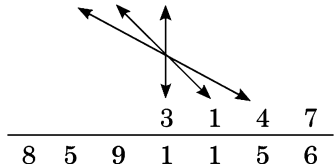
$(6 \times 7 + 8 \times 4 + 3 \times 1 + 4 \times 3 + \text{carry '10'} = 99. \text{ Write 9, carry '9'})$

Step VI: 5 6 8 3 4 8



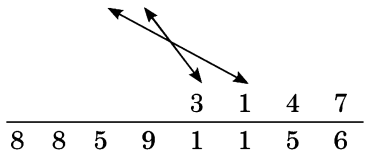
$(5 \times 7 + 6 \times 4 + 8 \times 1 + 3 \times 3 + \text{carry '9'} = 85. \text{ Write 5, carry '8'})$

Step VII: 5 6 8 3 4 8



$(5 \times 4 + 6 \times 1 + 8 \times 3 + \text{carry '8'} = 58. \text{ Write 8, carry '5'})$

Step VIII: 5 6 8 3 4 8



$(5 \times 1 + 6 \times 3 + \text{carry '5'} = 28. \text{ Write 8, carry '2'})$

Step IX:

$$\begin{array}{r}
 5 \quad 6 \quad 8 \quad 3 \quad 4 \quad 8 \\
 \swarrow \quad \searrow \\
 3 \quad 1 \quad 4 \quad 7 \\
 \hline
 1 \quad 7 \quad 8 \quad 8 \quad 5 \quad 9 \quad 1 \quad 1 \quad 5 \quad 6 \quad (5 \times 3 + \text{carry '2'} = 17)
 \end{array}$$

∴ The answer is 1788591156.

DIGIT-SUM CHECK FOR MULTIPLICATION

EXAMPLE 20 $27 \times 13 = 351$

Solution Digit-sum $9 \times 4 = 9$
 $36 = 9$
 $9 = 9$

Hence the calculation is correct.

It may be noted that if the calculation is correct, you will get digit-sums equal on both sides of the equation. (i.e., digit sum of product of two numbers on right-hand side and product of digit-sums of two numbers on left-hand side are equal). But if the digit-sums on both sides are equal, it does not mean that calculation is perfectly right.

EXAMPLE 21 Find the value of $343 \times 34 = 11662$

Solution Digit sum $1 \times 7 = 7$
 $7 = 7$

Hence the calculation is correct.

The above check shows that our answer is correct. But digit-sum check predicts that if the answer is correct, the digit-sum check will be correct. If the calculation is wrong, digit-sum check need not be wrong.

EXAMPLE 22 Find the value of $12 \times 13 = 516$

Solution Digit Sum $3 \times 4 = 12$
 $12 = 3$
 $3 = 3$

Hence the digit-sum check is correct. But the answer is false. Here the actual answer is 156. But we got it mistakenly as 516. Since digit-sum of 156 and 516 are the same, you will get digit-sum check as correct.

Therefore, if the answer is correct, digit-sum check will be correct. But we can't predict that if the digit-sum check is correct, the answer will be correct.



EXERCISES

1. 23145×11	2. 89068×11	3. 5776900×11	4. 1122334607×11
5. 35609×12	6. 457689×12	7. 22233344×12	8. 4488669911×12
9. 12768×13	10. 13489×13	11. 21438×13	12. 61346×13
13. 11434×13	14. 13922×13	15. $569 \times 9 = \dots\dots?$	16. $1328 \times 9 = \dots\dots?$

17. $56493 \times 9 = \dots\dots?$	18. $89273528 \times 9 = \dots\dots?$	19. $79 \times 9 = \dots\dots?$	20. 56×78
21. 79×84	22. 47×53	23. 92×86	24. 93×32
25. 89×32	26. 76×59	27. 87×67	28. 37×46
29. 68×47	30. 338×579	31. 974×687	32. 4374×346
33. 54678×243	34. 13478×3487	35. 387459×246	36. 984653×1347
37. 8652423×3145			



Solutions with Necessary Explanation

1. 254595	2. 979748	3. 63545900	4. 12345680677
5. 427308	6. 5492268	7. 266800128	8. 53864038932
9. 165984	10. 175357	11. 278694	12. 797498
13. 148642	14. 180986	15. 5121	16. 11952
17. 508437	18. 803461752	19. 711	20. 4368
21. 6636	22. 2491	23. 7912	24. 2976
25. 2848	26. 4484	27. 5829	28. 1702
29. 3196	30. 195702	31. 669138	32. 1513404
33. 13286754	34. 46997786	35. 95314914	36. 1326327591
37. 27211870335			

Divisibility

DIVISIBILITY BY 2 A number whose digit in the unit place is either even or zero is divisible by 2. For example,

- (a) 12786 and 6789580 are divisible by 2.
- (b) 18453 and 8678477 are not divisible by 2.

DIVISIBILITY BY 3: If the sum of the digits of a number is divisible by 3, then number is divisible by 3. For example,

- (a) 312: $3 + 1 + 2 = 6$ is divisible by 3;
 $\therefore$ 312 is divisible by 3.
- (b) 6894: $6 + 8 + 9 + 4 = 27$; $2 + 7 = 9$;
 $\therefore$ The number is divisible by 3.
- (c) 98562: $9 + 8 + 5 + 6 + 2 = 30$; $3 + 0 = 3$;
 $\therefore$ The number is divisible by 3.
- (d) 58763: $5 + 8 + 7 + 6 + 3 = 29$; $2 + 9 = 11$ is not divisibly 3.
 $\therefore$ The number is not divisible by 3.

DIVISIBILITY BY 4: If the last two digits of a number are divisible by 4, the number is divisible by 4. The number having two or more zeros at the end is also divisible by 4. For example,

- (a) 5848: 48 is divisible by 4. Hence the number is divisible by 4.
- (b) 4600: Last two digits are zero. Hence the number is divisible by 4.
- (c) 64000: There are more than 2 zeros at the end of the number. Therefore the number is divisible by 4.
- (d) 1426: 26 is not divisible by 4. Therefore the number is not divisible by 4.

DIVISIBILITY BY 5: If a number ends with '5' or '0', then the number is divisible by 5. For example,

- (a) 1365: The last digit is 5.
 $\therefore$ The number is divisible by 5.

- (b) 1480: Last digit is '0'.
∴ The number is divisible by 5.
- (c) 1573: Last digit is neither 5 nor zero.
∴ The number is not divisible by 5.

DIVISIBILITY BY 6: If a number is divisible by both 2 and 3, then the number is divisible by 6.

For a number to be divisible by 6.

- (i) The number should end with either an even number or a zero.
(ii) The sum of its digits should be divisible by 3.

For example,

- (a) 546: $5 + 4 + 6 = 15$; $1 + 5 = 6$; the last digit is an even number and sum of digits is divisible by 3.
∴ Therefore number is divisible by 6.
- (b) 58614: 4 is an even number. $5 + 8 + 6 + 1 + 4 = 24$ is divisible by 3. Therefore the number is divisible by 6.
- (c) 371: 1 is not an even number. Therefore the number is not divisible by 6.

DIVISIBILITY BY 8: If the last three digits of a number are divisible by 8, then the number is divisible by 8. Otherwise if the last three digits are zero, number is divisible by 8. For example,

- (a) 2448: ∴ 448 is divisible by 8. Therefore the number is divisible by 8.
- (b) 247952: ∴ 952 is divisible by 8; therefore the number is divisible by 8.
- (c) 34000: ∴ the last three digits of the number are zeros; therefore the number is divisible by 8.
- (d) 1246: ∴ 246 is not divisible by 8; therefore the number is not divisible by 8.

DIVISIBILITY BY 9: If the sum of all the digits of a number is divisible by 9, the number is divisible by 9. For example,

- (a) 36846: $3 + 6 + 8 + 4 + 6 = 27$ is divisible by 9 and hence the number is divisible by 9.

DIVISIBILITY BY 11: If the sum of digits at odd and even places are equal or differ by a number divisible by 11, then the number is divisible by 11. For example,

- (a) 27618701: The sum of odd positions, $S_o = 2 + 6 + 8 + 0 = 16$
Sum of even positions, $S_e = 7 + 1 + 7 + 1 = 16$
 $S_o = S_e$; therefore, the number is divisible by 11.
- (b) 264935: $S_o = 2 + 4 + 3 = 9$; $S_e = 6 + 9 + 5 = 20$;
 $S_o \sim S_e = 11$; therefore the number is divisible by 11.
- (c) 62849182: $S_o = 6 + 8 + 9 + 8 = 31$;
 $S_e = 2 + 4 + 1 + 2 = 9$;
 $S_o - S_e = 31 - 9 = 22$;
Therefore, the number is divisible by 11.

DIVISIBILITY BY 12: Any number which is divisible by both 3 and 4 is also divisible by 12.

To check the divisibility, by 12,

- (i) First check the last two digits of the number for divisibility by 4. If it is not divisible by 4, then the number is not divisible by 12.
- (ii) If it is divisible by 4, check whether the number is divisible by 3.

For example,

- (a) 246864: 64 is divisible by 4 and so the number is divisible by 4.
 $2 + 4 + 6 + 8 + 6 + 4 = 30$ is divisible by 3. Therefore, the number is divisible by 12.

DIVISIBILITY BY 14: Any number which is divisible by both 2 and 7 is divisible by 14.

To check the divisibility by 14

- (i) The last digit should be checked whether it is even or zero.
- (ii) The number should be divisible by 7.

For example,

- (a) 1834: 4 is divisible by 2 and therefore number is divisible by 2.
 $183 - 4 \times 2 = 175$;
 $17 - 5 \times 2 = 7$; therefore number is divisible by 7.
 Hence 1834 is divisible by 14.

DIVISIBILITY BY 15: Any number which is divisible by both 5 and 3 is divisible by 15.

To check divisibility by 15,

- (i) First check whether the last digit of number is 5 or zero. If not, then the number is not divisible by 15.
- (ii) Check for divisibility by 3, if it is divisible by 5. If the number is divisible by 3, then the number is divisible by 15.

For example,

- (a) 3645: Last digit of the number is 5 and therefore number is divisible by 5. Digit-sum of number = $3 + 6 + 4 + 5 = 18$. Since digit-sum is divisible by 3, number is divisible by 3. Since the given number is divisible by both 3 and 5, the number is divisible by 15.

DIVISIBILITY BY 16: Any number whose last 4 digit number is divisible by 16 or number having last 4 digits as zeros is divisible by 16. For example,

- (a) 31264: Since the last 4 digit of the number is divisible by 16, number is divisible by 16.

DIVISIBILITY BY 18: Any number which is divisible by 9 and has its unit-digit even or zero is divisible by 18. For example,

- (a) 18612: Unit digit 2 is even; therefore the number is divisible by 2.
 $1 + 8 + 6 + 1 + 2 = 18$, a multiple of 9. Hence the number is divisible by 18.
- (b) 298728: The unit-digit (8) is even, the digit sum is 36, a multiple of 9. Hence the number is divisible by 18.

SPECIAL CASES: Rules for divisibility by 7, 13, 17, 29 etc. are unique. For that you should know terms like *one-more osculator* and *negative osculator*.

One-more osculator means the number needs one more to be a multiple of 10.

For example,

- (a) 19 needs 1 to become 20, a multiple of 10. Therefore the osculator for 19 is 2 (i.e. $2 \times 10 = 20$). Similarly, the osculator for 49 is 5 (i.e., $5 \times 10 = 50$).
- (b) Negative osculator for 21 is 2 (i.e. $21 - 1 = 20 = 2 \times 10$). Similarly, the negative osculator for 51 is 5 (i.e. $51 - 1 = 50 = 5 \times 10$).

Negative osculator for 7

$7 \times 3 = 21$, one more than a multiple of 10.

$\therefore 21 - 1 = 20 = 2 \times 10$.

Hence the negative osculator for 7 is 2.

One-more osculator for 7

$7 \times 7 = 49$, which is one less than 50, a multiple of 10.

$7 \times 7 = 49 = 50 - 1$, $50 = 5 \times 10$.

Therefore, one-more osculator for 7 is 5.

One-more osculator for 13

$13 \times 3 = 39$, which is one less than 40, a multiple of ten.

Therefore, $13 \times 3 = 39 = 40 - 1$; $40 = 4 \times 10$;

One more osculator for 13 is 4.

One more osculator for 19

$19 \times 1 = 19$, which is one less than 20, a multiple of ten,

Therefore, $19 \times 1 = 19 = 20 - 1$; $20 = 2 \times 10$.

One more osculator for 19 is 2.

Negative osculator for 17

$17 \times 3 = 51$, which is one more than a multiple of ten.

$17 \times 3 = 51$; $= 51 - 1 = 50 = 5 \times 10$

Hence the negative osculator for 17 is 5.

DIVISIBILITY BY 7: The negative osculator for 7 is 2. This is used to check divisibility of a number by 7.

EXAMPLE 1 Is 112 divisible by 7?

Solution Step I: 11 2 : $11 - 2 \times 2 = 7$

As 7 is divisible by 7, the number 112 is divisible by 7.

EXAMPLE 2 Is 3871 divisible by 7?

Solution Step I: 387 1 : $387 - 1 \times 2 = 385$

Step II: 3 8 5 : $38 - 5 \times 2 = 28$

As 28 is divisible by 7, the given number is divisible by 7.

DIVISIBILITY BY 13: The osculator for 13 is 4, which is one more osculator. Therefore instead of subtraction done in the previous case, you have to add here.

EXAMPLE 3 Is 143 divisible by 13?

Solution 14 3: $14 + 3 \times 4 = 26$

Since 26 is divisible by 13, the given number divisible by 13.

EXAMPLE 4 Is 24167 divisible by 13?

Solution Step I: $24167 : 4 \times 7$ (from 24167) + 6 (from 24167) = 34

Step II: 4×4 (from 34) + 3 (from 34) + 1 (from 24167) = 20

Step III: 4×0 (from 20) + 2 (from 20) + 4 (from 24167) = 6

Step IV: 4×6 (from 6) + 2 (from 24167) = 26

Since 26 is divisible by 13, the number is divisible by 13.

DIVISIBILITY BY 17: The negative osculator for 17 is 5. Therefore the working principle is same as in the case of 7.

EXAMPLE 5 Is 1904 divisible by 17?

Solution 190 4: $190 - 4 \times 5 = 170$

Since 170 is divisible by 17, the number is divisible by 17.

EXAMPLE 6 Is 8971563 divisible by 17?

Solution 897156 3: $897156 - 3 \times 5 = 897141$

89714 1: $89714 - 1 \times 5 = 89709$

8970 9: $8970 - 9 \times 5 = 8925$

892 5: $892 - 5 \times 5 = 867$

86 7: $86 - 7 \times 5 = 51$

Since 51 is divisible by 17, the number is divisible by 17.

DIVISIBILITY BY 19: One more osculator for 19 is 2; the working principle is same as in the case of 13.

EXAMPLE 7 Is 149283 divisible by 19?

Solution Step I: 2×3 (from 149283) + 8 (from 149283) = 14

Step II: 2×4 (from 14) + 1 (from 14) + 2 (from 149283) = 11

Step III: 2×1 (from 11) + 1 (from 11) + 9 (from 149283) = 12

Step IV: 2×2 (from 12) + 1 (from 12) + 4 (from 149283) = 9

Step V: 2×9 (from 9) + 1 (from 149283) = 19

which is divisible by 19. Hence the number is divisible by 19.

SURDS

The roots of those quantities which cannot be exactly obtained are called surds; for example, $\sqrt{3}$, $\sqrt{6}$, etc.

MIXED SURDS: A rational factor and a surd multiplied together produce a mixed surd, for example, $2\sqrt{3}$, $4\sqrt{5}$, etc.

ORDER OF SURDS: $a^{1/n}$ is called a surd of the n th order.

Changing the surds into that of the same order:

EXAMPLE 1 Express $4^{1/3}$, $2^{1/5}$ and $5^{1/6}$ as surds of the same order and arrange them in the ascending order of magnitude.

Solution The LCM of 3, 5 and 6 (root indices) is 30. Reducing all surds into the 30th order, we get,

$$4^{1/3} = (4^{10})^{1/30} = (16^5)^{1/30}$$

$$2^{1/5} = (2^6)^{1/30} = (64)^{1/30}$$

$$5^{1/6} = (5^5)^{1/30} = (3125)^{1/30}$$

Hence, the order of magnitude is $((16^5)^{1/30} > (3125)^{1/30} > (64)^{1/30}$ or $4^{1/3} > 5^{1/6} > 2^{1/5}$

EXAMPLE 2 Express $4^{1/3}$, $3^{1/6}$ and $6^{1/4}$ as surds of the same order and arrange them in the ascending order of magnitude.

Solution The LCM of orders 4, 3 and 6 is 12. Now reduce them to the 12th order.

$$4^{1/3} = 4^{4/12} = (4^4)^{1/12} = (256)^{1/12}$$

$$3^{1/6} = 3^{2/12} = (3^2)^{1/12} = (9)^{1/12}$$

$$6^{1/4} = 6^{3/12} = (6^3)^{1/12} = (216)^{1/12}$$

Hence $(9)^{1/12} < (216)^{1/12} < (256)^{1/12}$ or $3^{1/6} < 6^{1/4} < 4^{1/3}$.

ADDITION AND SUBTRACTION OF SURDS: Similar surds like $3\sqrt{7}$, $5\sqrt{7}$, and $11\sqrt{7}$ can be added but dissimilar surds like $3\sqrt{5}$, $5\sqrt{2}$, $7\sqrt{3}$ cannot be added.

EXAMPLE 3 Simplify $\sqrt{75} + \sqrt{48}$

Solution $\sqrt{75} + \sqrt{48} = \sqrt{25 \times 3} + \sqrt{16 \times 3} = 5\sqrt{3} + 4\sqrt{3} = 9\sqrt{3}$

EXAMPLE 4 Simplify $\sqrt{48} + \sqrt{147}$

Solution $\sqrt{48} + \sqrt{147} = \sqrt{16 \times 3} + \sqrt{49 \times 3} = 4\sqrt{3} + 7\sqrt{3} = 11\sqrt{3}$

MULTIPLICATION OF SURDS

EXAMPLE 5 Find the product of $4^{1/3}$, $6^{1/6}$ and $\sqrt{5}$.

Solution $4^{1/3} \times 6^{1/6} \times 5^{1/2} = 4^{2/6} \times 6^{1/6} \times 5^{3/6} = (4^2 \times 6^1 \times 5^3)^{1/6} = (12000)^{1/6}$

EXAMPLE 6 Find the product of $4^{1/3}$, $3^{1/6}$ and $6^{1/4}$.

Solution $4^{1/3} \times 3^{1/6} \times 6^{1/4} = 4^{4/12} \times 3^{2/12} \times 6^{3/12} = (4^4 \times 3^2 \times 6^3)^{1/12} = (497664)^{1/12}$

DIVISION OF SURDS

EXAMPLE 7 Divide $12 \times 4^{1/3}$ by $3\sqrt{2}$.

Solution $\frac{12 \times 4^{1/3}}{3 \times 2^{1/2}} = 4 \times \frac{(4^2)^{1/6}}{(2^3)^{1/6}} = 4 \times \left(\frac{16}{8}\right)^{1/6} = 4 \times 2^{1/6}$

EXAMPLE 8 Divide $8 \times 6^{1/3}$ by $2\sqrt{2}$.

$$\begin{aligned}\text{Solution} \quad \frac{8 \times 6^{1/3}}{2\sqrt{2}} &= \frac{8 \times 6^{1/3}}{2 \times 2^{1/2}} = 4 \times \frac{6^{2/6}}{2^{3/6}} = 4 \times \left(\frac{6^2}{2^3}\right)^{1/6} = 4 \times \left(\frac{36}{8}\right)^{1/6} = 4 \times \left(\frac{9}{2}\right)^{1/6} \\ &= 2 \times 2^{5/6} \times 9^{1/6} = 2^{11/6} \times 9^{1/6}.\end{aligned}$$



Important Results

1. $\sqrt{a} \times \sqrt{a} = a$	2. $\sqrt{a} \times \sqrt{b} = \sqrt{ab}$
3. $\sqrt{a^2 \times b} = a\sqrt{b}$	4. $(\sqrt{a} + \sqrt{b})^2 = a + b + 2\sqrt{ab}$
5. $(\sqrt{a} - \sqrt{b})^2 = a + b - 2\sqrt{ab}$	6. $(\sqrt{a} + \sqrt{b})(\sqrt{a} - \sqrt{b}) = a - b$

SOME USEFUL ROOTS TO BE REMEMBERED

$$\sqrt{2} = 1.41421; \quad \sqrt{3} = 1.73205; \quad \sqrt{5} = 2.23607$$

$$\sqrt{6} = 2.44949; \quad \sqrt{7} = 2.64575; \quad \sqrt{8} = 2.82842$$

EXAMPLE 9 Find the value of $\sqrt{300}$.

$$\text{Solution} \quad \sqrt{300} = \sqrt{10 \times 10 \times 3} = 10\sqrt{3} = 10 \times 1.732 = \mathbf{17.32}$$

EXAMPLE 10 Find the value of $\sqrt{500}$.

$$\text{Solution} \quad \sqrt{500} = \sqrt{10 \times 10 \times 5} = 10\sqrt{5} = 10 \times 2.236 = \mathbf{22.36}$$

EXAMPLE 11 Evaluate the following:

$$(a) \sqrt{75} + \sqrt{147} \quad (b) \sqrt{80} + 3\sqrt{245} - \sqrt{125}$$

Solution

$$(a) \sqrt{75} + \sqrt{147} = \sqrt{5 \times 5 \times 3} + \sqrt{7 \times 7 \times 3} = 5\sqrt{3} + 7\sqrt{3} = 12\sqrt{3} = \mathbf{20.7846}$$

$$\begin{aligned}(b) \sqrt{80} + 3\sqrt{245} - \sqrt{125} &= \sqrt{4 \times 4 \times 5} + 3\sqrt{7 \times 7 \times 5} - \sqrt{5 \times 5 \times 5} \\ &= 4\sqrt{5} + 21\sqrt{5} - 5\sqrt{5} = 20\sqrt{5} = \mathbf{44.7214}\end{aligned}$$

EXAMPLE 12 Evaluate the following:

$$(a) \sqrt{108} + \sqrt{192} \quad (b) \sqrt{45} + 2\sqrt{405} - \sqrt{180}$$

Solution

$$\begin{aligned}(a) \sqrt{108} + \sqrt{192} &= \sqrt{6 \times 6 \times 3} + \sqrt{8 \times 8 \times 3} \\ &= 6\sqrt{3} + 8\sqrt{3} = 14\sqrt{3} = 14 \times 1.732 = \mathbf{24.248}\end{aligned}$$

$$\begin{aligned}(b) \sqrt{45} + 2\sqrt{405} - \sqrt{180} &= \sqrt{3 \times 3 \times 5} + 2\sqrt{9 \times 9 \times 5} - \sqrt{6 \times 6 \times 5} \\ &= 3\sqrt{5} + 2 \times 9\sqrt{5} - 6\sqrt{5} = 3\sqrt{5} + 18\sqrt{5} - 6\sqrt{5} \\ &= (3 + 18 - 6)\sqrt{5} = \mathbf{15\sqrt{5}}\end{aligned}$$

EXAMPLE 13 Evaluate the following:

(a) $\sqrt{2} \times \sqrt{3}$ (b) $\sqrt{6} \times \sqrt{150}$ (c) $\sqrt{242} \div \sqrt{72}$

Solution

(a) $\sqrt{2} \times \sqrt{3} = \sqrt{6} = \mathbf{2.4495}$

(b) $\sqrt{6} \times \sqrt{150} = \sqrt{6 \times 150} = \sqrt{900} = \mathbf{30}$

(c) $\sqrt{242} \div \sqrt{72} = \frac{\sqrt{121 \times 2}}{\sqrt{36 \times 2}} = \frac{11}{6} = \mathbf{1\frac{5}{6}}$

EXAMPLE 14 Evaluate the following:

(a) $\sqrt{2} \times \sqrt{3}$ (b) $\sqrt{8} \times \sqrt{288}$ (c) $\sqrt{243} \div \sqrt{75}$

Solution

(a) $\sqrt{2} \times \sqrt{3} = \sqrt{6} = 2.44949$

(b) $\sqrt{8} \times \sqrt{288} = \sqrt{8 \times 288} = \sqrt{16 \times 144} = 4 \times 12 = \mathbf{48}$

(c) $\sqrt{243} \div \sqrt{75} = \sqrt{9 \times 9 \times 3} \div \sqrt{5 \times 5 \times 3} = \frac{9\sqrt{3}}{5\sqrt{3}} = \frac{9}{5} = \mathbf{1\frac{4}{5}}$

EXAMPLE 15 Find the values of

(a) $\frac{1}{\sqrt{2}}$ (b) $\frac{1}{\sqrt{3}}$ (c) $\frac{1}{\sqrt{5}}$ (d) $\frac{1}{\sqrt{6}}$ (e) $\frac{1}{\sqrt{7}}$

Solution

(a) $\frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{\sqrt{2} \times \sqrt{2}} = \frac{\sqrt{2}}{2} = \mathbf{0.7071}$

(b) $\frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{\sqrt{3} \times \sqrt{3}} = \frac{\sqrt{3}}{3} = \mathbf{0.5773}$

(c) $\frac{1}{\sqrt{5}} = \frac{\sqrt{5}}{\sqrt{5} \times \sqrt{5}} = \frac{\sqrt{5}}{5} = \mathbf{0.4472}$

(d) $\frac{1}{\sqrt{6}} = \frac{\sqrt{6}}{\sqrt{6} \times \sqrt{6}} = \frac{\sqrt{6}}{6} = \mathbf{0.4082}$

(e) $\frac{1}{\sqrt{7}} = \frac{\sqrt{7}}{\sqrt{7} \times \sqrt{7}} = \frac{\sqrt{7}}{7} = \mathbf{0.3779}$

EXAMPLE 16 Evaluate the following:

(a) $\frac{1}{\sqrt{3} - 1}$ (b) $\frac{16}{2 + \sqrt{3}}$ (c) $\frac{\sqrt{3} + 1}{\sqrt{3} - 1}$ (d) $\frac{\sqrt{5} - 1}{\sqrt{5} + 1}$ (e) $\frac{2 + \sqrt{3}}{2 - \sqrt{3}}$

(f) $\frac{4 + \sqrt{2}}{\sqrt{2} + 1}$ (g) $\frac{\sqrt{3} - \sqrt{2}}{\sqrt{3} + \sqrt{2}}$

Solution

$$(a) \frac{1}{\sqrt{3}-1} = \frac{\sqrt{3}+1}{(\sqrt{3}-1)(\sqrt{3}+1)} = \frac{\sqrt{3}+1}{3-1} = \frac{\sqrt{3}+1}{2} = \mathbf{1.366}$$

$$(b) \frac{16}{2+\sqrt{3}} = \frac{16(2-\sqrt{3})}{(2+\sqrt{3})(2-\sqrt{3})} = \frac{16(2-\sqrt{3})}{4-3} \\ = 16(2-\sqrt{3}) = 16(2-1.732) = 16 \times 0.268 = \mathbf{4.288}$$

$$(c) \frac{\sqrt{3}+1}{\sqrt{3}-1} = \frac{(\sqrt{3}+1)(\sqrt{3}+1)}{(\sqrt{3}-1)(\sqrt{3}+1)} = \frac{(\sqrt{3}+1)^2}{3-1} = \frac{4+2\sqrt{3}}{2} = 2+\sqrt{3} = \mathbf{3.732}$$

$$(d) \frac{\sqrt{5}-1}{\sqrt{5}+1} = \frac{(\sqrt{5}-1)(\sqrt{5}-1)}{(\sqrt{5}+1)(\sqrt{5}-1)} = \frac{(\sqrt{5}-1)^2}{5-1} = \frac{6-2\sqrt{5}}{4} = \frac{3-\sqrt{5}}{2} = \frac{3-2.236}{2} = \mathbf{0.382}$$

$$(e) \frac{2+\sqrt{3}}{2-\sqrt{3}} = \frac{(2+\sqrt{3})(2+\sqrt{3})}{(2-\sqrt{3})(2+\sqrt{3})} = \frac{(2+\sqrt{3})^2}{4-3} = \frac{7+4\sqrt{3}}{1} \\ = 7+4 \times 1.732 = 7+6.928 = \mathbf{13.928}$$

$$(f) \frac{4+\sqrt{2}}{\sqrt{2}+1} = \frac{(4+\sqrt{2})(\sqrt{2}-1)}{(\sqrt{2}+1)(\sqrt{2}-1)} = \frac{2-4+3\sqrt{2}}{2-1} = \frac{3\sqrt{2}-2}{1} = 4.242-2 = \mathbf{2.242}$$

$$(g) \frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}} = \frac{(\sqrt{3}-\sqrt{2})(\sqrt{3}-\sqrt{2})}{(\sqrt{3}+\sqrt{2})(\sqrt{3}-\sqrt{2})} = \frac{(\sqrt{3}-\sqrt{2})^2}{3-2} = \frac{5-2\sqrt{6}}{1} \\ = 5-2(2.449) = 5-4.899 = \mathbf{0.101}$$



Short cut: (Direct Formula)

$$(i) \frac{\sqrt{a}-\sqrt{b}}{\sqrt{a}+\sqrt{b}} \quad \text{and} \quad (ii) \frac{\sqrt{a}+\sqrt{b}}{\sqrt{a}-\sqrt{b}}$$

Solution

$$(i) \frac{\sqrt{a}-\sqrt{b}}{\sqrt{a}+\sqrt{b}} = \frac{(\sqrt{a}-\sqrt{b})^2}{(\sqrt{a})^2-(\sqrt{b})^2} = \frac{a+b-2\sqrt{ab}}{a-b} = \frac{a+b}{a-b} - \frac{2\sqrt{ab}}{a-b}$$

$$(ii) \frac{\sqrt{a}+\sqrt{b}}{\sqrt{a}-\sqrt{b}} = \frac{(\sqrt{a}+\sqrt{b})^2}{(\sqrt{a})^2-(\sqrt{b})^2} = \frac{a+b+2\sqrt{ab}}{a-b} = \frac{a+b}{a-b} + \frac{2\sqrt{ab}}{a-b}$$

In the above expressions, the first and second terms are the same for both cases. They are added when numerator of surd has a '+' sign and subtracted when the numerator of the surd has a '-' sign.

EXAMPLE 17 Find the value of (a) $\frac{\sqrt{8}-\sqrt{7}}{\sqrt{8}+\sqrt{7}}$ (b) $\frac{\sqrt{8}+\sqrt{7}}{\sqrt{8}-\sqrt{7}}$

$$\text{Solution} \quad (a) \frac{\sqrt{8}-\sqrt{7}}{\sqrt{8}+\sqrt{7}} = \frac{8+7}{8-7} - \frac{2\sqrt{8 \times 7}}{8-7} = 15-2\sqrt{56} = \mathbf{15-4\sqrt{14}}$$

$$(b) \frac{\sqrt{8} + \sqrt{7}}{\sqrt{8} - \sqrt{7}} = \frac{8 + 7}{8 - 7} - \frac{2\sqrt{8 \times 7}}{8 - 7} = 15 + 2\sqrt{56} = \mathbf{15 + 4\sqrt{14}}$$

EXAMPLE 18 Find the value of the following expressions correct up to 3 decimal places.

$$(a) \frac{1}{\sqrt{6}} \quad (b) \frac{1}{\sqrt{11}} \quad (c) \frac{1}{\sqrt{2} - 1} \quad (d) \frac{1}{\sqrt{7} - 1} \quad (e) \frac{\sqrt{2} + 1}{\sqrt{2} - 1}$$

$$(f) \frac{\sqrt{5} + 1}{\sqrt{5} - 1} \quad (g) \frac{2 + \sqrt{3}}{2 - \sqrt{3}} \quad (h) \sqrt{\frac{\sqrt{5} + 1}{\sqrt{5} - 1}} \quad (i) \sqrt{\frac{2 + \sqrt{3}}{2 - \sqrt{3}}}$$

Solution

$$(a) \frac{1}{\sqrt{6}} = \frac{\sqrt{6}}{6} = \mathbf{0.4082}$$

$$(b) \frac{1}{\sqrt{11}} = \frac{\sqrt{11}}{11} = \mathbf{0.302}$$

$$(c) \frac{1}{\sqrt{2} - 1} = \frac{(\sqrt{2} + 1)}{(\sqrt{2} - 1)(\sqrt{2} + 1)} = \frac{\sqrt{2} + 1}{2 - 1} = \mathbf{2.414}$$

$$(d) \frac{1}{\sqrt{7} - 1} = \frac{(\sqrt{7} + 1)}{7 - 1} = \frac{3.64575}{6} = \mathbf{0.608}$$

$$(e) \frac{\sqrt{2} + 1}{\sqrt{2} - 1} = \frac{(\sqrt{2} + 1)^2}{2 - 1} = 3 + 2\sqrt{2} = \mathbf{5.828}$$

$$(f) \frac{\sqrt{5} + 1}{\sqrt{5} - 1} = \frac{(\sqrt{5} + 1)^2}{5 - 1} = \frac{6 + 2\sqrt{5}}{4} = \frac{3 + \sqrt{5}}{2} = \mathbf{2.618}$$

$$(g) \frac{2 + \sqrt{3}}{2 - \sqrt{3}} = \frac{(2 + \sqrt{3})^2}{4 - 3} = \frac{7 + 4\sqrt{3}}{1} = \mathbf{13.928}$$

$$(h) \sqrt{\frac{\sqrt{5} + 1}{\sqrt{5} - 1}} = \sqrt{\frac{(\sqrt{5} + 1)^2}{5 - 1}} = \frac{(\sqrt{5} + 1)}{2} = \mathbf{1.618}$$

$$(i) \sqrt{\frac{2 + \sqrt{3}}{2 - \sqrt{3}}} = \sqrt{\frac{(2 + \sqrt{3})^2}{4 - 3}} = 2 + \sqrt{3} = \mathbf{3.732}$$

EXAMPLE 19 Arrange the following in the ascending order

$$\sqrt{7} - \sqrt{6}, \sqrt{6} - \sqrt{5}, \sqrt{9} - \sqrt{7}, \sqrt{11} - \sqrt{9}$$

Solution $\sqrt{7} - \sqrt{6} = \frac{(\sqrt{7} - \sqrt{6})(\sqrt{7} + \sqrt{6})}{(\sqrt{7} + \sqrt{6})} = \frac{7 - 6}{\sqrt{7} + \sqrt{6}} = \frac{1}{\sqrt{7} + \sqrt{6}}$

$$\sqrt{6} - \sqrt{5} = \frac{(\sqrt{6} - \sqrt{5})(\sqrt{6} + \sqrt{5})}{(\sqrt{6} + \sqrt{5})} = \frac{6 - 5}{\sqrt{6} + \sqrt{5}} = \frac{1}{\sqrt{6} + \sqrt{5}}$$

$$\sqrt{9} - \sqrt{7} = \frac{(\sqrt{9} - \sqrt{7})(\sqrt{9} + \sqrt{7})}{(\sqrt{9} + \sqrt{7})} = \frac{9 - 7}{\sqrt{9} + \sqrt{7}} = \frac{2}{\sqrt{9} + \sqrt{7}}$$

$$\sqrt{11} - \sqrt{9} = \frac{(\sqrt{11} - \sqrt{9})(\sqrt{11} + \sqrt{9})}{(\sqrt{11} + \sqrt{9})} = \frac{11 - 9}{\sqrt{11} + \sqrt{9}} = \frac{2}{\sqrt{11} + \sqrt{9}}$$

$$\therefore \frac{1}{\sqrt{7} + \sqrt{6}} < \frac{1}{\sqrt{6} + \sqrt{5}}; \frac{2}{\sqrt{11} + \sqrt{9}} < \frac{2}{\sqrt{9} + \sqrt{7}} \text{ and } \frac{1}{\sqrt{6} + \sqrt{5}} < \frac{2}{\sqrt{11} + \sqrt{9}}$$

$$\therefore \frac{1}{\sqrt{7} + \sqrt{6}} < \frac{1}{\sqrt{6} + \sqrt{5}} < \frac{2}{\sqrt{11} + \sqrt{9}} < \frac{2}{\sqrt{9} + \sqrt{7}}$$

When the denominator increases, the value of fraction decreases.

Number System

NUMBER SYSTEM

CYCLIC NUMBER: This is an integer of n digits which on being multiplied by any number from 1 through n , gives a product containing the same n digits as in the original number and these digits are in the same cyclic order in the product. For example,

(a) $142857 \times 1 = 142857$

(b) $142857 \times 2 = 285714$

(c) $142857 \times 3 = 428571$

(d) $142857 \times 4 = 571428$

REAL NUMBER: Any number having a meaningful value is called a real number. It can be either rational or irrational.

PRIME NUMBER: A number which is divisible by one and the number itself is called a prime number. For example, 2, 3, 5, 7, 11, etc.

PERFECT NUMBER: If the sum of the divisors of a number excluding the number itself is equal to the number, then the number is called perfect number. For example, 6, 28, 496, 8128, etc.

i.e. $6 = 1 + 2 + 3$; $28 = 1 + 2 + 4 + 7 + 14$.

GOLDEN RATIO: The average of 1 and $\sqrt{5}$ is called the golden ratio.

i.e.
$$\frac{1 + \sqrt{5}}{2} = 1.61803.$$

The sum of the reciprocals of the divisors of a perfect number including that of its own is always equal to 2. For example,

for 6,
$$\frac{1}{1} + \frac{1}{2} + \frac{1}{3} + \frac{1}{6} = 2$$

for 28,
$$\frac{1}{1} + \frac{1}{2} + \frac{1}{4} + \frac{1}{7} + \frac{1}{14} + \frac{1}{28} = 2$$

Every even perfect number is of the form $2^{n-1}(2^n - 1)$ where $(2^n - 1)$ and n are prime numbers.

AMICABLE NUMBERS: If the sum of divisors of N_1 is N_2 and the sum of divisors of N_2 is N_1 , then N_1 and N_2 are amicable numbers. For example, 220 and 284.

The sum of divisors of 220

$$= 1 + 2 + 4 + 5 + 10 + 11 + 20 + 22 + 44 + 55 + 110 = 284$$

The sum of divisor of 284

$$= 1 + 2 + 4 + 71 + 142 = 220$$

$\therefore$ 220 and 284 are amicable numbers.

SOCIABLE: If the chain that leads back to the original number has more than two links the number is called *sociable*. For example, 12496, 14288, 15472, and 14536, 14264.

- (i) If n is odd, $n(n^2 - 1)$ is divisible by 24 and if n is an odd prime number greater than 3, $n^2 - 1$ is divisible by 24. For example,
 $n = 11$, $n^2 - 1 = 11^2 - 1 = 120 = 24 \times 5$, divisible by 24
- (ii) If n is odd, $2^n + 1$ is divisible by 3 and if n is even, $2^n - 1$ is divisible by 3.
- (iii) If n is prime to 5, then $n(n^4 - 1)$ is divisible by 30.
- (iv) If n is an odd number $2^{2n} + 1$ is divisible by 5 and if n is an even number, $2^{2n} - 1$ is divisible by 5.
- (v) If n is odd, $5^{2n} + 1$ is divisible by 13 and if n is even, $5^{2n} - 1$ is divisible by 13.
- (vi) If $a > b$, then $|a - b| = a - b$ and if $a < b$, then $|a - b| = b - a$.
- (vii) If $a < b$, then $a < \frac{a+b}{2} < b$. This is because $\frac{a+b}{2}$ is the average of a and b .
 The average of two numbers always lies between the two numbers. There are infinite number of rational numbers between any two rational numbers.
- (viii) $x^n + y^n$ is divisible by $x + y$ when n is odd and $x^n + y^n = (x + y)(x^{n-1} - x^{n-2}y + x^{n-3}y^2 - \dots + y^{n-1})$.
- (ix) $x^n - y^n$ is divisible by $x + y$ when n is even and by $x - y$ when n is either even or odd and $x^n - y^n = (x + y)(x^{n-1} - x^{n-2}y + x^{n-3}y^2 - \dots + y^{n-1})$
 when n is even and $x^n - y^n = (x - y)(x^{n-1} + x^{n-2}y + \dots + y^{n-1})$
 when n is either odd or even.

The fifth power of any single digit number has the same right hand digit as the number itself.

$$x^3 - x \text{ is divisible by } 3$$

$x^5 - x$ is divisible by 5

$x^7 - x$ is divisible by 7

$x^{11} - x$ is divisible by 11

$x^{13} - x$ is divisible by 13

i.e. if n is any prime number, then for any whole number $x^n - x$ is divisible by n .

Difference between the squares of two consecutive numbers is always an odd number and is equal to the sum of those two consecutive numbers.

When two numbers are divided by a third number, leave the same remainder, the difference of those two numbers must be perfectly divisible by the third number.

$$\text{Quotient} = \frac{\text{Dividend} - \text{Remainder}}{\text{Divisor}}$$

or

$$\text{Divisor} = \frac{\text{Dividend} - \text{Remainder}}{\text{Quotient}}$$

To find the product of two numbers when the sum and the difference of the two numbers are given.

$$\text{Product of numbers} = \frac{(\text{Sum} + \text{Difference})(\text{Sum} - \text{Difference})}{4}$$

EXAMPLE 1 The given sum and difference of two numbers are respectively 50 and 30. Find the product of those numbers.

Solution Product of numbers = $\frac{(50 + 30)(50 - 30)}{4} = \frac{80 \times 20}{4} = \mathbf{400}$

The numbers can also be found by the following formula

$$\text{First number} = \frac{\text{Sum} + \text{Difference}}{2}$$

$$\text{Second number} = \frac{\text{Sum} - \text{Difference}}{2}$$

The sum of the digits of a 2-digit number is S . If the digits are reversed, the number is decreased by N , then the number is $5\left(S + \frac{N}{9}\right) + \frac{1}{2}\left(S - \frac{N}{9}\right)$.

The sum of digits of a two-digit number is S . If the digits are reversed, the number is increased by N . Then the number is $5\left(S - \frac{N}{9}\right) + \frac{1}{2}\left(S + \frac{N}{9}\right)$.

EXAMPLE 2 The sum of digits of a two-digit number is 6 and if the digits are reversed the number is increased by 36. Find the number.

Solution Required number = $5\left(S - \frac{N}{9}\right) + \frac{1}{2}\left(S + \frac{N}{9}\right)$

$$= 5\left(6 - \frac{36}{9}\right) + \frac{1}{2}\left(6 + \frac{36}{9}\right)$$

$$\begin{aligned}
 &= 5(6 - 4) + \frac{1}{2}(6 + 4) \\
 &= 5 \times 2 + \frac{1}{2} \times 10 = \mathbf{15}
 \end{aligned}$$

If the ratio of sum and difference of two numbers is $x : y$, then the ratio of these two numbers is $\frac{x + y}{x - y}$.

EXAMPLE 3 The ratio of sum and difference of two numbers is $9 : 5$. Find the ratio of these two numbers.

Solution Required ratio = $\frac{9 + 5}{9 - 5} = \frac{14}{4} = \mathbf{7 : 2}$

If the sum of a number and its square is P , then the number is $\frac{\sqrt{1 + 4P} - 1}{2}$

EXAMPLE 4 If the sum of a number and its square is 2070, find the number.

Solution Let the number be x .

$$\begin{aligned}
 \therefore & \quad x^2 + x = 2070 \\
 \Rightarrow & \quad x^2 + x - 2070 = 0 \\
 \text{i.e.} & \quad (x + 46)(x - 45) = 0 \\
 \therefore & \quad x = \mathbf{45}
 \end{aligned}$$



Short cut: Required number = $\frac{\sqrt{1 + 4P} - 1}{2} = \frac{\sqrt{1 + 4 \times 2070} - 1}{2} = \frac{91 - 1}{2} = \mathbf{45}$

The ratio between a two-digit number and the sum of the digits of that number is $x : y$. If the digit in the unit place is P more than the digit in the tenth place, then

$$\text{Number} = \left(\frac{9x}{11y - 2x} \right) P$$

$$\text{Digit in unit place} = \left(\frac{10y - x}{11y - 2x} \right) P$$

$$\text{Digit in the tenth place} = \left(\frac{x - y}{11y - 2x} \right) P$$

EXAMPLE 5 The ratio between a two-digit number and the sum of the digits of that number is $4 : 1$. If the digit in the unit place is 4 more than digit in the tenth place, then find the number.

Solution Let the number be xy .

$$\therefore \quad \frac{10 \times x + y}{x + y} = \frac{4}{1}$$

$$\begin{aligned} \Rightarrow 10x + y &= 4x + 4y \\ \Rightarrow 6x &= 3y \quad \text{or} \quad 2x = y \\ \text{But } y &= x + 4 \\ \therefore 2x &= y \Rightarrow 2x = x + 4 \quad \text{or} \quad x = 4 \\ \therefore y &= x + 4 = 4 + 4 = 8 \\ \therefore \text{The required number is } 48. \end{aligned}$$



Short cut: Required number = $\left(\frac{9x}{11y - 2x}\right)P$

where $x : y$ = ratio of two-digit number to sum of digits.

P = amount by which unit place is more than tenth place.

$$\therefore \text{Required number} = \left(\frac{9 \times 4}{11 \times 1 - 2 \times 4}\right)4 = \frac{9 \times 4 \times 4}{3} = 48$$

To find the difference of the two digits of a two digit number, when the difference between two-digit number and the number obtained by interchanging the digits is given.

Solution Difference of two digits = $\frac{\text{Difference in original and interchanged numbers}}{9}$

The product of two numbers is P and their difference is q . Then the number are

$$\frac{\sqrt{q^2 + 4P} + q}{2} \quad \text{and} \quad \frac{\sqrt{q^2 + 4P} - q}{2}$$

EXAMPLE 6 The product of 2 numbers is 368 and the difference of these two numbers is 7. Find the numbers.

Solution Let the numbers be x and y . Given $x \times y = 368$; $x - y = 7$; $x = ?$; $y = ?$

$$\text{Required numbers are } \frac{\sqrt{7^2 + 4 \times 368} + 7}{2} \quad \text{and} \quad \frac{\sqrt{7^2 + 4 \times 368} - 7}{2} = 23 \text{ and } 16.$$

The product of 2 numbers is x and the sum of these 2 numbers is y then the numbers are $\frac{y + \sqrt{y^2 - 4x}}{2}$ and $\frac{y - \sqrt{y^2 - 4x}}{2}$.

EXAMPLE 7 The product of two numbers is 204 and their sum is 29. Find the numbers.

$$\text{Solution} \quad \text{Required numbers are } \frac{29 + \sqrt{29^2 - 4 \times 204}}{2} \quad \text{and} \quad \frac{29 - \sqrt{29^2 - 4 \times 204}}{2} = 17 \text{ and } 12.$$

The sum of squares of two numbers is x and the square of their difference is y , then the product of the two numbers is $\frac{x - y}{2}$.

EXAMPLE 8 The sum of squares of two numbers is 90 and the square of their difference is 36. Find the product of the numbers.

Solution Let the two numbers be x and y .

$$\text{Given } x^2 + y^2 = 90; (x - y)^2 = 36$$

$$\therefore \text{Product of number } xy = \frac{(x^2 + y^2) - (x - y)^2}{2} = \frac{90 - 36}{2} = 27$$

EXAMPLE 9 The product of two numbers is 120. The sum of their squares is 289. Find the sum of the two numbers and also find the difference of the two numbers.

$$\text{Sum of two numbers} = \sqrt{289 + 2 \times 120} = 23$$

$$\text{Difference of numbers} = \sqrt{289 - 2 \times 120} = \sqrt{49} = 7$$

The denominator of a rational number is ' D ' more than its numerator. If the numerator is increased by x and the denominator is decreased by y , the result obtained is N . Then the

$$\text{rational number is } \frac{x - N(D - y)}{x + (yN - D)}.$$

EXAMPLE 10 The denominator of a rational number is 3 more than its numerator. If the numerator is increased by 6 and denominator is decreased by 2, we obtain 2. Find the rational number.

$$\text{Solution Required answer} = \frac{6 - 2(3 - 2)}{6 + (2 \times 2 - 3)} = \frac{4}{7}$$

The sum of two numbers is P and their difference is q then the difference of their squares is Pq .

EXAMPLE 11 The sum of 2 numbers is 85 and their difference is 28. Find the difference of their squares.

$$\text{Solution Required answer} = 85 \times 28 = 2380$$

The difference between the squares of two consecutive numbers is x . Then the numbers are $\frac{x-1}{2}$ and $\frac{x+1}{2}$.

EXAMPLE 12 The difference between the squares of two consecutive numbers is 59. Find the numbers.

$$\text{Solution The numbers are } \frac{59+1}{2}, \frac{59-1}{2} = 30, 29. \text{ If two consecutive numbers are}$$

p and q , then the difference of their squares $= p^2 - q^2$. If the sum of 2 numbers is p and sum of their squares is q . Then

$$(i) \text{ Product of numbers} = \frac{p^2 - q}{2}$$

$$(ii) \text{ Numbers are } \left(\frac{p - \sqrt{2q - p^2}}{2} \right) \text{ and } \left(\frac{p + \sqrt{2q - p^2}}{2} \right)$$

EXAMPLE 13 The sum of two numbers is 15 and sum of their squares is 113. Find the numbers

Solution Required numbers are $= \frac{15 - \sqrt{2 \times 113 - 15^2}}{2}$ and $\frac{15 + \sqrt{2 \times 113 - 15^2}}{2}$

$$= \frac{15 - 1}{2} \text{ and } \frac{15 + 1}{2} = 7 \text{ and } 8$$

To find the number of zeros at the end of the product.

$$(2^m)(5^n) \text{ has } n \text{ zeros if } m > n \text{ or } m \text{ zeros if } m < n.$$

To find the numbers in the unit place for odd numbers.

$$(\dots 1)^n, (\dots 3)^{4n}, (\dots 7)^{4n}, (\dots 9)^{2n} \text{ ends with 1.}$$

For example,

$$\begin{aligned}(21)^1 &= 21, \text{ ends with 1} \\ (21)^2 &= 441, \text{ ends with 1.} \\ (13)^4 &= (169)^2 = 28561, \text{ ends with 1} \\ (17)^4 &= (289)^2 = 83521, \text{ ends with 1} \\ (9)^2 &= 81 \text{ ends with 1.}\end{aligned}$$

BINARY SYSTEM

A number system represents different numbers in different ways. There are many number systems. The number system with Base 2 is known as *binary number system*. The base denotes the number of symbols in the number system.

In the decimal system, for example, the base is 10 and it has 10 symbols (0, 1, 2, ..., 9). Some other examples are:

Number system	Base	Representation of symbols
Octal number system	8	0, 1, 2, 3, ..., 7
Hexadecimal number system	16	0, 1, 2, ..., 9, A(10), B (11), C(12) ... F(15)
Binary number system	2	0 and 1.

The binary number system was introduced by JV Newman in 1946.

CONVERSION OF DECIMAL NUMBERS INTO ITS BINARY EQUIVALENT: To find the binary equivalent of a decimal number, divide the given number by constant divisor 2 successively till last quotient 1 is obtained. For example convert 97 into its binary equivalent.

$$\begin{aligned}2 \overline{)97} \quad 2 \times 48 (q_1) &= 96; 97 - 96 = 1 (r_1) \\ 2 \overline{)48} \quad 2 \times 24 (q_2) &= 48; 48 - 48 = 0 (r_2) \\ 2 \overline{)24} \quad 2 \times 12 (q_3) &= 24; 24 - 24 = 0 (r_3) \\ 2 \overline{)12} \quad 2 \times 6 (q_4) &= 12; 12 - 12 = 0 (r_4)\end{aligned}$$

$$2 \overline{) 6} \quad 2 \times 3 (q_5) = 6; \quad 6 - 6 = 0 (r_5)$$

$$2 \overline{) 3} \quad 2 \times 1 (q_6) = 2; \quad 3 - 2 = 1 (r_6)$$

1 (last quotient)

$q_1, q_2, q_3, \dots, q_6$ are the first/second/third ... quotients and $r_1, r_2, \dots$ are the first/ second/ ... remainders. For all stages of division the common divisor 2 remains unchanged and the quotient obtained becomes next dividend. The process continues till the last quotient (1) is obtained.

Here 97 is the first dividend. After dividing it by 2, the first quotient ($q_1 = 48$) becomes the next dividend. Now after dividing 48 by 2, the second quotient ($q_2 = 24$) becomes the next dividend, and so on. Every time the remainder is noted. When the last quotient (1) is obtained, note down the remainders including the last quotient, 1, starting from the last quotient, then the last remainder, then the second last remainder ... and finally the first remainder.

That is binary equivalent of 97 is 1100001. i.e. $(97)_{10} = (1100001)_2$

CONVERSION OF BINARY NUMBER TO ITS DECIMAL EQUIVALENT

Convert 1011001 into its decimal equivalent.

The above number can be written inside each box in the following way and value of each box is written above it

2^6	2^5	2^4	2^3	2^2	2^1	2^0
1	0	1	1	0	0	1

Now $(1011001)_2$

$$\begin{aligned}
 &= 1 \times 2^6 + 0 \times 2^5 + 1 \times 2^4 + 1 \times 2^3 + 0 \times 2^2 + 0 \times 2^1 + 1 \times 2^0 \\
 &= 64 + 0 + 16 + 8 + 0 + 0 + 1 \\
 &= 89
 \end{aligned}$$

EXAMPLE 14 Convert 95 into its binary equivalent.

Solution $2 \overline{) 95} \quad 1 (r_1)$

$$\overline{) 47} \quad 1 (r_2)$$

$$\overline{) 23} \quad 1 (r_3)$$

$$\overline{) 11} \quad 1 (r_4)$$

$$\overline{) 5} \quad 1 (r_5)$$

$$\overline{) 2} \quad 0 (r_6)$$

1 (last quotient)

Here $(95)_{10} = (1011111)_2$

$$\begin{aligned}
 \text{Check: } (1011111)_2 &= 1 \times 2^6 + 0 \times 2^5 + 1 \times 2^4 + 1 \times 2^3 + 1 \times 2^2 + 1 \times 2^1 + 1 \times 2^0 \\
 &= 64 + 0 + 16 + 8 + 4 + 2 + 1 \\
 &= 95
 \end{aligned}$$

EXAMPLE 15 Convert 58 into its binary equivalent.

Solution $2 \overline{)58} \begin{array}{l} 0 \end{array}$

$\overline{)29} \begin{array}{l} 1 \end{array}$

$\overline{)14} \begin{array}{l} 0 \end{array}$

$\overline{)7} \begin{array}{l} 1 \end{array}$

$\overline{)3} \begin{array}{l} 1 \end{array}$

1

$$(58)_{10} = (111010)_2$$

$$\begin{aligned} \text{Check: } (111010)_2 &= 1 \times 2^5 + 1 \times 2^4 + 1 \times 2^3 + 0 \times 2^2 + 1 \times 2^1 + 0 \times 2^0 \\ &= 32 + 16 + 8 + 0 + 2 + 0 \\ &= 58 \end{aligned}$$

EXAMPLE 16 Convert the following numbers into their binary equivalents.

(a) 3

(b) 4

(c) 5

(d) 6

(e) 7

(f) 8

Solution

(a) $2 \overline{)3} \begin{array}{l} 1 \end{array}$

1

$$\text{i.e. } (3)_{10} = (11)_2$$

(b) $2 \overline{)4} \begin{array}{l} 0 \end{array}$

$\overline{)2} \begin{array}{l} 0 \end{array}$

1

$$\text{i.e. } (4)_{10} = (100)_2$$

(c) $2 \overline{)5} \begin{array}{l} 1 \end{array}$

$\overline{)2} \begin{array}{l} 0 \end{array}$

1

$$\text{i.e. } (5)_{10} = (101)_2$$

(d) $2 \overline{)6} \begin{array}{l} 0 \end{array}$

$\overline{)3} \begin{array}{l} 1 \end{array}$

1

$$\text{i.e. } (6)_{10} = (110)_2$$

(e) $2 \overline{)7} \begin{array}{l} 1 \end{array}$

$\overline{)3} \begin{array}{l} 1 \end{array}$

1

$$\text{i.e. } (7)_{10} = (111)_2$$

(f) $2 \overline{)8} \begin{array}{l} 0 \end{array}$

$\overline{)4} \begin{array}{l} 0 \end{array}$

$\overline{)2} \begin{array}{l} 0 \end{array}$

1

$$\text{i.e. } (8)_{10} = (1000)_2$$

EXAMPLE 17 Convert the following binary numbers into their decimal equivalent.

(a) 11001

(b) 100010

(c) 110010

(d) 1010110

(e) 1101101

Solution

$$(a) \quad (11001)_2 = 1 \times 2^4 + 1 \times 2^3 + 0 \times 2^2 + 0 \times 2^1 + 1 \times 2^0 \\ = 16 + 8 + 0 + 0 + 1 = \mathbf{25}$$

$$(b) \quad (100010)_2 = 1 \times 2^5 + 0 \times 2^4 + 0 \times 2^3 + 0 \times 2^2 + 1 \times 2^1 + 0 \times 2^0 \\ = 32 + 0 + 0 + 0 + 2 + 0 = \mathbf{34}$$

$$(c) \quad (110010)_2 = 1 \times 2^5 + 1 \times 2^4 + 0 \times 2^3 + 0 \times 2^2 + 1 \times 2^1 + 0 \times 2^0 \\ = 32 + 16 + 0 + 0 + 2 + 0 = \mathbf{50}$$

$$(d) \quad (1010110)_2 = 1 \times 2^6 + 0 \times 2^5 + 1 \times 2^4 + 0 \times 2^3 + 1 \times 2^2 + 1 \times 2^1 + 0 \times 2^0 \\ = 64 + 0 + 16 + 0 + 4 + 2 + 0 = \mathbf{86}$$

$$(e) \quad (1101101)_2 = 1 \times 2^6 + 1 \times 2^5 + 0 \times 2^4 + 1 \times 2^3 + 1 \times 2^2 + 0 \times 2^1 + 1 \times 2^0 \\ = 64 + 32 + 0 + 8 + 4 + 0 + 1 = \mathbf{109}$$

**EXERCISES**

- A certain number of two digits is three times the sum of its digits, and if 45 be added to it, the digit will be reversed. Then the number is: [LIC AAO]
 (a) 23 (b) 27 (c) 32 (d) 72
- A number of two digits has 3 for its unit's digit, and the sum of digits is $\frac{1}{7}$ of the number itself. The number is [LIC]
 (a) 43 (b) 53 (c) 63 (d) 73
- The sum of two numbers is 25 and their difference is 13. Find their product. [LIC]
 (a) 104 (b) 114 (c) 315 (d) 325
- Find the number which when multiplied by 15 is increased by 196. [LIC]
 (a) 14 (b) 20 (c) 26 (d) 28
- The numerator and denominator of a fraction are in the ratio of 2 : 3. If 6 is subtracted from the numerator, the result is a fraction that has a value $\frac{2}{3}$ of the original fraction. The numerator of the original fraction is: [SSC]
 (a) 6 (b) 18 (c) 27 (d) 36
- A number consists of two digits such that the digit in the ten's place is less by 2 than the digit in the unit's place. Three times the number added to $\frac{6}{7}$ times the number obtained by reversing the digits equals 108. The sum of digits in the number is: [SSC]
 (a) 6 (b) 7 (c) 8 (d) 9
- The sum of squares of three consecutive natural numbers is 2030. What is the middle number? [SSC]
 (a) 25 (b) 26 (c) 27 (d) 28
- If the sum of two numbers is 22 and the sum of their squares is 404, then the product of the numbers is: [SSC]
 (a) 40 (b) 44 (c) 80 (d) 88

9. If the sum and difference of two numbers are 20 and 8 respectively, then the difference of their squares is: [SSC]
(a) 12 (b) 28 (c) 160 (d) 180
10. The difference between two positive integers is 3. If the sum of their squares is 369, then the sum of the numbers is: [SSC]
(a) 25 (b) 27 (c) 33 (d) 81
11. Two numbers differ by 5. If their product is 336, then the sum of the two numbers is [SSC]
(a) 21 (b) 28 (c) 37 (d) 51
12. The product of two natural numbers is 17. Then the sum of the reciprocals of their squares is: [SSC]
(a) $\frac{1}{289}$ (b) $\frac{289}{290}$ (c) $\frac{290}{289}$ (d) 289
13. If doubling a number and adding 20 to the result gives the same answer as multiplying the number by 8 and taking away 4 from the product, the number is: [SSC]
(a) 2 (b) 3 (c) 4 (d) 6
14. Thrice the square of a natural number decreased by 4 times the number is equal to 50 more than the number. The number is: [SSC]
(a) 4 (b) 5 (c) 6 (d) 10
15. The sum of four numbers is 64. If you add 3 to the first number, 3 is subtracted from the second number, the third is multiplied by 3 and the fourth is divided by 3, then all the results are equal. What is the difference between the largest and the smallest of the original numbers? [SBI PO]
(a) 21 (b) 27 (c) 32
(d) Cannot be determined (e) None of these
16. In a two-digit number, the digit in the unit's place is more than twice the digit in ten's place by 1. If the digits in the unit's place and the ten's place are interchanged, difference between the new number and the original number is less than the original number by 1. What is the original number? [Bank PO]
(a) 25 (b) 37 (c) 49 (d) 52 (e) 73
17. If the digit in the unit's place of a two-digit number is halved and the digit in the ten's place is doubled, the number thus obtained is equal to the number obtained by interchanging the digits. Which of the following is definitely true? [Bank PO]
(a) Sum of the digits is a two-digit number
(b) Digit in the unit's place is twice the digit in the ten's place
(c) Digit in the unit's place and ten's place are equal
(d) Digit in the unit's place is half of the digit in the ten's place
(e) None of these
18. The sum of three consecutive odd numbers is 20 more than the first of these numbers. What is the middle number? [SBI PO]
(a) 7 (b) 9 (c) 11
(d) Data inadequate (e) None of these
19. One-fifth of a number is equal to $\frac{5}{8}$ of another number. If 35 is added to the first number, it becomes four times of the second number. The second number is: [Bank PO]
(a) 25 (b) 40 (c) 70 (d) 125

20. If the sum of a number and its square is 182, what is the number? **[Bank PO]**
 (a) 15 (b) 26 (c) 28 (d) 91 (e) None of these
21. If a number is decreased by 4 and divided by 6, the result is 8. What would be the result if 2 is subtracted from the number and then it is divided by 5? **[Bank PO]**
 (a) $9\frac{2}{3}$ (b) 10 (c) $10\frac{1}{5}$ (d) $11\frac{1}{5}$ (e) None of these
22. A two-digit number is such that the product of the digits is 8. When 18 is added to the number, then the digits are reversed. The number is: **[MBA]**
 (a) 18 (b) 24 (c) 42 (d) 81
23. The difference between a two-digit number and the number obtained by interchanging the digits is 36. What is the difference between the sum and the difference of the digits of the number if the ratio between the digits of the number is 1 : 2? **[MAT]**
 (a) 4 (b) 8 (c) 16 (d) None of these
24. Three times the first of three consecutive odd integers is 3 more than twice the third. The third integer is: **[MBA]**
 (a) 9 (b) 11 (c) 13 (d) 15
25. The sum of the digits of a two-digit number is 15 and the difference between the digits is 3. What is the two-digit number? **[BSRB]**
 (a) 69 (b) 78 (c) 96 (d) Cannot be determined
 (e) None of these
26. The product of two numbers is 45 and the sum of their squares is 106. The numbers are: **[RRB]**
 (a) 3 and 5 (b) 5 and 9 (c) 5 and 19 (d) 45 and 1
27. The product of two numbers is 120 and the sum of their square is 289. The sum of the numbers is: **[RRB]**
 (a) 20 (b) 23 (c) 169 (d) None of these
28. The sum of three numbers is 264. If the first number be twice the second and third number be one-third of the first, then the second number is: **[RRB]**
 (a) 48 (b) 54 (c) 72 (d) 84
29. If 50 is subtracted from two-third of a number, the result is equal to sum of 40 and one-fourth of that number. What is the number? **[RRB]**
 (a) 174 (b) 216 (c) 246 (d) 336
30. If $2\frac{1}{2}$ is added to a number and the sum is multiplied by $4\frac{1}{2}$ and 3 is added to the product and then dividing the sum by $1\frac{1}{5}$, the quotient becomes 25. What is the number? **[RRB]**
 (a) $2\frac{1}{2}$ (b) $3\frac{1}{2}$ (c) $4\frac{1}{2}$ (d) $5\frac{1}{2}$
31. In a two-digit number, if it is known that its unit's digit exceeds its ten's digit by 2, and that the product of the given number and the sum of its digits is equal to 144, then the number is: **[CBI]**
 (a) 24 (b) 26 (c) 42 (d) 46
32. The sum of two numbers is 22. Five times of one number is equal to 6 times the other. The bigger of the two numbers is: **[CBI]**
 (a) 10 (b) 12 (c) 15 (d) 16

33. If the sum of two numbers is 33 and their difference is 15, the smaller number is: [CBI]
 (a) 9 (b) 12 (c) 15 (d) 18
34. If the sum of one-half and one-fifth of a number exceeds one-third of that number by $7\frac{1}{3}$, the number is: [CBI]
 (a) 15 (b) 18 (c) 20 (d) 30
35. Two-third of a positive number and $\frac{25}{216}$ of its reciprocal are equal. The number is: [SSC]
 (a) $\frac{5}{12}$ (b) $\frac{12}{5}$ (c) $\frac{25}{144}$ (d) $\frac{144}{25}$



1. (b) 2. (c) 3. (b) 4. (a) 5. (b) 6. (b) 7. (b) 8. (a) 9. (c) 10. (b)
 11. (c) 12. (c) 13. (c) 14. (b) 15. (c) 16. (b) 17. (b) 18. (b) 19. (b) 20. (e)
 21. (b) 22. (b) 23. (b) 24. (d) 25. (d) 26. (b) 27. (b) 28. (c) 29. (b) 30. (b)
 31. (a) 32. (b) 33. (a) 34. (c) 35. (a)



Solutions with Necessary Explanation

1. Let the number be $10x + y$

Now, $10x + y = 3(x + y)$

$\Rightarrow 7x = 2y$

$\therefore \frac{x}{y} = \frac{2}{7}$

$\therefore$ Required number = 27

2. Let the number be $10x + 3$

$$x + 3 = \frac{1}{7}(10x + 3)$$

$\Rightarrow 7x + 21 = 10x + 3$

$\Rightarrow 3x = 18$

$\Rightarrow x = 6$

$\therefore$ Required number = 63

3. Let the number be x and y .

$\therefore x + y = 25$ (1)

$x - y = 13$ (2)

Now, $(25)^2 - (13)^2 = 4xy$

$$\begin{aligned} \therefore xy &= \frac{(25)^2 - (13)^2}{4} \\ &= \frac{(25 + 13)(25 - 13)}{4} \\ &= \frac{38 \times 12}{4} = 19 \times 6 = 114 \end{aligned}$$

4. Let number be x .

$$\begin{aligned}\therefore \quad 15x &= x + 196 \\ 14x &= 196 \\ x &= 14\end{aligned}$$

5. Let the fraction be $\frac{p}{q}$.

$$\text{Then} \quad \frac{p}{q} = \frac{2}{3} \quad (1)$$

$$\frac{p-6}{q} = \frac{4}{9} \quad (2)$$

$$\Rightarrow (1) \quad 3p = 2q \quad (A)$$

$$\Rightarrow (2) \quad 9(p-6) = 4q \quad (B)$$

From (A) and (B), we get

$$9(p-6) = 2 \times 3p$$

$$9p - 54 = 6p$$

$$3p = 54$$

$$\therefore \quad p = 18$$

6. Let the number be $10x + y$.

$$\text{Then} \quad x = y - 2 \quad (1)$$

$$3(10x + y) + \frac{6}{7}(10y + x) = 108$$

$$\Rightarrow 3\{10(y-2) + y\} + \frac{6}{7}\{10y + (y-2)\} = 108$$

$$\Rightarrow 3\{10y - 20 + y\} + \frac{6}{7}\{10y + y - 2\} = 108$$

$$\Rightarrow 33y - 60 + \frac{66}{7}y - \frac{12}{7} = 108$$

$$\Rightarrow \left(33 + \frac{66}{7}\right)y = 108 + 60 + \frac{12}{7}$$

$$\frac{297y}{7} = \frac{168 \times 7 + 12}{7}$$

$$\therefore \quad y = \frac{168 \times 7 + 12}{297}$$

$$= \frac{56 \times 7 + 4}{99} = \frac{396}{99} = 4$$

$$\therefore \quad x = y - 2 = 4 - 2 = 2$$

Sum of digits of number = $x + y = 2 + 4 = 6$

7. Let the numbers be $(x-1)$, x and $(x+1)$

$$\text{Then} \quad (x-1)^2 + x^2 + (x+1)^2 = 2030 \quad (1)$$

$$\Rightarrow 3x^2 + 2 = 2030$$

$$\Rightarrow 3x^2 = 2028$$

$$\Rightarrow x^2 = \frac{2028}{3}$$

$$\Rightarrow x^2 = 676$$

$$\therefore x = \sqrt{676}$$

$$= 26$$

$$\therefore \text{Middle number} = x = 26$$

8. Let the numbers be x and y .

$$\therefore x + y = 22 \quad (1)$$

$$x^2 + y^2 = 404 \quad (2)$$

$$(1)^2 - (2) \Rightarrow (x + y)^2 - (x^2 + y^2) = (22)^2 - 404$$

$$\Rightarrow 2xy = 484 - 404$$

$$\Rightarrow 2xy = 80$$

$$\Rightarrow xy = \frac{80}{2} = 40$$

9. Let the numbers be x and y .

$$\text{Then } x + y = 20 \quad (1)$$

$$x - y = 8 \quad (2)$$

$$\therefore x^2 - y^2 = (x + y)(x - y) = (1) \times (2)$$

$$(1) \times (2) \Rightarrow (x + y)(x - y) = 20 \times 8$$

$$\Rightarrow x^2 - y^2 = 160$$

10. Let the two positive integers be x and y .

$$\therefore x - y = 3 \quad (1)$$

$$x^2 + y^2 = 369 \quad (2)$$

$$(x + y) = ?$$

$$(x - y)^2 = (x^2 + y^2) - 2xy$$

$$\text{i.e. } 9 = 369 - 2xy$$

$$\Rightarrow 2xy = 360$$

$$\therefore (x + y)^2 = (x^2 + y^2) + 2xy$$

$$= 369 + 360$$

$$= 729$$

$$\therefore x + y = \sqrt{729} = 27$$

11. Let the numbers be x and y .

$$\text{Given } x - y = 5 \quad (1)$$

$$xy = 336 \quad (2)$$

$$\therefore x + y = \sqrt{(x - y)^2 + 4xy}$$

$$= \sqrt{5^2 + 4 \times 336}$$

$$= \sqrt{1369} = 37$$

12. Let the two natural numbers be x and y .

$$\text{Given } xy = 17$$

$$\therefore x = 1 \quad \text{and} \quad y = 17$$

$$\therefore \frac{1}{x^2} + \frac{1}{y^2} = \frac{1}{1^2} + \frac{1}{17^2} = 1 + \frac{1}{289} = \frac{290}{289}$$

13. Let the number be x .

$$\text{Then } 2x + 20 = 8x - 4$$

$$\Rightarrow 8x - 2x = 20 + 4$$

$$\Rightarrow 6x = 24$$

$$\Rightarrow x = \frac{24}{6} = 4$$

14. Let the natural number be x .

$$\text{Then } 3x^2 - 4x = x + 50$$

$$\Rightarrow 3x^2 - 5x - 50 = 0$$

$$\Rightarrow (3x + 10)(x - 5) = 0$$

$$\therefore x = \frac{-10}{3} \text{ or } 5$$

15. Let numbers be a , b , c and d .

$$\text{Now, } a + 3 = b - 3 = 3 \times c = \frac{d}{3} = k$$

$$\therefore a = k - 3; \quad b = k + 3; \quad c = \frac{k}{3}; \quad d = 3k;$$

$$\therefore a + b + c + d = 64$$

$$\Rightarrow k - 3 + k + 3 + \frac{k}{3} + 3k = 64$$

$$\Rightarrow \frac{15k + k}{3} = 64$$

$$\Rightarrow k = \frac{64 \times 3}{16} = 12$$

$$\therefore a = k - 3 = 9; \quad b = k + 3 = 12 + 3 = 15; \quad c = \frac{k}{3} = \frac{12}{3} = 4; \quad d = 3k \times 12 = 36$$

$\therefore$ Numbers are 9, 15, 4 and 36.

$$\text{Required difference} = (36 - 4) = 32$$

16. Let the two digit number be $10x + y$.

$$\text{Then } y = 2x + 1 \tag{1}$$

$$(10y + x) - (10x + y) = (10x + y) - 1$$

$$\Rightarrow (10y + x) + 1 = 2(10x + y)$$

$$\Rightarrow 10y - 2y + x - 20x + 1 = 0$$

$$\Rightarrow 8y - 19x + 1 = 0 \tag{2}$$

$\therefore$ Substituting value of y from (1), we get

$$8(2x + 1) - 19x + 1 = 0$$

$$\Rightarrow -3x + 9 = 0$$

$$\Rightarrow x = 3$$

$$\therefore y = 2x + 1 = 2(3) + 1 = 7$$

$$\therefore \text{Original number} = 10x + y = 10 \times 3 + 7 = 37$$

17. Let the number be $10x + y$.

$$\text{Then} \quad 10y + x = 10(2x) + \frac{y}{2}$$

$$\Rightarrow \quad 10y + x = 20x + \frac{y}{2}$$

$$\Rightarrow \quad 20y + 2x = 40x + y$$

$$\Rightarrow \quad 38x = 19y$$

$$\Rightarrow \quad 2x = y$$

18. Let the three consecutive odd numbers be $(x - 2)$, x and $(x + 2)$.

$$\text{Then} \quad (x - 2) + x + (x + 2) = (x - 2) + 20$$

$$\Rightarrow \quad 3x = x + 18$$

$$\Rightarrow \quad 2x = 18$$

$$\Rightarrow \quad x = 9$$

19. Let the numbers be x and y .

$$\text{Now} \quad \frac{1}{5}x = \frac{5}{8}y \quad (1)$$

$$x + 35 = 4y \quad (2)$$

$$\text{From (1),} \quad x = \frac{25}{8}y$$

Substituting value of x in (2), we get

$$\frac{25}{8}y + 35 = 4y$$

$$\Rightarrow \quad \left(4 - \frac{25}{8}\right)y = 35$$

$$\Rightarrow \quad \frac{7}{8}y = 35$$

$$\Rightarrow \quad y = \frac{35 \times 8}{7} = 40$$

$$\therefore \quad \text{Second number} = 40.$$

20. Let the number be x .

$$\therefore \quad x + x^2 = 182$$

$$\text{i.e.} \quad x(x + 1) = 182$$
$$= 13 \times 14$$

$$\therefore \quad x = 13$$

21. Let the number be x .

$$\text{Then} \quad \frac{x - 4}{6} = 8$$

$$\Rightarrow \quad x = 8 \times 6 + 4 = 52$$

$$\therefore \quad \frac{x - 2}{5} = \frac{52 - 2}{5} = 10$$

22. Let the number be $10x + y$.

$$\text{Now,} \quad xy = 8 \quad (1)$$

$$(10x + y) + 18 = 10y + x \quad (2)$$

$$\Rightarrow \quad 9y - 9x = 18$$

$$\Rightarrow \quad y - x = 2$$

$$\begin{aligned} & y = 4 \quad \text{and } x = 2 \\ \text{so that} & \quad xy = 8 \\ \therefore & \quad \text{Required number is } 10x + y = 10(2) + 4 = 24 \end{aligned}$$

23. Let the number be $10x + y$.

$$\begin{aligned} \therefore & \quad (10x + y) \sim (10y + x) = 36 \\ \text{i.e.} & \quad 9x \sim 9y = 36 \\ \Rightarrow & \quad x \sim y = 4 \\ \text{Given} & \quad y : x = 1 : 2 \\ \therefore \text{ When} & \quad y = 4, x = 8 \\ \text{so that} & \quad x \sim y = 4 \\ \therefore & \quad (x + y) \sim (x - y) = 2y = 8 \end{aligned}$$

24. Let the three consecutive odd numbers be $(x - 2)$, x and $(x + 2)$.

$$\begin{aligned} \therefore & \quad 3(x - 2) = 2(x + 2) + 3 \\ \Rightarrow & \quad 3x - 6 = 2x + 4 + 3 \\ \Rightarrow & \quad x = 4 + 3 + 6 = 13 \\ \therefore & \quad \text{Third integer} = x + 2 = 15 \end{aligned}$$

25. Let the digits of two-digit number be x and y .

$$\begin{aligned} \therefore & \quad x + y = 15 & (1) \\ & \quad x - y = 3 & (2) \end{aligned}$$

$$(1) + (2) \Rightarrow 2x = 18$$

$$\Rightarrow x = 9$$

$$\text{From (1), } y = 15 - 9 = 6$$

$\therefore$ 96 and 69 are possible answers for the required number, and both values are given as answer, correct answer is "cannot be determined". Hence answer is (d).

26. Let the numbers be x and y .

$$\begin{aligned} \therefore & \quad xy = 45 & (1) \\ & \quad x^2 + y^2 = 106 & (2) \end{aligned}$$

$$(2) + 2 \times (1) \Rightarrow x^2 + y^2 + 2xy = 106 + 90 = 196$$

$$\Rightarrow (x + y)^2 = 196$$

$$\Rightarrow x + y = 14$$

$\therefore$ Required numbers are **5 and 9**.

27. Let the numbers be x and y .

$$\therefore xy = 120 \quad (1)$$

$$x^2 + y^2 = 289 \quad (2)$$

$$(2) + 2 \times (1) \Rightarrow x^2 + y^2 + 2xy = 289 + 240 = 529$$

$$\therefore (x + y)^2 = 529$$

$$\Rightarrow x + y = \sqrt{529} = 23$$

28. Let the second number be x .

$$\therefore \text{ First number} = 2x$$

$$\text{Third number} = \frac{2}{3}x$$

$$\therefore 2x + x + \frac{2}{3}x = 264$$

$$\Rightarrow \frac{11}{3}x = 264$$

$$\Rightarrow x = \frac{264 \times 3}{11}$$

$$= 24 \times 3 = 72$$

29. Let the numbers be x .

$$\therefore \frac{2}{3}x - 50 = \frac{x}{4} + 40$$

$$\Rightarrow \frac{2}{3}x - \frac{x}{4} = 50 + 40 = 90$$

$$\Rightarrow \frac{8x - 3x}{12} = 90$$

$$\Rightarrow 5x = 12 \times 90$$

$$x = 12 \times \frac{90}{5}$$

$$= 12 \times 18 = 216$$

30. Let the numbers be x .

$$\text{Then } \left[\left(2\frac{1}{2} + x \right) 4\frac{1}{2} + 3 \right] \div 1\frac{1}{5} = 25$$

$$\Rightarrow \left[\left(\frac{5}{2} + x \right) \frac{9}{2} + 3 \right] \div \frac{6}{5} = 25$$

$$\Rightarrow \frac{25}{12} \times \frac{9}{2} + 3 \times \frac{5}{6} + x \times \frac{9}{2} \times \frac{5}{6} = 25$$

$$\Rightarrow \frac{75}{8} + \frac{5}{2} + \frac{15}{4}x = 25$$

$$\Rightarrow \frac{15}{4}x = 25 - \frac{75}{8} - \frac{5}{2}$$

$$\Rightarrow \frac{15}{4}x = \frac{25 \times 8 - 75 - 5 \times 4}{8}$$

$$\therefore x = \frac{4}{15} \times \frac{(200 - 75 - 20)}{8}$$

$$= \frac{4}{15} \times \frac{105}{8} = \frac{7}{2} = 3\frac{1}{2}$$

31. Let the number be $10x + y$.

Given

$$x + 2 = y$$

(1)

$$(10x + y)(x + y) = 144$$

(2)

Substituting value of y in terms of x in (2), we get

$$(10x + x + 2)(x + x + 2) = 144$$

$$\Rightarrow (11x + 2)(2x + 2) = 144$$

$$\Rightarrow (11x + 2)(x + 1) = 72$$

$$\Rightarrow 11x^2 + 13x + 2 = 72$$

i.e. $11x^2 + 13x - 70 = 0$

$\Rightarrow (11x + 35)(x - 2) = 0$

$\Rightarrow x = 2 \text{ or } \frac{-35}{11}$

$\therefore$ From (1) $y = x + 2 = 2 + 2 = 4$

$\therefore$ Required number $= 10x + y = 10(2) + 4 = 24$

32. Let the numbers be x and y .

Then $x + y = 22$ (1)

$5x = 6y$ (2)

From (2), $x = \frac{6}{5}y$

Substitute value of y in terms of x in (1), we get

$$x + \frac{5}{6}x = 22$$

$\Rightarrow \frac{11x}{6} = 22$

$\Rightarrow x = \frac{6 \times 22}{11} = 12$

$\therefore$ Bigger number $= 12$

33. Let numbers be x and y .

Then $x + y = 33$ (1)

$x - y = 15$ (2)

(1) - (2) $\Rightarrow 2y = 33 - 15 = 18$

$\therefore y = 9$

$\therefore$ Smaller number $= 9$

34. Let the number be x .

$$\frac{x}{2} + \frac{x}{5} = \frac{x}{3} + 7\frac{1}{3}$$

i.e. $\frac{x}{2} + \frac{x}{5} - \frac{x}{3} = \frac{22}{3}$

$$\frac{15x + 6x - 10x}{30} = \frac{22}{3}$$

$\Rightarrow \frac{11x}{30} = \frac{22}{3}$

$\Rightarrow x = \frac{22}{3} \times \frac{30}{11} = 20$

35. Let number be x .

Then $\frac{2}{3}x = \frac{25}{216} \times \frac{1}{x}$

$\Rightarrow x^2 = \frac{25}{216} \times \frac{3}{2} = \frac{25}{144}$

$\Rightarrow x = \sqrt{\frac{25}{144}} = \frac{5}{12}$

Arithmetical Reasoning

This chapter deals with questions of different types which require some typical arithmetical calculations to solve them. Some of these types of questions have already been included in Chapters, namely *Elementary Algebra*, *Problem on Ages*, *Venn Diagram*, etc. Therefore, this chapter includes some typical examples and exercises for readers.

TYPE I CALCULATION BASED PROBLEMS

EXAMPLE 1 How many different words of 5 letters can be formed with the letters of the word 'EVOLUTION', so that the vowels occupy odd places?

Solution The three odd places can be occupied by 4 vowels in ${}^4P_3 = 4 \times 3 \times 2 = 24$ ways.

The remaining two places can be occupied by 4 consonants in ${}^4P_2 = 4 \times 3 = 12$ ways.

$\therefore$

Required number of words = $24 \times 12 = 288$

EXAMPLE 2 Five players take part in a tennis match. Each player has to play with each other. How many match must they play?

Solution

- (i) The first player should play with 4 other players.
- (ii) The second player should play with 3 remaining players.
- (iii) The third player should play with 2 remaining players.
- (iv) The fourth player should play with 1 player other than first three players.

$\therefore$ Total number of matches = $4 + 3 + 2 + 1 = 10$

EXAMPLE 3 In a class, $1/3$ of the students are girls. $1/2$ of the girls passed with distinction and $1/3$ of the girls passed with distinction got admission to professional courses. If $3/4$ of the boys passed with distinction and $2/3$ of the boys passed with distinction got admission to professional courses, what part of the students are not admitted to professional courses?

Solution Let the number of students in the class be x .

$\therefore$ Number of girls = $x/3$; Number of boys = $2x/3$

$$\begin{aligned}\text{Number of girls got admission to professional courses} &= \frac{1}{3} \text{ of } \frac{1}{2} \text{ of } \frac{x}{3} \\ &= \frac{x}{18}\end{aligned}$$

$$\begin{aligned}\text{Number of boys got admission to professional courses} &= \frac{2}{3} \text{ of } \frac{3}{4} \text{ of } \frac{2x}{3} \\ &= \frac{x}{3}\end{aligned}$$

$$\therefore \quad \text{The number of students got admission to professional courses} = \frac{x}{18} + \frac{x}{3} = \frac{7x}{18}$$

$$\therefore \quad \text{The number of students who are not admitted to professional courses} = x - \frac{7x}{18} = \frac{11x}{18}$$

TYPE II DATA-BASED QUESTIONS

Examples (4–6) The following questions are based on the given data of an examination with three subjects.

(A) Candidates appeared	: 1000
(B) Candidates passed in all the three subjects	: 460
(C) Candidates passed in two subjects only	: 280
(D) Candidates passed in one subject only	: 240
(E) Candidates failed in Physics only	: 80
(F) Candidates failed in Chemistry only	: 60
(G) Candidates failed in Maths only	: 140

EXAMPLE 4 How many candidates failed in all the subjects?

Solution Candidates failed in all the subjects

$$\begin{aligned}&= \text{Candidates appeared} - (\text{Candidates passed in 1, 3 subject} \\ &\quad + \text{Candidates failed in 1 subject only}) \\ &= 1000 - [240 + 460 + (80 + 60 + 140)] \\ &= 1000 - [240 + 460 + 280] \\ &= 1000 - 980 = 20\end{aligned}$$

EXAMPLE 5 How many candidates passed in at least two subjects?

Solution Candidates passed in at least two subjects

$$\begin{aligned}&= \text{Candidates passed in 2 subjects} \\ &\quad + \text{Candidates passed in all the three subjects} \\ &= \text{Candidates failed in 1 subject} \\ &\quad + \text{Candidates passed in all the three subjects} \\ &= [(80 + 60 + 140) + 460] = 280 + 460 = 740\end{aligned}$$

EXAMPLE 6 How many candidates failed because of having failed in two or less subjects?

Solution Candidates failed in two or less subjects

$$\begin{aligned}
 &= \text{Candidates failed in 1 subject} + \text{Candidates failed in two subjects} \\
 &= \text{Candidates failed in 1 subject} + \text{Candidates passed in one subject} \\
 &= [(80 + 60 + 140) + 240] = 280 + 240 = \mathbf{520}
 \end{aligned}$$

TYPE III PROBLEMS ON AGES

EXAMPLE 7 The age of a father is 4 times that of his son. Four years ago, the father was six times as old as his son at that time. What is the present age of father?

Solution Let the age of son be x

$$\begin{aligned}
 \therefore \quad & \text{age of father} = 4x \\
 \therefore \quad & \frac{4x - 4}{x - 4} = 6 \\
 \Rightarrow \quad & 4x - 4 = 6x - 24 \\
 \Rightarrow \quad & 2x = 20 \\
 \Rightarrow \quad & x = 10 \text{ years} \\
 \text{i.e.} \quad & \text{age of son} = 10 \text{ years} \\
 \therefore \quad & \text{Present age of father} = 4x = \mathbf{40 \text{ years}}
 \end{aligned}$$

EXAMPLE 8 The ratio of ages of Sanal and Suresh is 3 : 4. Four years earlier, the ratio was 5 : 7. Find the present ages of Sanal and Suresh.

Solution Let the age of Sanal be x

$$\begin{aligned}
 \therefore \quad & \text{Age of Suresh} = \frac{4}{3}x = y \\
 \text{Given} \quad & \frac{x - 4}{\frac{4}{3}x - 4} = \frac{5}{7} \\
 \Rightarrow \quad & \frac{x - 4}{4\left(\frac{x}{3} - 1\right)} = \frac{5}{7} \\
 & \frac{3(x - 4)}{4(x - 3)} = \frac{5}{7} \\
 \Rightarrow \quad & 21(x - 4) = 20(x - 3) \\
 \Rightarrow \quad & x = 21 \times 4 - 20 \times 3 = 84 - 60 = \mathbf{24 \text{ years}} \\
 \therefore \quad & \text{Age of Sanal} = \mathbf{24 \text{ years}} \\
 & \text{Age of Suresh} = \frac{4}{3} \times 24 = \mathbf{32 \text{ years}}
 \end{aligned}$$

EXAMPLE 9 The ratio of ages of Ajith and Sreejith at present is 6 : 1. After 5 years the ratio will become 7 : 2. What is the present age of Sreejith?

Solution

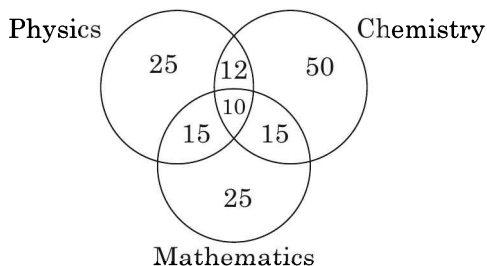
	Ajith	:	Sreejith
Present age	6	:	1
After 5 years	7	:	2

$$\therefore \text{Age of Sreejith} = 1 \times \frac{5(7-2)}{6 \times 2 - 7 \times 1} = \mathbf{5 \text{ years}}$$

$$\text{Age of Ajith} = 6 \times \frac{5(7-2)}{6 \times 2 - 7 \times 1} = \mathbf{30 \text{ years}}$$

TYPE IV VENN-DIAGRAM BASED QUESTIONS

EXAMPLE 10 Consider the following Venn diagram.



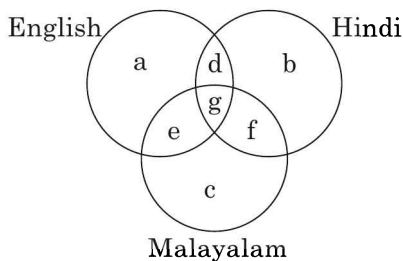
Three hundred candidates appeared in an examination with three subjects, Physics, Chemistry and Mathematics. The Venn diagram gives the number of candidates who failed in different subjects. What is the percentage of candidates who failed in at the most two subjects?

Solution Number of candidates who failed in at the most two subjects
 $=$ Candidates failed in one subject
 $+$ Candidates failed in two subjects
 $= (25 + 50 + 25) + (12 + 15 + 15)$
 $= 100 + 42 = \mathbf{142}$

$$\therefore \text{Required percentage} = \left(\frac{142}{300} \times 100 \right) \%$$

$$= 47\frac{1}{3} \% = \mathbf{47.33 \%}$$

EXAMPLE 11 A group of persons is attending a conference. Ten persons can speak English, fifteen persons can speak Hindi, twenty five persons can speak Malayalam. None of them can speak any other language. If two persons in the group can speak two languages and one person can speak all the three languages, how many persons are there in the group?



Solution Number of persons speaking English = $a + d + g + e = 10$ (1)

Number of persons speaking Hindi = $b + d + f + g = 15$ (2)

Number of persons speaking Malayalam = $c + e + f + g = 25$ (3)

Persons speaking 2 languages = $d + e + f = 2$ (4)

Persons speaking all three languages = $g = 1$ (5)

From Eqs. (1) and (5), we have $a + d + e = 9$ (6)

From Eqs. (2) and (5), we have $b + d + f = 14$ (7)

From Eqs. (3) and (5), we have $c + e + f = 24$ (8)

(6) - (4) $\Rightarrow a - f = 7$ (9)

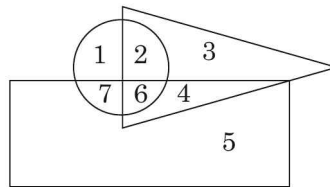
(7) + (8) $\Rightarrow b + c + d + e + 2f = 38$ (10)

(9) + (10) $\Rightarrow a + b + c + d + e + f = 45$ (11)

or $a + b + c + d + e + f + g = 46$

$\therefore$ Total number of persons in the group = **46**

In the figure given below, the circle represents government employed persons, the triangle represents lecturers and rectangle represents doctors.



Study the figure given above and answer the following questions.

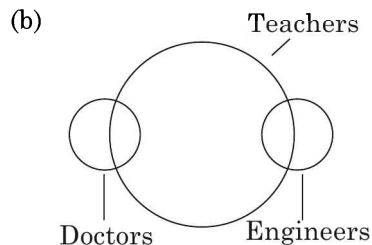
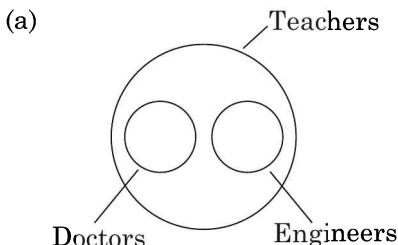
EXAMPLE 12 Which region represents lecturers working in non governmental institutions who are not professional?

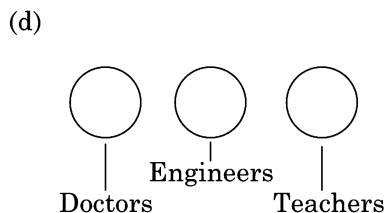
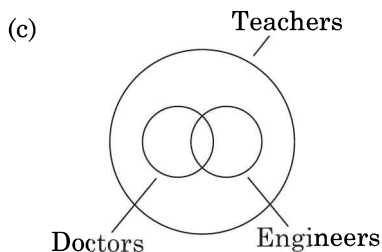
Solution Regions 3 and 4 represent lecturers in which 4 represents lecturer belonging to professional category. Therefore correct region is 3.

EXAMPLE 13 Which region represents government doctors, but not a lecturer?

Solution Here regions 7 and 6 denote government doctors. But region 6 represents a government doctor and a lecturer. So the region which represents a government doctor but not a lecturer is 7.

EXAMPLE 14 A group of doctors, engineers and teachers can be represented by Venn diagram as shown below:





(e) None of these

Solution Here doctors and engineers are two different professionals. Therefore, they can be represented by independent circles. Teacher is a different profession. But some of the doctors and engineers can be teachers. So it can be represented with a common intersection with circles representing doctors and engineers as given in (b).

Hence the answer is (b).



EXERCISES

- There are some cows and parrots in a group. The number of legs is 14 more than twice the number of heads. The number of cows is
(a) 5 (b) 7 (c) 10 (d) 12 (e) None of these
- A certain number of bullocks and an equal number of men are walking through a road. If the number of legs walking on the ground is 72, how many bullocks are there?
(a) 10 (b) 12 (c) 14 (d) 16 (e) None of these
- The sum of ages of Ajeesh, Abhishek, and Sajith is 74 years. What was the total of their ages three years ago?
(a) 63 years (b) 65 years (c) 67 years (d) 69 years (e) None of these
- Bineesh is 3 years older to Dinesh and 3 years younger to Gireesh, while Dinesh and Manoj are twins. How many years older is Gireesh to Manoj?
(a) 2 (b) 3 (c) 6 (d) 12 (e) None of these
- A person told his son, "I was of your present age when you were born". If the father is 42 now, how old was the boy five years back?
(a) 15 (b) 16 (c) 18 (d) 20 (e) None of these



1. (b) 2. (b) 3. (b) 4. (c) 5. (b)



Solutions with Necessary Explanation

1. Let the number of cows and parrots be x and y .

$$\text{Number of heads} = x + y$$

(1)

$\therefore$

$$\text{Number of legs} = 2(x + y) + 14$$

(2)

$$\text{i.e.} \quad 4x + 2y = 2(x + y) + 14 \quad (3)$$

$$\Rightarrow \quad 2x = 14$$

$$\text{or} \quad x = 7$$

$\therefore$ The number of cows is 7.

2. Let the number of bullocks and men be x each.

$$\therefore \quad 2x + 4x = 72$$

$$\Rightarrow \quad x = \frac{72}{6} = 12$$

$\therefore$ Number of bullocks = 12

3. Let the ages of Ajeesh, Abhishek and Sajith be x , y and z .

$$\text{Sum of ages} = x + y + z = 74 \quad (1)$$

$\therefore$ 3 years ago, sum of their ages is given by $(x - 3) + (y - 3) + (z - 3) = 74 - 9 = 65$ years.

4. Let the ages of Bineesh, Dinesh, Gireesh and Manoj be b , d , g and m respectively.

$$\therefore \quad b = d + 3 \quad (1)$$

$$b = g - 3 \quad (2)$$

$$d = m \quad (3)$$

Equating Eqs. (1) and (2), we get

$$d + 3 = g - 3$$

$$\text{or} \quad d = g - 6$$

$\therefore$ Gireesh is 6 years older to Manoj and Dinesh as Manoj and Dinesh are twins.

5. Here the age of father = 42 years

$$\therefore \quad \text{Age of son} = \frac{1}{2} \times 42 = 21 \text{ years}$$

$\therefore$ Five years ago, the age of son was $21 - 5 = 16$ years.

Simplification

This chapter deals with order of precedence of various arithmetical operators on numbers.

VBODMAS: VBODMAS is the rule applied to a given expression for its simplification. Here V stands for 'Virnaculum', B stands for 'Bracket', O stands for 'Of', D stands for 'Division', M stands for 'Multiplication', A stands for 'Addition' and S stands for 'Subtraction'.

Brackets must be removed in the order (), { } and []. After removing brackets, remaining operands are to be applied in the order.

(a) Of (b) Division (c) Multiplication (d) Addition and (e) Subtraction.

Virnaculum (or Bar): If an expression contains Virnaculum, simplify the expression under Virnaculum before applying 'BODMAS'.

EXAMPLE 1 Simplify $0.6 \times 0.6 + 0.6 \div 6$

[Bank PO]

Solution Given expression $= 0.6 \times 0.6 + \frac{0.6}{6}$
 $= 0.36 + 0.1 = 0.46$

EXAMPLE 2 Simplify $2004 - 1984 \div 2$

Solution Given expression $= 2004 - \frac{1984}{2}$
 $= 2004 - 992 = 1012$

EXAMPLE 3 Simplify $3660 \div 5 + 18$

Solution Given expression $= \frac{3660}{5} + 18$
 $= 732 + 18 = 750$

EXAMPLE 4 What will be the value in place of question mark in the given expression?

$$3\frac{4}{9} + 6\frac{2}{5} + ? = 5\frac{2}{3} + 3\frac{4}{5}$$

Solution Let the value in place of question mark be x .

$$\therefore 3\frac{4}{9} + 6\frac{2}{5} + x = 5\frac{2}{3} + 3\frac{4}{5}$$

$$\begin{aligned}\therefore x &= 5\frac{2}{3} + 3\frac{4}{5} - 6\frac{2}{5} - 3\frac{4}{9} \\ &= \left(5\frac{2}{3} + 3\frac{4}{5}\right) - \left(6\frac{2}{5} + 3\frac{4}{9}\right) \\ &= \left(8 + \frac{10+12}{15}\right) - \left(9 + \frac{18+20}{45}\right) \\ &= 9\frac{7}{15} - 9\frac{38}{45} \\ &= \frac{7}{15} - \frac{38}{45} \\ &= \frac{21-38}{45} \\ &= \frac{-17}{45}\end{aligned}$$

Hence value in place of question mark = $\frac{-17}{45}$

EXAMPLE 5 Find the value of $\left(1 - \frac{1}{3}\right)\left(1 - \frac{1}{4}\right)\left(1 - \frac{1}{5}\right) \dots \left(1 - \frac{1}{100}\right)$ [SSC]

Solution Given expression = $\frac{2}{3} \times \frac{3}{4} \times \frac{4}{5} \times \dots \times \frac{99}{100}$

$$= \frac{2}{100} = \frac{1}{50}$$

EXAMPLE 6 Two pens and three pencils costs ₹ 86. Four pens and a pencil cost ₹ 112. Find the cost of a pen and that of a pencil. [Bank PO]

Solution Let cost of pen and pencil be ₹ p and ₹ q .

$$\begin{aligned}\therefore 2p + 3q &= 86 & (1) \\ 4p + q &= 112 & (2)\end{aligned}$$

Solving (1) and (2), we get

$$(2) - 2 \times (1) \Rightarrow q - 6q = 112 - 172$$

$$\Rightarrow -5q = -60$$

$$\Rightarrow q = ₹ 12$$

From (2),
$$p = \frac{112 - 12}{4} = ₹ 25$$

∴ Cost of pen = ₹ 25; Cost of pencil = ₹ 12



EXERCISES

- The value of $1001 \div 11$ of 13 is: [SSC]
(a) 7 (b) 91 (c) 143 (d) 169
- The value of $25 - 5[2 + 3\{2 - 2(5 - 3) + 5\} - 10] \div 4$ is: [SSC]
(a) 5 (b) 23.25 (c) 23.75 (d) 25
- What mathematical operation should come at the place of '?' in the equation:
 $2 ? 6 - 12 \div 4 + 2 = 11$ [RRB]
(a) + (b) - (c) $\times$ (d) $\div$
- $3640 \div 14 \times 16 + 340 = ?$ [LIC]
(a) 0.70 (b) 3525 (c) 4480 (d) 9600 (e) None of these
- $(8 \div 88) \times 8888088 = ?$ [SSC]
(a) 808008 (b) 808080 (c) 808088 (d) 8008008
- $\frac{180 \times 15 - 12 \times 20}{140 \times 8 + 2 \times 55} = ?$ [BSRB]
(a) $\frac{1}{7}$ (b) $\frac{4}{5}$ (c) 2 (d) 4 (e) None of these
- Evaluate $\frac{8 - [5 - (-3 + 2)] \div 2}{|5 - 3| - |5 - 8| \div 3}$ [SSC]
(a) 2 (b) 3 (c) 4 (d) 5
- $1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{7} + \frac{1}{14} + \frac{1}{28}$ is equal to: [SSC]
(a) 2 (b) 2.5 (c) 3 (d) 3.5
- $1\frac{3}{4} + 5\frac{1}{3} + 3\frac{2}{5} = ?$ [Bank PO]
(a) $9\frac{2}{5}$ (b) $9\frac{29}{60}$ (c) $10\frac{2}{5}$ (d) $10\frac{29}{60}$
- $\frac{1}{\left(2\frac{1}{3}\right)} + \frac{1}{\left(1\frac{3}{4}\right)}$ is equal to: [RRB]
(a) $\frac{7}{14}$ (b) $\frac{12}{49}$ (c) $4\frac{1}{12}$ (d) None of these

11. $\frac{1}{3} + \frac{1}{2} + \frac{1}{x} = 4$. Then $x =$ [MBA]
 (a) $\frac{5}{18}$ (b) $\frac{6}{19}$ (c) $\frac{18}{5}$ (d) $\frac{24}{11}$
12. $\frac{3}{5}$ of $\frac{4}{7}$ of $\frac{5}{9}$ of $\frac{21}{24}$ of 504 = ? [Bank PO]
 (a) 63 (b) 69 (c) 96 (d) 109 (e) None of these
13. $\frac{3}{8}$ of $168 \times 15 \div 5 + ? = 549 \div 9 + 235$ [SBI PO]
 (a) 107 (b) 174 (c) 189 (d) 296 (e) None of these
14. When $\left(\frac{1}{2} - \frac{1}{4} + \frac{1}{5} - \frac{1}{6}\right)$ is divided by $\left(\frac{2}{5} - \frac{5}{9} + \frac{3}{5} - \frac{7}{18}\right)$, the result is: [SSC]
 (a) $2\frac{1}{18}$ (b) $3\frac{1}{6}$ (c) $3\frac{3}{10}$ (d) $5\frac{1}{10}$
15. Difference between $1\frac{3}{16}$ and its reciprocal is: [MAT]
 (a) $1\frac{1}{8}$ (b) $\frac{4}{3}$ (c) $\frac{15}{16}$ (d) None of these
16. Two-fifth of one-fourth of three-seventh of a number is 15. What is half of that number? [Bank PO]
 (a) 94 (b) 96 (c) 188 (d) 196 (e) None of these
17. If $x * y = x^2 + y^2 - xy$, then the value of $9 * 11$ is: [SSC]
 (a) 93 (b) 103 (c) 113 (d) 121
18. If $a * b = 2a - 3b + ab$, then $3 * 5 + 5 * 3$ is: [SSC]
 (a) 22 (b) 24 (c) 26 (d) 28
19. $4\frac{1}{2} \times 4\frac{1}{3} - 8\frac{1}{3} \div 5\frac{2}{3} = ?$ [Bank PO]
 (a) $\frac{7}{17}$ (b) $1\frac{33}{34}$ (c) 8 (d) $18\frac{1}{34}$
20. $\frac{\frac{1}{3} \div \frac{1}{3} \times \frac{1}{3}}{\frac{1}{3} \div \frac{1}{3} \text{ of } \frac{1}{3}} - \frac{1}{9} =$ [SSC]
 (a) 0 (b) $\frac{1}{9}$ (c) $\frac{1}{3}$ (d) 1
21. The value of $0.008 \times 0.01 \times 0.0072 \div (0.12 \times 0.0004)$ is: [SSC]
 (a) 0.012 (b) 0.12 (c) 1.02 (d) 1.2
22. $11.6 + 9.28 \div 0.464 - 0.2828 \div 0.07 = ?$ [RRB]
 (a) 9.2 (b) 9.56 (c) 27.2 (d) 27.56
23. If $\frac{x}{y} = \frac{4}{5}$, then the value of $\left(\frac{4}{7} + \frac{2y - x}{2y + x}\right)$ is: [RRB]
 (a) $\frac{3}{7}$ (b) 1 (c) $1\frac{1}{7}$ (d) 2

24. If $x = \frac{a}{a-1}$ and $y = \frac{1}{a-1}$, then: [Bank PO]
 (a) $x = y$ (b) $x = y$, if $a < 1$ (c) $x > y$
 (d) $x > y$, if $a < 1$ (e) $y > x$, if $a < 1$
25. If $3x + 7 = x^2 + P = 7x + 5$, what is the value of P ? [SBI PO]
 (a) $\frac{1}{2}$ (b) $8\frac{1}{4}$ (c) $8\frac{1}{2}$ (d) Cannot be determined
26. The value of $\frac{1}{2 + \frac{1}{2 + \frac{1}{2 - \frac{1}{2}}}}$ is: [SSC]
 (a) $\frac{3}{8}$ (b) $\frac{19}{8}$ (c) $\frac{8}{3}$ (d) $\frac{8}{19}$
27. If $2 = x + \frac{1}{1 + \frac{1}{3 + \frac{1}{4}}}$, then the value of x is: [SSC]
 (a) $\frac{12}{17}$ (b) $\frac{13}{17}$ (c) $\frac{18}{17}$ (d) $\frac{21}{17}$
28. If $4x + 5y = 83$ and $\frac{3x}{2y} = \frac{21}{22}$, then $y - x = ?$ [Bank PO]
 (a) 3 (b) 4 (c) 7 (d) 11
29. If $2x + y = 17$, $y + 2z = 15$ and $x + y = 9$, then the value of $4x + 3y + z$ equals: [SBI PO]
 (a) 41 (b) 43 (c) 45 (d) 55 (e) None of these
30. When simplified, the value of product $\left(1 - \frac{1}{2}\right)\left(1 - \frac{1}{3}\right)\left(1 - \frac{1}{4}\right) \dots \left(1 - \frac{1}{n}\right)$ equals: [SSC]
 (a) $\frac{1}{n}$ (b) $\frac{2}{n}$ (c) $\frac{2(n-1)}{n}$ (d) $\frac{2}{n(n+1)}$
31. The value of $999 \frac{995}{999} \times 999$ is: [SSC]
 (a) 990809 (b) 998996 (c) 998999 (d) 999824
32. The value of $\frac{3}{1^2 \cdot 2^2} + \frac{5}{2^2 \cdot 3^2} + \frac{7}{3^2 \cdot 4^2} + \frac{9}{4^2 \cdot 5^2} + \frac{11}{5^2 \cdot 6^2} + \frac{13}{6^2 \cdot 7^2} + \frac{15}{7^2 \cdot 8^2} + \frac{17}{8^2 \cdot 9^2} + \frac{19}{9^2 \cdot 10^2}$ is: [SSC]
 (a) $\frac{1}{100}$ (b) $\frac{99}{100}$ (c) 1 (d) $\frac{101}{100}$
33. Along a yard 225 metres long, 26 trees are planted at equal distances, one tree being at each end of the yard. What is the distance between two consecutive trees? [RRB]
 (a) 8 metres (b) 9 metres (c) 10 metres (d) 15 metres
34. A pineapple costs ₹ 7 each. A watermelon costs ₹ 5 each. X spends ₹ 38 on these fruits. The number of pineapples purchased is: [SSC]
 (a) 2 (b) 3 (c) 4 (d) Data inadequate

35. A printer numbers the pages of a book starting with 1 and uses 3189 digits in all. How many pages does the book have? [MAT]
 (a) 1000 (b) 1074 (c) 1075 (d) 1080
36. A sum of ₹ 1360 has been divided among A, B and C such that A gets $\frac{2}{3}$ of what B gets and B gets $\frac{1}{4}$ of what C gets. B's share is [MAT]
 (a) ₹ 120 (b) ₹ 160 (c) ₹ 240 (d) ₹ 300
37. $\frac{(469 + 174)^2 - (469 - 174)^2}{469 \times 174} = ?$ [MBA]
 (a) 2 (b) 4 (c) 295 (d) 643
38. Eight persons are planning to share equally the rent of a car. If one person withdraws from the arrangement and the others share equally the entire cost of the car, then the share of each of the remaining persons increased by [MBA]
 (a) $\frac{1}{7}$ (b) $\frac{1}{8}$ (c) $\frac{1}{9}$ (d) $\frac{7}{8}$
39. In an examination, a student scores 4 marks for every correct answer and loses 1 mark for every wrong answer. If he attempts in all 60 questions and secures 130 marks, the number of questions he attempts correctly is: [LIC AAO]
 (a) 35 (b) 38 (c) 40 (d) 42
40. A man has some hens and cows. If the number of heads be 48 and the number of legs equal 140, then the number of hens will be: [RRB]
 (a) 22 (b) 23 (c) 24 (d) 26



1. (a) 2. (c) 3. (c) 4. (e) 5. (a) 6. (c) 7. (d) 8. (a) 9. (d) 10. (d)
 11. (b) 12. (e) 13. (a) 14. (d) 15. (d) 16. (e) 17. (b) 18. (a) 19. (d) 20. (a)
 21. (a) 22. (d) 23. (b) 24. (d) 25. (b) 26. (d) 27. (d) 28. (b) 29. (e) 30. (a)
 31. (b) 32. (b) 33. (b) 34. (c) 35. (b) 36. (c) 37. (b) 38. (a) 39. (b) 40. (d)



Solutions with Necessary Explanation

1. $1001 \div 11 \text{ of } 13 = 1001 \div (11 \times 13)$
 $= 1001 \div 143 = 7$
2. $25 - 5 [2 + 3 \{2 - 2(5 - 3) + 5\} - 10] \div 4$
 $= 25 - 5[2 + 3 \{2 - 2 \times 2 + 5\} - 10] \div 4$
 $= 25 - 5[2 + 3 \times 3 - 10] \div 4$
 $= 25 - 5 \times 1 \div 4$
 $= 25 - \frac{5}{4} = 23.75$
3. Operator X in place of question mark satisfies the given equation according to order of precedence of operators.
 i.e. $2 \times 6 - 12 \div 4 + 2 = 2 \times 6 - 3 + 2 = 11$

$$\begin{aligned}
 4. \quad 3640 \div 14 \times 16 + 340 &= \frac{3640}{14} \times 16 + 340 \\
 &= 260 \times 16 + 340 = \mathbf{4500}
 \end{aligned}$$

$$\begin{aligned}
 5. \quad (8 \div 88) \times 8888088 &= \frac{1}{11} \times 8888088 \\
 &= \mathbf{808008}
 \end{aligned}$$

$$\begin{aligned}
 6. \quad \frac{180 \times 15 - 12 \times 20}{140 \times 8 + 2 \times 55} &= \frac{2700 - 240}{1120 + 110} \\
 &= \frac{2460}{1230} = \mathbf{2}
 \end{aligned}$$

$$\begin{aligned}
 7. \quad \frac{8 - [5 - (-3 + 2)] \div 2}{|5 - 3| - |5 - 8| \div 3} &= \frac{8 - [5 - (-1)] \div 2}{2 - 3 \div 3} \\
 &= \frac{8 - 6 \div 2}{2 - 3 \div 3} = \frac{8 - 3}{2 - 1} = \frac{5}{1} = \mathbf{5}
 \end{aligned}$$

$$\begin{aligned}
 8. \quad 1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{7} + \frac{1}{14} + \frac{1}{28} &= 1 + \frac{14 + 7 + 4 + 2 + 1}{28} \\
 &= 1 + \frac{28}{28} = 1 + 1 = \mathbf{2}
 \end{aligned}$$

$$\begin{aligned}
 9. \quad 1\frac{3}{4} + 5\frac{1}{3} + 3\frac{2}{5} &= (1 + 5 + 3) + \left(\frac{3}{4} + \frac{1}{3} + \frac{2}{5}\right) \\
 &= 9 + \left(\frac{3 \times 15 + 1 \times 20 + 2 \times 12}{60}\right) \\
 &= 9 + \frac{45 + 20 + 24}{60} = 9 + 1\frac{29}{60} = \mathbf{10\frac{29}{60}}
 \end{aligned}$$

$$\begin{aligned}
 10. \quad \frac{1}{\left(2\frac{1}{3}\right)} + \frac{1}{\left(1\frac{3}{4}\right)} &= \frac{1}{\frac{7}{3}} + \frac{1}{\frac{7}{4}} \\
 &= \frac{3}{7} + \frac{4}{7} = \frac{7}{7} = \mathbf{1}
 \end{aligned}$$

$$\begin{aligned}
 11. \quad \frac{1}{3} + \frac{1}{2} + \frac{1}{x} &= 4 \\
 \Rightarrow \quad \frac{1}{x} &= 4 - \left(\frac{1}{3} + \frac{1}{2}\right) \\
 &= 4 - \left(\frac{2+3}{6}\right) = 4 - \frac{5}{6} = \frac{19}{6} \\
 \therefore \quad x &= \frac{\mathbf{6}}{\mathbf{19}}
 \end{aligned}$$

$$12. \quad \frac{3}{5} \text{ of } \frac{4}{7} \text{ of } \frac{5}{9} \text{ of } \frac{21}{24} \text{ of } 504 = \frac{3}{5} \times \frac{4}{7} \times \frac{5}{9} \times \frac{21}{24} \times 504 = \frac{1}{6} \times 504 = \mathbf{84}$$

$$13. \quad \frac{3}{8} \text{ of } 168 \times 15 \div 5 + ? = 549 \div 9 + 235$$

Let number in place of question mark be x .

$$\therefore \quad \frac{3}{8} \times 168 \times 15 \div 5 + x = 549 \div 9 + 235$$

$$\text{i.e.} \quad \frac{3}{8} \times 168 \times \left(\frac{15}{5}\right) + x = \frac{549}{9} + 235$$

$$\therefore \quad \begin{aligned} x &= 61 + 235 - 3 \times 21 \times 3 \\ &= 296 - 189 = \mathbf{107} \end{aligned}$$

$$14. \quad \frac{1}{2} - \frac{1}{4} + \frac{1}{5} - \frac{1}{6} = \left(\frac{1}{2} + \frac{1}{5}\right) - \left(\frac{1}{4} + \frac{1}{6}\right)$$

$$= \frac{7}{10} - \frac{5}{12} = \frac{42 - 25}{60} = \frac{\mathbf{17}}{\mathbf{60}}$$

$$\frac{2}{5} - \frac{5}{9} + \frac{3}{5} - \frac{7}{18} = \left(\frac{2}{5} + \frac{3}{5}\right) - \left(\frac{5}{9} + \frac{7}{18}\right)$$

$$= 1 - \frac{17}{18} = \frac{\mathbf{1}}{\mathbf{18}}$$

$$\therefore \quad \text{Required result} = \frac{17/60}{1/18} = \frac{17}{60} \times 18 = \frac{\mathbf{51}}{\mathbf{10}} = \mathbf{5 \frac{1}{10}}$$

$$15. \quad \text{Required difference} = 1 \frac{3}{16} \sim \frac{1}{1 \frac{3}{16}}$$

$$= \frac{19}{16} \sim \frac{16}{19} = \frac{19^2 \sim 16^2}{19 \times 16}$$

$$= \frac{(19+16)(19-16)}{19 \times 16} = \frac{\mathbf{35 \times 3}}{\mathbf{19 \times 16}}$$

$$16. \quad \frac{2}{5} \text{ of } \frac{1}{4} \text{ of } \frac{3}{7} \text{ of } x = 15; \quad \frac{x}{2} = ?$$

$$\frac{2}{5} \times \frac{1}{4} \times \frac{3}{7} \times x = 15$$

$$\Rightarrow \quad x = 15 \times \frac{5 \times 4 \times 7}{2 \times 1 \times 3}$$

$$\Rightarrow \quad x = 5 \times 5 \times 2 \times 7$$

$$\Rightarrow \quad \frac{x}{2} = 5 \times 5 \times 7 = \mathbf{175}$$

17. Given $x * y = x^2 + y^2 - xy$

$$\begin{aligned} 9 * 11 &= 9^2 + 11^2 - 9 \times 11 \\ &= 81 + 121 - 99 \\ &= \mathbf{103} \end{aligned}$$

18. $a * b = 2a - 3b + ab$

$$\begin{aligned} \therefore 3 * 5 + 5 * 3 &= [(2 \times 3) - (3 \times 5) + (3 \times 5)] + [(2 \times 5) - (3 \times 3) + (5 \times 3)] \\ &= 6 + 16 = \mathbf{22} \end{aligned}$$

19. $4\frac{1}{2} \times 4\frac{1}{3} - 8\frac{1}{3} + 5\frac{2}{3} = ?$

Let the required value in place of question mark be x .

$$\begin{aligned} \therefore x &= \frac{9}{2} \times \frac{13}{3} - \left(\frac{25}{3} + \frac{17}{3} \right) = \frac{9}{2} \times \frac{13}{3} - \frac{25}{17} \\ &= \frac{39}{2} - \frac{25}{17} = 19\frac{1}{2} - 1\frac{8}{17} \\ &= (19 - 1) + \left(\frac{1}{2} - \frac{8}{17} \right) \\ &= 18 + \left(\frac{17 - 16}{34} \right) \\ &= \mathbf{18\frac{1}{34}} \end{aligned}$$

20. $\frac{\frac{1}{3} \div \frac{1}{3} \times \frac{1}{3}}{\frac{1}{3} \div \frac{1}{3} \text{ of } \frac{1}{3}} - \frac{1}{9} = \frac{1 \times \frac{1}{3}}{\frac{1}{3} \div \frac{1}{9}} - \frac{1}{9}$

$$= \frac{\frac{1}{3}}{\frac{1}{3} \times 9} - \frac{1}{9} = \frac{1}{9} - \frac{1}{9} = \mathbf{0}$$

21. $0.008 \times 0.01 \times 0.0072 \div (0.12 \times 0.0004) = \frac{0.008 \times 0.01 \times 0.0072}{0.12 \times 0.0004}$

$$= 0.001 \times 2 \times 1 \times 6 = \mathbf{0.012}$$

22. $11.6 + 9.28 \div 0.464 - 0.2828 \div 0.07 = 11.6 + \frac{9.28}{0.464} - \frac{0.2828}{0.07}$

i.e. Required value $= 11.6 + 20 - 4.04$

$$= 31.6 + 4.04 = \mathbf{27.56}$$

23. Given $\frac{x}{y} = \frac{4}{5}$

$$\therefore \frac{4}{7} + \frac{2y-x}{2y+x} = \frac{4}{7} + \frac{2-(x/y)}{2+(x/y)}$$

$$\begin{aligned} &= \frac{4}{7} + \frac{2 - (4/5)}{2 + (4/5)} \\ &= \frac{4}{7} + \frac{(6/5)}{(14/5)} = \frac{4}{7} + \frac{3}{7} = 1 \end{aligned}$$

24. Given $x = \frac{a}{a-1} = \frac{a-1+1}{a-1} = 1 + \frac{1}{a-1} = 1 + y$

$$\therefore x > y$$

25. $3x + 7 = 7x + 5$

$$\Rightarrow 7x - 3x = 7 - 5 = 2$$

$$\Rightarrow 4x = 2$$

$$\Rightarrow x = \frac{2}{4} = \frac{1}{2}$$

$$\therefore 3x + 7 = 3\left(\frac{1}{2}\right) + 7 = \frac{17}{2}$$

$$\therefore x^2 + P = \frac{17}{2}$$

$$\Rightarrow P = \frac{17}{2} - \left(\frac{1}{2}\right)^2 = \frac{17}{2} - \frac{1}{4} = \frac{34-1}{4} = \frac{33}{4} = 8\frac{1}{4}$$

26. $\frac{1}{2 + \frac{1}{2 + \frac{1}{2 - \frac{1}{2}}}}} = \frac{1}{2 + \frac{1}{2 + \frac{1}{\frac{3}{2}}}}$

$$= \frac{1}{2 + \frac{1}{2 + \frac{2}{3}}} = \frac{1}{2 + \frac{1}{\frac{8}{3}}} = \frac{1}{2 + \frac{3}{8}} = \frac{1}{\frac{19}{8}} = \frac{8}{19}$$

27. $\frac{1}{1 + \frac{1}{3 + \frac{1}{4}}} = \frac{1}{1 + \frac{1}{\frac{13}{4}}} = \frac{1}{1 + \frac{4}{13}}$

$$= \frac{1}{\frac{17}{13}} = \frac{13}{17}$$

$$\begin{aligned} \therefore x &= 2 - \frac{13}{17} = \frac{34-13}{17} \\ &= \frac{21}{17} \end{aligned}$$

$$28. \quad 4x + 5y = 83 \quad (1)$$

$$\frac{3x}{2y} = \frac{21}{22} \quad (2)$$

$$\text{From (2),} \quad \frac{x}{y} = \frac{7}{11} \quad (A)$$

$$\text{From (1),} \quad 4 \times \frac{7}{11}y + 5y = 83$$

$$\left(\frac{28}{11} + 5\right)y = 83$$

$$y = \frac{83 \times 11}{83} = 11$$

Substituting $y = 11$ in (A), we obtain

$$x = 7$$

$$\therefore y - x = 11 - 7 = 4$$



Short cut:

$$\therefore \frac{x}{y} = \frac{7}{11}, \quad y - x = (11 - 7)k = 4k.$$

i.e. $(y - x)$ is a constant multiple of 4.

From the given options, we are having only one multiple of 4.

$$29. \quad 2x + y = 17 \quad (1)$$

$$y + 2z = 15 \quad (2)$$

$$x + y = 9 \quad (3)$$

$$(1) \Rightarrow 2x = (17 - y)$$

$$\Rightarrow 4x = 34 - 2y \quad (A)$$

$$(2) \Rightarrow 2z = 15 - y$$

$$\text{or} \quad z = \frac{15 - y}{2} \quad (B)$$

$$\therefore (A) + (B) + 3y = 4x + 3y + z$$

$$= (34 - 2y) + \left(\frac{15 - y}{2}\right) + 3y$$

$$= \frac{83}{2} + \frac{y}{2}$$

Solving (1) and (3), we get

$$(1) - (3) \Rightarrow x = 17 - 9 = 8 \quad \text{and}$$

$$\therefore y = 1$$

$$\therefore \text{Required value} = \frac{83}{2} + \frac{1}{2} = 42$$

$$30. \quad \left(1 - \frac{1}{2}\right)\left(1 - \frac{1}{3}\right)\left(1 - \frac{1}{4}\right) \cdots \left(1 - \frac{1}{n}\right) = \frac{1}{2} \times \frac{2}{3} \times \frac{3}{4} \times \cdots \times \left(n - \frac{1}{n}\right) = \frac{1}{n}$$

31. $999 \frac{995}{999} \times 999 = 999^2 + 995$

$$\begin{aligned} &= (1000 - 1)^2 + 995 \\ &= 1000000 - 2000 + 1 + 995 \\ &= 998000 + 996 \\ &= \mathbf{998996} \end{aligned}$$

32. $\frac{3}{1^2 \cdot 2^2} + \frac{5}{2^2 \cdot 3^2} + \frac{7}{3^2 \cdot 4^2} + \frac{9}{4^2 \cdot 5^2} + \frac{11}{5^2 \cdot 6^2} + \frac{13}{6^2 \cdot 7^2} + \frac{15}{7^2 \cdot 8^2} + \frac{17}{8^2 \cdot 9^2} + \frac{19}{9^2 \cdot 10^2}$

$$= \left(\frac{1}{1^2} - \frac{1}{2^2} \right) + \left(\frac{1}{2^2} - \frac{1}{3^2} \right) + \left(\frac{1}{3^2} - \frac{1}{4^2} \right) + \dots + \left(\frac{1}{9^2} - \frac{1}{10^2} \right) = \frac{1}{1^2} - \frac{1}{10^2} = \frac{\mathbf{99}}{\mathbf{100}}$$

33. Required distance = $\frac{225}{25} = \mathbf{9 \text{ metres.}}$

34. Let number of pineapples be x and watermelon be y

$\therefore 7x + 5y = 38 \Rightarrow x = 4; y = 2$

35. Number of pages with single digit (i.e. from 1 to 9) = 9

Number of pages with two digit (i.e. from 10 to 99) = 90

Number of pages with three digit (i.e. from 100 to 999) = 900

Total number of pages = $9 + 90 + 900 = 999$

$\therefore$ Total number of digits in page numbers = $1 \times 9 + 2 \times 90 + 3 \times 900$
 $= 9 + 180 + 2700 = 2889$

Balance number of digits printed = $3189 - 2889 = 300$

$\therefore$ Required number of 4-digit page numbers = $\frac{300}{4} = \mathbf{75}$

$\therefore$ Total number of pages = $9 + 90 + 900 + 75 = \mathbf{1074}$

36. Let the share with C be ₹ x .

$\therefore$ C's share + B's share + A's share = $x + \frac{x}{4} + \frac{2}{3} \left(\frac{x}{4} \right) = 1360$

i.e. $x + \frac{x}{4} + \frac{x}{6} = 1360$

$\Rightarrow \frac{x(12 + 3 + 2)}{12} = 1360$

$\Rightarrow x = \frac{1360 \times 12}{17} = \mathbf{960}$

$\therefore$ B's share = $\frac{1}{4}x = \frac{960}{4} = \mathbf{₹ 240}$

37. $\frac{(469 + 174)^2 - (469 - 174)^2}{469 \times 174} = \frac{4 \times 469 \times 174}{469 \times 174} = \mathbf{4}$

$[\because (a + b)^2 - (a - b)^2 = 4ab]$

38. Rent/person (if persons are 8) = $\frac{x}{8}$

Rent/person (if persons are 7) = $\frac{x}{7}$

$$\text{Difference in rent/person} = \frac{x}{7} - \frac{x}{8} = \frac{x}{56}$$

$$\therefore \text{Increase in rent/person} = \frac{x/56}{x/8} = \frac{8}{56} = \frac{1}{7}$$

39. Let number of correct answers be x .

Marks scored by student =

$$\therefore x \times 4 + (60 - x)(-1) = 130$$

$$\Rightarrow 4x + x - 60 = 130$$

$$\Rightarrow x = \frac{130 + 60}{5} = 38$$

40. Let number of hens be x .

$$2 \times x + 4(48 - x) = 140$$

$$\Rightarrow 192 - 2x = 140$$

$$\Rightarrow x = 26$$

$\therefore$ Number of hens = 26.

Races and Games

RACES: A contest of speed in running, driving, sailing or rowing and so on is called a race.

RACE COURSE: The place, ground, field, path or track on which a contest is done is called the race course.

DEAD HEAT RACE: If all the persons contesting a race reach the goal exactly at the same time, then the race is said to be a dead heat race.

START: In a race with two contestants A and B, if before the start of the race, A is at the starting point and B is ahead of A by 10 metres, then we say that 'A gives B, a start of 10 metres'.

To cover a race of 100 m, A will have to cover 100 metres while B will have to cover only $(100 - 10) = 90$ metres.

Here, we can say that in a 100 m race, 'A can give B 10 m' or 'A can give B a start of 10 m' or 'A beats B by 10 m' means while A runs 100 m, B runs $(100 - 10) = 90$ m.

GAMES: In a game if A scores 100 points while B scores only 75 points, then we say that A can give B 25 points.



Important Facts

1. A is n times as fast as B. A gives B a start of x m. How long should the race course be so that both reach the winning post at the same time?

A's speed : B's speed = $n : 1$

A gains $(n - 1)$ metres in a race of n metres.

$\therefore$ A will gain x metres in a race of $\frac{nx \text{ metres}}{(n - 1)} = \frac{x}{\left(1 - \frac{1}{n}\right)}$ metres

$$\therefore \text{Length of race course} = x \left[\frac{1}{\left(1 - \frac{1}{n}\right)} \right] \text{ metres}$$

2. A can run x km race in t_1 minutes and B in t_2 minutes. Then A beats B by a distance
 $= \frac{x}{t_2} \times (t_2 - t_1)$ km.



Short cut: A beats B by a distance

$$= \frac{\text{Length of the race course}}{\text{Time taken by B to complete the race}} \times (\text{Difference between the race timings of A and B})$$

EXAMPLE 1 In a one-kilometre race A, B, and C are the three participants. A can give B a start of 50 m and C a start of 69 m. How many metres start can B give C?

Solution When A covers 1000 m, B covers $(1000 - 50) = 950$ m, and C covers $(1000 - 69) = 931$ m

When B covers 950 m, C covers 931 m

$$\therefore \text{When B covers 1000 m, C will cover } \frac{931}{950} \times 1000 = \mathbf{980 \text{ m}}$$

Therefore B can give C a start of 20 m.

EXAMPLE 2 In a one-kilometre race, A beats B by 30 seconds and B beats C by 15 seconds. A beats C by 180 m. Find the time taken by each to run a kilometre.

Solution A beats B by 30 seconds and B beats C by 15 seconds.

Therefore, A beats C by $(30 + 15) = 45$ seconds.

Also, A beats C by 180 m.

$$\text{Time taken by C to run 1 km} = \frac{45}{180} \times 1000 = \mathbf{250 \text{ seconds}}$$

$$\therefore \text{Time taken by B to run 1 km} = 250 - 15 = \mathbf{235 \text{ seconds}}$$

$$\text{Time taken by A to run 1 km} = 250 - 45 = \mathbf{205 \text{ seconds.}}$$

EXAMPLE 3 A can run 1 km in 3 min 10 sec and B can cover the same distance in 3 min 20 sec. By what distance can A beat B?

Solution Here A beat B by 10 sec.

$$\text{Distance covered by B in 10 sec} = \left(\frac{1000}{200} \times 10 \right) = \mathbf{50 \text{ m}}$$

$$\therefore \text{A beats B by } \mathbf{50 \text{ m.}}$$

EXAMPLE 4 P, Q and R are three contestants in a kilometre race. If P can give Q a start of 50 m and P can give R a start of 69 m, how many metre's start can Q give R?

Solution When P covers 1000 m, Q covers 950 m and R covers $(1000 - 69) = 931$ m

When Q covers 950 m, R covers 931 m.

$$\therefore \text{When Q covers 1000 m, R covers}$$

$$\left(\frac{931}{950} \times 1000\right) \text{ m} = 49 \times 20 = 980 \text{ m}$$

∴ Q can give R a start of $(1000 - 980) = 20 \text{ m}$

EXAMPLE 5 In a game of 100 points, P can give Q 10 points and R 20 points. Then how many points Q can give R in a game of 90 points?

Solution P : Q = 100 : 90

P : R = 100 : 80

$$\therefore \frac{Q}{R} = \frac{Q}{P} \times \frac{P}{R} = \frac{90}{100} \times \frac{100}{80} = \frac{90}{80}$$

∴ In a game of 90 points, Q can give R 10 points.



EXERCISES

- In a game of 100 points, P can give Q 10 points and R 28 points. Then Q can give R:
 - 10 points
 - 14 points
 - 16 points
 - 20 points
- In a race of 200 m, A can beat B by 31 m and C by 18 m. In a race of 280 m, C will beat B by:
 - 18 m
 - 20 m
 - 22 m
 - 25 m
- In a race of 100 m, A beats B by 10 m and C by 20 m. In a race of 180 m, B will beat C by:
 - 18 m
 - 20 m
 - 21 m
 - 22 m
- In a 500 m race, ratio of speeds of two contestants A and B is 4:5. A has a start of 140 m. Then A wins by
 - 60 m
 - 50 m
 - 40 m
 - 30 m
- A can run 23 m, while B runs 25 m. In a kilometre race, B beats A by:
 - 40 m
 - 60 m
 - 80 m
 - 100 m



1. (d) 2. (b) 3. (b) 4. (b) 5. (c)



Solutions with Necessary Explanation

1. P : Q = 100 : 90; P : R = 100 : 72

$$\therefore \frac{Q}{R} = \left(\frac{Q}{P} \times \frac{P}{R}\right) = \frac{90}{100} \times \frac{100}{72} = \frac{10}{8} = \frac{100}{80}$$

∴ Q can give R 20 points.

2. A : B = 200 : 169; A : C = 200 : 182

$$\frac{C}{B} = \left(\frac{C}{A} \times \frac{A}{B}\right) = \frac{182}{200} \times \frac{200}{169} = \frac{182}{169}$$

i.e., when C covers 182 m, B covers 169 m

∴ When C covers 280 m, B covers $\left(\frac{169}{182} \times 280\right)$ m = **260 m**

∴ C will beat B by 20 metres

3. A : B = 100 : 90; A : C = 100 : 80

$$\therefore \frac{B}{C} = \frac{B}{A} \times \frac{A}{C} = \frac{90}{100} \times \frac{100}{80} = \frac{90}{80}$$

∴ When B runs 90 m, C runs 80 m

∴ When B runs 180 m, C covers $\left(\frac{80}{90} \times 180\right)$ = 160 m

∴ B will beat C by 20 metres

4. To win, A has to cover a distance of $500 - 140 = 360$ m.

When A covers 4 m, B covers 5 m.

∴ When A covers 360 m, B covers $\left(\frac{5}{4} \times 360\right)$ m = **450 m**

∴ A wins by **50 m**

5. When B runs 25 m, A runs 23 m.

When B runs 1000 m, A runs $\left(\frac{23}{25} \times 1000\right)$ m = **920 m**

∴ B beats A by 80 m

Stocks and Shares

STOCK-CAPITAL: Total amount needed to run a company is called stock-capital.

SHARES OR STOCK: The whole capital is divided into small units, called shares or stock.

For each investment, company issues a share-certificate, showing the value of each share and the number of units held by a person.

The person who invests in shares or stock is called share holder or stock holder.

DIVIDEND: Annual profit of a company distributed among shareholders is called dividend.

Dividend is paid annually as per share or as a percentage.

FACE VALUE: The value of a share or stock printed on the share-certificate is called its face value or nominal value.

MARKET VALUE: The stocks of different companies are sold and bought in the open market through brokers at stock-exchanges. A share (stock) is said to be:

- (i) At premium or above par : If its market value is more than its face value.
- (ii) At par : If the market value is same as its face value.
- (iii) At discount or below par : If its market value is less than its face value.

Example: If a ₹ 100 stock is quoted at a premium of 8, then market value of the stock = ₹ $(100 + 8) = ₹ 108$

Similarly if a ₹ 100 stock is quoted at a discount of 6, then market value of stock = ₹ $(100 - 6) = ₹ 94$

BROKERAGE: Broker's charge is called brokerage.

- (i) When stock is purchased, brokerage is added to the cost price.
- (ii) When stock is sold, brokerage is subtracted from the selling price.

**Notes:**

- (a) Face value of a share is always a constant.
- (b) Market value of a share varies from time to time.
- (c) Dividend is always paid on the face value of a share.
- (d) Number of shares held by a person = $\frac{\text{Total investment}}{\text{Investment in one share}}$

By a ₹ 50; 8% stock at 60, we mean that

- (i) Face value of stock = ₹ 50.
- (ii) Market value of stock = ₹ 60.
- (iii) Annual dividend on 1 share = 8% of face value = 8% of ₹ 50 = ₹ 4.
- (iv) An investment of ₹ 50 gives an annual income of ₹ 4.
- (v) Rate of interest per annum = $\left(\frac{4}{60} \times 100\right)\% = 6\frac{2}{3}\%$

EXAMPLE 1 Find the cost of 96 shares of ₹ 10 each at $\frac{3}{4}$ discount, brokerage being $\frac{1}{4}$ per share. [LIC]

Solution Cost of 1 share = $\left[\left(10 - \frac{3}{4}\right) + \frac{1}{4}\right] = ₹ \frac{19}{2}$

∴ Cost of 96 shares = ₹ $\left(\frac{19}{2} \times 96\right) = ₹ 912$

EXAMPLE 2 Find the annual income derived by investing ₹ 7200 in 10% stock at 144.

Solution By investing ₹ 144, income obtained = ₹ 10.

By investing ₹ 7200, income obtained = $\left(\frac{10}{144} \times 7200\right) = ₹ 500$.

**EXERCISES**

- A man buys ₹ 50 shares in a company which pays 10% dividend. If the man gets 12.5% on his investment, at what price did he buy the shares? [LIC AAO]
 - (a) ₹ 37.50 (b) ₹ 40 (c) ₹ 48 (d) ₹ 52
- How many shares of market value ₹ 25 each can be purchased for ₹ 12750, brokerage being 2%? [MAT]
 - (a) 450 (b) 500 (c) 550 (d) 600
- The cost price of a ₹ 100 stock at 4 discount, when brokerage is $\frac{1}{4}\%$ is
 - (a) ₹ 95.75 (b) ₹ 96 (c) ₹ 96.25 (d) ₹ 104.25

4. A man bought 20 shares of ₹ 100 at 4 discount the rate of dividend being 12%. The rate of interest obtained is:

(a) $11\frac{1}{2}\%$ (b) 12% (c) 12.5% (d) 13%

5. A man invested ₹ 14,400 in ₹ 100 shares of a company at 20% premium. If the company declares 5% dividend at the end of the year, then how much does he get?

[Hotel Management]

(a) ₹ 500 (b) ₹ 600 (c) ₹ 650 (d) ₹ 720



1. (b) 2. (b) 3. (c) 4. (c) 5. (b)



Solutions with Necessary Explanation

1. Dividend on each share = $\frac{10}{100} \times ₹ 50 = ₹ 5$

Let the price at which he bought the share be ₹ x .

$$\therefore \frac{12.5}{100} \times x = 5$$

$$\therefore x = \frac{5 \times 100}{12.5} = ₹ 40$$

2. Selling price of each unit of share = $25 \times 1.02 = ₹ 25.50$

$\therefore$ Number of shares that can be purchased

$$= \frac{₹ 12750}{₹ 25.50} = 500$$

3. Cost price of stock at 4 discount and $\frac{1}{4}\%$ brokerage = ₹ $\left(100 - 4 + \frac{1}{4}\right) = ₹ 96.25$

4. Cost price of 1 unit of share = $(100 - 4) = ₹ 96$.

For 20 shares investment = $20 \times ₹ 96 = ₹ 1920$.

$$\text{Dividend} = \left(\frac{12}{100} \times 100\right) \times 20 = ₹ 240$$

$$\therefore \text{Rate of interest obtained} = \frac{240}{1920} \times 100 = 12.5\%$$

5. Price of 1 unit share = ₹ $100 + 20 = ₹ 120$.

$$\text{Dividend} = \frac{5}{100} \times 100 = ₹ 5 \text{ per unit}$$

$$\text{Number of units} = \frac{14400}{120} = 120$$

$$\therefore \text{Total dividend he got} = 120 \times 5 = ₹ 600$$

Discount

Let a person has to pay ₹ 140 after 5 years at an interest rate of 8% per annum ₹ 100 at 8% p.a. will amount to ₹ 140 in 5 years. Therefore we may say that:

Sum due = ₹ 140

Present worth = ₹ 100

True discount (T.D.) = ₹ (140 – 100) = ₹ 40

= Sum due – Present worth

Therefore, true discount may be defined as the interest on present worth. True discount is calculated on the amount.

Let $R\%$ be the rate of interest and T years be the period. Then,

$$(a) \text{ Present worth (P.W.)} = \frac{100 \times \text{Amount}}{100 + R \times T} = \frac{100 \times \text{T.D.}}{R \times T}$$

$$(b) \text{ True discount (T.D.)} = \frac{(\text{P.W.}) \times R \times T}{100} = \frac{\text{Amount} \times R \times T}{100 + (R \times T)}$$

$$(c) \text{ Sum} = \frac{\text{S.I.} \times \text{T.D.}}{(\text{S.I.} - \text{T.D.})}$$

$$(d) \text{ S.I.} - \text{T.D.} = \text{S.I. on T.D.}$$

$$(e) \text{ If the sum is put at compound interest, then P.W.} = \frac{\text{Amount}}{\left(1 + \frac{R}{100}\right)^T}$$

EXAMPLE 1 Find the present worth of ₹ 1341 due 4 years hence at 9% per annum.

$$\begin{aligned} \text{Solution P.W.} &= \frac{100 \times \text{Amount}}{100 + (R \times T)} = ₹ \frac{100 \times 1341}{100 + (9 \times 4)} \\ &= ₹ \frac{100 \times 1341}{136} = ₹ 950 \end{aligned}$$

EXAMPLE 2 The true discount on a bill due 10 months hence at 9% p.a. is ₹ 600. Find the amount of the bill and its present worth.

Solution Let the amount be ₹ A.

$$\therefore \text{T.D.} = \frac{A \times R \times T}{100 + (R \times T)} = \frac{A \times 9 \times \frac{10}{12}}{100 + \left(9 \times \frac{10}{12}\right)}$$

$$\text{i.e.} \quad 600 = \frac{A \times \frac{15}{2}}{100 + \frac{15}{2}}$$

$$\Rightarrow 600 \left(100 + \frac{15}{2}\right) = \frac{15}{2} \times A$$

$$\begin{aligned} \Rightarrow A &= 600 \left(100 + \frac{15}{2}\right) \times \frac{2}{15} \\ &= 80 \left(100 + \frac{15}{2}\right) \\ &= 8000 + 600 = \text{₹ } 8600 \end{aligned}$$

$$\begin{aligned} \text{P.W.} &= \text{Amount} - \text{T.D.} \\ &= 8600 - 600 = \text{₹ } 8000 \end{aligned}$$

EXAMPLE 3 True discount on a certain sum of a money due 3 years is ₹ 300, and the simple interest on the same sum for the same time and at the same rate is ₹ 450. Find the sum and the rate per cent.

Solution T.D. = ₹ 300 and S.I. = ₹ 450

$$\therefore \text{Sum due} = \frac{\text{S.I.} \times \text{T.D.}}{\text{S.I.} - \text{T.D.}} = \frac{450 \times 300}{450 - 300} = \text{₹ } 900$$

$$\text{Rate} = \frac{\text{S.I.} \times 100}{\text{Sum due} \times T} = \frac{450 \times 100}{900 \times 3} = \frac{50}{3} = 16\frac{2}{3}\%$$

EXAMPLE 4 The difference between the simple interest and true discount on a certain sum of money for 6 months at 10% p.a. is ₹ 15. Find the sum.

Solution Let the sum be ₹ x

$$\therefore \text{T.D.} = \frac{x \times 10 \times \frac{1}{2}}{100 + \left(10 \times \frac{1}{2}\right)} = \frac{x \times 5}{105} = \frac{x}{21}$$

$$\text{S.I.} = x \times 10 \times \frac{1}{2} \times \frac{1}{100} = \frac{x}{20}$$

$$\begin{aligned} \therefore \quad & \frac{x}{20} - \frac{x}{21} = 15 \\ \Rightarrow & \frac{x}{420} = 15 \\ \Rightarrow & x = 15 \times 420 = ₹ 6300 \\ \therefore & \text{Sum} = ₹ 6300 \end{aligned}$$

BANKER'S DISCOUNT: If trader A buys goods worth ₹ 20,000 from another trader B at a credit of 6 months, then A must allow B to withdraw the amount from bank account of A exactly after 6 months. The bill so prepared by B is called the bill of exchange.

The date exactly after 6 months is called nominally due date. Three days are added (grace days) to it to get a date, known as legally due date.

If B wishes to have the money before legally due date, he can get it from banker or broker, who deducts SI on the face value (here ₹ 20,000) for the period from date on which bill is discounted to the legally due date. This amount is called Banker's discount (B.D.).

i.e. B.D. = S.I. on face value for the period from date on which bill was discounted to the legally due date.

Banker's gain (B.G.) = B.D. – T.D. for the unexpired time



Important Formulae

1. B.D. = S.I. on bill for unexpired time
2. B.G. = (B.D.) – (T.D.) = S.I. on T.D. = $\frac{(T.D.)^2}{P.W.}$
3. T.D. = $\sqrt{P.W. \times B.G.}$
4. B.D. = $\left(\frac{\text{Amount} \times \text{Rate} \times \text{Time}}{100} \right)$
5. T.D. = $\left(\frac{\text{Amount} \times \text{Rate} \times \text{Time}}{100 + (\text{Rate} \times \text{Time})} \right)$
6. Amount = $\left(\frac{B.D. \times T.D.}{B.D. - T.D.} \right)$
7. T.D. = $\left(\frac{B.G. \times 100}{\text{Rate} \times \text{Time}} \right)$

EXAMPLE 5 The banker's discount on ₹ 2400 due certain time hence is ₹ 240. Find the true discount and banker's gain.

Solution Amount = $\frac{B.D. \times T.D.}{B.D. - T.D.} = \frac{B.D. \times T.D.}{B.G.}$

$$\therefore \quad \frac{T.D.}{B.G.} = \frac{\text{Amount}}{B.D.} = \frac{2400}{240} = \frac{10}{1}$$

If B.G. is ₹ 1, T.D. = ₹ 10 and if B.D. = ₹ 12, T.D. = B.D. – B.G. = 12 – 1 = ₹ 11.

$$\therefore \text{If B.D.} = ₹ 240, \text{T.D.} = ₹ \left(\frac{11}{12} \times 240 \right) = ₹ 220.$$

$$\text{B.G.} = \text{B.D.} - \text{T.D.} = 240 - 220 = ₹ 20$$

EXAMPLE 6 The banker's discount and the true discount on a sum of money due 6 months hence are ₹ 130 and ₹ 120 respectively. Find the sum and the rate per cent.

$$\text{Solution} \quad \text{Sum} = \frac{\text{B.D.} \times \text{T.D.}}{\text{B.D.} - \text{T.D.}} = ₹ \left(\frac{130 \times 120}{130 - 120} \right) = \frac{130 \times 120}{10} = ₹ 1560$$

But Banker's discount = S.I. on sum due.

$$\therefore \text{S.I. on ₹ 1560 for 6 months} = 130.$$

$$\therefore \quad \text{Rate} = \left(\frac{100 \times 130}{1560 \times \frac{1}{2}} \right) \% = \frac{100}{6} \% = 16 \frac{2}{3} \%$$

EXAMPLE 7 The present worth of a bill due certain time hence is ₹ 1300 and the true discount on the bill is ₹ 130. Find the banker's discount and the banker's gain.

$$\text{Solution} \quad \text{T.D.} = \sqrt{\text{P.W.} \times \text{B.G.}}$$

$$\text{or} \quad \text{B.G.} = \frac{(\text{T.D.})^2}{\text{P.W.}} = \frac{₹(130)^2}{1300} = ₹ 13$$

$$\therefore \quad \text{B.D.} = \text{T.D.} + \text{B.G.} = 130 + 13 = ₹ 143.$$

$$\therefore \quad \text{Banker's discount} = ₹ 143$$

$$\text{Banker's gain} = ₹ 13.$$



EXERCISES

- The true discount on a bill due 10 months hence at 15% per annum is ₹ 225. The amount of the bill is
(a) ₹ 1500 (b) ₹ 1750 (c) ₹ 1800 (d) ₹ 2000
- True discount on ₹ 3024 due 6 months hence is ₹ 144. The rate per cent is
(a) 6% (b) 8% (c) 10% (d) 12%
- The simple interest and the true discount on a certain sum for a given time and at a given rate are ₹ 85 and ₹ 80 respectively. The sum is
(a) ₹ 6800 (b) ₹ 3400 (c) ₹ 1700 (d) ₹ 1360
- A person bought an article for ₹ 2500 and sold it the same day for ₹ 3410, allowing him a credit of 2 years. If the rate of interest is 12% p.a., then the gain of man is:
(a) 0% (b) 5% (c) 8% (d) 10%
- ₹ 20 is the true discount on ₹ 210 due after a certain time. What will be the true discount on the same sum due after half of the former time, the rate of interest being the same?
(a) ₹ 10 (b) ₹ 10.20 (c) ₹ 10.40 (d) ₹ 10.50

6. The banker's discount on a certain sum of money is ₹ 84 and true discount on the same sum for the same time is ₹ 70. The sum due is:
 (a) ₹ 350 (b) ₹ 380 (c) ₹ 400 (d) ₹ 420
7. The banker's gain of a certain sum due 2 years hence at $12\frac{1}{2}\%$ per annum is ₹ 36. The present worth is
 (a) ₹ 280 (b) ₹ 350 (c) ₹ 576 (d) ₹ 630
8. The present worth of a certain sum due sometimes hence is ₹ 1700 and the true discount is ₹ 170. The banker's gain is
 (a) ₹ 20 (b) ₹ 18 (c) ₹ 17 (d) ₹ 16
9. The true discount on a bill of ₹ 720 is ₹ 80. The banker's discount is:
 (a) ₹ 80 (b) ₹ 90 (c) ₹ 100 (d) ₹ 120
10. The banker's gain on a bill due 1 year hence at 15% p.a. is ₹ 9. The true discount is:
 (a) ₹ 50 (b) ₹ 54 (c) ₹ 60 (d) ₹ 72
11. The banker's discount on ₹ 2400 at 12% p.a. is the same as true discount on ₹ 2520 for the same time and at the same rate. The time is:
 (a) 4 months (b) 5 months (c) 6 months (d) 8 months
12. The banker's discount on a sum of money for $1\frac{1}{2}$ years is ₹ 837 and the true discount on the same sum for 2 years is ₹ 900. The rate per cent is:
 (a) 10% (b) 11% (c) 12% (d) 13%
13. The present worth of a sum due certain time hence is ₹ 1024 and the banker's gain is ₹ 36. The true discount is:
 (a) ₹ 144 (b) ₹ 164 (c) ₹ 172 (d) ₹ 192
14. The banker's gain on a sum due 2 years hence at 10% p.a. is ₹ 120. The banker's discount is:
 (a) ₹ 240 (b) ₹ 360 (c) ₹ 540 (d) ₹ 720
15. A bill for ₹ 8000 is drawn on September 19 at 6 months. It is discounted on 8th January at 10%. Find the banker discount, true discount, banker's gain and the money that the holder of the bill receives.
 (a) ₹ 7600 (b) ₹ 7820 (c) ₹ 7840 (d) ₹ 7880



1. (c) 2. (c) 3. (d) 4. (d) 5. (d) 6. (d) 7. (c) 8. (c) 9. (b) 10. (c)
 11. (b) 12. (c) 13. (d) 14. (d) 15. (c)



Solutions with Necessary Explanation

1. Let P.W. be ₹ x .

∴ S.I. on ₹ x for 10 months at 15% p.a. = ₹ 225

$$\text{i.e.} \quad x \times \frac{15}{100} \times \frac{10}{12} = 225$$

$$\Rightarrow \quad x = \frac{225 \times 1200}{150} = 15 \times 120 = \text{₹ } 1800$$

2. P.W. = ₹ (3024 – 144) = ₹ 2880

∴ S.I. on ₹ 2880 for 6 months = ₹ 144.

$$\therefore \frac{144 \times 100}{2880 \times \frac{1}{2}} \% = 10\%$$

3. Sum = $\frac{\text{S.I.} \times \text{T.D.}}{\text{S.I.} - \text{T.D.}} = \frac{85 \times 80}{85 - 80} = \text{₹ } 1360$

4. C.P. = ₹ 2500; S.P. = ₹ $\left[\frac{3410 \times 100}{100 + (12 \times 2)} \right] = \text{₹ } 2750$

∴ Gain = $\frac{2750 - 2500}{2500} \times 100\% = 10\%$

5. S.I on ₹ (210 – 20) for a given time = ₹ 20.

S.I. on ₹ 190 for half the time = ₹ 10

∴ T.D. on ₹ 200 = ₹ 10

∴ T.D. on ₹ 210 = $\left(\frac{10}{200} \times 210 \right) = \text{₹ } 10.50$

6. Given B.D. = ₹ 84, T.D. = ₹ 70;

$$\frac{\text{B.D.} \times \text{T.D.}}{\text{B.D.} - \text{T.D.}} = \frac{84 \times 70}{84 - 70} = \text{₹ } 420$$

7. Given B.G. = ₹ 36; $R = 12\frac{1}{2}\%$ p.a.; $T = 2$ years;

$$\text{P.W.} = \frac{100 \times \text{T.D.}}{R \times T};$$

$$\text{T.D.} = \frac{\text{B.G.} \times 100}{R \times T} = \frac{36 \times 100}{12\frac{1}{2} \times 2} = \text{₹ } 144;$$

$$\therefore \text{P.W.} = \frac{100 \times 144}{12\frac{1}{2} \times 2} = \text{₹ } 576$$

8. Given P.W. = ₹ 1700; T.D. = ₹ 170

$$\therefore \text{B.G.} = \frac{(\text{T.D.})^2}{\text{P.W.}} = \frac{(170)^2}{1700} = \text{₹ } 17$$

9. P.W. = ₹ (720 – 80) = ₹ 640;

S.I. on ₹ 640 = ₹ 80;

$$\text{S.I. on ₹ } 720 = \left(\frac{80}{640} \times 720 \right) = \text{₹ } 90$$

∴ B.D. = ₹ 90

10. B.G. = ₹ 9; $T = 1$ year; $R = 15\%$ p.a.

$$\text{T.D.} = \frac{\text{B.G.} \times 100}{R \times T} = \frac{9 \times 100}{15 \times 1} = \text{₹ } 60$$

11. S.I. on ₹ 2400 = T.D. on ₹ 2520

∴ ₹ 2400 is the P.W. of ₹ 2520

i.e. ₹ 120 is S.I. on ₹ 2400 at 12%

$$\therefore \text{Time} = \frac{100 \times 120}{2400 \times 12} = \frac{5}{12} \text{ years} = \mathbf{5 \text{ months}}$$

12. Given B.D. for $1\frac{1}{2}$ years = ₹ 837;

$$\therefore \text{B.D. for 2 years} = ₹ 837 \times \frac{2}{3} \times 2 = ₹ 1116$$

T.D. for 2 years = 900;

$$\text{Sum} = \frac{\text{B.D.} \times \text{T.D.}}{\text{B.D.} - \text{T.D.}} = \frac{1116 \times 900}{1116 - 900} = \frac{1116 \times 900}{216} = ₹ 4650$$

∴ ₹ 1116 is S.I. on ₹ 4650

$$\therefore \text{Rate} = \left(\frac{1116 \times 100}{4650 \times 2} \right) = \frac{1116}{93} = \mathbf{12\%}$$

13. Given P.W. = ₹ 1024; B.G. = ₹ 36;

$$\therefore \text{T.D.} = \sqrt{\text{P.W.} \times \text{B.G.}} = \sqrt{1024 \times 36} = ₹ 192$$

14. B.G. = ₹ 120; $R = 10\%$; $T = 2$ years;

$$\text{T.D.} = \frac{\text{B.G.} \times 100}{R \times T} = \frac{120 \times 100}{10 \times 2} = ₹ 600$$

$$\therefore \text{B.D.} = ₹ (600 + 120) = ₹ 720$$

$$(\because \text{B.D.} = \text{T.D.} + \text{B.G.})$$

15. Face value of bill = ₹ 8000;

Date on which bill was drawn = September 19 at 6 months.

Nominally due date = March 19

Legally due date = March 22

Date on which bill was discounted = January 8.

Unexpired time:

Jan. Feb. March

$$23 \quad + \quad 28 \quad + \quad 22 \quad = 73 \text{ days} = \frac{1}{5} \text{ years.}$$

$$\therefore \text{B.D.} = \text{S.I. on ₹ 8000 for } \frac{1}{5} \text{ years}$$

$$= 8000 \times 10 \times \frac{1}{5} \times \frac{1}{100} = ₹ 160$$

$$\text{T.D.} = \frac{\text{B.D.} \times 100}{100 + R \times T} = \frac{160 \times 100}{100 + 10 \times \frac{1}{5}}$$

$$= \frac{16000}{102} = ₹ 156.86$$

$$\therefore \text{B.G.} = \text{B.D.} - \text{T.D.} = ₹ 160 - ₹ 156.86 = ₹ 3.14$$

$$\therefore \text{Money received by holder of bill} = ₹ (8000 - 160) = ₹ 7840$$

Logarithm

DEFINITION: If x is a positive real number other than 1 and $x^m = n$, then we write, $m = \log_x n$, and we say that value $\log n$ to the base x is m .

Example:

1. $10^2 = 100 \Rightarrow \log_{10} 100 = 2$
2. $5^4 = 625 \Rightarrow \log_5 625 = 4$
3. $3^{-2} = \frac{1}{9} \Rightarrow \log_3 \frac{1}{9} = -2$
4. $(0.1)^3 = 0.001 \Rightarrow \log_{(0.1)} 0.001 = 3$

PROPERTIES OF LOGARITHM:

1. $\log_x(ab) = \log_x a + \log_x b$
2. $\log_x \left(\frac{a}{b} \right) = \log_x a - \log_x b$
3. $\log_x x = 1$
4. $\log_x 1 = 0$
5. $\log_x a^m = m \log_x a$
6. $\log_x a = \frac{1}{\log_a x}$
7. $\log_x a = \frac{\log_b a}{\log_b x} = \frac{\log a}{\log x}$

COMMON LOGARITHM: Logarithm to the base 10 is known as common logarithm. If base is not mentioned in logarithm, by default, it is 10.

NATURAL LOGARITHM: Logarithm to the base e is called natural logarithm where $e = 2.718 \dots$.

CHARACTERISTIC: The integral part of the logarithm of a number is called its characteristic.

Case I. When the number is greater than 1.

Here, characteristic is a number which is one less than the number of digits to the left of decimal point in the given number.

Case II. When the number is less than 1.

Here, characteristic is one more than the number of zeros between the decimal point and the first significant digit of the number, and it is negative.

For such negative number like $-1, -2$, etc. we will write $\bar{1}$ (one bar) $\bar{2}$ (two bar) etc.

Example:

Number	Characteristic	Number	Characteristic
394.625	2	0.3162	$\bar{1}$
4963.849	3	0.0459	$\bar{2}$
27.348	1	0.0025	$\bar{3}$

MANTISSA: The decimal part of the logarithm of a number is known as its mantissa. For mantissa, you have to refer to logarithm table.

EXAMPLE 1 Evaluate $\log_2 64$

Solution Let $\log_2 64 = x$

$$\therefore 2^x = 64 = 2^6$$

$$\therefore x = 6$$

$$\therefore \log_2 64 = 6$$

EXAMPLE 2 Evaluate $\log_9 \left(\frac{1}{729} \right)$

Solution Let $\log_9 \left(\frac{1}{729} \right) = x$

$$\therefore 9^x = \frac{1}{729} = \frac{1}{9^3} = 9^{-3}$$

$$\therefore x = -3$$

$$\therefore \log_9 \left(\frac{1}{729} \right) = -3$$

EXAMPLE 3 Evaluate $49^{\log_7 6}$

Solution We have $x^{\log_x a} = a$

$$\therefore 49^{\log_7 6} = (7^2)^{\log_7 6} = 7^{2 \log_7 6} = 7^{\log_7 6^2} = 7^{\log_7 36} = 36$$

EXAMPLE 4 Simplify $\log \frac{75}{16} - 2 \log \frac{5}{9} + \log \frac{32}{243}$ [SSC]

Solution $\log \frac{75}{16} - 2 \log \frac{5}{9} + \log \frac{32}{243} = \log \frac{75}{16} - \log \left(\frac{5}{9}\right)^2 + \log \frac{32}{243}$

$$= \log \frac{75}{16} - \log \frac{25}{81} + \log \frac{32}{243}$$

$$= \log \left(\frac{75}{16} \times \frac{32}{243} \times \frac{81}{25} \right) = \log 2$$

EXAMPLE 5 Find the value of x which satisfies the relation: [MBA]

$$\log_{10} 3 + \log_{10}(4x + 1) = \log_{10}(x + 1) + 1$$

Solution $\log_{10} 3 + \log_{10}(4x + 1) = \log_{10}(x + 1) + 1$

$$\Rightarrow \log[3(4x + 1)] = \log_{10}(x + 1) + \log_{10} 10$$

$$\Rightarrow \log[3(4x + 1)] = \log_{10}[10(x + 1)]$$

$$\Rightarrow 3(4x + 1) = 10(x + 1)$$

$$\Rightarrow 2x = 10 - 3 = 7$$

$$\Rightarrow x = \frac{7}{2}$$

EXAMPLE 6 If $\log_{10} 2 = 0.30103$, find the value of $\log_{10} 50$ [CBI]

Solution $\log_{10} 50 = \log_{10} \left(\frac{100}{2} \right) = \log_{10} 100 - \log_{10} 2 = 2 - 0.30103 = 1.69897$



EXERCISES

- If $\log_{10} 2 = 0.3010$ and $\log_{10} 7 = 0.8451$, then the value of $\log_{10} 2.8$ is [SSC]
 (a) 0.4471 (b) 1.4471 (c) 2.4471 (d) None of these
- If $\log_{10} 5 + \log_{10}(5x + 1) = \log_{10}(x + 5) + 1$, then x is equal to: [CDS]
 (a) 1 (b) 3 (c) 5 (d) 10
- If $\log_{10} 7 = a$, then $\log_{10} \left(\frac{1}{70} \right)$ is equal to: [CDS]
 (a) $-(1 + a)$ (b) $(1 + a)^{-1}$ (c) $\frac{a}{10}$ (d) $\frac{1}{10a}$
- If $\log_a(ab) = x$, then $\log_b(ab)$ is [MAT]
 (a) $\frac{1}{x}$ (b) $\frac{x}{x + 1}$ (c) $\frac{x}{1 - x}$ (d) $\frac{x}{x - 1}$

5. $\frac{\log \sqrt{8}}{\log 8}$ is equal to: [MBA]

- (a) $\frac{1}{\sqrt{8}}$ (b) $\frac{1}{4}$ (c) $\frac{1}{2}$ (d) $\frac{1}{8}$

6. If $\log_x y = 100$ and $\log_2 x = 10$, then the value of y is: [SSC]

- (a) 2^{10} (b) 2^{100} (c) 2^{1000} (d) 2^{10000}

7. Value of $\log_2 16$ is [MBA]

- (a) $\frac{1}{8}$ (b) 4 (c) 8 (d) 16

8. If $\log_{10} 2 = 0.3010$, the numbr of digits in 2^{64} is: [CBI]

- (a) 18 (b) 19 (c) 20 (d) 21

9. If $\log 2 = 0.3010$ and $\log 3 = 0.4771$, then the value of $\log_5 512$ is: [MAT]

- (a) 2.870 (b) 2.967 (c) 3.876 (d) 3.912

10. $2 \log_{10} 5 + \log_{10} 8 - \frac{1}{2} \log_{10} 4 = ?$ [MAT]

- (a) 2 (b) 4 (c) $2 + 2 \log_{10} 2$ (d) $4 - 4 \log_{10} 2$

11. If $\log 2 = x$, $\log 3 = y$ and $\log 7 = z$, then the value of $\log(4 \sqrt[3]{63})$ is [SSC]

- (a) $2x + \frac{2}{3}y - \frac{1}{3}z$ (b) $2x + \frac{2}{3}y + \frac{1}{3}z$
(c) $2x - \frac{2}{3}y + \frac{1}{3}z$ (d) $-2x + \frac{2}{3}y + \frac{1}{3}z$

12. If $\log 3 = 0.477$ and $(1000)^x = 3$, then x equals: [SSC]

- (a) 0.0159 (b) 0.0477 (c) 0.159 (d) 10

13. If $a^x = b^y$, then: [Hotel Management]

- (a) $\log \frac{a}{b} = \frac{x}{y}$ (b) $\frac{\log a}{\log b} = \frac{x}{y}$ (c) $\frac{\log a}{\log b} = \frac{y}{x}$ (d) None of these

14. If $\log_4 x + \log_8 x = 5$, then x is equal to:

- (a) 16 (b) 32 (c) 48 (d) 64

15. If $\log_{10} 50 + \log_{10} 20 = x$, then x is equal to:

- (a) $\frac{1}{3}$ (b) -3 (c) 3 (d) 0.3



1. (a) 2. (b) 3. (a) 4. (d) 5. (c) 6. (c) 7. (b) 8. (c) 9. (c) 10. (a)
11. (b) 12. (c) 13. (c) 14. (d) 15. (c)



Solutions with Necessary Explanation

$$1. \log_{10} 2.8 = \log_{10} \left(\frac{28}{10} \right) = \log_{10} \left(\frac{2^2 \times 7}{10} \right)$$

$$= \log_{10} 2^2 + \log_{10} 7 - \log_{10} 10$$

$$= 2 \times 0.3010 + 0.8451 - 1$$

$$= 0.602 + 0.8451 - 1 = \mathbf{0.4471}$$

$$2. \log_{10} 5 + \log_{10} (5x + 1) = \log_{10} (x + 5) + 1$$

$$\Rightarrow \log_{10} [5(5x + 1)] = \log_{10} (x + 5) + \log_{10} 10$$

$$\Rightarrow \log_{10} [5(5x + 1)] = \log_{10} [10(x + 5)]$$

$$\Rightarrow 5(5x + 1) = 10(x + 5)$$

$$\Rightarrow 25x - 10x = 50 - 5$$

$$\Rightarrow 15x = 45$$

$$\Rightarrow x = 3$$

$$3. \text{ Given } \log_{10} 7 = \alpha;$$

$$\therefore \log_{10} \left(\frac{1}{70} \right) = \log_{10} 1 - \log_{10} 70$$

$$= \log_{10} 1 - \log_{10} (7 \times 10)$$

$$= 0 - [\log_{10} 7 + \log_{10} 10]$$

$$= -(\alpha + 1) = \mathbf{-(1 + \alpha)}$$

$$4. \text{ Given } \log_a (ab) = x;$$

$$\therefore \log_a (ab) = x \Rightarrow \log_a a + \log_a b = x$$

$$\Rightarrow \log_a b = x - 1$$

$$\therefore \log_b (ab) = \log_b a + \log_b b = \log_b a + 1$$

$$= \frac{1}{x - 1} + 1$$

$$= \frac{x}{x - 1}$$

$$\left(\because \log_b a = \frac{1}{\log_a b} \right)$$

$$5. \frac{\log \sqrt{8}}{\log 8} = \frac{\frac{1}{2} \log 8}{\log 8} = \frac{1}{2}$$

$$6. \text{ Given } \log_x y = 100; \log_2 x = 10;$$

$$\therefore \log_2 x \times \log_x y = 10 \times 100$$

$$\text{i.e. } \log_2 y = 1000$$

$$\therefore y = 2^{1000}$$

$$(\because \log_a x \times \log_x b = \log_a b)$$

$$7. \log_2 16 = \log_2 2^4 = 4 \log_2 2 = 4 \times 1 = 4$$

$$8. \log(2^{64}) = 64 \log 2 = 64 \times (0.3010) = 19.26592$$

$$\therefore \text{ Number of digits} = \text{Characteristic} + 1 = \mathbf{20}$$

9. Given $\log 2 = 0.3010$; $\log 3 = 0.4771$

$$\begin{aligned}\therefore \log_5 512 &= \log_5 (16^2 \times 2) \\ &= \log_5 ((2^4)^2 \times 2^1) = \log_5 (2^9) = 9 \log_5 2 \\ &= 9 \frac{\log_{10} 2}{\log_{10} 5} = \frac{9 \log_{10} 2}{\log_{10} 10 - \log_{10} 2} \\ &= \frac{9 \times 0.3010}{1 - 0.3010} = \frac{2.709}{0.699} = \mathbf{3.876}\end{aligned}$$

10. $2 \log_{10} 5 + \log_{10} 8 - \frac{1}{2} \log_{10} 4 = \log_{10} \left(\frac{5^2 \times 8}{2} \right) = \log_{10} (100) = \mathbf{2}$

11. Given $\log 2 = x$; $\log 3 = y$; $\log 7 = z$;

$$\begin{aligned}\therefore \log(4 \sqrt[3]{63}) &= \log(2^2 \times \sqrt[3]{3^2 \times 7}) = \log 2^2 + \frac{1}{3} \log(3^2 \times 7) \\ &= 2 \log 2 + \frac{1}{3} [2 \log 3 + \log 7] = \mathbf{2x + \frac{2}{3}y + \frac{1}{3}z}\end{aligned}$$

12. Given $\log 3 = 0.477$; $(1000)^x = 3$;

$$\begin{aligned}\therefore \log_{10} (1000)^x &= \log_{10} 3 \\ \Rightarrow \log_{10} (10^3)^x &= \log_{10} 3 \\ \Rightarrow 3x &= \log_{10} 3 \\ \Rightarrow x &= \frac{1}{3} \log_{10} 3 = \frac{1}{3} \times 0.477 = \mathbf{0.159}\end{aligned}$$

13. Given $a^x = b^y$

Applying logarithm on both sides.

$$\begin{aligned}\log a^x &= \log b^y \\ \Rightarrow x \log a &= y \log b \\ \Rightarrow \frac{\log a}{\log b} &= \frac{y}{x}\end{aligned}$$

14. Given $\log_4 x + \log_8 x = 5$

$$\begin{aligned}\text{i.e. } \frac{\log x}{\log 4} + \frac{\log x}{\log 8} &= 5 \\ \Rightarrow \frac{\log x}{2 \log 2} + \frac{\log x}{3 \log 2} &= 5 \\ \Rightarrow 5 \log x &= 5 \times 6 \log 2 \\ \Rightarrow \log x &= 6 \log 2 = \log 2^6 \\ \therefore x &= \mathbf{64}\end{aligned}$$

15. $\log_{10} 50 + \log_{10} 20 = x$

$$\begin{aligned}\Rightarrow \log_{10} (50 \times 20) &= x \\ \Rightarrow 10^x &= 1000 = 10^3 \\ \therefore x &= \mathbf{3}\end{aligned}$$

Part II REASONING

SECTION A GENERAL MENTAL ABILITY

Chapter 1

Analogy

In the question based on analogy, there will be a particular relationship between the given two words. You have to identify the pair of words which are having the similar relationship among them.

Some useful points on the basic knowledge required for these tests are given below.

STUDY AND TOPIC RELATIONSHIP

Example: Anthropology : Man

We know that 'anthropology' is the study of man. Similar examples are:

1. Astrology	:	Future	19. Occultism	:	Supernatural
2. Botany	:	Plants	20. Onomatology	:	Names
3. Bryology	:	Bryophytes	21. Ontology	:	Reality
4. Cardiology	:	Heart	22. Oology	:	Eggs
5. Concology	:	Shells	23. Ornithology	:	Birds
6. Craniology	:	Skull	24. Orography	:	Mountains
7. Eccrinology	:	Secretions	25. Paleography	:	Writings
8. Entomology	:	Insects	26. Paleontology	:	Fossils
9. Ethnology	:	Human races	27. Pathology	:	Diseases
10. Hematology	:	Blood	28. Pedology	:	Soil
11. Herpetology	:	Amphibians	29. Phycology	:	Algae
12. Histology	:	Tissues	30. Seismology	:	Earthquakes
13. Ichthyology	:	Fishes	31. Selenography	:	Moon
14. Malacology	:	Molluscs	32. Semantics	:	Language
15. Mycology	:	Fungi	33. Taxonomy	:	Classification
16. Nephrology	:	Kidney	34. Virology	:	Viruses
17. Neurology	:	Nerves	35. Zoology	:	Animals
18. Nidology	:	Nests			

WORKER AND TOOL RELATIONSHIP**Example:** Carpenter : Saw

A saw is a tool used by the carpenter.

Similar examples are:

1. Astronomer	:	Telescope	13. Labourer	:	Spade
2. Author	:	Pen	14. Lumberjack	:	Axe
3. Barber	:	Scissors	15. Mason	:	Plumbline
4. Blacksmith	:	Anvil	16. Painter	:	Brush
5. Bricklayer	:	Trowel	17. Sculptor	:	Chisel
6. Butcher	:	Chopper	18. Soldier	:	Gun
7. Chef	:	Knife	19. Surgeon	:	Scalpel
8. Cobbler	:	Awl	20. Tailor	:	Needle
9. Doctor	:	Stethoscope	21. Violinist	:	Bow
10. Farmer	:	Plough	22. Warrior	:	Sword
11. Gardener	:	Harrow	23. Wood cutter	:	Axe
12. Jockey	:	Tack			

TOOL AND ACTION RELATIONSHIP**Example:** Spade : Dig

A spade is used for digging.

Similar examples are:

1. Auger	:	Bore	10. Needle	:	Saw
2. Axe	:	Grind	11. Oar	:	Row
3. Chisel	:	Carve	12. Pen	:	Write
4. Filter	:	Purify	13. Shield	:	Guard
5. Gun	:	Shoot	14. Shovel	:	Scoop
6. Knife	:	Cut	15. Spanner	:	Grip
7. Loudspeaker	:	Amplify	16. Spoon	:	Feed
8. Mattock	:	Dig	17. Sword	:	Slaughter
9. Microscope	:	Magnify	18. Steering	:	Drive

WORK AND WORKING PLACE**Example:** Actor : Stage

An actor performs on a stage.

Similar examples are:

1. Artist	:	Theatre	9. Gambler	:	Casino
2. Beautician	:	Parlour	10. Grocer	:	Shop
3. Chef	:	Kitchen	11. Lawyer	:	Court
4. Clerk	:	Office	12. Mechanic	:	Garage
5. Doctor	:	Hospital	13. Painter	:	Gallery
6. Driver	:	Cabin	14. Pilot	:	Cockpit
7. Engineer	:	Site	15. Sailor	:	Ship
8. Farmer	:	Field	16. Scientist	:	Laboratory

17. Servant : House	20. Waiter : Restaurant
18. Teacher : School	21. Warrior : Battle field
19. Umpire : Pitch	22. Worker : Factory

WORKER AND PRODUCT

Example: Producer : Film

A producer makes movies.

Similar examples are:

1. Architect : Design	10. Farmer : Crop
2. Author : Book	11. Goldsmith : Ornaments
3. Butcher : Meat	12. Hunter : Prey
4. Carpenter : Furniture	13. Judge : Justice
5. Chef : Food	14. Mason : Wall
6. Choreographer : Ballet	15. Poet : Poem
7. Cobbler : Shoes	16. Tailor : Clothes
8. Dramatist : Play	17. Teacher : Education
9. Editor : Newspaper	

PRODUCT AND RAW MATERIAL

Example: Paper : Pulp

Paper is made from pulp.

Similar examples are:

1. Book : Paper	11. Omelette : Egg
2. Butter : Milk	12. Prism : Glass
3. Cloth : Fibre	13. Pullover : Wool
4. Fabric : Yarn	14. Road : Asphalt
5. Furniture : Wood	15. Rubber : Latex
6. Jaggery : Sugarcane	16. Sack : Jute
7. Jewellery : Gold	17. Shoes : Leather
8. Linen : Flax	18. Wall : Brick
9. Metal : Ore	19. Wine : Grapes
10. Oil : Seed	

INSTRUMENT AND MEASUREMENT

Example: Scale : Length

Similar examples are:

1. Ammeter : Current	
2. Anemometer : Wind vane	
3. Balance : Mass	
4. Barometer : Pressure	
5. Hygrometer : Humidity	
6. Odometer : Speed	
7. Rain gauge : Rain	
8. Screw gauge : Thickness	

9. Seismography	:	Earthquakes
10. Sphygmomanometer	:	Blood pressure
11. Taseometer	:	Strains
12. Thermometer	:	Temperature

QUANTITY AND UNIT

Example: Time : Seconds

Second is the unit of time.

Similar examples are:

1. Angle	:	Radians	10. Mass	:	Kilogram
2. Area	:	Hectare	11. Potential	:	Volt
3. Conductivity	:	Mho	12. Power	:	Watt
4. Current	:	Ampere	13. Pressure	:	Pascal
5. Energy	:	Joule	14. Resistance	:	Ohm
6. Force	:	Newton	15. Temperature	:	Degrees
7. Length	:	Metre	16. Volume	:	Litre
8. Luminosity	:	Candela	17. Work	:	Joule
9. Magnetic field	:	Oersted			

ANIMALS AND YOUNG ONES

Example: Man : Child

The child is the young one of man.

Similar examples are:

1. Butterfly	:	Catterpillar	8. Horse	:	Pony
2. Cat	:	Kitten	9. Insect	:	Larva
3. Cow	:	Calf	10. Lion	:	Cub
4. Dog	:	Puppy	11. Pig	:	Farrow
5. Duck	:	Duckling	12. Sheep	:	Lamb
6. Frog	:	Tadpole	13. Stallion	:	Colt
7. Hen	:	Chicken	14. Tortoise	:	Turtle

MALE AND FEMALE

Example: Horse : Mare

Mare is the female horse.

Similar examples are:

1. Dog	:	Bitch	6. Son	:	Daughter
2. Drone	:	Bee	7. Sorcerer	:	Sorceress
3. Gentleman	:	Lady	8. Stag	:	Doe
4. Lion	:	Lioness	9. Tiger	:	Tigress
5. Nephew	:	Niece			

WORD AND INTENSITY

Example: Anger : Rage

'Rage' is of higher intensity than 'anger'.

Similar examples are:

1. Crime	:	Sin	6. Quarrel	:	War
2. Error	:	Blunder	7. Refuse	:	Deny
3. Famous	:	Renowned	8. Sink	:	Drown
4. Kindle	:	Burn	9. Unhappy	:	Sad
5. Moist	:	Drench	10. Wish	:	Desire

WORD AND SYNONYM

Example: Abode : Dwelling

'Abode' means almost the same as 'dwelling'. Thus, 'dwelling' is the synonym of 'abode'.

Similar examples are:

1. Abduct	:	Kidnap	11. Flaw	:	Defect
2. Alight	:	Descend	12. Haughty	:	Proud
3. Assign	:	Allot	13. House	:	Home
4. Ban	:	Prohibition	14. Mend	:	Repair
5. Blend	:	Mix	15. Presage	:	Predict
6. Brim	:	Edge	16. Presume	:	Assume
7. Dearth	:	Scarcity	17. Sedate	:	Calm
8. Dissipate	:	Squander	18. Solicit	:	Request
9. Fallacy	:	Illusion	19. Substitute	:	Replace
10. Fierce	:	Violent	20. Vacant	:	Empty

WORD AND ANTONYM

Example: Gradual : Abrupt

'Abrupt' means sudden, which is the opposite of 'gradual'. Thus, 'abrupt' is the antonym of 'gradual'.

Similar examples are:

1. Advance	:	Retreat	11. Fresh	:	Stale
2. Affirm	:	Deny	12. Gentle	:	Harsh
3. Attack	:	Defend	13. Ignore	:	Notice
4. Best	:	Worst	14. Initial	:	Final
5. Chaos	:	Peace	15. Kindle	:	Extinguish
6. Condense	:	Expand	16. Lend	:	Borrow
7. Cordial	:	Hostile	17. Lethargy	:	Alertness
8. Create	:	Destroy	18. Mourn	:	Rejoice
9. Cruel	:	Kind	19. Robust	:	Weak
10. Deep	:	Shallow	20. Sink	:	Float

TYPE I: In this type of questions, two words are given. These words are related to each other in some way. Another word is also given. You have to find out the relationship between the first two words and choose the word from the given alternatives, which bear the same relationship to the third word.

EXAMPLE 1 Doctor : Diagnosis :: Judge : ?

- (a) Court (b) Punishment (c) Lawyer (d) Judgement

Solution As a doctor diagnoses a disease. Similarly, a judge give judgement. So, the answer is (d).

EXAMPLE 2 Bombay : Maharashtra : : Trivandrum?

- (a) Calcutta (b) Gujarat (c) Rajasthan (d) Kerala

Solution Bombay is the capital of Maharashtra. Similarly, Trivandrum is the capital of Kerala. So the answer is (d).

EXAMPLE 3 Conference : Chairman : : Newspaper : ?

- (a) Reporter (b) Distributor (c) Printer (d) Editor

Solution The chairman is the highest authority in a conference. Similarly, the editor is the highest authority in a newspaper agency. So the answer is (d).

EXERCISE 1

Directions: In each of the following questions, there is a certain relation between two words given on one side of : : and one word is given on the side of : : while another word is to be found from the given alternatives, having the same relation with this word as the given pair of words bear, choose the correct alternative.

- Moon : Satellite : : Earth : ?
(a) Sun (b) Planet (c) Solar system (d) Asteroid
- Melt : Liquid : : Freeze : ?
(a) Ice (b) Condense (c) Solid (d) Crystal
- Influenza : Virus : : Typhoid : ?
(a) Bacillus (b) Parasite (c) Protozoa (d) Bacteria
- Eye : Myopia : : Teeth : ?
(a) Pyorrhoea (b) Cataract (c) Trachoma (d) Eczema
- Scribble : Write : : Stammer : ?
(a) Walk (b) Play (c) Speak (d) Dance
- Muslims : Mosque : : Sikhs : ?
(a) Golden temple (b) Medina (c) Fire temple (d) Gurudwara
- Acting : Theatre : : Gambling : ?
(a) Casino (b) Club (c) Bar (d) Gym
- Breeze : Cyclone : : Drizzle : ?
(a) Earthquake (b) Storm (c) Flood (d) Downpour
- Water : Convection : : Space : ?
(a) Conduction (b) Transference (c) Vacuum (d) Radiation
- Grain : Stock : : Stick ?
(a) Heap (b) Bundle (c) Collection (d) String
- Oxygen : Burn : : Carbon dioxide : ?
(a) Isolate (b) Foam (c) Extinguish (d) Explode
- Planet : Orbit : : Projectile : ?
(a) Trajectory (b) Track (c) Milky way (d) Path
- Cobbler : Leather : : Carpenter : ?
(a) Furniture (b) Wood (c) Hammer (d) Chair

14. Genuine : Authentic :: Mirage : ?
(a) Image (b) Transpiration (c) Reflection (d) Illusion
15. Cub : Lion :: Colt : ?
(a) Doe (b) Stag (c) Leopard (d) Stallion
16. Drill : Bore :: Sieve : ?
(a) Thresh (b) Sift (c) Pry (d) Rinse
17. Country : President :: State : ?
(a) Governor (b) Minister (c) Chief Minister (d) Citizen
18. Car : Garage :: Aeroplane : ?
(a) Airport (b) Depot (c) Hangar (d) Harbour
19. Bread : Yeast :: Curd : ?
(a) Fungi (b) Bacteria (c) Germs (d) Virus
20. Ruby : Red :: Sapphire : ?
(a) Blue (b) White (c) Green (d) Silver
21. House : Garbage :: Ore : ?
(a) Rubbish (b) Gangue (c) Sand (d) Dregs
22. Steel : Rails :: Alnico : ?
(a) Aircraft (b) Machinery (c) Silver ware (d) Magnets
23. Naphthalene : Coal tar :: Dyes : ?
(a) Petroleum (b) Oils (c) Chemicals (d) Carbon
24. Darwin : Evolution :: Archimedes : ?
(a) Friction (b) Lubrication (c) Buoyancy (d) Liquids
25. Flower : Butterfly :: Dirt : ?
(a) Rats (b) Fly (c) Bugs (d) Sweeper
26. Glucose : Carbohydrate :: Soyabean : ?
(a) Proteins (b) Vitamins (c) Minerals (d) Legumes
27. Engineer : Map :: Bricklayer : ?
(a) Design (b) Templest (c) Mould (d) Cement
28. Microphone : Loud :: Microscope : ?
(a) Increase (b) Investigate (c) Examine (d) Magnify
29. Matricide : Mother :: Homicide : ?
(a) Human (b) Children (c) Father (d) Apes
30. Scrap : Food :: Lees : ?
(a) Bread (b) Tea (c) Wine (d) Rice
31. Quartz : Radio :: Gypsum : ?
(a) Glass (b) Porcelain (c) Cement (d) Powder
32. Cattle : Herd :: Sheep : ?
(a) Flock (b) Swarm (c) Crowd (d) Shoal
33. Newspaper : Press :: Cloth : ?
(a) Tailor (b) Mill (c) Fibre (d) Factory
34. Oceans : Deserts :: Waves : ?
(a) Sea (b) Dust (c) Sand dunes (d) Ripples
35. Giant : Dwarf :: Genius : ?
(a) Wicked (b) Gentle (c) Idiot (d) Tiny

ANSWERS

1. (b)	2. (a)	3. (d)	4. (a)	5. (c)	6. (d)	7. (a)	8. (d)	9. (d)	10. (b)
11. (c)	12. (a)	13. (b)	14. (d)	15. (d)	16. (b)	17. (a)	18. (c)	19. (b)	20. (a)
21. (b)	22. (d)	23. (a)	24. (c)	25. (b)	26. (a)	27. (b)	28. (d)	29. (a)	30. (c)
31. (c)	32. (a)	33. (b)	34. (c)	35. (c)					

Explanations

1. The moon is a satellite and earth is a planet.
2. The first is the process of formation of the second.
3. The first is the disease caused by the second.
4. The second is a disease of the first.
5. The first is an improper form of the second.
6. Muslims worship in a mosque and Sikhs worship in a gurudwara.
7. The second is the place for performing the first.
8. The second is more intense than the first.
9. The second is the mode of transfer of heat by the first.
10. The second is a collection of the first.
11. Oxygen helps in burning whereas carbon dioxide extinguishes fire.
12. The second is the path traced by the first.
13. The second is the raw material used by the first.
14. The given words are opposites of each other.
15. The first is a young one of the second.
16. The second denotes the function performed by the first.
17. The President is the head of the country. Similarly, the Governor is the head of the state.
18. The first is temporarily parked in the second.
19. The first is produced by the action of the second.
20. Ruby is a red precious stone and sapphire is a blue precious stone.
21. The waste of the house is called garbage. Similarly, the impurities in the ore are called gangue.
22. The first is used to make the second.
23. The first is obtained from the second.
24. Darwin gave the theory of evolution. Likewise, Archimedes gave the principle of buoyancy.
25. The first attracts the second.
26. Glucose is a source of carbohydrates and soyabean is a source of proteins.
27. The second gives the pattern to be followed by the first.
28. A microphone makes the sound louder and a microscope magnifies an object.
29. The first implies killing the second.
30. The first is the left over of the second.
31. The first is used to make the second.
32. The second is the group or collection of the first.
33. The second is the place where first is made.
34. If oceans were deserts, waves would be sand dunes.
35. The given words are opposites of each other.

EXERCISE 2

Directions: In each of the following questions, there is a certain relation between two given words on one side of :: and one word is given on the other side of :: while another word is to be found from the given alternatives, having the same relation with this word as the given pair of words bear. Choose the best alternative.

1. Tea : Cup :: Tobacco : ?
 (a) Leaves (b) Hookah (c) Toxin (d) Cheroot
2. Pesticide : Crop :: Antiseptic : ?
 (a) Wound (b) Clotting (c) Bandage (d) Bleeding
3. Atom : Matter :: Particle : ?
 (a) Proton (b) Electron (c) Molecule (d) Dust
4. Disease : Pathology :: Planet : ?
 (a) Astrology (b) Geology (c) Astronomy (d) Palaeontology
5. Fire : Ashes :: Explosion : ?
 (a) Sound (b) Debris (c) Fury (d) Flame
6. Tuberculosis : Lungs :: Cataract : ?
 (a) Ears (b) Throat (c) Skin (d) Eyes
7. Professor : Lecture :: Doctor : ?
 (a) Hospital (b) Disease (c) Medicine (d) Patient
8. Victory : Encouragement :: Failure : ?
 (a) Sadness (b) Defeat (c) Anger (d) Frustration
9. Fossils : Creatures :: Mummies : ?
 (a) Egypt (b) Human beings (c) Animals (d) Martyrs
10. Ornithologist : Birds :: Anthropologist : ?
 (a) Plants (b) Animals (c) Mankind (d) Environment
11. Ocean : Pacific :: Island : ?
 (a) Greenland (b) Netherlands (c) Island (d) Borneo
12. Meat : Vegetarian :: Liquor : ?
 (a) Insane (b) Introvert (c) Teetotaler (d) Foolish
13. Accommodation : Rent :: Journey : ?
 (a) Expense (b) Octroin (c) Freight (d) Fare
14. Head : Cap :: Finger : ?
 (a) Glove (b) Thimble (c) Nail (d) Thumb
15. Proteins : Growth :: Carbohydrates : ?
 (a) Energy (b) Strength (c) Resistance (d) Diseases
16. Cells : Tissues :: Atoms : ?
 (a) Elements (b) Molecules (c) Electrons (d) Organs
17. Girl : Beautiful :: Boy : ?
 (a) Smart (b) Heroic (c) Courageous (d) Handsome
18. Taxonomy : Classification :: Pedology : ?
 (a) Nature (b) Farming (c) Soil (d) Mountain
19. Aeroplane : Cockpit :: Train : ?
 (a) Wagon (b) Coach (c) Compartment (d) Engine

20. Palaeography : Writings :: Ichthyology : ?
 (a) Fishes (b) Whales (c) Oysters (d) Mammals
21. Bullet : Gun :: Smoke : ?
 (a) Factory (b) Cigarette (c) Chimney (d) Fire
22. Wrist : Elbow :: Ankle : ?
 (a) Heel (b) Fingers (c) Foot (d) Knee
23. Ottawa : Canada :: Canberra : ?
 (a) Argentina (b) Switzerland (c) Sri Lanka (d) Australia
24. Leather : Cobbler :: Wood : ?
 (a) Furniture (b) Cottage (c) Carpenter (d) Mason
25. Roentgen : X rays :: Becquerel : ?
 (a) Uranium (b) Radioactivity (c) Fission (d) Super conductivity

ANSWERS

1. (d) 2. (a) 3. (d) 4. (c) 5. (b) 6. (d) 7. (c) 8. (d) 9. (b) 10. (c)
 11. (a) 12. (c) 13. (d) 14. (b) 15. (a) 16. (b) 17. (d) 18. (c) 19. (d) 20. (a)
 21. (c) 22. (d) 23. (d) 24. (c) 25. (b)

Explanations

- Tea is contained in the cup. Similarly, tobacco is contained in cheroot.
- Pesticide protects crops from insects and antiseptic protects wounds from germs.
- The smallest unit of matter is atom and that of dust is particle.
- Pathology is the study of diseases. Similarly, astronomy is the study of planets.
- Ashes are the remains after a fire. Similarly, debris is the remains after an explosion.
- Tuberculosis is a disease of lungs. Similarly, cataract is a disease of eyes.
- Professor delivers lecture to his students. Similarly, Doctor gives medicine to his patients.
- Victory leads to encouragement and failure brings frustration.
- Fossils are the remains of creatures. Similarly, mummies are the remains of human beings.
- Ornithologist specialises in the study of birds. Similarly, anthropologist specialises in the study of mankind.
- The largest ocean is Pacific Ocean. Similarly, the largest island is Greenland.
- A vegetarian never eats meat. Similarly, a teetotaler never drinks liquor.
- Money paid for accommodation is called rent. Similarly, the money paid for a journey is called fare.
- Head is covered by a cap. Similarly, finger is covered by a thimble.
- Proteins are essential for growth. Similarly, carbohydrates are essential for energy.
- Cells constitute tissues and atoms constitute molecules.
- 'Beautiful' describes the quality of prettiness in girls while 'handsome' describes the quality of prettiness in boys.
- Taxonomy is the science dealing with classification. Similarly, pedology deals with the study of soil.
- The pilot of an aeroplane sits in the cockpit. Likewise, the driver of a train works in the engine.

20. Palaeography is the study of ancient writings. Similarly, ichthyology is the study of fishes.
21. The first comes out of the second.
22. Wrist is the lower part of elbow. Similarly, ankle is the lower part of knee.
23. Ottawa is the capital of Canada and Canberra is the capital of Australia.
24. The first is the raw material required by the second.
25. Roentgen discovered X-rays. Similarly, Becquerel discovered radioactivity.

EXERCISE 3

Directions: In each of the following questions, there is a certain relation between two given words on one side of :: and one word is given on the other side of :: while another word is to be found from the given alternatives, having the same relation with this word as the given pair of words bear. Choose the best alternative.

1. Tectonics : Building :: Taxidermy : ?
 (a) Classification (b) Conserving (c) Stuffing (d) Collecting
2. Visitor : Invitation :: Witness : ?
 (a) Subpoena (b) Permission (c) Assent (d) Document
3. Mash : Horse :: Mast : ?
 (a) Cow (b) Monkey (c) Chimpanzee (d) Pig
4. Penology : Punishment :: Seismology : ?
 (a) Law (b) Liver (c) Earthquakes (d) Medicine
5. Wine : Grapes :: Perry : ?
 (a) Whisky (b) Pears (c) Almonds (d) Pomegranates
6. Pericardium : Heart :: Rind : ?
 (a) Body (b) Head (c) Trees (d) Teak
7. Eccrinology : Secretions :: Selenography : ?
 (a) Sun (b) Moon (c) Crust (d) Mantle
8. Anaemia : Blood :: Anarchy : ?
 (a) Disorder (b) Monarchy (c) Government (d) Lawlessness
9. Anatomy : Zoology :: Paediatrics : ?
 (a) Chemistry (b) Medicine (c) Palaeontology (d) Mechanics
10. Winter : Hibernation :: Summer : ?
 (a) Survival (b) Activation (c) Aestivation (d) Cache
11. Vegetable : Chop :: Body : ?
 (a) Cut (b) Amputate (c) Peel (d) Prune
12. Misogamy : Marriage :: Misogyny : ?
 (a) Children (b) Husband (c) Relations (d) Women
13. Eye : Wink :: Heart : ?
 (a) Move (b) Throb (c) Pump (d) Quiver
14. Virology : Virus :: Semantics : ?
 (a) Amoeba (b) Language (c) Nature (d) Society
15. Mattock : Digging :: Shovel : ?
 (a) Break (b) Push (c) Scoop (d) Whittle

16. Yaws : Skin :: Thrush : ?
 (a) Legs (b) Eyes (c) Belly (d) Throat
17. Pituitary : Brain :: Thymus : ?
 (a) Larynx (b) Spinal cord (c) Throat (d) Chest
18. Roster : Duty :: Inventory : ?
 (a) Furnace (b) Exports (c) Goods (d) Produce
19. Xylograph : Engraving :: Diorama : ?
 (a) Painting (b) Exhibition (c) Colouring (d) Staging
20. Funk : Vitamins :: Curie : ?
 (a) Uranium (b) Radium (c) Radioactivity (d) Photography
21. Karnataka : Gold :: Madhya Pradesh : ?
 (a) Diamond (b) Iron (c) Copper (d) Gems
22. Dum Dum : Kolkata :: Palam : ?
 (a) Kerala (b) Delhi (c) Chennai (d) Mumbai
23. Seismography : Earthquakes :: Taseometer : ?
 (a) Volcanoes (b) Resistances (c) Landslides (d) Strains
24. Pig : Farrow :: Dog : ?
 (a) Mare (b) Puppy (c) Bitch (d) Colt
25. Line : Square :: Arc : ?
 (a) Ring (b) Sphere (c) Circle (d) Ball
26. Calendar : Dates :: Dictionary : ?
 (a) Vocabulary (b) Language (c) Words (d) Book
27. Shark : Fish :: Lavender : ?
 (a) Shrub (b) Tree (c) Herb (d) Climber
28. Applique : Ornament :: Impound : ?
 (a) Confiscate (b) Powder (c) Grab (d) Snatch
29. Oriel : Room :: Tendon : ?
 (a) Blood (b) Muscles (c) Veins (d) Liver
30. Probe : Wound :: Anemograph : ?
 (a) Humidity (b) Rainfall (c) Force (d) Pressure

ANSWERS

1. (c) 2. (a) 3. (d) 4. (c) 5. (b) 6. (c) 7. (b) 8. (c) 9. (b) 10. (c)
 11. (b) 12. (d) 13. (b) 14. (b) 15. (c) 16. (d) 17. (d) 18. (c) 19. (b) 20. (b)
 21. (a) 22. (b) 23. (d) 24. (b) 25. (c) 26. (c) 27. (a) 28. (a) 29. (b) 30. (c)

Explanations

1. Tectonics is the science dealing with the art of building. Similarly, taxidermy is the art of stuffing animals.
2. A visitor is given an invitation to attend an occasion. Similarly, a witness is delivered a subpoena providing for attendance at the court.
3. The first is a food for the second.
4. Penology is the study of punishment. Similarly, seismology is the study of earthquakes.

5. Wine is made from grapes and perry is made from pears.
6. The first is a thick covering over the second.
7. Eccrinology is the study of secretions and Selenography is the study of moon.
8. Anaemia is the lack of blood. Similarly, Anarchy is the lack of government.
9. Anatomy is a branch of zoology. Likewise, Paediatrics is a branch of medicine.
10. Winter sleep of animals is called hibernation and summer sleep is called aestivation.
11. Cutting of vegetables is called chopping. Cutting of a body part is called amputating.
12. The first is a hatred for the second.
13. The second is the movement of the first.
14. Virology deals with the effect of virus. Similarly, semantics deals with the effects of language.
15. Mattock is a tool for digging hard ground. Similarly, shovel is a tool to scoop.
16. The first is a disease of the second.
17. Pituitary is a gland in the brain. Similarly, thymus is a gland of the chest.
18. Roster is a list of duties and inventory is a list of goods.
19. The given words are synonyms of each other.
20. Funk discovered vitamins and Curie discovered radium.
21. Gold is mined in Karnataka. Likewise, diamonds are mined in Madhya Pradesh.
22. Dum Dum is an Airport in Kolkata and Palam is an Airport in Delhi.
23. Seismograph is an instrument to measure the intensity of an earthquake. Similarly, taseometer is an instrument to measure strains.
24. The second is the young one of the first.
25. The first is a part of the second.
26. Calendar is a list of dates. Likewise, dictionary is a collection of words.
27. Shark is a fish and lavender is a shrub.
28. The given words are synonyms of each other.
29. The first is a part of the second.
30. Probe is an instrument to examine a wound. Similarly, anemograph is an instrument for recording force.

TYPE II: In this type of questions, a pair of words is followed by four pairs of words as alternatives. You have to choose the pair of words in which the words bear the same relationship to each other.

EXAMPLE 4 Balance : Weigh

(a) Aeroplane : Height	(b) Radar : Detection
(c) Satellite : Revolution	(d) Television : Picture

Solution A balance is used to weigh. Similarly, a radar is used for detection.
Hence the answer is (b).

EXAMPLE 5 Cells : Cytology

(a) Worms : Ornithology	(b) Insects : Entomology
(c) Diseases : Physiology	(d) Tissues : Morphology

Solution The study of cells is called cytology. Similarly, the study of insects is called entomology.

Hence the answer is (b).

EXAMPLE 6 Chisel : Sculptor

(a) Scooter : Rider

(b) Brush : Teeth

(c) Scalpel : Surgeon

(d) Knife : Woodcutter

Solution Chisel is a tool used by a sculptor. Similarly, scalpel is a tool used by a surgeon. Hence the answer is (c).

EXERCISE 4

Directions: The following questions consist of two words each that have a certain relationship to each other, followed by four lettered pairs of words. Select the lettered pair that has the same relationship as the original pair of words.

1. Birds : Aves
 - (a) Fish : Water
 - (b) Whale : Fish
 - (c) Lizard : Insect
 - (d) Man : Homosapiens
2. Dusk : Night
 - (a) Afternoon : Evening
 - (b) Infant : Child
 - (c) Walk : Run
 - (d) Day : Light
3. Triangle : Hexagon
 - (a) Cone : Sphere
 - (b) Rectangle : Octagon
 - (c) Pentagon : Heptagon
 - (d) Angle : Quadrilateral
4. Shield : Soldier
 - (a) Stethoscope : Doctor
 - (b) Book : Author
 - (c) Advocate : Court
 - (d) Helmet : Rider
5. Pesticide : Plant
 - (a) Injection : Disease
 - (b) Vaccination : Body
 - (c) Medicine : Cure
 - (d) Teacher : Student
6. Cloth : Texture
 - (a) Body : Weigh
 - (b) Silk : Cloth
 - (c) Wood : Grains
 - (d) Ornaments : Gold
7. House : Ceiling
 - (a) Hut : Roof
 - (b) Building : Floor
 - (c) Bed : Bedsheet
 - (d) Grapes : Wine
8. Fish : Aquarium
 - (a) Teacher : Hostel
 - (b) Bee : Apiary
 - (c) Bird : Nest
 - (d) Child : School
9. Dog : Kennel
 - (a) Horse : Carriage
 - (b) Sheep : Flock
 - (c) Cow : Barn
 - (d) Sports : Stadium
10. Paper : Ream
 - (a) Eggs : Dozen
 - (b) Books : Pile
 - (c) Twigs : Bush
 - (d) Food : Packet
11. Explosion : Destruction
 - (a) Talk : Exaggeration
 - (b) Girl : Woman
 - (c) Success : Failure
 - (d) Engagement : Marriage

12. Identity : Anonymity
 - (a) Flaw : Perfection
 - (b) Careless : Mistake
 - (c) Truth : Lie
 - (d) Fear : Joy
13. Sigh : Relief
 - (a) Tear : Joy
 - (b) Trembling : Fear
 - (c) Carelessness : Accident
 - (d) Sweat : Hot
14. Ecstasy : Pleasure
 - (a) Hatred : Affection
 - (b) Joy : Grief
 - (c) Rage : Anger
 - (d) Mumble : Speak
15. Range : Mountain
 - (a) Point : Line
 - (b) Bouquet : Flower
 - (c) School : Class
 - (d) String : Bead
16. Spring : Summer
 - (a) Adolescence : Youth
 - (b) Fight : Battle
 - (c) Stagger : Walk
 - (d) Read : Learn
17. Teeth : Dentist
 - (a) Legs : Philanthropist
 - (b) Eyes : Occulist
 - (c) Operation : Surgeon
 - (d) Sight : Spectator
18. Textile : Mill
 - (a) Eggs : Hen
 - (b) Coal : Mine
 - (c) Food : Agriculture
 - (d) Brick : Kiln
19. Shoes : Cobbler
 - (a) Spectacles : Optician
 - (b) Education : Teacher
 - (c) Oxygen : Plant
 - (d) Food : Kitchen
20. Symphony : Music
 - (a) Mural : Painting
 - (b) Ode : Prose
 - (c) Preface : Book
 - (d) Editorial : Journal
21. Numismatist : Coins
 - (a) Philatelist : Stamps
 - (b) Jeweller : Jewels
 - (c) Cartographer : Maps
 - (d) Geneticist : Chromosomes
22. Embroider : Cloth
 - (a) Patch : Quilt
 - (b) Stain : Glass
 - (c) Carve : Knife
 - (d) Chase : Metal
23. Curtain : Drapery
 - (a) Cockroach : Insect
 - (b) Bedsheet : Bed
 - (c) Pillow : Cushion
 - (d) Mat : Floor
24. Theft : Confess
 - (a) Fight : Dare
 - (b) Fault : Admit
 - (c) Murder : Commit
 - (d) Mistake : Agree
25. Crown : Royal
 - (a) Throne : Regal
 - (b) Wrap : Ermine
 - (c) Pen : Author
 - (d) Crucifix : Religion

26. Cattle : Drove
 (a) Soldier : Crew (b) Grain : Bundle
 (c) Chicken : Brood (d) Bees : Heap
27. Meadow : Sheep
 (a) Stable : Horse (b) Hay : Insect
 (c) Grass : Grasshopper (d) Pasture : Cattle
28. Friendly : Inimical
 (a) Lithosphere : Hydrosphere (b) Condemnation : Approval
 (c) Abstain : Refrain (d) Disappointment : Embarrassment
29. Modesty : Arogance
 (a) Passion : Emotion (b) Practice : Perfection
 (c) Cause : Purpose (d) Debility : Strength
30. Traitor : Disloyalty
 (a) Executioner : Reliability (b) Rebel : Defiance
 (c) Manager : Administration (d) Hope : Pessimism

ANSWERS

1. (d) 2. (b) 3. (b) 4. (d) 5. (b) 6. (c) 7. (a) 8. (b) 9. (c) 10. (a)
 11. (d) 12. (a) 13. (b) 14. (c) 15. (d) 16. (a) 17. (b) 18. (d) 19. (a) 20. (a)
 21. (a) 22. (d) 23. (a) 24. (b) 25. (d) 26. (c) 27. (d) 28. (b) 29. (d) 30. (b)

Explanations

- Birds belong to the class of Aves. Similarly, man belongs to the class of Homasapiens.
- Dusk is the initial stage of night. Similarly, infant is the initial stage of child.
- The number of sides in the second figure in both the pairs is twice that in the first.
- Shield is a defensive instrument for a soldier. Likewise, helmet is a defensive instrument for a rider.
- Pesticide is meant to protect a plant from diseases. Similarly, vaccination is meant to protect the body from diseases.
- The quality of cloth is identified by its texture and that of wood by grains.
- Ceiling is the top part of a house. Likewise, roof is the top part of a hut.
- Fishes are kept and reared in an aquarium. Similarly, bees are reared in an apiary.
- Kennel is the home for a dog. Likewise, barn is the home for cows.
- The paper is bought in reams. Similarly, eggs are bought in dozen.
- Explosion is followed by destruction. Similarly, engagement is followed by marriage.
- A lack of identity is anonymity. Similarly, a lack of flaw is perfection.
- Sigh is a sign of relief. Similarly, trembling is a sign of fear.
- The first is the higher intensity of second.
- The first is the name given to a continuous chain of the second.
- Spring is followed by summer. Similarly, adolescence is followed by youth.
- The teeth are examined by a dentist. Likewise, eyes are examined by an oculist.
- Textiles are manufactured in a mill. Similarly, bricks are manufactured in kiln.

19. Shoes are made by a cobbler. Similarly, spectacles are designed by an optician.
20. Symphony is a type of music. Similarly, mural is a type of painting.
21. A numismatist collects coins. Similarly, a philatelist collects stamps.
22. A pattern is embroidered on a cloth and chased on a metal.
23. Curtain belongs to the class of drapery. Similarly, cockroach belongs to the class of insects.
24. Theft is confessed and fault is admitted.
25. Crown is a symbol of royalty. Similarly, crucifix is a mark of religion.
26. Drove is a group of cattle. Similarly, brood is a group of chickens.
27. Sheep graze in a meadow. Similarly, cattle graze in a pasture.
28. The words in each pair are antonyms of each other.
29. The words in each pair are antonyms of each other.
30. The second is the defining characteristic of the first.

EXERCISE 5

Directions: Each of the following questions consists of two words which have a certain relationship to each, followed by four pairs of words. Select the pair which has the same relationship as the original pair of words.

1. Magazine : Periodical
 - (a) Gun : Soldier
 - (b) Harvesting : Agriculture
 - (c) Pun : Joke
 - (d) Truck : Transport
2. Heart : Cardiology
 - (a) Brain : Psychology
 - (b) History : Histology
 - (c) Civics : Polity
 - (d) Fossils : Palaeontology
3. Doctor : Hospital
 - (a) Plumber : Wrench
 - (b) Chef : Kitchen
 - (c) Water : Reservoir
 - (d) Farmer : Village
4. Flag : Nation
 - (a) Emblem : Prosperity
 - (b) Insignia : Rank
 - (c) Wealth : Prestige
 - (d) Honour : Status
5. Coin : Mint
 - (a) Grain : Field
 - (b) Hay : Stable
 - (c) Wine : Brewery
 - (d) Book : Publisher
6. Infection : Illness
 - (a) Satisfaction : Appetite
 - (b) Applause : Audience
 - (c) Antidote : Disease
 - (d) Rehearsal : Performance
7. Sheep : Mutton
 - (a) Duck : Roast
 - (b) Hen : Poultry
 - (c) Deer : Venison
 - (d) Lamb : Veal
8. Yen : Currency
 - (a) Brass : Metal
 - (b) Flower : Fragrance
 - (c) Paper : Book
 - (d) Karnataka : State
9. Cricket : Football
 - (a) Solid : Liquid
 - (b) Hockey : Ball
 - (c) Prose : Poetry
 - (d) Shoe : Gloves

10. Geology : Earth	(b) Biology : Science
(a) Architect : Building	(d) Archaeology : Artifacts
(c) Aquarium : Fish	
11. Carpenter : Furniture	(b) King : Empire
(a) Teacher : Teaching	(d) Farmer : Agriculture
(c) Mason : Wall	
12. Traveller : Destination	(b) Accident : Hospital
(a) Beggar : Donation	(d) Refugee : Shelter
(c) Teacher : Education	
13. Horns : Bull	(b) Antlers : Stag
(a) Mane : Lion	(d) Wattles : Turkey
(c) Hoofs : Horse	
14. Taxonomist : Classify	(b) Doctor : Medicine
(a) Haggler : Bargain	(d) Engineer : Building
(c) Kind : Alms	
15. Horse : Equine	(b) Cat : Feline
(a) Lion : Carnivorous	(d) Dog : Vulpine
(c) Table : Furniture	
16. Wife : Marriage	(b) Nationality : Citizenship
(a) Bank : Money	(d) Attendance : Register
(c) Service : Qualification	
17. Sprain : Fracture	(b) Accident : Death
(a) Cool : Cold	(d) Fall : Slip
(c) Pneumonia : Fever	
18. Ampere : Current	(b) Speed : Time
(a) Sound : Waves	(d) Ohm : Resistance
(c) Distance : Kilometre	
19. Eyes : Tears	(b) Volcano : Lava
(a) Sea : Water	(d) Hunger : Bread
(c) Heart : Astery	
20. Bread : Flour	(b) Car : Engine
(a) Train : Wagon	(d) House : Wall
(c) Road : Asphalt	
21. Jupiter : Planet	(b) Yamuna : River
(a) Sparrow : Bird	(d) Everest : Peak
(c) Chilka : Lake	
22. Kitchen : Bedroom	(b) Vegetable : Fruit
(a) Woman : Man	(d) Gas : Cylinder
(c) Botany : Zoology	
23. Buoy : Channel	(b) Light house : Ship
(a) White line : High way	(d) Road map : Travel
(c) Red light : Street	

24. Knowledge : Ignorance
 (a) Cure : Health (b) Conceal : Hide
 (c) Breath : Suffocation (d) Construction : War
25. Dove : Peace
 (a) Crow : Scavenge (b) Knife : Cut
 (c) Lull : Storm (d) Pearl : Purity
26. Calligraphy : Writing
 (a) Music : Song (b) Lyric : Poem
 (c) Drama : Prose (d) Chapter : Stanza
27. Conciliatory : Friendliness
 (a) Cache : Hide (b) Garrulous : Old
 (c) Timid : Bold (d) Obvious : Explain
28. Wick : Candle
 (a) Lead : Pencil (b) Thread : Wool
 (c) Light : Darkness (d) Quick : Rapid
29. Gypsy : Caravan
 (a) Hare : Byre (b) Knight : Mansion
 (c) Monk : Temple (d) Convict : Cell
30. Plaintiff : Defendant
 (a) Judge : Jury (b) Court : Law
 (c) Attorney : Lawyer (d) Injured : Accused

ANSWERS

1. (c) 2. (d) 3. (b) 4. (b) 5. (c) 6. (d) 7. (c) 8. (d) 9. (c) 10. (d)
 11. (c) 12. (d) 13. (b) 14. (a) 15. (b) 16. (c) 17. (a) 18. (d) 19. (b) 20. (c)
 21. (d) 22. (c) 23. (a) 24. (c) 25. (d) 26. (b) 27. (a) 28. (a) 29. (b) 30. (d)

Explanations

- Magazine is a type of periodical. Likewise, pun is a type of joke.
- The study of heart is called cardiology. Similarly, the study of fossils is palaeontology.
- A doctor works in a hospital. Similarly, a chef works in a kitchen.
- Flag is the symbol of nation. Similarly, insignia is the symbol of rank.
- Coin is made in a mint. Similarly, wine is made in a brewery.
- Infection is followed by illness. Similarly, rehearsal is followed by performance.
- The flesh of sheep is called mutton. Similarly, the flesh of a deer is called venison.
- Yen is a currency. Likewise, Karnataka is a state.
- Both cricket and football belongs to the same class of sports. Similarly, both prose and poetry belong to the same class of literature.
- Geology is the study of earth. Likewise, archaeology is the study of artifacts.
- A carpenter makes furniture. Similarly, a mason builds a structure.
- A traveller seeks destination. Similarly, a refugee seeks shelter.
- A bull bears horns on its head. Likewise, a stag bears antlers on its head.
- A taxonomist classifies and a haggler bargains.

15. Equine is a horse like animal. Similarly, feline is a cat-like animal.
16. The second is necessary to acquire the first.
17. The second is more intensive form of the first.
18. Ampere is the unit of current. Similarly, Ohm is the unit of resistance.
19. Tears come out of the eye. Similarly, lava comes out of a volcano.
20. Bread is made of flour. Similarly, road is made of asphalt.
21. Jupiter is the largest planet. Similarly, Everest is the highest peak.
22. Both are parts of the same. Kitchen and bedroom are the parts of house. Similarly, botany and zoology are the branches of science.
23. A buoy indicates channel or a path for a ship to follow, white line shows the path on a highway to be followed by vehicles.
24. One is the opposite of the other.
25. Dove is a symbol of peace. Similarly, pearl is a symbol of purity.
26. Calligraphy is a type of writing. Similarly, lyric is a type of poem.
27. The words in both the pairs are synonyms of each other.
28. The uppermost working part of a candle is the wick. Likewise, the working part of a pencil is lead.
29. Gypsy stays in a caravan. Similarly, a knight stays in a mansion.
30. Injured is the plaintiff and the accused is the defendant.

EXERCISE 6

Directions: The following questions consist of two words each that have a certain relationship to each other followed by four lettered pairs of words. Select the lettered pair that has the same relationship as the original pair of words.

1. Read : Legible

(a) Hear : Audible	(b) Qualify : Eligible
(c) See : Illegible	(d) Require : Admissible
2. Locks : Warble

(a) Crows : Cackle	(b) Tiger : Yelp	(c) Owls : Hoot	(d) Camel : Bleat
3. Horse : Mare

(a) Duck : Geese	(b) Dog : Puppy	(c) Donkey : Pony	(d) Fox : Vixen
4. Water : Thirst

(a) Ripe : Harvest	(b) Book : Ignorance	(c) Needle : Stitch	(d) Rain : Drought
5. Iodine : Goitre

(a) Insuline : Diabetes	(b) Mango : Anaemia
(c) Hormones : Haemophilia	(d) Fat : Obesity
6. Waiter : Tip

(a) Student : Marks	(b) Worker : Bonus
(c) Employee : Wages	(d) Clerk : Bribe
7. Kangaroo : Australia

(a) Whale : River	(b) Elephant : Russia
(c) Penguin : Antartica	(d) India : Peacock
8. Hitler : Germany

(a) Shakespeare : England	(b) Mussolini : Italy
(c) Tulsidas : India	(d) Boris Yeltsin : Russia

9. Artist : Troupe
(a) Market : Crowd (b) Flowers : Garland (c) Singer : Chorus (d) Fishes : Pond
10. Pulp : Paper
(a) Rope : Hemp (b) Rayon : Cellulose (c) Thread : Needle (d) Yarn : Fabric
11. Hook : Fish
(a) Stadium : Games (b) Glove : Ball (c) Symphony : Music (d) Word : Alphabet
12. Graphite : Lubricant
(a) Movement : Friction (b) Iron : Steel
(c) Wool : Cloth (d) Diamond : Abrasive
13. Stage : Theatre
(a) Bedroom : House (b) Car : Road (c) Patient : Hospital (d) School : Education
14. Coffee : Beverages
(a) Bread : Butter (b) Milk : Tea (c) Burger : Snacks (d) Grapes : Wine
15. Cricket : Pitch
(a) Ship : Dock (b) Boat : Harbour (c) Wrestling : Track (d) Boxing : Ring
16. Monk : Monastery
(a) Noble : House (b) Lion : Hole (c) Nun : Convent (d) Peasant : Village
17. Lamb : Frisk
(a) Deer : Swoop (b) Cat : Steal (c) Bear : Leap (d) Lion : Stride
18. Laboratory : Germs
(a) School : Students (b) Playground : Games
(c) Library : Books (d) Observatory : Planets
19. Mongoose : Snake
(a) Milk : Goat (b) Fish : Crane (c) Whale : Crow (d) Water : Sky
20. Duralumin : Aircraft
(a) Brass : Alloy (b) Stone : Sculptor (c) Iron : Steel (d) Bronze : Statue
21. Termite : Wood
(a) Neem : Cotton (b) Fibre : Jute (c) Thread : Cloth (d) Moth : Wool
22. Introduction : Conclusion
(a) Salutation : Farewell (b) Deposit : Take
(c) Companion : Scoundrel (d) Mingle : Emerge
23. Plants : Coal
(a) Crops : Manure (b) Animals : Oil (c) Cow : Milk (d) Fire : Smoke
24. Lawn : Grass
(a) Wool : Sheep (b) Skin : Goat (c) Fur : Pelt (d) Rice : Farm
25. Cistern : Water
(a) House : Family (b) Cup : Tea (c) Vase : Water (d) Book : Knowledge
26. Preamble : Constitution
(a) Word : Dictionary (b) Contents : Magazine
(c) Explanation : Poetry (d) Preface : Book
27. Revolution : Change
(a) Disease : Medicine (b) Famous : Notorious
(c) Food : Energy (d) Treaty : Peace

28. Coronation : Reign
 (a) Vaccination : Immunity (b) Sculptor : Statue
 (c) Degree : Graduate (d) Summer : Rain
29. Leather : Milk
 (a) Cat : Mouse (b) Curd : Bacteria (c) Fruit : Rubber (d) Sun : Moon
30. Loud : Cape
 (a) House : Gate (b) Foot : Toe (c) Hand : Glove (d) Finger : Nails
31. Sailor : Pirate
 (a) Police : Robbers (b) Lion : Lamb (c) Plant : Fungus (d) Major : Sepoy
32. Pharaohs : Egypt
 (a) Socrates : Greece (b) Kings : India
 (c) Imperator : Rome (d) Government : State
33. Basement : Attic
 (a) Nadir : Zenith (b) Zenith : Apex (c) Zenith : Root (d) Apex : Pinnacle
34. Lok Sabha : Legislature
 (a) President : Executive (b) Minister : Meeting
 (c) Judge : Court (d) People : Election
35. Blister : Skin
 (a) Sore : Toe (b) Sty : Eye (c) Ball : Pitcher (d) Wound : Arm
36. Energy : Dissipate
 (a) Power : Generator (b) Food : Hunger
 (c) Money : Squander (d) Battery : Charge
37. Optimistic : Pessimistic
 (a) Difficult : Impossible (b) Study : Play
 (c) Tolerating : Disgusting (d) Export : Import
38. Acquire : Inherit
 (a) Profit : Loss (b) Learn : Discover
 (c) Instinct : Habit (d) Hierarchical : Succession
39. Protagonist : Character
 (a) Earth : Moon (b) Termite : Insect (c) Lizard : Housefly (d) Whale : Fish
40. Bird : Wings
 (a) Whale : Water (b) Dog : Lungs (c) Car : Wheel (d) Pen : Paper
41. War : Destruction
 (a) Fire : Burn (b) Court : Justice (c) Water : Drown (d) Food : Hunger
42. Elevation : Depression
 (a) Pen : Notebook (b) Intelligence : Stupidity
 (c) School : Education (d) Doctor : Medicine
43. Dawn : Morning
 (a) Morning : Evening (b) Sun : Rain
 (c) Lamp : Light (d) Autumn : Winter
44. Question : Question mark
 (a) Remark : Colon (b) Aside : Parentheses
 (c) Sentence : Period (d) Clause : Semicolon

45. Medicine : Capsule
 (a) Pearl : shell (b) Passenger : Bus (c) Heart : Lungs (d) Car : Vehicle
46. Liquor : Intoxicates
 (a) Oil : Smears (b) Medicine : Soothes
 (c) Drug : Addicts (d) Morphine : Sedates
47. Acoustics : Sound
 (a) Mathematics : Geometry (b) Radio : Song
 (c) Pathology : Disease (d) Communication : Phone
48. Large : Enormous
 (a) Big : Small (b) Plump : Fat (c) Less : Greater (d) Regal : Royal
49. Vaseline : Petrol
 (a) Tea : Leaves (b) Plant : Tree (c) Butter : Ghee (d) Cream : Milk
50. Jute : Sack
 (a) Shoe : Sock (b) Wool : Sweater (c) Cotton : Fibre (d) Mill : Cloth
51. Square : Cube
 (a) Triangle : Pyramid (b) Circle : Sphere
 (c) Line : Cylinder (d) Sphere : Earth
52. Coal : Thermal
 (a) Power : Energy (b) Bulb : Light (c) Air : Atmosphere (d) Water : Hydel
53. Carnivore : Herbivore
 (a) Animal : Bird (b) Flesh : Plant (c) Camel : Giraffe (d) Horse : Lion
54. Equator : Latitude
 (a) Visitor : Guest (b) Needle : Direction (c) Fish : Water (d) Pigeon : Bird
55. Calendar : Date
 (a) Time : Hour (b) Transport : Bus (c) Dictionary : Word (d) City : Pincode
56. Court : Justice
 (a) Police : Crime (b) Teacher : Study (c) Doctor : Sickness (d) Auditor : Accuracy
57. Fan : Sweat
 (a) Fire : Smoke (b) Rain : Drought
 (c) Wind : Evaporation (d) Crop : Harvest
58. Spider : Web
 (a) Ink : Pen (b) Cock : Hen (c) Teacher : Student (d) Poet : Poetry
59. Apostate : Religion
 (a) Potentate : Kingdom (b) Traitor : Country
 (c) Bureaucrat : Government (d) Jailor : Law
60. Ribs : Lungs
 (a) Heart : Artery (b) Fingers : Hands (c) Tongue : Mouth (d) Shell : Nut
61. Editor : Magazine
 (a) Director : Film (b) Novel : Writer
 (c) Psychiatrist : Neurotic (d) Librarian : Library
62. Hermit : Solitude
 (a) Warrior : Civility (b) Traitor : Loyalty
 (c) Ascetic : Self denial (d) Researcher : Finding

63. Taj Mahal : Agra
 (a) Eiffel Tower : Paris (b) Algevia : Africa
 (c) Cotton : Ahmedabad (d) Turkey : Asia
64. Jews : Synagogue
 (a) Parsis : Temple (b) Jains : Fire Temple
 (c) Buddhists : Pagoda (d) Hindus : Vedas
65. Script : Drama
 (a) Article : Magazine (b) Chapter : Book (c) Lyrics : Note (d) Score : Symphony

ANSWERS

1. (a)	2. (c)	3. (d)	4. (d)	5. (a)	6. (b)	7. (c)	8. (b)	9. (c)	10. (d)
11. (b)	12. (d)	13. (a)	14. (c)	15. (d)	16. (c)	17. (b)	18. (d)	19. (b)	20. (d)
21. (d)	22. (a)	23. (b)	24. (c)	25. (b)	26. (d)	27. (d)	28. (d)	29. (c)	30. (b)
31. (c)	32. (b)	33. (a)	34. (a)	35. (b)	36. (c)	37. (d)	38. (d)	39. (b)	40. (c)
41. (a)	42. (b)	43. (d)	44. (b)	45. (a)	46. (d)	47. (c)	48. (b)	49. (d)	50. (b)
51. (a)	52. (d)	53. (b)	54. (d)	55. (c)	56. (d)	57. (b)	58. (d)	59. (b)	60. (d)
61. (a)	62. (c)	63. (a)	64. (c)	65. (d)					

Explanations

- Legible means able to be read. Similarly, audible means able to be heard.
- The second is the sound produced by the first.
- The second is the female of the first.
- The lack of water is thirst. Similarly, the lack of rain is drought.
- The lack of first causes the second.
- The second is the additional money given to the first for good service.
- Kangaroo is the native of Australia. Similarly, penguin is the native of Antartica.
- Hitler was a tyrant belonging to Germany. Similarly, Mussolini was a tyrant belonging to Italy.
- A group of artists is called a troupe. Likewise, a group of singers is called chorus.
- Pulp is used to make paper. Likewise, yarn is used to make fabric.
- The first helps to catch the second.
- Graphite is used as a lubricant. Similarly, diamond is used as an abrasive.
- The first is a part of the second.
- The first one is specific and the second is the class to which it belongs.
- The game of cricket is played on a pitch. Similarly, the game of boxing is performed in a ring.
- The second is the dwelling place of the first.
- The second denotes the manner of walking of the first.
- A laboratory is a place to study germs. Similarly, observatory is a place of studying planets.
- The first is eaten by the second.
- The first is used to make the second.
- The first damages the second.
- The given words are opposite to each other.
- Plants on decay yield coal. Similarly, animals on decay produce oil.

24. The second grows on the first.
25. The first is used to hold the second.
26. Preamble is the introduction to the constitution and mentions its main ideals and objectives. Similarly, preface is the introduction to a book.
27. The first brings the second.
28. The first is followed by the second.
29. Leather and milk are both obtained from animals. Similarly, fruit and rubber are both obtained from trees.
30. Cape is the projection of land. Similarly, toe is the projection of foot.
31. Both belong to the same species but first one is useful and the second one is harmful.
32. The rulers of Egypt were known as pharaohs. Similarly, the rulers of India were known as kings.
33. Nadir (lowest point) and zenith (highest point) can be compared to basement and attic respectively.
34. Lok Sabha is a part of the Legislature. Likewise, President is a part of the Executive.
35. Blister is an infection of the skin. Similarly, sty is an infection of the eye.
36. 'Dissipate' and 'squander' are synonyms of 'waste'. Just as energy can be dissipated, money can be squandered.
37. The given words are opposite to each other.
38. Each pair consists of synonyms.
39. Protagonist is a character. Similarly, termite is an insect.
40. The second is used by the first for movement.
41. The first one causes the second.
42. The given words are opposites to each other.
43. Dawn is followed by morning. Similarly autumn is followed by winter.
44. The second is the symbol used at the end of first.
45. Medicine is contained in capsule. Similarly, pearl is contained in a shell.
46. The second is the effect of the first after consumption.
47. Acoustics is the science of sound. Similarly, pathology is the study of diseases.
48. Enormous is the extreme of large. Similarly, fat is the extreme of plump.
49. Vaseline is extracted from petrol. Similarly, cream is extracted from milk.
50. Sack is made up of jute. Similarly, sweater is made up of wool.
51. Each face of a cube is a square and each face of a pyramid is a triangle.
52. Coal produces thermal energy. Similarly, water produces hydel energy.
53. Flesh eating creatures are carnivore and plant eating creatures are herbivore.
54. The first one is specific and the second one is the class to which the first belongs. Equator is a line of latitude; pigeon is a bird.
55. Just as calendar consists of dates in an organised way; dictionary consists of words in an organised way.
56. The duty of court is to dispense justice. The duty of an auditor is to provide accuracy in financial matters.
57. The second is ceased due to the first.
58. As spider makes web, poet makes poetry.
59. The first rebels against the second.
60. Ribs are the bones, which protect the lungs. Similarly, shell protects the nut.

61. Just as all the articles of a magazine are interpreted by the editor, in the same way all the actions of film are interpreted by director.
62. A hermit leads a life of solitude. Likewise, an ascetic leads a life of self-denial.
63. The Taj Mahal is located in Agra. Similarly, Eiffel Tower is located in Paris.
64. The Jews worship in a synagogue. Likewise, the Buddhists worship in a pagoda.
65. Script is a written instruction of drama. Similarly, score is a written piece of symphony (type of music).

EXERCISE 7

1. A *sword* is related to *slaughter* in the same way *scalpel* is related to _____.
(a) murder (b) stab (c) surgery (d) chopping
2. *Ecology* is related to *environment* in the same way *histology* is related to _____.
(a) fossils (b) history (c) tissues (d) hormones
3. *Auger* is related to *carpenter* in the same way *awl* is related to _____.
(a) sculptor (b) cobbler (c) chef (d) mason
4. *Sirius* is related to *star* in the same way *cygnus* is related to _____.
(a) constellation (b) asteroid (c) galaxy (d) meteor
5. *Island* is related to *archipelago* in the same way *chapter* is related to _____.
(a) book (b) mystery (c) author (d) content
6. *Catalogue* is related to *library books* in the same way *index* is related to _____.
(a) chapters (b) books (c) preface (d) contents
7. *Bald* is related to *blond* in the same way *barren* is related to _____.
(a) vegetation (b) farm (c) fertile (d) inhabited
8. *Lion* is related to *prowl* in the same way *bear* is related to _____.
(a) frisk (b) lumber (c) stride (d) bound
9. *Aflatoxin* is related to *food poisoning* in the same way *histamine* is related to _____.
(a) allergy (b) headache (c) anthrax (d) contamination
10. *Ballworm* is related to *cotton* in the same way *ghundibug* is related to _____.
(a) wheat (b) rice (c) millet (d) tomato
11. *Naphthalene* is related to *woollen* in the same way *antibiotics* is related to _____.
(a) germs (b) immunity (c) diseases (d) body
12. *Drama* is related to *scene* in the same way *book* is related to _____.
(a) story (b) page (c) chapter (d) author
13. *Tungsten* is related to *filament* in the same way *bronze* is related to _____.
(a) copper (b) ships (c) tin (d) ornaments
14. *Tobacco* is related to *nerves* in the same way *alcohol* is related to _____.
(a) liver (b) liquor (c) intoxication (d) head
15. *Man* is related to *shout* in the same way *crow* is related to _____.
(a) caw (b) chirp (c) mutter (d) mob
16. *Gill* is related to *lamellae* in the same way *lung* is related to _____.
(a) ribs (b) trachea (c) alveoli (d) pharynx
17. *Dwell* is related to *demizen* in the same way *inherit* is related to _____.
(a) acquire (b) successor (c) outcast (d) heir

18. *Gardener* is related to *trowel* in the same way *seamstress* is related to _____.
(a) saw (b) scissors (c) sneakers (d) crowbar
19. *Wince* is related to *pain* in the same way *prostration* is related to _____.
(a) discomfiture (b) frustration (c) submissiveness (d) strained
20. *Solicitous* is related to *concern* in the same way *verbose* is related to _____.
(a) tiredness (b) wordiness (c) speech (d) deafness
21. *Physiology* is related to *biology* in the same way *metaphysics* is related to _____.
(a) Physics (b) Statistics (c) Mathematics (d) Philosophy
22. *Birds* are related to *aviary* in the same way *bees* are related to _____.
(a) aquarium (b) hive (c) brewery (d) apiary
23. *Anthropology* is related to *man* in the same way *Anthology* is related to _____.
(a) nature (b) trees (c) apes (d) poems
24. *Starvation* is related to *nutrition* in the same way *exhaustion* is related to _____.
(a) energy (b) bravery (c) freshness (d) courage
25. *Snowfall* is related to *precipitation* in the same way *grotto* is related to _____.
(a) throat (b) castle (c) cave (d) fort
26. *Condolence* is related to *loss* in the same way *congratulation* is related to _____.
(a) praise (b) achievement (c) accusation (d) reward
27. *Lumberjack* is related to *axe* in the same way *chef* is related to _____.
(a) bow (b) poker (c) chisel (d) colander
28. *Scrupulous* is related to *principles* in the same way *ethical* is related to _____.
(a) morals (b) virtues (c) religions (d) profits
29. *Annotate* is related to *text* in the same way *caption* is related to _____.
(a) novel (b) law (c) film (d) photograph
30. *Retirement* is related to *service* as *dismissal* is related to _____.
(a) agreement (b) communication (c) employment (d) adoption
31. *Betel* is related to *chew* in the same way *football* is related to _____.
(a) play (b) run (c) roll (d) kick
32. *Author* is related to *book* in the same way *choreographer* is related to _____.
(a) drama (b) ballet (c) masque (d) opera
33. *Book* is related to *magazine* in the same way *newspaper* is related to _____.
(a) journal (b) news (c) article (d) headline
34. *Indolence* is related to *work* in the same way *taciturn* is related to _____.
(a) observe (b) speak (c) cheat (d) act
35. *Brain* is related to *cranium* in the same way *pearl* is related to _____.
(a) box (b) oyster (c) sand (d) shore
36. *Wax* is related to *grease* in the same way *milk* is related to _____.
(a) drink (b) ghee (c) curd (d) protein
37. *Hooke* is related to *cells* in the same way *mulder* is related to _____.
(a) carbohydrates (b) minerals (c) vitamins (d) proteins
38. *Doctor* is related to *patient* in the same way *lawyer* is related to _____.
(a) customer (b) accused (c) magistrate (d) client

39. *Chef* is related to *restaurant* in the same way *druggist* is related to _____.
 (a) medicine (b) pharmacy (c) store (d) chemist
40. *Mirror* is related to *reflection* in the same way *water* is related to _____.
 (a) conduction (b) dispersion (c) immersion (d) refraction

ANSWERS

1. (c) 2. (c) 3. (b) 4. (a) 5. (a) 6. (d) 7. (c) 8. (b) 9. (a) 10. (b)
 11. (d) 12. (c) 13. (d) 14. (a) 15. (d) 16. (c) 17. (d) 18. (b) 19. (c) 20. (b)
 21. (d) 22. (d) 23. (d) 24. (a) 25. (c) 26. (b) 27. (d) 28. (a) 29. (d) 30. (c)
 31. (d) 32. (b) 33. (a) 34. (b) 35. (b) 36. (c) 37. (d) 38. (d) 39. (b) 40. (d)

Explanations

1. The second denotes the purpose for which the first is used.
2. Ecology deals with the study of environment. Similarly, histology deals with the study of tissues.
3. The first is the tool used by the second.
4. Sirius is a star and cygnus is a constellation.
5. The first is a part of the second.
6. Catalogue is an arranged list to find the names of the library books. Similarly, index is an arranged list of contents.
7. The given words are opposite to each other.
8. The second is the manner of walking of the first.
9. The first causes the second.
10. The first damages the second.
11. The first is used to protect the second from attack by germs and insects.
12. The second is a unit of the first.
13. The first is used to make the second. Thus tungsten is used to make filament and bronze is used to make ornaments.
14. The consumption of the first affects the second.
15. The second is the noisy sound made by the first.
16. The second is the oxygen absorbing part of the first.
17. When denizen dwells, he occupies the place. When heir inherits, he occupies the throne.
18. The second is the tool used by the first.
19. The first is the sign of the second.
20. A solicitous exhibits concern and a verbose exhibits wordiness.
21. Physiology is a branch of biology. Similarly, metaphysics is a branch of philosophy.
22. Birds are reared in an aviary. Similarly, bees are kept and reared in an apiary.
23. Anthropology deals with the study of man. Similarly, anthology deals with collection of poems.
24. A lack of nutrition is starvation. Similarly, a lack of energy is exhaustion.
25. Snowfall is a type of precipitation. Similarly, grotto is a type of cave.
26. Others offer condolence in a state of loss and congratulations when one make an achievement.
27. The second is the tool used by the first.

28. When one abides by the second, he is said to be the first by nature.
29. The first is a comment on the second.
30. The first interrupts the second.
31. The first is the object and the second is the action to be performed on it.
32. The first composes the second.
33. The second contains smaller articles of the same nature as the first.
34. The given words are antonyms to each other.
35. The first is enclosed by the second.
36. The first is used to prepare the second.
37. Hooke discovered the cells. Similarly, Mulder discovered the proteins.
38. The first works for the second.
39. The second is the working place for the first.
40. Light rays falling on mirror undergo reflection and those falling on water undergo refraction.

EXERCISE 8

1. *Trigonometry* is related to *triangles* in the same way *mensuration* is related to _____.
(a) geometry (b) circles (c) areas (d) polygons
2. *Lotus* is related to *cuticle* in the same way *fish* is related to _____.
(a) scales (b) gills (c) tail (d) wings
3. *Tapeworm* is related to *taeniasis* in the same way *plasmodium* is related to _____.
(a) malaria (b) constipation (c) diphtheria (d) diarrhoea
4. *Leaf* is related to *sap* in the same way *bone* is related to _____.
(a) fluid (b) blood (c) marrow (d) calcium
5. *Chlorophyll* is related to *chloroplast* in the same way *vulture* is related to _____.
(a) flesh (b) wings (c) air (d) bird
6. *Waves* are related to *air* in the same way *ripples* are related to _____.
(a) wind (b) water (c) tree (d) bud
7. *Rung* is related to *ladder* in the same way *twig* is related to _____.
(a) leaf (b) flower (c) tree (d) bud
8. *Circle* is related to *circumference* in the same way *square* is related to _____.
(a) area (b) volume (c) diagonal (d) perimeter
9. *Grain* is related to *granary* in the same way *curios* is related to _____.
(a) archives (b) museum (c) library (d) zoo
10. *Cat* is related to *kitten* in the same way *fish* is related to _____.
(a) fry (b) fawn (c) fin (d) foal
11. *Orthopaedist* is related to *bones* in the same way *chiroprapist* is related to _____.
(a) nails (b) sounds (c) feet (d) heart
12. *Formula* is related to *constituent* in the same way *equation* is related to _____.
(a) numbers (b) variables (c) term (d) constant
13. *Dog* is related to *kennel* in the same way *fowl* is related to _____.
(a) barn (b) cottage (c) nest (d) coop
14. *Honey* is related to *wax* in the same way *milk* is related to _____.
(a) cow (b) leather (c) eggs (d) butter

15. *Helm* is related to *rudder* in the same way *brain* is related to _____.
(a) heart (b) ribs (c) limbs (d) body
16. *Crumb* is related to *bread* in the same way *morsel* is related to _____.
(a) fruit (b) biscuit (c) food (d) cake
17. *Door* is related to *bang* in the same way *chain* is related to _____.
(a) thunder (b) clinch (c) tinkle (d) clank
18. *Hong Kong* is related to *China* in the same way *Vatican* is related to _____.
(a) Canada (b) Mexico (c) North America (d) Rome
19. *Horse* is related to *hay* in the same way *cow* is related to _____.
(a) leaves (b) fodder (c) milk (d) straw
20. *Earth* is related to *axis* in the same way *wheel* is related to _____.
(a) tyre (b) car (c) road (d) hub
21. *Gravity* is related to *pull* in the same way *magnetism* is related to _____.
(a) repulsion (b) separation (c) attraction (d) push
22. *Symphony* is related to *composer* in the same way *fresco* is related to _____.
(a) painter (b) inventor (c) singer (d) writer
23. *Bull* is related to *draught* in the same way as *cow* is related to _____.
(a) livestock (b) milch (c) farm (d) fodder
24. *Tooth* is related to *grit* in the same way *fist* is related to _____.
(a) blow (b) hand (c) open (d) clench
25. *Charminar* is related to *India* in the same way *Sphinx* is related to _____.
(a) England (b) Canada (c) Egypt (d) Vatican
26. *Labourer* is related to *wages* in the same way *entrepreneur* is related to _____.
(a) loan (b) interest (c) taxes (d) profit
27. *Land* is related to *cape* in the same way *water* is related to _____.
(a) strait (b) lagoon (c) bay (d) island
28. *Umbrella* is related to *rain* in the same way *goggles* are related to _____.
(a) light (b) glare (c) stare (d) sight
29. *Borrower* is related to *loan* in the same way *beggar* is related to _____.
(a) alms (b) mercy (c) money (d) gift
30. *Concert* is related to *theatre* in the same way as *banquet* is related to _____.
(a) hotel (b) party (c) feast (d) super
31. A *bird* is related to *cage* in the same way *man* is related to _____.
(a) house (b) field (c) room (d) prison
32. The *Green Revolution* is related to *plants* in the same way the *Silver Revolution* is related to _____.
(a) poultry (b) rubber (c) animals (d) forests
33. *Charcoal* is related to *wood* in the same way *coke* is related to _____.
(a) plastic (b) graphite (c) soot (d) coal
34. *Joule* is related to *energy* in the same way *Pascal* is related to _____.
(a) volume (b) pressure (c) density (d) purity
35. *Kindle* is related to *burn* in the same way *angry* is related to _____.
(a) annoyed (b) determined (c) resentful (d) furious

36. *Ostrich* is related to *antelope* in the same way *egret* is related to _____.
 (a) cow (b) buffalo (c) camel (d) zebra
37. *Blood* is related to *circulation* in the same way *hormone* is related to _____.
 (a) egestion (b) control (c) co-ordination (d) digestion
38. *Man* is related to *arms* in the same way *cockroach* is related to _____.
 (a) wings (b) pseudopodia (c) legs (d) antennae
39. *Transistor* is related to *radio* in the same way *television* is related to _____.
 (a) entertainment (b) cinema (c) video (d) cassette
40. *cobra* is related to *snake* in the same way *leopard* is related to _____.
 (a) tiger (b) lion (c) cat (d) zebra

ANSWERS

1. (c) 2. (a) 3. (a) 4. (c) 5. (d) 6. (b) 7. (c) 8. (d) 9. (b) 10. (a)
 11. (c) 12. (c) 13. (d) 14. (b) 15. (d) 16. (c) 17. (d) 18. (d) 19. (b) 20. (d)
 21. (c) 22. (a) 23. (b) 24. (d) 25. (c) 26. (d) 27. (c) 28. (b) 29. (b) 30. (a)
 31. (d) 32. (c) 33. (d) 34. (b) 35. (d) 36. (b) 37. (c) 38. (d) 39. (b) 40. (c)

Explanations

- Trigonometry is the study of triangles. Similarly, mensuration is the study of areas.
- The second is the means to protect the body of the first from water.
- The second is the disease caused by the first.
- The second is the fluid contained in the first.
- Chlorophyll is a type of chloroplast. Similarly, vulture is a type of bird.
- Waves travel in air; ripples travel in water.
- The first is a part of the second.
- The second is the measure of boundary of the first.
- Grain is stored in a granary. Similarly, curios (rare things to be collected) are kept in a museum.
- The second is the young one of the first.
- The first is a specialist of the second.
- The second is a unit of the first.
- The second is the living place of the first.
- Honey and wax are both obtained from the same organism i.e. bee. Similarly, milk and leather both are obtained from buffalo.
- Helm regulates the rudder and brain regulates the body.
- The first is a part of the second.
- The second is the sound made by the first.
- Hong Kong is a city of China. Similarly, Vatican is a city of Rome.
- The second is the food for the first.
- The first rotates about the second.
- The first draws things nearer through the second.
- The first is prepared by the second.
- The bull is a draught animal (beast of burden) and the cow is a milch animal (milk yielding).

24. The hold of tooth is called grit and the hold of the fist is called clench.
25. Charminar is situated in India. Similarly, Sphinx is a monument of Egypt.
26. The first earns in the form of the second.
27. The cape is the land projected into the water and the bay is the portion of water body projected into land.
28. The first provides protection from the second.
29. The first gets money in the form of the second.
30. The second is the place where the first is held.
31. The first is locked up in the second.
32. The first is the name given to increase in the production of the second.
33. The first is obtained from the second.
34. Joule is the unit of energy and Pascal is the unit of pressure.
35. The second is the larger intensity of the first.
36. Both live together to derive benefits from each other.
37. The second is the function of the first.
38. The first uses the second for the purpose of holding.
39. The second is the enlarged form of the first.
40. The first belongs to the family of the second.

EXERCISE 9

Directions: In each of the following questions, the first two words (given in *italics*) have a definite relationship. Choose one word out of the given four alternatives which will fill in the blank space and show the same relationship with the third word as between the first two.

1. *Hygrometer* is to *humidity* as *sphygmomanometer* is to _____.
 (a) pressure (b) blood pressure (c) precipitation (d) heart beat
2. *Steel* is to *Bokaro* as *hosiery* is to _____.
 (a) Chennai (b) Patna (c) Vishakhapatnam (d) Ludhiana
3. *Milk* is to *water* as *ghee* is to _____.
 (a) vanaspati (b) mustard oil (c) argemome (d) cream
4. *Insulin* is to *hormone* as *trypsin* is to _____.
 (a) juice (b) liver (c) enzyme (d) digestion
5. *Ploughing* is to *aeration* as *manuring* is to _____.
 (a) fertile (b) replenishment (c) earthing (d) agriculture
6. *Infestation* is to *food* as *infection* is to _____.
 (a) germs (b) diseases (c) body (d) microbes
7. *Book* is to *publisher* as *film* is to _____.
 (a) writer (b) editor (c) director (d) producer
8. *Bicycle* is to *pedal* as *boat* is to _____.
 (a) steering (b) water (c) oar (d) sail
9. *Latex* is to *rubber* as *flax* is to _____.
 (a) linen (b) wool (c) jute (d) cotton
10. *Cattle* is to *fodder* as *fish* is to _____.
 (a) hay (b) insects (c) feed (d) plankton

11. *Algae* is to *water* as *virus* is to _____.
 (a) man (b) host (c) surroundings (d) soil
12. *Insomnia* is to *lead* as *minamata* is to _____.
 (a) tobacco (b) mercury (c) alcohol (d) chromium
13. *Orange* is to *peel* as *tooth* is to _____.
 (a) gums (b) clove (c) enamel (d) joints
14. *Hear* is to *deaf* as *speak* is to _____.
 (a) quiet (b) silent (c) mumb (d) dumb
15. *Exercise* is to *obesity* as *water* is to _____.
 (a) thirst (b) alcohol (c) drink (d) purity
16. *Food* is to *fad* as *religion* is to _____.
 (a) crucification (b) notion (c) superstition (d) mythology
17. *Sulphur* is to *vulcanisation* as *chlorine* is to _____.
 (a) extraction (b) bleaching (c) metallurgy (d) allotropy
18. *Magnalium* is to *aluminium* as *brass* is to _____.
 (a) lead (b) magnetism (c) iron (d) copper
19. *Infrared* is to *heat* as *ultraviolet* is to _____.
 (a) cancer (b) blisters (c) mutation (d) ozone
20. *Zinc* is to *galvanisation* as *nickel* is to _____.
 (a) aircraft (b) corrosion (c) electroplating (d) filament
21. *Memory* is to *amnesia* as *movement* is to _____.
 (a) lubrication (b) lethargy (c) paralysis (d) hermit
22. *Liquid* is to *fluidity* as *comedian* is to _____.
 (a) ridicule (b) humour (c) solemnity (d) companion
23. *Kilometre* is to *distance* as *poundal* is to _____.
 (a) density (b) acceleration (c) momentum (d) force
24. *Truthfulness* is to *liar* as *loyalty* is to _____.
 (a) worker (b) traitor (c) diligent (d) faithful
25. *Preface* is to *book* as *overture* is to _____.
 (a) opera (b) ballad (c) novel (d) symphony
26. *Aluminium* is to *bauxite* as *iron* is to _____.
 (a) pyrite (b) magnesite (c) pyrolusite (d) haematite
27. *Tempest* is to *storm* as *slim* is to _____.
 (a) fat (b) plump (c) slender (d) beautiful
28. *Amorphousness* is to *definition* as *lassitude* is to _____.
 (a) energy (b) awareness (c) uniformity (d) companionship
29. *Tiff* is to *battle* as *frugal* is to _____.
 (a) sprint (b) vague (c) miserly (d) vital
30. *Exculpate* is to *acquit* as *precise* is to _____.
 (a) concise (b) conceal (c) brief (d) particular
31. *Burma* is to *Pagodas* as *Pakistan* is to _____.
 (a) Rivers (b) Canals (c) Agriculture (d) Dams
32. *Bhakra* is to *Sutlej* as *Aswan* is to _____.
 (a) Indus (b) Damodar (c) Volga (d) Nile

33. *Sparrow* is to *seed* as *silkworm* is to _____.
 (a) silk (b) maple (c) mulberry (d) pune
34. *Pineapple* is to *jelly* as *tomato* is to _____.
 (a) jam (b) pury (c) squash (d) pickles
35. *Aseel* is to *poultry* as *salmon* is to _____.
 (a) cow (b) camel (c) fish (d) horse

ANSWERS

1. (b) 2. (d) 3. (a) 4. (c) 5. (b) 6. (c) 7. (d) 8. (c) 9. (a) 10. (d)
 11. (b) 12. (b) 13. (c) 14. (d) 15. (a) 16. (c) 17. (b) 18. (d) 19. (a) 20. (c)
 21. (c) 22. (b) 23. (d) 24. (b) 25. (a) 26. (d) 27. (c) 28. (a) 29. (c) 30. (d)
 31. (b) 32. (d) 33. (c) 34. (b) 35. (c)

Explanations

1. The first is an instrument used to measure the second.
2. Bokaro is famous for steel industry and Ludhiana is famous for hosiery works.
3. The first is adulterated by using the second.
4. The second denotes the class to which the first belongs. Thus, insulin is a hormone and *trypsin* is an enzyme.
5. Ploughing is done for the aeration of soil and manuring is done for the replenishment of soil.
6. The contamination of food by germs is called infestation. Similarly, attack on body by germs is called infection.
7. The production of the first is done by the second.
8. The second is the tool which is acted upon to move the first.
9. The first is the raw material used to obtain the second.
10. The second is the food eaten by the first.
11. The second is the dwelling place for the first.
12. The poisoning by the second causes the first.
13. The second is the protective covering over the first.
14. One who cannot hear is deaf. Similarly, one who cannot speak is dumb.
15. The first eliminates the second.
16. The second is the name given to wrong notions about the first.
17. Sulphur is used for vulcanisation of rubber. Likewise, chlorine is used for bleaching.
18. Magnalium is an ore of aluminium and brass is an ore of copper.
19. The second is the effect produced by the first.
20. The second is the purpose for which first is used.
21. The lack of memory is amnesia and lack of movement is paralysis.
22. The second is the defining characteristic of the first.
23. Kilometre is a unit of distance and poundal is a unit of force.
24. The lack of first is the defining characteristic of the second.
25. The first is an opening comment on the second.
26. The second is the ore used for extraction of the first.

- 27. The first is of the higher intensity than the second.
- 28. The given words are opposite to each other.
- 29. The second is of higher intensity than the first.
- 30. The given words are synonyms of each other.
- 31. Burma is famous for pagodas and Pakistan is famous for canals.
- 32. Bhakra is a dam situated on the Sutlej river. Similarly, Aswan is a dam situated on Nile river.
- 33. The first feeds on the second.
- 34. The first is preserved in the form of the second.
- 35. Aseel is a breed of poultry and salmon is a breed of fish.

Classification

Classification means 'to assort the items of a given group on the basis of a certain common quality they possess and then spot the stranger out'.

In this test, a group of five items will be given, out of which four are similar to one another in the same manner and one is different. You have to find out this item which does not fit into the group.

Directions: Choose the word which is least like the other words in the group.

EXAMPLE 1 (a) Copper (b) Tin (c) Brass (d) Platinum (e) Zinc

Solution Here, all except brass are metal while brass is an alloy. Hence the answer is (c).

EXAMPLE 2 (a) Calf (b) Cub (c) Piglet (d) Duckling (e) Hireling

Solution Here, all except Hireling, are young ones of animals. Hence the answer is (e).

EXAMPLE 3 (a) Curd (b) Butter (c) Oil (d) Cheese (e) Cream

Solution Here, all except, oil are products obtained from milk. Hence the answer is (c).

EXAMPLE 4 (a) Magnalium (b) Germanium (c) Duralumin
(d) Bronze (e) Brass

Solution All, except germanium, are alloys, while germanium is a metal. So, the answer is (b).

EXAMPLE 5 (a) Garnet (b) Ruby (c) Graphite (d) Emerald (e) Topaz

Solution All, except graphite, are precious stones. So the answer is (c).

EXERCISES

Directions: In each of the following questions, five words are given, out of which four are alike in some manner and one is different. Choose the odd one out.

- (a) Nephrology (b) Entomology (c) Astrology (d) Mycology (e) Pathology
- (a) Whale (b) Dolphin (c) Shark (d) Cod (e) Starfish
- (a) Indigo (b) Orange (c) Yellow (d) Pink (e) Green

4. (a) Tarapur	(b) Kota	(c) Kalpakkam	(d) Paradeep	(e) Narora
5. (a) Brick	(b) Heart	(c) Diamond	(d) Spade	(e) Club
6. (a) Cataract	(b) Hypermetropia	(c) Trachoma	(d) Eczema	(e) Glaucoma
7. (a) Radium	(b) Thorium	(c) Sodium	(d) Polonium	(e) Uranium
8. (a) Bishop	(b) Knight	(c) Pawn	(d) Rook	(e) Jockey
9. (a) Mongolia	(b) China	(c) Burma	(d) Afghanistan	(e) Bangladesh
10. (a) Wise	(b) Gentle	(c) Honest	(d) Rude	(e) Arrogance
11. (a) Pituitary	(b) Pancreas	(c) Thalamus	(d) Adrenal	(e) Testis
12. (a) Nun	(b) Knight	(c) Monk	(d) Priest	(e) Padre
13. (a) Granite	(b) Lignite	(c) Peat	(d) Anthracite	(e) Bituminous
14. (a) Gasoline	(b) Methane	(c) Asphalt	(d) Paraffin wax	(e) Diesel
15. (a) Galileo	(b) Copernicus	(c) Columbus	(d) Bhaskara	(e) Aryabhatta
16. (a) Cricket	(b) Baseball	(c) Football	(d) Billiards	(e) Badminton
17. (a) Rigveda	(b) Yajurveda	(c) Atharvaveda	(d) Ayurveda	(e) Samaveda
18. (a) Producer	(b) Director	(c) Investor	(d) Financier	(e) Entrepreneur
19. (a) Flute	(b) Guitar	(c) Sitar	(d) Violin	(e) Veena
20. (a) Kiwi	(b) Eagle	(c) Emu	(d) Penguin	(e) Ostrich
21. (a) Tortoise	(b) Snail	(c) Turtle	(d) Spider	(e) Oyster
22. (a) Epicentre	(b) Seismology	(c) Focus	(d) Crater	(e) Richter Scale
23. (a) Arrow	(b) Missile	(c) Sword	(d) Bullet	(e) Spear
24. (a) Nerves	(b) Auricle	(c) Artery	(d) Valve	(e) Aorta
25. (a) Konark	(b) Madurai	(c) Ellora	(d) Khajuraho	(e) Dilwara
26. (a) Akbar	(b) Jahangir	(c) Shah Jahan	(d) Vikramaditya	(e) Aurangzeb
27. (a) Manipur	(b) Sikkim	(c) Maharashtra	(d) Haryana	(e) Lakshadweep
28. (a) Sial	(b) Mantle	(c) Core	(d) Sima	(e) Pengia
29. (a) Turtle	(b) Lamb	(c) Colt	(d) Bitch	(e) Farrow
30. (a) Mettur	(b) Aswan	(c) Hirakund	(d) Sutlej	(e) Pong

ANSWERS

1. (c)	2. (a)	3. (d)	4. (d)	5. (a)	6. (d)	7. (c)	8. (e)	9. (a)	10. (e)
11. (c)	12. (b)	13. (a)	14. (b)	15. (c)	16. (d)	17. (d)	18. (b)	19. (a)	20. (b)
21. (d)	22. (d)	23. (c)	24. (a)	25. (c)	26. (d)	27. (e)	28. (e)	29. (d)	30. (d)

Explanations

- All except astrology are concerned with biology.
- All except whale belong to the family of fishes, while whale is a mammal.
- All except pink are colours seen in a rainbow.
- All except Paradeep are atomic power stations, whereas Paradeep is a port.
- All except brick are suits of cards.
- All except eczema are eye infections, whereas eczema is a skin infection.
- All except sodium are radio isotopes, whereas sodium is a metal.
- All except jockey are chessmen, whereas jockey is a professional horse rider.

9. All except Mongolia are neighbouring countries of India.
10. All except arrogance are adjectives, whereas arrogance is a noun.
11. All except thalamus are hormone secreting glands.
12. All except knight are religious persons, whereas knight is a warrior.
13. All except granite are different types of coal, whereas granite is a rock.
14. All except methane are products obtained from petroleum.
15. All except Columbus were astronomers, whereas Columbus was an explorer.
16. All except billiards are outdoor games.
17. All except Ayurveda are names of holy scriptures, the four vedas, Ayurveda is a branch of medicine.
18. All except director spend money.
19. All except flute are string instruments.
20. All except eagle are flightless birds.
21. All except spider have hard protective sheels.
22. All except crater are terms associated with the earthquake.
23. All except sword strike the target at a distance.
24. All except nerves are parts of the heart.
25. All except Ellora are famous for temples whereas Ellora is famous for caves.
26. All except Vikramaditya were Mughal rulers.
27. All except Lakshadweep are states of India, whereas Lakshadweep is a Union Territory.
28. All except pengia are layers of earth.
29. All except bitch are young ones of animals, whereas bitch is a female dog.
30. All except Sutlej are dams, whereas Sutlej is a river.

Series Completion

This chapter deals with the questions in which a series of numbers or alphabetical letters is given, which are called *terms of the series*. These terms follow a certain pattern. You have to recognize this pattern and either complete the given series with the most suitable alternative or find the wrong term in the series.

TYPE 1: NUMBER SERIES

Case I: Completing the Given Series

EXAMPLE 1 Tick the number that will come next in the following sequence:

4, 6, 12, 14, 28, 30, (...)

(a) 32

(b) 64

(c) 62

(d) 60

Solution The given sequence is a combination of two series 4, 12, 28, (...) and 6, 14, 30. The number pattern followed in the first series is +8, +16, +32, ...

So, the missing number = $28 + 32 = 60$.

Hence, the answer is (d).

EXAMPLE 2 Complete the series: 4, -8, 16, -32, 64, (...)

(a) 128

(b) -128

(c) 192

(d) -192

Solution Each number in the series is got by multiplying the preceding number by -2.
 $\therefore$ Missing term = $64 \times (-2) = -128$

Hence, the answer is (b).

EXAMPLE 3 Which number would replace the question mark in the series 2, 7, 14, 23, ?, 47.

(a) 28

(b) 34

(c) 31

(d) 38

Solution The given sequence is +5, +7, +9, ... i.e. $2 + 5 = 7$; $7 + 7 = 14$; $14 + 9 = 23$.
 $\therefore$ Missing number = $23 + 11 = 34$.

Hence, the answer is (b).

EXERCISE 1

Directions: In each of the following questions, a number series is given with one term missing. Choose the correct alternative that will continue the same pattern and fill in the blank spaces.

1. 2, 6, 11, 17, (...), 32			[CBI Exam]
(a) 22	(b) 23	(c) 24	(d) 28
2. 3, 10, 20, 33, 49, 68, (...)			[Hotel Management]
(a) 75	(b) 85	(c) 90	(d) 91
3. 3, 7, 15, 31, 63, (...)			
(a) 92	(b) 127	(c) 115	(d) 131
4. 6, 24, 12, (...), 18, 8, 24			[Railways]
(a) 4	(b) 8	(c) 16	(d) 6
5. 212, 179, 146, 113, (...)			[Railways]
(a) 91	(b) 79	(c) 112	(d) 80
6. 2, 6, 3, 4, 20, 5, 6, (...), 7			[Railways]
(a) 25	(b) 30	(c) 42	(d) 28
7. 2, 5, 9, 19, 37, (...)			[Railways]
(a) 76	(b) 75	(c) 74	(d) 72
8. 2, 6, 12, 20, 30, 42, 56, (...)			[Railways]
(a) 60	(b) 64	(c) 70	(d) 72
9. 8, 24, 12, 36, 18, 54, (...)			
(a) 27	(b) 68	(c) 72	(d) 108
10. 165, 195, 255, 285, 345, (...)			
(a) 375	(b) 420	(c) 435	(d) 390
11. 71, 76, 69, 74, 67, 72, (...)			
(a) 65	(b) 76	(c) 77	(d) 80
12. 9, 12, 11, 14, 13, (...), 15			
(a) 12	(b) 16	(c) 10	(d) 17
13. 3, 15, 35, (...), 99, 143			
(a) 48	(b) 63	(c) 80	(d) 95
14. 3, 10, 20, 33, 49, 68, (...)			
(a) 75	(b) 85	(c) 90	(d) 91
15. 1, 3, 4, 8, 15, 27, (...)			
(a) 37	(b) 44	(c) 50	(d) 55
16. 66, 36, 18, (...)			
(a) 9	(b) 3	(c) 6	(d) 8
17. 11, 13, 17, 19, 23, 25, (...)			
(a) 25	(b) 27	(c) 29	(d) 31
18. 2, 4, 7, 11, 16, (...)			
(a) 18	(b) 20	(c) 22	(d) 25
19. 0, 2, 6, (...), 20, 30, 42			
(a) 8	(b) 10	(c) 12	(d) 14

20. 5, 16, 13, 26, 29, 58, 61, (...)
 (a) 122 (b) 125 (c) 128 (d) 64
21. 2, 9, 28, 65, 126, (...)
 (a) 137 (b) 223 (c) 217 (d) 199
22. 4, 9, 13, 22, 35, (...)
 (a) 57 (b) 70 (c) 63 (d) 75
23. 1, 8, 27, 64, 125, 216, (...)
 (a) 354 (b) 343 (c) 392 (d) 245
24. 1, 2, 3, 6, 9, 18, (...), 54
 (a) 18 (b) 36 (c) 81 (d) 27
25. 11, 13, 17, 19, 23, 29, 31, 37, 41, (...)
 (a) 43 (b) 47 (c) 51 (d) 53
26. 2, 5, 11, 23, 47, (...)
 (a) 49 (b) 52 (c) 95 (d) 106
27. 4, 9, 5, 12, 7, 15, 8, (...), 10
 (a) 25 (b) 18 (c) 21 (d) 24
28. 10, 5, 13, 10, 16, 20, 19, (...)
 (a) 22 (b) 23 (c) 38 (d) 40
29. 2, 6, 12, 20, 30, 42, 56, (...)
 (a) 60 (b) 64 (c) 70 (d) 72
30. 2, 3, 5, 7, 11, (...), 17
 (a) 12 (b) 13 (c) 14 (d) 15

ANSWERS

1. (c) 2. (c) 3. (b) 4. (c) 5. (d) 6. (c) 7. (b) 8. (d) 9. (a) 10. (c)
 11. (a) 12. (b) 13. (b) 14. (c) 15. (c) 16. (d) 17. (c) 18. (c) 19. (c) 20. (a)
 21. (c) 22. (a) 23. (b) 24. (d) 25. (a) 26. (c) 27. (b) 28. (d) 29. (d) 30. (b)

Explanations

- The sequence is +4, +5, +6 ...
 $\therefore$ Missing term = $17 + 7 = 24$.
- The sequence is +7, +10, +13, +16, +19 ...
 $\therefore$ Missing number = $68 + 22 = 90$
- Each number in the series is the preceding number multiplied by 2 and then increased by 1.
 Thus $(3 \times 2) + 1 = 7$; $(7 \times 2) + 1 = 15$; $(15 \times 2) + 1 = 31$ and so on.
- The given sequence is a combination of two series 6, 12, 18, 24, and 24, (...), 8.
 The first series consists of consecutive multiples of 6 and the second series consists of multiples of 8. Thus, the missing number is a multiple of 8 which lies between 8 and 24, which is 16.
- 33 is subtracted from each term of the series to obtain the next term.
 $\therefore$ Missing term = $113 - 33 = 80$
- The arrangement of series is as follows: $2 \times 3 = 6$; $4 \times 5 = 20$, and so on.
 Here from each three consecutive term, middle term = product of two end terms. So the missing number = $6 \times 7 = 42$.

7. Here, the numbers in the even position are equal to the numbers in the odd position multiplied by 2 and added by 1. The numbers in the odd positions are got by multiplying numbers in the even position by 2 and subtracting 1 from it. So the answer is $37 \times 2 + 1 = 75$.
8. The sequence is $1 \times 2, 2 \times 3, 3 \times 4, 4 \times 5, 5 \times 6, 6 \times 7, 7 \times 8$.
 $\therefore$ Next number = $8 \times 9 = 72$.
9. This series is got by alternatively multiplying by 3 and dividing by 2.
 So, $8 \times 3 = 24$; $24 \div 2 = 12$, $12 \times 3 = 36$; $36 \div 2 = 18$ and so on.
 $\therefore$ The missing term = $54 \div 2 = 27$.
10. This series is a multiple of consecutive prime numbers.
 i.e. $15 \times 11, 15 \times 13, 15 \times 17, 15 \times 19, 15 \times 23$.
 $\therefore$ The missing term = $15 \times 29 = 435$.
11. Here the series is got by addition of 5 and subtraction of 7 to terms alternately.
 Here, $71 + 5 = 76$; $76 - 7 = 69$; $69 + 5 = 74$; $74 - 7 = 67$ and so on.
 So the missing term = $72 - 7 = 65$.
12. Alternately, add 3 and subtract = 1.
 So $9 + 3 = 12$; $12 - 1 = 11$; $11 + 3 = 14$; $14 - 1 = 13$.
 So the missing term = $13 + 3 = 16$.
13. The terms of the series are $2^2 - 1, 4^2 - 1, 6^2 - 1, \dots, 10^2 - 1, 12^2 - 1$.
 So, the missing number = $8^2 - 1 = 64 - 1 = 63$.
14. The sequence is $+7 + 10, +13, +16, +19, \dots$
 So, the missing number = $68 + 22 = 90$.
15. The sum of any three consecutive terms of the series gives the next term.
 So, $1 + 3 + 4 = 8$; $3 + 4 + 8 = 15$; $4 + 8 + 15 = 27$ and so on.
 $\therefore$ The missing number = $8 + 15 + 27 = 50$.
16. Each number in the series is the product of the digits of the preceding number.
 i.e. $6 \times 6 = 36$; $3 \times 6 = 18$ and so on.
 $\therefore$ The missing number = $1 \times 8 = 8$.
17. The sequence is $+2, +4, +2, +4, +2, \dots$
 $\therefore$ The missing number = $25 + 4 = 29$.
18. The difference between consecutive numbers increases by 1.
 Thus, the sequence is $+2, +3, +4, +5 \dots$
 $\therefore$ The missing number = $16 + 6 = 22$.
19. The sequence is $+2, +4, \dots, \dots, +10, +12$
 So, the missing number = $6 + 6 = 12$.
20. The numbers are alternately multiplied by 2 and increased by 3.
 So, $5 \times 2 = 10$; $10 + 3 = 13$; $13 \times 2 = 26$; $26 + 3 = 29$ and so on.
 $\therefore$ The missing number = $61 \times 2 = 122$.
21. The sequence is $1^3 + 1, 2^3 + 1, 3^3 + 1, 4^3 + 1, 5^3 + 1, \dots$
 $\therefore$ The missing number = $6^3 + 1 = 216 + 1 = 217$.
22. The sum of two consecutive numbers of the series gives the next number.
 Thus, $4 + 9 = 13$; $9 + 13 = 22$; $13 + 22 = 35$ and so on.
 $\therefore$ The missing number = $22 + 35 = 57$.
23. The numbers are $1^3, 2^3, 3^3, 4^3, 5^3, 6^3$ and so on.
 $\therefore$ The missing number = $7^3 = 343$.

24. The numbers are alternately multiplied by 2 and $\frac{3}{2}$.

Thus, $1 \times 2 = 2$; $2 \times \frac{3}{2} = 3$; $3 \times 2 = 6$; $6 \times \frac{3}{2} = 9$; and so on.

$\therefore$ The missing number = $18 \times \frac{3}{2} = 27$.

25. The series consists of prime numbers.

$\therefore$ The missing number is the next prime number, which is 43.

26. The sequence is +3, +6, +12, +24 ...

$\therefore$ the missing number = $47 + 48 = 95$.

27. The given sequence is a combination of two series 4, 5, 7, 8, 10 and 9, 12, 15, (...)

$\therefore$ The missing term = $15 + 3 = 18$.

28. The given sequence consists of two series: 10, 13, 16, 19 and 5, 10, 20, (...)

$\therefore$ The missing number = $20 \times 2 = 40$.

29. The sequence is 1×2 , 2×3 , 3×4 , 4×5 , 5×6 , 6×7 , 7×8 .

$\therefore$ The missing term = $8 \times 9 = 72$.

30. The given series consists of prime numbers starting from 2. The prime number after 11 is 13. So, 13 is the missing number.

Case II: Finding the Wrong Term in the Given Series

EXAMPLE 4 Find the wrong number in the series.

3, 8, 15, 24, 34, 48, 63

- (a) 15 (b) 24 (c) 34 (d) 48 (e) 63

Solution The difference between consecutive terms of the given series are respectively 5, 7, 9, 11 and 13.

So 34 is the wrong number.

Answer is (c).

EXAMPLE 5 Find the wrong number in the series.

10, 26, 74, 218, 654, 1946, 5834

- (a) 26 (b) 74 (c) 218 (d) 654 (e) 1946

Solution Each term is obtained by multiplying the preceding term by 3 and then subtracting 4 from it.

So, $26 = (10 \times 3) - 4$; $74 = (26 \times 3) - 4$; $218 = (74 \times 3) - 4$ and so on.

So the wrong number is 654.

Answer is (d).

EXERCISE 2

Directions: In each of the following questions, one term in the number series is wrong. Find out the wrong term.

1. 8, 14, 26, 48, 98, 194, 386

- (a) 194 (b) 98 (c) 14 (d) 48 (e) 386

2. 8, 13, 21, 32, 47, 63, 83

- (a) 21 (b) 13 (c) 32 (d) 83 (e) 47

[Bank PO]

3. 3, 7, 15, 39, 63, 127, 255, 511
 (a) 39 (b) 15 (c) 7 (d) 63 (e) 127
4. 445, 221, 109, 46, 25, 11, 4
 (a) 221 (b) 109 (c) 46 (d) 25 (e) 11
5. 1, 2, 6, 15, 31, 56, 91
 (a) 31 (b) 15 (c) 56 (d) 91 (e) 2
6. 2, 5, 10, 17, 26, 37, 50, 64
 (a) 50 (b) 17 (c) 26 (d) 37 (e) 64
7. 46080, 3840, 384, 48, 24, 2, 1
 (a) 1 (b) 2 (c) 24 (d) 48 (e) 384
8. 52, 51, 48, 43, 34, 27, 16
 (a) 51 (b) 48 (c) 34 (d) 27 (e) 16
9. 325, 259, 202, 160, 127, 105, 94
 (a) 94 (b) 127 (c) 105 (d) 202 (e) 259
10. 125, 126, 124, 127, 123, 129
 (a) 123 (b) 124 (c) 126 (d) 127 (e) 129
11. 3, 4, 10, 32, 136, 685, 4116
 (a) 136 (b) 10 (c) 4116 (d) 685 (e) 32
12. 25, 36, 49, 81, 121, 169, 225
 (a) 36 (b) 49 (c) 121 (d) 169 (e) 225
13. 56, 72, 90, 110, 132, 150
 (a) 72 (b) 90 (c) 110 (d) 132 (e) 150
14. 6, 13, 18, 25, 30, 37, 40
 (a) 25 (b) 30 (c) 37 (d) 40 (e) 13
15. 10, 14, 28, 32, 64, 68, 132
 (a) 32 (b) 68 (c) 64 (d) 132 (e) 28

ANSWERS

1. (d) 2. (e) 3. (a) 4. (c) 5. (d) 6. (e) 7. (c) 8. (c) 9. (d) 10. (e)
 11. (e) 12. (a) 13. (e) 14. (d) 15. (d)

Explanations

- Each term in the series is less than twice the preceding term by 2.
 So, 48 is wrong, correct term = $(26)2 - 2 = 52 - 2 = 50$.
- The sequence is +5, +8, +11, +14, and so on. Here 47 is wrong. Correct term = $32 + 14 = 46$.
- Each number in the series is multiplied by 2 and the result increased by 1 to obtain the next number.
 $\therefore$ 39 is wrong. The correct term = $(15 \times 2) + 1 = 31$.
- Each number is obtained by dividing the preceding number by 2 after subtracting 3 from it.
 i.e. $221 = \left(\frac{445 - 3}{2}\right)$; $109 = \left(\frac{221 - 3}{2}\right)$ and so on.
 $\therefore$ 46 is wrong. The correct term is $\left(\frac{109 - 3}{2}\right) = 53$.

5. The sequence is $+1^2, +2^2, +3^2 + 4^2, +5^2, +6^2$
 $\therefore 91$ is wrong. The correct number is $56 + 6^2 = 56 + 36 = 92$.
6. The numbers are $1^2 + 1, 2^2 + 1, 3^2 + 1$ and so on.
 $\therefore 64$ is wrong. The correct term is $8^2 + 1 = 65$.
7. The terms are successively divided by 12, 10, 8, 6 etc. So, 24 is wrong. Correct term = $48/6 = 8$.
8. The sequence is $-1, -3, -5, -7, -9, -11$, etc.
 So, 34 is wrong. Correct term = $43 - 7 = 36$.
9. The sequence is $-66, -55, -44, -33, -22, -11$.
 So, 202 is wrong. Correct term = $259 - 55 = 204$.
10. Sequence is $+1, -2, +3, -4, +5$.
 So, 129 is wrong.
 Correct term = $123 + 5 = 128$.
11. Sequence is obtained by adding 1 to preceding term and then multiplying successively by 1, 2, 3, and so on.
 So, 32 is wrong. Correct term = $(10 + 1)3 = 33$.
12. The correct sequence is a set of squares of consecutive odd numbers.
 i.e. $5^2, 7^2, 9^2, 11^2, 13^2, 15^2$.
 So, 36 is wrong in the series.
13. The sequence is $+16, +18, +20, +22, +24$.
 So, 150 is wrong. Correct term = $132 + 24 = 156$.
14. This is an alternate series.
 i.e. $+7, +5, +7, +5$ and so on.
 So, 40 is wrong. Correct term = $37 + 5 = 42$.
15. Sequence is as follows:
 2nd term = 1st term + 4 = $10 + 4 = 14$;
 3rd term = 2nd term $\times 2 = 14 \times 2 = 28$;
 4th term = 3rd term + 4 = $28 + 4 = 32$;
 5th term = 4th term $\times 2 = 32 \times 2 = 64$ and so on.
 So, 132 is wrong. Correct term = $68 \times 2 = 136$.

TYPE 2: ALPHABET SERIES

EXAMPLE 6 What will be the next term in

BKS, DJT, FIU, HHV, (...)

- (a) IJX (b) IGX (c) JGW (d) IGU (e) JGU

Solution In each term, the first letter is moved forward by 2 letters the second letter backward by 1 letter and the third letter moved forward by 1 letter. So the missing term is JGW. So the answer is (c).

EXAMPLE 7 Choose the alternative that will complete the series:

aku, fpz, kue, (...), ueo, zjt

- (a) pzj (b) jtd (c) jue (d) kve (e) ukv

Solution The letters in each term in all the positions are moved forward by 5. So the missing term = pzj. So the answer is (a).

EXAMPLE 8 What terms will fill in the blank spaces?

Y W U S Q (...) (...)

- (a) N, J (b) M, L (c) J, R (d) L, M (e) O, M

Solution The given series consists of alphabet in the reverse order moved by 2 in each step. So the missing terms are O and M.

The answer is (e).

EXERCISE 3

Directions: In each of the following questions, various terms of a letter series are given with one term missing as shown by (?). Choose the missing term out of the given alternative.

- AZ, BY, CX, ? [Bank PO]
(a) EF (b) GH (c) IJ (d) DE (e) DW
- DEF HIJ MNO ? [Bank PO]
(a) STU (b) RST (c) RTV (d) SRQ (e) TUV
- BXJ ETL HPN KLP ? [SBI PO]
(a) NHR (b) MHQ (c) MIP (d) NIR (e) None of these
- AB DEF HIJK ? STUVWX [Bank PO]
(a) MNOPQ (b) LMNOP (c) LMNO (d) QRSTU (e) None of these
- A Z X B V T C R (?, ?) [RRB]
(a) P, D (b) E, O (c) Q, E (d) O, Q (e) Q, O
- G, H, J, M, (?, V) [Bank PO]
(a) T (b) S (c) R (d) U (e) Q
- P3C, R5F, T8I, V12L, ? [SBI PO]
(a) Y17O (b) X17M (c) X17O (d) X16O (e) None of these
- AYD BVF DRH ? KGL [AAO Exam]
(a) FMI (b) GMJ (c) HLK (d) GLJ (e) None of these
- CX FU IR ? OL RI
(a) LO (b) MN (c) NO (d) OP (e) OR
- DEB IJG NOL ? XYV
(a) RSP (b) STP (c) RSQ (d) STQ (e) STO
- A CD GHI ? UVWXY
(a) LMNO (b) MNO (c) NOPQ (d) NOP (e) MNOP
- EJOT DHL P CFIL ?
(a) BDFH (b) BHLH (c) DEIJ (d) DGKL (e) HFDB
- OTE PUF QVG RWH ?
(a) SYJ (b) TXI (c) SXJ (d) SXI (e) TYJ
- DCXW FEVU HGTS ?
(a) LKPO (b) ABYZ (c) JIRQ (d) LMRS (e) JRIQ
- R, M, (?, F, D, ?)
(a) C, B (b) J, H (c) B, H (d) H, C (e) I, C
- C, B, A, E, D, Z, G, F, ?
(a) X (b) Y (c) V (d) H (e) J

17. CMW, HRB, (?), RBL, WGQ, BLV
 (a) MWG (b) LVF (c) LWG (d) MXG (e) WMX
18. HS, JQ, LO, NM, (?)
 (a) PK (b) RH (c) PL (d) TG (e) RT
19. QPO, SRQ, UTS, WVU, (?)
 (a) XVZ (b) ZYA (c) YXW (d) VWX (e) AZY
20. ABP, CDQ, EFR, (?)
 (a) GHS (b) GHT (c) HGS (d) GHR (e) GSH

ANSWERS

1. (e) 2. (a) 3. (a) 4. (a) 5. (a) 6. (e) 7. (c) 8. (b) 9. (a) 10. (d)
 11. (e) 12. (a) 13. (d) 14. (c) 15. (e) 16. (b) 17. (a) 18. (a) 19. (c) 20. (a)

Explanations

- In each term, the first letter is moved 1 step forward and the second letter is moved 1 step backward to obtain the corresponding letters of the successive terms.
- The letters in each term are consecutive. There is a gap of one letter between the last letter of the first term and the first letter of the second term. In the successive terms the gap increases by one. So the last term is STU, which is having a gap of 3 letters from the last letter of the previous term.
- The first, second and third letters of each term are moved three steps forward, four steps backward and two steps forward respectively to obtain the corresponding letters of the successive terms.
- The number of letters in the terms increases by one at every step. Each term consists of letters in the alphabetical order. There is a gap of one letter between the last letter of a term and first letter of next term.
- The first, fourth and seventh letters are in alphabetical order. So the tenth letter would be the letter after C. i.e., D.
 The second and third are alternate and in the reverse order and so are the fifth and sixth and eighth and ninth. So 9th letter = $R - 2 = P$.
 So the answer is (a) P, D.
- The first, second, third, fourth and fifth terms are moved one, two, three, four and five steps respectively forward to obtain the successive terms so, $M + 4 = Q$.
- The first letters of the terms are alternate. The last letters of the terms are three steps ahead of last letter of the preceding term. The middle letter of the term follows the pattern +2, +3, +4, +5. So the next term = **X170**.
- The first letters of the term are moved one, two, three and four steps forward respectively to obtain the first letter of the successive terms. The second letters of the term are moved three, four, five, and six steps backward respectively to obtain the second letters of the successive terms. The last letters of the terms are alternate.
- The first letter of each term is moved three steps forward and the second letter is moved three steps backward to obtain the corresponding letters of the next term.
- The letters in each term are moved five steps forward to obtain the letters of the next term.
- The first term consists of one letter, the second term consists of two letters and the third letter consists of three letters. So, the required term consists of four letters. The last letter of first term and the first letter of the second term differ by two letters. Similarly, a gap of two

letters is there between the last letter of second term and the first letter of the third term. So the first letter of required term would be four steps ahead from the last letter of third term.

12. The letters of each term are moved one step backward to obtain the corresponding letters of the next term.
13. The first letters of the terms are in alphabetical order, and so are the second and third letters.
14. The first two letters of the term are in the reverse order. Similarly, the third and fourth letters of the term are also in the reverse order. Besides, the second letter of each term is the letter next to the first letter of the preceding term.
15. The letters are in the reverse order in which letters are moved 1, 2, 3, 4 and 5 steps from the last.
16. The third, sixth and ninth letters of the series are in reverse alphabetical order.
17. All the letters of each term are moved five steps forward to obtain the corresponding letters of the next term.
18. The first letter of each term is moved two steps forward and the second letter is moved two steps backward to obtain the corresponding letters of the next term.
19. Each term in the series consists of three consecutive letters in the reverse order. The first letter of each term and the last letter of the next term are the same.
20. The first and second letters of each term are moved two steps forward to obtain the corresponding letters of the next term. The third letter of the terms are consecutive terms in the alphabet.

TYPE 3: LETTER SERIES

This type of question usually consists of small letters which follow a certain pattern. Some letters are missing from the series. The missing letters are given in a proper sequence as one of the alternatives. You have to choose the correct alternative.

EXAMPLE 9 abca _____ bcaab _____ ca _____ bbc _____ a

- (a) ccaa (b) bbaa (c) abac (d) abba

[AAO Exam]

Solution Proceed step by step as follows:

1. The first blank space should be filled using 'a'; so that the first alphabet doubles in the first step after 'abc'.
2. The second blank space should be filled using 'b' so that in the second step both 'a' and 'b' doubles.
3. The third blank space should be filled by 'a' and the fourth blank space should be filled by 'c' so that in the third stage, all characters 'a', 'b' and 'c' doubles.
Thus, the answer is abac.
So, the answer is (c).

EXERCISE 4

Directions: In each of the following letter series, some of the letters are missing which are given in that order as one of the alternatives below it. Choose the correct alternative.

Questions 1 to 5

[CBI Exam]

1. abca _____ bcaab _____ aa _____ caa _____ c
(a) bbac (b) bbaa (c) acbb (d) acac
2. b _____ b _____ bb _____ bbb _____ bb _____ b
(a) bbbba (b) bbaaab (c) ababab (d) aabaab

3. aab _____ ab _____ cabcca _____ bcab _____ c
 (a) bbbc (b) bbab (c) cabc (d) cbab
4. ccbab _____ caa _____ bccc _____ a _____
 (a) babb (b) bbba (c) baab (d) babc
5. _____ abb _____ bb _____ a _____ bbab _____ ba
 (a) bababa (b) bbabbb (c) ababaa (d) aaaabb

Questions 6 to 10**[SSC]**

6. ba _____ ba _____ bac _____ acb _____ cbac
 (a) aacb (b) bbca (c) ccba (d) cbac
7. cc _____ ccbc _____ accbcc _____ c _____ b
 (a) acac (b) abac (c) abab (d) aabc
8. aaa _____ bb _____ aab _____ baaa _____ bb
 (a) abab (b) bbaa (c) babb (d) baab
9. acc _____ bc _____ a _____ ccbcc _____
 (a) abab (b) bcaa (c) aabc (d) bcab
10. aab _____ bbaaa _____ cbba _____ abc _____ ba
 (a) bcca (b) cbab (c) cbba (d) aabc

Questions 11 to 15**[AAO Exam]**

11. _____ aba _____ cabc _____ dcba _____ bab _____ a
 (a) abdca (b) bcadc (c) abcd (d) cbdaa
12. a _____ cdaab _____ cc _____ daa _____ bbb _____ ccddd
 (a) bdbda (b) bddca (c) dbbca (d) bbdac
13. a _____ abbb _____ ccccd _____ ddccc _____ bb _____ ba
 (a) abcd (b) abdbc (c) abdc (d) abcad
14. _____ bedbc _____ dcabd _____ bedbc _____ dc _____ bd
 (a) aaaaa (b) ccccc (c) bbbbbb (d) ddddd
15. adb _____ ac _____ da _____ cddcb _____ dbc _____ cbda
 (a) bccba (b) cbbaa (c) ccbba (d) bbbad

Questions 16 to 20**[Hotel Management]**

16. _____ aaba _____ bba _____ bba _____ abaa _____ b
 (a) aabab (b) ababa (c) baaba (d) bbaba
17. ab _____ bbc _____ c _____ ab _____ ab _____ b
 (a) ccaac (b) cbabc (c) cacac (d) bccab
18. _____ bca _____ cca _____ ca _____ b _____ c
 (a) aaaaa (b) bbbab (c) aabaa (d) bbabb
19. b _____ ac _____ cc _____ cb _____ ab _____ ac
 (a) cbaba (b) bbaac (c) abbbc (d) aabba
20. c _____ ac _____ aa _____ aa _____ bc _____ bcc
 (a) cabba (b) cbbbb (c) bbbbbb (d) cbacb

ANSWERS

1. (c) 2. (c) 3. (d) 4. (a) 5. (b) 6. (c) 7. (a) 8. (c) 9. (b) 10. (b)
 11. (a) 12. (d) 13. (c) 14. (a) 15. (b) 16. (a) 17. (c) 18. (b) 19. (d) 20. (b)

Explanations

1. The series is a / bcaa / bcaa / bcaa / bcaa / bc
Thus, the pattern bcaa is repeated.
12. The series is babb / bbab / bbba / bbbb
Thus, in each sequence 'a' moves one step forward and 'b' takes its place and finally to the fourth sequence, where 'a' is eliminated.
13. The series is aa / bcab / bcab / ccaa / bcab / bc
Thus, the pattern ccaa followed by bcab repeated twice, makes up the series.
4. The series is ccba / bbca / aabc / ccha / b.
Thus, the pattern consists of first two letters moved 1 step backward and the third letter moved 1 step forward and the last letter remains the same in the first stage.
In the second stage, i.e. from second to third term, first two letters moved 1 step backward and last two letters moved 1 step backward.
In the third stage, first two letters moved 1 step backward, third letter unchanged and fourth letter moved 1 step forward.
In the fourth stage, first letter moved 1 step backward.
5. The series is babb / babb / babb / babb / ba. Thus pattern babb is repeated.
6. The series is bac / bac / bac / bac / bac / bac. Thus, the pattern bac is repeated.
7. The series is ccac / ccac / ccac / ccac. Thus, the pattern ccac is repeated.
8. The series is aaa / bbb / aaa / bbb / aaa / bbb.
9. The series is ac / ac / ac / ac / ac / ac. Thus, the pattern ac is repeated.
10. The series is aab / aab / aab / aab / aab. Thus, the pattern aab is repeated.
11. The series is aab / aab / aab / aab / aab / aab. Thus, the letters equidistant from the beginning and the end of the series are the same.
12. The series is a / a / a / a / a / a / b / b / b / b / b / b / c / c / c / c / c / c / d / d / d / d / d / d. Thus, each letter of first sequence is repeated two times in the second sequence and three times in the third sequence.
13. The series is aaa / bbb / ccc / ddd / ccc / bbb / a.
14. The series is abcd / bcad / cabd / abcd / bcad / cabd. Thus, the pattern abcd / bcad / cabd is repeated twice.
15. The series is ad / ad / ad / ad / ad / ad / b / b / b / b / b / b / c / c / c / c / c / c / d / d / d / d / d / d. The letters equidistant from the beginning and end of the series are the same.
16. The series is aa / aa / aa / aa / aa / aa / b / b / b / b / b / b / a / a / a / a / a / a.
17. The series is abc / b / bca / c / cab / a / abc / b.
18. The series is bb / bb / bb / bb / bb / bb / a / a / a / a / a / a / b / b / b / b / b / b.
19. The series is ba / ba / ba / ba / ba / ba / a / a / a / a / a / a / b / b / b / b / b / b.
20. The series is cc / cc / cc / cc / cc / cc / a / a / a / a / a / a / b / b / b / b / b / b.

Coding and Decoding

Coding is a method of transmitting a message between the sender and the receiver without a third person knowing it.

The coding and decoding test is set up to judge the candidate's ability to decipher the law that codes a particular message and break the code to reveal the message. In these questions, the candidate is required to find out the original words from code values assigned to the word.

TYPE I: LETTER CODING: In these types of questions, code values are assigned to a word in terms of the alphabet and the requirement may be.

Case I: Coding: To form the code for the given word.

EXAMPLE 1 If in a certain language, TAP is coded as SZO, then how will FREEZE be coded ?

- (a) ESDFYF (b) GQFDYF (c) EQDFYG (d) EQDDYD

Solution Clearly, each letter in the code is the alphabet before the corresponding letter in the word.

S	Z	O
↑		
T	A	P

Thus, in FREEZE, F is coded as E, R as Q, E as D and Z as Y. So, the answer is (d).

EXAMPLE 2 If in a certain language GAMBLE is coded as FBLCKF, how will FLOWER be coded in that code?

- (a) GKPVFQ (b) EMNXDS (c) GMPVDS (d) HNQYGT (e) EKNVDQ

Solution The letters preceding the letters at odd places of the word and those succeeding the even places of the word in the alphabets form the code. Hence the answer is (b).

EXAMPLE 3 If in a certain language PENSION is coded as NEISNOP, how will FOLIAGE be coded in that code?

- (a) OFILGAE (b) EOAILGF (c) FGLIAOE (d) EGAILOF (e) FILOGAE

Solution In the code, first and last letters are reversed, second and second last letters are the same, third and third last letters are reversed and the middle letter remains the same. Hence the answer is (b).

EXERCISE 1

- If PLANE is coded as OKZMD in a certain language, how will TRAIN be coded?
(a) SQZHM (b) UQBHO (c) SQZJM (d) USBJM (e) USBJO
- If in a certain language; SPACE is coded as TQBDF, how will PURSE be coded in that code?
(a) QTSRF (b) OVQTD (c) QVSTF (d) ESRUP (e) OTQRD
- If in a certain language NATURE is coded as MASUQE, how is FAMINE coded in that code?
(a) FBMJND (b) FZMHND (c) GANIOE (d) EALIME (e) FZNJME
- If in a certain language SECURE is coded as ERUCES, how is SALINE coded in that code?
(a) SALIQE (b) EALINS (c) ENILAS (d) ERUCES (e) SLANIE
- If in a certain language MECHANICS is coded as HCEMASCIN, how is POSTER coded in that code?
(a) OPTSRE (b) SOPRET (c) RETSOP (d) TERPOS (e) POTSER
- If in a certain language DISPEL is coded as IDPSLE, how is EFFECT coded in that language?
(a) FEEFTC (b) CTFEFF (c) EFFTFC (d) ECTEFF (e) EEEFCT
- If BATCH is coded as ABSDG, how is FORSAKE coded in that code?
(a) ABDGS (b) EPQTZLD (c) EQPZLTD (d) GDSBA (e) None of these
- In a certain code, GOODNESS is coded as HNPCODTR. How is GREATNESS coded in that code?
(a) HQFZUODTR (b) HQFZUMFRT (c) HQFZSMFRT
(d) FSDBSODTR (e) HQFZUFRT
- If FOUGHT is coded as EQRKCZ, how will MALE be coded?
(a) LCH (b) NZMD (c) KCM I (d) NBIF (e) LBID
- If in a certain language BLEMISH is coded as AODPHVG, how will CHAPTER be coded in that code?
(a) DEBOVTDR (b) BKZSSHQ (c) CAHTPRE (d) BGAQMFP (e) ADGIQFS

ANSWERS

1. (a) 2. (c) 3. (d) 4. (c) 5. (b) 6. (a) 7. (b) 8. (b) 9. (a) 10. (b)

Explanations

- The letter preceding each letter of the given word in the alphabet is taken as the subsequent letter of its code.
- Each letter of the given word is moved one step ahead to obtain the subsequent letter of its code.
- The letters preceding the letters at odd places in the given word are taken as the corresponding letters of the code while those at even places remain the same.

4. The word is wholly reversed in the code.
5. In the code the first four and the last four letters are reversed in order.
6. In the code, every two letters are reversed in order.
7. In the code, the letters at odd places are one place before and those at even place are one place after the corresponding letter in the word.
8. In the code the letters at odd places are one alphabet ahead and those at even places are one alphabet before the corresponding letter in the word.
9. In the code the first letter is one place before, the second letter is two places ahead the third letter is three places before, the fourth letter is four places ahead, and so on.
10. In the code, the letters at odd places are one place before and those at even places are three places ahead of the corresponding letters in the word.

Case II: Decoding: To find the word from the given code. In decoding, instead of finding a code for a given word, you have to find the word from the given code.

EXAMPLE 4 If in a certain code, ALMIRAH is written as BNPMWGO, which word would be written as DNRWLUA?

- (a) COSGOLT (b) TOGSOLC (c) TOGCLOS (d) CLOSGOT (e) COLSTOG

Solution In the code, the first letter is one place ahead, the second letter is two places ahead and so on than the corresponding letter in the word.

So apply the same in the reverse direction to the given code so as to find the word. Therefore the answer is (d).

EXAMPLE 5 If in a certain code, SWITCH is written as TVJSDG, which word would be written as CQFZE?

- (a) BARED (b) BRAED (c) BREAD (d) BRADE (e) BRDAE

Solution Each letter in the code is one place ahead in odd places and one place before in even places from the corresponding letters of the word. So the answer is (c).

EXAMPLE 6 In a certain code language, REFRIGERATOR is coded as ROTAREGIRFER. Which word would be coded as NOITINUMMA?

- (a) ANMOMIUTNI (b) AMNTOMUIIN (c) AMMUNITION
(d) NMMUNITIOA (e) None of these

Solution The order of letters of the word is reversed in the code. So, reverse the letters in the code to get the word. So the answer is (c).

(EXERCISE 2)

1. If in a certain language, UTENSIL is coded as WVGPUKN, which word would be coded as DMSFXG?
(a) BKQEVE (b) BKQDWE (c) BKQDWF (d) BKQDVF (e) BKQDVE
2. If in a certain code language, REMOTE is coded as ROTEME, which word would be coded as PNIICC?
(a) NPIICC (b) PICCIN (c) PINCIC (d) PICNIC (e) PICINC
3. If in a certain language, SHIFT is coded as RFFBO, which word would be coded as LKUMB?
(a) MMXQG (b) MLVNC (c) KJVLA (d) MJVLC (e) KJTLA

4. If in a certain language, TRIANGLE is coded as SQHZMFKD, which word would be coded as DWZLOKD?
(a) EXAMPLE (b) FIGMENT (c) DISMISS (d) DISJOIN (e) None of these
5. If in a certain language GRASP is coded as BMVNK, which word would be coded as CRANE?
(a) FUDQH (b) HWFSJ (c) GVERI (d) XMVIZ (e) BQZMD
6. If in a certain language, MACHINE is coded as LBBIHOD, which word would be coded as SLTMFNB?
(a) RKSLEMA (b) TKULGMC (c) RMSNEOA (d) TMUNGOC (e) TMUNGMC
7. If in a certain language PORCELAIN is coded as QOSCFLBJO, which word is coded as BKMOSPP?
(a) ALTOLROPY (b) ALLOTROPY (c) ALOTROPY
(d) ATLOROPLY (e) None of these
8. If in a certain language LBAEHC is the code for the word BLEACH, then which of the following is coded as NBOLZKMH?
(a) OBNKLHM (b) LOBNHMKZ (c) OCPMALNI (d) MANKYJLG (e) BNLOKZHM
9. If in a certain language POPULAR is coded as QPQVMBS, which word would be coded as GBNPVT?
(a) FAMOSU (b) FAMOUS (c) FASOUM (d) FOSAUM (e) FAMSUO
10. If in a certain language, CRICKET is coded as FULFNHW, then EULGH is the code for which word?
(a) PRIDE (b) BRIDE (c) BLADE (d) BLIND (e) None of these

ANSWERS

1. (e) 2. (d) 3. (a) 4. (a) 5. (b) 6. (b) 7. (b) 8. (e) 9. (b) 10. (b)

Explanations

1. Each letter of the word is two place behind the corresponding letters of the code. So the answer is (e).
2. The groups of second and third letters and fourth and fifth letters in the word interchange places in the code.
3. The first, second, third, fourth and fifth letters of the word are respectively one, two, three, four and five places ahead of the corresponding letters of the code.
4. Each letter of word is one place ahead of corresponding letter of the code.
5. Each letter of the word is five place ahead of the corresponding letter of the code.
6. In the code, we have alternatively one letter one place behind and the other one place ahead of the corresponding letter in the word.
7. In the code, we have one letter one place ahead of the corresponding letter in the word and the other letter coming in the even places remain unchanged.
8. The word is formed into pairs of letters and letters in each pair is reversed.
9. Each letter of the word is one place behind the corresponding letter of the code.
10. Each letter of the word is three places behind the corresponding letter of the code.

TYPE II: NUMBER CODING: In these type of questions, either numerical code values are assigned to a word or alphabetical code values are assigned to numbers. You have to analyse the code as per the directions.

Case I: When numerical values are assigned to words.

EXAMPLE 7 If in a certain code language SISTER is coded as 535301, UNCLE as 84670 and BOY as 129, how will SON be coded in that code language?

- (a) 524 (b) 643 (c) 353 (d) 846 (e) None of these

Solution: Clearly, in the given code, the alphabets are coded as:

S	I	T	E	R	U	N	C	L	B	O	Y
5	3	3	0	1	8	4	6	7	1	2	9

So, S is coded as 5, O is coded as 2 and N is coded as 4. Hence correct code is 524. Therefore, the answer is (a).

EXAMPLE 8 If GIVE is coded as 5137 and BAT is coded as 924, how is GATE coded?

- (a) 5427 (b) 5724 (c) 5247 (d) 2547 (e) None of these

Solution: Here the alphabet are coded as follows:

G	I	V	E	B	A	T
5	1	3	7	9	2	4

So G is coded as 5, A is coded as 2, T is coded as 4 and E is coded as 7. Hence correct code is 5247. Therefore the answer is (c).

EXERCISE 3

- In a certain code, PALE is written as 2134, EARTH is written as 41590, how will PEARL be written in that code?
(a) 29530 (b) 24153 (c) 25413 (d) 25430 (e) None of these
- In a certain code, RIPPLE is written as 613382 and LIFE is written as 8192. How will RIFFLE be written in that code?
(a) 968812 (b) 869912 (c) 619982 (d) 269981 (e) None of these
- If ROSE is coded as 6821, CHAIR is coded as 73456 and PREACH is coded as 961473, what will be the code for SEARCH?
(a) 246173 (b) 214673 (c) 214763 (d) 216473 (e) None of these
- If in a certain code language, TWENTY is written as 863985 and ELEVEN is written as 323039, how will TWELVE be written in that code?
(a) 863203 (b) 863584 (c) 863903 (d) 863063 (e) None of these
- If A is coded as 1, B is coded as 2, and so on, how is HIGH coded in that code?
(a) 9879 (b) 7897 (c) 8978 (d) 8798 (e) None of these

Directions (Questions 6 to 8): If in a certain language ENTRY is coded as 12345 and STEADY is coded as 931785, then state which is the correct code for each of the given words.

6. NEATNESS

- (a) 2956169 (b) 21732199 (c) 21362199 (d) 21823698 (e) None of these

7. ARREST

- (a) 744589 (b) 744193 (c) 166479 (d) 745194 (e) 188924

8. ENDEAR

- (a) 524519 (b) 174189 (c) 128174 (d) 124179 (e) 164983

Directions (Questions 9 to 11): If in a certain language CHARCOAL is coded as 45164913 and MORALE is coded as 296137, how are the following words coded in that language?

9. REAL

- (a) 8519 (b) 6713 (c) 6513 (d) 6719 (e) None of these

10. COACH

- (a) 38137 (b) 49148 (c) 48246 (d) 49145 (e) None of these

11. COLLAR

- (a) 397758 (b) 497758 (c) 483359 (d) 493316 (e) None of these

Directions (Questions 12 to 15): If MISTAKE is coded as 9765412 and NAKED is coded as 84123, how are the following words coded?

12. DISTANT

- (a) 3765485 (b) 4798165 (c) 3697185 (d) 4768296 (e) None of these

13. ASSIST

- (a) 166762 (b) 466765 (c) 488976 (d) 435985 (e) 166872

14. INTIMATE

- (a) 89786145 (b) 79438163 (c) 78579452 (d) 78698365 (e) None of these

15. STAIN

- (a) 98175 (b) 89483 (c) 68194 (d) 65478 (e) None of these

16. If ROPE is coded as 6821 and CHAIR is coded as 73456 then what will be the code for CRAPE?

- (a) 73456 (b) 76421 (c) 77246 (d) 77123 (e) None of these

17. If PLAY is coded as 8123 and RHYME is coded as 49367, then how is MALE coded?

- (a) 6217 (b) 6198 (c) 6395 (d) 6285 (e) None of these

18. If in a certain language PRIVATE is coded as 1234567 and RIST is coded as 2396, then how is RIVETS coded in that language?

- (a) 687543 (b) 234769 (c) 496321 (d) 246598 (e) None of these

19. In a certain code language 24685 is written as 33776. How is 35791 written in that code?

- (a) 44882 (b) 44880 (c) 46682 (d) 44682 (e) None of these

20. In a certain code language 35796 is written as 44887. How is 46823 written in that code?

- (a) 57914 (b) 55914 (c) 55934 (d) 55714 (e) None of these

ANSWERS

1. (b) 2. (c) 3. (b) 4. (a) 5. (c) 6. (b) 7. (b) 8. (c) 9. (b) 10. (d)
11. (d) 12. (a) 13. (b) 14. (c) 15. (d) 16. (b) 17. (a) 18. (b) 19. (a) 20. (b)

Explanations

1. The alphabets in the word are coded as shown below:

P	A	L	E	R	T	H
↓	↓	↓	↓	↓	↓	↓
2	1	3	4	5	9	0

So PEARL is coded as 24153.

2. The alphabets in the word are coded as shown below:

R I P L E F
6 1 3 8 2 9

So RIFFLE is coded as 619982.

3. The alphabets in the word are coded as shown below:

R O S E C H A I P
6 8 2 1 7 3 4 5 9

So SEARCH is coded as 214673.

4. The alphabets in the word are coded as shown below:

T W E N Y L V
8 6 3 9 5 2 0

So TWELVE is written as 863203.

5. The alphabets in the word are coded as shown below:

A B C D E F G H I
↓ ↓ ↓ ↓ ↓ ↓ ↓ ↓ ↓
1 2 3 4 5 6 7 8 9

So HIGH is coded as 8978.

Questions 6 to 8: The alphabets in the word are coded as follows:

E N T R Y S A D
↓ ↓ ↓ ↓ ↓ ↓ ↓ ↓
1 2 3 4 5 9 7 8

6. So NEATNESS is coded as 21732199.

7. So ARREST is coded as 744193.

8. So ENDEAR is coded as 128174.

Questions 9 to 11: The alphabets in the word are coded as follows:

C H A R O L M E
4 5 1 6 9 3 2 7

9. So REAL is coded as 6713.

10. So COACH is coded as 49145.

11. So COLLAR is coded as 493316.

Questions 12 to 15: The alphabets in the word are coded as shown below:

M I S T A K E N D
9 7 6 5 4 1 2 8 3

12. So DISTANT is coded as 3765485.

13. So ASSIST is coded as 466765.

14. So INTIMATE is coded as 78579452.

15. So STAIN is coded as 65478.

16. The alphabets in the word are coded as shown below:

R O P E C H A I
6 8 2 1 7 3 4 5

So CRAPE is coded as 76421.

17. The alphabets are coded as shown below:

P L A Y R H M E
8 1 2 3 4 9 6 7

So MALE is coded as 6217.

18. The alphabets are coded as follows:

P R I V A T E S
1 2 3 4 5 6 7 9

So RIVETS is coded as 234769.

19. Here, in the code, the letters at odd places are one place ahead and those at even places are one place before the corresponding letter in the word.

2 4 6 8 5 → word
↓ ↓ ↓ ↓ ↓
3 3 7 7 6 → code

So 3 is coded as 4, 5 as 4, 7 as 8, 9 as 8 and 1 as 2. So 35791 is coded as 44882.

20. Here code follows same pattern as in Q. 19.

So 46823 is coded as 55914.

Blood Relations

TYPE 1

In these tests, the success of a candidate depends upon the knowledge of the blood relations.

Important Relations

Mother's or father's son	:	Brother
Mother's or father's daughter	:	Sister
Mother's or father's brother	:	Uncle
Mother's or father's sister	:	Aunt
Mother's or father's mother	:	Grandmother
Mother's or father's father	:	Grandfather
Son's wife	:	Daughter-in-law
Daughter's husband	:	Son-in-law
Husband's or wife's sister	:	Sister-in-law
Husband's or wife's brother	:	Brother-in-law
Brother's son	:	Nephew
Brother's daughter	:	Niece
Uncle or aunt's son or daughter	:	Cousin
Sister's husband	:	Brother-in-law
Brother's wife	:	Sister-in-law

EXAMPLE 1 Pointing to a photograph, a man tells his friend, 'She is the daughter of the only son of my father's wife'. How is the girl in the photograph related to the man?

- (a) Daughter (b) Cousin (c) Mother (d) Sister (e) Niece

Solution The relation may be represented as follows:

Father's wife—mother; mother's only son—himself. So the girl is man's daughter and the answer is (a).

EXAMPLE 2 Pointing to a lady, Suresh said, 'She is the daughter of the woman, who is the mother of the husband of my mother'? What is the relation of the lady to Suresh?

- (a) Aunt (b) Granddaughter (c) Daughter
(d) Sister (e) Sister-in-law

Solution The relation may be represented as follows:

Mother's husband—Father; Father's mother—Grandmother; Grandmother's daughter—Father's sister; Father's sister is Aunt. So the lady is Suresh's aunt and the answer is (a).

EXAMPLE 3 A girl introduced a boy as the son of the daughter of the father of her uncle. The boy is girl's _____.

- (a) Brother (b) Son (c) Uncle (d) Son-in-law (e) Nephew

Solution The relation may be represented as follows:

Father of uncle—Grandfather; Daughter of grandfather—Uncle's sister or mother; Son of mother—Brother. So the answer is (a).

EXERCISE 1

- Introducing a man to her husband, a woman said his brother's father is the only son of my grandfather. How is the woman related to this man? **[Bank PO]**
(a) Mother (b) Aunt (c) Sister (d) Daughter (e) Grandmother
- Pointing to a lady a girl said, 'She is the daughter-in-law of the grandmother of my father's only son'. How is the lady related to the girl?
(a) Sister-in-law (b) Mother (c) Aunt
(d) Mother-in-law (e) Cousin
- Introducing a man a woman said, He is the only son of my mother's mother. How is the woman related to the man? **[Officer's Grade]**
(a) Mother (b) Aunt (c) Sister (d) Niece (e) None of these
- Introducing a man, a woman said, 'His wife is the only daughter of my father'. How was that man related to the woman? **[AO Exam]**
(a) Brother (b) Father-in-law (c) Maternal uncle
(d) Husband (e) None of these
- Showing a lady in the park, Ashok said, 'She is the daughter of my grandfather's only son'. How is Ashok related to that lady? **[Bank PO]**
(a) Brother (b) Cousin (c) Father (d) Uncle (e) None of these
- Pointing to a man in a photograph, a woman said, 'His brother's father is the only son of my grandfather'. How is the woman related to the man in the photograph? **[SBI PO]**
(a) Mother (b) Aunt (c) Sister (d) Daughter (e) Grandmother
- Pointing to a person a man said to a woman, 'His mother is the only daughter of your father'. How was the woman related to the person?
(a) Aunt (b) Mother (c) Wife (d) Daughter (e) None of these
- Arun said, 'This girl is the wife of the grandson of my mother'. Who is Arun to the girl?
(a) Father (b) Grandfather (c) Husband
(d) Father-in-law (e) None of these
- Pointing to a lady, a man said, 'The son of her only brother is the brother of my wife'. How is the lady related to the man?
(a) Mother's sister (b) Grandmother (c) Mother-in-law
(d) Sister of father-in-law (e) Maternal aunt

10. Pointing to a lady on the platform, Anju said, 'She is the sister of the father of my mother's son'. Who is the lady to Anju?
 (a) Mother (b) Sister (c) Aunt (d) Niece (e) None of these

ANSWERS

1. (c) 2. (b) 3. (d) 4. (d) 5. (a) 6. (c) 7. (a) 8. (d) 9. (d) 10. (c)

Explanations

- Only son of her grandfather—her father; man's brother's father — man's father; So man's father is her father. i.e. She is man's sister.
- My father's only son—My brother; grandmother of my brother—My grandmother; Daughter-in-law of my grandmother—My mother. So the lady is girl's mother.
- My mother's mother—my grandmother; My grandmother's only son—my maternal uncle. So, the woman is man's niece.
- Only daughter of my father—myself; So, man is woman's husband.
- Grandfather's only son—father; Daughter of father—sister. So Ashok is lady's brother.
- Only son of woman's grandfather—woman's father; Man's brother's father—Man's father. So, the woman is man's sister.
- Only daughter of your father—your sister; Person's mother—woman's sister. So the woman is person's aunt.
- Mother's grandson—son; Son's wife—daughter-in-law. So, Arun is the father-in-law of the girl.
- Brother of my wife—My brother-in-law; Son of lady's brother is brother-in-law of man. So lady's brother is man's father-in-law. i.e. Lady is sister of man's father-in-law.
- Mother's son—brother; My brother's father—father; my father's sister—aunt; So the lady is Anju's aunt.

TYPE 2

In this type mutual blood relations or other information of more than two persons are mentioned and information about any two is questioned.

EXAMPLE 4 A and B are married couple. X and Y are brothers. X is the brother of A. How is Y related to B?

- (a) Brother-in-law (b) Brother (c) Son-in-law
 (d) Cousin (e) None of these

Solution A and B are husband and wife. Since X and Y are brothers, and X is the brother of A, Y is also the brother of A. Thus Y is the brother-in-law of B. Hence the answer is (a).

EXAMPLE 5 A and B are brothers, C and D are sisters. A's son is D's brother. How is B related to C?

- (a) Father (b) Brother (c) Grandfather
 (d) Uncle (e) None of these

Solution Clearly, the relations can be as follows. B is the brother of A. A's son is D's brother. i.e. D is the daughter of A and hence C is also the daughter of A. So, B is the uncle of C. Hence the answer is (d).

EXAMPLES 6–11

Read the following information carefully and answer the questions given below.

All the six members of a family A, B, C, D, E, and F are travelling together. B is the son of C but C is not the mother of B. A and C are a married couple. E is the brother of C. D is the daughter of A. F is the brother of B.

6. How many male members are there in the family?
 (a) 1 (b) 3 (c) 2 (d) 4 (e) 5
7. Who is the mother of B?
 (a) D (b) F (c) E (d) A (e) None of these
8. How many children does A have?
 (a) One (b) Two (c) Three (d) Four (e) None of these
9. Who is the wife of E?
 (a) A (b) F (c) B
 (d) Can't be determined (e) None of these
10. Which of the following is a pair of females?
 (a) A, E (b) B, D (c) D, F (d) A, D (e) None of these
11. How is E related to D?
 (a) Father (b) Brother (c) Uncle
 (d) Can't be determined (e) None of these

Solution B is the son of C but C is not the mother of B means C is the father of B. A is married to C means A is the mother of B. F is brother of B means F is the son of A and C. E is the brother of C. D is the daughter of A.

i.e. D is the daughter of A and C.

Therefore, E is the uncle of D.

6. A is the mother and hence female. B is the son and hence male. C is the husband and hence male. D is the daughter and hence female. E is the brother and hence male. F is the son and hence male. So there are four males. The answer is (d).
7. Clearly A is the mother of B. The answer is (d).
8. A has son B, son F and daughter D. i.e. three children. The answer is (c).
9. Clearly, from the data available, the wife of E cannot be determined. The answer is (d).
10. Clearly, the females are only the mother A and the daughter D. The answer is (d).
11. E is the brother of C who is the father of D. So, E is the uncle of D. The answer is (c).

EXERCISE 2

Read the following information and answer the questions that follow (Q. 1–4).

A is the son of B; C, B's sister has a son D and a daughter E; F is the maternal uncle of D.

1. How is A related to D?
 (a) Cousin (b) Nephew (c) Uncle (d) Brother (e) None of these
2. How is E related to F?
 (a) Sister (b) Daughter (c) Niece (d) Wife (e) None of these

3. How many nephews does F have?
(a) Nil (b) One (c) Two (d) Three (e) None of these
4. How is F related to E?
(a) Uncle (b) Brother (c) Maternal uncle
(d) Nephew (e) None of these
5. P is the son of Q while Q and R are the sisters to one another. T is the mother of R. If S is the son of T, which of the following statement is correct?
(a) T is the brother of Q (b) S is the cousin of P
(c) Q and S are sisters (d) S is the maternal uncle of P
(e) R is the grandfather of P
6. P is the brother of Q and R. S is R's mother. T is father of P. Which of the following statement cannot be definitely true?
(a) T is father of Q (b) S is mother of P (c) P is son of S
(d) T is husband of S (e) Q is son of T
7. A and B both are children of C. If C is the mother of A, A is the son of C but B is not the daughter of C, how are A and B mutually related?
(a) A is the brother of B (b) A is the sister of B (c) A is the cousin of B
(d) A is the nephew of B (e) None of these
8. E is the son of A. D is the son of B. E is married to C. C is daughter of B. How is D related to E?
(a) Brother (b) Uncle (c) Father-in-law
(d) Brother-in-law (e) None of these
9. A woman walking with a boy meets another woman and on being asked about her relationship with the boy, she says, 'My maternal uncle and his maternal uncle's maternal uncle are brothers'. How is the boy related to the woman?
(a) Nephew (b) Son (c) Grandson (d) Husband (e) Brother-in-law
10. P is the brother of D. X is the sister of P. A is the brother of F. F is the daughter of D. M is the father of X. Who is the uncle of A?
(a) X (b) P (c) F (d) M (e) None of these

Read the following information carefully and answer the questions that follow (Q. 11–14).

Five persons namely P, Q, X, Y, and Z are sitting on a park. P is the mother of X who is the wife of Z. Y is the brother of P and Q is the husband of P.

11. How is P related to Z?
(a) Mother (b) Aunt (c) Sister
(d) Mother-in-law (e) None of these
12. How is Y related to Q?
(a) Brother (b) Brother-in-law (c) Cousin
(d) Uncle (e) Son
13. How is X related to Q?
(a) Daughter (b) Daughter-in-law (c) Niece
(d) Aunt (e) Mother
14. How is Q related to Z?
(a) Father-in-law (b) Brother-in-law (c) Father
(d) Mother-in-law (e) None of these

Study the following information and answer the questions (15–19).

There is a family of six persons A, B, C, D, E, and F. They are lawyer, doctor, teacher, salesman, engineer and accountant. There are two married couples in the family. D, the salesman is married to the lady teacher. The doctor is married to the lawyer. F, the accountant is the son of B and brother of E. C, the lawyer is the daughter-in-law of A. E, is the unmarried engineer. A, is the grandmother of F.

15. How is E related to F?
 (a) Brother (b) Sister (c) Cousin
 (d) Can't be determined (e) None of these
16. What is the profession of B?
 (a) Teacher (b) Doctor (c) Lawyer
 (d) Can't be determined (e) None of these
17. Which is the profession of A?
 (a) Lawyer (b) Teacher (c) Doctor
 (d) Can't be determined (e) None of these
18. Which of the following is one of the couples?
 (a) F and D (b) D and B (c) E and A (d) A and C (e) None of these
19. How is D related to F?
 (a) Grandfather (b) Father (c) Uncle (d) Brother (e) None of these

Read the following information carefully and answer the questions (20–25).

A family consists of six members P, Q, R, X, Y, and Z. Q is the son of R but R is not the mother of Q. P and R are a married couple. Y is the brother of R. X is the daughter of P. Z is the brother of P.

20. Who is the brother-in-law of R?
 (a) P (b) Z (c) Y (d) X (e) None of these
21. Who is the father of Q?
 (a) P (b) Z (c) R
 (d) Can't be determined (e) None of these
22. How many children does P have?
 (a) Four (b) Three (c) Two (d) One (e) None of these
23. How many female members are there in the family?
 (a) One (b) Two (c) Three (d) Four (e) Five
24. How is Q related to X?
 (a) Uncle (b) Brother (c) Father (d) Husband (e) None of these
25. Which is a pair of brothers?
 (a) R and Y (b) Q and X (c) P and Z (d) P and X (e) None of these

Read the following information carefully and answer the questions (26–30).

A family consists of six members P, Q, R, S, T, and U. There are two married couples. Q is a doctor and the father of T. U is the grandfather of R and is a contractor. S is grandmother of T and is a housewife. There is one doctor, one contractor, one nurse, one housewife and two students in the family.

26. Who is the husband of P?
 (a) R (b) U (c) Q (d) S (e) T

27. Who is the sister of T?
 (a) R (b) U (c) T
 (d) Can't be determined (e) None of these
28. What is the profession of P?
 (a) Doctor (b) Nurse (c) Doctor or Nurse
 (d) Housewife (e) None of these
29. Which of the following are two married couples?
 (a) US, QT (b) US, QP (c) TS, RU (d) US, RP (e) None of these
30. Which of the following is definitely a group of male members?
 (a) QU (b) QUT (c) QUP (d) UT (e) None of these

ANSWERS

1. (a) 2. (c) 3. (c) 4. (a) 5. (d) 6. (e) 7. (a) 8. (d) 9. (b) 10. (b)
 11. (d) 12. (b) 13. (a) 14. (a) 15. (d) 16. (b) 17. (b) 18. (e) 19. (a) 20. (b)
 21. (c) 22. (c) 23. (b) 24. (b) 25. (a) 26. (c) 27. (a) 28. (b) 29. (b) 30. (a)

Explanations

- A is the son of B. D is the son of B's sister. So A is the cousin of D.
- E is the daughter of C and D is the son of C. So, F, who is the maternal uncle of D, is also maternal uncle of E. Thus, E is the niece of F.
- Clearly, F is the maternal uncle of D means F is the brother of D's mother, i.e. F is the brother of C. C is the sister of B. So, F is the brother of B, who is mother of A. Thus, F is the maternal uncle of A. So, A and D are the nephews of F. i.e. F has two nephews.
- From Q. 2, E is the niece of F, who is a male. So, F is the uncle of E.
- Q and R are sisters. So, T is the mother of R means T is the mother of Q and R. S is the son of T means S is the brother of Q. Thus, P is the son of Q means S is the maternal uncle of P.
- P, Q and R are children of the same parents. So, S who is mother of R and T, who is father of P will be mother and father of all three. However, it is not mentioned whether Q is male or female. So (e) can't be definitely true.
- B is the child of C, but not daughter means B is the son of C. Also, A is the son of C. So, A is the brother of B.
- C is B's daughter and D is B's son. So, D is the brother of C. E is a male married to C. So, E is the husband of C, whose brother is D. So, D is the brother-in-law of E.
- Boy's maternal uncle will be brother of boy's mother. Maternal uncle of mother's brother and maternal uncle of lady are brothers means lady is sister to mother's brother. i.e., lady is the mother of the boy. So, the boy is woman's son.
- A is the brother of F who is the daughter of D. So, A is the son of D. P is the brother of D. So, P is the uncle of A.
- P is the mother of X who is the wife of Z. So, P is the mother-in-law of Z.
- Q is the husband of P means P is the wife of Q. So, Y is the brother of P who is the wife of Q. Thus, Y is the brother-in-law of Q.
- P is the mother of X who is a female. So, X is the daughter of P. Q is the husband of P. Thus, X is the daughter of Q.
- P is the mother-in-law of Z. So, Q, the husband of P will be father-in-law of Z.

Questions (15 to 19)

Given E is the unmarried engineer. C is the daughter-in-law of A who is the grandmother of F means C is the mother of F. But F is the son of B. So, B is the husband of C. But C, the lawyer is married to the doctor. Thus C, the lawyer is the wife of B who is a doctor. F, the accountant will be the son of B and C. So, the other married couple can be that of grandmother of F. i.e. A and D. But D, the salesman is married to the lady teacher. So D, the salesman is the grandfather of F, father of B and the husband of A, the lady teacher.

15. Clearly, from the given data the relation between E and F cannot be determined.
16. Clearly, B is a doctor.
17. A is the lady teacher.
18. The two couples are C and B; and D and A which is none among the choices.
19. D is the grandfather of F.

Questions (20 to 25)

Q is the son of R but R is not the mother. So, R is the father of Q. P is married to R. So, P is the wife of R and the mother of Q. X is the daughter of P and hence of R and so it is the sister of Q. Y is the brother of R and Z is the brother of P.

20. R is the husband of P and Z is the brother of P. So, Z is the brother-in-law of R.
21. R is the father of Q.
22. Clearly, Q is the son of P and X is the daughter of P. So P has two children.
23. There are two females only—mother P and daughter X.
24. X is the sister of Q who is a male. So, Q is the brother of X.
25. Clearly, Y is the brother of R who is a male. So, Y and R are a pair of brothers.

Questions (26 to 30)

Q, the doctor, is the father of T. S, the housewife, is the grandmother of T and hence the mother of Q. Since there are only two married couples one being that of Q, the grandfather of R, i.e. U must be married to S. Thus, R and T will be both children of Q and these must be the students. So, P, who remains, shall be the wife of Q and she alone can be the nurse. Thus, U must be the contractor.

26. The husband of P will be Q.
27. Clearly, R and T are children of the same parents. So, R will be the sister of T.
28. P is the nurse.
29. The two married couples are Q, P and U, S.
30. Clearly, the males are Q, the father, and U, the grandfather.

TYPE 3**Coded Relationships**

In such questions, the relationships are represented by certain codes or symbols such as +, ×, −, ÷ etc. Then relationships between certain persons, given in the form of these codes, are to be analysed.

EXAMPLES (12–15)

Read the following information carefully and answer the questions that follow:

A + B means A is the son of B; A − B means A is the wife of B; A × B means A is the brother of B; A ÷ B means A is the mother of B and A = B means A is the sister of B.

12. What does P + R − Q means?
 - (a) Q is the father of P
 - (b) Q is the son of P
 - (c) Q is the uncle of P
 - (d) Q is the brother of P

13. What does $P \times R \div Q$ mean?
 (a) P is the brother of R (b) P is the father of Q
 (c) P is the uncle of Q (d) P is the nephew of Q
14. What does $P = R + Q$ mean?
 (a) P is the aunt of Q (b) P is the daughter of Q
 (c) P is the niece of Q (d) P is the sister of Q
15. What does $P = R \div Q$ mean?
 (a) P is the aunt of Q (b) P is the sister of Q
 (c) Q is the niece of P (d) Q is the daughter of P

Solution

12. $P + R - Q$ means P is the son of R who is the wife of Q. i.e. Q is the father of P. So, the answer is (a).
13. $P \times R \div Q$ means P is the brother of R who is the mother of Q. i.e., P is the uncle of Q. So the answer is (c).
14. $P = R + Q$ means P is the sister of R who is the son of Q. i.e. P is the daughter of Q. So, answer is (b).
15. $P = R \div Q$ means P is the sister of R who is the mother of Q. i.e., P is the aunt of Q. So the answer is (a).

EXERCISE 3

Read the following information carefully and answer the questions that follow: (Q. 1–8).

$A \div B$ means A is the father of B, $A - B$ means A is the wife of B; $A \times B$ means A is the brother of B, $A + B$ means A is the daughter of B.

- If $P \div R + S + Q$, which of the following is true?
 (a) P is the daughter of Q (b) Q is the aunt of P (c) P is the aunt of Q
 (d) P is the mother of Q (e) None of these
- If $P - R + Q$, which of the following statement is true?
 (a) P is the mother of Q (b) Q is the daughter of P (c) P is the aunt of Q
 (d) P is the sister of Q (e) P is the niece of Q
- If $P \times R \div Q$, which of the following statement is true?
 (a) P is the uncle of Q (b) P is the father of Q (c) P is the brother of Q
 (d) P is the son of Q (e) None of these
- If $P \times R - Q$, which of the following is true?
 (a) P is brother-in-law of Q (b) P is the brother of Q (c) P is the uncle of Q
 (d) P is the father of Q (e) None of these
- If $P + R \div Q$, which of the following is true?
 (a) P is the brother of Q (b) P is the son of Q (c) P is the husband of Q
 (d) P is the father of Q (e) P is the uncle of Q
- If $P \div R + Q$, which of the following is true?
 (a) P is the father of Q (b) P is the brother of Q (c) P is the mother of Q
 (d) P is the sister of Q (e) None of these

7. If $P \times R \div Q$, which of the following is true?
 (a) P is the uncle of Q (b) P is the father of Q (c) P is brother-in-law of Q
 (d) P is grandfather of Q (e) P is son-in-law of Q
8. If $P - R \times Q$, which of the following is true?
 (a) P is the sister of Q (b) Q is the husband of P (c) P is the sister-in-law of Q
 (d) Q is the sum of P (e) None of these
9. If $A + B$ means A is the mother of B; $A \div B$ means A is the brother of B; $A \times B$ means A is the son of B and $A - B$ means A is the daughter of B, which of the following means C is the niece of D?
 (a) $D - C$ (b) $D \times P - C$ (c) $C - P \div D$ (d) $P + D \div C$ (e) $D - P \div C$
10. If $A + B$ means A is the sister of B, $A - B$ means A is the brother of B, $A \times B$ means A is the daughter of B, which of the following shows the relation that E is the maternal uncle of D?
 (a) $D + F \times E$ (b) $D - F \times E$ (c) $D \times F + E$ (d) $D \times F - E$ (e) None of these

ANSWERS

1. (c) 2. (a) 3. (d) 4. (a) 5. (c) 6. (d) 7. (a) 8. (c) 9. (c) 10. (c)

Explanations

- $P \div R + S + Q$ means P is the daughter of R who is the father of S who is the father of Q i.e. P is the sister of S who is the father of Q i.e. P is the aunt of Q.
- $P - R + Q$ means P is the wife of R who is the father of Q, i.e. P is the mother of Q.
- $P \times R \div Q$ means P is the brother of R who is the daughter of Q, i.e. P is the son of Q.
- $P \times R - Q$ means P is the brother of R who is the wife of Q, i.e. P is the brother-in-law of Q.
- $P + R \div Q$ means P is the father of R who is the daughter of Q, i.e. P is the father of R and Q is the mother of R, i.e. P is the husband of Q.
- $P \div R + Q$ means P is the daughter of R who is the father of Q, i.e., P is the sister of Q.
- $P \times R + Q$ means P is the brother of R who is the father of Q, i.e. P is the uncle of Q.
- $P - R \times Q$ means P is the wife of R who is the brother of Q, i.e. P is the sister-in-law of Q.
- C is the niece of D means C is the daughter of the brother (say P) of D, i.e. $C - P \div D$.
- E is the maternal uncle of D means D is the daughter of the sister (say F) of E, i.e. $D \times F + E$.

Puzzle Test

This chapter includes questions in the form of puzzles involving certain number of items, which may be persons or things. You are required to analyse the given information, condense it in a suitable form and answer the questions asked.

EXAMPLE 1 Read the following information carefully and answer the questions given below:

1. There are five friends—Sonal, Ratheesh, Manoj, Ashok and Gireesh.
2. Sonal is shorter than Ratheesh but taller than Gireesh.
3. Manoj is the tallest.
4. Ashok is a little shorter than Ratheesh and little taller than Sonal.

Questions:

1. Who is the shortest?
(a) Gireesh (b) Sonal (c) Ashok (d) Ratheesh (e) None of these
2. If they stand in the order of their heights, who will be in the middle?
(a) Ratheesh (b) Gireesh (c) Sonal (d) Ashok (e) None of these
3. Who is the second tallest?
(a) Sonal (b) Ratheesh (c) Ashok (d) Gireesh (e) None of these
4. Who is taller than Ashok but shorter than Manoj?
(a) Manoj (b) Ratheesh (c) Gireesh (d) Ashok (e) Sonal
5. If they stand in the order of increasing heights, who will be the second?
(a) Ashok (b) Sonal (c) Gireesh (d) Ratheesh (e) None of these

Solution Let the friends be denoted by the first letter of each name, namely S, R, M, A and G.

Then

$$\begin{aligned} G &< S < R \\ S &< A < R \end{aligned}$$

Manoj is the tallest.

∴

$$G < S < A < R < M$$

1. (a) Gireesh is the shortest.
2. (d) Ashok is in the middle.
3. (b) Ratheesh is the second tallest.
4. (b) Ratheesh is taller than Ashok but shorter than Manoj.
5. (b) In the order of increasing heights, Sonal is in the second position.

EXERCISE 1

Directions (Q. 1 to 5): Read the following information carefully and answer the questions given below:

1. There is a group of six persons A, B, C, D, E and F from a family. They are Professor, Manager, Lawyer, Jeweller, Doctor and Engineer.
2. The Doctor is the grandfather of F who is a Professor.
3. The Manager D is married to A.
4. C, the Jeweller is married to the Lawyer.
5. B is the mother of F and E.
6. There are two married couples in the family.

[Bank PO]

Questions:

1. What is the profession of E?
(a) Doctor (b) Jeweller (c) Manager (d) Professor (e) None of these
2. How is A related to E?
(a) Brother (b) Uncle (c) Father (d) Grandfather (e) None of these
3. How many male members are there in the family?
(a) One (b) Three (c) Four
(d) Data inadequate (e) Can't be determined
4. What is the profession of A?
(a) Doctor (b) Lawyer (c) Jeweller (d) Manager (e) None of these
5. Which of the following is one of the pairs of couples in the family?
(a) AB (b) AC (c) AD
(d) Can't be determined (e) None of these

Directions (Q. 6 to 10): Study the following information carefully and answer the questions given below it:

Five friends A, B, C, D and E are sitting on a bench.

1. A is sitting next to B.
2. C is sitting next to D.
3. D is not sitting with E.
4. E is on the left end of the bench.
5. C is on second position from the right.
6. A is on the right side of B and to the right side of E.
7. A and C are sitting together.

Questions:

6. Where is A sitting?
(a) Between B and D (b) Between D and C (c) Between E and D
(d) Between C and E (e) Between B and C

7. Who is sitting in the centre?
 (a) A (b) B (c) C (d) D (e) E
8. C is sitting between
 (a) B and D (b) A and E (c) D and E
 (d) A and D (e) A and B
9. What is the position of D?
 (a) Extreme left (b) Extreme right (c) Third from left
 (d) Second from left (e) None of these
10. What is the position of B?
 (a) Second from right (b) Centre (c) Extreme left
 (d) Second from left (e) None of these

Directions (Q. 11 to 15): Study the following information carefully and answer the questions given below it: **[Bank PO]**

A training college has to conduct a refresher course for teachers of seven different subjects—Mechanics, Psychology, Philosophy, Sociology, Economics, Science, and Engineering from 22nd July to 29th July.

1. The course should start with Psychology.
2. 23rd July, being Sunday, should be holiday.
3. Science subject should be on the previous day of the engineering subject.
4. The course should end with Mechanics.
5. Philosophy should be immediately after the holiday.
6. There should be a gap of one day between Economics and Engineering.

Questions:

11. The refresher course will start with which one of the following subjects?
 (a) Psychology (b) Mechanics (c) Philosophy (d) Economics (e) None of these
12. Which subject will be on Tuesday?
 (a) Mechanics (b) Engineering (c) Economics (d) Psychology (e) None of these
13. Which subject precedes Mechanics?
 (a) Economics (b) Engineering (c) Philosophy (d) Psychology (e) None of these
14. How many day's gap is there between Science and Philosophy?
 (a) One (b) Two (c) Three (d) No gap (e) None of these
15. Which subject is followed by Science?
 (a) Engineering (b) Psychology (c) Philosophy (d) Economics (e) None of these

Directions (Q. 16 to 18): Read the following information carefully and answer the questions given below it.

1. Six friends A, B, C, D, E and F are sitting in a closed circle facing the centre.
2. E is to the left of D.
3. C is between A and B.
4. F is between E and A.

Questions:

16. Who is to the left of B?
 (a) A (b) C (c) D (d) E (e) None of these
17. Who is to the right of C?
 (a) A (b) B (c) D (d) E (e) F
18. Which of the above given statements is superfluous?
 (a) 1 (b) 2 (c) 3 (d) 4 (e) None of these

ANSWERS

1. (e) 2. (d) 3. (e) 4. (a) 5. (c) 6. (e) 7. (a) 8. (d) 9. (b) 10. (d)
 11. (a) 12. (c) 13. (e) 14. (a) 15. (d) 16. (c) 17. (a) 18. (e)

Explanations

Given F is a professor, B is the mother of F and E implies that E is a brother or sister of F.

There are only two married couples in the family.

Since D is married to A, C, the jeweller who is married to a lawyer who will be B.

Manager D is married to A means A is the doctor and grandfather of F and E. So E must be an engineer.

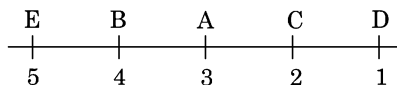
1. Here E is an engineer.

So the answer is 'None of these'

2. A is the grandfather of F and E is the brother or sister of F. So A is the grandfather of E.
 3. Since nothing is mentioned about E and F, the number of males cannot be determined.
 4. A, who is the grandfather of F is doctor.
 5. D is the manager married to A. So AD is one of the couples in the family.

Questions 6 to 10

6. A is sitting between B and C.
 7. A is sitting in the centre.
 8. C is sitting between A and D.
 9. D is on the extreme right.
 10. B is second from left.

**Questions 11 to 15**

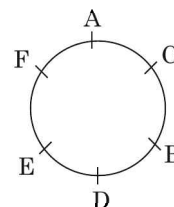
22	23	24	25	26	27	28	29
Psychology	Sunday	Philosophy	Economics	Science	Engineering	Sociology	Mechanics

11. The course will start with Psychology.
 12. Economics will be on Tuesday.
 13. Sociology precedes Mechanics.
 14. There is only one day's gap between Science and Philosophy.
 15. Economics is followed by Science.

Questions 16 to 18

Seating arrangement is as shown:

16. D is to the left of B.
 17. A is to the right of C.
 18. Here all statements are necessary to determine the arrangement.
 Therefore none of these is superfluous.

**EXERCISE 2**

Examine the following relationships among members of a family of 6 persons A, B, C, D, E, and F.

1. The number of males equals that of females.
 2. A and E are sons of F.

3. D is the mother of two, one boy and one girl.
4. B is the son of A.
5. There is only one married couple in the family at present.
1. Which one of the following references can be drawn from the above? [IAS]
 - (a) A, B and C are all females
 - (b) A is the husband of D
 - (c) E and F are the children of D
 - (d) D is the granddaughter of F
2. A, B, C, D, E, and F not necessarily in that order are sitting in 6 chairs regularly placed around a round table. It is observed that

A is between D and F

C is opposite to D

D and E are not on neighbouring chairs.

Which one of the following must be true? [IAS]

 - (a) A is the opposite to B
 - (b) D is the opposite to E
 - (c) C and B are neighbours
 - (d) B and E are neighbours
3. A, B, C, D, E and F, not necessarily in that order are sitting on six chairs regularly placed around a round table. It is observed that

A is between D and F

C is opposite to D and

D and E are not on neighbouring chairs.

Which one of the following pairs must be sitting on neighbouring chairs? [IAS]

 - (a) A and B
 - (b) C and E
 - (c) B and F
 - (d) A and C
4. A, B, C, D, E, F, and G are members of a family consisting of 4 adults and 3 children, two of whom, F and G are girls. A and D are brothers and A is a doctor. E is an engineer married to one of the brothers and has two children. B is married to D and G is their child. Who is C? [IAS]
 - (a) G's brother
 - (b) F's father
 - (c) E's daughter
 - (d) A's son.
5. Seven persons P, Q, R, S, T, U, and V participate in and finish all the events of a series of swimming races. There are no ties at the finish of any of the events. V always finishes somewhere ahead of P. P always finishes somewhere ahead of Q. Either R finishes first and T finishes last or S finishes first and U or Q finishes last. If in a particular race V finished fifth, then which one of the following would be true? [IAS]
 - (a) S finishes first
 - (b) R finishes second
 - (c) T finishes third
 - (d) R finishes fourth
6. Consider the following:
 1. Saxena, David, Jain and Kumar were District Collectors at places P, Q, R and S respectively in 1970.
 2. In 1972 they were transferred, Saxena and Jain interchanged places. Kumar and David also interchanged places.
 3. One year later in 1973 they were again transferred such that David and Jain interchanged places and Saxena and Kumar were also interchanged.

What should be the next round of transfers so that all the four persons could have been posted at all the four places? [IAS]

 - (a) Interchange Saxena and David as well as Jain and Kumar.
 - (b) Interchange Saxena and Kumar as well as David and Jain.
 - (c) Interchange David and Kumar as well as Saxena and Jain
 - (d) It is not possible for all the four persons to have been posted at all the four places.

7. In a group of six women, there are four dancers, four vocal musicians, one actress and three violinists. Girija and Vanaja are among the violinists while Jalaja and Shailaja do not know how to play the violin. Shailaja and Thanuja are among the dancers, Jalaja, Vanaja, Shailaja and Thanuja are all vocal musicians and two of them are also violinists. If Pooja is an actress, who among the following is both a dancer and a violinist? [IAS]
 (a) Jalaja (b) Shailaja (c) Thanuja (d) Pooja
8. In a group of six women there are four table tennis players, four postgraduates in economics, one postgraduate in commerce and three bank employees. Vimala and Kamala are among the bank employees while Amala and Komala are unemployed. Komala and Nirmala are among the table tennis players. Amala, Kamala, Komala and Nirmala are postgraduates in economics of whom two are bank employees. If Shyamala is a postgraduate in commerce who among the following is both a table tennis player and a bank employee? [IAS]
 (a) Nirmala (b) Vimala (c) Amala (d) Komala
9. A is seated between D and F at a round table. C and not E is seated opposite to D. Who sits opposite to B? [IAS]
 (a) A (b) D (c) C (d) F
10. There are 5 bus stops A, B, C, D, and E at equal intervals. C is not the middle stop. A and E are not terminal stops. C comes twice as many stops before D in upward journey as B comes after A. D is the first stop in the downward journey. Give the correct sequence of the stops in the downward journey [IAS]
 (a) DEACB (b) DAECB (c) DACEB (d) DCBAE
11. In a group of 5 people, K, L, and M are ambitious. M, N, and R are honest, L, M, and N are intelligent and K, N, and R are industrious. Among these, neither industrious nor ambitious persons would include [IAS]
 (a) K alone (b) L and R (c) M and N (d) None in the group
12. Six roads lead to a country. They may be indicated by letters X, Y, Z, and digits 1, 2, 3. When there is storm, Y is blocked. When there are floods X, 1 and 2 will be affected. When road 1 is blocked, Z is also blocked. At a time when there are floods and a storm also blows, which road(s) can be used? [IAS]
 (a) Z and 2 (b) Only Z (c) Only 3 (d) Only Y
13. A truck, a car and a motor cycle have equal kinetic energies. If equal stopping forces are applied and they stop after travelling a distance of X, Y, and Z respectively, then [IAS]
 (a) $X > Y > Z$ (b) $X < Y < Z$ (c) $X = Y = Z$ (d) $X = 4 Y = 8 Z$
14. Seven men A, B, C, D, E, F and G are standing in a queue in that order. Each one is wearing a cap of a different colour like violet, indigo, blue, green, yellow, orange and red, D is able to see in front of him green and blue, but not violet. E can see violet and yellow, but not red. G can see caps of all colours other than orange. If E is wearing an indigo coloured cap, then the colour of the cap worn by F is [IAS]
 (a) Blue (b) Violet (c) Red (d) Orange

In a group of 5 persons A, B, C, D, and E, there is a professor, a businessman and an artist. A and D are ladies who are unmarried and do not work. Of the married couple in the group E is the husband. B is the brother of A and is neither a businessman nor an artist. E's wife is an artist.

15. Who is the professor?
 (a) A (b) B (c) C (d) D
16. Who is the artist?
 (a) A (b) B (c) C (d) E

17. Who is the wife of E?
 (a) A (b) B (c) C (d) D
18. Which of the following groups include all the men?
 (a) BE (b) ABC (c) BCD (d) None of these

A, B, C, D, and E are five persons standing in a bus queue. A is a teacher standing to the right of D who is an advocate. B is a scientist and is standing to the left of E who is an electrician. In between B and A have two persons. C who is a banker stands to the right of A.

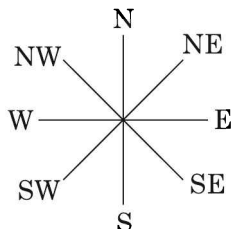
19. Who is in the middle?
 (a) A (b) B (c) C (d) D
20. Who is to the extreme left?
 (a) A (b) B (c) C (d) D
21. Who is to the extreme right?
 (a) A (b) B (c) C (d) D
22. How many persons are there to the right of E?
 (a) 4 (b) 3 (c) 2 (d) 1
23. Six persons A, B, C, D, E, and F are standing in a circle. B is between D and C. A is between E and C. F is to the right of D. Who is between A and F.
 (a) B (b) C (c) D (d) E
24. 300 persons are participating in a meeting out of which 120 are foreigners and the rest are Indians. Out of the Indians, there are 110 men who are not judges, 160 are men or judges, and 35 are women judges. There are no foreign judges. How many Indian women attended the meeting?
 (a) 45 (b) 55 (c) 35 (d) 60
25. In an examination question paper, there are 5 items each under list A and list B. The examinees are requested to match each item under list A with its corresponding item under list B. Further it was given that
 (i) No examinee has given the correct answer.
 (ii) Answers of no two examinees were identical. Find the number of examinees attended the test paper.
 (a) 24 (b) 26 (c) 119 (d) 129

ANSWERS

1. (b) 2. (d) 3. (b) 4. (d) 5. (a) 6. (c) 7. (c) 8. (a) 9. (a) 10. (a)
 11. (d) 12. (c) 13. (a) 14. (c) 15. (b) 16. (d) 17. (c) 18. (a) 19. (d) 20. (b)
 21. (c) 22. (b) 23. (d) 24. (b) 25. (a)

Direction Sense Test

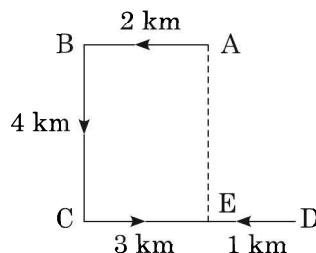
In this test, questions consist of a sort of direction puzzle. A successive follow-up of directions is formulated and the candidate is required to ascertain the final direction or the distance between two points. The test is meant to judge the candidate's ability to trace and follow correctly and sense the direction correctly.



EXAMPLE 1 Ravi walked 2 km west of his house and then turned south covering 4 km. Finally, he moved 3 km towards east and then again 1 km west. How far is he from his initial position?

- (a) 2 km (b) 4 km (c) 9 km (d) 10 km

Solution Ravi started from his house at A, moves 2 km west up to B, 4 km towards south up to C, 3 km east up to D and finally 1 km west up to E. Thus, his distance from the initial position $A = AE = BC = 4$ km. Hence the answer is (b).



EXAMPLE 2 A man walks 6 km to the east and then turns to the south and walks 5 km. Again he turns to the east and walks 6 km. Next, he turns towards north and walks 10 km. How far is he now from his starting point?

- (a) 5 km (b) 12 km (c) 13 km (d) 17 km (e) 11 km

Solution The man starts from the point A and walks 6 km, 5 km, 6 km and 10 km respectively towards east, south, east and north respectively.

Draw $BO \perp DE$

Now $BO = CD = 6$ km

$\therefore AO = AB + BO = 6 + 6 = 12$ km

$OE = DE - DO = 10 - 5 = 5$ km

$\therefore$ Man's distance from his starting point

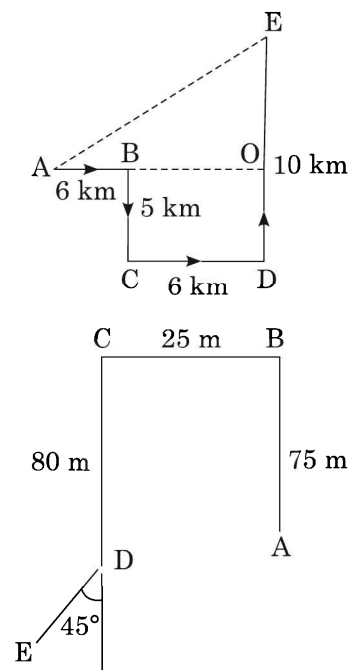
$$AE = \sqrt{AO^2 + OE^2} + \sqrt{12^2 + 5^2} = \sqrt{169} = 13 \text{ km}$$

Hence the answer is (c).

EXAMPLE 3 Arun moved a distance of 75 metres towards north. He then turned to the left and walked for about 25 metres, turned left again and walked 80 metres. Finally he turned to the right at an angle of 45° . In which direction was he moving finally?

- (a) North-east (b) North-west (c) South
(d) South-east (e) South-west

Solution Arun started from A, moved 75 m towards north up to B, turned left and walked 25 m up to C. He then turned left again and moved 80 m up to D. Turning to the right at an angle of 45° , she was finally moving in the direction DE, i.e. south-west. Hence, the answer is (e).



EXAMPLE 4 Sathesh left for his office in his car. He drove 15 km towards north and then 10 km towards west. He then turned to the south and covered 5 km. Further, he turned to the east and moved 8 km. Finally, he turned right and drove 10 km. How far and in which direction is he from his starting point?

- (a) 2 km west (b) 5 km east (c) 3 km north
(d) 6 km south (e) None of these

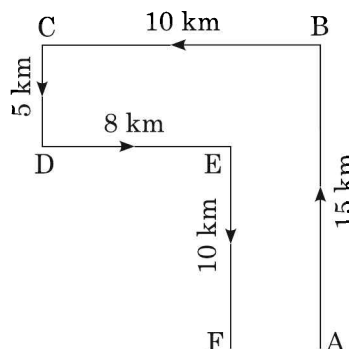
Solution Sathesh started from A towards B and moved a distance of 15 km and 10 km from B to C towards west. He then moved from C to D, 5 km towards south and 8 km towards E. Finally he turned right and moved a distance of 10 km up to F.

Now F and A are in the same straight line and F lies to the west of A.

$\therefore$ Distance from starting point A

$$= AF = BC - DE = 10 - 8 = 2 \text{ km}$$

Hence, the answer is (a).



EXERCISE 1

1. Kishore walks 10 km towards north. Then he walked 6 km towards south. Then he walks 3 km towards east. How far and in which direction is he with reference to his starting point?

- (a) 7 km east (b) 5 km west (c) 5 km north east (d) 7 km west

2. A man leaves for his office from his house. He walks towards east. After moving a distance of 20 m, he turns towards south and walks 10 m. Then he walks 35 m towards west and further 5 m towards north. He then turns towards east and walks 15 m. What is the straight distance in metres between his initial and final positions?
- (a) 0 (b) 5 (c) 10
(d) None of these (e) Can't be determined
3. James started from his house towards west. After walking a distance of 30 m, he turned towards right and walked 20 m. He then turned left and moved a distance of 10 m, turned to his left again and walked 40 m. He now turns to the left and walks 5 m. Finally, he turns to his left. In which direction is he walking now?
- (a) North (b) South (c) East (d) South-west (e) West
4. Sudheesh walked 10 metres towards south. He turned left and walked 20 metres and then moved to his right. Then he turned right and walked 20 metres. Finally he turned right and moved a distance of 10 metres. How far and in which direction is he from the starting point?
- (a) 10 m north (b) 20 m south (c) 20 m north (d) 10 m south (e) None of these
5. A man walked 30 m towards south. Then, turned to his right and walked 30 m. Then, turned to his left, he walked 20 m. Again, he turned to his left, walked 30 m. How far is he from his initial position?
- (a) 30 m (b) 20 m (c) 80 m (d) 60 m (e) None of these
6. Gopal started walking towards east. After moving a distance of 1 km, he turned southward and walked 5 km. Again, he turned to east and walked 2 km. Finally, he turned to the north and walked 9 km. How far is he from his starting point?
- (a) 7 km (b) 5 km (c) 4 km (d) 3 km (e) None of these
7. Sumesh went 15 km to the north from his house. Then he turned west and covered 10 km. Then he turned south and covered 5 km. Finally, turned to east, he covered 10 km. In which direction is he from his house?
- (a) East (b) West (c) North (d) South
8. After walking a distance of 50 m to the south of her house, Lakshmi turns left and walks another 20 m. Then she turns north and walks 30 m and then starts walking to her house. In which direction is she walking now?
- (a) North-west (b) North (c) South-east (d) East
9. Dinesh walks 20 m north. Then he turns right and walks 30 m. Now he turns right and walks 35 metres. Now, turning left, he walks 15 m. Again, he turns left and moves 15 metres. Finally, turning left, he again walks 15 metres. In which direction and how far is he from his original position?
- (a) 15 m east (b) 45 m east (c) 15 m west (d) 45 m west (e) None of these
10. Anil's house faces east. From the back-side of the house, he walks straight 50 metres; then turns to the right and walks 50 m again. Finally, he turns towards left and stops after walking 25 m. Now Anil is in which direction from the starting point?
- (a) South-east (b) North-east (c) South-west (d) North-west (e) None of these

ANSWERS

1. (c) 2. (b) 3. (a) 4. (b) 5. (e) 6. (b) 7. (c) 8. (a) 9. (e) 10. (d)

Explanations

1. The movements of Kishore are shown in the adjoining figure.

(A → B → C → D)

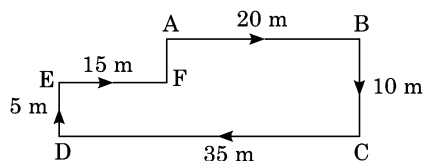
$$AC = AB - BC = 10 - 6 = 4 \text{ km}$$

$$CD = 3 \text{ km}$$

$$\therefore AD = \sqrt{AC^2 + CD^2} = \sqrt{4^2 + 3^2} = 5 \text{ km}$$

So, Kishore is 5 km north-east.

2. Movements of man shown in the adjoining figure.



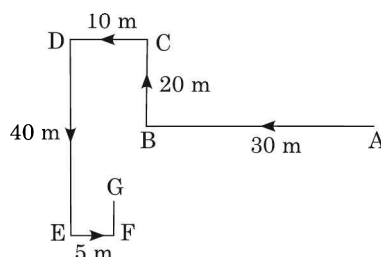
Here, $DC = AB + EF$

$\therefore$ F is in a line with A.

$$AF = BC - ED = 10 - 5 = 5 \text{ m}$$

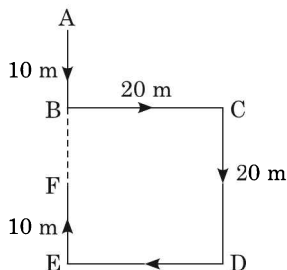
$\therefore$ The man is 5 metres away from his initial position.

3. The movements of James are shown in the adjoining figure from A to G.



James is walking in the direction FG, i.e. towards north.

4. The movement of Sudheesh are shown in the adjoining figure.

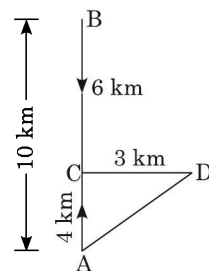


Distance from initial position of Sudheesh to final position

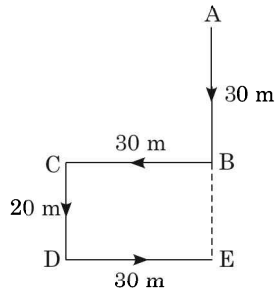
$$\begin{aligned} AF &= AB + BF \\ &= AB + (BE - FE) \\ &= 10 + (20 - 10) \\ &= 20 \text{ m} \end{aligned}$$

F lies in the south of A.

$\therefore$ 20 m south is the final position of Sudheesh from the starting point.

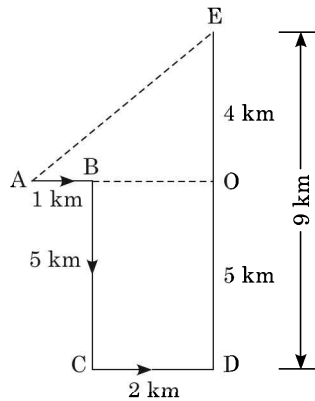


5. The movements of man are shown in the adjoining figure.



$$\begin{aligned}\therefore \text{Man's distance from the initial position} &= AE = AB + BE \\ &= AB + CD = 30 + 20 = 50 \text{ m.}\end{aligned}$$

6. The movements of Gopal are shown in the adjoining figure.



$$BO \perp DE$$

$$BO = CD = 2 \text{ km}$$

$$AO = AB + BO = 1 + 2 = 3 \text{ km}$$

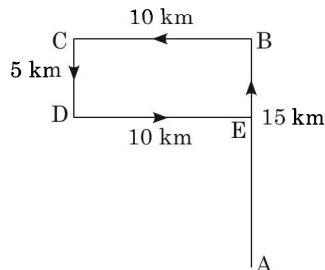
$$OE = DE - DO = DE - BC$$

$$= 9 - 5 = 4 \text{ km}$$

$\therefore$ Distance of Gopal from starting point A.

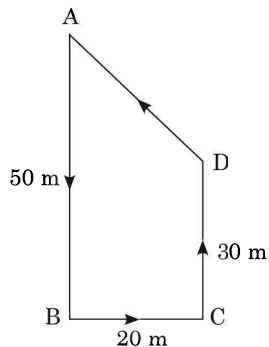
$$= AE = \sqrt{AO^2 + OE^2} = \sqrt{3^2 + 4^2} = 5 \text{ km}$$

7. The movements of Sumesh are shown in the adjoining figure.



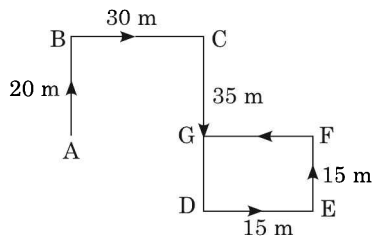
The final position E is north of Sumesh's house at A.

8. The movements of Lakshmi are shown in the adjoining figure.



She is moving in the direction DA towards her house.
i.e. north-west.

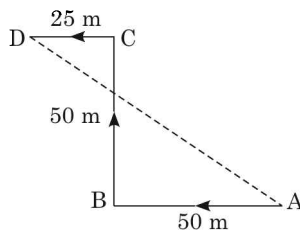
9. The movements of Dinesh are shown in the adjoining figure.



He is moving from A to G.

$\therefore$ Distance of Dinesh from initial position = AG = BC = 30 m east.

10. The movements of Anil are shown in the adjoining figure.



He starts walking from back of his house, i.e. towards west.

Now, the final position is D, which is to the north west of his starting point A.

EXERCISE 2

- Binoj's school bus is facing north when it reaches his school. After starting from Binoj's house, it turns right twice and then left before reaching the school. What direction was the bus facing when it left the bus stop in front of Binoj's house. **[Bank PO]**
 (a) South (b) North (c) East (d) West (e) None of these
- A, B, C, and D are playing cards. A and B are partners. D faces towards north. If A faces west, then who faces south? **[AO Exam]**
 (a) C (b) B (c) D
 (d) Data inadequate (e) None of these

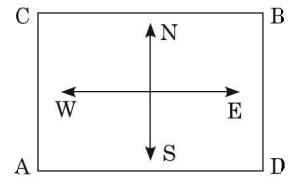
3. There are four persons P, Q, R and T. Q is to the south-west of P; R is to the east of Q and south-east of P, and T is to the north of R in line with QP. In which direction of P is T located?
(a) South-east (b) North (c) North-east (d) East
4. A and B start moving towards each other from two places 200 m apart. After walking 60 m, B turns left and goes 20 m, then he turns right and goes 40 m. He then turns right again and comes back to the road on which he had started walking. If A and B walk with the same speed, what is the distance between them now?
(a) 50 m (b) 40 m (c) 30 m (d) 20 m
5. Two ladies and two men are playing cards and are seated at north, east, south and west of a table. No lady is facing east. The persons sitting opposite to each other are not of the same sex. One man is facing south. Which directions are the ladies facing?

[RBI Officer's Exam]

- (a) East and west (b) South and east (c) North and east
(d) North and west (e) None of these
6. One morning after sunrise, Vishakh and Satheesh were standing in a lawn with their backs towards each other. Vishakh's shadow fell exactly towards left hand side. Which direction was Satheesh facing?
(a) East (b) West (c) North (d) South
7. The post office is to the east of the school while my house is to the south of the school. The market is to the north of the post office. If the distance of the market from the post office is equal to the distance of my house from the school, in which direction is the market with respect to my school?

[AAO Exam]

- (a) North (b) East (c) North-east (d) South-west
8. From the positions in original figure, A and C move diagonally to opposite corners and then one side each clockwise and anticlockwise respectively. D and B move two sides each clockwise and anticlockwise respectively. Where is A now?
(a) At the north-west corner
(b) At the north-east corner
(c) At the south-east corner
(d) At the south-west corner
(e) Midway between original position of B and D.
9. From the original position given in the above figure, A and B move one arm length clockwise and then cross over to the corner diagonally opposite, C and D move one arm length anticlockwise and cross over the corner diagonally opposite. The original configuration ADBC has now changed to
(a) CBDA (b) BDAC (c) DACB (d) ACBD (e) BCAD
10. Facing the east, Rajesh turned left and walked 10 m, then turned to his left again and walked 10 m. He then turned 45° towards his right and went straight to cover 25 m. In which direction from his starting point is he?
(a) South-west (b) South-east (c) North-west (d) North-east (e) East

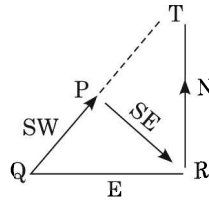
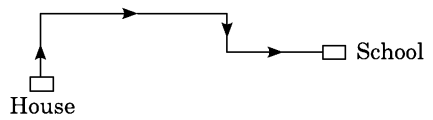


ANSWERS

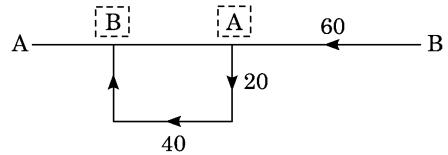
1. (d) 2. (a) 3. (c) 4. (b) 5. (d) 6. (d) 7. (c) 8. (c) 9. (c) 10. (d)

Explanations

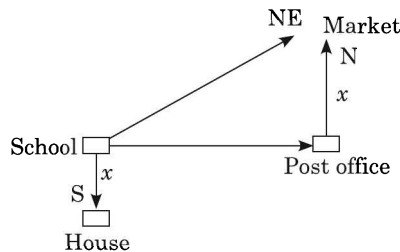
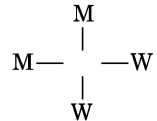
1. In the given diagram, the route of the bus from Binoj's house to the school is shown. As the bus faces north on reaching the school, from the diagram, it faces west in front of Binoj's house.
2. As per the data, D faces north. A faces towards west. So, its partner B will face towards A and hence towards east. So, C who will face D will face south.
3. Clearly, the arrangement according to the given directions is as shown. So, comparing the direction diagram, T is north east of P.



4. If both A and B move with the same speed, they will travel the same distance = $60 + 20 + 40 + 20 = 140$ m in the same time. So, A will travel 140 m in the horizontal direction. B travels 100 m horizontally. So distance between A and B will now be the difference = $140 - 100 = 40$ m.

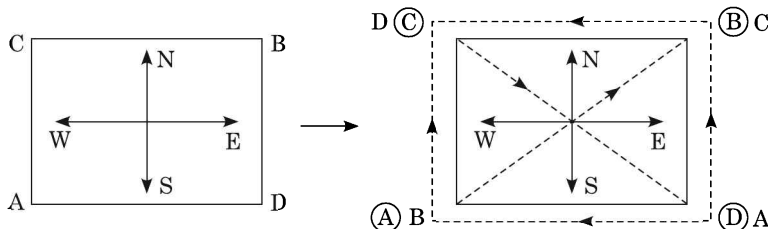


5. No lady is facing east means a man faces east. The persons opposite are not of same sex. So a woman will be facing west. Again a man faces south. So a woman will be facing north.
6. Since Vishakh's shadow falls towards left, therefore, Vishakh is facing north. So Satheesh with his back towards Vishakh, will be facing south.
7. Distance of house from school = x
= Distance of market from post office

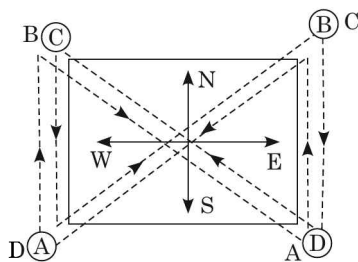


So, direction of market school with respect to school = **North-east**

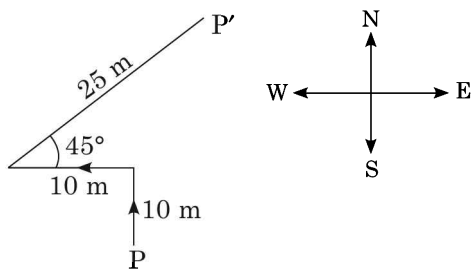
8. A is now at south-east corner.



9. ADBC is changed to DACB.



10. P' is North-east of P



Logical Venn Diagrams

This chapter deals with questions which aim at analysing a candidate's ability to relate a certain given group of items and illustrate it diagrammatically.

Certain types of Venn diagrams are given with its implication.

1. If *three* items are given belonging to three different groups, the same can be represented using Venn diagram as shown in Figure 7.1.

For example: Doctors, engineers, lawyers.

The above three bear no relationship with each other. So they are represented by 3 disjoint figures.

2. If *one* item belongs to the class of second and the second belongs to the class of third, then these items can be represented as three concentric circles as shown in Figure 7.2.

For example: Seconds, minutes, hours.

Here, seconds are part of minute and minutes are part of hours. So the Venn diagram would be as shown in Figure 7.2 with circle A representing seconds, B representing minutes and C representing hours.

3. If *two* separate items belong to the class of the third, they are represented by two disjoint circles inside a bigger circle as shown in Figure 7.3.

For example: Table, chair, furniture.

Here, table and chair are separate items both belonging to the class of furniture. So they would be represented as two disjoint circles A and B respectively representing table and chair within a circle C representing furniture.

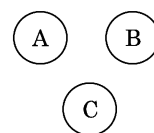


Figure 7.1

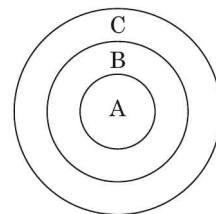


Figure 7.2

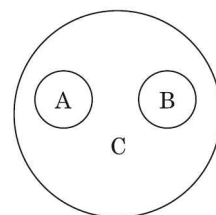


Figure 7.3

4. If *two* items belong to the class of third such that some items of each of these two groups are common in relationship, then they are represented by two intersecting circles enclosed within a bigger circle as shown in Figure 7.4.

For example: Males, Fathers, Brothers.

Here, some fathers may be brothers. So, fathers and brothers would be represented by two intersecting circles A and B. Also, both fathers and brothers are males. The bigger circle C represents males.

5. If *two* items are partly related to the third, and are themselves independent of each other they are represented by three intersecting circles in a line as shown in Figure 7.5.

For example: Dogs, Pets, Cats.

Here, some dogs and some cats are pets. But all the pets are not dogs or cats. Dogs and cats are not related to each other. So the given three items can be represented using three circles A, B, and C representing dogs, pets and cats respectively as shown in Figure 7.5.

6. If *three* items are partly related to each other, they are represented as shown in Figure 7.6.

For example: Clerks, Government employees, Educated persons.

Here, some clerks may be government employees and some may be educated. Similarly, some government employees may be clerks and some may be educated. Also, some educated persons may be clerks and some may be government employees.

7. If *one* item belongs to the class of second while the third item is entirely different from the two, then they may be represented by Figure 7.7.

For example: Engineers, Human beings, Rats.

Here, all engineers are human beings. This would be represented by two concentric circles. But the class of rats is entirely different from these two. In Figure 7.7, circle A represents engineers, circle B represents human beings and circle C represents rats.

8. If *one* item belongs to the class of second and the third item is partly related to these two, they can be represented as in Figure 7.8.

For example: Females, Mothers, Doctors.

All mothers are females. This would be represented by two concentric circle. But some females and some mothers can be doctors. So the circle representing doctors would intersect the two concentric circles. Thus in Figure 7.8, circle A represents mothers, circle B represents females and circle C represents doctors.

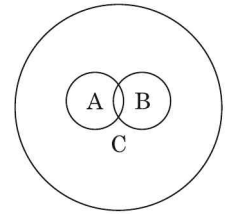


Figure 7.4

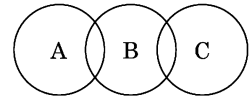


Figure 7.5

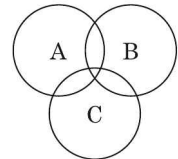


Figure 7.6

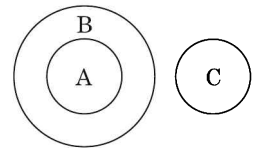


Figure 7.7

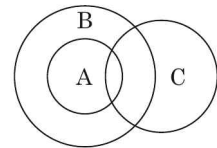


Figure 7.8

9. If *one* item belongs to the class of second and the third item in partly related to the second, they can be represented as in Figure 7.9.

For example: Grass-eating animals, cows, flesh eating animals.

Here, cows are grass-eating animals. So, they would be represented by two concentric circles. But some grass-eating animals are flesh-eating also. Thus the Venn diagram can be represented as in Figure 7.9 with circle A representing cows, circle B representing grass-eating animals and circle C representing flesh eating animals.

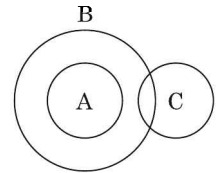
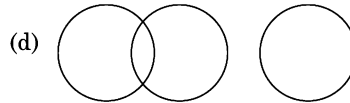
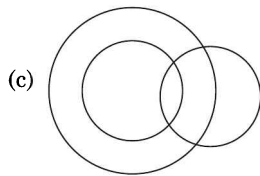
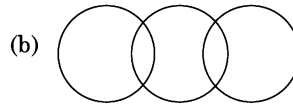
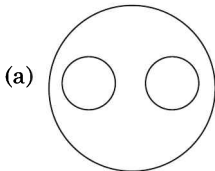


Figure 7.9

EXERCISES

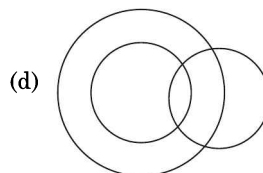
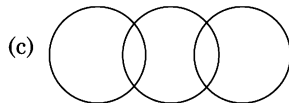
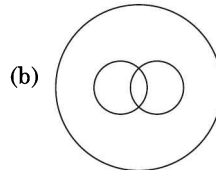
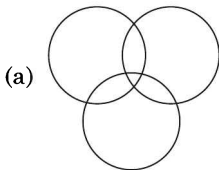
1. Which of the following diagrams correctly represents elephants, wolves, animals?

[IAS]



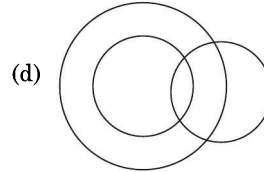
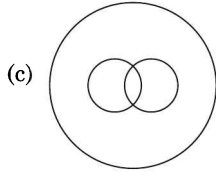
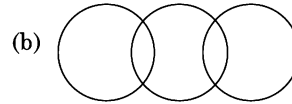
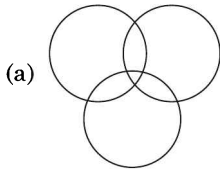
2. Which one of the following diagrams correctly represents the relationship among tennis fans, cricket players and students?

[IAS]

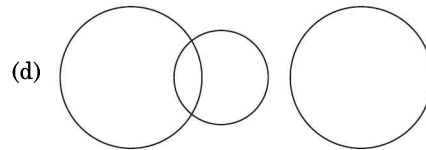
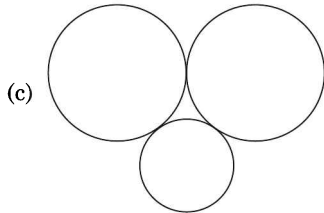
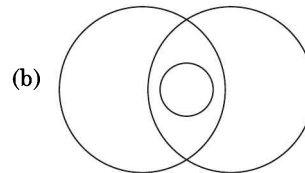
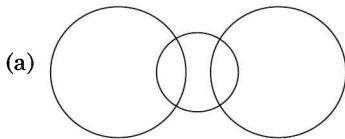


3. Which is the most suitable Venn diagram among the following, which represents interrelationship among anti-social elements, pickpockets and blackmailers?

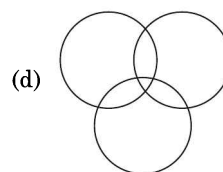
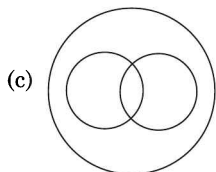
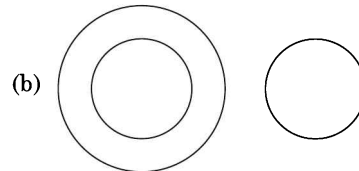
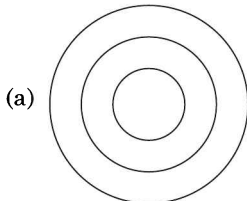
[IAS]



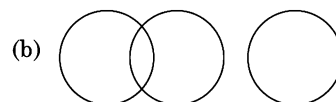
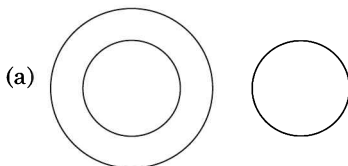
4. If animals that live on land and the animals that live in water are represented by two big circles and animals that live in water and on land are represented by small circle, the combination of these three can be represented as [IAS]



5. Which of the following gives the proper relation of tall men, black haired people, Indians? [IAS]



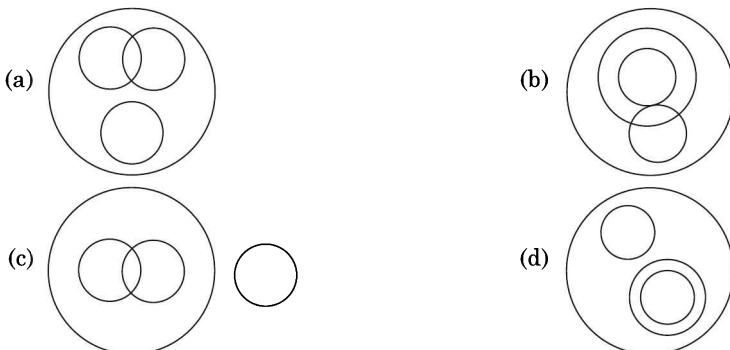
6. Choose from the four diagrams given below, the one that illustrates the relationship among languages, French, German.





7. In a dinner party both fish and meat were served. Some took only fish and some only meat. There were some vegetarians who did not accept either. The rest accepted both fish and meat. Which one of the following logic diagrams correctly reflects the above situation?

[IAS]

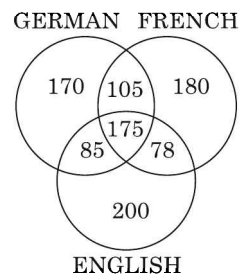


8. A survey was conducted on a sample of 1,000 persons with reference to their knowledge of English, French and German. The results of the survey are presented in the given Venn diagram. The ratio of the number of the persons who do not know any of the three languages to those who know all the three languages is

[IAS]

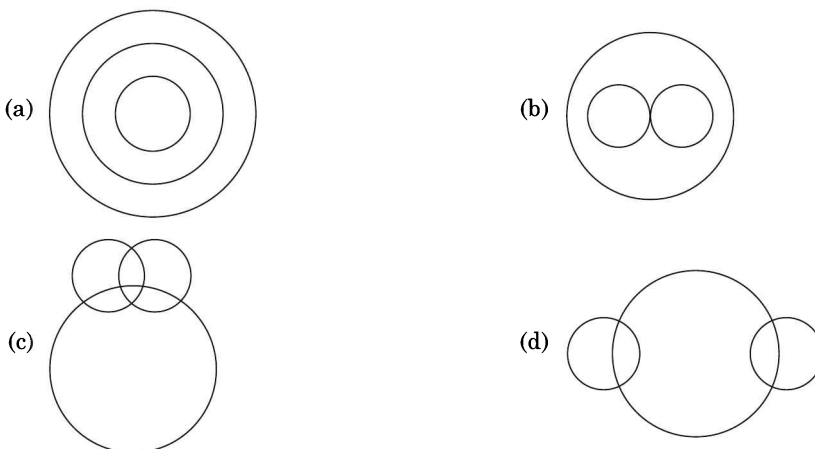
- (a) $1/27$
(c) $1/550$

- (b) $1/25$
(d) $175/1000$



9. Which one of the following Venn diagrams correctly illustrates the relationship among the classes: carrot, food, vegetable?

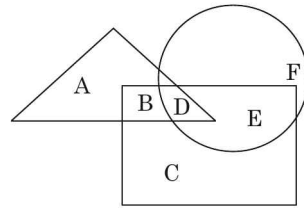
[IAS]



10. In the given figure, the triangle represents girls, the square represents sports persons and the circle represents coaches. The portion in the figure which represents girls who are sports persons but not coaches is the one labelled.

[IAS]

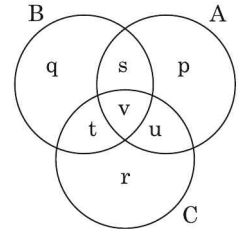
- (a) A (b) B
(c) D (d) E



11. In the given diagram, circle A represents teachers who can teach Physics, circle B represents teachers who can teach Chemistry and circle C represents those who can teach Mathematics. Among the regions marked p, q, r ... the one which represents teachers who can teach Physics and Mathematics but not Chemistry is

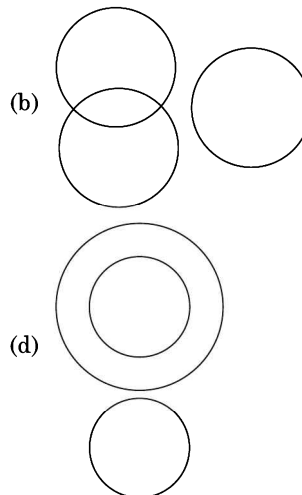
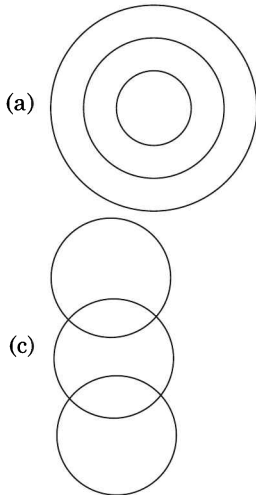
[IAS]

- (a) v (b) u
(c) s (d) t

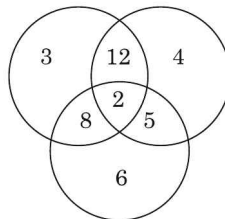


12. Which one of the following four logical diagrams represent correctly the relationship between Musicians, Instrumentalists, Violinists?

[IAS]



13. Consider the Venn diagram given below.

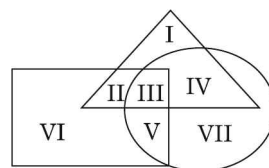


The numbers in the Venn diagram indicated are persons reading the newspapers. The diagram is drawn after surveying 50 persons. In a population of 10,000 how many can be expected to read at least two newspapers? Choose the correct answer from the following:

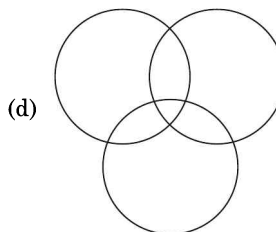
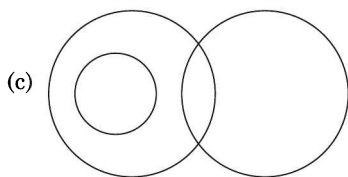
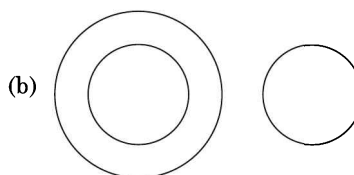
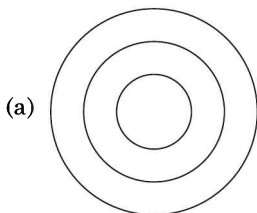
[IAS]

- (a) 5000 (b) 6250 (c) 6000 (d) 5400

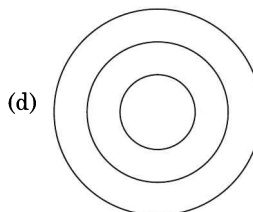
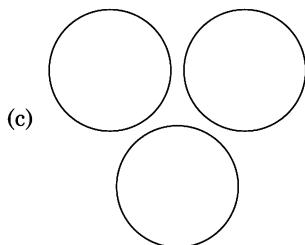
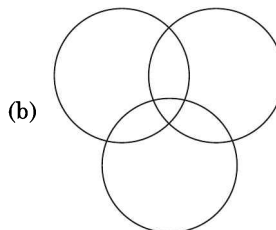
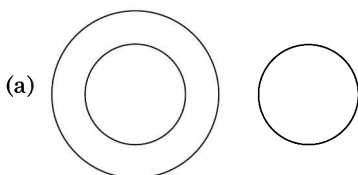
14. The triangle, square and circle shown below respectively represent the urban, hard working and educated people. Which one of the areas marked I–VII is represented by the urban educated who are not hardworking? [IAS]



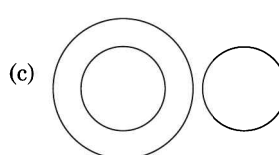
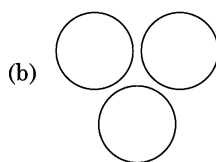
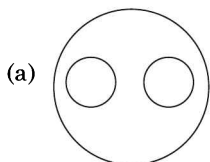
- (a) II
(b) I
(c) IV
(d) III
15. Which one of the following Venn diagrams best illustrates the three classes: rhombus, quadrilaterals and polygons?

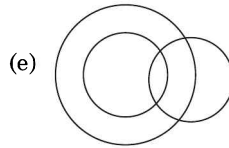
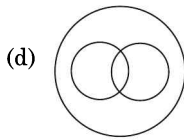


16. Select from the four alternative diagrams, the one that best illustrates the relationship among the three classes: pigeons, birds, dogs.



In each of the following questions, choose the Venn diagram which best illustrates the relationship among three given circles.





17. Diseases, leprosy, scurvy
18. Hockey, cricket, games
19. Yak, zebra, bear
20. Sun, moon, stars
21. Animals, men, plants
22. Mercury, mars, planets

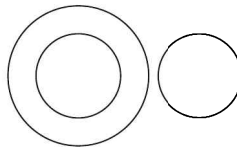
Choose the Venn diagram which best illustrates the three given classes in each question. Mark your answer as (a), (b), (c) or (d).

23. Protons, electrons, atoms
24. Sun, planets, earth
25. Dog, animal, pet
26. Science, physics, chemistry
27. Atmosphere, hydrogen, oxygen
28. Wheat, grains, maize
29. Machine, lathe, mathematics
30. Biology, botany, zoology

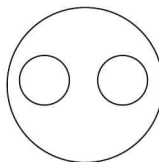
[CBI]

[Railways]

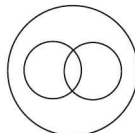
- (a) Indicates that one class is completely contained in the other but not the third.



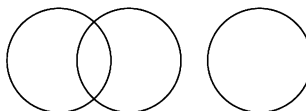
- (b) Indicates that two classes are completely contained in the third.



- (c) Indicates that neither class is completely contained in the other but the two have common members, forming one entity.



- (d) Indicates that two classes are inter-related and third one is not.

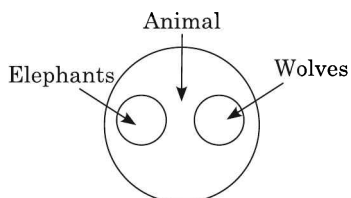


ANSWERS

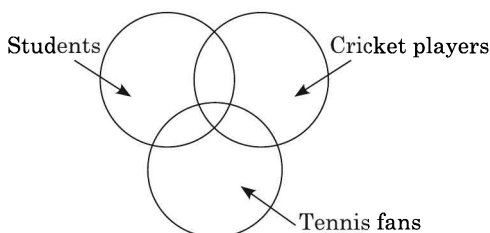
1. (a) 2. (a) 3. (c) 4. (a) 5. (d) 6. (c) 7. (c) 8. (b) 9. (a) 10. (b)
 11. (b) 12. (d) 13. (d) 14. (c) 15. (a) 16. (a) 17. (a) 18. (a) 19. (b) 20. (c)
 21. (c) 22. (a) 23. (b) 24. (a) 25. (c) 26. (b) 27. (b) 28. (b) 29. (a) 30. (b)

Explanations

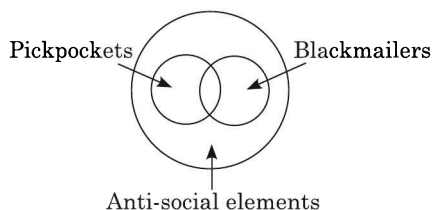
1. Elephants and wolves bear no relationship to each other. But, both of them are animals.



2. Some students can be cricket players. Some cricket players can be tennis fans. Some students can be tennis fans. So, the given items are partly related to each other.

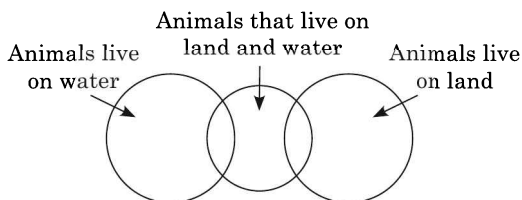


3. Both pickpockets and blackmailers are antisocial elements. But, some pickpockets can be blackmailers and vice versa.

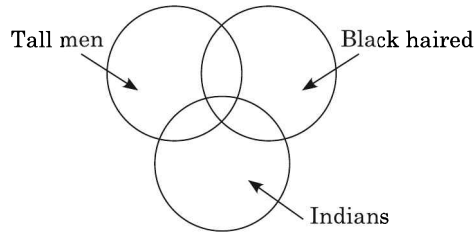


4. Here three classes are given. Two classes are not having any common characteristics. One class is having some common characteristic with the other two.

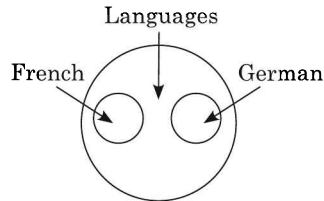
That is, 'Animals live on both land and water' is having common characteristic with the classes 'Animals live on land' and 'Animals live on water'.



5. Some tall men can be black haired. Some black haired persons can be Indians. Some tall men can be Indians. So, all the three are partly interrelated.

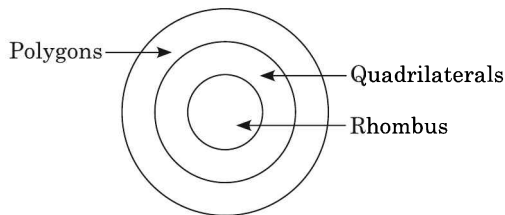


6. Both French and German are languages. But both of them are different from each other.

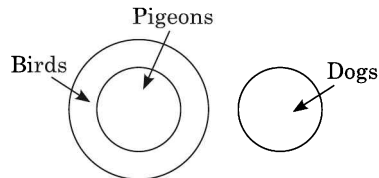


8. Required ratio = $\frac{7}{175} = \frac{1}{25}$

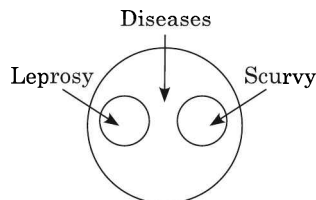
12. In the given three classes Musicians, Instrumentalists and Violinists,—Violinists come under the class of Instrumentalists. Musicians are different from two classes. Sometimes a musician may also be an Instrumentalist. But from the available choices only (d) is the most appropriate one.
15. All rhombus are quadrilaterals. All quadrilaterals are polygons.



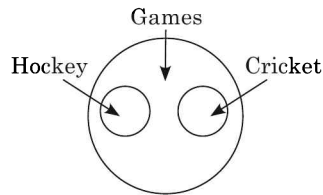
16. All pigeons are birds. But, dogs are entirely different.



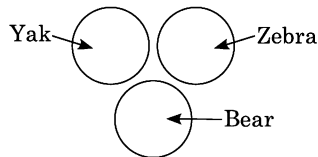
17. Both leprosy and scurvy are diseases. But, both are entirely different from each other.



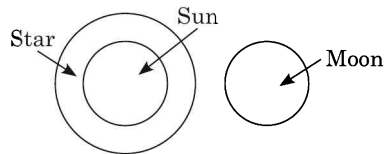
18. Both hockey and cricket are games. But, both are entirely different from each other.



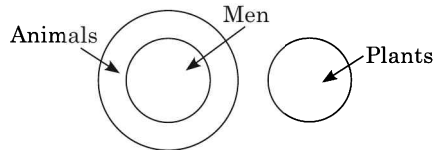
19. Yak, zebra and bear are all different from each other.



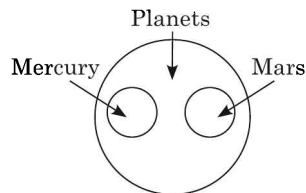
20. Sun is a star, moon is entirely different from the two.



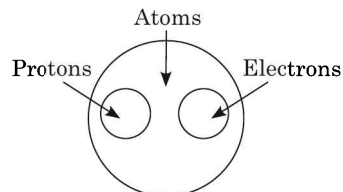
21. Men belongs to the class of animals. Plants are entirely different from the two.



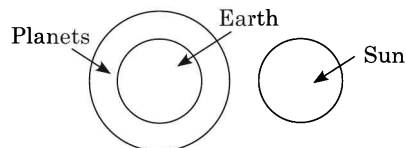
22. Mercury and mars are entirely different from each other. But both are planets.



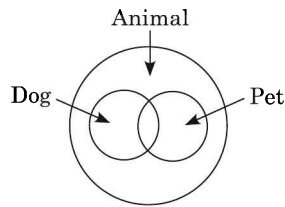
23. Protons and electrons are entirely different from each other. But, both are parts of atoms.



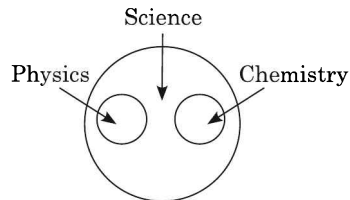
24. The earth belongs to the class of planets. But, sun is entirely different from the two.



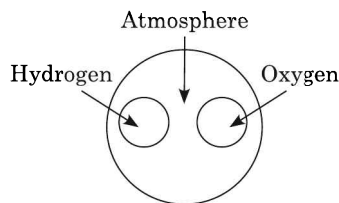
25. Some dogs are pets and some pets are dogs. Both dog and pet are animals.



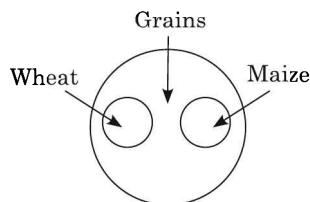
26. Physics and chemistry are entirely different from each other. But, both belong to the class of science.



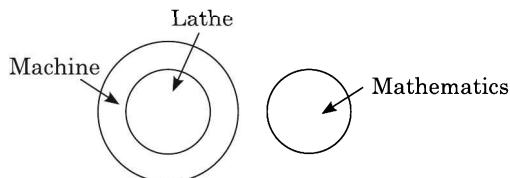
27. Hydrogen and oxygen are entirely different from each other. But, both are parts of atmosphere.



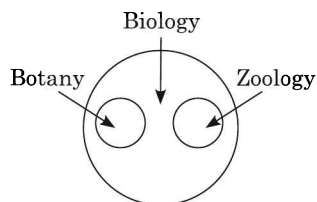
28. Wheat and maize are two different items. But, both belong to the class of grains.



29. Lathe is a type of machine. But, Mathematics is entirely different from the two.



30. Botany and zoology are entirely different from each other. But, both are branches of biology.



Number Ranking and Time Sequence Test

NUMBER TEST: In the number test, you are given a series of numbers. You have to find out how many times a number satisfying the conditions specified in the question occurs.

EXAMPLE 1 How many 7s immediately preceded by 6 but not immediately followed by 4 are there in the following series? **[AO Exam]**

7 4 2 7 6 4 3 6 7 5 3 5 7 8 4 3 7 6 7 2 4 0 6 7 4 3

- (a) 1 (b) 2 (c) 4 (d) 6 (e) None of these

Solution Clearly, the numbers satisfying the given conditions can be shown as follows:

7 4 2 7 6 4 3 6 ⑦ 5 3 5 7 8 4 3 7 6 ⑦ 2 4 0 6 7 4 3

Here there are two such 7s in the number series which is preceded by 6 and not followed by 4. Therefore the answer is (b).

EXAMPLE 2 How many 1s are there in the following sequence which are immediately preceded by 9 but not immediately followed by 7? **[Bank PO]**

7 1 9 1 1 7 1 8 9 1 7 1 2 1 3 1 4 5 7 1 3 9 1 7

- (a) One (b) Two (c) Three (d) Four (e) None of these

Solution The answer is (a).

EXAMPLE 3 In the following series of numbers, how many times have the figures 9, 1 and 8 appeared together, 1 being in the middle and 9 and 8 being on either side of 1? **[RRB Exam]**

2 1 9 8 1 9 8 3 7 1 9 7 8 1 2 9 1 9 8 1 8 2 1 2

- (a) One (b) Six (c) Three (d) Four (e) None of these

Solution There is only one digit between 9 and 8. Therefore, the answer is (a).

RANKING TEST: In the ranking test, generally the ranks of a person both from the top and from the bottom are mentioned and the total number of persons is asked. However, sometimes the question is put in the form of a puzzle of interchanging seats by two persons.

EXAMPLE 4 Sajith ranked thirteenth from the top and twenty sixth from the bottom among those who have passed in the annual examination in a class. If six students have failed in the annual examination, what was the total number of students in that class?

[Bank PO]

- (a) 38 (b) 44 (c) 45 (d) 50 (e) None of these

Solution Number of students passed = $(12 + 1 + 25) = 38$

Number of students failed = 6

∴ Total number of students = $38 + 6 = 44$

EXAMPLE 5 Sanal's position in a row is 12th from the front side and 7th from the back side. How many persons are standing in that row? [Railway Recruitment]

- (a) 17 (b) 18 (c) 19 (d) 20 (e) 21

Solution Number of persons in the row = $11 + 6 + 1 = 18$

TIME SEQUENCE TEST: In the time sequence test, you will be asked to find a day which falls between two pair of dates. From that you have to locate the exact day.

EXAMPLE 6 The Managing Director entered the conference room ten minutes before 12.30 hours for interviewing. He came 20 minutes before the chairman who was 30 minutes late. At what time, the interviews were scheduled? [AO Exam]

- (a) 12.50 (b) 12.40 (c) 12.20 (d) 12.10 (e) 12.00 Noon

Solution The Managing Director came at 12.20. Thus chairman came at 12.40. Since the chairman was late by 30 minutes, interviews were scheduled to be held at 12.10. The answer is (c).

EXERCISES

1. How many 8s are there in the following number sequence which are preceded by 7 but not immediately followed by 4? [SBI PO]

2 3 8 2 5 7 8 3 7 8 4 6 9 8 4 3 2 7 8 9 5 7 8 1 5 2 9

- (a) One (b) Two (c) Three (d) Four (e) None of these

2. How many 3s are there in the following number sequence which are immediately preceded by 6 but not immediately followed by 7? [Bank PO]

2 3 7 4 3 5 6 3 7 4 6 3 8 9 6 3 5 1 8 3 7 2 4 2 8 6 3 9

- (a) One (b) Two (c) Three (d) Four (e) More than four

3. How many numbers from 11 to 50 are there which are exactly divisible by 7 but not by 3? [Bank PO]

- (a) Two (b) Four (c) Five (d) Six (e) Seven

4. How many numbers from 1 to 100 are there each of which is not only exactly divisible by 4 but also has 4 as a digit? [SBI PO]

- (a) 7 (b) 10 (c) 20 (d) 21 (e) More than 21

5. Abhishek ranks thirteenth in a class of thirty one. What is his rank from the last?

[RBI Exam]

- (a) 15th (b) 17th (c) 19th (d) 20th (e) None of these

6. In a row of girls, if Savitha who is tenth from the left and Anita who is ninth from the right interchange their places, Savitha becomes fifteenth from the left. How many girls are there in the row?

[Bank PO]

- (a) 16 (b) 18 (c) 19 (d) 22 (e) None of these

7. How many days will there be from 26th January, 1988 to 15th May, 1988 (both days included)?

[Railway Recruitment]

- (a) 110 (b) 111 (c) 112 (d) 113 (e) None of these

8. Ram remembers that Lakshman's birthday is after 19th but before 22nd November, whereas Anil remembers that Lakshman's birthday is after 20th but before 24th November. Which day is Lakshman's birthday?

- (a) 20th November (b) 21st November (c) 22nd November
(d) 23rd November (e) None of these

9. In the series given below, count each 5 which is not immediately preceded by 3 but is immediately followed by 7. How many such 5s are there?

1 5 7 3 5 7 4 7 3 7 2 5 6 5 8 5 7 4 5 6 5 5 7 1 5 7 7 5 5

- (a) 1 (b) 2 (c) 3 (d) 4 (e) 5

10. Below is given a number series

1 8 5 7 2 9 8 4 3 6 2 7 5 1 8 9 4 3 6 5 9

How many instances are there in which an even number is followed by two odd numbers?

- (a) Nil (b) One (c) Two (d) Three (e) None of these

11. A number is greater than 3 but less than 8. Also, it is greater than 6 but less than 10. The number is _____.

- (a) 5 (b) 6 (c) 7 (d) 8 (e) 9

12. $16 \times 85 = 8651$, what is 73×42 ?

- (a) 4372 (b) 3723 (c) 4327 (d) 4732 (e) 2734

13. In a row of 16 boys, when Pramod was shifted by two places towards the left, he became 7th from the left end. What was his earlier position from the right end of the row?

- (a) 7th (b) 8th (c) 9th (d) 10th (e) None of these

14. A bus for Trivandrum leaves every 30 minutes from a bus stand. On enquiring, the clerk told a passenger that the bus had already left ten minutes ago and the next bus will leave at 9.35 am. At what time did the clerk give this information to the passenger?

- (a) 9.10 am (b) 8.55 am (c) 9.08 am (d) 9.05 am (e) 9.15 am

15. In a row of 21 girls, when Anita was shifted by four places towards the right, she became 12th from the left end. What was her earlier position from the right end of the row?

- (a) 9th (b) 10th (c) 11th (d) 12th (e) 14th

ANSWERS

1. (c) 2. (c) 3. (b) 4. (a) 5. (c) 6. (e) 7. (b) 8. (b) 9. (d) 10. (d)
11. (c) 12. (c) 13. (b) 14. (e) 15. (e)

Explanations

1. There are three such 8s which are preceded by 7 but not immediately followed by 4.
2. There are three such 3s which are immediately preceded by 6 but not immediately followed by 7.
3. No. of numbers which are divisible by 7 from 11 to 50 = $\frac{49 - 14}{7} + 1 = 6$. But out of these which are divisible by 3 = 21 and 42.
 $\therefore$ Required numbers = 14, 28, 35, 49.
 $\therefore$ Required no. of numbers = 4.
4. Numbers from 1 to 100 which are divisible by 4 and has 4 as a digit are 4, 24, 40, 44, 48, 64, 84.
 $\therefore$ Required number of numbers = 7.
5. Rank of Abhishek from last = $31 - 13 + 1 = 19$.
6. Savitha's present position = 15th from left = Anita's previous position = 9th from right
 $\therefore$ No. of girls in the row = $15 + 9 - 1 = 23$.
7. Number of days = $6 + 29 + 31 + 30 + 15 = 111$
 ($\because$ Number of days in February in the year 1988 is 29).
8. According to Ram, Lakshman's birthday is on one of the days among 20th and 21st November. According to Anil, Lakshman's birthday is on one of the days among 21st, 22nd and 23rd November.
 The day common in both is 21st November. Therefore, Lakshman's birthday is on 21st November.
9. There are four such 5s in the given series.
10. There are three such instances in which an even number is followed by two odd numbers.
11. As per the first condition, if x is the number, $8 > x > 3$. As per second condition $10 > x > 6$
 Taking two conditions together, $8 > x > 6$.
 $\therefore$ The number is 7.
12. Given, $16 \times 85 = 8651$
 Similarly $73 \times 42 = 4327$
 i.e. by placing from unit's place of product, digits in ten's place of first number, unit's place of second number, unit's place of first number and ten's place of second number.
13. 1 2 3 4 5 6 P 8 9 10 11 12 13 14 15 16
 $\uparrow$
 P denotes Pramod's new position.
 $\therefore$ His earlier position from right end of row = $16 - 9 + 1 = 8$
14. Bus leaves to Trivandrum every 30 minutes
 The last bus left 10 minutes before
 The next bus will leave at 9.35 am
 $\therefore$ Time at which information got = $9.35 - 0.30 + 0.10 = 9.15$ am
15. Anita's earlier position = $12 - 4 = 8$ th from left end.
 $\therefore$ Position from right = $21 - 8 + 1 = 14$ th

Decision Making

In this type of test, a vacancy is declared. The necessary qualifications required in the candidates coming up to fill the vacancy are provided and the merits of the candidates mentioned. The decision about each candidate has to be made from among the five choices named (a), (b), (c), (d), and (e) which state the courses of action to be taken as per the candidate's potentials.

EXAMPLE (1–5): Study the following information carefully and answer the questions given below it.

Following are the qualifications necessary for the recruitment of a librarian in a state university.

The candidate must:

1. have a master's degree in Library Science with at least 55% marks or the equivalent grade and a consistently good academic record.
2. have one year specialisation in an area of Information Technology/Archives and manuscript keeping master's degree in an area of thrust in the institution.
3. have at least ten year's experience as a deputy librarian in a university.
4. bear an evidence of innovative library service and organisation of published work.

In the case of a candidate who:

5. has a 15 year's experience as a college librarian, the case may be referred to the Vice Chancellor.
6. having obtained less than 55% marks in library science but has 13 years' experience as a Deputy Librarian in a University, the case may be referred to the Registrar of the University for his consideration.
7. having an M.Phil./Ph.D. degree in Library Science/ Information Science/ Documentation/Archives and manuscript keeping but has only ten year's experience as a college librarian, the condition at (1) may be waived.

EXERCISE 1

Based on the above conditions and the information provided against each of the candidates in the questions given below, decide which of the following courses of action should be taken against each candidate.

Mark the answer as (a) if the candidate is to be selected; (b) if the candidate is not to be selected; (c) if the data are inadequate; (d) if the case is to be referred to the Registrar and (e) if the case is to be referred to the Vice Chancellor.

1. Ashok having Master's degree in Library Science with 70% marks and with one year specialisation in an area of Information Technology joined as a Librarian in the Indian College on 22nd February 1985. He also holds a certificate of innovative library science in the college.
2. Satheesh, an M.Phil in Library Science has been a Deputy Librarian in the MG University since 27th September 1988. He has also obtained master's degree in Archives and manuscript keeping. He holds the evidence of innovative organisation of published work of the college students doing Ph.D.
3. Gopala Krishnan has been a Deputy Librarian in the MS University since 1981. He holds an evidence of contributing library service in the same institution. He has a master's degree in Library Science with 53% marks.
4. Roy holding a Ph.D. degree in Library Science has one year specialisation in the Archives and manuscript keeping. He has been a Deputy Librarian in the Agriculture University since 11 May 1990. He also has a certificate of innovative Library service in a public library for three years.
5. Suresh Kumar has been a college librarian since 15 January, 1983. He holds an M.Phil Degree in Library Science.

ANSWERS

1. (e) 2. (a) 3. (b) 4. (a) 5. (c)

Explanations

1. Ashok fulfills conditions (1), (2) and (4). Being a college librarian for 16 years, he satisfies condition (5). So, the answer is (e).
2. Satheesh, being an M.Phil degree holder, satisfies condition (7) so that condition (1) may be waived. He satisfies conditions (2), (3) and (4). So he is selected and the answer is (a).
3. Gopala Krishnan satisfies condition (4). Being a Deputy Librarian for last 20 years, he satisfies condition (3). As the marks scored by him are less than 55%, he violates condition (1). So the answer is (b).
4. Roy, being a Ph.D. holder, satisfies condition (7) so that (1) is waived. He satisfies condition (2) and (4) and being a Deputy Librarian for eleven years, he satisfies condition (3). So the answer is (a).
5. As the candidate Suresh Kumar, being an M.Phil, satisfies condition (7) and so condition (1) is waived. He being a college librarian for 18 years satisfied condition (3). As there is no information regarding condition (2) and (4) are not mentioned, answer is (c).

EXERCISE 2

Study the following information carefully and answer the questions given below it.

[Bank PO]

The following are the criteria for the admission to medical courses in a college.

The student must:

1. have passed XII standard examination in science with at least 60 % marks.
2. be at least 18 years old as on 23.3.93.
3. have obtained at least 71% marks in the entrance examination.
4. be able to pay monthly tuition fee of ₹ 560.
5. be able to pay one time deposit of ₹ 18000.

In the case of a student who:

6. satisfies all other conditions except (5) and can pay one time deposit only upto ₹ 12,000, the case may be referred to the Director of the Institute.
7. satisfies all criteria except that at (1), the case may be referred to the chairman.
8. satisfies all criteria but has not yet received the result of the final examination of XII std. may be provisionally admitted.

On the basis of the above conditions and the information provided against each student in the questions given below, decide which of the following courses of action should be taken against each candidate.

Mark answer as (a) if the student is to be admitted; (b) if the student is not to be admitted; (c) if the case is to be referred to the Director; (d) if the case is to be referred to the Chairman; (e) if the student is to be provisionally admitted.

1. Nitin Sharma has passed XII standard examination with 62% marks and was 20 years old on 9th January, 1993. He has secured 75% marks in the entrance test. He can pay a tuition fee of ₹ 560 and a one time deposit of only ₹ 12000.
2. Rajendra Kumar has passed XII standard examination in Science with 55% marks. He was born on 26th March, 1974. He has secured 83% marks in the entrance examination. He can pay one time deposit of ₹ 18000 and monthly tuition fee of ₹ 560.
3. Prakash Mehra secured 60% and 72% marks in the XII standard and entrance examination respectively. He was 19 years old on 2nd December, 1992. He is able to pay the stipulated one time deposit and monthly tuition fee.
4. Mukesh Maheshwari was born on 15th February, 1974. He attained 51 % marks in XII standard examination and 75% marks in entrance test. He is able to pay monthly tuition fee of ₹ 560 and one time deposit of ₹ 18000.
5. Peeyush Yadav has appeared for the final exam of XII standard in Science stream. He was 19 years old on 25.7.92. He has secured 75 % in the entrance examination. He is able to pay one time deposit of ₹ 18000 and monthly tuition fee of ₹ 560.
6. Deepak Gupta passed XII standard in Science with 63% marks and was born on 1.6.73. He has secured 74% marks in the entrance examination and can pay monthly tuition fees of ₹ 560. He can pay one time deposit of ₹ 9000 only.
7. Harpal Singh, born on 8th September, 1974, passed XII standard examination in Science with 68% marks. He attained 69% marks in the Entrance Test and is able to pay the stipulated monthly tuition fee and one time deposit.
8. Rakesh Sharma, who attained 59% marks in XII std. examination and 73.5% marks in the entrance test, is able to pay the monthly tuition fee of ₹ 560 and one time deposit of ₹ 18000. He is 22 years of age.

9. Nisha Gupta, a 21 year old girl, passed the XII standard examination with 64% marks and the entrance test with 77% marks. She is able to pay the tuition fee of ₹ 560 and a one time deposit of ₹ 18000.
10. Jatin Narang was born on 7.9.74. He secured 72% marks in the entrance test. He has appeared for XII standard examination and can pay the stipulated tuition fee and one time deposit.

ANSWERS

1. (c) 2. (d) 3. (a) 4. (d) 5. (e) 6. (b) 7. (b) 8. (d) 9. (a) 10. (e)

Explanations

1. The candidate satisfies condition (6) instead of condition (5).
2. The candidate satisfies condition (7) instead of condition (1).
3. All conditions of eligibility are satisfied.
4. The candidate satisfies condition (7) instead of condition (1).
5. The candidate satisfies condition (8).
6. Condition (5) is not fulfilled.
7. Condition (3) is not fulfilled.
8. Condition (7) is not satisfied instead of condition (1).
9. All conditions of eligibility are satisfied.
10. As the candidate has not yet received the result of the final examination of XII standard, he satisfies condition (8).

Assertion and Reason

In this type of questions two statements will be given: Assertion (A) and Reason (R). You have to go through these statements and check whether the statement given as (A) and statement given as (R) are as stated in the following manner and you have to answer either (a), (b), (c) or (d).

- (a) Both (A) and (R) are individually true and (R) is the correct explanation of (A).
- (b) Both (A) and (R) are individually true, but (R) is not the correct explanation of (A)
- (c) (A) is true but (R) is false.
- (d) (A) is false but (R) is true.

EXERCISES

1. Assertion (A): The same face of the Moon is always presented to the Earth.
Reason (R): The moon rotates about its own axis in $23\frac{1}{2}$ days which is about the same time that it takes to orbit the earth. [IAS]
2. Assertion (A): Existence of human life on Venus is highly improbable.
Reason (R): Venus has extremely high level of CO_2 in its atmosphere [IAS]
3. Assertion (A): All the proteins in our food are digested in small intestine only.
Reason (R): The protein-digesting enzymes from pancreas are released into small intestine. [IAS]
4. Assertion (A): Amoeba reproduces by fission.
Reason (R): All unicellular organisms reproduce by asexual methods. [IAS]
5. Assertion (A): Bangalore receives much higher annual rainfall than that of Mangalore.
Reason (R): Bangalore has the benefit of receiving rainfall both from south-west and north-east monsoons. [IAS]
6. Assertion (A): The Central Rural Sanitation Programme was launched in 1986 to improve the quality of life of rural people in India.
Reason (R): The rural sanitation is a subject in the concurrent list in the constitution of India. [IAS]

7. Assertion (A): The west flowing rivers of peninsular India have no deltas.
Reason (R): These rivers do not carry any alluvial sediments. [IAS]
8. Assertion (A): The thickness of the atmosphere is maximum over the equator.
Reason (R): High insulation and strong convection currents occur over the equator. [IAS]
9. Assertion (A): In our houses, the current in AC electricity line changes directions 50 times per second.
Reason (R): The frequency of alternating voltage supplied is 50 Hz.
10. Assertion (A): Fatty acids should be a part of the balanced human diet.
Reason (R): The cells of the human body can't synthesize any fatty acids.
11. Assertion (A): India does not export natural rubber.
Reason (R): About 97% of India's demand for natural rubber is met from domestic production.
12. Assertion (A): For the first time, India had no trade deficit in the year 2002–03.
Reason (R): For the first time, India's exports crossed \$50 billion in the year 2002–03. [IAS]
13. Assertion (A): To dilute sulphuric acid, acid is added to water and not water to acid.
Reason (R): Specific heat of water is quite large. [IAS]
14. Assertion (A): Devaluation of currency may promote export.
Reason (R): The price of the country's product in the international market may fall due to devaluation.
15. Assertion (A): The fiscal deficit is greater than the budgetary deficit.
Reason (R): The fiscal deficit is the borrowing from RBI plus other liabilities of government to meet its expenditure.
16. Assertion (A): According to statistics, more female children are born each year than male children in India.
Reason (R): In India, the death rate of male child is higher than that of the female child.
17. Assertion (A): Insect resistant transgenic cotton has been produced by inserting Bt gene.
Reason (R): The Bt gene is derived from a bacterium.
18. Assertion (A): Information technology is fast becoming a very important field of activity in India.
Reason (R): Software is one of the major exports of the country and India has a very strong base in hardware.
19. Assertion (A): Chile continues to be an important producer of copper in the world.
Reason (R): Chile is endowed with the world's largest deposit of porphyry copper.
20. Assertion (A): Dolly was the first cloned mammal.
Reason (R): Dolly was produced by *in-vitro* fertilization.
21. Assertion (A): During the time of Akbar, for every ten cavalrymen, the mansabdars had to maintain twenty horses.
Reason (R): Horses had to be rested while on march and replacements were necessary in times of war.
22. Assertion (A): Lord Linlithgow described the August Movement of 1942 as the most serious rebellion since the Sepoy Mutiny.
Reason (R): There was massive upsurge of the peasantry in certain areas. [IAS]

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23. Assertion (A): The Gaudham School of Art bears the mark of Hellenistic influence.
Reason (R): Hinayana form was influenced by that art. [IAS]
24. Assertion (A): Formic acid is a stronger acid than acetic acid.
Reason (R): Formic acid is an organic acid. [IAS]
25. Assertion (A): At first the Turkish administration in India was essentially military.
Reason (R): The country was parcelled out as Iqtas among leading military leaders. [IAS]
26. Assertion (A): According to Asoka's edicts social harmony among the people was more important than religious devotion.
Reason (R): He spreaded ideas of equity instead of promotion of religion. [IAS]
27. Assertion (A): The temperature of a metal wire rises when an electric current is passed through it.
Reason (R): Collission of metal atoms with each other releases heat energy.
28. Assertion (A): The Khilafat movement did bring the urban Muslims into the fold of the National Movement.
Reason (R): There was a predominant elements of anti-imperialism in both the National and Khilafat Movement.
29. Assertion (A): Phenyl is used as a household germicide.
Reason (R): Phenyl is a phenol derivative and phenol is an effective germicide.
30. Assertion (A): Partition of Bengal in 1905 brought to an end the Moderate's role in the Indian Freedom Movement.
Reason (R): The Surat session of Indian National Congress separated the Extremists from the Moderates.
31. Assertion (A): The first ever bill to make primary education compulsory in India was rejected in 1911.
Reason (R): Discontent would have increased if everybody could read.
32. Assertion (A): Sodium metal is stored under kerosene.
Reason (R): Metallic sodium melts when exposed to air.
33. Assertion (A): The congress rejected the Cripps proposals.
Reason (R): The Cripps Mission consisted solely of the whites.
34. Assertion (A): The United States of America has threatened to ask the World Trade Organisation (WTO) to apply sanctions against the developing countries for the non-obsergance of ILO conventions.
Reason (R): The United States of America itself has adopted and implemented those ILO conventions.
35. Assertion (A): During the reign of Shahjahan, Dara Sikh was sent on an expedition to Balkha, Badakhshan and Qandahar.
Reason (R): The expedition sent by Shahjahan to the Middle-east was a marvellous success. [IAS]
36. Assertion (A): Gandhi stopped the Non-Co-operation movement in 1922.
Reason (R): Violence at Chauri Chaura led him to stop the movement. [IAS]
37. Assertion (A): The reservation of thirty-three per cent of seats for women in Parliament and State Legislatures does not require constitutional amendment.
Reason (R): Political parties contesting elections can allocate thirty-three per cent of seats they contest to women candidates without any constitutional amendment. [IAS]

38. Assertion (A): Wilful disobedience or non-compliance of court orders and use of derogatory language about judicial behaviour amount to contempt of court.
Reason (R): Judicial activism can't be practised without aiming the judiciary with punitive powers to punish contemptuous behaviour. [IAS]
39. Assertion (A): The emergence of economic globalisation does not imply the decline of socialist ideology.
Reason (R): The ideology of socialism believes universalism and globalism. [IAS]
40. Assertion (A): A diamond sparkles more than a glass imitation cut to the same shape.
Reason (R): The refractive index of diamond is less than that of glass. [IAS]
41. Assertion (A): The monsoonal rainfall decreases as one goes towards the west and north-west in the Ganga plain.
Reason (R): The moisture bearing monsoonal winds go higher up as one moves up in the Ganga plain. [IAS]
42. Assertion (A): A lock of Einstein's hair, if scientists could locate it and extract its DNA, could help in producing another Einstein, by cloning.
Reason (R): The DNA extracted from the cell of an embryo at an early stage of development can be transferred to individual eggs which in turn can be implanted into the uterus of a surrogate mother to give birth to an identical offspring.
43. Assertion (A): The USA re-emerged as India's single largest import source in the early nineties.
Reason (R): With swift political developments in the erstwhile Soviet Union, India gradually began to rely on the USA for its defence requirement. [IAS]
44. Assertion (A): In India, the political parties which formed the governments represented the majority of seats secured in the elections to the House of the people at the centre and the Legislative Assemblies in the states but not the majority of votes.
Reason (R): The elections based on the majority-vote-system decides the result on the basis of relative majority of votes secured. [IAS]
45. Assertion (A): A mixture of salt and ice gives temperature below 0°C.
Reason (R): The salt raises the freezing point of ice. [IAS]
46. Assertion (A): Hong Kong is to revert to China from British control in a few years.
Reason (R): The people of Hong Kong have opted for it in a referendum. [IAS]
47. Assertion (A): Balan wrote his memoirs in Turki.
Reason (R): Turki was the official language of the Mughal Court. [IAS]
48. Assertion (A): Minimum wages in India are fixed in accordance with the levels of living and the labour participation ratios.
Reason (R): All workers covered by the Minimum Wages Acts are above the poverty line. [IAS]
49. Assertion (A): Italy, Switzerland, Sweden and Norway have abundant power resources.
Reason (R): They have the largest coal deposits in Europe. [IAS]
50. Assertion (A): The Quit India movement marked the culmination of Indian national movement.
Reason (R): After the Quit India movement it was matter of time to find a suitable mechanisms for transfer of power. [IAS]

51. Assertion (A): The form of government in Rigvedic period was monarchy.
Reason (R): Priest enjoyed both social and political status and influenced administration. [IAS]
52. Assertion (A): Rainfall is scanty on east of western ghats.
Reason (R): The east of western ghats is on the lee side. [IAS]
53. Assertion (A): Insects are not affected by pesticides.
Reason (R): Insects are killed by pesticides. [IAS]
54. Assertion (A): The finance commission aims at safeguarding the fiscal autonomy of the states.
Reason (R): The finance commission is constituted every fifth year. [IAS]
55. Assertion (A): Soap removes oil and dirt.
Reason (R): Soap increases the surface tension of water. [IAS]
56. Assertion (A): Red phosphorus is used in matchsticks.
Reason (R): Red phosphorus is less dangerous compared to the white one. [IAS]
57. Assertion (A): Earthworm is a friend to the man.
Reason (R): It decreases the soil erosion. [IAS]
58. Assertion (A): Dry battery can't be recharged.
Reason (R): The chemical reaction is reversible. [IAS]
59. Assertion (A): India adopted UK's Parliamentary system.
Reason (R): The Upper House has judicial power. [IAS]
60. Assertion (A): Colonialism has started in India during the 19th century.
Reason (R): Industrial revolution demanded market places. [IAS]

ANSWERS

1. (c)	2. (b)	3. (b)	4. (c)	5. (a)	6. (c)	7. (b)	8. (b)	9. (a)	10. (c)
11. (a)	12. (a)	13. (a)	14. (a)	15. (a)	16. (c)	17. (a)	18. (a)	19. (a)	20. (b)
21. (d)	22. (a)	23. (c)	24. (b)	25. (b)	26. (b)	27. (b)	28. (d)	29. (c)	30. (d)
31. (b)	32. (a)	33. (b)	34. (a)	35. (c)	36. (a)	37. (d)	38. (b)	39. (b)	40. (c)
41. (a)	42. (c)	43. (b)	44. (b)	45. (a)	46. (c)	47. (c)	48. (c)	49. (c)	50. (d)
51. (b)	52. (a)	53. (d)	54. (b)	55. (a)	56. (a)	57. (c)	58. (c)	59. (c)	60. (a)

Situation Reaction Test

In this test, a certain situation is described and the candidate is required to choose the most suitable reaction to the given situation from among the alternatives given. The test judges the reasoning power of the candidate and his ability to get correctly and promptly to a situation that may arise in emergency.

EXAMPLE 1 You are visiting a place for first time and are travelling in a bus. Suddenly you realise that the driver is taking the bus to a lonely place with no right intentions. You would:

- (a) with the help of some other passengers, try to baffle the driver and take over the bus.
- (b) sit and wait to face repercussions.
- (c) jump out of the running bus.
- (d) console the worried passengers.

Solution When you expect wrong doing of the driver, the immediate action to prevent it is the need. So the answer is **(a)**.

EXAMPLE 2 While playing cricket in the school, suddenly when you hit the ball, it strikes your classmate on the forehead and blood starts oozing out. You would:

- (a) run away from the field.
- (b) start fighting with the boy why he came in the way.
- (c) blame somebody else for the accident.
- (d) take the boy to the first-aid room.

Solution In the above situation, the urgent need is to provide first aid to the boy so that bleeding may stop. So the answer is **(d)**.

EXAMPLE 3 While you board a train at the station, you find a suitcase beneath your seat. You would:

- (a) report the matter to the police.
- (b) open the suitcase to look through its contents.

- (c) try to find out the address of the owner from the papers, etc. in the suitcase.
- (d) finding no one to claim it, take it into your own possession.

Solution As the case is a loss of a valuable article for the concerned owner, necessary steps have to be taken to hand over the article, by reporting the matter to the police. So the answer is (a).

EXERCISES

To each of the following questions, four probable answers have been given. Select the most appropriate alternative as the answer.

1. You are living in a college hostel. The *dal* served to you in the mess has a lot of stones. What would you do?
 - (a) leave eating the *dal* altogether.
 - (b) bring the matter to the notice of mess incharge.
 - (c) speak to the cook about changing the *dal*.
 - (d) buy your own *dal* and cook it in your room.
2. You are entrusted with the job of taking care of a child. If the child insists on doing something which you would not allow, you would:
 - (a) beat the child.
 - (b) threaten the child and make him quiet.
 - (c) try to make him understand why you will not allow the action.
 - (d) let the child cry.
3. You are driving your car on the road when you hit against a fruit vendor's cart. You would:
 - (a) escape from the site by driving away.
 - (b) abuse the fruit vendor for putting his cart on the way.
 - (c) pay the fruit vendor for the damage done to him.
 - (d) insist that it was not your fault.
4. While travelling in a train, you notice a man from the coach behind yours fall off the train. You would:
 - (a) pull the alarm chain so that the train may stop and the man may be helped.
 - (b) shout at the falling man asking him to get up quickly and entrain.
 - (c) jump off the train to assist the falling man.
 - (d) wait till the train stops at the next station and inform the railway authorities there.
5. You find that the person whom you call your friend has been cheating you. What would you do?
 - (a) break relations with him.
 - (b) give him tit for tat.
 - (c) make him realise his mistake.
 - (d) tell other friends about him.
6. You are alone in the house and there is quite a danger of thieves around. Just then, you hear a knock at the door. You would:
 - (a) open the door to see who is there.
 - (b) first peep out from the window to confirm whether you know the person.
 - (c) not open the door.
 - (d) ask the servant to see who is there.

7. You are returning from school. On the way, you find a sealed envelope in a street, fully addressed with unused stamps on it. You would:
 - (a) leave it there as it was and walk away.
 - (b) remove the stamps and destroy the envelope.
 - (c) open the envelope, find out who has dropped it by mistake, and send it to him if possible.
 - (d) post it at the nearest letter box.
8. While firing crackers, a child gets severe burns on the hand. What would you do?
 - (a) dip the child's hands in cold water till there is no more burning sensation.
 - (b) wash the hands with dettol.
 - (c) send some one to call the doctor.
 - (d) apply some ointment on the affected area.
9. If in the examination hall, you find that the question paper is too tough to be answered satisfactorily by you, the best thing to do for you is to:
 - (a) tell the examiner that the questions are out of course.
 - (b) provoke the candidates to walk out of the examination hall.
 - (c) try to know something from your neighbour.
 - (d) try to solve the questions as much as you know with a cool head.
10. On reaching the railway station, you find that the train you wanted to catch is just to start and there is hardly any time for purchasing the ticket. The best thing for you is to:
 - (a) rush to the train rather than miss it and inform the TTI at the next stoppage about your inability to purchase the ticket.
 - (b) rush to the train and perform your journey quietly.
 - (c) first purchase the ticket and then catch the train if it is there.
 - (d) miss the train rather than take the risk of boarding the moving train.
11. Your bathroom tap is leaking and is a constant source of irritating noise. You would:
 - (a) sleep with pillows upon your ears.
 - (b) put a bucket underneath.
 - (c) try to put up a cork upon the mouth of the tap.
 - (d) call a plumber to repair the tap.
12. While attending your friend's party, you see your friend's muffler catching fire from the candle on the table behind him. You would:
 - (a) ask your friend to see behind him.
 - (b) rush to call friend's mother.
 - (c) rush and take out the muffler from his neck, drop it and pour water on it.
 - (d) take out the muffler and throw it away.

ANSWERS

1. (b) 2. (c) 3. (c) 4. (a) 5. (c) 6. (b) 7. (d) 8. (a) 9. (d) 10. (a)
11. (d) 12. (c)

Mathematical Operations

This chapter deals with the questions on simple mathematical operations. Here, the four fundamental operations: plus (+), minus (−), multiplication (×), and division (÷) and also, statement such as 'less than', 'greater than', 'equal to', 'not equal to', etc. are represented by symbols, different from the usual ones. The questions involving these operations are set using artificial symbols. The candidate has to substitute the real signs and solve the questions accordingly, to get the answer. Here are a few examples.

EXAMPLE 1 If '+' means '÷', '−' means '×', '÷' means '+' and '×' means '−', then

$$36 \times 12 + 4 \div 6 + 2 - 3 = ?$$

- (a) 2 (b) 18 (c) 42 (d) $6\frac{1}{2}$ (e) None of these

Solution Putting the proper sign in the given expression, we get

$$\begin{aligned} 36 - 12 \div 4 + 6 \div 2 \times 3 &= 36 - 3 + 3 \times 3 \\ &= 36 - 3 + 9 = 42 \end{aligned}$$

So, the answer is (c).

EXAMPLE 2 If '+' means 'minus', '−' means 'multiplied by', '÷' means 'plus' and '×' means 'divided by', then

$$10 \times 5 \div 3 - 2 + 3 = ?$$

- (a) 5 (b) $53/3$ (c) 18 (d) 21 (e) None of these

Solution Using the proper signs, we get

$$\begin{aligned} 10 \div 5 + 3 \times 2 - 3 &= 2 + 3 \times 2 - 3 \\ &= 2 + 6 - 3 = 5 \end{aligned}$$

So, the answer is (a).

EXAMPLE 3 If 'P' denotes '+', 'Q' denotes '−', 'R' denotes '÷' and 'S' denotes '×', then

$$1 \ 8 \ S \ 3 \ 6 \ R \ 1 \ 2 \ Q \ 6 \ P \ 7 = ?$$

- (a) 115 (b) 55 (c) $648/13$ (d) 25 (e) None of these

Solution Using proper signs, we get

$$18 \times 36 \div 12 - 6 + 7 = 18 \times 3 - 6 + 7 \\ = 54 - 6 + 7 = 55$$

So, the answer is **(b)**.

EXAMPLE 4 If ' $>$ ' denotes '+', ' $<$ ' denotes '-', ' $\div$ ' denotes ' $\times$ ', ' $\times$ ' denotes ' $\div$ ', ' $=$ ' denotes 'less than' and ' $\times$ ' denotes 'greater than', find which of the following statement is correct.

- (a) $3 + 2 > 4 = 9 + 3 < 2$ (b) $3 > 2 > 4 = 18 + 3 < 1$
 (c) $3 > 2 < 4 \times 8 + 4 < 2$ (d) $3 + 2 < 4 \times 9 + 3 < 3$

Solution Using proper notations, we have

- (a) $3 \div 2 + 4 < 9 \div 3 - 2$ or $\frac{11}{2} < 1$, which is false.
 (b) $3 + 2 + 4 < 18 \div 3 - 1$ or $9 < 5$, which is false.
 (c) $3 + 2 - 4 > 8 \div 4 - 2$ or $1 > 0$, which is true.
 (d) $3 \div 2 - 4 > 9 \div 3 - 3$ or $-\frac{5}{2} > 0$, which is false.

So, the statement **(c)** is true.

EXERCISES

- If ' $-$ ' means ' $\times$ ', ' $\times$ ' means '+', '+' means ' $\div$ ' and ' $\div$ ' means ' $-$ ', then what will be the value of $40 \times 12 + 3 - 6 \div 60$?
[Bank PO]
 (a) 44 (b) 7.95 (c) 16 (d) 479.95 (e) None of these
- If '+' means ' $\div$ ', ' $\div$ ' means ' $-$ ', ' $-$ ' means ' $\times$ ' and ' $\times$ ' means '+', what will be the value of the following expression $8 + 4 \div 3 \times 5 - 9$?
[Bank PO]
 (a) 44 (b) $5\frac{2}{3}$ (c) $6\frac{1}{3}$ (d) 46 (e) None of these
- If '+' means ' $\div$ ', ' $\times$ ' means ' $-$ ', ' $\div$ ' means ' $\times$ ' and ' $-$ ' means '+', then $9 + 3 \div 4 - 8 \times 2 = ?$
[Bank PO]
 (a) $-6\frac{1}{4}$ (b) $6\frac{3}{4}$ (c) $-1\frac{3}{4}$ (d) 18 (e) None of these
- If '+' means ' $\div$ ', ' $-$ ' means ' $\times$ ', ' $\div$ ' means ' $-$ ' and ' $\times$ ' means '+', what will be the value of $8 + 6 \div 4 - 7 \times 3$?
[SBI PO]
 (a) $-\frac{23}{2}$ (b) 14 (c) $-\frac{71}{3}$ (d) 12 (e) None of these
- If '+' means ' $\times$ ', ' $-$ ' means '+' and ' $\times$ ' means ' $\div$ ', find the value of $5 + 4 - 18 \times 3$.
[Bank PO]
 (a) -34 (b) 6 (c) 26 (d) 14 (e) None of these
- If '+' means ' $\div$ ', ' $-$ ' means '+', ' $\times$ ' means ' $-$ ' and ' $\div$ ' means ' $\times$ ', then $8 \div 4 - 6 + 3 \times 4 = ?$
[AAO Exam]
 (a) 8 (b) 46 (c) 4 (d) 13 (e) None of these
- If ' $\div$ ' means '+', ' $-$ ' means ' $\div$ ', ' $\times$ ' means ' $-$ ' and '+' means ' $\times$ ', then $\frac{(36 \times 4) - 8 \times 4}{4 + 8 \times 2 + 16 \div 1} = ?$
[Railway Recruitment]
 (a) 16 (b) 12 (c) 8 (d) 0

8. If '+' stands for 'division', '-' stands for 'equal to', '×' stands for 'addition', '÷' stands for 'greater than', '=' stands for 'less than', '>' stands for 'multiplication' and '<' stands for 'subtraction', then which of the following alternatives is correct?
- (a) $5 + 2 \times 1 = 3 + 4 > 1$ (b) $5 > 2 \times 1 - 3 > 4 < 1$
 (c) $5 \times 2 < 1 - 3 < 4 \times 1$ (d) $5 < 2 \times 1 \div 3 > 4 \times 1$
9. If 'L' denotes '×', 'M' denotes '÷', 'P' denotes '+' and 'Q' denotes '-', then
 $1\ P\ 2\ 4\ M\ 8\ Q\ 6\ M\ 2\ L\ 3 = ?$
- (a) $13/6$ (b) $-\frac{1}{6}$ (c) $14\frac{1}{2}$ (d) 10 (e) None of these
10. If 'P' denotes '+', 'Q' denotes '×', 'R' denotes '÷', 'S' denotes '-' and '=' denotes greater than, then which of the following statement is true?
- (a) $4\ P\ 8\ R\ 2\ S\ 1\ Q\ 6 = 1$ (b) $3\ S\ 1\ Q\ 8\ P\ 6\ R\ 2 = 0$
 (c) $8\ R\ 2\ S\ 3\ Q\ 4\ P\ 6 = 2$ (d) $9\ P\ 2\ Q\ 6\ S\ 4\ R\ 2 = 21$

ANSWERS

1. (e) 2. (a) 3. (d) 4. (c) 5. (c) 6. (e) 7. (d) 8. (b) 9. (d) 10. (c)

Explanations

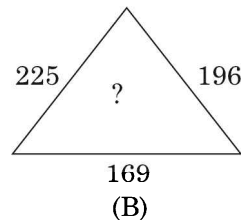
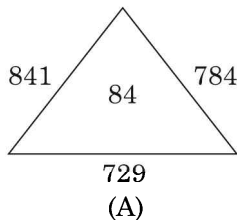
1. Using the correct symbols, we have:
 Given expression = $40 + 12 \div 3 \times 6 - 60 = 40 + 4 \times 6 - 60 = 40 + 24 - 60 = 4$
2. Using correct symbols, we have:
 $8 \div 4 - 3 + 5 \times 9 = 2 - 3 + 5 \times 9 = 2 - 3 + 45 = 44$
3. Using correct symbols, we have:
 $9 \div 3 \times 4 + 8 - 2 = 3 \times 4 + 8 - 2 = 12 + 8 - 2 = 18$
4. Using correct symbols, we get:
 $8 \div 6 - 4 \times 7 + 3 = \frac{8}{6} - 28 + 3 = \frac{-71}{3}$
5. Using correct symbols, we have:
 $5 \times 4 + 18 \div 3 = 20 + 6 = 26$
6. Using correct symbols, we have:
 $8 \times 4 + 6 \div 3 - 4 = 32 + 2 - 4 = 30$
7. Using correct symbols, we have
 $\frac{(36 - 4) \div 8 - 4}{4 \times 8 - 2 \times 16 + 1} = \frac{32 \div 8 - 4}{32 - 32 + 1} = \frac{4 - 4}{1} = 0$
8. Using correct symbols in (b), we get
 $5 \times 2 + 1 = 3 \times 4 - 1$ or $11 = 11$, which is true.
9. Using correct symbols, we get:
 $16 + 24 \div 8 - 6 \div 2 \times 3 = 16 + 3 - 3 \times 3 = 16 + 3 - 9 = 10$
10. Using correct symbols we get:
 (a) $4 + 8 \div 2 - 1 \times 6 = 1$ or $4 + 4 - 6 = 1$ or $2 = 1$, which is false.
 (b) $3 - 1 \times 8 + 6 \div 2 = 0$ or $3 - 8 + 3 = 0$ or $-2 = 0$, which is false.
 (c) $8 \div 2 - 3 \times 4 + 6 = -2$ or $4 - 12 + 6 = -2$ or $-2 = -2$, which is true.
 (d) $9 + 2 \times 6 - 4 \div 2 = 21$ or $9 + 12 - 2 = 21$ or $+19 = 21$, which is false.
 So, the answer is (c).

Inserting the Missing One

In this type of problems, a figure is given which is subdivided into a certain number of parts and each one bears a number except one subdivision. The numbers in the figure follow a particular pattern. You have to analyse the given figure and then fill in the blank space with the most suitable alternative.

Directions: Find the missing number from among the given alternatives.

EXAMPLE 1



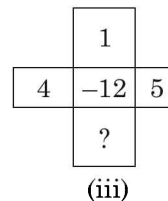
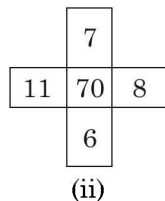
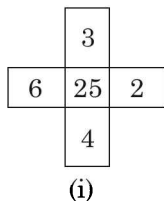
- (a) 32 (b) 42 (c) 62 (d) 82

Solution In figure (A), $27^2 = 729$, $28^2 = 784$, $29^2 = 841$ and $27 + 28 + 29 = 84$.

Similarly, in figure (B), $13^2 = 169$, $14^2 = 196$, $15^2 = 225$ and $13 + 14 + 15 = 42$.

So, the answer is **(b)**.

EXAMPLE 2



- (a) 10 (b) 6 (c) 2 (d) 1

Solution In figure (i), $(3^2 + 6^2) - (2^2 + 4^2) = (9 + 36) - (4 + 16) = 45 - 20 = 25$

In figure (ii), $(7^2 + 11^2) - (6^2 + 8^2) = (49 + 121) - (36 + 64) = 170 - 100 = 70$

In figure (iii), $(1^2 + 4^2) - (5^2 + x^2) = -12$

or $(1 + 16) - (25 + x^2) = -12$

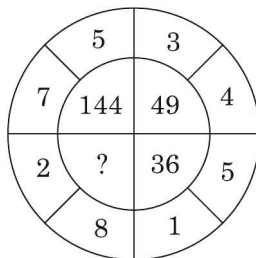
or $17 - 25 - x^2 = -12$

or $x^2 = 17 - 25 + 12 = 4$

$\therefore x = 2$

So, the answer is (c).

EXAMPLE 3



(a) 100

(b) 81

(c) 64

(d) 121

Solution Here, $(7 + 5)^2 = 144$; $(3 + 4)^2 = 49$; $(5 + 1)^2 = 36$;

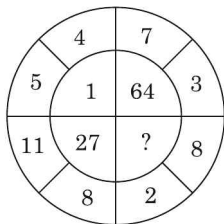
Similarly $(2 + 8)^2 = 100$

$\therefore$ Missing number is **100**.

EXERCISES

Directions: Find the missing number in each of the following questions:

1.



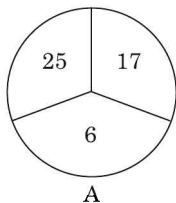
(a) 125

(b) 8

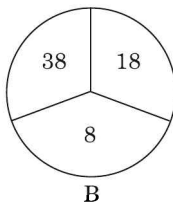
(c) 0

(d) 216

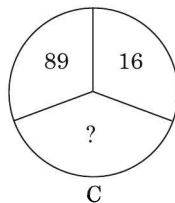
2.



(a) 19



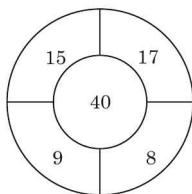
(b) 17



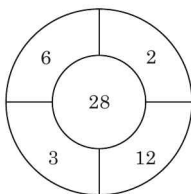
(c) 15

(d) 13

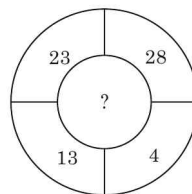
3.



A



B



C

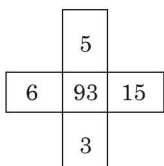
(a) 1

(b) 8

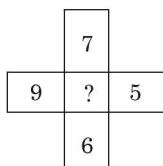
(c) 12

(d) 27

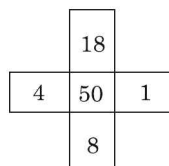
4.



A



B



C

(a) 27

(b) 19

(c) 89

(d) 5

5.

7B	5C	6B
3C	9B	19A
15A	17A	?

(a) 10C

(b) 12C

(c) 14C

(d) 16C

6.

9	A	12
B	10	?
8	C	11

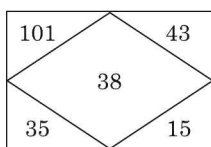
(a) A = 13, B = 11, C = 9

(b) A = 13, B = 9, C = 11

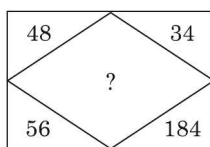
(c) A = 9, B = 11, C = 13

(d) A = 9, B = 13, C = 11

7.



A



B

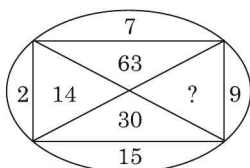
(a) 198

(b) 158

(c) 142

(d) 127

8.



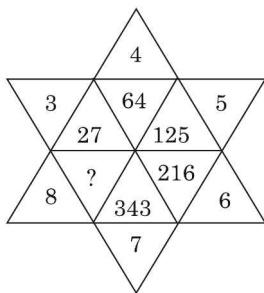
(a) 33

(b) 145

(c) 135

(d) 18

9.



(a) 64

(b) 512

(c) 16

(d) 24

10.

3	8	10	2	?	1
6	56	90	2	20	0

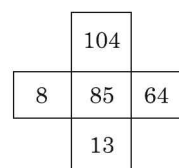
(a) 0

(b) 3

(c) 5

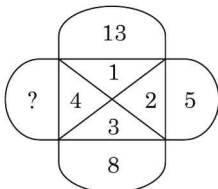
(d) 7

Directions: In each of the following questions, numbers have been arranged according to the pattern shown in the sample figure given below. Find the most correct alternative to fill in the space provided by question mark.

[Railways]

Sample figure

11.



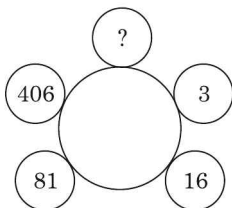
(a) 10

(b) 11

(c) 12

(d) 13

12.



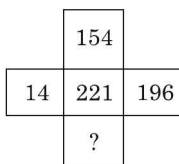
(a) 2031

(b) 731

(c) 1625

(d) 1

13.



(a) 11

(b) 13

(c) 15

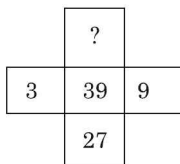
(d) 17

14.

BD ₃	CE ₅	DF ₁₅
EG ₂	FH ₄	GI ₈
HJ ₄	IK ₆	?

(a) JL₂₄(b) IJ₁₈(c) JK₁₈(d) JL₁₂

15.



(a) 33

(b) 81

(c) 243

(d) 42

ANSWERS

1. (d) 2. (c) 3. (a) 4. (c) 5. (d) 6. (d) 7. (c) 8. (c) 9. (b) 10. (c)
 11. (c) 12. (a) 13. (a) 14. (a) 15. (b)

Explanations

1. $(5 - 4)^3 = 1$; $(7 - 3)^3 = 64$; $(11 - 8)^3 = 27$; $(8 - 2)^3 = 216$.
 $\therefore$ The missing number is **216**.
2. The sum of the numbers in the upper two parts of each circle is seven times the number in the third part.
 In figure A, $(25 + 17) = 42 = 7 \times (6)$
 In figure B, $(38 + 18) = 56 = 7 \times (8)$
 In figure C, $(89 + 16) = 105 = 7 \times (15)$
 $\therefore$ The missing number is **15**.
3. The difference of the product of the numbers in two lower parts and the sum of the numbers in two upper parts is the number inside the smaller circle.
 In figure A, $(9 \times 8) - (15 + 17) = 72 - 32 = 40$
 In figure B, $(3 \times 12) - (6 + 2) = 36 - 8 = 28$
 Similarly, in figure C, $(13 \times 4) - (23 + 28) = 52 - 51 = 1$
 $\therefore$ The missing number is **1**.
4. In figure A, $(6 \times 3) + (5 \times 15) = 18 + 75 = 93$
 In figure C, $(4 \times 8) + (18 \times 1) = 32 + 18 = 50$
 Similarly, in figure B, $(9 \times 6) + (7 \times 5) = 54 + 35 = 89$
 $\therefore$ The missing number is **89**.
5. In each column, out of the letters A, B and C, each of these must appear once. Along the diagonal, the sum of two numbers is equal to the third number.
 $\therefore$ The missing number will be $7 + 9 = 16$ and the letter will be C as A and B already appears once in that column.
6. The total of numbers in each row and each column is 30.
7. In figure (A), $(101 + 15) - (35 + 43) = 116 - 78 = 38$.
 The same pattern would be followed in figure (B).
 $\therefore$ The missing number $= (48 + 184) - (56 + 34) = 232 - 90 = 142$
8. We have $15 \times 2 = 30$; $2 \times 7 = 14$; $7 \times 9 = 63$;
 $\therefore$ Missing number $= 9 \times 15 = 135$.
9. We have, $3^3 = 27$; $4^3 = 64$; $5^3 = 125$; $6^3 = 216$; $7^3 = 343$.
 $\therefore$ Missing number $= 8^3 = 512$.

10. Let X denotes the number in the first row and Y that in the second row.

$$\therefore Y = X(X - 1)$$

$$\begin{aligned}\therefore 6 &= 3(3 - 1); 56 = 8(8 - 1); \\ 90 &= 10(10 - 1); 2 = 2(2 - 1); \\ 0 &= 1(1 - 1);\end{aligned}$$

$$\text{So, } X(X - 1) = 20$$

$$\therefore X = 5$$

11. Arrangement is as follows:

Taking clockwise;

$$5 + 3 = 8; 8 + 4 = 12; 12 + 1 = 13;$$

So, the missing number is **12**.

12. Here taking the arrangement clockwise,

$$(3 \times 5) + 1 = 16; (16 \times 5) + 1 = 81; (81 \times 5) + 1 = 406;$$

So $(406 \times 5) + 1 = 2031$.

$\therefore$ The missing number is **2031**.

13. Let the missing number be x

From the given sample figure, $8 \times 13 = 104$; $8^2 = 64$; $8 + 13 + 64 = 85$.

Similarly, here $14^2 = 196$;

$$14 + x + 196 = 221$$

$$\therefore x = 11$$

$$14 \times 11 = 154$$

So, the missing number is **11**.

14. The number in the third column in each row is obtained by multiplying the numbers in the first and second column in the same row.

$\therefore$ In first row, $3 \times 5 = 15$ and in second row, $2 \times 4 = 8$.

$\therefore$ Number to be filled in blank space $= 4 \times 6 = 24$.

$\therefore$ From the given alternatives only (a) contains suffix 24.

So, the answer is (a).

15. Here, $3 + 27 + 9 = 39$

$$? = 3 \times 27 = 81 \text{ i.e., missing number} = 3 \times 27 = 81.$$

So the answer is (b).

Logical Sequence of Words

In these type of questions, certain inter-related words are given and numbered, followed by various sequences of numbers denoting them, as alternatives. The candidate is required to arrange these words in a logical sequence based on a common property and then choose the correctly graded sequence from the given alternatives.

Some common sequences have been mentioned as follows:

I. Sequence of occurrence of events or various stages in a process:

EXAMPLE 1 Arrange the following in a logical order.

1. Birth 2. Marriage 3. Death 4. Funeral 5. Education

(a) 1, 4, 2, 5, 3

(b) 1, 5, 2, 3, 4

(c) 3, 4, 2, 5, 1

(d) 2, 5, 4, 1, 3

Solution The given words when arranged in the order of various events occur in a man's life, form the sequence: Birth, Education, Marriage, Death, Funeral.

Therefore, the correct order becomes 1, 5, 2, 3, 4.

Hence, the answer is (b).

EXAMPLE 2 Arrange the following in a meaningful sequence:

1. Doctor 2. Fever 3. Prescribe 4. Diagnose 5. Medicine

(a) 1, 4, 3, 2, 5

(b) 2, 1, 3, 4, 5

(c) 2, 1, 4, 3, 5

(d) 2, 4, 3, 5, 1

[CBI]

Solution Here fever occurs first. One then goes to a doctor. After diagnosing he prescribes medicine. Therefore, the correct order is: Fever, Doctor, Diagnose, Prescribe and Medicine.

Hence, the answer is (c).

II. Sequence of objects in a class or group, from part to the whole:

EXAMPLE 3 Arrange the following in a logical order.

1. Leaf 2. Fruit 3. Stem 4. Root 5. Flower

(a) 3, 4, 5, 1, 2

(b) 4, 1, 3, 5, 2

(c) 4, 3, 1, 2, 5

(d) 4, 3, 1, 5, 2

[SSC]

Solution The given words are part of a plant. Arranging them in order from bottom to top in the order of their occurrence, we get Root, Stem, Leaf, Flower and Fruit.

Hence the sequence is 4, 3, 1, 5, 2.

Hence, the correct answer is (d).

EXAMPLE 4 Arrange the following in a meaningful order, from particular to general.

1. District 2. Village 3. Country 4. Town 5. State

(a) 2, 4, 1, 5, 3

(b) 2, 1, 4, 5, 3

(c) 5, 3, 2, 1, 4

(d) 2, 5, 3, 4, 1

Solution Here the village is the smallest area. A number of villages are combined to form a town. A number of towns are combined to form a district. So many districts are combined to form a state. So many states are clubbed to form a country. Hence the sequence is Village, Town, District, State, Country.

Thus, the correct order is 2, 4, 1, 5, 3.

Hence, the answer is (a).

III. Sequence of increasing/decreasing size, value, intensity, etc.

EXAMPLE 5 Arrange the following in a logical order.

1. Gold 2. Iron 3. Silver 4. Platinum 5. Diamond

(a) 3, 4, 2, 5, 1

(b) 2, 3, 1, 5, 4

(c) 4, 5, 1, 2, 3

(d) 5, 4, 2, 3, 1

Solution The given names when arranged in the order of increasing values, (i.e. from cheapest to costly) form the sequence:

Iron, Silver, Gold, Diamond, Platinum.

Hence, the answer is (b).

EXAMPLE 6 Arrange the following in a logical sequence from small to big.

1. Elephant 2. Cat 3. Mosquito 4. Tiger 5. Whale

(a) 1, 3, 5, 4, 2

(b) 2, 5, 1, 4, 3

(c) 3, 2, 4, 1, 5

(d) 5, 3, 1, 2, 4

[CBI]

Solution Here the given names can be arranged (from small to big) in a sequence as:

Mosquito, Cat, Tiger, Elephant, Whale.

Hence, the answer is (c).

IV. Sequence in which a chain of given objects is formed:

EXAMPLE 7 Arrange the following in a logical order.

1. Book 2. Pulp 3. Timber 4. Jungle 5. Paper

(a) 2, 5, 1, 4, 3

(b) 3, 2, 5, 1, 4

(c) 4, 3, 2, 5, 1

(d) 5, 4, 3, 1, 2

[SSC]

Solution Here the names can be arranged in the order in which they are used for the manufacture of a book. It can be written as a sequence:

Jungle, Timber, Pulp, Paper, Book

Hence the sequence is 4, 3, 2, 5, 1

Hence, the answer is (c).

EXAMPLE 8 Arrange the following in a meaningful sequence:

1. Yarn 2. Plant 3. Saree 4. Cotton 5. Cloth

(a) 2, 4, 1, 5, 3

(b) 2, 4, 3, 5, 1

(c) 2, 4, 5, 1, 3

(d) 2, 4, 5, 3, 1

[CBI]

Solution Here the names given are the stages in the manufacture of saree. Therefore the sequence can be formed as: Plant, Cotton, Yarn, Cloth, Saree.

Hence the sequence is 2, 4, 1, 5, 3

Hence, the answer is (a).

EXERCISES

Directions (Q.1 to 10): In each of the following questions, arrange the given words in a meaningful sequence and then choose the most appropriate sequence from amongst the alternatives provided below each question:

1. 1. Sentence 2. Chapter 3. Letter 4. Book 5. Word 6. Paragraph

(a) 4, 2, 1, 6, 5, 3

(b) 4, 2, 6, 1, 5, 3

(c) 4, 6, 1, 2, 3, 5

(d) 4, 6, 2, 5, 1, 3

[CBI]

2. 1. Police 2. Punishment 3. Crime 4. Justice 5. Judgement

(a) 1, 2, 3, 4, 5

(b) 3, 1, 2, 4, 5

(c) 3, 1, 4, 5, 2

(d) 5, 4, 3, 2, 1

[SSC]

3. 1. College 2. Child 3. Salary 4. School 5. Employment

(a) 1, 2, 4, 3, 5

(b) 2, 4, 1, 5, 3

(c) 4, 1, 3, 5, 2

(d) 5, 3, 2, 1, 4

[RRB]

4. 1. Mother 2. Child 3. Milk 4. Cry 5. Smile

(a) 1, 5, 2, 4, 3

(b) 2, 4, 1, 3, 5

(c) 2, 4, 3, 1, 5

(d) 3, 2, 1, 5, 4

5. 1. Atomic Age 2. Metallic Age 3. Stone Age 4. Alloy Age

(a) 1, 3, 4, 2

(b) 2, 3, 1, 4

(c) 3, 2, 4, 1

(d) 4, 3, 2, 1

[MAT]

6. 1. Heel 2. Shoulder 3. Skull 4. Neck 5. Knee 6. Chest 7. Thigh 8. Stomach
9. Face 10. Hand

(a) 2, 4, 7, 10, 1, 5, 8, 9, 6, 3

(b) 3, 4, 7, 9, 2, 5, 8, 10, 6, 1

(c) 4, 7, 10, 1, 9, 6, 3, 2, 5, 8

(d) 3, 9, 4, 2, 10, 6, 8, 7, 5, 1

[SSC]

7. 1. Rain 2. Monsoon 3. Rescue 4. Flood 5. Shelter 6. Relief

(a) 1, 2, 3, 4, 5, 6

(b) 1, 2, 4, 5, 3, 6

(c) 2, 1, 4, 3, 5, 6

(d) 4, 1, 2, 3, 5, 6

[SSC]

8. 1. Never 2. Sometimes 3. Generally 4. Seldom 5. Always

(a) 5, 2, 1, 3, 4

(b) 5, 2, 4, 3, 1

(c) 5, 3, 1, 2, 4

(d) 5, 3, 2, 4, 1

[SSC]

9. 1. Butterfly 2. Cocoon 3. Egg 4. Worm

(a) 1, 3, 4, 2

(b) 2, 4, 1, 3

(c) 1, 4, 3, 2

(d) 3, 4, 2, 1

[CBI]

10. 1. Site 2. Plan 3. Rent 4. Money 5. Building 6. Construction

(a) 1, 2, 3, 6, 5, 4

(b) 2, 3, 6, 5, 1, 4

(c) 3, 4, 2, 6, 5, 1

(d) 4, 1, 2, 6, 5, 3

[Hotel Management]

ANSWERS

1. (b) 2. (c) 3. (b) 4. (b) 5. (c) 6. (d) 7. (c) 8. (d) 9. (d) 10. (d)

Solutions and Hints

- In this words are arranged such that each successive term constitute part of first word.
i.e., Book → Chapter → Paragraph → Sentence → Word → Letter.
Chapter constitutes part of a book.
Paragraph constitutes part of a chapter etc.
- In this words are arranged according to the order of occurrence.
Here *crime* was done first. Interference of *Police* occurs as second. After the criminal was caught by the police he will be brought to the count before the *Justice*. After the hearing is over, *judgement* will be issued. Based on that *punishment* will be given to the criminal. Therefore crime, police, justice, judgement and punishment is the required order.
- The sequence of words here represents different stages from birth of a person to his final stage in his carrer. Here Child → School → College → Employment → Salary the sequence mention that first stage of a person is *child*. After some time that child will go to *school*, then *college*. After completing his study in the college, he will appear for various competitive examinations for a job. Then on getting success in the examination, he will get an *employment*. After getting employed he will get *salary*. Therefore the sequence is 2, 4, 1, 5, 3.
Hence answer is (b).
- The sequence here represents various expression of a child in different situations.
Here *child* first *cries*. While crying *mother* will give *milk* to the child. Then the child will smile on getting milk.
Hence sequence is Child → Cry → Mother → Milk → Smile.
Hence answer is (b).
- The sequence here represents different periods in order. i.e., Stone age → Metallic age → Alloy age → Atomic age.
Hence answer is (c).
- This sequence consists of body parts from head to toe in order. Therefore sequence is Skull → Face → Neck → Shoulder → Hand → Chest → Stomach → Thigh → Knee → Heel.
Hence answer is (d).
- This sequence consists of climatic changes from monsoon and subsequent processes. Therefore sequence in Monsson → Rain → Flood → Rescue → Shelter → Relief.
Hence answer is (c).
- This sequence contains the same word with different intensity in the decreasing order of intensity. i.e., Always → Generally → Sometimes → Seldom → Never.
Hence answer is (d).

9. This sequence consists of different stages of formation of Butterfly from its egg.

i.e., Egg → Worm → Cocoon → Butterfly.

Hence answer is (d).

10. This sequence consists of different names related to construction of a building in the order of execution/priority. i.e., Money → Site → Plan → Construction → Building → Rent.

i.e., First priority is given for money. After arranging money, place or land is bought for construction which is the site. After having the site, plan is prepared for construction of building. After construction is over building got ready for occupying. Then the building is given for rent.

Hence answer is (d).

SECTION B LOGICAL DEDUCTION

Chapter 16

Logic

Logic is the science of thought as expressed in language. Questions based on logic are to be solved considering the given statements as they are true without considering the formal validity or the truth of the statements. i.e., the conclusion should follow directly from the statements given.

LOGICAL REASONING: In logic, a statement of certain relation between two or more terms is analogous to a sentence in grammar.

The proposition consists of three parts, namely subject, predicate and copula.

1. **Subject:** The subject is about which something is said.
2. **Predicate:** The predicate is the part of the proposition denoting which is affirmed or denied about the subject.
3. **Copula:** The copula is that part of the proposition which denotes the relation between the subject and the predicate.

EXAMPLE 1 Consider the proposition 'Man is intelligent'. Here the information is given about the man. So 'Man' is the subject.

'Intelligent' is the quality affirmed for this subject. So it is the predicate. 'Is' denotes the relation between the subject and the predicate. So, it is the copula.

CLASSIFICATION OF PROPOSITIONS: 'Propositions' can be classified into four types:

1. **Universal Affirmative Proposition (denoted by A):** This distributes only the subject i.e. the predicate is not interchangeable with the subject while maintaining the validity of the proposition.

EXAMPLE 2 All books are pens.

This is proposition A since we cannot say 'All pens are books'.

2. Universal Negative Proposition (denoted by E): This distributes both the subject and the predicate, i.e. an entire class of predicate term is denied to the entire class of the subject term, as in the proposition. For example,
No pen is pencil.

3. Particular Affirmative Proposition (denoted by I): This distributes neither the subject nor the predicate. For example,
Some pupil are brilliant.

Here subject term 'some pupil' is used not for all but only for some students and similarly the predicate term 'brilliant' is affirmed for a part of subject class. So both are undistributed.

4. Particular Negative Proposition (denoted by O): This distributes only the predicate. For example,
Some cats are not dogs.

Here, the subject term 'some cats' is used only for a part of its class and hence is undistributed while the predicate term 'dogs' is denied in entity to the subject term and hence is distributed.

The above four types of proposition can be summarised as follows:

Proposition	Type
(a) (A) distributes subject only	All S is P
(b) (E) distributes both subject and predicate	No S is P
(c) (I) distributes neither	Some S is P
(d) (O) distributes predicate only	Some S is not P

Statement Form	Type of Proposition	Distributes
(A) All S is P	Universal affirmative	S only
(B) No S is P	Universal negative	Both S and P
(I) Some S is P	Particular affirmative	Neither S nor P
(O) Some S is not P	Particular negative	P only

LOGICAL DEDUCTION: The phenomenon of deriving a conclusion from a single proposition or a set of given proposition is known as **logical deduction**. The given proposition are also referred to as the premise.

There are two inferential processes of deduction.

Immediate Deductive Inference: Here the conclusion is deduced from one of the given propositions, by any of the three ways—conversion, obversion and contraposition.

Conversion: The conversion proceeds with interchanging the subject term and predicate term, i.e. the subject term of the premise becomes the predicate term of the conclusion and the predicate term of the premise becomes the subject of the conclusion.

The given proposition is called **convertend**, whereas the conclusion drawn from it is called its **converse**.

Valid conversions

Convertend	Converse
A: All S is P Example: All books are pens.	I: Some P is S Some pens are books.
E: No S is P Example: No cat is dog.	E: No P is S No dog is cat.
I: Some S is P Example: Some boys are foolish.	I: Some P is S Some fools are boys.
O: Some S is not P	No valid conversion

In a conversion, quality remains the same and the quantity may change.

Obversion: In obversion, we change the quality of the proposition and replace the predicate term by its complement.

Valid obversions

Obvertend	Obverse
A: All birds are mammals. E: No poets are singers. I: Some men are engineers. O: Some politicians are not honest.	E: No birds are non-mammals. A: All poets are non-singers O: Some men are not non-engineers. I: Some politicians are non-honest.

Contraposition: In contraposition, we first replace the subject and predicate terms in the proposition and then exchange both these terms with their complements.

Valid contrapositives

Proposition	Contrapositive
A: All humans are animals. I: Some humans are animals.	A: All non-animals are non-humans. I: Some non-animals are non-humans.

MEDIATE DEDUCTIVE INFERENCE (SYLLOGISM): A syllogism is a deductive argument in which the conclusion has to be drawn from two propositions referred to as the premises.

Example: 1. All girls are students.
2. All students are brilliant.
3. All girls are brilliant.

Here, the propositions 1 and 2 are the premises and the proposition 3, which follows from the first two propositions, is called the *conclusion*.

In logic, a *term* is a word or a combination of words, which by itself can be used as a subject or predicate of a proposition.

Syllogism is concerned with three terms:

1. **Major Term:** This is the predicate of the conclusion and is denoted by **P** (first letter of 'Predicate').
2. **Minor Term:** This is the subject of the conclusion and is denoted by **S** (first letter of 'Subject').
3. **Middle Term:** This is the term common to both the premises and is denoted by **M** (first letter of 'Middle').

Example: Premises: 1. All cats are animals.
2. All tigers are cats.

Conclusion: All tigers are animals.

Here, 'Animals' is the predicate of the conclusion and so it is the major term P.

'Tigers' is the subject of the conclusion and so, it is the minor term, S.

'Cats' is the term common to both the premises and so it is the middle term, M.

Major and Minor Premises: Of the two premises, the major premise is that in which the middle term is the subject and the minor premise is that in which the middle term is the predicate.

RULES FOR DERIVING THE CONCLUSION FROM TWO GIVEN PREMISES

1. **The conclusion does not contain the middle term.**

EXAMPLE 1

Statements : 1. All desks are tables.
2. Some tables are chairs.

Conclusions: 1. All tables are desks.
2. Some tables are not chairs.

Since both the conclusions 1 and 2 contain the middle term 'tables', neither of them can follow.

2. **No term can be distributed in the conclusion unless it is distributed in the premises.**

EXAMPLE 2

Statements : 1. Some students are boys.
2. All boys are fools.

Conclusions: 1. All fools are boys.
2. Some students are fools.

Statement 1 is an I-type proposition which distributes neither the subject nor the predicate. Statement 2 is an A-type proposition which distributes the subject i.e., 'boys' only.

Conclusion 1 is an A-type proposition which distributes the subject 'fools' only.

Since the term 'fools' is distributed in conclusion 1 without being distributed in the premise, so conclusion 1 cannot follow.

3. The middle term (M) should be distributed at least once in the premise. Otherwise, the conclusion cannot follow.

For the middle term to be distributed in a premise,

- (i) M must be the subject if premise is an A type proposition.
- (ii) M must be subject or predicate if premise is an E type proposition.
- (iii) M must be predicate if premise is an O type proposition.

In an I type proposition, which distributes neither the subject nor the predicate, the middle term cannot be distributed.

EXAMPLE 3

- Statements : 1. All clocks are watches.
2. Some watches are red.
- Conclusions: 1. All watches are clocks.
2. Some clocks are red.

In the premises, the middle term is 'watches'. Clearly, it is not distributed in the first premise, which is an A type proposition as it does not form its subject. It is not distributed in the second premise which is an I type proposition. Since the middle term is not distributed even in both the premises, so no conclusion follows:

4. No conclusion follows:

- (a) *if both the premises are particular.*

EXAMPLE 4

- Statements : 1. Some pens are pencils.
2. Some pencils are rulers.
- Conclusions: 1. All pens are rulers.
2. Some rulers are pens.

Since both the premises are particular, no definite conclusion follows:
(b) *if both the premises are negative.*

EXAMPLE 5

- Statements : 1. No mango is pineapple.
2. No pineapple is coconut.
- Conclusions: 1. No mango is coconut.
2. Some coconuts are pineapples.

Since both the premises are negative, neither conclusion follows:
(c) *if the major premise is particular and the minor premise is negative.*

EXAMPLE 6

- Statements : 1. Some cows are bulls.
2. No cats are cows.
- Conclusions: 1. No cows are cats.
2. Some bulls are cats.

Here, the first premise containing the middle term 'cows' as the subject is the major premise and the second premise containing the middle term 'cows' as

the predicate in the minor premise. Since the major premise is particular and the minor premise is negative, no conclusion follows.

5. If the middle term is distributed twice, the conclusion cannot be universal.

EXAMPLE 7

Statements : 1. All tables are cots.
2. No chairs are tables.

Conclusions: 1. No chairs are cots.
2. Some chairs are cots.

Here, the first premise is an A type proposition and so, the middle term 'tables' forming the subject is distributed. The second premise is an E type proposition and so the middle term 'tables' forming the predicate is distributed.

Since the middle term is distributed twice, the conclusion cannot be universal.

6. If one premise is negative, the conclusion must be negative.

EXAMPLE 8

Statements : 1. All pens are rubbers.
2. No rubber is pencil.

Conclusions: 1. No pens are pencils.
2. Some pencils are pens.

Since one premise is negative, the conclusion must be negative. So conclusion 2 cannot follow.

7. If one premise is particular, the conclusion must be particular.

EXAMPLE 9

Statements : 1. Some men are thieves.
2. All thieves are intelligent persons.

Conclusions: 1. Some men are intelligent persons.
2. All intelligent persons are men.

Since, one premise is particular, the conclusion must be particular. so, conclusion 2 cannot follow.

8. If both the premises are affirmative, the conclusion must be affirmative.

EXAMPLE 10

Statements : 1. All cows are cats.
2. All cats are bulls.

Conclusions: 1. All cows are bulls.
2. Some cows are not bulls.

Since both the premises are affirmative, the conclusion must be affirmative. So, conclusion 2 cannot follow.

9. If both the premises are universal, the conclusion must be universal.

Complementary pair: A pair of contradictory statements, i.e. a pair of statements such that if one is true, the other is false and when no definite conclusion can be drawn, either of them is bound to follow, is called a *complementary pair*.

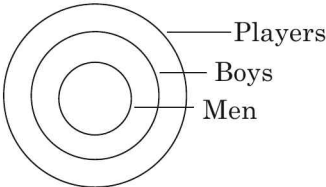
E and I type propositions together form a complementary pair and usually either of them follows, in a case where we cannot arrive at a definite conclusion, using the rules of syllogism.

Study the various possible cases and draw all possible inferences in each case, along with verification through Venn diagrams.

Case 1: All men are boys. All boys are players.

Immediate Deductive Inferences: The converse of first premise 'Some boys are men' and the converse of second premise 'Some players are boys' both hold good.

Mediate Deductive Inferences: Since both the premises are universal and affirmative, the conclusion must be universal and affirmative. Also, the conclusion should not contain the middle term. Therefore, it follows that 'All men are players'. The converse of this conclusion 'Some players are men' also holds good.

Venn Diagram	Inferences
	1. Some boys are men. 2. Some players are boys. 3. All men are players. 4. Some players are men.

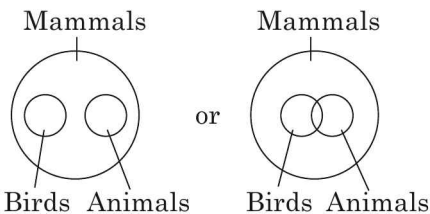
Case 2: All birds are mammals.

All animals are mammals.

Immediate Deductive Inferences: The converse of first premise, 'Some mammals are birds' and the converse of second premise 'Some mammals are animals' both hold good.

Mediate Deductive Inferences: Both, being A-type propositions, distribute subject only.

Thus, the middle term 'mammals' is not distributed even once in the premises. Therefore, no definite conclusion follows.

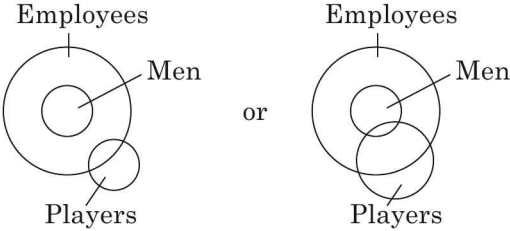
Venn Diagram	Inferences
	1. Some mammals are birds. 2. Some mammals are animals. 3. Either 'No bird is animal' or 'Some birds are animals' as E and I-type propositions form a complementary pair.

Case 3: All men are employees.

Some employees are players.

Immediate Deductive Inferences: The converse of first premise 'Some employees are men' and the converse of the second premise 'Some players are employees', both hold good.

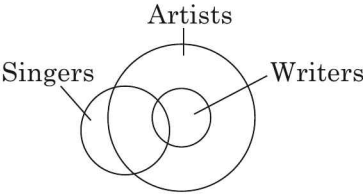
Mediate Deductive Inferences: The first premise is an A-type proposition which distributes the subject only while the second premise is an I-type proposition which distributes neither subject nor predicate. Since the middle term 'boys', is not distributed even once in the premises, no definite conclusion can be drawn.

Venn Diagram	Inferences
	1. Some employees are men. 2. Some players are employees. 3. Either 'No men is player' or 'Some men are players' as E and I-type proposition form a complementary pair.

Case 4: Some singers are writers.
All writers are artists.

Immediate Deductive Inferences: The converse of the first premise 'Some writers are singers' and the converse of the second premise 'Some artists are writers', both hold good.

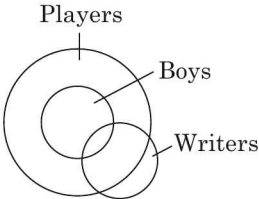
Mediate Deductive Inferences: Since one premise is particular the conclusion must be particular and should not contain the middle term. so it follows that 'Some singers are artists.' The converse of this conclusion 'Some artists are singers' also holds good.

Venn Diagram	Inferences
	1. Some writers are singers. 2. Some artists are writers. 3. Some singers are artists. 4. Some artists are singers.

Case 5: All boys are players. Some boys are writers.

Immediate Deductive Inferences: The converse of the first premise 'Some players are boys' and the converse of the second premise 'Some writers are boys' both hold good.

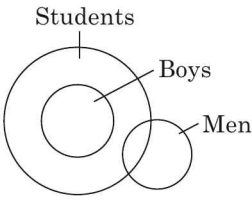
Mediate Deductive Inferences: Since the premise is particular, the conclusion must be particular and should not contain the middle term. So, it follows that 'Some players are writers' and the converse of this conclusion 'Some writers are players' also holds good.

Venn Diagram	Inferences
	1. Some players are boys. 2. Some writers are boys 3. Some players are writers. 4. Some writers are players.

Case 6: All boys are students. Some men are students.

Immediate Deductive Inferences: The converse of the first premise 'Some students are boys' and the converse the second premise 'Some students are men' both hold good.

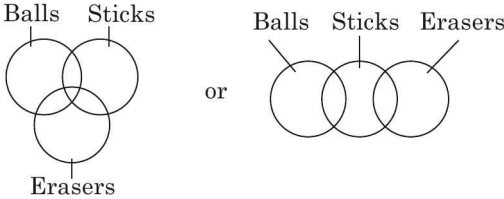
MEDIATE DEDUCTIVE INFERENCES: The first premise is an A-type proposition, which distributes subject only. The second premise is an I-type proposition, which distributes neither subject not predicate. So the middle term 'students' is not distributed even once in the premises. Hence, no definite conclusion can be drawn.

Venn Diagram	Inferences
	1. Some students are boys. 2. Some students are men. 3. Either 'No boy is man' or 'some boys are men' follows.

Case 7: Some balls are sticks. Some sticks are erasers.

Immediate Deductive Inferences: The converse of the first premise 'Some sticks are balls' and the converse of the second premise 'Some erasers are sticks' both hold good.

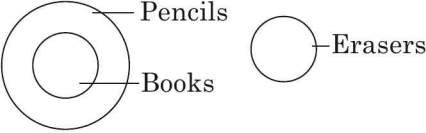
Mediate Deductive Inferences: Since both premises are particular, no definite conclusion follows.

Venn Diagram	Inferences
	1. Some sticks are balls. 2. Some erasers are sticks. 3. Either 'Some balls are erasers' or 'No ball is eraser' follows as I and E-type propositions form a complementary pair.

Case 8: All books are pencils. No pencil is eraser.

Immediate Deductive Inferences: The converse of the first premise 'Some pencils are books' and the converse of the second premise 'No eraser is pencil', both hold good.

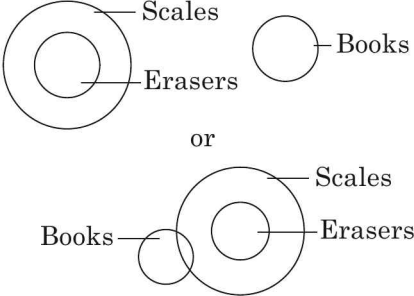
Mediate Deductive Inferences: Since both premises are universal, the conclusion must be universal. Since one premise is negative, the conclusion must be negative. So 'No book is eraser' follows. The converse of the conclusion 'No eraser is book' also holds good.

Venn Diagram	Inferences
	1. Some pencils are books. 2. No eraser is pencil. 3. No book is eraser. 4. No eraser is book.

Case 9: No book is eraser. All erasers are scales.

Immediate Deductive Inferences: The converse of the first premise 'No eraser is book' and the converse of the second premise 'Some scales are erasers', both hold good.

Mediate Deductive Inferences: The first premise, an E-type proposition, distributes both the subject and the predicate. Second premise, an A-type proposition, distributes the subject. Thus, the middle term 'eraser' is distributed twice in the premises. So conclusion cannot be universal. One premise is negative. So the conclusion must be negative. Thus the conclusion must be particular negative (i.e., O-type). Therefore 'Some scales are not books' follows.

Venn Diagram	Inferences
	1. No eraser is book. 2. Some scales are erasers. 3. Some scales are not books. 4. Either 'Some books are scales' or 'No book is scale' follows as E and I-type propositions form a complementary pair.

Case 10: Some fans are heaters. No heater is refrigerator.

Immediate Deductive Inferences: The converse of the first premise 'Some heaters are fans' and the converse of the second premise 'No refrigerator is heater', both hold good.

Mediate Deductive Inferences: Since the first premise is particular and the second premise negative, the conclusion must be particular and negative or O-type. Therefore, it follows that 'Some fans are not refrigerators.'

Venn Diagram	Inferences
or	1. Some heaters are fans. 2. No refrigerator is heater. 3. Some fans are not refrigerators. 4. Either 'Some fans are refrigerators' or 'No fan is refrigerator' follows as E and I-type propositions form a complementary pair.

Note: While deriving logical conclusions, always remember that the following conclusions hold:

- (i) The converse of each of the given premises;
- (ii) The conclusion that directly follows from the given premises in accordance with the rules of syllogism; and
- (iii) The converse of the derived conclusions.

TYPE I: TWO-PREMISE ARGUMENTS (a) WITH TWO CONCLUSIONS

Directions (1 to 10): In each question below are given two statements followed by two conclusions numbered I and II. You have to take the given two statements to be true even if they seem to be at variance from the commonly known facts and then decide which of the given conclusions logically follows from the given two statements, disregarding commonly known facts.

Give answer (a) if only conclusion I follows; (b) if only conclusion II follows; (c) if either I or II follows; (d) if neither I nor II follows; and (e) if both I and II follow.

1. Statements: All cakes are ice creams.
All ice creams are toffees.

Conclusions: I. All cakes are toffees.
II. All toffees are ice creams.

[RRB Exam]

2. Statements: Some pearls are gems.
Some gems are ornaments.

Conclusions: I. Some gems are pearls.
II. Some ornaments are gems.

[Bank PO]

3. Statements: All lights are trucks.
Some trucks are jeeps.

Conclusions: I. All jeeps are lights.
II. Some lights are jeeps.

[Bank PO]

4. Statements: Lawyers married only fair girls.
Sobha is very fair.

Conclusions: I. Sobha was married to a lawyer.
II. Sobha was not married to a lawyer.

[Railway Recruitment]

5. Statements: All pencils are bricks.
All bricks are bottles.
Conclusions: I. All pencils are bottles.
II. All bricks are pencils. **[RBI Exam]**
6. Statements: Some books are pencils.
Some pencils are pens.
Conclusions: I. Some books are pens.
II. Some pens are books. **[SBI PO]**
7. Statements: All trays are erasers.
All pens are erasers.
Conclusions: I. All trays are pens.
II. Some pens are trays. **[BSRB Exam]**
8. Statements: All birds are trees.
Some trees are hens.
Conclusions: I. Some birds are hens.
II. Some hens are trees.
9. Statements: Some cooks are lazy.
All boys are lazy.
Conclusions: I. Some boys are cooks.
II. Some cooks are boys. **[Bank PO]**
10. Statements: Sohan is a good sportsmen.
Sportsmen are healthy.
Conclusions: I. All healthy person are sportsmen.
II. Sohan is healthy. **[Bank PO]**

ANSWERS

1. (a) 2. (d) 3. (d) 4. (c) 5. (a) 6. (d) 7. (d) 8. (a) 9. (d) 10. (b)

Solutions and Hints

- Since both the statements are affirmative, the conclusion must be affirmative. Conclusion II cannot follow as it contains the middle term. So, only conclusion I follows.
- Since both the premises are particular, no conclusion follows.
- The first premise, being an A type proposition distributes subject only. So middle term 'trucks' forming the predicate is not distributed.
The second premise, being an I type proposition distributes neither the subject nor the predicate. So, the middle term 'trucks' forming the subject is not distributed.
- The data does not mention whether all fair girls were married to lawyers. So, either of the two conclusions may follow.
- Since both the premises are affirmative, the conclusion must be affirmative and only conclusion I follows.
- Since, both the premises are particular, no conclusion follows.
- Both the premises are A type propositions. So, in either, the middle term 'erasers' forming the predicate is not distributed.
Since the middle term is not distributed at least once in the premises, no conclusion follows.

8. Since one premise is particular, the conclusion must be particular. Conclusion II cannot follow as it contains the middle term. So only conclusion I follows.
9. The first premise is an I type proposition and distributes neither the subject nor the predicate. So the middle term 'lazy' forming the predicate is not distributed.

The second premise is an A type proposition and distribute subject only. So, the middle term 'lazy' forming the predicate is not distributed. Since the middle term is not distributed at least once in the premises, no conclusion follows.

10. Conclusion I cannot follow as it contains the middle term. So only conclusion II follows.

TWO-PREMISE ARGUMENTS (b) WITH MORE THAN TWO CONCLUSIONS

In these type of questions, two statements called *premises* are given followed by four *conclusions*. The candidate has to find out which of the conclusions follow from the given premises. First the given statements are analysed. If the middle term is not distributed at least once, no conclusion follows. No conclusion follows when

- (i) both the premises are particular.
- (ii) both the premises are negative.

EXAMPLE 1

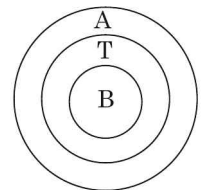
- Statements: Some mangoes are apples.
Some apples are lemons.
- Conclusions: I. No mango is lemon.
II. All apples are lemons.
III. Some lemons are mangoes
IV. No apple is mango.

Solution Since both the premises are particular, (i.e. of type 'Some S is P'), no conclusion follows.

EXAMPLE 2

- Statements: All birds are tigers.
All tigers are animals.
- Conclusions: I. All birds are animals.
II. Some tigers are birds.
III. Some animals are birds.
IV. Some animals are tigers.

Solution Region for birds 'B' lies entirely within the region for animals 'A'. So conclusion I follows. Since T has a common area with B, A has a common area with B and A has a common area with T, conclusions II, III and IV follow. Thus all the conclusions follow.



EXAMPLE 3

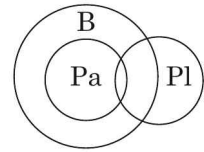
- Statements: All books are papers.
Some papers are pencils.
- Conclusions: I. All pencils are papers.
II. All pencils are books.

III. Some books are pencils.

IV. No book is pencil.

Solution Clearly, the middle term is 'Papers' denoted by 'Pa'.

The first premise is an A type proposition which distributes subject only. So the middle term 'Papers', forming the predicate is not distributed. The second premise is an I type proposition and distributes neither the subject nor the predicate. So, the middle term 'Papers', forming the subject is not distributed. Since the middle term is not distributed at least once in the premises, no conclusion follows.



EXERCISES

Directions (Q.1 to 5): In each question below are given two statements followed by four conclusions numbered I, II, III and IV. You have to consider the given statements to be true even if they seem to be at variance from the commonly known facts. Then decide which of the given conclusion logically follow from the given two statements, disregarding the commonly known facts.

1. Statements: Some teachers are students.
All students are girls.

- Conclusions: I. All teachers are girls.
II. Some girls are teachers.
III. Some girls are students.
IV. All students are teachers.

- (a) Only I follows (b) Only I, II and III follow
(c) All follow (d) Only II and III follow
(e) None follows

[Bank PO]

2. Statements: All soaps are clean.
All clean are wet.

- Conclusions: I. Some clean are soaps.
II. No clean is soap.
III. Some wet are soaps.
IV. All wet are soaps.

- (a) Only I follows (b) Either I or II follows
(c) Either III or IV follows (d) Only I and III follow
(e) None follows

[AFO Exam]

3. Statements: All typists are stenographers.
Some stenographers are boys.

- Conclusions: I. All boys are stenographers.
II. All boys are teachers.
III. Some teachers are boys.
IV. No teacher as boy.

- (a) Only I follows (b) Only II and III follow
(c) Either II or III follows (d) Either I or IV follows
(e) None follows

[Bank PO]

4. Statements: Some green are blue.
No blue is white.

Conclusions: I. Some blue are green.
 II. Some white are green.
 III. Some green are not white.
 IV. All white are green.

- (a) Only I follows (b) Only II and III follow
 (c) Only I and II follow (d) Only I and III follow
 (e) Either III or IV follows

[AFO Exam]

5. Statements: All scientists are fools.
 All fools are illiterate.

Conclusions: I. All scientists are illiterate.
 II. All illiterates are scientists.
 III. All illiterates are fools.
 IV. Some illiterates would be scientists.

- (a) Only I and IV follow (b) Only II follows
 (c) Only III and IV follow (d) Only IV follows
 (e) None follows

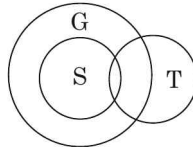
[UTI Exam]

ANSWERS

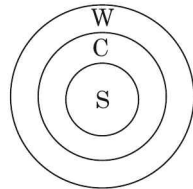
1. (d) 2. (d) 3. (e) 4. (d) 5. (a)

Solutions and Hints

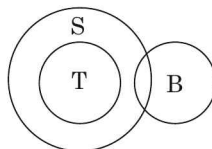
1. From the Venn diagram, it is clear that some girls are teachers and some girls are students, i.e. conclusions II and III follow. But conclusions I and IV cannot follow.



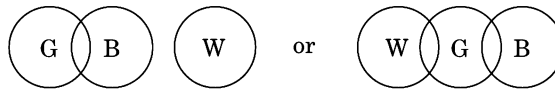
2. From the Venn diagram, C and S have a common area. So some clean are soaps. Similarly W and S have a common area. So some wet are soaps. Therefore, only conclusions I and III follow.



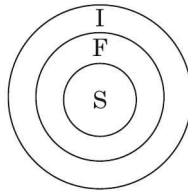
3. From the Venn diagram, none of the conclusions follow.



4. From the Venn diagram, conclusion I and III follow, i.e. 'Some blue are green' and 'Some green are not white'



5. From the Venn diagram, area of scientists 'S' lie entirely within the area of illiterates 'I'. So conclusion I follows. The area 'I' does not lie completely within 'S' or 'F'. Therefore, conclusion II and III cannot follow. But illiterates and scientists have a common area. So conclusion IV follows. Therefore, only conclusion I and IV follow.



TYPE 2: THREE-PREMISE ARGUMENTS: Here, you are given three premises, containing four terms in all, followed by three or four conclusions. In such type of questions, first check for the converse of each of the premises amongst the given conclusions. Then apply the rules of syllogism to the given premises taking two inter-related premises at a time. Then use the conclusion so derived as a premise together with another premise to infer yet another conclusion.

Directions: In each of the following questions, three statements are given followed by four conclusions numbered I, II, III and IV. You have to take the given statements to be true even if they seem to be at variance with commonly known facts and then decide which of the given conclusions logically follow from the given statements disregarding the commonly known facts.

EXAMPLE 1

Statements: Some flowers are rods.
Some rods are doors. Some doors are houses.

Conclusions: I. Some houses are flowers.
II. Some doors are flowers.
III. Some flowers are doors.
IV. No house is flower.

- (a) Either I or II follows (b) Either I or IV follows
(c) Only II and III follow (d) Only I and IV follow
(e) None of these

[SBI PO]

Solution Each combination of premises shall contain two particular premises and as such no definite conclusion follows.

Here, I and IV are contradictory statements involving only the extreme terms and none of the middle term 'rods' or 'doors'. Thus they form a complementary pair. So either I or IV follows.

Hence, the answer is (b).

EXAMPLE 2

Statements: All buildings are rains. All papers are buildings. All dogs are papers.

Conclusions: I. All dogs are rains.
 II. Some papers are rains.
 III. Some rains are buildings
 IV. Some rains are papers.

- (a) Only I and II follow (b) Only II and III follow
 (c) Only I, II and III follow (d) All follow
 (e) None of these

[Bank PO]

Solution Conclusion III is the converse of first premise and so it holds.

All papers are buildings. All buildings are rains.

Since both the premises are universal and affirmative, the conclusion must also be universal and affirmative and should not contain the middle term. Therefore, it follows that 'All papers are rains' and IV is the converse of this conclusion and so it holds.

All dogs are papers. All papers are buildings.

Clearly it follows that 'All dogs are buildings'.

All dogs are buildings. All buildings are rains.

Clearly it follows that 'All dogs are rains' and so conclusion I follows. Hence I, III and IV follow from the given premises.

Hence, the answer is (e).

EXAMPLE 3

Statements: Some fruits are flowers.

No flower is a boat.

All boats are rivers.

Conclusions: I. Some fruits are rivers.
 II. Some rivers are boats.
 III. Some rivers are fruits.
 IV. Some flowers are fruits.

- (a) Only I and III follow (b) Only II and III follow
 (c) Only II and IV follow (d) All follow
 (e) None of these

[SBI PO]

Solution IV is the converse of first premise and so it holds.

II is the converse of third premise and so it holds.

Some fruits are flowers. No flower is a boat

Since one premise is particular and one negative, conclusion must be particular and negative and should not contain the middle term. So it follows that 'Some fruits are not boats'.

No flower is a boat. All boats are rivers.

Since the middle term is distributed twice, the conclusion cannot be universal. Again, since one premise is negative, the conclusion must be negative and shouldn't contain the middle term. Therefore, it follows that 'Some flowers are not rivers'.

Therefore, only II and IV follow.

Hence, the answer is (c).

EXERCISES

Directions (Q.1 to 5): In each of the following questions, three statements are followed by three conclusions numbered I, II and III. You have to take the given statements to be true even if they seem to be at variance from the commonly known facts. You have to decide which of the given conclusions logically follow from the given statements disregarding the commonly known facts.

1. Statements: Some blades are hammers.
Some hammers are knives.
Some knives are axes.

Conclusions: I. Some axes are hammers.
II. Some knives are blades.
III. Some axes are blades.

- (a) Only I follows (b) Only II follows
(c) Only III follows (d) None follows
(e) None of these

[Bank PO]

2. Statements: All dolls are windows.
All bottles are windows.
All cars are bottles.

Conclusions: I. All cars are windows.
II. Some cars are dolls.
III. Some windows are cars.

- (a) Only I and II follow (b) Only II and III follow
(c) Only I and III follow (d) All follow
(e) None of these

[Bank PO]

3. Statements: Some hills are rivers.
Some rivers are deserts.
All deserts are roads.

Conclusions: I. Some roads are rivers.
II. Some roads are hills.
III. Some deserts are hills

- (a) Only I follows (b) Only I and II follow
(c) Only II and III follow (d) All follow
(e) None follow

[Bank PO]

4. Statements: All fruits are vegetables.
All pens are vegetables.
All vegetables are rains.

Conclusions: I. All fruits are rains.
II. All pens are rains.
III. Some rains are vegetables.

- (a) Only I and II follow (b) Only II and III follow
(c) Only I and III follow (d) All follow
(e) None follows

[Bank PO]

5. Statements: All dogs are fruits.
No chair is fruit.
Some chairs are clowns.

- Conclusions:
- I. Some clowns are dogs.
 - II. Some chairs are dogs.
 - III. No chair is dog.
 - IV. No dog is clown.

- (a) Either I or IV follows
- (b) Either I or IV and III follow
- (c) Either I or IV and II follow
- (d) Either II or III follows and either I or IV follows.

[SBI PO]

Directions (Q. 6): In the following question, select one alternative in which the third statement is implied by the first two statements.

6. (a) All dogs are mad. All sick persons are mad. So, all sick persons are dogs.
 (b) All oranges are black. All figs are oranges. So, all figs are black.
 (c) All windows are dogs. Some doors are dogs. So, all windows are doors.
 (d) No man can fly. No kite can fly. So, all men are kites.

Directions (Q. 7 to 10): Each question given below has a set of three or four arguments. Each set of statements is further divided into three segments. Choose the alternative when the third segment in the statement can be logically deduced using both the preceding two, but not just from one of them.

7. (A) All good people are knights. All warriors are good people. All knights are warriors.
 (B) No footballers are ministers. All footballers are tough. Some ministers are players.
 (C) All pizzas are snacks. Some meals are pizzas. Some meals are snacks.
 (D) Some students are boys. All students are girls. Some girls are boys.

- (a) A only (b) B and C (c) C only (d) C and D

[CAT]

8. (A) All beautiful things are sad. She is beautiful. She is sad.
 (B) All nice things are flat. TVs are flat. TVs are nice things.
 (C) Potatoes are stems. All stems are fruits. Potatoes are fruits.

- (a) A only (b) A and B (c) C only (d) A and C

[MAT]

9. (A) P is an artist. Some artists are pretty. P is pretty.
 (B) All actors are brave. Some men are actors. Some men are brave.
 (C) All actors are pretty. P is not an actor. P is not pretty.
 (D) Some men are employees. All employees are efficient. Some efficient people are employees.

- (a) A only (b) B and D (c) B only (d) C only

10. (A) Ravens are black. Ravens are evil. All evil are black.
 (B) Horses are faster than eagles. All eagles are hawks. Horses are faster than hawks.
 (C) No priest is a saint. Peter is a priest. Peter is a saint.

- (a) A only (b) B only (c) C only (d) None of these.

[MAT]

ANSWERS

1. (d) 2. (c) 3. (a) 4. (d) 5. (b) 6. (b) 7. (c) 8. (c) 9. (c) 10. (b)

Solutions and Hints

1. Clearly each combination of premises shall contain two particular premises and as such no definite conclusion follows. Here,

Some blades are hammers. Some hammers are knives.

Since both the premises are particular, conclusion must be particular and should not contain the middle term 'hammers'. So, it follows that 'Some blades are knives'.

Some blades are knives. Some knives are axes

Since both the premises are particular, conclusion must be particular and should not contain the middle term 'knives'. So, it follows that 'Some blades are axes'. From the given conclusions, none of them follows.

Hence, the answer is (d).

2. *All cars are bottles. All bottles are windows.*

Clearly it follows that 'All cars are windows'. Therefore conclusion I follows.

Conclusion III is the converse of I. So III follows.

Thus only I and III follows.

Hence, the answer is (c).

3. *Some hills are rivers. Some rivers are deserts*

Since both the premises are particular, no definite conclusion follows.

Some rivers are deserts. All deserts are roads

Since one premise is particular, conclusion should be particular and should not contain the middle term. So it follows that 'Some rivers are roads'. I is the converse of this conclusion. So I follows.

Hence, the answer is (a).

4. *All fruits are vegetables. All vegetables are rains*

Since both the premises are universal and affirmative, the conclusion must also be universal and affirmative and should not contain the middle term. Therefore, it follows that 'All fruits are rains'. So I follows.

All pens are vegetables. All vegetables are rains.

Since both the premises are universal and affirmative, the conclusion must also be universal and affirmative and should not contain the middle term. Therefore, it follows that 'All pens are rains'. So II follows. Conclusion III is the converse of the third premise and so it holds. So III follows.

Hence, the answer is (d).

5. *All dogs are fruits. No chair is fruit.*

Since one premise is negative, conclusion must be negative. Since both the premises one universal, the conclusion must be universal. Therefore, it follows that 'No dog is chair', which is the converse of this conclusion and so it holds. Hence III follows.

No dog is chair. Some chairs are clowns.

Since one premise is negative and one particular, conclusion must be particular and negative. Therefore it follows that 'Some dogs are not clowns'. I and IV involves only the extreme terms and not the middle term 'chairs' and form complementary pair. So either I or IV follows.

Hence, the answer is (b).

6. Only the statement (b) in which third statement is implied by first two statements.

Hence, answer is (b).

7. Only the statement (c) in which third segment can be logically deduced from the first two segment. i.e. All pizzas are snacks. Some meals are Pizzas.

Hence, the answer is (c).

8. Only the statement (c) in which third segment can be logically deduced from the first two segment.

i.e., Potatoes are stems.

All stems are fruits.

Potatoes are fruits.

Hence, answer is (c).

9. Only statement (c) in which third segment can be logically deduced from the first two segment.
i.e., All actors are pretty.

P is not an actor.

P is not pretty.

Hence answer is (c).

10. Only statement (b) in which third segment can be logically deduced from the first two segment.
i.e., Horses are faster than eagles.

All eagles are hawks.

Horses are faster than hawks.

Hence, answer is (b).

Statements and Arguments

In this type of questions, a statement is given and two arguments for or against or both follow. Analyse the statement first and then decide which of the given arguments is strong with respect to the statement.

If argument I is strong, then answer (a); if argument II is strong, then answer (b); if either of the arguments I or II is strong then answer (c); if none of the arguments I or II is strong, then answer (d) and if both the arguments I and II are strong, then answer (e).

EXAMPLE 1

Statement: Should colleges be given the status of a university in India?

- Arguments:**
- I. Yes. Colleges are in a better position to assess the students' performance and therefore the degrees will be more valid.
 - II. No. It is utopian to think that there will not be nepotism and corruption in awarding degrees by colleges.

Solution At the college level, all the students are assessed according to their performance in the university exams and not on the basis of any criteria of a more intimate dealing with the students. So, argument I is vague. At this level, the awarding of degrees is impartial and simply based on his performance. So, argument II also does not hold. So, the answer is (d).

EXAMPLE 2

Statement: Should government jobs in rural areas have more incentives?

- Arguments:**
- I. Yes. Incentives are essential for attracting government servants there.
 - II. No. Rural areas are already cheaper, healthier and less complex than urban areas. So, why offer extra incentives?

Solution Government jobs in rural areas are having several difficulties as mentioned in the statement. In lieu of these, extra incentives are needed. So only argument I is strong. So the answer is (a).

EXAMPLE 3

Statement: Should computers be used in all possible sectors in India?

Arguments: I. Yes. It will bring efficiency and accuracy in the work.

II. No. It will be an injustice to the monumental human resources which are at present underutilised.

Solution It is clear that underutilised human resources have to be put together for better use. Computers with better and speedy efficiency can accomplish this. So, only argument I is strong. So the answer is (a).

EXERCISES

1. **Statement:** Should all news be controlled by government in a democracy?

[Bank PO]

Arguments: I. Yes. A variety of news only confuses people.

II. No. The controlled news loses credibility.

2. **Statement:** Should higher education be completely stopped for sometime?

[Bank PO]

Arguments: I. No. It will hamper the country's future progress.

II. Yes. It will reduce the educated unemployment.

3. **Statement:** Should workers be allowed to participate in the management of factories in India?

[Bank PO]

Arguments: I. Yes. It is the present management theory.

II. No. Many workers are illiterate and so their contributions will not be of any value.

4. **Statement:** Should the private sector be permitted to operate telephone services?

[Bank PO]

Arguments: I. Yes. They are operated in advanced western countries.

II. No. It is risky to put them in private hands.

5. **Statement:** Should new big industries be started in Bombay?

[Bank PO]

Arguments: I. Yes. It will create new job opportunities.

II. No. It will further add to the pollution of the city.

6. **Statement:** Should India manufacture atom bombs?

[Bank PO]

Arguments: I. Yes. It is imperative to protect the sovereignty and integrity of the country.

II. No. This will create imbalance in the power of nations in this region.

7. **Statement:** Should non-vegetarian food be totally banned in our country?

[Bank PO]

Arguments: I. Yes. It is expensive and therefore it is beyond the means of most people in our country.

II. No. Nothing should be banned in a democratic country like ours.

8. **Statement:** Should luxury hotels be banned in India?

[Bank PO]

Arguments: I. Yes. They are places from where international criminals operate.

II. No. Affluent foreign tourists will have no place to stay.

9. **Statement:** Should adult education programme be given priority over compulsory education programmes?

[Bank PO]

- Arguments: I. No. It will also help in success of compulsory education programme.
II. Yes. It will help to eliminate the adult illiteracy.
10. Statement: Should India go in for computerisation in industry? **[Bank PO]**
Arguments: I. I. No. Computerisation demands a lot of money. We should not waste money on it.
II. Yes. When advanced countries are introducing computers in various areas, how can India afford to lag behind?
11. Statement: Should telecasting feature films be stopped? **[UTI Exam]**
Arguments: I. Yes. Young children are misguided by the feature films.
II. No. This is the only way to educate the masses.
12. Statement: Should all the remote parts of a country be connected by road? **[RBI Exam]**
Arguments: I. No. It will disturb peaceful simple life of the villagers.
II. Yes. It must be done immediately.
13. Statement: Should there be only one university throughout India? **[UTI Exam]**
Arguments: I. Yes. This is the only way to bring about uniformity in the educational standards.
II. No. This is administratively impossible.
14. Statement: Should election expenses to central and state legislatures be met by the government? **[AAO Exam]**
Arguments: I. Yes. It will put an end to political corruption.
II. No. It is not good in any country.
15. Statement: Should fashionable dresses be banned? **[AAO Exam]**
Arguments: I. Yes. Fashions keep changing and hence consumption of cloth increases.
II. No. Fashionable clothes are a persons's self expression and therefore his/her fundamental right.
16. Statement: Should political parties be banned? **[AAO Exam]**
Arguments: I. Yes. It is necessary to teach a lesson to the politicians.
II. No. It will lead to an end of democracy.
17. Statement: Should a total ban be put on trapping wild animals? **[AAO Exam]**
Arguments: I. Yes. Trappers are making a lot of money.
II. No. Bans on hunting and trapping are not effective.
18. Statement: Should the illiterate be debarred from voting?
Arguments: I. Yes. They are easily misguided.
II. No. It is their constitutional right.
19. Statement: Should religion be taught in our schools?
Arguments: I. No. Ours is a secular state.
II. Yes. Teaching religion helps inculcate moral values among children.
20. Statement: Should there be no place of interview in selection?
Arguments: I. Yes. It is very subjective in assessment.
II. No. It is the only instrument to judge candidates motives and personality.
21. Statement: Should articles of only deserving authors be allowed to be published? **[Bank PO]**

- Arguments: I. Yes. It will save a lot of paper which is in short supply.
II. No. It is not possible to draw a line between the deserving and the undeserving.
22. Statement: Should India encourage exports, when most things are insufficient for internal use itself? **[Bank PO]**
Arguments: I. Yes. We have to earn foreign exchange to pay for our imports.
II. No. Even selective encouragement would lead to shortages.
23. Statement: Should income tax be evaded by people? **[Bank PO]**
Arguments: I. Yes. Taxes are excessively high.
II. No. It is anti-national to do so.
24. Statement: Should personal tax be abolished in India? **[Bank PO]**
Arguments: I. Yes. It will motivate people to earn more.
II. No. Individuals must learn to share their wealth with other people.
25. Statement: Should India have no military force at all? **[Bank PO]**
Arguments: I. No. Other countries in the world do not believe in non-violence.
II. Yes. Many Indians believe in non-violence.
26. Statement: Should selection tests be of the objective rather than of the descriptive type? **[Bank PO]**
Arguments: I. Yes. The assessment of answers to objective type questions is fair and impartial.
II. No. The descriptive type test is certainly a better tool than the objective type test.
27. Statement: Should those who receive dowry, despite the law prohibiting it, be punished? **[Bank PO]**
Arguments: I. Yes. Those who violate the law, must be punished.
II. No. Dowry system is firmly rooted in the society since time immemorial.
28. Statement: Should smoking be prohibited? **[BSRB Exam]**
Arguments: I. Yes. It is wrong to smoke away millions of money.
II. No. It will throw thousands of workers in the tobacco industry out of employment.
29. Statement: Should taxes on colour television be further increased? **[AAO Exam]**
Arguments: I. Yes. Colour television is a luxury item and only rich people buy them.
II. No. Televisions are bought by the poor too.
30. Statement: Should loyalty be the only criterion for promotion in any organisation? **[Officer's Grade]**
Arguments: I. Yes. Without loyal men, no organisation can function.
II. No. It leads to hypocrisy and partiality.

ANSWERS

1. (b)	2. (a)	3. (b)	4. (c)	5. (e)	6. (a)	7. (b)	8. (b)	9. (b)	10. (d)
11. (d)	12. (b)	13. (b)	14. (a)	15. (b)	16. (d)	17. (d)	18. (b)	19. (b)	20. (a)
21. (b)	22. (a)	23. (b)	24. (d)	25. (d)	26. (a)	27. (a)	28. (d)	29. (d)	30. (d)

Explanations

1. A variety of news helps people to develop their own news. So argument I is strong. The controlled news shall be partial. So it loses credibility. Thus argument II is strong.
2. Higher education is not the cause of unemployment. In fact, it has created greater job opportunities. So argument II is vague. Besides higher education promotes the country's development. So argument I is strong.
3. The argument I in support does not provide a valid reason for the pursuance of the policy. So it is vague. Argument II provides a valid reason that illiterate workers will create wastefulness in management. So only argument II is strong.
4. In a developing country, a policy cannot be followed merely on the pretext that it is pursued in the western countries. So argument I does not hold good. Putting telephone services in private hands may even bring out greater efficiency. So, argument II is also vague.
5. With big industries, pollution is always a problem. But it is having some advantage as there are more openings in the field of employment. So, both the arguments are strong.
6. For attaining nuclear power, manufacture of atom bombs is an inevitability to protect the country from the threat of nuclear power. So, argument I is strong. Argument II against the statement is, however, weak.
7. Restriction on the diet of people will be denying them from their basic human rights. So, only argument II holds.
8. Luxury hotels are a mark of country's standard and a place for staying the affluent foreign tourists. So, argument II is strong. Ban on hotels is not a way to end the success of international criminals. So argument I is weak.
9. Argument I gives a reason in support of the statement and so it is not strong. The adult education programme needs to be given priority because it shall eliminate adult illiteracy and thus help in further spread of education. So argument II is strong.
10. Development in a new field is not a matter of merely following other countries. So, argument II is not strong. Computerisation is a much beneficial project and investment in it is not at all a waste. So argument I is also not strong enough.
11. Argument I is not strong because films also educate masses. Similarly, argument II against the statement is weak because it is not the only way to educate the masses.
12. Connecting remote parts by roads will only help the people there. So, argument II alone holds strong.
13. To bring uniformity in educational standards, we can have many universities all following same curricular and policies under one board. But one university throughout the country makes impossible the management of education. So, argument II is strong.
14. The policy will end political corruption to extract these amounts. The second argument is vague. So only argument I is strong.
15. Imposing ban on fashionable dresses will be a restriction on the personal choice and hence the right to freedom of an individual. So, only argument II is strong.
16. The ban on political parties, candidates can independently contest elections. So, it will not end democracy. So, argument II is not strong. Argument I does not give a strong reason.
17. Ban is necessary to protect our natural environment. So, none of the arguments is strong enough.
18. Argument I is not strong as no one can be debarred from their constitutional right even if they cannot practise it. In the same context argument II is strong.
19. Ours is a secular state does not mean that religion and religious values are to be eradicated. These inculcate moral values. So, argument I is vague and only argument II is strong.

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20. Besides interview, there can be written examination to judge candidate's motives. So, the second argument is not strong enough. However, interview is a subjective assessment. So argument I is strong.
 21. The first argument is not a strong reason in support of the statement. It is not possible to analyse the really deserving and not deserving. So argument II is strong.
 22. India can export only the surplus and those which can be saved from the luxury needs to pay for its import. Encouragement to export cannot lead to shortages as it shall provide the resources for imports. So, only argument I is strong.
 23. Income tax taken from the people is utilised for the welfare of the people. So it will be anti-national on their part to evade taxes. Thus, only argument II is strong.
 24. Abolishing the tax will increase the people's income and make them passive. So, argument I is vague. The personal tax is no way of sharing wealth with other people. So, argument II also does not hold.
 25. India needs to have military force to defend itself against the threat of other military powers in the world. So, both the arguments are weak.
 26. Judgement being subjective tests depends on the individual who judges while that in objective tests is fair and impartial. So, argument I is strong and argument II is weak.
 27. Laws are made to ensure that no person pursues the practice. So, person, who violate the laws, is to be punished. So, argument I is strong. Even though wrong practice is firmly rooted, it should be ended. So, argument II is not strong.
 28. Smoking is to be abolished as it is injurious to health and not only to save money. So argument I is vague. To provide employment one cannot continue a hazardous task. So, argument II is also vague.
 29. The colour television that has now become more of a means of entertainment and television is bought by all those who can afford it and not only by rich. So, argument I is vague. Poor people cannot buy televisions. So, argument II is not strong.
 30. The argument in support of the statement is vague. When loyalty is considered, hypocrisy does not matter much as the fact that efficiency is neglected. So, the arguments are not strong enough.

Statements and Assumptions

An assumption is something that can be supposed on considering a given statement.

Generally, a statement is given, followed by two assumptions. You have to assess the given statement and then decide which of the given assumptions is implicit in the statement and choose the correct alternative provided.

Directions: In the questions of the type, a statement is given followed by two assumptions numbered I and II. You have to consider the statement and decide which of the given assumptions is implicit.

Give answer (a) if only assumption I is implicit; (b) if only assumption II is implicit; (c) if either I or II is implicit; (d) if neither I nor II is implicit and (e) if both I and II are implicit.

EXAMPLE 1

Statement : A good book, even if costly is sold.

Assumptions : I. Some books are better than others.
II. Most of the books are costly.

Solution Statement mentions a 'good' book. This means some books may not be good. So assumption I is implicit. The words 'if costly' show that most books are not costly. So assumption II is not implicit. Hence the answer is (a).

EXAMPLE 2

Statement : "You must learn to refer the dictionary if you want to become a good writer"—A advises B.

Assumptions : I. Only writers refer to the dictionary.
II. All writers good or bad refer to the dictionary.

Solution It does not follow from the statement that only writers and nobody else refer to the dictionary. Nothing is mentioned about the bad writers. So, both the assumptions I and II are not implicit and therefore the answer is (d).

EXAMPLE 3

Statement : “All are cordially invited to attend the entertainment programme. It is free”—An announcement in a newspaper.

Assumptions : I. A negligible number of readers read announcements in a newspaper.
II. People generally do not go to entertainment programme which are free.

Solution Nothing is mentioned in the statement about the number of people who read announcements or whether people do or do not go to free entertainment. Hence, the answer is (d).

EXERCISES

Directions: In the following question given a statement is followed by two assumptions numbered I and II. An assumption is something supposed or taken for granted. You have to consider the statement and the following assumptions and decide which of the assumption is implicit in the statement.

Give answer (a) if only assumption I is implicit; (b) if only assumption II is implicit; (c) if either I or II is implicit; (d) if neither I nor II implicit and (e) if both I and II are implicit.

1. **Statement** : The boy is too honest to speak the truth.
Assumptions : I. Very honest boys also tell lies.
II. Dishonest boys also speak the truth.
2. **Statement** : Of all the radio sets manufactured in India, the ‘X’ brand has the largest sale.
Assumptions : I. The sale of all the radio sets manufactured in India is known.
II. The manufacturing of no other radio set in India is as large as ‘X’ brand radio.
3. **Statement** : Warning: Cigarette smoking is injurious to health. **[Bank PO]**
Assumptions : I. Non-smoking promotes health.
II. Really, this warning is not necessary.
4. **Statement** : The next meeting of the Governing Board of the Institute will be held after one year. **[Bank PO]**
Assumptions : I. The Institute will remain in function after one year.
II. The Governing Board will be dissolved after one year.
5. **Statement** : Let us change the eligibility criteria to reduce the number of applicants. **[RBI Exam]**
Assumptions : I. The present eligibility criteria is very low.
II. A large number of applications are not desirable.
6. **Statement** : The integrated steel plants in India would no longer have to depend on imports for continuous casting refractories. **[Bank PO]**
Assumptions : I. Continuous casting refractories are needed by India.
II. Continuous casting refractories are in demand.
7. **Statement** : A sentence in the letter to the candidates called for written examination. “You have to bear your expenses on travel etc.” **[SBI PO]**
Assumptions : I. If not clarified all the candidates may claim reimbursement of expenses.
II. Many organisation7s reimburse expenses on travel to candidate called for written examination.

8. Statement : It is through participative management policy alone that indiscipline in our industries can be contained and a quality of life ensured to the worker.
[SBI PO]
Assumptions : I. Quality of life in our industry is better.
II. Indiscipline results in poor quality of life.
9. Statement : “Blue tie would help us identify our staff from others”—A suggestion in a company.
[Bank PO]
Assumptions : I. The company needs to identify its staff.
II. Blue tie is the latest fashion.
10. Statement : “A car is required on rent”—An advertisement.
[Bank PO]
Assumptions : I. All types of vehicles are available on rent.
II. People will respond to the advertisement.
11. Statement : Read this notice before entering the club.
Assumptions : I. People are literate.
II. No blind person comes to the club.
12. Statement : Postal rates have been increased to meet the deficit.
Assumptions : I. The present rates are very low.
II. If the rates are not increased the deficit cannot be met.
13. Statement : “Present day education is in shambles and the country is going to the dogs”.
[AEO]
Assumptions : I. A good education system is essential for the well being of a nation.
II. A good education alone is sufficient for the well being of a nation.
14. Statement : “Computer education should start at schools itself”.
Assumptions : I. Learning computers is easy.
II. Computer education fetches jobs easily.
15. Statement : The new education policy envisages major modifications in the education system.
Assumptions : I. The present education system is inconsistent with national needs.
II. The present education system needs change.
16. Statement : “Do not lean out of the door of the bus”—A warning in a school bus.
Assumptions : I. Leaning out of a running bus is dangerous.
II. Children do not pay any heed to such warnings.
17. Statement : Be humble even after being victorious.
Assumptions : I. Many people are humble after being victorious.
II. Generally, people are not humble.
18. Statement : Opening a library in Rambli will be a wastage.
Assumptions : I. Inhabitants of Rambli are illiterate.
II. Inhabitants of Rambli are not interested in reading.
19. Statement : Films have become indispensable for the entertainment of people.
Assumptions : I. Films are the only media of entertainment.
II. People enjoy films.
20. Statement : Children are influenced more by their teachers now-a-days.
Assumptions : I. The children consider teachers as their models.
II. A large amount of children’s time is spent in school.

21. Statement : "In order to bring punctuality in our office, we must provide conveyance allowance to our employees"—Incharge of a company tells Personnel Manager. **[Bank PO]**
Assumptions : I. Conveyance allowance will not help in bringing punctuality.
II. Discipline and reward should always go hand in hand.
22. Statement : A warning in a train compartment—"To stop train, pull chain. Penalty for improper use ₹ 500".
Assumptions : I. Some people misuse the alarm chain.
II. On certain occasions, people may want to stop a running train.
23. Statement : "You should not grant him leave in this week due to exigency of work"—A supervisor advises the administrative officer. **[Officer's Grade]**
Assumptions : I. Request for leave can be turned down also.
II. The supervisor has reviewed the work required to be done during the said period.
24. Statement : "We offer best training in the field of computers"—An advertisement. **[Bank PO]**
Assumptions : I. People are interested in getting training in computers.
II. People want best training.
25. Statement : "Use aluminium—The versatile metal for packaging"—An advertisement. **[Bank PO]**
Assumptions : I. Aluminium is the only versatile metal.
II. Some companies use metallic packing.
26. Statement : The president assured the people that elections will be held here after every five years. **[SBI PO]**
Assumptions : I. People are afraid that the elections may not be held at all.
II. People are afraid that elections may not be held after five years.
27. Statement : If it is easy to become an engineer, I don't want to be an engineer. **[SBI PO]**
Assumptions : I. An individual aspires to be professional.
II. One desires to achieve a thing which is hard earned.
28. Statement : You can hereby appointed as a programmer with a probation period of one year and your performance will be reviewed at the end of the period for confirmation"—A line in an appointment letter. **[Bank PO]**
Assumptions : I. The performance of an individual generally is not known at the time of appointment offer.
II. Generally an individual tries to prove his worth in the probation period.
29. Statement : Lock your valuables in a cupboard and call everybody gentleman. **[SBI PO]**
Assumptions : I. Valuables locked in cupboard cannot be stolen.
II. Stealing is a crime.
30. Statement : "Banking services are fine-tuned to meet growing business needs"—An advertisement. **[SBI PO]**
Assumptions : I. Banking is a part of business activity.
II. Industrialists prefer better banking services.

31. Statement : The successful man has the ability to judge himself correctly. [AAO Exam]
 Assumptions : I. To judge others is of no use to a successful man.
 II. The successful man cannot make a wrong judgement.
32. Statement : Television X-the neighbour's envy, the owner's pride—A TV advertisement. [RBI Exam]
 Assumptions : I. People are envious of their neighbours superior possessions.
 II. People want to be envied by their neighbours.
33. Statement : "The function will start at 3 PM. You are requested to take your seats before 3 PM"—Last sentence in an invitation card.
 Assumptions : I. If the invitee is not in his seat before 3 PM, the function will not start.
 II. Function will start as scheduled.
34. Statement : If you have any problems, bring them to me.
 Assumptions : I. You have some problems.
 II. I can solve any problem.
35. Statement : Vitamin E tablets keep your complexion in a glowing condition by improving circulation.
 Assumptions : I. People like a glowing complexion.
 II. Complexion becomes dull in the absence of circulation.
36. Statement : "Ensure a good night's sleep for your family with safe and effective X mosquito coil"—An advertisement. [UTI Exam]
 Assumptions : I. X mosquito coil is better than any other mosquito coil.
 II. A good night's sleep is desirable.
37. Statement : All existing inequalities can be reduced, if not utterly eradicated by action of government or by revolutionary change of government. [LIC]
 Assumptions : I. Inequality is a man-made phenomenon.
 II. No person would voluntarily part with what he possesses.
38. Statement : The taste of food contributes to the intake of nourishment which is essential for the survival of human beings. [Management Trainee's Exam]
 Assumptions : I. Human beings take food for the enjoyment of its taste.
 II. Human beings experience the taste of food.
39. Statement : Every year doctors, scientists and engineers migrate from India to greener pastures. [Income Tax and Central Excise]
 Assumptions : I. Brain drain has affected India adversely.
 II. Better scales and better standards of living act as a bait to lure them.
40. Statement : The present examination system needs overhauling thoroughly. [Investigator's Exam]
 Assumptions : I. The present examination system is obsolete.
 II. Overhauling results in improvement.

(ANSWERS)

1. (d)	2. (a)	3. (d)	4. (e)	5. (d)	6. (e)	7. (e)	8. (b)	9. (a)	10. (b)
11. (e)	12. (b)	13. (e)	14. (a)	15. (e)	16. (a)	17. (d)	18. (c)	19. (b)	20. (a)
21. (b)	22. (e)	23. (e)	24. (e)	25. (e)	26. (e)	27. (b)	28. (e)	29. (a)	30. (e)
31. (e)	32. (a)	33. (b)	34. (b)	35. (e)	36. (e)	37. (a)	38. (b)	39. (d)	40. (e)

Explanations

1. The statement means that boy is so honest that he will definitely speak the truth. i.e. very honest boy will not tell lies. So assumption I is not implicit. The statement mentions nothing about dishonest boys. So, assumption II is not implicit.
2. Comparison could not be made without knowing the sale of all the radio sets. So assumption I is implicit. The statement mentions only that the sale is largest and nothing is mentioned about the manufacture. So, assumption II is not implicit.
3. The statement confirms that smoking affects health adversely and so non-smoking will keep the health as it is. Thus, assumption I is not implicit. The warning is necessary to tell the people that injury would be caused by smoking. So, assumption II is not implicit.
4. The meeting of the Governing Board will be held after one year means the institute will start functioning at that time. So, assumption I is implicit. The board cannot be dissolved at the time when its meeting starts. So, assumption II is implicit.
5. The statement shows that the eligibility criteria needs to be raised but not because they are very low but only to reduce the number of applicants. So, assumption I is not implicit. The statement does not mention anything about the worth of applicants. So, assumption II is also not implicit.
6. The statement mentions the self-sufficiency of India in continuous casting refractories. This means that they are needed in the country. So, assumption I is implicit. Since continuous casting refractories are needed in integrated steel plants, it means that they are in demand. So assumption II is implicit.
7. Not mentioning the condition may provoke all the candidates to demand their claim. So, assumption I is implicit. The condition is mentioned because some companies do reimburse the travel expenses. So, assumption II is also implicit.
8. The statement mentions that participative management policy will provide quality life to the workers. So, assumption I is not implicit. The statement mentions that participative management will contain the indiscipline and ensure quality life to workers. So, assumption II is implicit.
9. Here, the suggestion is given for providing a method of identification. This means that the staff needs to be identified. So assumption I is implicit. The statement does not mention anything about the fashion. So, assumption II is not implicit.
10. The advertisement clearly mentions that a car is required on rent. So, assumption I is not implicit. An advertisement is given for the people to respond it. So, assumption II is implicit.
11. The statement is meant for the people to read. So, it is assumed that the people are literate and assumption I is implicit. Since the notice is to be read by everyone entering the club, so it is assumed that no blind person comes to the club. Thus, assumption II is also implicit.
12. The statement shows that the present postal rates are not sufficient to meet the deficit but they may still be high. So, assumption I is not implicit. The rates are increased to meet the deficit. So, assumption II is implicit.
13. The statement mentions the degradation of the country with the disruption of the education system. So, assumption I is implicit. Clearly, from the statement it does not follow whether education alone is required or whether other things go with it for the well being of the nation. So assumption II is also implicit.
14. The computer education can be started at the school level only if it is easy. So, assumption I is implicit. In the statement nothing is mentioned about the link between the job and the computer education. So, assumption II is not implicit.

15. Modifications are made in the present education system finding that it was inconsistent with the needs and required to be changed. So, both the assumptions I and II are implicit.
16. The warning against leaning out of running bus is made to heed against the dangers involved. So, assumption I is implicit. The warning is meant to be heeded but nothing is mentioned whether children pay heed to it or not. So, assumption II is not implicit.
17. Nothing is mentioned about the nature of the people in the statement. It simply gives advice. So, both the assumptions I and II are not implicit.
18. A library will be a wastage only when the people of the place cannot read or they are not interested in reading. So, either of the assumptions I or II is implicit.
19. 'Films are indispensable' doesn't mean that they are the only means of entertainment. So, assumption I is not implicit. Assumption II follows the statement. So assumption II is implicit.
20. Children consider teachers as their model as they are more influenced by them. So, assumption I is implicit. It is not necessary that the children are influenced by teachers because they spend much time in school. So, assumption II is not implicit.
21. Assumption I is against the statement. So it is not implicit. The allowance will serve as a reward to the employees and shall provoke them to come on time. So, assumption II is implicit.
22. Penalty is imposed to prevent people from misusing the alarm chain. This means that some people misuse it. So, assumption I is implicit. The alarm chain is provided to stop the running train in case of emergency. So, assumption II is also implicit.
23. The advice is given to turn down the request for leave. So assumption I is implicit. The mention of the exigency of work makes assumption II implicit.
24. The advertisement is meant to cater to the people's demand of computer training. So, assumption I is implicit. The offer of 'best training' makes assumption II implicit.
25. 'Versatility' is mentioned as the special quality of aluminium. So, assumption I is implicit. The advertisement is meant for those who use metallic packing. So, assumption II is also implicit.
26. The statement is made to eliminate the fear of the people that the elections may not be held at all or they may not be held after five years. So, both assumptions I and II are implicit.
27. Nothing is mentioned about the professional nature of the job. So, assumption I is not implicit. From the statement, it is clear that one rejects a thing that is easy to achieve. So, assumption II is implicit.
28. The performance of the individual has to be tested over a span of time as the statement mentions. So, assumption I is implicit. The statement mentions that the individual's worth shall be reviewed before confirmation. So assumption II is also implicit.
29. The statement points out that a person who keeps his things locked away shall feel that every person is good because he has no danger for his things. So, assumption I is implicit. The statement mentions nothing about the lawful nature of the act of stealing. So, assumption II is not implicit.
30. According to the statement, banking is connected to business activity. So, assumption I is implicit. Banking is adjusted in a way to promote business needs. This means that business is promoted by better banking. So, assumption II is also implicit.
31. The basic quality of a successful man is that he can judge himself. This means that he need not judge others. So, assumption I is implicit. The word 'correctly' makes assumption II implicit.
32. The advertisement attacks the particular quality mentioned in the assumption. So, assumption I is implicit. The statement mentions that the thing shall be coveted because it shall be his superior possession. So, assumption II is not implicit.
33. In the statement, it is mentioned that the function will start at 3 PM and not that the invitees will be waited for. So, assumption I is not implicit and only assumption II is implicit.

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34. The statement doesn't show that you have problems. So, assumption I is not implicit. The statement states that problems will be solved by 'me'. So, assumption II is implicit.
 35. The advice is given for people who like glowing complexion. So, assumption I is implicit. Since complexion glows if circulation is improved. So, assumption II is also implicit.
 36. The advertisement mentions the coil safe and effective pointing to the best qualities. So, assumption I is implicit. The advertisement is meant to enable people to have a good night's sleep. So, assumption II is also implicit.
 37. Since inequality can be reduced, it means that it is not natural but created. So, assumption I is implicit. Nothing is mentioned about the people's response. So, assumption II is not implicit.
 38. It is mentioned that nourishment is essential for survival. So, this is the basic cause of intake of food. Hence, assumption I is not implicit. Since taste of food affects the intake of nourishment, it means that human beings are affected by taste. So, assumption II is implicit.
 39. Nothing is mentioned about the effect of migration on the country. So, assumption I is not implicit. Nothing is mentioned about the reason for the migration. So, assumption II is also not implicit.
 40. 'Thorough' overhauling is needed only in case of an obsolete system. So, assumption I is implicit. Overhauling is done for improvement. So, assumption II is also implicit.

Statements and Course of Action

In such questions, a statement mentioning a particular problem is given and is followed by two courses of action numbered I and II. You have to analyse the statement and then decide which of the courses of action should be a step or decision taken for improvement, follow-up or further action in regard to the given statement. Everything in the statement is assumed to be true and logical courses of action should follow from it.

EXAMPLE 1

Statement : Many cases of cholera were reported from a nearby village.

Courses of action : I. The question should be raised in the Legislative Assembly.
II. A team of doctors should be rushed to the village.

Solution The disease has to be eradicated. For this proper and immediate medication and preventive measures by doctors are necessary. So only course II follows.

EXAMPLE 2

Statement : People residing in some remote tribal areas are far from education.

Courses of action : I. Schools for children and adults should be opened there.
II. Social workers should be entrusted with the job of educating them.

Solution Clearly, to make permanent arrangements for education in remote tribal areas, schools have to be opened in those areas. Education by social workers shall be a temporary remedy. So only course of action I follows.

EXAMPLE 3

Statement : The librarian finds some cases in which the pages from certain books issued from the library, are torn.

Courses of action : I. The librarian should keep a record of books issued to each student, and if the pages are found torn, strict measures should be taken against the student who had been issued that book.
II. Some funds should be collected from the students collectively to renovate the library.

Solution Clearly, precaution should be taken to catch the guilty person and punish him for the act. This alone will help curb the wrong practice. So, only course I will follow.

EXERCISES

Directions: In the following question a statement is followed by two courses of action numbered I and II. A course of action is a step or decision to be taken for improvement, follow-up, or further action in regard to the problem policy, etc. on the basis of the information given in the statement. You have to assume everything in the statement to be true, then decide which of the two given courses of actions logically follows for pursuing.

Give answer (a) if only I follows; (b) if only II follows; (c) if either I or II follows; (d) if neither I nor II follows; and (e) if both I and II follow.

1. Statement : A leading US multinational engineering and construction firm is keen to invest in India in a variety of sectors ranging from power to land management. **[Bank PO]**
 Courses of action : I. Such multinational companies should not be allowed to operate in India.
 II. India should encourage multinational companies from other developed countries to invest in the power sector to bring in competitive climate.
2. Statement : A shopkeeper was reported to be selling adulterated grains.
 Courses of action : I. He should be fined and his shop sealed.
 II. He should be asked to leave the town and open a shop elsewhere.
3. Statement : A train has derailed near a station while moving over a bridge and fell into the river.
 Courses of action : I. The railway authorities should clarify the reason of the accident to the government.
 II. The government should allocate funds to compensate the destruction caused.
4. Statement : The Central Bureau of Investigation receives the complaint of an officer taking bribe to do the duty he is supposed to.
 Courses of action : I. The CBI should try to catch the officer red handed and then take a strict action against him.
 II. The CBI should wait for some more complaints about the officer to be sure about the matter.
5. Statement : Most of the children in India are not able to get education, because they get employed to earn livelihood in their childhood only.
 Courses of action : I. Education should be made compulsory for all children up to the age of 14.
 II. Employment of children below the age of 14 years, should be banned.
6. Statement : The Finance Minister submits his resignation a month before the new budget is to be presented in the Parliament.
 Courses of action : I. The resignation should be accepted and another person should be appointed as the Finance Minister.
 II. The resignation should not be accepted.
7. Statement : Courts take too long in deciding important disputes of various departments.
 Courses of action : I. Courts should be ordered to speed up matters.
 II. Special powers should be granted to officers to settle disputes concerning their department.

8. Statement : A group of school students was reported to be enjoying at a picnic spot during school hours.
 Courses of action : I. The principal should contact the parents of those students and tell them about the incident with a real warning for future.
 II. Some disciplinary action must be taken against those students and all other students should be made aware of it.
9. Statement : Footpaths of a busy road are crowded with vendors selling cheap items.
 Courses of action : I. The help of police should be sought to drive them away.
 II. Some space should be provided to them where they can earn their bread without blocking the footpaths.
10. Statement : The sale of a particular product has gone down considerably causing great concern to the company.
 Courses of action : I. The company should make a proper study of rival products in the market.
 II. The price of the product should be reduced and quality improved.
11. Statement : A recent study shows that children below five die in the cities of the developing countries mainly from diarrhoea and parasitic intestinal worms. **[Bank PO]**
 Courses of action : I. The government of developing countries should take adequate measures to improve the hygienic conditions in the cities.
 II. Children below five years in the cities of the developing countries need to be kept under constant medication.
12. Statement : The Doordarshan is concerned about the quality of its programmes particularly in view of stiff competition it is facing from Star and other satellite TV channels and is contemplating various measures to attract talent for its programmes. **[Bank PO]**
 Courses of action : I. In an effort to attract talent, the Doordarshan has decided to revise its fee structure for the artists.
 II. The fee structure should not be revised until other electronic media also revise it.
13. Statement : The Indian electronic component industry venturing into the West European markets faces tough competition from the Japanese.
 Courses of action : I. India should search for other international markets for its products.
 II. India should improve the quality of the electronic components to compete with the Japanese in capturing these markets.
14. Statement : Exporters in the capital are alleging that commercial banks are violating the Reserve Bank of India directive to operate a post shipment export credit denominated in foreign currency at international interest rates from January this year. **[Bank PO]**
 Courses of action : I. The officers concerned in the commercial banks are to be suspended.
 II. The RBI should be asked to stop giving such directives to commercial banks.
15. Statement : One of the problems facing the food processing industry is the irregular supply of raw material. The producers of raw material are not getting a reasonable price. **[Bank PO]**
 Courses of action : I. The government should regulate the supply of raw materials to other industries also.
 II. The government should announce an attractive package to ensure regular supply of raw materials for food processing industry.

- 16. Statement** : India's performance in the recent Olympic Games was very poor. Not even a single medal could be bagged by the players. The government has spent ₹ 5 crores in training and deputing a team of players to participate in the Olympic Games. **[Bank PO]**
- Courses of action** : I. India should stop sending players to the future Olympic Games.
II. Government should immediately set up an enquiry commission to find out the reason for India's dismal performance.
- 17. Statement** : The government will slap legally enforceable penalties on coal companies defaulting on quality and quantity of coal supplies to bulk consumers, especially to the thermal power station. **[Bank PO]**
- Courses of action** : I. The requirement of coal for thermal power stations should be assessed realistically.
II. The coal companies should introduce welfare measures for their employees.
- 18. Statement** : Some serious blunders were detected in the accounts section of a factory.
- Courses of action** : I. An efficient team of auditors should be appointed to check the accounts.
II. A show cause notice should be issued to all the employees involved in the irregularity.
- 19. Statement** : The police department has come under a cloud with recent revelations that at least two senior police officials are suspected to have been involved in the illegal sale of a large quantity of weapons from the state police armoury.
- Courses of action** : I. A thorough investigation should be ordered by the state government to bring out all those who are involved into the illegal sale of arms.
II. State police armoury should be kept under central government's control.
- 20. Statement** : Orissa and Andhra Pradesh have agreed in principle to set up a joint control board for better control, management and productivity of several interstate multipurpose projects. **[Bank PO]**
- Courses of action** : I. Other neighbouring states should setup such control boards.
II. The proposed control board should not be allowed to function as such joint boards are always ineffective.
- 21. Statement** : The committee has criticised the institute for the failure to implement a dozen of regular programmes despite an increase in the staff strength and not drawing up a firm action plan for studies and research. **[SBI PO]**
- Courses of action** : I. The broad objectives of the institute should be redefined to implement a practical action plan.
II. The institute should give a report on reasons for not having implemented the planned programmes.

ANSWERS

1. (b)	2. (a)	3. (d)	4. (a)	5. (e)	6. (b)	7. (e)	8. (e)	9. (e)	10. (a)
11. (e)	12. (a)	13. (b)	14. (d)	15. (b)	16. (b)	17. (d)	18. (a)	19. (a)	20. (a)
21. (e)									

(Explanations)

1. Clearly, financing is the major problem in starting any project. The investment by multinational companies shall, therefore, be a way to development. So only II follows.
2. It is clear that if shopkeeper is allowed to continue without being punished, he would create the problem elsewhere. So only I follows.
3. It is necessary to take preventive measures so as to protect the passengers and pay them adequate compensation. So none of the two courses follows.
4. One complaint is enough for a wrong doing. This should be confirmed by catching the guilty one and then strict action is to be taken against him. So only course I follows.
5. To educate all children, enforcement of education is necessary. Also, the reason is that they are employed. So ban on such employment is also needed. Thus, both the courses follow.
6. The present finance minister shall know better all the plans and resources of the government and he alone can present a suitable budget. So course II should be followed.
7. For quick disposal of cases, either the matters in the court should be speeded up or the matters should be cleared up in their respective departments to prevent the delay. So both the courses follow.
8. Both warning and future prevention are necessary. So both the courses follow.
9. The footpaths are meant for an entirely different purpose. So, they need to be kept empty. For this, police help has to be sought. Also, the vendors cannot be deprived of a living. So, both the courses follow.
10. A study of rival products in the market will help assess the cause for the lowering down of the prices and then a suitable action will be taken. So only I follow.
11. The two diseases mentioned are caused by unhygienic conditions. So, improving the hygienic conditions is a step towards their eradication. Also, constant medication will help timely detection of the disease and hence a proper treatment. So both I and II follow.
12. The decision to revise its fee structure for artists is taken by Doordarshan as a remedy to the challenging problem that had arisen before it. It cannot wait till other media take action. So only course I follows.
13. An escapist attitude does not help much. The need is to compete and emerge successful. So, only course II follows.
14. The statement mentions that the commercial banks violate a directive issued by the RBI. The remedy is only to make the banks implement the act. So none of the courses follows.
15. In order to solve the problem of food industry, a regular supply of raw materials should be ensured. So course II shall follow.
16. To compete against a challenge, the first step must be to find out where the lackening is. So, only course II follows.
17. None of the courses of action is a suitable follow up of the government's act against defaulters. So, none of the courses of action follows.
18. Detect the blunder and its improvement is the urgent need. So, only course I follows.
19. Here the first course of action is suitable follow up of the problem.
20. The effectiveness of such control boards is established by the fact that Orissa and A.P. have agreed to it for better control of its multipurpose projects. So only course of action I follows.
21. The problem is that despite an increase in staff strength, the institute has failed in its objective of implementing its plan. So, either there should be reasons for the lack or the plans are a failure and must be revised for practical implementation. Thus, both the courses follow.

Statements and Conclusions

In this type of questions, one or more statements are given. You have to analyse the given statements, understand their indirect implications and then decide which of the given conclusion follows.

Directions: In each of the following questions, a statement is given followed by two conclusions I and II. Give answer (a) if only conclusion I follows; (b) if only conclusion II follows; (c) if either I or II follows; (d) if neither I nor II follows and (e) if both I and II follow.

EXAMPLE 1

Statement : The distance of 900 km by road between Bombay and Jafra will be reduced to 280 km by sea. This will lead to a saving of ₹ 7.92 crores per annum on fuel.

Conclusions : I. Transportation by sea is cheaper than that by road.
II. Fuel must be saved to the greatest extent.

Solution According to the statement, the sea transport is cheaper than the road transport in the case of route from Bombay to Jafra, not in all the cases. So, conclusion I does not follow. The statement stresses on the saving of fuel. So, conclusion II follows. Hence, the answer is (b).

EXAMPLE 2

Statement : Sealed tenders are invited from competent contractors experienced in executing construction jobs.

Conclusions : I. Tenders are invited only from experienced contractors.
II. It is difficult to find competent tenderers in construction jobs.

Solution According to the statement, tenders are invited from contractors experienced in executing construction jobs. So, conclusion I follows. The availability (c) of competent tenderers in construction is not mentioned. So, conclusion II does not follow. Hence the answer is (a).

EXAMPLE 3

Statement : Any young man who makes dowry as a condition for marriage discredits himself and dishonours womanhood.

Conclusions : I. Those who take dowry should be condemned by society.
II. Those who do not take dowry respect womanhood.

Solution Clearly, the statement declares dowry as an evil practice and reflects its demerits. Thus, conclusion I follows. Also, it is given that those who take dowry, dishonour womanhood. This implies that those who do not take dowry respect womanhood. So, conclusion II follows. Hence the answer is (e).

EXERCISES

Directions: In each question below are given certain statements followed by some conclusions. Choose the conclusion which follows from the given statements.

1. Statements:

1. None but the rich can afford air-travel.
2. Some of those who travel by air become sick.
3. Some of those who become sick require treatment.

Conclusions:

- (a) All the rich persons travel by air.
- (b) Those who travel by air become sick.
- (c) All the rich persons become sick.
- (d) All those who travel by air are rich.

2. Statements:

1. Only students can participate in the race.
2. Some participants in the race are females.
3. All female participants in the race are invited for coaching.

Conclusions:

- (a) All participants in the race are invited for coaching.
- (b) All participants in the race are males.
- (c) All students are invited for coaching.
- (d) All participants in the race are students.

3. Statements:

1. A forest has as many sandal trees as it has Ashoka trees.
2. Three-fourths of the trees are old one and half of the trees are at the flowering stage.

Conclusions:

- (a) All Ashoka trees are at the flowering stage.
- (b) All sandal trees are at the flowering stage.
- (c) At least one-half of the Ashoka trees are old.
- (d) One-half of the sandal trees are at the flowering stage.
- (e) None of these.

4. Statements:

1. Shyam is not the father of Hari.
2. Hari is the son of Suresh.
3. Suresh has three sons.

Conclusions:

- (a) Shyam is son of Suresh.
- (b) Hari is the brother of Shyam.
- (c) Suresh is the father of Hari.
- (d) Shyam has no children.

[Railways]

Directions: In each of the following questions, a statement is followed by two conclusions. Give answer (a) if the conclusion I follows; (b) if conclusion II follows; (c) if both I and II follow; (d) if either I or II follows; and (e) if neither I nor II follows.

5. Statement : Of all the television sets manufactured in India, is the 'solar' brand has the largest sale. [Bank PO]

- Conclusions : I. The volume of sales of all the brands of television sets manufactured in India, is known.
II. The production of no other television set in India is as large as that of 'solar'.

6. Statement : Parents are prepared to pay any price for an elite education to their children.

- Conclusions : I. All parents these days are very well-off.
II. Parents have an obsessive passion for a perfect development of their children through good schooling.

7. Statements : 1. The TV programmes, telecast specially for women are packed with a variety of recipes and household hints.
2. A major portion of magazines for women also contains the items mentioned above.

- Conclusions : I. Women are not interested in other things.
II. An average woman's primary interest lies in home and specially in the kitchen.

8. Statement : Patients with minor ailments usually do not go to eminent doctors.

- Conclusions : I. Eminent doctors remain too busy with patients suffering from serious complications.
II. Their charges are rather high.

9. Statement : From all available cultural records, it is evident that even in ancient India, both the masters and disciples valued not the quantity but the quality of knowledge.

- Conclusions : I. Giving importance to quantity of knowledge is meaningless.
II. There was an identity of educational values between teachers and students in ancient India.

10. Statement : The doctors nursed only those bleeding wounds, which were caused by bullets. A patient X was bleeding profusely.

- Conclusions : I. X was nursed by the doctors.
II. The doctors did not nurse X.

11. Statement : In a one day cricket match, the total runs scored by a team were 200. Out of these 160 runs were made by spinners.

- Conclusions : I. 80% of the team consists of spinners.
II. The opening batsman was spinner.

12. Statement : Players who break various records in a fair way get special prizes. Player X broke the word record but was found to be under the influence of a prohibited drug.

Conclusions : I. X will get the special prize.
II. X will not get the special prize.

13. Statement : National Aluminium Company has moved India from a position of shortage to self sufficiency in the metal.

Conclusions : I. Previously, India had to import aluminium.
II. With this speed, it can soon become a foreign exchange earner.

14. Statement : The domestic demand has been increasing faster than the production of indigeneous crude oil.

Conclusions : I. Crude oil must be imported.
II. The domestic demand should be reduced.

15. Statement : He stressed the need to step the present examination system and its replacement by other methods which would measure the real merit of the students.

Conclusions : I. Examinations should be abolished.
II. The present examination system does not measure the real merit of the students.

Directions: In each of the following questions, a statement has been given followed by two conclusions. Give answer (a) if conclusion I is true and II is wrong; (b) if both the conclusions are true; (c) if both the conclusions are wrong and (d) if conclusion II is true and I is wrong.

16. Statement : The percentage of the national income shared by the top 10 per cent of households in India is 35.

Conclusions : I. When an economy grows fast, concentration of wealth in certain packets of population takes place.
II. The national income is unevenly distributed in India.

17. Statement : The use of non-conventional sources of energy will eliminate the energy crisis in the world.

Conclusions : I. Modern technology is gradually replacing the conventional sources of energy.
II. The excessive exploitation of environment has led to depletion of conventional sources of energy.

18. Statement : In India, the fruits of development have not been equitably distributed between our rural and urban sectors.

Conclusions : I. Rural poverty is substantially higher than urban poverty.
II. A large family size in India may be a consequence rather than a cause of poverty.

Directions: In each of the following questions, a statement is given followed by two conclusions I and II. Mark answer (a) if both the conclusions can be drawn from the statement; (b) if only I can be drawn, (c) if only II can be drawn, (d) if neither I nor II can be drawn and (e) if either I or II can be drawn.

19. Statement : The TV staff deserves an applaud for showing booth capturing.

Conclusions : I. TV always aims at showing things in their true perspective.
II. People involved in booth-capturing have been recognized and are being tried by law.

20. Statement : Smoking is one of those human weakness which tends to test the will power of the smoker to the edge.

Conclusions : I. It is very difficult for the smokers to give up smoking even if they want to do so.
II. Human beings have other weaknesses as well.

21. Statement : Video libraries are flourishing very much these days.
Conclusions : I. People in general have got a video craze.
II. It is much cheaper to see as many movies as one likes on videos rather than going to the cinema hall.
22. Statement : All those political prisoners were released on bail who had gone to jail for reasons other than political dharnas. Bail was not granted to persons involved in murders.
Conclusions : I. No political prisoner had committed murder.
II. Some politicians were not arrested.
23. Statement : No country is absolutely self-dependent these days.
Conclusions : I. It is impossible to grow and produce all that a country needs.
II. Countrymen in general have become lazy.
- Directions:** In each question below is given a statement followed by two conclusions I and II. Give answer (a) if conclusion I is implicit; (b) if conclusion II is implicit; (c) if both I and II are implicit; (d) if either I or II is implicit and (e) if neither I nor II is implicit.
24. Statement : The best way to escape from a problem is to solve it.
Conclusions : I. Your life will be dull, if you don't face a problem.
II. To escape from problems, you should always have some solutions with you.
25. Statement : Good is the enemy of the best.
Conclusions : I. You should be good to your best enemy.
II. The best do not like the good.

ANSWERS

1. (d)	2. (d)	3. (e)	4. (c)	5. (c)	6. (b)	7. (b)	8. (c)	9. (b)	10. (d)
11. (e)	12. (b)	13. (c)	14. (d)	15. (b)	16. (d)	17. (b)	18. (a)	19. (b)	20. (a)
21. (a)	22. (b)	23. (b)	24. (e)	25. (a)					

Deriving Conclusions from Passages

This section comprises some questions on logical deduction. Here, each question consists of a passage followed by five inferences based on it. Readers are required to analyse the passage and grasp desirable facts from it. Then, he has to consider each inference in context of the given passage, decides its degree of truth or falsity and then choose the best alternative provided in the question.

Directions: Read the following passage and examine each inference given below it in the context of this passage. Mark answer (a) if the inference is definitely true; (b) if the inference is probably true; (c) if the data is inadequate; (d) if the inference is probably false; and (e) if the inference is definitely false.

EXAMPLE 1 The bauxite deposits in India are widely distributed. Recently, deposits in Orissa have been developed and the largest plant of its kind in Asia has been set up to produce alumina and aluminium. Its annual capacity is 800,000 tonnes of alumina and 2,25,000 tonnes of aluminium. It uses the latest French technology. The ore is exported to Japan and European countries. In 1987, the output of bauxite was 2.8 million tonnes. The country's reserves are estimated at 270 million tonnes, of which 73 million tonnes are of high quality.

- (i) The plant set up in Orissa is the largest in India.
- (ii) Aluminium is exported to Japan and European countries.
- (iii) Orissa is the largest producer of bauxite in India.
- (iv) The plant in Orissa was set up by financial aid from France.
- (v) About 27% of total reserves of bauxite are of high quality.

Solution

- (i) (a) Since, the plant set up in Orissa is the largest in Asia, it is evident that it is also the largest in India.
- (ii) (e) Not aluminium, but its ore is exported to Japan and European countries.
- (iii) (b) Since the largest bauxite producing plant is in Orissa, it is much possible that Orissa is the largest producer of bauxite.

- (iv) (e) The plant set up in Orissa uses French technology; it was not set up by financial aid from France.
- (v) (a) Total reserves of bauxite = 270 million tonnes. High quality reserves = 73 million

tonnes. Percentage of high quality bauxite reserves = $\left(\frac{73}{270} \times 100\right)\% = 27\%$.

EXAMPLE 2 According to trade estimates, the black pepper crop will be in the range of 75,000 to 95,000 tonnes during the pepper season from November 1989 to October 1990. The crop will be thus the double of that for the 1988–89 season. Besides good monsoon, the record production is attributed to the crash programme launched in Kerala by the State Government last June. Although 80% of the country's total black pepper production is grown in Kerala, the yield per hectare is low at 300 kg as compared to 550 kg in Indonesia and 1200 kg in Brazil.

- (i) India exports black pepper more than that of Brazil.
- (ii) In no other state in the country black pepper yield is more than 300 kg/hectare.
- (iii) Good water supply is needed for cultivating black pepper.
- (iv) The black pepper crop during 1988–89 was about 30,000 tonnes.
- (v) The Central Government helped the Kerala Government in implementing the crash programme. **[RBI Exam]**

Solution

- (i) (c) Nothing is mentioned about the exports of India and Brazil in the passage.
- (ii) (b) As mentioned in the passage, Kerala is the largest pepper producing state and the yield per hectare is 300 kg. But other states show a low yield and so a less production of black pepper.
- (iii) (a) The good monsoon has led to a high production of black pepper as mentioned in the passage.
- (iv) (e) The total produce during 1989–90 was estimated to be 75,000 to 95,000 tonnes and the produce during 1988–89 was half of this, which is more than 30,000 tonnes.
- (v) (c) It is mentioned in the passage that crash programme was launched by Kerala Government. But there is no mention of the efforts or aid of the Central Government to the programme.

EXERCISES

Directions: In each question below is given a passage followed by several inferences. You have to examine each inference separately in the context of the passage and decide upon its degree of truth or falsity.

Mark answer (a) if the inference is 'definitely true'; (b) if the inference is 'probably true' though not definitely true in the light of facts given; (c) if the data are inadequate; (i.e. from facts given you cannot say whether the inference is likely to be true or false); (d) if the inference is 'probably false' though not definitely false in the light of the facts given; and (e) if the inference is 'definitely false'.

1. In the forties, nationalisation was considered the panacea for all socio-economic ills. Today, privatisation has become the buzzword that has been sweeping both the developed and the developing world for more than a decade now. Even in India, the idea has been gathering momentum. But before the idea is transplanted in India, there are several aspects of privatisation that need to be understood. It would be worthwhile, in this context, to look at the experiences of other countries.

- (i) Other countries which have adopted privatisation are considering now to change over to industrialisation.
- (ii) Privatisation has been practiced in the USA since long.
- (iii) India is also thinking of privatisation.
- (iv) Nationalisation has failed to improve substantially the socio-economic situation of some countries.
- (v) India is the first country in Asian subcontinent to adopt privatisation.

[Bank PO]

2. The Ministry of Environment and Forest has granted environmental clearance to the Karkatla open-cast expansion project of the Central Coal fields Ltd. in Bihar that envisages exploitation of non-cocking coal reserves. The present production level of 0.8 million tonnes is proposed to be expanded to 1.5 million tonnes per annum at an estimated cost of 67.82 crores under the project. The total land area requirement for the proposed mining activities is about 651 hectares which includes about one sixth of it as forest land.

- (i) The expansion plan would require about 100 hectares of forest land.
- (ii) Karkatla open-cast mine is the only one of non-cocking coal in the country.
- (iii) There is no demand for non-cocking coal.
- (iv) The production cost of one tonne of non-cocking coal from Karkatla mine will be about ₹ 450.
- (v) Environmental concerns get less priority over the need of the coal.

[Bank PO]

3. There is more bad news on food front. It now appears certain that there will be a shortfall of about 9 million tonnes in the food production achieved in the last *kharif* season. However, rice procurement may only be partially affected since West Bengal and Andhra Pradesh have had sufficient rainfall while Punjab, the major contributor to the central pool is less dependent on rainfall. Still, the overall availability of rice may go down by more than four million tonnes. There may be worst news ahead.

- (i) There is no canal water facility in West Bengal and Andhra Pradesh.
- (ii) The procurement price of rice will increase this year.
- (iii) Rice is mainly produced in the *kharif* season.
- (iv) In the last year, there was a deficit production of rice by five million tonnes.
- (v) It is likely that production of rice will be below the normal level in the next year.

[Bank PO]

ANSWERS

1. (i) (e) (ii) (c) (iii) (a) (iv) (a) (v) (e) 2. (i) (a) (ii) (c) (iii) (a) (iv) (a) (v) (c)
 3. (i) (a) (ii) (b) (iii) (a) (iv) (e) (v) (b)

Explanations

1. (i) As mentioned in the passage, the idea of privatisation has been sweeping both the developed and the developing world for more than a decade.
 (ii) In the passage, nothing has been mentioned about the USA.
 (iii) It has been clearly mentioned in the passage that the idea of privatisation has been gathering momentum in India.

- (iv) It is mentioned in the passage that earlier nationalisation was considered the remedy for socio-economic ills whereas at present, the idea of privatisation is dominant. This implies that nationalisation failed to improve the socio-economic situations of some countries.
- (v) As mentioned in the passage, the idea of privatisation is being promoted all over the world and has not been successfully transplanted in India.
2. (i) According to the passage, land required for expansion plan = 651 hectares.

$$\text{Forest land} = \left(\frac{1}{6} \text{ of total land} \right) = \frac{1}{6} \times 651$$

$$= 108.5 = 100 \text{ (approximately)}$$

- (ii) It is mentioned only that Karkatla mine deals with exploitation of non-cocking coal reserves. But, it is not given that it is the only such mine.
- (iii) The granting of environmental clearance to Karkatla mine shows that there is a demand for non-cocking coal.
- (iv) Total estimated production = 1.5 million tonnes = (1.5×10^6) tonnes.

$$\text{Total estimated cost} = ₹ 67.82 \text{ crores} = ₹ (67.82 \times 10^7)$$

$$\text{Cost per tonne of coal} = ₹ \left[\frac{67.82 \times 10^7}{1.5 \times 10^6} \right]$$

$$= ₹ 452.13 = ₹ 450 \text{ (approximately)}$$

- (v) The given fact is neither mentioned nor can be derived from the passage.
3. (i) As mentioned in the passage, the rice production in West Bengal and Andhra Pradesh would not be affected since they had sufficient rainfall. This implies that the farming there is dependent mainly on rain and no other irrigation facilities are available.
- (ii) As mentioned in the passage, the rice production has gone down which may lead to a rise in procurement price of rice.
- (iii) The given fact is clearly evident from the given passage.
- (iv) As mentioned in the passage, there is a deficit production of rice by five million tonnes in the present year.
- (v) It is mentioned in the passage that 'There may be worst news ahead'. There is no surety about the given fact, but it appears to be true according to the present trends.

SECTION C NONVERBAL REASONING

Chapter 22

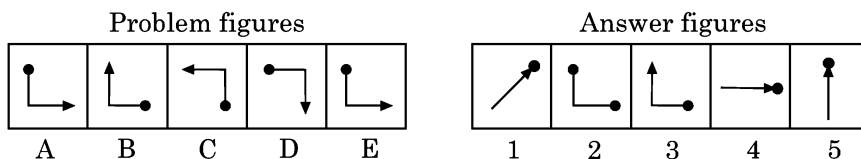
Series

In this chapter, four types of series of figures are given. There will be a set of answer figure also. You have to identify the most appropriate figure from the set of answer figures.

TYPE 1: FIVE FIGURE SERIES

This type of problems on series consists of five figures numbered A, B, C, D and E forming *the problem set*, followed by five other figures numbered 1, 2, 3, 4 and 5 forming *the answer set*. The five consecutive problem figures form a definite sequence and it is required to choose one of the figures from the answer set which will continue the same sequence.

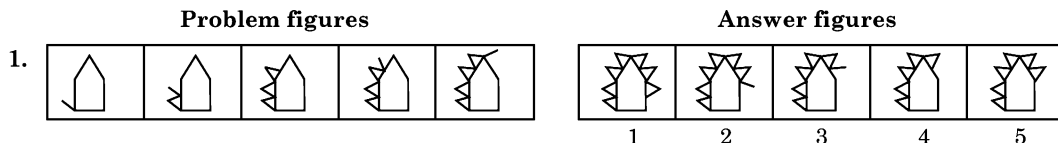
EXAMPLE 1

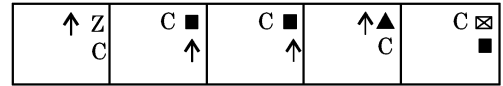


Solution In this case, the pin rotates 90° clockwise and the arrow rotates 90° anticlockwise in each step. Hence, the answer is figure (3).

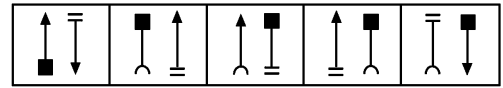
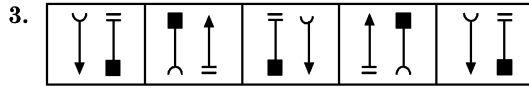
EXERCISE 1

Directions (Q. 1–15): In each of the questions given below which one of the five answer figures on the right should come after the problem figures on the left, if the sequence were continued?

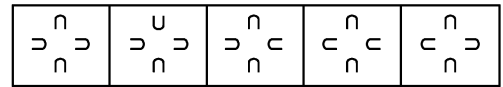
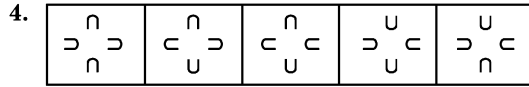




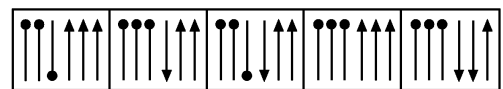
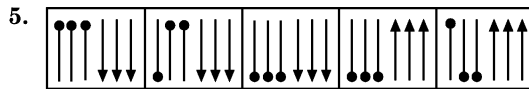
1 2 3 4 5



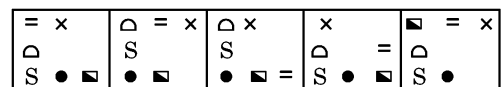
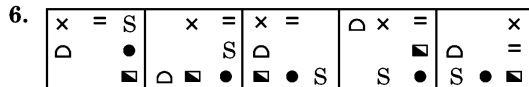
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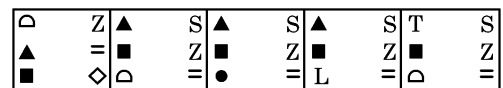
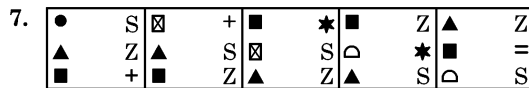
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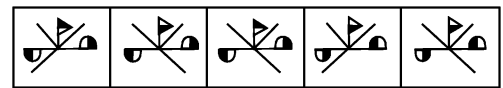
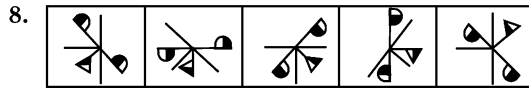
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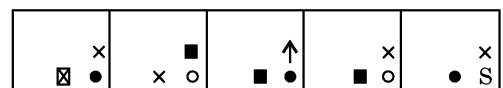
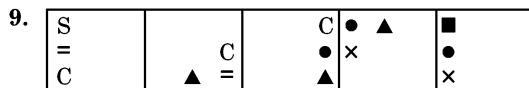
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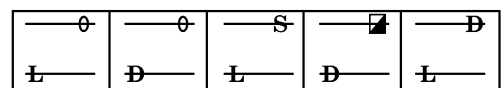
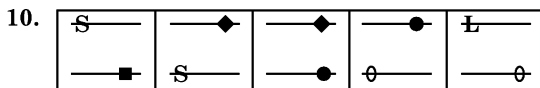
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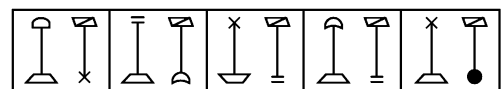
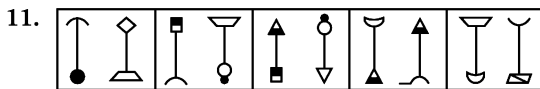
1 2 3 4 5



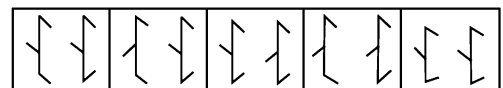
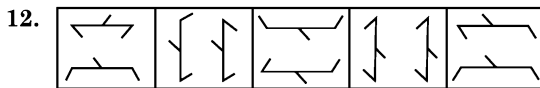
1 2 3 4 5



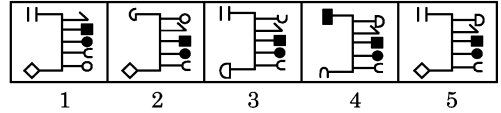
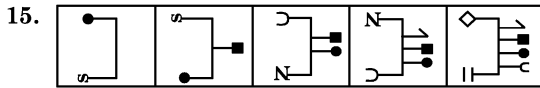
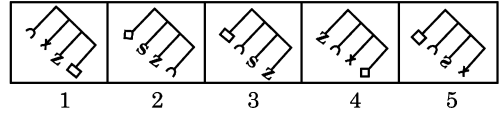
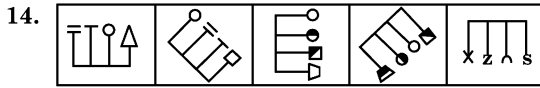
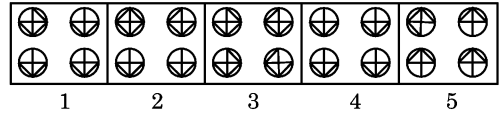
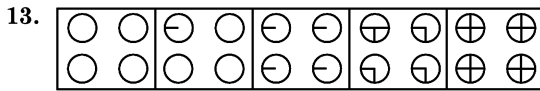
1 2 3 4 5



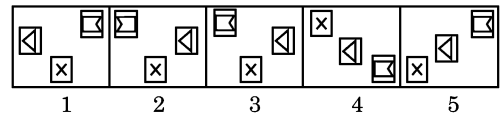
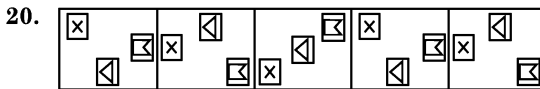
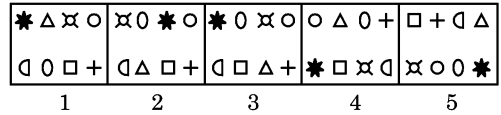
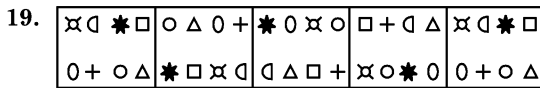
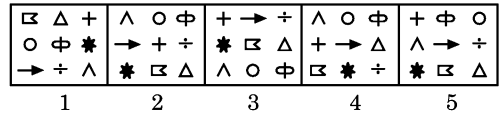
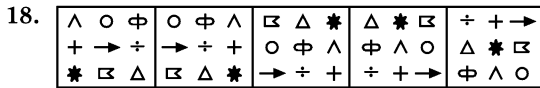
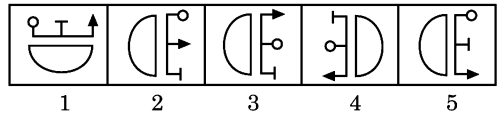
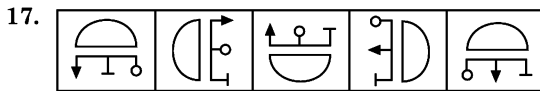
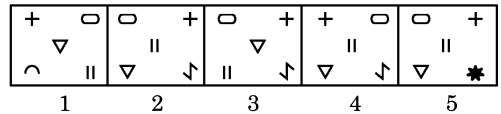
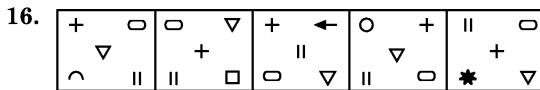
1 2 3 4 5



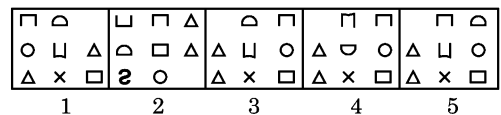
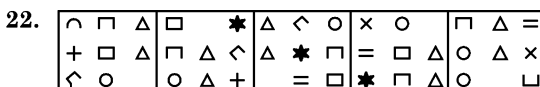
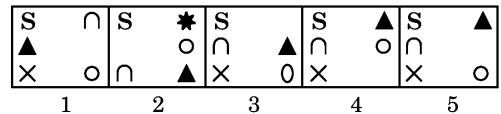
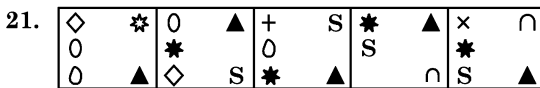
1 2 3 4 5

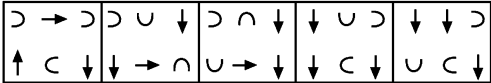
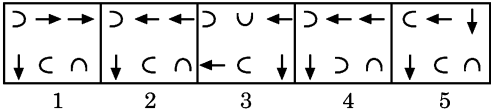
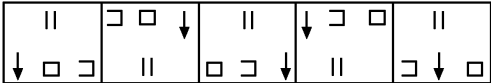
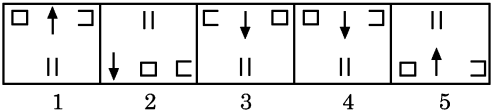
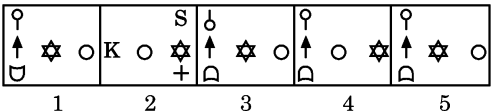


Directions (Q. 16–20): Each of the following questions consists of unmarked figures followed by figures marked 1, 2, 3, 4 and 5. Select a figure from the marked figures which will continue the series established by the unmarked figures.

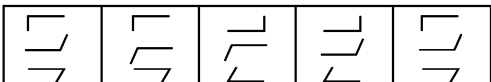
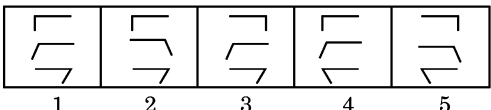
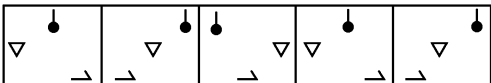
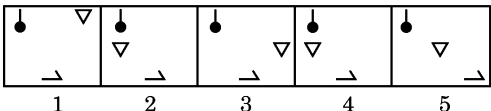
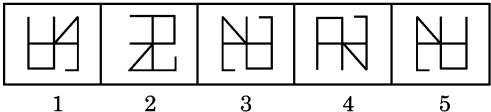
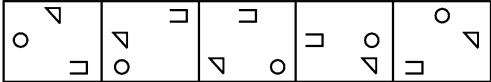
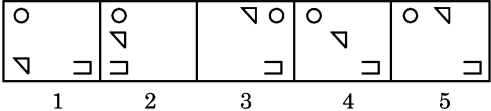
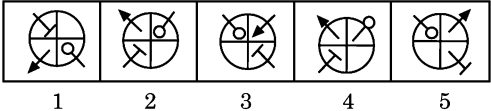


Directions (Q. 21–25): Each of the following questions consists of unmarked figures followed by five figures marked 1, 2, 3, 4 and 5. Select a figure from the marked figures which will continue the series established by the unmarked figures.



23.  
24.  
25.  

Directions (Q. 26–30): Each of the following questions consists of unmarked figures followed by five figures marked 1, 2, 3, 4 and 5. Select a figure from the marked figures which will continue the series established by the unmarked figures.

26.  
27.  
28.  
29.  
30.  

ANSWERS

1. (5) 2. (3) 3. (2) 4. (1) 5. (4) 6. (2) 7. (4) 8. (3) 9. (4) 10. (5)
 11. (3) 12. (5) 13. (1) 14. (3) 15. (1) 16. (2) 17. (5) 18. (3) 19. (4) 20. (5)
 21. (5) 22. (4) 23. (5) 24. (3) 25. (1) 26. (1) 27. (3) 28. (4) 29. (5) 30. (2)

Explanations

- In subsequent steps one, two three one line segments are added.
- In subsequent steps, the ACW-end element shifts one-and-a-half side ACW. The CW-end element shifts one side CW while a new element appears in the middle.

3. In each step the elements get inverted while the arrows and the end-elements interchange places in alternate steps.
4. Two end one arcs get inverted in alternate steps.
5. One, two, three, elements get inverted in subsequent steps in succession.
6. In subsequent steps 5, 4, 3, 5, ... elements from CW and shift half-a-side CW while others shift half-a-side ACW.
7. In alternate steps the elements of LHS and RHS columns shift one step downward while one of the elements in the other column gets replaced by a new one.
8. In each step whole figure rotates by 45° ACW. The triangular element gets inverted and its shading is changed while the semi-circular elements change their side and shading in alternate steps.
9. In alternate steps the elements shift one-and-a-half sides and half a side ACW while one element is replaced by a new one in each step. The replacement happens in succession.
10. In each step one element is replaced by a new one, alternately on upper and lower parts, while the other interchanges place in one step and gets inverted in the next step.
11. In subsequent steps the upper part of LHS element becomes lower without getting inverted while the lower part of RHS becomes the upper after getting inverted.
12. The whole figure rotates by 90° ACW. All the three dashes on lower figure change direction while the two dashes on upper figure change direction in each step.
13. In subsequent steps 1, 3, 5, 7, 9 lines are added beginning from the upper-left.
14. The whole figure rotates by 45° CW in each step. In alternate steps three elements shift one step in cyclic order while one end element is replaced by a new one.
15. In alternate steps the LHS elements interchange position while a new element is added alternately at upper and lower ends of RHS.
16. The central element and two other elements move one step ACW in cyclic order. The other two interchange positions and one of them is replaced by a new one. The cyclic movement rotates one step in ACW order.
17. The whole figure rotates by 90° ACW. The middle tip becomes lowest tip, lowest becomes upper and upper becomes middle in alternate steps.
18. In the first step elements move one step from right to left, the left most becoming the rightmost. In the next step the row of elements moves one step downwards. This goes on alternately.
19. In the first step the pairs of elements interchange diagonally. In the next step the elements on the outer side move one side CW while those on the inner side move one side ACW. This goes on alternately.
20. In each step elements move half a side downwards. The lower most moves to upper most.
21. In the first step the upper-and-lower left elements interchange places. The lower-right moves to upper-right, the upper-right moves to middle left while a new element appears at lower right. In the next step the upper and lower-right elements interchange place. The elements in the left column move one step downwards while a new element appears at upper-left position.
22. Two elements of the middle column interchange places sequentially while the upper two and lower two elements of the first column interchange places and the third is replaced by a new one. The columns of elements move one step towards left while the left most element moves to the rightmost position. The column with only two elements moves up and down in subsequent steps.
23. In the first step the rightmost element of each row moves to the left most while other two rotate by 90° CW and move one step towards right. In the next step the end elements of the

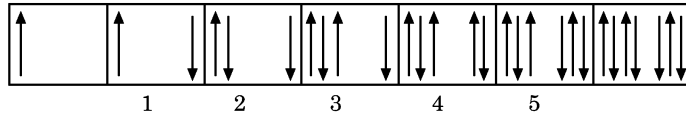
lower row and the middle element of the upper row move one step ACW in cyclic order while others remain unchanged.

24. In every step middle two elements get inverted and two end elements moved CW by two sides in first step and half-a side and two-and-a half side each respectively CW in the next step. Then the process repeats.
25. In the first step the odd-numbered elements from top rotate by 90° CW. In the next step the even-numbered elements from top rotate by 90° ACW.
26. The middle and side elements rotate by 180° alternately.
27. The elements move half-a-side in rows from left to right in each step. After reaching the rightmost position they shift to the left most position.
28. The rectangle moves one step ACW while the dash moves one step CW in each step.
29. Elements move one side and half-a-side ACW alternately in each step.
30. The elements move one step ACW and change their direction in each step.

TYPE 2: FIND OUT THE WRONG FIGURE IN THE SERIES

This type of questions begins with an un-numbered figure followed by five figures numbered from 1 to 5 and then again an un-numbered figure on the extreme right. These seven figures together form a series which starts at the first (un-numbered) figure and ends at the last (un-numbered) figure. However, only one of these figures does not fit into the series. The number of that figure is the answer.

EXAMPLE 2



Solution In the above set of figures, the arrows are added to the right and left sides alternately. But in the third figure the arrow which was to be added to the right side, has been added to the left side. Hence figure (3) is the answer.

EXERCISE 2

Directions (Q. 1–5): In each of the following problems, a related pair of figures is given followed by five numbered pairs of figures. Four out of five pairs have similar relationship as that of the original pair. Select a pair that does not have a relationship similar to that in the original pair.

- 1.
- 2.

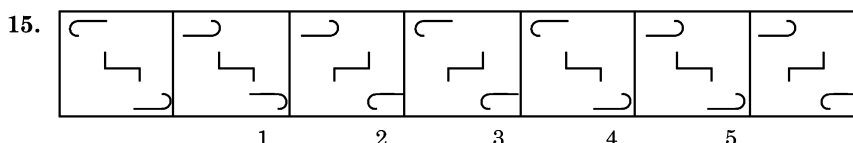
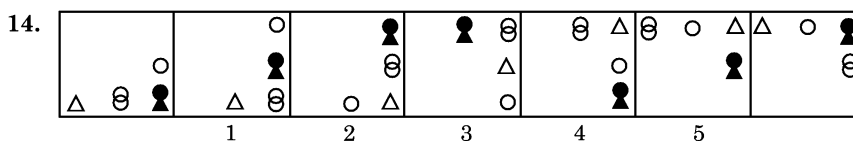
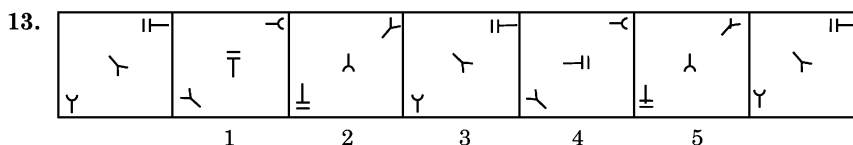
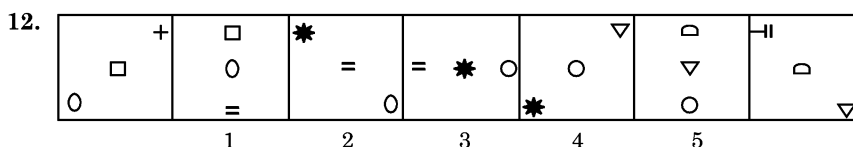
- 3.
- 4.
- 5.

Directions (Q. 6–10): Each of the following series consists of seven figures, two of which at the ends are unnumbered. One of the five numbered figures does not fit inot the series. Find-out the figure.

- 6.
- 7.
- 8.
- 9.
- 10.

Directions (Q. 11–15): Each of the following series consists of seven figures, two of which at the ends are unnumbered. One of the five numbered figures does not fit into the series. Find out the figure.

- 11.



ANSWERS

1. (4) 2. (5) 3. (2) 4. (3) 5. (1) 6. (3) 7. (2) 8. (4) 9. (2) 10. (3)
11. (3) 12. (2) 13. (4) 14. (5) 15. (1)

Explanations

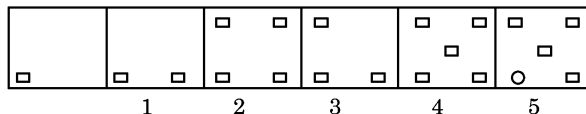
1. The elements move one step ACW. The element reaching the upper part increases by one while that reaching lower part is replaced by a new one.
2. In alternate steps the upper-right and the upper-left pair move downwards diagonally. While a new element appears alternately on upper-right and upper-left. The circle in the lower row moves from one end to the other in each step. The other two elements in the lower row interchange places and move upwards diagonally.
3. In each step three elements move diagonally while other two interchange places diagonally. Two elements—one at centre and one at a corner—are replaced by new ones. This movement rotates by 90° CW.
4. In each step one and two line segments alternately from left and right side of the upper element are lost while two line segments—one each in the middle and the lower element are added alternately on right and left side.
5. In the first step elements at positions 1 and 4 interchange places. So do elements at positions 6 and 9. 3 moves to $2 \rightarrow 8 \rightarrow 7$ while a new element appears at position 3. In the next step 3 and 6 and 4 and 7 interchange their places. 9 moves to $8 \rightarrow 2 \rightarrow 1$ while a new element appears at 9.
6. In each step one arm gets rotated by 45° CW. This movement proceeds in ACW direction.
7. The whole figure rotates by 45° CW and 90° ACW alternately while end elements get inverted in each step.

8. The whole figure rotates through 90° ACW in each step and symbols inside a square gets interchanged within it in every step.
9. The whole figure rotates by 90° and 180° ACW alternately. 'x' moves from vertex to base.
10. The lower dash changes direction in each step while the upper elements change direction alternately.
11. The outermost and innermost sides rotate by 90° ACW while the middle one rotates by 90° CW.
12. The elements shift one step in cyclic order and one element is replaced by a new one. The line of orientation rotates by 45° ACW.
13. The uppermost element rotates 90° CW and moves to the middle. The middle element rotates by 180° and moves to lower position while the lower element rotates by 90° CW and moves to the upper position.
14. The elements move one step ACW in one step while in the next the element on extreme ACW position moves to CW and others move one step ACW.
15. The upper and the lower element rotate by 180° while the middle one gets inverted vertically in alternate steps.

TYPE 3: DETECTING THE INCORRECT ORDER IN A SERIES

The third type of questions on series consists of an un-numbered figure followed by five other figures numbered as 1, 2, 3, 4 and 5. All the six figures together form a series. The un-numbered figure marks the beginning of the series and so its position is fixed. However, the positions of two figures in the series are incorrect and the series would be complete if these figures are interchanged. The earlier of the two numbered figures whose positions are interchanged is the answer. In case, the position of no two figures is to be interchanged, then the answer is 5.

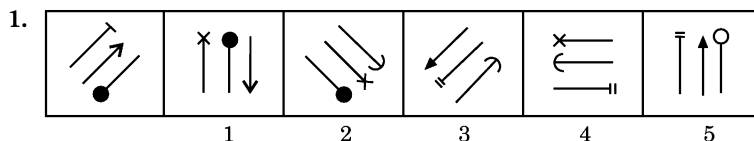
EXAMPLE 3

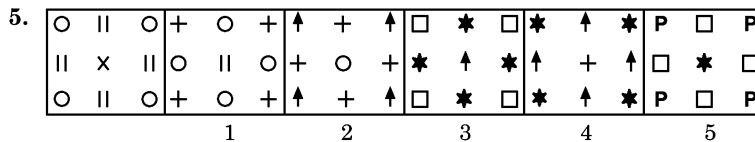
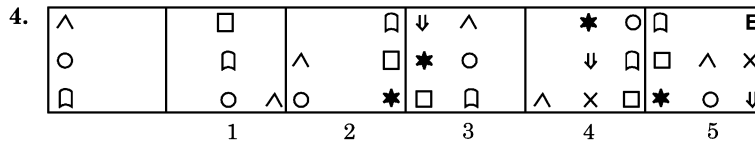
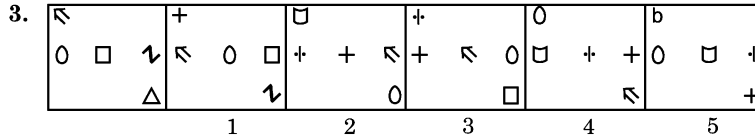
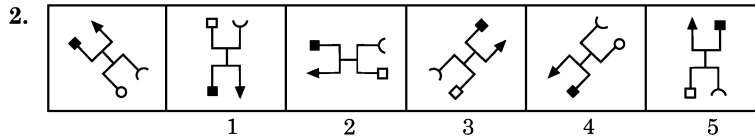


Solution Here, the number of squares increases in step by one and then these squares change into circles stepwise. But this series will be established only if figure (2) and figure (3) are interchanged. So, the answer is (2).

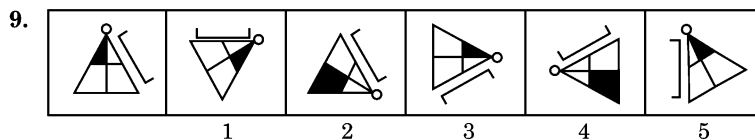
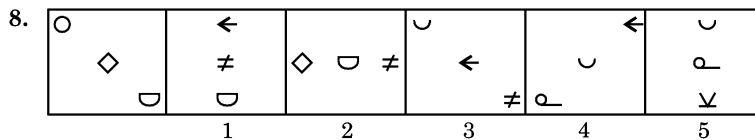
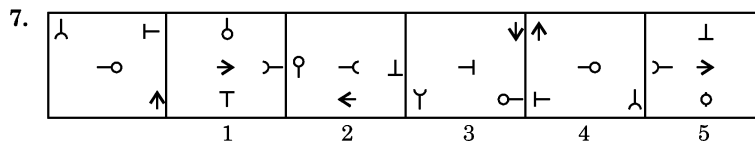
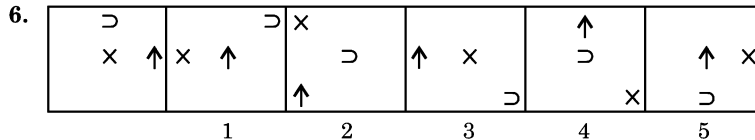
EXERCISE 3

Directions (Q. 1–5): Each of the following questions consists of six figures, the first of which is unnumbered and marks the beginning of the series continued in the successive figures numbered from 1 to 5. However, the series will be established only if the positions of two of the numbered figures are interchanged. The number of the first of the two figures is the answer. If no figure need to be interchanged then the answer is 5.

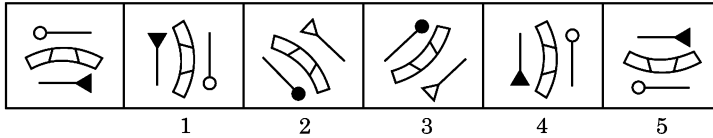




Directions (Q. 6–10): Each of the following questions consists of six figures, the first of which is unnumbered and marks the beginning of the series continued in the successive figures numbered from 1 to 5. However, the series will be established only if the positions of two of the numbered figures are interchanged. The number of the first of the two figures is the answer. If no figure need to be interchanged then the answer is 5.



10.



1

2

3

4

5

ANSWERS

1. (3) 2. (1) 3. (2) 4. (5) 5. (3) 6. (3) 7. (5) 8. (4) 9. (5) 10. (1)

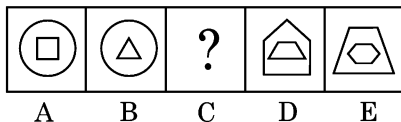
Explanations

- The central element interchanges place alternately with the element on either side and gets inverted while the third one is replaced by a new one. The whole figure rotates by 45° ACW in each step.
- The whole figure rotates by 45° , 90° , 135° , 180° ACW in subsequent steps while the end elements interchange their places alternately on either end in subsequent steps.
- In each step the elements move one step downwards and towards right. A new element appears at the top-left position while the lower one is lost.
- The column of elements moves half-a-side from left to right. The central element interchanges alternately with upper and lower elements and the third element is placed at the lower end of the next column.
- In each step the similar corner elements move to the middle of the sides while one of the middle elements moves to the centre. New elements appear at the corners.
- The horizontal arrow rotates by 90° ACW and is placed on LHS. The vertical arrow on the RHS rotates by 90° ACW and its head gets inverted while the LHS arrow rotates by 180° and its head gets inverted.
- The outermost and innermost elements interchange places and sizes. The second element from outside rotates by 180° while the second from inside rotates by 90° CW.
- The upper-right and lower-left elements get vertically inverted and move to middle-right and centre respectively. The central element gets laterally inverted and moves to the lower right.
- The LHS and RHS elements get enlarged. The RHS element becomes the outer one while the LHS becomes the inner one. The number of sides of LHS element increases by one. The middle element gets reduced in size rotates by 135° CW and becomes the innermost.
- The elements move two sides. The end elements get reversed while the middle one remains unchanged.

TYPE 4: CHOOSING THE MISSING FIGURE IN A SERIES

In this type of questions, you are given a set of four or five figures (labelled A, B, C, D and E) following a certain sequence and hence forming a series. However, one of figures in the problem set will be missing. You have to choose this figure from the five alternatives given in the Answer set.

EXAMPLE 4



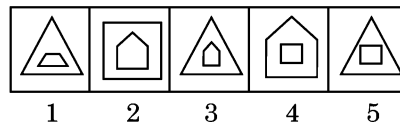
A

B

C

D

E



1

2

3

4

5

Solution Here, the inner small figure becomes the outer large figure and a new small figure appears inside it in every step. Hence, the answer is (3).

EXERCISE 4

Directions (Q. 1–5): The second figure in the first unit of the problem figures bears a certain relationship to the first figure. Similarly, one of the figures in the answer figures bears the same relationship to the second figure in the second unit of the problem figures. You are therefore to locate the figure which would replace the question mark.

Problem Figures

Answer Figures

		?	

1	2	3	4	5
		?	

1	2	3	4	5
		?	

1	2	3	4	5
		?	

1	2	3	4	5
		?	

1	2	3	4	5

Directions (Q. 6–10): The first figure in the first unit of the problem figures bears a certain relationship to the second figures. Similarly, one of the figures in the answer figures bears the same relationship to the first figure in the second unit of the problem figures. You are therefore to locate the figure which would replace the question mark.

		?	

1	2	3	4	5
		?	

1	2	3	4	5

8. 1 2 3 4 5
9. 1 2 3 4 5
10. 1 2 3 4 5

Directions (Q. 11–15): The first figure in the first unit of the problem figures bears a certain relationship to the second figure. Similarly, one of the figures in the answer figures bears the same relationship to the second figure in the second unit of the problem figures. You are therefore to locate the figure which would replace the question mark.

11. 1 2 3 4 5
12. 1 2 3 4 5
13. 1 2 3 4 5
14. 1 2 3 4 5
15. 1 2 3 4 5

ANSWERS

1. (2) 2. (2) 3. (4) 4. (5) 5. (3) 6. (3) 7. (4) 8. (1) 9. (2) 10. (5)
11. (3) 12. (1) 13. (3) 14. (4) 15. (3)

Explanations

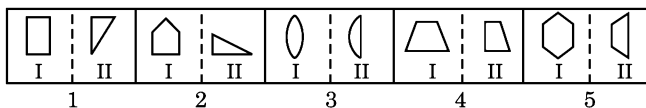
1. From figure II to I: The inner element rotates by 135° ACW, the arrowhead gets inverted, while the other end changes its shading. The outer element rotates by 45° ACW and assumes new form.
2. From figure II to I: The middle element moves to the opposite side and gets enlarged. The CW element moves $1\frac{1}{2}$ sides ACW while the ACW element moves $1\frac{1}{2}$ sides CW.
3. From figure II to I: The main figure rotates by 90° ACW and the curve attached to it gets inverted. The middle element rotates by 45° while the other two elements move one side ACW.
4. From figure II to I: The elements at positions 1, 3 and 8 move one step CW while elements at positions 2, 7 and 9 move one step ACW.
5. From figure II to I: The first and third columns have their mirror image while the middle column has its water image in the LHS figure.
6. From figure II to I: Each element moves one side and a half ACW without getting inverted.
7. From figure II to I: One side increases in each element except in the upper-left, in which one side is lost. The middle row element changes side.
8. From figure II to I: The middle figure rotates by 90° CW and is placed on the upper and the lower part while the innermost is placed in the middle.
9. From figure II to I: The first is laterally inverted form of the second.
10. From figure II to I: The upper three elements in the column go two steps downward. The right element in the row goes down in third row of the column and left element in the lower. The lowermost goes to the second row and the second row to right of the upper row.
11. From figure I to II: The elements interchange positions. The LHS rotates by 90° ACW while the RHS rotates by 90° CW.
12. From figure I to II: The RHS remains constant, the lower element gets vertically inverted while the upper one rotates by 90° ACW and shifts to left.
13. From figure I to II: The shaded petal with the dot rotates by 90° ACW. The unshaded petal with dot rotates by 45° CW while the other two rotate by 135° CW.
14. From figure I to II: The arrow rotates by 90° CW, the pin rotates by 45° ACW while the middle bar rotates by 45° ACW.
15. From figure I to II: The central element rotates by 135° CW gets inverted and shifts to the lower-left. The lower-left rotates by 45° CW gets inverted and shifts to the upper-right while the upper-right rotates by 90° CW and shifts to the centre.

Analogy

This chapter is dealt with identifying the odd pair of figures which are not having a particular relation between them in one way or another.

CHOOSING THE ODD RELATIONSHIP: In this type of questions, five pairs of figures are given. The two figures in four out of five pairs are related in a particular manner. You have to select the pair which does not show the relationship.

EXAMPLE 1

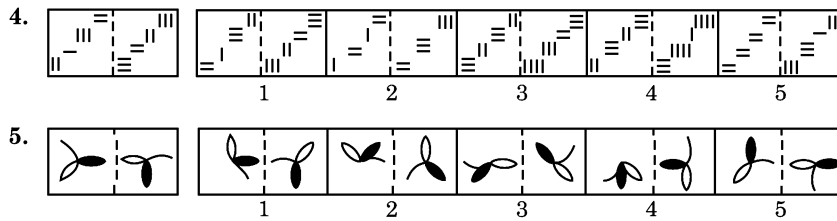


Solution In all the pairs except (2), the second element is half of the first element.

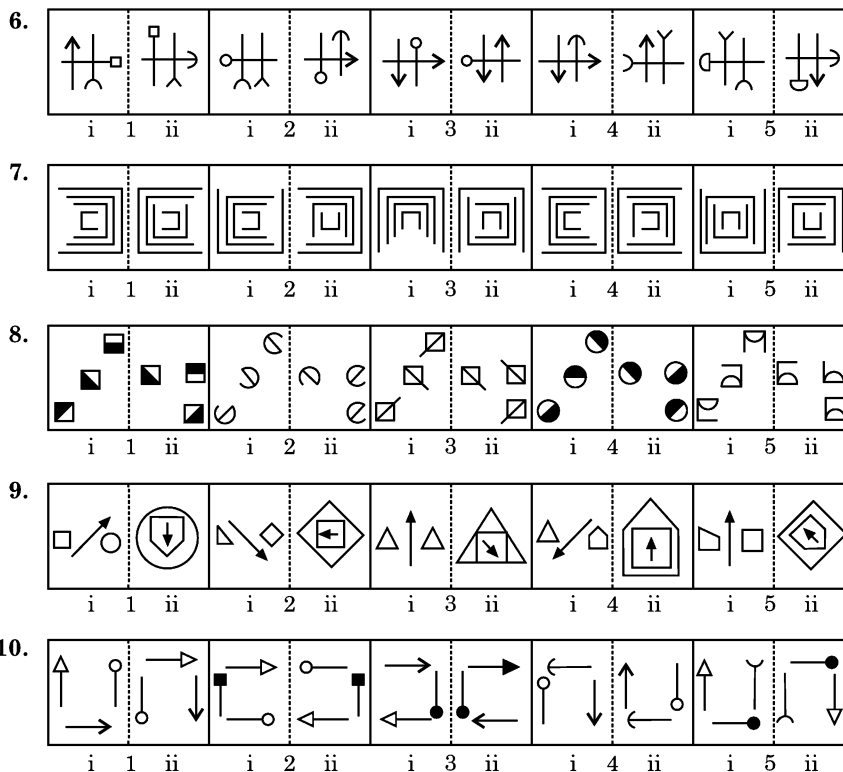
EXERCISE 1

Directions (Q. 1–5): In each of the following problems, a related pair of figures is given followed by five numbered pairs of figures. Four out of five pairs have similar relationship as that of the original pair. Select a pair that does not have a relationship similar to that in the original pair.

- 1
2
3
4
5
- 1
2
3
4
5
- 1
2
3
4
5



Directions (Q. 6–10): In each of the following questions in four out of the five figures, element I is related to element II in the same particular way. Find out the figures in which element I is not related to element II.



ANSWERS

1. (1) 2. (3) 3. (3) 4. (4) 5. (5) 6. (4) 7. (2) 8. (1) 9. (3) 10. (5)

Explanations

- The upper-left and the lower-right elements rotate by 90° CW while the upper-right and the lower-left elements rotate by 90° ACW.
- The upper-left element shifts to lower-right position. The upper-right and lower-right shift to upper-left and lower-left position respectively and rotate by 90° ACW. The central element shifts to the upper-right position and rotates by 90° CW. The lower-left element shifts to centre and changes its shape.

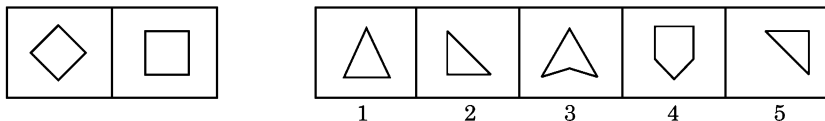
3. The upper-left element rotates by 90° CW and shifts to the centre. The central element moves to the lower-left. The lower-right element rotates by 90° ACW and shifts to the upper-right position while a new element appears at lower-right position.
4. The uppermost and the lowermost sets of dashes rotate by 90° and an additional dash is added to each of them. One dash is lost from second upper set of dashes while one dash is added to the second from the bottom.
5. The unshaded leaf rotates by 45° CW. The shaded leaf rotates by 90° CW. The half-leaf rotates by 135° CW and changes its direction.
6. One of the elements move half-a-side CW one by one while the other two move one step once in row and twice diagonally and again in row in subsequent steps.
7. The CW element shifts to the centre while the central element becomes the ACW element. Other two elements move one-and-a half sides CW. Elements rotate by 90° CW.
8. In each step the elements move from right to left and one of the elements is replaced by a new one. The line on which the elements are arranged rotates by 45° and 90° ACW.
9. The triangle rotates by 45° , 90° , 135° , 180° ... CW in subsequent steps. The shading moves one step CW. The bar outside moves one step CW. The bar outside moves from one side of the outer circle to the other in alternate steps.
10. In the first step the whole figure rotates by 90° CW while the elements on the ends further rotate by 180° . In the next step the whole figure rotates by 45° ACW, elements on the ends interchange positions and their shading also changes. The process goes on.

Classification

In this chapter, problem figures are having same characteristics. You have to identify the most appropriate figure from answer figures, which show some characteristics.

CHOOSING THE FIGURE WITH SAME PROPERTIES: This type of questions contain two figures forming the problem set followed by five other figures forming the answer set. The problem figures have same common characteristics. You are required to select one of the figures from the answer set which also exhibits the same characteristics.

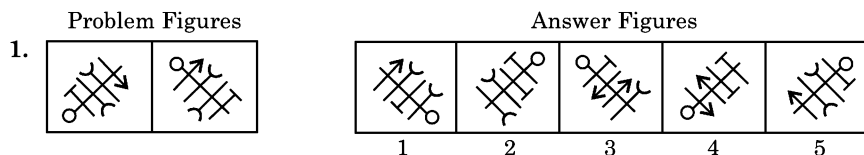
EXAMPLE 1

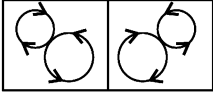
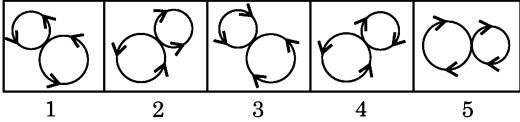
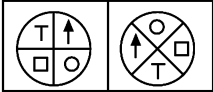
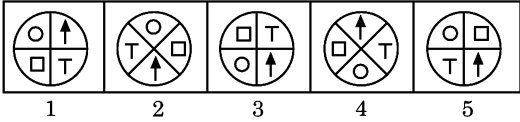
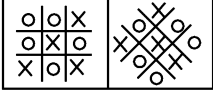
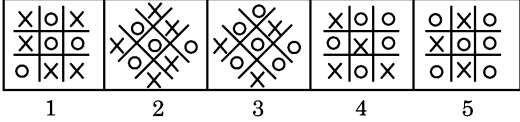
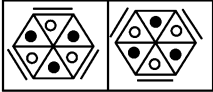
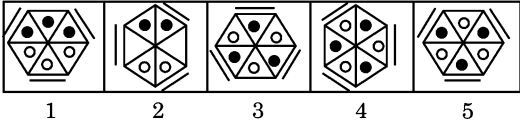


Solution Each one of the two figures on the left has been formed by four distinct lines. Similarly, figure (3) is made up of four lines. Hence figure (3) is the answer.

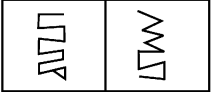
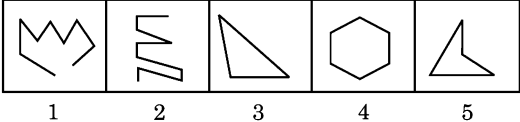
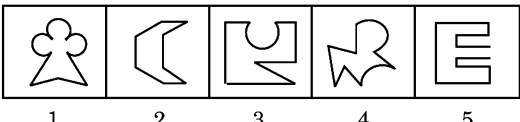
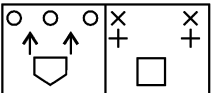
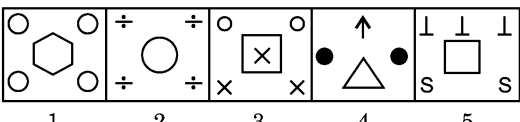
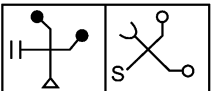
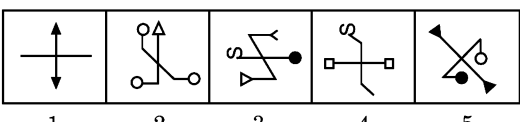
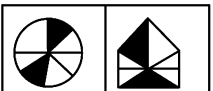
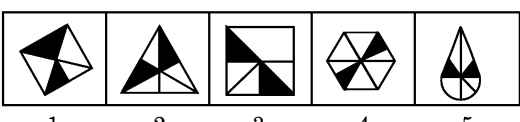
EXERCISE 1

Directions (Q. 1–5): In each question below, two problem figures, followed by five answer figures numbered (1), (2), (3), (4) and (5) are given. The two problem figures have some common characteristics/features. You have to find out one figure out of the five answer figures which has the same features/characteristics. The number of that figure is the answer.



2.  
3.  
4.  
5.  

Directions (Q. 6–10): The first figure in the first unit of the problem figures bears a certain relationship to the second figure. Similarly, one of the figures in the answer figures bears the same relationship to the second figure in the second unit of the problem figures. You are therefore to locate the figure which would replace the question mark.

6.  
7.  
8.  
9.  
10.  

ANSWERS

1. (5) 2. (4) 3. (3) 4. (4) 5. (4) 6. (1) 7. (5) 8. (4) 9. (2) 10. (5)

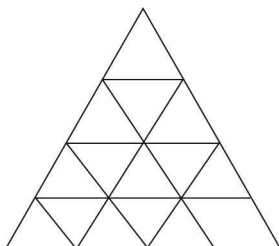
Explanations

1. Two identical elements are placed in the middle and are in the opposite direction.
2. Directions of arrows are CW in one and ACW in the other.
3. Positions of the elements are similar in respect of other elements.
4. Element '×' is placed in the centre.
5. Shaded and unshaded circles are placed in a definite sequence and outer line segments lie adjacent to unshaded circles.
6. The figures have only one right angle.
7. The figures are open-ended.
8. The number of lines of the enclosed figure is equal to the number of smaller elements.
9. The titled lines have similar shapes on them.
10. The shaded parts have three unshaded parts on one side and two on the other.

Analytical Reasoning

The chapter on analytical reasoning involves the problems relating to the counting of geometrical figures in a given complex figure. You have to analyse the complex figure and determine the number of any particular type of figure as shown in the examples.

EXAMPLE 1 How many straight lines are there in the following figure?



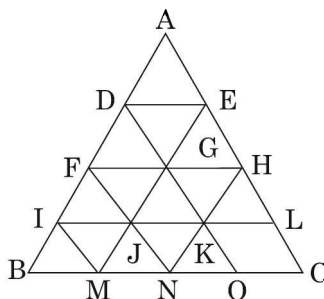
(a) 9

(b) 11

(c) 15

(d) 48

Solution The given figure is as shown below.

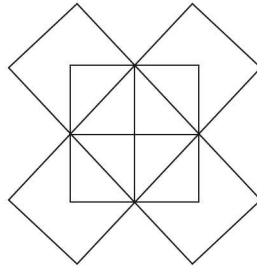


Horizontal lines are DE, FH, IL and BC. i.e., 4 nos.

Slanting lines are IM, FN, DO, AC, AB, EM and HN. i.e. 7 nos.

∴ Total number of lines = 4 + 7 = 11.

EXAMPLE 2 How many rectangles does the following figure have?



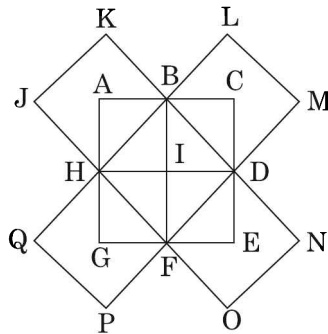
(a) 10

(b) 12

(c) 13

(d) 14

Solution The given figure can be labelled as shown below:

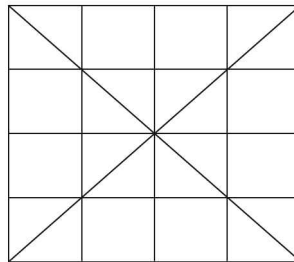


The above figure consists of:

- (i) rectangle JKNO composed of four components JKBH, DNOF, JKDF and BNOH i.e. altogether 5 in numbers;
- (ii) rectangle QLMP composed of four components QHFP, BLMD, QBDP and HLMF i.e. altogether 5 in numbers;
- (iii) square ACEG composed of four components ACDH, BCEF, DEGH and GABF i.e. altogether 4 in numbers.

Hence, total number of rectangles = $5 + 5 + 4 = 14$.

EXAMPLE 3 How many triangles does the following figure have?



(a) 36

(b) 40

(c) 44

(d) 48

Solution The figure may be labelled as shown below.

The simplest triangles are APQ, QTU, UXY, YKC, AEQ, QRU, UVY, YJC, BGS, SRU, UTW, WND, BHS, SVU, UXW and WMD i.e. in all 16 numbers.

The triangles having two components each are QSU, SYU, YWU and WQU i.e. in all 4 numbers.

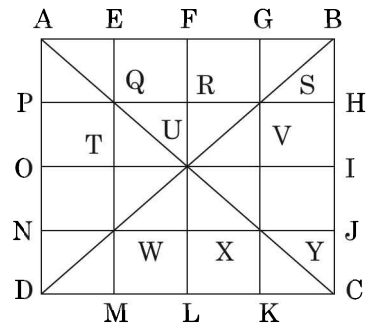
The triangles having three components each are AFU, FBU, BIU, CLU, LDU, DOU OAU i.e. in all 8 numbers.

The triangles having four components each are QSY, SYW, YWQ, WQS i.e. in all 4 numbers.

The triangles having six components each are ABU, BCU, CDU, DAU i.e. in all 4 numbers.

The triangles having seven components each are ANY, GAY, BEW, WJB, HQC, CQM, KSD and DPS i.e. in all 8 numbers.

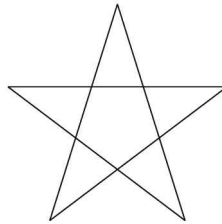
∴ Total number of triangles = $16 + 4 + 8 + 4 + 4 + 8 = 44$. Answer is (c)



EXERCISES

1. How many triangles are there in the figure given below?

[IAS]



(a) 5

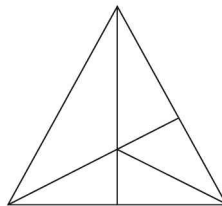
(b) 6

(c) 8

(d) 10

2. How many triangles are there in the following figure?

[IAS]



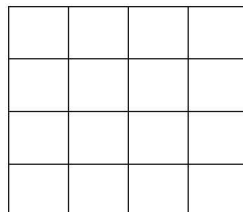
(a) 9

(b) 10

(c) 11

(d) 12

3. How many squares are there in the following figure?



(a) 16

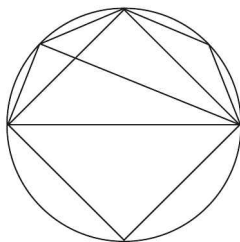
(b) 17

(c) 25

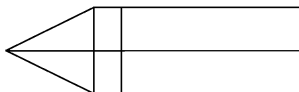
(d) 27

(e) 30

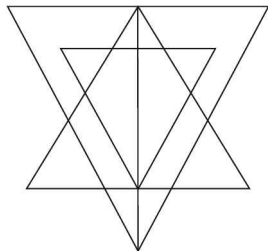
4. Count the number of triangles in the following figure.



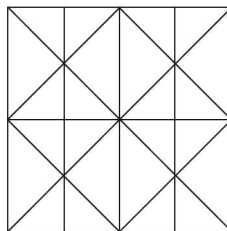
- (a) 8 (b) 10 (c) 11 (d) 12
5. How many rectangles are there in the following figure? [SSC]



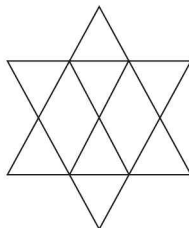
- (a) 6 (b) 7 (c) 8 (d) 9
6. Count the number of triangles in the following figure. [Hotel Management]



- (a) 27 (b) 25 (c) 23 (d) 21
7. What is the number of straight lines in the following figure?

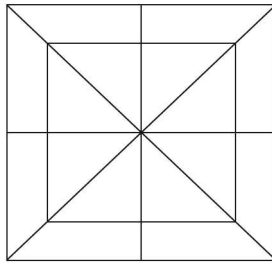


- (a) 11 (b) 14 (c) 16 (d) 17
8. Determine the number of pentagons in the following figure?



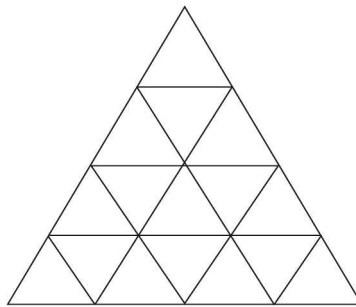
- (a) 5 (b) 6 (c) 8 (d) 10

9. What is the number of triangles and squares in the following figure?



- (a) 28 triangles, 10 squares (b) 28 triangles, 8 squares
(c) 32 triangles, 10 squares (d) 32 triangles, 8 squares

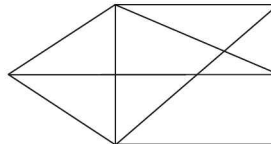
10. How many straight lines are there in the following figure?



- (a) 9 (b) 11 (c) 15 (d) 48

11. How many triangles are there in the following figure?

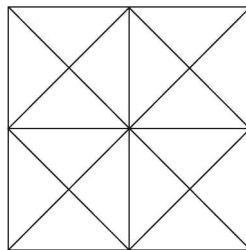
[SSC]



- (a) 12 (b) 13 (c) 14 (d) 15

12. How many triangles and squares are there in the given figure?

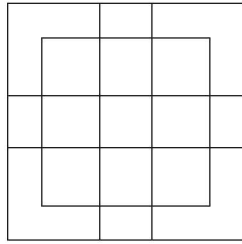
[Railways]



- (a) 44 triangles, 10 squares (b) 14 triangles, 16 squares
(c) 24 triangles, 6 squares (d) 24 triangles, 9 squares

13. How many squares does the following figure contain?

[Hotel Management]



(a) 18

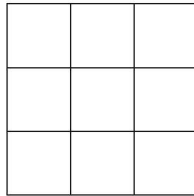
(b) 19

(c) 25

(d) 27

14. The maximum number of squares in the following figure is _____.

[Railways]



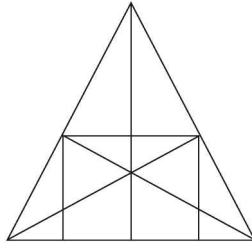
(a) 14

(b) 13

(c) 10

(d) 9

15. What is the number of straight lines and the number of triangles in the given figure?



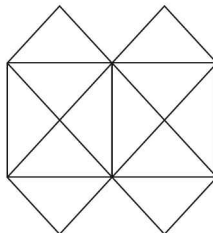
(a) 10 straight lines and 34 triangles

(b) 9 straight lines and 34 triangles

(c) 9 straight lines and 36 triangles

(d) 10 straight lines and 36 triangles.

Directions: Study the following figure and answer the questions (16 to 18).



16. What is the minimum number of straight lines that is needed to construct the figure?

(a) 11

(b) 13

(c) 15

(d) 21

17. Count the number of triangles in the figure

(a) 12

(b) 16

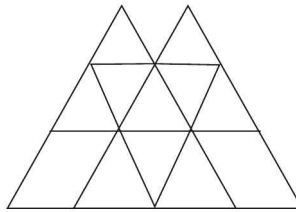
(c) 20

(d) 24

18. How many squares does the figure contain?

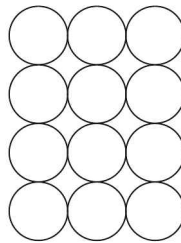
- (a) 5 (b) 6 (c) 7 (d) 8

19. Count the number of triangles and parallelogram in the figure given below.



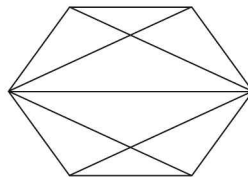
- (a) 16, 22 (b) 18, 23 (c) 14, 20 (d) 15, 21

20. In the following figure, if the centres of all the circles are joined by horizontal and vertical lines, then find the number of squares that can be formed.



- (a) 6 (b) 7 (c) 8 (d) 10

Directions: Analyse the following figure and answer the questions (21 and 22).



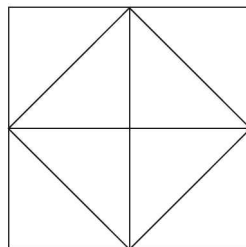
21. Find the number of quadrilaterals

- (a) 5 (b) 7 (c) 9 (d) 10

22. Find the number of pentagons

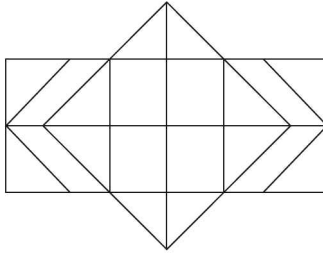
- (a) 2 (b) 3 (c) 4 (d) 6

23. Count the number of pentagons in the following figure



- (a) 16 (b) 14 (c) 12 (d) 10

24. Determine the number of rectangles and hexagons in the following figure.



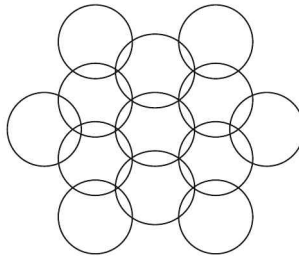
(a) 8 rectangles, 3 hexagons

(b) 15 rectangles, 3 hexagons

(c) 24 rectangles, 5 hexagons

(d) 30 rectangles, 5 hexagons

25. How many circles are there in the figure given below?



(a) 11

(b) 12

(c) 13

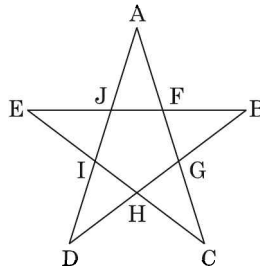
(d) 14

ANSWERS

1. (d) 2. (d) 3. (e) 4. (b) 5. (d) 6. (a) 7. (b) 8. (d) 9. (c) 10. (b)
 11. (d) 12. (a) 13. (d) 14. (a) 15. (c) 16. (b) 17. (c) 18. (c) 19. (b) 20. (c)
 21. (c) 22. (d) 23. (c) 24. (d) 25. (c)

Explanations

1. The given figure can be represented as shown below.



The simplest triangles are AJF, BFG, CGH, DHI and EIJ i.e. in total 5 numbers.

The triangles having 3 components are AIC, ADG, BEH, BJD and CFE i.e. in total 5 numbers.

∴ Total number of triangles = 5 + 5 = 10.

2. The given figure can be labelled as follows:

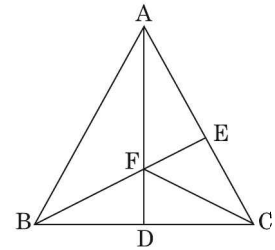
The simplest triangles are AFE, EFC, CFD, DFB and BFA i.e. 5 numbers.

The triangles having two components are AFC, CFB, ABD and EAB i.e. 4 numbers.

The triangles having three components are ADC and EBC i.e. 2 numbers.

The triangles having five components is ABC i.e. 1 number.

So total number of triangles = $5 + 4 + 2 + 1 = 12$



3. The figure can be labelled as follows:

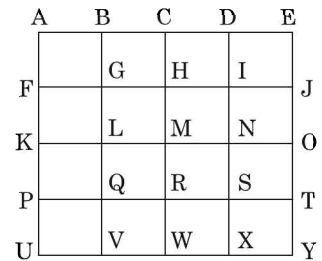
The simplest squares are ABGF, BCHG, DIHC, EJID, FGLK, GHML, GLKF, OTSN, NSRM, MRQL, LQPK, TYXS, SXWR, RWVQ and QVUP i.e. 16 numbers.

The squares composed of four simple squares are ACMK, DNLB, EOMC, HRPF, ISQG, JTRH, KMWU, LNXV and MOYW i.e. 9 numbers.

The squares composed of nine simple squares are ADSP, BETQ, FIXU and GJYV i.e. 4 numbers.

The square composed of sixteen simple squares is AEYU i.e. 1 number.

$\therefore$ Total number of squares = $16 + 9 + 4 + 1 = 30$

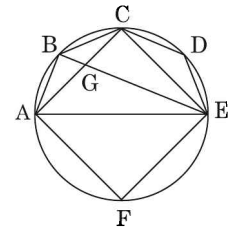


4. The figure can be labelled as follows:

The simplest triangles are ABG, BCG, CDE, GCE, AGE and EFA i.e. 6 numbers.

The triangles composed of two triangles each are ABC, ABE, BCE and ACE i.e. 4 numbers.

$\therefore$ There are $6 + 4 = 10$ triangles.

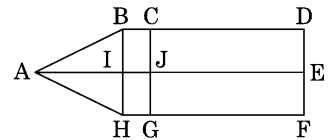


5. The given figure can be labelled as follows:

The simplest rectangles are BCJI, CDEJ, EFGJ and JGHI i.e. 4 numbers.

The rectangles having two components are BDEI, EFHI, CDFG and BCGH i.e. 4 numbers. The only rectangle composed of four components is BDFH.

$\therefore$ Total number of rectangles = $4 + 4 + 1 = 9$



6. The figure can be labelled as follows:

The simplest triangles are GKL, MNH, DLJ, DJM, QRE, OFP, PAI and RIA i.e. 8 numbers.

The triangles having two components each are BOD, CDQ, ENI, RPA, IKF, HJI, JGI, DLM, KPI and DKI i.e. 10 numbers.

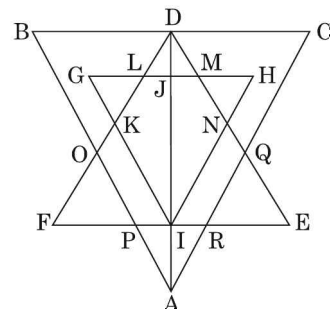
The triangles having four components each are DIE, DFI, DOA, DAQ and HGI i.e. 5 numbers.

The triangles having six components each are DBA and CDA i.e. 2 numbers.

Only triangle having eight components is DEF.

The only triangle having twelve components is ABC

$\therefore$ Total number of triangles = $8 + 10 + 5 + 2 + 1 + 1 = 27$



7. The given figure may be labelled as follows:

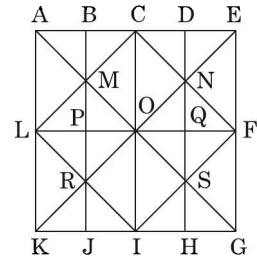
There are 3 horizontal lines, namely AE, LF and KG.

There are 5 vertical lines i.e. AK, BJ, CI, DH and EG.

There are 6 slanting lines i.e. AG, KE, LC, CF, LI and IF.

$\therefore$ Total number of straight lines = $3 + 5 + 6 = 14$.

$\therefore$ The answer is (b)



8. The figure may be labelled as follows:

Here six pentagons are there having three triangles and two rhombuses as components: i.e. AJHFD, KHFD, IFDBL, GDBLJ, EBLJH and CLJHF.

Four pentagons are formed which are composed of three triangles and one rhombus.

i.e. LCFHM, BEHML, KHFM, and IFMBL

$\therefore$ Total number of pentagons = $6 + 4 = 10$.

9. The figure may be labelled as follows:

Triangles

The simplest triangles are IJQ, JKQ, KLQ, LMQ, MNQ, NOQ, OPQ and PIQ i.e. 8 numbers.

Triangles composed of two components are ABQ, BCQ, CDQ, DEQ, EFQ, FGQ, GHQ, HAQ, IKQ, KMQ, MOQ and OIQ i.e. 12 numbers.

The triangles composed of four components are ACQ, CEQ, EGQ, GAQ, MOI, OIK, IKM and KMO i.e. 8 numbers.

The triangles composed of eight components are ACE, CEG, EGA and GAC i.e. 4 numbers.

Therefore, total number of triangles = $8 + 12 + 8 + 4 = 32$.

Squares

The squares composed of two components are IJQP, JKLQ, LMNO and NOPQ i.e. 4 numbers.

The squares composed of four components are ABQH, BCDQ, DEFQ and QFGH i.e. 4 numbers.

The only square composed of eight components is IKMO.

Only square composed of sixteen components is ACEG.

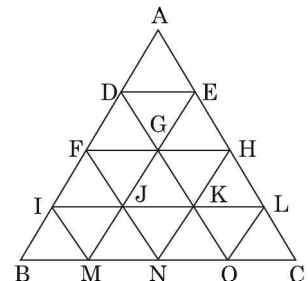
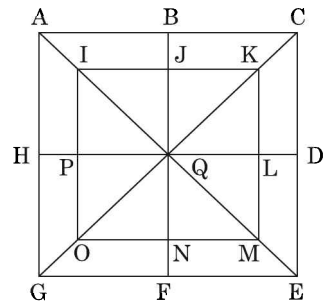
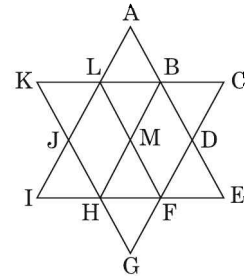
$\therefore$ Total number of squares = $4 + 4 + 1 + 1 = 10$.

10. The figure may be labelled as follows:

The horizontal lines are DE, FH, IL and BC i.e., 4 numbers.

The slanting lines are IM, FN, DO, AC, AB, ME and NH i.e. 7 numbers.

$\therefore$ Total number of lines is $4 + 7 = 11$.



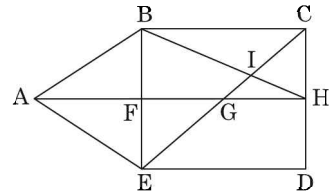
11. The figure may be labelled as follows:

The simplest triangles are AFE, ABF, BCI, CHI, GIH and FGE i.e. 6 numbers.

The triangles composed of two components are ABE, BHF, BIE, BCH, CHG and AGE i.e. 6 numbers.

The triangles composed of three components are ABH, BCE and CDE i.e. 3 numbers.

Hence total number of triangles = $6 + 6 + 3 = 15$.



12. The figure may be labelled as follows:

Triangles

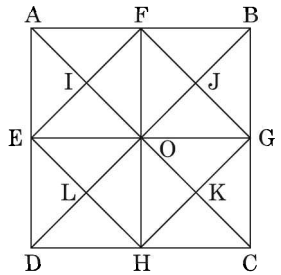
The simplest triangles are AFI, FBG, BGJ, JGO, FJO, OGK, GCK, KCH, OKH, OHL, LHD, ELD, EOL, IOE, FOI and AIE i.e. 16 numbers.

The triangles having two simple triangles each are AFE, AFO, FOE, AOE, FBG, BGO, GOF, OFB, OGC, GCH, CHO, HOG, EOH, OHG, HDE and DEO i.e. 16 numbers.

The triangles having four simple triangles each are AOB, BOC, COD, DOA, GFE, FEH, EHG and GFH i.e. 8 numbers.

The triangles having eight simple triangles each are ABC, BCD, CDA and DAB i.e. 4 numbers.

$\therefore$ Number of triangles in the figure = $16 + 16 + 8 + 4 = 44$.



Squares

The squares containing two triangles each are FJOI, JGKO, KHLO and OLEI i.e. 4 numbers.

The squares containing four triangles each are AFOE, FBGO, OGCH and OHDE i.e. 4 numbers.

The square containing eight triangles is EFGH.

The square containing sixteen triangles is ABCD.

$\therefore$ Total number of squares = $4 + 4 + 1 + 1 = 10$.

13. The figure may be labelled as follows:

The simplest squares are EFRQ, RNSZ, SGHT, QRZY, MQYX, A₁TOU, B₁A₁UV, VUIJ, WB₁VP, XYB₁W, YZA₁B₁ and ZSTA₁ i.e. 13 numbers.

The squares having two components each are AEYL, FBGZ, A₁HCI and KB₁JD i.e. 4 numbers.

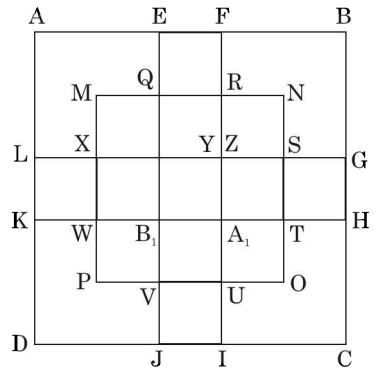
The squares having four components each are XZUP, YSOV, QNTB₁, and MRA₁W i.e. 4 numbers.

The squares having seven components each are AFA₁K, EBHB₁, YGCJ and LYJD i.e. 4 numbers.

The square having nine components is MNOP.

The square having seventeen components is ABCD.

$\therefore$ Total number of squares = $13 + 4 + 4 + 4 + 1 + 1 = 27$.



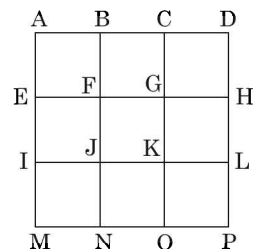
14. Given figure is labelled as follows:

The simplest squares are ABFE, BCGF, CDHG, HLKG, GKJF, FJIE, LPOK, KONJ and JNMI i.e. 9 numbers.

The squares composed of four components are ACKI, BDLJ, EGOM and FHPN i.e. 4 numbers.

The square composed of nine components is ADPM.

$\therefore$ Total number of squares = $9 + 4 + 1 = 14$



15. The given figure may be labelled as follows:

The horizontal lines are DF and BC i.e. 2 numbers.

The vertical lines are DG, AH and FI i.e. 3 numbers.

The slanting lines are DC, BF, AB and AC i.e. 4 numbers.

$\therefore$ Total number of straight lines = $2 + 3 + 4 = 9$ numbers.

The simplest triangles are AED, AFE, DEK, EFK, DKJ, KFL, DJB, FCL, BJG, and LCI i.e. 10 numbers.

The triangles composed of two components each are AFD, DFK, AKD, AFK, KFC, DKB, FCI, DGB, BKH and KCH i.e. 10 numbers.

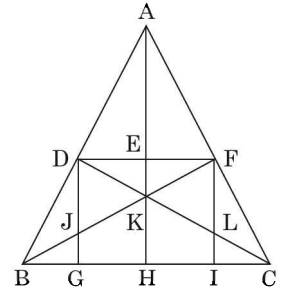
The triangles composed of three components each are DFJ and DFL i.e. 2 numbers.

The triangles composed of four components each are AKB, ACK, BKC, BDF, DFC, DGC and FIB i.e. 7 numbers.

The triangles composed of six components each are AHB, AHC, DAC, DFB, BDC and BFC i.e. 6 numbers.

The triangle composed of twelve components is ABC.

$\therefore$ Total number of triangles = $10 + 10 + 2 + 7 + 6 + 1 = 36$



16. The figure may be labelled as follows:

The straight lines are:

Horizontal: AE and JF i.e. 2 numbers.

Vertical: AJ, CH and EF i.e. 3 numbers.

Slanting: JL, AG, BF, DE, AB, DJ, EI and FG i.e. 8 numbers.

i.e. Total number of straight lines = $2 + 3 + 8 = 13$

17. The simplest triangles in the given figure are ABC, CDE, ACK, CEL, AKJ, CHK, JKH, CLH, EFL, LFH, FGH and HIJ i.e. 12 numbers.

The triangles composed of two components are AHJ, ACH, CFH, CEH, ACJ, CHJ, EFH and CEF i.e. 8 numbers.

Total number of triangles in the given figure = $12 + 8 = 20$

18. The squares composed of two components are ABCK, CDEL, LFGH, HIJK and CLHK i.e. 5 numbers.

The squares composed of four components are ACHJ and CEFH i.e. 2 numbers.

$\therefore$ Number of squares = $5 + 2 = 7$

19. The figure may be represented as follows:

Triangles

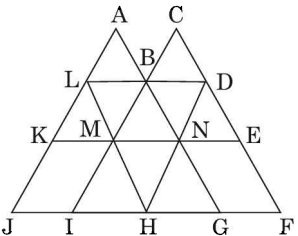
The simplest triangles are ABL, CDB, BML, DNB, KLM, MBN, NDE, IMH, MNH and NGH i.e. 10 numbers.

The triangles composed of three components are LHJ and DFH i.e. 2 numbers.

The triangles composed of four components are: ANK, CEM, BGI and HLD i.e. 4 numbers.

The triangles composed of eight components are AGJ and CFI i.e. 2 numbers.

$\therefore$ Total number of triangles = $10 + 2 + 4 + 2 = 18$.



Parallelograms

The simplest parallelograms are KMIJ and NEFG i.e. 2 numbers.

The parallelograms composed of two components are BMKL, BNML, BDEN, BDNM, MNHI, MNGH, BNHM, ABML and CDNB i.e. 9 numbers.

The parallelograms composed of three components are KNHJ, MEFH, LBIJ and BDFG i.e. 4 numbers.

The parallelograms composed of four components are ANHL, CDHM, LBGH, BDHI, LDNK and LDEM i.e. 6 numbers.

The parallelograms composed of seven components are LDFH and LDHJ i.e. 2 numbers.

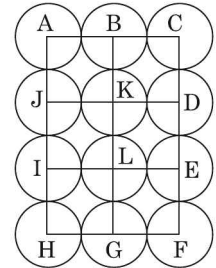
∴ Total number of parallelogram = 2 + 9 + 4 + 6 + 2 = 23.

20. The centres of all the circles are joined and all the vertices are labelled as follows:

The simplest squares are ABKJ, BCDK, JKLI, KDEL, ILGH and LEFG i.e. 6 numbers.

The squares composed of four simple squares each are ACEI and JDFH i.e. 2 numbers.

Thus, total number of squares formed = 6 + 2 = 8.



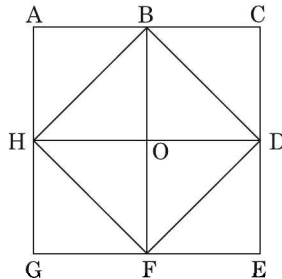
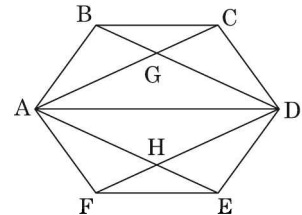
21. The figure may be labelled as follows:

The quadrilaterals in the figure are ABDE, ABDF, ACDF, ACDE, ACDH, AGDH, ABDH, AGDE and AGDF.

∴ Total number of quadrilaterals = 9

22. The pentagons in the above figure are ABCDE, CDEFA, DEFAB, FABCD, AGDEF and ABCDH i.e. 6 numbers.

23. The figure may be labelled as follows:



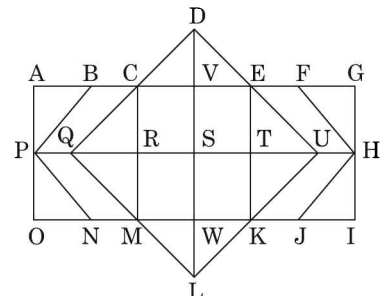
The pentagons in the above figure are ABDFH, BCDHF, DEFHB, FGHBD, ABDFG, GHBDE, ACDFH, BCEFH, GABDE, GACDF, HACEF and BCEGH i.e. 12 numbers.

24. The simplest rectangles are CVSR, VETS, STKW and RSWM i.e. 4 numbers.

The rectangles having two components each are: CVWM, VEKW, CETR and RTKM i.e. 4 numbers.

The rectangles having three components each are: ACRP, PRMO, EGHT and THIK i.e. 4 numbers.

The rectangles having four components each are: AVSP, PSWO, CEKM, VGHS and SHIW i.e. 5 numbers.



The rectangles having five components each are AETP, PTKO, CGHR and RHIM i.e. 4 numbers.

The rectangles having six components each are ACOM and EGIK i.e. 2 numbers.

The rectangles having eight components each are AVWO, VGIW, AGHP and PHIO i.e. 4 numbers.

The rectangles having ten components each are: AEKO and CGIM i.e. 2 numbers.

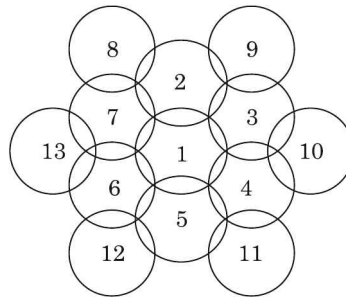
The rectangle having sixteen components each is AGIO.

$\therefore$ Total number of rectangles = $4 + 4 + 4 + 5 + 4 + 2 + 4 + 2 + 1 = 30$.

The hexagons in the given figure are CEUKMQ, BFHJNP, CDEKLM, CFHJMQ, and BEUKNP i.e. 5 numbers.

Total number of hexagons = 5.

25. The given figure may be labelled as follows:



Total number of circles = 13.

Problems on Cubes and Dice

In this chapter you will be given with a dice marked from 1 to 6 in different faces placed in 4 or 5 positions. You have to identify the face coming opposite to a specified face or a face coming in between two different specified faces.

Or you will be given with a cube painted with different colours in different faces. If the cube is cut into smaller cubes. You have to find out the number of cubes, which are:

- (i) at least one face painted
- (ii) at least two faces painted
- (iii) at least three faces painted
- (iv) no faces painted, etc.

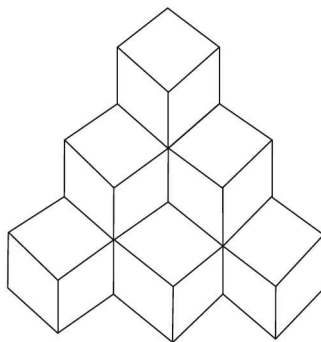
PROBLEMS ON DICE: In this test, you are given figures showing the same die in different positions. Study the figures carefully and you have to find the number opposite a given number on the die.

PROBLEMS ON CUBES: In this test, you have to study the given figure carefully and answer the questions that follow. You may have to find either of the following:

- (i) Count the number of cubes.
- (ii) Count the number of cubes that are painted with one or more particular colour.
 - (a) in one face only, (b) in more than one faces
- (iii) Find the colour of a face which is opposite to another face painted with a particular colour.
- (iv) Count the number of cubes with specific number of faces painted.
- (v) (a) no face painted, (b) one face painted, (c) two faces painted, (d) three faces painted, (e) four faces painted, (f) five faces painted, (g) all faces painted

EXAMPLE 1 Find the number of cubes in the given figure.

[IAS]



Solution There are three columns containing 1 cube each, two columns containing 2 cubes each and one column containing 3 cubes.

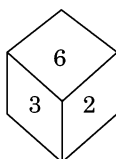
$\therefore$ Total number of cubes in the figure = $3 \times 1 + 2 \times 2 + 1 \times 3 = 3 + 4 + 3 = 10$.

EXERCISES

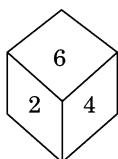
Observe the figures carefully and find the number opposite a given number on the dice.

1. Consider the following pictures of a dice.

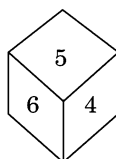
[IAS]



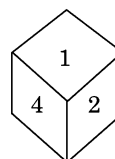
(i)



(ii)



(iii)



(iv)

What is the number opposite to 3?

(a) 1

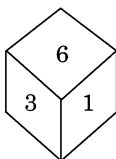
(b) 4

(c) 5

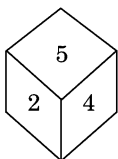
(d) Data insufficient

2. The number opposite to 3 is

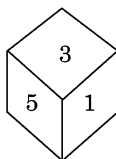
[IAS]



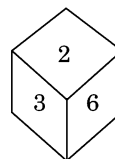
(i)



(ii)



(iii)



(iv)

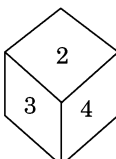
(a) 2

(b) 3

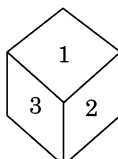
(c) 4

(d) 6

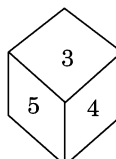
3. A die is thrown four times and its four different positions are given below. Find the number on the face opposite the face showing 2.



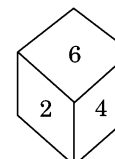
(i)



(ii)



(iii)



(iv)

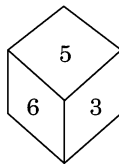
(a) 3

(b) 4

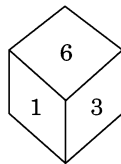
(c) 5

(d) 6

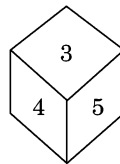
4. Shown below are four different positions of the same dice. Find the number on the face opposite the face showing 6.



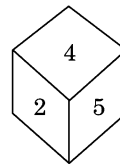
(i)



(ii)



(iii)



(iv)

(a) 1

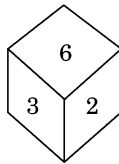
(b) 2

(c) 4

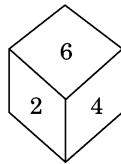
(d) 5

5. Which number is on the face opposite 6?

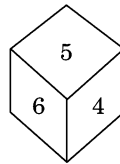
[IAS]



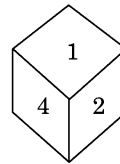
(i)



(ii)



(iii)



(iv)

(a) 1

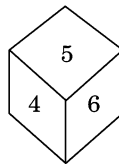
(b) 2

(c) 3

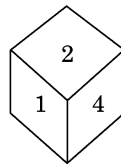
(d) 4

6. Which number is opposite to 3?

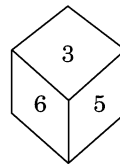
[IAS]



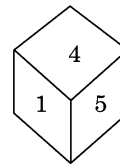
(i)



(ii)



(iii)



(iv)

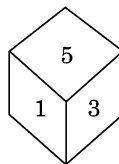
(a) 1

(b) 2

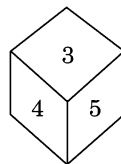
(c) 4

(d) 6

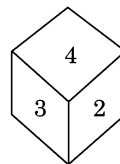
7. Which number is opposite to 4?



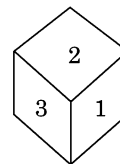
(i)



(ii)



(iii)



(iv)

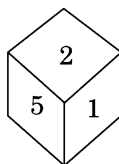
(a) 1

(b) 2

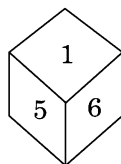
(c) 5

(d) 6

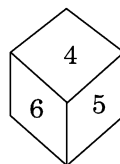
8. Which number is on the face opposite 4?



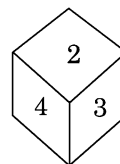
(i)



(ii)



(iii)



(iv)

(a) 1

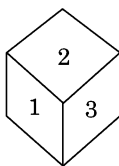
(b) 2

(c) 3

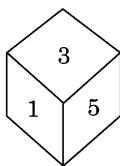
(d) 5

9. What should be the number opposite 3?

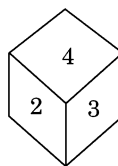
[IAS]



(i)



(ii)



(iii)

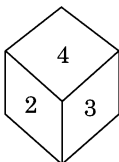
(a) 1

(b) 6

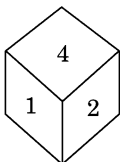
(c) 5

(d) 4

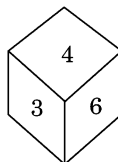
10. What numbers occur at the bottom face in the three positions of the same die?



(i)



(ii)



(iii)

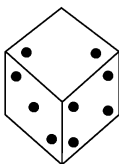
(a) 6, 6, 2

(b) 5, 6, 1

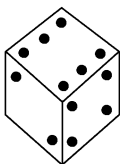
(c) 5, 5, 5

(d) 6, 5, 2

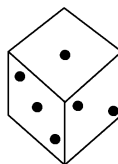
11. Find the number of dots on the face opposite the face bearing 3 dots.



(i)



(ii)



(iii)

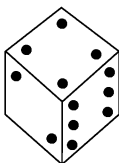
(a) 5

(b) 6

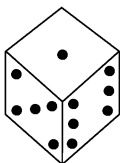
(c) 4

(d) Cannot be determined

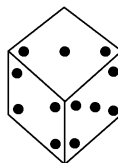
12. How many dots lie opposite 2 dots?



(i)



(ii)



(iii)

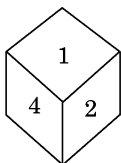
(a) 1

(b) 3

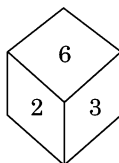
(c) 5

(d) 6

13. What will be the number at the bottom if 5 is at the top; the two positions of the dice being as given below.



(i)



(ii)

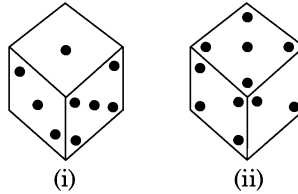
(a) 1

(b) 2

(c) 3

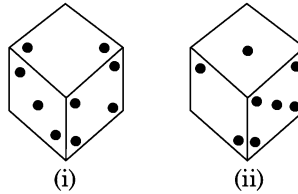
(d) 6

14. What is the number of dots on the face opposite 2 dots?



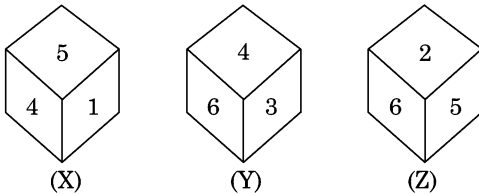
- (a) 1 (b) 3 (c) 4 (d) 6

15. What is the number of dots at the bottom face of the left hand side dice?



- (a) 3 (b) 4 (c) 5 (d) 6

16.



- (i) Which number lies at the bottom face of the die X?
(a) 1 (b) 2 (c) 3 (d) 4
- (ii) Which number lies at the bottom face of the die Y?
(a) 6 (b) 5 (c) 2 (d) 1
- (iii) Which number lies opposite 6?
(a) 1 (b) 2 (c) 4 (d) 5
- (iv) Which numbers are hidden behind the numbers 6 and 5 in the die Z?
(a) 1 and 4 (b) 1 and 3 (c) 4 and 3 (d) 1 and 2
- (v) Which of the hidden numbers adjacent to 5 in die X are common to the hidden numbers adjacent to 5 in die Z?
(a) 1 and 4 (b) 2 (c) 6 (d) None of these

ANSWERS

1. (b) 2. (c) 3. (c) 4. (c) 5. (a) 6. (c) 7. (a) 8. (a) 9. (b) 10. (c)
11. (b) 12. (c) 13. (b) 14. (b) 15. (d) 16. (i) (c) (ii) (c) (iii) (a) (iv) (b) (v) (d)

Explanations

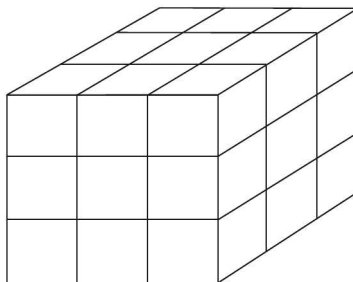
2. From figure (i), (iii) and (iv) it is clear that the numbers 6, 1, 5 and 2 lie adjacent to 3. So, 4 lies opposite 3.

3. From figure (i) and (iv) 3 is opposite to 6. From figure (i) and (ii) 1 is opposite to 4. So number opposite to 2 is 5.
4. From figure (i) and (iii), 4 is opposite to face showing 6.
5. From figure (ii) and (iv) 1 is opposite to face showing 6.
6. From figure (i) and (iii) number opposite 3 is 4.
7. From figure (iii) and (iv) face opposite 4 is 1.
8. From figure (ii) and (iii) face opposite 4 is 1.
9. From figure (i), (ii) and (iii), face opposite 3 is 6.
10. From figure (i), (ii) and (iii) 1, 2, 3 and 6 appears on sides of dice and 4 appears on top. So number at bottom on each dice is 5. i.e. 5, 5, 5.
11. From figure (i) and (iii) 1 dot appears opposite 4. From figure (ii) and figure (i), 6 appears opposite to 3.
12. From figure (i) and (ii), 5 lies opposite 2.
13. From figure (i) and (ii) number opposite to 2 is 5.
i.e. if 5 is at the top, number at bottom will be 2.
14. From figure (i) and (ii) 3 dots lies opposite 2.
15. From figure (i) and (ii), 3, 4, 5 and 1 lies by the side of 2. So 6 lies opposite to 2 i.e. bottom of the left hand side dice.
16. (i) From figure (X) and (Z), 4, 1, 6 and 2 lies by the side of 5. So number lies at the bottom face of die X is 3.
(ii) From figure (X) and (Y), 1, 5, 6 and 3 lies by the side of 4. So number that lies at bottom face of Y is 2.
(iii) From figure (Y) and (Z), 2, 5, 4 and 3 lies by the side of 6. So number opposite to 6 is 1.
(iv) From figure (Y) and (Z), number 1 is opposite to 6. From figure (X) and (Z), number 3 is opposite to 5.
 $\therefore$ So 1 and 3 is the answer.
(v) From figure (X) and (Z), number adjacent to 5 are 4, 1, 2 and 6. No numbers are in common to dice X and Z which are hidden and adjacent to 5.

PAINTING A STACK OF CUBES

EXERCISES

Directions: A wooden cube is painted blue on all the four adjoining sides and green on two opposite sides, i.e. top and bottom. It is then cut at equal distances at right angles four times vertically (top and bottom) and two times horizontally (along the sides) as shown in the figure. Study the diagram and answer the following questions:



1. How many cubes will have one face painted only in blue?
(a) 1 (b) 2 (c) 3 (d) 4 (e) 5
2. How many cubes will have one face painted only in green?
(a) 1 (b) 2 (c) 3 (d) 4 (e) 5
3. How many cubes are formed in all?
(a) 16 (b) 24 (c) 27 (d) 32 (e) 48
4. How many cubes will have at least three sides painted?
(a) 8 (b) 6 (c) 3 (d) 2 (e) 1
5. How many cubes will have no face painted at all?
(a) 1 (b) 2 (c) 3 (d) 4 (e) 5

Directions (6–8): A cube is painted blue on all faces is cut into 125 cubes of equal size. Now, answer the following questions. [MBA]

6. How many cubes are not painted on any face?
(a) 8 (b) 16 (c) 18 (d) 27 (e) 54
7. How many cubes are painted on one face only?
(a) 8 (b) 16 (c) 36 (d) 54 (e) None of these
8. The minimum number of colours required to paint all the sides of a cube that no two adjacent faces may have the same colour is [MBA]
(a) 1 (b) 2 (c) 3 (d) 4 (e) 6

Directions (9–13): A cube painted red on two adjacent faces and black on the faces opposite to the red faces and green on the remaining faces is cut into 64 smaller cubes of equal size.

[CBI]

9. How many cubes are there which have no faces painted?
(a) 0 (b) 4 (c) 8 (d) 16
10. How many cubes have only one face painted?
(a) 8 (b) 16 (c) 24 (d) 32
11. How many cubes have less than 3 faces painted?
(a) 8 (b) 24 (c) 28 (d) 48
12. How many cubes are there with three faces painted?
(a) 4 (b) 8 (c) 16 (d) 24
13. How many cubes have one face green and one of the adjacent faces black or red?
(a) 8 (b) 16 (c) 24 (d) 28

Directions (14–20): A solid cube of each side 8 cm has been painted red, blue and black on pairs of opposite faces. It is then cut into cubical blocks of each side 2 cm.

14. How many cubes have no faces painted?
(a) 0 (b) 4 (c) 8 (d) 12
15. How many cubes have three faces painted with different colours?
(a) 0 (b) 4 (c) 8 (d) 12
16. How many cubes have two faces painted red and black and all other faces unpainted?
(a) 4 (b) 8 (c) 16 (d) 32
17. How many cubes have to faces painted black?
(a) 2 (b) 4 (c) 8 (d) None of these

18. How many cubes have one face painted blue and one face painted red? (the other faces may be painted or not painted)
 (a) 16 (b) 12 (c) 8 (d) 0
19. How many cubes have only one face painted red and all other faces unpainted?
 (a) 4 (b) 8 (c) 12 (d) 16
20. How many cubes have three faces painted?
 (a) 0 (b) 4 (c) 6 (d) 8

ANSWERS

1. (d) 2. (b) 3. (c) 4. (a) 5. (a) 6. (d) 7. (d) 8. (c) 9. (c) 10. (c)
 11. (d) 12. (b) 13. (c) 14. (c) 15. (c) 16. (b) 17. (d) 18. (a) 19. (b) 20. (d)

Explanations

Note: The figure may be analyzed by dividing it into three horizontal layers.

- There are four cubes in the middle layer, which have one face painted only in blue.
- There is only one cube each at top and bottom layer which has one face painted only in green.
- There are 9 cubes in each layer. Therefore, altogether in all the three layers there are 27 cubes.
- Four cubes in the top layer and four cubes in the bottom layer have three sides painted. Hence there are 8 such cubes.
- There is only one central cube in the middle layer, which has no faces painted at all.
- There will be 5 cubes width for the given large cube as length, width and depth. So in the middle layers there will be $3 \times 3 = 9$ cubes/layer. So in all, there will be $9 \times 3 = 27$ cubes unpainted on all faces.
- Number of cubes which are one side painted – 9 cubes at top + 9 cubes at bottom + (12 cubes/layer $\times$ 3 for middle layers).
 $\therefore$ Total number of cubes with only one face painted = $9 + 12 \times 3 + 9 = 54$.
- Adjacent faces are avoided for same colours, opposite faces may be provided with same colours. So as a cube is having 3 pair of faces, minimum number of colours required to paint all sides of cube = 3.
- There are 8 cubes with no face painted.
- There are 24 cubes having only one face painted.
- There are 24 cubes having only one face painted and 24 cubes having only two faces painted. So the number of cubes having less than 3 faces painted = $24 + 24 = 48$.
- There are 8 cubes having three faces painted.
- There are $8 + 4 + 4 + 8 = 24$ such cubes which have one face green and one of the adjacent faces black or red.
- Four central cubes each in 2 middle layers have no face painted. i.e. total number of such cubes = 8.
- Four corners cubes in top and bottom layers have three faces painted with different colours. Thus, there are 8 such cubes.
- There are four cubes in top layer and four cubes in bottom layer which have two faces painted red and black and all other faces unpainted. So, there are 8 such cubes.

17. There will be no cubes which have two of its faces both painted black. So number of cubes = 0. The answer is (d).
18. There are 8 cubes in top layer and 8 cubes in bottom layer which have one face painted blue and one face painted red. So, the total number of such cubes = $8 + 8 = 16$.
19. There are 4 cubes in top layer and 4 cubes in bottom layer which have only one face painted red and all other faces unpainted.
 $\therefore$ Total number of such cubes = $4 + 4 = 8$.
20. Four corner in top layer and four corner cubes in bottom layer have faces painted. So there are 8 such cubes.

Mirror Images

Image of an object, as seen in a mirror, is called the *mirror image* or *mirror reflection*.

The right side of an object appears on the left side in its image and vice versa. Phenomenon of getting a letter inverted in its mirror image is called the *lateral inversion*.

Note: Letters having identical mirror images are : A H I M O T U V W X and Y

EXAMPLE 1 Find out the mirror image of the word 'SECOND'.

<i>Object</i>		<i>Image</i>
SECOND		DNOƆ2

Image of word SECOND will be as seen on the right side after dotted line.

EXERCISES

Directions (Q. 1–5): In each of the following questions, you are given a combination of letters or/and numbers followed by four alternatives (a), (b), (c) and (d). Choose the alternative which most closely resembles the mirror-image of the given word.

1. ABILITY

(a) YTILIBA	(b) ABLITY	(c) YTLIIBA	(d) LITYABI
2. 97563

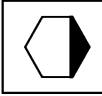
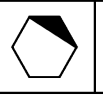
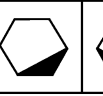
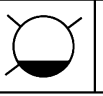
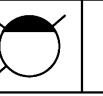
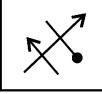
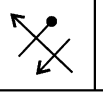
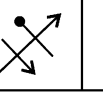
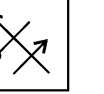
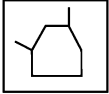
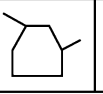
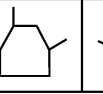
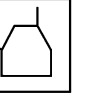
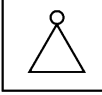
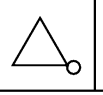
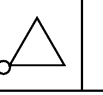
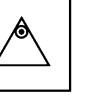
(a) 63975	(b) 36579	(c) 36975	(d) 57963
3. mineral

(a) eralmin	(b) lsɹɛnim	(c) ɹɹɹɹɹ	(d) larenim
4. A1B2C3

(a) 3C2B1A	(b) 2C3A1B	(c) 3C2B1A	(d) 2C3A1B
5. GANDHI1948

(a) 8491IHNDAG	(b) 1948 GANDHI
(c) 8491IHNDAG	(d) 1948 GANDHI

Directions (Q. 6–10): In each one of the following questions, choose the correct mirror-image of the figure (X) from among the four alternatives (a), (b), (c) and (d) given along with it.

6.  (X)  (a)  (b)  (c)  (d)
7.  (X)  (a)  (b)  (c)  (d)
8.  (X)  (a)  (b)  (c)  (d)
9.  (X)  (a)  (b)  (c)  (d)
10.  (X)  (a)  (b)  (c)  (d)

ANSWERS

1. (c) 2. (d) 3. (b) 4. (c) 5. (a) 6. (b) 7. (b) 8. (b) 9. (c) 10. (c)

Water Images

The reflection of an object as seen in water is called its **water image**. It is the inverted image obtained by turning the object upside down.

Note: The letters whose water images remain unchanged are: C, D, E, H, I, K, O, and X.

EXAMPLE 1 Find out the water image of the word 'SERIES'.

- (a) SEIKES (b) SEIEES (c) SEIEES (d) IESSER

The answer is (b).

EXERCISES

Directions: In each one of the following questions, you are given a combination of letters or/and numbers followed by four alternatives (a), (b), (c), and (d). Choose the alternative which most clearly resembles the water image of the given combination.

1. NEW DELHI

- (a) DEГHI IEM (b) WEN IHJED (c) IEM DEГHI (d) IHГED WEN

2. escape

- (a) eƆsɹɐɐ (b) sƆeɐɐɐ (c) ɐɹɐɐɐ (d) ɐɹɐɐɐ

3. 15 AUG 1947

- (a) I᠙᠙᠕ ᠕᠒᠒ I᠙ (b) I᠙ ᠕᠒᠒ I᠙᠙᠕ (c) ᠕᠙᠙᠕ ᠒᠒᠕ I᠙ (d) ᠑I ᠒᠒᠕ ᠕᠙᠕

4. PERMANENT

- (a) ƆM᠙᠕᠕᠕᠕᠕᠕ (b) ƆM᠙᠕᠕᠕᠕᠕᠕ (c) M᠙᠙᠕᠕᠕᠕᠕᠕ (d) ƆM᠙᠕᠕᠕᠕᠕᠕

5. NEHRU 1964

- (a) I᠙᠙᠕ I᠙᠙᠕᠕ (b) 19᠖᠕ I᠙᠙᠕᠕ (c) I᠙᠙᠕ NEHRU (d) I᠙᠙᠕᠕ I᠙᠙᠕

ANSWERS

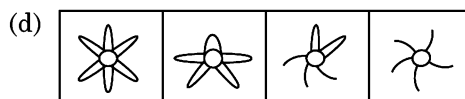
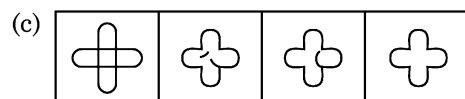
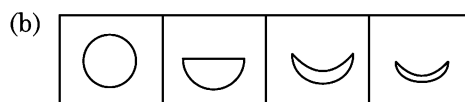
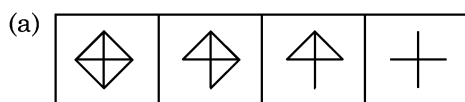
1. (c) 2. (d) 3. (b) 4. (d) 5. (d)

Rule Detection

In this chapter, we will solve problems in which a particular rule is given and it is required to select from the given sets of figures, a set of figures which obeys the rule and thus forms a series.

EXAMPLE 1 Which one of the given sets of figures satisfies the following rule?

Rule: Closed figure becomes more and more open.

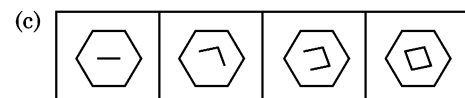
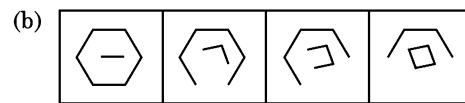
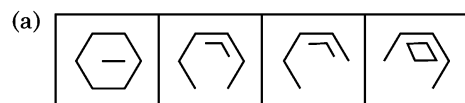


Solution The answer is (a).

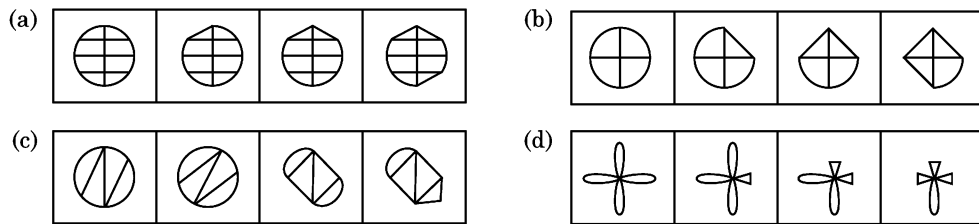
EXERCISES

Directions: In each of the following questions, choose the set of figures which follows the given rule.

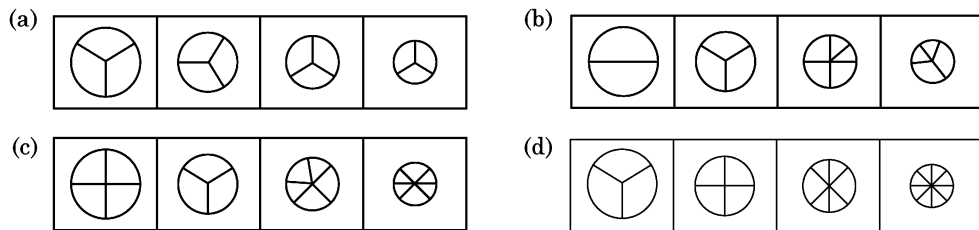
1. Rule: Closed figures losing their sides and open figures gaining their sides.



2. Rule: Sectors get converted to triangles one by one.



3. Rule: As the circle decreases in size, its sector increases in number.



ANSWERS

1. (b) 2. (b) 3. (d)

Model Test Paper 1

1. Find the least number which, upon being divided by 2, 3, 4, 5 and 6 leaves in each case a remainder of 1, but when divided by 7 leaves no remainder.
(a) 271 (b) 291 (c) 301 (d) 311 (e) None of these
2. What is the greatest number that will divide 676 and 942 and will leave remainders 4 and 18 respectively?
(a) 64 (b) 46 (c) 24 (d) 42 (e) None of these
3. The sum of two number is 528 and their HCF is 33. How many pairs of such numbers can be formed?
(a) 1 (b) 2 (c) 3 (d) 4 (e) None of these
4. The product of two numbers is 2646 and their HCF is 21. Find the number of possible pair of numbers.
(a) 1 (b) 2 (c) 3 (d) 4 (e) None of these
5. Find the value of $\frac{1}{2 + \frac{1}{3 + \frac{1}{1 + \frac{1}{4}}}} =$
(a) $\frac{17}{43}$ (b) $\frac{18}{43}$ (c) $\frac{19}{43}$ (d) $\frac{21}{43}$ (e) None of these
6. A person is having some amount with him. He gave $\frac{1}{3}$ rd of the amount as loan to A and took back 54 more. He gave $\frac{1}{2}$ of the amount with him as loan to B and took back 25 more. When he met C, the amount left was ₹ 262. What was the amount with the person when he met A?
(a) 620 (b) 640 (c) 630 (d) 650 (e) None of these

7. How many number of five digits can be formed with the digits 0, 1, 2, 4, 6 and 8?
(a) 800 (b) 600 (c) 500 (d) 400 (e) None of these
8. There are five students P, Q, R, S, and T. In how many ways can they sit so that Q and R do not sit together?
(a) 120 (b) 48 (c) 66 (d) 72 (e) None of these
9. From a pack of 52 playing cards 2 cards are drawn. What is the probability that it has one ace and one jack?
(a) $\frac{4}{663}$ (b) $\frac{8}{663}$ (c) $\frac{16}{663}$ (d) $\frac{32}{663}$ (e) None of these
10. There are twenty students in a class. If 8 of them are girls and a team of 2 boys and 2 girls is to be selected from the class for its anniversary, in how many ways it can be done?
(a) 3696 (b) 1848 (c) 924 (d) 1386 (e) None of these
11. Two dices are thrown. What is the probability that the sum of numbers appeared is less than 8?
(a) $\frac{3}{12}$ (b) $\frac{5}{12}$ (c) $\frac{7}{12}$ (d) $\frac{11}{12}$ (e) None of these
12. The sum of three numbers is 98. If the ratio between the first and second and second and third are respectively 2 : 3 and 5 : 8, then find the second number.
(a) 48 (b) 28 (c) 30 (d) 36 (e) None of these
13. If 15 men or 24 women or 36 boys can do a piece of work in 12 days, working 8 hours a day, how many men must be associated with 12 women and 6 boys to do another piece of work $2\frac{1}{4}$ times as great in 30 days working 6 hrs a day?
(a) 24 (b) 18 (c) 10 (d) 8 (e) None of these
14. The ratio between two numbers is 4 : 5. If each number is increased by 9, the ratio becomes 5 : 6. Find the numbers.
(a) 48, 60 (b) 24, 30 (c) 36, 45 (d) 12, 15 (e) None of these
15. The salary of an employee is first increased by 10% and then decreased by 10%. What was the change in his salary?
(a) 0% (b) +1% (c) -1% (d) +5% (e) None of these
16. Tax on a commodity is increased by 20% and its consumption is decreased by 15%. Find the effect on revenue.
(a) +5% (b) -5% (c) +2% (d) -2% (e) None of these
17. A man drives a car to a certain place at a speed of 48 km/h and returns at a speed of 24 km/h. Find the average speed of his whole journey.
(a) 24 km/h (b) 36 km/h (c) 32 km/h (d) 40 km/h (e) None of these
18. The sum of ages of a son and father is 56 years. After 4 years, the age of father will be three times that of son. What is the age of son?
(a) 10 years (b) 12 years (c) 16 years (d) 18 years (e) None of these
19. A man purchased an article at $\frac{3}{4}$ th of the listed price and sold at half more than the listed price. What was his gain percentage?
(a) 25% (b) 50% (c) 75% (d) 100% (e) None of these

20. Two tables were bought at ₹ 2700. One of them was sold at a loss of 6% and other at a gain of $7\frac{1}{2}\%$. On the whole neither loss nor gain was the result of transaction. What did each table cost?

(a) 1350, 1350 (b) 1400, 1300 (c) 1500, 1200 (d) 1600, 1100 (e) None of these

21. $\frac{\sqrt[3]{456533} \times \sqrt{256}}{\sqrt{64} \times \sqrt[3]{1331}} = ?$

(a) 7 (b) 14 (c) 21 (d) 28 (e) None of these

Directions (22–24): In each of the following questions a number series is given in which one number is wrong. You have to find out that number and have to follow the new series which will be started by that number. Then what will be the third number of the new series?

22. 2 9 5 36 125 648 3861

(a) 12 (b) 11 (c) 75 (d) 72 (e) None of these

23. 3 4 12 45 190 1005 6066

(a) 98 (b) 96 (c) 384 (d) 386 (e) None of these

24. 2 7 19 43 99 209 431

(a) 181 (b) 183 (c) 87 (d) 85 (e) None of these

Directions (25–27): In each of the following questions a number series is given. After the series, a number is given followed by (a), (b), (c), (d) and (e). You have to complete the series starting with the number given, following the sequence of the given series.

25. 2 10 27 60

5 (a) (b) (c) (d) (e)

Which of the following number will come in place of (b)?

(a) 39 (b) 13 (c) 34 (d) 38 (e) None of these

26. 15 16 25 50

189 (a) (b) (c) (d) (e)

Which of the following numbers will come in place of (e)?

(a) 354 (b) 273 (c) 394 (d) 426 (e) None of these

27. 5 149 49 113

146 (a) (b) (c) (d) (e)

Which of the following number will come in place of (d)?

(a) 290 (b) 234 (c) 254 (d) 218 (e) None of these

28. If $x + \frac{1}{x} = 3$, then find the value of $x^2 + \frac{1}{x^2}$?

(a) 3 (b) 5 (c) 7 (d) 9 (e) None of these

29. If $a + b + c = 0$, find the value of $\frac{a^2 + b^2 + c^2}{c^2 - ab}$

(a) 0 (b) 1 (c) 2 (d) 3 (e) None of these

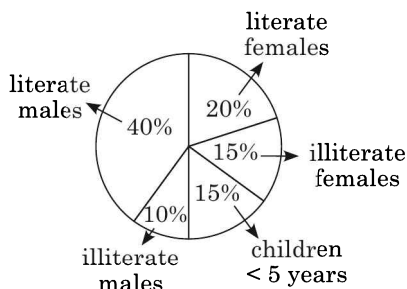
30. Find the value of $\frac{2 + \sqrt{3}}{2 - \sqrt{3}}$
- (a) $4 + 7\sqrt{3}$ (b) $7 - 4\sqrt{3}$ (c) $7 + 4\sqrt{3}$ (d) $4 - 7\sqrt{3}$ (e) None of these
31. A and B enter into a partnership with capitals 4 : 5, and at the end of 8 months, A withdraws. If they receive profits in the ratio 8 : 15, find how long B's capital was used?
- (a) 4 months (b) 8 months (c) 10 months (d) 12 months (e) None of these
32. Out of ₹ 7000, some amount was lent at 6% per annum and the remaining at 4% per annum. If the total simple interest from both the fractions in 5 years was ₹ 1600, find the sum lent at 6% per annum.
- (a) ₹ 5000 (b) ₹ 4000 (c) ₹ 3000 (d) ₹ 2000 (e) None of these
33. Two equal amounts of money are deposited in two banks, each at 15% per annum, for $3\frac{1}{2}$ years and 5 years respectively. If the difference between their interests is ₹ 144, find the sum.
- (a) ₹ 400 (b) ₹ 300 (c) ₹ 640 (d) ₹ 720 (e) None of these
34. A certain sum grows upto ₹ 4840 in 2 years and upto ₹ 5324 in 3 years on compound interest. Find the rate per cent.
- (a) 5% (b) 10% (c) 15% (d) 20% (e) None of these
35. A certain number of men complete a piece of work in 60 days. If there were 8 men more, the work could be finished in 10 days less. How many men were originally there?
- (a) 20 men (b) 30 men (c) 40 men (d) 50 men (e) None of these
36. P, Q and R together do a piece of work for ₹ 535. P working alone can do it in 5 days. Q alone can do it in 6 days and R alone can do it in 7 days. Then what will be the share of R for its work.
- (a) ₹ 100 (b) ₹ 150 (c) ₹ 200 (d) ₹ 250 (e) None of these
37. A cistern has a leak which would empty it in 8 hours. A tap is turned on which admits 6 litres a minute in the cistern, and it is now emptied in 12 hours. How many litres does the cistern hold?
- (a) 8460 L (b) 4320 L (c) 8640 L (d) 4230 L (e) None of these
38. Two trains are moving in the opposite direction on parallel tracks at speeds of 64 km/h and 96 km/h respectively. The first train passes a telegraph post in 5 seconds whereas the second train passes the post in 6 seconds. Find the time taken by the trains to cross each other completely.
- (a) $\frac{28}{11}$ s (b) $\frac{14}{11}$ s (c) $\frac{28}{5}$ s (d) $\frac{14}{5}$ s (e) None of these
39. A train passes two persons who are walking in the direction opposite which the train is moving, at the rate of 5 m/s and 10 m/s in 6 seconds and 5 seconds respectively. Find the length of the train and speed of the train.
- (a) 140 m, 10 m/s (b) 150 m, 20 m/s (c) 150 m, 30 m/s
(d) 140 m, 40 m/s (e) None of these

40. A boat travels upstream from B to A and downstream from A to B in 3 hours. If the speed of boat in still water is 9 km/h and speed of the current is 3 km/h, find the distance between A and B in km.
 (a) 6 km (b) 9 km (c) 12 km (d) 18 km (e) None of these
41. A hall, whose length is 16 metres and breadth twice its height takes 168 metres of paper 2 m wide for sticking on its four walls. Find the area of floor.
 (a) 384 m^2 (b) 192 m^2 (c) 576 m^2 (d) 288 m^2 (e) None of these
42. The radius of a sphere is increased by 5%. Find the percentage increase in its surface area.
 (a) 20 (b) 15.5 (c) 10.25 (d) 5.25 (e) None of these
43. A man covers a certain distance by car at a speed of 60 km/h and returns on a scooter at a speed of 40 km/h. Find his average speed for the whole journey.
 (a) 40 km/h (b) 48 km/h (c) 50 km/h (d) 60 km/h (e) None of these
44. The upper part of a tree broken by wind makes an angle of 30° with the ground and the distance from the root of the point where the top of the tree touches the ground is 10 m. What is the height of the tree?
 (a) 30 m (b) 40 m (c) 50 m (d) 60 m (e) None of these

Directions (45–46): The questions below consist of a question followed by two statements labelled I and II. You have to decide whether these statements are sufficient to answer the question. Give answer

- (a) if statement I alone is sufficient to answer the question but statement II alone is not sufficient to answer the question;
 (b) if statement II alone is sufficient to answer the question but statement I alone is not sufficient to answer the question.
 (c) if you can get answer from I and II together.
 (d) if statement I alone or II alone is sufficient.
 (e) if both statements taken together are not sufficient.
45. What is the value of a two-digit number?
 I. The sum of the two digits is 4.
 II. The difference between the two digits is 2.
46. If x greater than y ?
 I. x is greater than 145.
 I. y is greater than 140.

Directions (47–49): Study the following pie chart and answer the questions given below.



Total population of a town in 2011 = 10 lakhs.

47. What is the total number of literate males in the town?
 (a) 500000 (b) 400000 (c) 200000 (d) 150000 (e) None of these
48. What is the ratio of literate females to literate males?
 (a) 1 : 2 (b) 2 : 3 (c) 1 : 4 (d) 1.5 : 4 (e) None of these
49. What is the total strength of literate people to total population expressed as percentage?
 (a) 40% (b) 30% (c) 50% (d) 60% (e) None of these
50. 6 September 1986 was Saturday. What will be the day on 15 August 2012?
 (a) Monday (b) Tuesday (c) Wednesday (d) Thursday (e) None of these

ANSWERS

1. (c) 2. (d) 3. (d) 4. (b) 5. (c) 6. (c) 7. (b) 8. (d) 9. (b) 10. (b)
 11. (c) 12. (c) 13. (d) 14. (c) 15. (c) 16. (c) 17. (c) 18. (b) 19. (d) 20. (c)
 21. (b) 22. (e) 23. (d) 24. (b) 25. (a) 26. (a) 27. (d) 28. (c) 29. (c) 30. (c)
 31. (d) 32. (d) 33. (c) 34. (b) 35. (c) 36. (b) 37. (c) 38. (c) 39. (b) 40. (c)
 41. (b) 42. (c) 43. (b) 44. (e) 45. (e) 46. (e) 47. (b) 48. (a) 49. (d) 50. (c)

Solutions and Hints

7. Required number of numbers = $5 \times {}^5P_4 = 5 \times 5! = 5 \times 120 = 600$
8. No. of ways in which Q and R sit together = $2 \times 4! = 48$
 $\therefore$ No. of ways in which Q and R do not sit together = $5! - 48 = 120 - 48 = 72$
9. Required probability = $\frac{{}^4C_1 \times {}^4C_1}{{}^{52}C_2} = \frac{4 \times 4 \times 2}{52 \times 51} = \frac{8}{13 \times 51} = \frac{8}{663}$
10. Required number of ways = ${}^8C_2 \times {}^{12}C_2 = \frac{8 \times 7}{1 \times 2} \times \frac{12 \times 11}{1 \times 2} = 28 \times 66 = 1848$
11. Desired sum of number are 2, 3, 4, 5, 6 and 7.
 $n(s) = 1 + 2 + 3 + 4 + 5 + 6 = 21$
 $\therefore$ Required Probability = $\frac{21}{36} = \frac{7}{12}$
12. $x : y = 2 : 3$ and $y : z = 5 : 8$
 $x : y : z = 10 : 15 : 24$
- | | | |
|-----|-----|-----|
| x | y | z |
| 2 | 3 | |
| | 5 | 8 |
| 10 | 15 | 24 |
- $\therefore$ Second number $y = \frac{15}{10 + 15 + 24} \times 98 = \frac{15}{49} \times 98 = 30$
13. $\left. \begin{array}{l} 30 \text{ days} : 12 \text{ days} \\ 6 \text{ h} : 8 \text{ h} \\ 1 \text{ work} : 2\frac{1}{4} \text{ work} \end{array} \right\} \therefore 15 \text{ men} : \text{required no. of men}$

$$\therefore \text{Required no. of men} = \frac{15 \times 12 \times 8 \times 2.25}{30 \times 6 \times 1} = 18$$

$$24 \text{ women} = 15 \text{ men}$$

$$\therefore 12 \text{ women} = 7.5 \text{ men}$$

$$36 \text{ boys} = 15 \text{ men}$$

$$\therefore 6 \text{ boys} = \frac{15}{6} = 2.5 \text{ men}$$

$$\therefore 12 \text{ women} + 6 \text{ boys} = 7.5 + 2.5 = 10 \text{ men}$$

$$\therefore 18 - 10 = 8 \text{ men must be associated.}$$

14. Ratio of numbers = 4 : 5

i.e. $4k : 5k$

After increasing, ratio = $(4k + 9) : (5k + 9) = 5 : 6$

$$\Rightarrow k = 9$$

$\therefore$ The numbers are **36 and 45**

15. Net change = $\frac{-x^2}{100}\% = \frac{-10^2}{100} = -1\%$

16. Net revenue = $20\% - 15\% - \frac{20 \times 15\%}{100} = (20 - 15 - 3)\% = 2\%$

i.e. increase of **2%**.

17. Average speed = $\frac{2 \times 48 \times 24}{48 + 24} = \frac{2 \times 48 \times 24}{72} = 2 \times 16 = \mathbf{32 \text{ km/h}}$

18. $x + (56 - x) = 56$

$$\frac{(x + 4)}{(60 - x)} = 1/3 \text{ or } 60 - x = 3x + 12$$

$$\Rightarrow x = \mathbf{12 \text{ years.}}$$

19. Let the listed price of article be ₹ 100.

$$\therefore \text{Cost price} = \frac{3}{4} \times 100 = ₹ 75$$

$$\text{Selling price} = \frac{3}{2} \times 100 = ₹ 150$$

$$\therefore \text{Profit} = \frac{\text{SP} - \text{CP}}{\text{CP}} \times 100 = \frac{150 - 75}{75} \times 100 = \mathbf{100\%}$$

20. Let the cost of first table be ₹ x .

$$\therefore x \times \left(\frac{94}{100}\right) + (2700 - x) \frac{107.5}{100} = 2700$$

$$x \left(\frac{107.5 - 94}{100}\right) = 2700 \left(\frac{107.5 - 100}{100}\right)$$

$$\therefore x = \frac{2700 \times 7.5}{107.5 - 94} = \frac{2700 \times 7.5}{13.5} = ₹ \mathbf{1500}$$

$\therefore$ Prices of tables are **₹ 1500 and ₹ 1200**

21. $\frac{77 \times 16}{8 \times 11} = 14$

22. The series is $\times 1 + 7, \times 2 - 11, \times 3 + 15, \dots$

Here the wrong number in the series is 5. Therefore, the series starting with 5 is

$$5, (5 \times 1 + 7), (5 \times 1 + 7) \times 2 - 11$$

i.e., 5, 12, 3.

$\therefore$ The third term of series is 3.

Hence the answer here is (e).

23. The series is $\times 1 + 1^2, \times 2 + 2^2, \times 3 + 3^2, \dots$

Here the wrong number in the series is 190. Therefore, series starting with 190 is

$$190, (190 \times 1 + 1^2), (190 \times 1 + 1^2) \times 2 + 2^2$$

i.e., 190, 191, 386.

$\therefore$ The third term of series is 386.

Hence the answer is (d).

24. The series is $\times 2 + 3, \times 2 + 5, \times 2 + 7, \times 2 + 9, \dots$

Here the wrong number in the series is 43. Therefore, series starting with 43 is

$$43, (43 \times 2 + 3), (43 \times 2 + 3) \times 2 + 5$$

i.e., 43, 89, 183.

$\therefore$ The third term of series is 183.

Hence the answer is (b).

25. The series is $\times 2 + 6, \times 2 + 7, \times 2 + 6, \dots$

Here the required term in place of (b)

$$= (5 \times 2 + 6) \times 2 + 7 = 39$$

Hence the answer is (a).

26. The series is $+ 1^2, + 3^2, + 5^2, + 7^2, \dots$

Here the required term in place of (e)

$$= 189 + 1^2 + 3^2 + 5^2 + 7^2 + 9^2 = 354$$

Hence the answer is (a).

27. The series is $+ 12^2, - 10^2, + 8^2, - 6^2, \dots$

Here the required number in place of (d)

$$= 146 + 12^2 - 10^2 + 8^2 - 6^2 = 146 + 44 + 28 = 218.$$

Hence the answer is (d).

28. $x^2 + \frac{1}{x^2} = \left(x + \frac{1}{x}\right)^2 - 2 = 3^2 - 2 = 7$

29. $a + b + c = 0 \Rightarrow (a + b + c)^2 = 0 \Rightarrow a^2 + b^2 + c^2 + 2(ab + bc + ca) = 0$

$$\Rightarrow a^2 + b^2 + c^2 = -2(ab + bc + ca) = -2[ab + c(a + b)]$$

$$= -2[ab + c(-c)] = 2[c^2 - ab]$$

$$\therefore \frac{a^2 + b^2 + c^2}{c^2 - ab} = 2$$

$$30. \frac{2 + \sqrt{3}}{2 - \sqrt{3}} = \frac{(2 + \sqrt{3})^2}{(2 - \sqrt{3})(2 + \sqrt{3})} = \frac{4 + 3 + 4\sqrt{3}}{4 - 3} = \mathbf{7 + 4\sqrt{3}}$$

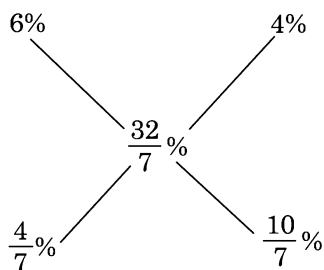
$$31. \text{Ratio of capitals} = 4 : 5; \text{ratio of time} = 8 : x$$

$$\Rightarrow 4 \times 8 : 5 \times x = 8 : 15$$

$$\Rightarrow \frac{32}{5x} = \frac{8}{15}$$

$$\Rightarrow x = \frac{32 \times 15}{5 \times 8} = 4 \times 3 = \mathbf{12 \text{ months}}$$

$$32. \text{Overall rate of interest} = \frac{1600 \times 100}{5 \times 7000} = \frac{32}{7} \%$$



$$\text{Ratio of two amounts} = \mathbf{2 : 5}$$

$$\therefore \text{Amount lent at } 6\% = 7000 \times \frac{2}{7} = \mathbf{₹ 2000}$$

$$33. \text{Sum} = \frac{\text{Difference in interests}}{\text{Rate} \times \text{Difference in period}} \times 100$$

$$= \frac{144 \times 100}{15 \times 1.5} = \mathbf{₹ 640}$$

$$34. \text{Rate} = \frac{\text{Difference in amounts of } n\text{th and } (n + 1)\text{ years}}{\text{Amount after } n \text{ years}} \times 100$$

$$= \frac{5324 - 4840}{4840} \times 100 = \mathbf{10\%}$$

$$35. 60 \times x = 50(x + 8)$$

$$\Rightarrow x = \frac{50 \times 8}{60 - 50} = \frac{400}{10} = \mathbf{40 \text{ men}}$$

$$36. \text{P's share} : \text{Q's share} : \text{R's share}$$

$$= 6 \times 7 : 5 \times 7 : 5 \times 6$$

$$= \mathbf{42 : 35 : 30}$$

$$\begin{aligned}\therefore \text{R's share} &= \frac{30}{42 + 35 + 30} \times 535 \\ &= \frac{30}{107} \times 535 = \text{₹ } 150\end{aligned}$$

37. The filler tap can fill the tank in $\frac{12 \times 8}{12 - 8} = 24$ hours

$$\therefore \text{Capacity of tank} = 6 \times 60 \times 24 = 8640 \text{ litres}$$

38. Ratio of speeds = 64 : 96 = 2 : 3

Time to cross telegraph posts = 5 seconds and 6 seconds

$$\text{Required time to cross each other} = \frac{ax + by}{x + y} = \frac{2 \times 5 + 3 \times 6}{2 + 3} = \frac{28}{5} \text{ s}$$

39. Length of train = Difference in speed $\times \frac{\text{time 1} \times \text{time 2}}{\text{time 1} - \text{time 2}}$

$$= \frac{(10 - 5) \times 6 \times 5}{6 - 5} = 150 \text{ m}$$

Speed of train be x m/s

$$(x + 5) \times 6 = (x + 10)5$$

$$\Rightarrow 6x + 30 = 5x + 50$$

$$\Rightarrow x = 50 - 30 = 20 \text{ m/s}$$

40. $u = 9$ km/h; $v = 3$ km/h

Let AB = x km

$$x = (u + v)t_1 = (u - v)t_2$$

$$t_1 + t_2 = 3 \text{ hours}$$

$$\therefore (9 + 3)t_1 = (9 - 3)(3 - t_1)$$

$$\Rightarrow 12t_1 = 18 - 6t_1$$

$$\Rightarrow t_1 = \frac{18}{12 + 6} = 1 \text{ hour}$$

$$\therefore x = (u + v)t_1 = (9 + 3) \times 1 = 12 \text{ km}$$

$$= \text{Total time} \times \frac{(\text{Speed in still water})^2 - (\text{Speed of current})^2}{2 \times \text{Speed in still water}}$$

$$= 3 \times \frac{9^2 - 3^2}{2 \times 9} = \frac{3 \times 72}{18} = 12 \text{ km}$$

41. Let height be h metres

$$\therefore b = 2h$$

$$\therefore 2(16 + 2h)h = 168 \times 2$$

$$\Rightarrow (16 + 2h)h = 168$$

$$\Rightarrow h^2 + 8h - 84 = 0$$

$$(h + 14)(h - 6) = 0$$

$$\therefore h = 6 \text{ m}$$

$$\therefore \text{Floor area} = 16 \times 2h = 16 \times 12 = \mathbf{192 \text{ m}^2}$$

$$\begin{aligned} 42. \text{ Required percentage value} &= 2x + \frac{x^2}{100} \\ &= 2 \times 5 + \frac{5^2}{100} = 10 + 0.25 = \mathbf{10.25} \end{aligned}$$

$$43. \text{ Average speed} = \frac{2 \times 60 \times 40}{60 + 40} = \mathbf{48 \text{ km/h}}$$

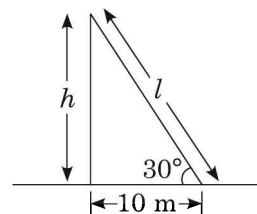
$$44. \tan 30^\circ = \frac{h}{10} = \frac{1}{\sqrt{3}}$$

$$\text{or } h = \frac{10}{\sqrt{3}}$$

$$l \sin 30^\circ = h = \frac{10}{\sqrt{3}} = \frac{l}{2}$$

$$\text{or } l = \frac{20}{\sqrt{3}}$$

$$\therefore \text{Total height of tree} = l + h = \frac{20}{\sqrt{3}} + \frac{10}{\sqrt{3}} = \frac{30}{\sqrt{3}} = 10\sqrt{3} \text{ m} = \mathbf{17.32 \text{ m}}$$



45. Even both statements taken together will not give the correct answer.

46. x can take values from 146 and y can take values from 141. y can also take values more than 146 such as 147, 148, ... So you can't clearly state that x is greater than y .

$\therefore$ The answer is (e).

47. 40% of 1000000 = **400000**

$$48. \frac{20\% \text{ of } 10 \text{ lakh}}{40\% \text{ of } 10 \text{ lakh}} = \frac{1}{2}$$

$\therefore$ Required ratio is **1 : 2**

49. Total strength of literates = 40 + 20 = **60%**

50. 6 September 1986 – Saturday.

6 September 1986 – 6 September 2011

$$= 25 \text{ years} = 6 \times 4 + 1 \text{ years}$$

$$= 6 \times 5 + 1 \text{ odd days} = 31 \text{ odd days}$$

$$31 \% 7 = 3 \text{ odd days.}$$

6 September 2011 – 15 August 2012

$$= 24 + 31 + 30 + 31 + 31 + 29 + 31 + 30 + 31 + 30 + 31 + 15$$

$$= 344 \text{ days}$$

$$344 \% 7 = 1 \text{ odd days}$$

$\therefore$ Total odd days = 3 + 1 = **4 odd day**

$\therefore$ 15th August 2012 is Saturday + 4 = **Wednesday**

Model Test Paper 2

Each question (1–3) given below consists of a statement followed by two arguments numbered I and II. You have to decide which of the arguments is a ‘strong’ argument and which is a ‘weak’ argument.

Give answer (a) if only argument I is strong; (b) if only argument II is strong; (c) if either I or II is strong; (d) if neither I nor II is strong and (e) if both I and II are strong.

1. Statement : Should state lotteries be stopped?

Arguments : I. Yes. Government should not promote gambling habits.
II. No. Government will lose a large amount of revenue.

2. Statement : Should the illiterate be debarred from voting?

Arguments : I. Yes. They are easily misguided.
II. No. It is their constitutional right.

3. Statement : Is monarchy better than democracy?

Arguments : I. Yes. If the chair has one confirmed ruler, there are no ambitious aspirants fighting for it.

II. No. People are more contented and happy in democracy.

In each question (4 – 6) below is given a statement followed by two assumptions numbered I and II. Consider the statement and decide which of the given assumptions is implicit.

Give answer (a) if only assumption I is implicit; (b) if only assumption II is implicit; (c) if either I or II is implicit; (d) if neither I nor II is implicit and (e) if both I and II are implicit.

4. Statement : Please consult me before making any decision on exports from the company.

Assumptions : I. You may take a wrong decision if you don’t consult me.
II. It is important to take a right decision.

5. Statement : “If you are beautiful, we will catch your beauty. If you are not, we will make you beautiful”—An advertisement of a photo studio.

Assumptions : I. How to look beautiful, is a problem of youngster.
II. Nobody desires to be beautiful.

6. Statement : A warning in a train compartment—"To stop train, pull chain. Penalty for use without sufficient cause attracts 6 months imprisonment and/or fine up to ₹ 1000".

Assumptions : I. Some people misuse the alarm chain.
II. On certain occasions, people may want to stop a running train.

In each question (7–9) below is given a statement followed by two courses of action numbered I and II. A course of action is a step or administrative decision to be taken for improvement, follow-up or further action in regard to the problem policy, etc. on the basis of the information given in the statement. You have to assume everything in the statement to be true, then decide which of the two given suggested courses of actions logically follows for pursuing.

Give answer (a) if only I follows; (b) if only II follows; (c) if either I or II follows; (d) if neither I nor II follows and (e) if both I and II follow:

7. Statement : A shopkeeper was reported to be selling adulterated grains.
Courses of Action : I. He should be fined and his shop sealed.
II. He should be asked to leave the town and open a shop elsewhere.
8. Statement : A train has derailed near a station while moving over a bridge and fell into the river.
Courses of Action : I. The railway authorities should clarify the reason of the accident to the government.
II. The government should allocate funds to compensate the destruction caused.
9. Statement : The Courts take too long in deciding important disputes of various departments.
Courses of Action : I. The Courts should be ordered to speed up matters.
II. Special powers should be granted to officers to settle disputes concerning their department.

In each of the following questions (10–12) a statement is given followed by two conclusions I and II. Give answer (a) if only conclusion I follows; (b) if only conclusion II follows; (c) if either I or II follows; (d) if neither I nor II follows and (e) if both I and II follow.

10. Statement : Domestic demand has been increasing faster than the production of indigenous crude oil.
Conclusions : I. Crude oil must be imported.
II. Domestic demand should be reduced.
11. Statement : Patients with minor ailments usually do not go to eminent doctors.
Conclusions : I. Eminent doctors remain too busy with patients suffering from serious complications.
II. Their charges are rather high.
12. Statement : Parents are prepared to pay any price for an elite education to their children.

Conclusions : I. All parents these days are very well off.

II. Parents have an obsessive passion for a perfect development of their children through good schooling.

In each of the following questions (13–19) there is a certain relation between two given words on one side of :: and one word is given on the another side of :: while another word is to be found from the given alternatives, having the same relation with this word as the words of the given pair bear. Choose the correct alternative.

13. Melt : Liquid :: Freeze : ?

- (a) Ice (b) Condense (c) Solid (d) Crystal

14. Acting : Theatre :: Gambling : ?

- (a) Casino (b) Club (c) Bar (d) Gym

15. Dungeon : Confinement :: Asylum : ?

- (a) Refuge (b) Mercy (c) Truancy (d) Remorse

16. Ruby : Red :: Sapphire : ?

- (a) Blue (b) White (c) Green (d) Silver

17. Scrap : Food :: Lees : ?

- (a) Bread (b) Tea (c) Wine (d) Rice

18. Wimbledon Trophy : Tennis :: Walker's Cup : ?

- (a) Hockey (b) Polo (c) Golf (d) Wrestling

19. Meat : Vegetarian :: Liquor : ?

- (a) Insane (b) Introvert (c) Teetotaller (d) Foolish

In each of the following questions (20–22) a group of three inter-related words is given. Choose a word from the given alternatives, that belongs to the same group.

20. Peas : Gram : Pulses

- (a) Rice (b) Barley (c) Beans (d) Coconut

21. Potato : Carrot : Radish

- (a) Tomato (b) Spinach (c) Sesame (d) Groundnut

22. Marble : Slate : Gneiss

- (a) Quartzite (b) Limestone (c) Coal (d) Sandstone

Directions (23–27): Choose the word which is least like the other words in the group.

23. (a) Whale (b) Dolphin (c) Shark (d) Cod (e) Starfish

24. (a) Indigo (b) Orange (c) Yellow (d) Pink (e) Green

25. (a) Pituitary (b) Pancreas (c) Thalamus (d) Adrenal (e) Testis

26. (a) Nerves (b) Auricle (c) Artery (d) Valve (e) Aorta

27. (a) Metre (b) Furlong (c) Yard (d) Mile (e) Acre

28. A number series is given with one term missing. Choose the correct alternative that will continue the same pattern and fill in the blank spaces.

3, 7, 25, _____ 721.

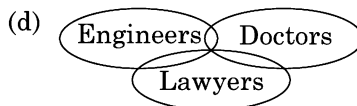
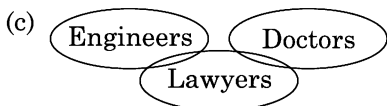
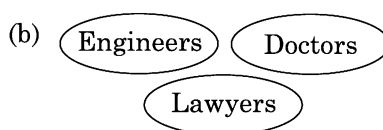
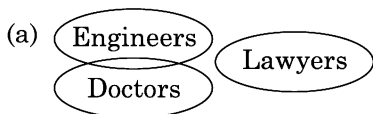
- (a) 144 (b) 166 (c) 121 (d) 112 (e) None of these

29. Find the wrong term in the series

2, 6, 12, 24, 30, 42

- (a) 6 (b) 12 (c) 24 (d) 30 (e) 42

30. If BATCH is coded as ABCTH, how is SEVEN coded in that code?
 (a) FTUGP (b) RFUFN (c) ESEVN (d) TFGUP (e) None of these
31. If in a certain language 'TABLE' is coded as 'FMCBU' then which word is coded as 'TOWER' in the same language?
 (a) SFXPU (b) QDVNS (c) UPXFS (d) SNVDQ (e) None of these
32. If 'book' is called 'pencil', 'pencil' is called 'pen', 'pen' is called 'rubber' and 'rubber' is called 'sky'. Then what is used to draw figure?
 (a) pencil (b) pen (c) book (d) rubber (e) None of these
33. In a certain language, '45' means 'Good morning', '357' means 'Buy good items'. '268' means 'Man is handsome'. What stands for 'morning' in that code?
 (a) 3 (b) 4 (c) 5 (d) 6 (e) 8
34. Pointing to a photograph, a woman said—"He is the only son of my mother's father." Then the relation of the person in the photograph to the woman is
 (a) Aunt (b) Uncle (c) Father (d) Son (e) None of these
35. A person started walking from a point P for a distance of 3 m towards east. Then he turned left and walked 10 m. Then he turned right and walked 2 m. Then he turned left and walked 2 m and reach the point Q. What is the present position of the person with reference to its starting point?
 (a) 17 m NE (b) 7 m NE (c) 13 m NE (d) 13 m SW (e) None of these
36. A group of persons consisting of engineers, doctors and lawyers are represented suitably using one of the following Venn diagram. The best representation of it is



(e) None of these

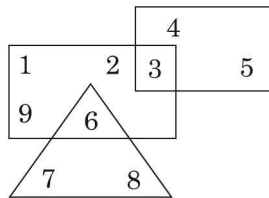
37. Dinesh is ranked 4th from top among boys and 19th from top among all students. If the total number of girls in the class is 22, then find the number of girls behind Dinesh in ranking.
 (a) 3 (b) 7 (c) 18 (d) 15 (e) None of these
- (38–42) Six persons A, B, C, D, E and F are seated around a round table.
 (i) C is seated to the left of E and opposite to B.
 (ii) A is seated opposite to D and left of B.
 (iii) F is between B and D.
38. Who is seated in between A and F?
 (a) C (b) D (c) E (d) B
39. Who is seated to the right of C?
 (a) A (b) B (c) E (d) F

40. Who is seated opposite to E?
 (a) A (b) B (c) D (d) F
41. Who is seated to the right of E?
 (a) A (b) B (c) C (d) D
42. Who is seated between D and E?
 (a) A (b) B (c) C (d) F
43. How many numbers are there between 1 and 100 which is having at least one digit as 4 and the number divisible by 4?
 (a) 3 (b) 5 (c) 7 (d) 9 (e) None of these
44. How many days will there be from 15 August 1988 to 5 June 1989 both days included?
 (a) 272 (b) 293 (c) 294 (d) 295 (e) None of these
45. How many 2 are there in the following number sequence which are immediately preceded by 5 but not followed by 9?

1 3 5 2 8 6 4 2 7 3 9 5 2 3 8 5 2 8

- (a) 1 (b) 2 (c) 3 (d) 4 (e) None of these

(46–48) In the following figure, rectangle, square and triangle represent male, civil servants and professionals in a society. On the basis of this figure answer the following questions.



46. Who among the following is a male and a professional?
 (a) 7 (b) 8 (c) 6 (d) 5 (e) 4
47. Who among the following is a female civil servant?
 (a) 3 (b) 4 (c) 2 (d) 1 (e) 9
48. Who among the following is neither a civil servant nor a professional?
 (a) 6 (b) 5 (c) 4 (d) 2 (e) 7

Directions (49–53): Study the following information carefully and answer the questions given below it. The following are the criteria to get employment in a pharmaceutical company. The applicant must

- have passed graduation with science subject with at least 50% marks.
- have completed 21 years (in case of males) and 19 years (in case of females) of age as on 1 August 2010.
- Pay deposit as follows:

Marks at graduation	If son/daughter of trustee or staff	If not son/daughter of trustee or staff
75% and above	₹ 5000	₹ 10000
61% to 74%	₹ 10000	₹ 20000
60% and below	₹ 20000	₹ 40000

However,

- (iv) If the candidate has secured more than 90% marks at graduation, he should be referred to the Managing Director for consideration for further concession in deposit.
- (v) In the case of SC/ST candidates, deposit payable is 20% less in each of the above cases.
- (vi) If the candidate remits in cash the amount of deposit immediately on the date of first call, the candidate can be selected, provided he fulfils conditions (i) and (ii).
- (vii) If, on the date of first call, the amount of deposit brought by a candidate eligible as per (i) and (ii) is less than the total amount required but more than $\frac{3}{4}$ th the candidate can be provisionally selected, provided he deposits the balance amount within next ten days.
- (viii) If on the date of first call, the amount of deposit brought is less than $\frac{3}{4}$ th but more than $\frac{1}{2}$, the candidate eligible as per (i) and (ii) can be sent to the Managing Director.
- (ix) If on the date of first call, the amount of deposit brought is less than $\frac{1}{2}$, the candidate though eligible as per (i) and (ii), cannot be selected.

On the basis of the above criteria, decide which of the following courses is described in each question.

Mark answer (a) if the candidate can be selected; (b) if the candidate can be provisionally selected; (c) if the candidate should be referred to the Managing Director; (d) if the candidate cannot be selected; (e) if the data provided is inadequate.

- 49. Satheesh, son of a trustee of the organisation, obtained 68% marks in B.Sc. He brings a deposit of ₹ 8200 on the day of the first call, but intends to pay the balance amount within ten days. He is a general candidate and his date of birth is 2nd November 1988.
- 50. Sujatha is an SC candidate. Her date of birth is 26 January 1990. She passed her graduation in Science with 67% marks. She can deposit ₹ 18000.
- 51. Mahesh, a 22 year old general category candidate, secured 92% marks in graduation with Science. He can pay only ₹ 3000 as deposit. He is not a son of a staff or trustee of the organisation.
- 52. Ramesh, born on 21 August, 1985 is a son of an employee working in the organisation secured 66% marks in M.Sc. and is in a position to deposit only ₹ 2000. He is an SC category candidate.
- 53. Suresh Varma, a general category candidate, is a graduate in Science with 57% marks. His date of birth is 13 December 1987.

Directions (54–58): In the following questions, two statements are given namely Assertion (A) and Reason (R). Choose the correct alternative from the following:

- (a) Both A and R are true and R is the correct explanation of A.
 - (b) Both A and R are true but R is not the correct explanation of A.
 - (c) A is true but R is false.
 - (d) A is false but R is true.
 - (e) Both are false.
54. Assertion (A) : An atom is neutral despite the charged particles in it.
Reason (R) : Neutrons do not have any charge.

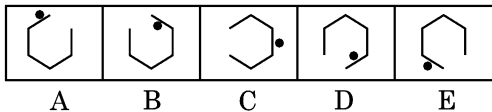
55. Assertion (A) : Glass tumbler breaks in winter when hot water is poured in it.
Reason (R) : When hot water is poured, the outer surface of glass expands.
56. Assertion (A) : Silver is not used to make electric wires.
Reason (R) : Silver is a bad conductor.
57. Assertion (A) : Carbon forms the largest number of compounds.
Reason (R) : Carbon has the catenation property.
58. Assertion (A) : Cut fruits and vegetables should not be kept open for long time.
Reason (R) : Their vitamin content is ruined.

Directions (59–60): To each of the following questions four probable answers have been given. Select the most appropriate alternative as the answer.

59. While firing crackers, a child gets severe burns on the hand. What would you do?
(a) Dip the child's hand in cold water till there is no more burning sensation.
(b) Wash the hands with dettol.
(c) Send some one to call the doctor.
(d) Apply some ointment on the affected area.
60. You are moving across the road on a scooter when you observe that two boys on a bike snatch a lady's gold chain and ride away. You would
(a) Console the woman.
(b) Chose the boys to catch hold of them.
(c) Inform the police about the matter.
(d) Stand and see what happens next.
61. If '+' means '×', '-' means '÷' and '×' means '-', then the value of $6 + 3 \times 5 - 1$ is
(a) 44 (b) 13 (c) 20 (d) 24 (e) None of these
62. If '×' means '-', '-' means '+', '+' means '÷' and '÷' means '×', then the value of $\frac{12 \times 2 - 7 \div 5}{8 - 2 \times 5}$ is
(a) 1 (b) 3 (c) 5 (d) 7 (e) 9

Directions (63–66): Each of the following questions consists of five figures marked A, B, C, D and E called problem figures followed by five other figures marked 1, 2, 3, 4 and 5 called answer figures. Select a figure from amongst the answer figures which will continue the same series as established by the five problem figures.

63.



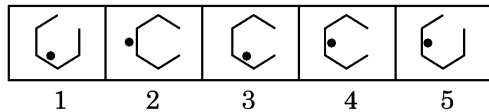
A

B

C

D

E



1

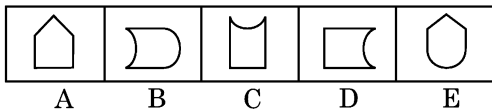
2

3

4

5

64.



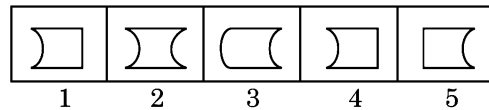
A

B

C

D

E



1

2

3

4

5

65.

x	o	c	=	x	=	c	s	=	o	s
=	c	s	s	=	o	o	s	x	c	=

 A B C D E
66.

A	B	C	D	E

1	2	3	4	5

Directions (67–70): Each of the following questions consists of two sets of figures. Figures A, B, C and D constitute problem figures while figures 1, 2, 3, 4 and 5 constitute answer figures. There is a definite relationship between figures A and B. Establish a similar relationship between figures C and D by choosing a suitable figure (D) from answer figures.

67.

A	B	C	D

1	2	3	4	5
68.

A	B	C	D

1	2	3	4	5
69.

A	B	C	D

1	2	3	4	5
70.

A	B	C	D

1	2	3	4	5

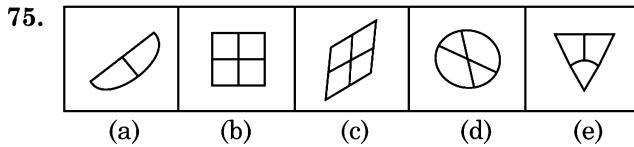
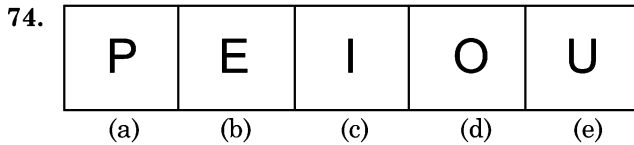
Directions (71–80): Out of the five figures (a), (b), (c), (d) and (e), given in each problem, four are similar in a certain way. However, one figure is not like the other four. Choose the figure which is different from the rest.

71.

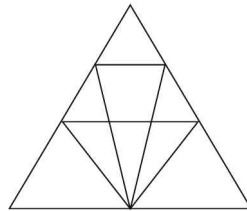
(a)	(b)	(c)	(d)	(e)
72.

A	F	Z	E	N
(a)	(b)	(c)	(d)	(e)
73.

D	H	L	O	T
(a)	(b)	(c)	(d)	(e)

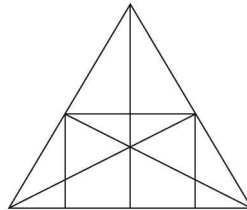


76. Count the number of triangles in the following figure.



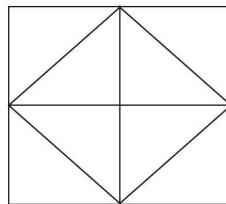
- (a) 12 (b) 18 (c) 22 (d) 26

77. How many straight lines are there in the given figure?



- (a) 10 (b) 9 (c) 8 (d) 6

78. Count the number of pentagons in the following figure.



- (a) 16 (b) 14 (c) 12 (d) 10

79. A cube is painted blue on all faces is cut into 125 cubes of equal size. How many cubes are there which are painted on one face only?

- (a) 8 (b) 16 (c) 36 (d) 54 (e) None of these

80. A cube is painted on all faces is cut into 64 cubes of equal size. How many cubes are there which are painted on at least one face?

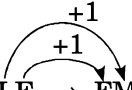
- (a) 40 (b) 48 (c) 56 (d) 64 (e) None of these

ANSWERS

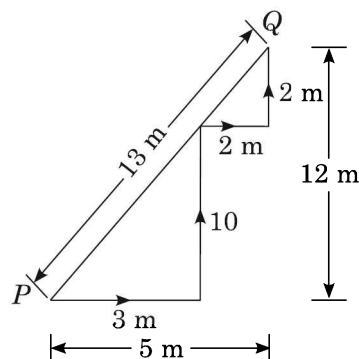
1. (d)	2. (b)	3. (d)	4. (e)	5. (d)	6. (e)	7. (a)	8. (d)	9. (e)	10. (c)
11. (c)	12. (b)	13. (c)	14. (a)	15. (a)	16. (a)	17. (c)	18. (c)	19. (c)	20. (c)
21. (d)	22. (a)	23. (a)	24. (d)	25. (c)	26. (a)	27. (e)	28. (c)	29. (c)	30. (c)
31. (b)	32. (b)	33. (b)	34. (b)	35. (c)	36. (b)	37. (b)	38. (d)	39. (c)	40. (d)
41. (a)	42. (c)	43. (c)	44. (d)	45. (c)	46. (c)	47. (b)	48. (d)	49. (b)	50. (a)
51. (c)	52. (d)	53. (e)	54. (b)	55. (c)	56. (c)	57. (d)	58. (a)	59. (a)	60. (b)
61. (b)	62. (e)	63. (4)	64. (4)	65. (4)	66. (5)	67. (4)	68. (4)	69. (4)	70. (1)
71. (e)	72. (d)	73. (d)	74. (a)	75. (d)	76. (b)	77. (b)	78. (c)	79. (d)	80. (c)

Solutions and Hints

- Clearly, none of the reasons is strong enough in favour of or against the statement. So none of these holds.
- Argument I is not strong enough because no one can be debarred from their constitutional rights even if they cannot practise it to their benefit. But argument II is strong.
- The success of a government depends on its outlook and policies. So argument I is not strong enough. Argument II is also vague because a democracy is coveted for the reason that in it, the voice of the people is above all.
- The statement was spoken for fear that the other person may not take a wrong decision. So, assumption I is implicit. The statement confirms that it was important to take the right decision. So assumption II is also implicit.
- Nothing is mentioned in the advertisement about the problem of youngsters. So assumption I is not implicit. The advertisement is meant for persons who desire to look beautiful. So assumption II is also not implicit.
- Penalty is imposed to prevent people from misusing the alarm chain. So assumption I is implicit. The alarm chain is provided to stop the running train in times of urgency. So assumption II is also implicit.
- If he is allowed to continue without being punished, he would create the problem elsewhere. So only I follows.
- Preventive measures to protect the passengers and pay them adequate compensation are necessary in this case. So none of the courses follows.
- For quick disposal of cases, either the methods in the court should be speeded up or the matters should be cleared up in their respective departments to prevent the delay. So, both the courses follow.
- As given in the statement, domestic demand is increasing and for having a balance between production of crude oil and consumption of oil and oil by-products either of the two conclusions should follow.
- As given in the statement, patients with minor ailments do not go to eminent doctors. This may be either of the two reasons stated in conclusions I and II. Therefore, either I or II follows.

12. Most parents wish to give the best education to their children as they can. This can be done only through good schooling. Therefore, only conclusion II follows. But conclusion I does not follow.
13. The first is the process of formation of the second.
14. The second is the place for performing the first.
15. A prisoner is confined within the dungeon, and an unsheltered person takes refuge within the asylum.
16. Ruby is a red precious stone and sapphire is a blue precious stone.
17. The first is the left over the second.
18. The Wimbledon Trophy is associated with Tennis while the Walker's Cup is associated with Golf.
19. A vegetarian never eats meat. Similarly, a teetotaler never drinks liquor.
23. All except whale belong to the family of fishes.
24. All except pink are colours seen in a rainbow.
25. All except Thalamus are hormone secreting glands.
26. All except nerves are parts of the heart.
27. All except acre are units for measuring distance, while acre is a unit of area.
28. The series is obtained by multiplying with 3 and subtracting 2, multiplying with 4 and subtracting 3 and so on. So $25 \times 5 - 4 = 121$ is the missing term.
29. The series obtained is $1^2 + 1, 2^2 + 2, 3^2 + 3, \dots$
 Next term = $4^2 + 4 = 20$
 Here it is 24
 $\therefore 24$ is wrong.
30. BATCH $\rightarrow$ ABCETH
 Every pair of letters is reversed in the code. Therefore 'SEVEN' is coded as 'ESEVN'
31. 
 Word is reversed and 1 is added to every character.
 Similarly ? $\rightarrow$ 'TOWER'
 The required word is obtained by reversing the code 'TOWER' and subtracting 1 from each character.
 $\therefore$ The answer is 'QDVNS'
32. From the given items, pencil is the one which can be used for drawing a figure. But, here 'pencil' is called 'pen'. Therefore 'pen' is the answer.
33. Here '45' means 'good morning', '357' means 'Buy good items'. Above two codes consists of '5' as a code for the common word 'good'.
 $\therefore$ In '45', '5' stands for 'good' and '4' stands for 'morning'
 $\therefore$ The answer is (b).
34. Only son of my mother's father means brother of my mother.
 $\therefore$ The person is brother of woman's mother, i.e. **uncle**.

35. $PQ = \sqrt{(3+2)^2 + (10+2)^2}$
 $= \sqrt{5^2 + 12^2} = \sqrt{169} = 13 \text{ m}$
 $\therefore Q$ is **13 m NE** of P .



36. The 3 groups of persons are of 3 different profession and is not having any common property. Therefore, it can be represented by the choice (b).

37. Number of girls in front of Dinesh = $19 - 4 = 15$

Total number of girls = 22

$\therefore$ Number of girls behind Dinesh = $22 - 15 = 7$

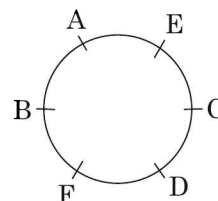
38. B is seated between A and F.

39. E is seated to the right of C.

40. F is seated opposite to E.

41. A is seated to the right of E.

42. C is seated between D and E.



43. There are seven such numbers which are having at least one digit as 4 and number fully divisible by 4. They are 4, 24, 40, 44, 48, 64 and 84.

44. Number of days = $17 + 30 + 31 + 30 + 31 + 31 + 28 + 31 + 30 + 31 + 5 = 295$ **days**

45. 1 3 5 2 8 6 4 2 7 3 9 5 2 3 8 5 2 8

Number '2', which is underlined is given as per the condition given in the question. The total number of such 2s is 3.

46. Only 6 is a male and a professional.

47. Among the civil servants only one person is male. 4 and 5 are female civil servants
 $\therefore$ The answer is (b).

48. Among the choices given, number '2' is the only person who is neither a civil servant nor a professional.

49. As he intends to pay the balance within ten days condition (vii) is fulfilled.

50. The candidate is supposed to deposit ₹ 16000 only as she being an SC candidate gets 20% deduction. So all conditions of eligibility are fulfilled.

51. The candidate has secured more than 90% marks. So, condition (iv) is fulfilled.

52. The candidate does not satisfy condition (iii), as he is supposed to pay ₹ 8000.

53. It is not mentioned whether the candidate is a son of a trustee or not.

54. An atom contains positively charged protons and an equal number of negatively charged electrons. So it is neutral. However, it is true that neutrons in the nucleus of an atom are neutral.

55. When hot water is poured in the glass tumbler in winter, its inner surface tends to expand while the outer surface in contact with cold atmosphere does not. This opposite interaction causes the tumbler to break.
56. Silver is a good conductor of electricity but it is not used to make electric wires because it is expensive.
57. Carbon forms a large number of compounds due to its tendency to form chains and rings of varying sizes, called its catenation property. However, the largest number of compounds are formed by Hydrogen.
58. When cut fruits and vegetables are kept in open, the vitamins in them get oxidised and remain of no use.

61. $6 + 3 \times 5 - 1 = ?$

Putting the actual sign in the above expression, we get,

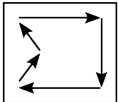
$$6 \times 3 - 5 \div 1 = 18 - 5 = 13$$

62. $\frac{12 \times 2 - 7 \div 5}{8 - 2 \times 5} = ?$

Putting the actual sign in the above expression, we get

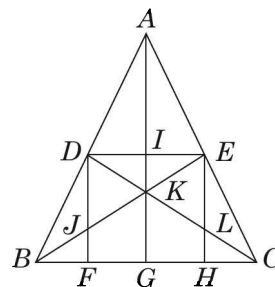
$$\frac{12 - 2 + 7 \times 5}{8 + 2 - 5} = \frac{12 - 2 + 35}{5} = \frac{45}{5} = 9$$

63. The main figure rotates one step ACW in each turn and dot moves one step CW in each turn and also gets inside and outside the main figure alternately.
64. The figure is rotated 90° CW in each step. Then, two elements, one element, no element, again two elements and one element change their shape.

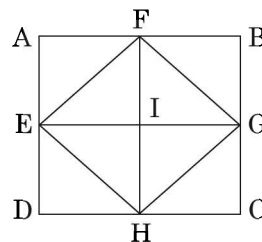
65. The symbols move in the order  in each step.

66. A similar figure appears every third step and the symbols inside the figure are replaced by new ones in each step.
67. The pin and the arrow moves to the adjacent side in ACW direction. Out of these two, the one which was inside, comes out and the other which was outside gets in.
68. The half arrow rotates 90° CW, moves to the adjacent side CW and gets inverted. The bent pin rotates 90° ACW and moves to the adjacent side CW.
69. The main figure is replaced by a figure with the number of sides less by one. The black circles inside the figure comes out on either sides of the figure and a white circle is introduced inside it.
70. The two figures approach each other and gets overlapped.
71. All except figure (e) are punctuation marks.
72. It is the only letter having four lines. The rest of all have three lines.
73. All other letters are fourth starting from the previous one. So figure (d) should be 'P'.

74. All except figure (a) are vowels.
75. In all other cases, lines drawn inside the figure divide it into equal parts.
76. Simplest triangles = 8
 Triangles with two components = 5
 Triangles with three components = 3
 Triangles with four components = 1
 Triangles with nine components = 1
 $\therefore$ Number of triangles = $8 + 5 + 3 + 1 + 1 = 18$
77. Horizontal lines are DE and BC i.e. 2 Nos.
 Vertical lines are DF, EH and AG. i.e. 3 Nos.
 Starting lines are DC, BE, AB and AC. i.e. 4 Nos.
 $\therefore$ Total number of straight lines = $2 + 3 + 4 = 9$ Nos.



78. From the figure,
 DEFGC, AFGHD, BGHEA, CHEFB are 4 pentagons.
 AFGHE, BGHEF, CHEFG and DEFGH are 4 pentagons.
 DAFGC, DEFBC, AFGCD and ABGHD are 4 pentagons.
 In all, total number of pentagons = $4 + 4 + 4 = 12$.



79. $9 + 12 \times 3 + 9 = 54$, such cubes are painted on one face only.
80. Number of cubes with at least one face painted
 = Total number of cubes – Number of cubes which are not painted
 = $64 - 8 = 56$

Model Test Paper 3

- On a railway route between two places A and B, there are 10 stations on the way. If 4 new stations are to be added, how many types of new tickets will be required if each ticket is issued for a one way journey?
(a) 14 (b) 48 (c) 96 (d) 108
- Aryan runs at a speed of 40 m/min. Rahul follows him after an interval of 5 minutes and runs at a speed of 50 m/min. Rahul's dog runs at a speed of 60 m/min and starts along with Rahul. The dog reaches Aryan and then comes back to Rahul, and continues to do so till Rahul reaches Aryan. What is the total distance covered by the dog?
(a) 600 m (b) 750 m (c) 980 m (d) 1200 m
- A big rectangular plot of area 4320 m^2 is divided into 3 square shaped smaller plots by fencing parallel to the smaller side of the plot. However, some area of land was still left as a square could not be formed. So, 3 more square shaped plots were formed by fencing parallel to the larger side of the original plot such that no area of the plot was left surplus. What are the dimensions of original plot?
(a) $160 \text{ m} \times 27 \text{ m}$ (b) $240 \text{ m} \times 18 \text{ m}$ (c) $120 \text{ m} \times 36 \text{ m}$ (d) $135 \text{ m} \times 32 \text{ m}$
- Two men and one woman board a bus in which 5 seats are vacant. One of these five seats is reserved for ladies. A woman may or may not sit on the seat reserved for ladies but a man cannot sit on the seat reserved for ladies. In how many different ways can the five seats be occupied by these three passengers?
(a) 15 (b) 36 (c) 48 (d) 60
- A square is divided into 9 identical smaller squares. Six identical balls are to be placed in these smaller squares such that each of the three rows gets at least one ball (one ball in one square only). In how many different ways can this be done?
(a) 27 (b) 36 (c) 54 (d) 81
- There are six persons A, B, C, D, E and F. They are to be seated in a row such that B never sits anywhere ahead of A, and C never sits anywhere ahead of B. In how many different ways can this be done?
(a) 60 (b) 72 (c) 120 (d) None of these

7. An equilateral triangular plate is to be cut into n number of identical small equilateral triangular plates. Which one of the following can be possible value of n ?
 (a) 196 (b) 216 (c) 256 (d) 296
8. There are 10 identical coins and each one of them has 'H' engraved on its one face and 'T' engraved on its other face. These 10 coins are lying on a table and each one of them has 'H' face as the upper face.
 In one attempt, exactly four (neither more nor less) coins can be turned upside down. What is the minimum total number of attempts in which the 'T' faces of all the 10 coins can be brought to be the upper faces?
 (a) 4 (b) 7 (c) 8 (d) Not possible
9. 300 persons are participating in a meeting, out of which 120 are foreigners, and the rest are Indians. Out of the Indians, there are 110 men who are not judges, 160 are men or judges, and 35 are women judges. There are no foreign judges. How many Indian women attended the meeting?
 (a) 35 (b) 45 (c) 55 (d) 60
10. There are six persons : A, B, C, D, E and F.
 A has 3 items more than C.
 D has 4 items less than B.
 E has 6 items less than F.
 C has 2 items more than F.
 F has 3 items more than D.
 Which one of the following figures cannot be equal to the total number of items possessed by all the 6 persons?
 (a) 41 (b) 47 (c) 53 (d) 58

ANSWERS

1. (d) 2. (d) 3. (c) 4. (b) 5. (b) 6. (c) 7. (c) 8. (d) 9. (c) 10. (d)

Solutions and Hints

1. Including A and B, there are twelve stations in all. From each station tickets to 11 other stations are required.
 $\therefore$ Total number of tickets required for a one way journey = ${}^{12}P_2 = \frac{12!}{(12-2)!} = \frac{12!}{10!}$
 $= 12 \times 11 = 132$
 When four more stations are added, the total number of tickets required for a one-way journey from 16 stations in all is given by
 Number of tickets required = ${}^{16}P_2 = \frac{16!}{(16-2)!} = \frac{16!}{14!} = 16 \times 15 = 240$
 $\therefore$ Number of new types of tickets = $240 - 132 = 108$.
2. Speed of Aryan = 40 m/min; Speed of Rahul = 30 m/min
 Speed of Rahul's dog = 60 m/min

Distance travelled by Aryan in 5 minutes = $40 \times 5 = 200$ m.

Relative speed of Aryan with respect to Rahul = $40 - 50 = 10$ m/min

or relative speed of Rahul with respect to Aryan = 10 m/min

$$\therefore \text{Time taken to reach Rahul with Aryan} = \frac{200}{10} = 20 \text{ min}$$

$$\therefore \text{Distance covered by dog in 20 minutes} = 60 \times 20 = \mathbf{1200 \text{ m}}$$

■► **Short cut:** Distance covered by the dog = Speed of the dog $\times$ time taken

$$= \text{Speed of the dog} \times \frac{\text{Relative distance of Aryan w.r.t. Rahul}}{\text{Relative speed of Rahul w.r.t. Aryan}}$$

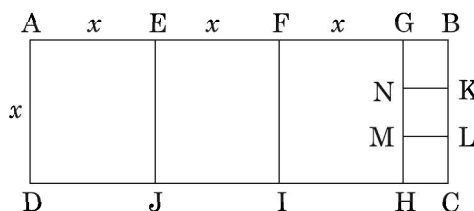
$$= 60 \times \frac{200}{10} = \mathbf{1200 \text{ m}}$$

3. Here ABCD is divided into 3 squares by fencing EJ, GH and FI. The remaining rectangle GBCH is divided into 3 squares by fencing KN and LM.

Let AD be x .

$$\therefore AE = EF = FG = DJ = JI = IH = x$$

$$\text{Similarly } AD = EJ = FI = GH = BC = x$$



$$\therefore BK = KL = LC, BK = \frac{AD}{3} = \frac{x}{3}$$

$$\therefore GB = NK = ML = HC = \frac{x}{3} \quad (\because \text{GBKN is a square})$$

$$\therefore \text{Total area of rectangle ABCD} = 3x^2 + 3 \times \left(\frac{x}{3}\right)^2 = 4320 \text{ m}^2$$

$$\text{i.e.} \quad 3x^2 + 3 \times \frac{x^2}{9} = 4320 \text{ m}^2$$

$$\text{i.e.} \quad 3x^2 + \frac{x^2}{3} = 4320 \text{ m}^2$$

$$\text{i.e.} \quad \frac{10x^2}{3} = 4320 \text{ m}^2$$

$$\text{or} \quad x^2 = 3 \times 432 = 1296 \text{ m}^2$$

$$\therefore x = \mathbf{36 \text{ m}}$$

$$\therefore \text{Size of plot} = \left(3x + \frac{x}{3}\right) \times x$$

$$= \mathbf{120 \text{ m} \times 36 \text{ m}}$$

4. If the lady takes the seat reserved for ladies, it can be filled in ${}^1C_1$ ways = 1 way.

Remaining seats can be filled in by the remaining two men using 4 seats available in

$${}^4C_2 \text{ ways} = \frac{4!}{(4-2)!} = 4 \times 3 = 12 \text{ ways}$$

$\therefore$ Total = $1 \times 12 = 12$ ways.

If the lady doesn't take the seat reserved for ladies, that seat can be filled in ${}^1C_0 = 1$ way.

The remaining seats can be filled by the three passengers in ${}^4P_3 = \frac{4!}{(4-3)!}$
 $4 \times 3 \times 2 = 24$ ways.

$\therefore$ Total = $1 \times 24 = 24$ ways

$\therefore$ Total number of ways = $12 + 24 = 36$ ways.

5. The square is divided into 9 smaller squares as shown in the figure.

The squares can be filled in ${}^6C_1, {}^3C_1, {}^2C_1, {}^1C_1, {}^5C_1, {}^0C_1, {}^0C_0, {}^0C_0$ and ${}^4C_1$ ways respectively as shown in the figure subject to the given condition.

$\therefore$ Maximum number of ways in which a row can be filled is given by ${}^6C_1 \times {}^3C_1 \times {}^2C_1 = 6 \times 3 \times 2 = 36$ ways.

So the answer is **(b)**.

${}^6C_1$	${}^3C_1$	${}^2C_1$
${}^1C_1$	${}^5C_1$	${}^0C_0$
${}^0C_0$	${}^0C_0$	${}^4C_1$

6. Taking the given condition, take A, B and C are together in the order, then number

of ways of arrangement = $\frac{6!}{3!} = 120$ ways

7. An equilateral triangle can be cut into smaller equilateral triangles in 4 numbers.

So here the area of triangle = $\frac{\sqrt{3}}{4} a^2 = \frac{\sqrt{3}}{4} a_1^2 \times n$ where

a = side of larger equilateral triangle.

a_1 = side of smaller equilateral triangle.

$\therefore n$ will be a power of 4.

$\therefore$ The answer is **(c)**.

8. **Not possible**. Because you can't turn 'T' faces of all the coins in an attempt.

9. Given total participants = 300

Number of foreigners = 120

Number of Indian participants = $300 - 120 = 180$

Number of men or judges = 160

$\therefore$ Women who are not judges = $180 - 160 = 20$

Number of women judges = 35

$\therefore$ Total number of Indian women attended the meeting = $20 + 35 = 55$.

10. Put $d = 0$; $b = d + 4 = 0 + 4 = 4$; $f = d + 3 = 0 + 3 = 3$; $c = f + 2 = 3 + 2 = 5$; $e = f - 6 = 3 - 6 = -3$; $a = c + 3 = 5 + 3 = 8$.

$\therefore a = 8$; $b = 4$; $c = 5$; $d = 0$; $e = -3$; $f = 3$

$\therefore a + b + c + d + e + f = 8 + 4 + 5 + 0 - 3 + 3 = 17$

Adding multiples of 6 to 17, we get, $17 + 4 \times 6 = 41$; $41 + 6 = 47$; $47 + 6 = 53$; $53 + 6 = 59 \neq 58$.

So the answer is **(d)**.

Model Test Paper 4

Directions: Four alternatives are given for the following questions. You have to choose the most correct answer from the given alternatives.

1. A person travels 12 km due north, then 15 km due east, after that 15 km due west and then 18 km due south. How far is he from the starting point?
(a) 6 km (b) 12 km (c) 33 km (d) 60 km
2. Six persons A, B, C, D, E and F are standing in a row. C and D are standing close to each other alongside E. B is standing beside A only. A is fourth from F. Who are standing on the extremes?
(a) A and F (b) B and D (c) B and F (d) None of these
3. A person has 4 coins each of different denominations. What is the number of different sums of money the person can form (using one or more coins at a time)?
(a) 16 (b) 15 (c) 12 (d) 11
4. How many numbers lie between 300 and 500 in which 4 comes only one time?
(a) 99 (b) 100 (c) 110 (d) 120
5. How many letters of the English alphabet (capitals) appear same when looked at in a mirror?
(a) 9 (b) 10 (c) 11 (d) 12
6. How many three-digit numbers can be generated from 1, 2, 3, 4, 5, 6, 7, 8, 9 such that the digits are in the ascending order?
(a) 80 (b) 81 (c) 83 (d) 84
7. There are four persons A, B, C, D; and A has some coins. A gave half of the coins to B and 4 more besides. B gave half of the coins to C and 4 more besides. C gave half of the coins to D and 4 more besides. Both B and D end up with same number of coins. How many coins did A have originally?
(a) 96 (b) 84 (c) 72 (d) 64
8. While adding the first few continuous natural numbers, candidate missed one of the numbers and wrote the answer 177. Which was the number missed?
(a) 11 (b) 12 (c) 13 (d) 14
9. Four metal rods of lengths 78 cm, 104 cm, 117 cm and 169 cm are to be cut into parts of equal length. Each part must be as long as possible. What is the maximum number of pieces that can be cut?
(a) 27 (b) 36 (c) 43 (d) 480

10. In an examination, there are three subjects A, B and C. A student has to pass in each subject. 20% students failed in A, 22% students failed in B and 16% failed in C. The total number of students passing the whole examination lies between
 (a) 42% and 84% (b) 42% and 78% (c) 58% and 78% (d) 58% and 84%
11. How many times are an hour hand and a minute hand of a clock will be at right angles during their motion from 1.00 pm to 10.00 pm?
 (a) 9 (b) 10 (c) 18 (d) 20
12. There are 240 balls and n number of boxes $B_1, B_2, B_3, \dots, B_n$. The balls are to be placed in the boxes such that B_1 should contain 4 balls more than B_2 , B_2 should contain 4 balls more than B_3 , and so on. Which one of the following cannot be the possible value of n ?
 (a) 4 (b) 5 (c) 6 (d) 7
13. In a carrom board game competition, m boys and n girls ($m > n > 1$) of a school participate in which every student has to play exactly one game with all other student. Out of the total games played, it was found that in 221 games one player was a boy and the other player was a girl.

Consider the following statements:

1. The total number of students that participated in the competition was 30.
2. The number of games in which both players were girls was 78.

Which of the statements given above is/are correct?

- (a) 1 only (b) 2 only (c) Both 1 and 2 (d) Neither 1 nor 2
14. There are three cans A, B and C. The capacities of A, B and C are 6 litres, 10 litres and 16 litres respectively. The can C contains 16 litres of milk. The milk has to be divided in them using these three cans only.

Consider the following statements:

1. It is possible to have 6 litres of milk each in can A and can B.
2. It is possible to have 8 litres of milk each in can B and can C.

Which of the statements given above is/are correct?

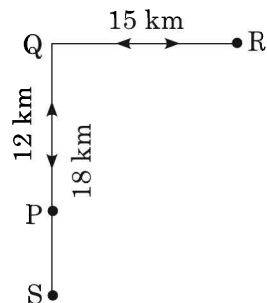
- (a) 1 only (b) 2 only (c) Both (a) and (b) (d) Neither (a) nor (b)
15. There is a family of 6 persons A, B, C, D, E and F. There are two married couples in the family. The family members are lawyer, teacher, salesman, engineer, accountant and doctor. D, the salesman is married to the lady teacher. The doctor is married to the lawyer. F, the accountant is the son of B and brother of E. C, the lawyer is the daughter-in-law of A. E is the unmarried engineer. A is the grandmother of F. How is E related to F?
 (a) Brother (b) Sister
 (c) Father (d) Cannot be established (determined)

ANSWERS

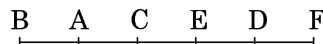
1. (a) 2. (c) 3. (b) 4. (d) 5. (b) 6. (d) 7. (c) 8. (c) 9. (b) 10. (b)
 11. (c) 12. (d) 13. (d) 14. (a) 15. (d)

(Solutions and Hints)

1. The person starts from P goes through Q, R, return to S. So the distance of the person from the starting point = PS = QS
 $- QP = 18 - 12 = 6 \text{ km}$
 So the answer is **(a)**.



2. The arrangement of six persons is given on the right side
 $\therefore$ B and F are standing on the extremes



So the answer is **(c)**

3. The number of denominations that can be formed using the given 4 coins
 (i) When only one coin was used, the required number = 4

(ii) When two coins were used, the required number = ${}^4C_2 = \frac{4 \times 3}{1 \times 2} = 6$

(iii) When three coins were used, the required number = ${}^4C_3 = {}^4C_1 = 4$

(iv) When all the four coins were used, the required number = 1

$\therefore$ Total number of denominations = $4 + 6 + 4 + 1 = 15$

4. Numbers which are having 4 in unit place only = $20 \times 1 = 20$ (once in 10 numbers)
 Numbers which are having 4 in tenth place = $2 \times 10 = 20$
 Numbers which are having 4 in hundredth place = $100 \times 1 = 100$
 Number of 4s repeating in one's and tenth's place = 1 number
 Number of 4s repeating in one's and hundredth place = 9 numbers
 Number of 4s repeating in tenth and hundredth place = 9 numbers
 Number of 4s repeating in all three positions = 1 number
 $\therefore$ Number of numbers with '4' in one place only = $(20 + 20 + 100) - (1 + 9 + 9 + 1) = 120$.
 So the answer is **(d)**.

6. (i) If 1 and 2 are provided in the first two places, then the third place can be filled in ${}^7C_1 = 7$ ways

So the number of numbers so formed = 7

- (ii) If 1 and 3 are filled in first two position, then the third place can be filled in ${}^6C_1 = 6$ ways and so on. Total number of numbers so formed with '1' as 1st place = $7 + 6 + 5 + 4 + 3 + 2 + 1 = 28$.

Similarly with 2 in the first place and 3 in the second place, third place can be filled in ${}^6C_1 = 6$ ways. So total number of numbers with '2' as 1st place = $6 + 5 + 4 + 3 + 2 + 1 = 21$ and so on.

Similarly with 7 in the first place and 8 in the second place, third place can be filled in ${}^1C_1 = 1$ way. So the total number of numbers with '7' as 1st digit and '8' as 2nd digit is 1.

So the total number of three digit numbers thus formed with all the given 9 digits = $28 + 21 + 15 + 10 + 6 + 3 + 1 = 84$.

7. Let the number of coins with A , B , C and D be a , b , c and d respectively.

No. of coins with $A = 'a'$

No. of coins with $B = b = \frac{a}{2} + 4$

No. of coins with $C = c = \frac{b}{2} + 4 = \frac{1}{2}\left(\frac{a}{2} + 4\right) + 4 = \frac{a}{4} + 2 + 4 = \frac{a}{4} + 6$

No. of coins with $D = d = \frac{c}{2} + 4 = \frac{1}{2}\left(\frac{a}{4} + 6\right) + 4 = \frac{a}{8} + 3 + 4 = \frac{a}{8} + 7$

Given $b = d$ at the end

$$\text{i.e.} \quad \left(\frac{a}{2} + 4\right) - \left(\frac{a}{4} + 6\right) = \frac{a}{8} + 7$$

$$\text{or} \quad \frac{a}{4} - 2 = \frac{a}{8} + 7$$

$$\frac{a}{8} = 2 + 7 = 9$$

$$\text{or} \quad a = 72$$

Hence the answer is (c).

8. The sum of first 19 natural numbers comes to $\frac{19 \times 20}{2} = 190$

$\therefore$ Addition of 13 makes it possible to get the correct sum.

Hence the answer is (c)

9. To find the maximum possible length, find HCF of given numbers.

HCF of 78, 104, 117 and 169

HCF of 78 and 104 is 26 (i.e. last divisor)

$$\begin{array}{r} 1 \\ 78 \overline{) 104} \\ \underline{78} \\ 26 \overline{) 78} (3 \\ \underline{78} \\ 0 \end{array}$$

HCF of 117 and last HCF (26) is 13.

$$\begin{array}{r} 4 \\ 26 \overline{) 117} \\ \underline{104} \\ 13 \overline{) 26} (2 \\ \underline{26} \\ 0 \end{array}$$

HCF of 169 and 13 is 13.

$$\begin{array}{r} 13 \\ 13 \overline{) 169} \\ \underline{169} \\ 0 \end{array}$$

Maximum number of pieces that can be cut, which are having 13 cm length is given by

$$\text{required number} = \frac{78}{13} + \frac{104}{13} + \frac{117}{13} + \frac{169}{13} = 6 + 8 + 9 + 13 = 36$$

So, the answer is **(b)**.

10. Subjects are A, B and C

Students passed in A = $100 - 20 = 80\%$

Students passed in B = $100 - 22 = 78\%$

Students passed in C = $100 - 16 = 84\%$

Minimum pass percentage = 78% and that can be maximum possible value of whole pass percentage. If students failed in A, B and C independently in the respectively given percentages, total failure percentage can be of maximum possible upto $20 + 22 + 16 = 58\%$ or pass percentage = 42%

$\therefore$ Pass percentage lies between **42% and 78%**

Hence the answer is **(b)**.

11. In every hour there will be two positions of hour hand and minute hand coming at right angles. Therefore in 9 hours there will be 18 such positions.

Hence the answer is (c).

12. If m is the number of balls in the first box, then in the second box, it will be $(m - 4)$, in the third box it will be $(m - 8)$ and so on.

$$\begin{aligned}\therefore \text{Sum of balls in } n \text{ boxes together} &= \frac{n}{2} [(m - 4) + (m - 4) + (n - 1) - 4] \\ &= \frac{n}{2} [2m - 8 - 4n + 4] \\ &= \frac{n}{2} [2m - 4n - 4] = n[m - 2n - 2] = 240\end{aligned}$$

$\therefore$ Sum of balls obtained above is a multiple of n , value of n which is not possible from the given alternatives is 7.

$\therefore$ n is a factor of 240, the value of n which is not possible from the given alternatives is 7. So the answer is (d).

13. It is given that number of boys (m) > Number of girls (n)

Given in 221 games plays, one player was boy and the other was girl

So, minimum number of boys = 221

So, minimum number of girls = 221

However, the number of boys is greater than that of girls.

So neither of the 2 given statements are correct. Hence the answer is **(d)**.

14. We are having cans A, B and C with measurements 6 litres, 10 litres and 16 litres respectively. We can take 6 litres each in can A and can B. So statement 1 is correct. We are not having can for measuring 8 litres of milk. So second statement is false. Hence **1 only** is correct.

15. It is mentioned that F is the brother of E and E is the unmarried engineer. So it is not clear that whether E is male or female. Hence the relation of E to F **cannot be established**. Hence the answer is **(d)**.

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