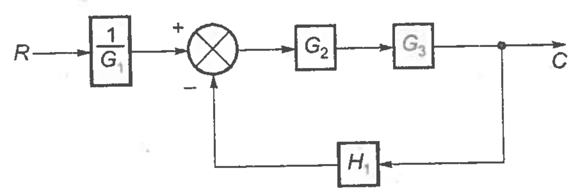
Student's Assignments 1

Q.1 A feedback control system is shown below. Find the transfer function for this system.

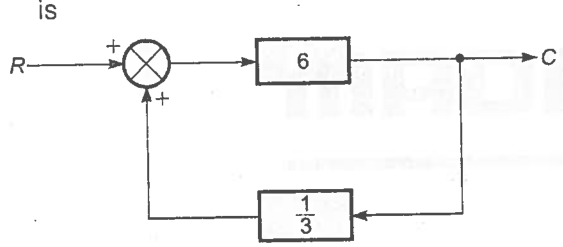


Q.2 The step response of a system is given as

$y = 1 - \frac{7}{3}e^{-t} + \frac{3}{2}e^{-2t} - \frac{1}{6}e^{-4t}$. If the transfer function of this system is $\frac{(s+a)}{(s+b)(s+c)(s+d)}$ then $a + b + c + d$ is ________.

Q.3 A system has the transfer function $\frac{(1-s)}{(1+s)}$. Its gain at ω = 1 rad/sec is ______.

Q.4 The close loop gain of the system shown below is



Student's Assignments 1 Explanations

1. $\left(\frac{G_2G_3}{G_1(1+H_1G_2G_3)}\right)$

Multiply G_2 and G_3 and apply feedback formula and then again multiply with $\frac{1}{G_1}$

$$T(s) = \frac{G_2G_3}{G_1(1+G_2G_3H_1)}$$

2. (15)

$$y = 1 - \frac{7}{3}e^{-t} + \frac{3}{2}e^{-2t} - \frac{1}{6}e^{-4t}$$

$$p(t) = \frac{dy}{dt}$$

$$= \frac{7}{3}e^{-t} + \frac{3}{2}\times(-2)\times e^{-2t} - \left(\frac{1}{6}\right)(-4)e^{-4t}$$

Laplace transform of $p(t)$,

$$p(s) = \frac{7/3}{s+1} + \frac{-3}{s+2} + \frac{2/3}{s+4}$$

$$= \frac{s+8}{(s+1)(s+2)(s+4)}$$

$$\Rightarrow \quad a+b+c+d = 15$$

3. (1)

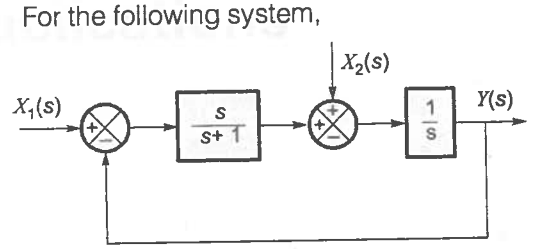
For all pass system, gain = '1' at all frequencies.

4. (−6)

$$\text{C.L.T.F.} = \frac{6}{1-6\times\frac{1}{3}} = \frac{6}{-1} = -6$$

Student's Assignments 2

Q.1 For the following system,

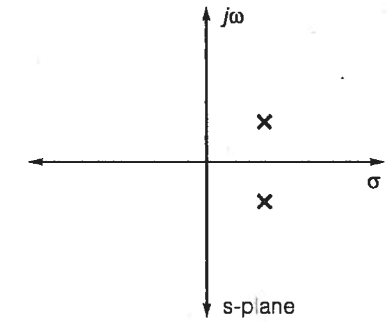


when $X_1(s) = 0$, the transfer function $\frac{Y(s)}{X_2(s)}$ is

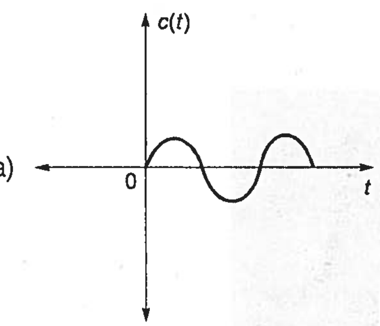
(a) $\frac{s+1}{s^2}$ (b) $\frac{1}{s+1}$

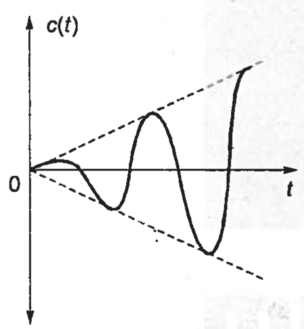
(c) $\frac{s+2}{s(s+1)}$ (d) $\frac{s+1}{s(s+2)}$

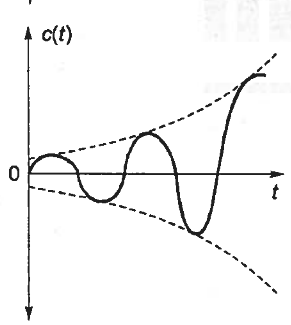
Q.2 If closed-loop transfer function poles shown below

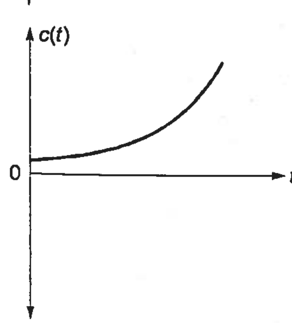


Impulse response is

(a)


(b)


(c)


(d)


Q.3 The impulse response of several continuous systems are given below. Which is/are stable?

1. $h(t) = te^{-t}$
2. $h(t) = 1$
3. $h(t) = e^{-t}\sin 3t$
4. $h(t) = \sin\omega t$

(a) 1 only (b) 1 and 3

(c) 3 and 4 (d) 2 and 4

Q.4 Ramp response of the transfer function

$F(s) = \frac{s+1}{s+2}$ is

(a) $\frac{1}{4} - \frac{1}{4}e^{-2t} + \frac{1}{2}t$ (b) $\frac{1}{4}e^{-2t} + \frac{1}{4} + \frac{1}{2}t$

(c) $\frac{1}{2} - \frac{1}{2}e^{-2t} + t$ (d) $\frac{1}{2}e^{-2t} + \frac{1}{2} - t$

Q.5 Which of the following statements are correct?

1. Transfer function can be obtained from the signal flow graph of the system.
2. Transfer function typically characterizes to linear time invariant systems.
3. Transfer function gives the ratio of output to input in frequency domain of the system.

(a) 1 and 2 (b) 2 and 3

(c) 1 and 3 (d) 1, 2 and 3

Q.6 Which of the following is not a desirable feature of a modern control system?

(a) Quick response
(b) Accuracy
(c) Correct power level
(d) Oscillations

Q.7 In regenerating feedback, the transfer function is given by

(a) $\frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)H(s)}$

(b) $\frac{C(s)}{R(s)} = \frac{G(s)H(s)}{1-G(s)H(s)}$

(c) $\frac{C(s)}{R(s)} = \frac{G(s)H(s)}{1+G(s)H(s)}$

(d) $\frac{C(s)}{R(s)} = \frac{G(s)}{1-G(s)H(s)}$

Q.8 In a continuous data system
(a) data may be a continuous function of time at all points in the system.
(b) data is necessarily a continuous function of time at all points in the system.
(c) data is continuous at the input and output parts of the system but not necessarily during intermediate processing of the data.
(d) only the reference signal is a continuous function of time.

Q.9 The principle of homogeneity and superposition are applied to
(a) linear time variant systems
(b) non-linear time variant systems
(c) linear time invariant systems
(d) non-linear time invariant systems

Q.10 Consider the following statements regarding the advantages of closed loop negative feedback control systems over open-loop systems:
1. The overall reliability of the closed loop systems is more than that of open-loop system.
2. The transient response in the closed loop system decays more quickly than in open-loop system.
3. In an open-loop system, closing of the loop increases the overall gain of the system.
4. In the closed-loop system, the effect of variation of component parameters on its performance is reduced.

Of these statements:
(a) 1 and 3 are correct
(b) 1, 2 and 4 are correct
(c) 2 and 4 are correct
(d) 3 and 4 are correct

Q.11 Match **List-I (Time function)** with **List-II (Laplace transforms)** and select the correct answer using the codes given below lists:

List-I	List-II
A. $[af_1(t)+bf_2(t)]$	1. $aF_1(s)+bF_2(s)$
B. $[e^{-at}f(t)]$	2. $sF(s)+f(0)$
C. $\left[\frac{df(t)}{dt}\right]$	3. $\frac{1}{s}F(s)$
D. $\left[\int_0^t f(x)dx\right]$	4. $sF(s)-f(0^-)$
	5. $F(s+a)$

Codes:

	A	B	C	D
(a)	5	2	3	4
(b)	1	5	4	3
(c)	2	1	3	4
(d)	1	5	3	4

Q.12 If a system is represented by the differential equation, is of the form $\frac{d^2y}{dt^2}+6\frac{dy}{dt}+9y=r(t)$

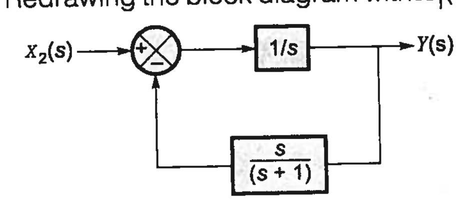
(a) $k_1e^{-t}+k_2e^{-9t}$ (b) $(k_1+k_2)e^{-3t}$
(c) $ke^{-3t}\sin(t+\phi)$ (d) $te^{-3t}u(t)$

Q.13 A linear system initially at rest, is subject to an input signal $r(t)=1-e^{-t}(t\geq 0)$. The response of the system for $t>0$ is given by $c(t)=1-e^{-2t}$. The transfer function of the system is

(a) $\frac{(s+2)}{(s+1)}$ (b) $\frac{(s+1)}{(s+2)}$
(c) $\frac{2(s+1)}{(s+2)}$ (d) $\frac{(s+1)}{2(s+2)}$

Student's Assignments 2 Explanations

1. (d)
Redrawing the block diagram with $X_1(s)=0$



The transfer function

$$T(s)=\frac{Y(s)}{X_2(s)}=\frac{G(s)}{1+G(s)H(s)} \quad ...(i)$$

Here, $G(s)=\frac{1}{s}$ and $H(s)=\frac{s}{s+1}$

$$\frac{Y(s)}{X_2(s)}=\frac{1/s}{1+\frac{1}{s}\times\frac{s}{s+1}}=\frac{(s+1)}{s(s+2)}$$

2. (c)

$$\text{T.F.}=\frac{1}{[s-(\sigma+j\omega)][s-(\sigma-j\omega)]}$$

$$=\frac{1}{[(s-\sigma)-j\omega)][(s-\sigma)+j\omega)]}$$

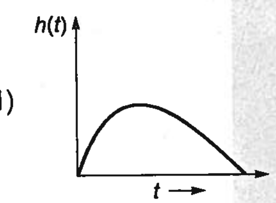
$$=\frac{1}{[(s-\sigma)^2-(j\omega)^2]}=\frac{1}{[(s-\sigma)^2+\omega^2]}$$

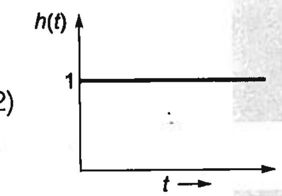
For impulse response, taking its inverse Laplace transformation we get,

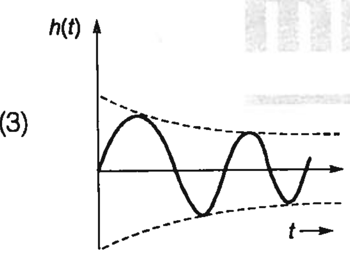
$$c(t)=e^{\sigma t}\sin\omega t$$

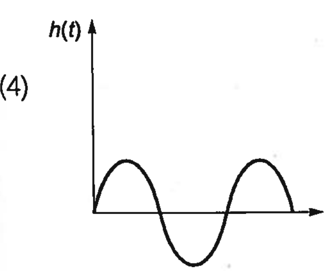
hence option (c) is correct.

3. (b)
If the impulse response decays to zero as time approaches infinity, the system is stable.

(1)


(2)


(3)


(4)


4. (a)

$$\frac{C(s)}{R(s)}=\frac{s+1}{s+2}$$

$$\therefore\quad C(s)=R(s)\cdot\frac{s+1}{s+2}$$

$$=\frac{1}{s^2}\cdot\frac{s+1}{s+2}=\frac{1}{s^2}\left(1-\frac{1}{s+2}\right)=\frac{1}{s^2}-\frac{1}{s^2(s+2)}$$

$$\frac{1}{s^2}\cdot\left(\frac{s+1}{s+2}\right)=\frac{A}{s}+\frac{B}{s^2}+\frac{C}{s+2}$$

$$s+1=As(s+2)+B(s+2)+Cs^2$$

$$=As^2+2As+Bs+2B+Cs^2$$

$\therefore$ $A+C=0$, $2A+B=1$ and $2B=1$

$$\therefore\quad A=\frac{1}{2},\quad B=\frac{1}{4},\quad C=-\frac{1}{4}$$

$$\therefore\quad \frac{C(s)}{R(s)}=\frac{1}{4s}+\frac{1}{2s^2}+\left(-\frac{1}{4}\right)\frac{1}{s+2}$$

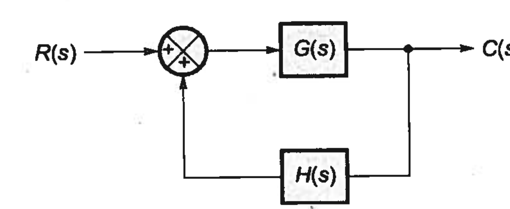
$$=\frac{1}{4}u(t)+\frac{1}{2}tu(t)-\frac{1}{4}e^{-2t}u(t)$$

5. (d)
(i) Transfer function can be obtained from signal flow graph of the system.
(ii) Transfer function typically characterizes to LTI systems.
(iii) Transfer function gives the ratio of output to input in s-domain of system.

$$\text{TF}=\left.\frac{L[\text{Output}]}{L[\text{Input}]}\right|_{\text{Initial conditions}=0}$$

6. (d)

7. (d)
Block diagram of regenerating feedback system



$$\frac{C(s)}{R(s)}=\frac{G(s)}{1-G(s)H(s)}$$

8. (b)

9. (c)

10. (b)
Closed loop control system with negative feedback will have
- Less gain
- More accuracy
- More bandwidth hence more speed

11. (b)

12. (d)
Let $R(s)$ is the input Laplace transform of given differential equation is

$s^2Y(s) + 6sY(s) + 9Y(s) = R(s)$

$(s^2 + 6s + 9)\, Y(s) = R(s)$

$$TF = \frac{Y(s)}{R(s)} = \frac{1}{(s+3)^2}$$

$$IR = L^{-1}\left[\frac{1}{(s+3)^2}\right] = te^{-3t}u(t)$$

13. (c)
Given that $r(t) = 1 - e^{-t}$,

$c(t) = 1 - e^{-2t}$

$$R(s) = \frac{1}{s} - \frac{1}{s+1} = \frac{1}{s(s+1)}$$

$$C(s) = \frac{1}{s} - \frac{1}{(s+2)} = \frac{2}{s(s+2)}$$

$$TF = \frac{C(s)}{R(s)} = \frac{2s(s+1)}{s(s+2)} = \frac{2(s+1)}{s+2}$$

■■■■

CHAPTER 03

Block Diagrams

3.1 Block Diagrams : Fundamentals

A block diagram of a system is a pictorial representation of the functions performed by each component and of the flow of signals. Such a diagram depicts the interrelationships that exist among the various components. Differing from a purely abstract mathematical representation, a block diagram has the advantage of indicating more realistically the signal flows of the actual system.

In a block diagram, all system variables are linked to each other through functional blocks. The **functional block** or simple **block** is a symbol for the mathematical operation on the input signal to the block that produces the output. The transfer functions of the components are usually entered in the corresponding blocks, which are connected by arrows to indicate the direction of the flow of signals. Note that the signal can pass only in the direction of the arrows. Thus, a block diagram of a control system explicitly shows a unilateral property.

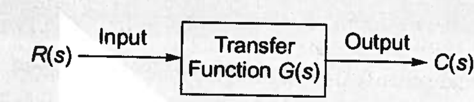


***Figure-3.1:** Element of a block diagram*

$$C(s) = R(s) \cdot G(s)$$

$$G(s) = \frac{C(s)}{R(s)}$$

Note: The dimension of the output signal from the block is the dimension of the input signal multiplied by the dimension of the transfer function in the block.

The advantages of the block diagram representation of a system are that it is easy to form the overall block diagram for the entire system by merely connecting the blocks of the components according to the signal flow and it is possible to evaluate the contribution of each component to the overall performance of the system. If the mathematical and functional relationships of all the system elements are known, the block diagram can be used as a tool for the analytic or computer solution of the system. In general, block diagrams can be used to model linear as well as non-linear systems.

It should be noted that the block diagram of a given system is not unique. A number of different block diagrams can be drawn for a system, depending on the point of view of the analysis.

Take off Point or Branch Point

A take off point is a point from which the signal from a block goes concurrently to other blocks or summing points. It should be noted that such taking off from any signal does not alter the parent signal. This permits the signal to proceed unaltered along several different paths to several destinations.

Summing Point

A circle with a cross, is the symbol that indicates a summing point. The plus or minus sign at each arrowhead indicates whether that signal is to be added or subtracted.

3.2 Block Diagram of a Closed-loop System

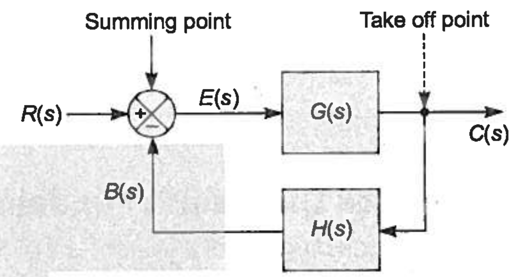


Figure-3.2: *Block diagram of closed-loop system*

Above figure shows the block diagram of a negative feedback system. With reference to this figure, the terminology used in block diagrams of control systems is given below.

$R(s)$ = reference input signal

$C(s)$ = output signal or controlled variable

$B(s)$ = feedback signal

$E(s)$ = actuating signal or error signal

$G(s) = \frac{C(s)}{E(s)}$ = forward path transfer function

$H(s)$ = feedback path transfer functions

$G(s)\,H(s) = \frac{B(s)}{E(s)}$ = open-loop transfer function

$T(s) = \frac{C(s)}{R(s)}$ = closed-loop transfer function

$\frac{E(s)}{R(s)}$ = Error ratio

$\frac{B(s)}{R(s)}$ = Primary feedback ratio

From figure,

$$C(s) = E(s)\,G(s) \quad ...(i)$$

$$E(s) = R(s) - B(s) = R(s) - H(s)\,C(s) \quad ...(ii)$$

Eliminating $E(s)$ from equation (i) and (ii) we have

$$C(s) = G(s)\,R(s) - G(s)\,H(s)\,C(s)$$

or

$$\frac{C(s)}{R(s)} = T(s) = \frac{G(s)}{1+G(s)H(s)}$$

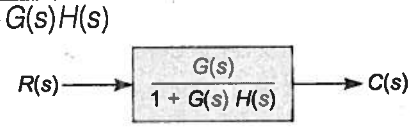


Figure-3.3: *Reduced block diagram*

Mutliple-input-multiple-output Systems

When multiple inputs are present in a linear system, each input can be treated independent of the others. Complete output of the system can then be obtained by superposition, i.e. outputs corresponding to each input alone are added together.

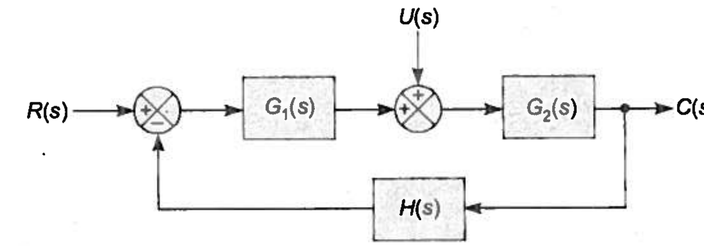


Figure-3.4: *Block diagram of two-input system*

Consider a two-input linear system shown in above figure. The response to the reference input can be obtained by assuming $U(s) = 0$. The corresponding block diagram is shown in below figure,

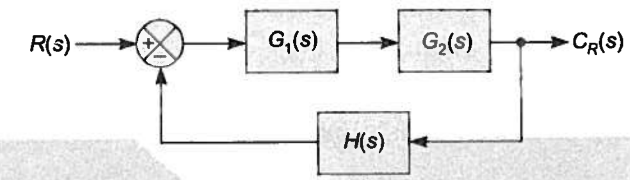


Figure-3.5: *Block diagram when R(s) acting alone*

which gives, $C_R(s)$ = output due to R(s) acting alone

$$= \frac{G_1(s)\,G_2(s)}{1+G_1(s)G_2(s)H(s)}R(s) \quad ...(i)$$

Similarly, the response to the input $U(s)$ is obtained by assuming $R(s) = 0$. The block diagram corresponding to this case is shown in below figure,

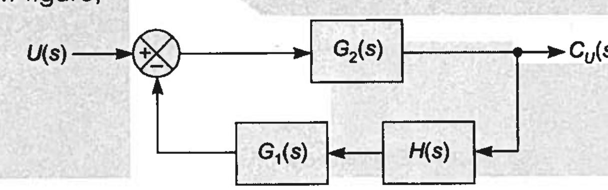


Figure-3.6: *Block diagram when U(s) acting alone*

which gives, $C_U(s)$ = input due to $U(s)$ acting alone

$$= \frac{G_2(s)}{1+G_1(s)G_2(s)H(s)}U(s) \quad ...(ii)$$

The overall response due to the simultaneous application of $R(s)$ and $U(s)$ can be obtained by adding the two individual responses.

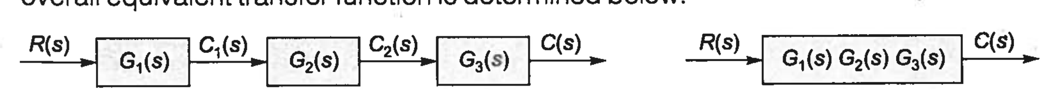
Adding equation (i) and (ii), we get

$$C(s) = C_R(s) + C_U(s)$$

$$= \frac{G_2(s)}{1+G_1(s)G_2(s)H(s)}[G_1(s)R(s)+U(s)] \quad ...(iii)$$

3.3 Block Diagram Transformation Theorems

1. **Blocks in Cascade:** When several blocks are connected in cascade (as shown in below figure), the overall equivalent transfer function is determined below:



(a) (b)

Figure-3.7: *Blocks in cascade and equivalence*

$$\frac{C_1(s)}{R(s)} = G_1(s) \quad ...(i)$$

$$\frac{C_2(s)}{C_1(s)} = G_2(s) \quad ...(ii)$$

$$\frac{C(s)}{C_2(s)} = G_3(s) \quad ...(iii)$$

while connecting the blocks in cascade it should be noted that the forward block does not alter the output signal of the preceding block.

Hence, multiplying equations (i), (ii) and (iii)

$$\frac{C_1(s)}{R(s)} \cdot \frac{C_2(s)}{C_1(s)} \cdot \frac{C(s)}{C_2(s)} = G_1(s) \cdot G_2(s) \cdot G_3(s)$$

Therefore, the overall equivalent transfer function is,

$$\frac{C(s)}{R(s)} = G_1(s) \cdot G_2(s) \cdot G_3(s)$$

2. **Blocks in Parallel:** When one or more blocks are connected in parallel as shown in below figure,

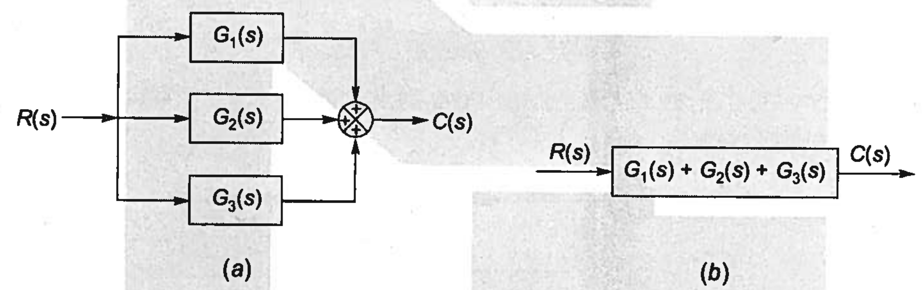


Figure-3.8: *Equivalence of blocks in parallel*

The overall equivalent transfer function is determined below,

$$C(s) = R(s)\, G_1(s) + R(s)\, G_2(s) + R(s)\, G_3(s)$$

or $$C(s) = R(s)\, [G_1(s) + G_2(s) + G_3(s)]$$

Therefore, the overall equivalent transfer function is

$$\frac{C(s)}{R(s)} = [G_1(s) + G_2(s) + G_3(s)]$$

3. Consecutive summing points can be interchanged, as this interchange does not alter the output signal as shown in figure given below.

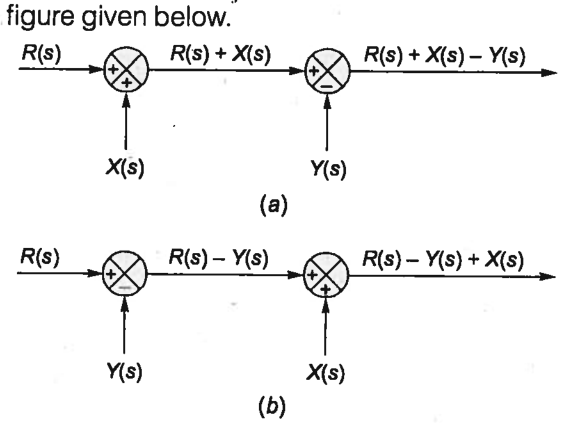


Figure-3.9: *Interchange of consecutive summing points*

4. Shifting of a take off point from a position before a block to a position after the block as shown in figure given below.

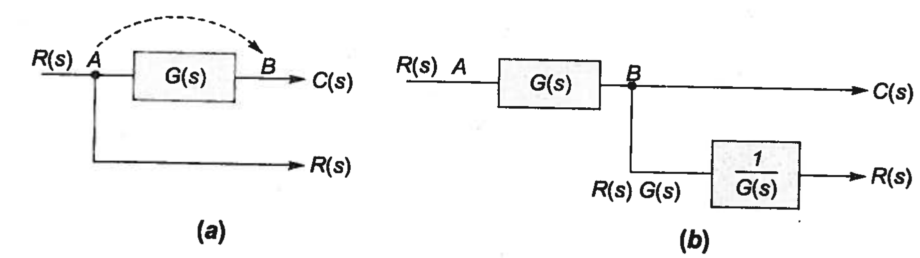


Figure-3.10: *Take off point at position A shifted to position B*

5. Shifting of a take off point from a position after a block to a position before the block as shown in figure given below.

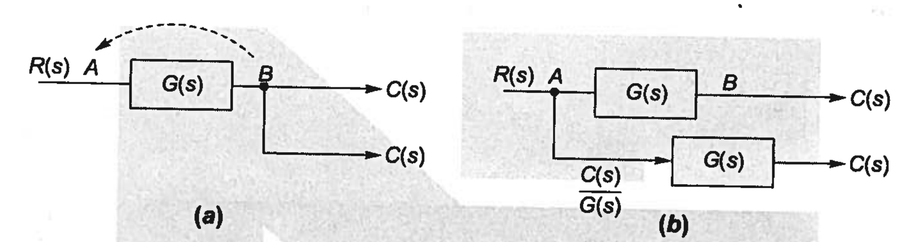


Figure-3.11: *Take off point at position B is shifted to position A*

6. Shifting of a summing point from a position before a block to a position after the block as shown in figure given below.

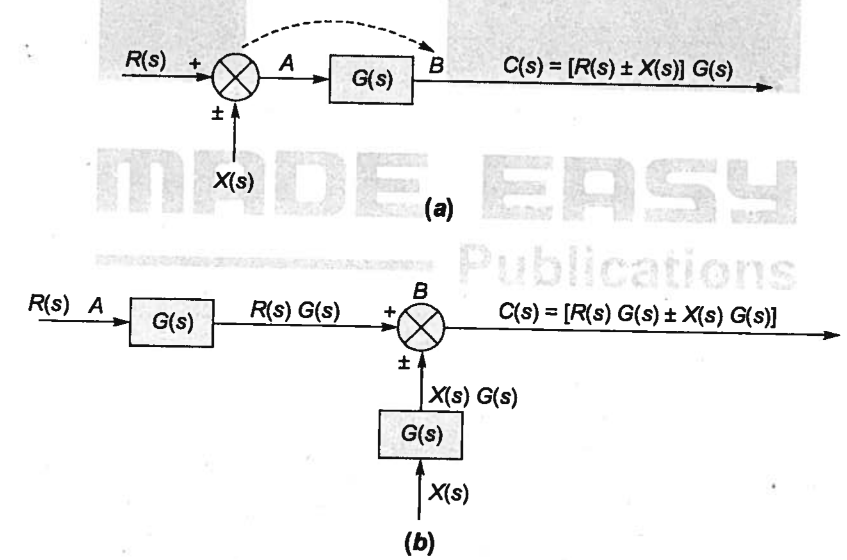


Figure-3.12: *Summing at position A is shifted to position B*

7. Shifting of a summing point from a position after a block to a position before the block as shown in figure given below.

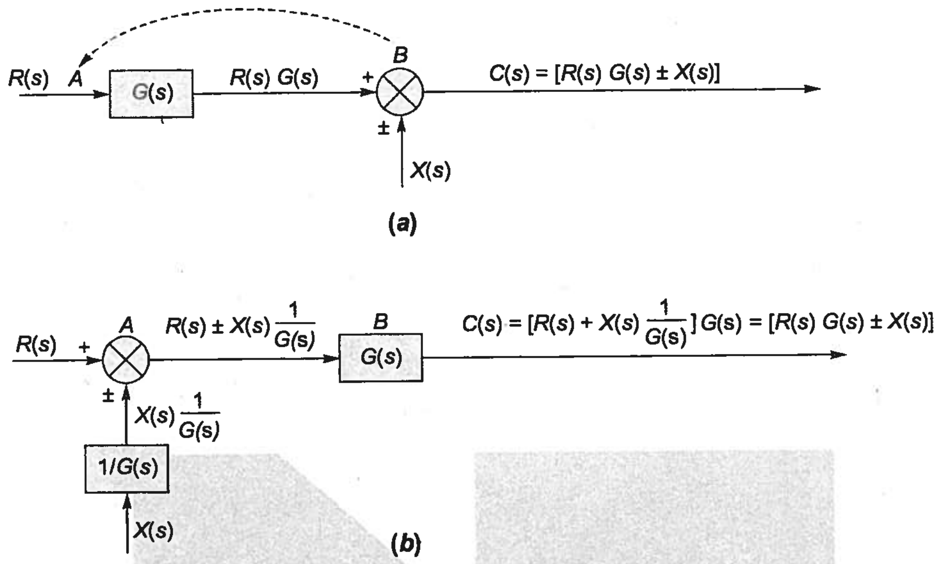


Figure-3.13: *Summing point at position B is shifted to position A*

8. Shifting of a take off point from position before a summing point to a position after the summing point as shown in figure given below.

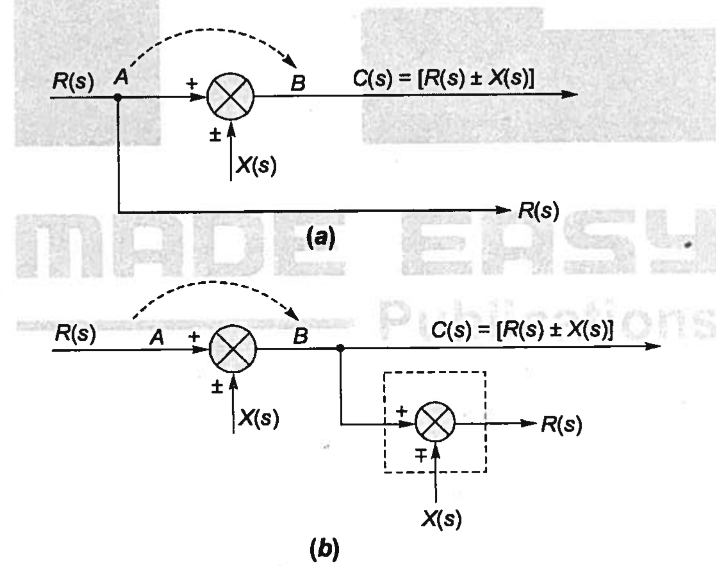


Figure-3.14: *Take off point at position A is shifted to position B*

9. Shifting of a take off point from a position after a summing point to a position before the summing point as shown in figure given below.

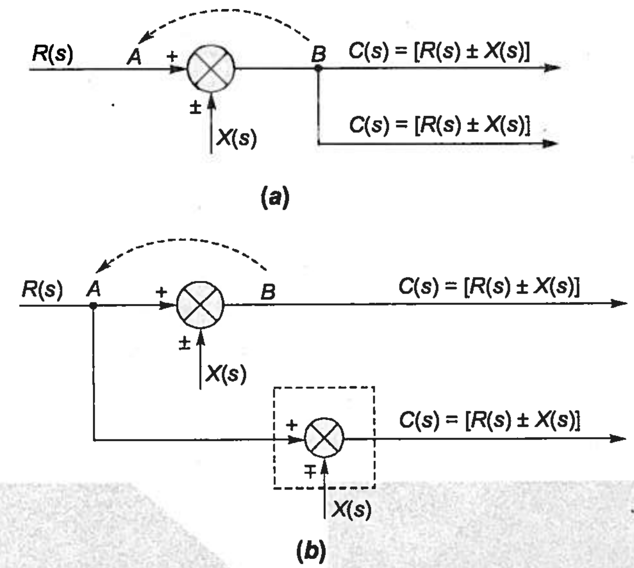


Figure-3.15: *Take off point at position B is shifted to position A*

10. (*a*) Elimination of a summing point in a closed-loop system is explained below:

A closed-loop control system is represented by a block diagram as shown in figure given below, wherein a fraction of the output $B(s) = C(s)\,H(s)$ is compared with the input $R(s)$ which results in an error $E(s)$ given by

$$E(s) = [R(s) \pm B(s)]$$

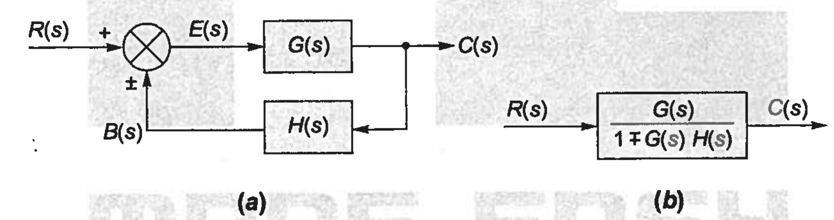


Figure-3.16: *Elimination of a summing point in a closed-loop (feedback) system*

From Fig. 3.16 (a) $\quad \dfrac{C(s)}{E(s)} = G(s) \quad$ ...(i)

$G(s)$ is termed as forward path transfer function and

$$\frac{B(s)}{C(s)} = H(s) \quad \text{...(ii)}$$

$H(s)$ is termed as feedback path transfer function.

The output of the summing point is given by

$$E(s) = [R(s) \pm B(s)] \quad \text{...(iii)}$$

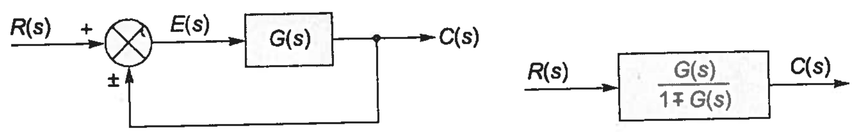
Substituting for $E(s)$ and $B(s)$ in equation (iii) from equations (i) and (ii) the following equation is obtained:

$$\frac{C(s)}{G(s)} = R(s) \pm C(s)\,H(s)$$

Simplifying the above equation, the overall transfer function is denoted as M(s) is given by

$$M(s) = \frac{C(s)}{R(s)} = \frac{G(s)}{1 \mp G(s)\,H(s)}$$

(b) A unity feedback control system is shown in Fig. 3.17 (a), wherein feedback path transfer function is unity and the equivalence of a unity feedback control system to a single block representation is shown in Fig. 3.17 (b).



(a) (b)

Figure-3.17 : *Elimination of a summing point in a unity feedback control system*

Since H(s) = 1, the transfer function of a unity feedback control system relating the output and input is thus given by

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1 \pm G(s)\cdot 1}$$

or

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1 \pm G(s)}$$

(c) The transfer function relating $E(s)$ and $R(s)$ for a closed-loop control system shown in figure given below,

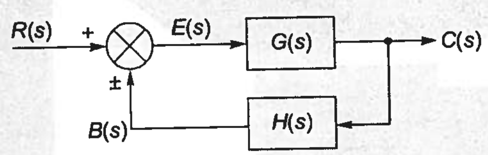


Figure-3.18 : *Block diagram of a closed-loop control system*

As $E(s) = [R(s) \pm B(s)]$

and $B(s) = C(s)\,H(s)$

Therefore, $E(s) = [R(s) \pm C(s)\,H(s)]$

It is also seen that $C(s) = E(s)\,G(s)$

Therefore, $E(s) = [R(s) \pm E(s)\,G(s)\,H(s)]$

Simplifying equation, the transfer function relating $E(s)$ and $R(s)$ is obtained below:

$$\frac{E(s)}{R(s)} = \frac{1}{1 \pm G(s)H(s)}$$

11. When two or more inputs act on a system, the total output is obtained by adding the effect of each individual input separately.

A case of two inputs acting on a system is shown in figure given below.

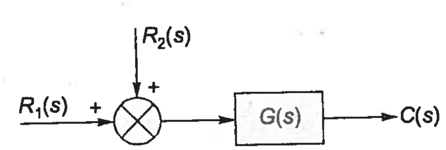


Figure-3.19 : *Two inputs acting on a system*

In order to calculate the output due to $R_1(s)$, $R_2(s)$ is considered as zero as shown in figure given below.

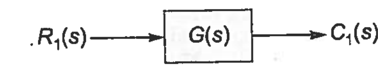


Figure-3.20 : *Considering only $R_1(s)$ as input*

The output is given by $C_1(s) = R_1(s)\,G(s)$

Similarly, while determining the output due to $R_2(s)$, $R_1(s)$ is considered as zero as shown in figure given below.

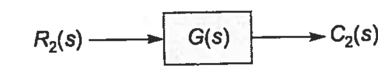


Figure-3.21 : *Considering only $R_2(s)$ as input*

Output for this case is given by

$$C_2(s) = R_2(s)\,G(s)$$

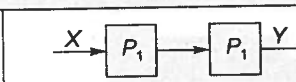
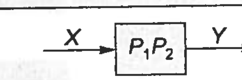
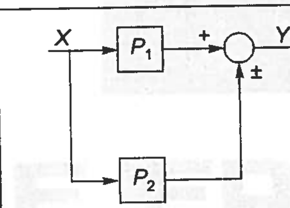
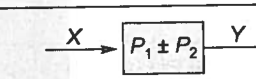
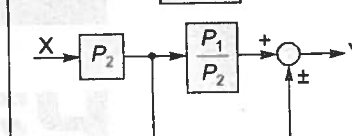
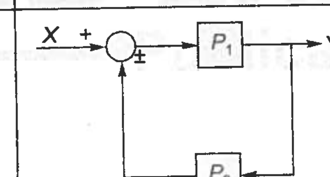
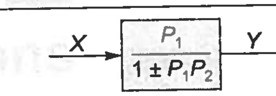
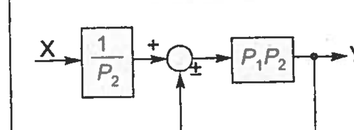
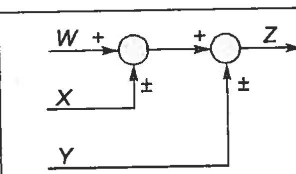
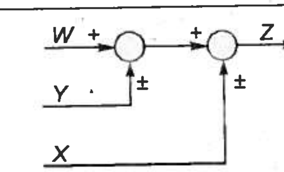
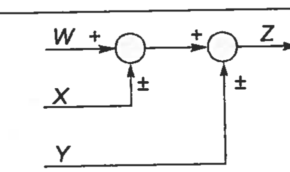
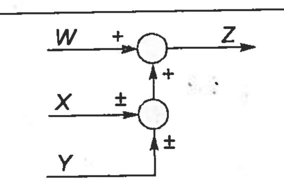
The total output is given by

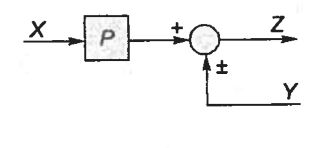
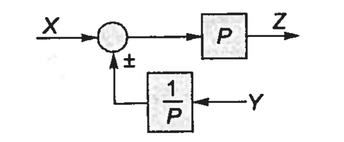
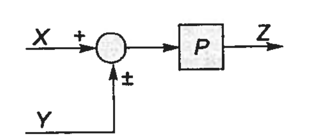
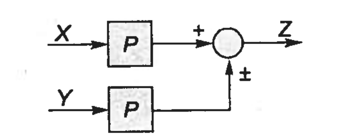
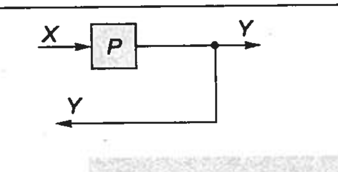
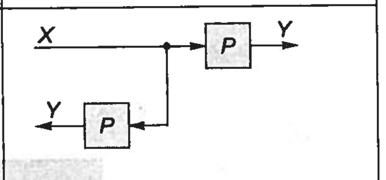
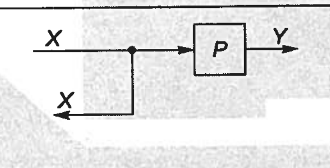
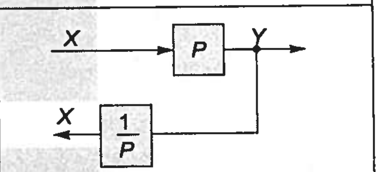
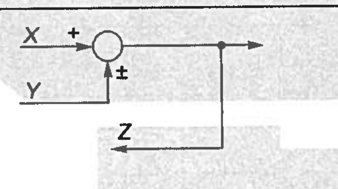
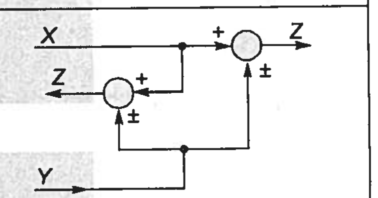
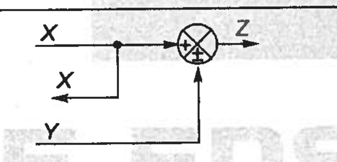
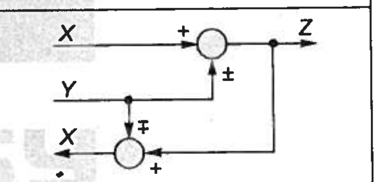
$$C(s) = C_1(s) + C_2(s)$$

or

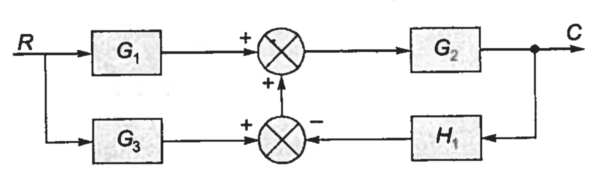
$$C(s) = R_1(s)\,G(s) + R_2(s)\,G(s)$$

The following chart is illustrating ***the transformation theorems***. The letter P is used to represent any transfer function, and W, X, Y, Z denote any transformed signals.

	Transformation	Equation	Block Diagram	Equivalent Block Diagram
1.	Combining blocks in cascade	$Y = (P_1 P_2)X$		
2.	Combining blocks in parallel; or Eliminating a forward loop	$Y = P_1X \pm P_2X$		
3.	Removing a block from a forward path	$Y = P_1X \pm P_2X$		
4.	Eliminating a feedback loop	$Y = P_1(X \mp P_2Y)$		
5.	Removing a block from a feedback loop	$Y = P_1(X \mp P_2Y)$		
6a	Rearranging summing points	$Z = W \pm X \pm Y$		
6b	Rearranging summing points	$Z = W \pm X \pm Y$		

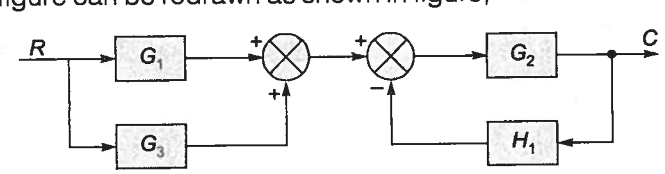
7.	Moving a summing point before a block	$Z = PX \pm Y$		
8.	Moving a summing point beyond a block	$Z = P[X \pm Y]$		
9.	Moving a takeoff point before a block	$Y = PX$		
10.	Moving a takeoff point beyond a block	$Y = PX$		
11.	Moving a takeoff point before a summing point	$Y = X \pm Y$		
12.	Moving a takeoff point beyond a summing point	$Z = X \pm Y$		

Example-3.1 For the block diagram shown in figure, determine the overall trasnfer function C/R.

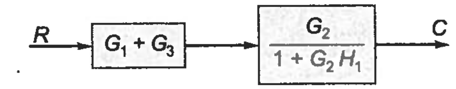


Solution:

The block diagram of figure can be redrawn as shown in figure,



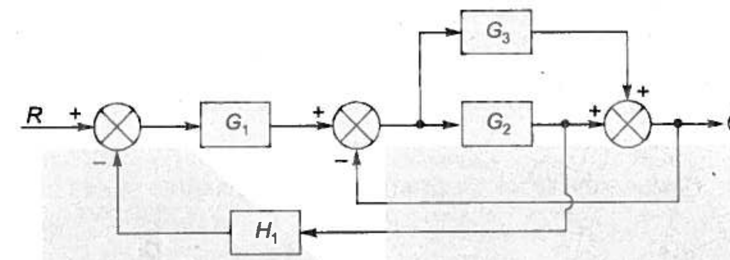
Eliminate two summing points,



Therefore,

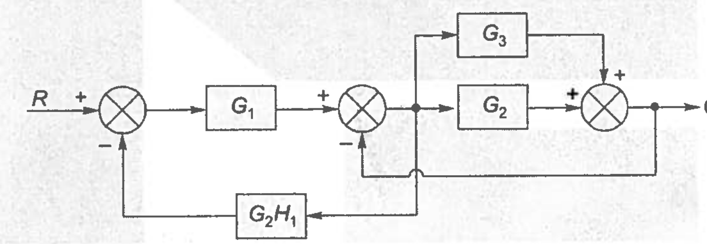
$$\frac{C}{R} = \frac{(G_1 + G_3)G_2}{1 + G_2 H_1} = \frac{G_1 G_2 + G_2 G_3}{1 + G_2 H_1}$$

Example-3.2 Determine the transfer function C/R from the block diagram shown in figure.



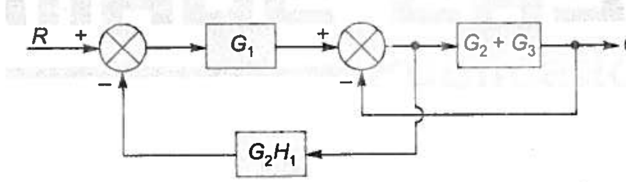
Solution:

Shift the takeoff point after block G_2 to a position before block G_2



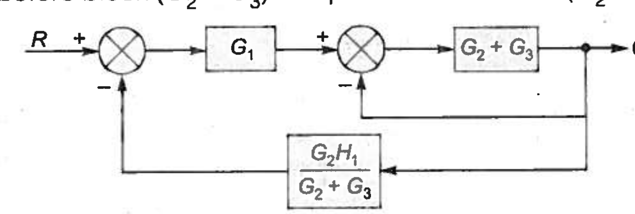
(a)

Eliminate the summing point after block G_2



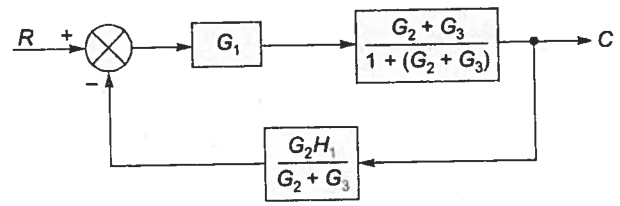
(b)

Shift the takeoff point before block ($G_2 + G_3$) to a position after block ($G_2 + G_3$)



(c)

Eliminate the summing point before block $(G_2 + G_3)$

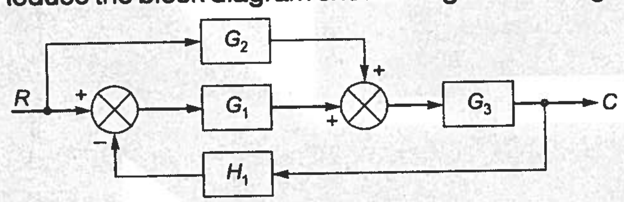


(d)

The overall transfer function is

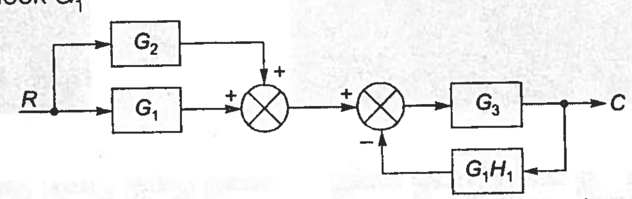
$$\frac{C}{R} = \frac{\frac{G_1(G_2+G_3)}{[1+(G_2+G_3)]}}{1+\frac{G_1(G_2+G_3)}{[1+(G_2+G_3)]}\cdot\frac{G_2H_1}{(G_2+G_3)}} = \frac{G_1G_2+G_1G_3}{1+G_2+G_3+G_1G_2H_1}$$

Example-3.3 **Reduce the block diagram shown in figure into single block representation form.**



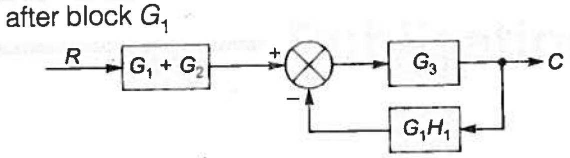
Solution:

Shift the summing point before block G_1 to a position after block G_1 and interchange consecutive summing points after block G_1



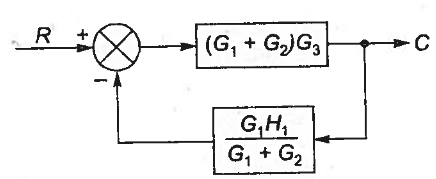
(a)

Eliminate summing point after block G_1



(b)

Shift the summing point after block $(G_1 + G_2)$ to a position before block $(G_1 + G_2)$ and combine blocks $(G_1 + G_2)$ and G_3



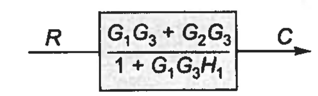
(c)

Figure: *Block diagram reduction of system*

Thus, the overall transfer function is

$$\frac{C}{R} = \frac{(G_1+G_2)G_3}{1+(G_1+G_2)G_3\cdot\frac{G_1H_1}{(G_1+G_2)}} = \frac{G_1G_3+G_2G_3}{1+G_1G_3H_1}$$

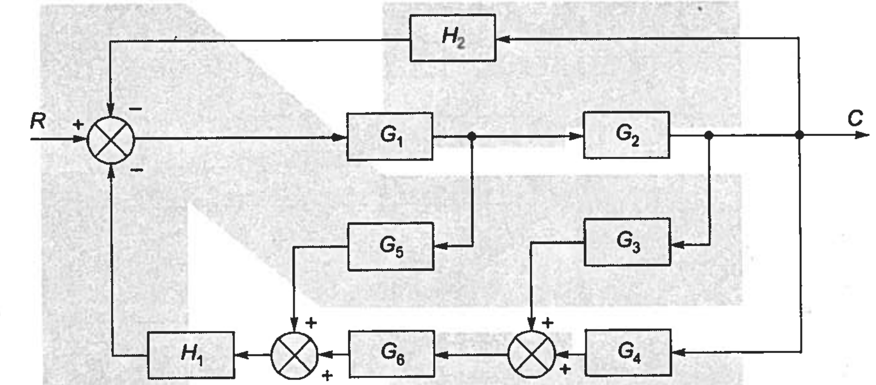
The single block diagram representation is



(d)

Figure: *Single block diagram representation*

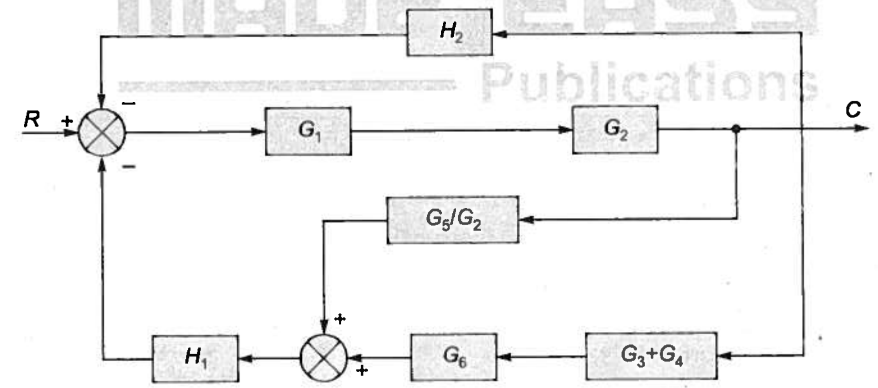
Example-3.4 **Determine the overall transfer function for the block diagram shown below.**



Solution:

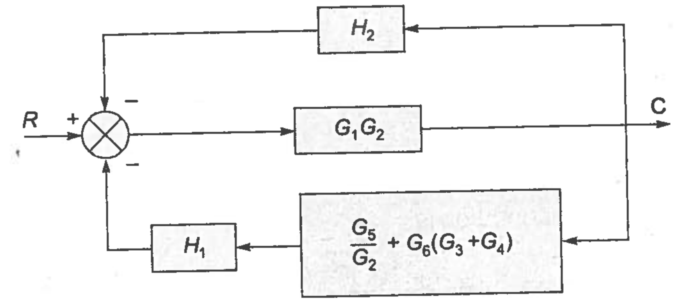
Shift the takeoff point placed before block G_2 towards right side of block G_2

Also, the blocks G_3 and G_4 are in parallel, the equivalence is $(G_3 + G_4)$.



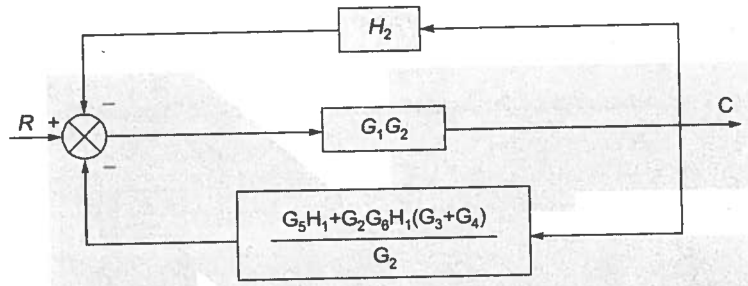
(a)

Therefore, the blocks G_1 and G_2 are in cascade and the blocks G_5/G_2 and $G_6(G_3 + G_4)$ are in parallel, the equivalence is $\left\{\left[\frac{G_5}{G_2} + G_6(G_3+G_4)\right]\right\}$,



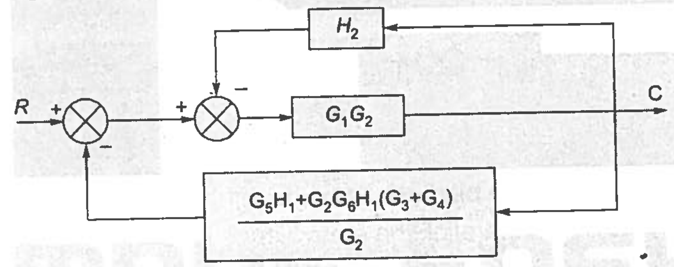
(b)

The blocks H_1 and $\frac{G_5}{G_2} + G_6(G_3 + G_4)$ are in cascade



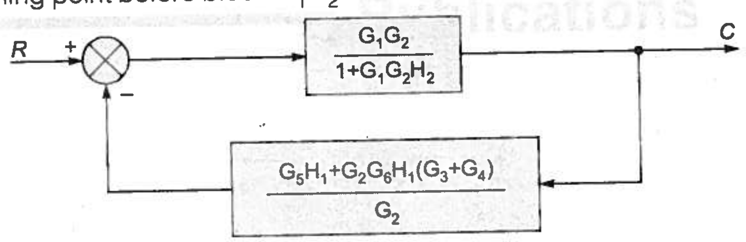
(c)

Spliting the summing point,



(d)

Eliminate the summing point before block G_1G_2



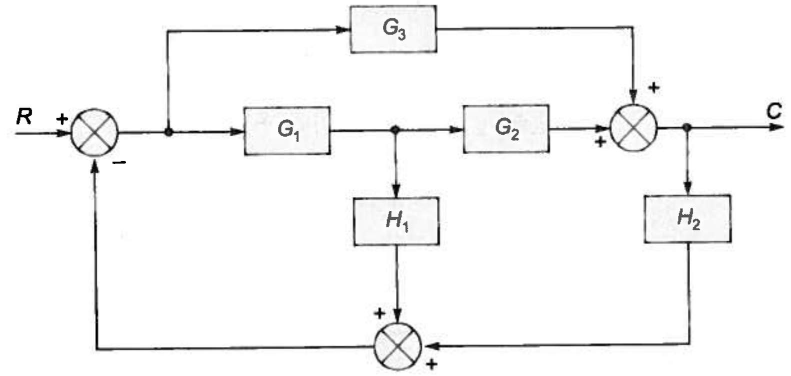
(e)

Thus, the overall transfer function is:

$$\frac{C}{R} = \frac{\dfrac{G_1G_2}{1+G_1G_2H}}{1+\dfrac{G_1G_2}{1+G_1G_2H_2} \times \dfrac{G_5H_1 + G_2G_6H_1(G_3+G_4)}{G_2}}$$

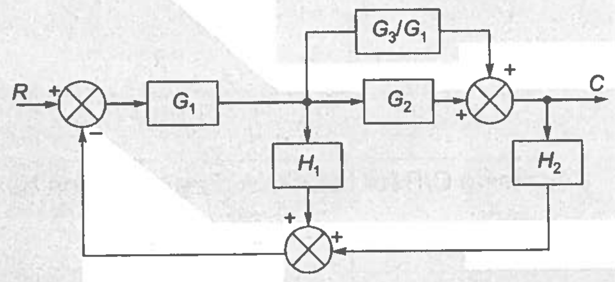
$$\Rightarrow \quad \frac{C}{R} = \frac{G_1G_2}{1+G_1G_2H_2 + G_1G_5H_1 + G_1G_2G_3G_6H_1 + G_1G_2G_4G_6H_1}$$

Example-3.5 The block diagram of a control system is shown below. Determine the overall transfer function.



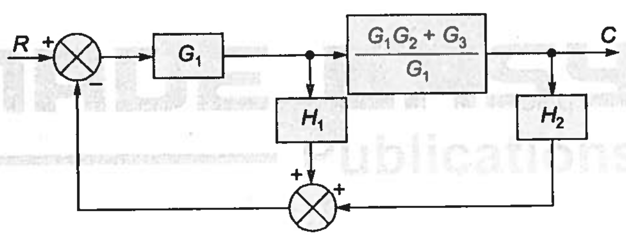
Solution:

Shift the take off point before block G_1 to a position after block G_1



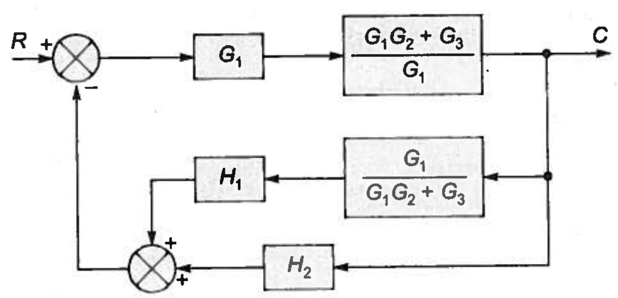
(a)

The blocks G_3/G_1 and G_2 are in parallel, the equivalence is $\frac{G_3}{G_1} + G_2 = \frac{G_1G_2 + G_3}{G_1}$



(b)

Shift the take off point located before block $\frac{(G_1G_2 + G_3)}{G_1}$ towards right side of the same block

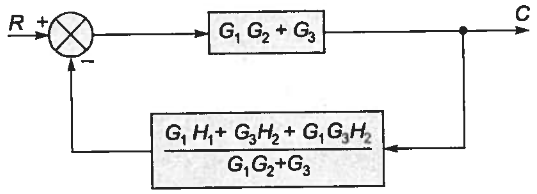


(c)

The blocks G_1 and $\frac{(G_1G_2+G_3)}{G_1}$ are in cascade, the equivalence is $\frac{G_1\times(G_1G_2+G_3)}{G_1} = G_1G_2+G_3$

Also, the blocks H_2 and $\frac{H_1G_1}{(G_1G_2+G_3)}$ are in parallel, the equivalence is

$$H_2+\frac{H_1G_1}{G_1G_2+G_3} = \frac{G_1H_1+G_3H_2+G_1G_2H_2}{G_1G_2+G_3}$$



(d)

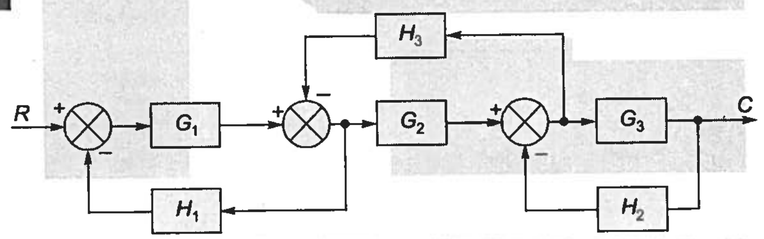
Eliminating the summing point

$$\frac{C}{R} = \frac{G_1G_2+G_3}{1+(G_1G_2+G_3)\times\frac{(G_1H_1+G_3H_2+G_1G_2H_2)}{G_1G_2+G_3}}$$

Simplifying,

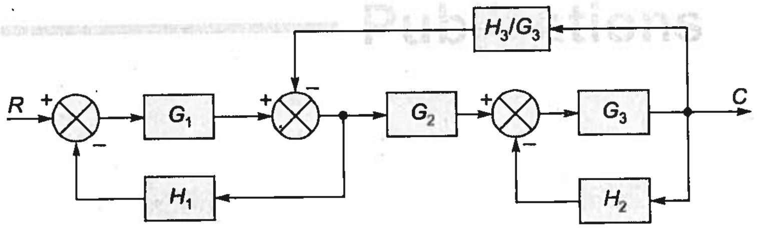
$$\frac{C}{R} = \frac{G_1G_2+G_3}{1+G_1H_1+G_3H_2+G_1G_2H_2}$$

Example-3.6 **Determine C/R for the block diagram given below.**



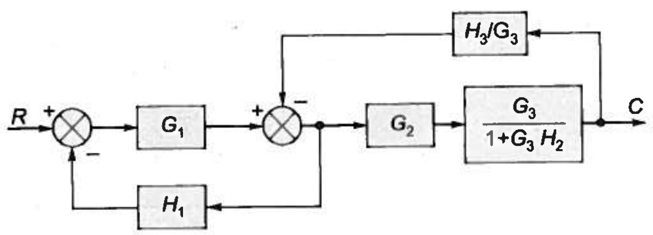
Solution:

Shift the take off point placed before block G_3 to a position after block G_3



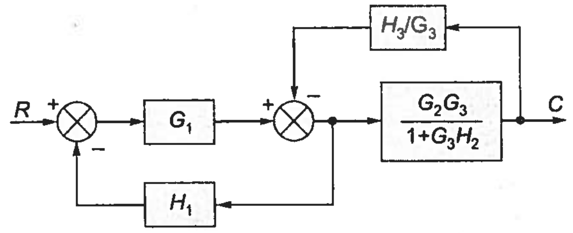
(a)

Eliminate the summing point placed before block G_3



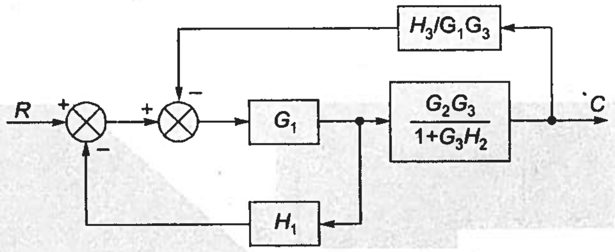
(b)

The blocks G_2 and $\frac{G_3}{(1+G_3H_2)}$ are in cascade,



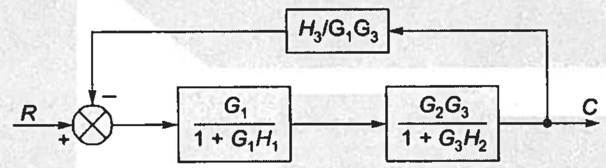
(c)

Shift the summing point after block G_1 to a location before block G_1



(d)

Interchange consecutive summing points and then eliminate the summing point before block G_1



(e)

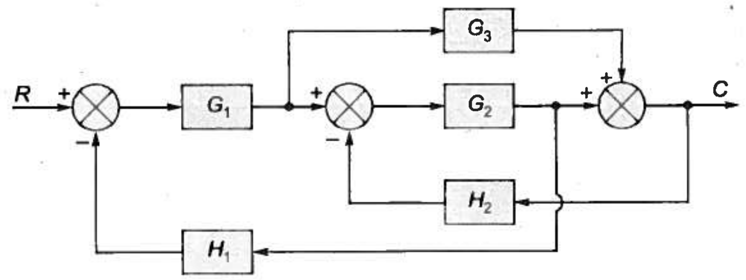
Hence, the forward path transfer function is $\frac{G_1G_2G_3}{(1+G_1H_1)(1+G_3H_2)}$

and the feedback path transfer function is $\frac{H_3}{G_1G_3}$

Therefore,

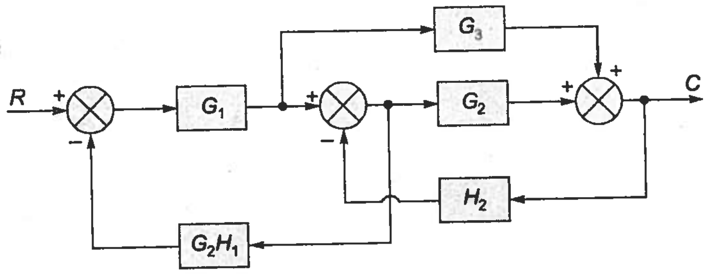
$$\frac{C}{R} = \frac{G_1G_2G_3}{1+G_1H_1+G_2H_3+G_3H_2+G_1G_3H_1H_2}$$

Example-3.7 **Determine the overall transfer function relating C and R for the system whose block diagram is shown in figure.**



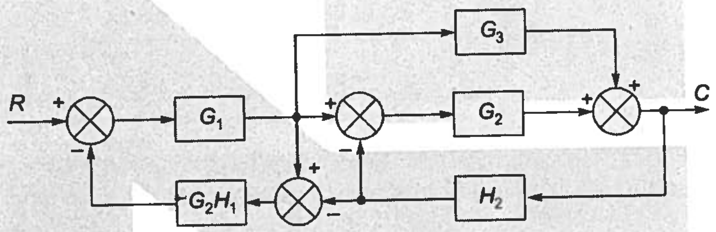
Solution:

Shift the take off point after block G_2 to a position before block G_2



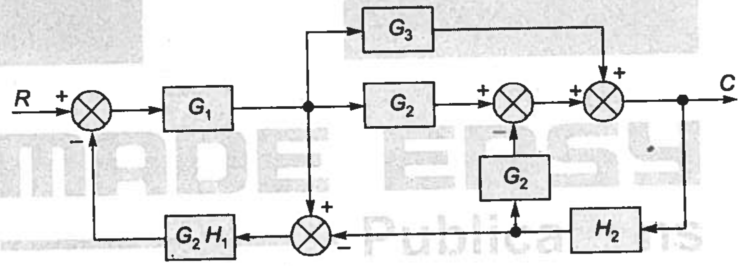
(a)

Shift the take off point before block G_2 to a position after block G_1



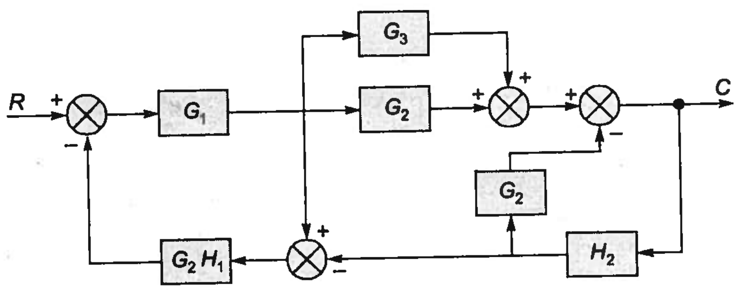
(b)

Shift summing point before block G_2 to a position after block G_2



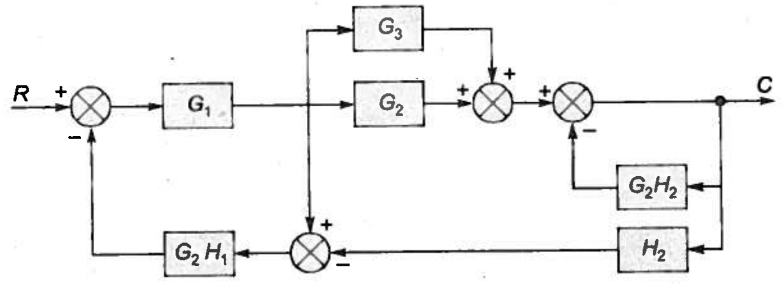
(c)

Interchange consecutive summing points after block G_2



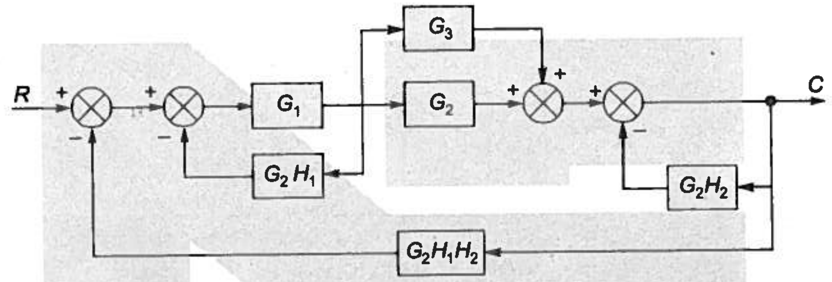
(d)

Shift the take off point after block H_2 to a position before block H_2



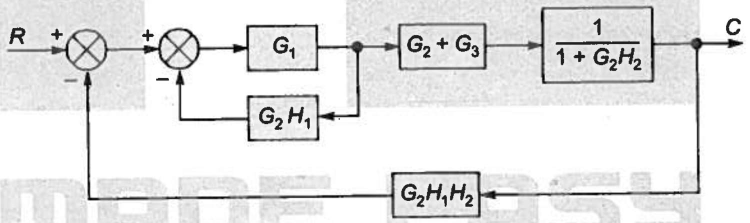
(e)

The block diagram can be redrawn as,



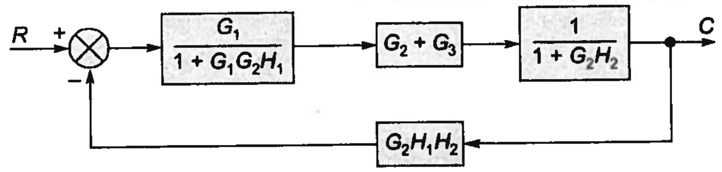
(f)

Eliminate two consecutive summing points after block G_2



(g)

Eliminate the summing point before block G_1



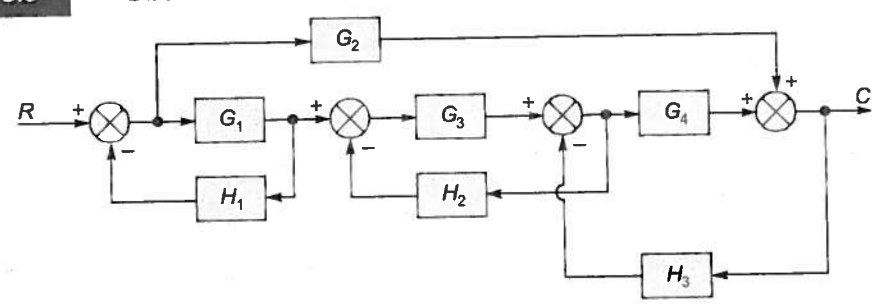
(h)

Thus, the overall transfer function is

$$\frac{C}{R} = \frac{\frac{G_1}{1+G_1G_2H_1}\cdot(G_2+G_3)\cdot\frac{1}{(1+G_2H_2)}}{1+\frac{G_1}{(1+G_1G_2H_1)}\cdot(G_2+G_3)\cdot\frac{1}{(1+G_2H_2)}\cdot(G_2H_1H_2)}$$

$$= \frac{G_1G_2+G_1G_3}{1+G_1G_2H_1+G_2H_2+G_1G_2H_1H_2(2G_2+G_3)}$$

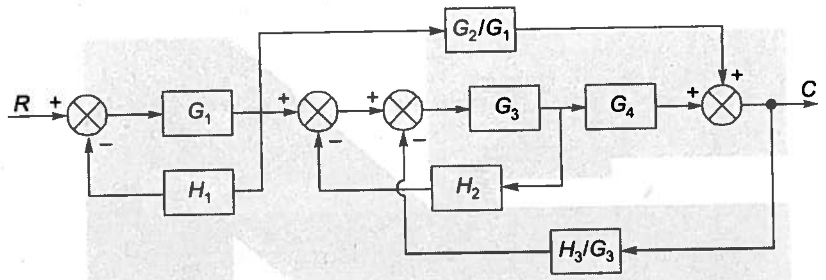
Example-3.8 Obtain the overall transfer function for the block diagram



Solution:

Shift the take off point placed before G_1 towards right side of block G_1

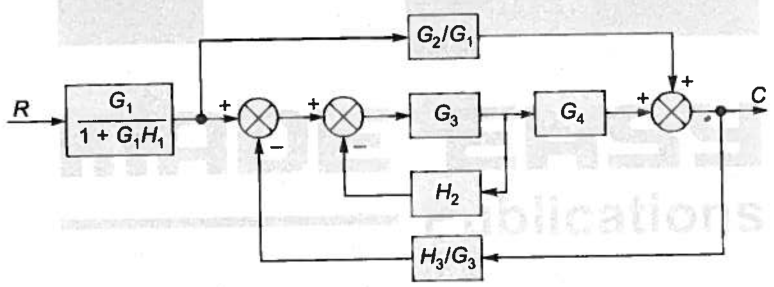
Also, shift the summing point located after block G_3 towards left side of block G_3



(a)

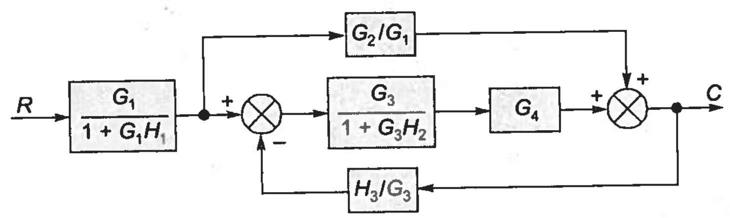
Eliminate summing point before block G_1, the equivalence is $\frac{G_1}{(1+G_1H_1)}$

Interchange consecutive summing points placed before block G_3



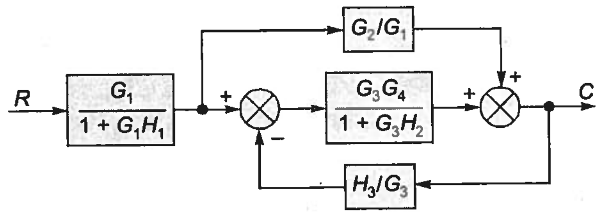
(b)

Eliminate the summing point located before block G_3, the equivalence is $\frac{G_3}{(1+G_3H_2)}$



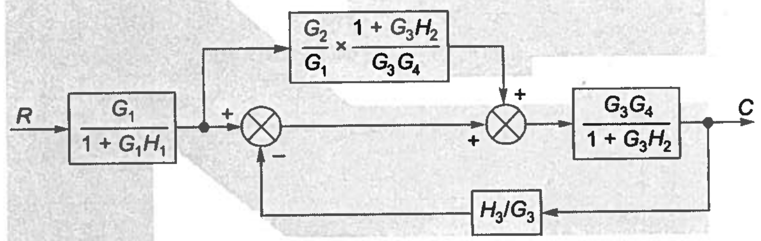
(c)

The block G_4 and $\frac{G_3}{(1+G_3H_2)}$ are in cascade, the equivalence is $\frac{G_3G_4}{(1+G_3H_2)}$



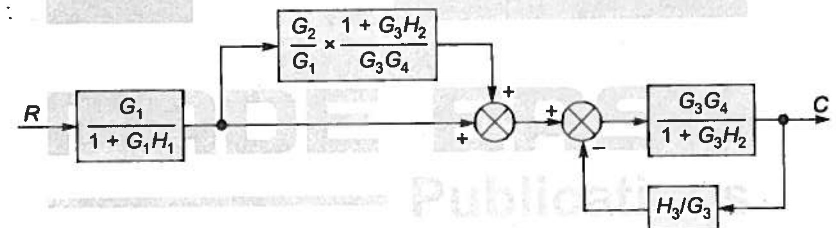
(d)

Shift the summing point located after block $\frac{G_3G_4}{(1+G_3H_2)}$ towards its left side



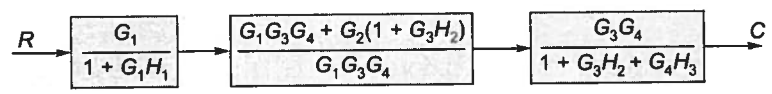
(e)

Interchange consecutive summing points



(f)

Eliminate the summing points,



(g)

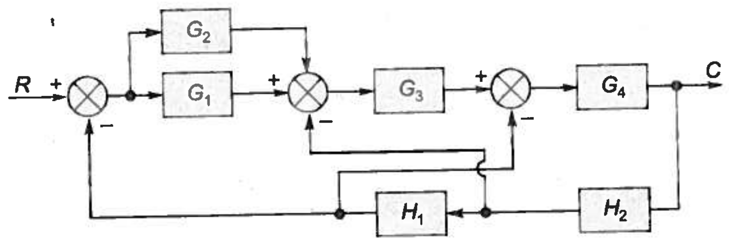
Thus,

$$\frac{C}{R} = \frac{G_1}{1+G_1H_1} \times \frac{G_1G_3G_4+G_2(1+G_3H_2)}{G_1G_3G_4} \times \frac{G_3G_4}{1+G_3H_2+G_4H_3}$$

On simplifying,

$$\frac{C}{R} = \frac{G_1G_3G_4+G_2(1+G_3H_2)}{1+G_1H_1+G_3H_2+G_4H_3+G_1G_3H_1H_2+G_1G_4H_1H_3}$$

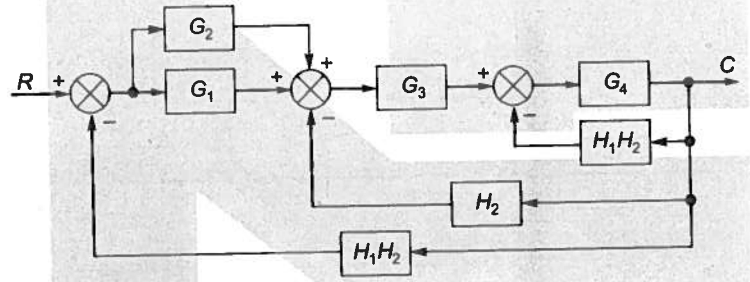
Example-3.9 Obtain the overall transfer function for a system represented by the block diagram given below.



Solution:

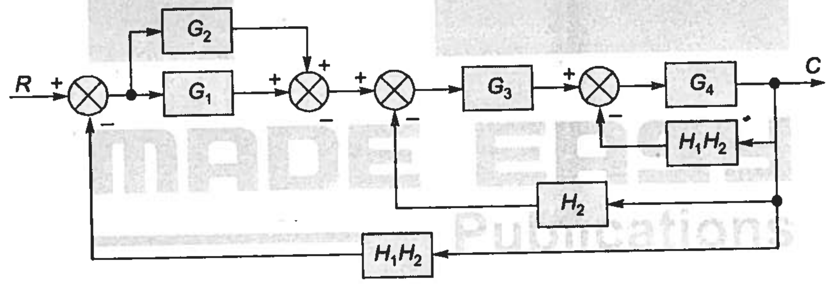
Shift the take-off point located after block H_2 towards right side of H_2

Also, shift the take-off point placed after block H_1 towards right side of block H_2



(a)

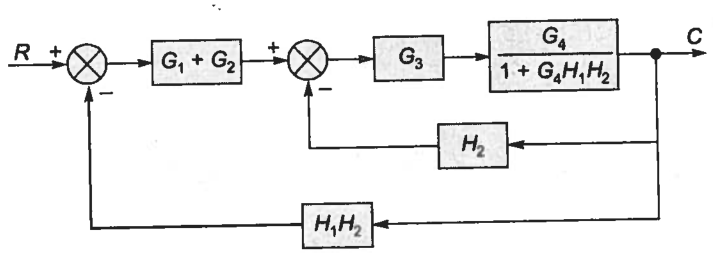
Split the summing point located after block G_1



(b)

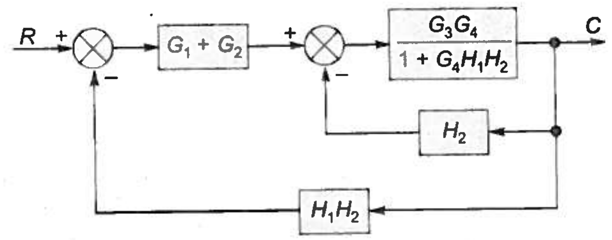
The blocks G_1 and G_2 are parallel, the equivalence is $(G_1 + G_2)$

Also, eliminate the summing point placed before block G_4, the equivalence is $\frac{G_4}{(1+G_4H_1H_2)}$.



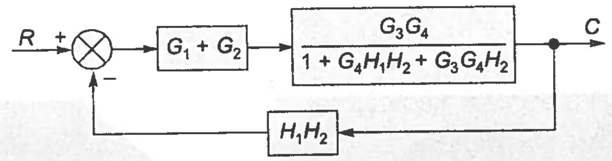
(c)

The blocks G_3 and $\frac{G_4}{1+G_4H_1H_2}$ are in cascade, the equivalence is $\frac{G_3G_4}{(1+G_4H_1H_2)}$.



(d)

Eliminate summing point placed after block $(G_1 + G_2)$



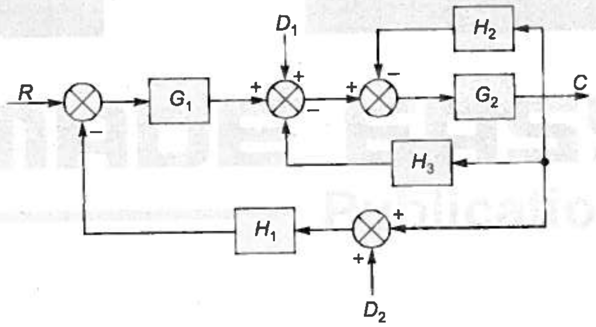
(e)

$$\therefore \quad \frac{C}{R} = \frac{\dfrac{(G_1+G_2)G_3G_4}{1+G_4H_1H_2+G_3G_4H_2}}{1+\dfrac{(G_1+G_2)G_3G_4}{1+G_4H_1H_2+G_3G_4H_2}\times H_1H_2}$$

On simplifying,

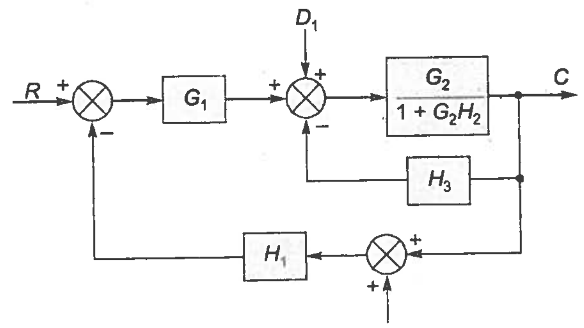
$$\frac{C}{R} = \frac{G_1G_3G_4 + G_2G_3G_4}{1+G_3G_4H_2+G_4H_1H_2+G_1G_3G_4H_1H_2+G_2G_3G_4H_1H_2}$$

Example-3.10 Determine the expression for the output for the system block diagram.



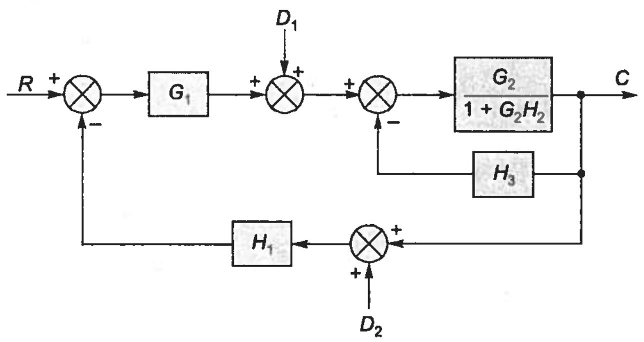
Solution:

Eliminate the summing point located before block G_2



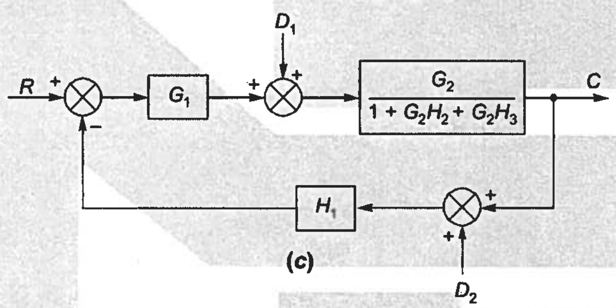
(a)

Split the summing point placed before block $\frac{G_2}{1+G_2H_2}$.



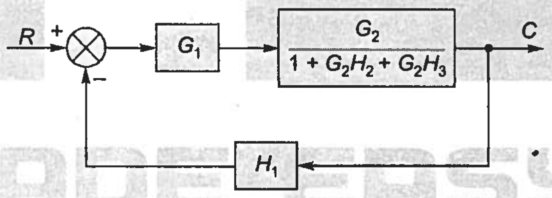
(b)

Eliminate the summing point located before block $\frac{G_2}{(1+G_2H_2)}$



(c)

Consider $D_1 = 0$, $D_2 = 0$ and the output as C_r for the input R acting alone



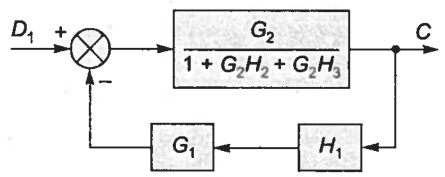
(d)

Eliminate summing point before block G_1

$$\therefore \quad \frac{C_r}{R} = \frac{\frac{G_1G_2}{1+G_2H_2+G_2H_3}}{1+\frac{G_1G_2}{1+G_2H_2+G_2H_3}\times H_1}$$

On simplifying, $$C_r = \left(\frac{G_1G_2}{1+G_2H_2+G_2H_3+G_1G_2H_1}\right)\cdot R$$

Consider $R = 0$, $D_2 = 0$ and the output as C_{d1} for the input D_1 acting alone

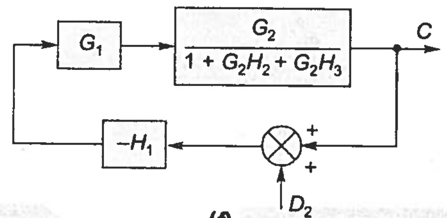


(e)

$$\therefore \quad \frac{C_{d1}}{D_1} = \frac{\frac{G_2}{1+G_2H_2+G_2H_3}}{1+\frac{G_2}{1+G_2H_2+G_2H_3}\times G_1H_1}$$

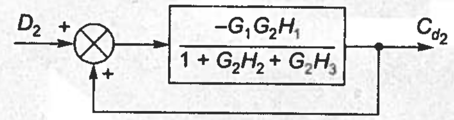
On simplifying, $$C_{d1} = \left(\frac{G_2}{1+G_2H_2+G_2H_3+G_1G_2H_1}\right)D_1$$

Consider $R = 0$, $D_1 = 0$ and the output as C_{d2} for the input D_2 acting alone



(f)

The blocks $-H_1$, G_1 and $\frac{G_2}{(1+G_2H_2+G_2H_3)}$ are in cascade, the equivalence is $\frac{-G_1G_2H_1}{1+G_2H_2+G_2H_3}$.



(g)

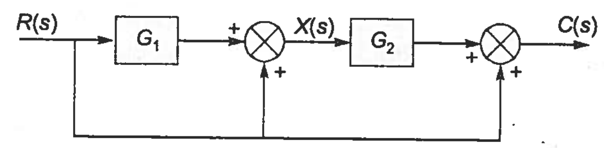
$$\therefore \quad \frac{C_{d2}}{D_2} = \frac{\frac{-G_1G_2H_1}{1+G_2H_2+G_2H_3}}{1-\frac{-G_1G_2H_1}{1+G_2H_2+G_2H_3}\times 1}$$

On simplifying, $$C_{d2} = \left(-\frac{G_1G_2H_1}{1+G_2H_2+G_2H_3+G_1G_2H_1}\right)\cdot D_2$$

The output $C = C_r + C_{d1} + C_{d2}$

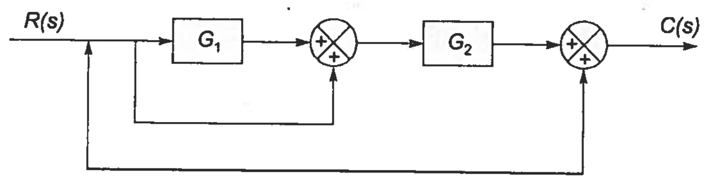
or $$C = \frac{G_1G_2}{1+G_2H_2+G_2H_3+G_1G_2H_1}\cdot R + \frac{G_2}{1+G_2H_2+G_2H_3+G_1G_2H_1}\cdot D_1 + \frac{-G_1G_2H_1}{1+G_2H_2+G_2H_3+G_1G_2H_1}\cdot D_2$$

Example-3.11 **Simplify the block diagram shown in figure and obtain the transfer function relating $C(s)$ and $R(s)$.**



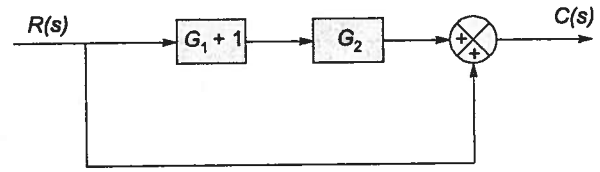
Solution:

The block diagram can be modified as,



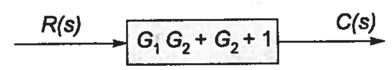
(a)

Eliminating the minor feedforward path,



(b)

$$\frac{C(s)}{R(s)} = G_1 G_2 + G_2 + 1$$



(c)

The same result can also be obtained by proceeding as follows. Since signal $X(s)$ is the sum of two signals $G_1 R(s)$ and $R(s)$, we have

$$X(s) = G_1 R(s) + R(s)$$

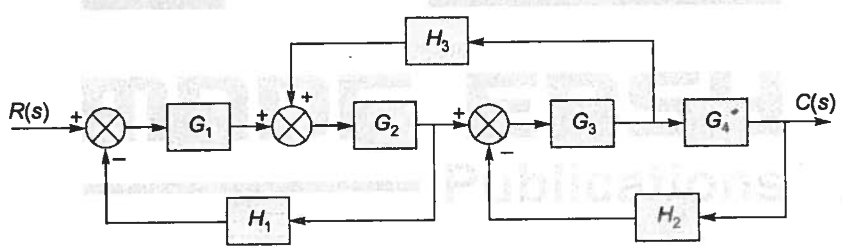
The output signal $C(s)$ is the sum of $G_2 X(s)$ and $R(s)$. Hence

$$C(s) = G_2 X(s) + R(s)$$
$$= G_2[G_1 R(s) + R(s)] + R(s)$$

So, we have the same result as before

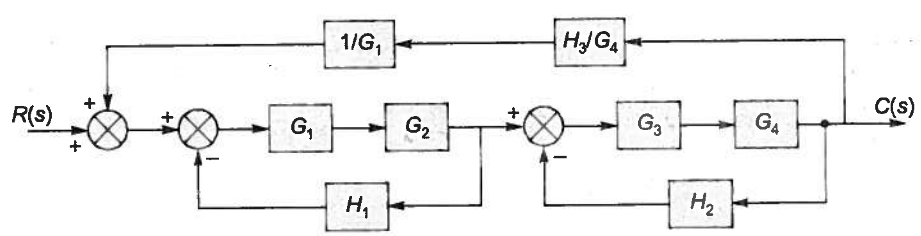
$$\frac{C(s)}{R(s)} = G_1 G_2 + G_2 + 1$$

Example-3.12 **Simplify the block diagram shown in figure and obtain the closed-loop transfer function $C(s)/R(s)$.**



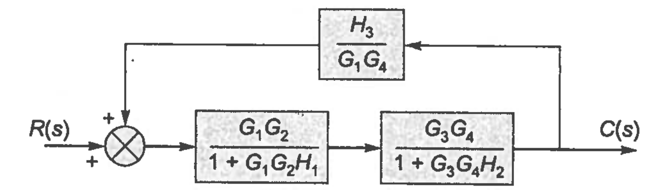
Solution:

Move the branch point between G_3 and G_4 to the right-hand side of the loop containing G_3, G_4 and H_2. Also, move the summing point between G_1 and G_2 to the left-hand side of the first summing point.

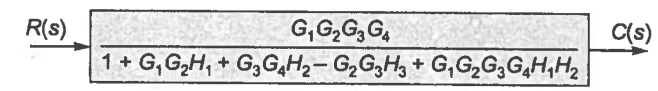


(a)

By simplifying each loop,



(b)



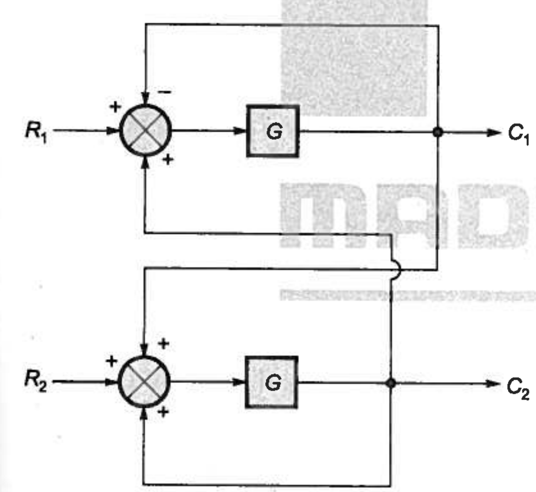
(c)

$$\frac{C(s)}{R(s)} = \frac{G_1 G_2 G_3 G_4}{1 + G_1 G_2 H_1 + G_3 G_4 H_2 - G_2 G_3 H_3 + G_1 G_2 G_3 G_4 H_1 H_2}$$

Student's Assignments 1

Common Data Q.1 and Q.2

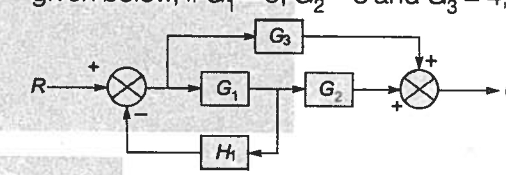
A block diagram of feedback control system is shown in following figure



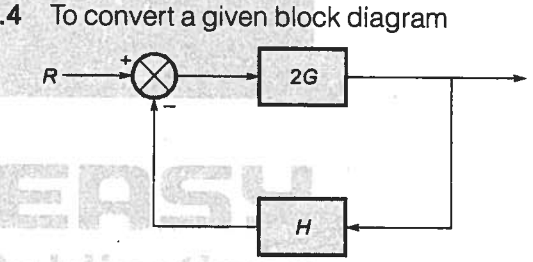
Q.1 The transfer function $\left.\frac{C_1}{R_1}\right|_{R_2=0}$ is $\frac{aG(1-bG)}{1-2G^2}$ then $a + b$ = ________.

Q.2 The transfer function $\left.\frac{C_1}{R_2}\right|_{R_1=0}$ is $\frac{dG^2}{1-eG^2}$ then $d + e$ = ________.

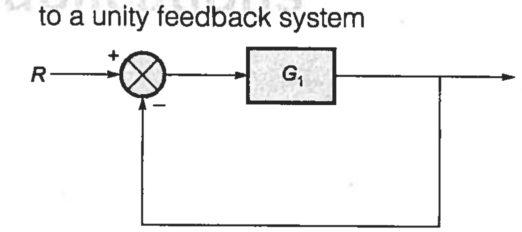
Q.3 ________ is the value of $\frac{C}{R}$ for the system given below, if $G_1 = 5$, $G_2 = 8$ and $G_3 = 4$, $H_1 = 2$.



Q.4 To convert a given block diagram



to a unity feedback system

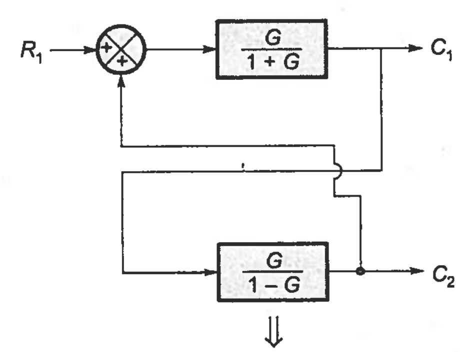


Find the value of G_1.

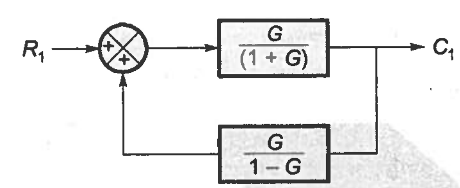
Student's Assignments 1 Explanations

1. (2)

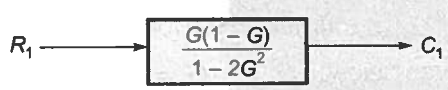
$\left.\frac{C_1}{R_1}\right|_{R_2=0}$



⇓



⇓



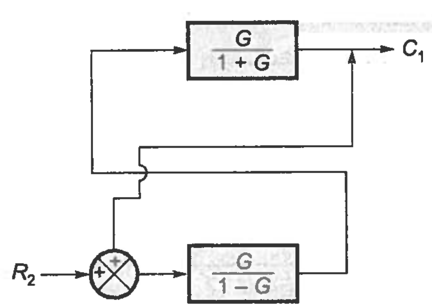
$$\left.\frac{C_1}{R_1}\right|_{R_2=0} = \frac{G(1-G)}{1-2G^2}$$

It compares with $\frac{aG(1-b^n)}{1-2G^2}$

$a = 1, \quad b = 1, \quad a + b = 1 + 1 = 2$

2. (3)

$$\left.\frac{C_1}{R_2}\right|_{R_1=0}$$



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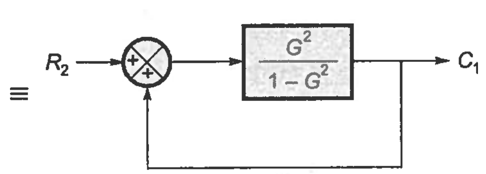
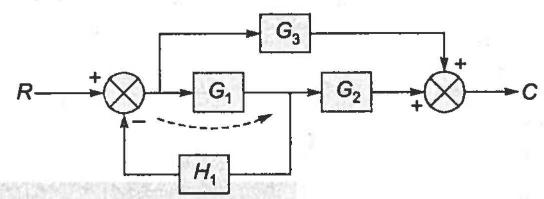


$$\left.\frac{C_1}{R_2}\right|_{R_1=0} = \frac{\frac{G}{1+G}\cdot\frac{G}{1-G}}{1-\frac{G^2}{1-G^2}} = \frac{G^2}{1-2G^2}$$

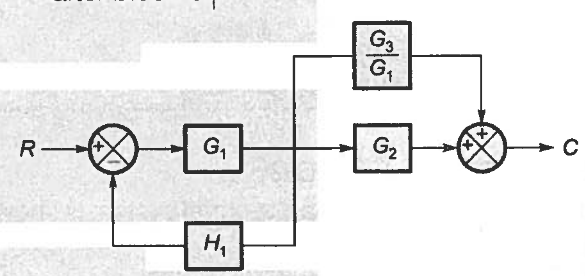
It compares with $\frac{dG^2}{1-eG^2}$

$$d + e = 1 + 2 = 3$$

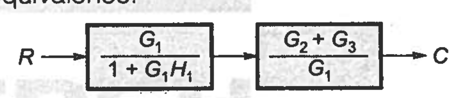
3. (4)

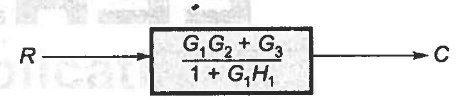


Shift the take-off point before G_1 to a position after block G_1.



Eliminate summing point + before G_1, the equivalence and summing point after G_2, the equivalence.





$$\frac{C}{R} = \frac{G_1G_2+G_3}{1+G_1H_1}$$

Put the value of G_1, G_2, G_3 and H_1

$$\frac{C}{R} = \frac{5.8+4}{1.+5.2} = \frac{44}{11} = 4$$

4. $\left(\frac{2G}{1-2G+2GH}\right)$

$$\frac{2G}{1+2GH} = \frac{G_1}{1+G_1}$$

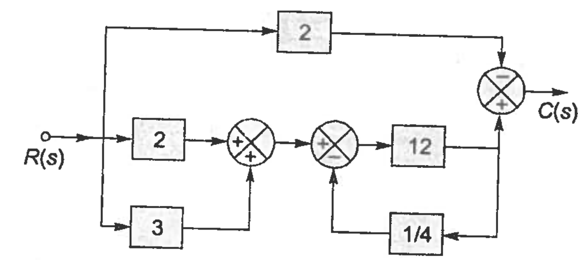
$$2G + 2GG_1 = G_1 + 2GHG_1$$

$$G_1(2G - 1 - 2GH) = -2G$$

$$G_1 = \frac{2G}{2GH+1-2G}$$

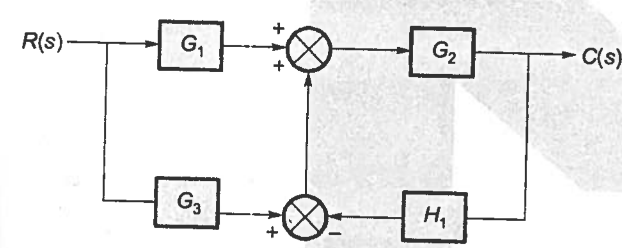
Student's Assignments 2

Q.1 What is the overall gain of the system given below?



(a) –10 (b) 13
(c) 17 (d) 36

Q.2 Consider the block diagram shown below.



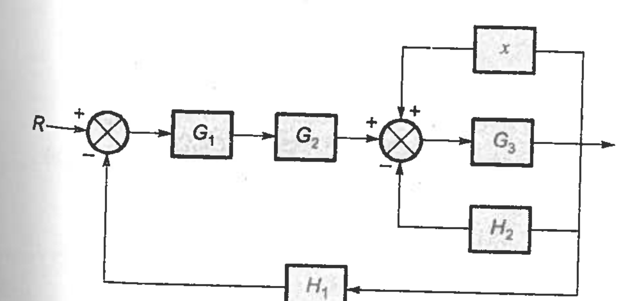
If the transfer function of the system is given by

$$T(s) = \frac{G_1G_2 + G_2G_3}{1+X}. \text{ Then 'X' is}$$

(a) $G_2G_3H_1$ (b) G_2H_1
(c) $G_1G_2H_1$ (d) G_3H_1

Q.3 A system block diagram is shown in the given figure. The overall transfer function of the system is

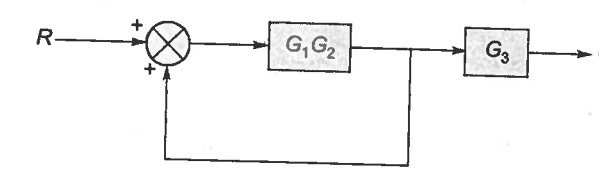
$$\frac{C}{R} = \frac{G_1G_2G_3}{1+G_1G_2G_3H_1+G_3H_2-G_2G_3H_3}$$



Determine the value of x.

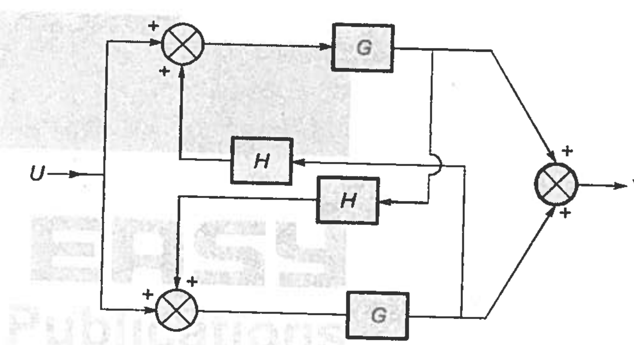
(a) H_3 (b) G_2H_3
(c) G_1H_3 (d) G_3H_3

Q.4 For the block diagram given in the following figure, the expression of $\frac{C}{R}$ is



(a) $\frac{G_1G_2G_3}{1-G_2G_1}$ (b) $\frac{G_1G_2}{1-G_1G_2G_3}$
(c) $\frac{G_1G_2G_3}{1-G_1G_2G_3}$ (d) $\frac{G_1G_2}{G_3(1-G_1G_2)}$

Q.5 The overall transfer function of the system shown in given figure is



(a) $\frac{G}{1-GH}$ (b) $\frac{GH}{1-GH}$
(c) $\frac{2G}{1-2GH}$ (d) $\frac{2G}{1-GH}$

Q.6 The transfer function $C(s)/R(s)$ of the system, whose block diagram is given below is