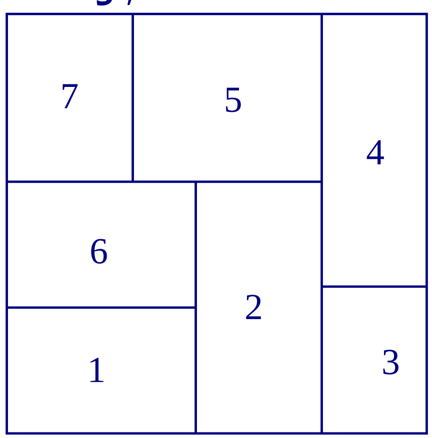
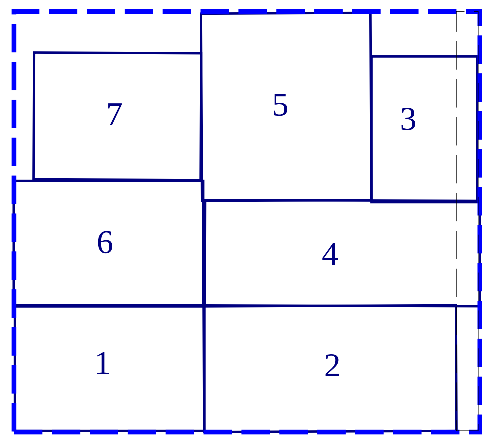
Floorplanning

- Inputs to the floorplanning problem:
  - A set of blocks, fixed or flexible.
  - Pin locations of fixed blocks.
  - A netlist.
- Objectives: **Minimize area**, **reduce wirelength** for (critical) nets, **maximize routability**, determine shapes of flexible blocks

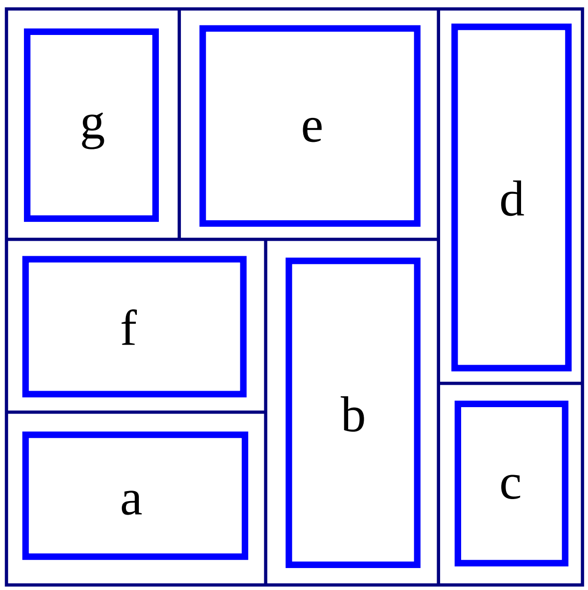


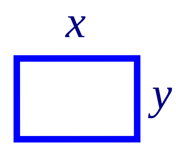
An optimal floorplan, in terms of area

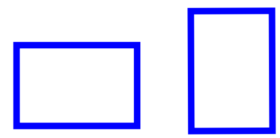


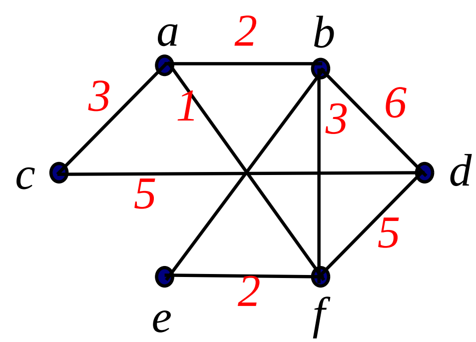
A non−optimal floorplan

Floorplan Design



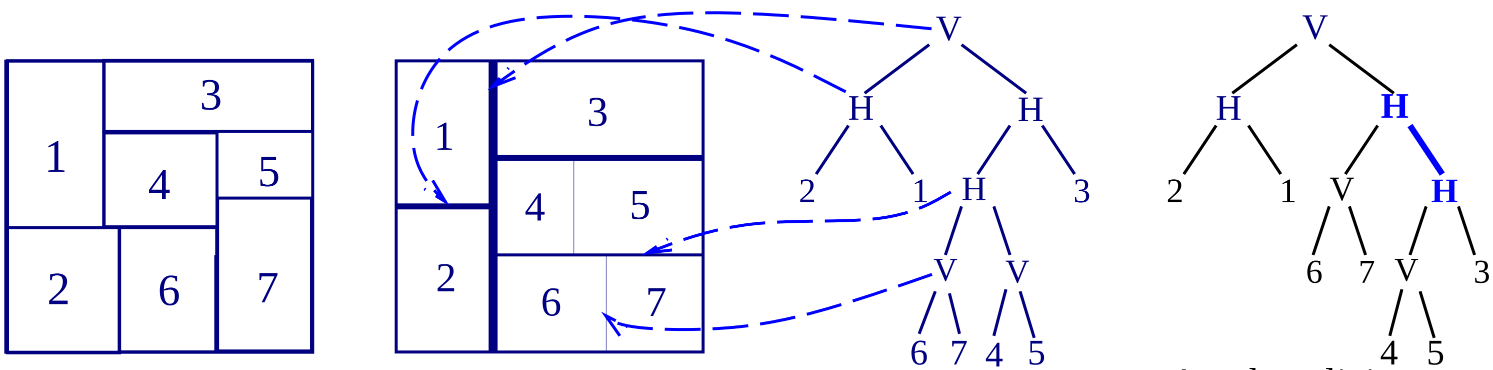
- *Modules:* 

- *Area: A=xy*
- *Aspect ratio: r <= y/x <= s*
- *Rotation:* 
- *Module connectivity*



Floorplanning: Terminology

- **Rectangular dissection:** Subdivision of a given rectangle by a finite # of horizontal and vertical line segments into a finite # of non-overlapping rectangles.
- **Slicing structure:** a rectangular dissection that can be obtained by repetitively subdividing rectangles horizontally or vertically.
- **Slicing tree:** A binary tree, where each internal node represents a vertical cut line or horizontal cut line, and each leaf a basic rectangle.
- **Skewed slicing tree:** One in which no node and its **right** child are the same.



Non−slicing floorplan

Slicing floorplan

A slicing tree (skewed)

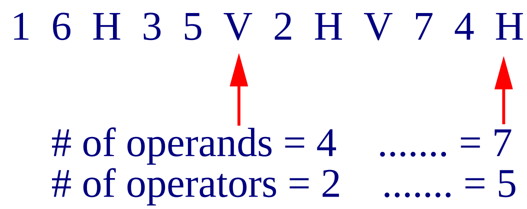
Another slicing tree (non−skewed)

Floorplan Design by Simulated Annealing

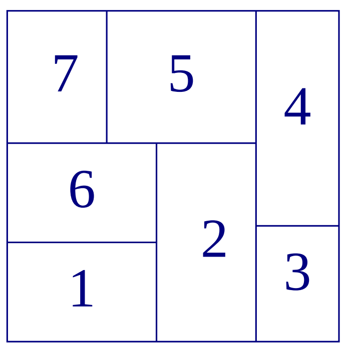
- Related work
  - Wong & Liu, "A new algorithm for floorplan design," DAC'86.
    * Consider slicing floorplans.
  - Wong & Liu, "Floorplan design for rectangular and L-shaped modules," ICCAD'87.
    * Also consider L-shaped modules.
  - Wong, Leong, Liu, Simulated Annealing for VLSI Design, pp. 31–71, Kluwer academic Publishers, 1988.
- Ingredients: solution space, neighborhood structure, cost function, annealing schedule?

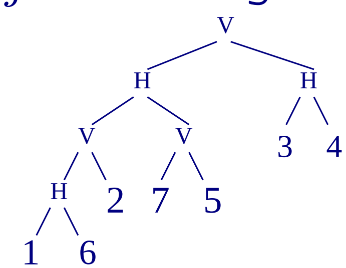
Solution Representation

- An expression $E = e_1 e_2 \ldots e_{2n-1}$, where $e_i \in \{1, 2, \ldots, n, H, V\}, 1 \leq i \leq 2n-1$, is a **Polish expression** of length $2n - 1$ iff
  1. every operand j, $1 \leq j \leq n$, appears exactly once in E;
  2. (**the balloting property**) for every subexpression $E_i = e_1 \ldots e_i, 1 \leq i \leq 2n-1$, #operands > #operators.



- Polish expression $\longleftrightarrow$ Postorder traversal.
- ijH: rectangle i on bottom of j; ijV: rectangle i on the left of j.



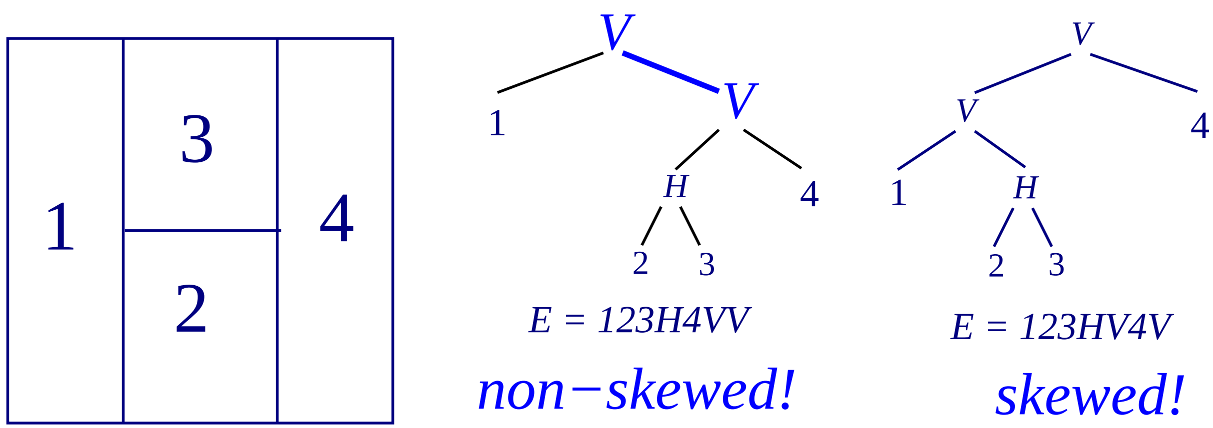


E = 16H2V75VH34HV

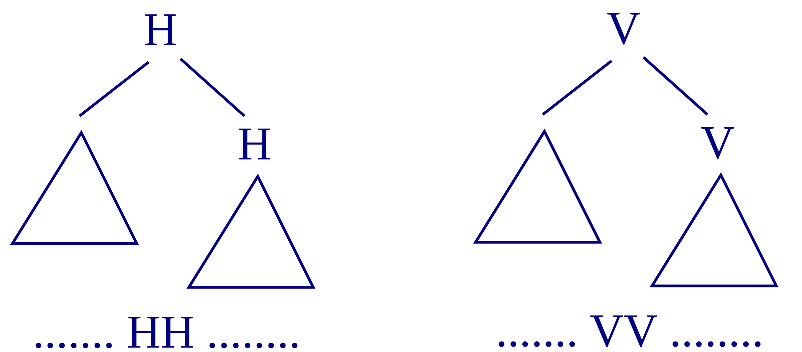
*E = 16+2*75*+34+**

Postorder traversal of a tree!

Solution Representation (cont'd)



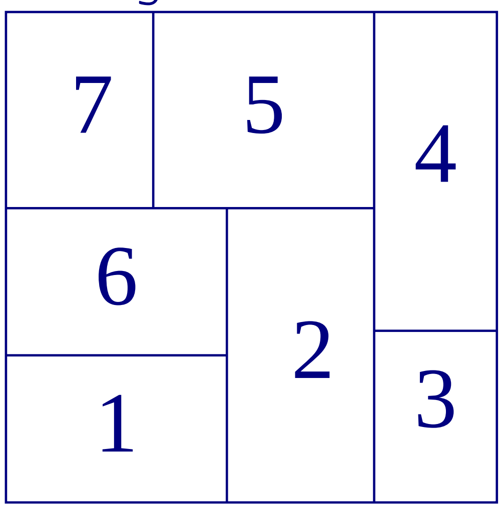
Non−skewed cases

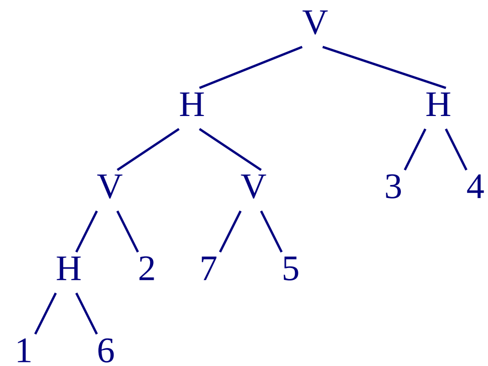


- **Question:** How to eliminate ambiguous representation?

Normalized Polish Expression

- A Polish expression $E = e_1 e_2 \dots e_{2n-1}$ is called **normalized** iff E has no consecutive operators of the same type (H or V).
- Given a **normalized** Polish expression, we can construct a **unique** rectangular slicing structure.



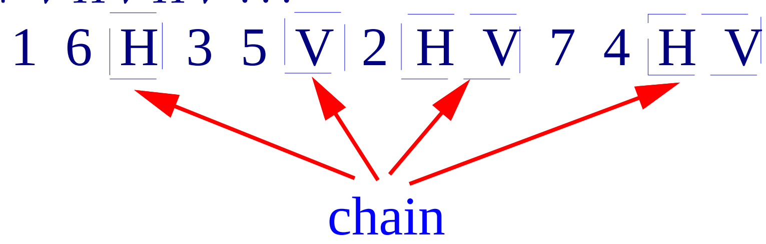


E = 16H2V75VH34HV

A normalized Polish expression

Neighborhood Structure

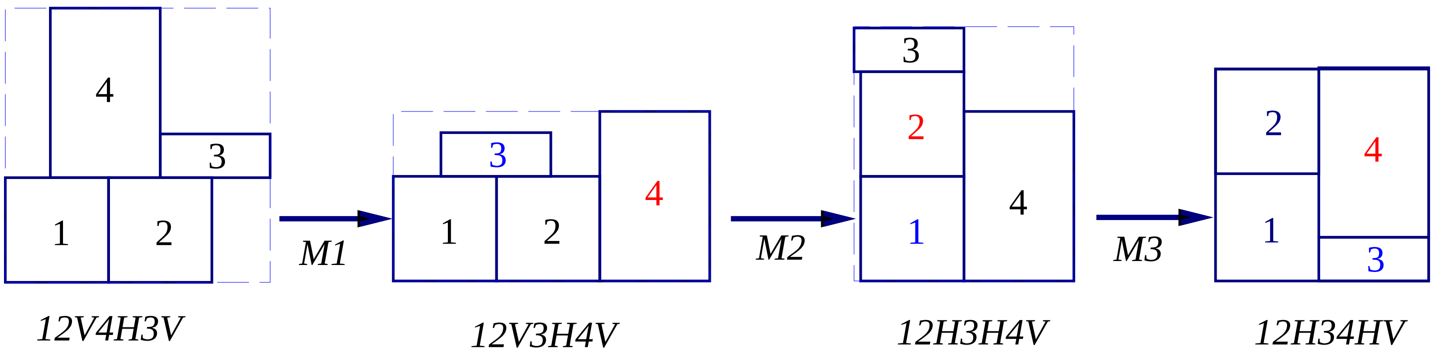
- **Chain:** $HVHVH\ldots$ or $VHVHV\ldots$



- **Adjacent:** 1 and 6 are adjacent operands; 2 and 7 are adjacent operands; 5 and V are adjacent operand and operator.

- 3 types of moves:
  - $M1$ **(Operand Swap)**: Swap two adjacent operands.
  - $M2$ **(Chain Invert)**: Complement some chain ($\overline{V} = H, \overline{H} = V$).
  - $M3$ **(Operator/Operand Swap)**: Swap two adjacent operand and operator.

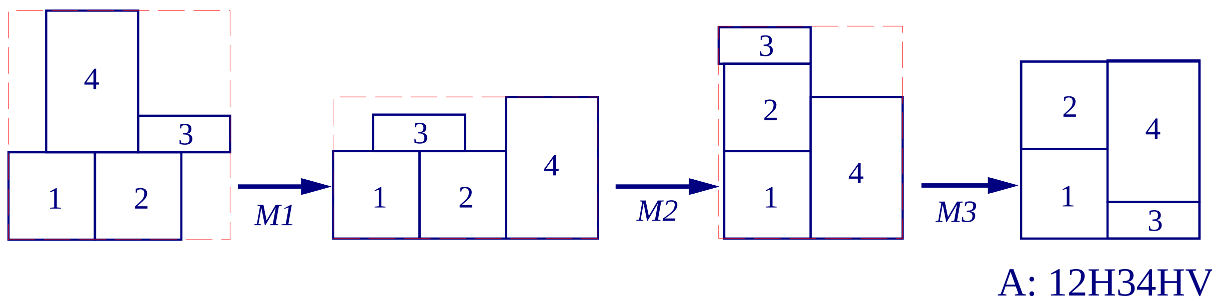
Effects of Perturbation



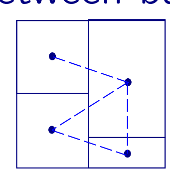
- **Question:** The balloting property holds during the moves?
  - $M1$ and $M2$ moves are OK.
  - **Check the $M3$ moves!** Reject "illegal" $M3$ moves.
- **Check $M3$ moves:** Assume that the M_3 move swaps the operand e_i with the operator e_{i+1}, $1 \leq i \leq k-1$. Then, the swap will not violate the balloting property iff $2N_{i+1} < i$.
  - N_k: # of operators in the Polish expression $E = e_1 e_2 \ldots e_k, 1 \leq k \leq 2n-1$.

Cost Function

- $\Phi = A + \lambda W$.
  - A: area of the smallest rectangle
  - W: overall wiring length
  - λ: user-specified parameter

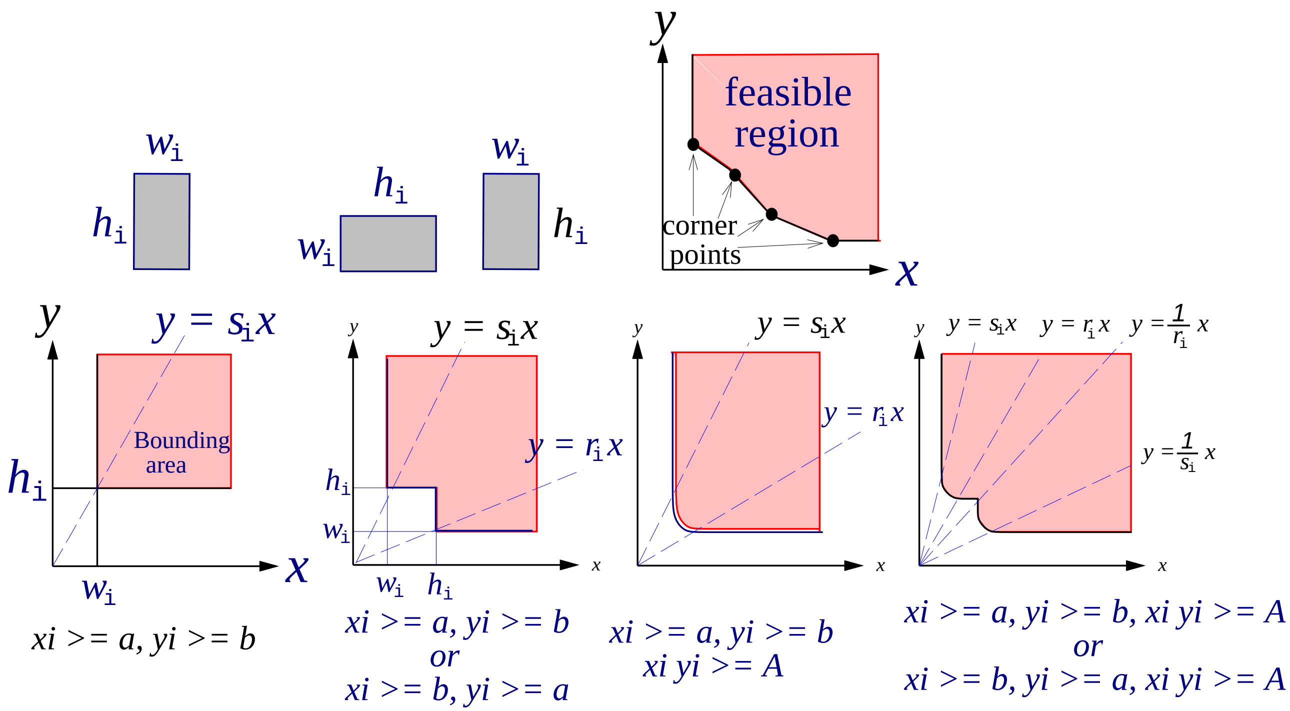


- $W = \sum_{ij} c_{ij} d_{ij}$.
  - c_{ij}: # of connections between blocks i and j.
  - d_{ij}: center-to-center distance between basic rectangles i and j.



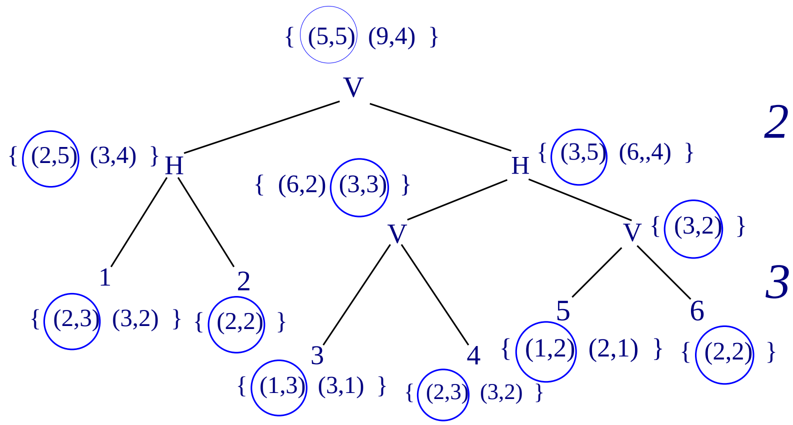
Cost Evaluation: Shape Curves

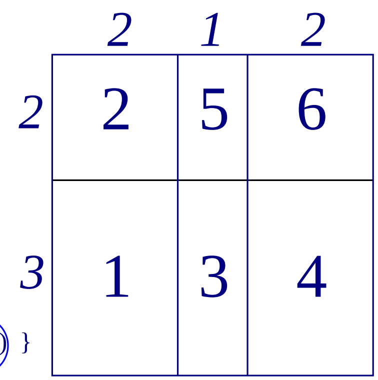
- Shape curves correspond to different kinds of constraints where the shaded areas are feasible regions.

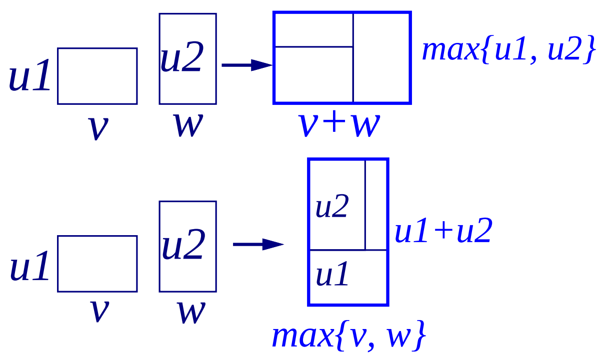


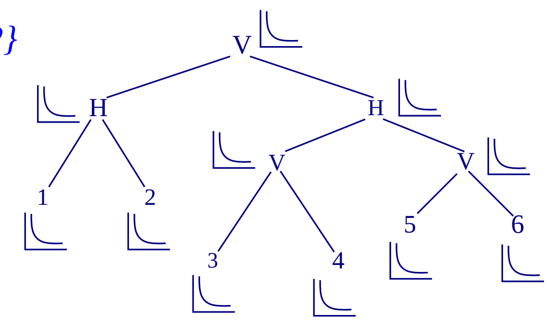
(a) rigid, fixed orientation (b) rigid, free orientation (c) flexible, fixed orientation (d) flexible, free orientation

Area Computation





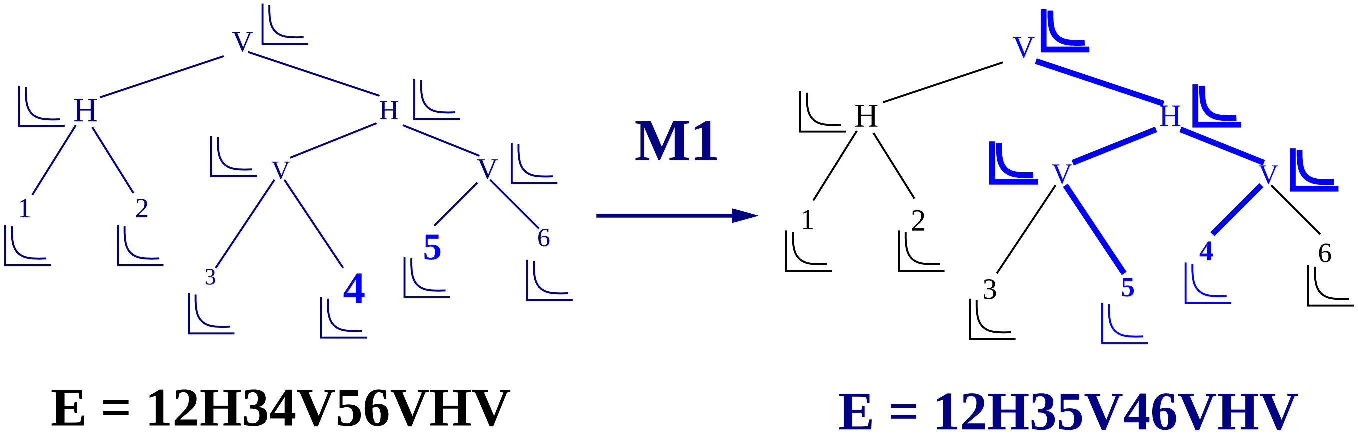




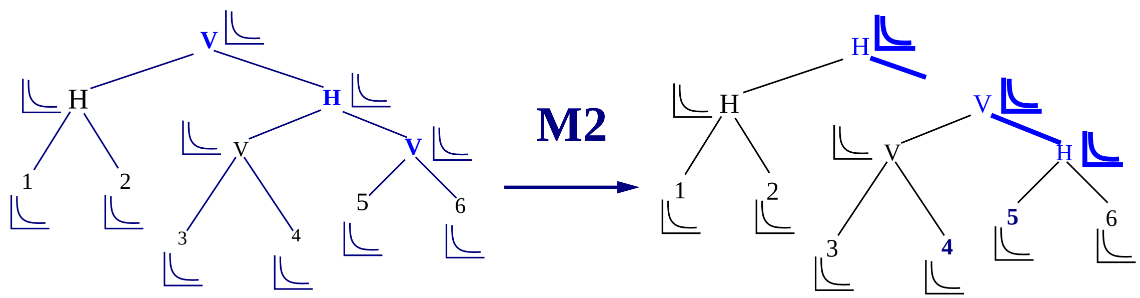
- Wiring cost?

Incremental Computation of Cost Function

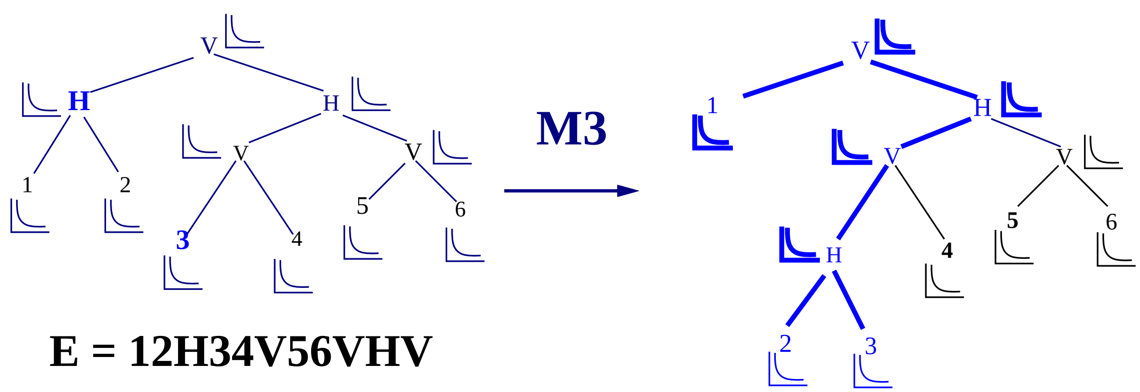
- Each move leads to only a minor modification of the Polish expression.
- At most **two paths** of the slicing tree need to be updated for each move.



Incremental Computation of Cost Function (cont'd)



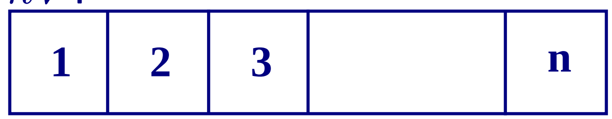
E = 12H34V56VHV → **E = 12H34V56HVH**



E = 12H34V56VHV → **E = 123H4V56VHV**

Annealing Schedule

- Initial solution: $12V3V \ldots nV$.



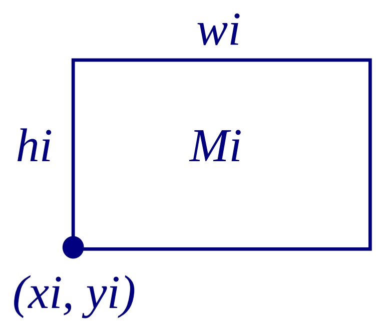
- $T_i = r^i T_0, i = 1, 2, 3, \ldots;\ r = 0.85$.
- At each temperature, try kn moves (k = 5–10).
- Terminate the annealing process if
  - # of accepted moves $<$ 5%,
  - temperature is low enough, or
  - run out of time.

Algorithm: Simulated_Annealing_Floorplanning(P, ϵ, r, k)

begin
$E \leftarrow 12V3V4V \ldots nV$; /* initial solution */
$Best \leftarrow E$; $T_0 \leftarrow \frac{\Delta_{avg}}{ln(P)}$; $M \leftarrow MT \leftarrow uphill \leftarrow 0$; $N = kn$;
repeat
  $MT \leftarrow uphill \leftarrow reject \leftarrow 0$;
  repeat
    SelectMove(M);
    Case M of
    M_1: Select two adjacent operands e_i and e_j; $NE \leftarrow Swap(E, e_i, e_j)$;
    M_2: Select a nonzero length chain C; $NE \leftarrow Complement(E, C)$;
    M_3: $done \leftarrow FALSE$;
      while not $(done)$ **do**
        Select two adjacent operand e_i and operator e_{i+1};
        if $(e_{i-1} \neq e_{i+1})$ **and** $(2N_{i+1} < i)$ **then** $done \leftarrow TRUE$;
      $NE \leftarrow Swap(E, e_i, e_{i+1})$;
    $MT \leftarrow MT + 1$; $\Delta cost \leftarrow cost(NE) - cost(E)$;
    if $(\Delta cost \leq 0)$ **or** $(Random < e^{\frac{-\Delta cost}{T}})$
    then
      if $(\Delta cost > 0)$ **then** $uphill \leftarrow uphill + 1$;
      $E \leftarrow NE$;
      if $cost(E) < cost(best)$ **then** $best \leftarrow E$;
    else $reject \leftarrow reject + 1$;
  until $(uphill > N)$ **or** $(MT > 2N)$;
  $T = rT$; /* reduce temperature */
  until $(\frac{reject}{MT} > 0.95)$ **or** $(T < \epsilon)$ **or** $OutOfTime$;
  end

Floorplanning by Mathematical Programming

- Sutanthavibul, Shragowitz, and Rosen, "An analytical approach to floor-plan design and optimization," 27th DAC, 1990.
- Notation:
  - w_i, h_i: width and height of module M_i.
  - (x_i, y_i): coordinate of the lower left corner of module M_i.
  - $a_i \leq w_i/h_i \leq b_i$: aspect ratio w_i/h_i of module M_i. (Note: We defined aspect ratio as h_i/w_i before.)
- Goal: Find a mixed **integer linear programming (ILP)** formulation for the floorplan design.
  - **Linear** constraints? Objective function?



*Area = hi * wi*
Aspect ratio = wi / hi

Nonoverlap Constraints

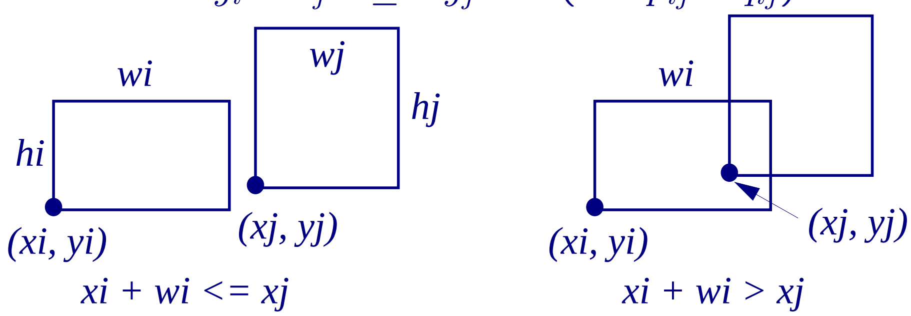
- Two modules M_i and M_j are nonoverlap, if at least one of the following linear constraints is satisfied (cases encoded by p_{ij} and q_{ij}):

		p_{ij}	q_{ij}
M_i to the left of M_j:	$x_i + w_i \leq x_j$	0	0
M_i below M_j:	$y_i + h_i \leq y_j$	0	1
M_i to the right of M_j:	$x_i - w_j \geq x_j$	1	0
M_i above M_j:	$y_i - h_j \geq y_j$	1	1

- Let W, H be upper bounds on the floorplan width and height, respectively.

- Introduce two 0,1 variables p_{ij} and q_{ij} to denote that one of the above inequalities is enforced; e.g., $p_{ij} = 0, q_{ij} = 1 \Rightarrow y_i + h_i \leq y_j$ is satisfied.

$$
\begin{aligned}
x_i + w_i &\leq x_j + W(p_{ij} + q_{ij}) \\
y_i + h_i &\leq y_j + H(1 + p_{ij} - q_{ij}) \\
x_i - w_j &\geq x_j - W(1 - p_{ij} + q_{ij}) \\
y_i - h_j &\geq y_j - H(2 - p_{ij} - q_{ij})
\end{aligned}
$$



Cost Function & Constraints

- Minimize $Area = xy$, **nonlinear!** (x, y: width and height of the resulting floorplan)
- How to fix?
  - Fix the width W and minimize the height y!
- Four types of constraints:
  1. no two modules overlap ($\forall i, j : 1 \leq i < j \leq n$);
  2. each module is enclosed within a rectangle of width W and height H ($x_i + w_i \leq W, y_i + h_i \leq H, 1 \leq i \leq n$);
  3. $x_i \geq 0$, $y_i \geq 0, 1 \leq i \leq n$;
  4. $p_{ij}, q_{ij} \in \{0, 1\}$.
- w_i, h_i are known.

Mixed ILP for Floorplanning

Mixed ILP for the floorplanning problem with rigid, fixed modules.

$$\min \quad y$$

subject to

$$x_i + w_i \leq W, \qquad 1 \leq i \leq n \qquad (1)$$

$$y_i + h_i \leq y, \qquad 1 \leq i \leq n \qquad (2)$$

$$x_i + w_i \leq x_j + W(p_{ij} + q_{ij}), \qquad 1 \leq i < j \leq n \qquad (3)$$

$$y_i + h_i \leq y_j + H(1 + p_{ij} - q_{ij}), \qquad 1 \leq i < j \leq n \qquad (4)$$

$$x_i - w_j \geq x_j - W(1 - p_{ij} + q_{ij}), \qquad 1 \leq i < j \leq n \qquad (5)$$

$$y_i - h_j \geq y_j - H(2 - p_{ij} - q_{ij}), \qquad 1 \leq i < j \leq n \qquad (6)$$

$$x_i, y_i \geq 0, \qquad 1 \leq i \leq n \qquad (7)$$

$$p_{ij}, q_{ij} \in \{0, 1\}, \qquad 1 \leq i < j \leq n \qquad (8)$$

- Size of the mixed ILP: for n modules,
  - # continuous variables: $O(n)$; # integer variables: $O(n^2)$; # linear constraints: $O(n^2)$.
  - Unacceptably huge program for a large n! (How to cope with it?)
- Popular LP software: LINDO, lp_solve, etc.

Mixed ILP for Floorplanning (cont'd)

Mixed ILP for the floorplanning problem: rigid, freely oriented modules.

$$
\begin{aligned}
& \text{min} \quad y \\
& \textit{subject to} \\
& x_i + r_i h_i + (1 - r_i) w_i \le W, && 1 \le i \le n && (9) \\
& y_i + r_i w_i + (1 - r_i) h_i \le y, && 1 \le i \le n && (10) \\
& x_i + r_i h_i + (1 - r_i) w_i \le x_j + M(p_{ij} + q_{ij}), && 1 \le i < j \le n && (11) \\
& y_i + r_i w_i - (1 - r_i) h_i \le y_j + M(1 + p_{ij} - q_{ij}), && 1 \le i < j \le n && (12) \\
& x_i - r_j h_j + (1 - r_j) w_j \ge x_j - M(1 - p_{ij} + q_{ij}), && 1 \le i < j \le n && (13) \\
& y_i - r_j w_j - (1 - r_j) h_j \ge y_j - M(2 - p_{ij} - q_{ij}), && 1 \le i < j \le n && (14) \\
& x_i, y_i \ge 0, && 1 \le i \le n && (15) \\
& p_{ij}, q_{ij} \in \{0, 1\}, && 1 \le i < j \le n && (16)
\end{aligned}
$$

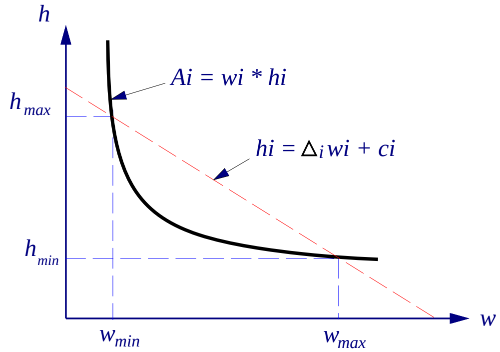
- For each module i with free orientation, associate a 0-1 variable r_i:
  - $r_i = 0$: 0° rotation for module i.
  - $r_i = 1$: 90° rotation for module i.
- $M = \max\{W, H\}$.

Flexible Modules

- Assumptions: w_i, h_i are unknown; area lower bound: A_i.
- Module size constraints: $w_i h_i \geq A_i$; $a_i \leq \frac{w_i}{h_i} \leq b_i$.
- Hence, $w_{min} = \sqrt{A_i a_i}$, $w_{max} = \sqrt{A_i b_i}$, $h_{min} = \sqrt{\frac{A_i}{b_i}}$, $h_{max} = \sqrt{\frac{A_i}{a_i}}$.
- $w_i h_i \geq A_i$ nonlinear! How to fix?
  - Can apply a first-order approximation of the equation: a line passing through (w_{min}, h_{max}) and (w_{max}, h_{min}).

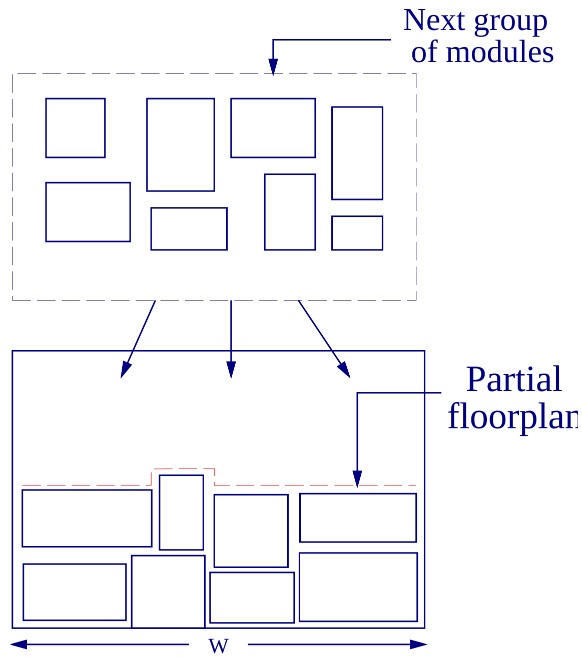
$$\begin{aligned} h_i &= \Delta_i w_i + c_i & /* \;\; y = mx + c \;\; */ \\ \Delta_i &= \frac{h_{max} - h_{min}}{w_{min} - w_{max}} & /* \;\; slope \;\; */ \\ c_i &= h_{max} - \Delta_i w_{min} & /* \;\; c = y_0 - mx_0 \;\; */ \end{aligned}$$

  - Substitute $\Delta_i w_i + c_i$ for h_i to form linear constraints (x_i, y_i, w_i are unknown; Δ_i, Δ_j, c_i, c_j can be computed as above).


h
Ai = wi * hi
h max
hi = Δi wi + ci
h min
w
w min
w max

Reducing the Size of the Mixed ILP

- Time complexity of a mixed ILP: exponential!
- Recall the large size of the mixed ILP: # variables, # constraints: $O(n^2)$.
  - How to fix it?
- Key: Solve a partial problem at each step (successive augmentation)
- Questions:
  - How to select next subgroup of modules? $\Rightarrow$ linear ordering based on connectivity.
  - How to minimize the # of required variables?


Next group
of modules
Partial
floorplan
W

Reducing the Size of the Mixed ILP (cont'd)

- Size of each successive mixed ILP depends on (1) # of modules in the next group; (2) "size" of the partially constructed floorplan.
- Keys to deal with (2)
  - Minimize the problem size of the partial floorplan.
  - Replace the already placed modules by a set of covering rectangles.
  - # rectangles is usually much smaller than # placed modules.