Notes On Plane Electromagnetic Waves

We are reaching one of the most important parts of this course, electromagnetic radiation. After Thanksgiving we will do a lot of math to show you that electromagnetic waves propagate at the speed of light. But first we show you that with what we already have plus a few plausible assumptions, we can get an intuitive understanding for how electromagnetic waves are generated, and some sense of their nature. We will return to this subject in more mathematical detail in a bit, but first the intuitive approach.

Creation and Reflection of Electromagnetic Plane Waves

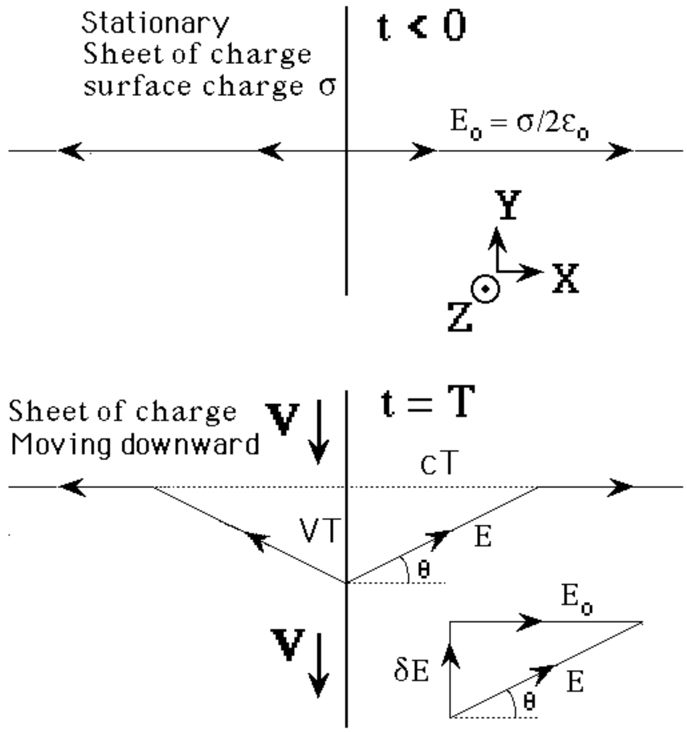
Creation: Electromagnetic plane waves propagate in empty space at the speed of light, $c = 1/\sqrt{\mu_o \varepsilon_o}$. Here we want to demonstrate how one would create such waves in a particularly simple geometry--planar. Although physically this is not particularly applicable to the real world, it is reasonably easy to treat, and we can see directly how electromagnetic plane waves are generated, *why it takes work to make them,* and how much energy they carry away with them.

To make an electromagnetic plane wave, we do much the same thing we do when we make waves on a string. We grab the string somewhere and shake it, and thereby generate a wave on the string. We do work against the tension in the string when we shake it, and that work is carried off as an energy flux in the wave. Electromagnetic waves are much the same proposition. The electric field line serves as the "string". As we will see below, there is a tension associated with an electric field line, in that when we shake it (try to displace it from its initial position), there is a restoring force that resists the shake, and a wave propagates along the field line as a result of the shake. To understand in detail what happens in this process will involve using most of the electromagnetism we have learned thus far, from Gauss's Law to Ampere's Law plus the reasonable assumption that electromagnetic information propagates at c in a vacuum (we will show this is the case mathematically after Thanksgiving).

The first obvious question is...how in the world do we shake an electric field line? What do we grab on to? Well, what we do is shake the electric charges that they are attached to. After all, it is these charges that produce the electric field, and in a very real sense the electric field is "rooted" in the electric charges that produce them. Knowing this, and assuming that in a vacuum, electromagnetic signals propagate at the speed of light, we can pretty much puzzle out how to make a plane electromagnetic wave by shaking charges. Let's first figure out how to make a *kink* in an electric field line, and then we'll go on to make sinusoidal waves.

Suppose we have an infinite sheet of charge located in the yz-plane, initially at rest, with surface charge density σ (see sketch next page). From Gauss's Law, we know that this surface charge will give rise to a static electric field $\boldsymbol{E}_o$ given by $\boldsymbol{E}_o = +\hat{x}\, \sigma/2\varepsilon_o$ for $x > 0$, and $\boldsymbol{E}_o = -\hat{x}\, \sigma/2\varepsilon_o$ for $x < 0$ ($\hat{x}$ is a unit vector in the x-direction).

Now, at $t = 0$, we grab the sheet of charge and start pulling it *downward* with *constant* velocity $\boldsymbol{V} = -V\hat{y}$. Let's look at how things will then appear at a later time $t = T$.



In particular, let's look at the field line that goes through $y = 0$ for $t < 0$ (before the sheet starts moving--see the top panel to the immediate right). The "foot" of this electric field line, that is, where it is anchored, is rooted in the electric charge that generates it, and that "foot" must move downward with the sheet of charge, at the same speed as the charges move downward. Thus the "foot" of our electric field line, which was initially at $y = 0$ at $t = 0$, will be a distance $-VT$ down the y-axis at time $t = T$.

However, we have assumed that the information that this field line is being dragged downward can only propagate outward from $x = 0$ with the speed of light, c. Thus the portion of our field line outside of a distance along the x-axis of cT from the origin doesn't know the charges are moving, and thus has not yet begun to move downward. Our field line therefore must appear at time $t = T$ as we show it in the lower sketch. Nothing has happened outside of $|x| > cT$; the foot of the field line at $x = 0$ is a distance $-VT$ down the y-axis, and we have guessed about what the field line must look like for $0 < |x| < cT$ by simply connecting the two positions on the field line that we know about at time T ($x = 0$ and $|x| = cT$) by a straight line. This is exactly the guess we would make if we were dealing with a string instead of an electric field. This is a reasonable thing to do, and it turns out to be the right guess.

If you think about what the shape of this field line implies, it implies the following. What we have done by pulling down on the charged sheet is to generate a perturbation in the electric field, $\delta\boldsymbol{E}$, in addition to the static field $\boldsymbol{E_o}$. If we look at the total field $\boldsymbol{E}$ for $|x| < cT$, we have $\boldsymbol{E} = \boldsymbol{E_o} + \delta\boldsymbol{E}$, as shown in the lower right inset in our figure above. The vector $\boldsymbol{E}$ must be parallel to the line connecting the foot of the field line and the position of the field line at $|x| = cT$, just by the geometry of the situation. Thus, the two triangles to the right of the y-axis, as shown on the lower sketch, must be congruent. Therefore, we must have that $\tan\theta = \delta E/E_o = VT/cT = V/c$, where the angle θ is as shown. Thus we have that $\delta E = (V/c)E_o$, or, using our result from Gauss's Law for E_o,

$$\delta\mathbf{E} = +\hat{y}\, V\sigma/2\varepsilon_o c. \qquad (1)$$

We have generated an electric field perturbation, and this expression tells us how large the perturbation field $\delta\boldsymbol{E}$ is for a given speed of the sheet of charge, V.

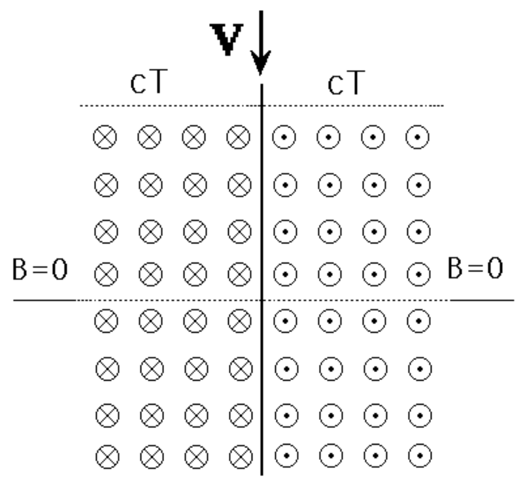
Now we understand why we say that the electric field line has a *tension* associated with it, just as a string does. The direction of our perturbation $\delta\boldsymbol{E}$ is such that the forces it exerts on the charges in the sheet *resist* the motion of the sheet--that is, there is an *upward* electric force on the sheet when we try to move it *downward*. That means that because of the presence of the electric field, we have to exert an additional *downward* force on an area dA of the sheet containing charge $dq = \sigma\, dA$. This additional force is the downward force *we* have to exert to overcome the upward "tension" associated with the electric field. This tension is given by $+dq\, \delta\mathbf{E} = \hat{y}\, (\sigma dA)(V\sigma/2\varepsilon_o c)$, where we have used equation (1) above for $\delta\mathbf{E}$. This is just like the restoring tension we must work against when we perturb a string. Thus we have to do work against the electric field to create the perturbation in the electric field, applying a force sufficient to overcome the electric force, that is, applying a force $\mathbf{dF}_{us} = -dq\, \delta\mathbf{E}$. The amount of work per unit time that we must do (joules per second) is therefore

$$dW/dt = \mathbf{dF}_{us} \cdot \mathbf{V} = [-\hat{y}\,(\sigma dA)(V\sigma/2\varepsilon_o c)] \bullet [-\hat{y}\, V] = +(V^2\sigma^2/2\varepsilon_o c)\, dA$$

or, dividing by dA to get the work we must do per unit time per unit area of the sheet,

$$dW/dtdA = +V^2\sigma^2/2\varepsilon_o c \qquad (2)$$

What else has happened in this process of moving the charged sheet down? Well, once the charged sheet is in motion, we have created a sheet of current with current per unit length $\eta = \sigma V$ (the units of this quantity are Amps/meter, as they must be). From Ampere's Law, we know that we will therefore have created a *magnetic field*, in addition to our δE. Such a current sheet will produce a magnetic field of magnitude $\delta B = \mu_o \eta/2 = \mu_o V\sigma/2$, and this magnetic field will reverse across the current sheet. We show the configuration of the field appropriate to a downward current in the sketch below. Again, the information that the charged sheet has started moving, producing a current sheet and associated magnetic field, can only propagate outward from $x = 0$ with speed c, and therefore outside of a distance along the x-axis of $\pm cT$, the magnetic field is still zero, while inside that distance it has the configuration shown in the sketch, with the magnitude given above. Note that



MAGNETIC FIELD AT t = T

$$\delta E/\delta B = (V\sigma/2\varepsilon_o c)/(\mu_o V\sigma/2) = 1/(c\varepsilon_o\mu_o) = c \qquad (3)$$

where we have used $c = 1/\sqrt{\mu_o\varepsilon_o}$. Not only does our current sheet generate a magnetic field that is perpendicular to our electric field perturbation electric field δE, as we must have for an electromagnetic wave, but we also have the relationship between the magnitudes that we expect to see for a transverse electromagnetic wave, which we will derive in detail from Maxwell's equations after Thanksgiving (see also Chapter 14 of *EMI*).

Now, lets discuss the energy carried away by these perturbation fields. The energy flux associated with an electromagnetic field is given by the Poynting vector $\mathbf{S} = \mathbf{E} x \mathbf{B}/\mu_0$. If we compute this quantity for our fields for $x > 0$, we get an energy flow to the right ($\delta\mathbf{E} x \delta\mathbf{B}$ is in the $+\hat{x}$ direction for $x > 0$) whose magnitude δS (in joules per sec per square meter) is

$$\delta S = \delta E \delta B/\mu_o = (V\sigma/2\varepsilon_o c)(\mu_o V\sigma/2)/\mu_o = V^2\sigma^2/4\varepsilon_o c \quad (4)$$

This is only 1/2 of the work we do per unit time per unit area to pull the sheet down, as given by equation (2). *But*, the fields on the left carry exactly the same amount of energy flux *to the left*, (the magnetic field reverses across $x = 0$, whereas the electric field does not, so the Poynting flux also reverses across $x = 0$). So the total energy flux carried off by the perturbation electric and magnetic fields we have generated is *exactly equal* to the rate at which we do work per unit area to pull the charged sheet down against the tension in the electric field (see equation (2) above). Thus we have generated perturbation electromagnetic fields which carry off energy, and they carry off energy at exactly the rate that it takes us to create them.

This is where the energy comes from that is carried by an electromagnetic wave. The agent who originally "shook" the charge to produce the wave had to do work to shake it, against the perturbation electric field the shaking produces, and that agent is the ultimate source of the energy carried by the wave. An exactly analogous situation exists when one asks where the energy carried by a wave on a string comes from. The agent who originally shook the string to produce the wave had to do work to shake it against the restoring tension in the string, and that agent is the ultimate source of energy carried by a wave on a string.

That takes care of generating a kink. How about generating a sinusoidal wave with frequency ω, like the waves considered in the text? To do this, instead of pulling the charge sheet down at constant speed, we just shake it up and down with a velocity $\mathbf{V}(t) = -\hat{\mathbf{y}} V \cos \omega t$. Then this oscillating sheet of charge will generate fields which are given by:

$$x>0: \delta\mathbf{E}(x,t) = +\hat{\mathbf{y}}\,(c\mu_o\sigma V/2)\cos\omega(t - x/c); \quad \delta\mathbf{B}(x,t) = +\hat{\mathbf{z}}\,(\mu_o\sigma V/2)\cos\omega(t - x/c) \quad (5)$$

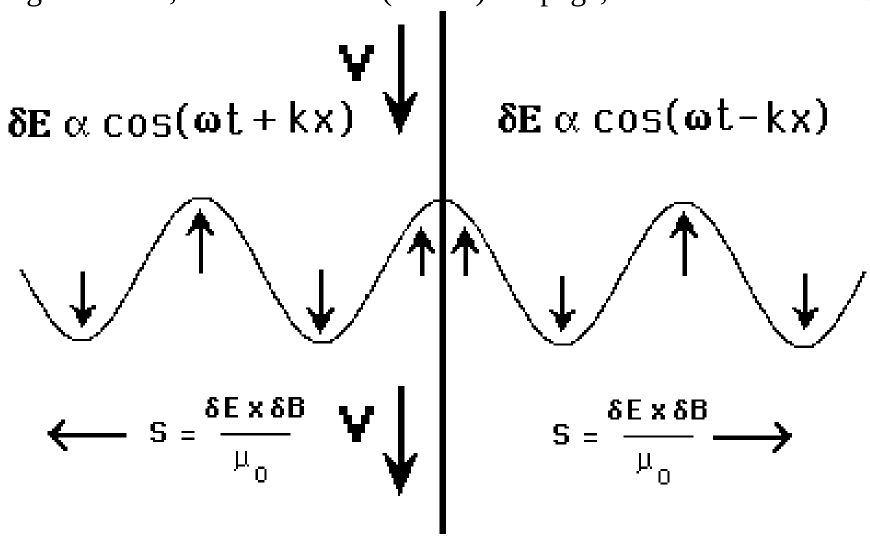
$$x<0: \delta\mathbf{E}(x,t) = +\hat{\mathbf{y}}\,(c\mu_o\sigma V/2)\cos\omega(t + x/c); \quad \delta\mathbf{B}(x,t) = -\hat{\mathbf{z}}\,(\mu_o\sigma V/2)\cos\omega(t + x/c) \quad (6)$$

It's clear in equations (5) and (6) why we have chosen the *amplitudes* of these terms--these are just the $\delta B = \mu_o\sigma V/2 = \delta E/c$ amplitudes of the kink generated above for constant speed of the sheet, but now allowing for the fact that the speed is varying sinusoidally in time with frequency ω. But why have we put the $(t-x/c)$ and $(t+x/c)$ arguments in equations (5) and (6)?

Consider first $x > 0$. If we are sitting at some $x > 0$ at time t, and are measuring an electric field there, the field we are observing should not depend on what the current sheet is doing *at that* observation time ***t***. Information about what the current sheet is doing takes a time x/c to propagate out to the observer at $x > 0$. Thus what the observer at $x > 0$ sees at time t depends on what the current sheet was doing *at an earlier time*, namely $t-x/c$. The electric field as a function of time should reflect that time delay due to the finite speed of propagation from the origin to some $x > 0$, and this is the reason the $(t-x/c)$ appears in equation (5), and not t itself. For $x < 0$, the argument is exactly the same, except if $x < 0$, $t+x/c$ is the expression for the earlier time, and not $t-x/c$. This is exactly the time-delay effect one gets when one measures waves on a string. If we are measuring wave amplitudes on a string some distance away from the agent who is shaking the string to generate the waves, what we measure at time t depends on what the agent was doing at an earlier time, allowing for the wave to propagate from the agent to the observer.

Let $k = \omega/c$ (or $\omega/k = c$). Then if we note that $\cos \omega(t-x/c) = \cos(\omega t - kx) = \cos(kx - \omega t)$, we see that we have generated in equations (5) and (6) precisely the kinds of plane electromagnetic waves we will describe in lecture, other than the (unimportant) fact that we have cosines instead of sines in our expressions. Note that we can also easily arrange to get rid of our static field $\boldsymbol{E_o}$ by simply putting a stationary charged sheet with charge per unit area $-\sigma$ at $x = 0$. That charged sheet will cancel out the static field due to the positive sheet of charge, but will not affect the perturbation field we have calculated, since it is not moving. In reality, that is how electromagnetic waves are generated--with an overall neutral medium where charges of one sign (actually the electrons) are accelerated while an equal number of charges of the opposite sign essentially remain at rest. Thus an observer only sees the wave fields, and not the static fields. In the following summary, we will assume that we have set $\boldsymbol{E_o}$ to zero in this way.

Thus we obtain a picture of the electric field generated by the oscillation of our current sheet as shown below. We show this configuration at a time when the sheet is moving *down*--at that time the perturbation electric field is *up*, which is what we expect from our initial discussion of how to generate a kink. For simplicity, in the figure below we do not show the magnetic field, which is out of (or into) the page, in the usual manner.



Let's summarize what we have done. Using Maxwell's equations in free space, with no charges or currents present, we will show in a bit (and our text shows) that waves propagate in vacuum at the speed of light, carrying energy flux $\boldsymbol{S} = \boldsymbol{E}x\boldsymbol{B}/\mu_0$, that the electric and magnetic fields are perpendicular to each other, and to the direction of propagation, and that the ratio of the electric field magnitude to the magnetic field magnitude is the speed of light.

What we have accomplished in the construction here, which really only assumes that the feet of the electric field lines move with the charges, and that information propagates at c, is to show we can generate such a wave by shaking a plane of charge sinsoidally. The wave we generate has electric and magnetic fields perpendicular to one another, and transverse to the direction of propagation, with the ratio of the electric field magnitude to the magnetic field magnitude equal to the speed of light. Moreover, we see directly where the energy flux $\boldsymbol{S} = \boldsymbol{E}x\boldsymbol{B}/\mu_0$ carried off by the wave comes from. It is put in by the agent who shakes the charges, and thereby generates the electromagnetic wave. If we go to more complicated geometries, these statements become much more complicated in detail, but the overall picture remains as we have presented it.

Finally, before going on to the reflection of electromagnetic waves, let us rewrite slightly the expressions given in equations (5) and (6) for the fields generated by our oscillating charged sheet, in terms of the current per unit length in the sheet, $\eta(t)$. The quantity $\eta(t)$ is given by $\sigma\boldsymbol{V}(t)$ and since here we have $\boldsymbol{V}(t) = -\hat{y}V\cos\omega t$, then it follows that $\eta(t) = -\hat{y}\sigma V\cos\omega t$. Thus we can rewrite equations (5) and (6) for the electric and magnetic fields generated by an oscillating current sheet with charge per unit length in the sheet $\eta(t)$ as follows:

Electromagnetic fields $\delta\mathbf{E}_\eta$ ***and*** $\delta\mathbf{B}_\eta$

generated by a time-varying current sheet $\eta(t)$ located in the *yz* plane at $x = 0$:

$x > 0$ $\quad \delta\mathbf{E}_\eta(x,t) = -\, c\mu_o\eta(t-x/c)\,/2; \quad \delta\mathbf{B}_\eta(x,t) = +\,\hat{\mathbf{x}} \times \delta\mathbf{E}(x,t)/c$ (7)

$x < 0$ $\quad \delta\mathbf{E}_\eta(x,t) = -\, c\mu_o\eta(t+x/c)\,/2; \quad \delta\mathbf{B}_\eta(x,t) = -\,\hat{\mathbf{x}} \times \delta\mathbf{E}(x,t)/c$ (8)

Note that $\delta\boldsymbol{B}_\eta(x,t)$ reverses across the current sheet, with a jump of $\mu_o\eta(t)$ at the sheet, as it must from Ampere's Law. *Any* oscillating sheet of current *must* generate the plane electromagnetic waves described by these equations, just as *any* stationary electric charge *must* generate a Coulomb electric field. That's just the way things work.

Note: To avoid possible future confusion, we point out that if you go on to a more advanced course in electromagnetism, you will study the radiation fields generated by a *single* oscillating charge, and find that they are proportional to the *acceleration* of the charge. This is very different from the case here, where the radiation fields of our oscillating sheet of charge are proportional to the *velocity* of the charges. However, there is no contradiction, because when you add up the radiation fields due to all the single charges making up our sheet, you recover the same result we give in equations (7) and (8) (see Chapter 30, Section 7, of Feynman, Leighton, and Sands, *The Feynman Lectures on Physics*, *Vol 1*, Addison-Wesley, 1963).

Reflection:

How does a very good conductor reflect an electromagnetic wave falling on it? In words, what happens is the following. The time-varying electric field of the incoming wave drives an oscillating current on the surface of the conductor, following Ohm's Law. That oscillating current sheet, of necessity, must generate waves propagating in both directions from the sheet. One of these waves is the reflected wave. The other wave cancels out the incoming wave inside the conductor. Let us make this qualitative description quantitative.

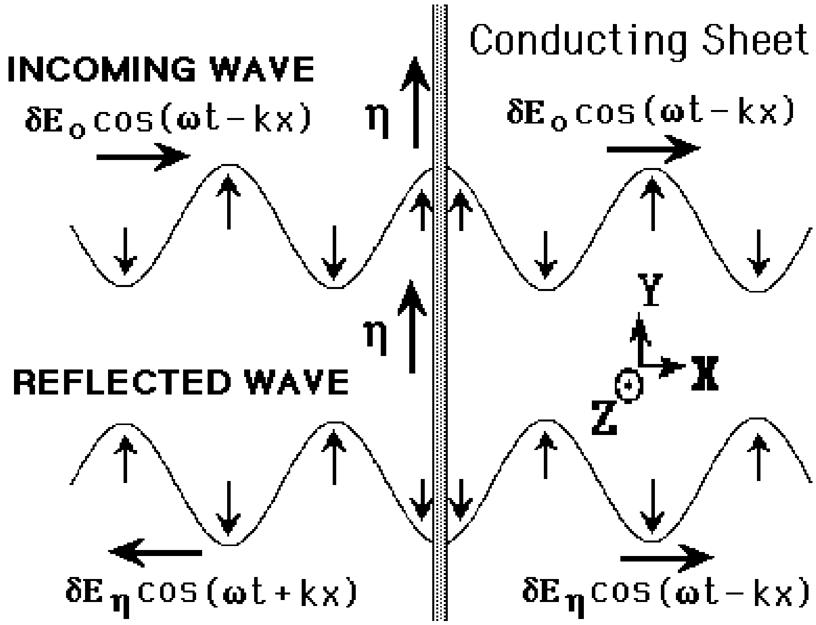
Suppose we have an infinite plane wave propagating to the right, generated by currents far to the left and not shown. Suppose that the electric field of this wave is

$$\delta\boldsymbol{E}_o(x,t) = +\,\hat{\mathbf{y}}\;\delta E_o \cos(\omega t - kx)$$

and that the magnetic field is

$$\delta\boldsymbol{B}_o(x,t) = +\,\hat{\mathbf{z}}\;\delta B_o \cos(\omega t - kx)$$

as shown in the top wave form in the sketch on the next page. We put at the origin ($x = 0$) a conducting sheet with width D, which we assume is small compared to a wavelength of our incoming wave. This conducting sheet will *reflect* our incoming wave. How? The



electric field of the incoming wave will cause a current $\boldsymbol{j} = \delta \boldsymbol{E}/\rho$ to flow in the sheet, where ρ is the resistivity (if σ is the conductivity, then ρ = 1/σ). Moreover, the direction of ***j*** *will be in the same direction as the electric field of the incoming wave*, as shown in the sketch. Thus our incoming wave sets up an oscillating sheet of current with current per unit length $\eta = jD$. As in our discussion of the generation of plane electromagnetic waves above, this current sheet *will also generate electromagnetic waves*, moving both to the right and to the left (see sketch, lower wave form) away from the oscillating sheet of charge. Using equation (7) above, for $x > 0$ the wave generated by the current will be $\delta \boldsymbol{E}_\eta = -\hat{y}\, c\mu_o(jD)/2c \cos(\omega t - kx)$. For $x<0$, we will have a similar expression, except that the argument will be $(\omega t+kx)$ (see sketch). **Note** the sign of this electric field $\delta \boldsymbol{E}_\eta$ at $x = 0$; it is *down* when the sheet of current (and $\delta \boldsymbol{E}_o$) is *up*, and vice-versa, just as we saw before. Thus, for $x > 0$, the generated electric field $\delta \boldsymbol{E}_\eta$ will always be *opposite* the direction of the electric field of the incoming wave, and *it will tend to cancel out the incoming wave for* $x > 0$. For a very good conductor, in fact, we can show (see Appendix) that $\eta = jD$ will be equal to $2\delta E_o/c\mu_o$, so that for $x > 0$ we will have $\delta \boldsymbol{E}_\eta = -\hat{y}\, \delta E_o \cos(\omega t - kx)$. That is, for a very good conductor, the electric field of the wave generated by the current *will exactly cancel* the electric field of the incoming wave *for* $x > 0$*!* And that's what a very good conductor does. It supports exactly the amount of current per unit length η needed to cancel out the incoming wave for $x > 0$ ($2\delta E_o/c\mu_o$, or equivalently $2\delta B_o/\mu_o$) and for $x < 0$, *this same current* generates a "reflected" wave propagating back in the direction from which the original incoming wave came, *with the same amplitude as the original incoming wave*. This is how a very good conductor *totally* reflects electromagnetic waves. In the Appendix, we show that η will in fact approach value needed to accomplish this in the limit that the resistivity ρ approaches 0.

In the process of this reflection, there is a force per unit area exerted on the conductor. This is just the ***VxB*** force due to the current ***j*** flowing in the presence of the magnetic field of the incoming wave, or a force per unit volume of $\boldsymbol{j} \times \delta \boldsymbol{B}_o$. If we calculate the total force ***dF*** acting on an cylindrical volume with area dA and length D of the conductor, we find that it is in the $+x$ direction, with magnitude

$$dF = dA\, D\, |\mathbf{j} \times \delta \mathbf{B}_o| = dA\ jD\, \delta B_o = dA\, (2\delta E_o/c\mu_o)\, \delta B_o = dA\, (2\, \delta E_o\, \delta B_o/c\mu_o)$$

so that the force per unit area, *dF/dA*, or radiation pressure, is just twice the Poynting flux divided by the speed of light *c*.

APPENDIX

We show here that a perfect conductor will perfectly reflect an incident wave. To approach the limit of a perfect conductor, we first consider the finite resistivity case, and then let the resistivity go to zero. As above, the electric field of the incoming wave will, by Ohm's Law, cause a current $\boldsymbol{j} = \boldsymbol{E}/\rho$ to flow in the sheet, where ρ is the resistivity. Since the sheet is assumed thin compared to a wavelength, we can assume that the entire sheet sees essentially the same electric field, so that $\boldsymbol{j}$ will be uniform across the thickness of the sheet, and outside of the sheet we will see fields appropriate to a equivalent surface current $\eta(t) = D\boldsymbol{j}(t)$. This current sheet will generate additional electromagnetic waves, moving both to the right and to the left, away from the oscillating sheet of charge. The total electric field, $\delta\boldsymbol{E}_{total}(x,t)$, will be the sum of the incident electric field and the electric field generated by the current sheet. Using equations (7) and (8) above, we thus have for the total electric field the following expressions:

$$x > 0 \qquad \delta\mathbf{E}_{total}(x,t) = \delta\mathbf{E}_o(x,t) + \delta\mathbf{E}_\eta(x,t) = \delta\mathbf{E}_o(x,t) - c\mu_o\eta(t-x/c)/2 \qquad (9)$$

$$x < 0 \qquad \delta\mathbf{E}_{total}(x,t) = \delta\mathbf{E}_o(x,t) + \delta\mathbf{E}_\eta(x,t) = \delta\mathbf{E}_o(x,t) - c\mu_o\eta(t+x/c)/2 \qquad (10)$$

We also have a relation between the current density $\boldsymbol{j}$ and $\delta\boldsymbol{E}_{total}$ from the microscopic form of Ohm's Law, to wit $\boldsymbol{j}(t) = \delta\boldsymbol{E}_{total}(0,t)/\rho$, where ρ is the resistivity, and $\delta\boldsymbol{E}_{total}(0,t)$ is the total electric field at the position of the conducting sheet. Note that is is appropriate to use the total electric field in Ohm's Law--the currents arise from the total electric field, irrespective of the origin of that field. So we have

$$\eta(t) = D\mathbf{j}(t) = D\,\delta\mathbf{E}_{total}(0,t)/\rho \qquad (11)$$

If we look at either (9) or (10) at $x = 0$, we have

$$\delta\mathbf{E}_{total}(0,t) = \delta\mathbf{E}_o(0,t) + \delta\mathbf{E}_\eta(0,t) = \delta\mathbf{E}_o(0,t) - c\mu_o\eta(t)/2 \qquad (12)$$

or using (11)

$$\delta\mathbf{E}_{total}(0,t) = \delta\mathbf{E}_o(0,t) - c\mu_o D\,\delta\mathbf{E}_{total}(0,t)/2\rho \qquad (13)$$

We can now solve equation (13) for $\delta\boldsymbol{E}_{total}(0,t)$, with the result that

$$\delta\mathbf{E}_{total}(0,t) = \delta\mathbf{E}_o(0,t)/(1+c\mu_o D/2\rho) \qquad (14)$$

and therefore, using equation (11)

$$\eta(t) = D\mathbf{j}(t) = D\,\delta\mathbf{E}_{total}(0,t)/\rho = D\delta\mathbf{E}_o(0,t)/(\rho+c\mu_o D/2) \qquad (15)$$

If we take the limit that $\rho \rightarrow 0$ (no resistance, a perfect conductor), then we can easily see using equation (14) that $\delta \boldsymbol{E}_{total}(0,t) \rightarrow 0$, and using equation (15) that $\eta(t) \rightarrow 2\delta \boldsymbol{E}_o(0,t)/c\mu_o = +\hat{y}\ 2\delta E_o \cos(\omega t)/c\mu_o = +\hat{y}\ 2\delta B_o \cos(\omega t)/\mu_o$. In this same limit equations (9) and (10) become

$$x > 0 \quad \delta \mathbf{E}_{total}(x,t) = \delta \mathbf{E}_o(x,t) - c\mu_o\eta(t-x/c)/2 = +\hat{\mathbf{y}}\,[\delta E_o - \delta E_o]\cos(\omega t-kx) \equiv 0 \qquad (16)$$

$$x < 0 \quad \delta \mathbf{E}_{total}(x,t) = \delta \mathbf{E}_o(x,t) - c\mu_o\eta(t+x/c)/2 = +\hat{\mathbf{y}}\,\delta E_o\,[\cos(\omega t-kx)-\cos(\omega t+kx)]$$
$$= +\hat{\mathbf{y}}\,\delta E_o\, 2 \sin kx \sin \omega t \qquad (17)$$

Again in the same limit of zero resistivity, our total magnetic fields become

$$x > 0 \quad \delta \mathbf{B}_{total}(x,t) \equiv 0 \qquad (18)$$

$$x < 0 \quad \delta \mathbf{B}_{total}(x,t) = +\hat{\mathbf{z}}\,\delta V_o[\cos(\omega t-kx)+\cos(\omega t+kx)]/c$$
$$= +\hat{\mathbf{z}}\, 2\delta B_o \cos kx \cos \omega t \qquad (19)$$

Thus, from equation (16) and (17) we see that we get no electromagnetic wave for $x > 0$, and standing electromagnetic waves for $x < 0$. Note that right at $x = 0$, the total electric field vanishes. The current per unit length $\eta(t) = +\hat{y}\,(2\delta B_o/\mu_o)\cos(\omega t)$ at $x = 0$ is just the current per length we need to bring the magnetic field down from its value at $x < 0$ to zero for $x > 0$.

You may be perturbed by the fact that in the limit of a perfect conductor, the electric field vanishes at $x = 0$, since it is the electric field at $x = 0$ that is driving the current there! In the limit of very small resistance, the electric field required to drive any finite current is very small. In the limit that the resistivity is zero, the electric field is zero, but as we approach that limit, we can still have a perfectly finite and well determined value of $\boldsymbol{j} = \boldsymbol{E}/\rho$, as we found by taking this limit in equation (14) and (15) above.