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6.013 Electromagnetics and Applications
Spring 2009

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ELECTROMAGNETICS AND APPLICATIONS

6.013 Content:

Fundamentals and Applications

Fields, media, and boundaries

Circuits

Motors, generators, and MEMS

Limits to computation speed

Microwave communications and radar

Wireless communications and waves

Optical devices and communications

Acoustics

Prerequisites:

6.003 or 6.02+6.007

[6.002+8.02+18.02]

Appendices B-E

Handouts:

Administration sheet, Equations

Subject outline, lecture notes

Prob. set 1, Objectives & Outcomes

WHAT ARE \bar{E} AND \bar{H} ?

Lorentz Force Law:

$$\bar{f} = q(\bar{E} + \bar{v} \times \mu_o \bar{H}) \text{ [Newtons]}$$

$4\pi \times 10^{-7}$

Velocity [m/s]

Charge [Coulombs]

“EM fields were invented to explain forces”

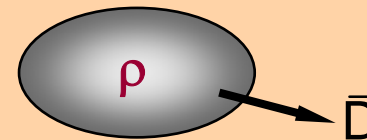
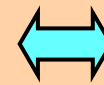
- Find \bar{E} by setting $\bar{v} = 0$ and measuring \bar{f}
- Find \bar{H} by setting $\bar{v} = 0$, $\bar{E} = 0$, and measuring \bar{f}
(requires 2 \bar{f} measurements using 2 \bar{v} tests)

INTEGRAL MAXWELL'S EQUATIONS

Graphical Equations:

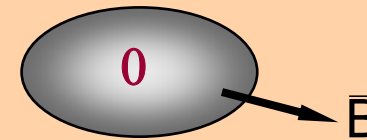
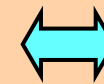
$$\oiint_S \bar{D} \cdot \hat{n} da = \iiint_V \rho dv$$

Gauss



$$\oiint_S \bar{B} \cdot \hat{n} da = 0$$

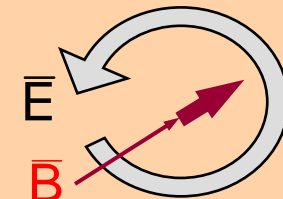
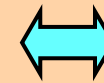
Gauss



$$\oint_C \bar{E} \cdot d\bar{s} = - \frac{\partial}{\partial t} \iint_A \bar{B} \cdot \hat{n} da$$

0 (statics)

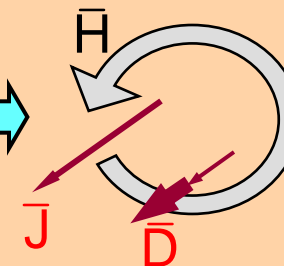
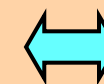
Faraday



$$\oint_C \bar{H} \cdot d\bar{s} = \iint_A \bar{J} \cdot \hat{n} da + \frac{\partial}{\partial t} \iint_A \bar{D} \cdot \hat{n} da$$

0 (statics)

Ampere



$$\bar{D} = \epsilon \bar{E}, \quad \bar{B} = \mu \bar{H}$$

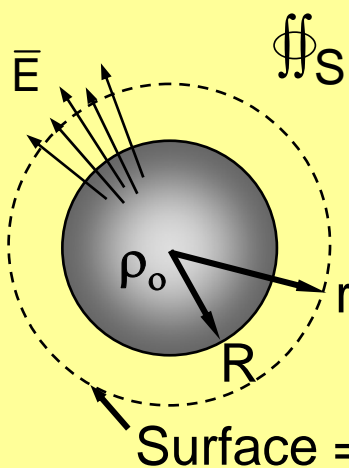
Constitutive relations

EXAMPLE: SPHERICAL CHARGE

Example I. Sphere of radius R , charge density ρ_o

Spherical symmetry precludes θ and ϕ components for \vec{E}

Solution:

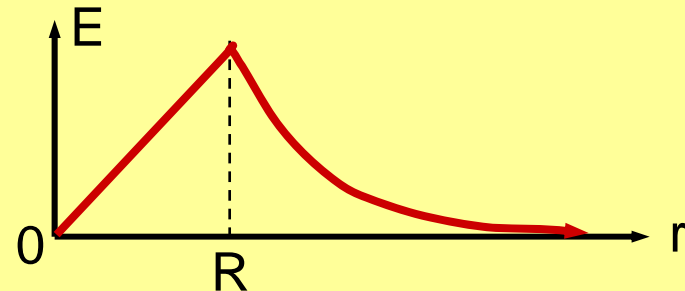


$$\oiint_S \vec{D} \cdot \hat{n} da = \iiint_V \rho dv = Q \quad (\text{Gauss's Law})$$

$$r > R \Rightarrow \epsilon E_r (4\pi r^2) = Q \Rightarrow E_r = \frac{Q}{4\pi\epsilon r^2} \quad [\text{V/m}]$$

$$r < R \Rightarrow \epsilon E_r (4\pi r^2) = \rho_o \left(\frac{4}{3}\pi r^3\right) \Rightarrow E_r = \frac{\rho_o r}{3\epsilon} \quad [\text{V/m}]$$

$$\text{Equivalently, } r > R \Rightarrow E_r = \frac{\rho_o R^3}{3\epsilon r^2}$$



EXAMPLE: CYLINDRICAL CHARGE

Example 2. Cylinder of radius R , charge density ρ_o

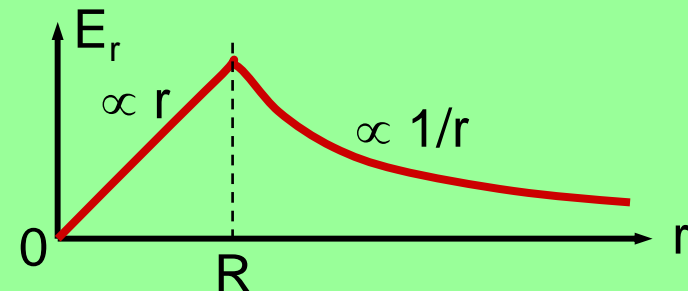
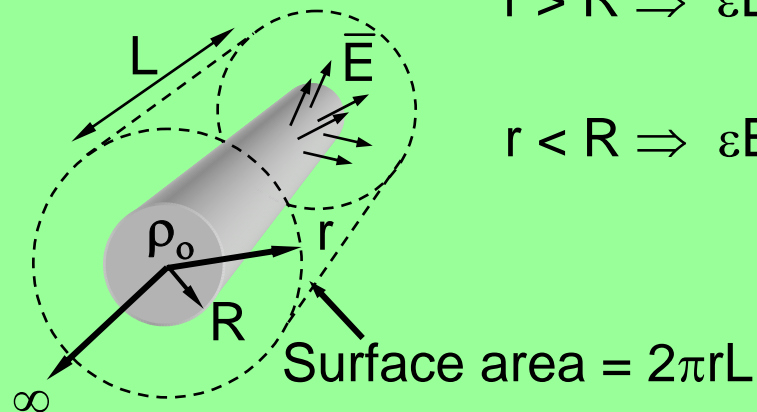
Cylindrical symmetry precludes θ and z components for \vec{E}

Solution:

$$\oiint_S \vec{D} \cdot \hat{n} da = \iiint_V \rho dv \quad (\text{Gauss})$$

$$r > R \Rightarrow \epsilon E_r (2\pi r L) = \rho_o (\pi R^2 L) \Rightarrow E_r = \frac{\rho_o R^2}{2\epsilon r} \quad [\text{V/m}]$$

$$r < R \Rightarrow \epsilon E_r (2\pi r L) = \rho_o (\pi r^2 L) \Rightarrow E_r = \frac{\rho_o r}{2\epsilon} \quad [\text{V/m}]$$



HOLLOW SPHERICAL CHARGE

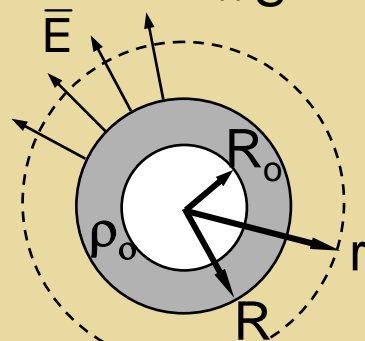
Example 3:

Hollow sphere of inner radius R_o , charge density ρ_o

Spherical symmetry precludes θ and ϕ components for \vec{E}

Solution:

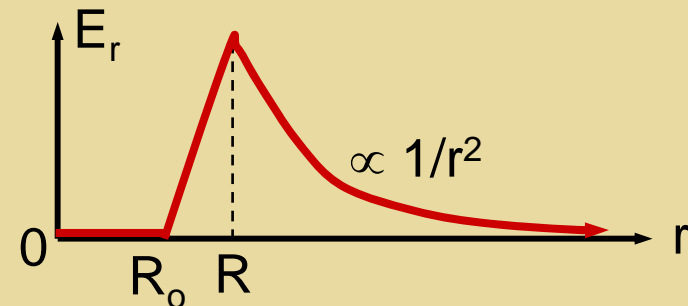
$$\oiint_S \vec{D} \cdot \hat{n} da = \iiint_V \rho dv = Q \quad (\text{Gauss's Law})$$



$$r > R \Rightarrow \epsilon E_r (4\pi r^2) = Q \Rightarrow E_r = \frac{Q}{4\pi\epsilon r^2} \quad [\text{V/m}]$$

$$r < R_o \Rightarrow \epsilon E_r (4\pi r^2) = 0 \Rightarrow E_r = 0 \quad [\text{V/m}]$$

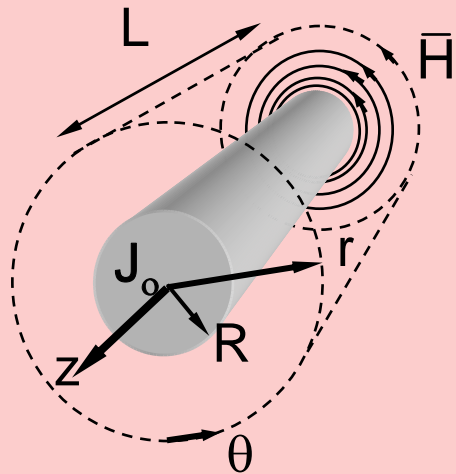
Surface = $4\pi r^2$



Hollow cylinders also have zero \vec{E}

CYLINDRICAL CURRENT \bar{J} [A/m²]

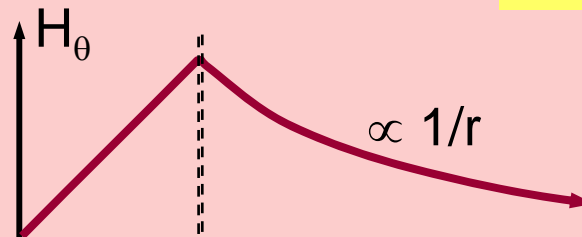
Example 4. Uniform current J_z for $r < R$:



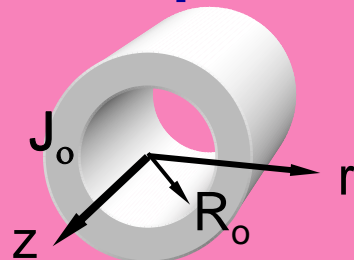
$$\oint_C \bar{H} \cdot d\bar{s} = \iint_A \bar{J} \cdot \hat{n} da + \frac{\partial}{\partial t} \iint_A \bar{D} \cdot \hat{n} da$$

$$r > R: 2\pi r H_\theta = I = J_0 \pi R^2 \Rightarrow H_\theta = I / 2\pi r \text{ [A/m]}$$

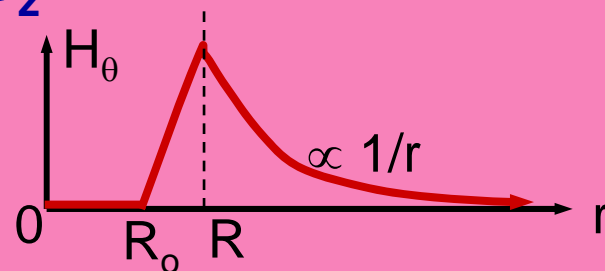
$$r < R: 2\pi r H_\theta = J_0 \pi r^2 \Rightarrow H_\theta = J_0 r / 2 \text{ [A/m]}$$



Example 5. Hollow current J_z :



Hollow cylinders also have zero H for $r < R_0$

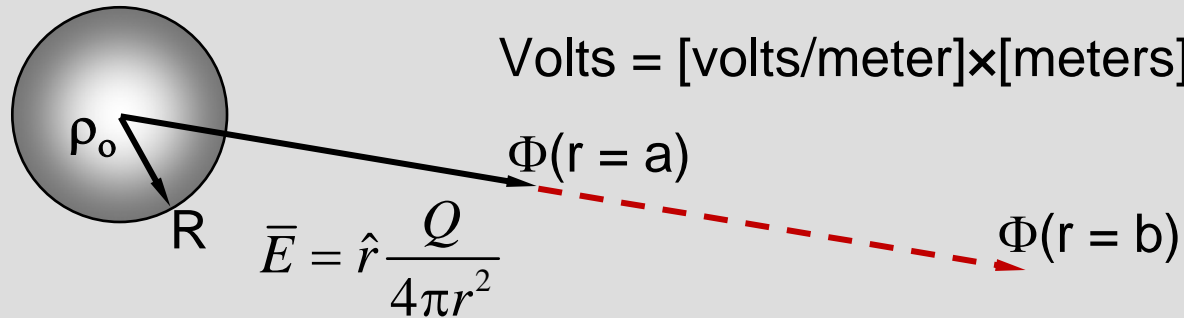


STATIC POTENTIALS Φ (Volts)

Example 6. Spherical charge density ρ_o , radius R:

What is the electric potential $\Phi(r)$ [volts] for $r > R$?

Volts = [volts/meter] × [meters]



Solution:

$$\Phi_a - \Phi_b = \int_a^b \vec{E} \cdot d\vec{r} \quad (\text{Volts})$$

$$\Phi(r) - \Phi_\infty = \int_r^\infty \vec{E} \cdot d\vec{r} = \Phi(r)$$

$$\Phi(r) = \int_r^\infty \frac{Q}{4\pi} r^{-2} \hat{r} \cdot d\vec{r} = -\frac{Q}{4\pi} r^{-1} \Big|_r^\infty$$

$$\Phi(r) = \frac{Q}{4\pi r} \quad \text{Volts, } (r > R)$$

Absolute potentials are defined relative to infinity: $\Phi(\infty) \triangleq 0$