

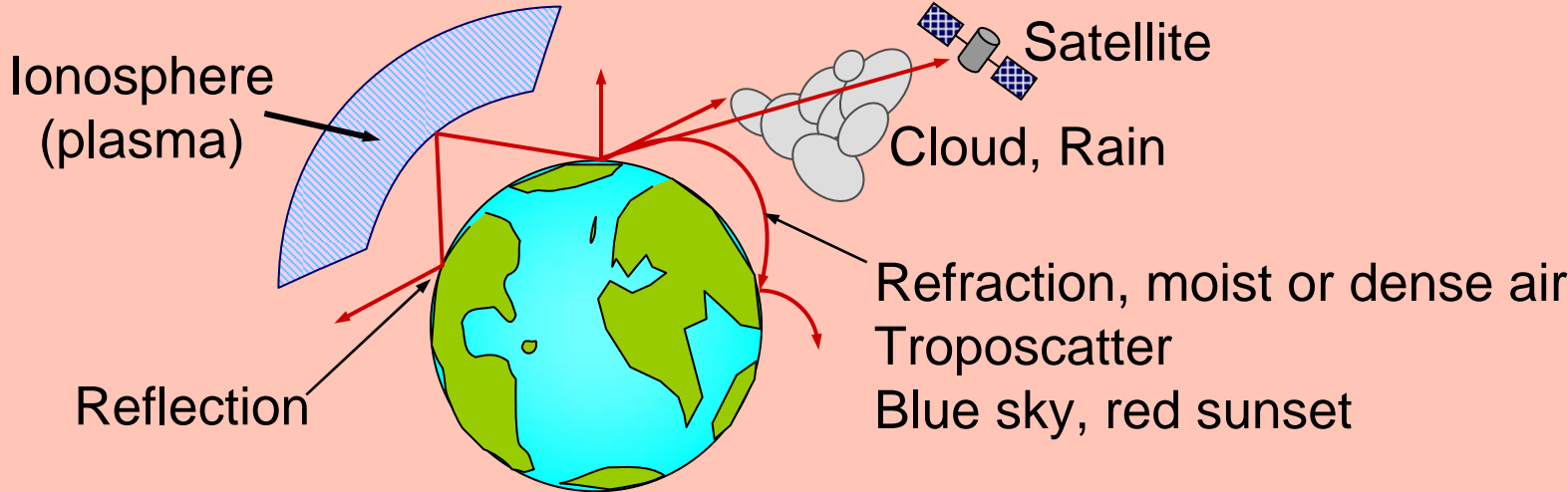
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6.013 Electromagnetics and Applications
Spring 2009

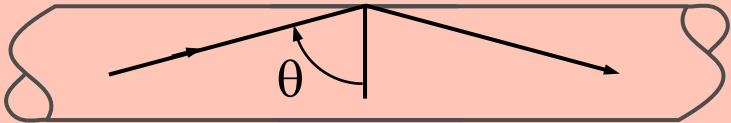
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WAVES IN MEDIA

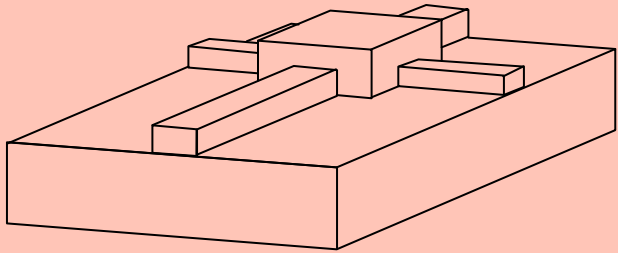
Radio Communications



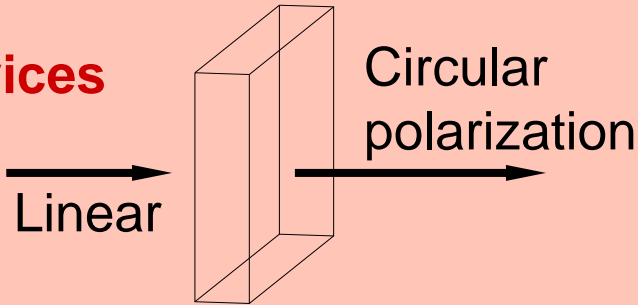
Optical Fibers



Optoelectronics on chips



Devices



WAVES IN MEDIA – Constitutive Relations

Vacuum: $\bar{D} = \epsilon_0 \bar{E}$ $\nabla \cdot \bar{D} = \rho_f$
 ρ_f = free charge density

Dielectric Materials: $\bar{D} = \epsilon \bar{E} = \epsilon_0 \bar{E} + \bar{P}$

$$\nabla \cdot \epsilon_0 \bar{E} = \rho_f + \rho_p$$

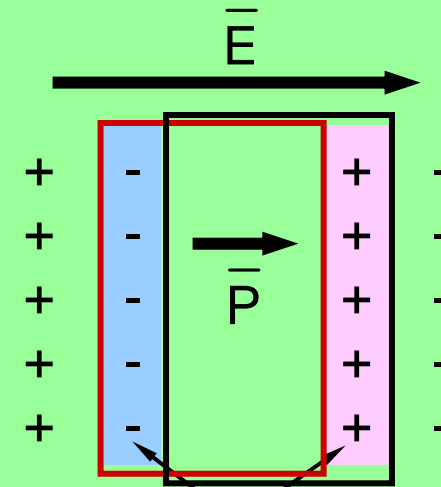
$$\nabla \cdot \bar{P} = -\rho_p$$

\bar{P} = "Polarization Vector"

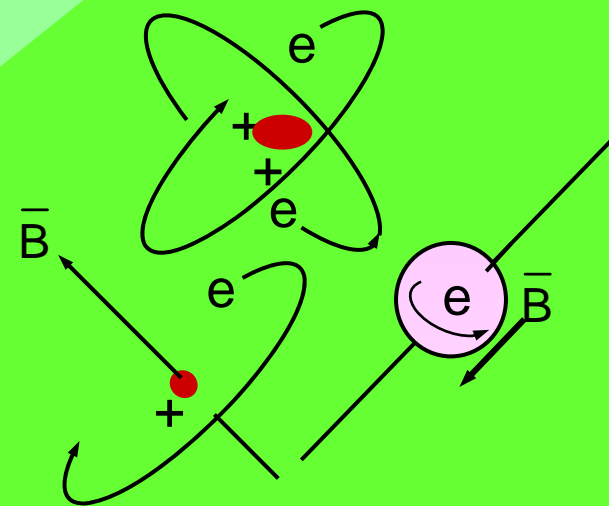
Magnetic Materials: $\nabla \cdot \bar{B} = 0$
 $\bar{B} = \mu_0 \bar{H}$ in vacuum

$$\bar{B} = \mu \bar{H} = \mu_0 (\bar{H} + \bar{M})$$

\bar{M} = "Magnetization Vector"



polarization charge density ρ_p



TYPES OF MEDIA

Properties are function of:	Designation:
Field direction	Anisotropic $\underline{\underline{D}} = \underline{\underline{\epsilon}}\underline{\underline{E}}, \underline{\underline{B}} = \underline{\underline{\mu}}\underline{\underline{H}}$
Position	Inhomogeneous
Time: $\neq f(t)$ $\neq f(\text{history})$	Stationary Amnesic
Frequency	Dispersive
$\underline{\underline{E}}$ or $\underline{\underline{H}}$	Non-linear
Temperature	Temperature dependent
Pressure	Compressive

ANISOTROPIC DIELECTRICS

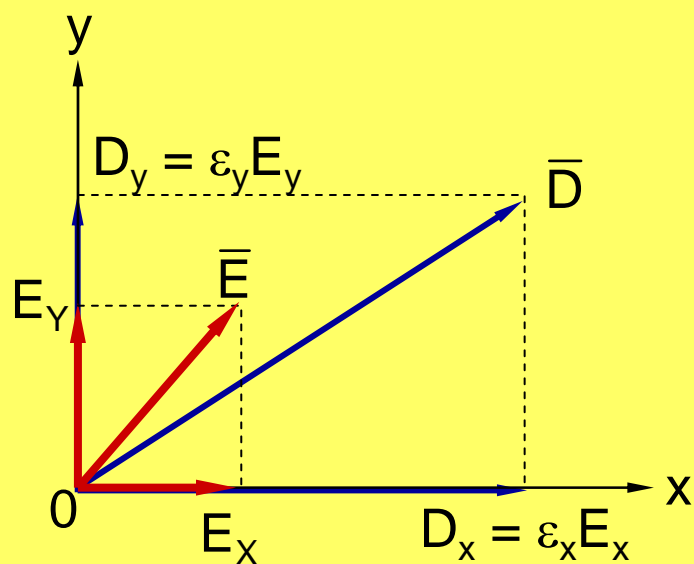
$$\underline{\bar{D}} = \underline{\bar{\epsilon}} \underline{\bar{E}}$$

$$D_x = \epsilon_{xx} E_x + \epsilon_{xy} E_y + \epsilon_{xz} E_z$$

$$D_y = \epsilon_{yx} E_x + \epsilon_{yy} E_y + \epsilon_{yz} E_z$$

$$D_z = \epsilon_{zx} E_x + \epsilon_{zy} E_y + \epsilon_{zz} E_z$$

$$\text{Let } \underline{\bar{\epsilon}} = \begin{bmatrix} \epsilon_x & 0 & 0 \\ 0 & \epsilon_y & 0 \\ 0 & 0 & \epsilon_z \end{bmatrix} \quad \longrightarrow$$



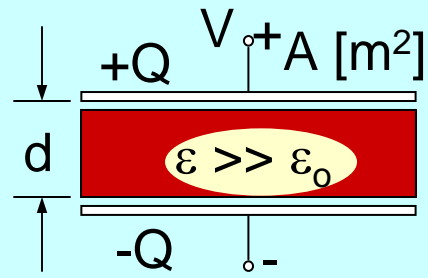
x,y,z are “principal axes”

Note: When $\epsilon_x \neq \epsilon_y \neq \epsilon_z$, $\underline{\bar{D}} \parallel \underline{\bar{E}}$ iff $\underline{\bar{E}} \parallel \hat{x}, \hat{y}$, or \hat{z}

Real $\underline{\bar{\epsilon}}, \underline{\bar{\mu}} \Rightarrow$ Lossless medium

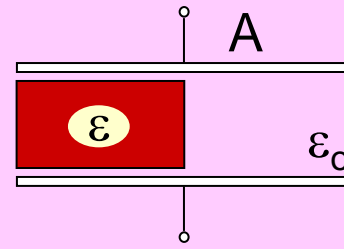
MAKING ANISOTROPIC MATERIALS

$$C = Q/V$$



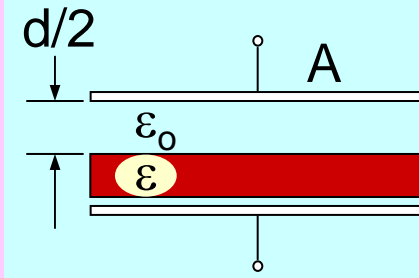
$$C = \frac{\epsilon_{\text{eff}} A}{d}$$

$$\epsilon_{\text{eff}} = \epsilon$$



$$C \cong \frac{\epsilon (A/2)}{d}$$

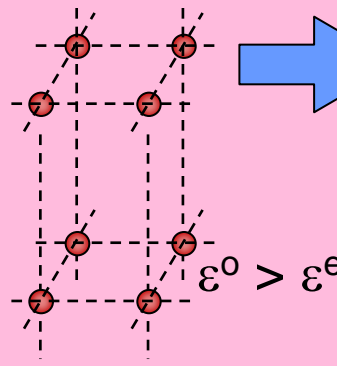
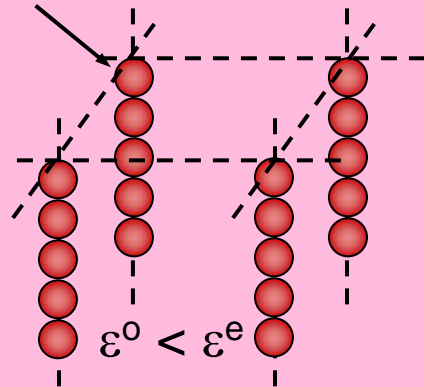
$$\epsilon_{\text{eff}} \cong \epsilon/2$$



$$C \cong \frac{\epsilon_0 A}{(d/2)}$$

$$\epsilon_{\text{eff}} \cong 2\epsilon_0$$

Atom or molecule



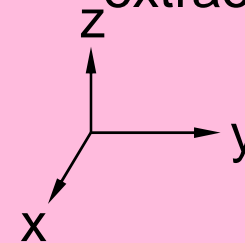
“Uniaxial Medium”

$$\epsilon_x = \epsilon_y = \epsilon^o$$

$$\epsilon_z = \epsilon^e$$

“ordinary”

“extraordinary”



WAVES IN UNIAXIAL MEDIA

Derive wave equation:

$$\begin{aligned}\nabla \times \underline{\bar{\mathbf{E}}} &= -j\omega \underline{\bar{\mathbf{B}}} & \nabla \cdot \underline{\bar{\mathbf{D}}} &= \rho_f = 0 \\ \nabla \times \underline{\bar{\mathbf{H}}} &= j\omega \underline{\bar{\mathbf{D}}} & \nabla \cdot \underline{\bar{\mathbf{B}}} &= 0\end{aligned}$$

Assume:

$$\begin{aligned}\underline{\bar{\mathbf{D}}} &= \underline{\bar{\boldsymbol{\epsilon}}}\underline{\bar{\mathbf{E}}} \\ \sigma &= 0, \underline{\bar{\boldsymbol{\mu}}} = \underline{\bar{\boldsymbol{\mu}}}_0\end{aligned} \quad \underline{\bar{\boldsymbol{\epsilon}}}_{||} = \begin{bmatrix} \epsilon^e & 0 & 0 \\ 0 & \epsilon & 0 \\ 0 & 0 & \epsilon \end{bmatrix}$$

$$\nabla \times (\nabla \times \underline{\bar{\mathbf{E}}}) = \nabla (\nabla \cdot \underline{\bar{\mathbf{E}}}) - \nabla^2 \underline{\bar{\mathbf{E}}} = -j\omega \mu \nabla \times \underline{\bar{\mathbf{H}}} = \omega^2 \underline{\bar{\boldsymbol{\mu}}}\underline{\bar{\boldsymbol{\epsilon}}}\underline{\bar{\mathbf{E}}}$$

Can show $\nabla \cdot \underline{\bar{\mathbf{E}}} = 0$ (can also test final solution)

Therefore:

$$\left[\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \right] [\hat{x}\underline{\bar{E}}_x + \hat{y}\underline{\bar{E}}_y + \hat{z}\underline{\bar{E}}_z] + \omega^2 \underline{\bar{\boldsymbol{\mu}}}\underline{\bar{\boldsymbol{\epsilon}}}\underline{\bar{\mathbf{E}}} = 0$$

Assume = 0 (assume UPW in z direction)

Yields 3 decoupled equations (x,y,z components)

BIREFRINGENT MEDIA

Decoupled wave equations:

$$\left[\frac{\partial^2}{\partial z^2} + \underbrace{\omega^2 \mu \epsilon^e}_{\triangleq (k^e)^2} \right] \underline{E}_x = 0, \quad k^e = \omega \sqrt{\mu \epsilon^e} \quad (\text{x-polarization equation})$$

$$\left[\frac{\partial^2}{\partial z^2} + \underbrace{\omega^2 \mu \epsilon}_\triangleq (k^o)^2 \right] \underline{E}_y = 0, \quad k^o = \omega \sqrt{\mu \epsilon} \quad (\text{y-polarization equation})$$

Solutions:

$$\underline{E}_x \propto e^{-jk^e z} = e^{-j(\omega/v^e)z} \quad \text{where} \quad \begin{cases} v^e = 1/\sqrt{\mu \epsilon^e} \text{ "extraordinary" velocity} \\ v^o = 1/\sqrt{\mu \epsilon} \text{ "ordinary" velocity} \end{cases}$$

Thus x- and y-polarized waves propagate independently at different velocities

If $v^e < v^o$ then $v^e \rightarrow$ "slow-axis velocity"

BIREFRINGENT MEDIA

Example:

Input: $\underline{\bar{E}}_1 = E_0 (\hat{x} + \hat{y}) \Rightarrow 45^\circ$ linear polarization

Output: $\underline{\bar{E}}_2 = E_0 (\hat{x} e^{-jk^e d} + \hat{y} e^{-jk^o d})$
 What pol.?

$$\Delta\phi \triangleq \phi^e - \phi^o = (k^e - k^o)d$$

$\pi/2$ "Quarter-wave plate"

$= \pi$ "Half-wave plate"

$=$

