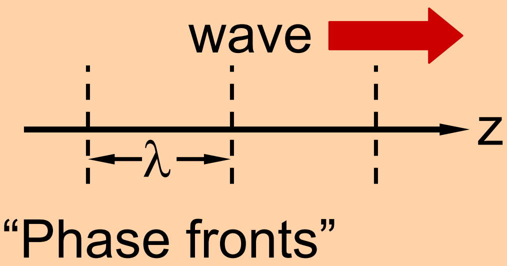
6.013 Electromagnetics and Applications
Spring 2009

UNIFORM PLANE WAVES

Z-Directed Wave:

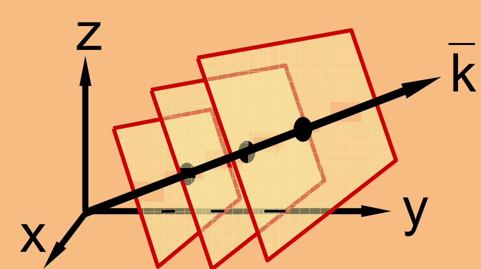
$$\overline{\overline{E}} = \overline{\overline{E}}_o e^{-jkz} = \overline{\overline{E}}_o e^{-j\underbrace{2\pi z/\lambda}_{\phi}}$$

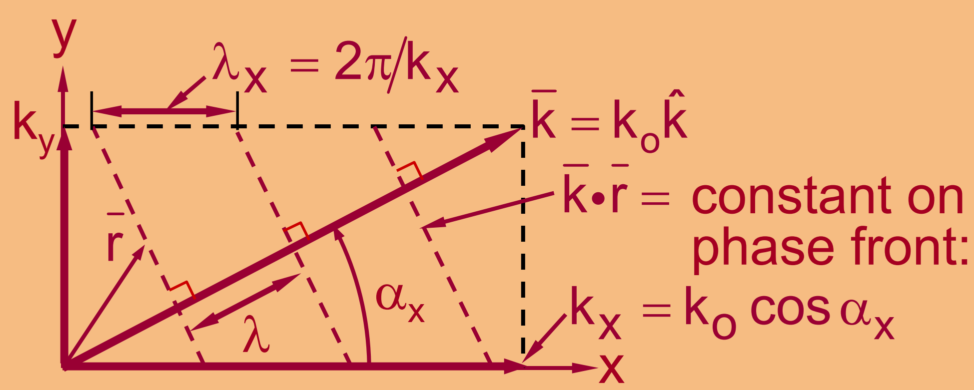


3-D Wave:

$$\overline{\overline{E}} = \overline{\overline{E}}_o e^{-jk_x x - jk_y y - jk_z z}$$

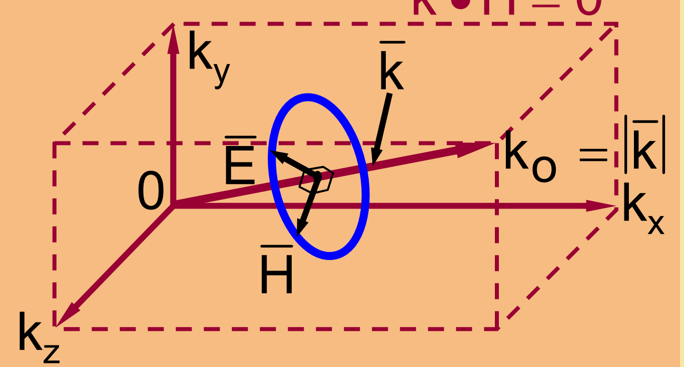
$$\overline{\overline{E}} = \overline{\overline{E}}_o e^{-j\overline{k}\cdot\overline{r}} \quad \text{where} \begin{cases} \overline{k} \triangleq \hat{x}k_x + \hat{y}k_y + \hat{z}k_z \\ \overline{r} \triangleq \hat{x}x + \hat{y}y + \hat{z}z \end{cases}$$





$$\overline{k} \bullet \overline{\overline{E}} = 0$$

$$\overline{k} \bullet \overline{\overline{H}} = 0$$



PLANE WAVES PROPAGATING AT ANGLES

Dispersion Relation:

Substitute $\underline{\bar{E}}_o e^{-j\bar{k}\bullet\bar{r}}$ into wave equation $\left(\nabla^2 + k_o{}^2\right)\underline{\bar{E}} = 0$:

$$\left.\begin{array}{l} \underline{\bar{E}}_o e^{-j\bar{k}\bullet\bar{r}} = \underline{\bar{E}}_o e^{-j\left(k_x x + k_y y + k_z z\right)} \\ \nabla^2 = \dfrac{\partial^2}{\partial x^2} + \dfrac{\partial^2}{\partial y^2} + \dfrac{\partial^2}{\partial z^2} \end{array}\right\} \Rightarrow k_x^2 + k_y^2 + k_z^2 = \omega^2\mu\varepsilon \triangleq k_o^2$$

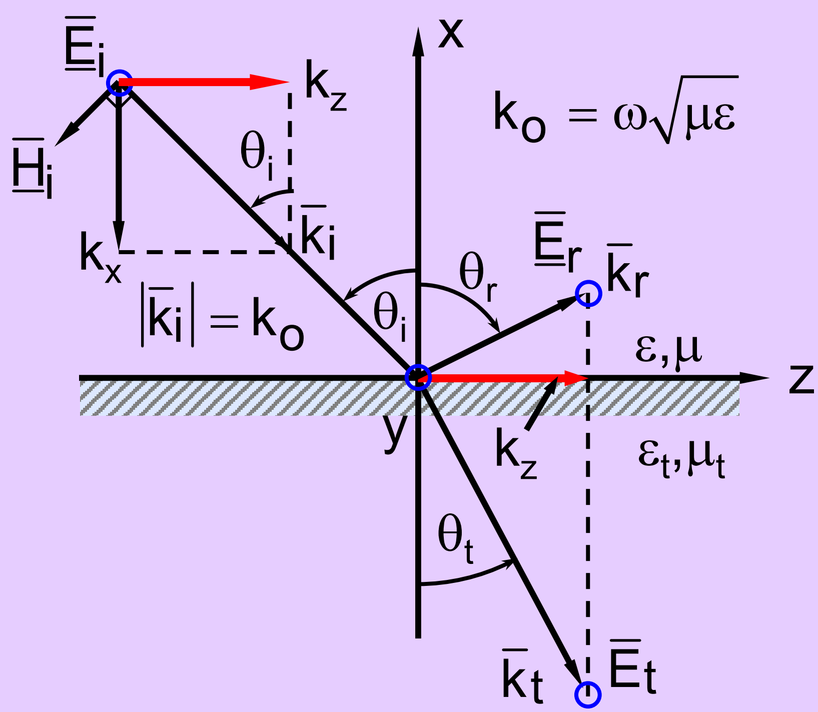
Wave Vector $\bar{k}$:

Perpendicular to uniform plane wave phase front,

Therefore perpendicular to $\bar{E}$ and $\bar{H}$

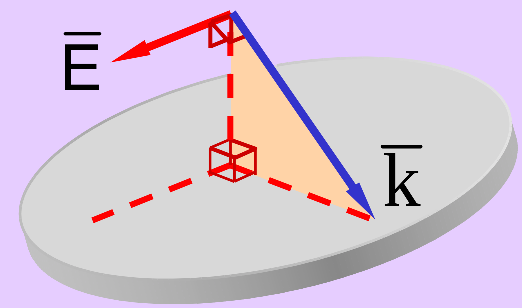
UPW AT PLANAR BOUNDARY

Case I: TE Wave



"Transverse Electric"

$\triangleq \bar{E} \perp$ Plane of incidence



Trial Solutions:

Incident: $\underline{\bar{E}}_i = \hat{y}E_o e^{+jk_x x - jk_z z} = \hat{y}E_o e^{+j(k_o\cos\theta_i)x - j(k_o\sin\theta_i)z}$

Reflected: $\underline{\bar{E}}_r = \hat{y}\underline{\Gamma}E_o e^{-j(k_o\cos\theta_r)x - j(k_o\sin\theta_r)z}$

Transmitted: $\underline{\bar{E}}_t = \hat{y}\underline{T}E_o e^{+j(k_t\cos\theta_t)x - j(k_t\sin\theta_t)z}$

IMPOSE BOUNDARY CONDITIONS

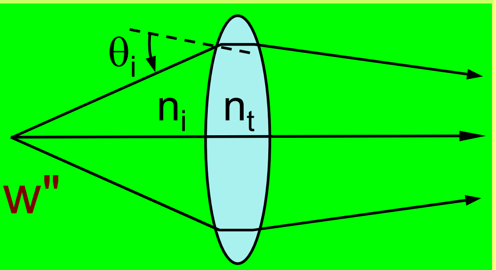
$\overline{\underline{E}}_{//}$ is continuous at x = 0:

$$E_o e^{-jk_o \sin\theta_i z} + \underline{\Gamma} E_o e^{-jk_o \sin\theta_r z} = \underline{T} E_o e^{-jk_t \sin\theta_t z} \quad \text{for all } z$$

Therefore:

$$\begin{cases} \underbrace{k_o \sin\theta_i}_{k_{i_z}} = \underbrace{k_o \sin\theta_r}_{k_{r_z}} = \underbrace{k_t \sin\theta_t}_{k_{t_z}} = k_z \\ \theta_r = \theta_i \quad \text{Angle of incidence equals angle of reflection} \end{cases}$$

Snell's Law:



$$\frac{\sin\theta_t}{\sin\theta_i} = \frac{k_o}{k_t} = \frac{\omega\sqrt{\mu\varepsilon}}{\omega\sqrt{\mu_t\varepsilon_t}} = \frac{v_t}{v_i} = \frac{n_i}{n_t} \quad \text{"Snell's Law"}$$

where $n \triangleq c/v_{phase} = c\sqrt{\mu\varepsilon}$ "Refractive Index"

$n_{vacuum} = 1$

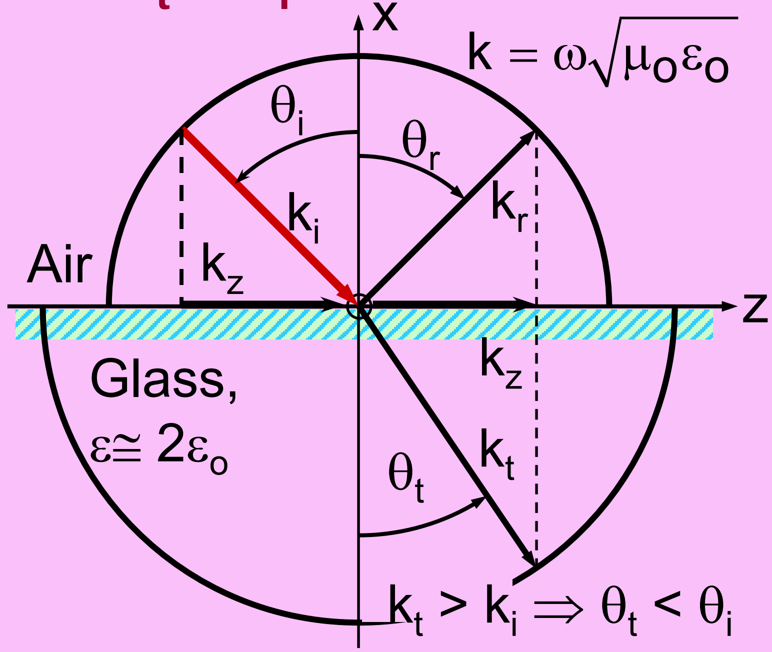
$n_{glass} \cong 1.5 - 1.66$

$n_{water} \cong 1.3$ at visible wavelengths

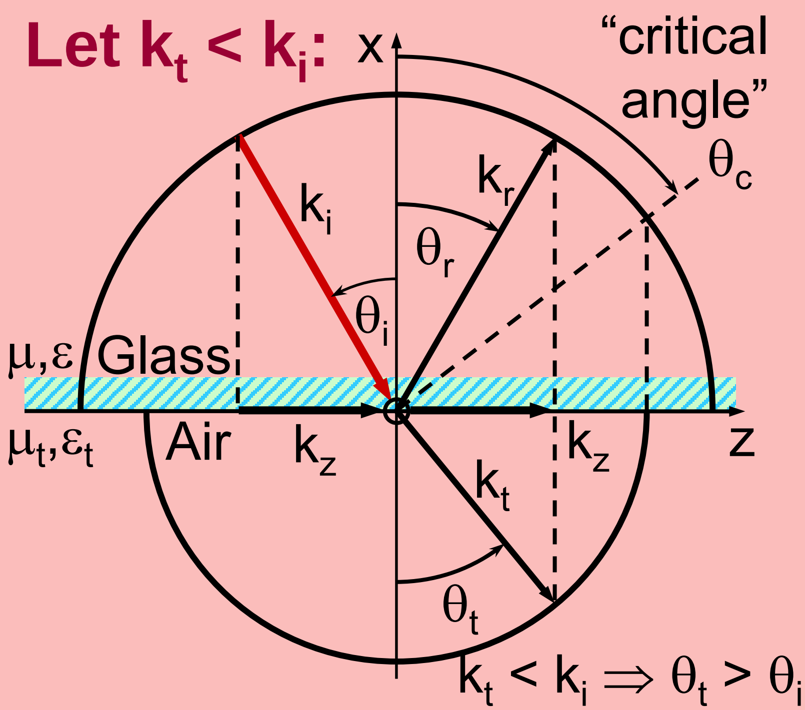
$\lesssim 9$ at audio-radio frequencies

SNELL'S LAW: $k_o \sin \theta_i = k_t \sin \theta_t$

Let $k_t > k_i$:



Let $k_t < k_i$:



Critical Angle θ_c:

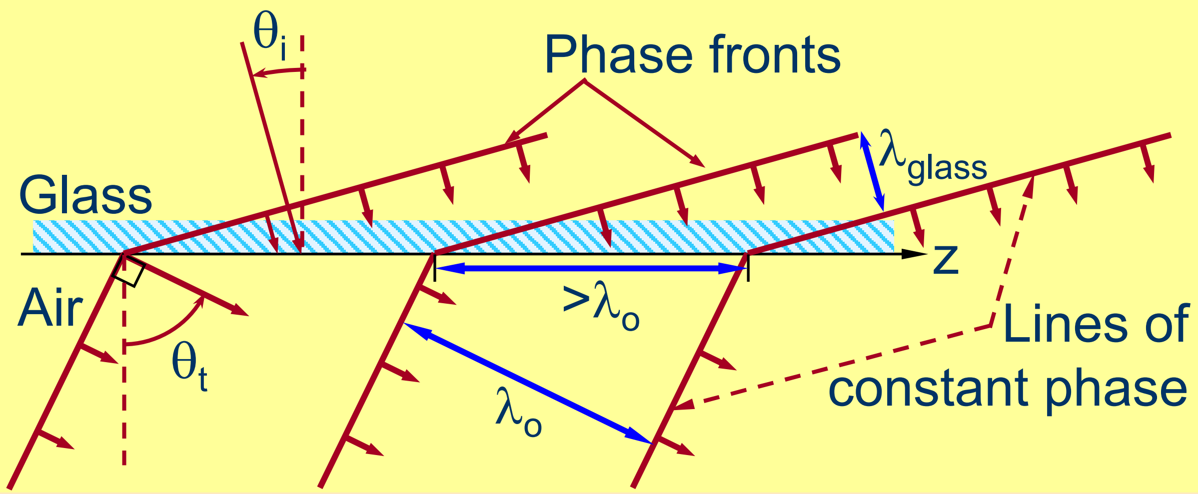
$$\frac{\sin\theta_t}{\sin\theta_i} = \frac{n_i}{n_t}$$

Therefore:

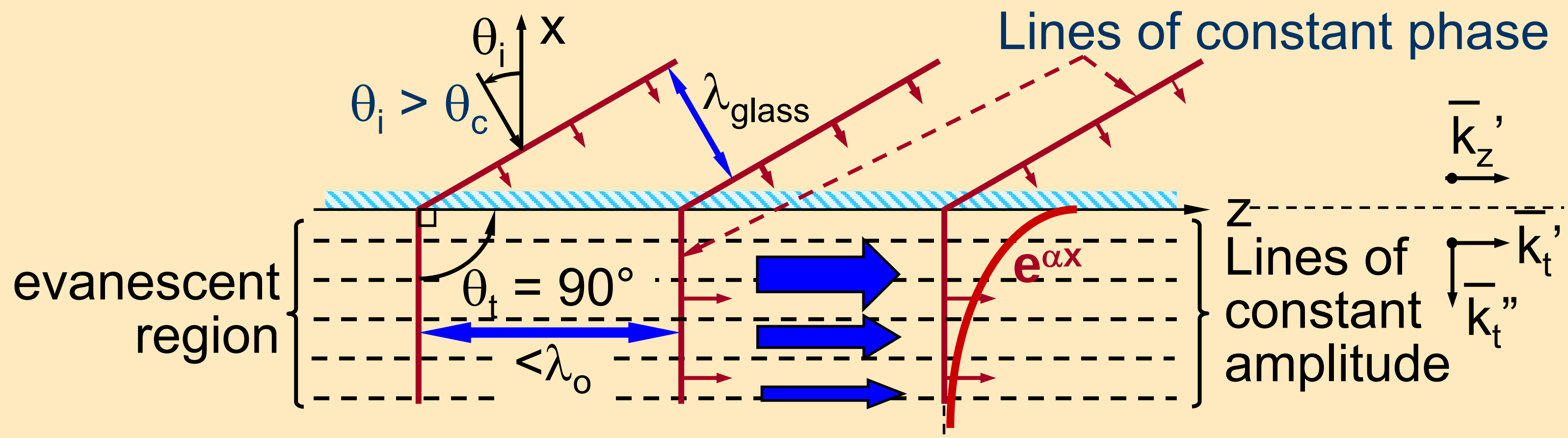
$$\begin{cases} \theta_t \rightarrow 90^o \Rightarrow \sin\theta_t \rightarrow 1 \text{ as } \theta_i \rightarrow \theta_c \\ \theta_c = \sin^{-1}(n_t/n_i) \quad \text{"critical angle"} \\ \text{e.g., } [\varepsilon_i = 2\varepsilon_o] \Rightarrow [n_i = \sqrt{2}] \Rightarrow [\theta_c = 45^\circ] \end{cases}$$

NON-UNIFORM PLANE WAVES (NUPW)

Normal refraction: $\theta_i < \theta_c$



Beyond the critical angle, evanescence:



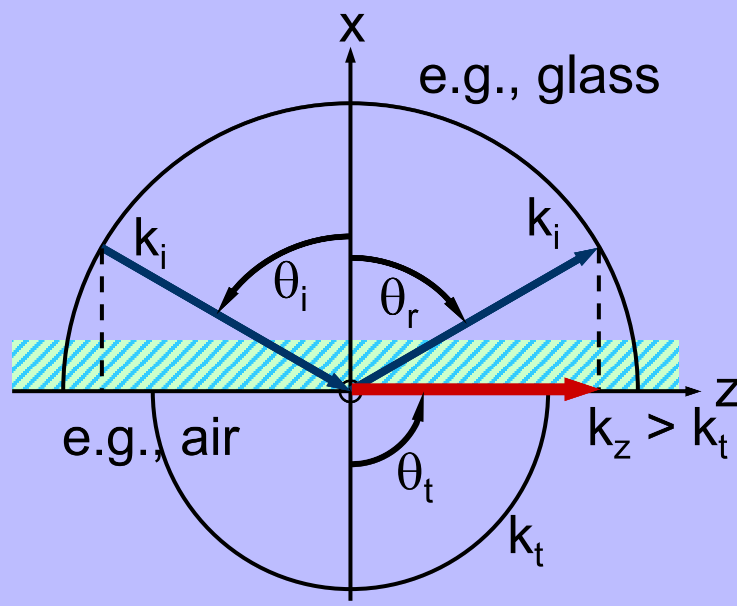
EVANESCENT WAVES

k_{tx} when $\theta > \theta_c$:

Since: $k_t^2 = k_z^2 + k_{tx}^2 = \omega^2\mu_t\varepsilon_t$

Therefore: $\begin{cases} k_{tx}^2 = k_t^2 - k_z^2 < 0 \\ \underline{k}_{tx} = \pm j\alpha = \pm j\sqrt{k_z^2 - k_t^2} \end{cases}$

Where: $k_z^2 = \omega^2\mu_i\varepsilon_i \sin^2\theta_i$



Fields when $\theta > \theta_c$:

$$\underline{\overline{E}}_t = \hat{y}\underline{T}E_o e^{-jk_z z + jk_{tx}x} = \hat{y}\underline{T}E_o e^{+\alpha x - jk_z z} \quad (x < 0)$$

More generally: $\underline{\overline{E}}_t = \hat{y}\underline{T}E_o\, e^{-j\underline{\overline{k}}_t \bullet \overline{r}}$

where: $\underline{\overline{k}}_t = k_z\hat{z} - j\alpha\hat{x} \triangleq \overline{k}' - j\overline{k}''$

If lossless medium, $\overline{k}' \bullet \overline{k}'' = 0$ $\qquad \underline{\overline{E}}, \underline{\overline{H}} \propto e^{-j(\overline{k}' - j\overline{k}'')\bullet\overline{r}}$

EVANESCENT WAVES -- SUMMARY

Names:

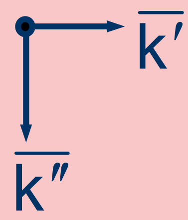
"non-uniform plane wave"
"evanescent wave" ($\langle\langle S(t)\rangle = 0$ in direction of decay)
"surface wave"
"inhomogeneous plane wave"

Lossless medium:

$$\underline{E},\underline{H} \ \alpha \ e^{-j(\bar{k}'-j\bar{k}'')\bullet\bar{r}}$$

$$\bar{k}' \bullet \bar{k}'' = 0$$

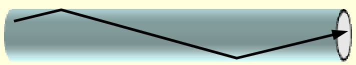
$$\underline{k}_{tx} = \pm j\alpha = \pm j\sqrt{k_z^2 - k_t^2}$$



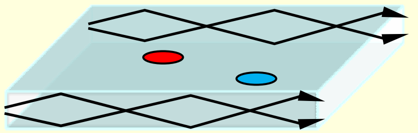
Lossy medium:

$$\bar{k}' \bullet \bar{k}'' \neq 0$$

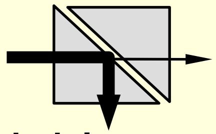
Applications:



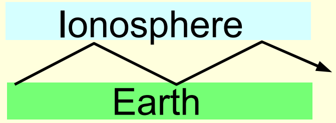
Optical fibers



Flat panel displays



Variable couplers



Global communications