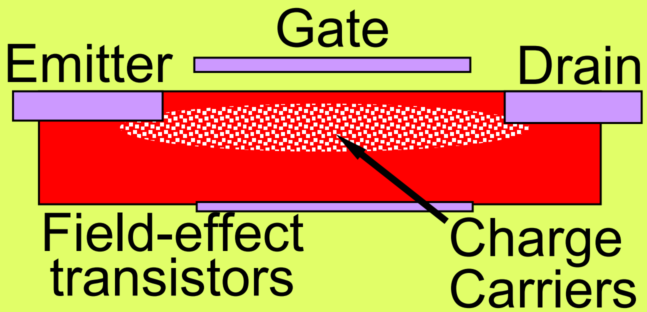
6.013 Electromagnetics and Applications
Spring 2009

LIMITS TO COMPUTATION SPEED

Devices:



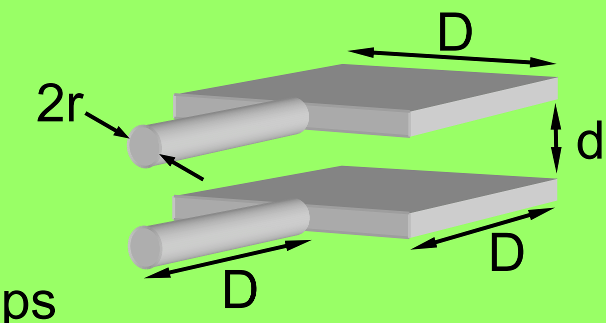
Carrier transit and diffusion times

($f = ma$, $v < c$)

$RC \cong \varepsilon/\sigma$; RL, LC time constants

Beyond scope of 6.013 (read Section 8.2)

Interconnect, short lines $<<\lambda$:



Wire resistance $R \propto D/r^2$

Capacitance $C = \varepsilon A/d \propto D^2/d$

$\tau = RC \propto D^3/r^2 d \cong$ const. if D:r:d = const.

R is high for polysilicon, C is high for thin gaps

L/R and $\tau = \sqrt{LC}$ scale well with size and do not limit speed

Interconnect, long lines $>\sim\lambda/8$:

Propagation delay: $c = 1/\sqrt{\mu\varepsilon} < 3 \times 10^8$ [m s^{-1}] (ε might be $\sim 2\varepsilon_o$)

Reflections at wire and device junctions, unless carefully designed

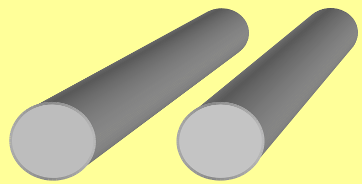
Resistive loss

Radiation and cross-talk (3-GHz clocks imply 30-GHz harmonics)

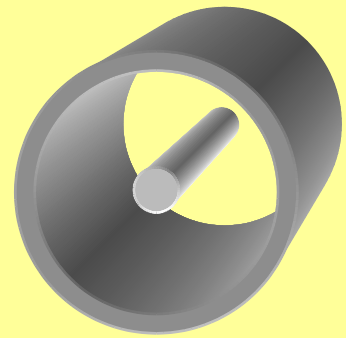
WIRED INTERCONNECTIONS

Transverse EM Transmission Lines:

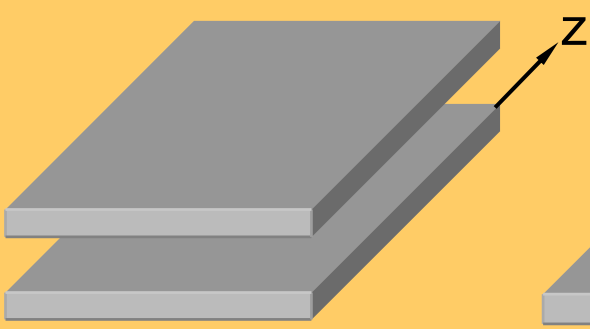
TEM: $\overline{E}_z = \overline{H}_z = 0$



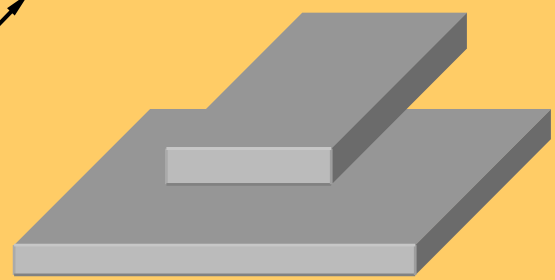
Parallel wires



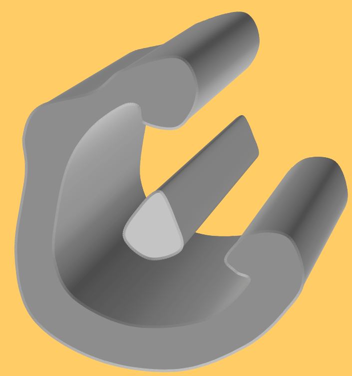
Coaxial cable



Parallel plates



Stripline



Arbitrary cross-section $\neq f(z)$

PARALLEL-PLATE TRANSMISSION LINE

Boundary Conditions:

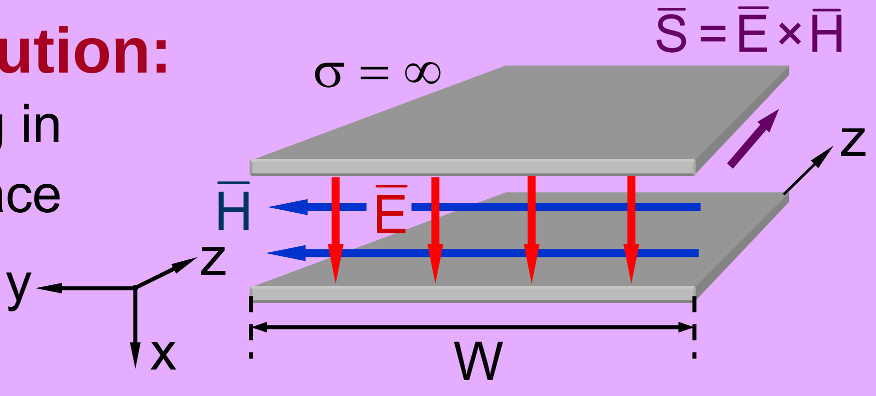
$\overline{E}_{//} = \overline{H}_{\perp} = 0$ at perfect conductors

Uniform Plane Wave Solution:

x-polarized wave propagating in +z direction in free space

$$\overline{E} = \hat{x}\, E_{+}\left(t - \frac{z}{c}\right)$$

$$\overline{H} = \hat{y}\, \left(\frac{1}{\eta_o}\right) E_{+}\left(t - \frac{z}{c}\right)$$



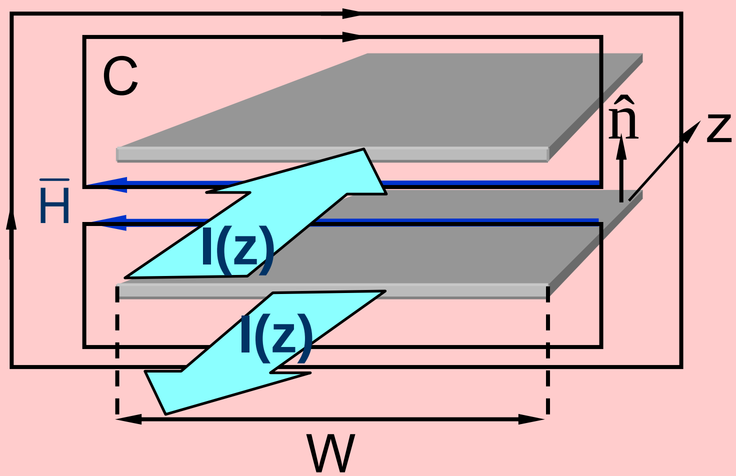
Currents in Plates:

$$\oint_C \overline{H} \bullet d\overline{s} = \iint_A \overline{J} \bullet d\overline{a} = I(z)$$

I(z) = H(z)W, independent of path C

Surface Currents J_s(A m^{-1}):

$$\overline{J}_s(z) = \hat{n} \times \overline{H}(z) \quad [\text{A m}^{-1}]$$



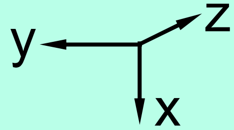
TRANSMISSION LINE VOLTAGES

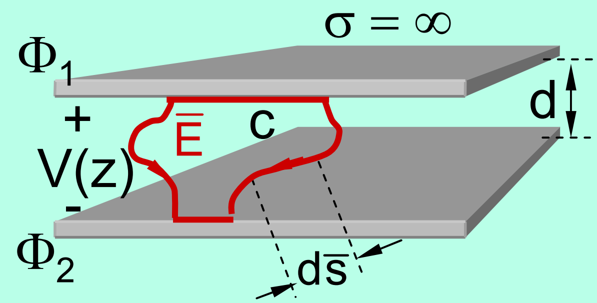
Voltages between plates:

Since $H_z = 0 \Rightarrow \oint_c \overline{E} \bullet d\overline{s} = 0$ at fixed z,

$\int_1^2 \overline{E} \bullet d\overline{s} = \Phi_1 - \Phi_2 = V(z)$

V(z) is uniquely defined





Surface charge density $\rho_s(z)$ [C m^{-2}]:

$\hat{n} \bullet \varepsilon \overline{E}(z) = \rho_s(z)$ (Boundary condition; from $\nabla \bullet \overline{D} = \rho$

Integrate $\overline{E}, \overline{H}$ to find v(t, z), i(t, z)

$v(t,z) = \int_1^2 \overline{E} \bullet d\overline{s} = d \times E_+(t - z/c)$ here, where $\overline{E} = \hat{x}E_+(t - z/c)$

$i(t,z) = \oint_c \overline{H} \bullet d\overline{s} = (W/\eta_o)E_+(t - z/c)$, where $\overline{H} = \hat{y}E_+(t - z/c)$

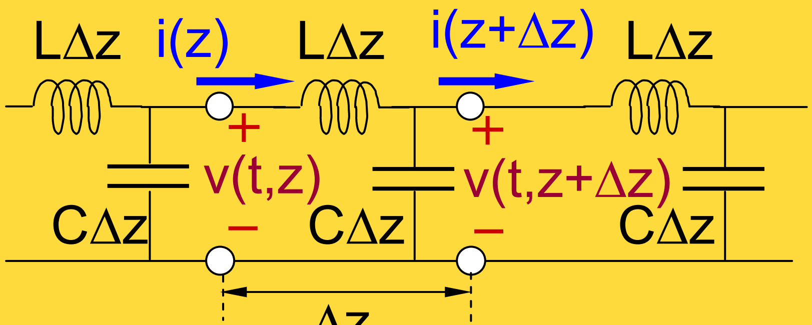
$v(t,z) = Z_o\, i(t,z)$ [if there is no backward propagating wave]

$Z_o = \eta_o d/W$ [ohms] "Characteristic impedance"

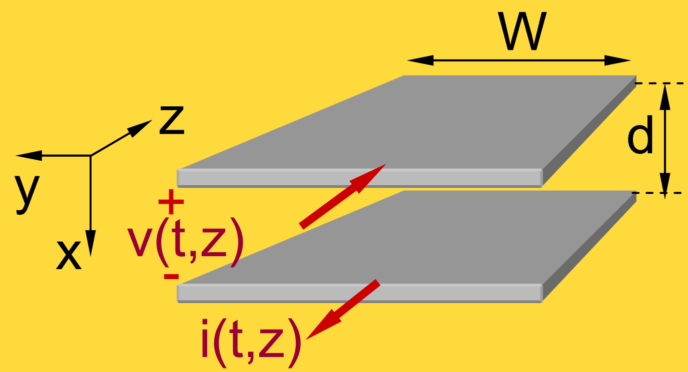
Note: v(z) violates KVL, and i(z) violates KCL

TELEGRAPHER'S EQUATIONS

Equivalent Circuit:



L [Henries m^{-1}], C [Farads m^{-1}]



$\overline{E}\bullet\hat{z} = \overline{H}\bullet\hat{z} = 0$ (TEM)

Difference Equations:

$$v(z+\Delta z) - v(z) = -L\Delta z\frac{di(z)}{dt}$$

$$i(z+\Delta z) - i(z) = -C\Delta z\frac{dv(z)}{dt}$$

Limit as $\Delta z \rightarrow 0$:

$$\frac{dv(z)}{dz} = -L\frac{di(z)}{dt}$$

$$\frac{di(z)}{dz} = -C\frac{dv(z)}{dt}$$

Wave Equation:

$$\frac{d^2v}{dz^2} = LC\frac{d^2v}{dt^2}$$

SOLUTION: TELEGRAPHER'S EQUATIONS

Wave Equation: $$\frac{d^2 v}{dz^2} = LC\frac{d^2 v}{dt^2}$$

Solution:

$$v(z,t) = f_+(t - z/c) + f_-(t + z/c)$$

f_+ and f_- are arbitrary functions

Substituting into Wave Equation:

$$(1/c^2)\,[f_+''(t - z/c) + f_-''(t + z/c)] = LC\,[f_+''(t - z/c) + f_-''(t + z/c)]$$

Therefore: $$c = 1/\sqrt{LC} = 1/\sqrt{\mu\varepsilon}$$

Current I(z,t):

Recall: $$\frac{di(z)}{dz} = -C\frac{dv(z)}{dt} = -C[f_+'(t - z/c) + f_-'(t + z/c)]$$

Therefore: $i(z,t) = cC\,[f_+(t - z/c) - f_-(t + z/c)]$

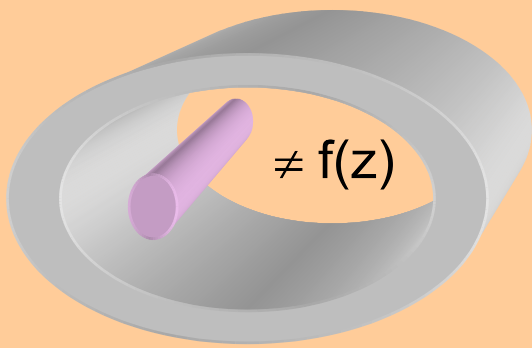
Where: $cC = C/\sqrt{LC} = \sqrt{C/L} = Y_o$ "Characteristic admittance"

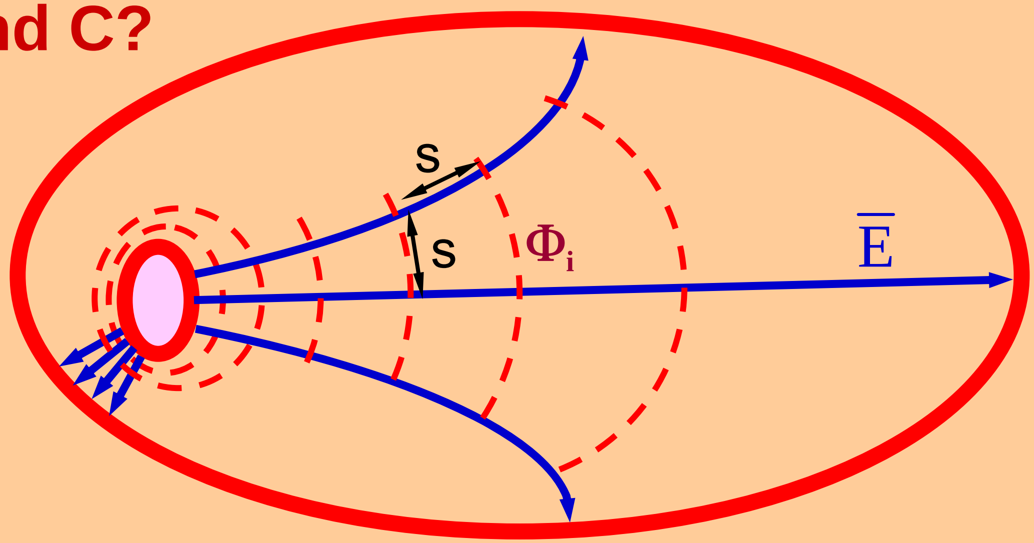
$Z_o = 1/Y_o = \sqrt{L/C}$ ohms "Characteristic impedance"

Therefore: $$i(z,t) = Y_o[f_+(t - z/c) - f_-(t + z/c)]$$

ARBITRARY TEM LINES

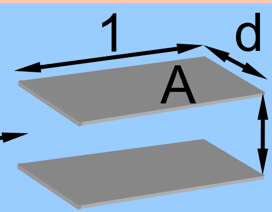
Can we estimate L and C?





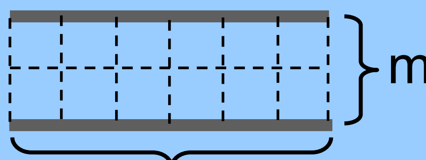
$Z_o = \sqrt{\frac{L}{C}} = \frac{\sqrt{LC}}{C}$ where $LC = \mu\varepsilon$

$C_\square$ = capacitance/m = $\frac{\varepsilon A}{d} =$ ε



C = n$C_\square$ for n parallel square incremental capacitors ($C_\square = \varepsilon$ [Fm^{-1}])

C = $C_\square$/m for m capacitors $C_\square$ in series

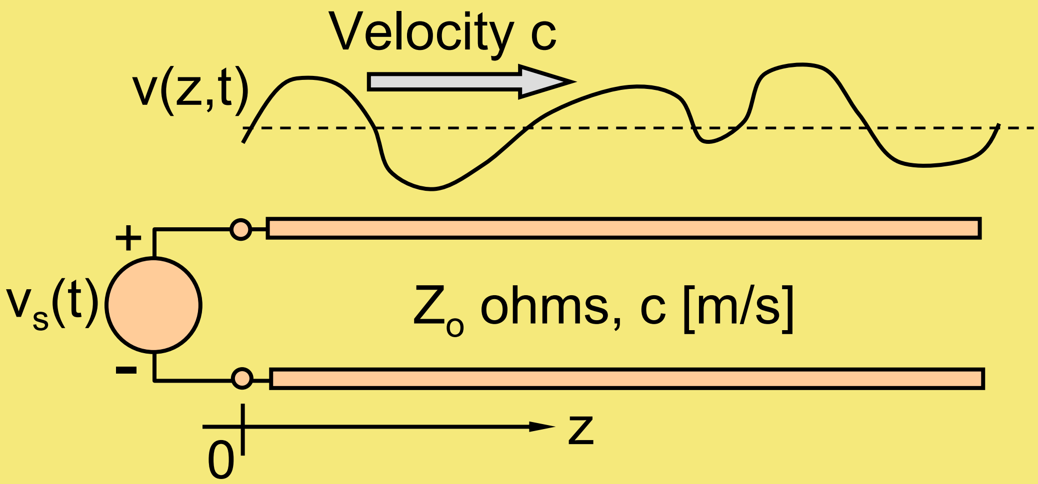


Therefore:

C = n$C_\square$/m = nε/m [Farads/m]

$Z_o = \frac{\sqrt{LC}}{C} = \frac{\sqrt{\mu\varepsilon}}{n\varepsilon/m} = \eta_o \frac{m}{n}$ ohms [$\eta_o = \sqrt{\mu/\varepsilon}$]; $Z_{o\,single\,cell} = 377\Omega$

TRANSMISSION LINE VOLTAGES



Matching boundary conditions:

v(t) and I(t) are continuous at z = 0

$$\Rightarrow \quad v(z,t) = v_s(t - z/c)$$

$$i(z,t) = \frac{1}{Z_o} v_s(t - z/c)$$