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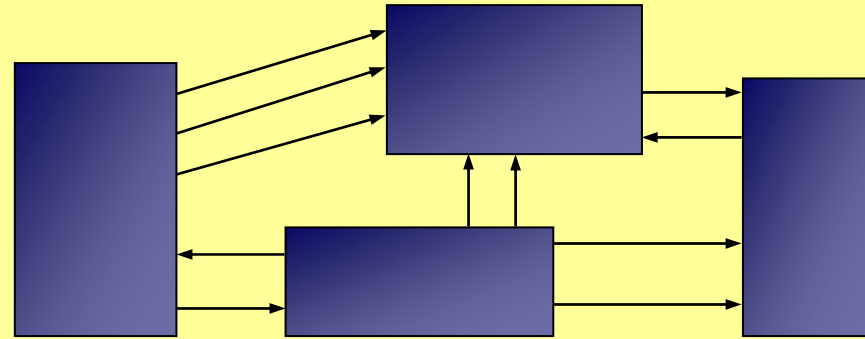
6.013 Electromagnetics and Applications
Spring 2009

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TRANSIENT SIGNALS IN COMPUTERS

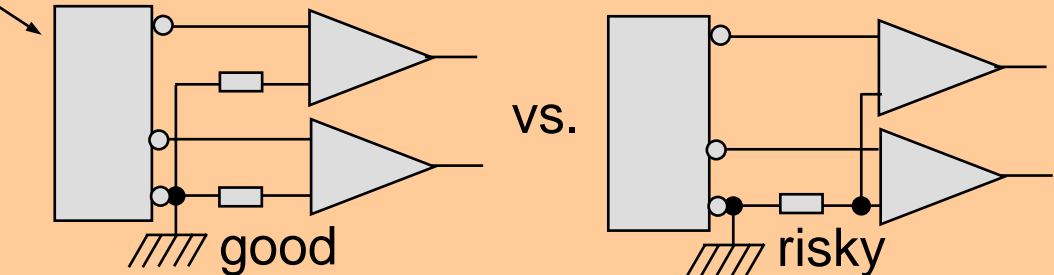
Ideal World:

- Only 1's and 0's
- Instantaneous links



Reality:

- Voltages exhibit propagation delay, decay, reflections
- Spurious transients can superimpose to flip bits erroneously
- RFI generated and picked up by wires can flip bits
- Ground loops matter



TEM LINE THEVENIN EQUIVALENT

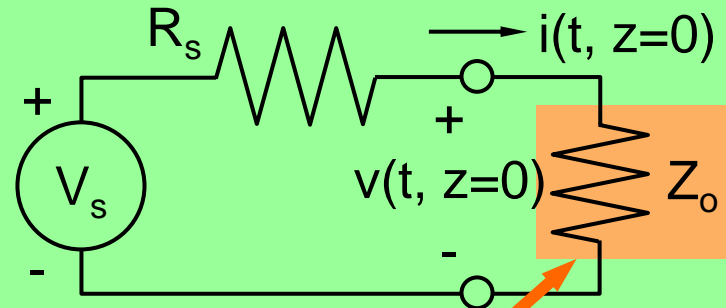
Example: Given $V_s(t)$:

$$v(z, t) = v_+(t - \frac{z}{c}) + v_-(t + \frac{z}{c})$$

$$i(z, t) = \frac{1}{Z_o} [v_+(t - \frac{z}{c}) - v_-(t + \frac{z}{c})]$$

Assume $v_- = 0$ (no other sources)

Then $v(t, z=0) = Z_o i(t, z=0)$ yields equivalent circuit



Solution at terminals:

$$v_+(t, z=0) = V_s(t) \frac{Z_o}{Z_o + R_s} \text{ (voltage divider equation)}$$

Solution for all t, z :

$$v_+(t, z) = V_s(t - \frac{z}{c}) \frac{Z_o}{Z_o + R_s} \text{ (forward propagating wave only)}$$

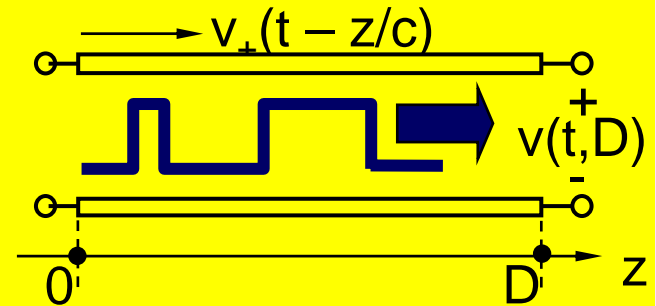
TEM LINE THEVENIN EQUIVALENT

Voltages at an Open Circuit:

$$i(t,D) = Y_o[v_+(t,D) - v_-(t,D)] = 0 \text{ (open circuit)}$$

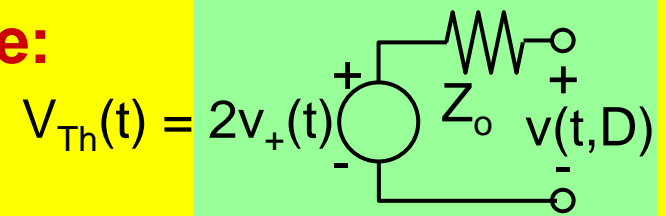
$$\Rightarrow v_-(t,D) = v_+(t,D)$$

$$\Rightarrow v(t,D) = v_+(t,D) + v_-(t,D) = 2v_+(t,D)$$



Thevenin Equivalent for TEM source:

$$V_{\text{open circuit}}(t) = 2v_+(t) = V_{\text{Th}}(t)$$



Example—Resistive Load:

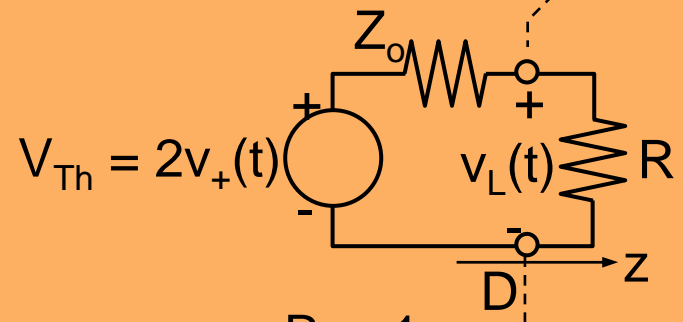
$$\text{At } z = D: v_-(t) = v_L(t) - v_+(t)$$

$$\text{But: } v(t) = 2v_+(t) \frac{R}{R + Z_o}$$

$$\text{Thus: } v_-(t) = v_+(t) \frac{R - Z_o}{R + Z_o} = v_+(t) \Gamma$$

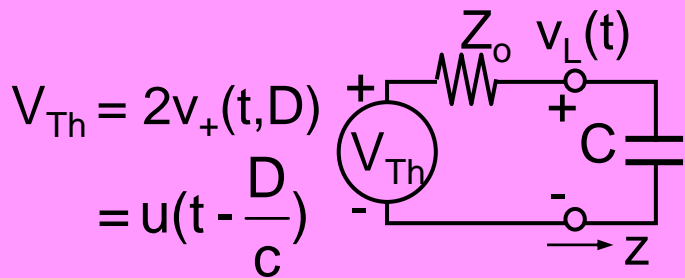
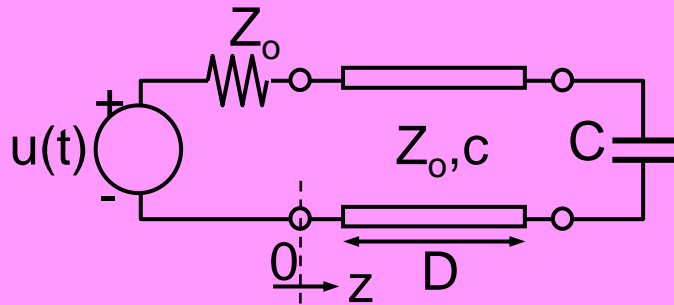
$$\text{Define: } \Gamma = \text{“reflection coefficient”} = v_-/v_+ \text{ at load} = \frac{R_n - 1}{R_n + 1}$$

(= 0 if $R = Z_o$, = -1 if $R = 0$, and = +1 if $R = \infty$)



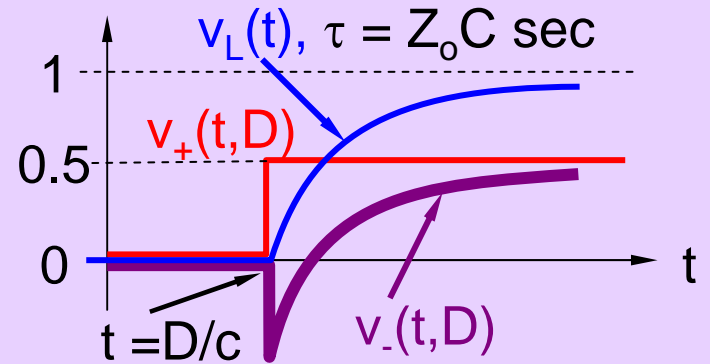
CAPACITIVELY TERMINATED TEM LINE

Example: Capacitive Load

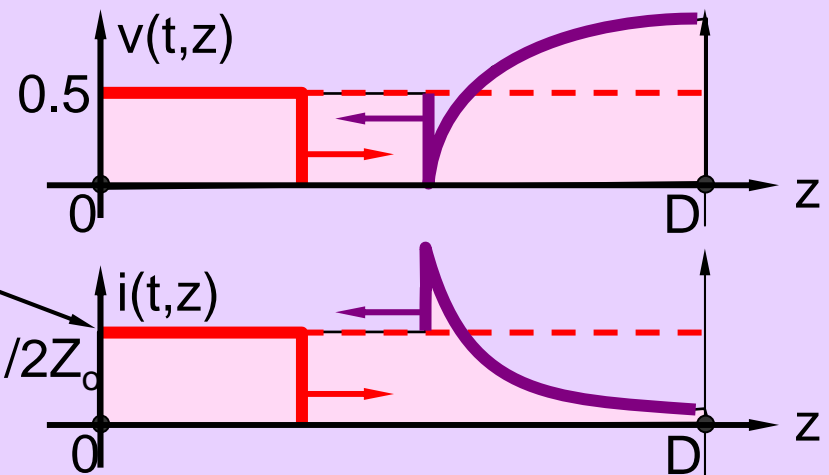


$$i(z, t) = Y_0 \left[v_+ \left(t - \frac{z}{c} \right) - v_- \left(t + \frac{z}{c} \right) \right]$$

$t = 0+$, short-circuit response
 $t \rightarrow \infty$, open-circuit response

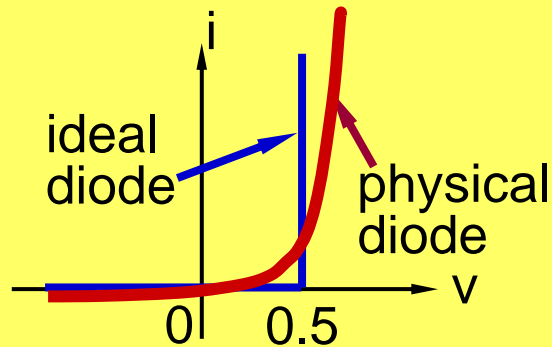


$v_L = v_- + v_+$ at the load \Rightarrow
 $v_-(t, D) = v_L(t, D) - v_+(t, D)$

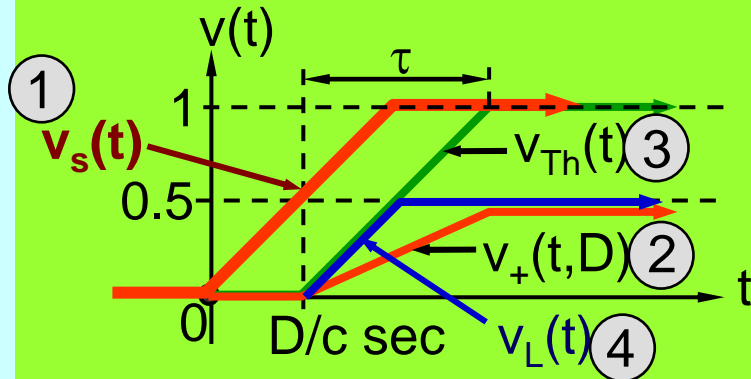
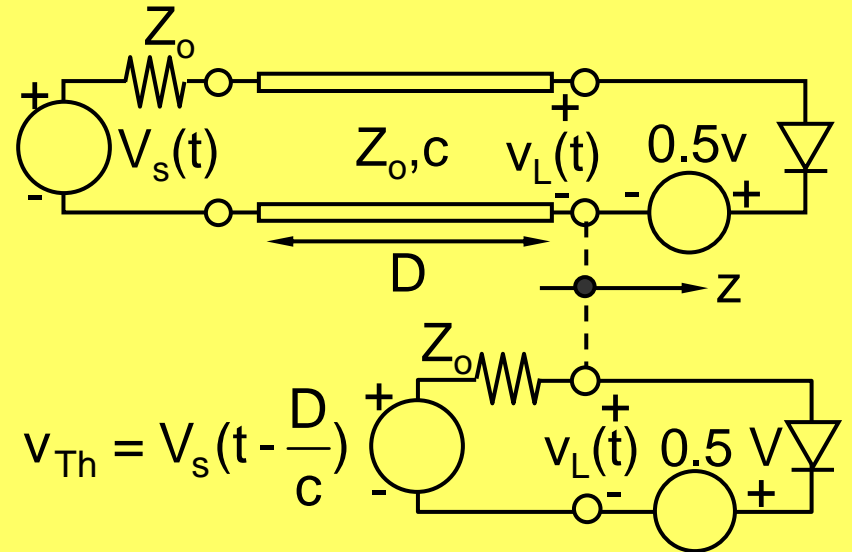


DIODE-TERMINATED TEM LINE

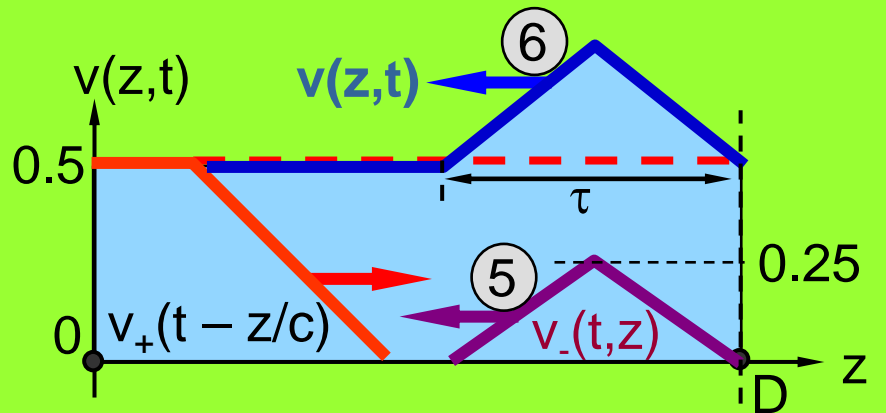
Example -- Logic Circuit:



Let $v_s(t) = 1$ -volt ramped step



$$v_L(t) = v_+(t, D) + v_-(t, D)$$



INITIAL CONDITIONS

Example – charged TEM line:

$$v(z,t) = v_+(z - ct) + v_-(z + ct)$$

$$i(z,t) = \frac{1}{Z_0} [v_+(z - ct) - v_-(z + ct)]$$

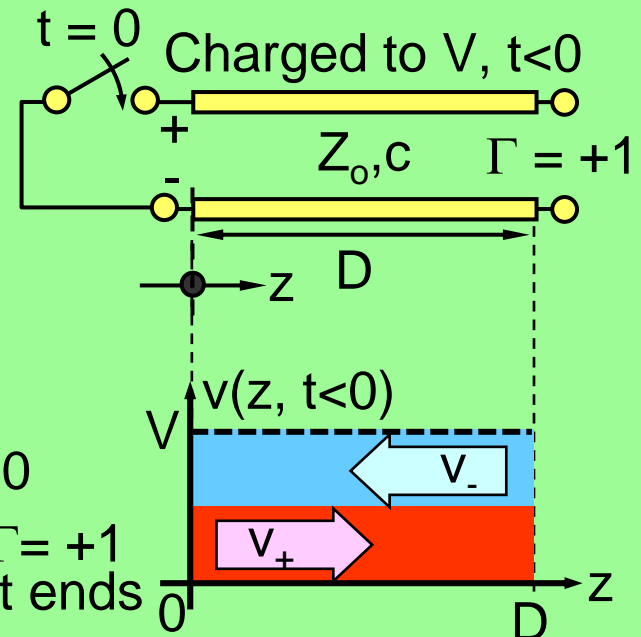
Initial voltage and currents:

$$v_+(z,t) = \frac{1}{2} [v(z,t) + Z_0 i(z,t)]$$

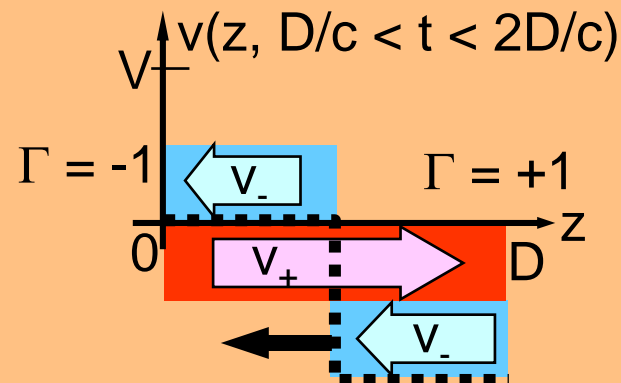
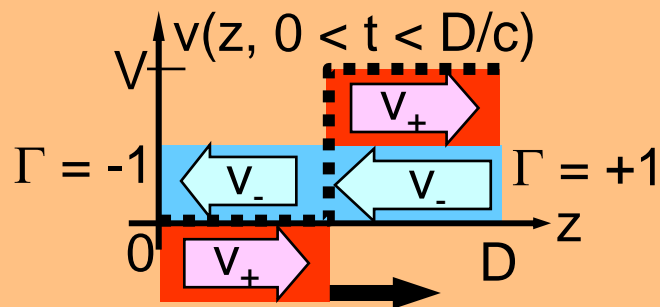
$$v_-(z,t) = \frac{1}{2} [v(z,t) - Z_0 i(z,t)]$$

$$\Rightarrow i(t < 0) = 0$$

$\Gamma = +1$
at ends



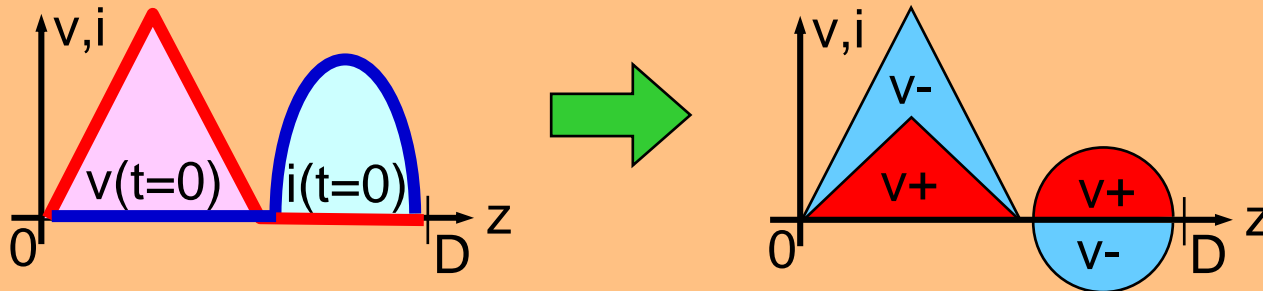
Subsequent voltages and currents:



Lossless
system,
rattles
forever

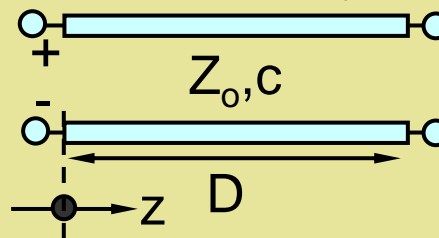
INITIAL CONDITIONS

Arbitrary initial conditions:

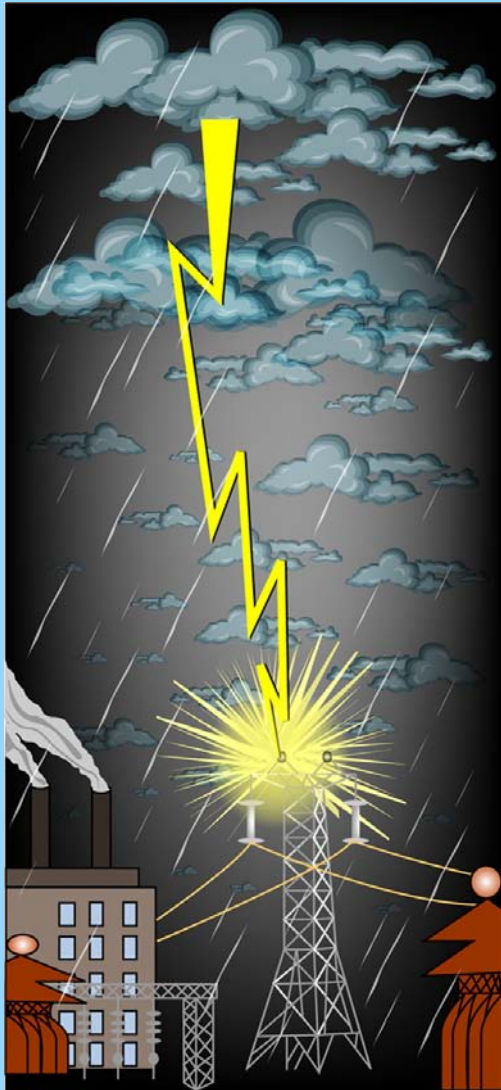


Arbitrary circuits:

Put switches anywhere



LIGHTNING STRIKE EXAMPLE



Lightning strikes the midpoint

What happens?

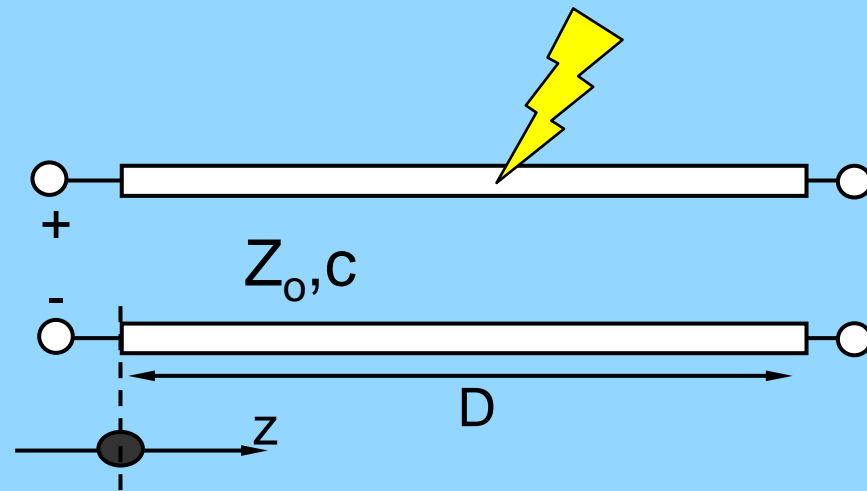
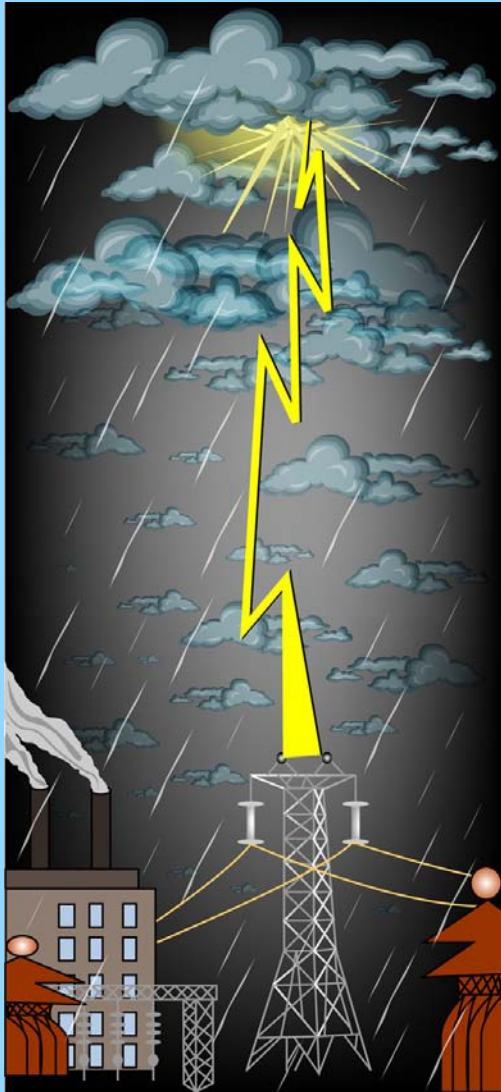


Figure by MIT OpenCourseWare.

LIGHTNING STRIKE EXAMPLE



Revenge?

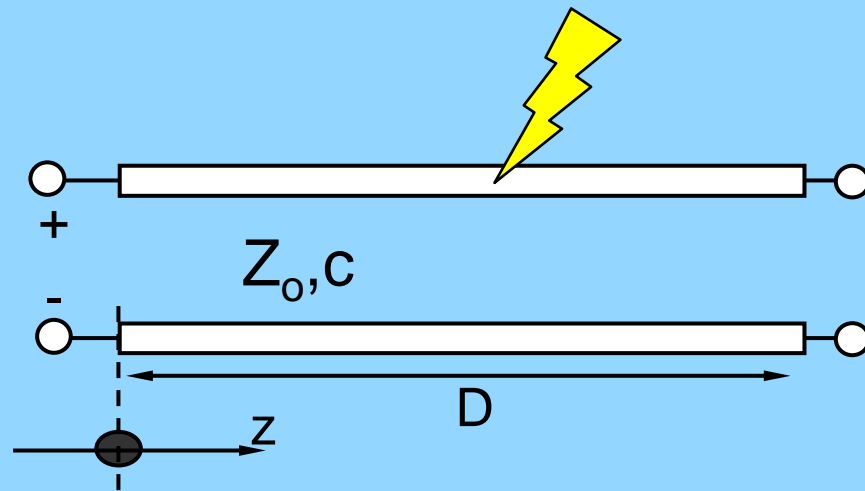
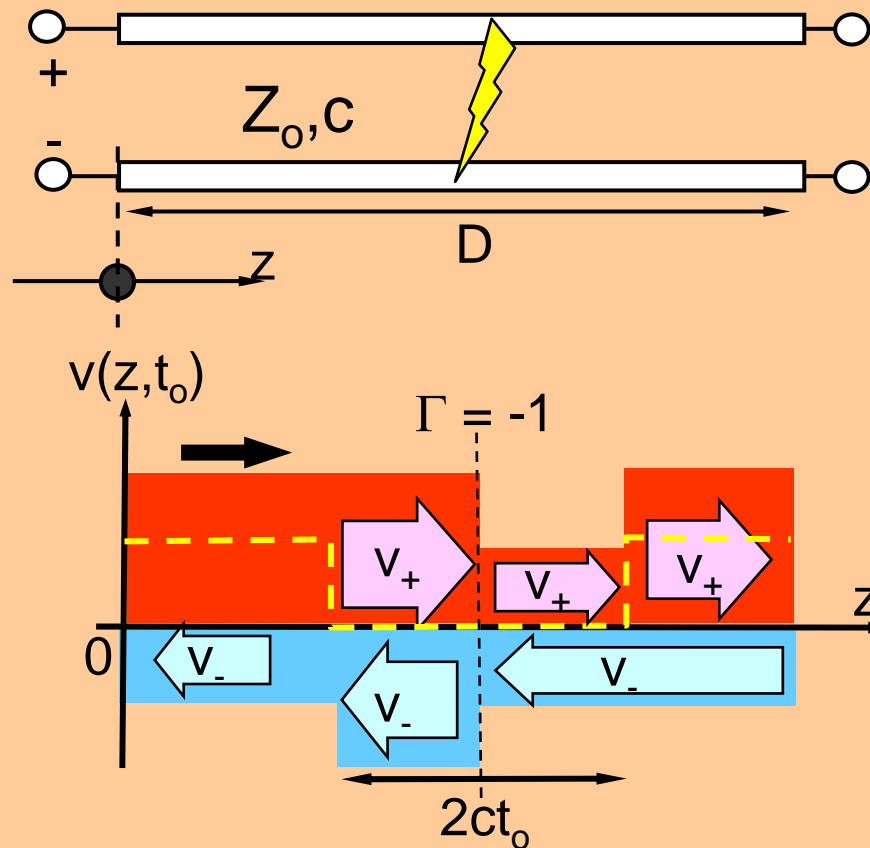


Figure by MIT OpenCourseWare.

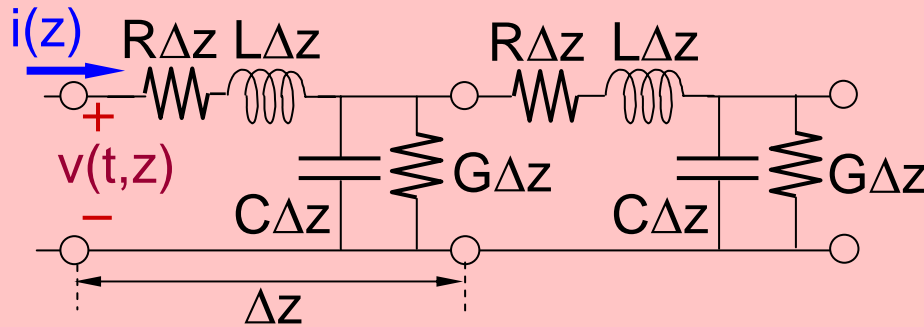
LIGHTNING STRIKE EXAMPLE

One possibility:
Temporary short through ionized channel



LOSSY TRANSMISSION LINES

Equivalent Circuit:



$$L \text{ [Hy m}^{-1}\text{]}, C \text{ [F m}^{-1}\text{]}, \\ R \text{ [}\Omega \text{ m}^{-1}\text{]}, G \text{ [S m}^{-1}\text{]}$$

Equations for $\Delta z \rightarrow 0$:

$$\left. \begin{aligned} \frac{d\underline{V}(z)}{dz} &= -(R + j\omega L)\underline{I}(z) \\ \frac{d\underline{I}(z)}{dz} &= -(G + j\omega C)\underline{V}(z) \end{aligned} \right\} \Rightarrow \begin{cases} \frac{d^2\underline{V}(z)}{dz^2} = (R + j\omega L)(G + j\omega C)\underline{V}(z) \\ \underline{V}(z) = \underline{V}_+ e^{-j\underline{k}z} + \underline{V}_- e^{+j\underline{k}z} \\ \underline{k} = \sqrt{-(R + j\omega L)(G + j\omega C)} = \underbrace{k' - jk''}_{\text{decay}} \end{cases}$$

$$\left. \begin{aligned} \underline{V}(z) &= \underline{V}_+ e^{-j\underline{k}'z - \underline{k}''z} \\ \underline{I}(z) &= Y_0 \underline{V}_+ e^{-j\underline{k}'z - \underline{k}''z} \end{aligned} \right\} \text{ for forward wave: } Y_0 = \sqrt{(G + j\omega C)/(R + j\omega L)}$$

Propagation $e^{-j\underline{k}z}$: decay rate ($e^{-\underline{k}''z}$) and phase velocity (v_p) = $f(\omega)$

(Exception: "distortionless line" for which $R/L = G/C$)