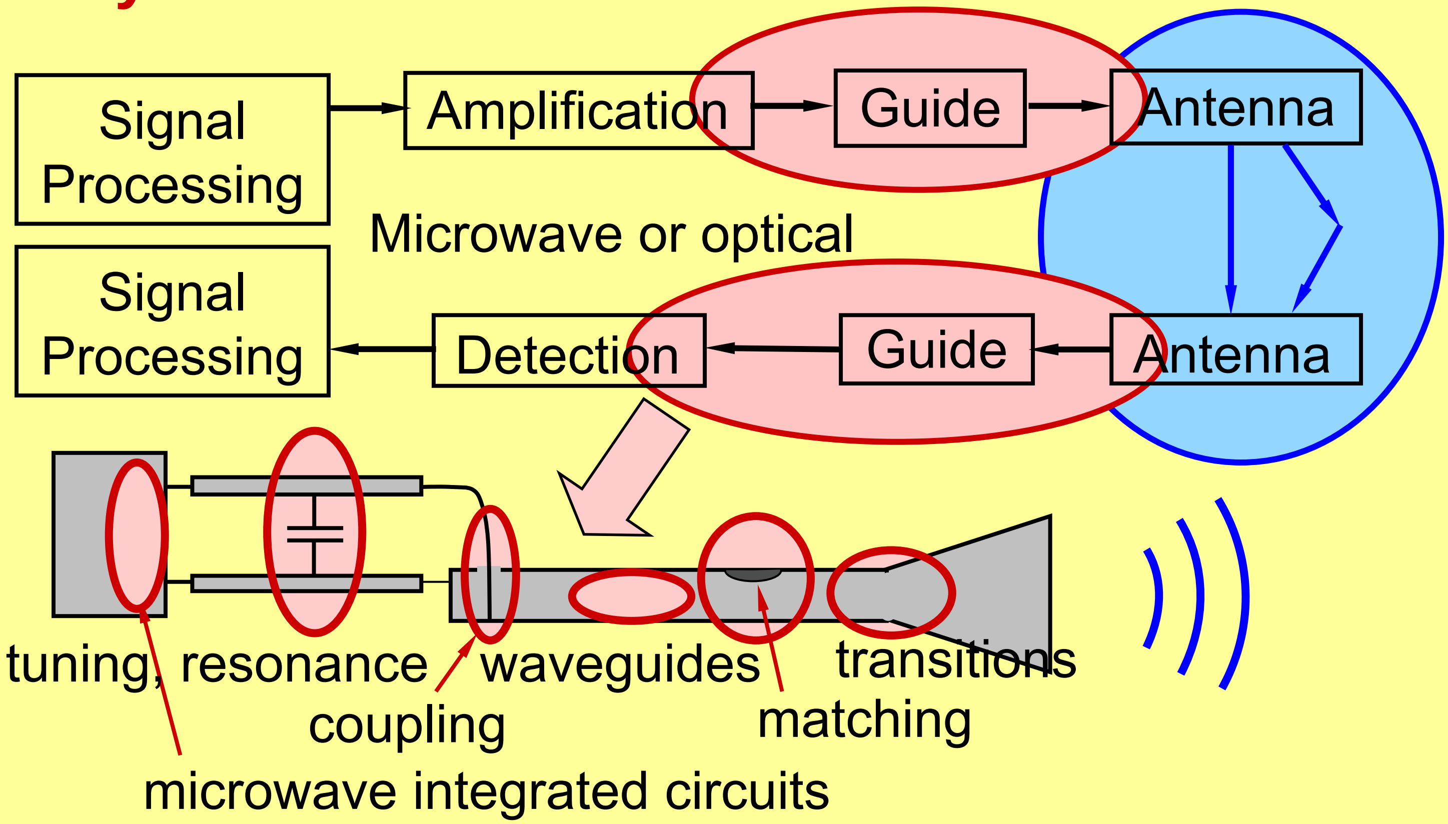
6.013 Electromagnetics and Applications
Spring 2009

EM GUIDANCE AND FILTERING

Generic System Architecture:



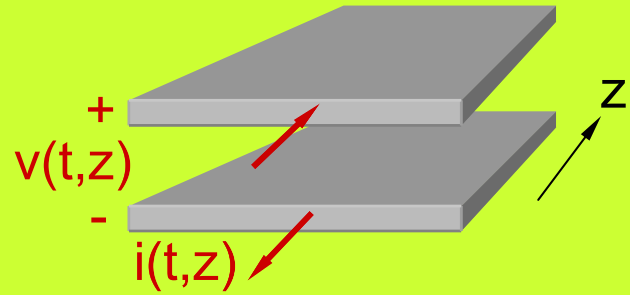
Systems fail at weakest link, so understand all parts

Communications, bi-static radar—separately located systems
Radar, lidar, data recording—co-located systems
Passive sensing—uses receiver side only

MICROWAVE CIRCUITS

Printed Circuits :

"TEM" $\Rightarrow \bar{E} \bullet \hat{z} = \bar{H} \bullet \hat{z} = 0$



Difference Equations:

$\underline{V}(z+\Delta z) - \underline{V}(z) = -j\omega\, L\Delta z\, \underline{I}(z)$

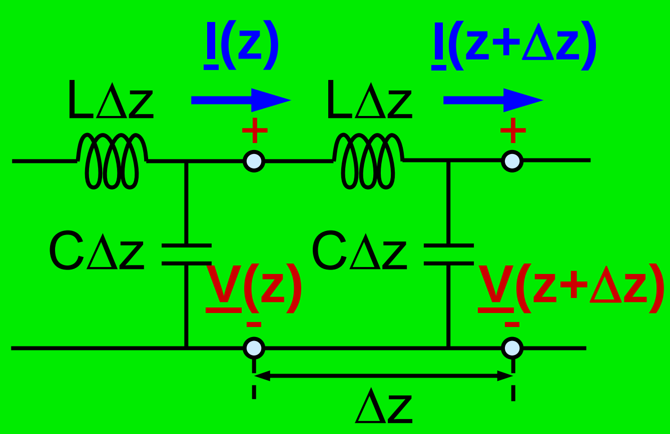
$\underline{I}(z+\Delta z) - \underline{I}(z) = -j\omega\, C\Delta z\, \underline{V}(z)$

Limit as $\Delta z \rightarrow 0$:

$$\frac{d\underline{V}}{dz} = -j\omega L\ \underline{I}$$

$$\frac{d\underline{I}}{dz} = -j\omega C\ \underline{V}$$

Equivalent TEM line circuit:



$$\Rightarrow \frac{d^2\underline{V}(z)}{dz^2} + \omega^2 LC\ \underline{V}(z) = 0 \quad \text{Wave Equation}$$

TEM PHASOR EQUATIONS

Wave Equation: $\left(\frac{d^2}{dz^2} + \omega^2 LC\right)\underline{V}(z) = 0$

Voltage Solution: $\underline{V}(z) = \underline{V}_+ e^{-jkz} + \underline{V}_- e^{jkz}$

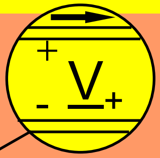
Test solution: $[(-jk)^2 \underline{V}_+ e^{-jkz} + (jk)^2 \underline{V}_- e^{jkz}] +$

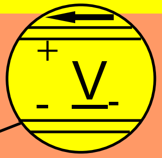
$\omega^2 LC[\underline{V}_+ e^{-jkz} + \underline{V}_- e^{jkz}] = 0$

Passes test iff: $k^2 = \omega^2 LC$

Current $\underline{I}(z)$:

Since: $\frac{\partial \underline{V}(z)}{\partial z} = -j\omega L\ \underline{I}(z)$





Therefore: $\underline{I}(z) = \frac{1}{j\omega L} jk(\underline{V}_+ e^{-jkz} - \underline{V}_- e^{+jkz})$

$= Y_o(\underline{V}_+ e^{-jkz} - \underline{V}_- e^{+jkz})$

Line admittance: $Y_o = \frac{k}{\omega L} = \frac{\omega\sqrt{LC}}{\omega L} = \sqrt{\frac{C}{L}} = \frac{1}{Z_o}$

TEM Equations:

$\underline{V}(z) = \underline{V}_+ e^{-jkz} + \underline{V}_- e^{jkz}$

$\underline{I}(z) = Y_o(\underline{V}_+ e^{-jkz} - \underline{V}_- e^{jkz})$

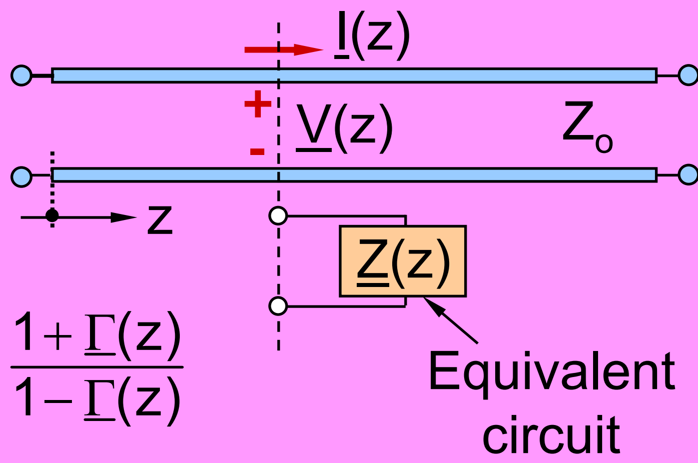
COMPLEX LINE IMPEDANCE $\underline{Z}(z)$

Impedance:

$$\underline{Z}(z) = \frac{\underline{V}(z)}{\underline{I}(z)} = R(z) + jX(z)$$

Resistance Reactance

$$\underline{Z}(z) = \frac{Z_o\left(\underline{V}_+ e^{-jkz} + \underline{V}_- e^{jkz}\right)}{\underline{V}_+ e^{-jkz} - \underline{V}_- e^{jkz}} = Z_o \frac{1+\underline{\Gamma}(z)}{1-\underline{\Gamma}(z)}$$



Complex Reflection Coefficient $\underline{\Gamma}(z)$:

$$\underline{\Gamma}(z) \triangleq \frac{\underline{V}_- e^{+jkz}}{\underline{V}_+ e^{-jkz}} = \underline{\Gamma}_L e^{2jkz}$$ where $\underline{\Gamma}_L = \underline{\Gamma}(z=0) = \frac{\underline{V}_-}{\underline{V}_+}$

Examples: $\underline{\Gamma} = 0 \Rightarrow \underline{Z}(z) = Z_o$ $\underline{\Gamma} = 1 \Rightarrow \underline{Z} = \infty$ $\underline{\Gamma} = -1 \Rightarrow \underline{Z} = 0$

Normalized Impedance $\underline{Z}_n(z)$:

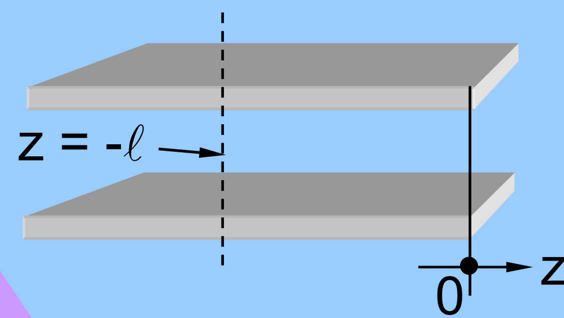
Definition: $\underline{Z}_n(z) \triangleq \frac{\underline{Z}(z)}{Z_o} = \frac{[1+\underline{\Gamma}(z)]}{[1-\underline{\Gamma}(z)]}$ ⇒ $\underline{\Gamma}(z) = \frac{[\underline{Z}_n(z)-1]}{[\underline{Z}_n(z)+1]}$

$\underline{Z}(z)$ TRANSFORMATIONS

$\underline{Z}(z) = f(\underline{Z}_L, Z_o, k, z)$:

Substituting: $\underline{\Gamma}_L = \frac{[\underline{Z}_L - Z_o]}{[\underline{Z}_L + Z_o]}$ into $\underline{Z}(z) = Z_o \frac{[1 + \underline{\Gamma}(z)]}{[1 - \underline{\Gamma}(z)]}$; $\underline{\Gamma}(z) = \underline{\Gamma}_L e^{2jkz}$

Yields: $$\underline{Z}(z) = Z_o \frac{\underline{Z}_L - jZ_o \tan kz}{Z_o - j\underline{Z}_L \tan kz}$$



Example: Open Circuit, $Z_L = \infty$:

$\underline{Z}(-\ell) = -jZ_o \cot k\ell \cong -jZ_o/k\ell$ for $k\ell << 1$

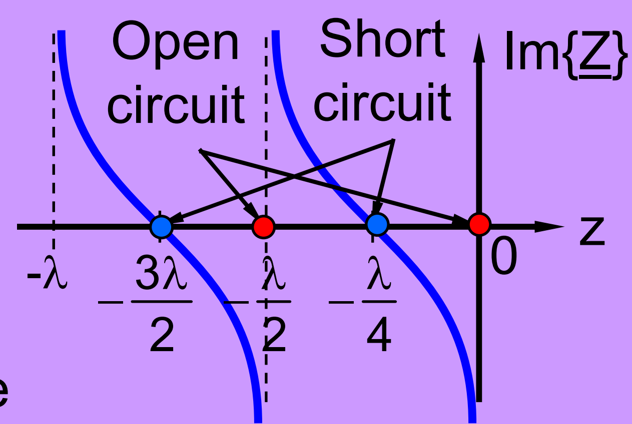
$= -j\frac{\sqrt{L/C}}{\omega\sqrt{LC}\ell} = \frac{1}{j\omega \underbrace{C\ell}_{\text{capacitor } C_o}}$

$= 0$ when z = -λ/4, -3λ/4,…

$= \infty$ when z = 0, -λ/2,…

In general: -j∞ < $\underline{Z}(-\ell)$ < +j∞

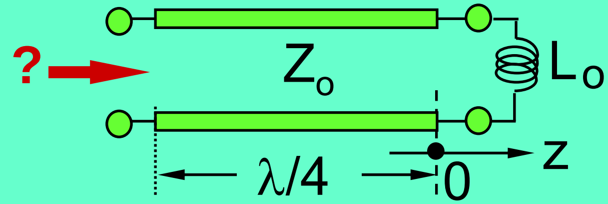
(Yields ANY capacitance or inductance at a SINGLE ω)



EXAMPLES: $\underline{Z}(z)$ TRANSFORMATIONS

Example—Inductive Load, $Z_L = j\omega L_o$ for $z = -\lambda/4$:

Recall: $$\underline{Z}(z) = Z_o \frac{\underline{Z}_L - jZ_o \tan kz}{Z_o - j\underline{Z}_L \tan kz}$$



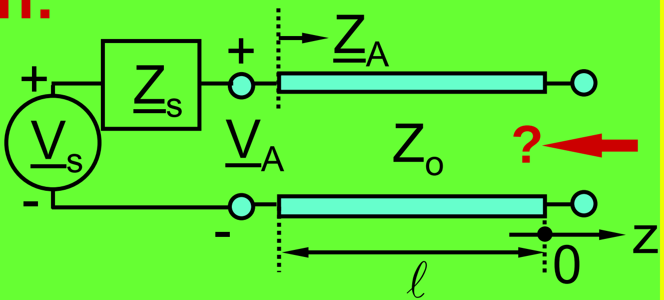
Since: $$kz = -k\ell = -\frac{2\pi}{\lambda}\frac{\lambda}{4} = -\frac{\pi}{2}, \text{ therefore } \tan(kz) = -\infty$$

Therefore: $$\underline{Z}(-\ell) = \frac{Z_o{}^2}{\underline{Z}_L} = \frac{L/C}{j\omega L_o} = \frac{1}{j\omega C_o} \quad (C_o = CL_o/L)$$

Note: $\underline{Z}(z) = j\omega L_o$ if $\ell = \lambda/2, \lambda, \ldots [\tan(-2\pi\ell/\lambda) = 0]$

Example – Source Transformation:

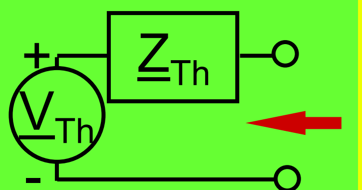
$$\underline{Z}_{Th} = Z_o \frac{\underline{Z}_S + jZ_o \tan k\ell}{Z_o - j\underline{Z}_S \tan k\ell}$$



$$\underline{V}_A = \underline{V}_S \frac{\underline{Z}_A}{\underline{Z}_S + \underline{Z}_A} \text{ where } \underline{Z}_A = -jZ_o \cot k\ell$$

$$= \underline{V}_+ (e^{jk\ell} + e^{-jk\ell}) = 2\underline{V}_+ \cos k\ell$$

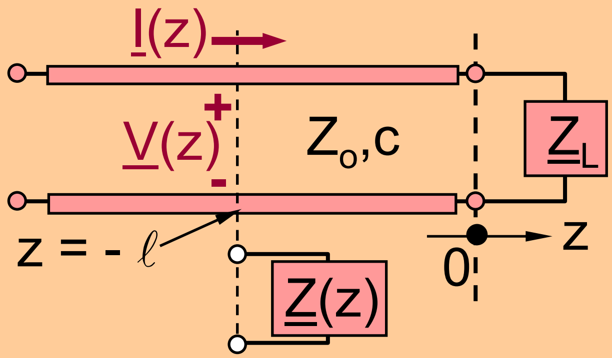
Therefore: $\underline{V}_{Th} = 2\underline{V}_+ = \underline{V}_S \underline{Z}_A / [(\underline{Z}_S + \underline{Z}_A)\cos k\ell]$



ALTERNATE APPROACH TO FINDING $\underline{Z}(z)$

Algorithmic, rotate $\underline{\Gamma}(z)$:

$$\underline{Z}(z) = \underline{V}(z)/\underline{I}(z) = Z_o \frac{\underline{V}_+ e^{-jkz} + \underline{V}_- e^{jkz}}{\underline{V}_+ e^{-jkz} - \underline{V}_- e^{jkz}}$$



(1)

$$\underline{Z}(z) = Z_o \frac{1+\underline{\Gamma}(z)}{1-\underline{\Gamma}(z)}$$

(2)

$$\underline{\Gamma}(z) = \frac{\underline{V}_- e^{+jkz}}{\underline{V}_+ e^{-jkz}} = \underline{\Gamma}_L e^{2jkz} = \underline{\Gamma}(z), \quad \text{where } \underline{\Gamma}_L = \underline{\Gamma}(z=0) = \frac{\underline{V}_-}{\underline{V}_+}$$

$$\underline{\Gamma}(z) = \frac{[\underline{Z}_n - 1]}{[\underline{Z}_n + 1]} \quad (3) \quad \underline{Z}_n(z) \triangleq \frac{\underline{Z}(z)}{Z_o}$$

Γ-Plane Solution Method:

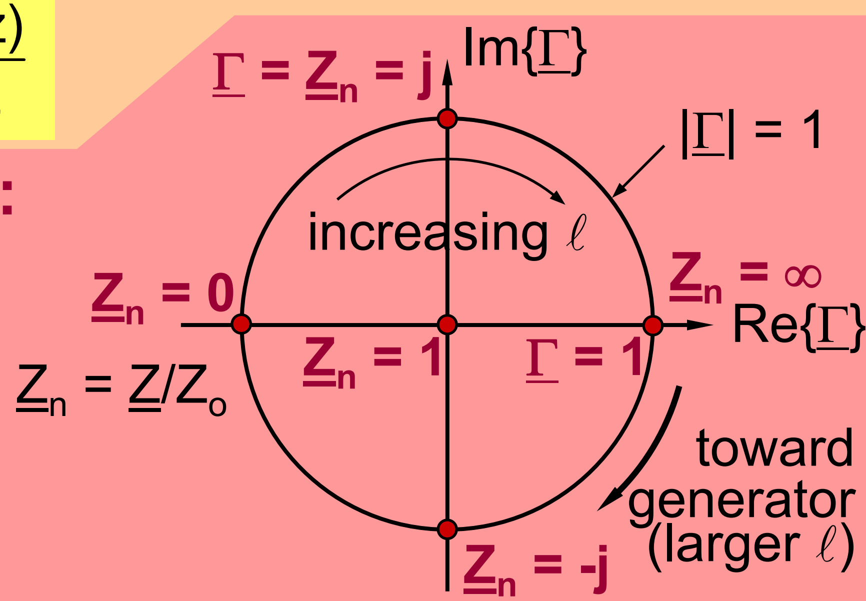
$\underline{Z}_L \Leftrightarrow \underline{\Gamma}_L \Leftrightarrow \underline{\Gamma}(z) \Leftrightarrow \underline{Z}(z)$

(3) (2) (1)

($\lambda/2 \Rightarrow$ full rotation)

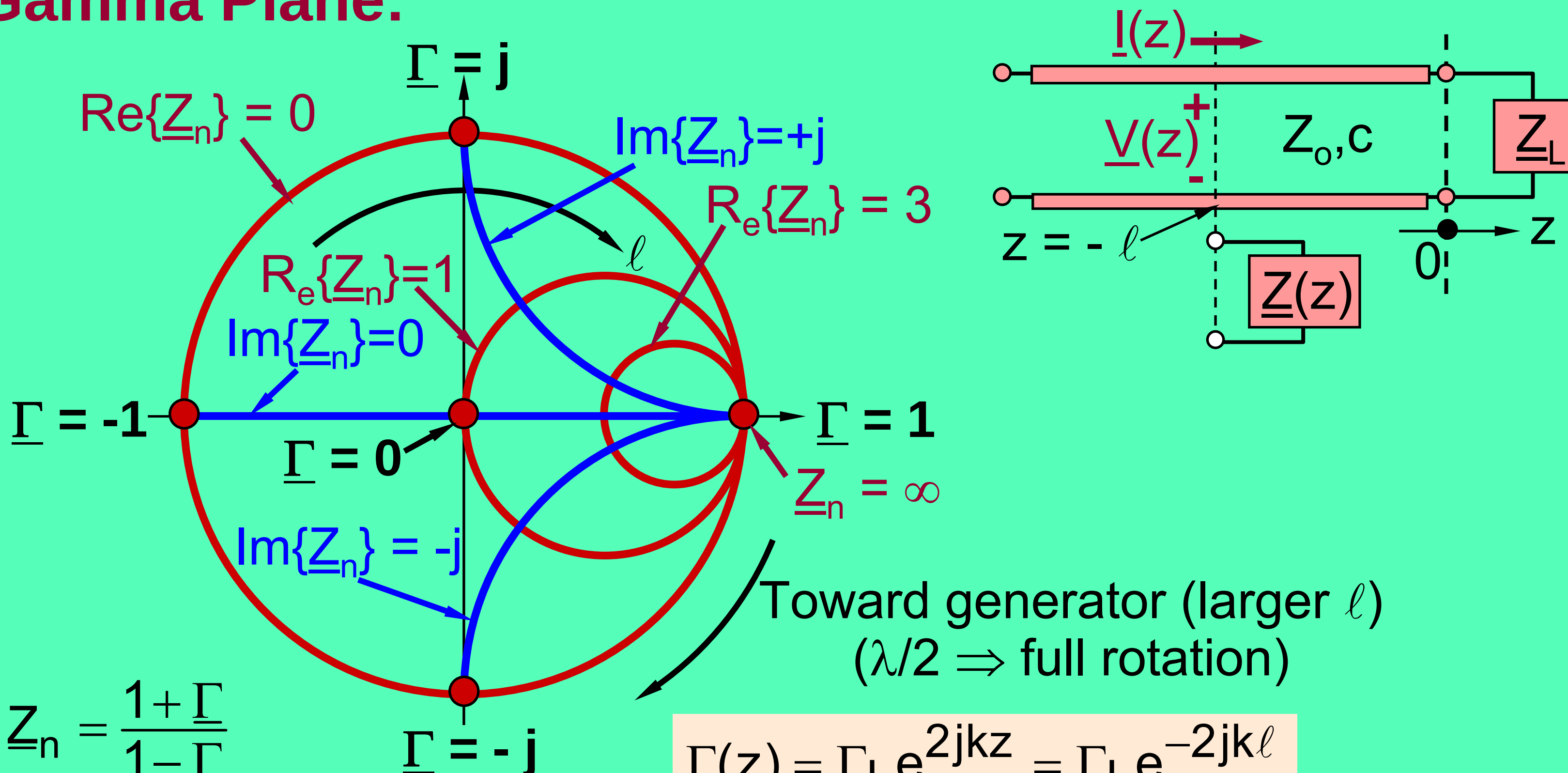
$e^{-2jk\ell}$ goes clockwise as $\ell \rightarrow \infty$

$(e^{j\phi} = \cos\phi + j\sin\phi)$



GAMMA PLANE ⇒ SMITH CHART

Gamma Plane:



$$\underline{Z}_n = \frac{1+\underline{\Gamma}}{1-\underline{\Gamma}}$$

$$\underline{\Gamma}(z) = \underline{\Gamma}_L e^{2jkz} = \underline{\Gamma}_L e^{-2jk\ell}$$

Smith Chart = Gamma Plane + $\underline{Z}_n(z)$:

$$\frac{[\underline{Z}_n(z)-1]}{[\underline{Z}_n(z)+1]} = \underline{\Gamma}(z) \qquad \underline{Z}_n(z) \triangleq \frac{\underline{Z}(z)}{Z_o} = \frac{[1+\underline{\Gamma}(z)]}{[1-\underline{\Gamma}(z)]}$$

$$\underline{Z}_L \Leftrightarrow \underline{Z}_{Ln} \Leftrightarrow \underline{\Gamma}_L \Leftrightarrow \underline{\Gamma}(z) \Leftrightarrow \underline{Z}_n(z) \Leftrightarrow \underline{Z}(z)$$