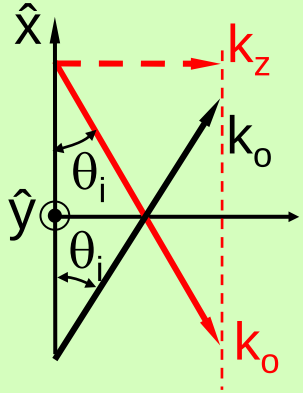
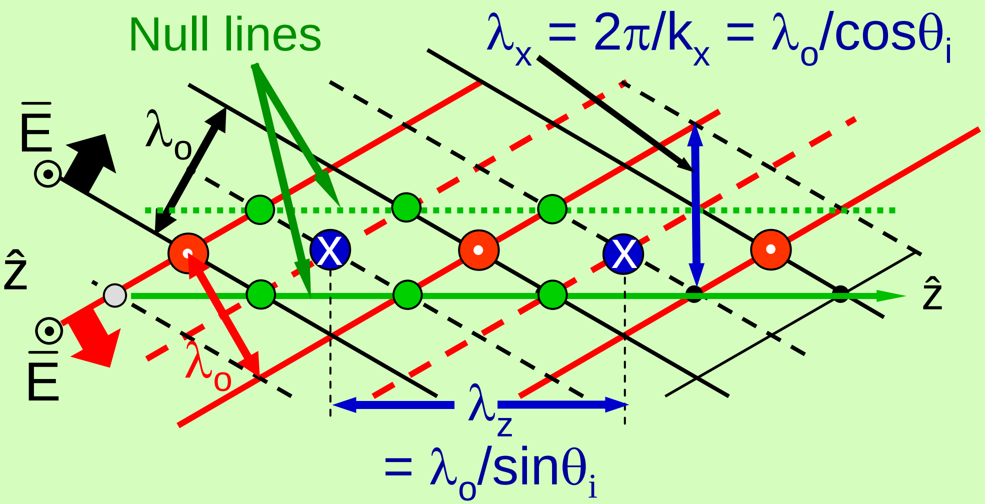
6.013 Electromagnetics and Applications
Spring 2009

WAVEGUIDES AND SYSTEMS

Parallel-plate waveguide: TE case

Plane wave interference satisfies boundary conditions





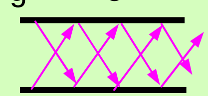
$k_o = \omega\sqrt{\mu_o\varepsilon_o}$

$k_z = k_o \sin\theta_i$

$k_x = k_o \cos\theta_i$

$v_p = v_o/\sin\theta_i$

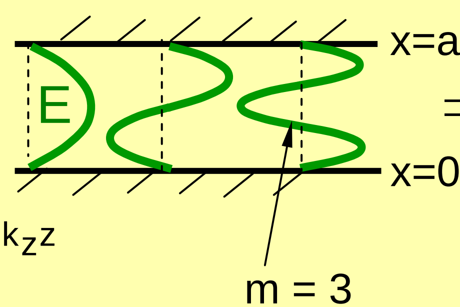
$v_g = v_o \sin\theta_i$



$$\underline{\overline{E}} = \hat{y}\left(E_o e^{jk_x x - jk_z z} - E_o e^{-jk_x x - jk_z z}\right) = \hat{y}2jE_o \sin k_x x \cdot e^{-jk_z z} \quad \Rightarrow \quad k_x = m\frac{\pi}{a}$$

$$\underline{\overline{H}} = -\frac{\nabla\times\underline{\overline{E}}}{j\omega\mu} = -\frac{1}{j\omega\mu}\left(\hat{z}\frac{\partial \underline{E}_y}{\partial x} - \hat{x}\frac{\partial \underline{E}_y}{\partial z}\right) \quad \Rightarrow \quad m\frac{\lambda_x}{2} = a$$

$$= \frac{2E_o}{j\omega\mu}(\hat{x}\cdot k_z \sin k_x x - \hat{z}\cdot jk_x \cos k_x x)e^{-jk_z z}$$



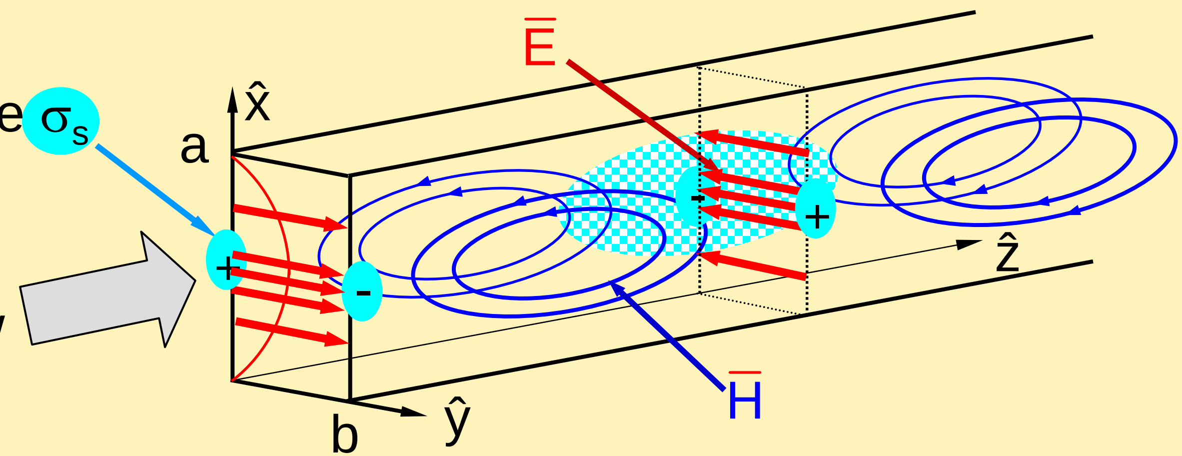
TE_{10} WAVEGUIDE MODE

Add Sidewalls to TE_1 Parallel Plate Waveguide $\Rightarrow$ TE_{10}:

Surface charge σ_s

$$\sigma_s = \hat{n} \cdot \varepsilon \overline{E}$$

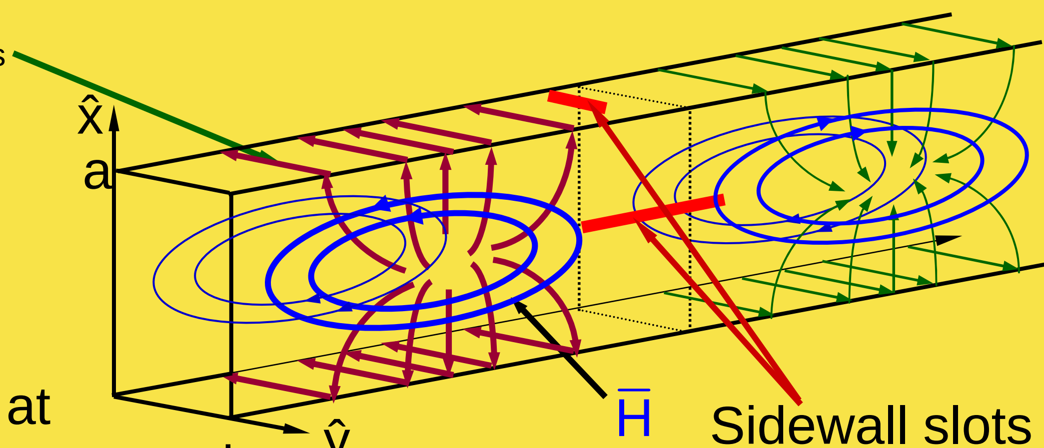
Power flow



Surface current $\overline{J}_s$

$$\overline{J}_s = \hat{n} \times \overline{H}$$

Antennas can be coupled through holes or slots; slots at angles can radiate

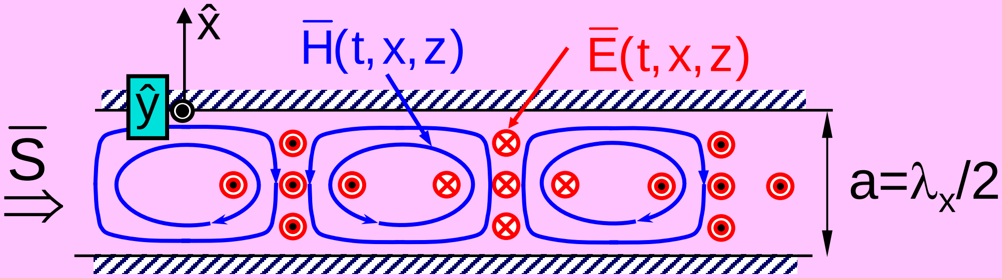


Sidewall slots that don't block current

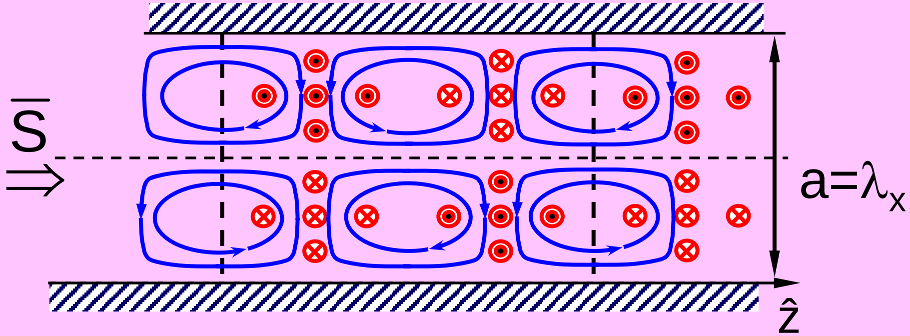
MODE PATTERNS - TE

TE modes: $\bar{E} = \hat{y}2jE_o \sin\left(m\frac{\pi}{a}x\right) \cdot e^{-jk_z z}$

TE_1 mode:

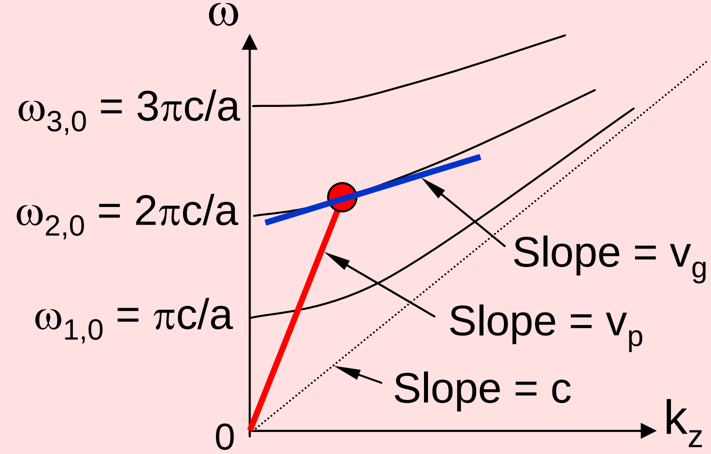


TE_2 mode:



$$k_z = \sqrt{\left(\frac{\omega}{c}\right)^2 - k_x^2} = \sqrt{\left(\frac{\omega}{c}\right)^2 - \left(\frac{m\pi}{a}\right)^2}$$

= 0 for 'cutoff frequency' ω_{mn}

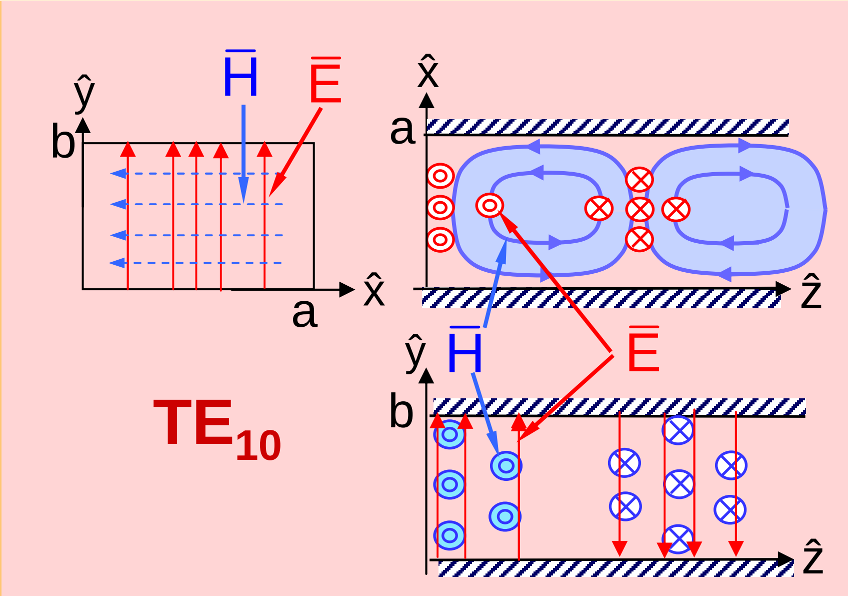


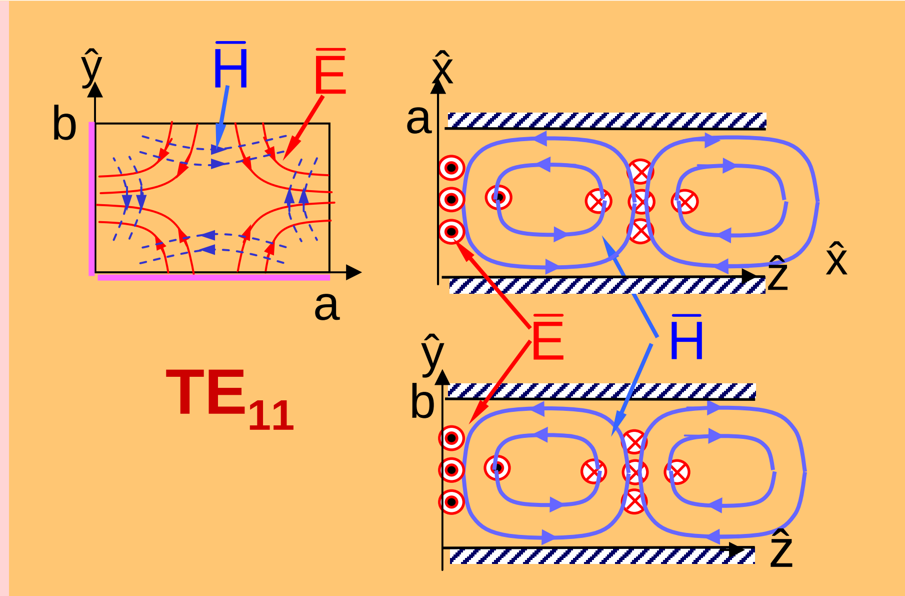
$$v_{phase} = \frac{\omega}{k_z}$$

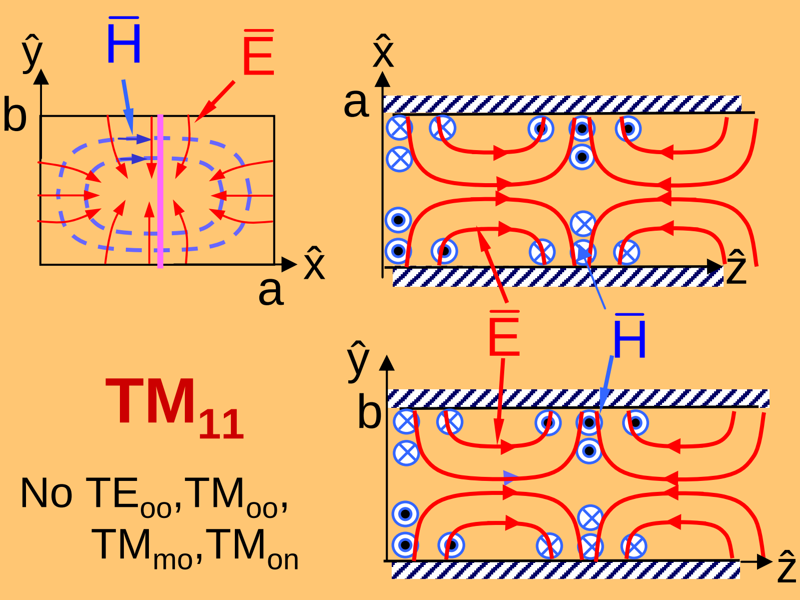
$$v_{group} = \frac{d\omega}{dk_z} \rightarrow 0 \quad \text{at } \omega_{co} = m\frac{\pi c}{a}$$

$\omega < \omega_{co} \rightarrow k_z = j\alpha$ ⇨ Evanescent

RECTANGULAR WAVEGUIDE MODES

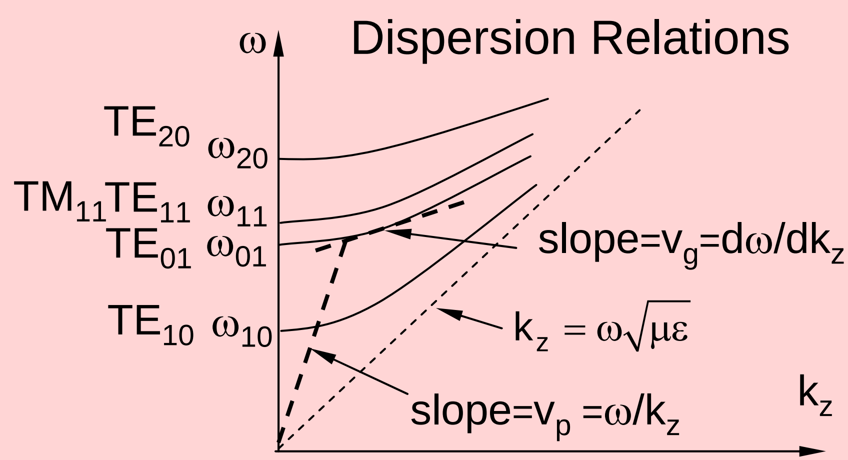






No TE_{oo}, TM_{oo}, TM_{mo}, TM_{on}

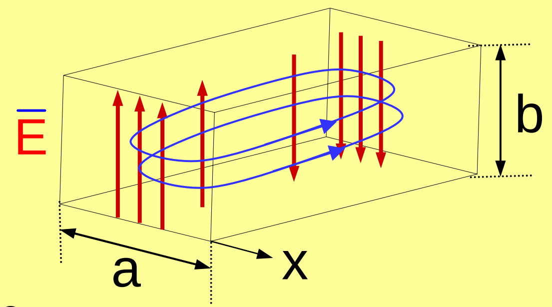
$$k_z = \sqrt{\left(\frac{\omega}{c}\right)^2 - \left(\frac{m\pi}{a}\right)^2 - \left(\frac{n\pi}{b}\right)^2} \quad TE(M)_{mn} : a < b$$



RECTANGULAR WAVEGUIDE DESIGN

Modes and Dimensions:

TE_{m0} modes: $a = m\lambda_x/2$



$$\overline{E}_{m0} = \hat{y}\cdot E_o 2j\sin k_x x\ e^{-jk_z z}$$

$$k_z = \sqrt{(k_o{}^2 - k_x{}^2)} = \sqrt{\frac{\omega_o^2}{c^2} - \left(\frac{m\pi}{a}\right)^2} = 0 \text{ if } \omega_o = \frac{mc\pi}{a}$$

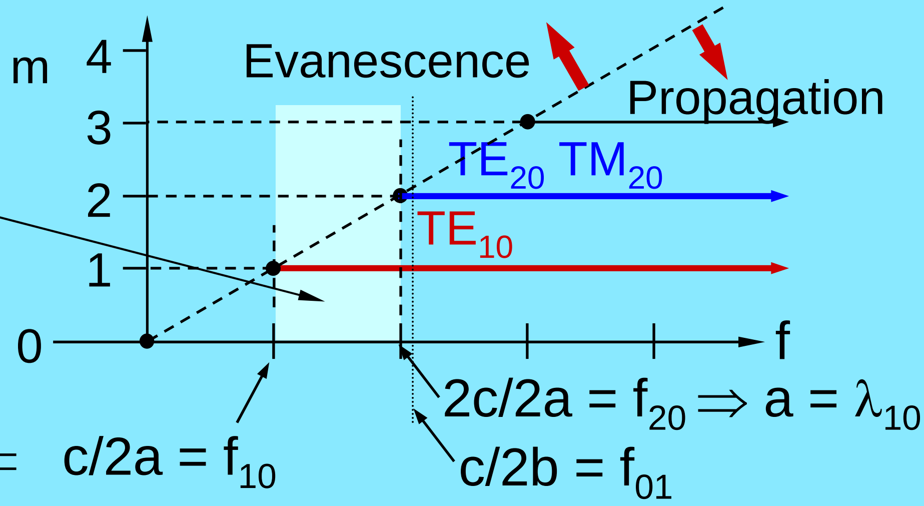
TE_{mn} modes: $k_x = m\pi/a$, $k_y = n\pi/b \Rightarrow k_z = \sqrt{k_o^2 - k_x^2 - k_y^2} = \sqrt{\frac{\omega_o^2}{c^2} - \left(\frac{m\pi}{a}\right)^2 - \left(\frac{n\pi}{b}\right)^2}$

Cutoff Frequencies: $f_{m0} = \frac{mc}{2a}$; $f_{0n} = \frac{nc}{2b}$

Want $b \le a/2$ so that

$f_{01} \ge f_{20} = c/a$ ($f_{01} = c/2b$)

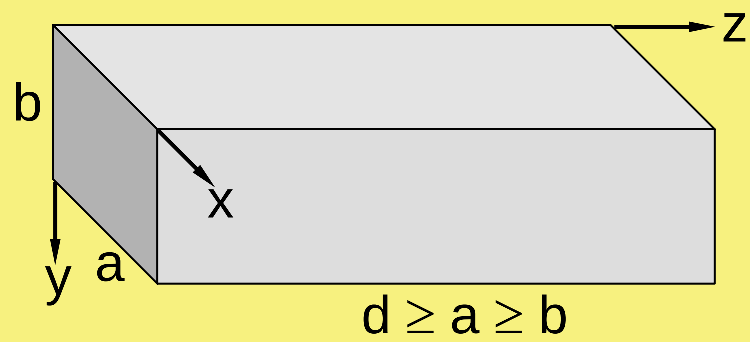
Single propagating mode
(if $a \ge 2b$) [TE_{10}]
(Two modes would interfere,
producing nulls in frequency)



$a = \lambda_{10}/2 \Leftarrow$ $c/2a = f_{10}$

CAVITY RESONATOR DESIGN

Derivation of resonant frequencies f_{mnp}



TE_{mn} waveguide modes:

$$a = m\frac{\lambda_x}{2},\ b = n\frac{\lambda_y}{2} \quad \Rightarrow \quad k_z = \sqrt{k_o^2 - k_x^2 - k_y^2} = \sqrt{\frac{\omega_o^2}{c^2} - \left(\frac{m\pi}{a}\right)^2 - \left(\frac{n\pi}{b}\right)^2}$$

Shortest side Longest side

TE_{mnp} resonator modes: $a = m\lambda_x/2$, $b = n\lambda_y/2$, $d = p\lambda_z/2$

$$\Rightarrow \ k_z = \sqrt{k_o^2 - k_x^2 - k_y^2} = \frac{2\pi}{\lambda_z} = \frac{p\pi}{d} \quad \Rightarrow \quad \sqrt{\frac{\omega_{mnp}^2}{c^2} - \left(\frac{m\pi}{a}\right)^2 - \left(\frac{n\pi}{b}\right)^2 - \left(\frac{p\pi}{d}\right)^2} = 0$$

$$\Rightarrow \ \omega_{mnp} = c\sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 + \left(\frac{p\pi}{d}\right)^2} \quad \Rightarrow \quad f_{mnp} = \sqrt{\left(\frac{mc}{2a}\right)^2 + \left(\frac{nc}{2b}\right)^2 + \left(\frac{pc}{2d}\right)^2}$$

CAVITY RESONATOR PERTURBATION

Work = force • distance = Δw

F_e [N/m^2] = $\rho_s E/2$ (attractive)

$= \varepsilon E^2/2$; $\langle F_e \rangle = \varepsilon_o E^2/4 = \langle W_e \rangle$[J/m^3]

F_m [N/m^2] = $\bar{J}_s \times \bar{H}\mu_o/2$(repulsive)

$= \mu H^2/2$; $\langle F_m \rangle = \mu_o H^2/4 = \langle W_m \rangle$[J/m^3]

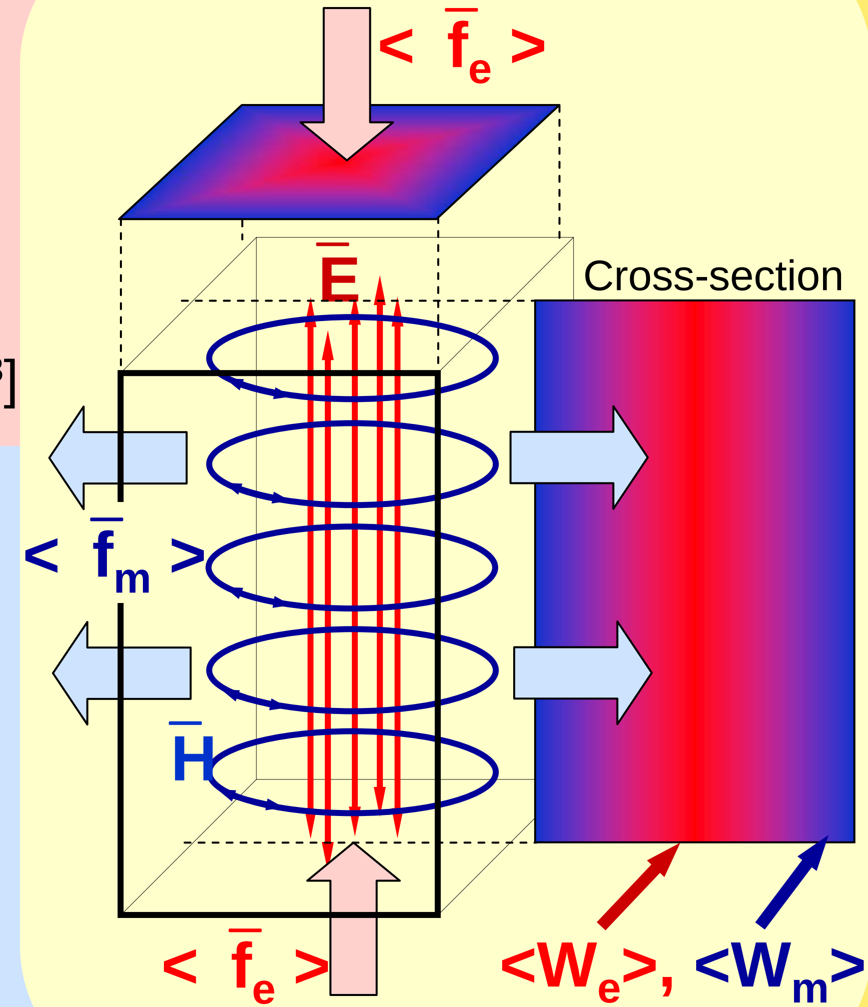
At resonance $\langle w_e \rangle = \langle w_m \rangle = w_T/2$

$w_T = n\,hf$, so $\Delta w_T = nh\Delta f$

Δf [Hz] = $\Delta w_T/nh = f\,(\Delta w_T)/w_T$

Δf [Hz] = $f \bullet (\Delta w_m - \Delta w_e)/w_T$

Thus pushing in the wall where W_m dominates does work and raises f_{mnp}; where W_e dominates f_{mnp} drops

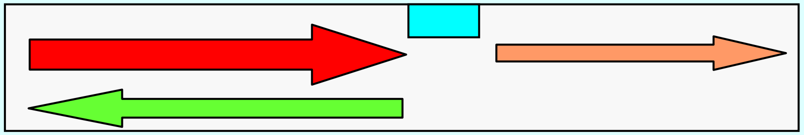


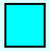
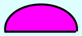
WAVEGUIDE SYSTEMS

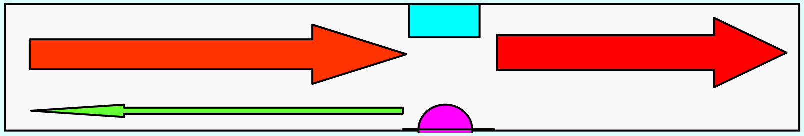
Physics of Matching:

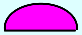
Choose waveguide that propagates only one mode at f.
Structural discontinuities cause reflections; tuning cancels them.

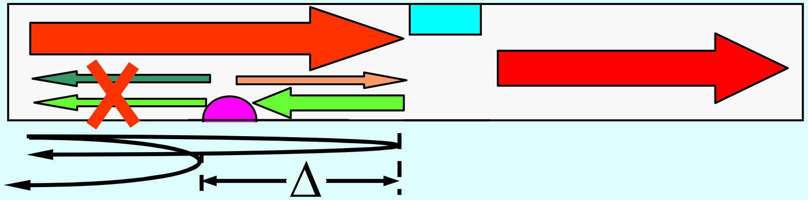
Given some reflecting obstacle:



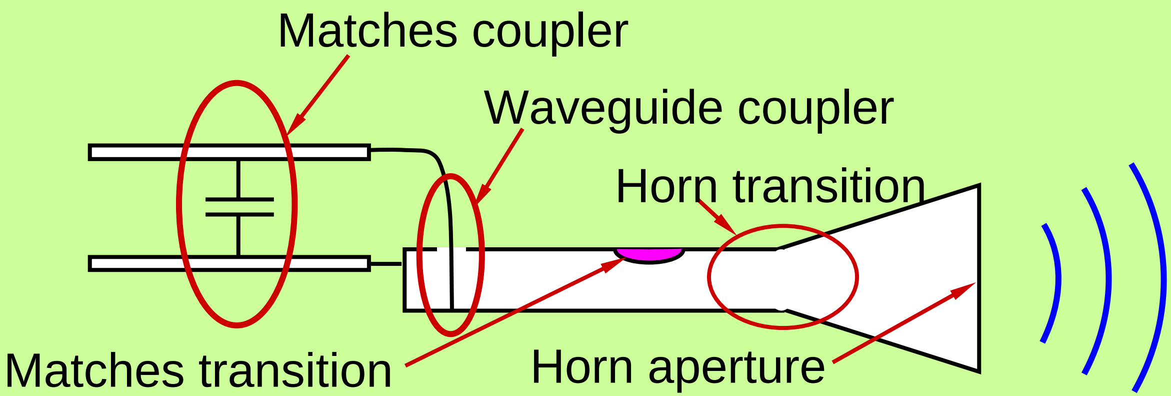
Cancel reactance  with
 [= f(freq.)] near obstacle



Cancel reflection with offset
[magnitude and phase (Δ)]



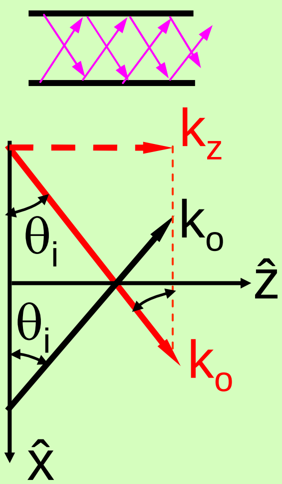
Example:

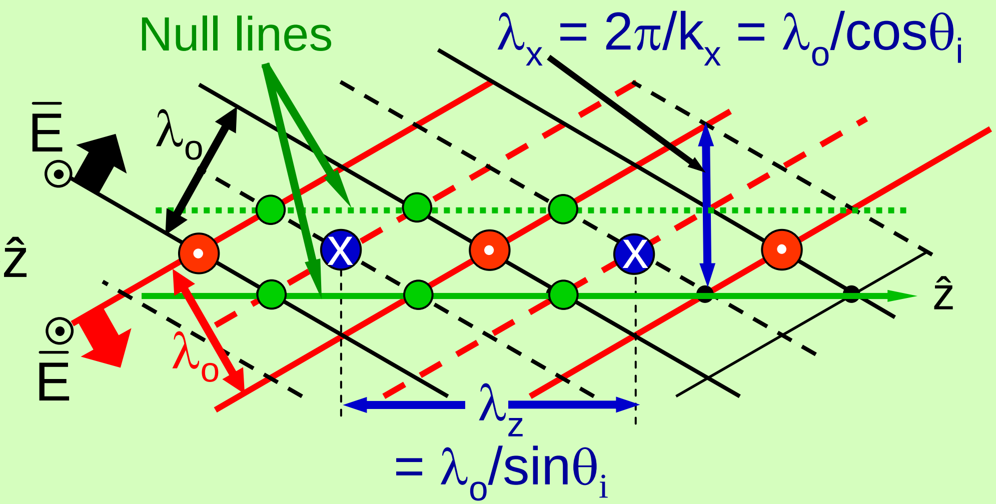


WAVEGUIDES AND SYSTEMS

Parallel-plate waveguide: TE case

Plane wave interference satisfies boundary conditions





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