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6.013 Electromagnetics and Applications  
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# WIRELESS COMMUNICATIONS

<b>RANGE</b>	<b>ACTIVE</b>	<b>PASSIVE</b>
Arm's Length	Pills, hearing aids, computer peripherals	Faucets, CD's, thermometers,
<100 m	Wireless phones, remote controllers, computer links	Cameras, doors
<100 km	Radio, television, cell phones, UWB, 802.11	Multispectral remote sensing
Global	Ham radio, communications satellites, radar, lidar	Weather satellites
Cosmic	Radio & optical interplanetary communications, radar, lidar	Radio & optical astronomy

# COMMUNICATION REQUIRES ENERGY AND POWER

**Typical receivers need:**

$$E_b > \sim 10^{-20} \text{ Joules/bit}$$

**Received power required:**

$$P_{\text{rec}} = M_{\text{bps}} E_b \text{ [Watts]}$$

( $M_{\text{bps}}$  is data rate, bits/sec)

# RADIATED POWER

Transmitted Intensity:  $I(\theta, \phi, r)$  [W/m<sup>2</sup>]

For isotropic radiation:  $I(\theta, \phi, r) = P_R / 4\pi r^2$  [Wm<sup>-2</sup>]

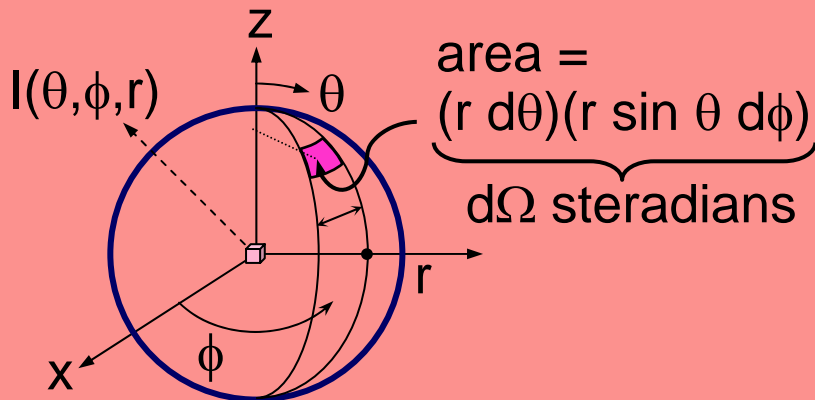


Total power radiated [W]

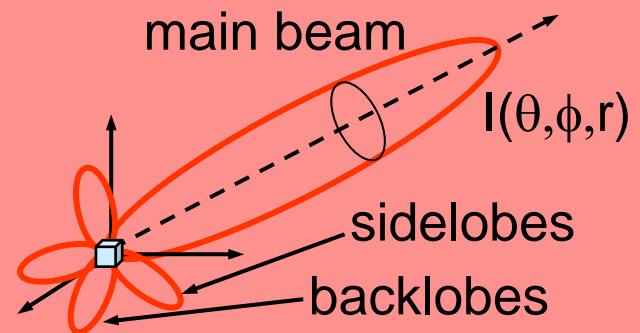
$$P_R = \int_0^{2\pi} \int_0^\pi I(\theta, \phi, r) \underbrace{r^2 \sin\theta \, d\theta \, d\phi}_{d\Omega \text{ (steradians)}}$$

**Steradian:** unit of solid angle  
 $d\theta, d\phi$ : units of radians.  
 Spheres: span  $4\pi$  steradians

Isotropic:



Antenna pattern:



# ANTENNA GAIN $G(\theta, \phi)$

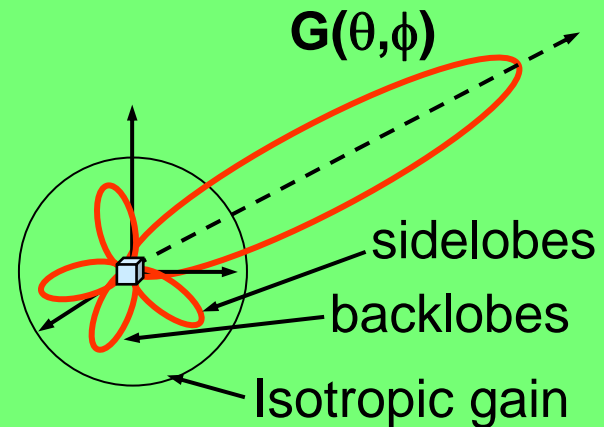
## Gain over Isotropic, $G(\theta, \phi)$ :

$$G(\theta, \phi) = \frac{I(\theta, \phi, r)}{(P_R/4\pi r^2)}$$

← Intensity actually radiated [ $\text{Wm}^{-2}$ ]  
← Intensity if  $P_R$  were radiated isotropically

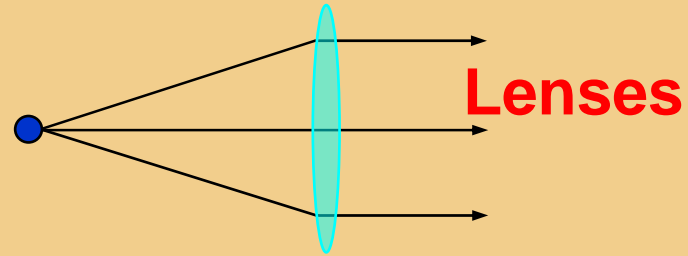
## Intensity at receiver:

$$I(\theta, \phi, r) = G(\theta, \phi) (P_R/4\pi r^2) \quad [\text{Wm}^{-2}]$$

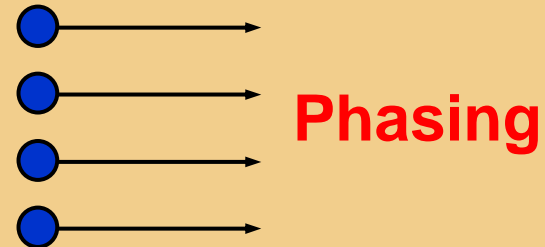
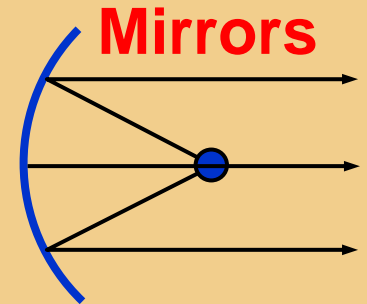


# HOW TO INCREASE ANTENNA GAIN $G(\theta, \phi)$ ?

**Focus the energy**



Photographs illustrating lenses, mirrors, and phasing removed due to copyright restrictions.



# ANTENNA EFFECTIVE AREA $A_e(\theta,\phi)[\text{m}^2]$

**Intensity radiated in a particular direction**

$$I(\theta,\phi,r) = G_t(\theta,\phi) (P_R/4\pi r^2) \quad [\text{W}/\text{m}^2]$$

**Power Received from a particular direction**

$$P_{\text{rec}} = I(\theta,\phi) A(\theta,\phi) \quad [\text{W}]$$

**Antenna Effective Area and Gain**

$$A(\theta,\phi) = G(\theta,\phi) (\lambda^2/4\pi) \quad [\text{m}^2]$$

**Power Received from a particular direction**

$$P_{\text{rec}} = P_t G_t G_r (\lambda/4\pi r)^2 \quad [\text{W}] \Rightarrow \text{“reciprocity”}$$

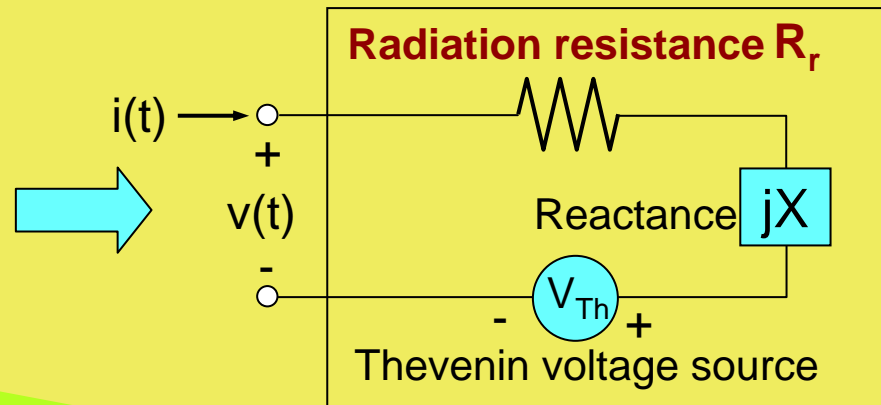
# CIRCUIT PROPERTIES OF ANTENNAS

## When transmitting:

Power radiated =  $P_R$

$$P_R = \langle i^2(t) \rangle R_r \text{ [W]}$$

Equivalent circuit of antenna

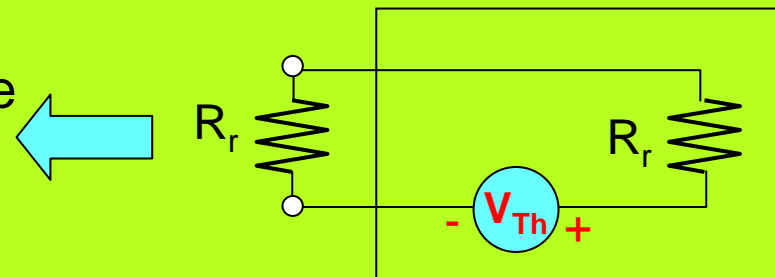


## When receiving:

Thevenin voltage  $V_{Th}$  is induced by incoming waves

Maximum power extractable from the antenna:

$$P_{rec} = \langle (V_{Th}(t)/2)^2 \rangle / R_r$$



Reactive elements are tuned out



# SUMMARY

## Wireless communications are ubiquitous

$G(\theta, \phi) = I(\theta, \phi, r) / (P_t / 4\pi r^2) = \text{Antenna gain over isotropic}$

Boost antenna gain using lenses, mirrors, or phasing

$A_r = G_r (\lambda^2 / 4\pi) = \text{Antenna effective area [m}^2\text{]}$

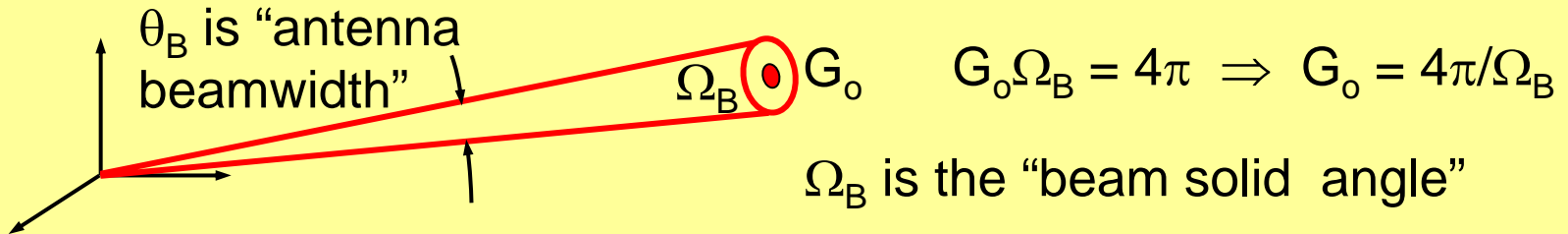
$M \text{ [bps]} = P_{\text{rec}} / E_b = I A_e / E_b = P_t G_t G_r (\lambda / 4\pi r)^2 / E_b = \text{data rate}$

$E_b > \sim 10^{-20} \text{ [J]} \text{ at the receiver}$  (see footnote 39 on p360)

Antennas have Thevenin equivalent circuits, radiation resistance

# EXAMPLE – INTERSTELLAR COMM.

$$P_{\text{Rad}} = \int_0^{2\pi} \int_0^\pi G(\theta, \phi) \frac{P_{\text{Rad}}}{4\pi r^2} r^2 \underbrace{\sin\theta \, d\theta \, d\phi}_{d\Omega \text{ steradians}} \Rightarrow \int_{4\pi} G(\theta, \phi) d\Omega = 4\pi$$



Best microwave antennas:  $\theta_B \cong 1 \text{ arc min} = (1/60)^\circ (1/57) \text{ radians} \cong 2^{-12} \text{ rad}$

$G_o = 4\pi / \Omega_B \cong 2^3 / 2^{-24} \cong 2^{27} \cong 10^8 \text{ (or 80 dB)}$

Strongest transmitters  $\sim 10^6 \text{ Watts}$

Nearest stars  $\sim 1 \text{ light year} = 3 \times 10^7 \text{ sec} \times 3 \times 10^8 \text{ m/s} \cong 10^{16} \text{ m}$

$$P_{\text{rec}} = \frac{P_{\text{rad}}}{4\pi r^2} G_t G_r \frac{\lambda^2}{4\pi} \cong \frac{10^6 \ 10^8 \ 10^8 \ 0.03^2}{10 \ 10^{32} \ 10} \cong 10^{-15} \text{ [W] [J/s]}$$

Data rate  $R \cong P_{\text{rec}} [\text{J/s}] / 10^{-20} [\text{J/bit}] = 10^{-15} / 10^{-20} = 10^5 \text{ bits/sec}$