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6.013 Electromagnetics and Applications
Spring 2009

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RECEIVING ANTENNAS

Reciprocity:

$$A_e(\theta, \phi) = \frac{\lambda^2}{4\pi} G(\theta, \phi)$$

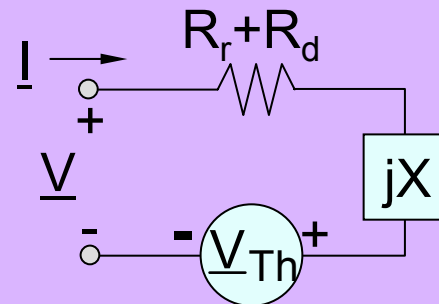
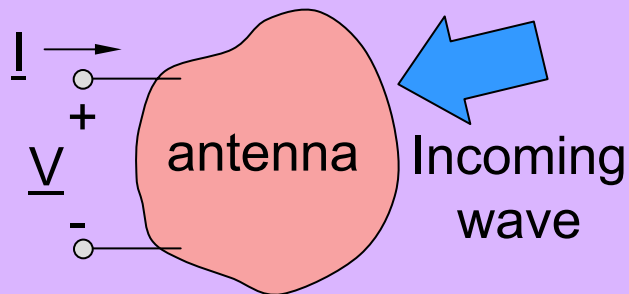
Not yet proved here; sometimes untrue (when?)

Approach:

Study small simple antennas; $d \ll \lambda \Rightarrow$ quasistatics

Seek V_{Th} in equivalent circuit = open-circuit voltage

Examples: small loop antenna and short dipole



SMALL LOOP ANTENNA

Quasistatic Limit:

$$\nabla \times \bar{\mathbf{E}} = -j\omega\bar{\mathbf{B}} \quad (\text{Faraday's Law})$$

$$\Rightarrow \oint_C \bar{\mathbf{E}} \cdot d\bar{\mathbf{s}} = -j\omega \iint_A \bar{\mathbf{B}} \cdot d\bar{\mathbf{a}} = \underline{V}_{Th}$$

Open Circuit Voltage:

$$\underline{V}_{Th} = -j\omega AN\mu_0 \bar{\mathbf{H}} \cdot \hat{\mathbf{z}} \propto \sin\theta$$

θ = angle of incidence relative to z axis

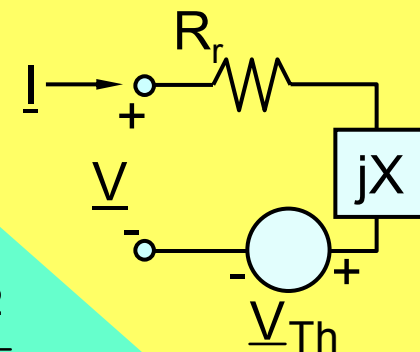
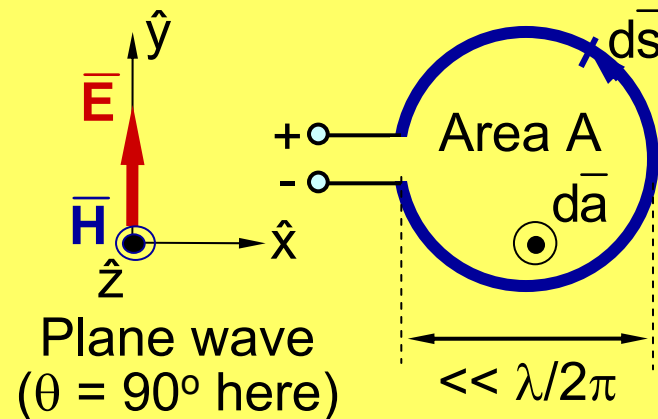
Radiation Resistance:

$$P_{rec}(\theta) = \frac{1}{8R_r} |\underline{V}_{Th}|^2 \text{ [W]} \Rightarrow R_r = \frac{|\underline{V}_{Th}|^2}{8P_{rec}}$$

$$P_{rec}(\theta) = I(\theta)A_e(\theta) [\propto \sin^2\theta] = \frac{1}{2} \eta_0 |\bar{\mathbf{H}}(\theta)|^2 G(\theta) \frac{\lambda^2}{4\pi}$$

$$G(\theta) = \frac{3}{2} \sin^2\theta \quad [\propto A_e(\theta) \propto \sin^2\theta; \int_{4\pi} G(\theta)d\Omega = 4\pi]$$

$$\therefore R_r = \frac{|-j\omega AN\mu_0 \bar{\mathbf{H}} \sin\theta|^2}{8[\frac{1}{2}\eta_0 |\bar{\mathbf{H}}|^2][\frac{3}{2}\sin^2\theta \frac{\lambda^2}{4\pi}]} = \frac{2\pi}{3\eta_0} \left(\frac{\omega\mu_0 AN}{\lambda} \right)^2 \text{ ohms for N-turn coil}$$



SHORT DIPOLE RADIATION

Far field radiation:

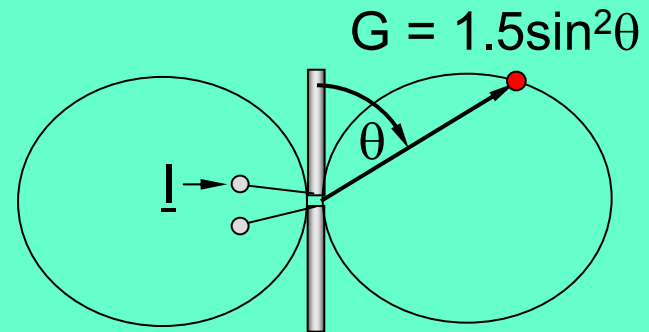
$$\bar{\mathbf{E}}_{\text{ff}} = \hat{\theta} \frac{j\eta_0}{2\lambda r} \mathbf{I} d_{\text{eff}} \sin\theta e^{-jkr} \quad [\text{V m}^{-1}]$$

Far field intensity:

$$\bar{T}(r, \theta) = \hat{r} \frac{|\bar{\mathbf{E}}_{\text{ff}}|^2}{2\eta_0} \quad [\text{W m}^{-2}]$$

Total power radiated:

$$\begin{aligned} P_T &= \int_0^{2\pi} \int_0^\pi [\bar{T}(r, \theta) \cdot \hat{r}] r^2 \sin\theta \, d\theta \, d\phi = \pi\eta_0 \left| \frac{\mathbf{I} d_{\text{eff}}}{2\lambda} \right|^2 \int_0^\pi \sin^3\theta \, d\theta \\ &= \frac{\eta_0 \pi}{3} \left| \frac{\mathbf{I} d_{\text{eff}}}{\lambda} \right|^2 \quad [\text{W}] \end{aligned}$$



Antenna gain:

$$G(\theta, \phi) \triangleq \frac{I(r, \theta)}{[P_T / 4\pi r^2]} = \frac{3}{2} \sin^2\theta$$

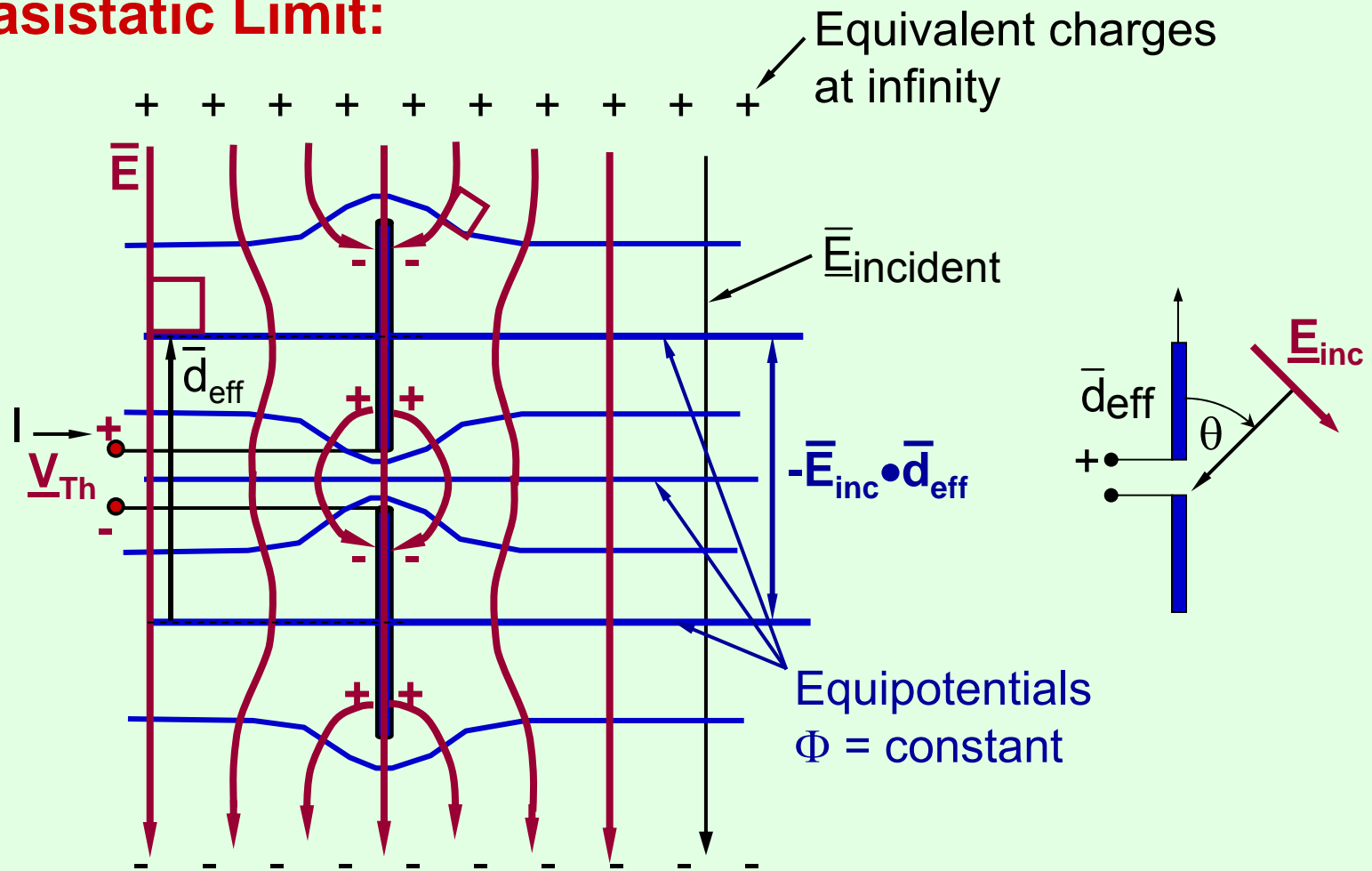
Radiation resistance:

$$P_T = \frac{1}{2} |\mathbf{I}|^2 R_r$$

$$R_r = \frac{2\pi\eta_0}{3} \left(\frac{d_{\text{eff}}}{\lambda} \right)^2 \quad [\Omega] \quad (\text{short dipole antenna})$$

SHORT DIPOLE ANTENNA

Quasistatic Limit:



$$\underline{V}_{\text{Th}} = -\bar{E}_{\text{inc}} \cdot \bar{d}_{\text{eff}} = -E_{\text{inc}} d_{\text{eff}} \sin \theta$$

ANTENNA EFFECTIVE AREA A_e

Antenna Equivalent Circuit:

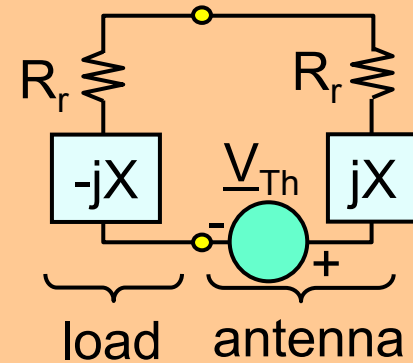
$$P_{\text{rec}} = \frac{|\underline{V}_{\text{Th}}/2|^2}{2R_r}$$

$$= |\underline{E}_{\text{inc}}|^2 \frac{d_{\text{eff}}^2}{8R_r} \sin^2\theta$$

$$= I(\theta)\eta_0 \sin^2\theta \frac{d_{\text{eff}}^2}{4R_r} \quad [I(\theta) = \frac{|\underline{E}_{\text{inc}}|^2}{2\eta_0}]$$

$$R_r = \frac{2\pi\eta_0}{3} \left(\frac{d_{\text{eff}}}{\lambda}\right)^2$$

$$P_{\text{rec}} = I(\theta)\sin^2\theta \frac{3\lambda^2}{8\pi}$$



$$\underline{V}_{\text{Th}} = -\underline{E}_{\text{inc}} d_{\text{eff}} \sin\theta$$

Antenna Effective Area A_{eff} :

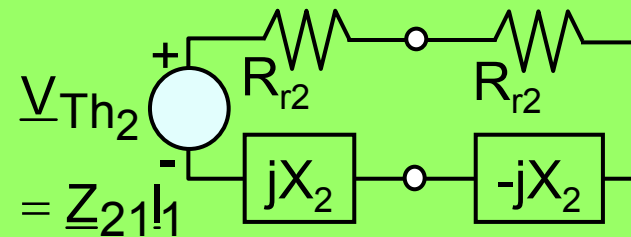
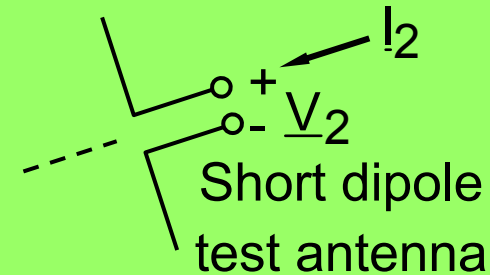
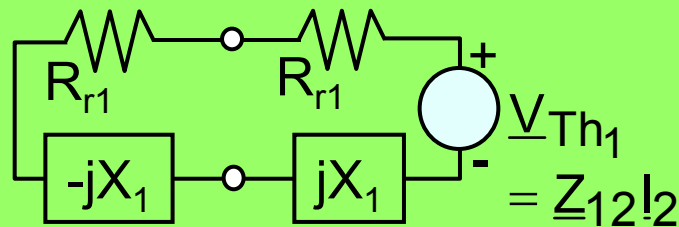
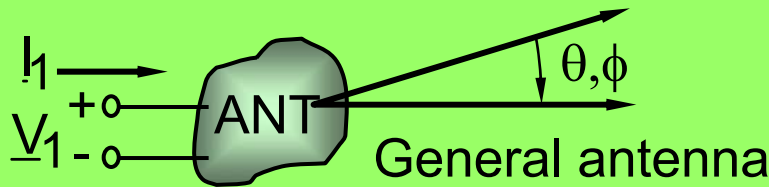
$$A_e(\theta) \triangleq \frac{P_{\text{rec}}}{I(\theta)}$$

$$A_e(\theta) = \frac{\lambda^2}{4\pi} 1.5 \sin^2\theta = \frac{\lambda^2}{4\pi} G(\theta) = A_e(\theta)$$

True for
most antennas

PROOF: $A = G\lambda^2/4\pi$ (If reciprocity)

Test Range:



Proof:

$$P_{\text{rec}1} = \frac{|\underline{Z}_{12}I_2|^2}{8R_{r1}} = P_{t2} \frac{G_2}{4\pi r^2} A_1$$

$$P_{\text{rec}2} = \frac{|\underline{Z}_{21}I_1|^2}{8R_{r2}} = P_{t1} \underbrace{\frac{G_1}{4\pi r^2}}_{\text{Wm}^{-2}} A_2$$

$$\frac{P_{\text{rec}2}}{P_{\text{rec}1}} = \frac{G_1 A_2 P_{t1}}{G_2 A_1 P_{t2}} \Rightarrow \frac{A_1}{G_1} = \frac{A_2}{G_2} \left[\frac{P_{t1}}{P_{t2}} \cdot \frac{P_{\text{rec}1}}{P_{\text{rec}2}} \right]$$

$$\text{But } \frac{P_{\text{rec}1}}{P_{\text{rec}2}} = \frac{|\underline{Z}_{12}I_2|^2}{|\underline{Z}_{21}I_1|^2} \cdot \frac{R_{r2}}{R_{r1}} = \frac{|\underline{Z}_{12}|^2}{|\underline{Z}_{21}|^2} \cdot \frac{P_{t2}}{P_{t1}}$$

If $|\underline{Z}_{12}|^2 = |\underline{Z}_{21}|^2$, then

$$\frac{A_1}{G_1} = \frac{A_2}{G_2} = \frac{\lambda^2}{4\pi} \quad \text{Q.E.D.}$$

NON-RECIPROCAL DEVICES

Reciprocity:

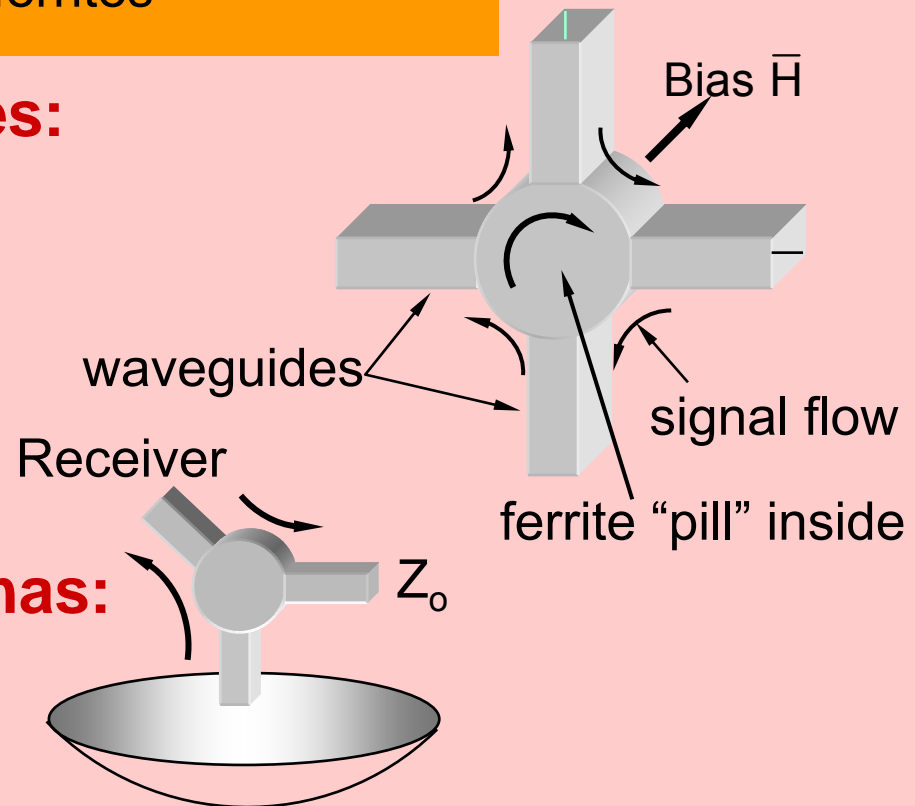
$$|Z_{12}|^2 = |Z_{21}|^2 \text{ if } \bar{\epsilon} = \bar{\epsilon}^t, \bar{\mu} = \bar{\mu}^t \text{ everywhere.}$$

Exceptions: magnetized plasmas,
magnetized ferrites

Non-reciprocal Devices:

4-Port Circulators
2 and 3-port circulators
Isolators

Non-reciprocal Antennas:

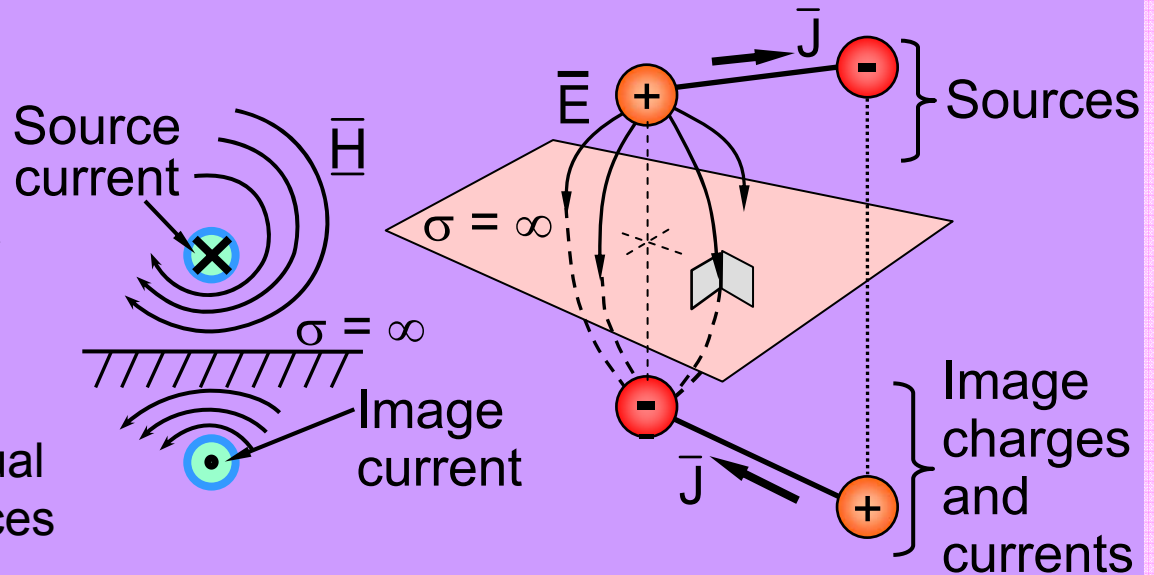


MIRROR IMAGES

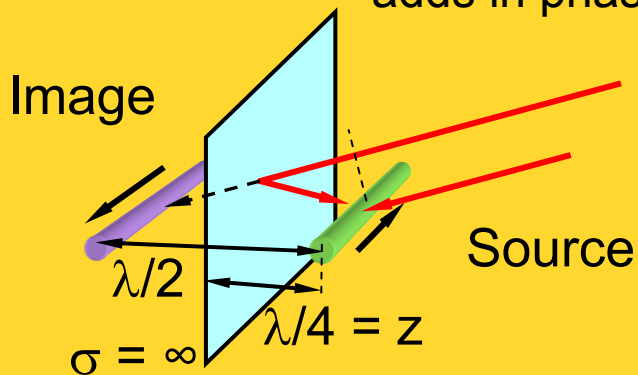
Mirror Images:

\vec{E} must be \perp to mirror
 \vec{H} must be parallel

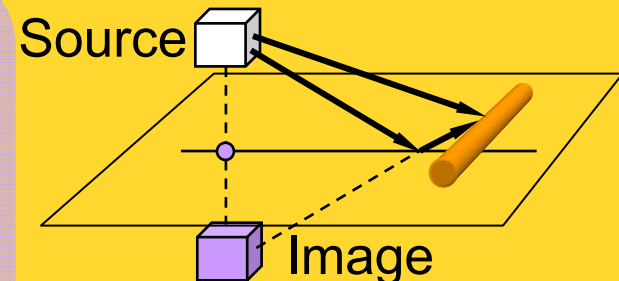
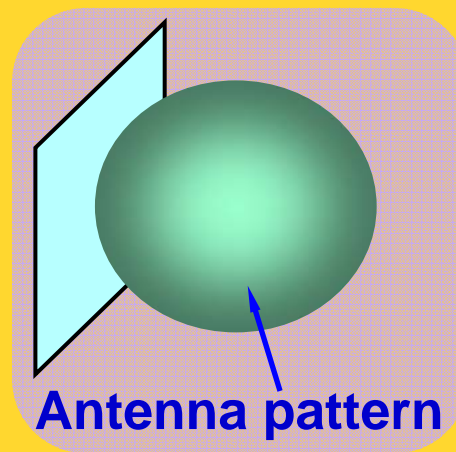
Fields above mirror equal
 Fields with image sources



Examples: Path at normal incidence is $\lambda/2$ longer, but adds in phase due to mirror's phase reversal



Normal gain has nulls
 at $z = 0, \lambda/2, \lambda,$ etc.



Antenna height above
 ground matters!