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6.013 Electromagnetics and Applications
Spring 2009

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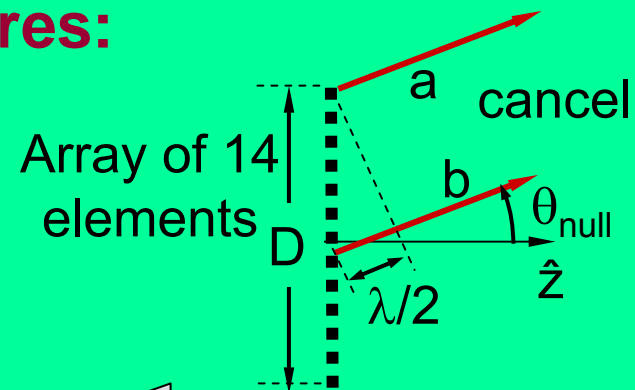
APERTURE ANTENNAS

Dense phased arrays ~apertures:

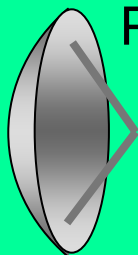
Pairs of radiators cancel (e.g., a,b)

Sum of pairs therefore = 0

First null at $\theta = \sim \lambda/D$



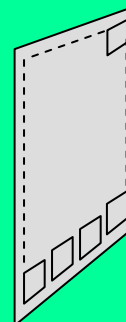
Aperture antennas:



Parabolic reflector



Aperture screen



Phased array

Derivation of far-field aperture radiation:

Find surface current \bar{J}_s that could produce the same aperture fields

Find the far fields radiated by an infinitesimal element of \bar{J}_s

Integrate the contributions from the entire aperture

Simplify the expressions by using small-angle approximations

Note the Fourier relationship between aperture and far fields

APERTURE ANTENNAS

Assume UPW in aperture:

$$\underline{\bar{E}}_+ = \hat{z} E_o e^{-jky} \Rightarrow \underline{\bar{H}}_+ = \hat{x} \frac{E_o}{\eta_o} e^{-jky}$$

$$\underline{\bar{E}}_- = \hat{z} E_o e^{+jky} \Rightarrow \underline{\bar{H}}_- = -\hat{x} \frac{E_o}{\eta_o} e^{+jky}$$

Satisfies $E_{//}$ continuous at $y = 0$

Equivalent surface current:

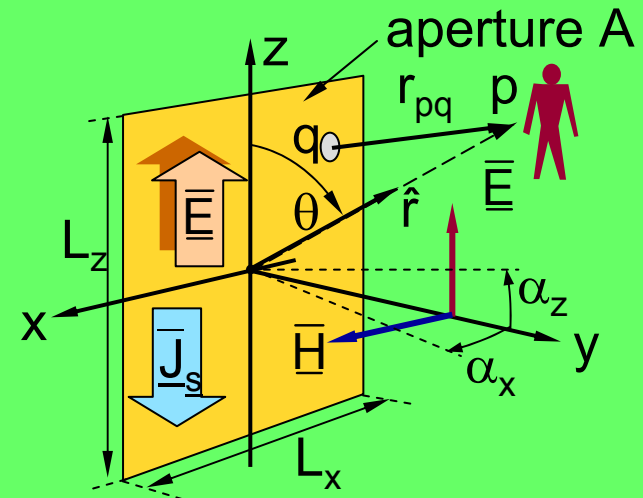
$$\begin{aligned} \underline{\bar{J}}_s &= \hat{y} \times [\underline{\bar{H}}(y=0_+) - \underline{\bar{H}}(y=0_-)] \\ &= \hat{y} \times \hat{x} [E_o/\eta_o + E_o/\eta_o] \end{aligned}$$

$$\underline{\bar{J}}_s = -\hat{z} 2E_o/\eta_o \text{ [A m}^{-1}\text{]}$$

$$\hat{z} l dz = \underline{\bar{J}}_s dx dz \text{ (Hertzian dipole radiator)}$$

Radiated far fields:

$$\begin{aligned} \underline{\bar{E}}_{ff}(\theta, \phi) &= \hat{\theta} \frac{j\eta_o}{2\lambda r} \sin\theta \iint_A \underline{\bar{J}}_{zs}(x, z) e^{-jkr_{pq}} dx dz \\ &= -\hat{\theta} \frac{j}{\lambda r} \sin\theta \iint_A E_{oz}(x, z) e^{-jkr_{pq}} dx dz \end{aligned}$$



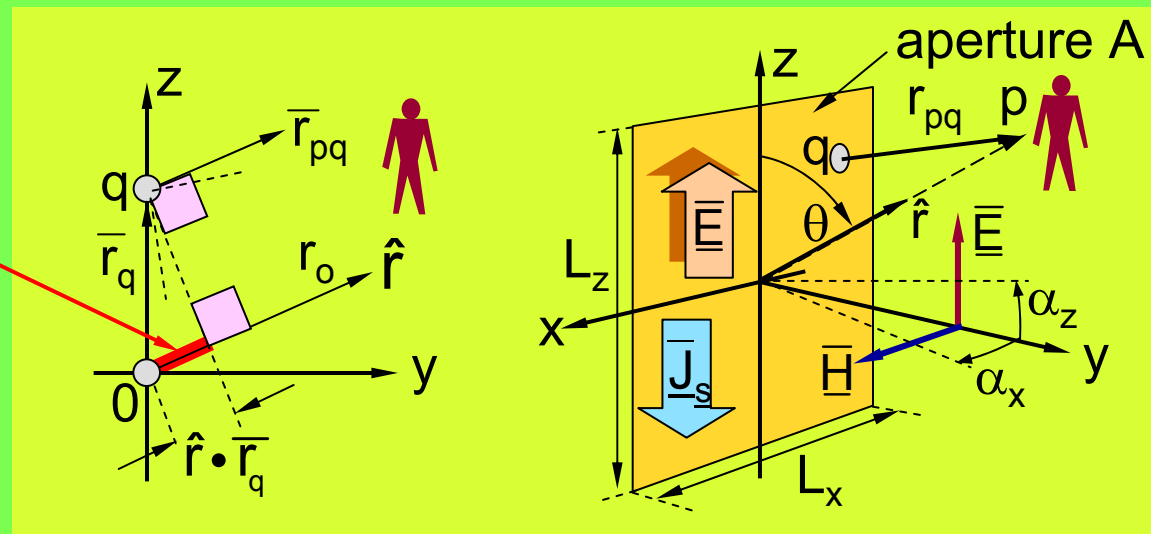
FAR-FIELD RADIATION

Assume UPW in aperture:

$$\bar{\mathbf{E}}_{\text{ff}}(\theta, \phi) = -\hat{\theta} \frac{j}{\lambda r} \sin\theta \iint_A \mathbf{E}_{\text{oz}}(x, z) e^{-jkr_{pq}} dx dz$$

$$e^{-jkr} \cong e^{-jkr_o + j\mathbf{k}\hat{\mathbf{r}} \cdot \bar{\mathbf{r}}_q}$$

Fraunhofer approximation
(else, Fresnel)



$$\bar{\mathbf{E}}_{\text{ff}}(\alpha_x, \alpha_z) \cong -\hat{\theta} \frac{j}{\lambda r} e^{-jkr_o} \iint_A \mathbf{E}_{\text{oz}}(x, z) e^{+j\frac{2\pi}{\lambda}(x\alpha_x + z\alpha_z)} dx dz$$

~Fourier (angle \Leftrightarrow space) Close to the y axis

Fourier relationship:

$$\underline{\mathbf{S}}(f) = \int_{-\infty}^{\infty} \mathbf{s}(t) e^{-j2\pi ft} dt \quad \mathbf{s}(t) = \int_{-\infty}^{\infty} \underline{\mathbf{S}}(f) e^{+j2\pi ft} df \quad (\text{frequency} \Leftrightarrow \text{time})$$

EXAMPLE: RECTANGULAR APERTURE

Assume UPW in aperture: (Observer close to the y axis)

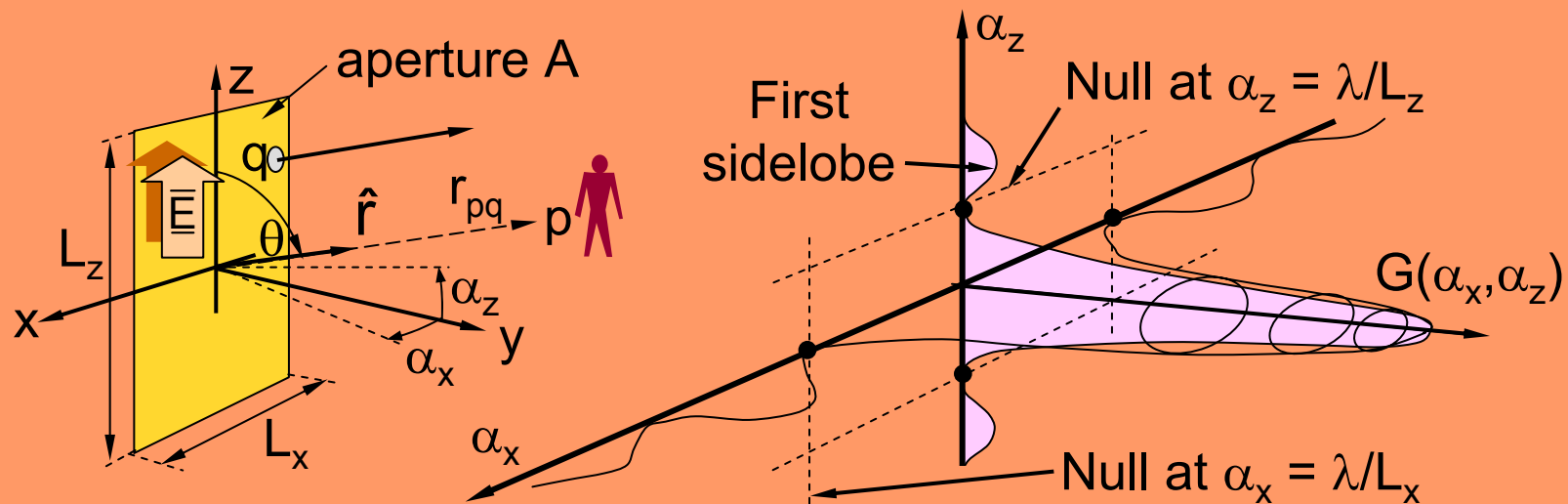
$$\bar{E}_{ff}(\alpha_x, \alpha_z) \cong -\hat{\theta} \frac{j}{\lambda r} e^{-jk r_0} \int_{-L_z/2}^{L_z/2} e^{+j2\pi\alpha_z \frac{z}{\lambda}} \int_{-L_x/2}^{L_x/2} \underline{E}_{oz}(x, z) e^{+j2\pi\alpha_x \frac{x}{\lambda}} dx dz$$

$$\int_{-L_x/2}^{L_x/2} \underline{E}_{oz}(x, z) e^{+j2\pi\alpha_x \frac{x}{\lambda}} dx = \underline{E}_{oz} \frac{\lambda}{j2\pi\alpha_x} (e^{+j\pi\alpha_x \frac{L_x}{\lambda}} - e^{-j\pi\alpha_x \frac{L_x}{\lambda}}) = \underline{E}_{oz} L_x \frac{\sin \frac{\pi\alpha_x L_x}{\lambda}}{\frac{\pi\alpha_x L_x}{\lambda}}$$

$$\triangleq \underline{E}_{oz} L_x \operatorname{sinc} \frac{\pi\alpha_x L_x}{\lambda} *$$

$$\bar{E}_{ff}(\alpha_x, \alpha_z) \cong -\hat{\theta} \frac{j}{\lambda r} \underline{E}_{oz} e^{-jk r_0} L_x L_z \operatorname{sinc} \frac{\pi\alpha_x L_x}{\lambda} \operatorname{sinc} \frac{\pi\alpha_z L_z}{\lambda}$$

Antenna Gain: $G(\alpha_x, \alpha_z) \propto |\bar{E}_{ff}|^2 \propto \operatorname{sinc}^2(\pi\alpha_x L_x / \lambda) \operatorname{sinc}^2(\pi\alpha_z L_z / \lambda)$



* $\operatorname{sinc} \theta \equiv (\sin \theta) / \theta$ [= 1 for $\theta = 0$]

RECTANGULAR APERTURE (2)

Antenna Gain $G(\alpha_x, \alpha_z)$:

$$G(\alpha_x, \alpha_z) = \frac{I(\alpha_x, \alpha_z, r)}{(P_t / 4\pi r^2)}$$

$$P_t = A \frac{|\underline{E}_{oz}|^2}{2\eta_0} \quad [\text{W}] \quad (A=L_x L_z)$$

$$I(\alpha_x, \alpha_z, r) = \frac{|\underline{E}_{ff}|^2}{2\eta_0} [\text{W/m}^2] = \frac{1}{2\eta_0} \left(\frac{1}{\lambda r}\right)^2 |\underline{E}_{oz}|^2 A^2 \text{sinc}^2\left(\frac{\pi\alpha_x L_x}{\lambda}\right) \text{sinc}^2\left(\frac{\pi\alpha_z L_z}{\lambda}\right)$$

$$G(\alpha_x, \alpha_z) = A(4\pi/\lambda^2) \text{sinc}^2(\pi\alpha_x L_x/\lambda) \text{sinc}^2(\pi\alpha_z L_z/\lambda)$$

$$G_o = A \frac{4\pi}{\lambda^2} \Rightarrow A = A_e = G_o \frac{\lambda^2}{4\pi} \quad \left[\frac{\sin\theta}{\theta} \rightarrow 1 \text{ as } \theta \rightarrow 0\right]$$

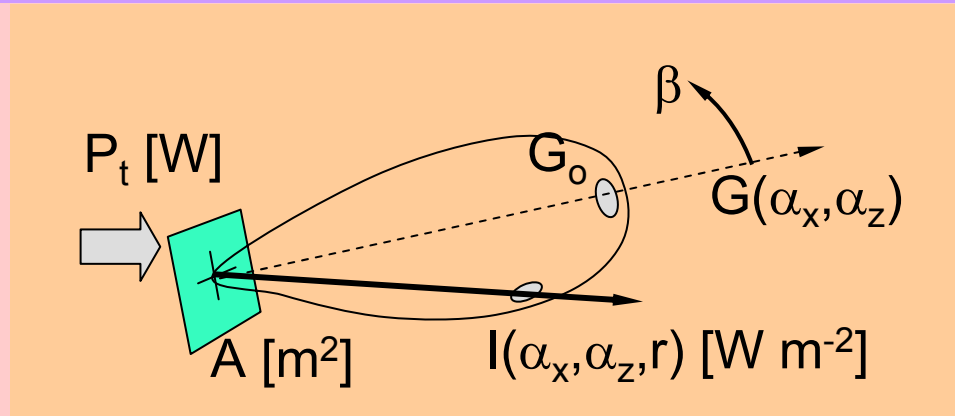
η_A = "aperture efficiency" $\equiv A_e/A \approx 0.65$ typically

(non-uniform aperture amplitude and/or phase $\Rightarrow A_e < A$)

Huygen's approximation:

$$\underline{E}_{ff} \cong \hat{\theta} (j/2\lambda r) (1+\cos\beta) \iint_A \underline{E}_{oz}(x,z) e^{-jk_r \rho} dx dz$$

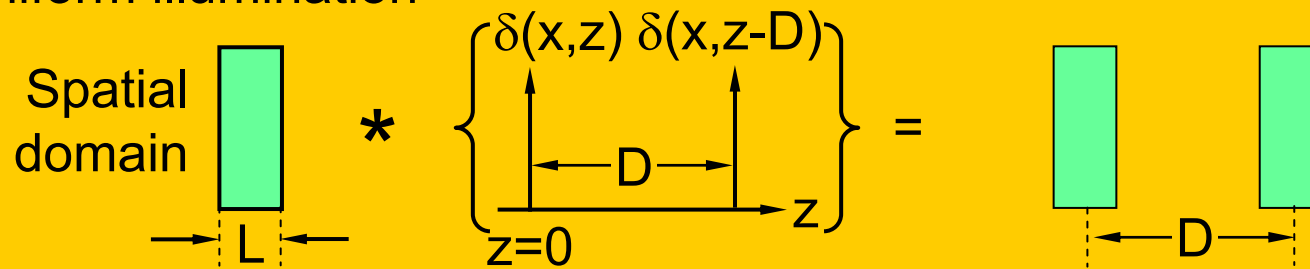
*True for any uniformly illuminated aperture



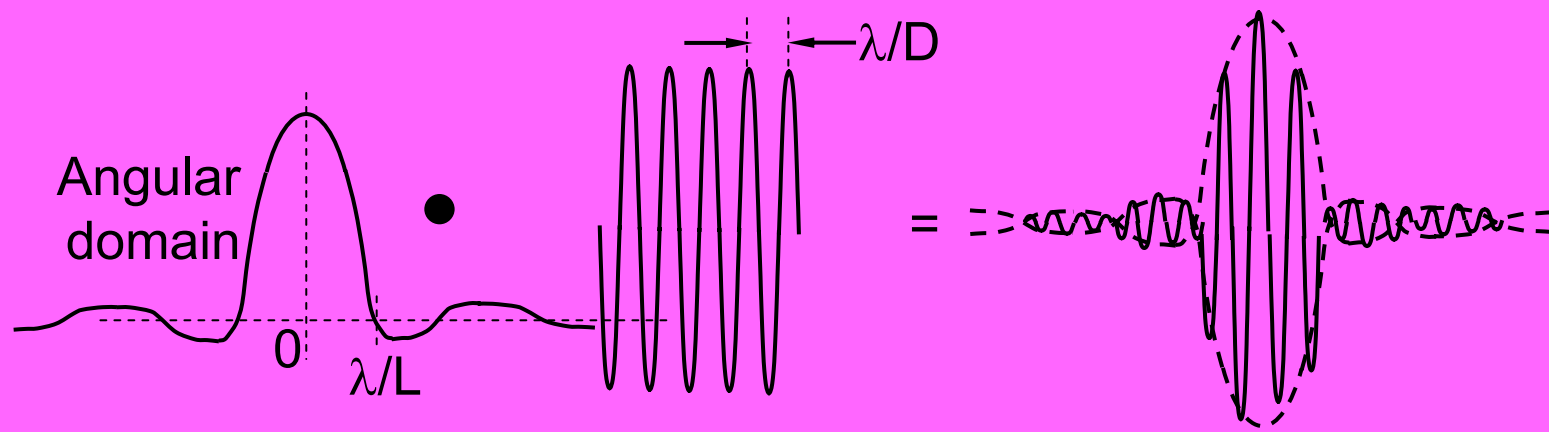
EXAMPLE: SLIT DIFFRACTION

Pair of rectangular slots:

Uniform illumination

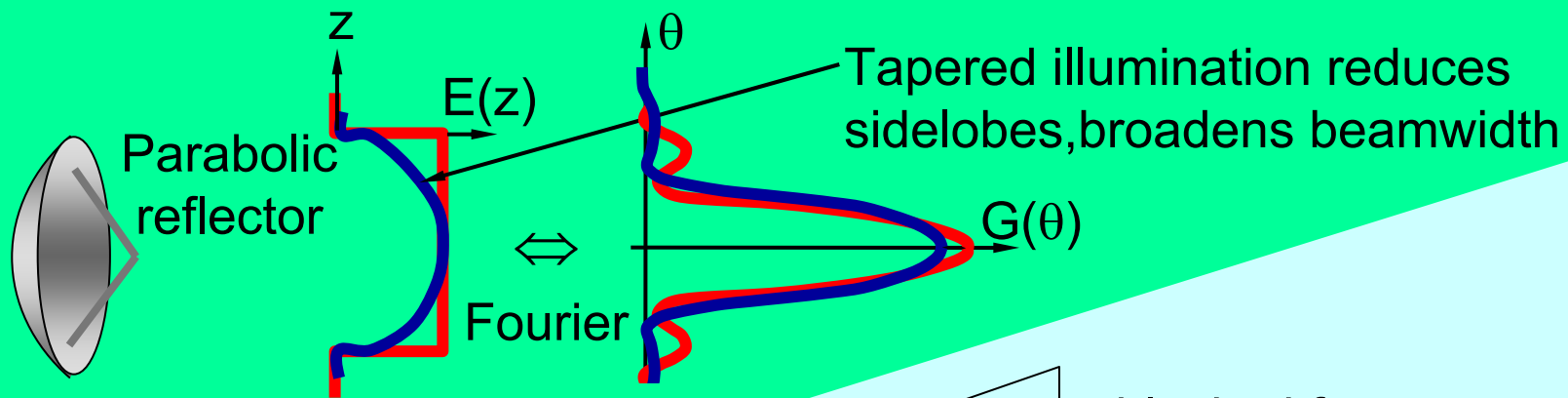


Fourier transforms



DIFFRACTION

Aperture tapering: $\bar{E}_{ff} \cong \hat{\theta} (j/2\lambda r) (1+\cos\beta) \iint_A E_{oz}(x,z) e^{-jkr_{pq}} dx dz$

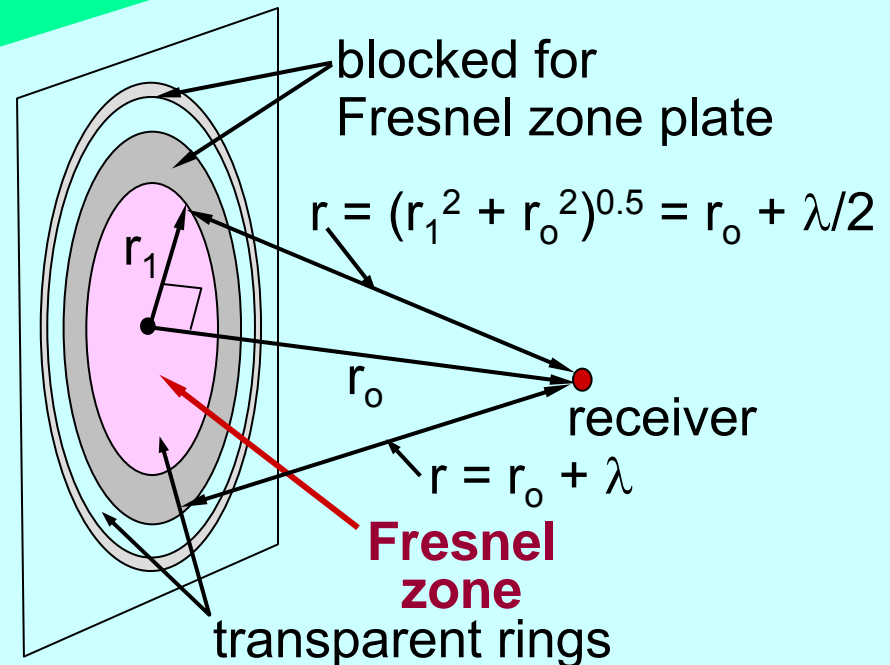


Fresnel zone plate:

Blocks rings of negative phase, maximizing Huygen's integral

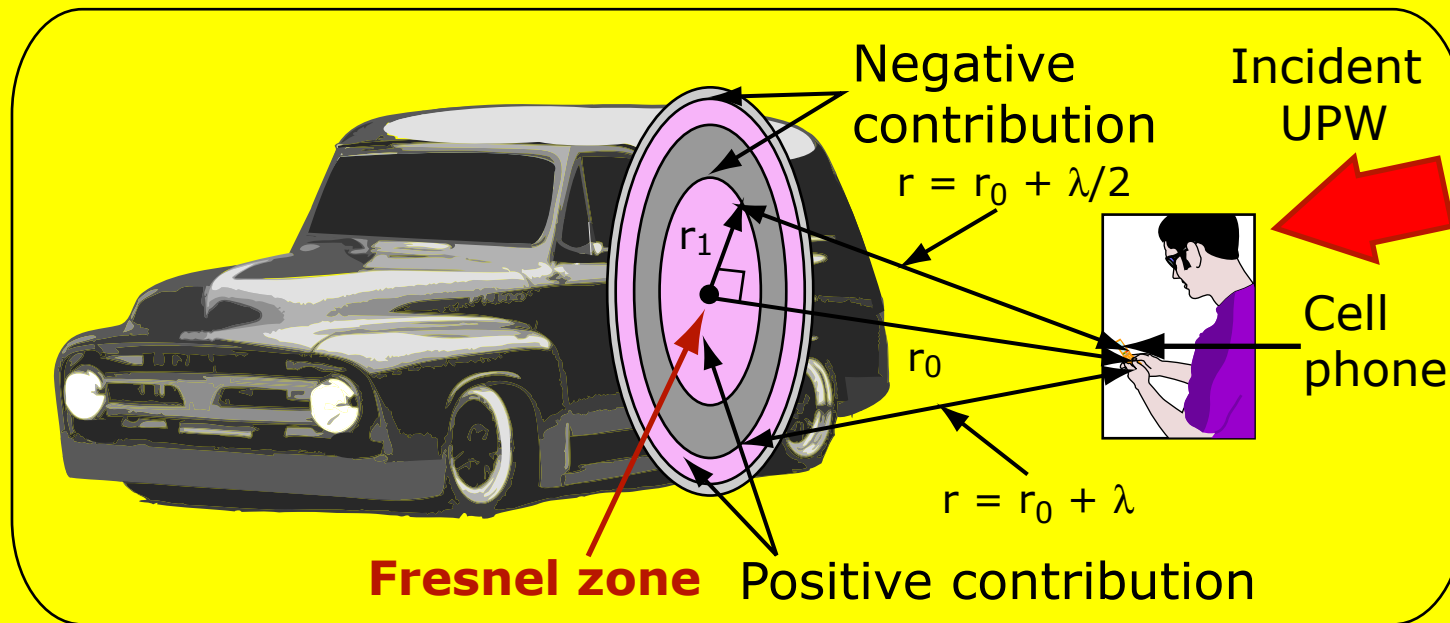
Aperture radius = r_1 maximizes received signal. Additional transparent rings increase it further, yielding lense behavior

Adjacent rings cancel if $\beta \approx 1$



FRESNEL ZONE SIZE, REFLECTIONS

Fresnel zone radius $r_1 = \sqrt{(r_0 + \frac{\lambda}{2})^2 - r_0^2} \cong \sqrt{r_0 \lambda}$ for $r_0 \gg \lambda$



When the truck is much larger than the Fresnel zone, it acts like a large mirror. When the truck is smaller, the power reflected declines with illuminated zone area on the truck. Thus the reflecting power of a mirror depends on its size and distance relative to a wavelength.